SPACE RESEARCH COORDINATION CENTER

BOUNDS FOR THE EIGENVALUES OF A SIMPLY-SUPPORTED RECTANGULAR PLATE UNDER A COMPRESSIVE STRESS VARYING LINEARLY IN THE DIRECTION OF LOADING

BY

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ABSTRACT

The effects of constant edge loads and of a uniformly distributed in-plane load on the transverse vibrational frequencies and elastic stability characteristics of a simply-supported plate are investigated. Bounds for the eigenvalues are obtained for various plate width-to-length ratios as functions of two parameters associated with the distributed in-plane load and the edge loads. The upper bounds are calculated by the Rayleigh-Ritz method and the lower bounds by the Second Projection Method of Bazley and Fox. In all instances, the gap between the bounds over their average is less than are half of one per cent. Buckling load combinations are obtained by determining the values of the loading parameters making the first eigenvalue equal to zero.

NOMENCLATURE

Latin Symbols

a	Length of the plate
aij	Matrix elements (equation 46)
b	Width of the plate
Ci	Constants of linear combination (equation 20)
D	Flexural rigidity of the plate
$D_L, D_L{}^o, D_{L'}$	Respective domains of the operators L, L^o and L'
f	Function of class C^4 defined over $0 \leq x \leq 1$ (equation 16)
G	Function of time (equation 6)
g	Function of class C^4 defined over $0 \leq x \leq 1$ (equation 16)
g	Acceleration due to gravity
H	Hilbert space
Jo, J_1	Bessel functions of the first kind of orders zero and one
jo,i	Zeros of Jo(x)
L	Differential operator in eigenvalue problem (equation 13)
L^o	Base operator (equation 27)
L'	Additional operator (equation 28)
$L^k, L^{1,k}$	Intermediate operators
m	Plate density per unit area
Nx,Ny,Nxy	In-plane loads per unit of length
P_1	Compressive thrust per unit of length at the edge x=o
Pcr	Uniform critical compressive load for a simply supported plate
P^k	Projection operator (equation 36)
Q^1	Projection operator (equation 44)
r	Aspect ratio of the plate (equation 12)
t	Time

u	Function in the domain of L
u_i	Function in D_L (equation 23)
v	Function in D_L (equation 17)
$\bar{w}$	Lateral deflection of the plate (equation 1)
X	Body force component per unit area
$\bar{x}$	Actual coordinate
x	Non-dimensional coordinate $x = \bar{x}/a$
Y	Body force component per unit area
$\bar{y}$	Actual coordinate
y	Non-dimensional coordinate $y = \bar{y}/b$

Greek Symbols

α	Distributed in-plane load parameter (equation 12)
α_i	Constants of linear combination (equation 44)
β	Parameter (equation 12)
β_i	Constants of linear combination (equation 59)
γ	Edge load parameter (equation 12)
γ^*	Positive number (equation 42)
δ_{ij}	Kronecker delta
ϵ	Circular frequency of vibration (equation 7)
$\phi(\bar{x},\bar{y})$	Mode shape of deflection (equation 6)
ϕ	Function
$\psi(x)$	Eigenfunction of the operator L (equation 11)
ψ_i^o	Eigenfunction of the operator L^o (equation 30)
ψ_i'	Eigenfunction of the operator L' (equation 40)
$\psi^{l,k}$	Eigenfunction of the operator $L^{l,k}$ (equation 56)

λ_i	Eigenvalue of the operator L (equation 13)
$\hat{\lambda}_i$	Upper bound for the eigenvalue
λ_i^o	Eigenvalue of the operator L^o (equation 30)
λ_i'	Eigenvalue of the operator L' (equation 41)
$\lambda_i^{l,k}$	Eigenvalue of the intermediate operator $L^{l,k}$ (equation 56)

I. INTRODUCTION

For many heavily loaded structural elements used in aerospace applications, inertia forces are of great significance. For instance, the body forces developed in the mid-plane of a plate by an acceleration component in this plane affect the potential energy of the plate and, consequently, its natural frequencies of vibration and its stability characteristics.

The effect of in-plane loads on the deflection, natural frequencies and buckling stresses for plates appears to have been considered mainly for the case where these loads are uniform throughout the plate.

Bickley[(1)*] considered a clamped circular plate under tension and investigated the effect of this tension on the normal displacements under pressure, and on the natural frequencies of the plate.

Conway and his associates [(2,3,4)] determined the effect of combinations of uniform tensile or compressive in-plane loads on the deflection and stress distribution of simply supported and clamped rectangular plates.

Stein and Neff[(5)] determined the buckling stresses for a simply supported rectangular plate in shear using the Rayleigh-Ritz method, while McKenzie[(6)] considered the buckling of a rectangular plate under combined bi-axial compression, bending and shear with two edges simply supported while the other two edges were arbitrarily supported. His solution was obtained by an approximate variational method.

Johns[(7)] determined the static instability of rectangular orthotropic panels subjected to bi-axial in-plane compression and lateral loads dependent on the panel deflection. The panels were considered to be resting on an elastic foundation and their edges were elastically

*Numbers in parentheses placed superior to the line of the text refer to the bibliography.

restrained against rotation.

Wang and Sussman(8) considered the elastic stability of a simply supported plate under linearly variable compressive loads. Using the Rayleigh-Ritz method, they determined the buckling coefficients and presented them in graphical form for different length-to-width ratios. They concluded that the average buckling stress in the plate is less than the uniform buckling stress and that the body force is a contributing factor to the compressive stresses.

Weeks and Shideler(9) investigated the effect of constant bi-axial in-plane loads on the vibration characteristics of rectangular plates with various boundary conditions. Their solutions are exact if one pair of opposite edges is simply supported, otherwise the solutions are approximate.

Further results for rectangular plates subjected to uniform in-plane loads can be found in references 10 through 12, and in references 13 and 14 for skew plates.

Hermann(15) appears to be the only author having considered the effect of a body force and uniaxial in-plane compression on the fundamental frequency of vibration of a simply supported rectangular plate. The body force acting in the plane of the plate may be due to its weight or to an acceleration of the plate in its plane. Using the Rayleigh method, Hermann calculated an approximation to the fundamental frequency and to the buckling load for a plate with aspect ratio equal to 3. As a first approximation, the linear term was replaced by its average. The frequency decrease was found to be piecewise linear with increasing fractions of the critical loading.

In summary, one finds that in nearly all the cases treated in the literature, the in-plane loads have been taken as being constant, and that in the only two instances where the body force has been considered the effect of the distributed load on the buckling load and on the fundamental frequency is obtained by approximate techniques.

The object of the present investigation is then to determine the effect of a linearly varying compressive load on the natural frequencies of a simply supported rectangular plate, taking into account the influence of the plate aspect ratio. The type of loading considered here gives rise to a differential equation with variable coefficients whose solution is impossible to obtain. In order to find approximate solutions, two methods are used:

The Rayleigh-Ritz method is utilized to calculate upper bounds for the natural frequencies. However, since the quality of these upper bounds is unknown without calculations of error estimates or, what is equivalent, of lower bounds, the Second Projection method of Bazley and Fox[(16)] is used to complete the prediction of the frequencies. Brief descriptions of these mathematical methods are given in Section III.

The determination of the buckling loads is accomplished by finding the values of the loading parameters for which the first eigenvalue of the plate goes to zero. Some of the results obtained here are compared to the results of Wang and Sussman[(8)].

Section II is concerned with the derivation of the plate eigenvalue problem for the determination of the plate natural frequencies, and the results are presented and discussed in Section IV.

II. FORMULATION OF THE PROBLEM

Consider a rectangular plate having uniform thickness, h, small in comparison to its length a in the $\bar{x}$-direction and its width b in the $\bar{y}$-direction.

The plate is subjected to in-plane loads Nx, Ny, Nxy given per unit of length, and to body forces X and Y given per unit area and acting in the middle plane.

The differential equation governing the lateral deflection $\bar{w}$ $(\bar{x},\bar{y},t)$ of the plate can readily be obtained from reference 11 (p. 335) by inclusion of the inertia term. In the absence of lateral loads, it reads

$$m\frac{\partial^2 \bar{w}}{\partial t^2} + D\nabla^4 \bar{w} - N_x \frac{\partial^2 \bar{w}}{\partial \bar{x}^2} - N_y \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} - 2N_{xy}\frac{\partial^2 \bar{w}}{\partial \bar{x}\partial \bar{y}} + X\frac{\partial \bar{w}}{\partial \bar{x}} + Y\frac{\partial \bar{w}}{\partial \bar{y}} = 0 \qquad (1)$$

where m denotes the plate density per unit area and D is the flexural rigidity

$$D = \frac{Eh^3}{12(1-\nu^2)} \qquad (2)$$

The in-plane loads must satisfy the equilibrium equations

$$\frac{\partial N_x}{\partial \bar{x}} + \frac{\partial N_{xy}}{\partial \bar{y}} + X = 0$$

$$\frac{\partial N_{xy}}{\partial \bar{x}} + \frac{\partial N_y}{\partial \bar{y}} + Y = 0 \qquad (3)$$

For the plate subjected to a uniform compressive thrust P_1, along the edge $\bar{x} = 0$ and to a distributed body force mg per unit area, the in-plane loads are given by

$$N_x = -\left[P_1 + mg\bar{x} \right] \quad , \quad N_x = N_{xy} = 0 \quad , \quad X = mg \quad , \quad Y = 0 \tag{4}$$

The loading and the geometry are illustrated in Figure 1. That such a state of stress is possible can be seen by substitution of these expressions in the equilibrium equations (3). However, this stress distribution involves the assumption that the plate is free to deform in its own plane.

Upon substitution of the in-plane loads (4) into equation (1), the lateral deflection of the plate is governed by

$$m\frac{\partial^2 \bar{w}}{\partial t^2} + D\nabla^4 \bar{w} + \left(P_1 + mg\bar{x}\right)\frac{\partial^2 \bar{w}}{\partial \bar{x}^2} + mg\frac{\partial \bar{w}}{\partial \bar{x}} = 0 \tag{5}$$

The search for eigenvibrations of the form

$$\bar{w}(\bar{x},\bar{y},t) = \phi(\bar{x},\bar{y})\, G(t) \tag{6}$$

yields the pair of equations

$$\frac{d^2 G}{dt^2} + \epsilon^2 G = 0 \tag{7}$$

$$\nabla^4 \phi + \frac{1}{D}\left(P_1 + mg\bar{x}\right)\frac{\partial^2 \phi}{\partial \bar{x}^2} + \frac{mg}{D}\frac{\partial \phi}{\partial \bar{x}} - \frac{m\epsilon^2 \phi}{D} = 0 \tag{8}$$

where ϵ^2 , the separation constant, represents the square of the circular frequency of the motion.

At this stage, separation of the variables $\bar{x}$ and $\bar{y}$ can be accomplished only if the plate is simply supported along the edges $\bar{y} = 0$ and $\bar{y} = b$. In this case, ϕ can be written in the form,

$$\phi(\bar{x},\bar{y}) = \Psi(x)\,\Omega(y) \tag{9}$$

where $x = \frac{\bar{x}}{a}$ and $y = \frac{\bar{y}}{b}$ and

$$\Omega_n(y) = C_n \sin n\pi y \tag{10}$$

while Ψ must satisfy the differential equation

$$\frac{d^4\psi}{dx^4} + \left(\frac{P_1 a^2}{D} + \frac{mga^3x}{D} - \frac{2n^2\pi^2a^2}{b^2}\right)\frac{d^2\psi}{dx^2} + \frac{mga^3}{D}\frac{d\psi}{dx} - \left(\frac{m\epsilon^2a^4}{D} - \frac{n^4\pi^4a^4}{b^4}\right)\psi = 0 \tag{11}$$

Introduction of the parameters

$\alpha = mga^3/D$

$a/b = r$

$\gamma = P_1/P_{cr} = P_1 b^2/k\pi^2 D$ where k is a numerical factor depending on r as defined by Timoshenko and Gere,[11]

$$\beta = \frac{\pi^2}{\alpha}\left[k\gamma r^2 - 2(nr)^2\right]$$

$$\lambda = a^4\left[\frac{\epsilon^2 m}{D} - \frac{n^4\pi^4}{b^4}\right] \tag{12}$$

permits equation (11) to be written in the form

$$\frac{d^4\psi}{dx^4} + \alpha\frac{d}{dx}\left[(\beta + x)\frac{d\psi}{dx}\right] = \lambda\psi \tag{13}$$

with the boundary conditions (for simple supports at $\bar{x} = 0$ and $\bar{x} = a$):

$$\psi = \frac{d^2\psi}{dx^2} = 0 \quad \text{at} \quad x=0 \text{ and } x=1. \tag{14}$$

The solution of equation (13) with boundary conditions (14) constitute the eigenvalue problem to be solved. The natural frequencies of the plate are related to the eigenvalues λ by the relation

$$\epsilon^2_{pq} = \frac{D}{ma^4}\left[\lambda_p + \frac{q\pi^4 a^4}{b^4}\right] \tag{15}$$

where p and q give the mode of vibration, i.e. the numbers of half waves in which the vibrating plate deforms in the $\bar{x}$ and $\bar{y}$ directions respectively.

The determination of the buckling loads will be discussed in Section IV. Presently attention will be restricted to the determination of the upper and lower bounds to the natural frequencies, i.e. to the bounds to the eigenvalues specified by equations (13) and (14). It should be noted that an eigenvalue problem of the form found here was considered previously by one of the authors in a different application.[17]

To establish the basis for the estimation of the eigenvalues λ , the mathematical problem will be cast in variational form.

The differential operator appearing in equation (12), and henceforth denoted by L, has a domain D_L consisting of the set of functions of class C^4 defined over the range $0 \leq x \leq 1$ and satisfying the boundary conditions (14). Over this range, the inner product between two functions f and g is denoted by and given by the Lebesgue integral

$$(f, g) = \int_0^1 f(x)g(x)\,dx \tag{16}$$

It can be shown by integration by parts that the operator L is self-adjoint, i.e.

$$(Lu, v) = (u, Lv) \tag{17}$$

for any two functions u and v in D_L.

Now, as is well known, a variational principle can always be constructed from a self-adjoint operator in such a way that the corresponding Euler equation is the given differential equation. If we assume that the eigenvalues of L are ordered in the non-decreasing sequence

$$\lambda_1 \leqslant \lambda_2 \leqslant \lambda_3 \leqslant \ldots \tag{18}$$

Courant's maximum-minimum principle[18] gives the following characterization for the j^{th} eigenvalue:

$$\lambda_j = \max_{\{u_i\}} \left\{ \min_{(\phi, u_i)=0} \frac{\int_0^1 \left[\left(\frac{d^2\phi}{dx^2} \right)^2 - \alpha(\beta + x) \left(\frac{d\phi}{dx} \right)^2 \right] dx}{\int_0^1 \phi^2 \, dx} \right\}_{i=1 \text{ to } j-1} \tag{19}$$

where ϕ and the set of functions $\{u_i\}$ belong to D_L.

The problem is now reduced to finding approximations to the stationary values of the Rayleigh quotient shown in equation (19).

III. DISCUSSION OF THE SOLUTION

The number and variety of techniques that have been used to estimate eigenvalues of self-adjoint operators is enormous. For review of the literature on this subject, the reader is referred to references 19 through 22.

The methods used here to bracket the eigenvalues are the Rayleigh-Ritz method and the Second Projection method of Bazley and Fox[16].

A. Upper Bounds (Rayleigh-Ritz method)

The central idea in the Rayleigh-Ritz method consists in determining the stationary values of the Rayleigh quotient over the linear manifold spanned by a set of n linearly independent functions $\{u_i\}$ satisfying the prescribed boundary conditions of the operator L. The problem consists, then, in finding the functions u of the form

$$u = \sum_{i=1}^{n} c_i u_i \tag{20}$$

i.e. in finding the constants ci, making the quotient stationary, and its corresponding value. The result is the general matrix eigenvalue problem

$$\left[(u_i, Lu_j)\right]\left[c_j\right] = \hat{\lambda} \left[(u_i, u_j)\right]\left[c_j\right] \tag{21}$$

Now since the set of functions u was restricted to the finite-dimensional manifold, it is apparent that the eigenvalues $\hat{\lambda}_j$ are upper bounds to the eigenvalues of the operator L, i.e.

$$\hat{\lambda}_j \geqslant \lambda_j \quad , \quad j = 1, 2, \ldots n \tag{22}$$

Furthermore, it follows that as n increases, the upper bounds will be improved since the Rayleigh quotient will take its stationary value over a larger subspace.

In the present study, the functions u_i were chosen as follows:

$$u_i = \sqrt{2}\,\sin i\pi x \tag{23}$$

They are orthonormal, i.e.

$$(u_i, u_j) = \delta_{ij} \tag{24}$$

and they yield the following inner products:

$$\begin{aligned} (u_i, Lu_j) &= 0 && \text{for } (i \pm j) \text{ even} \\ &= 2\alpha i j\left[\frac{1}{(i-j)^2} + \frac{1}{(i+j)^2}\right] && \text{for } (i \pm j) \text{ odd} \\ &= (i\pi)^2\left[(i\pi)^2 - \alpha\beta - \tfrac{1}{2}\alpha\right] && \text{for } i = j \end{aligned} \tag{25}$$

The inner products were used to solve numerically the matrix eigenvalue problem specified by equation (21). The results are presented and discussed in Section IV.

B. Lower Bounds (Bazley-Fox Second Projection method)

In the determination of lower bounds of a self-adjoint operator L, the method of intermediate problems introduced by Weinstein and Aronszajn[20] was modified by Bazley and Fox by the introduction of a second projection in order to reduce their determination to finite, linear algebraic computations. Only the outline of the method will be given here. For further details and proofs, the reader is referred to reference 16.

We consider the set of all square integrable functions defined on the interval $[0,1]$ along with the inner product defined in equation (16). This set, along with the inner product defined, constitutes a separable Hilbert space which will be denoted by H. The self-adjoint operator L in H is assumed to be decomposable as the sum of two operators

$$L = L^o + L' \tag{26}$$

where L^o is a self-adjoint operator with easily solved eigenvalue problem and L' is positive definite and self-adjoint.

We take for L^o the operator

$$L^o = \frac{d^4}{dx^4} + \alpha(\beta+1)\frac{d^2}{dx^2} \tag{27}$$

and for L'

$$L' = -\frac{d}{dx}\left[\alpha(1-x)\frac{d}{dx}\right] \tag{28}$$

L^o is easily shown to be self-adjoint, and its eigenvalue problem, viz

$$\frac{d^4\psi^o}{dx^4} + \alpha(\beta+1)\frac{d^2\psi^o}{dx^2} = \lambda^o\psi^o$$

$$\psi^o = \frac{d^2\psi^o}{dx^2} = 0 \quad \text{at } x=0 \text{ and } x=1 \tag{29}$$

has the solution

$$\psi_n^o = \sqrt{2}\,\sin n\pi x$$

and

$$\lambda_n^o = (n\pi)^2\left[(n\pi)^2 - \alpha(\beta+1)\right] \tag{30}$$

The domain of L', $D_{L'}$, consists of the set of functions of class C^2 vanishing at x = 0 and x = 1. For any function ϕ in $D_{L'}$,

$$(L'\phi,\phi) = -\int_0^1 \frac{d}{dx}\left[\alpha(1-x)\frac{d\phi}{dx}\right]\phi\,dx = \int_0^1 \alpha(1-x)\left(\frac{d\phi}{dx}\right)^2 dx - \left[\alpha(1-x)\,\phi\,\frac{d\phi}{dx}\right]_0^1 \tag{31}$$

Since the boundary term vanishes,

$$(L'\phi, \phi) > 0 \qquad \text{for } \phi \neq 0 \tag{32}$$

i.e. L' is positive definite. Furthermore, it can be shown by integration by parts that L' is self-adjoint.

