PROPELLANT SLOSH COUPLING WITH BENDING

INTERIM REPORT

June 1969

Lockheed

MISSILES & SPACE COMPANY
A GROUP DIVISION OF LOCKHEED AIRCRAFT CORPORATION
SUNNYVALE, CALIFORNIA

HREC-1485-1
LMSC/HREC D148988

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Contract NAS8-21485

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FOREWORD

This document presents an interim report of a research program performed by the Lockheed Missiles & Space Company, Huntsville Research & Engineering Center (Lockheed/Huntsville), while under contract to the National Aeronautics & Space Administration, Marshall Space Flight Center (MSFC), Contract NAS8-21485. This report summarizes the derivation and computational procedures of a new method for studying the vibrational characteristics of a large liquid-propellant space vehicle.

Technical coordinator of the contract was Mr. Harry J. Buchanan, Aero-Astrodynamics Laboratory, NASA/MSFC.

ACKNOWLEDGMENT

The authors wish to express their sincere appreciation to Mr. W. D. Whetstone, Lockheed/Huntsville, for his valuable suggestions concerning the formulation of a coupled elastic and fluid problem.

SUMMARY

A new approach is used to study the vibrational characteristics of a large liquid-propellant space vehicle. This approach permits taking the higher slosh modes into account and using the dynamic behavior of liquid propellant contained in a tank as determined experimentally. The developed program will provide a set of system bending modes including the effect of liquid propellant in the vehicle. Lateral force distribution coefficients due to the dynamics of the liquid propellant can be computed.

Lagrange's equation is employed to formulate a coupled elastic and fluid problem. The vehicle is modeled as a series of non-uniform beams interconnected by elastic interstages. The engines of each stage may be represented as branch beams attached to the lower ends of the beams. Generalized coordinates of the system are beam end displacements, coefficients of beam deflection functions, branch beam deflection angles and coefficients of slosh modes. The kinetic and potential energies associated with the hardware of a vehicle are computed by summation along its longitudinal axis. The energies associated with liquid propellant are obtained by performing volume integration over the tank volumes occupied by the propellant.

In this study, a program called SLOSH was developed. This program computes the mass and stiffness matrices of the system associated with the liquid propellant and provides the lateral force distribution coefficients due to the dynamics of the propellant. A Lockheed/Huntsville-developed bending program (Ref. 1) was modified to a two-dimensional case in order to accomodate the additional slosh modes used in the present model. This program computes the mass and stiffness matrices of the system associated with the hardware of a vehicle. After a combination of the two sets of matrices, the bending program will solve the constructed eigenvalue problem.

A study of the vibrational characteristics of Saturn V during its first-stage flight is included as an example of the new method. A user's guide of the developed digital SLOSH program is also provided.

CONTENTS

NOMENCLATURE

$\underline{A}$	mass matrix of a system
a_k	k^{th} tank radius at the liquid propellant surface (m)
$\underline{B}$	stiffness matrix of a system
c_{km}^{n}	eigenvectors of the n^{th} slosh mode associated with the k^{th} tank (dimensionless)
d_{1k}	distance between lower beam end and tank coordinate system of the k^{th} tank (m)
G_{3k}	distance between beam end and center of mass of the k^{th} tank (m)
$J_1(j_{km}R)$	Bessel-function of the first kind associated with the k^{th} tank
j_{km}	m^{th} root of equation $J_1'(j_k) = 0$
L_i	length of the i^{th} beam (m)
ℓ_k	distance between propellant surface and center of mass of the k^{th} tank (m)
M_{mk}	propellant mass in the first m layers of the k^{th} tank (kg)
N_i	number of fundamental deflection functions of the i^{th} beam
N_k	number of slosh modes of the k^{th} tank
P	potential energy of a system (kg-m^2/sec^2)
P_k	potential energy of the propellant contained in the k^{th} tank (kg-m^2/sec^2)
$\underline{Q}$	generalized coordinate vector
$\underline{Q}^T$	transposed matrix of $\underline{Q}$

NOMENCLATURE (Continued)

q_i	i^{th} generalized coordinate
T	kinetic energy of a system ($kg\text{-}m^2/sec^2$)
T_k	kinetic energy of the propellant contained in the k^{th} tank ($kg\text{-}m^2/sec^2$)
u_i^ℓ	lower end displacement of the i^{th} beam (m)
u_i^r	upper end displacement of the i^{th} beam (m)
$\underline{X}$	eigenvector of a system
Y_{ij}	j^{th} fundamental deflector function of the i^{th} beam (dimensionless)

Symbols

α_{2i}	lateral acceleration of the i^{th} beam (m/sec^2)
α_3	longitudinal acceleration of a vehicle (m/sec^2)
η	wave height of liquid propellant (m)
λ_{kn}	eigenvalues of a fluid system (dimensionless)
ω	natural frequencies of a system (rad/sec)
ϕ_k	velocity potential of the propellant contained in the k^{th} tank (m^2/sec^2)
ϕ_{kn}	eigenfunctions of a fluid system (dimensionless)
$\ddot{\psi}_i$	angular acceleration of the i^{th} beam (rad/sec^2)
ρ_k	mass density of the propellant contained in the k^{th} tank (kg/m^3)
ξ_{kn}	n^{th} slosh coefficient of the k^{th} tank (m)
ζ_{ij}	j^{th} beam deflection coefficient of the i^{th} beam (m)

Section 1
INTRODUCTION

To ensure a successful flight of a liquid-propellant space vehicle, the fundamental frequencies of the control system, liquid propellant and the hardware of the vehicle should be designed such that they are fairly widely separated. Hence, the oscillations of a vehicle will not be excited by the function of the control system. In case the vehicle is excited by a forcing function, interaction between propellant sloshing and bending must not present any large amplitude dynamic response.

In order to simulate the dynamic response of a vehicle, it has been traditional to model the vehicle as a non-uniform beam (Ref. 2). Liquid propellant contained in each tank is replaced by a mass-spring-dashpot system attached to the beam. Due to the limitations of computation time on a digital computer and the capacity of a hybrid computer (Ref. 3), only the first slosh modes for some of the propellant tanks can be taken into account. The above assumptions provide a good mathematical model of a vehicle during its early portion of flight. However, when the propellant level becomes relatively low or in case of a shallow tank, the dynamic behavior of liquid propellant cannot be adequately represented by its first mode alone. Therefore, a new mathematical model is needed which includes higher slosh modes and can be computed within the capabilities of a computer.

A new approach for studying the interaction between vehicle bending and propellant sloshing is introduced in this report. Lagrange's equation is used to formulate the problem. A digital computer program which solves a maximum of 60 degrees of freedom was developed. The mass and stiffness matrices of the coupled elastic and fluid system can be accurately calculated. System bending modes with the presence of liquid propellant of a vehicle will be provided. If a mathematical model for atmospheric flight simulation is needed, the model can be properly defined by taking into account only the first few of these modes. In general, this approach will lead to a smaller set of differential equations than the conventional approach. Consequently, the objectives of the study are to define an accurate model for analyzing the coupling between vehicle bending and propellant sloshing and to present a set of system bending modes for flight simulation.

