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GENERATION OF UNCERTAINTY BOUNDARY FOR ARCASONDE 1A TEMPERATURE SENSOR SYSTEM

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DEPARTMENT OF ELECTRICAL ENGINEERING
UNIVERSITY OF UTAH
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ABSTRACT

A method of computing error bars, i.e., a running estimate of the overall uncertainty, for the temperature profile from a thermistor-type meteorological rocketsonde is presented. The resultant uncertainty is derived from estimated uncertainties in the parameter values assumed in the mathematical data correction relations. The "uncertainty boundary" is defined and the method of combining uncertainties is discussed.

A computer program in FORTRAN V is developed which computes corrections and uncertainties for real flight data. Simulated flight data is generated and used for illustration. Nominal values and uncertainty estimates for the parameters are those associated with the ARCASONDE 1A film-mounted thermistor sensor system.

Quantitative results reveal the relative sensitivity of the corrected temperature to the various parameters. Sensor improvement with the use of a radiation shield is illustrated in terms of reduced uncertainty boundary.

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GLOSSARY

a	=	albedo
A	=	total body surface
c	=	specific heat of bead, wire, or film
c_p	=	specific heat of air
D	=	diameter of bead, wire, or thickness of film
D_{es}	=	distance between the earth and the sun
$E_{b\lambda}(T)$	=	plank radiant energy spectral distribution function for the source in $d\Omega$ at temperature T
$f_{i,j}$	=	geometric factor for ith body with respect to jth source
g	=	gravitational acceleration
h	=	convective heat transfer coefficient
I_j	=	radiant emittance of jth source
k	=	thermal conductivity of the body
ℓ	=	length of the wire
m	=	mass of the parachute and the rocketsonde
q	=	radiation heat input
r	=	recovery factor
R	=	radius of the earth
R_s	=	radius of the sun
S	=	solar constant
T	=	temperature of bead, wire, or film
T_{air}	=	temperature of the air
T_m	=	measured value of T_b

T_s = sensor temperature

T'_{wb} = temperature gradient at the bead wire junction

T_r = recovery temperature

v = volume of bead, wire, or film

V = air speed

W = electric power dissipation

α = long-wave absorptivity

α_s = solar absorptivity

$\alpha_{i, j}$ = absorptivity of ith body with respect to jth source

α_λ = spectral absorptivity

$\bar{\alpha}_j$ = mean absorptivity relative to the jth source

β_j = radiation input perturbation factor

ε = emissivity

ε_λ = spectral emissivity

Ω = solid angle subtended by the environment at the source

θ = angle between sensor surface element dA and the direction toward $d\Omega$

λ = radiation wavelength

σ = Boltzmann constant

σ_{p_ℓ} = standard deviation of parameter p_ℓ

η_{p_ℓ} = mean value of parameter p_ℓ

ρ = mass density of the sensor

$\mu_{\ell, m}$ = correlation coefficient between p_ℓ and p_m

Subscript

$i = 1$	bead
$i = 2$	wire
$i = 3$	film
$j = 1$	sun
$j = 2$	albedo
$j = 3$	long-wave radiation from the earth
$j = 4$	sonde parts in view of the sensor as a long-wave source
b	bead
w	wire
f	film
fm	Mylar part of the film
fs	silver part of the film

I. INTRODUCTION

In order to increase the altitude capability of rocketsonde atmospheric temperature sensors, many different sensor configurations have been considered and developed.

Evaluation of sensor performance is necessary in order to compare sensors and improve the sensor system. This study employs the mathematical modeling approach to the theoretical study of immersion-type thermometer sensors used in current meteorological rocketsonde systems.

Figure 1 represents the system block diagram of a temperature measurement system. Input to the system is air temperature, T_{air}, which is transformed to the temperature of the sensor, T_s. The value of T_s is different from T_{air} due to heat flux associated with the following error sources:

1. Radiation
2. Aerodynamic heating
3. Heat conduction
4. Self heating
5. Thermal time lag

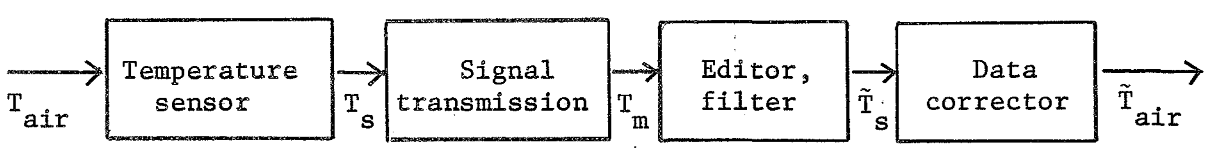

Fig. 1. Temperature measurement system.

T_s is then converted to the frequency of a blocking oscillator and relayed to a ground station where the signal is detected and recorded. This temperature, T_m, in general, contains measurement noise. Commonly, the data is filtered and edited to reject noise and spurious values and to obtain an improved representation, $\tilde{T}_s$, of the sensor temperature. Finally, computed corrections based on physical knowledge of the sensor and environment leads to the corrected air temperature, $\tilde{T}_{air}$.

Physical knowledge of the sensor system is embodied in a mathematical model of the form

$$T_s = f\left(T_{air}, \underset{\sim}{p}\right) \tag{1}$$

where $\underset{\sim}{p}$ is a vector notation representing the set of parameters used in the thermistor heat equation. From Eq. 1, an inverse function is derived and is used for data reduction using parameter estimates $\tilde{\underset{\sim}{p}}$.

$$\tilde{T}_{air} = f^{-1}\left(\tilde{T}_s, \tilde{\underset{\sim}{p}}\right) \tag{2}$$

The better system is generally considered as the system which has the smaller difference between T_s and T_{air}. However, if accurate information is known about each parameter, $\tilde{T}_{air}$ can be computed with small uncertainty by Eq. 2, regardless of the difference between T_s and T_{air}. The better system is actually the one which has the smaller uncertainty in $\tilde{T}_{air}$.

In the following, a method is developed for computing the uncertainty in $\tilde{T}_{air}$ due to parameter uncertainties, and is applied to the ARCASONDE 1A meteorological rocketsonde temperature sensor.

II. DESCRIPTION OF THE ARCASONDE 1A TEMPERATURE MEASUREMENT SYSTEM

In the following two sections, a mathematical model of ARCAsonde 1A temperature sensor and data correction system will be derived. The configuration of the ARCASONDE 1A sensor is shown in Fig. 2. Detail dimensions are listed in Table 1, page 27.

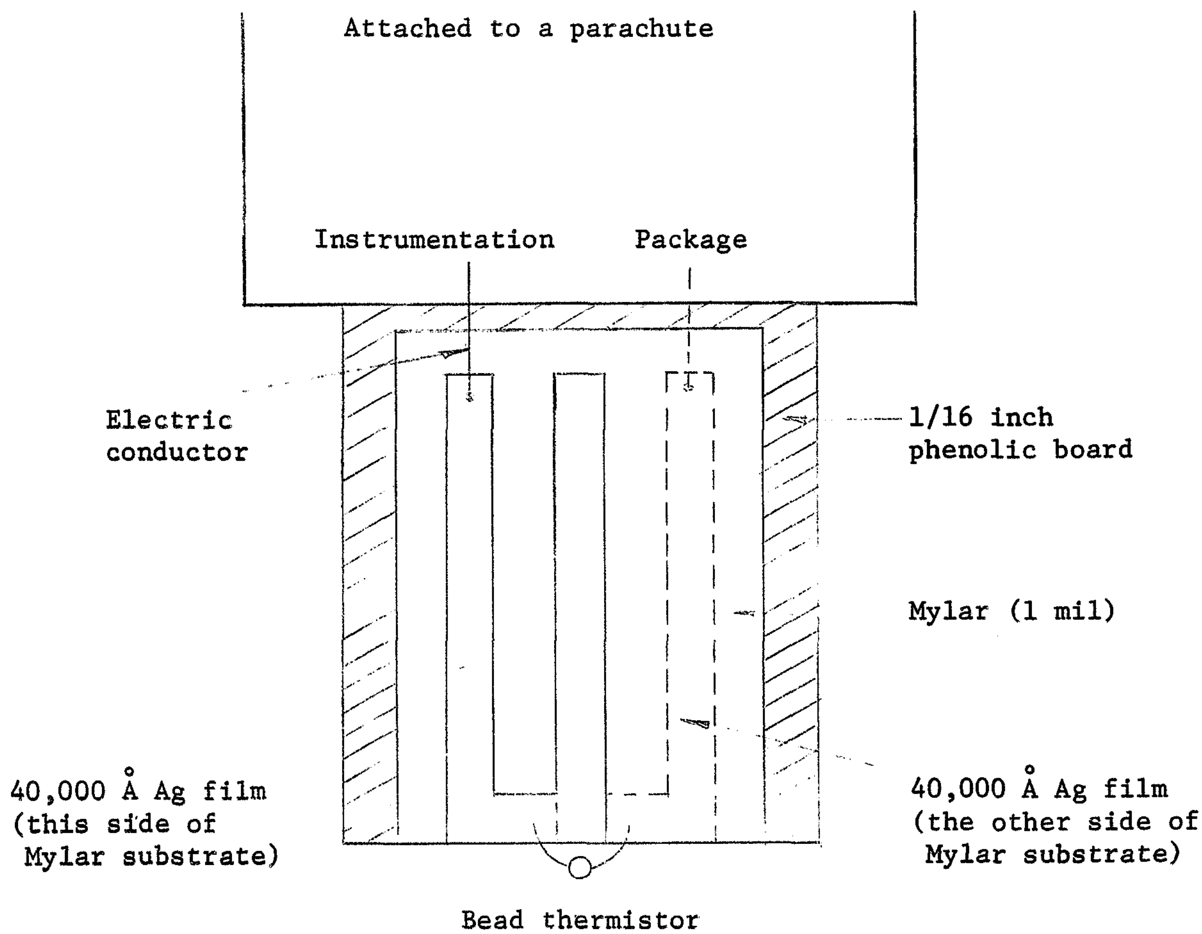

Fig. 2. ARCASONDE 1A thermistor sensor.

2.1 Mathematical Model of ARCASONDE 1A Sensor System

The temperature which is read by the electronic circuit in the sonde is that of the thermistor bead, i.e., T_s of Fig. 1 is T_b in the present discussion. The bead temperature is influenced by heat conduction from the wire, which is proportional to the temperature gradient in the wire at the point of contact with the bead. Using the bead and film temperatures as boundary conditions, the temperature gradient in the wire can be obtained.

The heat balance equations for the bead, wire, and film are:

(Bead)

$$(\rho c v)_b \frac{\partial T_b}{\partial t} = h_b A_b \left(T_{air} + r_b \frac{V^2}{2c_p} - T_b \right) + q_b A_b - A_b \sigma \varepsilon_b T_b^4 + W_b$$

$$+ 2k_w \frac{\pi}{4} D_w^2 T'_{wb} \qquad (3.a)$$

(Wire)

$$(\rho c v)_w \frac{\partial T_w}{\partial t} = h_w A_w \left(T_{air} + r_w \frac{V^2}{2c_p} - T_w \right) + q_w A_w - A_w \sigma \varepsilon_w T_w^4 + W_w$$

$$+ k_w v_w \frac{\partial^2 T_w}{\partial x^2} \qquad (3.b)$$

(Film)

$$(\rho c v)_f \frac{\partial T_f}{\partial t} = h_f A_f \left(T_{air} + r_f \frac{V^2}{2c_p} - T_f \right) + q_f A_f - A_f \sigma \varepsilon_f T_f^4 + W_f$$

$$+ k_f v_f \left(\frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2} \right) \qquad (3.c)$$

where subscript b, w, f indicate bead, wire, and film, respectively, where

ρ = mass density of an element

c = specific heat of an element

v = volume of an element

h = convective heat transfer coefficient

T'_{wb} = temperature gradient at the bead and the wire junction

r = recovery factor

V = relative speed of air

c_p = specific heat of air

q = radiation heat input

A = surface of an element

W = self heating

k = thermal conductivity

σ = Stefan-Boltzmann constant

D_b, D_w, D_f = diameter of bead, diameter of wire, thickness of film, respectively

ε = emissivity

These equations are coupled by conductive boundary conditions at surfaces of contact.

The radiation environment is considered in four parts, designated by the subscript j = 1,..., 4, according to radiant heat sources as follows:

- j = 1   direct solar illumination
- j = 2   indirect solar illumination
- j = 3   earth radiation
- j = 4   sonde radiation

The radiation input to a surface element of the sensor system is represented by

$$q_i = \sum_{j=1}^{4} \alpha_{i,j} f_{i,j} I_j$$

where

- α = radiation absorptivity
- f = geometric factor
- I = radiant emittance

and subscript i = 1, 2, 3 indicates the sensor part (body) b, w, f, respectively. More complete discussion concerning radiation terms is given in Appendix D.

It is not practical to solve the nonlinear simultaneous partial differential equation. The following assumptions have been made in order to simplify the computation:

1. Time constant of the wire is very small compared with that of the film and the bead. Therefore, we can assume

$$\frac{\partial T_w}{\partial t} = 0$$

2. Temperature of the film is not influenced by heat exchange with the wire.

3. Temperature distribution of the film is assumed to be uniform near the film-wire junction.

By the above assumptions, Eqs. 3.a, 3.b, and 3.c are simplified as follows:

(Bead and wire)

$$\frac{(\rho c D)_b}{6} \frac{dT_b}{dt} = h_b \left(T_{air} + r_b \frac{V^2}{2c_p} - T_b \right) + q_b + \frac{W_b}{A_b} - \sigma \varepsilon_b T_b^4 + X \qquad (4.a)$$

(Film)

$$\frac{(\rho c D)_f}{2} \frac{dT_f}{dt} = h_f \left(T_{air} + r_f \frac{V^2}{2c_p} - T_f \right) + q_f - \sigma \varepsilon_f T_f^4 \qquad (4.b)$$

where

$$X = H_k \left(K_1 T_{air} + P + QT_f - T_b \right)$$

$$H_k = c_2 \lambda_w \coth \lambda_w \ell$$

$$K_1 = \frac{h_w \left(1 - \operatorname{sech} \lambda_w \ell \right)}{h_w + 4\sigma\varepsilon_w T_{aw}^3}$$

$$P = \frac{\left(1 - \operatorname{sech} \lambda_w \ell \right) \left(h_w r_w \frac{V^2}{2c_p} + 3\sigma\varepsilon_w T_{aw}^4 \right)}{h_w + 4\sigma\varepsilon_w T_{aw}^3}$$

$$Q = \operatorname{sech} \lambda_w \ell$$

$$\lambda_w = \left(\frac{4\left(h_w + 4\sigma\varepsilon_w T_{aw}^3 \right)}{(kD)_w} \right)^{1/2}$$

$$c_2 = \frac{(kD)_w D_w}{2D_b^2}$$

These constitute a set of two simultaneous differential equations representing the sensor system.

2.2 Data Correction System

From Fig. 1, the desired air temperature, T_{air}, is obtained by correcting the data, $\tilde{T}_b$, representing the sensor temperature, T_b. The required mathematical expressions are obtained from Eq. 4.a and 4.b, which expressed in one-step finite difference form are:

$$\tilde{T}^i_{air} = \frac{1}{\tilde{h}^i_b + \tilde{H}^i_k \tilde{K}^i_1} \left[\frac{(\tilde{\rho}\tilde{c}\tilde{D})_b}{6} \dot{\tilde{T}}^i_b + \left(\tilde{h}^i_b + 4\sigma\tilde{\varepsilon}_b \left(\tilde{T}^i_b\right)^3 + \tilde{H}^i_k \right) \tilde{T}^i_b \right.$$

$$\left. - \left(\tilde{h}^i_b \tilde{r}^i_b \frac{\left(\tilde{V}^i\right)^2}{2c_p} + 3\sigma\tilde{\varepsilon}_b \left(\tilde{T}^i_b\right)^4 + \frac{\tilde{W}^i_b}{A_b} + \tilde{q}^i_b + \tilde{H}^i_k \tilde{p}^i \right) - \tilde{H}^i_k \tilde{Q}^i \tilde{T}^i_f \right] \tag{5.a}$$

$$T^i_f = \left[1 - \frac{2\Delta t}{(\tilde{\rho}\tilde{c}\tilde{D})_f} \left(\tilde{h}^{i-1}_f + \sigma\tilde{\varepsilon}_f \left(\tilde{T}^{i-1}_f \right)^3 \right) T^{i-1}_f \right]$$

$$+ \frac{2\Delta t}{(\tilde{\rho}\tilde{c}\tilde{D})_f} \left(\tilde{q}^{i-1}_f + \tilde{h}^{i-1}_f \tilde{r}^{i-1}_f \frac{\left(\tilde{V}^{i-1}\right)^2}{2c_p} \right) + \tilde{h}^{i-1}_f \frac{2\Delta t}{(\tilde{\rho}\tilde{c}\tilde{D})_f} \tilde{T}^{i-1}_{air} \tag{5.b}$$

where superscript i indicates the time, and

$$\tilde{H}^i_k = \tilde{c}_2 \tilde{\lambda}^i_w \coth \tilde{\lambda}^i_w \tilde{\ell}$$

$$\tilde{K}_1^i = \frac{\tilde{h}_w^i \left(1 - \text{sech}\ \tilde{\lambda}_w^i \tilde{\ell}\right)}{\tilde{h}_w^i + 4\sigma\tilde{\varepsilon}_w \left(\tilde{T}_{air}^i\right)^3}$$

$$\tilde{P}^i = \frac{\left(1 - \text{sech}\ \tilde{\lambda}_w^i \tilde{\ell}\right)\left(\tilde{h}_w^i \tilde{r}_w^i \frac{\left(\tilde{V}^i\right)^2}{2c_p} + 3\sigma\tilde{\varepsilon}_w \left(\tilde{T}_{air}^i\right)^4\right)}{\tilde{h}_w^i + 4\sigma\tilde{\varepsilon}_w \left(\tilde{T}_{air}^i\right)^3}$$

$$\tilde{Q}^i = \text{sech}\ \tilde{\lambda}_w^i \tilde{\ell}$$

$$\tilde{\lambda}_w^i = \left\{ \frac{4\left(\tilde{h}_w^i + 4\sigma\tilde{\varepsilon}_w \left(\tilde{T}_{air}^i\right)^3\right)}{(\tilde{k}\tilde{D})_w} \right\}^{1/2}$$

$$\tilde{c}_2 = \frac{(\tilde{k}\tilde{D})_w \tilde{D}_w}{2\tilde{D}_b^2}$$

Δt = sampling interval of $\tilde{T}_s$ (uniform)

Assume the entire sensor system is initially uniform, i.e., $\tilde{T}_f^0 = \tilde{T}_b^0$.

Solution proceeds in time steps by computing alternately, T_f^i, then T_{air}^i. A schematic representation of the data correction process, together with the simulated sensors as it was programmed for use in this study, is shown in Fig. 3.

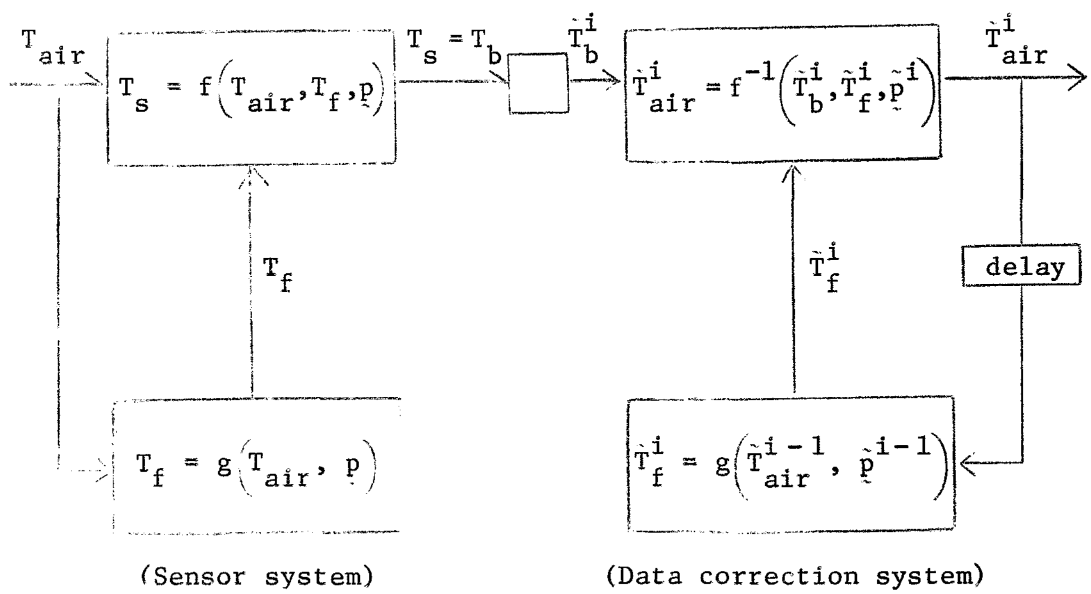

Fig. 3. Complete system description of ARCASONDE 1A measurement system.

III. PROPAGATION OF UNCERTAINTIES

A method of generating the uncertainty boundaries for the corrected air temperature function, $\tilde{T}_{air}$, discussed in the previous section is developed as follows. Successful assessment of variance in $\tilde{T}_{air}$ will require error examination in four categories:

1. Dimensions and properties (manufacturing variability).
2. Data handling (measurement errors).
3. Environmental parameters.
4. Approximations in the thermal analysis (model errors).

Uncertainty associated with error in the basic mathematical model of the sensor is assumed negligible.

3.1 Uncertainty Definition

The corrected air temperature is a function of a set of parameters, $\tilde{p}$.

$$\tilde{T}_{air} = f\left(\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \ldots, \tilde{p}_n\right) \tag{6}$$

Estimated value for the parameters are obtained from:

1. Specific laboratory measurements (such as for sensor emissivity, wire length).
2. In-flight measurement (such as for air speed).
3. Past experimental data (such as for earth albedo).

4. Theoretical calculations (such as for geometric factors, convective coefficients).

Suppose the error in a parameter estimate is defined as the difference between the estimated value and the true value of the parameter. Suppose further, that estimated values for a given flight are derived independently by many sufficiently qualified investigators. The estimated value, and therefore the error, would exhibit some statistical distribution, analogous to the results from a multiple sample experiment. The standard deviation and the mean of the estimated value may be defined as they are for a multiple sample experiment. If $\tilde{p}_i$ is the estimated value of a parameter from many different investigators, then the mean and standard deviation is defined as

$$\eta_{p_i} = \int_{-\infty}^{+\infty} p_i \, f\left(p_i\right) dp_i = \tilde{p} \tag{7}$$

$$\sigma_{p_i}^2 = \int_{-\infty}^{+\infty} \left(p_i - \eta_{p_i}\right)^2 f\left(p_i\right) dp_i \tag{8}$$

where $f\left(p_i\right)$ is a density function of p_i.

If η_{p_i} and σ_{p_i} are given for each parameter in Eq. 6, $\eta_{T_{air}}$ and $\sigma_{T_{air}}$ can be computed. If $\dot{T}_{air}$ is distributed normally, one can expect to find, or can be confident of finding, true T_{air} lying in the interval, $T_{air} \pm 2\sigma_{T_{air}}$, 95.45 percent of the time. It is expected that, in fact, T_{air} tends to be distributed normally because of the central limit theorem. The uncertainty boundary (or simply "uncertainty") is defined as ± 2σ of the nominal value.

3.2 Uncertainty Distribution

Though the distribution of the estimated value of a parameter may be conceptually defined, it is really known quantitatively for the parameters in the present discussion. For some parameters the uncertainty is given in the form of $p_i \pm \Delta p_i$. If only limiting values are known, the worst case method is sometimes used. The worst case method is a nonstatistical approach that employs the possible extremes of the parameters.

$$\Delta T_{air} = \sum_{i=1}^{n} \left| \left(\frac{\partial f}{\partial p_i} \right)_{p_i} \right| \Delta p_i \tag{9}$$

This usually gives an unrealistically pessimistic result. Even if they are strongly correlated, the assumption that the algebraic signs of all terms are the same is not statistically justified.

In order to obtain a more realistic uncertainty boundary, the concept of "uncertainty distribution" will be introduced. Kline and McClintock [9, 1953] applied this concept to describe uncertainties in single-sample experiments.

