ALIGNMENT OF OBLIQUE ROTATORS

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Abstract

Pulsars may be slowed down by radiation field torques. If so, there is also an accompanying torque that aligns the magnetic moment with the spin axis. The general implication of the alignment effect is that most pulsars would have to have been formed either with periods not too different from what are now observed or with magnetic moments almost precisely orthogonal to the spin axis. Alternatively, neutral sheet formation in the pulsar magnetosphere, or other plasma effects, may suppress the alignment torque.

I. INTRODUCTION

The discovery of the pulsar phenomenon has generated considerable interest in the properties of rapidly rotating, strongly magnetized, and highly condensed objects. We will call such objects "spinars", following Morrison (1969), and we will be concerned with those spinars that are oblique rotators (i.e., that have a sensible angle between their spin axis and magnetic moment). Exactly why spinars might also be pulsars, presumably pulsing at their frequency of rotation, remains a topic of wide speculation. What we wish to report here is the effect of magnetic dipole radiation from spinars in decreasing the angle ("magnetic obliquity") between the spin axis and the magnetic moments. In other words, the torque exerted on a spinar not only decreases its rotational frequency but also aligns the magnetic moment and spin axis. This alignment effect has important consequences for any theory that relies on dipole radiation to account for the rotational history of the pulsars, and may be an essential consideration to other theories as well.

II. TORQUES

Traditionally, the torque on a rotating dipole is calculated from the energy lost to the radiation field; however, only the torque component parallel to the spin axis does work. To calculate the net torque that acts on a rotating dipole, it is preferable to integrate the Maxwell stresses exerted at the surface of the rotator. The i^{th} component of the torque is then given by

$$T_i = \int \epsilon_{ijk}\, r_j\, T_{kn}\, dS_n \qquad (1)$$

where T_{kn} is the Maxwell Stress tensor and dS_n is the element of rotator surface area, the normal component being n. Writing the torque in cartesian coordinates, but the stress in spherical coordinates, gives

$$T_x = -\int r\,[T_{r\theta} \sin\varphi + T_{r\varphi} \cos\theta \cos\varphi]\, dS$$

$$T_y = \int r\,[T_{r\theta} \cos\varphi - T_{r\varphi} \cos\theta \sin\varphi]\, dS \qquad (2)$$

$$T_z = \int r\,[T_{r\varphi} \sin\theta]\, dS$$

where rotation is about the z axis, the magnetic moment lies in the x-z plane, and the integration is over the surface of the sphere.

Since we are calculating the stress at the surface of the star, where the magnetic field is fixed in the highly conducting stellar material, the stresses are

$$T_{r\theta} = H_r B_\theta \tag{3}$$

$$T_{r\varphi} = H_r B_\varphi \quad ,$$

(electrostatic terms give only a small contribution, which we will neglect).

Using the exact expressions for the magnetic field of an oblique rotator in a vacuum (Deutsch, 1955) we find

$$T_x = T \sin \chi \cos \chi \quad ,$$

and (4)

$$T_z = - T \sin^2 \chi \quad ,$$

where

$$T = 2\pi\, a^6 B^2 \Omega^3 / 3\, \mu_0 c^3 \quad .$$

The torque along the spin axis (z) reproduces the standard result. Here B is the surface magnetic field at the poles.

It is easy to understand the origin of these two torque components. Figure 1 illustrates the spinar in the x-z plane. Rotation causes the field lines, which emanate mainly near the magnetic poles, to be swept back; consequently, the tension on the field lines has a component opposite the direction of rotation. There is therefore a net force applied near the poles that gives a torque component opposite to $\underset{\sim}{L}$, which reduces the angular momentum, and towards the closest magnetic pole, which reduces the magnetic obliquity.

The equations of motion are

$$I \frac{d\Omega}{dt} = T_z$$

and (5)

$$I\Omega \frac{d\chi}{dt} = - T_x \quad ,$$

thus we have immediately the constant of motion

$$\Omega \cos \chi = \Omega_0 \cos \chi_0 \quad . \tag{6}$$

Thus if the slow pulsars ($P \sim 1$ sec) have evolved from fast pulsars ($P \sim 10^{-2}$ to 10^{-3} sec), predominantly as a result of such electromagnetic torques, then they must have been formed with their magnetic moments almost exactly orthogonal to the spin axes. Note that equation (6) is valid even if the dipole field changes with time.

III. DECELERATION PARAMETER

The time variation of the pulsar frequency can be described by (Goldwire and Michel 1969)

$$\dot{\Omega} = - K' \Omega^n \quad , \tag{8}$$

where the deceleration parameter (n) is observationally given by

$$\ddot{\Omega}\, \Omega / (\dot{\Omega})^2 \equiv n \quad . \tag{9}$$

Including the effects of changing χ , the torque equations (5) give (here assuming constancy of the magnetic field strength)

$$n = 3 + 2 \cot^2 \chi \quad , \tag{10}$$

The minimum value being $n = 3$, a result sometimes quoted without restriction on χ . Determination of n would then determine χ on the vacuum radiation hypothesis. For NP0532, the preliminary value of $n = 5 \pm 3$ only requires $32° < \chi < 90°$, but the value of n should eventually be known with much higher precision.