Section 2
MATHEMATICAL MODEL

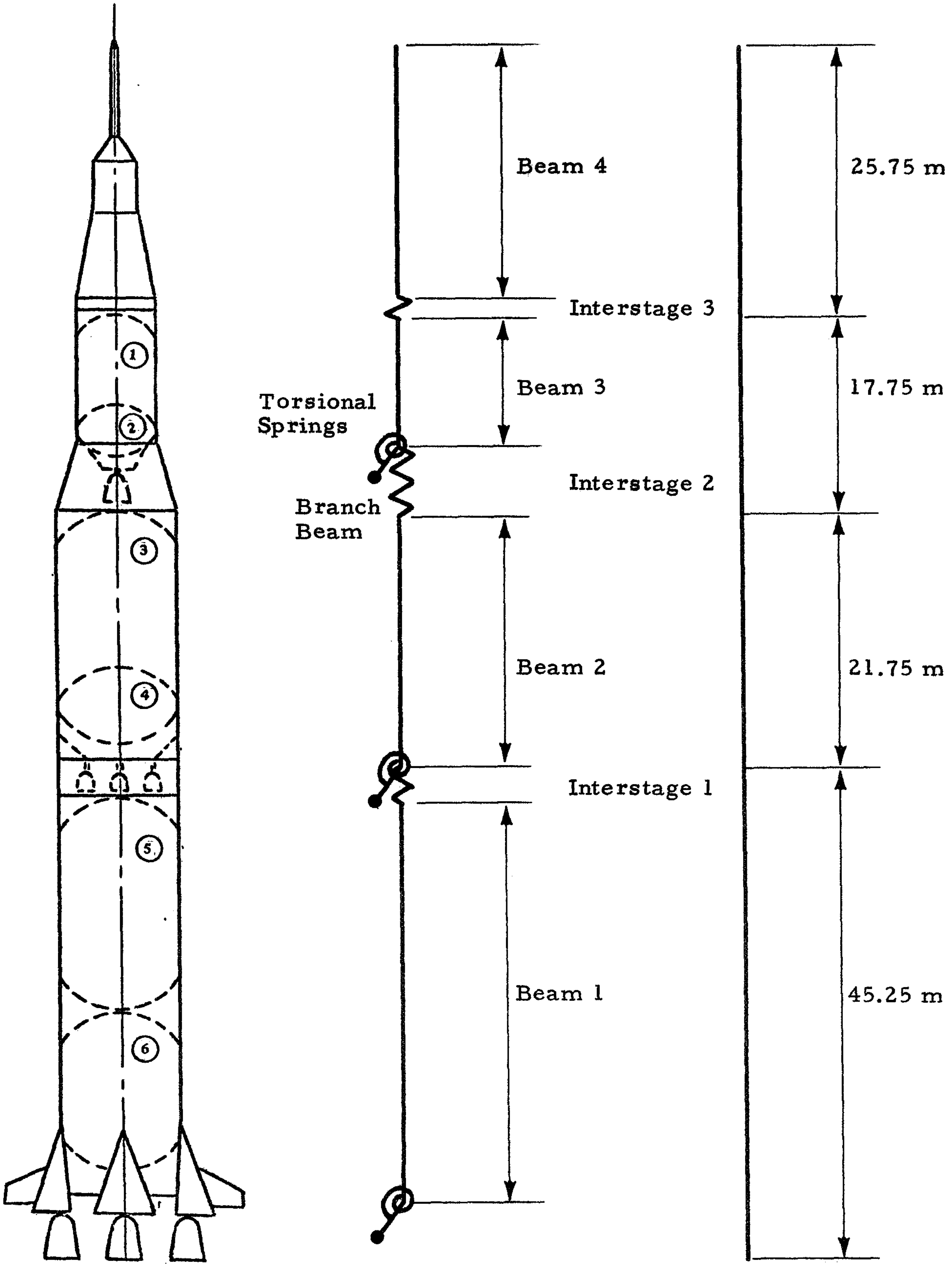
Saturn V is used as a typical vehicle in this study. With minor modifications, the presented method and the developed digital computer program may be applied to analyze other liquid-propellant space vehicles. As shown in Fig. 1b, the vehicle may be modeled as a system of four non-uniform beams interconnected by elastic interstages. The engines of each stage may be modeled as branch beams attached to the lower ends of the beams. The deflection of each beam is represented by a linear combination of four fundamental deflection functions. Dynamic behavior of liquid propellant contained in each tank is described by the first one to three slosh modes. Hence, the generalized coordinates of the system are beam end displacements, engine deflection angles, coefficients of beam deflection functions and coefficients of slosh modes.

The hardware mass of the vehicle is distributed along the longitudinal axis of the vehicle. Tank configurations are approximated by simple functions such that volume integrals can be readily performed (see Figs. 2 to 5).

Suppose that $\underline{A}$, $\underline{B}$ and $\underline{Q}$ are the mass matrix, stiffness matrix and the generalized coordinates vector of the system, respectively. The kinetic energy of the system may be expressed as

$$T = 1/2\ \dot{\underline{Q}}^T \underline{A}\, \dot{\underline{Q}} \tag{2.1}$$

where $\dot{\underline{Q}}^T$ is the transposed matrix of $\dot{\underline{Q}}$ and $\dot{\underline{Q}} \equiv \frac{dQ}{dt}$. The potential energy of the system is



(1a) (1b) (1c)