Suppose the variable p_i is expressed by $p_i \pm \Delta p_i$, where the value Δp_i is an estimation. An equivalent expression is the following:

$$P\left(p_i \leq p_i + \Delta p_i\right) \approx 1$$

$$P\left(p_i \leq p_i - \Delta p_i\right) \approx 0 \tag{10}$$

If we define $F(X) = P\left\{p_i \leq X\right\}$, the above expression can be shown graphically as in Fig. 4.

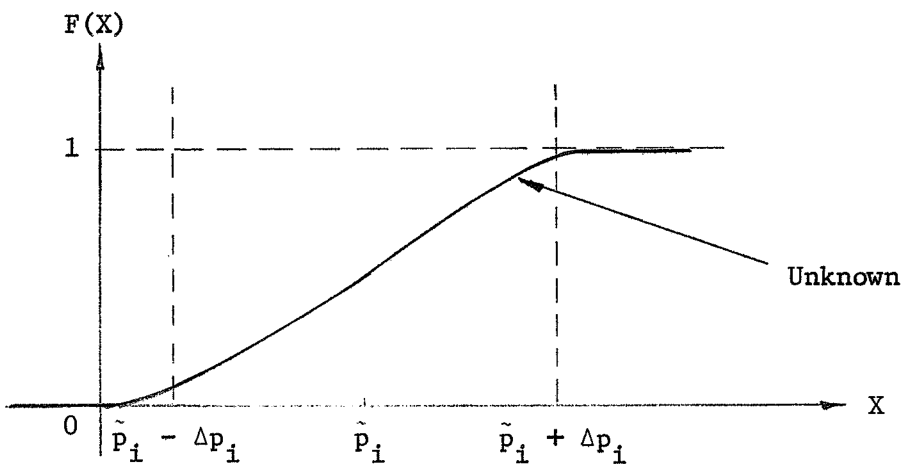

Fig. 4. Pseudo distribution curve.

Even if, between $p_i - \Delta p_i$ and $p_i + \Delta p_i$, there is no information, a pseudo distribution may be imagined.

The corresponding pseudo density function may then be defined as $f(X) = dF(X)/dX$. One might feel that the pseudo distribution is

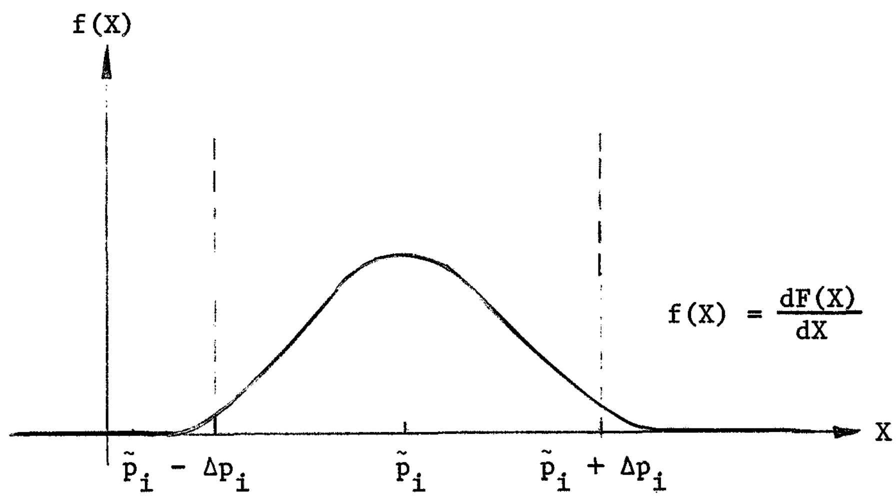

Fig. 5. Pseudo density function.

uniform for some parameters. For other parameters, however, intuition may suggest the errors are more likely near the center than near the ends of their respective ranges. Therefore, one might attempt to simulate this feeling by assuming the density function to be approximately normal, and Δp_i to be $2\sigma_{p_i}$ [7, Eisenhart 1963].

3.3 Propagation of Uncertainties

Now that we have estimated (or imagined) σ_{p_i}, the next step is to relate to the σ_{p_i} in accordance with $\sigma_{T_{air}}$, Eq. 6. Papoulis [10, 1965] discussed an approximation for this relation for the case of two parameters. The idea is easily extended to n parameters (Appendix A).

$$\sigma^2_{T_{air}} = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial p_i}\right)_{p_i} \sigma^2_{p_i} + \sum_{\substack{i=1 \\ i \neq j}} \sum_{j=1} \left(\frac{\partial f}{\partial p_i}\right)\left(\frac{\partial f}{\partial p_j}\right) \frac{\mathrm{cov}\left(p_i, p_j\right)}{\sigma_{p_i} \sigma_{p_j}} \tag{11}$$

where

$$\mathrm{cov}\left(p_i, p_j\right) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(p_i - \tilde{p}_i\right)\left(p_j - \tilde{p}_j\right) f\left(p_i, p_j\right) dp_i dp_j$$

$$= \mu_{ij} \sigma_{p_i} \sigma_{p_j}$$

The validity of the above equation depends on the nonlinearity of $f\left(p_1, p_2, \ldots, p_n\right)$ and the distribution of the parameters, but the approximation error within a sufficiently small range of the parameters is assumed to be small compared with the estimation error in the σ_{p_i}.

Thorough mathematical study of the approximation error is beyond the scope of the present discussion, but Monte-Carlo simulation studies indicate the error is small in the ARCASONDE 1A system for a considerably large range of the parameters.

IV. UNCERTAINTY BOUNDARY OF ARCASONDE 1A SYSTEM

The method of generating the uncertainty boundary in $\tilde{T}_{air}$, which was discussed in Chapter III, will be applied to the ARCASONDE 1A system which was introduced in Chapter II.

4.1 Expression of Uncertainty Boundary

Equations 5.a and 5.b can be expressed in the following form:

$$T^{i}_{air} = f^{i}\left(p^{k}_{j}\right) \qquad \begin{matrix} j = 1,\ldots, n \\ k = 1,\ldots, i \end{matrix} \tag{12}$$

where the input parameters include the sensor temperature $\tilde{T}^{k}_{b}$. The subscript j denotes the particular parameter, and k denotes the time.

Applying Eq. 11 to Eq. 12 gives

$$\sigma^{2}_{T_{air}} = \sum_{j=0}^{i} \sum_{m=1}^{n} \left(\frac{\partial f^{i}}{\partial p^{j}_{m}}\right)^{2} \sigma^{2}_{p^{j}_{m}} + \sum_{j=0}^{i} \sum_{k=0}^{i} \sum_{m=1}^{n} \sum_{\ell=1}^{n} \left(\frac{\partial f^{i}}{\partial p^{j}_{m}}\right)\left(\frac{\partial f^{i}}{\partial p^{k}_{\ell}}\right) \frac{\mathrm{cov}\left(p^{j}_{m}, p^{k}_{\ell}\right)}{\sigma_{p^{j}_{m}} \sigma_{p^{k}_{\ell}}}$$

$$\ell \neq m, \; j \neq k \tag{13}$$

The term $\left(\frac{\partial f^{i}}{\partial p^{j}_{m}}\right)$ is called the sensitivity coefficient [16, Tomovic 1962] and contains information about the system. $\sigma_{p^{j}_{m}}$ and $\mathrm{cov}\left(p^{j}_{m}, p^{k}_{\ell}\right)$ are independent of the sensor system and the values are estimated according to conditions at each point in time.

4.2 Sensitivity Analysis

In order to obtain the value of $\sigma^i_{T_{air}}$, the sensitivity coefficients have to be computed for each parameter at each preceding time point. Therefore, at each time i, we have to compute i x n sensitivity coefficients. One might consider this a very time consuming process for a system of many parameters and over many points in time, but actually $\frac{\partial f^i}{\partial p^j_\ell} \rightarrow 0$ when $(i - j) \rightarrow \infty$, therefore, older terms become negligible.

The sensitivity coefficient $\frac{\partial T^i_{air}}{\partial p^j_\ell}$ indicates the effect on T_{air} at time i of variation in p_ℓ at time j. As shown in Fig. 6, an error

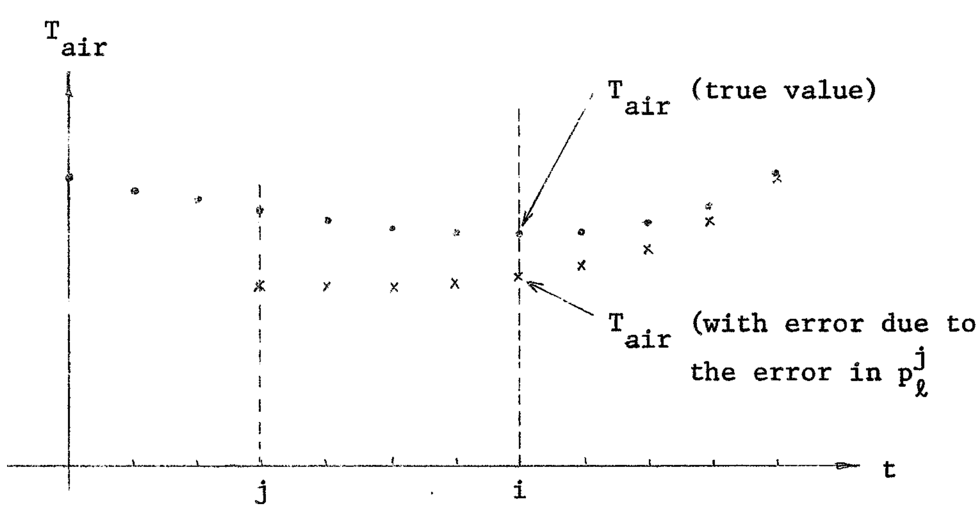

Fig. 6. Graphical description of $\frac{\partial T^i_{air}}{\partial p^j_\ell}$.

in p_ℓ at time j will have a diminishing effect on succeeding values of T_{air}.

Sensitivity coefficients are computed as follows. Equations 5.a and 5.b may be written in the form

$$T^i_{air} = f\left(\underset{\sim}{p}^i,\ T^i_f\right) \tag{14.a}$$

$$T^i_f = g\left(\underset{\sim}{p}^{i-1},\ T^{i-1}_f,\ T^{i-1}_{air}\right) \tag{14.b}$$

where

$$\underset{\sim}{p}^i = \begin{bmatrix} p^i_1 \\ p^i_2 \\ \cdot \\ \cdot \\ \cdot \\ p^i_n \end{bmatrix}$$

Assume the sensitivity coefficients have been computed up to the time (i - 1). Let already computed quantities be enclosed in parentheses. Only the bracketed quantities need be computed at the ith point in time. For the parameter, p_ℓ;

$\underline{j = i}$

$$\frac{\partial T^i_{air}}{\partial p^j_\ell} = \frac{\partial T^i_{air}}{\partial p^j_\ell} = \left[\frac{\partial f}{\partial p_i}\right]$$

Expression for the $\frac{\partial f^i}{\partial p_i}$ for the ARCASONDE 1A are listed in Appendix B.

$\underline{j = i - 1}$

$$\frac{\partial T^i_{air}}{\partial p^j_\ell} = \frac{\partial T^i_{air}}{\partial p^{i-1}_\ell} = \left[\frac{\partial f^i}{\partial T^i_f}\right]\left[\frac{\partial T^i_f}{\partial p^{i-1}_\ell}\right]$$

where

$$\frac{\partial f^i}{\partial T^i_f} = \left[\frac{\partial f^i}{\partial \underset{\sim}{G}^i}\right]\left[\frac{\partial \underset{\sim}{G}^i}{\partial T^i_f}\right] + \left[\frac{\partial f^i}{\partial T^i_f}\right]$$

$$\frac{\partial T^i_f}{\partial p^{i-1}_\ell} = \left[\frac{\partial g^i}{\partial p^{i-1}_\ell}\right] + \left[\frac{\partial g^i}{\partial T^{i-1}_{air}}\right]\left(\frac{\partial T^{i-1}_{air}}{\partial p^{i-1}_\ell}\right)$$

Computation of $\frac{\partial f^i}{\partial \underset{\sim}{G}^i}$ and $\frac{\partial \underset{\sim}{G}^i}{\partial T^i_f}$ are shown in Appendix B.

j = i - 2

$$\frac{\partial T_{air}^{i}}{\partial p_\ell^{j}} = \frac{\partial T_{air}^{i}}{\partial p_\ell^{1-2}} = \left(\frac{\partial f^{i}}{\partial T_f^{i}}\right) \left[\frac{\partial T_f^{i}}{\partial p_\ell^{i-2}}\right]$$

$$\frac{\partial T_f^{i}}{\partial p_\ell^{i-2}} = \left(\frac{\partial g^{i}}{\partial T_{air}^{i-1}}\right)\left(\frac{\partial T_{air}^{i-1}}{\partial p_\ell^{i-2}}\right) + \left[\frac{\partial g^{i}}{\partial T_f^{i-1}}\right]\left(\frac{\partial T_f^{i-1}}{\partial p_\ell^{i-2}}\right)$$

Computation of $\frac{\partial g^{i}}{\partial T_f^{i-1}}$ is shown in Appendix B.

j = i - 3

$$\frac{\partial T_{air}^{i}}{\partial p_\ell^{j}} = \frac{\partial T_{air}^{i}}{\partial p_\ell^{i-3}} = \left(\frac{\partial f^{i}}{\partial T_f^{i}}\right)\left[\frac{\partial T_f^{i}}{\partial p_\ell^{i-3}}\right]$$

$$\frac{\partial T_f^{i}}{\partial p_\ell^{i-3}} = \left(\frac{\partial g^{i}}{\partial T_{air}^{i-1}}\right)\left(\frac{\partial T_{air}^{i-1}}{\partial p_\ell^{i-3}}\right) + \left(\frac{\partial g^{i}}{\partial T_f^{i-1}}\right)\left(\frac{\partial T_f^{i-1}}{\partial p_\ell^{i-3}}\right)$$

j = 0

$$\frac{\partial T_f^i}{\partial p_\ell^0} = \left(\frac{\partial f^i}{\partial T_f^i}\right)\left(\frac{\partial T_f^i}{\partial p_\ell^0}\right)$$

$$\frac{\partial T_f^i}{\partial p_\ell^0} = \left(\frac{\partial g^i}{\partial T_{air}^{i-1}}\right)\left(\frac{\partial T_{air}^{i-1}}{\partial p_\ell^0}\right) + \left(\frac{\partial g^i}{\partial T_f^{i-1}}\right)\left(\frac{\partial T_f^{i-1}}{\partial p_\ell^0}\right)$$

Notice that only simple multiplication is needed for derivatives with respect to parameter values older than i - 3.

The correlation coefficient in time for a truly constant parameter p_ℓ is $\mu_\ell = 1$. The contribution of p_ℓ to $\sigma_{T_{air}^i}$, $\sigma_{T_{air}^i(p_\ell)}$ is

$$\sigma_{T_{air}^i(p_\ell)} = \frac{\partial T_{air}^i}{\partial p_\ell} \sigma_{p_\ell}$$

For constant parameters, the above procedure for computing the sensitivity coefficients is simplified by solving a set of difference equations.

From Eqs. 14.a and 14.b

$$\frac{\partial T^{i}_{air}}{\partial p_{\ell}} = \frac{\partial f^{i}}{\partial p_{\ell}} + \frac{\partial f^{i}}{\partial T^{i}_{f}} \frac{\partial T^{i}_{f}}{\partial p_{\ell}}$$

$$\frac{\partial T^{i}_{f}}{\partial p_{\ell}} = \frac{\partial g}{\partial p_{\ell}} + \frac{\partial g}{\partial T^{i-1}_{f}} \frac{\partial T^{i-1}_{f}}{\partial p_{\ell}} + \frac{\partial g}{\partial T^{i-1}_{air}} \frac{\partial T^{i-1}_{air}}{\partial p_{\ell}}$$

Substituting $U^{i} = \frac{\partial T^{i}_{air}}{\partial p_{\ell}}$ and $V^{i} = \frac{\partial T^{i}_{f}}{\partial p_{\ell}}$,

$$U^{i} = \frac{\partial f^{i}}{\partial p_{\ell}} + \frac{\partial f^{i}}{\partial T^{i}_{f}} V^{i} \tag{15.a}$$

$$V^{i} = \frac{\partial g^{i}}{\partial p_{\ell}} + \frac{\partial g^{i}}{\partial T^{i-1}_{f}} V^{i-1} + \frac{\partial g^{i}}{\partial T^{i-1}_{air}} U^{i-1} \tag{15.b}$$

which is a special case of the more general procedure given above

(Appendix C). The required computation is obviously much less for constant parameters.

4.3 Estimated Nominal Values and Uncertainties of Parameters

Sensor Properties

A list of input parameters for the ARCASONDE 1A temperature sensor is presented in Table 1. Since the film is a composition of silver and Mylar, properties of both must be used. Subscript fm, fs indicate the Mylar and silver parts of the film, respectively. Subscript f indicates the effective value.

A. D_b, D_w, D_{fm}, D_{fs} (diameter of bead, wire, and the thickness of the film). Uncertainty in these quantities is due to manufacturing variability and imperfections in shape. The uncertainty is estimated to be 10 percent.

B. $(\rho c)_b$, $(\rho c)_w$, $(\rho c)_{fm}$, $(\rho c)_{fs}$, $(\rho c)_f$ (density times heat capacity). The effective value of ρc for the film is given by

$$(\rho c)_f = \frac{D_s(\rho c)_s + D_m(\rho c)_m}{D_s + D_m}$$

The uncertainties for $(\rho c)_b$, $(\rho c)_w$ are estimated to be 5 percent due to lack of knowledge of composition, and 10 percent for $(\rho c)_f$ due to an estimated variability in Mylar thickness.

TABLE 1

Uncertainties and Nominal Value of Parameters

	Nominal Value	Uncertainty	Reference
D_b	$.28 \times 10^{-3}$ m (11 mil)	10%	Drews [4]
D_w	$.25 \times 10^{-4}$ m	10%	Drews
D_{fm}	$.25 \times 10^{-4}$ m		Drews
D_{fs}	$.4 \times 10^{-5}$ m		Drews
D_f	$.31 \times 10^{-4}$ m	7%	
$(\rho c)_b$	1.95×10^6 J/m^3 - °K	5%	Wright [19]
$(\rho c)_w$	2.79×10^6 J/m^3 - °K	5%	Wright
$(\rho c)_{fs}$	2.45×10^6 J/m^3 - °K		Weast [18]
$(\rho c)_{fm}$	1.84×10^6 J/m^3 - °K		Dupont [5]
$(\rho c)_f$	1.96×10^6 J/m^3 - °K	10%	
k_w	30.98 watt/m - °K	5%	Weast
k_{fm}	.152 watt/m - °K	5%	Dupont
k_{fs}	408 watt/m - °K	5%	Weast
α_b	.11	25%	Thompson [15]
α_w	.10	10%	Thompson
α_{fs}	.02		Weast
α_{fm}	.80		Drews
α_f	.65	50%	
α_{sb}	.16	40%	Thompson
α_{sw}	.19	50%	Thompson
α_{sfs}	.07		Weast
α_{sfm}	.06		Drews
α_{sf}	.22	50%	
ℓ	3.2×10^{-3} m	50%	Drews

C. k_w, k_{fm}, k_{fs}, k_f (thermal conductivity). The uncertainty for k is estimated to be 5 percent because of the impurity of materials and experimental error in published data [11, Powell, Ho, Liley]. Effective value of k is k_{fs} because $(kD)_s >> (kD)_m$.

D. α_b, α_w, α_{fs}, α_{fm} (absorptivity of long wave radiation). The effective absorptivity of the Mylar-exposed side of a silver-plated region is taken as

$$\alpha = \alpha_m + \left[\alpha_s + \alpha_m \left(1 - \alpha_s\right)\right]\left(1 - \alpha_m\right)$$

which assumes the reflectivity of the Mylar to be small. Similarly, the emissivity is assumed to be

$$\varepsilon = \varepsilon_m + \left(1 - \alpha_m\right)\left[\left(1 - \alpha_s\right) \varepsilon_m + \varepsilon_s\right]$$

which includes approximately the emission of the Mylar forward that emitted backward and reflected by the silver, and the emission of silver through the Mylar and, which incidentally, is the same value as the above absorptivity.

The effective emissivity of the film is computed by averaging that of the inner and outer film strips, and using their lengths as weighting factors.

Uncertainty in α_b is estimated to be about 25 percent because of the condition of coating the sphere surface [15, Thompson 1966]. Uncertainty of ε_f is estimated to be 50 percent due to a nonuniform plastic coating of unknown composition over the silver.

E. α_{sb}, α_{sw}, α_{sfs}, α_{sfm}, α_{sf} (absorptivity of short wave radiation). Uncertainty of α_{sb} and α_{sw} are assumed to be 40 percent due to the manufacturing variation in the surface [15, Thompson 1966]. The uncertainty in α_{sf} is estimated to be 50 percent due to the plastic coating.

F. ℓ (length of the wire). Uncertainty in the length of the wire is estimated to be about 50 percent due to manufacturing variation.

Convective Environment

Convective coefficient h and recovery factor r are computed using the interpolation formula [13, Staffanson and Alsaj 1968].

$$h = \frac{1}{\frac{1}{h_1} + \frac{1}{h_2}}, \quad r = r_1 + K_n \frac{r_2 - r_1}{Kn + Kn_0}$$

Subscript 1, 2 denote continuum and free molecular values, respectively. The uncertainty increases in the transition flow region where knowledge is least reliable. The uncertainty in recovery factor is estimated as shown in Fig. 7.

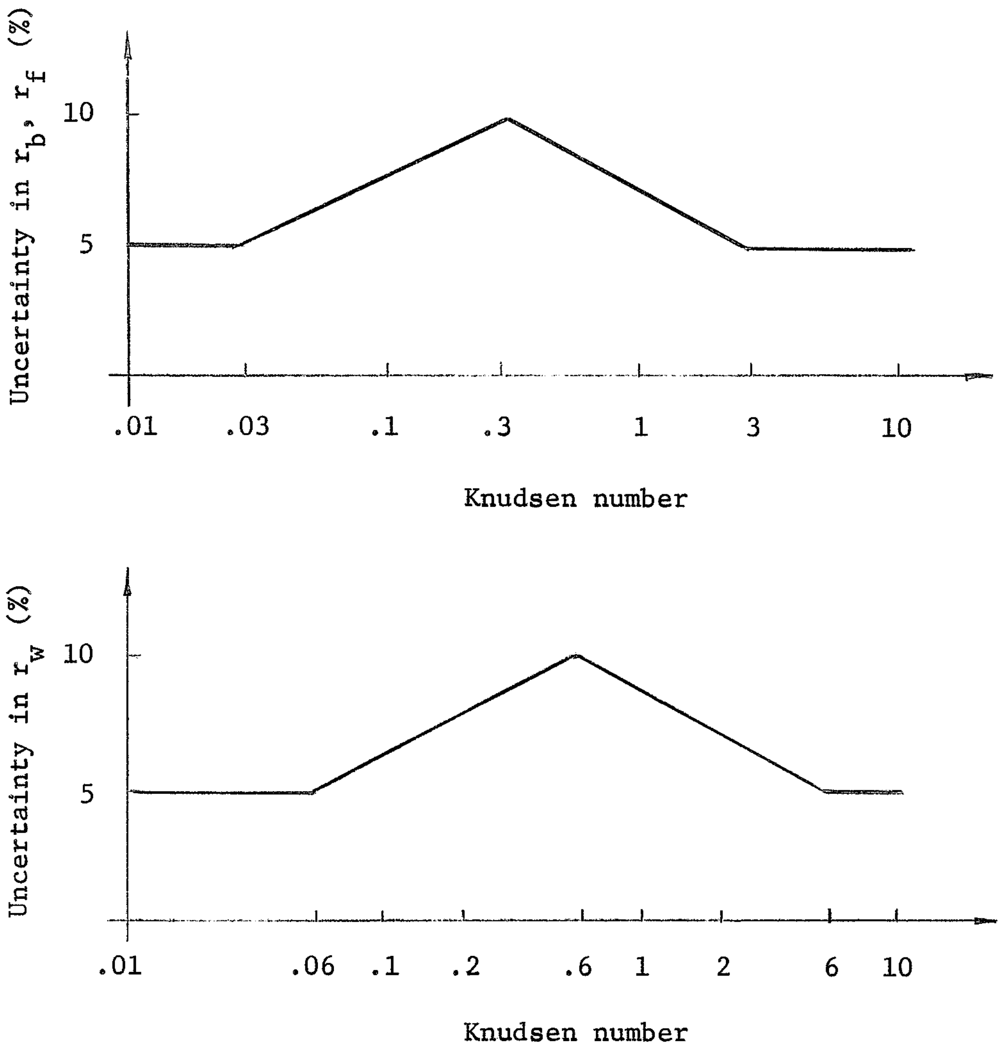

Fig. 7. Percent uncertainty of recovery factor.