Now since L' is non-negative and D_L coincides with D_{L^o},

$$(u, L^o u) \leqslant (u, Lu) \tag{33}$$

and consequently the ordered eigenvalues of L^o and L satisfy the inequalities:

$$\lambda_i^o \leqslant \lambda_i \qquad i = 1, 2, 3 \ldots \tag{34}$$

Thus the eigenvalues of L^o are lower bounds to those of L. However, in most instances (including in this application) these lower bounds are far removed from the true eigenvalues λ_i . In order to improve these rough bounds, a sequence of self-adjoint operators L^k is constructed so that they have the same domain as L^o and their eigenvalues satisfy the inequalities

$$\lambda_i^o \leqslant \lambda_i^k \leqslant \lambda_i^{k+1} \leqslant \lambda_i \tag{35}$$

In the Weinstein-Aronszajn construction, the intermediate operators L^k are constructed using linearly independent vectors in $D_{L'}$. Here, we select as these vectors the eigenfunctions ψ_i' of the operator L'. The projection of any function ϕ in $D_{L'}$ on the span of the first k eigenfunctions of L' is given by

$$P^k\phi = \sum_{i=1}^{k} (\phi, \psi_i')\psi_i' \tag{36}$$

The k-th operator L^k is then defined by

$$L^k \phi = L^o \phi + L' P^k \phi = L^o \phi + \sum_{i=1}^{k} (\phi, \psi_i') L' \psi_i' \tag{37}$$

Its domain is D_{L^o} and it is self-adjoint as can easily be shown.
The inequalities

$$(\phi, L^o \phi) \leq (\phi, L^k \phi) \leq (\phi, L^{k+1} \phi) \leq (\phi, L\phi) \tag{38}$$

being satisfied, the parallel inequalities (35) are also satisfied.
The determination of the eigenfunctions ψ_i' requires the solution of the eigenvalue problem

$$-\frac{d}{dx}\left[\alpha(1-x)\frac{d\psi'}{dx}\right] = \lambda'\psi' \tag{39}$$

with $\psi'(0) = \psi'(1) = 0$

The normalized eigenfunctions for this problem are

$$\psi_i' = \frac{1}{J_1(j_{o,i})} J_o\left(j_{o,i}\sqrt{1-x}\right) \tag{40}$$

where J_o and J_1 are respectively the Bessel functions of the first kind of order zero and one, and $j_{o,i}$ are the zeros of $J_o(x)$. The corresponding eigenvalues are

$$\lambda_i' = \frac{\alpha\, j_{o,i}^2}{4} \tag{41}$$

It is readily apparent that the determination of the spectrum of the intermediate operator L^k gives lower bounds to those of L. In general, however, the eigenvalue problem for L^k presents difficulties, and in order to overcome them, Bazley and Fox [16] have modified the intermediate operator L^k by introducing smaller operators $L^{l,k}$ whose spectra can always be determined by finite algebraic computations.

For every positive number γ^*, the operator L^k may be written as

$$L^k = \left(L^0 - \gamma^*\right) + \left(L' P^k + \gamma^*\right) \tag{42}$$

Consider the inner product $\langle u, v \rangle$ defined for any two functions u and v of H by

$$\langle u, v \rangle = \left(\left[L' P^k + \gamma^* \right] u, v \right) \tag{43}$$

Consider a set $\{q_i\}$ of linearly independent functions in H. The projection Q^l of an element ϕ of H on the linear manifold spanned by the first l vectors of the set is given by

$$Q^l \phi = \sum_{i=1}^{l} \alpha_i q_i \tag{44}$$

where

$$\alpha_i = \sum_{j=1}^{l} a_{ij} \left(\phi, \left[L' P^k + \gamma^* \right] q_j \right) \tag{45}$$

and where the a_{ij} are the elements of the matrix inverse to that with $\langle q_i, q_j \rangle$, i.e.

$$\left[a_{ij} \right] = \left[\left(\left[L' P^k + \gamma^* \right] q_i , q_j \right) \right]^{-1} \tag{46}$$

Hence the projection $Q^l \phi$ can be written as

$$Q^l \phi = \sum_{i=1}^{l} \sum_{j=1}^{l} a_{ij} \left(\phi, \left[L' P^k + \gamma^* \right] q_j \right) q_i \tag{47}$$

As l increases, the projection $Q^{\ell}\phi$ increases, and for any ϕ in H,

$$0 \leq \left(\phi, [L'P^k + \gamma^*] Q^{\ell}\phi\right) \leq \left(\phi, [L'P^k + \gamma^*] Q^{\ell+1}\phi\right) \leq \left(\phi, [L'P^k + \gamma^*]\phi\right) \tag{48}$$

The operators $L^{l,k}$ are now defined by

$$L^{\ell,k} = [L^0 - \gamma^*] + [L'P^k + \gamma^*] Q^{\ell} \tag{49}$$

They have the explicit representation

$$L^{\ell,k}\phi = [L^0 - \gamma^*]\phi + \sum_{i=1}^{\ell}\sum_{j=1}^{\ell} a_{ij}\left(\phi, [L'P^k + \gamma^*] q_j\right)[L'P^k + \gamma^*] q_i \tag{50}$$

In view of equations (48) and (50) we have

$$\left(\phi, [L^0 - \gamma^*]\phi\right) \leq \left(\phi, L^{\ell,k}\phi\right) \leq \left(\phi, L^{\ell+1,k}\phi\right) \leq \left(\phi, L^k\phi\right) \leq \left(\phi, L\phi\right) \tag{51}$$

for any ϕ in D_L, and the corresponding eigenvalues satisfy then the parallel inequalities

$$\lambda_i^0 - \gamma^* \leq \lambda_i^{\ell,k} \leq \lambda_i^{\ell+1,k} \leq \lambda_i^k \leq \lambda_i \tag{52}$$

The original problem now reduces to the solution of the eigenvalue problem

$$L^{\ell,k}\psi = \lambda^{\ell,k}\psi \tag{53}$$

for the determination of lower bounds to the eigenvalues of the operator L.

To facilitate the solution of this problem, the following special choice of the functions q_i can always be made:

$$q_i = \left[L' P^k + \gamma^* \right]^{-1} \psi_i^o \tag{54}$$

where ψ_i^o are the eigenfunctions of L^o given by equation (30). With this choice, the operator $L^{l,k}$ takes the form

$$L^{l,k} \phi = \left[L^o - \gamma^* \right] \phi + \sum_{i=1}^{l} \sum_{j=1}^{l} a_{ij} \left(\phi, \psi_i^o \right) \psi_j^o \tag{55}$$

and the eigenvalue problem under consideration becomes

$$L^{l,k} \psi^{l,k} = \left[L^o - \gamma^* \right] \psi^{l,k} + \sum_{i=1}^{l} \sum_{j=1}^{l} a_{ij} \left(\psi^{l,k}, \psi_i^o \right) \psi_j^o = \lambda^{l,k} \psi^{l,k} \tag{56}$$

Its solution is accomplished as follows:

If the eigenfunction $\psi^{l,k}$ is orthogonal to the span of $\left\{ \psi_i^o \right\}_{i=1}^{l}$, equation (56) reduces to

$$\left[L^o - \gamma^* \right] \psi^{l,k} = \lambda^{l,k} \psi^{l,k} \tag{57}$$

which means that $L^{l,k}$ has the same eigenvalues and eigenfunctions as $\left[L^o - \gamma^* \right]$; or

$$\begin{aligned} \lambda_i^{l,k} &= \lambda_i^o - \gamma^* \\ \psi_i^{l,k} &= \psi_i^o \end{aligned} \qquad i > l \tag{58}$$

If, however, $\psi^{l,k}$ is in the span of $\left\{ \psi_i^o \right\}_{i=1}^{l}$, it may be written as

$$\psi^{l,k} = \sum_{n=1}^{l} \beta_n \psi_n^o \tag{59}$$

Substitution in equation (56) yields

$$\sum_{i=1}^{\ell} \beta_i (\lambda_i^o - \gamma^*) \psi_i^o + \sum_{i=1}^{\ell} \sum_{j=1}^{\ell} a_{ij} \beta_j \psi_i^o = \lambda^{\ell,k} \sum_{i=1}^{\ell} \beta_i \psi_i^o \tag{60}$$

which in view of the linear independence of the eigenfunctions ψ_i^o yields

$$\sum_{j=1}^{\ell} \left\{ (\lambda_i^o - \gamma^*) \delta_{ij} + a_{ij} - \lambda^{\ell,k} \delta_{ij} \right\} \beta_j = 0 \qquad i=1,2,\dots \ell \tag{61}$$

For non-trivial eigenfunctions $\psi^{\ell,k}$, the determinant of this system of equations must vanish:

$$\det \left| (\lambda_i^o - \gamma^*) \delta_{ij} + a_{ij} - \lambda^{\ell,k} \delta_{ij} \right| = 0 \tag{62}$$

This characteristic equation gives the eigenvalues of $L^{l,k}$ corresponding to the eigenfunctions in the span of $\left\{ \psi_i^o \right\}_{i=1}^{\ell}$. These can then be ordered with the other eigenvalues obtained from equation (58).

The major labor involved in the determination of the eigenvalues $\lambda^{\ell,k}$ lies in the computation of the elements a_{ij}. From Bazley and Fox[16] and Fauconneau[17] we find that they can be computed from the equation

$$[a_{ij}] = \left[\frac{1}{\gamma^*} \left\{ \delta_{ij} - \sum' \frac{\lambda'_m}{\lambda'_m + \gamma^*} (\psi_i^o, \psi'_m)(\psi_j^o, \psi'_m) \right\} \right]^{-1} \tag{63}$$

The positive parameter γ^* appearing in the formation of $L^{l,k}$ has been shown in reference 16 to have an optimum value for the determination of $\lambda_j^{l,k}$ such that $\lambda_{l+1}^{0} - \gamma^* = \lambda_j^{l,k}$. Since $\lambda_j^{l,k}$ is not known, γ^* must be chosen from an estimate of $\lambda_j^{l,k}$. Such an estimate was supplied here by the Rayleigh-Ritz procedure.

In the following section, the details of the numerical procedure used are reviewed, and the results are presented and discussed.

IV. RESULTS and DISCUSSION

The procedure used in the calculation of the eigenvalues of the plate was as follows: For a given aspect ratio a/b, and for given values of the parameters α and γ, the bounds for the eigenvalues λ defined by equation (12) were computed for fixed values of the parameter n. This parameter indicates the number of half waves in the mode shape in the y-direction. The range of discrete values of n considered was from 1 through 5. The circular frequencies were then calculated using equation (5).

The upper bounds were computed using 15 x 15 matrix sizes, and the lower bounds were obtained from intermediate operators with k=l=8. Thus, for fixed aspect ratio and fixed loading, a sequence of eigenvalue problems (one for each value of n) was solved to take into account the possible combinations of half waves in the x - and y- directions giving the mode shapes of the plate. Consequently, for each case considered, the upper bound method yielded a matrix giving upper approximations to the eigenvalues of the plate for combinations of its first 5 half waves in the y-direction and its first 15 half waves in the x-direction. Similarly, the Second Projection Method yielded lower approximations to the eigenvalues of the plate for combinations of its first 5 half waves in the y- direction and its first 8 half-waves in the x- direction. By inspection of the two resulting matrices, it was then possible to order the bounds to the eigenvalues in ascending order of magnitude, and to observe their corresponding mode orders.

This procedure was followed for fixed values of the parameter α and increasing values of γ to that value of γ for which the lowest eigenvalue became equal to zero. This critical value of γ gives an indication of the reduction of the critical constant edge loading brought about by the distributed in-plane load of magnitude corresponding to the given value of α.

The following geometrical and loading conditions were considered: a/b = 0.5 for a range of α from 5.0 to 30.0; a/b = 1.0 for α ranging from 5.0 to 65.0; a/b = 2.0 for α ranging from 5.0 to 60.0, and a/b = 3.0 for α ranging from 5.0 to 60.0.

The effect of the in-plane loads on the two lowest natural frequencies of the plate is illustrated in Figures 2 through 9 for the various aspect ratios and loading conditions studied.

Examination of the figures indicates that the variation of the frequencies with the loading depends markedly on the aspect ratio of the plate. Two distinct types of behavior are evidenced, depending on whether the aspect ratio is less than or greater than 1.0.

1) Aspect ratio less or equal to 1.0:

As illustrated in figures 2 through 4, for a/b $\leqslant$ 1.0, the natural frequencies decrease linearly with increasing values of γ, ie with increasing values of the edge loads, for fixed values of the distributed in-plane load parameter α. It should be noted that in Figures 3 and 5, which represent the variation of the second natural frequencies, the curves have been terminated at those values of γ for which the plate became elastically unstable.

For plates with aspect ratios 0.5 and 1.0, the lowest frequency always corresponds to the (1,1) mode which is also the buckling mode of the plate.

The large effect of the distributed in-plane load on the frequency variations and on the critical values of the edge load parameter γ is illustrated by the wide spread between the curves corresponding to fixed values of the parameter α .

2) Aspect ratio greater than 1.0

For aspect ratios greater than 1.0, the frequency variations with the loads are markedly different than those exhibited by plates with aspect ratios less or equal to 1.0. Figures 6 through 9 illustrate the variations of the first two frequencies of plates with a/b = 2.0 and 3.0. As the graphs indicate, the mode (1,1) is not always the one corresponding to the lowest frequency of the plate. Indeed, for small values of γ the first mode of vibration is the (1,1) mode and the frequency variation is essentially linear for γ up to approximately 0.5. However, as γ increases beyond this value, the lowest frequency switches to another mode. Figure 10 illustrates this phenomenon for a plate with a/b = 2.0 and $\alpha = 50$. In that figure, the variations of the frequencies associated with the (1,1) mode and the (2,1) mode are illustrated. For γ less than 0.64, the lowest frequency is associated with the (1,1) mode, and it decreases with increasing values of γ . The frequency associated with the (2,1) mode also decreases with increasing values of γ. However, for $\gamma = 0.64$, the two modes suddenly exchange roles: the lowest frequency now corresponds to the (2,1) mode which eventually becomes the buckling mode as γ reaches its critical value. It should be noted that after this mode interchange, each one continues the variation trend initiated by the other one.

The dashed line in Figure 6 indicates the locus of the mode transition points for the various values of α plotted.

Figure 11 shows how the plate with a/b = 3.0 exhibits two such mode shifts. First, the lowest frequency corresponds to the (1,1) mode, then it switches to the (2,1) mode and, eventually, to the (3,1) mode which is then the buckling mode. Qualitatively, similar behaviors are exhibited by the higher order frequencies. For instance, in Figure 9, the second frequency for the plate with a/b = 3.0 shifts mode three times before buckling occurs. Similar behaviors are to be expected for plates with higher aspect ratios. These results appear to be in qualitative agreement with the case handled by Herrmann[15] for a/b = 3.0.

As illustrated by Figures 6 and 8, the distributed in-plane load appears to have a much smaller effect on plates with a/b greater than 1.0 than as plates with aspect ratios less or equal to 1.0.

The effect of the distributed load on the critical edge load is illustrated in Figure 12 which permits the determination of the critical edge load for a given value of the aspect ratio and of the distributed in-plane load parameter. The results obtained here compare favorably with those of Wang and Sussman.[8] For instance, for a square plate with $\gamma = 0$, the present study indicates that the value of α for buckling is equal to 63.8. Using the graphical results of Wang and Sussman for the same conditions, the critical value of α is calculated to be 64.15 (within the limits of accuracy of their graph).

The coupling of the Second Projection method with the Rayleigh-Ritz method yielded excellent results: in all instances, even near buckling, the gap between the bounds for the eigenvalues over their average was less than 0.5 per cent. Table I shows some of the numerical results for the bounds for the eigenvalues of a square plate.

(Complete tabulated results are presented in Appendices A through D.) The results show once more the quality of the Rayleigh-Ritz method (at least for the trial functions used in this application). Of course, this quality could not be established a priori, and only through the calculation of lower bounds can confidence be established to use the method in predicting the natural frequencies of plates with other aspect ratios and other values of the loading parameters.

Finally, it should be noted that one of the advantages in the methods used here resides in the fact that the shift in the modes of vibration is extremely easy to observe. In other methods, such as finite difference methods and finite element methods, such observations are rather difficult, if not impossible, to make.

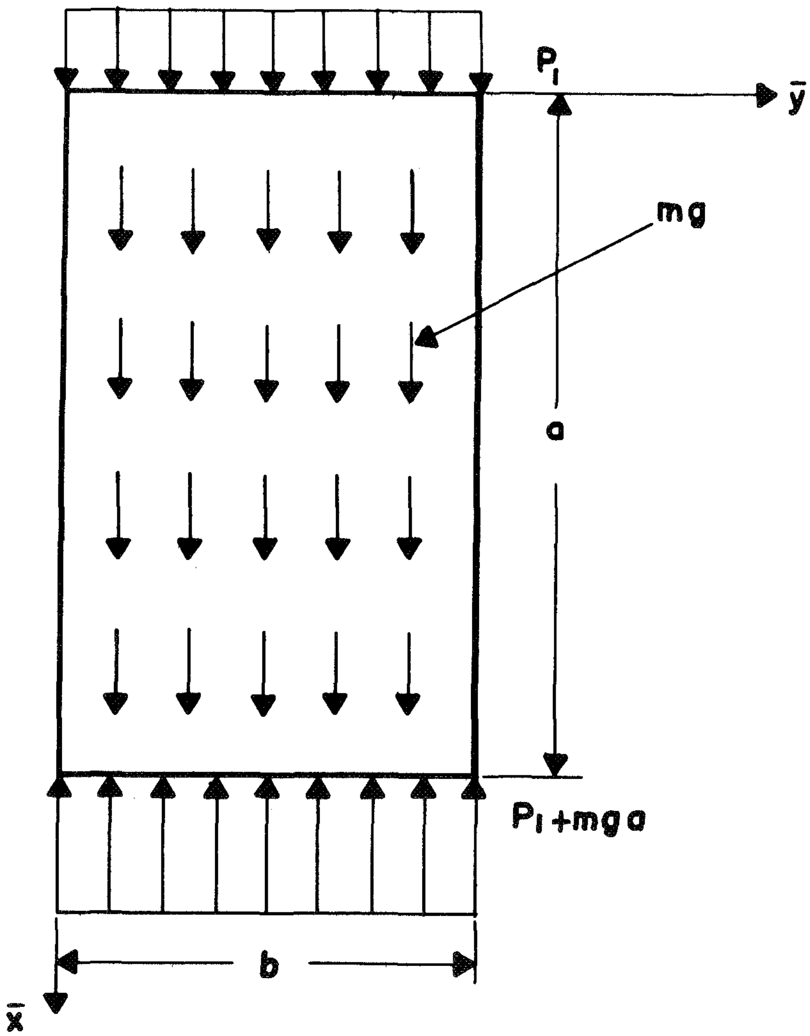


Figure 1 PLATE SUBJECTED TO LINEARLY VARYING COMPRESSIVE LOADS

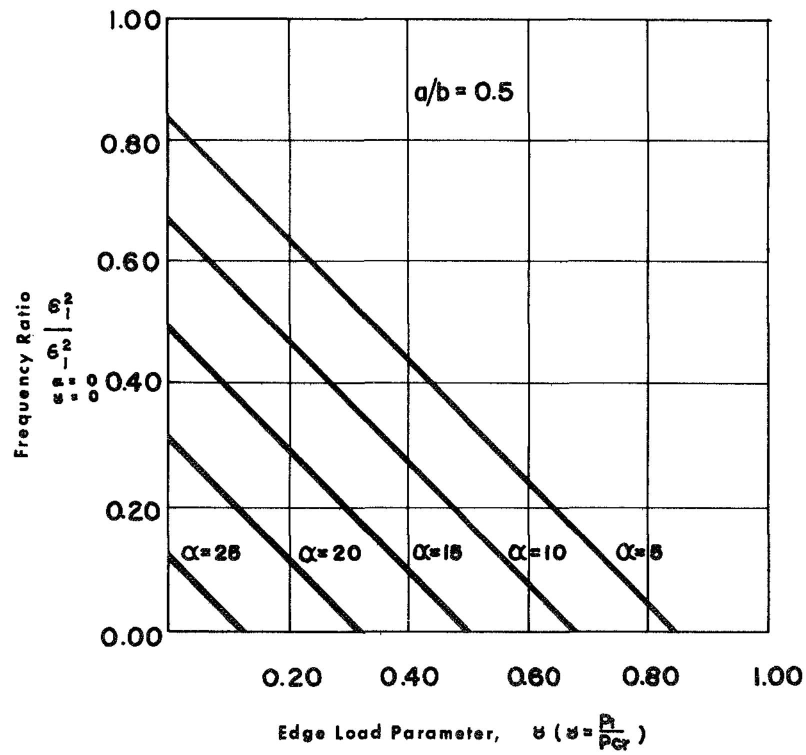


Figure 2 FIRST FREQUENCY VARIATION WITH IN-PLANE LOADS

(Aspect Ratio = 0.5)