Fig. 1 - Saturn V Vehicle Configuration

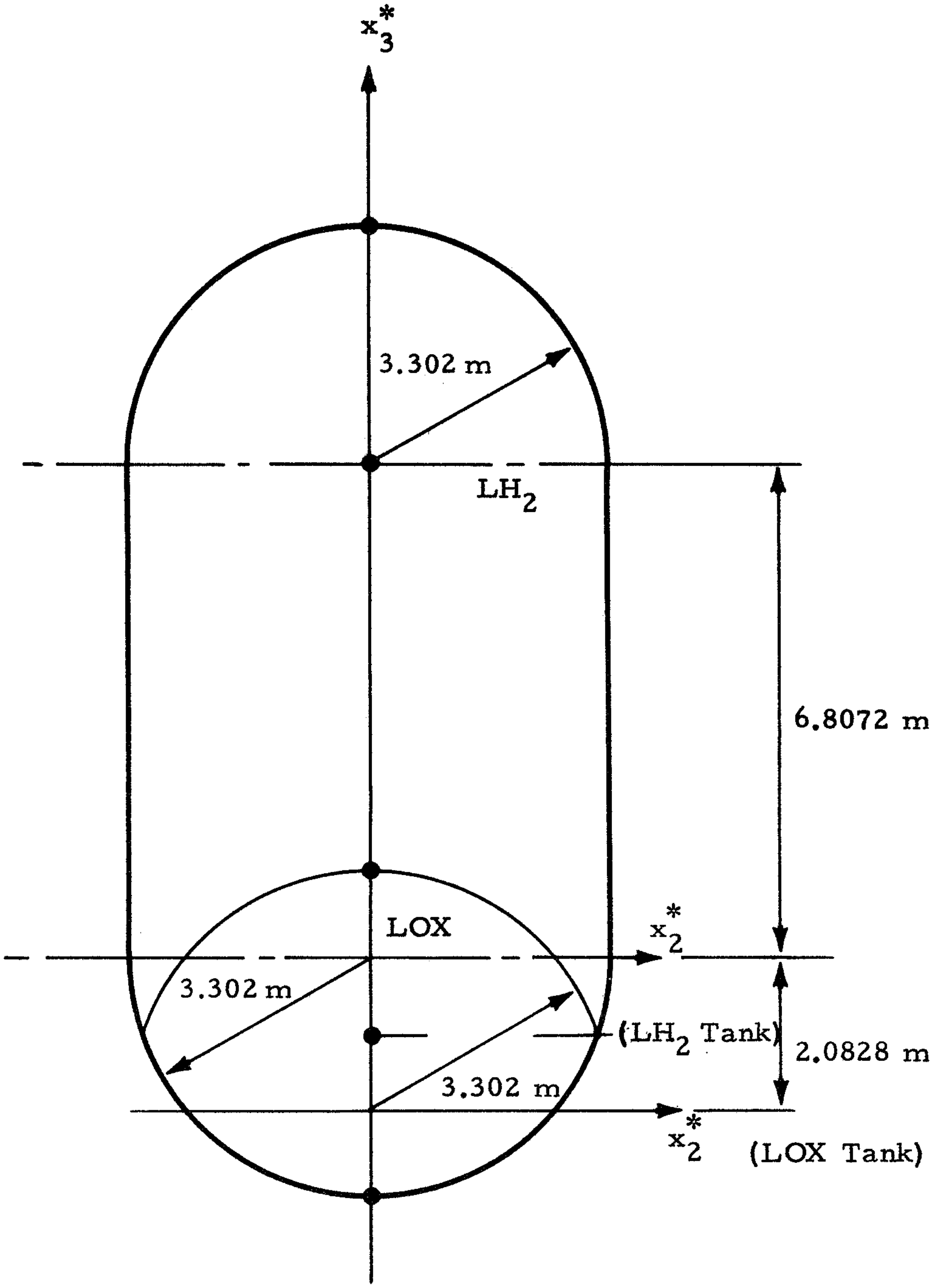


Fig. 2 - S-IVB Tank Configuration

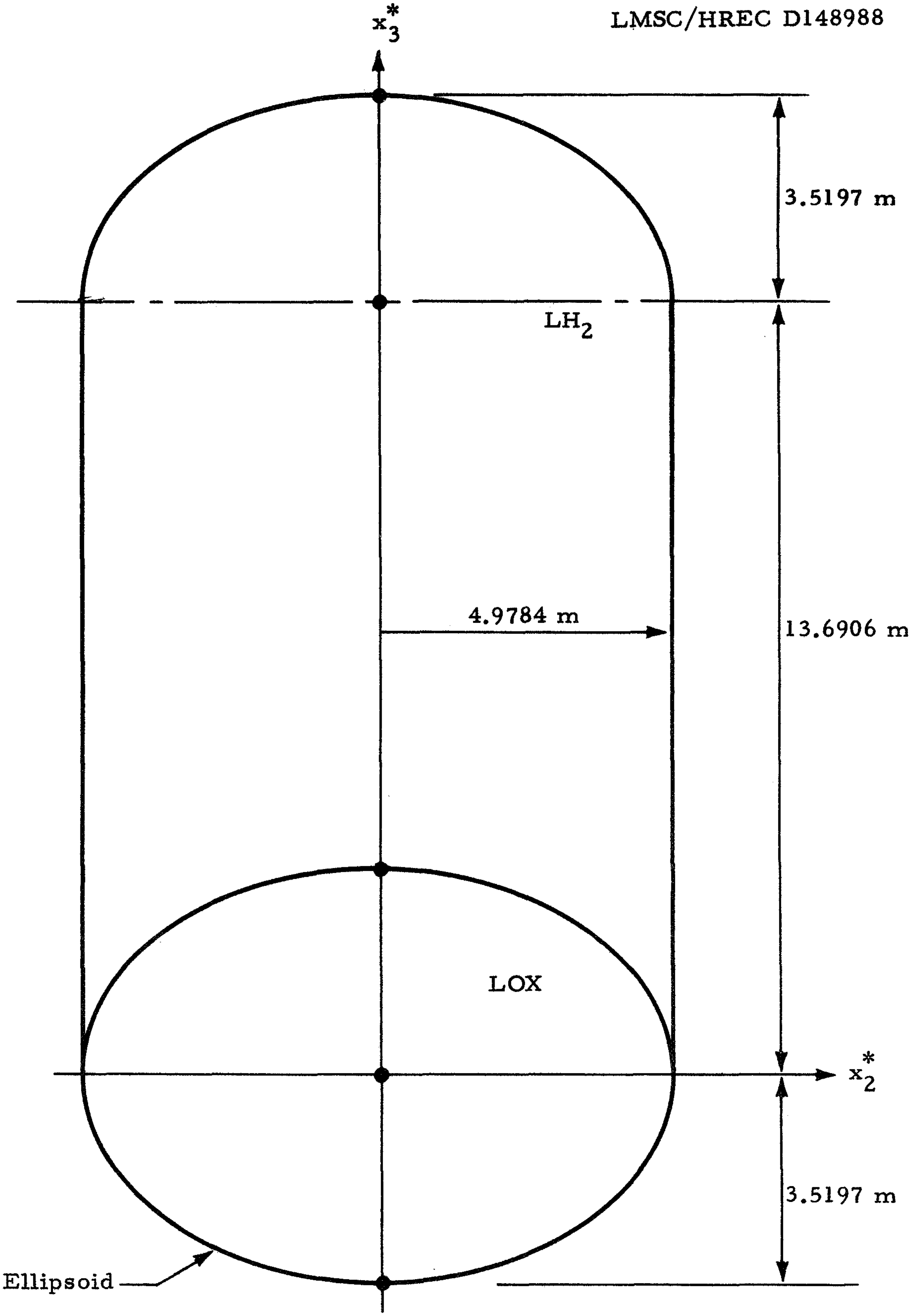


Fig. 3 - S-II Tank Configuration

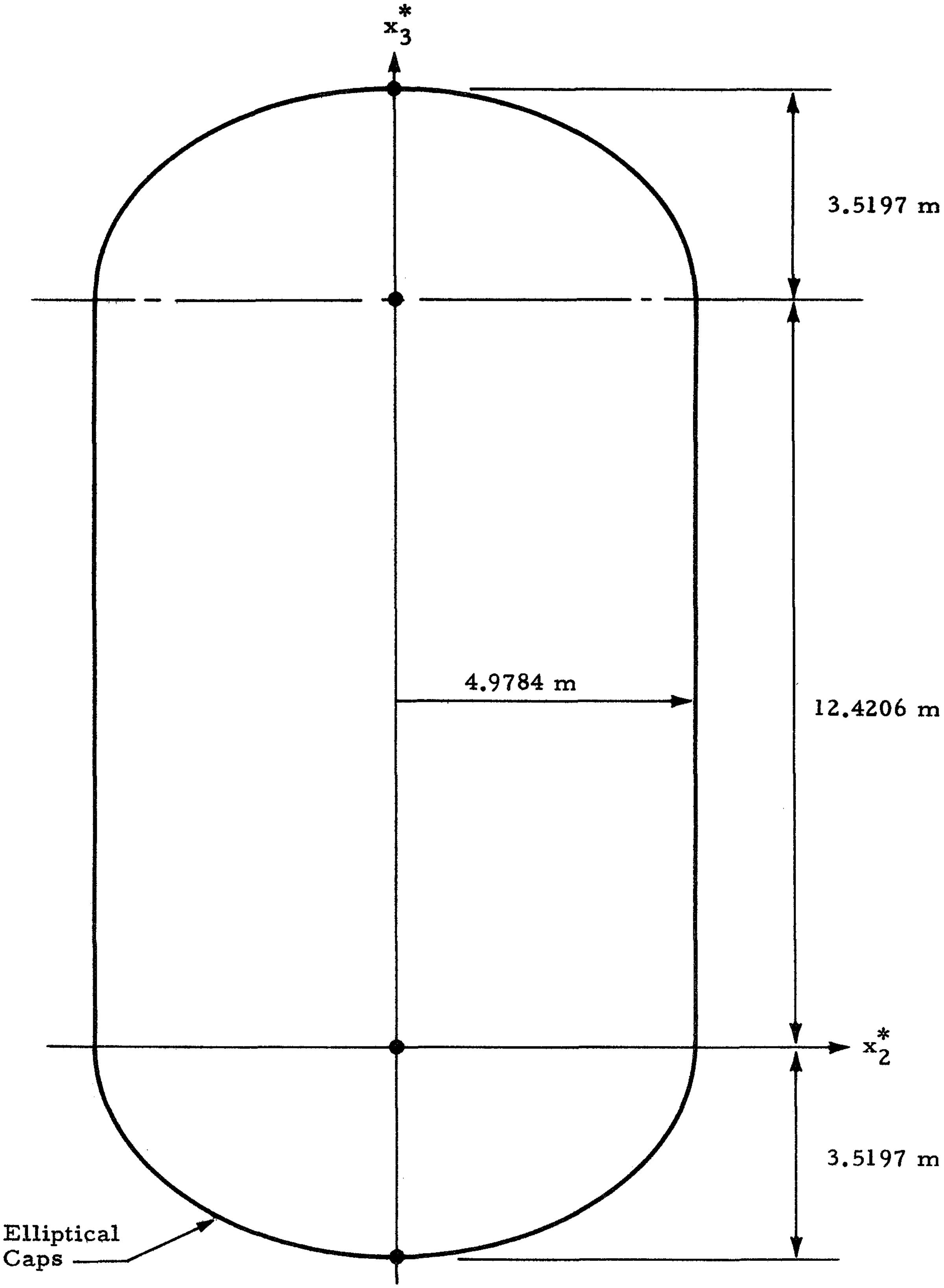


Fig. 4 - S-IC LOX Tank Configuration

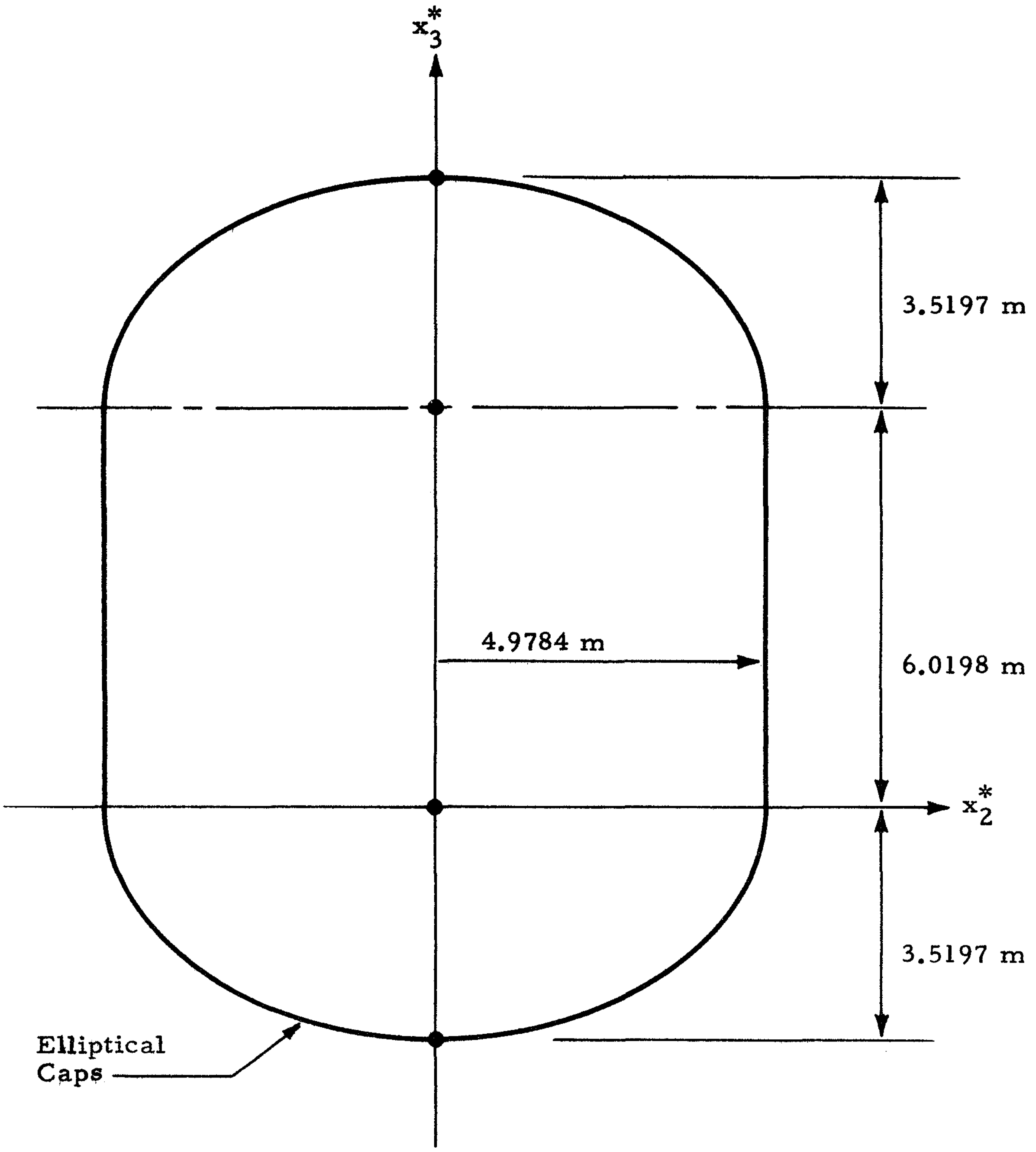


Fig. 5 - S-IC Fuel Tank Configuration

$$P = 1/2\, \underline{Q}^T \underline{B}\, \underline{Q} \tag{2.2}$$

For a conservative system, Lagrange's equation is

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) + \frac{\partial P}{\partial q_i} = 0 \qquad i = 1, 2, \ldots, n \tag{2.3}$$

where q_i is the i^{th} element of $\underline{Q}$. If Eqs. (2.1) and (2.2) are used, Eq. (2.3) becomes

$$\underline{A}\, \ddot{\underline{Q}} + \underline{B}\, \underline{Q} = 0 \tag{2.4}$$