Table 2 indicates the assumed uncertainty in h with respect to Reynolds number. Linear interpolation gives the uncertainty at any Reynolds number.

TABLE 2

Uncertainty of Convective Coefficient

Reynolds Number	Uncertainty (%)
$\leq 10^{-2}$	5%
10^{-1}	7.5%
1	10%
10	15%
10^{2}	18%
10^{3}	15%
10^{4}	10%
10^{5}	7.5%
$\geq 10^{6}$	5%

Radiation Environment

The geometric factor, $f_{i, j}$, is a quantity which depends on the shape of the sensor surfaces, the solid angle subtended by the source, and the orientation of the sensor relative to the source. The method of obtaining the values of $f_{i, j}$ is presented in Appendix D and the nominal values are presented in Table 3.

TABLE 3

Nominal Values and Uncertainties of the Geometric Factor

	Sphere	Cylinder	Plate
Sun	$.272 \times 10^{-5} \pm 100\%$	$.347 \times 10^{-5} \pm 100\%$	$.535 \times 10^{-5} \pm 100\%$
Albedo	0.40 ± 15%	0.40 ± 15%	0.40 ± 15%
Earth	0.42 ± 5%	0.43 ± 10%	0.41 ± 10%
Sonde	0.087 ± 3%	0.108 ± 3%	0.044 ± 3%

The radiant emittance, I_j (total radiant power emitted per unit area), of a solid surface depends on its emissivity, ε_λ, and its absolute temperature, T. I_j is computed in Appendix D. The nominal values of I_j are presented in Table 4.

TABLE 4

Nominal Values and Uncertainties of Radiant Emittance

Sun	6.4558×10^7 watt/m^2 ± 1%
Albedo	460.7 watt/m^2 ± 36%
Earth	233.8 watt/m^2 ± 20%
Sonde	458 watt/m^2 ± 15%

A. $f_{1,1}$, $f_{2,1}$, $f_{3,1}$ (geometric factors with respect to the sun). Uncertainties of $f_{2,1}$ are 100 percent because there is, in general, no knowledge as to sensor aspect to the sun. The sensor might be completely in the shade or exposed "broadside" to the sun.

B. $f_{1,3}$, $f_{2,3}$, $f_{3,3}$ (geometric factors with respect to the earth long-wave radiation). As indicated in Appendix D, $f_{3,3}$ varies as the sensor rotates relative to the earth.

If we assume the parachute has less than 45° coning motion, the variation of $f_{2,3}$ and $f_{3,3}$ due to the motion is about 10 percent.

The geometric factor of a truly spherical bead does not depend on the motion of the parachute. However, an estimated 5 percent uncertainty in $f_{1,3}$ is assumed for deviations due to the presence of the wires and nonsphericity.

C. $f_{1,2}$, $f_{2,2}$, $f_{3,2}$ (geometric factor with respect to the earth albedo). The albedo geometric factor is dependent on the position of the sun and cloud distribution, as well as to sensor attitude. The uncertainties in $f_{1,2}$ are estimated to be 15 percent.

D. $f_{1,4}$, $f_{2,4}$, $f_{3,4}$ (geometric factor with respect to the sonde surfaces or shield). The errors in estimated nominal values, $f_{1,4}$, are estimated to be 3 percent.

E. I_1, I_2, I_3, I_4 (radiant emittance). The uncertainty of

I_1 is due to the seasonal variation of the distance between the sun and the earth and is estimated to be 1 percent [8, Johnson 1954].

The uncertainty of I_2 is due to the cloud cover variability and is estimated to be 36 percent (see Appendix D).

The uncertainty of I_3 is due to variability of earth surface matter and temperature. The analysis of Tiros II (1960) data by Bandeen [2, 1961] gives about ± 20 percent variation in earth temperature at a given time over the North American continent. Assuming the uncertainty in the effective black body, temperature of the local region of the earth is 5 percent, uncertainty in I_3 is 20 percent.

The uncertainty of I_4 is due to the variability in the sonde temperature and emissivity. Uncertainty in I_4 is arbitrarily estimated to be 15 percent.

Self Heating

An estimation of the power dissipated by the thermistor is about 20×10^{-6} watts [4, Drews 1966]. The uncertainty is due to the temperature dependency of the power dissipation and is estimated to be 15 percent.

Data

Measurement error of T_b is estimated to be 2°K, and the uncertainty in T_b is assumed to be 10 percent. The uncertainty of the relative velocity, V, is about 5 percent based on the assumed angular motion about the center of mass trajectory recorded by the tracking radar.

Correlation Coefficient $\mu_{m, \ell}$

For the present study, all parameters are assumed that independent so $\text{cov}\left(p_m^j, p_\ell^k\right)$ and, therefore, $\mu_{m, \ell}$ are set equal to zero if $m \neq \ell$.

4.4 Simulation Study of ARCASONDE 1A System

A. Simulation

Sensor air flow is simulated by computing parachute motion based on the ballistic coefficient and drag [6, Eddy 1965] properties of the ARCASONDE system.

Total parachute-sonde mass	$m = 2.33$ kg
Parachute reference area	$A = 16.4$ m^2

The U.S. Standard Atmosphere (1962) will be used for density of the air, ρ.

Initial conditions of the parachute motion are arbitrarily chosen as

Initial altitude	70 km
Horizontal velocity	30 m/s
Vertical velocity	-180 m/s

At every second, the altitude and the relative speed with respect to the air are computed. At each altitude, the properties of the air are obtained from the atmosphere table, which already has been read in, including T_{air}. Values of h and r are obtained, based on the given altitude and the air speed. Those values are used in Eq. 4.a and 4.b in order to obtain T_b.

The simulated T_b is used as input $\tilde{T}_b$ to the data corrector. Parameter values, p_j, used in the simulator are also used as input $\tilde{p}_j$ in the data corrector. Therefore, $\tilde{T}_{air}$ is equal to T_{air}. Now the objective of the study is to produce the uncertainty boundary of $\tilde{T}_{air}$.

The uncertainties in the parameters which were discussed in Section 4.3 are combined with Eq. 13, where sensitivity coefficients are computed by the method discussed in Section 4.2. The computational procedure is shown in Fig. 8.

Notice in Fig. 8 that part A corresponds to the "sensor system" in Fig. 2, and part B corresponds to "data reduction system" in Fig. 2.

B. Results

As shown in Fig. 9, the uncertainty boundary increases rapidly with altitude. This is due to the decrease of convective coupling with the air, while the radiation input remains constant.

As h_b and h_w decrease with altitude, there is more conductive heat flow from the film. The sensor is, therefore, more sensitive to the film temperature as altitude increases. This is clear if you compare the effect of h_b and h_f at 70 and 50 km in Table 5. The uncer-

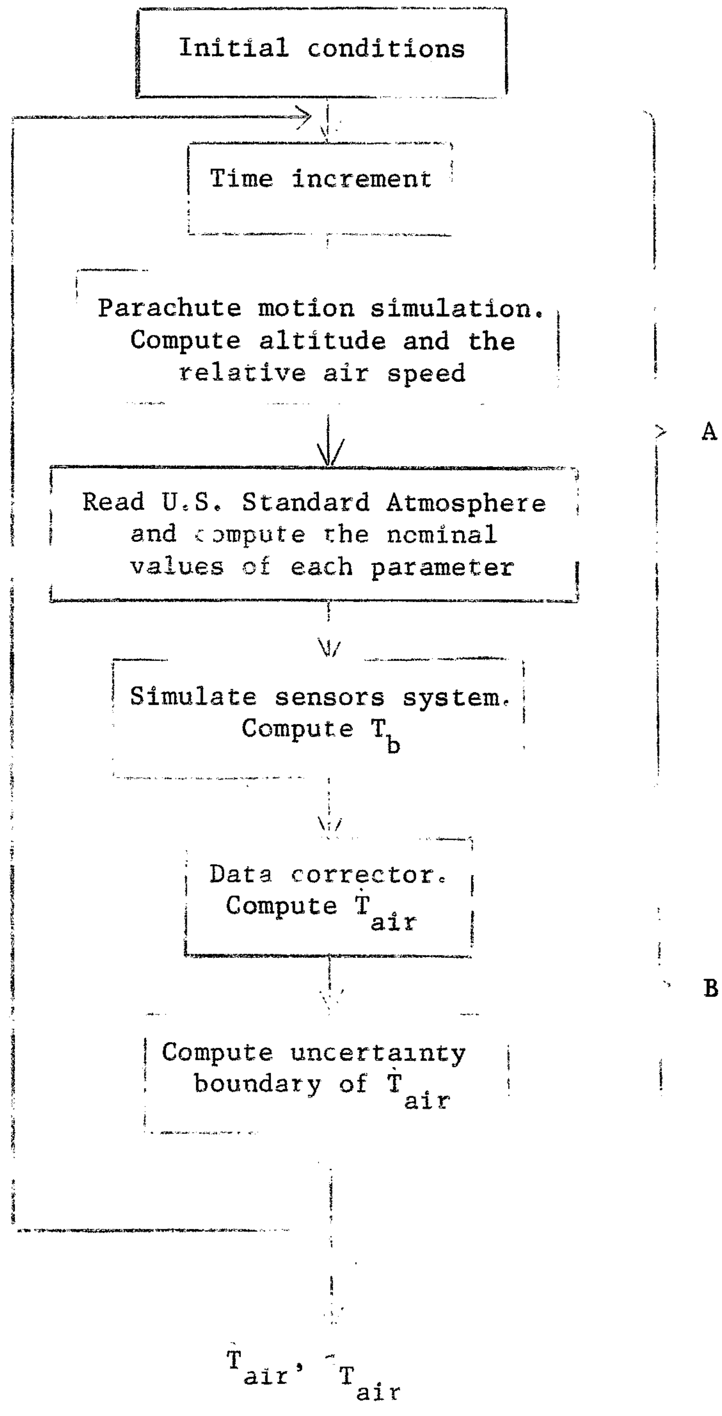

Fig. 8. Block diagram of simulation study.

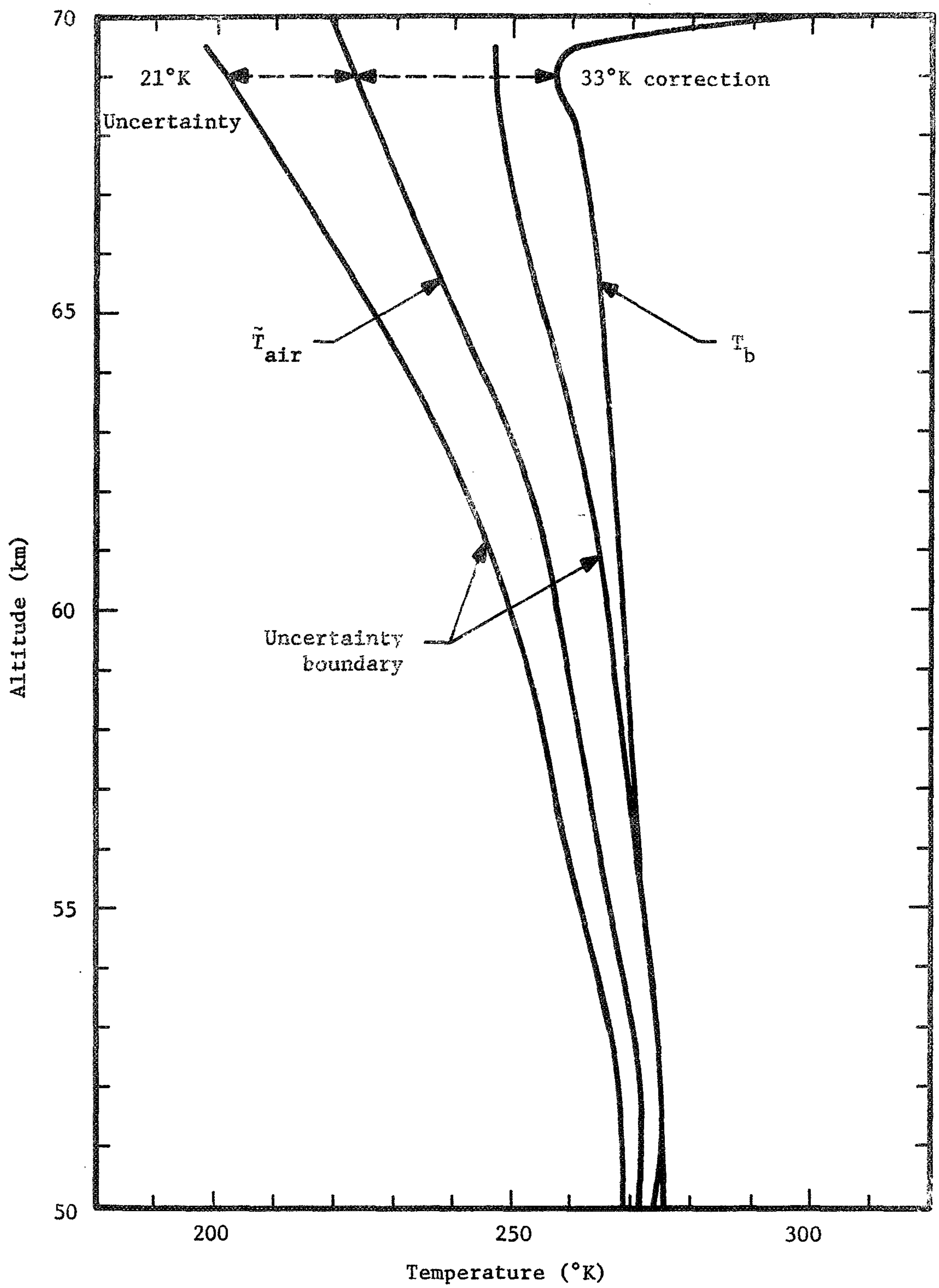

Fig. 9. Uncertainty boundary.

TABLE 5

Uncertainty Components

Altitude (km)	70	65	60	55	50
h_b	.27	.86-1	.63-1	.43-1	.30-1
r_b	.32	.33	.19	.11	.57-1
h_w	.14	.78-2	.37-2	.41-2	.25-2
r_w	.38	.32	.16	.76-1	.48-1
h_f	.35	.11	.29-1	.46-3	.50-2
r_f	.76-1	.51-1	.17-1	.53-2	.16-2
V	1.38	1.16	.56	.27	.13
T_b	3.47	2.34	1.71	1.37	1.19
$\dot{T}_b$	.23	.59-2	.12-2	.29-3	.18-2
$f_{1,1}$	.92	.53	.31	.19	.12
$f_{1,2}$	.15	.83-1	.48-1	.29-1	.19-1
$f_{1,3}$	.18-1	.10-1	.60-2	.36-2	.23-2
$f_{2,1}$	1.38	.77	.43	.24	.14
$f_{2,2}$	.17	.95-1	.53-1	.30-1	.17-1
$f_{2,3}$	.33-1	.18-1	.10-1	.57-2	.33-2
$f_{3,1}$	4.17	2.26	1.24	.65	.32
$f_{3,2}$	.33	.18	.99-1	.52-1	.26-1
$f_{3,3}$	.34	.18	.10	.53-1	.26-1
w_b	.40	.23	.13	.80-1	.51-1
α_b	.48-1	.43-1	.34-1	.26-1	.19-1
α_w	.41-1	.30-1	.21-1	.14-1	.93-2

TABLE 5

(continued)

Uncertainty Components

Altitude (km)	70	65	60	55	50
D_b	.19-1	.16	.94-1	.60-1	.31-1
$(\rho c)_b$	.40-1	.15-2	.42-2	.26-3	.93-3
D_w	.11	.24	.14	.93-1	.54-1
k_w	.41-1	.79-1	.49-1	.32-1	.19-1
ℓ	.66	1.18	.74	.48	.29
$(\rho c)_f$	.30	.35-2	.10-2	.97-3	.58-4
D_f	.21	.25-2	.70-3	.68-3	.40-4
$f_{1,4}$	.23-2	.15-2	.10-2	.71-3	.50-3
$f_{2,4}$	.25-2	.16-2	.11-2	.70-3	.44-3
$f_{3,4}$	.51-1	.29-1	.16-1	.85-2	.42-2
α_{sb}	.40	.27	.18	.13	.87-1
α_{sw}	.53	.34	.23	.15	.91-1
α_{sf}	7.59	4.37	2.42	1.26	.62
I_1	.11	.65-1	.37-1	.20-1	.10-1
I_2	2.30	1.35	.78	.43	.23
I_3	1.70	.98	.55	.29	.15
I_4	.28	.96	.92-1	.50-1	.25-1
resultant	23.98	14.06	8.36	4.98	3.20

tainty components listed in Table 5 are the terms

$$\left\{ \sum_{j=0}^{i} \left(\frac{\partial T^{i}_{air}}{\partial p^{j}_{\ell}} \right)^{2} \sigma^{2}_{p^{j}_{\ell}} \right\}^{1/2}$$

Quantities listed in Tables 5, 8, 9, and 10, pages 39, 46, 48, and 50, respectively, at 70 km are actually those computed at 69.5 km. Those computed at the initial altitude, 70 km, are those at ejection from the rocket and are not of interest here. Values tabulated, nevertheless, exhibit some effect of the initial transient, e.g., the small values at 70 km of ℓ, k_w, D_w.

The sensitivity to error in T_b (measurement error) approaches unity at low altitudes and increases rapidly at high altitudes. As the convective coefficients, h_i, decrease, sensitivity to air temperature decreases; i.e., a given variation in T_b corresponds to increasingly larger variations in T_{air}.

Notice that the uncertainty contribution associated with direct solar heating, $\alpha_s f_{i,1} I_1$, is large compared with other parameters. This suggests that a solar shield would significantly decrease the overall uncertainty. In the following, the ARCASONDE 1A sensor is assumed to be placed one radius deep into a cylindrical tube. The tube is oriented with its axis parallel to the flow, and with radius sufficiently large so that the boundary layer is away from the sensor.

C. With Shield

The purpose of the shield is to replace a sufficient part of the highly variable natural radiation environment with an environment whose influence on sensor temperature is both small enough and well enough known to enable precise correction of the sensor data.

A shield with a downward view half angle of $\theta = 45°$ is used, and the inside of the shield is painted black. The black painted wall will minimize the effect of radiation arising from reflections within the shield, which would cause large uncertainties. The emissivity of the black paint is assumed to be 1, and the geometric factors are computed by the method discussed in Appendix D. The temperature of the shield is assumed to be 300 ± 2°K. Input values are listed in Tables 6 and 7.

TABLE 6

Nominal Values and Uncertainties of the Geometric Factor (with Shield)

	Sphere	Cylinder	Plate
Sun	0	0	0
Albedo	.147 ± 15%	.16 ± 15%	.09 ± 15%
Earth	.147 ± 5%	.16 ± 10%	.09 ± 10%
Sonde	.853 ± 3%	.84 ± 3%	.91 ± 3%

TABLE 7

Nominal Values and Uncertainties of Radiant Emittance (with Shield)

Sun	0	
Albedo	460.7 watt/m^2 ± 36%	
Earth	233.8 watt/m^2 ± 20%	
Sonde	458.0 watt/m^2 ± 2%	T_{shield} = 300°K ± 2°K
	221.0 watt/m^2 ± 4%	T_{shield} = 250°K ± 2°K

Figure 10 shows the distinct improvement of performance at high altitude. Comparison of Tables 5 and 8 shows the increase of the effect of uncertainty in h. Increased heating due to the shield increases the sensitivity to h and to other parameters such as ℓ. This suggests that a colder shield would significantly improve the sensor.

D. Cold Shield

The benefit of a cool shield is investigated by letting the shield temperature be 250°K. The corresponding value of I_4 is included in Table 7.

The results are especially significant at higher altitude as shown in Fig. 11. Notice in Table 8 that uncertainties due to ℓ, h are much smaller for the cold shield case.

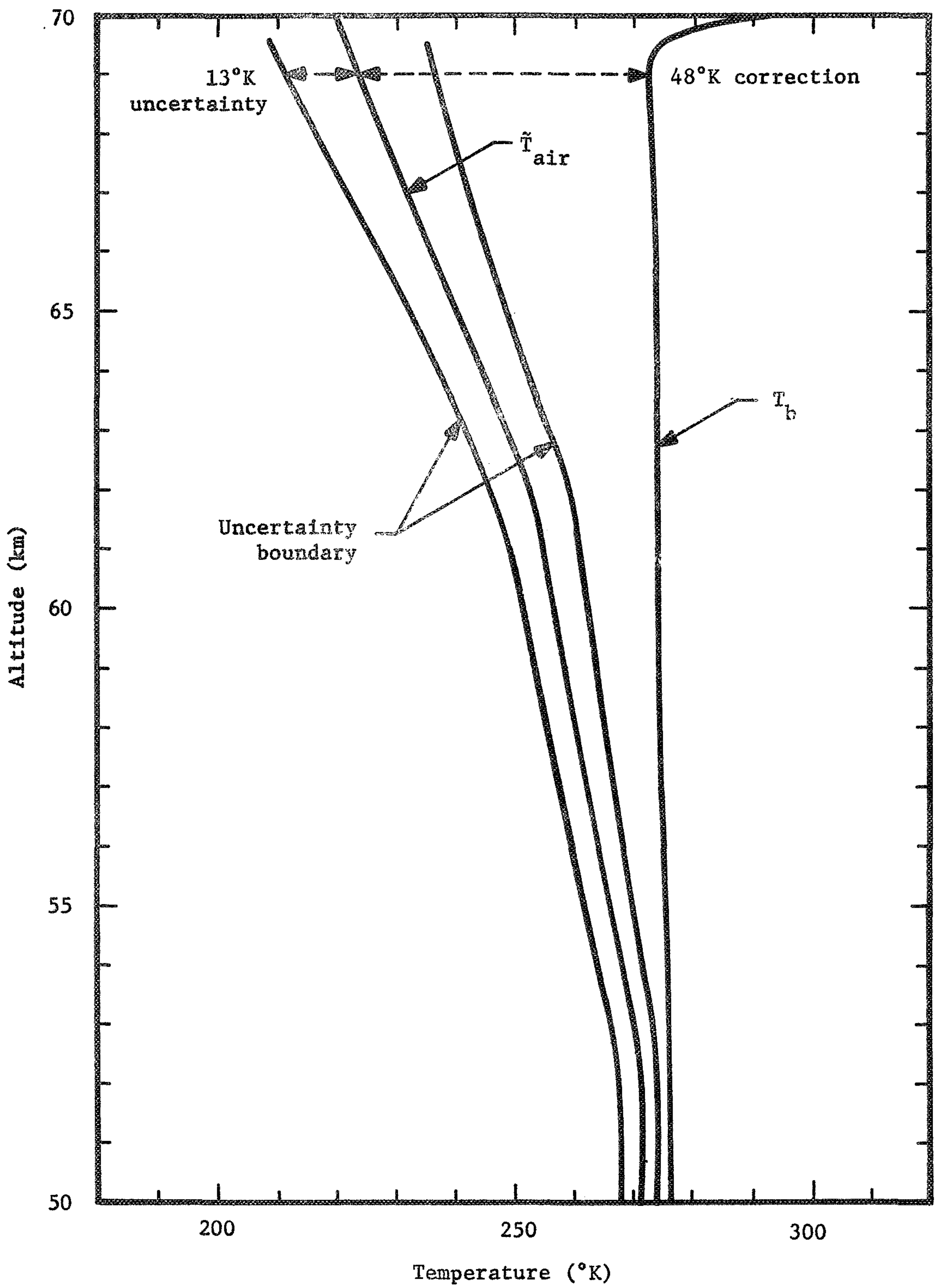

Fig. 10. Uncertainty boundary with a shield.