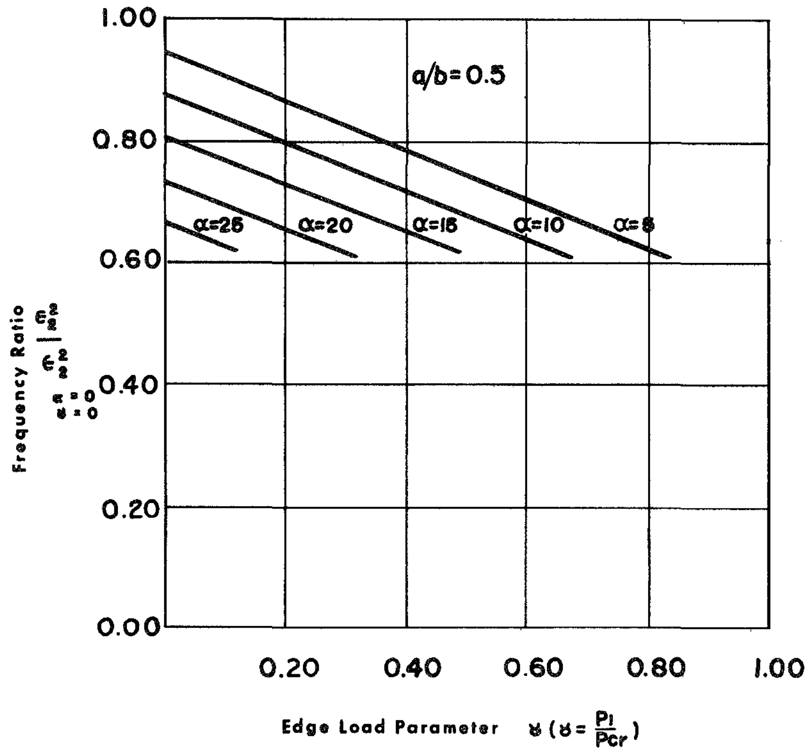


Figure 3 SECOND FREQUENCY VARIATION WITH IN-PLANE LOADS

(Aspect Ratio = 0.5)

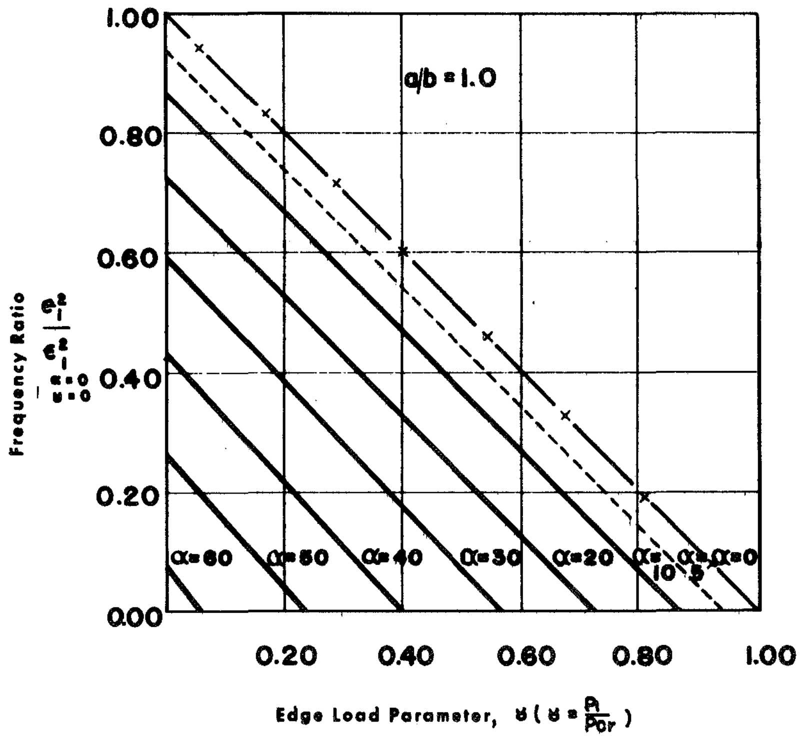


Figure 4 FIRST FREQUENCY VARIATION WITH IN-PLANE LOADS

(Aspect Ratio = 1.0)

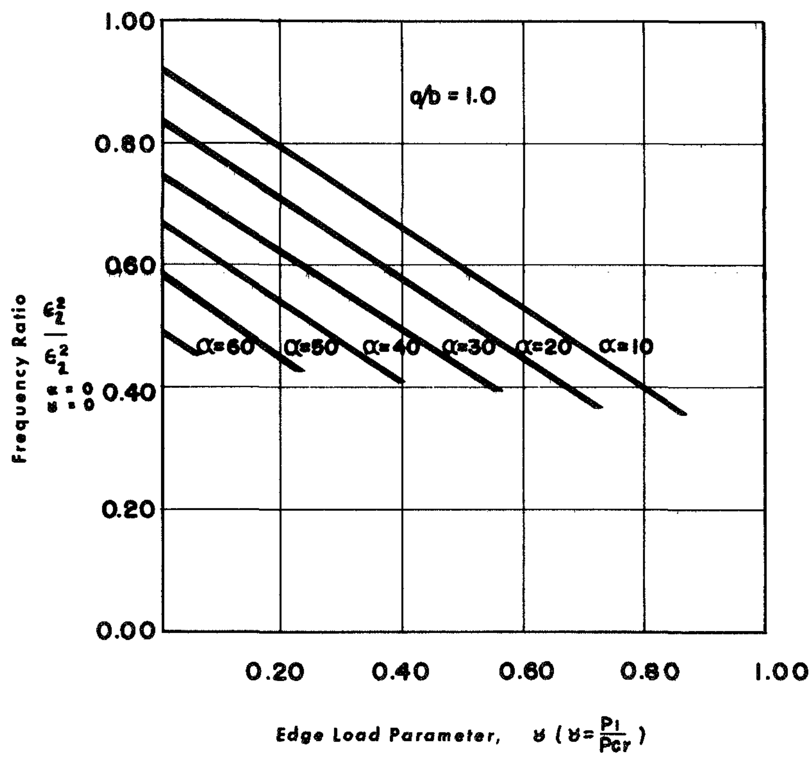


Figure 5 SECOND FREQUENCY VARIATION WITH IN-PLANE LOADS

(Aspect Ratio = 1.0)

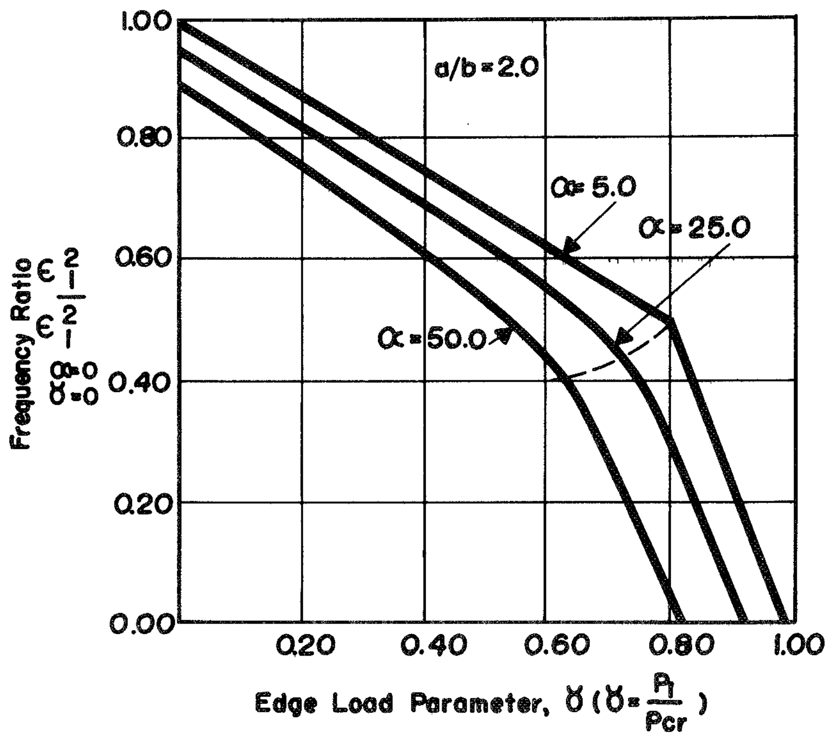
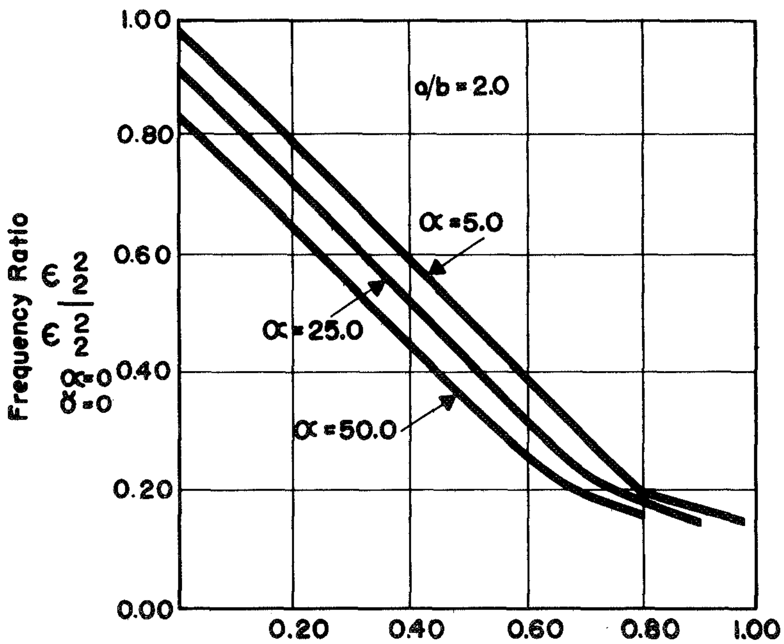


Figure 6 FIRST FREQUENCY VARIATIONS WITH IN-PLANE LOADS (Aspect Ratio = 2.0)



Edge Load Parameter, γ ($\gamma = \frac{P_1}{P_{cr}}$)

Figure 7 SECOND FREQUENCY VARIATION WITH IN-PLANE LOADS
(Aspect Ratio = 2.0)

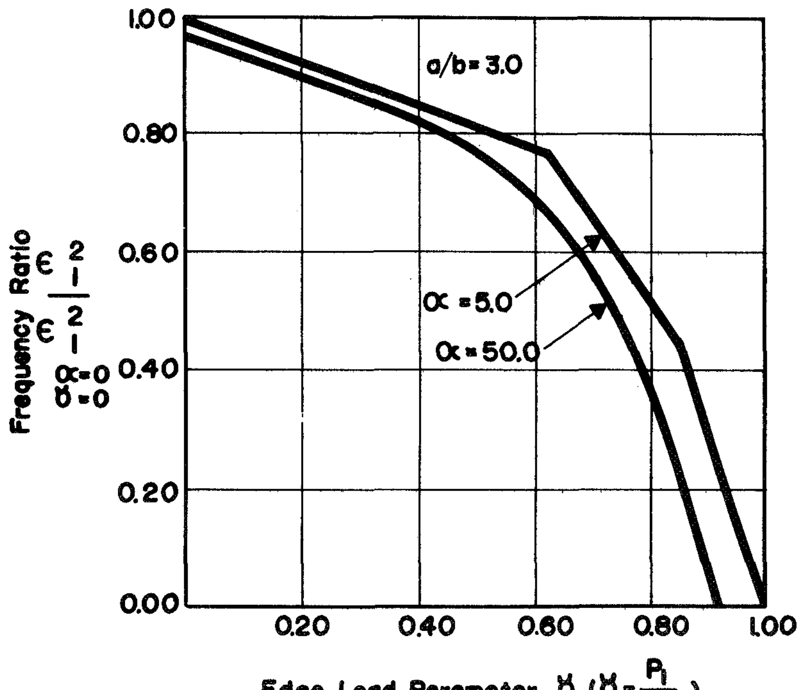


Figure 8 FIRST FREQUENCY VARIATION WITH IN-PLANE LOADS (Aspect Ratio = 3.0)

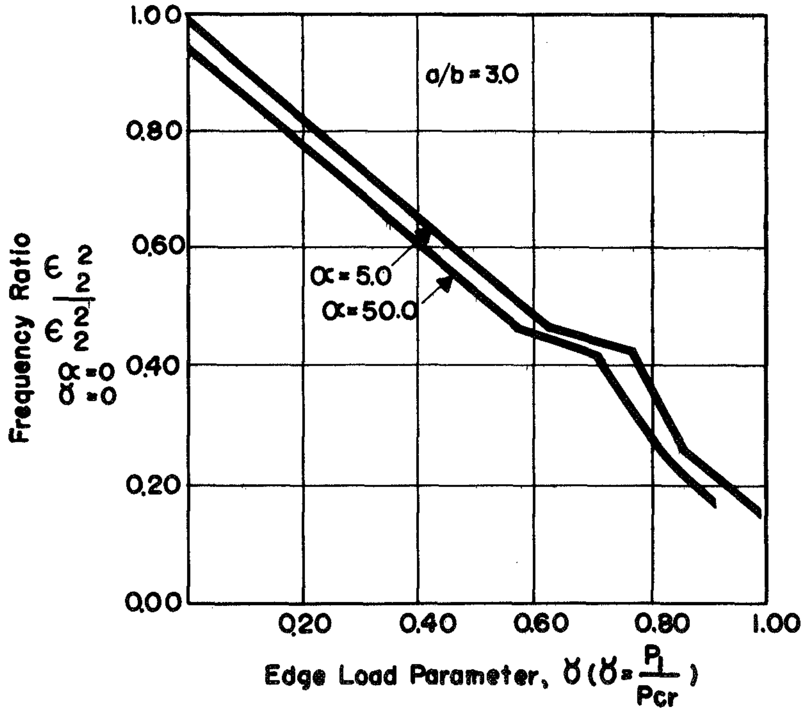


Figure 9 SECOND FREQUENCY VARIATION WITH IN-PLANE LOADS (Aspect Ratio = 3.0)

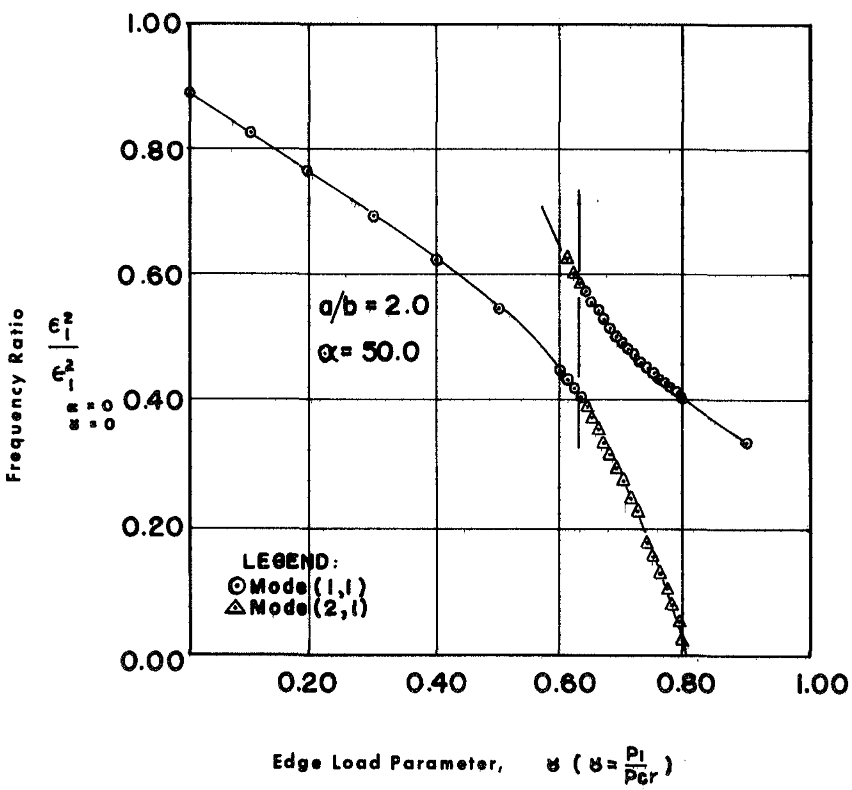


Figure 10 MODE SHIFT IN THE FIRST FREQUENCY VARIATION OF A PLATE SUBJECTED TO IN-PLANE LOADS (Aspect Ratio = 2.0)

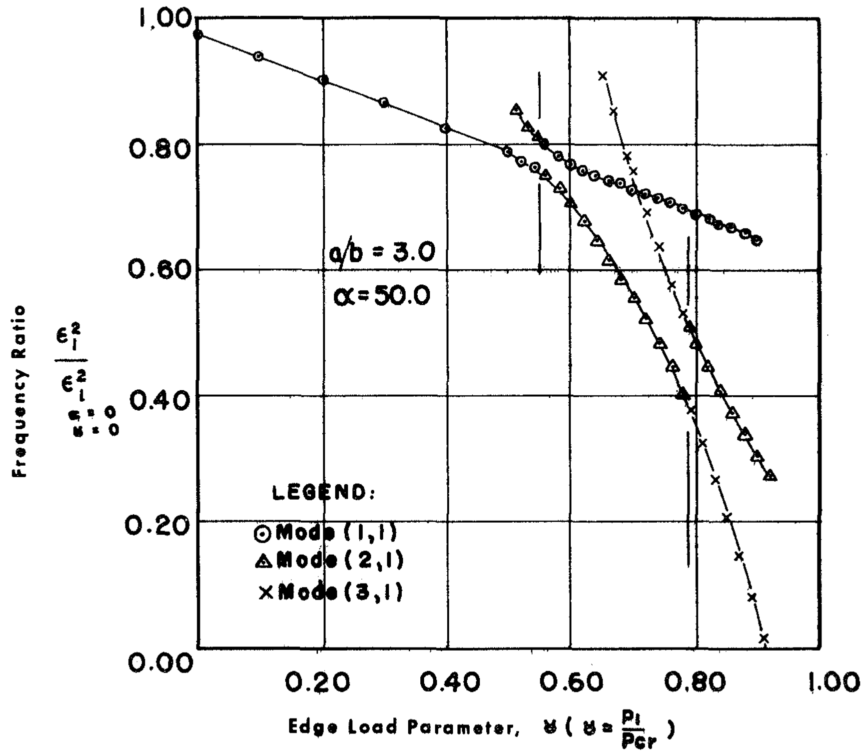


Figure 11 MODE SHIFT IN THE FIRST FREQUENCY VARIATION OF A PLATE SUBJECTED TO IN-PLANE LOADS (Aspect Ratio = 3.0)

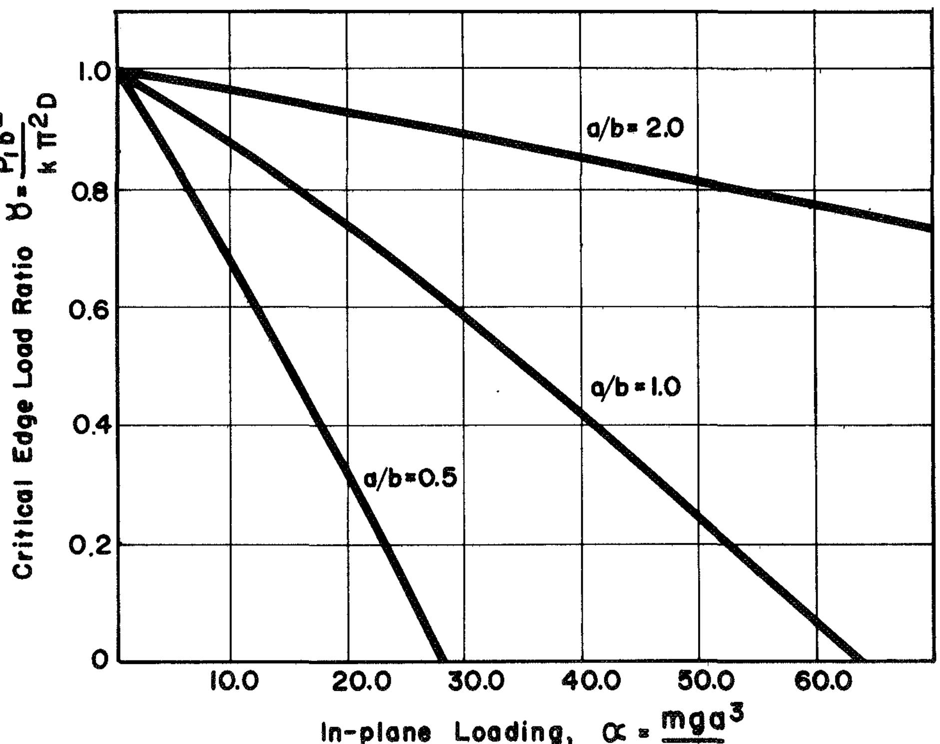


Figure 12 CRITICAL LOAD BUCKLING RATIOS

Table I

UPPER and LOWER BOUNDS to the FREQUENCIES OF A SQUARE PLATE WITH COMPRESSIVE AND IN-PLANE LOADING

$\alpha = 10.0$

γ	Order	Upper Bound	Lower Bound
	1	339.24	339.21
	2	2236.66	2236.41
0.00	3	2385.33	2385.31
	4	6035.76	6035.52
	5	9295.64	9292.32
	1	300.29	300.18
	2	2080.81	2080.57
0.10	3	2346.35	2346.31
	4	5879.89	5879.65
	5	8944.98	8941.66
	1	261.17	261.14
	2	1924.97	1924.73
0.20	3	2307.37	2307.34
	4	5724.02	5723.78
	5	8594.32	8591.00
	1	222.11	222.08
	2	1769.13	1768.89
0.30	3	2268.38	2268.36
	4	5568.15	5567.92
	5	8243.66	8240.35
	1	183.05	183.02
	2	1613.30	1613.08
0.40	3	2229.40	2229.37
	4	5412.29	5412.04
	5	7893.01	7889.72
	1	143.96	143.93
	2	1457.49	1457.26
0.50	3	2190.41	2190.38
	4	5256.42	5256.18
	5	7542.36	7539.06
	1	104.85	104.81
	2	1301.70	1301.47
0.60	3	2151.42	2151.40
	4	5100.55	5100.32
	5	7191.71	7188.43
	1	65.71	65.68
	2	1145.94	1145.70
0.70	3	2112.43	2112.40
	4	4944.69	4944.44
	5	6841.07	6837.79