The solution of the above equation may be taken as

$$\underline{Q} = \underline{X} \sin \omega t \tag{2.5}$$

where $\underline{X}$ and ω are the eigenvectors and the natural frequencies of a system, respectively. Substituting Eq. (2.5) into Eq. (2.4), one finds

$$(\omega^2 \underline{A} - \underline{B})\, \underline{X} = 0 \tag{2.6}$$

Detailed derivations of matrices $\underline{A}$ and $\underline{B}$ are given in Section 3 and the Appendix. Subprograms which compute these matrices are discussed in Section 4. Routines which solve the system of Eq. (2.6) are described in Ref. 1.

Section 3
DERIVATION

Lagrange's equation is used to define a mathematical model for studying the vibrational characteristics of Saturn V. The structure of the vehicle is considered as a series of elastic non-uniform beams interconnected by elastic interstages. The engines of each stage may be represented by a mass which is attached to the beam through a rigid rod and a torsional spring (see Fig. 1b). The vehicle is assumed to be axisymmetric and restricted to plane motion. Liquid propellants contained in the tanks of Saturn V are considered incompressible, inviscid fluids. Furthermore, small oscillations and irrotational flow are assumed.

The kinetic and potential energies of a vehicle are computed in the following manner. Energies associated with the hardware are obtained by integrating along the longitudinal axis of the vehicle. Energies associated with the liquid propellant are computed by performing volume integration over the tank volumes occupied by the propellant. The total energy of a vehicle is represented by the sum of these energies. The kinetic and potential energies of the hardware of a vehicle are defined in Ref. 1. Except that a few notations may differ from those used in this report, the reader will not have any communication difficulty.

As shown in Fig. 6, the kinetic energy of the propellant contained in the k^{th} tank can be expressed as a volume integral

$$T_k = 1/2\ \rho_k \int_V \left[v_r^2 + v_\theta^2 + v_z^2 \right] dV \tag{3.1}$$

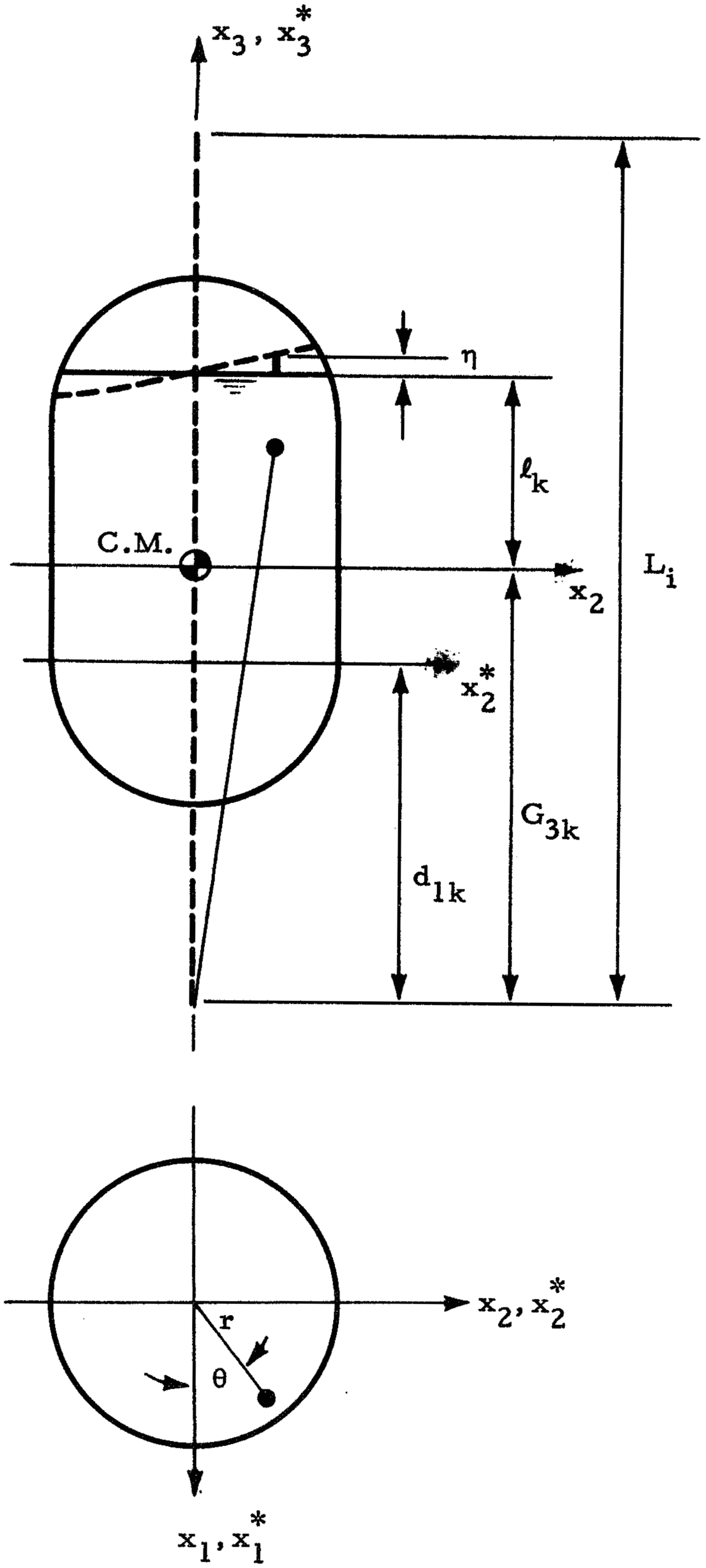


Fig. 6 - Coordinate Systems of k^{th} Tank

where

$$\begin{cases} v_r = \left[\dot{u}_i^{\ell} + \dfrac{\dot{u}_i^r - \dot{u}_i^{\ell}}{L_i} (d_{1k} + x_3^*) + \displaystyle\sum_{j=1}^{N_i} \dot{\zeta}_{ij} Y_{ij} \right] \sin\theta - \dfrac{\partial}{\partial r} \displaystyle\int \phi_k \, dt \\[2ex] v_\theta = \left[\dot{u}_i^{\ell} + \dfrac{\dot{u}_i^r - \dot{u}_i^{\ell}}{L_i} (d_{1k} + x_3^*) + \displaystyle\sum_{j=1}^{N_i} \dot{\zeta}_{ij} Y_{ij} \right] \cos\theta - \dfrac{1}{r} \dfrac{\partial}{\partial \theta} \displaystyle\int \phi_k \, dt \\[2ex] v_z = - \dfrac{\dot{u}_i^r - \dot{u}_i^{\ell}}{L_i} x_2 - \dfrac{\partial}{\partial x_3} \displaystyle\int \phi_k \, dt \end{cases} \tag{3.2}$$

and

$$\phi_k = - \sin\theta \sum_{n=1}^{N_k} \frac{a_k}{\lambda_{kn}} \ddot{\xi}_{kn} \phi_{kn} \tag{3.3}$$

Notations used in the above equations are defined below:

ρ_k	mass density of the propellant contained in the k^{th} tank
u_i^{ℓ}	lower end displacement of the i^{th} beam
u_i^r	upper end displacement of the i^{th} beam
L_i	length of the i^{th} beam
d_{1k}	distance between lower beam end and tank coordinate system of the k^{th} tank
N_i	number of fundamental deflection functions of the i^{th} beam
ζ_{ij}	j^{th} beam deflection coefficient of the i^{th} beam

Y_{ij}	j^{th} fundamental deflection function of the i^{th} beam
ϕ_k	velocity potential of the propellant contained in the k^{th} tank
N_k	number of slosh modes of the k^{th} tank
a_k	k^{th} tank radius at the liquid propellant surface
λ_{kn}	eigenvalues of a fluid system
ξ_{kn}	n^{th} slosh coefficient of the k^{th} tank
ϕ_{kn}	eigenfunctions of a fluid system