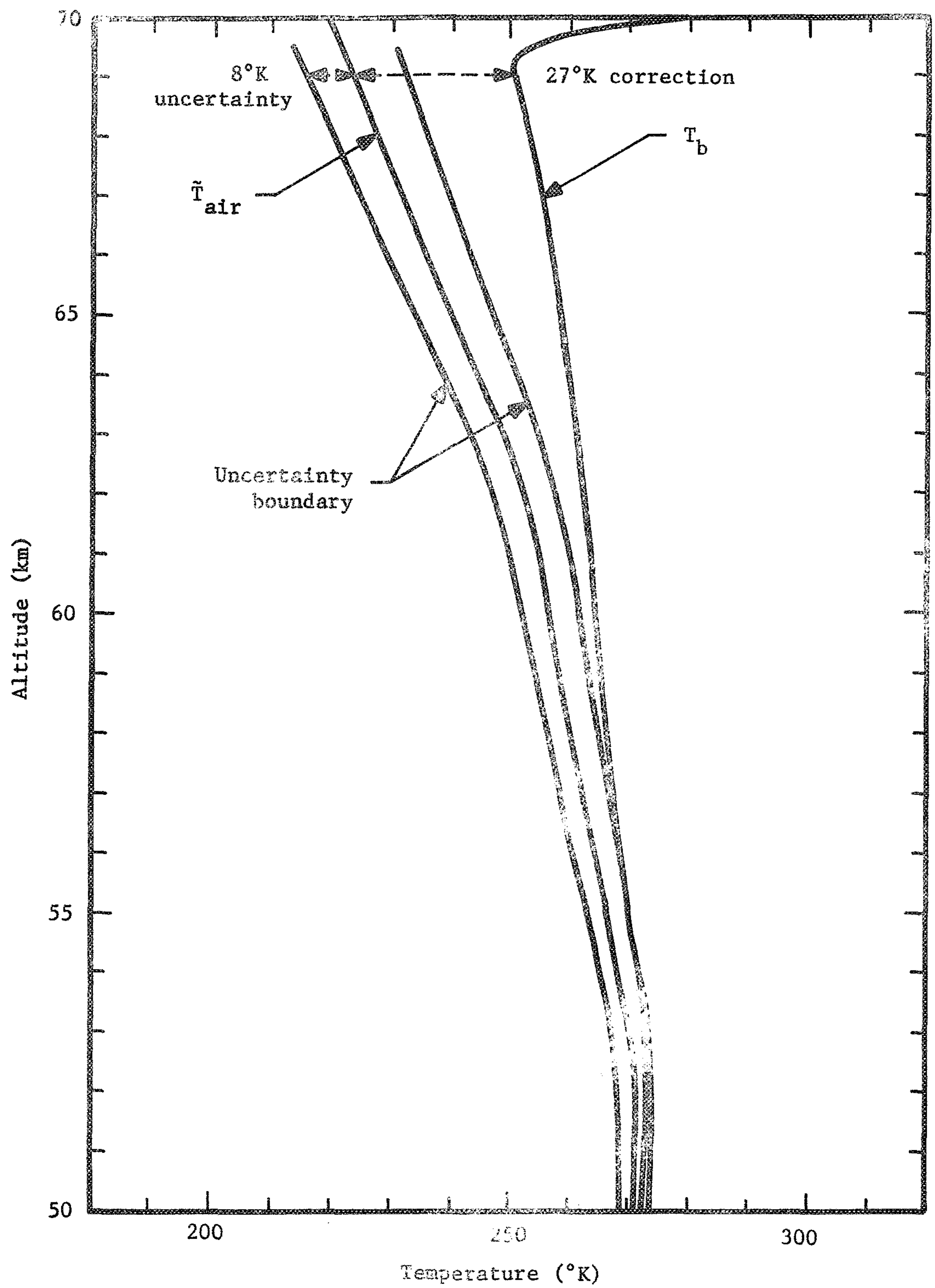

Fig. 11. Uncertainty boundary with cold shield.

TABLE 8

Uncertainty Components (with Shield)

Altitude (km)	70	65	60	55	50
h_b	.55	.31	.20	.12	.72-1
r_b	.32	.33	.19	.11	.57-1
h_w	.38	.19	.14	.91-1	.60-1
r_w	.38	.32	.16	.76-1	.48-1
h_f	.67	.35	.17	.84-1	.40-1
r_f	.74-1	.49-1	.16-1	.52-2	.16-2
V	1.38	1.16	.56	.27	.13
T_b	3.48	2.34	1.71	1.37	1.19
$\dot{T}_b$	.96-1	.94-3	.32-3	.18-3	.26-2
$f_{1,1}$	.0	.0	.0	.0	.0
$f_{1,2}$	.53-1	.30-1	.18-1	.11-1	.68-2
$f_{1,3}$	.62-2	.35-2	.21-2	.12-2	.79-3
$f_{2,1}$	.0	.0	.0	.0	.0
$f_{2,2}$	.68-1	.34-1	.21-1	.12-1	.69-2
$f_{2,3}$	.12-1	.67-2	.38-2	.21-2	.12-2
$f_{3,1}$	.0	.0	.0	.0	.0
$f_{3,2}$	.73-1	.40-1	.22-1	.11-1	.57-2
$f_{3,3}$	.73-1	.40-1	.22-1	.11-1	.57-2
w_b	.40	.23	.13	.80-1	.51-1
α_b	.60-1	.35-1	.23-1	.15-1	.10-1
α_w	.56-1	.37-1	.25-1	.16-1	.98-2

TABLE 8

(continued)

Uncertainty Components (with Shield)

Altitude (km)	70	65	60	55	50
α_f	3.47	2.13	1.05	.49	.23
D_b	.41	.17	.15	.10	.59-1
$(\rho c)_b$	.25-1	.51-3	.16-3	.12-3	.14-2
D_w	.62	.33	.28	.20	.12
k_w	.22	.12	.10	.73-1	.47-1
ℓ	3.56	1.97	1.70	1.21	.79
$(\rho c)_f$	.13	.11-2	.17-3	.53-3	.17-3
D_f	.88-1	.76-3	.12-3	.37-3	.12-3
$f_{1,4}$	.23-1	.15-1	.10-1	.71-2	.49-2
$f_{2,4}$	.21-1	.13-1	.86-2	.55-2	.34-2
$f_{3,4}$	1.0	.57	.32	.17	.82-1
α_{sb}	.79-1	.51-1	.35-1	.24-1	.06-1
α_{sw}	.10	.64-1	.42-1	.27-1	.17-1
α_{sf}	.56	.32	.18	.93-1	.46-1
I_1	.0	.0	.0	.0	.0
I_2	.56	.33	.20	.11	.63-1
I_3	.36	.21	.12	.36-1	.33-1
I_4	.69	.40	.22	.12	.60-1
resultant	13.12	8.16	5.54	3.92	2.94

TABLE 9

Uncertainty Components (Cold Shield)

Altitude (km)	70	65	60	55	50
h_b	.12	.43-1	.26-1	.14-1	.42-2
r_b	.32	.33	.19	.11	.57-1
h_w	.17-1	.11	.80-1	.37-1	.38-1
r_w	.38	.32	.16	.76-1	.48-1
h_f	.22	.12-1	.32-1	.34-1	.23-1
r_f	.77-1	.51-1	.17-1	.53-2	.16-2
V	1.38	1.16	.56	.27	.13
T_b	3.47	2.33	1.71	1.37	1.19
$\dot{T}_b$	.29	.85-2	.19-2	.35-3	.12-2
$f_{1,1}$	.0	.0	.0	.0	.0
$f_{1,2}$	.53-1	.30-1	.18-1	.11-1	.70-2
$f_{1,3}$	.62-2	.35-2	.21-2	.12-2	.79-3
$f_{2,1}$	.0	.0	.0	.0	.0
$f_{2,2}$	.68-1	.38-1	.21-1	.12-1	.69-2
$f_{2,3}$	.12-1	.68-2	.38-2	.21-2	.12-2
$f_{3,1}$	.0	.0	.0	.0	.0
$f_{3,2}$	.76-1	.42-1	.23-1	.12-1	.58-2
$f_{3,3}$	.76-1	.42-1	.23-1	.12-1	.58-2
w_b	.40	.23	.13	.80-1	.51-1
α_b	.75-2	.95-2	.12-1	.11-1	.94-2
α_w	.34-1	.27-1	.20-1	.14-1	.90-2

TABLE 9

(continued)

Uncertainty Components (Cold Shield)

Altitude (km)	70	65	60	55	50
α_f	.81	.23	.42	.34	.21
D_b	.37-1	.24	.17	.12	.66-1
$(\rho c)_b$	.47-1	.19-2	.51-3	.36-3	.61-3
D_w	.21	.34	.26	.18	.11
k_w	.73-1	.12	.88-1	.63-1	.40-1
ℓ	1.14	1.75	1.33	.98	.64
$(\rho c)_f$	.40	.48-2	.14-2	.12-2	.21-5
D_f	.28	.34-2	.99-3	.83-3	.15-5
$f_{1,4}$	.11-1	.69-2	.49-2	.34-2	.23-2
$f_{2,4}$	.93-2	.60-2	.40-2	.26-2	.16-2
$f_{3,4}$	.52	.30	.17	.87-1	.42-1
α_{sb}	.73-1	.48-1	.34-1	.23-1	.16-1
α_{sw}	.94-1	.60-1	.41-1	.26-1	.16-1
α_{sf}	.61	.35	.19	.10	.49-1
I_1	.0	.0	.0	.0	.0
I_2	.59	.35	.21	.12	.65-1
I_3	.39	.23	.13	.68-1	.35-1
I_4	.72	.42	.23	.12	.62-1
resultant	8.60	6.64	4.72	3.56	2.78

TABLE 10

Comparison of Uncertainty Boundary

Altitude(km)	Without Shield(°K)	With Hot Shield(°K)	With Cold Shield(°K)
70	23.98	13.12	8.60
65	14.06	8.16	6.64
60	8.36	5.54	4.72
55	4.98	3.92	3.56
50	3.20	2.94	2.78

V. CONCLUSIONS

The automatic computation of the running error of a measurement system, according to time-varying estimates of error in assumed parameter values, provides a useful quantitative basis for system evaluation and improvement. A plot of the uncertainty boundary about the nominal output value from a simulated system provides a clear graphical model of operational capability. The uncertainty envelope computed along with the reduction of real data provides the user with a convenient indicator of the quality of measurement results.

Results obtained for the current ARCASONDE 1A sensor indicate that the system uncertainty exceeds the magnitude of the correction for altitudes below about 53 km, so corrections tend to be meaningless for this sensor in the stratosphere and below. The uncertainty appears to remain greater than half the correction throughout the mesosphere (50-80 km). The chief contributing factors to the overall uncertainty are uncertainties in film absorptivity, emissivity, and aspect with respect to the sun. The contribution of an assumed 1°K error in acquiring the sensor temperature is the major contributor at low altitudes.

The hypothetical addition of a simple cylindrical shield to the ARCASONDE 1A sensor, while increasing its error, considerably improves its performance in terms of greater accuracy after corrections. Corrections may be needed to lower altitude, however, when a shield is used. The increased radiant heat input from the relatively hot shield at 300°K limits the benefit of a shield. Reducing the shield temperature to 250°K decreases error flux into the sensor sufficient to considerably decrease sensitivity to h_i and other parameters, as well as to direct solar parameters.

APPENDIX A

APPROXIMATION OF THE RESULTANT STANDARD DEVIATION

The following is a brief examination of the assumptions underlying the relation (Eq. 13) used in this dissertation to compute the overall uncertainty in the corrected temperature.

Expansion of a function of n variables in Taylor's series

$$f(\underset{\sim}{p}) = \sum_{k=0}^{\infty} \frac{1}{k!} \left[\sum_{j=1}^{n} \Delta p_j \frac{\partial}{\partial p_j} \right]^k f(\underset{\sim}{p}) \tag{A.1}$$

Neglecting third-order terms and higher,

$$= f_o + \sum_{j=1}^{n} \frac{\partial f}{\partial p_j}\bigg|_0 \Delta p_j + \frac{1}{2} \sum_{j=1}^{n} \frac{\partial^2 f}{\partial p_j^2}\bigg|_0 \Delta p_j$$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial^2 f}{\partial p_i \partial p_j}\bigg|_0 \Delta p_i \Delta p_j \quad (i \neq j) \tag{A.2}$$

where Δp_j is the variation of the jth variable from its expected value.

$$\Delta p_j = p_j - \eta_j \;,\; \eta_j = p_j\big|_0 = E\left[p_j\right]$$

The coefficients

$$f_o = a \ , \ \left.\frac{\partial f}{\partial p_j}\right|_0 = b_j \ , \ \left.\frac{\partial^2 f}{\partial p_j^2}\right|_0 = c_j \ , \ \left.\frac{\partial^2 f}{\partial p_i \partial p_j}\right|_0 \ d_{ij}$$

are independent of the variation. The expected value of f is then

$$E[f] = a + \frac{1}{2} \sum_{j=1}^{n} c_j \sigma_j^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} \operatorname{cov}\left(p_i, p_j\right) \tag{A.3}$$

The terms of the variance, $\sigma_f^2 = E\left[f^2\right] - E^2[f]$ of $f(\underset{\sim}{p})$ in the two-variable case are found as follows:

$$f = a + b_1 \Delta p_1 + b_2 \Delta p_2 + \frac{1}{2} c_1 \Delta p_1^2 + \frac{1}{2} c_2 \Delta p_2^2 + d_{12} \Delta p_1 \Delta p_2$$

$$E[f] = a + \frac{1}{2} c_1 \sigma_1^2 + \frac{1}{2} c_2 \sigma_2^2 + d_{12} \operatorname{cov}\left(p_1, p_2\right)$$

$E^2(f)$ contains 10 terms and $E\left(f^2\right)$ has 19 terms. Their difference after cancellation contains 21 terms.

$$\sigma^2 = E\left(f^2\right) - E^2(f) = b_1^2\sigma_1^2 + b_2^2\sigma_2^2 + 2b_1b_2 \text{ cov}\left(p_1, p_2\right)$$

$$+ b_1c_1 E\left(\Delta p_1^3\right) + b_2c_2 E\left(\Delta p_2^3\right)$$

$$+ \left(2b_1d_{12} + b_2c_1\right) E\left(\Delta p_1^2 \Delta p_2\right)$$

$$+ \left(2b_2d_{12} + b_1c_2\right) E\left(\Delta p_1 \Delta p_2^2\right)$$

$$+ \frac{1}{2} c_1^2 \left[E\left(\Delta p_1^4\right) - \sigma_1^4\right] + \frac{1}{2} c_2^2 \left[E\left(\Delta p_2^4\right) - \sigma_2^4\right]$$

$$+ 2d_{12}^2 \left[E\left(\Delta p_1 \Delta p_2^2\right) - \text{cov}^2\left(p_1, p_2\right)\right]$$

$$+ \frac{1}{2} c_1c_2 \left[E\left(\Delta p_1^2 \Delta_2^2\right) - \sigma_1^2\sigma_2^2\right]$$

$$+ c_1d_{12} \left[E\left(\Delta p_1^3 \Delta p_2\right) - \sigma_1^2 \text{ cov}\left(p_1, p_2\right)\right]$$

$$+ c_2d_{12} \left[E\left(\Delta p_1 \Delta p_2^3\right) - \sigma_2^2 \text{ cov}\left(p_1, p_2\right)\right]$$

If, in addition to the implied initial assumptions that higher order terms in the Taylor's expansion are negligible, it is assumed that the distribution of the p_j's are symmetrical so that $E\left(\Delta p_1^3\right)$ and $E\left(\Delta p_2^3\right)$ are small, that c, d << b, and that third and fourth-order moments are small, then the above rather lengthy expression reduces to

$$\sigma^2 = b_1^2\sigma_1^2 + b_2^2\sigma_2^2 + 2b_1b_2 \text{ cov}\left(p_1, p_2\right)$$

The corresponding relation for the n-variable case is

$$\sigma_f^2 = \sum_{j=1}^{n} b_j^2\sigma_j^2 + 2\sum_{i=1}^{n}\sum_{j=1}^{n} b_ib_j \text{ cov}\left(p_i, p_j\right) \quad (i \neq j)$$

as can be seen by introducing additional variables from the beginning and dropping small terms. An indication of the validity of Eq. 13 was found by a Monte Carlo computation in which the $\tilde{p}$'s were generated as random variables. The resulting standard deviation of T_{air}, using normal and uniform distributions for the parameters, with $\sigma_j = \Delta p_j/2$ and $p_j - \Delta p_j < p_j < p_j + \Delta p_j$, respectively, is compared with $\sigma_{T_{air}}$ from Eq. 13 in Table A.1. Recall that uncertainty in T_{air} is comparable to twice the standard deviation. The tabulated results are those of the unshielded ARCASONDE 1A, discussed in Section 4.B. The data corrector routine was repeated 300 times, using subroutine (RANDN) from the UNIVAC 1108 Math Pack as a source of random numbers. Correlation between parameters was small. The comparison is considered quite good indicating that the system equations accommodate the magnitude of the given variation is such that the magnitude of the given variation in parameters does not exceed limits of validity.

TABLE A.1

Comparison of Approximated and Simulated $\sigma_{T_{air}}$

Altitude	Approximation (°K)	Simulation (normal distribution) (°K)	Simulation (uniform distribution) (°K)
70	11.99	11.71	11.65
65	7.03	7.00	6.98
60	4.18	4.16	4.14
55	2.49	2.48	2.47
50	1.60	1.60	1.59

APPENDIX B

EXPRESSIONS FOR SENSITIVITY COEFFICIENTS

A. The $\frac{\partial f^i}{\partial p^i}$ are computed as follows. Superscript i is ommited in the right-hand members of the following statements since it is obvious.

$$\frac{\partial f^i}{\partial h_b^i} = \frac{1}{\left(h_b + H_K K_1\right)^2} \left[\left(h_b + H_K K_1\right)\left(T_b - r_b \frac{V^2}{2c_p}\right) - E1\right]$$

where

$$E1 = \frac{(\rho c D)_b}{6} \dot{T}_b + \left(h_b + 4\sigma\varepsilon_b T_b^3 + H_K\right) T_b$$

$$- \left(h_b\ r_b \frac{V^2}{2c_p} + 3\sigma\varepsilon_b T_b^4 + q_b + H_K P\right) - H_K Q\ T_f$$

$$\frac{\partial f^i}{\partial r_b^i} = - \frac{h_b \frac{V^2}{2c_p}}{h_b + H_K K_1}$$

$$\frac{\partial f^i}{\partial h_w^i} = - E2 \frac{H_K \frac{\partial K_1}{\partial h_w} + K_1 \frac{\partial H_K}{\partial h_w}}{\left(h_b + H_K K_1\right)^2} + \frac{\frac{\partial H_K}{\partial h_w}\left(h_b + H_K K_1\right) - H_K\left(H_K \frac{\partial K_1}{\partial h_w} + K_1 \frac{\partial H_K}{\partial h_w}\right)}{\left(h_b + H_K K_1\right)^2} T_b$$

$$- \frac{\left(\frac{\partial H_K}{\partial h_w} P + \frac{\partial P}{\partial h_w} H_K\right)\left(h_b + H_K K_1\right) - H_K P \left(H_K \frac{\partial K_1}{\partial h_w} + K_1 \frac{\partial H_K}{\partial h_w}\right)}{\left(h_b + H_K K_1\right)^2}$$

$$- \frac{\left(\frac{\partial H_K}{\partial h_w} Q + \frac{\partial Q}{\partial h_w} H_K\right)\left(h_b + H_K K_1\right) - H_K Q \left(H_K \frac{\partial K_1}{\partial h_w} + K_1 \frac{\partial H_K}{\partial h_w}\right)}{\left(h_b + H_K K_1\right)^2} T_f$$

where

$$\frac{\partial \lambda_w}{\partial h_w} = \frac{2}{(kd)_w \lambda_w}$$

$$\frac{\partial H_K}{\partial h_w} = \left(\frac{\partial \lambda_w}{\partial h_w}\right)\left(- C_2 \lambda_w \ell \operatorname{cosech}^2 \lambda_w \ell + C_2 \coth \lambda_w \ell\right)$$

$$\frac{\partial K_1}{\partial h_w} = \frac{\left(h_w + 4\sigma\varepsilon_w T_{aw}^3\right) h_w \ell_w \operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell \left(\frac{\partial \lambda}{\partial h_w}\right)}{\left(h_w + 4\sigma\varepsilon T_{aw}^3\right)^2}$$

$$+ \frac{4\sigma\varepsilon_w T_{aw}^3 \left(1 - \operatorname{sech} \lambda_w \ell\right)}{\left(h_w + 4\sigma\varepsilon_w T_{aw}^3\right)^2}$$

$$A\left(h_w\right) = 1 - \text{sech}\ \lambda_w \ell$$

$$B\left(h_w\right) = h_w r_w \frac{V^2}{2c_p} + \frac{q_w}{A_w} + 3\sigma\varepsilon_w T_{aw}^4$$

$$\frac{\partial P}{\partial h_w} = \frac{1}{\left(h_w + 4\sigma\varepsilon_w T_{aw}^2\right)^2} \left[\left\{A\left(h_w\right) r_w \frac{V^2}{2c_p} + B\left(h_w\right)\ \ell\ \text{sech}\ \lambda_w \ell\right.\right.$$

$$\left.\left.\tanh\ \lambda_w \ell \left(\frac{\partial \lambda_w}{\partial h_w}\right)\right\}\left(h_w + 4\sigma\varepsilon_w T_{aw}^3\right) - A\left(h_w\right) B\left(h_w\right)\right]$$

$$\frac{\partial Q}{\partial h_w} = - \ell\ \text{sech}\ \lambda_w \ell\ \tanh\ \lambda_w \ell \left(\frac{\partial \lambda_w}{\partial h_w}\right)$$

$$E_2 = \frac{(\rho c D)_b}{6} \dot{T}_b + \left(h_b + 4\sigma\varepsilon_b T_{ab}^3\right) T_b - \left(h_b r_b \frac{V^2}{2c_p} + 3\sigma\varepsilon_b T_{ab}^4 \frac{W}{A_b} + q_b\right)$$

$$\frac{\partial f^i}{\partial r_w^i} = - \frac{H_K}{h_b + H_K K_1} \frac{h_w \frac{V^2}{2c_p}\left(1 - \text{sech}\ \lambda_w \ell\right)}{h_w + 4\sigma\varepsilon_w T_{aw}^3}$$

$$\frac{\partial f^i}{\partial h_f^i} = 0$$

$$\frac{\partial f^i}{\partial r_f^i} = 0$$

$$\frac{\partial f^i}{\partial V^i} = - \frac{1}{h_b + H_K K_1} \left(h_b r_b \frac{V}{c_p} + H_K \frac{\partial P}{\partial V} \right)$$

where

$$\frac{\partial P}{\partial V} = \frac{h_w r_w V}{c_p} \left(\frac{1 - \operatorname{sech} \lambda_w \ell}{h_w + 4\sigma\varepsilon_w T_{aw}^3} \right)$$

$$\frac{\partial f^i}{\partial T_b^i} = \frac{1}{\left(h_b + H_K K_1\right)^2} \left\{ \left[h_b + 4\sigma\varepsilon_b T_{ab}^3 + \frac{\partial H_K}{\partial T_b} T_b + H_K - \frac{\partial H_K}{\partial T_b} P \right. \right.$$

$$\left. \left. - \frac{\partial P}{\partial T_b} H_K - T_f \left(H_K \frac{\partial Q}{\partial T_b} + Q \frac{\partial H_K}{\partial T_b} \right) \right] \left(h_b + H_K K_1 \right) - El \left(\frac{\partial H_K}{\partial T_b} K_1 + \frac{\partial K_1}{\partial T_b} H_K \right) \right\}$$

where

$$\frac{\partial T_{aw}}{\partial T_b} = \frac{1}{2}$$

$$\frac{\partial \lambda_w}{\partial T_b} = \frac{12\sigma\varepsilon_w T_{aw}^2}{\lambda_w (kd)_w}$$

$$\frac{\partial H_K}{\partial T_b} = C_2 \left\{ \frac{\partial \lambda_w}{\partial T_b} \coth \lambda_w \ell - \lambda_w \left(\text{cosech}^2 \lambda_w \ell \right) \ell \frac{\partial \lambda_w}{\partial T_b} \right\}$$

$$\frac{\partial T_{aw}^3}{\partial T_b} = \frac{3}{2} T_{aw}^2$$

$$\frac{\partial K_1}{\partial T_b} = \frac{1}{\left(h_w + 4\sigma\varepsilon_w T_{aw}^3\right)^2} \left[\left(h_w + 4\sigma\varepsilon_w T_{aw}^3\right)\left(h_w \text{ sech } \lambda_w \ell \right.\right.$$

$$\left.\left. \tanh \lambda_w \ell\right) \ell \frac{\partial \lambda_w}{\partial T_b} - h_w\left(1 - \text{sech } \lambda_w \ell\right) 4\sigma\varepsilon_w \frac{\partial T_{aw}^3}{\partial T_b} \right]$$