Table I (continued)

γ	Order	Upper Bound	Lower Bound
	1	26.52	26.50
	2	990.21	989.97
0.80	3	2073.44	2073.41
	4	4788.82	4788.57
	5	6490.43	6487.15

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APPENDIX A

BOUNDS FOR THE FREQUENCIES OF A RECTANGULAR PLATE WITH IN-PLANE AND EDGE LOADS

ASPECT RATIO = 0.5

$\alpha = 5.0$

$\gamma = 0.0$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	127.20	127.19
2	(1,2)	364.71	364.70
3	(1,3)	1004.02	1004.01
4	(2,1)	1660.47	1660.34
5	(2,2)	2336.23	2336.11

$\alpha = 5.0$

$\gamma = 0.1$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	111.97	111.96
2	(1,2)	349.48	349.47
3	(1,3)	988.80	988.79
4	(2,1)	1599.59	1599.46
5	(2,2)	2275.35	2275.22

$\alpha = 5.0$

$\gamma = 0.2$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	96.743	96.730
2	(1,2)	334.26	334.25
3	(1,3)	973.58	973.57
4	(2,1)	1538.71	1538.58
5	(2,2)	2214.47	2214.35

$\alpha = 5.0$

$\gamma = 0.3$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	81.512	81.500
2	(1,2)	319.03	319.01
3	(1,3)	958.35	958.34
4	(2,1)	1477.84	1477.70
5	(2,2)	2153.59	2153.47

$\alpha = 5.0$

$\gamma = 0.4$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	66.280	66.269
2	(1,2)	303.80	303.79
3	(1,3)	943.13	943.12
4	(2,1)	1416.96	1416.83
5	(2,2)	2092.71	2092.59

$\alpha = 5.0$

$\gamma = 0.5$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	51.047	51.038
2	(1,2)	288.58	288.56
3	(1,3)	927.91	927.89
4	(2,1)	1356.08	1355.96
5	(2,2)	2031.83	2031.70

$\alpha = 5.0$

$\gamma = 0.6$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	35.813	35.797
2	(1,2)	273.35	273.33
3	(1,3)	912.68	912.67
4	(2,1)	1295.21	1295.08
5	(2,2)	1970.96	1970.83

$\alpha = 5.0$

$\gamma = 0.7$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	20.578	20.564
2	(1,2)	258.12	258.11
3	(1,3)	897.46	897.44
4	(2,1)	1234.34	1234.21
5	(2,2)	1910.08	1909.96

$\alpha = 5.0$

$\gamma = 0.8$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	5.341	5.329
2	(1,2)	242.89	242.88
3	(1,3)	882.24	882.22
4	(2,1)	1173.46	1173.34
5	(2,2)	1849.20	1849.08

α = 10.0

γ = 0.0

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	101.49	101.47
2	(1,2)	339.24	339.21
3	(1,3)	978.78	978.75
4	(2,1)	1560.94	1560.70
5	(2,2)	2236.66	2236.41

α = 10.0

γ = 0.1

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	86.229	86.205
2	(1,2)	323.99	323.97
3	(1,3)	963.54	963.51
4	(2,1)	1500.07	1499.83
5	(2,2)	2175.78	2175.54

α = 10.0

γ = 0.2

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	70.962	70.939
2	(1,2)	308.75	308.72
3	(1,3)	948.31	948.28
4	(2,1)	1439.21	1938.97
5	(2,2)	2114.90	2114.66

$\alpha = 10.0$

$\gamma = 0.3$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	55.692	55.655
2	(1,2)	293.50	293.47
3	(1,3)	933.08	933.05
4	(2,1)	1378.35	1378.11
5	(2,2)	2054.02	2053.79

$\alpha = 10.0$

$\gamma = 0.4$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	40.418	40.399
2	(1,2)	278.25	278.22
3	(1,3)	917.84	917.82
4	(2,1)	1317.49	1317.25
5	(2,2)	1993.15	1992.91

$\alpha = 10.0$

$\gamma = 0.5$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	25.140	25.115
2	(1,2)	263.00	262.97
3	(1,3)	902.61	902.58
4	(2,1)	1256.63	1256.40
5	(2,2)	1932.27	1932.04

$\alpha = 10.0$

$\gamma = 0.6$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9.8584	9.8304
2	(1,2)	247.74	247.72
3	(1,3)	887.37	887.34
4	(2,1)	1195.78	1195.54
5	(2,2)	1871.40	1871.16

$\alpha = 15$

$\gamma = 0.0$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	74.952	94.842
2	(1,2)	313.16	313.05
3	(1,3)	953.11	953.00
4	(2,1)	1460.90	1460.45
5	(2,2)	2136.50	2136.06

$\alpha = 15.0$

$\gamma = 0.1$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	59.623	59.514
2	(1,2)	297.88	297.77
3	(1,3)	937.86	937.75
4	(2,1)	1400.06	1399.60
5	(2,2)	2075.62	2075.18

$\alpha = 15.0$

$\gamma = 0.2$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	44.258	44.178
2	(1,2)	282.59	282.49
3	(1,3)	922.60	922.48
4	(2,1)	1339.22	1338.77
5	(2,2)	2014.75	2014.30

$\alpha = 15.0$

$\gamma = 0.3$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	28.940	28.841
2	(1,2)	267.30	267.20
3	(1,3)	907.35	907.23
4	(2,1)	1278.39	1277.94
5	(2,2)	1953.88	1953.42

$\alpha = 15.0$

$\gamma = 0.4$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	13.583	13.472
2	(1,2)	252.01	251.90
3	(1,3)	892.09	891.98
4	(2,1)	1217.57	1217.12
5	(2,2)	1893.01	1892.56

$\alpha = 20.0$

$\gamma = 0.0$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	47.440	47.410
2	(1,2)	286.39	286.35
3	(1,3)	926.98	926.94
4	(2,1)	1360.41	1360.00
5	(2,2)	2035.77	2035.36

$\alpha = 20.0$

$\gamma = 0.1$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	32.001	31.966
2	(1,2)	271.05	271.01
3	(1,3)	911.70	911.65
4	(2,1)	1299.61	1299.22
5	(2,2)	1974.91	1974.50

$\alpha = 20.0$

$\gamma = 0.2$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	16.546	16.505
2	(1,2)	255.70	255.67
3	(1,3)	896.41	896.37
4	(2,1)	1238.83	1238.43
5	(2,2)	1914.05	1913.64

$\alpha = 20.0$

$\gamma = 0.3$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1.0736	1.0423
2	(1,2)	240.34	240.31
3	(1,3)	881.13	881.09
4	(2,1)	1178.07	1177.67
5	(2,2)	1853.19	1852.79

$\alpha = 25.0$

$\gamma = 0.0$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	18.775	18.735
2	(1,2)	258.84	258.80
3	(1,3)	900.34	900.28
4	(2,1)	1259.56	1259.10
5	(2,2)	1934.50	1934.03

APPENDIX B

BOUNDS FOR THE FREQUENCIES OF A RECTANGULAR PLATE WITH IN-PLANE AND EDGE LOADS

ASPECT RATIO = 1.0

$\alpha = 5.00$

$\gamma = 0.00$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	364.710	364.697
2	(2,1)	2336.24	2336.11
3	(1,2)	2410.42	2410.40
4	(2,2)	6135.23	6135.10
5	(3,1)	9518.55	9516.82

$\alpha = 5.00$

$\gamma = 0.05$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	345.221	345.205
2	(2,1)	2258.31	2258.19
3	(1,2)	2390.94	2390.92
4	(2,2)	6057.30	6057.17
5	(3,1)	9343.22	9341.49

$\alpha = 5.00$

$\gamma = 0.10$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	325.731	325.721
2	(2,1)	2180.38	2180.26
3	(1,2)	2371.45	2371.44
4	(2,2)	5979.37	5979.24
5	(3,1)	9167.88	9166.15

$\alpha = 5.00$

$\gamma = 0.15$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	306.240	306.228
2	(2,1)	2102.46	2102.33
3	(1,2)	2351.97	2351.95
4	(2,2)	5901.44	5901.31
5	(3,1)	8992.55	8990.82

$\alpha = 5.00$

$\gamma = 0.20$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	286.749	286.734
2	(2,1)	2024.53	2024.40
3	(1,2)	2332.48	2332.47
4	(2,2)	5823.51	5823.37
5	(3,1)	8817.21	8815.49

$\alpha = 5.00$

$\gamma = 0.25$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	267.258	267.240
2	(2,1)	1946.60	1946.48
3	(1,2)	2313.00	2312.99
4	(2,2)	5745.58	5745.44
5	(3,1)	8641.88	8640.16

$\alpha = 5.00$

$\gamma = 0.30$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	247.765	247.753
2	(2,1)	1868.68	1868.55
3	(1,2)	2293.51	2293.50
4	(2,2)	5667.65	5667.53
5	(3,1)	8466.54	8464.82

$\alpha = 5.00$

$\gamma = 0.35$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	228.272	228.257
2	(2,1)	1790.75	1790.62
3	(1,2)	2274.03	2274.02
4	(2,2)	5589.73	5589.60
5	(3,1)	8291.21	8289.49

$\alpha = 5.00$

$\gamma = 0.40$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	208.778	208.768
2	(2,1)	1712.83	1712.70
3	(1,2)	2254.55	2254.53
4	(2,2)	5511.80	5511.67
5	(3,1)	8115.88	8114.15

$\alpha = 5.00$

$\gamma = .45$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	189.283	189.270
2	(2,1)	1634.91	1634.78
3	(1,2)	2235.06	2235.05
4	(2,2)	5433.87	5433.73
5	(3,1)	7940.54	7938.82

$\alpha = 5.00$

$\gamma = 0.50$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	169.786	169.772
2	(2,1)	1556.98	1556.85
3	(1,2)	2215.58	2215.56
4	(2,2)	5355.94	5355.81
5	(3,1)	7765.21	7763.49

$\alpha = 5.00$

$\gamma = 0.55$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	150.290	150.272
2	(2,1)	1479.06	1478.93
3	(1,2)	2196.09	2196.08
4	(2,2)	5378.01	5277.88
5	(3,1)	7589.88	7588.16

$\alpha = 5.00$

$\gamma = 0.60$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	130.790	130.778
2	(2,1)	1401.14	1401.02
3	(1,2)	2176.61	2176.58
4	(2,2)	5200.08	5199.95
5	(3,1)	7414.54	7412.82

$\alpha = 5.00$

$\gamma = 0.65$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	111.289	111.275
2	(2,1)	1323.22	1323.10
3	(1,2)	2157.12	2157.10
4	(2,2)	5122.15	5122.02
5	(3,1)	7239.21	7237.49

$\alpha = 5.00$

$\gamma = 0.70$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	91.787	91.778
2	(2,1)	1245.30	1245.17
3	(1,2)	2137.64	2137.62
4	(2,2)	5044.22	5044.09
5	(3,1)	7063.88	7062.16

$\alpha = 5.00$

$\gamma = 0.75$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	72.282	72.271
2	(2,1)	1167.39	1167.26
3	(1,2)	2118.15	2118.14
4	(2,2)	4966.29	4966.16
5	(3,1)	6888.54	6886.83

$\alpha = 5.00$

$\gamma = 0.80$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	52.775	52.761
2	(2,1)	1089.48	1089.35
3	(1,2)	2098.67	2098.65
4	(2,2)	4888.37	4888.24
5	(3,1)	6713.21	6711.50

$\alpha = 5.00$

$\gamma = 0.85$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	33.265	33.256
2	(2,1)	1011.56	1011.44
3	(1,2)	2079.18	2079.17
4	(2,2)	4810.44	4810.31
5	(3,1)	6537.88	6536.16

$\alpha = 5.00$

$\gamma = 0.90$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	13.751	13.740
2	(2,1)	933.66	933.53
3	(1,2)	2059.69	2059.67
4	(2,2)	4732.51	4732.37
5	(3,1)	6362.55	6360.83

α =10.00

γ = 0.00

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	339.241	339.210
2	(2,1)	2236.66	2236.41
3	(1,2)	2385.33	2385.31
4	(2,2)	6035.76	6035.52
5	(3,1)	9295.64	9292.32

α =10.00

γ = 0.05

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	319.726	319.692
2	(2,1)	2158.73	2158.50
3	(1,2)	2365.84	2365.81
4	(2,2)	5957.82	5957.60
5	(3,1)	9120.31	9116.99

α =10.00

γ = 0.10

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	300.209	300.181
2	(2,1)	2080.81	2080.57
3	(1,2)	2346.35	2346.31
4	(2,2)	5879.89	5879.65
5	(3,1)	8944.98	8941.66

$\alpha = 10.00$

$\gamma = 0.15$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	280.689	280.659
2	(2,1)	2002.89	2002.64
3	(1,2)	2326.86	2326.83
4	(2,2)	5801.96	5801.71
5	(3,1)	8769.65	8766.33

$\alpha = 10.00$

$\gamma = 0.20$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	261.167	261.142
2	(2,1)	1924.97	1924.73
3	(1,2)	2307.37	2307.34
4	(2,2)	5724.02	5723.78
5	(3,1)	8594.32	8591.00

$\alpha = 10.00$

$\gamma = 0.25$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	241.642	241.607
2	(2,1)	1847.05	1846.80
3	(1,2)	2287.88	2287.85
4	(2,2)	5646.09	5645.84
5	(3,1)	8418.99	8415.68

α =10.00

γ = 0.30

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	222.113	222.084
2	(2,1)	1769.13	1768.89
3	(1,2)	2268.38	2268.36
4	(2,2)	5568.15	5567.92
5	(3,1)	8243.66	9240.35

α =10.00

γ = 0.35

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	202.581	202.557
2	(2,1)	1691.22	1690.98
3	(1,2)	2248.89	2248.86
4	(2,2)	5490.22	5489.98
5	(3,1)	8068.34	8065.03

α = 10.00

γ = 0.40

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	183.045	183.018
2	(2,1)	1613.30	1613.08
3	(1,2)	2229.40	2229.37
4	(2,2)	5412.29	5412.04
5	(3,1)	7893.01	7889.72

$\alpha = 10.00$

$\gamma = 0.45$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	163.504	163.467
2	(2,1)	1535.40	1535.17
3	(1,2)	2209.91	2209.87
4	(2,2)	5334.35	5334.11
5	(3,1)	7717.68	7714.39

$\alpha = 10.00$

$\gamma = 0.50$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	143.958	143.927
2	(2,1)	1457.49	1457.26
3	(1,2)	2190.41	2190.38
4	(2,2)	5256.42	5256.18
5	(3,1)	7542.36	7539.06

$\alpha = 10.00$

$\gamma = 0.55$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	124.406	124.372
2	(2,1)	1379.60	1379.36
3	(1,2)	2170.92	2170.89
4	(2,2)	5178.49	5178.24
5	(3,1)	7367.04	7363.75

$\alpha = 10.00$

$\gamma = 0.60$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	104.848	104.811
2	(2,1)	1301.70	1301.47
3	(1,2)	2151.42	2151.40
4	(2,2)	5100.55	5100.31
5	(3,1)	7191.71	7188.43

$\alpha = 10.00$

$\gamma = 0.65$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	85.281	85.250
2	(2,1)	1223.81	1223.58
3	(1,2)	2131.93	2131.90
4	(2,2)	5022.62	5022.38
5	(3,1)	7016.39	7013.10

$\alpha = 10.00$

$\gamma = 0.70$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	65.705	65.679
2	(2,1)	1145.94	1145.70
3	(1,2)	2112.43	2112.40
4	(2,2)	4944.69	4944.44
5	(3,1)	6841.07	6837.79

α =10.00

γ = 0.75

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	46.118	46.091
2	(2,1)	1068.07	1067.83
3	(1,2)	2092.93	2092.90
4	(2,2)	4866.75	4866.50
5	(3,1)	6665.75	6662.46

α =10.00

γ = 0.80

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	26.519	26.497
2	(2,1)	990.21	989.97
3	(1,2)	2073.44	2073.41
4	(2,2)	4788.82	4788.57
5	(3,1)	6490.43	6487.15

α =10.00

γ = 0.85

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	6.904	6.880
2	(2,1)	912.36	912.13
3	(1,2)	2053.94	2053.90
4	(2,2)	4710.89	4710.65
5	(3,1)	6315.11	6311.84

α = 15.00

γ = 0.00

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	313.161	313.122
2	(2,1)	2136.50	2136.16
3	(1,2)	2359.95	2359.91
4	(2,2)	5935.75	5935.40
5	(3,1)	9072.17	9067.42

α = 15.00

γ = 0.05

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	293.598	293.557
2	(2,1)	2058.58	2058.25
3	(1,2)	2340.45	2340.40
4	(2,2)	5857.81	5857.47
5	(3,1)	8896.85	8892.11

α = 15.00

γ = 0.10

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	274.030	273.994
2	(2,1)	1980.66	1980.23
3	(1,2)	2320.95	2320.90
4	(2,2)	5779.87	5779.52
5	(3,1)	8721.53	8716.79

α =15.00

γ = 0.15

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	254.455	254.417
2	(2,1)	1902.75	1902.42
3	(1,2)	2301.44	2301.40
4	(2,2)	5701.93	5701.59
5	(3,1)	8546.21	8541.48

α =15.00

γ = 0.20

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	234.874	234.834
2	(2,1)	1824.85	1824.52
3	(1,2)	2281.94	2281.89
4	(2,2)	5623.99	5623.65
5	(3,1)	8370.89	8366.16

α =15.00

γ = 0.25

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	215.285	215.250
2	(2,1)	1746.94	1746.62
3	(1,2)	2262.43	2262.39
4	(2,2)	5546.04	5545.70
5	(3,1)	8195.58	8190.84

$\alpha = 15.00$

$\gamma = 0.30$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	195.688	195.651
2	(2,1)	1669.05	1668.72
3	(1,2)	2242.92	2242.89
4	(2,2)	5468.10	5467.77
5	(3,1)	8020.26	8015.54

$\alpha = 15.00$

$\gamma = 0.35$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	176.081	176.042
2	(2,1)	1591.16	1590.84
3	(1,2)	2223.42	2223.37
4	(2,2)	5390.16	5389.82
5	(3,1)	7844.95	7840.22

$\alpha = 15.00$

$\gamma = 0.40$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	156.463	156.430
2	(2,1)	1513.28	1512.95
3	(1,2)	2203.91	2203.87
4	(2,2)	5312.22	5311.88
5	(3,1)	7669.64	7664.92

α =15.00

γ = 0.45

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	136.833	136.798
2	(2,1)	1435.41	1435.08
3	(1,2)	2184.40	2184.36
4	(2,2)	5234.28	5233.93
5	(3,1)	7494.32	7489.62