The eigenfunctions ϕ_{kn} of Eq. (3.3) can be chosen in the following form

$$\phi_{kn} = \sum_{m=1}^{5} c_{km}^{n} \left(\frac{r}{a_k}\right)^{2m-1} + \sum_{m=6}^{10} c_{km}^{n} J_1\left(j_{km}\frac{r}{a_k}\right) e^{j_{km}\left(\frac{x_3}{a_k} - \frac{\ell_k}{a_k}\right)} \tag{3.4}$$

where c_{km}^{n} and $J_1(j_{km}\frac{r}{a_k})$ are the eigenvectors of the n^{th} slosh mode and Bessel function of the first kind associated with the k^{th} tank, respectively. The notation ℓ_k is the distance between propellant surface and center of mass of the k^{th} tank and j_{km} is the m^{th} root of the equation

$$J_1'(j_k) = 0 \tag{3.5}$$

The superscript "prime" of J_1 denotes a differentiation with respect to its argument. The functions of Eq. (3.4) give an excellent solution for liquid contained in an axisymmetric tank with arbitrary height. Substituting Eqs. (3.2) – (3.4) into Eq. (3.1), the kinetic energy T_k may be written in terms of the generalized coordinates u_i^{ℓ}, u_i^{r}, ζ_{ij} and ξ_{kn}.

$$\begin{aligned}
T_k = -\pi a_k^3 \rho_k \Bigg\{ & V_k^{pp}(\dot{u}_i^{\ell})^2 + V_k^{pq}\,\dot{u}_i^{\ell}\,\dot{u}_i^{r} + V_k^{qq}(\dot{u}_i^{r})^2 \\
& + \sum_{j=1}^{N_i} (ULB)_{ij}\,\dot{u}_i^{\ell}\,\dot{\zeta}_{ij} + \sum_{j=1}^{N_i} (URB)_{ij}\,\dot{u}_i^{r}\,\dot{\zeta}_{ij} \\
& + \sum_{j=1}^{N_i}\sum_{m=1}^{N_i} (BB)_{ijm}\,\dot{\zeta}_{ij}\,\dot{\zeta}_{im} + \sum_{n=1}^{N_k} \frac{1}{\lambda_{kn}}\, s_{kn}^{p}\,\dot{u}_i^{\ell}\,\dot{\xi}_{kn} \\
& + \sum_{n=1}^{N_k} \frac{1}{\lambda_{kn}}\, s_{kn}^{q}\,\dot{u}_i^{r}\,\dot{\xi}_{kn} + \sum_{j=1}^{N_i}\sum_{n=1}^{N_k} \frac{1}{\lambda_{kn}} (BS)_{kjn}\,\dot{\zeta}_{ij}\,\dot{\xi}_{kn} \\
& + \frac{1}{2}\sum_{n=1}^{N_k}\sum_{m=1}^{N_k} \frac{1}{\lambda_{kn}\,\lambda_{km}}\, s_{knm}\,\dot{\xi}_{kn}\,\dot{\xi}_{km} \Bigg\}
\end{aligned} \qquad (3.6)$$

Detailed derivations and expressions of V_k^{pp}, V_k^{pq}, V_k^{qq}, $(ULB)_{ij}$, $(URB)_{ij}$, $(BB)_{ijm}$, s_{kn}^{p}, s_{kn}^{q}, $(BS)_{kjn}$ and s_{kmn} are given in the Appendix.

The potential energy of the propellant contained in the k^th tank is expressed as a surface integral integrated over the static free surface of the propellant (Ref. 4).

$$P_k = \frac{1}{2}\rho_k\,\alpha_3 \int_S \eta^2\, r\, d\theta\, dr \qquad (3.7)$$

where

$$\eta = \sin\theta \sum_{n=1}^{N_k} \xi_{kn} \phi_{kn}$$

is the wave height of liquid propellant (see Fig. 6) and α_3 is the longitudinal acceleration of a vehicle. Similarly, Eq. (3.7) can be written in terms of the generalized coordinates of the system.

$$P_k = \frac{\pi}{2} \rho_k \alpha_3 a_k^2 \sum_{n=1}^{N_k} \sum_{m=1}^{N_k} p_{knm} \xi_{kn} \xi_{km} \tag{3.8}$$

The quantities p_{kmn} and the intermediate steps which lead to Eq. (3.8) are given in the Appendix.

If Eqs. (3.6) and (3.8) are substituted into Eq. (2.3) and differentiated with respect to the appropriate coordinates, the elements of the mass and stiffness matrices associated with the propellant will be found. After a combination with the corresponding matrices associated with the hardware, a matrix equation which governs the vibrational characteristics of a system (Eq. (2.6)) will be obtained. The method of solving this equation is given in Ref. 1.

Lateral force distribution coefficients due to the dynamics of the liquid propellant are computed in the same fashion as in Ref. 4. However, a maximum of three slosh modes per tank can now be included. It can be shown that the lateral force exerted on the k^{th} tank wall due to the motion of the first m layers of the propellant (measured from the tank bottom) is

$$({}_mF'_2)_k = ({}_mA_\alpha)_k \alpha_{2i} + ({}_mB_\psi)_k \ddot{\psi}_i + \sum_{n=1}^{N_k} ({}_mC_\xi)_{kn} \xi_{kn} \alpha_3 \tag{3.9}$$

where

$$({}_mA_\alpha)_k = -M_{mk} + \pi a_k^2 \rho_k \sum_{n=1}^{N_k} \ell_k b_n \left(\sum_{j=1}^{10} c_{kj}^n I_{kj} \right) \tag{3.10}$$

$$\begin{aligned} ({}_mB_\psi)_k = & -G_{3k} M_{mk} - \pi a_k^4 \rho_k \oint Z^2 R \, dR \\ & - \pi a_k^2 \ell_k^2 \rho_k \sum_{j=1}^{9} q_j \int_{{}_mA} \left(\frac{\partial f_j}{\partial R} + \frac{f_j}{R} \right) RdRdZ \\ & + \pi a_k^2 \rho_k \sum_{n=1}^{N_k} \ell_k \left[G_{3k} b_n - \ell_k (b_n - h_n) \right] \left(\sum_{j=1}^{10} c_{kj}^n I_{kj} \right) \end{aligned} \tag{3.11}$$

$$({}_mC_\xi)_{kn} = \pi a_k^2 \rho_k \sum_{j=1}^{10} c_{kj}^n I_{kj} \tag{3.12}$$

$$I_{kj} = \int_{{}_mA} \left(\frac{\partial \phi_{kn}}{\partial R} + \frac{\phi_{kn}}{R} \right) RdRdZ \tag{3.13}$$

$$R \equiv \frac{r}{a_k}$$

$$Z \equiv \frac{x_3}{a_k}$$

and ${}_{m}A$ denotes an integration over the first m layers of the propellant in a tank. Notations α_{2i} and $\ddot{\psi}_i$ are the lateral and angular accelerations of the i^{th} beam, respectively. G_{3k} and M_{mk} are the distance between center of mass and beam end and the propellant mass in the first m layers of the k^{th} tank, respectively. Definitions of b_m, q_j, k_m and f_j which are all related to the k^{th} tank can be found in Ref. 4. Consequently, the lateral force in the m^{th} layer of the k^{th} tank is calculated by subtracting $({}_{m-1}F'_2)_k$ from $({}_{m}F'_2)_k$ or

$$({}_{m}F^{*}_2)_k = ({}_{m-1}F'_2)_k - ({}_{m}F'_2)_k \tag{3.14}$$

Section 4
DIGITAL COMPUTER PROGRAM

4.1 PROGRAM ORGANIZATION

A program called SLOSH was developed in this study. Detailed discussions are presented in Section 4.2. This program computes the lateral dynamic force distribution coefficients due to the propellant of a vehicle, and provides the information with regard to the dynamic behavior of the liquid propellant contained in a rigid tank. If the SLOSH program is used as a subprogram of a modified Lockheed/Huntsville bending program (Ref. 1), it will compute the mass and stiffness matrices associated with the liquid propellant of a vehicle. With these matrices, the bending program will solve the constructed eigenvalue program and provide the free vibrational characteristics of a vehicle.