$$\frac{\partial T_{aw}^4}{\partial T_b} = 2 T_{aw}^3$$

$$\frac{\partial P}{\partial T_b} = \frac{1}{\left(h_w + 4\sigma\varepsilon_w T_{aw}^3\right)^2} \left\{ \left[\left(\text{sech } \lambda_w \ell \ \tanh \lambda_w \ell\right) \ell \frac{\partial \lambda_w}{\partial T_b} \right. \right.$$

$$\left. \left(h_w r_w \frac{V^2}{2c_p} + q_w + 3\sigma\varepsilon_w T_{aw}^4\right) + 6\left(1 - \text{sech } \lambda_w \ell\right) \sigma\varepsilon_w T_{aw}^3 \right] \left(h_w + 4\sigma\varepsilon_w T_{aw}^3\right)$$

$$\left. - \left(1 - \text{sech } \lambda_w \ell\right)\left(h_w r_w \frac{V^2}{2c_p} + q_w + 3\sigma\varepsilon_w T_{aw}^4\right) 6\sigma\varepsilon_w T_{aw}^2 \right\}$$

$$\frac{\partial Q}{\partial T_b} = \left(\text{sech } \lambda_w \ell \ \tanh \lambda_w \ell\right) \ell \frac{\partial \lambda_w}{\partial T_b}$$

$$\frac{\partial f^i}{\partial \dot{T}_b^i} = \frac{\rho c D_b}{6\left(h_b + H_K K_1\right)}$$

$$\frac{\partial f^i}{\partial f_{11}^i} = \frac{\partial f^i}{\partial q_b} I_1 \alpha_{11}$$

where

$$\frac{\partial f^i}{\partial q_b} = - \frac{1}{h_b + H_K K_1}$$

$$\frac{\partial f^i}{\partial f^i_{12}} = \frac{\partial f^i}{\partial q_b} I_2 \alpha_{12}$$

$$\frac{\partial f^i}{\partial f^i_{13}} = \frac{\partial f^i}{\partial q_b} I_3 \alpha_{13}$$

$$\frac{\partial f^i}{\partial f^i_{21}} = \frac{\partial f^i}{\partial q_w} I_1 \alpha_{21}$$

where

$$\frac{\partial f^i}{\partial q_w} = - \frac{H_K}{h_b + H_K K_1} \frac{1 - \text{sech } \lambda_w \ell}{h_w + 4\sigma\varepsilon_w T^3_{aw}}$$

$$\frac{\partial f^i}{\partial f^i_{22}} = \frac{\partial f^i}{\partial q_w} I_2 \alpha_{22}$$

$$\frac{\partial f^i}{\partial f^i_{23}} = \frac{\partial f^i}{\partial q_w} I_3 \alpha_{23}$$

$$\frac{\partial f^i}{\partial f^i_{31}} = 0$$

$$\frac{\partial f^i}{\partial f^i_{32}} = 0$$

$$\frac{\partial f^i}{\partial f^i_{33}} = 0$$

$$\frac{\partial f^i}{\partial W^i_b} = - \frac{1}{h_b + H_K K_1} + \left(\frac{\partial f^i}{\partial T_b}\right)\left(\frac{\partial T_b}{\partial W^i_b}\right)$$

$$\frac{\partial f^i}{\partial \varepsilon_b} = \frac{\partial f^i}{\partial q_b} \left(f_{13} I_3 + f_{14} I_4\right) + \frac{\sigma T_b^4}{h_b + H_K K_1}$$

$$\frac{\partial f^i}{\partial \varepsilon_w} = \frac{\partial f^i}{\partial H_K} \frac{\partial H_K}{\partial \lambda_w} \frac{\partial \lambda_w}{\partial \varepsilon_w}$$

$$+ \frac{\partial f^i}{\partial K_1} \left(\frac{\partial K_1}{\partial \varepsilon_w} + \frac{\partial K_1}{\partial \lambda_w} \frac{\partial \lambda_w}{\partial \varepsilon_w}\right)$$

$$+ \frac{\partial f^i}{\partial P} \left(\frac{\partial P}{\partial \varepsilon_w} + \frac{\partial P}{\partial \lambda_w} \frac{\partial \lambda_w}{\partial \varepsilon_w}\right) + \frac{\partial f^i}{\partial Q} \frac{\partial Q}{\partial \lambda_w} \frac{\partial \lambda_w}{\partial \varepsilon_w}$$

where

$$\frac{\partial f^i}{\partial H_K} = \frac{1}{\left(h_b + H_K K_1\right)^2} \left[\left(T_b - P - QT_f\right)\left(h_b + H_K K_1\right) - El\ K_1\right]$$

$$\frac{\partial f^i}{\partial K_1} = \frac{1}{\left(h_b + H_K K_1\right)^2} - \left[El\ H_K\right]$$

$$\frac{\partial f^i}{\partial P} = \frac{-H_K}{h_b + H_K K_1}$$

$$\frac{\partial f^i}{\partial Q} = \frac{-H_K T_f}{h_b + H_K K_1}$$

$$\frac{\partial H_K}{\partial \lambda_w} = C_2 \left(\coth \lambda_w \ell - \lambda_w \ell\ \text{cosech}^2\ \lambda_w \ell\right)$$

$$\frac{\partial K_1}{\partial \lambda_w} = \frac{h_w \ell\ \text{sech}\ \lambda_w \ell\ \tanh \lambda_w \ell}{h_w + 4\sigma\varepsilon_w T_{aw}^3}$$

$$\frac{\partial Q}{\partial \lambda_w} = -\ \ell\ \text{sech}\ \lambda_w \ell\ \tanh \lambda_w \ell$$

$$\frac{\partial \lambda_w}{\partial \varepsilon_w} = \frac{8\sigma T_{aw}^3}{\lambda_w (kd)_w}$$

$$\frac{\partial K_1}{\partial \varepsilon_w} = \frac{-h_w \left(1 - \operatorname{sech} \lambda_w \ell\right) 4\sigma T_{aw}^3}{\left(h_w + 4\sigma \varepsilon_w T_{aw}^3\right)^2}$$

$$\frac{\partial P}{\partial \varepsilon_w} = \frac{1 - \operatorname{sech} \lambda_w \ell}{\left(h_w + 4\sigma \varepsilon_w T_{aw}^3\right)^2} \left[3\sigma T_{aw}^4 \left(h_w + 4\sigma \varepsilon_w T_{aw}^3\right) \right.$$

$$\left. - \left(h_w r_w \frac{V^2}{2c_p} + q_w + 3\sigma \varepsilon_w T_{aw}^4\right) 4\sigma T_{aw}^3 \right]$$

$$\frac{\partial f^i}{\partial \varepsilon_f} = 0$$

$$\frac{\partial f^i}{\partial D_b} = \frac{1}{\left(h_b + H_K K_1\right)^2} \left[\left(\frac{\rho c}{6} \dot{T}_b + T_b \frac{\partial H_K}{\partial D_b} - P \frac{\partial H_K}{\partial D_b} - QT_f \frac{\partial H_K}{\partial D_b}\right) \right.$$

$$\left. \left(h_b + H_K K_1\right) - K_1 \frac{\partial H_K}{\partial D_b} E1 \right]$$

where

$$\frac{\partial C_2}{\partial D_b} = \frac{-\ (kD)_w\ D_w}{D_b^3}$$

$$\frac{\partial H_K}{\partial D_b} = \lambda_w \ \coth \lambda_w \ell \left(\frac{\partial C_2}{\partial D_b}\right)$$

$$\frac{\partial f^i}{(\rho c)_b} = \frac{D_b\ \dot{T}_b}{6\left(h_b + H_K K_1\right)}$$

$$\frac{\partial f^i}{\partial D_w} = \frac{1}{\left(h_b + H_K K_1\right)^2}\left[\left\{T_b \frac{\partial H_K}{\partial D_w} - H_K \frac{\partial P}{\partial D_w} - P \frac{\partial H_K}{\partial E_w} - T_f \left(\frac{\partial H_K}{\partial D_w} Q + \frac{\partial Q}{\partial h_w} H_K\right)\right\}\right.$$

$$\left.\left(h_b + H_K K_1\right) - El \left(\frac{\partial H_K}{\partial D_w} K_1 + \frac{\partial K_1}{\partial D_w} H_K\right)\right]$$

where

$$\frac{\partial C_2}{\partial D_w} = \frac{k_w D_w}{D_b^2}$$

$$\frac{\partial \lambda_w}{\partial D_w} = - \frac{\lambda_w}{2D_w}$$

$$\frac{\partial H_K}{\partial D_w} = \left(\frac{\partial C_2}{\partial D_w} \lambda_w + C_2 \frac{\partial \lambda_w}{\partial D_w} \right) \coth \lambda_w \ell - C_2 \lambda_w \ell \frac{\partial \lambda_w}{\partial D_w} \operatorname{cosech}^2 \lambda_w \ell$$

$$\frac{\partial K_1}{\partial D_w} = \frac{h_w}{h_w + 4\sigma \varepsilon_w T_{aw}^3} \left(\operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell \right) \ell \frac{\partial \lambda_w}{\partial D_w}$$

$$\frac{\partial P}{\partial D_w} = \frac{\left(h_w r_w \frac{V^2}{2c_p} + q_w + 3\sigma \varepsilon_w T_{aw}^4 \right)}{h_w + 4\sigma \varepsilon_w T_{aw}^3} \left(\operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell \right) \ell \frac{\partial \lambda_w}{\partial D_w}$$

$$\frac{\partial Q}{\partial D_w} = - \left(\operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell \right) \ell \frac{\partial \lambda_w}{\partial D_w}$$

$$\frac{\partial f^i}{\partial k_w} = \frac{1}{\left(h_b + H_K K_1 \right)^2} \left[\left\{ T_b \frac{\partial H_K}{\partial k_w} - H_K \frac{\partial P}{\partial k_w} - P \frac{\partial H_K}{\partial k_w} - T_f \left(\frac{\partial H_K}{\partial k_w} Q + \frac{\partial Q}{\partial k_w} H_K \right) \right\} \right.$$

$$\left. \left(h_b + H_K K_1 \right) - El \left(\frac{\partial H_K}{\partial k_w} K_1 + \frac{\partial K_1}{\partial k_w} H_K \right) \right]$$

where

$$\frac{\partial C_2}{\partial k_w} = \frac{d_w^2}{2d_b^2}$$

$$\frac{\partial \lambda_w}{\partial k_w} = -\frac{\lambda_w}{2k_w}$$

$$\frac{\partial H_K}{\partial k_w} = \left(\frac{\partial C_2}{\partial k_w}\lambda_w + C_2\frac{\partial \lambda_w}{\partial k_w}\right)\coth \lambda_w \ell - C_2 \lambda_w \ell \frac{\partial \lambda_w}{\partial k_w} \operatorname{cosech}^2 \lambda_w \ell$$

$$\frac{\partial K_1}{\partial k_w} = \frac{h_w}{h_w + 4\sigma\varepsilon_w T_{aw}^3}\left(\operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell\right) \ell \frac{\partial \lambda_w}{\partial k_w}$$

$$\frac{\partial P}{\partial k_w} = \frac{\left(h_w r_w \frac{V^2}{2c_p} + q_w + 3\sigma\varepsilon_w T_{aw}^4\right)}{h_w + 4\sigma\varepsilon_w T_{aw}^3}\left(\operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell\right) \ell \frac{\partial \lambda_w}{\partial k_w}$$

$$\frac{\partial Q}{\partial k_w} = -\left(\operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell\right) \ell \frac{\partial \lambda_w}{\partial k_w}$$

$$\frac{\partial f^i}{\partial \ell} = \frac{1}{\left(h_b + H_K K_1\right)^2} \left[\left\{ \left(T_b - P\right) \frac{\partial H_K}{\partial \ell} - H_K \frac{\partial P}{\partial \ell} - T_f \left(\frac{\partial H_K}{\partial \ell} Q + \frac{\partial Q}{\partial \ell} H_K \right) \right\} \left(h_b + H_K K_1\right) - E1 \left(\frac{\partial H_K}{\partial \ell} K_1 + H_K \frac{\partial K_1}{\partial \ell} \right) \right]$$

where

$$\frac{\partial H_K}{\partial \ell} = - C_2 \lambda_w^2 \operatorname{cosech}^2 \lambda_w \ell$$

$$\frac{\partial K_1}{\partial \ell} = \frac{h_w r_w}{h_w + 4\sigma\varepsilon_w T_{aw}^3} \operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell$$

$$\frac{\partial P}{\partial \ell} = \frac{\left(h_w r_w \frac{V^2}{2c_p} + q_w + 3\sigma\varepsilon_w T_{aw}^4 \right) \lambda_w \operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell}{h_w + 4\sigma\varepsilon_w T_{aw}^3}$$

$$\frac{\partial Q}{\partial \ell} = - \lambda_w \operatorname{sech} \lambda_w \ell \tanh \lambda_w \ell$$

$$\frac{\partial f^i}{\partial (\rho c)_f} = 0$$

$$\frac{\partial f^i}{\partial D_f} = 0$$

$$\frac{\partial f^i}{\partial f_{14}} = \frac{\partial f^i}{\partial q_b} \alpha_{14} I_4$$

$$\frac{\partial f^i}{\partial f_{24}} = \frac{\partial f^i}{\partial q_w} \alpha_{24} I_4 = \left(\frac{H_K}{h_b + H_K K_1}\right)\left(\frac{1 - \text{sech } \lambda_w \ell}{h_w + 4\sigma\varepsilon_w T_{aw}^3}\right) \alpha_{24} I_4$$

$$\frac{\partial f^i}{\partial f_{34}} = 0$$

$$\frac{\partial f^i}{\partial \alpha_{11}} = \frac{\partial f^i}{\partial q_b} \left(f_{11} I_1 + f_{12} I_2\right)$$

$$\frac{\partial f^i}{\partial \alpha_{21}} = \frac{\partial f^i}{\partial q_w} \left(f_{21} I_1 + f_{22} I_2\right)$$

$$\frac{\partial f^i}{\partial \alpha_{31}} = 0$$

$$\frac{\partial f^i}{\partial I_1} = \frac{\partial f^i}{\partial q_b} f_{11}\ \alpha_{11} + \frac{\partial f^i}{\partial q_w} f_{21}\ \alpha_{21}$$

$$\frac{\partial f^i}{\partial I_2} = \frac{\partial f^i}{\partial q_b} f_{12}\ \alpha_{12} + \frac{\partial f^i}{\partial q_w} f_{22}\ \alpha_{22}$$

$$\frac{\partial f^i}{\partial I_3} = \frac{\partial f^i}{\partial q_b} f_{13}\ \alpha_{13} + \frac{\partial f^i}{\partial q_w} f_{23}\ \alpha_{23}$$

$$\frac{\partial f^i}{\partial I_4} = \frac{\partial f^i}{\partial q_b} f_{14}\ \alpha_{14} + \frac{\partial f^i}{\partial q_w} f_{24}\ \alpha_{24}$$

B. Computation of $\frac{\partial f^i}{\partial T_f^i}$. Grouping the set of quantities H_K^i, K_1^i, P^iQ^i as elements of a vector $\underset{\sim}{G}$

$$G^i \left(T_f, \underset{\sim}{p}\right) = \begin{bmatrix} H_K^i \left(T_f^i, \underset{\sim}{p}^i\right) \\ K_1^i \left(T_f^i, \underset{\sim}{p}^i\right) \\ P^i \left(T_f^i, \underset{\sim}{p}^i\right) \\ Q^i \left(T_f^i, \underset{\sim}{p}^i\right) \end{bmatrix}$$

$$\frac{\partial f^i}{\partial T_f^i} = \frac{\partial f^i}{\partial \underset{\sim}{G}^i} \frac{\partial \underset{\sim}{G}^i}{\partial T_f^i}$$

where $\frac{\partial f^i}{\partial \underset{\sim}{G}^i}$ and $\frac{\partial \underset{\sim}{G}^i}{\partial T^i}$ are given as follows:

$$\frac{\partial f^i}{\partial \underset{\sim}{G}^i} = \left(\frac{\partial f}{\partial H_K} , \frac{\partial f}{\partial K_1} , \frac{\partial f}{\partial Q} , \frac{\partial f}{\partial P} \right)$$

$$\frac{\partial f}{\partial H_K} = \frac{1}{\left(h_b + H_K K_1\right)} \left[\left(T_b - P - Q\ T_f\right)\left(h_b + H_K K_1\right) - E1\ K_1 \right]$$

$$\frac{\partial f}{\partial K_1} = - \frac{1}{\left(h_b + H_K K_1\right)^2} \left[E1\ H_K \right]$$

$$\frac{\partial f}{\partial P} = - \frac{H_K}{h_b + H_K K_1}$$

$$\frac{\partial f}{\partial Q} = - \frac{H_K\ T_f}{h_b + H_K K_1}$$

$$\frac{\partial \underset{\sim}{G}^i}{\partial T_f^i} = \begin{bmatrix} \dfrac{\partial H_K}{\partial T_f} \\ \\ \dfrac{\partial K_1}{\partial T_f} \\ \\ \dfrac{\partial P}{\partial T_f} \\ \\ \dfrac{\partial Q}{\partial T_f} \end{bmatrix}$$

$$\frac{\partial H_K}{\partial T_f} = C_2 \left(\coth \lambda_w \ell - \lambda_w \ell \ \text{cosech}^2 \ \lambda_w \ell \right) \left(\frac{\partial \lambda_w^i}{\partial T_f^i} \right)$$

$$\frac{\partial K_1}{\partial T_f} = \frac{1}{\left(h_w + 4\sigma\varepsilon_w T_{aw}^3 \right)^2} \left[\left(h_w + 4\sigma\varepsilon_w T_{aw}^3 \right) \left(h_w \ \text{sech} \ \lambda_w \ell \right. \right.$$

$$\left. \left. \tanh \lambda_w \ell \right) \ell \ \frac{\partial \lambda_w}{\partial T_f} - h_w \left(1 - \text{sech} \ \lambda_w \ell \right) 6\sigma\varepsilon_w T_{aw}^2 \right]$$

$$\frac{\partial P}{\partial T_f} = \frac{1}{\left(h_w + 4\sigma\varepsilon_w T_{aw}^3\right)^2} \left\{ \left[\left(\text{sech } \lambda_w \ell \ \tanh \lambda_w \ell\right) \ell \frac{\partial \lambda_w}{\partial T_f} \right. \right.$$

$$\left. \left(h_w r_w \frac{V^2}{2c_p} + q_w + 3\sigma\varepsilon_w T_{aw}^4\right) + 6\left(1 - \text{sech } \lambda_w \ell\right) \sigma\varepsilon_w T_{aw}^3 \right]$$

$$\left(h_w + 4\sigma\varepsilon_w T_{aw}^3\right) - \left(1 - \text{sech } \lambda_w \ell\right)\left(h_w r_w \frac{V^2}{2c_p} + q_w\right.$$

$$\left. \left. + 3\sigma\varepsilon_w T_{aw}^4\right) 6\sigma\varepsilon_w T_{aw}^2 \right\}$$

$$\frac{\partial Q}{\partial T_f} = - \ell \text{ sech } \lambda_w \ell \ \tanh \lambda_w \ell \left(\frac{\partial \lambda_w}{\partial T_f}\right)$$

where

$$\frac{\partial \lambda_w}{\partial T_f} = \frac{12\sigma\varepsilon_w T_{aw}^2}{(kd)_w \lambda_w}$$

C. $\frac{\partial g^i}{\partial T_f^{i-1}}$, $\frac{\partial g^i}{\partial T_{air}^{i-1}}$, and $\frac{\partial g^i}{\partial p_\ell^{i-1}}$ are computed as follows, this case omitting superscript $i - 1$ in the right-hand members.

$$\frac{\partial g^i}{\partial T_f^{i-1}} = 1 - \frac{2\Delta t}{(\rho c D)_f} h_f - \frac{8\Delta t \sigma \varepsilon_f}{(\rho c D)_f} T_f^3$$

$$\frac{\partial g^i}{\partial T_{air}^{i-1}} = 2h_f \frac{\Delta t}{(\rho c D)_f}$$

$$\frac{\partial g^i}{\partial h_f^{i-1}} = \frac{2\Delta t}{(\rho c D)_f} \left(r_f \frac{V^2}{2c_p} - T_f + T_{air} \right)$$

$$\frac{\partial g^i}{\partial r_f^{i-1}} = \frac{2\Delta t}{(\rho c D)_f} h \frac{V^2}{2c_p}$$

$$\frac{\partial g^i}{\partial V} = \frac{2\Delta t}{(\rho c D)_f} h_f r_f \frac{V}{c_p}$$

$$\frac{\partial g^i}{\partial (\rho c)_f} = - \frac{E3}{(\rho c)_f^2 D_f}$$

$$\frac{\partial g^i}{\partial D_f} = - \frac{E3}{(\rho c)_f D_f^2}$$

where

$$E_3 = -2\Delta t\left(h_f + \sigma\varepsilon_f T_f^3\right) T_f + 2\Delta t\left(q_f + h_f r_f \frac{V^2}{2c_p}\right) + 2\Delta t\ h_f\ T_{air}$$

$$\frac{\partial g^i}{\partial f_{31}} = \left(\frac{\partial g^i}{\partial q_f}\right)\left(\alpha_{31}\ I_1\right)$$

$$\frac{\partial g^i}{\partial f_{32}} = \left(\frac{\partial g^i}{\partial q_f}\right)\left(\alpha_{32}\ I_2\right)$$

$$\frac{\partial g^i}{\partial f_{33}} = \left(\frac{\partial g^i}{\partial q_f}\right)\left(\alpha_{33}\ I_3\right)$$

$$\frac{\partial g^i}{\partial f_{34}} = \left(\frac{\partial g^i}{\partial q_f}\right)\left(\alpha_{34}\ I_f\right)$$

where

$$\frac{\partial g^i}{\partial q_f} = \frac{2\Delta t}{(\rho c D)_f}$$

$$\frac{\partial g^i}{\partial \varepsilon_f} = - \frac{2\Delta t}{\rho c \ D_f} \sigma \ T_f^4 + \frac{\partial g^i}{\partial q_f} \left(f_{33} \ I_3 + f_{34} \ I_4 \right)$$

$$\frac{\partial g^i}{\partial I_1} = \frac{\partial g^i}{\partial q_f} f_{31} \ \alpha_{31}$$

$$\frac{\partial g^i}{\partial I_2} = \frac{\partial g^i}{\partial q_f} f_{32} \ \alpha_{32}$$

$$\frac{\partial g^i}{\partial I_3} = \frac{\partial g^i}{\partial q_f} f_{33} \ \alpha_{33}$$

$$\frac{\partial g^i}{\partial I_4} = \frac{\partial g^i}{\partial q_f} f_{34} \ \alpha_{34}$$

$$\frac{\partial g^i}{\partial \alpha_{31}} = \frac{\partial g^i}{\partial q_f} \left(f_{31} \ I_1 + f_{32} \ I_2 \right)$$

APPENDIX C

SENSITIVITY OF TIME INVARIANT PARAMETERS

The computation of the sensitivity coefficient for constant parameters is a special case of the computation for time variant parameters.