α =15.00

γ = 0.50

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	117.190	117.152
2	(2,1)	1357.55	1357.22
3	(1,2)	2164.89	2164.84
4	(2,2)	5156.34	5155.99
5	(3,1)	7319.02	7314.32

α =15.00

γ = 0.55

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	97.530	97.498
2	(2,1)	1279.70	1279.37
3	(1,2)	2145.38	2145.34
4	(2,2)	5078.39	5078.05
5	(3,1)	7143.71	7139.01

$\alpha = 15.00$

$\gamma = 0.60$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	77.852	77.817
2	(2,1)	1201.87	1201.54
3	(1,2)	2125.87	2125.82
4	(2,2)	5000.45	5000.12
5	(3,1)	6968.40	6963.71

$\alpha = 15.00$

$\gamma = 0.65$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	58.152	58.115
2	(2,1)	1124.05	1123.74
3	(1,2)	2106.35	2106.31
4	(2,2)	4922.51	4922.18
5	(3,1)	6793.10	6788.41

$\alpha = 15.00$

$\gamma = 0.70$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	38.427	38.396
2	(2,1)	1046.26	1045.94
3	(1,2)	2086.84	2086.80
4	(2,2)	4844.57	4844.23
5	(3,1)	6617.80	6613.14

α =15.00

γ = 0.75

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	18.672	18.640
2	(2,1)	968.50	968.18
3	(1,2)	2067.32	2067.28
4	(2,2)	4766.63	4766.29
5	(3,1)	6442.51	6437.84

α = 20.00

γ = 0.00

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	286.391	286.345
2	(2,1)	2035.77	2035.36
3	(1,2)	2334.25	2334.21
4	(2,2)	5835.21	5834.76
5	(3,1)	8848.17	8842.16

α = 20.00

γ = 0.05

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	266.752	266.713
2	(2,1)	1957.87	1957.45
3	(1,2)	2314.73	2314.69
4	(2,2)	5757.25	5756.81
5	(3,1)	8672.86	8666.85

α = 20.00

γ = 0.10

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	247.102	247.061
2	(2,1)	1879.97	1879.56
3	(1,2)	2295.21	2295.16
4	(2,2)	5679.30	5678.86
5	(3,1)	8497.56	8491.56

α =20.00

γ = 0.15

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	227.440	227.404
2	(2,1)	1802.08	1801.68
3	(1,2)	2275.69	2275.64
4	(2,2)	5601.35	5600.91
5	(3,1)	8322.25	8316.27

α =20.00

γ = 0.20

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	207.764	207.726
2	(2,1)	1724.20	1723.80
3	(1,2)	2256.17	2256.13
4	(2,2)	5523.39	5522.95
5	(3,1)	8146.96	8140.97

α =20.00

γ = 0.25

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	188.072	188.032
2	(2,1)	1646.34	1645.94
3	(1,2)	2236.64	2236.59
4	(2,2)	5445.44	5445.01
5	(3,1)	7971.66	7965.68

α =20.00

γ= 0.30

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	168.363	168.321
2	(2,1)	1568.48	1568.07
3	(1,2)	2217.11	2217.07
4	(2,2)	5367.49	5367.06
5	(3,1)	7796.36	7790.39

α =20.00

γ= 0.35

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	148.633	148.597
2	(2,1)	1490.64	1490.24
3	(1,2)	2197.58	2197.54
4	(2,2)	5289.53	5289.10
5	(3,1)	7621.07	7615.11

α =20.00

γ= 0.40

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	128.882	128.844
2	(2,1)	1412.82	1412.42
3	(1,2)	2178.05	2178.00
4	(2,2)	5211.58	5211.15
5	(3,1)	7445.78	7439.82

α =20.00

γ = 0.45

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	109.104	109.072
2	(2,1)	1335.02	1334.61
3	(1,2)	2158.52	2158.47
4	(2,2)	5133.63	5133.20
5	(3,1)	7270.49	7264.56

α =20.00

γ = 0.50

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	89.298	89.264
2	(2,1)	1257.24	1256.84
3	(1,2)	2138.98	2138.94
4	(2,2)	5055.68	5055.25
5	(3,1)	7095.21	7089.28

α =20.00

γ = 0.55

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	69.457	69.421
2	(2,1)	1179.49	1179.09
3	(1,2)	2119.44	2119.40
4	(2,2)	4977.72	4977.30
5	(3,1)	6919.93	6914.02

$\alpha = 20.00$

$\gamma = 0.60$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	49.576	49.546
2	(2,1)	1101.77	1101.37
3	(1,2)	2099.90	2099.86
4	(2,2)	4899.77	4899.35
5	(3,1)	6744.65	6738.74

$\alpha = 20.00$

$\gamma = 0.65$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	29.649	29.617
2	(2,1)	1024.10	1023.69
3	(1,2)	2080.36	2080.32
4	(2,2)	4821.82	4821.39
5	(3,1)	6569.38	6563.48

$\alpha = 20.00$

$\gamma = 0.70$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9.666	9.633
2	(2,1)	946.47	946.08
3	(1,2)	2060.82	2060.76
4	(2,2)	4743.87	4743.44
5	(3,1)	6394.12	6388.23

α =25.00

γ = 0.00

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	258.836	258.803
2	(2,1)	1934.50	1934.03
3	(1,2)	2308.22	2308.16
4	(2,2)	5734.11	5733.58
5	(3,1)	8623.65	8616.51

α =25.00

γ = 0.05

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	239.087	239.051
2	(2,1)	1856.62	1856.15
3	(1,2)	2288.68	2288.61
4	(2,2)	5656.14	5655.62
5	(3,1)	8448.36	8441.23

α =25.00

γ = 0.10

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	219.316	219.279
2	(2,1)	1778.76	1778.30
3	(1,2)	2269.13	2269.07
4	(2,2)	5578.17	5577.66
5	(3,1)	8273.08	8265.96

α = 25.00

γ = 0.15

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	199.524	199.484
2	(2,1)	1700.92	1700.45
3	(1,2)	2249.58	2249.52
4	(2,2)	5500.20	5499.68
5	(3,1)	8097.80	8090.71

α = 25.00

γ = 0.20

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	179.706	179.673
2	(2,1)	1623.09	1622.63
3	(1,2)	2230.03	2229.97
4	(2,2)	5422.23	5421.71
5	(3,1)	7922.52	7915.44

α = 25.00

γ = 0.25

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	159.861	159.817
2	(2,1)	1545.28	1544.82
3	(1,2)	2210.48	2210.42
4	(2,2)	5344.26	5343.75
5	(3.1)	7747.25	7740.18

$\alpha = 25.00$

$\gamma = 0.30$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	139.983	139.946
2	(2,1)	1467.50	1467.03
3	(1,2)	2190.92	2190.86
4	(2,2)	5266.30	5265.77
5	(3,1)	7571.98	7564.91

$\alpha = 25.00$

$\gamma = 0.35$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	120.068	120.030
2	(2,1)	1389.74	1389.28
3	(1,2)	2171.36	2171.30
4	(2,2)	5188.33	5187.82
5	(3,1)	7396.72	7389.67

$\alpha = 25.00$

$\gamma = 0.40$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	100.113	100.080
2	(2,1)	1312.02	1311.56
3	(1,2)	2151.79	2151.73
4	(2,2)	5110.36	5109.85
5	(3,1)	7221.46	7214.42

$\alpha = 25.00$

$\gamma = 0.45$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	80.109	80.075
2	(2,1)	1234.34	1233.88
3	(1,2)	2132.23	2132.17
4	(2,2)	5032.40	5031.89
5	(3,1)	7046.21	7039.19

$\alpha = 25.00$

$\gamma = 0.50$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	60.050	60.014
2	(2,1)	1156.70	1156.24
3	(1,2)	2112.66	2112.60
4	(2,2)	4954.43	4953.92
5	(3,1)	6870.97	6863.96

$\alpha = 25.00$

$\gamma = 0.55$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	39.926	39.889
2	(2,1)	1079.12	1078.66
3	(1,2)	2093.09	2093.02
4	(2,2)	4876.47	4875.96
5	(3,1)	6695.73	6688.75

α = 25.00

γ = 0.60

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	19.727	19.696
2	(2,1)	1001.61	1001.15
3	(1,2)	2073.51	2073.45
4	(2,2)	4798.50	4797.99
5	(3,1)	6520.50	6513.52

$\alpha = 30.00$

$\gamma = 0.00$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	230.384	230.324
2	(2,1)	1832.72	1832.20
3	(1,2)	2281.83	2281.76
4	(2,2)	5632.44	5631.85
5	(3,1)	8398.63	8390.51

$\alpha = 30.00$

$\gamma = 0.05$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	210.478	210.424
2	(2,1)	1754.90	1754.36
3	(1,2)	2262.25	2262.19
4	(2,2)	5554.45	5553.86
5	(3,1)	8223.38	8215.29

$\alpha = 30.00$

$\gamma = 0.10$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	190.538	190.491
2	(2,1)	1677.09	1676.58
3	(1,2)	2242.67	2242.61
4	(2,2)	5476.46	5475.88
5	(3,1)	8048.12	8040.04

α = 30.00

γ = 0.15

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	170.562	170.520
2	(2,1)	1599.32	1598.79
3	(1,2)	2223.09	2223.02
4	(2,2)	5398.48	5397.89
5	(3,1)	7872.87	7864.81

α = 30.00

γ = 0.20

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	150.543	150.499
2	(2,1)	1521.57	1521.05
3	(1,2)	2203.50	2203.43
4	(2,2)	5320.49	5319.90
5	(3,1)	7697.63	7689.59

α = 30.00

γ = 0.25

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	130.476	130.439
2	(2,1)	1443.87	1443.36
3	(1,2)	2183.91	2183.85
4	(2,2)	5242.50	5241.91
5	(3,1)	7522.39	7514.37

α = 30.00

γ = 0.30

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	110.355	110.317
2	(2,1)	1366.20	1365.69
3	(1,2)	2164.32	2164.24
4	(2,2)	5164.52	5163.93
5	(3,1)	7347.17	7339.17

α = 30.00

γ = 0.35

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	90.172	90.140
2	(2,1)	1288.59	1288.08
3	(1,2)	2144.71	2144.65
4	(2,2)	5086.54	5085.95
5	(3,1)	7171.95	7163.96

α = 30.00

γ = 0.40

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	69.918	69.877
2	(2,1)	1211.03	1210.53
3	(1,2)	2125.11	2125.05
4	(2,2)	5008.55	5007.97
5	(3,1)	6996.73	6988.76

α =30.00

γ = 0.45

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	49.581	49.547
2	(2,1)	1133.54	1133.03
3	(1,2)	2105.50	2105.44
4	(2,2)	4930.57	4930.01
5	(3,1)	6821.53	6813.59

α =30.00

γ = 0.50

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	29.147	29.120
2	(2,1)	1056.13	1055.63
3	(1,2)	2085.88	2085.83
4	(2,2)	4852.59	4852.02
5	(3,1)	6646.34	6638.42

α =30.00

γ = 0.55

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8.600	8.564
2	(2,1)	978.83	978.34
3	(1,2)	2066.26	2066.19
4	(2,2)	4774.61	4774.03
5	(3,1)	6471.16	6463.26

$\alpha = 35.00$

$\gamma = 0.00$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	200.896	200.850
2	(2,1)	1730.50	1729.93
3	(1,2)	2255.05	2254.99
4	(2,2)	5530.18	5529.53
5	(3,1)	8173.14	8164.12

$\alpha = 35.00$

$\gamma = 0.05$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	180.777	180.730
2	(2,1)	1652.75	1652.18
3	(1,2)	2235.44	2235.36
4	(2,2)	5452.18	5451.53
5	(3,1)	7997.92	7988.99

$\alpha = 35.00$

$\gamma = 0.10$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	160.605	160.557
2	(2,1)	1575.04	1574.48
3	(1,2)	2215.81	2215.74
4	(2,2)	5374.17	5373.51
5	(3,1)	7822.70	7813.80

α =35.00

γ = 0.15

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	140.374	140.332
2	(2,1)	1497.38	1496.83
3	(1,2)	2196.19	2196.12
4	(2,2)	5296.16	5295.51
5	(3,1)	7647.50	7638.62

α =35.00

γ = 0.20

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	120.076	120.025
2	(2,1)	1419.77	1419.21
3	(1,2)	2176.55	2176.48
4	(2,2)	5218.16	5217.51
5	(3,1)	7472.30	7463.44

α = 35.00

γ = 0.25

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	99.701	99.649
2	(2,1)	1342.21	1341.66
3	(1,2)	2156.92	2156.85
4	(2,2)	5140.15	5139.50
5	(3,1)	7297.11	7288.29

$\alpha = 35.00$

$\gamma = 0.30$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	79.239	79.200
2	(2,1)	1264.73	1264.19
3	(1,2)	2137.27	2137.21
4	(2,2)	5062.15	5061.50
5	(3,1)	7121.94	7113.14

$\alpha = 35.00$

$\gamma = 0.35$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	58.675	58.637
2	(2,1)	1187.33	1186.79
3	(1,2)	2117.62	2117.55
4	(2,2)	4984.14	4983.50
5	(3,1)	6946.78	6938.01

$\alpha = 35.00$

$\gamma = 0.40$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	37.995	37.956
2	(2,1)	1110.03	1109.49
3	(1,2)	2097.96	2097.89
4	(2,2)	4906.14	4905.51
5	(3,1)	6771.63	6762.88

α =35.00

γ = 0.45

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	17.179	17.138
2	(2,1)	1032.85	1032.31
3	(1,2)	2078.29	2078.23
4	(2,2)	4828.14	4827.50
5	(3,1)	6596.49	6587.77

α =40.00

γ = 0.00

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	170.208	170.154
2	(2,1)	1627.90	1627.30
3	(1,2)	2227.86	2227.79
4	(2,2)	5427.34	5426.62
5	(3,1)	7947.20	7937.54

α =40.00

γ = 0.05

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	149.802	149.762
2	(2,1)	1550.28	1549.67
3	(1,2)	2208.20	2208.12
4	(2,2)	5349.31	5348.61
5	(3,1)	7772.02	7762.40

α =40.00

γ = 0.10

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	129.317	129.268
2	(2,1)	1472.71	1472.11
3	(1,2)	2188.52	2188.44
4	(2,2)	5271.28	5270.58
5	(3,1)	7596.86	7587.29

$\alpha = 40.00$

$\gamma = 0.15$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	108.740	108.691
2	(2,1)	1395.22	1394.64
3	(1,2)	2168.84	2168.76
4	(2,2)	5193.25	5192.54
5	(3,1)	7421.71	7412.16

$\alpha = 40.00$

$\gamma = 0.20$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	88.060	88.018
2	(2,1)	1317.81	1317.23
3	(1,2)	2149.15	2149.08
4	(2,2)	5115.22	5114.52
5	(3,1)	7246.58	7237.05

$\alpha = 40.00$

$\gamma = 0.25$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	67.262	67.219
2	(2,1)	1240.49	1239.91
3	(1,2)	2129.45	2129.38
4	(2,2)	5037.19	5036.49
5	(3,1)	7071.46	7061.98

α =40.00

γ = 0.30

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	46.328	46.284
2	(2,1)	1163.29	1162.71
3	(1,2)	2109.75	2109.67
4	(2,2)	4959.17	4958.48
5	(3,1)	6896.35	6886.91

α =40.00

γ = 0.35

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	25.235	25.197
2	(2,1)	1086.22	1085.65
3	(1,2)	2090.03	2089.96
4	(2,2)	4881.15	4880.46
5	(3,1)	6721.26	6711.85

α = 40.00

γ = 0.40

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	3.957	3.919
2,	(2,1)	1009.31	1008.74
3	(1,2)	2070.30	2070.23
4	(2,2)	4803.12	4802.43
5	(3,1)	6546.19	6536.81

α = 45.00

γ = 0.00

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	138.115	138.062
2	(2,1)	1525.04	1524.42
3	(1,2)	2200.23	2200.15
4	(2,2)	5323.89	5323.14
5	(3,1)	7720.84	7710.61

α = 45.00

γ = 0.05

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	117.329	117.284
2	(2,1)	1447.60	1446.98
3	(1,2)	2180.50	2180.42
4	(2,2)	5245.84	5245.09
5	(3,1)	7545.73	7535.54

α = 45.00

γ = 0.10

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	96.425	96.379
2	(2,1)	1370.25	1369.65
3	(1,2)	2160.76	2160.68
4	(2,2)	5167.78	5167.03
5	(3,1)	7370.64	7360.48

α =45.00

γ = 0.15

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	75.386	75.346
2	(2,1)	1293.01	1292.40
3	(1,2)	2141.01	2140.94
4	(2,2)	5089.73	5088.97
5	(3,1)	7195.56	7185.45

α =45.00

γ = 0.20

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	54.191	54.151
2	(2,1)	1215.89	1215,30
3	(1,2)	2121.26	2121.19
4	(2,2)	5011.67	5010.94
5	(3,1)	7020.50	7010.43

α = 45.00

γ = 0.25

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	32.817	32.769
2	(2,1)	1138.92	1138.33
3	(1,2)	2101.48	2101.41
4	(2,2)	4933.62	4932.90
5	(3,1)	6845.46	6835.44

α =45.00

γ = 0.30

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	11.236	11.202
2	(2,1)	1062.13	1061.56
3	(1,2)	2081.70	2081.63
4	(2,2)	4855.58	4854.85
5	(3,1)	6670.44	6660.47

α = 50.00

γ = 0.00

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	104.368	104.317
2	(2,1)	1422.05	1421.42
3	(1,2)	2172.11	2172.03
4	(2,2)	5219.84	5219.06
5	(3,1)	7494.11	7483.43

α = 50.00

γ = 0.05

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	83.083	83.031
2	(2,1)	1344.88	1344.26
3	(1,2)	2152.31	2152.23
4	(2,2)	5141.75	5140.97
5	(3,1)	7319.08	7308.45

α = 50.00

γ = 0.10

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	61.625	61.580
2	(2,1)	1267.84	1267.22
3	(1,2)	2132.50	2132.41
4	(2,2)	5063.67	5062.89
5	(3,1)	7144.07	7133.49

α =50.00

γ= 0.15

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	39.968	39.915
2	(2,1)	1190.97	1190.37
3	(1,2)	2112.67	2112.58
4	(2,2)	4985.59	4984.82
5	(3,1)	6969.08	6958.56

α=50.00

γ= 0.20

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	18.084	18.038
2	(2,1)	1114.28	1113.69
3	(1,2)	2092.82	2092.74
4	(2,2)	4907.51	4906.74
5	(3,1)	6794.12	6783.64

α =55.0

γ = 0.00

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	68.663	68.614
2	(2,1)	1319.14	1318.53
3	(1,2)	2143.48	2143.39
4	(2,2)	5115.16	5114.34
5	(3,1)	7267.05	7256.04

α =55.0

γ = 0.05

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	46.722	46.673
2	(2,1)	1242.35	1241.74
3	(1,2)	2123.59	2123.50
4	(2,2)	5037.04	5036.23
5	(3,1)	7092.11	7081.16

α =55.0

γ = 0.10

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	24.535	24.477
2	(2,1)	1165.76	1165.16
3	(1,2)	2103.68	2103.59
4	(2,2)	4958.93	4958.13
5	(3,1)	6917.21	6906.32

$\alpha = 55.0$

$\gamma = 0.15$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2.0612	2.0186
2	(2,1)	1089.41	1088.81
3	(1,2)	2083.76	2083.68
4	(2,2)	4880.83	4880.04
5	(3,1)	6742.33	6731.50

α =60.0

γ= 0.00

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	30.620	30.565
2	(2,1)	1216.57	1215.95
3	(1,2)	2114.28	2114.19
4	(2,2)	5009.85	5009.04
5	(3,1)	7039.70	7028.49

α=60.0

γ= 0.05

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	7.8258	7.7783
2	(2,1)	1140.31	1139.71
3	(1,2)	2094.28	2094.21
4	(2,2)	4931.71	4930.89
5	(3,1)	6864.89	6853.75

APPENDIX C

BOUNDS FOR THE FREQUENCIES OF A RECTANGULAR PLATE WITH IN-PLANE AND EDGE LOADS

ASPECT RATIO = 2.0

$\alpha = 5.0$

$\gamma = 0.0$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2410.42	2410.40
2	(2,1)	6135.23	6135.10
3	(3,1)	16,239.77	16,238.03
4	(1,2)	28,126.50	28,126.49
5	(4,1)	38,568.54	38,537.30