The overlay configuration of the modified bending program is shown in Fig. 7. The functions of the principal subroutines are briefly outlined below:

Deck Name	Function
MAINX	Directs logic flow to other routines
BDAT	Sets up control information
C56Y	Generates fundamental beam deflection functions
C56D	Computes integral terms associated with the hardware
NPUT	Reads in data and sets up problem definition
C56A	Computes mass and stiffness arrays associated with the hardware
SLOSH	Computes mass and stiffness arrays associated with the liquid propellant and the lateral force distribution coefficients due to the dynamics of propellant

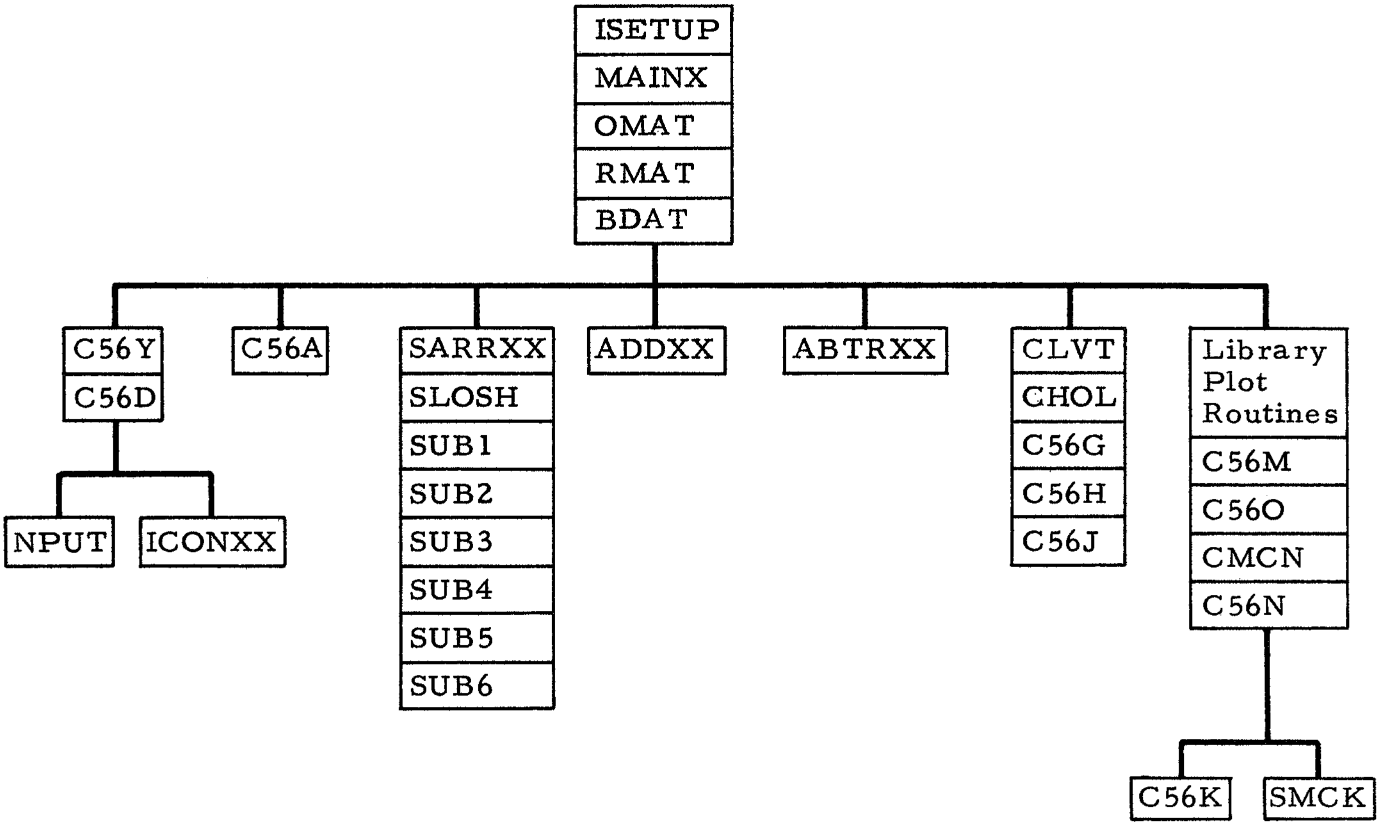


Fig. 7 - Program Overlay Configuration

Deck Name	Function
SUB2	Generates slosh normal modes
SUB3	Performs line integrals associated with the propellant
SUB4	Describes tank configurations
CLVT	Solves the constructed eigenvalue problem
C56K	Plots beam properties, displacement functions and mode shapes.

4.2 THE SLOSH PROGRAM

The SLOSH program serves the following purposes:

1. To read in certain output and intermediate data of Lomen's program (Ref. 4) such that the computation time of the modified bending program can be greatly reduced. In the future, if the SHARE simultaneous equation package (SOLVE) and the SHARE eigenvalue routine (RWEG2F), which are in the MAP symbolic language, of Lomen's program could be coded in the Fortran IV language or substituted by other standard routines, the function of Lomen's program may be readily replaced by the SLOSH program. It is expected that the SLOSH program will take less computer time than the Lomen program.

2. To compute the mass and stiffness matrices associated with the liquid propellant of a vehicle.

3. To provide the lateral force distribution coefficients due to the propellant dynamics.

Input data which are now furnished by the Lomen program are listed on the following page.

Variable Name (Lomen program)	Variable Name (SLOSH program)	Description
TEMP(2)	DBTBCM(I)	Distance between tank bottom and center of mass
VALP	PEV(I, J)	Eigenvalues of a fluid system
CNK(J, K)	C(I, J, K)	Eigenvectors of a fluid system
HN(J)	HN(I, J)	see p. 3 - 9 of Ref. 4
ARG2	TTFCB(I, J)	$\pi a_k^2 \ell_k^2 \sum_{j=1}^{9} q_j \int_{{}_mA} \left(\frac{\partial f_j}{\partial R} + \frac{f_i}{R}\right)$ RdRdZ (see second term of Eq. (3.11))

Note that the Lomen program has some errors in the computation of the lateral force distribution coefficients. His program has also been slightly modified in order to obtain the above variables.

Unfortunately, due to the complexity of the bending program, it will not be able to evaluate the integrals of Eq. (A-4) in the bending program for the moment. Currently, the fundamental deflection functions which are generated from subroutine C56Y are approximated by straight-line segments in order to perform the integrations of Eq. (A-4) in the SLOSH program. Each deflection function associated with a tank is represented by three straight-line segments. The ordinates of the terminals of the straight lines are shown by the dots along the x_3^* axes in Figs. 2 through 5. In general, these segments should be chosen to give the best approximation of the portion of a deflection curve where the integration over the propellant volume is actually taking place.

The tank configurations of Saturn V are specified by simple curves (Figs. 2 through 5). Bessel functions which are represented by series form (Ref. 5) are included as a subroutine of the SLOSH program. Finally, the line integrals of Eqs. (A-3) through (A-6) are evaluated by the Gauss mechanical quadrature formula (Ref. 6).

4.3 PROGRAM LIMITATIONS

The present IBM 7094 version of the modified bending program is designed for a four-beam model, see Fig. 1-C and Section 5. The DIMENSION statements which are used in the SLOSH program require the following limitations to be observed:

Variable Name	Description	Maximum Capacity
NSMODE	Number of slosh modes per tank	3
NPT(I)	Number of partitions per tank	32
NCOR	Total number of degrees of freedom	60

4.4 USER'S GUIDE FOR THE FOUR-BEAM MODEL

4.4.1 Input

The sequence and format of an input data deck for a particular flight time are shown below:

Data Set	No. of Cards	Format	Description		
1	1	12A6	Title Card		
2	1	12I6	Column	Value	
			1-6	1	Rotary inertia is included.
				0	Rotary inertia is not included.
			7-12	1	Fundamental deflection functions are printed.
				0	Fundamental deflection functions are not printed.
			13-18	1	Shear deflections are included.
				0	Shear deflections are not included.

Data Set	No. of Cards	Format			Description
2	1	12I6	Column	Value	
			19-24	0	Only problem definition data printed.
				1	Integral terms are printed.
				2	Mass and stiffness arrays are printed.
				3	Intermediate eigenproblem array and all accuracy check arrays are printed.
			25-30		Not operational
			31-36	N	N modes are plotted.
			37-42		Not operational
			43-48		Not operational
			49-54		Not operational
			55-60	1	Output tape in Stodola format is generated.
				0	Tape is not generated.
			61-66		Not operational
			67-72	0	Slosh coordinates are not included.
				3	Slosh coordinates are included.
3	Variable	4E18.8			Mass distribution of a vehicle
4	Variable	7E11.7			Mass distribution of the hardware of a vehicle
5	1	8I10	1-10	N	N^{th} stage of flight
			11-20	N	N cases
			21-30	N	N slosh modes per tank
			31-40	0	Propellant mass is included in the bending program.

Data Set	No. of Cards	Format	Description		
5	1	8I10	Column	Value	
				1	Otherwise
			41-50	0	No lateral force distribution coefficients printout
				1	Otherwise
			51-60	0	Zero length interstage
				1	Otherwise
			61-70	0	No normalized slosh normal modes printout
				1	Otherwise
			71-80	0	No intermediate computation printout
				1	Otherwise
6	1	8E10.6	Column		
			1-10	Flight time	
			11-20	Longitudinal acceleration of a vehicle	
			21-80	Propellant levels	
7	1	8E10.6	Propellant mass densities		
8	1	8E10.6	Distance between tank bottom and center of mass for all tanks		
9	1	8E10.6	Beam lengths		
10	1	8E10.6	Distance between tank coordinate system and beam end for all tanks		
11	Variable	8E10.6	First three eigenvalues of a fluid system for all tanks		
12	Variable	10E8.4	First three eigenvectors of a fluid system for all tanks		
13	Variable	8E10.6	Abscissas of approximate fundamental deflection functions		
14	Variable	8E10.6	Ordinates of approximate fundamental deflection functions		
15	1	8I10	Number of partitions of each tank		

Data Set	No. of Cards	Format	Column
16	1	8E10.6	Partition heights of all tanks
17	Variable	8E10.6	HN(I, J), see Section 4.2
18	Variable	8E10.6	TTFCB(I, J), see Section 4.2

In case of multiple runs, input data decks for different flight times may be stacked together. In addition, the Stodola tape is used to generate the fundamental deflection functions. As an example, the data deck for Saturn V at flight time t = 0 is given on the following pages.