Suppose the system is described as

$$X^{i+1} = f^i\left(X^i, p^i\right)$$

and if $p^i \neq p^j$ when $i \neq j$, then

$$\frac{\partial X^{i+1}}{\partial p^i} = \frac{\partial f^i}{\partial p^i}$$

$$\frac{\partial X^{i+1}}{\partial p^{i-1}} = \frac{\partial f^i}{\partial X^i} \frac{\partial X^i}{\partial p^{i-1}}$$

$$\frac{\partial X^{i+1}}{\partial p^{i-2}} = \frac{\partial f^i}{\partial X^i} \frac{\partial X^i}{\partial X^{i-1}} \frac{\partial X^{i-1}}{\partial p^{i-2}}$$

$$\vdots \qquad\qquad \vdots$$

$$\frac{\partial X^{i+1}}{\partial p^0} = \frac{\partial f^i}{\partial X^i} \frac{\partial X^i}{\partial X^{i-1}} \frac{\partial X^{i-1}}{\partial X^{i-2}} \cdots \frac{\partial X^1}{\partial p^0}$$

Now if $p^i = p^j$ for $i \neq j$, then p^i and p^j are completely correlated, then

$$\sigma_{T_{air}^{i+1}} = \sum_{j=0}^{i} \frac{\partial X^{i+1}}{\partial p^j} \sigma_{p^j}$$

Assuming $\sigma_{p^j} = \sigma_p$

$$\sigma_{T_{air}^{i+1}} = \sum_{j=0}^{i} \frac{\partial X^{i+1}}{\partial p^j} \sigma_p$$

$$= \left(\frac{\partial f^i}{\partial p^i} + \frac{\partial f^i}{\partial X^i} \frac{\partial X^i}{\partial p^{i-1}} \right.$$

$$+ \frac{\partial f^i}{\partial X^i} \frac{\partial X^i}{\partial X^{i-1}} \frac{\partial X^{i-1}}{\partial p^{i-2}} + \frac{\partial f^i}{\partial X^i} \frac{\partial X^i}{\partial X^{i-1}} \frac{\partial X^{i-1}}{\partial X^{i-2}} \frac{\partial X^{i-2}}{\partial p^{i-3}}$$

$$\left. + \frac{\partial f^i}{\partial X^i} \frac{\partial X^i}{\partial X^{i-1}} \frac{\partial X^{i-1}}{\partial X^{i-2}} \cdots \frac{\partial X^2}{\partial X^1} \frac{\partial X^1}{\partial p} \right) \sigma_p$$

$$= \left(\frac{\partial f^i}{\partial p^i} + \frac{\partial f^i}{\partial X^i} \left\{ \frac{\partial f^{i-1}}{\partial p^{i-1}} + \frac{\partial f^{i-1}}{\partial X^{i-1}} \left[\frac{\partial X^{i-1}}{\partial p^{i-2}} + \frac{\partial X^{i-1}}{\partial X^{i-2}} \frac{\partial X^{i-2}}{\partial p^{i-3}} \right. \right. \right.$$

$$\left. \left. \left. + \cdots + \frac{\partial X^{i-1}}{\partial X^{i-2}} \cdots \frac{\partial X^2}{\partial X^1} \frac{\partial X^1}{\partial p} \right] \right\} \right) \sigma_p$$

$$= \left(\frac{\partial f^i}{\partial p} + \frac{\partial f^i}{\partial X^i} \left\{ \frac{\partial f^{i-1}}{\partial p} + \frac{\partial f^{i-1}}{\partial X^{i-1}} \left[\frac{\partial X^{i-1}}{\partial p} + \frac{\partial X^{i-1}}{\partial X^{i-2}} \frac{\partial X^{i-2}}{\partial p} \right. \right. \right.$$

$$\left. \left. \left. + \cdot \cdot \cdot + \frac{\partial X^{i-1}}{\partial X^{i-2}} \cdot \cdot \cdot \frac{\partial X^2}{\partial X^1} \frac{\partial X^1}{\partial p} \right] \right\} \right) \sigma_p$$

The coefficient of σ_p is the solution of the following difference equation:

$$\frac{\partial X^{i+1}}{\partial p} = \frac{\partial f^i}{\partial p} + \frac{\partial f^i}{\partial X^i} \frac{\partial X^i}{\partial p}$$

Therefore

$$\sigma_{T_{air}^{i+1}} = \frac{\partial X^{i+1}}{\partial p} \sigma_p = \sum_{j=0}^{i} \frac{\partial X^{i+1}}{\partial p^j} \sigma_p$$

APPENDIX D

RADIATION HEAT TRANSFER

The radiative heat input power to the sensor is given by the general expression,

$$Aq_R = \int_A \int_\Omega \int_\lambda \frac{\cos\,\theta}{\pi} \alpha_\lambda \; \varepsilon_\lambda \; E_{b\lambda}(T) \; d\lambda \; d\Omega \; dA \qquad \text{(D.1)}$$

where

α_λ = spectral absorptivity of the body

ε_λ = spectral emissivity of the source in $d\Omega$

$E_{b\lambda}(T)$ = plank radiant energy spectral distribution function for the source in $d\Omega$ at temperature T

λ = solid angle subtended by the environment

A = total sensor surface area

θ = angle between sensor surface element dA and the direction toward $d\Omega$

λ = radiation wavelength

Consider the four principal environmental radiation sources seen by the sensor:

j = 1 sun

j = 2 earth and atmosphere as a source of reflected solar radiation

$j = 3$ earth and atmosphere as a long wave source

$j = 4$ sonde parts (including shield) in view of the sensor

Assuming the radiant emittance

$$I_j = \int_0^\infty \varepsilon_{\lambda j} \, E_{b\lambda}\left(T_j\right) d\lambda \tag{D.2}$$

and the mean absorptivity

$$\bar{\alpha}_j = \frac{\int_0^\infty \alpha_{\lambda j} \, \varepsilon_{\lambda j} \, E_{b\lambda}\left(T_j\right) \, d\lambda}{I_j}$$

are independent of the angle (taken as an appropriate mean value, if necessary and practical, for this assumption), then the geometric factor, f_j

$$f_j = \frac{1}{A} \int_A \int_{\Omega_j} \cos\theta \, dA \, \frac{d\Omega_j}{\pi} \tag{D.3}$$

may be calculated separately and treated as a multiplicative factor, and the radiation input term takes the form

$$q_R = \sum_j \bar{\alpha}_j f_j I_j \qquad \text{(D.4)}$$

Geometric Factor f_j

A. $f_{1,1}$, $f_{2,1}$, $f_{3,1}$ (geometric factor with respect to the sun). The solid angle subtended by the sun is $\pi R_s^2/D_{es}$, where R_s is the radius of the sun and D_{es} is the distance between the earth and the sun. Then, referring to Eq. D.3

$$f_{i1} = \frac{R_s^2}{D_{es} A} \int_A \cos\theta \, dA$$

The computed value of $f_{i,1}$ is listed in Table 3.

B. $f_{1,4}$, $f_{2,4}$, $f_{3,4}$ (geometric factor with respect to the sonde). Figure D.1 shows $f_4(\theta_0)$ for the three shapes when the sonde surfaces occupy a "polar cap" with half-angle θ_0 as shown in the figure [14, Staffanson 1969]. The curves are given by

$$f_4 = 0.5\left(1 - \cos\theta_0\right)$$

$$= \left(\theta_0 - 0.5 \sin 2\theta_0\right)/\pi$$

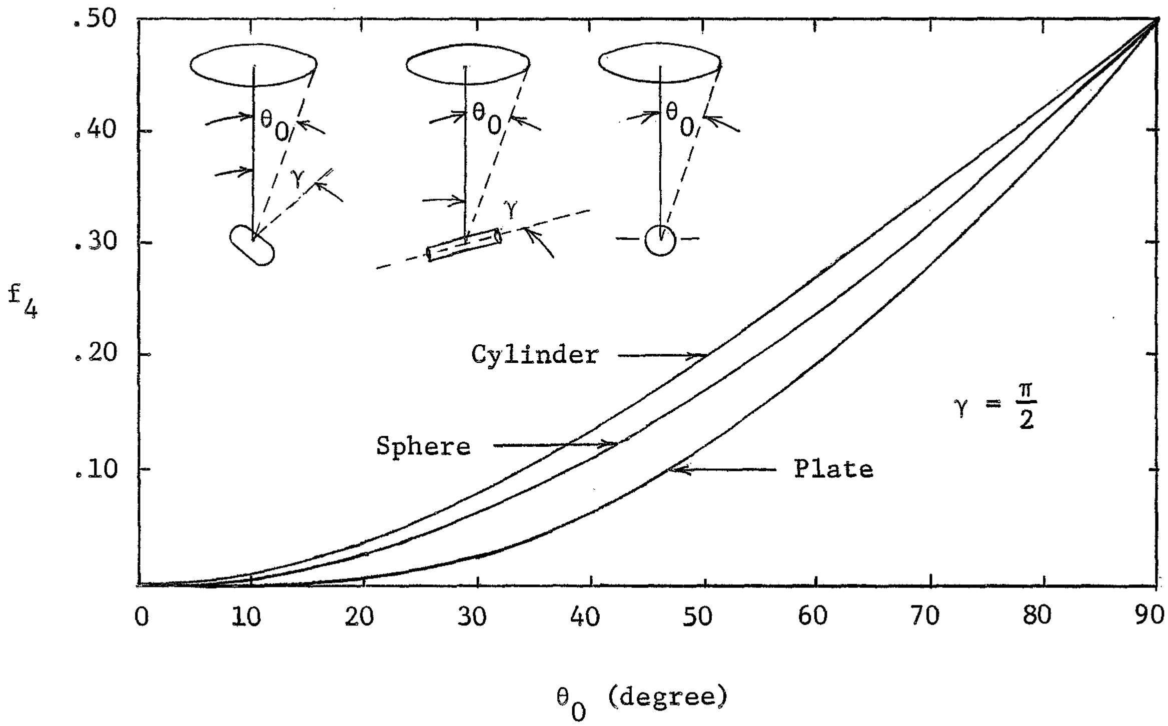

Fig. D.1. Geometric factor for the three sensor shapes of a circular region located 90 degrees from the sensor axis subtending half-angle θ_0.

for the sphere and plate, respectively, and by numerical tables from Ballinger [1, 1960]. A half angle of 35° approximates that for the ARCASONDE 1A.

C. $f_{1,3}$, $f_{2,3}$, $f_{3,3}$ (geometric factor for thermal radiation from the earth). The geometric factors associated with the earth long-wave radiation are computed by the method used for $f_{i,4}$. Nominal value of $f_{i,4}$ is computed, based on $\gamma = \frac{\pi}{2}$ and the uncertainty is due to the fact that the parachute might have a coning motion which varies γ.

D. $f_{1,2}$, $f_{2,2}$, $f_{3,2}$ (geometric factor with respect to earth albedo). Unlike the thermal geometric factor, the albedo geometric factor is dependent on the position of the sun. The values presented in Table 3 are based on the assumption that the sun is high enough to illuminate essentially all of the earth under the sensor.

Radiant Emittance I_j

Values for the radiant emittance, I_j, together with an estimated uncertainty for each source, j, are as follows:

A. $$I_1 = \left(\frac{r_{es}}{r_s}\right)^2 S = 6.456 \times 10^7 \text{ watts/m}^2 \pm 1\%$$

In the calculation of I_1, r_{es} is the mean earth-to-sun distance 9.29×10^7 mi, r_s is the radius of the sun 4.32×10^5 mi, and S is the solar constant,

$$S = \int_0^\infty \varepsilon_\lambda E_{b\lambda} \, d\lambda = 1396 \text{ watts/m}^2 \pm 1\% \quad [8, \text{Johnson } 1954]$$

B. $I_2 = aS = 460.7 \text{ watts/m}^2 \pm 36\%$

Here, a is the albedo of the earth. It has been estimated that clouds can reflect back to space 50 percent or more of the solar flux and absorb another 20 percent. The portions of the earth covered with water reflect about 5 percent of the total radiation reaching them, and the land masses, on the average, reflect slightly more [12, Snoddy 1965].

Therefore, cloud cover becomes a very important factor in determining the earth's albedo. P. F. Clapp [3, 1962] presents cloud cover data using Tiro's nephanalysis, showing cloud cover for various seasons at different latitudes on the earth. By averaging cloudiness for the four seasons, assuming values of reflectance of clouds and the surface of the earth, curves of albedo versus latitude were obtained. From this information it was possible to make some estimation of the effect of cloud cover on albedo. Assuming that clouds reflect 50 percent and the surface of the earth reflects 5 percent, the average albedo is about 0.33, with a variability of ± 0.12 or 36 percent.

C. $I_3 = 233.8$ watts/m^2 ± 20%

The earth's long-wave emittance depends on the surface temperature and it's emission characteristics. Neglecting details of the planet surface, it is possible to compute the average energy radiated by a planet using a thermal balance based on the solar radiation absorbed by the planet.

As the temperature of most planets do not vary appreciably over extended periods, it can be concluded that the thermally radiated energy is equivalent to the absorbed solar energy.

Using S as the solar heat flux per unit projected area of the planet (as seen from the sun), a as the planetary albedo, R as the planet radius, and I as the thermal energy radiated per average unit planet area and time, the energy balance is

$$(1 - a)\ S\pi R^2 = f\pi R^2 I$$

or

$$I = \frac{1 - a}{4} S$$

I_3 for the earth computed by this method equals 233.8 watts/m^2 ± 20%, using 36 percent variability in a. Actual measurements of earth long-wave radiation have been made by Tiros II (1960) and Tiros IV (1966). Bandeen [2, 1961] analyzed the Tiros II data, and the results fall within this 20 percent uncertainty.

D. I_4 = 458 watts/m^2 ± 15, ± 3% (shield)

The radiant emittance of the sonde can be found by using the relation $I_4 = \sigma\varepsilon T_4^4$. T_4 is assumed to be 300°K and

$\varepsilon_4 = 1.0$. Small f_4 renders this magnitude essentially insignificant. In the case of the shield, however, f_4 is much larger, but T_4 is assumed measured to within 2°K, and the shield interior is assumed black, both by its coating and by the effect of reflections within its concave interior surfaces.

APPENDIX E

SIMULATION PROGRAM

Fortran V was used to program the simulation study discussed in Chapter IV. The organization of the programming is summarized in the flow diagram, Fig. E.1.

In the main program the environmental conditions are established, and thermal properties of the sensors are assigned. Initial conditions and uncertainties of each parameter are also stated in the main program.

Subroutine TRAJ generates the motion of the parachute with given initial conditions and parachute dimensions. Subroutine ATMO is called from TRAJ to find the necessary atmosphere conditions at a given altitude. Computation will be terminated when the parachute reaches to a lower limit in altitude.

Subroutine SIMULA is called and the temperature of the sensor is computed. Subroutine HANDR is called from SIMULA to calculate necessary values of h and r.

HANDR subprogram calls ATMO for necessary atmosphere conditions. INTRE and INTKN are called from HANDR to calculate uncertainties in h and r.

After SIMULA computes T_b, subroutine REDUCT is called and $\tilde{T}_{air}$ is computed. The uncertainty boundary of $\tilde{T}_{air}$ is also computed in REDUCT.

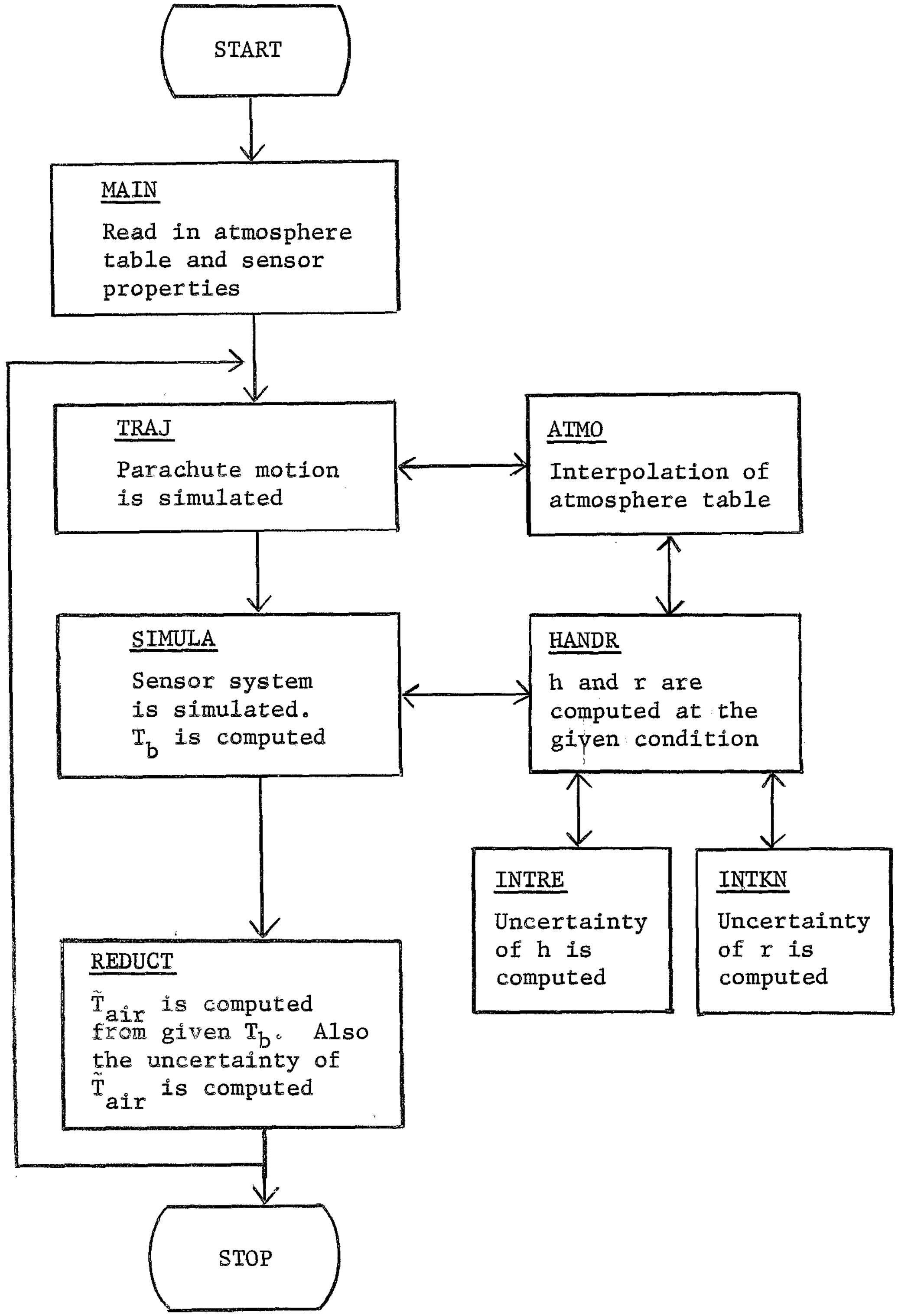

Fig. E.1. Block diagram.

MAIN PROGRAM

```
C.....INPUT QUANTITIES ARE READ IN
C.....LIST OF SYMBOLS
C      ZSR,ZRB,ZRW          OPPENHEIMS TABLE
C      BESS                 BESSEL FUNCTION
C      IC,IS,IT,IF          FLOW REGIME EXPRESSION
C      ALPHA,GEOMF          ABSORPTIVITY,GEOMETRIC FACTOR
C      D,RHOC,EPS           DIAMETER,DENSITY*SPECIFIC HEAT,EMISSITIVITY
C      W,ZKD                SELF HEATING,CONDUCTIVITY*DIAMETER
C      ZL,BETA              WIRE LENGTH,PERTURBATION FACTOR
C      CLB,CLW,CLF          CHARACTERISTIC LENGTH OF BEAD,WIRE,FILM
C      Z0,VV0,VH0           INITIAL ALTITUDE,VERTICAL AND HORIZONTAL SPEED
C      AREA,MASS            AREA AND MASS OF PARACHUTE
C      TB,TBD,TF            TEMPERATURE OF BEAD,TIME DERIVETIVE,FILM
C      JACK,JAZZ,JOEY       COUNTER
       COMMON /ATMOS/ ALT(250), TENV(250), R(250), CS(250), ZMU(250)
       COMMON /ATMOS/ ZKK(250), ZMFP(250)
       COMMON /WIND/ WX(250), WZ(250)
       COMMON /RFAC/ ZSR(15), ZRB(15), ZRW(15)
       COMMON /CLEN/ CLB, CLW, CLF
       COMMON /REG/ IC, IS, IT, IF
       COMMON /PP/ Z0, VV0, VH0, AREA, WMASS
       COMMON /DAC/ ALPHA(3,4),GEOMF(3,4),FLUX(4),BETA(4)
       COMMON /DIC/ EPS(3),RHOC(3),D(3),W(3),ZKD(3)
       COMMON /DOC/ R0,ZL,BESS(50)
       COMMON/MM/ MMP
       COMMON/EIK/ TB,TBD,JACK,JOEY,TP,JAZZ
       COMMON /UUU/ UNC(39),TF
     1 FORMAT (4A1)
     2 FORMAT (3F10.3)
     3 FORMAT (7E10.4)
     4 FORMAT (2E15.5)
     5 FORMAT (3E15.5)
     6 FORMAT (4E15.5)
     7 FORMAT (5E15.5)
C.....WIND STRUCTURE IS GIVEN
       DO 10 I=1,131
       WZ(I)=0.
       WX(I)=0.
    10 CONTINUE
C.....READ ATMOSPHERE TABLE AND THE SENSOR CHARACTERISTICS
       READ 3, (ALT(I),TENV(I),R(I),CS(I),ZMU(I),ZKK(I),ZMFP(I), I=1,131)
       READ 2, (ZSR(I), ZRB(I), ZRW(I), I=1,13)
       READ 6, (BESS(I), I=1,32)
       READ 1, IC, IS, IT, IF
       READ 6, ((ALPHA(I,J), J=1,4), I=1,3)
       READ 6, ((GEOMF(I,J), J=1,4), I=1,3)
       READ 6, (FLUX(J), J=1,4)
       READ 5, (D(I), I=1,3)
       READ 5, (RHOC(I), I=1,3)
       READ 5, (EPS(I), I=1,3)
       READ 5, (W(I), I=1,3)
       READ 5, (ZKD(I), I=1,3)
       READ 4, R0, ZL
       MMP=1
       READ 6, (BETA(J), J=1,4)
       READ 5, CLB, CLW, CLF
       READ 7, Z0, VV0, VH0, AREA, WMASS
       PRINT 100
   100 FORMAT (1H1,31HSONDE CONFIGURATION PARAMETERS:,/)
       DO 101 I=1,3
   101 PRINT 102, I, (ALPHA(I,J), J=1,4)
   102 FORMAT (1H ,6HALPHA(,I1,4H,J):,9X,4E15.5)
       DO 103 I=1,3
   103 PRINT 104, I, (GEOMF(I,J), J=1,4)
   104 FORMAT (1H ,2HF(,I1,4H,J):,13X,4E15.5)
       PRINT 105, (FLUX(J), J=1,4)
   105 FORMAT (1H ,5HI(J):,15X,4E15.5)
       PRINT 106, (BETA(J), J=1,4)
   106 FORMAT (1H ,8HBETA(J):,12X,4E15.5)
       PRINT 107, (D(I), I=1,3)
   107 FORMAT (1H ,5HD(I):,15X,3E15.5)
C.....PRINT OUT OF INPUT QUANTITIES
       PRINT 108, (RHOC(I), I=1,3)
   108 FORMAT (1H ,9HRHO-C(I):,11X,3E15.5)
       PRINT 109, (EPS(I), I=1,3)
   109 FORMAT (1H ,11HEPSILON(I):,9X,3E15.5)
       PRINT 110, (W(I), I=1,3)
   110 FORMAT (1H ,5HW(I):,15X,3E15.5)
       PRINT 111, (ZKD(I), I=1,3)
   111 FORMAT (1H ,6HKD(I):,14X,3E15.5)
       PRINT 112, R0
   112 FORMAT (1H ,3HR0:,17X,E15.5)
       PRINT 113, ZL
   113 FORMAT (1H ,12HWIRE LENGTH:,8X,E15.5)
C.....W/A IS COMPUTED
       PAI=3.1416
       W(1)=W(1)/(PAI*D(1)**2)
C.....UNCERTAINTIES FOR EACH PARAMETERS
       DATA (UNC(I),I=7,16)/.025,0.0,.05,.5,.075,.025,.5,.075,.05,.5/
       DATA (UNC(I),I=17,25)/.075,.05,.075,.125,.05,.25,.05,.025,.05/
       DATA (UNC(I),I=26,34)/.025,.25,.05,.035,.015,.015,.015,.20,.20/
       DATA (UNC(I),I=35,39)/.25,.005,.18,.10,.020/
C.....INITIAL CONDITIONS
       TB=300.
       TBD=0.0
       TF=300.
       JACK=0
       JAZZ=100
       JOEY=0
       CALL TRAJ
       STOP
       END
```