$\alpha = 5.0$

$\gamma = 0.1$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2254.55	2254.53
2	(2,1)	5511.80	5511.67
3	(3,1)	14,837.08	14,835.34
4	(1,2)	27,970.65	27,970.64
5	(4,1)	36,074.87	36,043.64

$\alpha = 5.0$

$\gamma = 0.2$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2098.67	2098.65
2	(2,1)	4888.37	4888.24
3	(3,1)	13,434.39	13,432.65
4	(1,2)	27,814.79	27,814.78
5	(4,1)	33,581.21	33,550.00

$\alpha = 5.0$

$\gamma = 0.3$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1942.78	1942.77
2	(2,1)	4264.94	4264.82
3	(3,1)	12,031.70	12,029.98
4	(1,2)	27,658.93	27,658.92
5	(4,1)	31,087.54	31,056.35

$\alpha = 5.0$

$\gamma = 0.4$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1786.87	1786.86
2	(2,1)	3641.52	3641.39
3	(3,1)	10,629.02	10,627.29
4	(1,2)	27,503.07	27,503.06
5	(4,1)	28,593.87	28,562.70

$\alpha = 5.0$

$\gamma = 0.5$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1630.92	1630.91
2	(2,1)	3018.12	3017.99
3	(3,1)	9226.35	9224.62
4	(4,1)	26,100.21	26,069.06
5	(1,2)	27,347.22	27,347.20

$\alpha = 5.0$

$\gamma = 0.6$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1474.89	1474.88
2	(2,1)	2394.79	2394.67
3	(3,1)	7823.68	7821.97
4	(4,1)	23,606.55	23,575.42
5	(1,2)	27,191.36	27,191.35

$\alpha = 5.0$

$\gamma = 0.7$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1318.48	1318.46
2	(2,1)	1771.81	1771.68
3	(3,1)	6421.04	6419.32
4	(4,1)	21,112.90	21,081.79
5	(1,2)	27,035.50	27,035.49

$\alpha = 5.0$

$\gamma = 0.8$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	1131.71	1131.66
2	(1,1)	1179.14	1179.05
3	(3,1)	5018.44	5016.74
4	(4,1)	18,619.26	18,588.16
5	(1,2)	26,879.64	26,879.63

$\alpha = 5.0$

$\gamma = 0.9$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	522.44	522.33
2	(1,1)	1008.88	1008.86
3	(3,1)	3615.91	3614.21
4	(4,1)	16,125.62	16,094.56
5	(1,2)	26,723.78	26,723.77

$\alpha = 10.0$

$\gamma = 0.0$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2385.33	2385.31
2	(2,1)	6035.75	6035.52
3	(3,1)	16,016.83	16,013.46
4	(1,2)	28,101.69	28,101.65
5	(4,1)	38,172.87	38,111.29

$\alpha = 10.0$

$\gamma = 0.1$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2229.40	2229.37
2	(2,1)	5412.29	5412.04
3	(3,1)	14,614.14	14,610.78
4	(1,2)	27,945.82	27,945.79
5	(4,1)	35,679.21	35,617.68

$\alpha = 10.0$

$\gamma = 0.2$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2073.44	2073.41
2	(2,1)	4788.82	4788.57
3	(3,1)	13,211.45	13,208.10
4	(1,2)	27,789.96	27,789.94
5	(4,1)	33,185.56	33,124.11

α = 10.0

γ = 0.3

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1917.43	1917.41
2	(2,1)	4165.36	4165.12
3	(3,1)	11,808.77	11,805.44
4	(1,2)	27,634.10	27,634.06
5	(4,1)	30,691.91	30,630.52

α = 10.0

γ = 0.4

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1761.34	1761.32
2	(2,1)	3541.94	3541.70
3	(3,1)	10,406.11	10,402.79
4	(1,2)	27,478.23	27,478.20
5	(4,1)	28,198.27	28,136.95

α = 10.0

γ = 0.5

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1605.09	1605.06
2	(2,1)	2918.63	2918.40
3	(3,1)	9003.50	9000.20
4	(4,1)	25,704.64	25,643.40
5	(1,2)	27,322.36	27,322.34

$\alpha = 10.0$

$\gamma = 0.6$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1448.41	1448.38
2	(2,1)	2295.67	2295.43
3	(3,1)	7600.93	7597.66
4	(4,1)	23,211.03	23,149.87
5	(1,2)	27,166.49	27,166.46

$\alpha = 10.0$

$\gamma = 0.7$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1289.69	1289.67
2	(2,1)	1674.61	1674.37
3	(3,1)	6198.46	6195.20
4	(4,1)	20,717.44	20,656.38
5	(1,2)	27,010.62	27,010.60

$\alpha = 10.0$

$\gamma = 0.8$

ORDER	MODE	UPPER BOUND	LOWER BOUND
	(2,1)	1027.57	1027.42
	(1,1)	1156.74	1156.64
	(3,1)	4796.14	4792.93
	(4,1)	18,223.88	18,162.92
	(1,2)	26,854.75	26,854.71

$\alpha =$ 10.0

$\gamma =$ 0.9

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	417.28	417.10
2	(1,1)	986.66	986.61
3	(3,1)	3394.15	3390.95
4	(4,1)	15,730.37	15,669.50
5	(1,2)	26,698.88	26,698.85

$\alpha = 15.0$

$\gamma = 0.0$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2359.95	2359.91
2	(2,1)	5935.75	5935.10
3	(3,1)	15,793.31	15,788.44
4	(1,2)	28,076.78	28,076.73
5	(4,1)	37,776.63	37,685.63

$\alpha = 15.0$

$\gamma = 0.1$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2203.91	2203.87
2	(2,1)	5312.22	5311.88
3	(3,1)	14,390.60	14,385.77
4	(1,2)	27,920.90	27,920.85
5	(4,1)	35,282.98	35,192.11

$\alpha = 15.0$

$\gamma = 0.2$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2047.80	2047.76
2	(2,1)	4688.69	4688.35
3	(3,1)	12,987.92	12,983.10
4	(1,2)	27,765.03	27,764.98
5	(4,1)	32,789.36	32,698.63

α = 15.0
γ = 0.3

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1891.58	1891.55
2	(2,1)	4065.20	4064.86
3	(3,1)	11,585.27	11,580.50
4	(1,2)	27,609.15	27,609.11
5	(4,1)	30,295.74	30,205.16

α = 15.0
γ = 0.4

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1735.17	1735.13
2	(2,1)	3441.80	3441.47
3	(3,1)	10,182.67	10,177.92
4	(1,2)	27,453.27	27,453.22
5	(4,1)	27,802.14	27,711.74

α = 15.0
γ = 0.5

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1578.33	1578.29
2	(2,1)	2818.68	2818.36
3	(3,1)	8780.15	8775.45
4	(4,1)	25,308.57	25,218.35
5	(1,2)	27,297.38	27,297.34

α = 15.0

γ = 0.6

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1420.30	1420.27
2	(2,1)	2196.55	2196.24
3	(3,1)	7377.77	7373.11
4	(4,1)	22,815.04	22,725.02
5	(1,2)	27,141.50	27,141.46

α = 15.0

γ = 0.7

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1255.90	1255.88
2	(2,1)	1580.48	1580.16
3	(3,1)	5975.61	5971.00
4	(4,1)	20,321.57	20,231.75
5	(1,2)	26,985.61	26,985.57

α = 15.0

γ = 0.8

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	918.41	918.22
2	(1,1)	1136.98	1136.86
3	(3,1)	4573.85	4569.31
4	(4,1)	17,828.16	17,738.57
5	(1,2)	26,829.71	26,829.67

α = 15.0

γ = 0.9

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	308.17	307.95
2	(1,1)	965.29	965.24
3	(3,1)	3172.89	3168.40
4	(4,1)	15,334.84	15,245.48
5	(1,2)	26,673.81	26,673.77

α = 20.0

γ = 0.0

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1.1)	2334.25	2334.21
2	(2,1)	5835.21	5834.76
3	(3,1)	15,569.20	15,562.97
4	(1,2)	28,051.77	28,051.70
5	(4,1)	37,379.81	37,260.31

α = 20.0

γ = 0.1

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2178.05	2178.00
2	(2,1)	5211.58	5211.15
3	(3,1)	14,166.49	14,160.30
4	(1,2)	27,895.88	27,895.82
5	(4,1)	34,886.18	34,766.93

α = 20.0

γ = 0.2

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2021.72	2021.67
2	(2,1)	4587.97	4587.54
3	(3,1)	12,763.81	12,757.68
4	(1,2)	27,739.98	27,739.93
5	(4,1)	32,392.59	32,273.61

$\alpha = 20.0$

$\gamma = 0.3$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1865.18	1865.14
2	(2,1)	3964.44	3964.02
3	(3,1)	11,361.21	11,355.13
4	(1,2)	27,584.09	27,584.02
5	(4,1)	29,899.01	29,780.32

$\alpha = 20.0$

$\gamma = 0.4$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1708.24	1708.20
2	(2,1)	3341.11	3340.69
3	(3,1)	9958.69	9952.69
4	(4,1)	27,405.48	27,287.09
5	(1,2)	27,428.18	27,428.12

$\alpha = 20.0$

$\gamma = 0.5$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1550.43	1550.40
2	(2,1)	2718.38	2718.97
3	(3,1)	8556.34	8550.41
4	(4,1)	24,912.01	24,793.93
5	(1,2)	27,272.28	27,272.21

$\alpha = 20.0$

$\gamma = 0.6$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1390.05	1390.02
2	(2,1)	2097.84	2097.45
3	(3,1)	7154.25	7148.40
4	(4,1)	22,418.60	22,300.88
5	(1,2)	27,116.36	27,116.30

$\alpha = 20.0$

$\gamma = 0.7$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1213.63	1213.62
2	(2,1)	1492.74	1492.37
3	(3,1)	5752.58	5746.83
4	(4,1)	19,925.29	19,807.94
5	(1,2)	26,960.44	26,960.39

$\alpha = 20.0$

$\gamma = 0.8$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	806.41	806.19
2	(1,1)	1117.44	1117.30
3	(3,1)	4351.72	4346.07
4	(4,1)	17,432.12	17,315.16
5	(1,2)	26,804.51	26,804.46

$\alpha = 20.0$

$\gamma = 0.9$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	195.09	194.87
2	(1,1)	944.40	944.31
3	(3,1)	2952.43	2946.89
4	(4,1)	14,939.10	14,822.59
5	(1,2)	26,648.57	26,648.51

$\alpha = 25.0$

$\gamma = 0.0$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2308.22	2308.16
2	(2,1)	5734.11	5733.58
3	(3,1)	15,344.50	15,337.01
4	(1,2)	28,026.66	28,026.59
5	(4,1)	36,982.41	36,835.40

$\alpha = 25.0$

$\gamma = 0.1$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2151.79	2151.73
2	(2,1)	5110.36	5109.85
3	(3,1)	13,941.78	13,934.39
4	(1,2)	27,870.75	27,870.68
5	(4,1)	34,488.81	34,342.18

$\alpha = 25.0$

$\gamma = 0.2$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1995.15	1995.09
2	(2,1)	4486.66	4486.17
3	(3,1)	12,539.13	12,531.81
4	(1,2)	27,714.83	27,714.77
5	(4,1)	31,995.26	31,849.05

$\alpha = 25.0$

$\gamma = 0.3$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1838.15	1838.10
2	(2,1)	3863.09	3862.60
3	(3,1)	11,136.59	11,129.36
4	(1,2)	27,558.91	27,558.85
5	(4,1)	29,501.76	29,355.98

$\alpha = 25.0$

$\gamma = 0.4$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1680.45	1680.41
2	(2,1)	3239.90	3239.43
3	(3,1)	9734.22	9727.10
4	(4,1)	27,008.32	26,863.03
5	(1,2)	27,402.98	27,402.92

$\alpha = 25.0$

$\gamma = 0.5$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1521.19	1521.15
2	(2,1)	2617.84	2617.38
3	(3,1)	8332.11	8325.10
4	(4,1)	24,514.97	24,370.20
5	(1,2)	27,247.04	27,246.98

$\alpha = 25.0$

$\gamma = 0.6$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1356.93	1356.91
2	(2,1)	2000.12	1999.69
3	(3,1)	6930.43	6923.56
4	(4,1)	22,021.74	21,877.52
5	(1,2)	27,091.09	27,091.03

$\alpha = 25.0$

$\gamma = 0.7$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1157.89	1157.87
2	(2,1)	1416.19	1415.79
3	(3,1)	5529.49	5522.75
4	(4,1)	19,528.66	19,385.02
5	(1,2)	26,935.13	26,935.07

$\alpha = 25.0$

$\gamma = 0.8$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	691.61	691.38
2	(1,1)	1097.79	1097.65
3	(3,1)	4129.92	4123.34
4	(4,1)	17,035.79	16,892.80
5	(1,2)	26,779.15	26,779.09

$\alpha = 25.0$

$\gamma = 0.9$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	77.823	77.600
2	(1,1)	923.71	923.63
3	(3,1)	2733.14	2726.73
4	(4,1)	14,543.20	14,400.89
5	(1,2)	26,623.16	26,623.10

$\alpha = 30.0$

$\gamma = 0.0$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2281.83	2281.76
2	(2,1)	5632.44	5631.85
3	(3,1)	15,119.22	15,110.60
4	(1,2)	28,001.44	28,001.36
5	(4,1)	36,584.44	36,410.83

$\alpha = 30.0$

$\gamma = 0.1$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2125.11	2125.05
2	(2,1)	5008.55	5007.97
3	(3,1)	13,716.50	13,708.01
4	(1,2)	27,845.51	27,845.43
5	(4,1)	34,090.89	33,917.86

$\alpha = 30.0$

$\gamma = 0.2$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1968.05	1968.00
2	(2,1)	4384.74	4384.18
3	(3,1)	12,313.89	12,305.51
4	(1,2)	27,689.56	27,689.48
5	(4,1)	31,597.39	31,424.97

α = 30.0

γ = 0.3

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1810.42	1810.36
2	(2,1)	3761.16	3760.62
3	(3,1)	10,911.44	10,903.21
4	(1,2)	27,533.61	27,533.52
5	(4,1)	29,103.97	28,932.20

α = 30.0

γ = 0.4

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1651.67	1651.63
2	(2,1)	3138.23	3137.71
3	(3,1)	9509.26	9501.17
4	(4,1)	26,610.65	26,439.58
5	(1,2)	27,377.64	27,377.57

α = 30.0

γ = 0.5

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1490.28	1490.26
2	(2,1)	2517.27	2516.77
3	(3,1)	8107.48	8099.55
4	(4,1)	24,117.47	23,947.14
5	(1,2)	27,221.67	27,221.58

$\alpha = 30.0$

$\gamma = 0.6$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1319.92	1319.90
2	(2,1)	1904.28	1903.81
3	(3,1)	6706.37	6698.64
4	(4,1)	21,624.46	21,454.95
5	(1,2)	27,065.67	27,065.59

$\alpha = 30.0$

$\gamma = 0.7$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1084.67	1084.62
2	(2,1)	1354.63	1354.24
3	(3,1)	5306.44	5298.91
4	(4,1)	19,131.69	18,963.04
5	(1,2)	26,909.66	26,909.59

$\alpha = 30.0$

$\gamma = 0.8$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	573.80	573.57
2	(1,1)	1077.93	1077.78
3	(3,1)	3908.68	3901.37
4	(4,1)	16,639.23	16,471.52
5	(1,2)	26,753.63	26,753.57

$\alpha = 35.0$

$\gamma = 0.0$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2255.05	2254.99
2	(2,1)	5530.18	5529.53
3	(3,1)	14,893.34	14,883.72
4	(1,2)	27,976.12	27,976.04
5	(4,1)	36,185.90	35,986.70

$\alpha = 35.0$

$\gamma = 0.1$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2097.96	2097.89
2	(2,1)	4906.14	4905.51
3	(3,1)	13,490.63	13,481.18
4	(1,2)	27,820.16	27,820.07
5	(4,1)	33,692.40	33,493.96

$\alpha = 35.0$

$\gamma = 0.2$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1940.37	1940.31
2	(2,1)	4282.23	4281.62
3	(3,1)	12,088.08	12,078.80
4	(1,2)	27,664.18	27,664.10
5	(4,1)	31,198.98	31,001.39

α = 35.0

γ = 0.3

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1781.92	1781.85
2	(2,1)	3658.68	3658.09
3	(3,1)	10,685.77	10,676.67
4	(1,2)	27,508.19	27,508.10
5	(4,1)	28,705.67	28,508.99

α = 35.0

γ = 0.4

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1621.74	1621.69
2	(2,1)	3036.18	3035.62
3	(3,1)	9283.84	9274.94
4	(4,1)	26,212.51	26,016.78
5	(1,2)	27,352.18	27,342.09

α = 35.0

γ = 0.5

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1457.34	1457.30
2	(2,1)	2416.94	2416.41
3	(3,1)	7882.50	7873.81
4	(4,1)	23,719.53	23,524.84
5	(1,2)	27,196.15	27,196.07

$\alpha = 35.0$

$\gamma = 0.6$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1277.61	1277.60
2	(2,1)	1811.58	1811.08
3	(3,1)	6482.17	6473.72
4	(4,1)	21,226.81	21,033.23
5	(1,2)	27,040.11	27,040.03

$\alpha = 35.0$

$\gamma = 0.7$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	994.73	994.62
2	(1,1)	1307.08	1306.74
3	(3,1)	5083.58	5075.41
4	(4,1)	18,734.42	18,542.04
5	(1,2)	26,884.04	26,883.95

$\alpha = 35.0$

$\gamma = 0.8$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	452.67	452.45
2	(1,1)	1057.78	1057.63
3	(3,1)	3688.26	3680.39
4	(4,1)	16,242.47	16,051.37
5	(1,2)	26,727.93	26,727.84

$\alpha = 40.0$

$\gamma = 0.0$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2227.86	2227.79
2	(2,1)	5427.34	5426.62
3	(3,1)	14,666.88	14,656.39
4	(1,2)	27,950.70	27,950.59
5	(4,1)	35,786.79	35,562.96

$\alpha = 40.0$

$\gamma = 0.1$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2070.30	2070.23
2	(2,1)	4803.12	4802.43
3	(3,1)	13,264.20	13,253.90
4	(1,2)	27,794.69	27,794.59
5	(4,1)	33,293.36	33,070.56

$\alpha = 40.0$

$\gamma = 0.2$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1912.04	1911.98
2	(2,1)	4179.11	4178.44
3	(3,1)	11,861.73	11,851.65
4	(1,2)	27,638.67	27,638.58
5	(4,1)	30,800.03	30,578.34

α = 40.0

γ = 0.3

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1752.53	1752.46
2	(2,1)	3555.67	3555.04
3	(3,1)	10,459.60	10,449.76
4	(1,2)	27,482.64	27,482.54
5	(4,1)	28,306.86	28,086.36

α = 40.0

γ = 0.4

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1590.45	1590.40
2	(2,1)	2933.85	2933.25
3	(3,1)	9058.00	9048.42
4	(4,1)	25,813.89	25,594.65
5	(1,2)	27,326.58	27,326.49

α = 40.0

γ = 0.5

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1421.84	1421.81
2	(2,1)	2317.23	2316.68
3	(3,1)	7657.24	7647.95
4	(4,1)	23,321.18	23,103.30
5	(1,2)	27,170.50	27,170.41

$\alpha = 40.0$

$\gamma = 0.6$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1228.13	1228.11
2	(2,1)	1723.69	1723.18
3	(3,1)	6257.90	6248.91
4	(4,1)	20,828.81	20,612.41
5	(1,2)	27,014.39	27,014.29

$\alpha = 40.0$

$\gamma = 0.7$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	892.14	892.01
2	(1,1)	1269.20	1268.90
3	(3,1)	4861.06	4852.41
4	(4,1)	18,336.88	18,122.07
5	(1,2)	26,858.24	26,858.15

$\alpha = 40.0$

$\gamma = 0.8$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	327.89	327.70
2	(1,1)	1037.27	1037.13
3	(3,1)	3468.96	3460.70
4	(4,1)	15,845.58	15,632.48
5	(1,2)	26,702.06	26,701.97

α =45.0

γ = 0.0

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2200.23	2200.15
2	(2,1)	5323.89	5323.14
3	(3,1)	14,439.84	14,428.58
4	(1,2)	27,925.16	27,925.06
5	(4,1)	35,387.12	35,139.66