4.1.2 Output

All of the input information will be provided. By taking the proper options in the input data cards, the following output may be obtained:

1. Mass and stiffness properties of a vehicle
2. Frequencies, mode shapes and generalized mass printout and plots
3. Lateral force distribution coefficients due to the propellant dynamics
4. Other pertinent intermediate computations.

```
$DATA
SA-503  4-BEAM MODEL     T = 0.0                                           5/08/69
    1     0     1     1            13                                            3
   0.23626649E 04     0.23573295E 04     0.27036256E 04     0.31628707E 04
   0.33285230E 04     0.34410596E 04     0.36767982E 04     0.39405403E 04
   0.42356412E 04     0.45399702E 04     0.48011578E 04     0.50480263E 04
   0.53577202E 04     0.56919917E 04     0.61586480E 04     0.66828657E 04
   0.77538636E 04     0.90879822E 04     0.11009531E 05     0.13229081E 05
   0.18052180E 05     0.23924706E 05     0.29495270E 05     0.32815892E 05
   0.21687931E 05     0.11980989E 05     0.12246694E 05     0.12512399E 05
   0.12673966E 05     0.12834600E 05     0.11839444E 05     0.10833241E 05
   0.10083304E 05     0.93333669E 04     0.94514519E 04     0.15194759E 05
   0.21486629E 05     0.26989582E 05     0.33300475E 05     0.39198215E 05
   0.44646836E 05     0.49588704E 05     0.53733486E 05     0.57266059E 05
   0.60288021E 05     0.62788426E 05     0.64890358E 05     0.66530950E 05
   0.66973726E 05     0.66521650E 05     0.66191993E 05     0.66122411E 05
   0.66091067E 05     0.66095638E 05     0.66103289E 05     0.66114230E 05
   0.66146886E 05     0.66205900E 05     0.66233432E 05     0.66217429E 05
   0.66169973E 05     0.66070654E 05     0.65995812E 05     0.65959879E 05
   0.65973279E 05     0.66089760E 05     0.66166635E 05     0.66160175E 05
   0.66151226E 05     0.66136185E 05     0.66123054E 05     0.66115715E 05
   0.66174439E 05     0.66472948E 05     0.66426489E 05     0.65664463E 05
   0.64245011E 05     0.23282243E 05     0.28176086E 04     0.27943169E 04
   0.27733561E 04     0.27992799E 04     0.28295992E 04     0.30277514E 04
   0.32259036E 04     0.34105756E 04     0.35947303E 04     0.31216408E 04
   0.26293969E 04     0.27270104E 04     0.47518060E 04     0.15427260E 05
   0.26450445E 05     0.36523800E 05     0.45739597E 05     0.54427097E 05
   0.62322824E 05     0.69070327E 05     0.74929092E 05     0.79899129E 05
   0.84006128E 05     0.87567783E 05     0.90360886E 05     0.91766075E 05
   0.92142841E 05     0.92046022E 05     0.91846694E 05     0.91866706E 05
   0.91998785E 05     0.92114057E 05     0.92214917E 05     0.92259683E 05
   0.92266713E 05     0.92281804E 05     0.92303079E 05     0.92297454E 05
   0.92266346E 05     0.92152712E 05     0.91982504E 05     0.91911584E 05
   0.91943937E 05     0.91984893E 05     0.92037847E 05     0.92069631E 05
   0.92071696E 05     0.92074386E 05     0.92078077E 05     0.92111420E 05
   0.92199349E 05     0.92174018E 05     0.91908011E 05     0.91728349E 05
   0.91742394E 05     0.91770318E 05     0.91843259E 05     0.91898471E 05
   0.91895702E 05     0.91892676E 05     0.91888633E 05     0.91883332E 05
```

```
0.91871894E 05    0.91834450E 05    0.91647684E 05    0.91483038E 05
0.91471163E 05    0.91463544E 05    0.91549088E 05    0.91632633E 05
0.91626657E 05    0.91620681E 05    0.91626246E 05    0.91631931E 05
0.91657832E 05    0.91684851E 05    0.92248433E 05    0.52461524E 05
0.18609846E 04    0.15439439E 04    0.13505210E 04    0.11570981E 04
0.11902729E 04    0.13364321E 04    0.15454375E 04    0.17831982E 04
0.20697527E 04    0.23883941E 04    0.20667154E 04    0.14749879E 04
0.14943984E 04    0.17038439E 04    0.21107256E 04    0.26943844E 04
0.29260625E 04    0.29489598E 04    0.31466561E 04    0.36524189E 04
0.39769297E 04    0.40988808E 04    0.42410328E 04    0.44110898E 04
0.51752899E 04    0.68769500E 04    0.79648893E 04    0.61198972E 04
0.51822236E 04    0.47136198E 04    0.44939153E 04    0.15776188E 05
0.26984787E 05    0.37564240E 05    0.47332613E 05    0.56460783E 05
0.64506386E 05    0.71460281E 05    0.77381727E 05    0.82266733E 05
0.86181260E 05    0.89146151E 05    0.91270739E 05    0.92815909E 05
0.93218636E 05    0.92273187E 05    0.90354288E 05    0.87510861E 05
0.83617176E 05    0.79413260E 05    0.74021925E 05    0.55650749E 05
0.46615170E 04    0.50170264E 04    0.53247137E 04    0.57640376E 04
0.61267679E 04    0.61522581E 04    0.61345309E 04    0.61744359E 04
0.61517924E 04    0.62348036E 04    0.62672083E 04    0.62447421E 04
0.62179283E 04    0.62635377E 04    0.63153897E 04    0.63018345E 04
0.62800830E 04    0.62810884E 04    0.62859023E 04    0.62518303E 04
0.62100632E 04    0.62204508E 04    0.62453366E 04    0.62298808E 04
0.62037739E 04    0.62248390E 04    0.62643871E 04    0.62684572E 04
0.62617956E 04    0.62378630E 04    0.61993787E 04    0.61987057E 04
0.62219718E 04    0.62192002E 04    0.61913320E 04    0.61967866E 04
0.62296903E 04    0.62310693E 04    0.61931635E 04    0.61850534E 04
0.62158768E 04    0.62314718E 04    0.62162052E 04    0.62345662E 04
0.63128748E 04    0.64054938E 04    0.65534583E 04    0.66091667E 04
0.64970885E 04    0.63271238E 04    0.61058152E 04    0.25290804E 04
0.94003768E 03    0.99834408E 03    0.12032820E 04    0.13818518E 04
0.14292407E 04    0.14904870E 04    0.16841668E 04    0.18778466E 04
0.14571715E 04    0.10092639E 04    0.98426457E 03    0.96501241E 03
0.10560612E 04    0.11445960E 04    0.10922031E 04    0.10398102E 04
0.11145262E 04    0.11955305E 04    0.11738495E 04    0.11521687E 04
0.12504126E 04    0.13867420E 04    0.12890850E 04    0.11402718E 04
0.13260284E 04    0.15117849E 04    0.15271377E 04    0.52020798E 04
0.10390113E 05    0.15185668E 05    0.20038562E 05    0.24551768E 05
```

```
0.28178075E 05    0.31128205E 05    0.33816591E 05    0.36041375E 05
0.35202442E 05    0.32670067E 05    0.29944018E 05    0.26763094E 05
0.23129185E 05    0.19029560E 05    0.45328135E 04    0.23745145E 04
0.24588235E 04    0.24734155E 04    0.25360910E 04    0.26290373E 04
0.26963036E 04    0.27377726E 04    0.27520569E 04    0.27532770E 04
0.27610743E 04    0.27787396E 04    0.28037536E 04    0.28389924E 04
0.28705272E 04    0.28940089E 04    0.29095729E 04    0.29113218E 04
0.29122960E 04    0.29116402E 04    0.29119510E 04    0.29147473E 04
0.29128981E 04    0.28983819E 04    0.28835419E 04    0.28661635E 04
0.28413736E 04    0.28012610E 04    0.27365241E 04    0.26617008E 04
0.25590780E 04    0.24055935E 04    0.21954058E 04    0.53624876E 03
0.55948942E 03    0.72317525E 03    0.90184326E 03    0.12280898E 04
0.15552980E 04    0.18247921E 04    0.19843665E 04    0.13906609E 04
0.82056584E 03    0.52050165E 03    0.22043746E 03    0.20741816E 03
0.21052520E 03    0.28821405E 03    0.37107688E 03    0.36509304E 05
0.37320694E 03    0.28283148E 03    0.19245603E 03    0.17498977E 03
0.18212763E 03    0.17595043E 03    0.16524366E 03    0.15799631E 03
0.15193528E 03    0.14851008E 03    0.14631296E 03    0.14526146E 03
0.14482639E 03    0.15075475E 03    0.16096359E 03    0.16390047E 03
0.16170335E 03    0.15926271E 03    0.15662617E 03    0.15421908E 03