SUBROUTINE TRAJ

```
      SUBROUTINE TRAJ
C.....THE PARACHUTE MOTION IS SIMULATED
C.....LIST OF SYMBOLS
C     T,H                 TIME,TIME INCREMENT
C     VEL,ZOT             VELOCITY,ALTITUDE
      DIMENSION  ZK(4),VVK(4),XK(4),VHK(4),XXX(1,500),YYY(1,500)
      COMMON /PP/ Z0, VV0, VH0, AREA, WMASS
      COMMON /AP/ ZOT, TINF, RHO, VS, AMU, AK, AMPP, VEL
      COMMON /W/ VWV, HWV
      COMMON/MM/ MMP
      COMMON/EIK/ TB,TBD,JACK,JOEY,TP,JAZZ
      PRINT 1, AREA, WMASS
    1 FORMAT (1H1,50HFOR A PARACHUTE SYSTEM WITH CROSS SECTIONAL AREA =,
     1F6.2,27H METERS SQUARED, AND MASS =,F5.2,10H KILOGRAMS)
      PRINT 2, Z0
    2 FORMAT (1H ,5X,35HAND INITIAL CONDITIONS:  ALTITUDE =,F9.2,
     17H METERS)
      PRINT 3, VV0, VH0
    3 FORMAT (1H ,30X,20HVERTICAL VELOCITY = ,F7.2,14H METERS/SECOND,
     124H, HORIZONTAL VELOCITY = ,F7.2,14H METERS/SECOND)
      PRINT 4
    4 FORMAT (1H ,5X,26HAND A STANDARD ATMOSPHERE:)
      PRINT 5
    5 FORMAT (1H ,5X,24HAND A ZERO WIND PROFILE:)
      N=0
      A=200.
      T=0.
      X0=0.
      V=SQRT(VV0**2+VH0**2)
      H=1.
      INKA=0
C.....ALTITUDE
   10 ZOT=Z0
      TZOT=TZ0
      INKA=INKA+1
      VVOT=VV0
      XOT=X0
      VHOT=VH0
      DO 20 J=1,4
      IATE=0
C.....ATMOSPHERIC CONDITION IS FOUND AT THIS ALTITUDE
      CALL ATMO (IATE)
      IF (IATE .EQ. 1) GO TO 21
C.....RELATIVE WIND SPEED
      VZ=VVOT-VWV
      VX=VHOT-HWV
      TVEL=VEL
C.....TOTAL VELOCITY IS FOUND
      VEL=SQRT(VZ**2+VX**2)
      C=(AREA/WMASS)*(1.-ZOT*(5.33E-6))
      G=9.80665-ZOT*(3.04E-6)
      IF (J .NE. 1) GO TO 17
      G=0.5*RHO*VEL**2
      XXX(1,INKA)=VEL
      Z00T=ZOT/1000.
      YYY(1,INKA)=Z00T
   17 ZK(J)=H*VVOT
      CALL SIMULA
      VVK(J)=H*(-0.5*C*RHO*VEL*VZ-G)
      XK(J)=H*VHOT
      VHK(J)=H*(-0.5*C*RHO*VEL*VX)
      IF (J .NE. 1) GO TO 18
      ZOT=Z0+0.5*ZK(1)
      VVOT=VV0+0.5*VVK(1)
      XOT=X0+0.5*XK(1)
      VHOT=VH0+0.5*VHK(1)
      GO TO 20
   18 IF (J .NE. 2) GO TO 19
      ZOT=Z0+0.5*ZK(2)
      VVOT=VV0+0.5*VVK(2)
      XOT=X0+0.5*XK(2)
      VHOT=VH0+0.5*VHK(2)
      GO TO 20
   19 IF (J .NE. 3) GO TO 20
      ZOT=Z0+ZK(3)
      VVOT=VV0+VVK(3)
      XOT=X0+XK(3)
      VHOT=VH0+VHK(3)
   20 CONTINUE
      TZ0=Z0
C.....NEXT ALTITUDE
      Z0=Z0+(ZK(1)+2.*ZK(2)+2.*ZK(3)+ZK(4))/6.
      VV0=VV0+(VVK(1)+2.*VVK(2)+2.*VVK(3)+VVK(4))/6.
      X0=X0+(XK(1)+2.*XK(2)+2.*XK(3)+XK(4))/6.
      VH0=VH0+(VHK(1)+2.*VHK(2)+2.*VHK(3)+VHK(4))/6.
      IF (ABS(VH0) .LT. 1.E-10) VH0=0.
      V=SQRT(VV0**2+VH0**2)
      IF (V .GT. 100.) N=1
      T=T+H
      IF (N .EQ. 0) H=1.
      IF (N .EQ. 0) GO TO 10
      H=A/V
C.....COMPUTATION IS TERMINATED AT 50 KM
      IF(ZOT.LT.50000.) GO TO 23
      IF(INKA.GT.1000) GO TO 23
      GO TO 10
   21 PRINT 22
   22 FORMAT (1H0,25HATMOSPHERE TABLE EXCEEDED)
   23 CONTINUE
      RETURN
      END
```

SUBROUTINE ATMO

```
      SUBROUTINE ATMO (IATE)
C.....LINEAR INTERPOLATION OF THE ATMOSPHERE TABLE IS DONE
      COMMON /ATMOS/ ALT(250), TENV(250), R(250), CS(250), ZMU(250)
      COMMON /ATMOS/ ZKK(250), ZMFP(250)
      COMMON /WIND/ WX(250), WZ(250)
      COMMON /AP/ ZOT, TINF, RHO, VS, AMU, AK, AMFP, VEL
      COMMON /W/ VWV, HWV
      COMMON/MM/ MMP
      IF (ZOT .LT. ALT(1) .OR. ZOT .GT. ALT(131)) GO TO 2
      DO 1 I=1,131
      DZ=ALT(I)-ZOT
      IF (DZ .LT. 0.) GO TO 1
      ERR=DZ/(ALT(I)-ALT(I-1))
      DIFFTE=TENV(I)-TENV(I-1)
      DIFFR=R(I)-R(I-1)
      DIFFCS=CS(I)-CS(I-1)
      DIFFMU=ZMU(I)-ZMU(I-1)
      DIFFK=ZKK(I)-ZKK(I-1)
      DIFFMP=ZMFP(I)-ZMFP(I-1)
      DIFFWZ=WZ(I)-WZ(I-1)
      DIFFWX=WX(I)-WX(I-1)
      CORRTE=ERR*DIFFTE
      CORRR=ERR*DIFFR
      CORRCS=ERR*DIFFCS
      CORRMU=ERR*DIFFMU
      CORRK=ERR*DIFFK
      CORRMP=ERR*DIFFMP
      CORRWZ=ERR*DIFFWZ
      CORRWX=ERR*DIFFWX
      TINF=TENV(I)-CORRTE
      RHO=R(I)-CORRR
      VS=CS(I)-CORRCS
      AMU=ZMU(I)-CORRMU
      AK=(ZKK(I)-CORRK)*4185.
      AMFP=ZMFP(I)-CORRMP
      VWV=WZ(I)-CORRWZ
      HWV=WX(I)-CORRWX
      GO TO 3
    1 CONTINUE
    2 IATE=1
    3 CONTINUE
      RETURN
      END
```

SUBROUTINE HANDR

```
      SUBROUTINE HANDR
C.....CONVECTIVE COEFFICINTS AND RECOVERY FACTORS ARE FOUND
C.....LIST OF SYMBOLS
C     AM,SR,PR          MACH NUMBER,SPEED RATIO,PRANDTLE NUMBER
C     REB,REW,REF       REYNOLDS NUMBER OF BEAD,WIRE,FILM
C     AKNB,AKNW,AKNF    KNUDSEN NUMBER OF BEAD WIRE FILM
C     RB,RW,RF          RECOVERY FACTOR OF BEAD,WIRE,FILM
C     HB,HW,HF          CONVECTIVE COEFFICIENT OF BEAD WIRE FILM
C     RBC,RWC,RFC       RECOVERY FACTOR IN CONTINUUM FLOW
C     RBF,RWF,RFF      RECOVERY FACTOR IN FREE MOLECULAR FLOW
C     HBC,HWC,HFC       CONVECTIVE COEFFICIENT IN CONTINUUM FLOW
C     HBF,HWF,HFF       CONVECTIVE COEFFICIENT IN FREE MOLECULAR FLOW
C     UHB,UHW,UHF       UNCERTAINTIES OF HB,HW,HF
C     URB,URW,URF       UNCERTAINTIES OF RB,RW,RF
      COMMON /AP/ ZOT, TINF, RHO, VS, AMU, AK, AMFP, VEL
      COMMON /RFAC/ ZSR(15), ZRB(15), ZRW(15)
      COMMON /CLEN/ CLB, CLW, CLF
      COMMON /REG/ IC, IS, IT, IF
      COMMON /DIC/ EPS(3),RHOC(3),D(3),W(3),ZKD(3)
      COMMON /UUU/ UNC(39),TF
      COMMON /HR/ H(3),RECF(3)
      COMMON/MM/ MMP
      COMMON/EIK/ TB,TBD,JACK,JOEY,TP,JAZZ
      DATA CP/1000./
      DATA ACB, ACW, ACF/0.9, 0.9, 0.9/
C.....COMPUTATION FOR SPHERE
C.....FREE MOLECULAR FLOW RECOVERY FACTOR
      AM=VEL/VS
      SR=0.837*AM
      PR=CP*AMU/AK
      ARB=1.167
      ARW=1.167
      IF (SR .GT. 10.) GO TO 2
      DO 1 L=1,13
      DSR=ZSR(L)-SR
      IF (DSR .LT. 0.) GO TO 1
      ERRR=DSR/(ZSR(L)-ZSR(L-1))
      DIFFRB=ZRB(L)-ZRB(L-1)
      DIFFRW=ZRW(L)-ZRW(L-1)
      CORRRB=ERRR*DIFFRB
      CORRRW=ERRR*DIFFRW
      ARB=ZRB(L)-CORRRB
      ARW=ZRW(L)-CORRRW
      GO TO 2
    1 CONTINUE
    2 REB=RHO*VEL*CLB/AMU
      AMRB=AM/REB
      AMSRB=AM/SQRT(REB)
C.....FLOW REGIME SEARCH
      IRB=IC
      IF (REB .LE. 1. .AND. AMRB .GT. 0.01) IRB=IS
      IF (REB .LE. 1. .AND. AMRB .GT. 0.1) IRB=IT
      IF (REB .GT. 1. .AND. AMSRB .GT. 0.01) IRB=IS
      IF (REB .GT. 1. .AND. AMSRB .GT. 0.1) IRB=IT
      IF (AMRB .GT. 3.) IRB=IF
      AKNB=AMFP/CLB
C.....RECOVERY FACTOR
      RB=0.845+(AKNB*(ARB-0.845))/(AKNB+0.3)
C.....FREE MOLECULAR FLOW CONVECTIVE COEFFICIENT
      FHB=289.*ACB*VS*RHO
      IF (SR .LT. 1.E-8) GO TO 3
      T=1./(1.+0.3275911*SR)
      ERFA=0.254829592*T-0.284496736*T**2+1.421413741*T**3
      ERFB=-1.453152027*T**4+1.061405429*T**5
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```
      ERF=1.-(ERFA+ERFB)*EXP(-SR**2)
      FHB=0.44311*FHB*((2.*SR+1./SR)*ERF+1.12838*EXP(-SR**2))
    3 HB=FHB
      HBF=FHB
C.....CONTINUUM FLOW CONVECTIVE COEFFICIENT
    4 ANUCB=2.+0.37*REB**0.6*PR**0.333
      HB=ANUCB*AK/CLB
      HBC=HB
C.....CONVECTIVE COEFFICIENT
      HB=HBC*HBF/(HBC+HBF)
      DKN=AKNB
      INU=1
      REYN=REB
C.....UNCERTAINTIES OF CONVECTIVE COEFFICIENT
      CALL INTRE(REYN,UNCH)
C.....UNCERTAINTIES OF RECOVERK FACTOR
      CALL INTKN(DKN,UNCR,INU)
      URB=RB*UNCR
      UHB=HB*UNCH
C.....COMPUTATION FOR CYLINDER
    5 REW=RHO*VEL*CLW/AMU
      AMRW=AM/REW
      AMSRW=AM/SQRT(REW)
      IRW=IC
C.....FLOW REGIME SEARCH
      IF (REW .LE. 1. .AND. AMRW .GT. 0.01) IRW=IS
      IF (REW .LE. 1. .AND. AMRW .GT. 0.1) IRW=IT
      IF (REW .GT. 1. .AND. AMSRW .GT. 0.01) IRW=IS
      IF (REW .GT. 1. .AND. AMSRW .GT. 0.1) IRW=IT
      IF (AMRW .GT. 3.) IRW=IF
      AKNW=AMFP/CLW
C.....RECOVERY FACTOR
      RW=0.845+(AKNW*(ARW-0.845))/(AKNW+0.6)
C.....FREE MOLECULAR FLOW CONVECTIVE COEFFICIENT
      T=0.5*SR**2
      Z=T/3.75
      IF (Z .GT. 1.) PRINT 6
    6 FORMAT(1H ,51HVELOCITY TOO LARGE FOR BESSEL FUNCTION APPROXIMATOR)
      BM0A=1.+3.5156229*Z**2+3.0899424*Z**4+1.2067492*Z**6
      BM0B=0.2659732*Z**8+0.0360768*Z**10+0.0045813*Z**12
      BM0=BM0A+BM0B
      BM1TA=0.5+0.87890594*Z**2+0.51498869*Z**4+0.15084934*Z**6
      BM1TB=0.02658733*Z**8+0.00301532*Z**10+0.00032411*Z**12
      BM1T=BM1TA+BM1TB
      HWA=289.*ACW*VS*RHO*EXP(-T)
      HWB=(1.+2.*T)*BM0+2.*T**2*BM1T
      FHW=HWA*HWB
      HWF=FHW
C.....CONTINUUM FLOW CONVECTIVE COEFFICIENT
    8 IF (REW .GT. 4000.) GO TO 9
      CHW=(AK/CLW)*(0.43+0.48*REW**0.5)
      GO TO 11
    9 IF (REW .GT. 40000.) GO TO 10
      CHW=(AK/CLW)*(0.43+0.174*REW**0.618)
      GO TO 11
   10 CHW=(AK/CLW)*(0.43+0.0239*REW**0.805)
   11 HW=CHW
      HWC=CHW
C.....CONVECTIVE COEFFICIENT
      HW=HWC*HWF/(HWC+HWF)
      INU=2
      DKN=AKNW
      REYN=REW
C.....UNCERTAINTIES OF CONVECTIVE COEFFICIENT
      CALL INTRE(REYN,UNCH)
C.....UNCERTAINTIES OF RECOVERK FACTOR
      CALL INTKN(DKN,UNCR,INU)
      UHW=HW*UNCH
      URW=RW*UNCR
C.....COMPUTATION FOR PLATE
C.....CHARACTERISTIC LENGTH COMPUTATION
      SIG=5.67E-8
      IF(JACK.EQ.0) GO TO 33
      CLF=SQRT(ZKD(3)/(2.*(H(3)+4.*SIG*EPS(3)*TF**3)))
      CLF=CLF*3.
   33 CONTINUE
      NEK=0
   35 CONTINUE
      IF(CLF.GT..03) CLF=.03
   12 REF=RHO*VEL*CLF/AMU
      AMRF=AM/REF
      AMSRF=AM/SQRT(REF)
      IRF=IC
C.....FLOW REGIME SEARCH
      IF (REF .LE. 1. .AND. AMRF .GT. 0.01) IRF=IS
      IF (REF .LE. 1. .AND. AMRF .GT. 0.1) IRF=IT
      IF (REF .GT. 1. .AND. AMSRF .GT. 0.01) IRF=IS
      IF (REF .GT. 1. .AND. AMSRF .GT. 0.1) IRF=IT
      IF (AMRF .GT. 3.) IRF=IF
C.....FREE MOLECULAR FLOW RECOVERY FACTOR
      RFF=1.167
      RF=RFF
C.....FREE MOLECULAR FLOW CONVECTIVE COEFFICIENT
      FHF=289.*ACF*VS*RHO
      HFF=FHF
C.....RECOVERY FACTOR
   13 AKNF=AMFP/CLF
      AKNFT=AKNF*(SQRT(REF)/7.2)
      RFT=.845+(AKNFT*.322)/(AKNFT+0.3)
      RF=RFT
C.....CONTINUUM FLOW CONVECTIVE COEFFICIENT
      HF=(AK/CLF)*.664*(REF**.5)*(PR**.333)
      HFC=HF
C.....CONVECTIVE COEFFICIENT
      HF=HFC*HFF/(HFC+HFF)
      AKNF=AMFP/CLF
      AKNFT=AKNF*(SQRT(REF)/7.2)
      IF(JACK.NE.0) GO TO 34
      IF(CLF.GT..0299) GO TO 34
      CLF1=CLF
      CLF=SQRT(ZKD(3)/(2.*(HF+4.*SIG*EPS(3)*TF**3)))
      CLF=CLF*3.
      IF(ABS(CLF-CLF1).LT..001) GO TO 34
      IF(NEK.GE.15) GO TO 34
      NEK=NEK+1
      GO TO 35
   34 CONTINUE
      INU=1
      REYN=REF
      DKN=AKNFT
C.....UNCERTAINTIES OF CONVECTIVE COEFFICIENT
      CALL INTRE(REYN,UNCH)
C.....UNCERTAINTIES OF RECOVERK FACTOR
      CALL INTKN(DKN,UNCR,INU)
      UHF=HF*UNCH
      URF=RF*UNCR
   50 CONTINUE
      H(1)=HB
      H(2)=HW
      H(3)=HF
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      RECF(1)=RB
      RECF(2)=RW
      RECF(3)=RF
      UNC(1)=UHB/2.
      UNC(2)=URB/2.
      UNC(3)=UHW/2.
      UNC(4)=URW/2.
      UNC(5)=UHF/2.
      UNC(6)=URF/2.
  333 FORMAT(1H ,9E10.3)
      RETURN
      END
```

SUBROUTINE SIMULA

```
      SUBROUTINE SIMULA
C.....SENSOR SYSTEM IS SIMULATED
C.....LIST OF SYMBOLS
C     QRA                  TOTAL RADIATION INPUT
C     TW                   TEMPERATURE OF WIRE
C     DELT                 TIME INCREMENT
      DIMENSION RAD(3,4),QRA(3)
      COMMON /AP/ ZOT, TINF, RHO, VS, AMU, AK, AMFP, VEL
      COMMON /HR/ H(3), RECF(3)
      COMMON /DAC/ ALPHA(3,4),GEOMF(3,4),FLUX(4),BETA(4)
      COMMON /DIC/ EPS(3),RHOC(3),D(3),W(3),ZKD(3)
      COMMON /DOC/ R0,ZL,BESS(50)
      COMMON /UUU/ UNC(39),TF
      COMMON/MM/ MMP
      COMMON /CLEN/ CLB, CLW, CLF
      COMMON/EIK/ TB,TBD,JACK,JOEY,TP,JAZZ
      DATA CP,PI,SIG/1000.,3.1416,5.6687E-8/
      CALL HANDR
      IF(JACK.EQ.0) TF=TB
      IF(JACK.EQ.0) TB1=TB
      TB=TB1
    5 CONTINUE
      TW=(TB+TF)/2.
C.....RADIATION HEAT INPUT
      DO 2 I=1,3
      QRA(I)=0.
      DO 2 J=1,4
      RAD(I,J)=ALPHA(I,J)*GEOMF(I,J)*FLUX(J)*BETA(J)
    2 QRA(I)=RAD(I,J)+QRA(I)
      IF(JACK.NE.JOEY) GO TO 15
      PRINT 209,JACK
      PRINT 110,T,ZOT,TINF,VEL,TF,TB,TBD
      PRINT 115,(H(I),I=1,3),(RECF(I),I=1,3),(QRA(I),I=1,3)
      PRINT 115,(UNC(I),I=1,6),CLF
   15 CONTINUE
      QRA(1)=QRA(1)+W(1)
      DELT=1.0
C.....TIME
      T=T+DELT
C.....CALCULATION OF BEAD TEMPERATURE
C.....PRELIMINARY COMPUTATION 1
      TT4B=4.*SIG*EPS(1)*(TB**3)
      TT4W=4.*SIG*EPS(2)*(TW**3)
      TT3W=3.*SIG*EPS(2)*(TW**4)
      TT3B=3.*SIG*EPS(1)*(TB**4)
      VCP=VEL*VEL/(2.*CP)
      HRVB=H(1)*RECF(1)*VCP
      HRVW=H(2)*RECF(2)*VCP
      HRVF=H(3)*RECF(3)*VCP
C.....PRELIMINARY COMPUTATION2
      C2=(ZKD(2)*D(2))/(2.*D(1)*D(1))
      RAMW=(H(2)+TT4W)/ZKD(2)
      RAMW=RAMW*4.
      RAMW=SQRT(RAMW)
C.....PRELIMINARY COMPUTATION 3 HYPERBOLIC FUNCTION
      EE=EXP(RAMW*ZL)
      EEM=1./EE
      PSECH=2./(EE+EEM)
      PTANH=PSECH*(EE-EEM)/2.
      PCOTH=1./PTANH
      PCOSEH=2./(EE-EEM)
C.....PRELIMINARY COMPUTATION 4
      HK=C2*RAMW*PCOTH
      P=(1.-PSECH)*(HRVW+QRA(2)+TT3W)/(H(2)+TT4W)
      Q=PSECH
      XK1=H(2)*(1.-PSECH)/(H(2)+TT4W)
C.....BEAD TEMPERATURE COMPUTATION
      CAL=6.*DELT/(RHOC(1)*D(1))
      CAT=1.-CAL*(SIG*EPS(1)*(TB**3)+HK+H(1))
      DOG1=(H(1)+HK*XK1)*TINF+HK*Q*TF
      DOG2=HRVB+QRA(1)+HK*P
      DOG=DOG1+DOG2
      TB=CAT*TB+CAL*DOG
C.....CALCULATION OF FILM TEMPERATURE
      COL=2.*DELT/(RHOC(3)*D(3))
      PIG=1.-COL*(H(3)+SIG*EPS(3)*(TF**3))
      WOLF=COL*(H(3)*TINF+HRVF+QRA(3))
      TF=PIG*TF+WOLF
C.....TIME DERIVETIVE OF BEAD
      TBD=(TB-TB1)/DELT
      TP=TB1
      TB1=TB
      CALL REDUCT
  110 FORMAT(1H ,F10.2,E15.6,4F15.6,E15.6)
  115 FORMAT(1H ,10G12.4)
  111 FORMAT(1H ,8F12.5)
  209 FORMAT(/,1H ,5HJACK=,I4)
      RETURN
      END
```