α =45.0

γ = 0.1

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2042.10	2042.02
2	(2,1)	4699.49	4698.79
3	(3,1)	13,037.19	13,026.17
4	(1,2)	27,769.11	27,769.00
5	(4,1)	32,893.77	32,647.63

α =45.0

γ = 0.2

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1882.99	1882.93
2	(2,1)	4075.40	4074.71
3	(3,1)	11,634.84	11,624.09
4	(1,2)	27,613.05	27,612.93
5	(4,1)	30,400.56	30,155.83

α = 45.0

γ = 0.3

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1722.13	1722.06
2	(2,1)	3452.17	3451.52
3	(3,1)	10,232.94	10,222.48
4	(1,2)	27,456.96	27,456.86
5	(4,1)	27,907.56	27,664.35

α = 45.0

γ = 0.4

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1557.56	1557.51
2	(2,1)	2831.38	2830.79
3	(3,1)	8831.77	8821.62
4	(4,1)	25,414.82	25,173.24
5	(1,2)	27,300.84	27,300.74

α = 45.0

γ = 0.5

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1383.15	1383.12
2	(2,1)	2218.65	2218.10
3	(3,1)	7431.76	7421.94
4	(4,1)	22,922.43	22,682.59
5	(1,2)	27,144.70	27,144.60

α = 45.0

γ = 0.6

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1169.39	1169.36
2	(2,1)	1642.53	1642.05
3	(3,1)	6033.69	6024.28
4	(4,1)	20,430.48	20,192.53
5	(1,2)	26,988.51	26,988.41

α = 45.0

γ = 0.7

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	780.61	780.46
2	(1,1)	1236.99	1236.73
3	(3,1)	4639.08	4630.10
4	(4,1)	17,939.14	17,703.23
5	(1,2)	26,832.28	26,832.18

α = 45.0

γ = 0.8

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	199.07	198.91
2	(1,1)	1016.31	1016.20
3	(3,1)	3251.12	3242.63
4	(4,1)	15,448.59	15,214.90
5	(1,2)	26,676.00	26,675.89

$\alpha = 50.0$

$\gamma = 0.0$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2172.11	2172.03
2	(2,1)	5219.84	5219.06
3	(3,1)	14,212.21	14,200.29
4	(1,2)	27,899.51	27,899.40
5	(4,1)	34,986.89	34,716.79

$\alpha = 50.0$

$\gamma = 0.1$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2013.29	2013.21
2	(2,1)	4595.25	4594.51
3	(3,1)	12,809.62	12,797.99
4	(1,2)	27,743.42	27,743.31
5	(4,1)	32,493.64	32,225.18

$\alpha = 50.0$

$\gamma = 0.2$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1853.15	1853.09
2	(2,1)	3971.11	3970.40
3	(3,1)	11,407.42	11,396.11
4	(1,2)	27,587.30	27,587.17
5	(4,1)	30,000.58	29,733.90

$\alpha = 50.0$

$\gamma = 0.3$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1690.58	1690.51
2	(2,1)	3348.24	3347.57
3	(3,1)	10,005.84	9994.87
4	(1,2)	27,431.15	27,431.04
5	(4,1)	27,507.79	27,242.99

$\alpha = 50.0$

$\gamma = 0.4$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1522.76	1522.72
2	(2,1)	2728.98	2728.36
3	(3,1)	8605.20	8594.63
4	(4,1)	25,015.33	24,752.56
5	(1,2)	27,274.96	27,274.85

$\alpha = 50.0$

$\gamma = 0.5$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1340.41	1340.39
2	(2,1)	2121.89	2121.33
3	(3,1)	7206.12	7195.97
4	(4,1)	22,523.31	22,262.74
5	(1,2)	27,118.74	27,118.64

α = 50.0

γ = 0.6

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1099.47	1099.43
2	(2,1)	1569.78	1569.34
3	(3,1)	5809.64	5799.95
4	(4,1)	20,031.87	19,773.67
5	(1,2)	26,962.47	26,962.36

α = 50.0

γ = 0.7

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	662.32	662.17
2	(1,1)	1207.95	1207.73
3	(3,1)	4417.84	4408.68
4	(4,1)	17,541.21	17,285.56
5	(1,2)	26,806.14	26,806.04

α = 50.0

γ = 0.8

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	65.765	65.629
2	(1,1)	994.81	994.71
3	(3,1)	3035.18	3026.60
4	(4,1)	15,051.57	14,798.70
5	(1,2)	26,649.75	26,649.64

$\alpha = 55.0$

$\gamma = 0.0$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2143.48	2143.39
2	(2,1)	5115.16	5114.34
3	(3,1)	13,984.00	13,971.53
4	(1,2)	27,873.75	27,873.62
5	(4,1)	34,586.11	34,294.39

$\alpha = 55.0$

$\gamma = 0.1$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1983.82	1983.73
2	(2,1)	4490.39	4489.62
3	(3,1)	12,581.50	12,569.37
4	(1,2)	27,717.60	27,717.47
5	(4,1)	32,092.98	31,803.27

$\alpha = 55.0$

$\gamma = 0.2$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1822.41	1822.33
2	(2,1)	3866.27	3865.56
3	(3,1)	11,179.51	11,167.75
4	(1,2)	27,561.42	27,561.29
5	(4,1)	29,600.10	29,312.54

$\alpha = 55.0$

$\gamma = 0.3$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1657.70	1657.65
2	(2,1)	3243.96	3243.30
3	(3,1)	9778,30	9766.97
4	(4,1)	27,107.55	26,822.30
5	(1,2)	27,405.20	27,405.08

$\alpha = 55.0$

$\gamma = 0.4$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1485.67	1485.61
2	(2,1)	2626.90	2626.30
3	(3,1)	8378.34	8367.45
4	(4,1)	24,615.42	24,332.67
5	(1,2)	27,248.94	27,248.82

$\alpha = 55.0$

$\gamma = 0.5$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1292.60	1292.57
2	(2,1)	2027.83	2027.29
3	(3,1)	6980.40	6970.02
4	(4,1)	22,123.84	21,843.77
5	(1,2)	27,092.63	27,092.52

$\alpha = 55.0$

$\gamma = 0.6$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1017.42	1017.35
2	(2,1)	1506.18	1505.79
3	(3,1)	5585.91	5576.07
4	(4,1)	19,633.00	19,355.83
5	(1,2)	26,936.26	26,936.14

$\alpha = 55.0$

$\gamma = 0.7$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	538.36	538.22
2	(1,1)	1180.64	1180.45
3	(3,1)	4197.61	4188.40
4	(4,1)	17,143.14	16,869.11
5	(1,2)	26,779.82	26,779.71

α = 60.0

γ = 0.0

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	2114.28	2114.19
2	(2,1)	5009.85	5009.04
3	(3,1)	13,755.22	13,742.29
4	(1,2)	27,847.87	27,847.73
5	(4,1)	34,184.80	33,872.47

α = 60.0

γ = 0.1

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1953.62	1953.53
2	(2,1)	4384.92	4384.13
3	(3,1)	12,352.84	12,340.32
4	(1,2)	27,691.66	27,691.53
5	(4,1)	31,691.80	31,381.89

α = 60.0

γ = 0.2

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1790.66	1790.59
2	(2,1)	3760.91	3760.18
3	(3,1)	10,951.11	10,939.02
4	(1,2)	27,535.41	27,535.28
5	(4,1)	29,199.12	28,891.80

$\alpha = 60.0$

$\gamma = 0.3$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1623.29	1623.22
2	(2,1)	3139.45	3138.80
3	(3,1)	9550.38	9538.78
4	(4,1)	26,706.86	26,402.32
5	(1,2)	27,379.11	27,378.99

$\alpha = 60.0$

$\gamma = 0.4$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1445.82	1445.78
2	(2,1)	2525.48	2524.88
3	(3,1)	8151.25	8140.17
4	(4.1)	24,215.12	23,913.57
5	(1,2)	27,222.77	27,222.64

$\alpha = 60.0$

$\gamma = 0.5$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	1238.47	1238.43
2	(2,1)	1937.52	1937.02
3	(3,1)	6754.72	6744.22
4	(4,1)	21,724.07	21,425.76
5	(1,2)	27,066.36	27,066.24

$\alpha = 60.0$

$\gamma = 0.6$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	923.48	923.40
2	(1,1)	1451.21	1450.87
3	(3,1)	5362.64	5352.78
4	(4,1)	19,233.92	18,939.10
5	(1,2)	26,909.88	26,909.77

$\alpha = 60.0$

$\gamma = 0.7$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	409.15	409.01
2	(1,1)	1154.20	1154.04
3	(3,1)	3978.66	3969.53
4	(4,1)	16,745.00	16,453.98
5	(1,2)	26,753.31	26,753.20

APPENDIX D

BOUNDS FOR THE FREQUENCIES OF A RECTANGULAR PLATE WITH IN-PLANE AND EDGE LOADS

ASPECT RATIO = 3.0

$\alpha = 5.0$

$\gamma = 0.0$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9716.16	9716.15
2	(2,1)	16,363.25	16,363.11
3	(3,1)	31,338.21	31,336.46
4	(4,1)	60,485.59	60,454.27
5	(5,1)	111,987.1	111,701.1

$\alpha = 5.0$

$\gamma = 0.1$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9365.47	9365.46
2	(2,1)	14,960.54	14,960.41
3	(3,1)	28,182.14	28,180.38
4	(4,1)	54,874.82	54,843.52
5	(5,1)	103,220.9	102,934.3

$\alpha = 5.0$

$\gamma = 0.2$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9014.78	9014.77
2	(2,1)	13,557.82	13,557.69
3	(3,1)	25,026.08	25,024.33
4	(4,1)	49,264.05	49,232.79
5	(5,1)	94,454.09	94,167.51

$\alpha = 5.0$

$\gamma = 0.3$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8,664.07	8664.06
2	(2,1)	12,155.10	12,154.9
3	(3,1)	21,870.02	21,868.28
4	(4,1)	43,653.29	43,622.07
5	(5,1)	85,687.28	85,400.71

$\alpha = 5.0$

$\gamma = 0.4$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8313.34	8313.32
2	(2,1)	10,752.38	10.752.25
3	(3,1)	18,713.96	18,712.24
4	(4,1)	38,042.55	38,011.35
5	(5,1)	76,920.46	76,633.90

$\alpha = 5.0$

$\gamma = 0.5$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	7962.51	7962.50
2	(2,1)	9349.71	9349.58
3	(3,1)	15,557.94	15,556.21
4	(4,1)	32,431.80	32,400.65
5	(5,1)	68,153.68	67,867.11

$\alpha = 5.0$

$\gamma = 0.6$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	7610.73	7610.71
2	(2,1)	7947.89	7947.76
3	(3,1)	12,401.98	12,400.26
4	(4,1)	26,821.08	26,789.98
5	(5,1)	59,386.88	59,100.30

$\alpha = 5.0$

$\gamma = 0.7$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	6542.47	6542.35
2	(1,1)	7262.21	7262.19
3	(3,1)	9246.28	9244.58
4	(4,1)	21,210.40	21,179.35
5	(5,1)	50,620.10	50,333.50

$\alpha = 5.0$

$\gamma = 0.8$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	5137.54	5137.46
2	(3,1)	6092.55	6090.85
3	(1,1)	6911.13	6911.11
4	(4,1)	15,599.80	15,568.81
5	(5,1)	41,853.34	41,566.71

$\alpha = 5.0$

$\gamma = 0.9$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(3,1)	2927.06	2925.55
2	(2,1)	3743.84	3743.63
3	(1,1)	6560.34	6560.33
4	(4,1)	9989.38	9958.45
5	(5,1)	33,086.61	32,799.94

α = 10.0

γ = 0.0

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9691.26	9691.22
2	(2,1)	16,263.98	16,263.72
3	(3,1)	31,115.36	31,111.94
4	(4,1)	60,089.94	60,028.05
5	(5,1)	111,370.0	110,797.3

α = 10.0

γ = 0.1

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9340.53	9340.49
2	(2,1)	14,861.21	14,860.95
3	(3,1)	27,959.26	27,955.84
4	(4,1)	54,479.16	54,417.36
5	(5,1)	102,603.2	102,030.5

α = 70.0

γ = 0.2

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8989.77	8989.74
2	(2,1)	13,458.42	13,458.17
3	(3,1)	24,803.16	24,799.78
4	(4,1)	48,868.39	48,806.70
5	(5,1)	93,836.36	93,263.65

$\alpha = 10.0$

$\gamma = 0.3$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8638.96	8638.93
2	(2,1)	12,055.61	12,055.37
3	(3,1)	21,647.07	21,643.71
4	(4,1)	43,257.63	43,196.08
5	(5,1)	85,069.56	84,496.86

$\alpha = 10.0$

$\gamma = 0.4$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8,288.03	8288.00
2	(2,1)	10,652.81	10,652.58
3	(3,1)	18,491.03	18,487.70
4	(4,1)	37,646.92	37,585.51
5	(5,1)	76,302.78	75,730.07

$\alpha = 10.0$

$\gamma = 0.5$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	7936.68	7936.65
2	(2,1)	9250.22	9249.99
3	(3,1)	15,335.09	15,331.79
4	(4,1)	32,036.24	31,974.99
5	(5,1)	67,536.04	66,963.30

$\alpha = 10.0$

$\gamma = 0.6$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	7580.06	7580.05
2	(2,1)	7852.44	7852.21
3	(3,1)	12,179.45	12,176.19
4	(4,1)	26,425.64	26,364.59
5	(5,1)	58,769.32	58,196.54

$\alpha = 10.0$

$\gamma = 0.7$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	6437.39	6437.21
2	(1,1)	7239.30	7239.27
3	(3,1)	9024.96	9021.77
4	(4,1)	20,815.21	20,754.40
5	(5,1)	50,002.65	49,429.79

$\alpha = 10.0$

$\gamma = 0.8$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	5023.56	5023.49
2	(3,1)	5880.09	5876.93
3	(1,1)	6887.24	6887.20
4	(4,1)	15,205.11	15,144.57
5	(5,1)	41,236.05	40,663.08

α = 10.0

γ = 0.9

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(3,1)	2686.86	2684.37
2	(2,1)	3656.14	3655.64
3	(1,1)	6536.14	6536.11
4	(4,1)	9595.80	9535.54
5	(5,1)	32,469.58	31,896.42

α = 15.0

γ = 0.0

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9666.19	9666.16
2	(2,1)	16,164.33	16,163.96
3	(3,1)	30,892.00	30,887.00
4	(4,1)	59,693.72	59,602.00
5	(5,1)	110,751.7	109,893.2

α = 15.0

γ = 0.1

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9315.39	9315.35
2	(2,1)	14,761.45	14,761.09
3	(3,1)	27,735.83	27,730.86
4	(4,1)	54.082.92	53,991.43
5	(5,1)	101,984.9	101,126.4

α = 15.0

γ = 0.2

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8964.52	8964.49
2	(2,1)	13,358.53	13,358.18
3	(3,1)	24,579.67	24,574.75
4	(4,1)	48,472.14	48,380.91
5	(5,1)	93,218.06	92,359.66

α =15.0

γ = 0.3

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8613.53	8613.48
2	(2,1)	11,955.58	11,955.24
3	(3,1)	21,423.55	21,418.69
4	(4,1)	42,861.40	42,770.45
5	(5,1)	84,451.29	83,592.88

α =15.0

γ = 0.4

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8262.24	8262.20
2	(2,1)	10,552.65	10,552.32
3	(3,1)	18,267.52	18,262.74
4	(4,1)	37,250.73	37,160.12
5	(5,1)	75,684.56	74,826.13

α =15.0

γ = 0.5

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	7909.92	7909.88
2	(2,1)	9150.27	9149.95
3	(3,1)	15,111.74	15,107.04
4	(4,1)	31,640.16	31,549.94
5	(5,1)	66,917.90	66,059.41

$\alpha = 15.0$

$\gamma = 0.6$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	7540.73	7540.72
2	(2,1)	7763.64	7763.33
3	(3,1)	11,956.71	11,952.13
4	(4,1)	26,029.80	25,940.03
5	(5,1)	58,151.30	57,292.72

$\alpha = 15.0$

$\gamma = 0.7$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	6327.92	6327.72
2	(1,1)	7217.14	7217.09
3	(3,1)	8804.56	8800.12
4	(4,1)	20,419.82	20,330.57
5	(5,1)	49,384.82	48,526.06

$\alpha = 15.0$

$\gamma = 0.8$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	4894.68	4894.66
2	(3,1)	5678.59	5674.27
3	(1,1)	6863.80	6863.77
4	(4,1)	14,810.62	14,722.00
5	(5,1)	40,618.52	39,759.47

$\alpha = 15.0$
$\gamma = 0.9$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(3,1)	2438.00	2434.97
2	(2,1)	3571.97	3571.16
3	(1,1)	6512.20	6512.18
4	(4,1)	9203.33	9115.38
5	(5,1)	31,852.48	30,992.99

$\alpha = 20.0$

$\gamma = 0.0$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9640.97	9640.91
2	(2,1)	16,064.28	16,063.82
3	(3,1)	30,668.10	30,661.63
4	(4,1)	59,296.93	59,176.19
5	(5,1)	110,132.8	108,989.1

$\alpha = 20.0$

$\gamma = 0.1$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9290.05	9289.99
2	(2,1)	14,661.24	14,660.78
3	(3,1)	27,511.84	27,505.44
4	(4,1)	53,686.09	53,565.75
5	(5,1)	101,366.0	100,222.3

$\alpha = 20.0$

$\gamma = 0.2$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8939.02	8938.97
2	(2,1)	13,258.14	13,257.70
3	(3,1)	24,355.60	24,349.29
4	(4,1)	48,075.30	47,955.41
5	(5,1)	92,599.17	91,455.50

$\alpha = 20.0$

$\gamma = 0.3$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8587.76	8587.72
2	(2,1)	11,854.99	11,854.54
3	(3,1)	21,199.43	21,193.21
4	(4,1)	42,464.58	42,345.22
5	(5,1)	83,832.45	82,688.77

$\alpha = 20.0$

$\gamma = 0.4$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8235.93	8235.89
2	(2,1)	10,451.90	10,451.47
3	(3,1)	18,043.44	18,037.35
4	(4,1)	36,853.99	36,735.22
5	(5,1)	75,065.79	73,922.06

$\alpha = 20.0$

$\gamma = 0.5$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	7882.02	7881.99
2	(2,1)	9049.97	9049.56
3	(3,1)	14,887.93	14,882.00
4	(4,1)	31,243.60	31,125.52
5	(5,1)	66,299.25	65,155.41

$\alpha = 20.0$

$\gamma = 0.6$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	7485.68	7485.66
2	(2,1)	7688.35	7687.99
3	(3,1)	11,733.86	11,728.13
4	(4,1)	25,633.58	25,516.32
5	(5,1)	57,532.83	56,388.79

$\alpha = 20.0$

$\gamma = 0.7$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	6213.78	6213.57
2	(1,1)	7195.45	7195.39
3	(3,1)	8585.53	8580.03
4	(4,1)	20,024.27	19,907.98
5	(5,1)	48,766.62	47,622.27

$\alpha = 20.0$

$\gamma = 0.8$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	4746.39	4746.35
2	(3,1)	5492.40	5487.28
3	(1,1)	6840.78	6840.71
4	(4,1)	14,416.44	14,301.28
5	(5,1)	40,000.75	38,855.89

α = 20.0

γ = 0.9

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(3,1)	2181.80	2178.57
2	(2,1)	3489.64	3488.57
3	(1,1)	6488.50	6488.47
4	(4,1)	8812.25	8698.31
5	(5,1)	31,235.36	30,089.67

α = 25.0

γ = 0.0

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9615.57	9615.52
2	(2,1)	15,963.82	15,963.25
3	(3,1)	30,443.66	30,435.82
4	(4,1)	58,899.56	58,750.55
5	(5,1)	109,513.3	108,084.7

α = 25.0

γ = 0.1

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9264.51	9264.45
2	(2,1)	14,560.59	14,560.03
3	(3,1)	27,287.29	27,279.56
4	(4,1)	53,288.68	53,140.30
5	(5,1)	100,746.5	99,317.94

α = 25.0

γ = 0.2

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8913.26	8913.20
2	(2,1)	13,157.25	13,156.72
3	(3,1)	24,130.95	24,123.34
4	(4,1)	47,677.89	47,530.23
5	(5,1)	91,979.72	90,551.21