IV. AGE/LIFETIME RELATIONSHIP

The slowing down can be parameterized by a characterisitic time ("lifetime") defined by

$$\tau \equiv - \Omega/\dot{\Omega} \quad , \tag{11}$$

and if the actual age (t) of the pulsar is known, the relationship

$$\sin \chi/\sin \chi_0 = \exp(- \frac{t}{\tau} \cot^2 \chi) \tag{12}$$

interrelates χ and χ_0. These quantities are also interrelated by the spin ratio (Ω_0/Ω) from equation (6) and can thereby be solved for. The Crab pulsar is presumably 915 years old, whereas the lifetime is observed to be 2484±2 years, thus we can give the magnetic obliquities and deceleration parameter as a function of spin ratio for NP0532, as has been plotted in Figure 2. Note that (1) the deceleration parameter n varies between 3 and 4.58, (2) the minimum initial obliquity is 78° (at $\Omega_0/\Omega = 2.55$), and (3) the minimum magnetic obliquity at present would be 48°. No solution exists for a spin ratio less than 1.95 (in general: $\Omega_0/\Omega \geq (1 - 2t/\tau)^{-1/2}$). The surface magnetic field required to give a fixed value of τ varies as

$$B \sim \csc \chi$$

and varies only by about a factor of 1.34 over the entire range of possible values for Ω_0/Ω .

V. LARGE SPIN RATIOS

If one supposes that pulsars are spinars formed with nearly 90° magnetic obliquities, then the age/lifetime relationship can be written

$$t/\tau = -\ln(\sin \chi) \tan^2 \chi$$

thus

$$t = \frac{\tau}{(n-3)} \ln[\tfrac{1}{2}(n-1)] \quad .$$

The lifetime would then be determined entirely from observable parameters, as shown in Table 1. For small spin ratios, the right hand side of the above equations must be multiplied by $(1 - \Omega^2/\Omega_0{}^2)^{1/2}$. For all conditions, we have

$$t < \tau/2 \quad .$$

VI. NEUTRAL SHEET MODIFICATION

The existence of plasma surrounding the spinar (c.f. Michel 1969) leads one to question the validity of the vacuum electromagnetic field solutions. No detailed solutions have been worked out that include the plasma effects in a self-consistent way. One speculation (Michel and Tucker 1969) is that the field structure will be substantially uniform, with only localized rapid changes from inward to outward radial flux ("neutral sheets"). Such a model also exhibits a slowing down torque (c.f. Michel 1969), but we see from Figure 1 that there would be no alignment torque (neglecting the finite thickness of the neutral sheets), since the surface stresses are then symmetrical about the z axis. We can describe the effect of suppressing the alignment torque by introducing a phenomenological parameter $\eta (< 1)$ multiplying the upper equation (4). Then the constant of motion becomes

$$\Omega^{\eta} \cos \chi = \text{constant}$$

hence

$$\Omega_{min} = \Omega_0 (\cos \chi_0)^{1/\eta}$$

For example, to produce a spin ratio of 10^2 with $\chi_0 \sim 60°$ would require $\eta \approx 0.15$, etc. In the neutral sheet picture, η would be of the order of the sheet thickness divided by the stellar radius, which is argued (Michel and Tucker 1969) to be much less than 10^{-1}.

The point to be emphasized is that, if spinars are pulsars, they must either have a significantly different field structure than suggested by the vacuum dipole analysis or be constrained in one of two essential ways: their spin ratios are either small or the initial magnetic obliquities were preferentially quite near 90°.

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TABLE 1

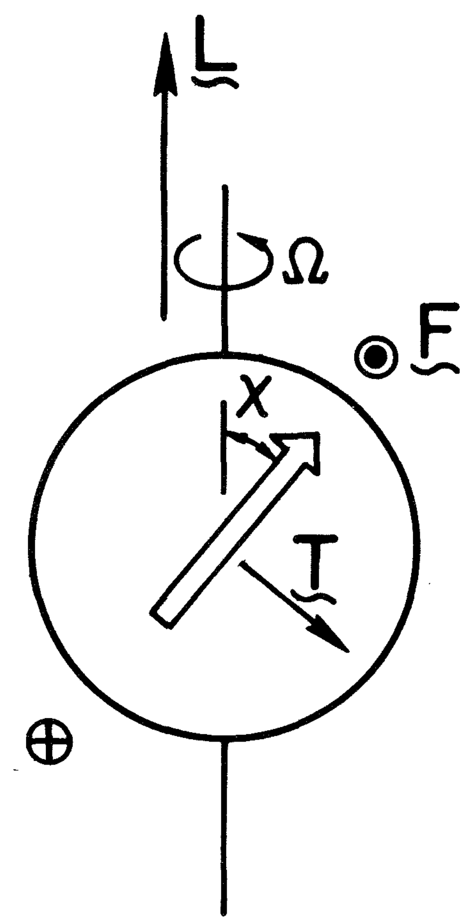
Age/Lifetime and Magnetic Obliquity as a Function of Deceleration Parameter

n	t/τ	χ (degrees)
3	.500	90
4	.406	54.74
5	.347	45
6	.305	39.23
7	.275	35.26
8	.251	32.31
9	.231	30
10	.215	28.13
30	.099	15.23
100	.040	8.17
1000	.006	2.56