SUBROUTINE REDUCT

```
      SUBROUTINE REDUCT
C.....TEMPERATURE REDUCTION AND UNCERTAINTY BOUNDARY IS FOUND
C.....LIST OF SYMBOLS
C     NG,NP              NUMBER OF TOTAL PARAMETER,TIME VARIANT PARAMETER
C     SDVZ               STANDARD DEVIATION OF EACH PARAMETER
C     S,SSS              SENSITIVITY COEFFICIENT WITH RESPECT TO TB
C     DGPL,SS            SENSITIVITY COEFFICIENT WITH RESPECT TO TF
C     STDV               STANDARD DEVIATION OF REDUCED AIR TEMPERATURE
C     TI                 REDUCED TEMPERATURE OF THE AIR
      DIMENSION RAD(3,4),GRA(3)
      COMMON /AP/ ZOT, TINF, RHO, VS, ANU, AK, AMPP, VEL
      COMMON /HR/ H(3), RECF(3)
      COMMON /DAC/ ALPHA(3,4),GEOMF(3,4),FLUX(4),BETA(4)
      COMMON /DIC/ EPS(3),RHOC(3),D(3),W(3),ZKD(3)
      COMMON /DOC/ R0,ZL,BESS(50)
      COMMON /UUU/ UNC(39),TF
      COMMON/MM/ MMP
      COMMON/EIK/ TB,TBD,JACK,JOEY,TP,JAZZ
      DIMENSION S(40),SI1(40),SS(40,40),SSI1(40,40),SSS(40,40),SSSI1(40,
     140),DSF(40)
      DIMENSION PR(40),SDV(40,40),SDVZ(40),STN(40,40),STNZ(40),STE(40),S
     1T6(40)
      DIMENSION SAV(40),SAU(40),SAV1(40),SAU1(40),SAAA(40),PGR(40)
      DIMENSION STP(40,40),STPP(40)
C.....NUMBER OF PARAMETER IS SET
      JIN=9
      NP=39
      NG=19
      NG1=NG+1
      DATA CP,PI,SIG/1000.,3.1416,5.6687E-8/
      TB=TP
      IF(JACK.EQ.0) TF=TB
      DELT=1.0
C.....TIME
      T=T+DELT
C.....RADIATION INPUT
      DO 2 I=1,3
      GRA(I)=0.
      DO 2 J=1,4
      RAD(I,J)=ALPHA(I,J)*GEOMF(I,J)*FLUX(J)*BETA(J)
    2 GRA(I)=RAD(I,J)+GRA(I)
      GRA(1)=GRA(1)+W(1)
C.....PRODUCTION OF STANDARD DEVIATION
      PR(1)=H(1)
      PR(2)=RECF(1)
      PR(3)=H(2)
      PR(4)=RECF(2)
      PR(5)=H(3)
      PR(6)=RECF(3)
      PR(7)=VEL
      PR(8)=TB
      PR(9)=TBD
      PR(10)=GEOMF(1,1)
      PR(11)=GEOMF(1,2)
      PR(12)=GEOMF(1,3)
      PR(13)=GEOMF(2,1)
      PR(14)=GEOMF(2,2)
      PR(15)=GEOMF(2,3)
      PR(16)=GEOMF(3,1)
      PR(17)=GEOMF(3,2)
      PR(18)=GEOMF(3,3)
      PR(19)=W(1)
      PR(20)=ALPHA(1,3)
      PR(21)=ALPHA(2,3)
      PR(22)=ALPHA(3,3)
      PR(23)=D(1)
      PR(24)=RHOC(1)
      PR(25)=D(3)
      PR(26)=ZKD(2)/D(2)
      PR(27)=ZL
      PR(28)=RHOC(3)
      PR(29)=D(3)
      PR(30)=GEOMF(1,4)
      PR(31)=GEOMF(2,4)
      PR(32)=GEOMF(3,4)
      PR(33)=ALPHA(1,1)
      PR(34)=ALPHA(2,1)
      PR(35)=ALPHA(3,1)
      PR(36)=FLUX(1)
      PR(37)=FLUX(2)
      PR(38)=FLUX(3)
      PR(39)=FLUX(4)
      DO 9 NP1=1,NP
      SDVZ(NP1)=UNC(NP1)*PR(NP1)
    9 CONTINUE
      SDVZ(8)=1.0
      DO 11 I0=1,6
      SDVZ(I0)=UNC(I0)
   11 CONTINUE
   10 CONTINUE
C.....PRELIMINARY COMPUTATION 1
      TW=(TB+TF)/2.
      TT4B=4.*SIG*EPS(1)*(TB**3)
      TT4W=4.*SIG*EPS(2)*(TW**3)
      TT3W=3.*SIG*EPS(2)*(TW**4)
      TT3B=3.*SIG*EPS(1)*(TB**4)
      VCP=VEL*VEL/(2.*CP)
      HRVB=H(1)*RECF(1)*VCP
      HRVW=H(2)*RECF(2)*VCP
      HRVF=H(3)*RECF(3)*VCP
C.....PRELIMINARY COMPUTATION2
      C2=(ZKD(2)*D(2))/(2.*D(1)*D(1))
      RAMW=4.*(H(2)+TT4W)/ZKD(2)
      RAMW=SQRT(RAMW)
C.....PRELIMINARY COMPUTATION 3 HYPERBOLIC FUNCTION
      EE=EXP(RAMW*ZL)
      EEM=1./EE
      PSECH=2./(EE+EEM)
      PTANH=PSECH*(EE-EEM)/2.
      PCOTH=1./PTANH
      PCOSEH=2./(EE-EEM)
C.....PRELIMINARY COMPUTATION 4
      O1=1.-PSECH
      O2=HRVW+GRA(2)+TT3W
      O3=H(2)+TT4W
      O5=PSECH*PTANH
      O6=RAMW*ZL*PCOSEH**2
      OO3=O3**2
C.....PRELIMINARY COMPUTATION 5
      HK=C2*RAMW*PCOTH
      P=(O1*O2)/O3
      Q=PSECH
      XK1=H(2)*O1/O3
      O4=H(1)+HK*XK1
      OO4=O4**2
      TOM2=RHOC(1)*D(1)*TBD/6.
      TOM3=(H(1)+TT4B+HK)*TB
      TOM4=HRVB+TT3B+GRA(1)+HK*P
      TOM5=HK*Q*TF
```

```
      07=TOM2+TOM3-TOM4-TOM5
      08=TOM2+TOM3-TOM4-HK*(TB-P)
      DHKDR=C2*(PCOTH-06)
      DK1DR=H(2)*ZL*05/03
      DPDR=02*ZL*05/03
      DQDR=-ZL*05
      DRDTF=TT3W/(ZKD(2)*RAMW*TW**2)
      DRDTF=DRDTF*4.
      DFDHK=((TB-P-Q*TF)*04-XK1*07)/004
      DFDK1=-07*HK/004
      DFDP=-HK/04
      DFDQ=-HK*TF/04
      DFDQB=-1./04
      DFDQW=-HK*01/(04*03)
C.....FOR STEP IA=0
      IA=0
C.....S1   HB
      S(1)=(04*(TB-HRVB/H(1))-07)/004
C.....S2   RB
      S(2)=-H(1)*VCP/04
C.....S3         HW
      C41=2./(ZKD(2)*RAMW)
      C42=C41*(-C2*06+C2*PCOTH)
      C43=(03*H(2)*ZL*05*C41+TT4W*01)/003
      C44=((01*RECF(2)*VCP+02*ZL*05*C41)*03-01*02)/003
      C45=-ZL*05*C41
      WAC=HK*C43+XK1*C42
      WHITE=-08*WAC/004+(C42*04-HK*WAC)*TB/004
      BLACK=((C42*P+C44*HK)*04-HK*P*WAC)/004
      PINK=((C42*Q+C45*HK)*04-HK*Q*WAC)*TF/004
      S(3)=WHITE-BLACK-PINK
C.....S4  RW
      S(4)=-HK*H(2)*VCP*01/(04*03)
C.....S5,S6,S22 HF,RF,ALFF
      S(5)=0.0
      S(6)=0.0
      S(22)=0.0
C.....S7  VEL
      DPDV=01*H(2)*RECF(2)*VEL/(03*CP)
      S(7) =-(HRVB*2./VEL+HK*DPDV)/04
C.....S8  TB
      TOY=6.*SIG*EPS(2)*TW**2
      DRDT=TT3W/(RAMW*ZKD(2)*TW**2)
      DRDT=DRDT*4.
      DHDT=C2*DRDT*(PCOTH-06)
      DKDT=((03*05*ZL*DRDT-01*TOY)*H(2))/003
      DPDT=((05*ZL*DRDT*02+01*TOY)*03-01*02*TOY)/003
      DQDT=-05*ZL*DRDT
      S(8)= ((H(1)+TT4B+DHDT*(TB-P)+HK-HK*DPDT-TF*(HK*DQDT+Q*DHDT))*04-0
     17*(DHDT*XK1-DKDT*HK))/004
C.....S9  TBD
      S(9)= RHOC(1)*D(1)/(6.*04)
      S(28)=0.0
      S(29)=0.0
C.....S10,S11,S12,S30  F1B,F2B,F3B,F4B
      S(10)=DFDQB*ALPHA(1,1)*FLUX(1)
      S(11)=DFDQB*ALPHA(1,2)*FLUX(2)
      S(12)=DFDQB*ALPHA(1,3)*FLUX(3)
      S(30)=DFDQB*ALPHA(1,4)*FLUX(4)
C.....S13,S14,S15,S31
      S(13)=DFDQW*ALPHA(2,1)*FLUX(1)
      S(14)=DFDQW*ALPHA(2,2)*FLUX(2)
      S(15)=DFDQW*ALPHA(2,3)*FLUX(3)
      S(31)=DFDQW*ALPHA(2,4)*FLUX(4)
C.....S16,S17,S18,S32  F1F,F2F,F3F,F4F
      S(16)=0.0
      S(17)=0.0
      S(18)=0.0
      S(32)=0.0
C.....S19 W(1)
      S(19)=-1./04
C.....S20  ALFB
      SABC=DFDQB*(GEOMF(1,3)*FLUX(3)+GEOMF(1,4)*FLUX(4))+TT3B/((3.*EPS(1
     1))*04)
      S(20)=SABC
C.....S21 ALFW
      DQDEW=GEOMF(2,3)*FLUX(3)+GEOMF(2,4)*FLUX(4)
      DLDEW=TT4W/(2.*EPS(2)*ZKD(2)*RAMW)
      DLDEW=DLDEW*4.
      DKDEW=-H(2)*01*TT4W/(EPS(2)*003)
      DPDEW=01*((03*TT3W/EPS(2))-(02*TT4W/EPS(2)))/003
      A1=DFDHK*DHKDR*DLDEW
      A2=DFDK1*(DKDEW+DK1DR*DLDEW)
      A3=DFDP*(DPDEW+DPDR*DLDEW)
      A4=DFDQ*DQDR*DLDEW
      S(21)=A1+A2+A3+A4
C.....S23 DB
      CT1=-ZKD(2)*D(2)/(D(1)**3)
      CT2=RAMW*PCOTH*CT1
      S(23)=((RHOC(1)*TBD/6.+CT2*(TB-P-Q*TF))*04-XK1*CT2*07)/004
C.....S24 ROCB
      S(24)=D(1)*TBD/(6.*04)
C.....S25 DW
      CE1=ZKD(2)/(D(1)**2)
      CE2=-RAMW/(2.*D(2))
      CE3=(CE1*RAMW+C2*CE2)*PCOTH-C2*CE2*06
      CE4=H(2)*05*ZL*CE2/03
      CE5=CE4*02/H(2)
      CE6=-05*ZL*CE2
      S(25)=(1./004)*((TB*CE3-HK*CE5-P*CE3-TF*(CE3*Q+CE6*HK))*04-07*(CE3
     1*XK1+CE4*HK))
C.....S26 KW
      CE1=(D(2)**2)/(2.*D(1)**2)
      CE2=-(RAMW*D(2))/(2.*ZKD(2))
      CE3=(CE1*RAMW+C2*CE2)*PCOTH-C2*CE2*06
      CE4=H(2)*05*ZL*CE2/03
      CE5=CE4*02/H(2)
      CE6=-05*ZL*CE2
      S(26)=(1./004)*((TB*CE3-HK*CE5-P*CE3-TF*(CE3*Q+CE6*HK))*04-07*(CE3
     1*XK1+CE4*HK))
C.....S27 ZL
      CE3=-C2*06*RAMW/ZL
      CE4=05*H(2)*RAMW/03
      CE5=02*05*RAMW/03
      CE6=-05*RAMW
      S(27)=(1./004)*((TB*CE3-HK*CE5-P*CE3-TF*(CE3*Q+CE6*HK))*04-07*(CE3
     1*XK1+CE4*HK))
C.....S33,S34,S35 ALFSB,ALFSW,ALFSF
      S(33)=DFDQB*(GEOMF(1,1)*FLUX(1)+GEOMF(1,2)*FLUX(2))
      S(34)=DFDQW*(GEOMF(2,1)*FLUX(1)+GEOMF(2,2)*FLUX(2))
      S(35)=0.0
C.....S36,S37,S38 I1,I2,I3,I4
      S(36)=DFDQB*GEOMF(1,1)*ALPHA(1,1)+DFDQW*GEOMF(2,1)*ALPHA(2,1)
      S(37)=DFDQB*GEOMF(1,2)*ALPHA(1,2)+DFDQW*GEOMF(2,2)*ALPHA(2,2)
      S(38)=DFDQB*GEOMF(1,3)*ALPHA(1,3)+DFDQW*GEOMF(2,3)*ALPHA(2,3)
      S(39)=DFDQB*GEOMF(1,4)*ALPHA(1,4)+DFDQW*GEOMF(2,4)*ALPHA(2,4)
      IF(JACK.EQ.0) GO TO 92
C     STANDARD DEVIATION
      DO 72 NP1=1,NQ
      STMZ(NP1)=S(NP1)*SDVZ(NP1)
```

```
   72 CONTINUE
C.....FOR STEP IA=1
      IA=IA+1
      DFDTF=-HK*G/04
      DFDTF=DFDTF+(DFDHK*DHKDR+DFDK1*DK1DR+DFDG*DGDR+DFDP*DPDR)*DRDTF
      COW=2.*DELT/(RHOC(3)*D(3))
      DGPL(5)=COW*(TII1-TFI1+RFI1*(VI1**2)/(2.*CP))
      DGPL(6)=(COW*VI1**2)*HFI1/(2.*CP)
      DGPL(7)= COW*RFI1*VI1*HFI1/CP
      CF=2.*DELT/(RHOC(3)*D(3))
      DGDGF=COW
      DGPL(22)=-CF*SIG*(TFI1**4)+DGDGF*(GEOMF(3,3)*FLUX(3)+GEOMF(3,4)*FL
     1UX(4))
C.....RHOC  DF
      R3R=2.*DELT*(-(HFI1+SIG*EPS(3)*TFI1**3)*TFI1+GRA(3)+HFI1*RFI1*(VI1
     1**2)/(2.*CP)+HFI1*TII1)
      DGPL(28)=-R3R/((RHOC(3)**2)*D(3))
      DGPL(29)=-R3R/(RHOC(3)*(D(3)**2))
C.....F1F F2F F3F F4F
      DGPL(16)=DGDGF*(FLUX(1)*ALPHA(3,1))
      DGPL(17)=DGDGF*(FLUX(2)*ALPHA(3,2))
      DGPL(18)=DGDGF*(FLUX(3)*ALPHA(3,3))
      DGPL(32)=DGDGF*(FLUX(4)*ALPHA(3,4))
C.....ALFSF
      DGPL(35)=DGDGF*(GEOMF(3,1)*FLUX(1)+GEOMF(3,2)*FLUX(2))
C.....I1=I2,I3,I4
      DGPL(36)=DGDGF*GEOMF(3,1)*ALPHA(3,1)
      DGPL(37)=DGDGF*GEOMF(3,2)*ALPHA(3,2)
      DGPL(38)=DGDGF*GEOMF(3,3)*ALPHA(3,3)
      DGPL(39)=DGDGF*GEOMF(3,4)*ALPHA(3,4)
      DGDTIN=HFI1*COW
      DGDTF=1.-COW*HFI1-4.*EPS(3)*SIG*(TFI1**3)*COW
      DO 83 IMO=1,NG
      SS(IMO,IA)=DGPL(IMO)+DGDTIN*SI1(IMO)
      SI1(IMO)=S(IMO)
   83 CONTINUE
      DO 84 IMO=1,NG
      SSS(IMO,IA)=DFDTF*SS(IMO,IA)
   84 CONTINUE
      IF(JACK.EQ.1) GO TO 80
C     STANDARD DEVIATION
      JAY=JACK-IA
      DO 73 NP1=1,NG
      STM(NP1,IA)=SSS(NP1,IA)*SDV(NP1,IA)
   73 CONTINUE
C.....FOR STEP IA=2
      IA=IA+1
      DO 85 IMO=1,NG
      SS(IMO,IA)=DGDTIN*SSSI1(IMO,IA)+DGDTF*SSI1(IMO,IA)
   85 CONTINUE
      DO 86 IMO=1,NG
      SSS(IMO,IA)=DFDTF*SS(IMO,IA)
   86 CONTINUE
      IF(JACK.EQ.2) GO TO 80
C     STANDARD DEVIATION
      JAY=JACK-IA
      DO 74 NP1=1,NG
      STM(NP1,IA)=SSS(NP1,IA)*SDV(NP1,IA)
   74 CONTINUE
C.....FOR STEP IA GREATER OR EQUAL 3
   87 IA=IA+1
      DO 88 IMO=1,NG
      SS(IMO,IA)=DGDTIN*SSSI1(IMO,IA)+DGDTF*SSI1(IMO,IA)
      SSS(IMO,IA)=SS(IMO,IA)*DFDTF
   88 CONTINUE
C     STANDARD DEVIATION
      JAY=JACK-IA
      IF(JAY.EQ.0) GO TO 80
      DO 89 IMO=1,NG
      STM(IMO,IA)=SSS(IMO,IA)*SDV(IMO,IA)
   89 CONTINUE
      IF(IA.GE.JIN) GO TO 69
      IF(JACK.GE.IA) GO TO 87
   80 CONTINUE
C.....COMPUTATION OF OVER ALL STANDARD DEVIATION
      DO 76 NP1=1,NG
      STM(NP1,IA)=SSS(NP1,IA)*SDV(NP1,IA)
   76 CONTINUE
   69 CONTINUE
      DO 41 NP1=NG,NP
      SAV(NP1)=DGPL(NP1)+DGDTF*SAV1(NP1)+DGDTIN*SAU1(NP1)
      SAU(NP1)=S(NP1)+DFDTF*SAV(NP1)
      SAAA(NP1)=SAU(NP1)*SDVZ(NP1)
   41 CONTINUE
      DO 42 NP1=NG,NP
      SAV1(NP1)=SAV(NP1)
      SAU1(NP1)=SAU(NP1)
   42 CONTINUE
C.....COMPUTATION OF STE
      STDV=0.0
      DO 77 IA=1,JIN
      STE(IA)=0.0
   77 CONTINUE
      DO 79 IA=1,JIN
      DO 78 NP1=1,NG
      EEE=STM(NP1,IA)**2
      STP(NP1,IA)=EEE
      STE(IA)=STE(IA)+EEE
   78 CONTINUE
      STG(IA)=SQRT(STE(IA))
      STDV=STDV+STE(IA)
   79 CONTINUE
      DO 44 NP1=NG,NP
      STDV=STDV+SAAA(NP1)**2
   44 CONTINUE
      CORN=0.0
      DO 91 NP1=1,NG
      CORN=CORN+STMZ(NP1)**2
   91 CONTINUE
      CORN=SQRT(CORN)
      STDV=STDV+CORN
      STDV=SQRT(STDV)
      DO 46 NP1=1,NG
      PGR(NP1)=0.0
      DO 45 IA=1,JIN
      PGR(NP1)=STM(NP1,IA)**2+PGR(NP1)
   45 CONTINUE
      PGR(NP1)=PGR(NP1)+STMZ(NP1)**2
      PGR(NP1)=SQRT(PGR(NP1))
   46 CONTINUE
      IF(JACK.NE.JOEY) GO TO 13
      PRINT 527
      PRINT 521,01,02,03,04,05,06,07,08
      PRINT 521,P,G,HK,XK1,DFDTF,DGDTF,DGDTIN
      PRINT 521,(DGPL(I),I=5,7),(DGPL(I),I=16,18),DGPL(22),DGPL(28)
      PRINT 521,DGPL(29),DGPL(32),(DGPL(I),I=35,39)
      PRINT 524
      PRINT 211,(S(IP),IP=1,39)
      PRINT 523
      PRINT 211,CORN,(STE(II),II=1,JIN)
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      PRINT 520
      PRINT 521,(PGR(NP1),NP1=1,NG)
      PRINT 521,(SAAA(NP1),NP1=NG1,NP)
   13 CONTINUE
      IF(JACK.NE.JAZZ) GO TO 515
      DO 501 ITA=1,NG
      STMP(ITA)=STNZ(ITA)**2.
      PRINT 491,ITA
      PRINT 492,STMP(ITA),(STP(ITA,IA),IA=1,JIN)
  501 CONTINUE
      JAZZ=JAZZ+100
  515 CONTINUE
      DO 63 II=1,JIN
      III=II+1
      DO 63 IMO=1,NG
      SSI1(IMO,III)=SS(IMO,II)
      SSSI1(IMO,III)=SSS(IMO,II)
   63 CONTINUE
      GO TO 95
   92 STDV=0.0
      DO 93 NP1=1,NP
      STDV=STDV+(S(NP1)*SDVZ(NP1))**2
   93 CONTINUE
      STDV=SQRT(STDV)
      DO 61 IMO=1,NP
   61 SI1(IMO)=S(IMO)
   95 CONTINUE
      IF(JACK.NE.JOEY) GO TO 14
      PRINT 210,STDV
   14 CONTINUE
C.....DATA REDUCTION
      TREDUC=07/04
      TG=TREDUC
      TI=TREDUC
C.....CALCULATION OF FILM TEMPERATURE
      COL=2.*DELT/(RHOC(3)*D(3))
      PIG=1.-COL*(H(3)+SIG*EPS(3)*(TF**3))
      WOLF=COL*(H(3)*TG+HRVF+QRA(3))
      TF=PIG*TF+WOLF
C.....SAVE PREVIOUS PARAMETER VALUES
      TFI1=TF
      TII1=TI
      HFI1=H(3)
      VI1=VEL
      RFI1=RECF(3)
      IF(JACK.NE.JOEY) GO TO 15
      PRINT 140,TINF,TREDUC,TF
      JOEY=JOEY+10
   15 CONTINUE
      JACK=JACK+1
C.....COMPUTATION OF PARAMETER SHIFT
      JEEP=JIN
      IF(JACK.LE.JIN) JEEP=JACK
      IF(JEEP.LE.1) GO TO 52
      DO 50 KEY=2,JEEP
      KKEY=KEY-1
      DO 50 NP1=1,NP
      SDV(NP1,KEY)=SDV(NP1,KKEY)
   50 CONTINUE
   52 CONTINUE
      DO 51 NP1=1,NP
      SDV(NP1,1)=SDVZ(NP1)
   51 CONTINUE
  137 FORMAT(50H  NOT CONVERGE
  210 FORMAT(1H ,19HSTANDARD DEVIATION=,E15.6)
  215 FORMAT(50H PARAMETER DEVIATION
  211 FORMAT(1H ,9E14.6)
  212 FORMAT(50H SENSITIVITY
  140 FORMAT(1H ,9F12.6)
  491 FORMAT(/,3H ,10HPARAMETER=,I3)
  492 FORMAT(1H ,9E14.6)
  520 FORMAT(50H PPP
  521 FORMAT(1H ,9E14.6)
  523 FORMAT(50H QQQ
  525 FORMAT(50H RRR
  527 FORMAT(50H SSS
      RETURN
      END
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SUBROUTINE INTRE

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      SUBROUTINE INTRE(REYN,UNCH)
      DIMENSION APE(9),DPE(9)
      P=1.E-3
C.....UNCERTAINTY TABLE FOR DIFFERENT REYNOLDS NUMBER
      DO 2 I=1,9
      P=P*10.
    2 APE(I)=P
      DPE(1)=.05
      DPE(2)=.075
      DPE(3)=.10
      DPE(4)=.15
      DPE(5)=.18
      DPE(6)=.15
      DPE(7)=.10
      DPE(8)=.075
      DPE(9)=.05
      IF(REYN.GT.APE(9)) GO TO 50
      IF(REYN.LT.APE(1)) GO TO 50
C.....LINEAR INTERPOLATION
      DO 10 NANA=2,9
      NON=NANA
      IF(REYN.LT.APE(NANA)) GO TO 12
   10 CONTINUE
   12 A2=APE(NON)
      B2=DPE(NON)
      A1=APE(NON-1)
      B1=DPE(NON-1)
      RCCC=REYN/A1
      UNCH=B1+(B2-B1)*ALOG10(RCCC)/(ALOG10(A2)-ALOG10(A1))
      GO TO 51
   50 UNCH=0.05
   51 CONTINUE
      RETURN
      END
```

SUBROUTINE INTKN

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      SUBROUTINE INTKN(DKN,UNCR,INU)
      DIMENSION BWA(5),BWB(5)
C.....UNCERTAINTY TABLE FOR DIFFERENT KNUDSEN NUMBER
      IF(INU.EQ.2) GO TO 5
C.....FOR BEAD AND PLATE
      BWA(1)=3.E-2
      BWA(2)=0.1
      BWA(3)=0.3
      BWA(4)=1.0
      BWA(5)=3.0
      GO TO 7
C.....FOR WIRE
    5 CONTINUE
      BWA(1)=6.E-2
      BWA(2)=0.2
      BWA(3)=0.6
      BWA(4)=2.0
      BWA(5)=6.0
    7 CONTINUE
      BWB(1)=0.05
      BWB(2)=0.075
      BWB(3)=0.10
      BWB(4)=0.075
      BWB(5)=0.05
      IF(DKN.GT.BWA(5)) GO TO 50
      IF(DKN.LT.BWB(1)) GO TO 50
C.....LINEAR INTERPOLATION
      DO 10 NANA=2,5
      NON=NANA
      IF(DKN.LT.BWA(NANA)) GO TO 12
   10 CONTINUE
   12 CONTINUE
      A2=BWA(NON)
      B2=BWB(NON)
      A1=BWA(NON-1)
      B1=BWB(NON-1)
      DDD=DKN/A1
      UNCR=B1+(B2-B1)*ALOG10(DDD)/(ALOG10(A2)-ALOG10(A1))
      GO TO 51
   50 CONTINUE
      UNCR=.05
   51 CONTINUE
      RETURN
      END
```

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