α =25.0

γ = 0.3

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8561.62	8561.56
2	(2,1)	11,753.82	11,753.30
3	(3,1)	20,974.73	20,967.27
4	(4,1)	42,067.20	41,920.37
5	(5,1)	83,213.05	81,784.51

α =25.0

γ = 0.4

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8209.04	8208.98
2	(2,1)	10,350.55	10,350.06
3	(3,1)	17,818.81	17,811.56
4	(4,1)	36,456.72	36,310.83
5	(5,1)	74,446.49	73,017.88

α = 25.0

γ = 0.5

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	7852.78	7852.74
2	(2,1)	8949.43	8948.97
3	(3,1)	14,663.70	14,656.69
4	(4,1)	30,846.56	30,701.79
5	(5,1)	65,680.11	64,251.30

α =25

γ = 0.6

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	7409.56	7409.48
2	(2,1)	7631.73	7631.39
3	(3,1)	11,511.02	11,504.32
4	(4,1)	25,237.01	25,093.54
5	(5,1)	56,913.94	55,484.81

α = 25.0

γ = 0.7

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	6094.54	6094.37
2	(1,1)	7173.97	7173.90
3	(3,1)	8368.39	8362.07
4	(4,1)	19,628.63	19,486.67
5	(5,1)	48,148.10	46,718.47

α = 25.0

γ = 0.8

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	4575.27	4575.08
2	(3,1)	5324.78	5319.30
3	(1,1)	6818.11	6818.03
4	(4,1)	14,022.69	13,882.53
5	(5,1)	39,382.79	37,952.31

$\alpha = 25.0$

$\gamma = 0.9$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(3,1)	1918.91	1915.75
2	(2,1)	3408.11	3406.87
3	(1,1)	6464.96	6464.93
4	(4,1)	8422.88	8284.77
5	(5,1)	30,618.28	29,186.48

$\alpha = 30.0$

$\gamma = 0.0$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9590.00	9589.93
2	(2,1)	15,862.95	15,862.29
3	(3,1)	30,218.69	30,209.56
4	(4,1)	58,501.60	58,325.12
5	(5,1)	108,893.2	107,180.1

$\alpha = 30.0$

$\gamma = 0.1$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9238.75	9238.69
2	(2,1)	14,459.46	14,458.82
3	(3,1)	27,062.17	27,053.20
4	(4,1)	52,890.69	52,715.13
5	(5,1)	100,126.4	98,413.43

$\alpha = 30.0$

$\gamma = 0.2$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8887.21	8887.15
2	(2,1)	13,055.83	13,055.21
3	(3,1)	23,905.72	23,896.93
4	(4,1)	47,279.88	47,105.37
5	(5,1)	91,359.70	89,646.73

$\alpha = 30.0$

$\gamma = 0.3$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8535.09	8535.02
2	(2,1)	11,652.08	11,651.49
3	(3,1)	20,749.44	20,740.89
4	(4,1)	41,669.24	41,495.93
5	(5,1)	82,593.10	80,880.10

$\alpha = 30.0$

$\gamma = 0.4$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8181.50	8181.44
2	(2,1)	10,248.61	10,248.07
3	(3,1)	17,593.62	17,585.36
4	(4,1)	36,058.91	35,886.96
5	(5,1)	73,826.67	72,113.54

$\alpha = 30.0$

$\gamma = 0.5$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	7821.87	7821.85
2	(2,1)	8848.86	8848.36
3	(3,1)	14,439.07	14,431.14
4	(4,1)	30,449.06	30,278.73
5	(5,1)	65,060.49	63,347.06

$\alpha = 30.0$

$\gamma = 0.6$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	7315.36	7315.23
2	(1,1)	7590.54	7590.23
3	(3,1)	11,288.33	11,280.85
4	(4,1)	24,840.13	24,671.70
5	(5,1)	56,294.62	54,580.74

$\alpha = 30.0$

$\gamma = 0.7$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	5969.65	5969.51
2	(1,1)	7152.44	7152.38
3	(3,1)	8153.85	8146.91
4	(4,1)	19,232.96	19,066.78
5	(5,1)	47,529.26	45,814.60

$\alpha = 30.0$

$\gamma = 0.8$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	4380.75	4380.27
2	(3,1)	5176.10	5170.62
3	(1,1)	6795.77	6795.68
4	(4,1)	13,629.51	13,465.99
5	(5,1)	38,764.67	37,048.78

α = 30.0

γ = 0.9

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(3,1)	1649.58	1646.70
2	(2,1)	3326.70	3325.39
3	(1,1)	6441.50	6441.47
4	(4,1)	8035.64	7875.20
5	(5,1)	30,001.27	28,283.41

$\alpha = 35.0$

$\gamma = 0.0$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9564.24	9564.16
2	(2,1)	15,761.65	15,760.90
3	(3,1)	29,993.17	29.982.86
4	(4,1)	58,103.06	57,899.90
5	(5,1)	108,272.5	106,275.4

$\alpha = 35.0$

$\gamma = 0.1$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9212.77	9212.68
2	(2,1)	14,357.87	14,357.15
3	(3,1)	26,836.49	26,826.41
4	(4,1)	52,492.11	52,290.20
5	(5,1)	99,505.76	97,508.74

$\alpha = 35.0$

$\gamma = 0.2$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8860.88	8860.80
2	(2,1)	12,953.88	12,953.19
3	(3,1)	23,679.90	23,670.07
4	(4,1)	46,881.29	46,680.82
5	(5,1)	90,739.10	88,742.09

α = 35.0

γ = 0.3

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8508.14	8508.07
2	(2,1)	11,549.75	11,549.10
3	(3,1)	20,523.56	20,514.03
4	(4,1)	41,270.72	41,071.91
5	(5,1)	81,972.60	79,975.51

α = 35.0

γ = 0.4

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8153.23	8153.17
2	(2,1)	10,146.10	10,145.49
3	(3,1)	17,367.91	17,358.76
4	(4,1)	35,660.57	35,463.67
5	(5,1)	73,206.35	71,209.06

α = 35.0

γ = 0.5

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	7788.93	7788.89
2	(2,1)	8748.53	8748.00
3	(3,1)	14,214.09	14,205.40
4	(4,1)	30,051.12	29,856.43
5	(5,1)	64,440.39	62,442.72

α = 35.0

γ = 0.6

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	7209.53	7209.37
2	(1,1)	7558.09	7557.81
3	(3,1)	11,065.94	11,057.84
4	(4,1)	24,442.97	24,250.91
5	(5,1)	55,674.91	53,676.57

α = 35.0

γ = 0.7

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	5838.44	5838.33
2	(1,1)	7130.49	7130.45
3	(3,1)	7942.80	7935.42
4	(4,1)	18,837.34	18,648.38
5	(5,1)	46,910.14	44,910.70

α = 35.0

γ = 0.8

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	4165.29	4164.51
2	(3,1)	5043.69	5038.51
3	(1,1)	6773.71	6773.60
4	(4,1)	13,237.05	13,051.83
5	(5,1)	38,146.44	36,145.25

$\alpha = 35.0$

$\gamma = 0.9$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(3,1)	1373.85	1371.38
2	(2,1)	3244.89	3243.60
3	(1,1)	6417.96	6417.89
4	(4,1)	7651.05	7470.29
5	(5,1)	29,384.42	27,380.48

$\alpha = 40.0$

$\gamma = 0.0$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9538.30	9538.21
2	(2,1)	15,659.92	15,659.10
3	(3,1)	29,767.10	29,755.70
4	(4,1)	57,703.94	57,474.92
5	(5,1)	107,651.3	105,370.5

$\alpha = 40.0$

$\gamma = 0.1$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9186.54	9186.46
2	(2,1)	14,255.78	14,255.01
3	(3,1)	26,610.23	26,599.11
4	(4,1)	52,092.94	51,865.56
5	(5,1)	98,884.54	96,603.87

$\alpha = 40.0$

$\gamma = 0.2$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8834.23	8834.14
2	(2,1)	12,851.38	12,850.63
3	(3,1)	23,453.49	23,442.70
4	(4,1)	46,482.12	46,256.64
5	(5,1)	90,117.94	87,837.27

$\alpha = 40.0$

$\gamma = 0.3$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8480.74	8480.67
2	(2,1)	11,446.81	11,446.11
3	(3,1)	20,297.11	20,286.70
4	(4,1)	40,871.64	40,648.34
5	(5,1)	81,351.55	79,070.77

$\alpha = 40.0$

$\gamma = 0.4$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8124.16	8124.10
2	(2,1)	10,043.02	10,042.38
3	(3,1)	17,141.68	17,131.76
4	(4,1)	35,261.73	35,040.91
5	(5,1)	72,585.49	70,304.42

$\alpha = 40.0$

$\gamma = 0.5$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	7753.43	7753.40
2	(2,1)	8648.82	8648.27
3	(3,1)	13,988.83	13,979.59
4	(4,1)	29,652.77	29,434.89
5	(5,1)	63,819.84	61,538.22

α = 40.0

γ = 0.6

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	7096.15	7095.97
2	(1,1)	7530.00	7529.76
3	(3,1)	10,844.05	10,835.49
4	(4,1)	24,045.59	23,831.19
5	(5,1)	55,054.82	52,772.30

α = 40.0

γ = 0.7

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	5700.13	5700.06
2	(1,1)	7107.45	7107.43
3	(3,1)	7736.51	7728.89
4	(4,1)	18,441.84	18,231.60
5	(5,1)	46,290.74	44,006.75

α = 40.0

γ = 0.8

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(3,1)	3932.71	3931.68
2	(2,1)	4923.50	4918.77
3	(1,1)	6751.89	6751.77
4	(4,1)	12,845.49	12,640.27
5	(5,1)	37,528.12	35,241.77

α = 40.0

γ = 0.9

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(3,1)	1091.59	1089.64
2	(2,1)	3162.26	3161.11
3	(1,1)	6393.91	6393.54
4	(4,1)	7269.93	7071.16
5	(5,1)	28,767.76	26,477.69

α = 45.0

γ = 0.0

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9512.15	9512.05
2	(2,1)	15,557.75	15,556.87
3	(3,1)	29,540.47	29,528.08
4	(4,1)	57,304.23	57,050.12
5	(5,1)	107,029.5	104,465.4

α = 45.0

γ = 0.1

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9160.07	9159.98
2	(2,1)	14,153.20	14,152.36
3	(3,1)	26,383.39	26,371.34
4	(4,1)	51,693.18	51,441.19
5	(5,1)	98,262.75	95,698.80

α = 45.0

γ = 0.2

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8807.25	8807.15
2	(2,1)	12,748.32	12,747.52
3	(3,1)	23,226.49	23,214.85
4	(4,1)	46,082.38	45,832.81
5	(5,1)	89,496.21	86,932.26

α = 45.0

γ = 0.3

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8452.85	8452.78
2	(2,1)	11,343.26	11,342.53
3	(3,1)	20,070.07	20,058.93
4	(4,1)	40,472.02	40,225.20
5	(5,1)	80,729.95	78,165.84

α = 45.0

γ = 0.4

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8094.17	8094.11
2	(2,1)	9939.43	9938.76
3	(3,1)	16,914.95	16,904.42
4	(4,1)	34,862.38	34,618.77
5	(5,1)	71,964.12	69,399.61

α = 45.0

γ = 0.5

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	7714.74	7714.71
2	(2,1)	8550.24	8549.69
3	(3,1)	13,763.35	13,753.53
4	(4,1)	29,254.02	29,014.18
5	(5,1)	63,198.83	60,633.59

α = 45.0

γ = 0.6

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	6977.10	6976.93
2	(1,1)	7504.07	7503.86
3	(3,1)	10,622.86	10,614.00
4	(4,1)	23,648.01	23,412.63
5	(5,1)	54,434.36	51,867.92

α = 45.0

γ = 0.7

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	5553.89	5553.85
2	(1,1)	7081.81	7081.80
3	(3,1)	7537.17	7529.52
4	(4,1)	18,046.55	17,816.54
5	(5,1)	45,671.11	43,102.73

α = 45.0

γ = 0.8

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(3,1)	3686.58	3685.39
2	(2,1)	4811.72	4807.50
3	(1,1)	6730.28	6730.16
4	(4,1)	12,455.02	12,231.53
5	(5,1)	36,909.76	34,338.30

α = 45.0

γ = 0.9

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(3,1)	802.59	801.16
2	(2,1)	3078.41	3077.47
3	(1,1)	6367.87	6364.98
4	(4,1)	6894.26	6681.80
5	(5,1)	28,151.38	25,575.06

α = 50.0

γ = 0.0

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9485.80	9485.70
2	(2,1)	15,455.12	15,454.18
3	(3,1)	29,313.27	29,299.98
4	(4,1)	56,903.94	56,625.59
5	(5,1)	106,407.1	103,560.1

α = 50.0

γ = 0.1

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9133.35	9133.25
2	(2,1)	14,050.11	14,049.23
3	(3,1)	26,155.96	26,143.09
4	(4,1)	51,292.83	51,017.10
5	(5,1)	97,640.36	94,793.52

α = 50.0

γ = 0.2

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8779.91	8779.83
2	(2,1)	12,644.69	12,643.86
3	(3,1)	22,998.89	22,986.50
4	(4,1)	45,682.06	45,409.32
5	(5,1)	88,873.92	86,027.05

$\alpha = 50.0$

$\gamma = 0.3$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8424.41	8424.33
2	(2,1)	11,239.10	11,238.33
3	(3,1)	19,842.46	19,830.70
4	(4,1)	40,071.84	39,802.54
5	(5,1)	80,107.82	77,260.73

$\alpha = 50.0$

$\gamma = 0.4$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8063.14	8063.08
2	(2,1)	9835.34	9834.67
3	(3,1)	16,687.75	16,676.71
4	(4,1)	34,462.55	34,197.26
5	(5,1)	71,342.26	68,494.62

$\alpha = 50.0$

$\gamma = 0.5$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	7672.00	7671.98
2	(2,1)	8453.48	8452.92
3	(3,1)	13,537.71	13,527.56
4	(4,1)	28,854.90	28,594.33
5	(5,1)	62,577.38	59,728.81

$\alpha = 50.0$

$\gamma = 0.6$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	6853.14	6852.99
2	(1,1)	7479.17	7478.99
3	(3,1)	10,402.64	10,393.62
4	(4,1)	23,250.28	22,995.26
5	(5,1)	53,813.55	50,963.41

$\alpha = 50.0$

$\gamma = 0.7$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	5398.84	5398.82
2	(1,1)	7049.02	7048.87
3	(3,1)	7350.04	7342.68
4	(4,1)	17,651.57	17,403.35
5	(5,1)	45,051.27	42,198.65

$\alpha = 50.0$

$\gamma = 0.8$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(3,1)	3429.54	3428.32
2	(2,1)	4705.42	4701.73
3	(1,1)	6708.84	6708.73
4	(4,1)	12,065.84	11,825.91
5	(5,1)	36,291.41	33,434.85

$\alpha = 50.0$

$\gamma = 0.9$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(3,1)	506.58	505.66
2	(2,1)	2992.93	2992.26
3	(1,1)	6328.26	6247.49
4	(4,1)	6536.15	6387.61
5	(5,1)	27,535.33	24,672.60

α = 55.0	γ = 0.0

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9459.24	9459.13
2	(2,1)	15,352.03	15,351.04
3	(3,1)	29,085.51	29,071.40
4	(4,1)	56,503.06	56,201.26
5	(5,1)	105,784.1	102,654.6

α = 55.0	γ = 0.1

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9106.34	9106.24
2	(2,1)	13,946.50	13,945.57
3	(3,1)	25,927.95	25,914.34
4	(4,1)	50,891.90	50,593.31
5	(5,1)	97,017.74	93,888.04

α = 55.0	γ = 0.2

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8752.21	8752.12
2	(2,1)	12,540.48	12,539.59
3	(3,1)	22,770.70	22,757.69
4	(4,1)	45,281.16	44,986.23
5	(5,1)	88,251.07	85,121.63

$\alpha = 55.0$

$\gamma = 0.3$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8395.40	8395.32
2	(2,1)	11,134.32	11,133.53
3	(3,1)	19,614.29	19,601.98
4	(4,1)	39,671.11	39,380.37
5	(5,1)	79,485.16	76,355.41

$\alpha = 55.0$

$\gamma = 0.4$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8030.92	8030.86
2	(2,1)	9730.84	9730.17
3	(3,1)	16,460.10	16,448.67
4	(4,1)	34,062.24	33,776.41
5	(5,1)	70,719.91	67,589.46

$\alpha = 55.0$

$\gamma = 0.5$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	7624.19	7624.16
2	(2,1)	8359.42	8358.88
3	(3,1)	13,312.00	13,301.61
4	(4,1)	28,455.44	28,175.36
5	(5,1)	61,955.51	58,823.86

$\alpha = 55.0$

$\gamma = 0.6$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	6724.47	6724.33
2	(1,1)	7454.71	7454.55
3	(3,1)	10,183.68	10,174.63
4	(4,1)	22,852.45	22,579.26
5	(5,1)	53,192.42	50,058.79

$\alpha = 55.0$

$\gamma = 0.7$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	5234.13	5234.09
2	(1,1)	6992.04	6990.77
3	(3,1)	7192.81	7186.88
4	(4,1)	17,256.99	16,992.12
5	(5,1)	44,431.26	41,294.51

$\alpha = 55.0$

$\gamma = 0.8$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(3,1)	3163.36	3162.22
2	(2,1)	4602.52	4599.39
3	(1,1)	6687.54	6687.43
4	(4,1)	11,678.17	11,423.64
5	(5,1)	35,673.10	32,531.45

$\alpha = 55.0$

$\gamma = 0.9$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(3,1)	203.24	202.73
2	(2,1)	2905.34	2904.92
3	(4,1)	6115.28	5889.74
4	(1,1)	6356.04	6340.71
5	(5,1)	26,919.68	23,770.31

$\alpha = 60.0$

$\gamma = 0.0$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9432.46	9432.34
2	(2,1)	15,248.47	15,247.44
3	(3,1)	28,857.18	28,842.33
4	(4,1)	56,101.59	55,777.18
5	(5,1)	105,160.5	101,748.8

$\alpha = 60.0$

$\gamma = 0.1$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	9079.06	9078.95
2	(2,1)	13,842.36	13,841.39
3	(3,1)	25,699.35	25,685.09
4	(4,1)	50,490.38	50,169.82
5	(5,1)	96,393.89	92,982.32

$\alpha = 60.0$

$\gamma = 0.2$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8724.11	8724.01
2	(2,1)	12,435.67	12,434.76
3	(3,1)	22,541.91	22,528.35
4	(4,1)	44,879.69	44,563.50
5	(5,1)	87,627.67	84,215.99

$\alpha = 60.0$

$\gamma = 0.3$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	8365.75	8365.68
2	(2,1)	11,028.91	11,028.11
3	(3,1)	19,385.56	19,372.83
4	(4,1)	39,269.85	38,958.71
5	(5,1)	78,861.98	75,449.88

$\alpha = 60.0$

$\gamma = 0.4$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	7997.34	7997.27
2	(2,1)	9626.00	9625.32
3	(3,1)	16,232.03	16,220.29
4	(4,1)	33,661.47	33,356.22
5	(5,1)	70,097.08	66,684.09

$\alpha = 60.0$

$\gamma = 0.5$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(1,1)	7570.06	7570.02
2	(2,1)	8269.11	8268.61
3	(3,1)	13,086.31	13,075.81
4	(4,1)	28,055.66	27,757.35
5	(5,1)	61,333.23	57,918.74

$\alpha = 60.0$

$\gamma = 0.6$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	6591.02	6590.89
2	(1,1)	7430.28	7430.14
3	(3,1)	9966.30	9957.36
4	(4,1)	22,454.59	22,164.58
5	(5,1)	52,570.98	49,154.02

$\alpha = 60.0$

$\gamma = 0.7$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(2,1)	5059.00	5058.93
2	(3,1)	6876.20	6872.51
3	(1,1)	7100.72	7097.66
4	(4,1)	16,862.92	16,583.03
5	(5,1)	43,811.10	40,390.30

$\alpha = 60.0$

$\gamma = 0.8$

ORDER	MODE	UPPER BOUND	LOWER BOUND
1	(3,1)	2889.18	2888.18
2	(2,1)	4501.59	4498.99
3	(1,1)	6666.32	6666.22
4	(4,1)	11,292.28	11,025.05
5	(5,1)	35,054.89	31,628.05