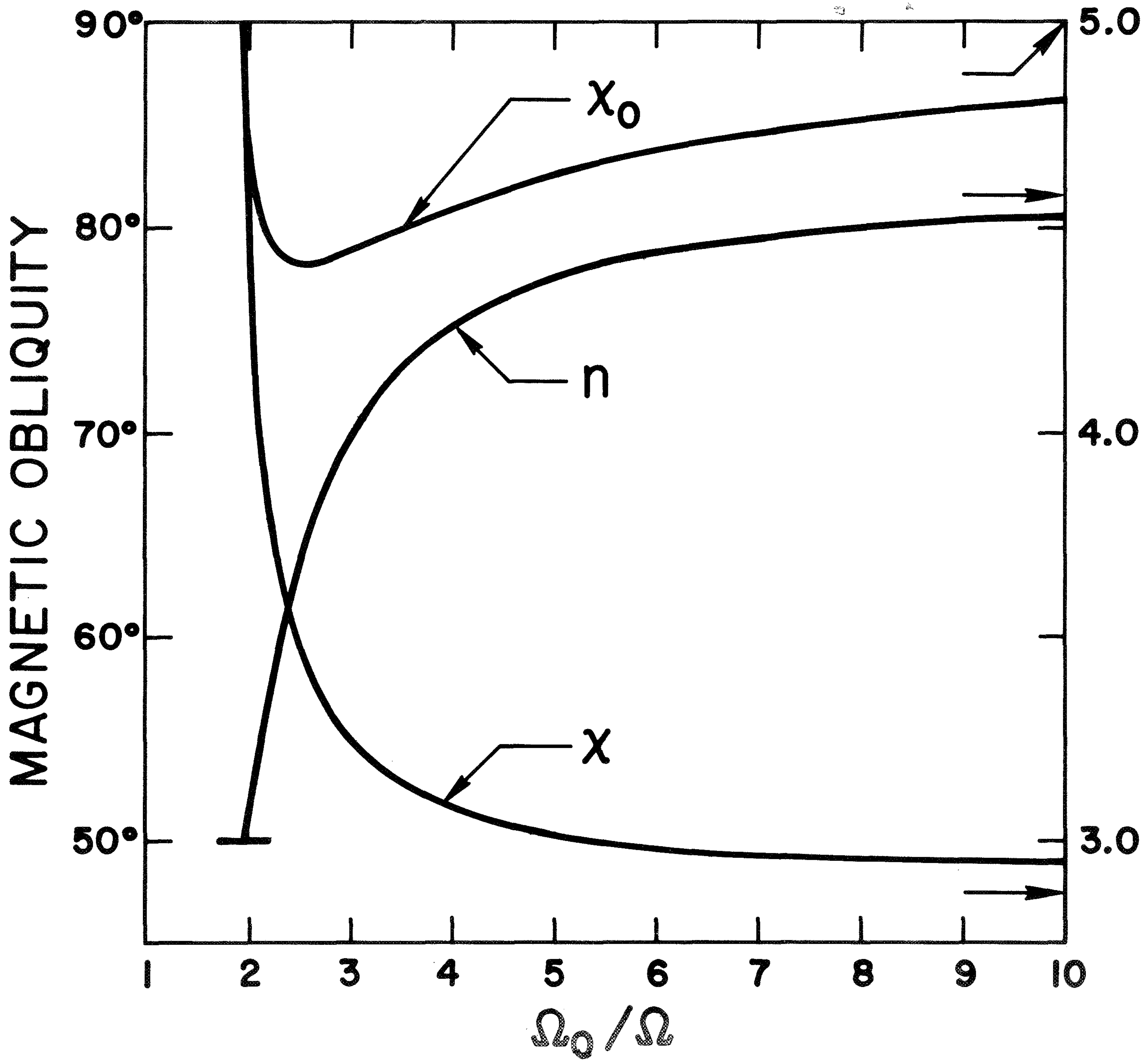
CAPTIONS

Figure 1 Origin of the alignment and slowing down torques. Sweeping back of field lines by retardation effects gives a force component opposite to rotational velocity and concentrated near magnetic pole. Resultant torque vector ($\underset{\sim}{T}$) has component opposite angular momentum vector ($\underset{\sim}{L}$) and towards nearest pole.

Figure 2 Initial and final magnetic obliquity angles for NP0532, as a function of the spin ratio (Ω_0/Ω), required to give the observed age/lifetime ratio. The right hand scale applies to n . Asymptotic values (infinite spin ratio) are indicated by the arrow nearest the curve in question.



L
Ω
F
χ
T



MAGNETIC OBLIQUITY
90°
80°
70°
60°
50°
5.0
4.0
3.0
χ_0
n
χ
1
2
3
4
5
6
7
8
9
10
Ω_0 / Ω

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