0.15202196E 03    0.16987196E 03    0.20857351E 03    0.39405461E 03
0.96594968E 03    0.20251109E 04    0.51284455E 04    0.66107659E 04
0.64443820E 04    0.63541707E 04    0.63768922E 04    0.63823495E 04
0.63511213E 04    0.63704042E 04    0.64977736E 04    0.66031736E 04
0.66285334E 04    0.65193910E 04    0.57682072E 04    0.47213021E 04
0.27521835E 04    0.10250668E 04    0.17785502E 04    0.25220336E 04
0.26600031E 04    0.27421946E 04    0.24391747E 04    0.21121795E 04
0.16727557E 04    0.13304935E 04    0.13228143E 04    0.12501937E 04
0.75084723E 03    0.29740779E 03    0.17932668E 03    0.75859934E 02
0.62762536E 02    0.50345443E 02    0.51958788E 02    0.53572134E 02
0.98268116E 02    0.14556211E 03    0.34015844E 03    0.65754948E 03
0.65802542E 03    0.60639845E 03    0.58140813E 03    0.56502956E 03
0.56083677E 03    0.56083677E 03    0.56111835E 03    0.56151383E 03
0.56220813E 03    0.56304304E 03    0.56276742E 03    0.56188857E 03
0.53101644E 03    0.48144759E 03    0.43271751E 03    0.38369088E 03
0.30107227E 03    0.21845365E 03    0.18900776E 03    0.25514547E 03
0.35930001E 03    0.52928470E 03    0.66920247E 03    0.75184672E 03
0.50964312E 03    0.13377658E 03    0.42684788E 02
```

```
2362664+04 2357329+04 2703625+04 3162870+04 3328523+04 3441059+04 3676798+04
3940540+04 4233641+04 4539970+04 4801157+04 5048026+04 5357720+04 5691991+04
6158648+04 6682865+04 7753863+04 9087982+04 1100953+05 1322908+05 1805218+05
2392470+05 2949527+05 3281589+05 2168793+05 1198098+05 1224669+05 1251239+05
1267396+05 1283460+05 1183944+05 1083324+05 1008330+05 9333366+04 9451451+04
9729587+04 8016660+04 6041035+04 5475306+04 5098347+04 4874232+04 4745298+04
4421236+04 4086905+04 3843915+04 3681305+04 3722174+04 3903649+04 3489253+04
2755130+04 2425473+04 2355891+04 2324547+04 2329117+04 2336769+04 2347711+04
2380366+04 2439381+04 2466912+04 2450909+04 2403453+04 2304134+04 2229292+04
2193359+04 2206759+04 2323240+04 2400114+04 2393655+04 2384705+04 2369664+04
2356534+04 2349195+04 2407920+04 2706428+04 2983622+04 3122108+04 3205112+04
3001454+04 2817608+04 2794316+04 2773356+04 2799279+04 2829599+04 3027751+04
3225903+04 3410575+04 3594730+04 3121640+04 2629396+04 2727010+04 2849200+04
2763669+04 2638750+04 2421537+04 2204325+04 2316368+04 2494173+04 2381301+04
2237248+04 2061996+04 1881272+04 2012736+04 2233198+04 1923295+04 1442513+04
1202769+04 1003441+04 1023453+04 1155532+04 1270804+04 1371664+04 1416429+04
1423460+04 1438551+04 1459826+04 1454201+04 1423093+04 1309459+04 1139251+04
1068331+04 1100685+04 1141640+04 1194593+04 1226378+04 1228443+04 1231133+04
1234824+04 1268167+04 1356095+04 1330764+04 1064757+04 8850965+03 8991409+03
9270659+03 1000005+04 1055217+04 1052449+04 1049423+04 1045380+04 1040079+04
1028641+04 9911980+03 8044316+03 6397859+03 6279107+03 6202911+03 7058353+03
7893804+03 7834043+03 7774281+03 7829938+03 7886778+03 8145798+03 8415978+03
1405180+04 2031468+04 1860984+04 1543943+04 1350521+04 1157098+04 1190272+04
1336432+04 1545437+04 1783198+04 2069752+04 2388394+04 2066715+04 1474987+04
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3652418+04 3976929+04 4098880+04 4241032+04 4411089+04 5175289+04 6876950+04
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1452153+04 1454892+04 1443585+04 1157585+04 8909167+03 7444281+03 5979395+03
6196536+03 6422464+03 6197802+03 5929664+03 6385757+03 6904278+03 6768726+03
6551211+03 6561265+03 6609403+03 6268683+03 5851013+03 5954889+03 6203747+03
6049189+03 5788119+03 5998770+03 6394252+03 6434952+03 6368337+03 6129011+03
5744168+03 5737438+03 5970099+03 5942383+03 5663701+03 5718247+03 6047284+03
6061074+03 5682015+03 5600915+03 5909148+03 6065099+03 5912433+03 6096042+03
6879129+03 7805318+03 9284964+03 9996209+03 9588475+03 9227360+03 8978282+03
8889571+03 9400376+03 9983440+03 1203282+04 1381851+04 1429240+04 1490487+04
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1684166+04 1877846+04 1457171+04 1009263+04 9842645+03 9650124+03 1056061+04
1144596+04 1092203+04 1039810+04 1114526+04 1195530+04 1173849+04 1152168+04
1250412+04 1386742+04 1289085+04 1140271+04 1326028+04 1511784+04 1527137+04
1459741+04 1287666+04 1114166+04 1446960+04 1889024+04 1893141+04 1670041+04
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5594894+03 7231752+03 9018432+03 1228089+04 1555298+04 1824792+04 1984366+04
1390660+04 8205658+03 5205016+03 2204374+03 2074181+03 2105252+03 2882140+03
3710768+03 3650930+05 3732069+03 2828314+03 1924560+03 1749897+03 1821276+03
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1448263+03 1507547+03 1609635+03 1639004+03 1617033+03 1592627+03 1566261+03
1542190+03 1520219+03 1698719+03 2085735+03 3940546+03 9659496+03 2025110+04
5128445+04 6610765+04 6444382+04 6354170+04 6376892+04 6382349+04 6351121+04
6370404+04 6497773+04 6603173+04 6628533+04 6519391+04 5768207+04 4721302+04
2752183+04 1025066+04 1778550+04 2522033+04 2660003+04 2742194+04 2439174+04
2112179+04 1672755+04 1330493+04 1322814+04 1250193+04 7508472+03 2974077+03
1793266+03 7585993+02 6276253+02 5034544+02 5195878+02 5357213+02 9826811+02
1455621+03 3401584+03 6575494+03 6580254+03 6063984+03 5814081+03 5650295+03
5608367+03 5608367+03 5611183+03 5615138+03 5622081+03 5630430+03 5627674+03
5618885+03 5310164+03 4814475+03 4327175+03 3836908+03 3010722+03 2184536+03
1890077+03 2551454+03 3593000+03 5292847+03 6692024+03 7518467+03 5096431+03
1337765+03 4268478+02
          1         1         1         1         0         0         0         0
  0.0       11.37137  9.957054  3.84683   15.153894 5.42925   16.543782 10.898632
70.8016    1143.3979 70.8016   1143.3979 1143.3979 799.3213
5.436664   2.15754   8.683117  3.140649  8.833208  5.96866
 45.25      21.75     17.75
 7.9714     5.8892    4.68      4.67968   26.1197   12.22968
2.2925     5.6755    8.8979    2.5206    5.8236    9.0631    2.0642    5.555
8.7901     2.0947    5.635     8.8779    1.8842    5.388     8.6083    2.0329
 5.5301     8.764
1141-01 2881-01-6537-01 4385-01-9301-02 1    +01 1275+00-5035-01 2848-01-185 -01
-6518-02-2353-01 4346-01-3072-01 74  -02-4334-01 1   +01 1345+00-6384-01 4028-01
4218-02 9576-02-1421-01 1028-01-2739-02 9889-02-757 -01 1   +01 1429+00-6945-01
```

```
 6504-01 431 -01-1274+00 7754-01-138 -01 1   +01 181 +00-5529-01 2703-01-16  -01
-2649-01-2267-01 421 -01-2798-01 58  -02-3501-01 1   +01 1929+00-8158-01 4701-01
 1515-01-361 -02 1501-01-7955-02 11  -02-396 -03-102 +00 1   +01 2141+00-9559-01
 148 -01-3301-01 9027-01-9992-01 3644-01 1   +01 7743-01-3942-01 2714-01-1685-01
-1002-01 159 -01-5267-01 5974-01-2195-01-2196-01 1   +01 9263-01-476 -01 3183-01
 6845-02-1003-01 3483-01-3827-01 1371-01 3311-02-5231-01 1   +01 1005+00-5164-01
 1218+00-588 -01 153 +00-2013+00 762 -01 1   +01 1243+00-5412-01 2932-01-1888-01
-3816-01 2975-02-5893-01 9119-01-3838-01 1406-01 1   +01 1272+00-6207-01 3991-01
 2207-01 1785-01-1007-01-6117-02 6377-02-2456-01-6294-01 1   +01 1393+00-6944-01
 4516-02-2398-01 7   -01-7858-01 2993-01 1   +01 1733-01-1083-01 7282-02-5712-02
-1852-02 2223-02-7058-02 6448-02-1759-02-5345-02 1   +01 2467-01-1432-01 1042-01
 1474-02-2025-02 6347-02-5581-02 1485-02 1015-02-146 -01 1   +01 2861-01-1603-01
 2035-01-4811-01 1393+00-1569+00 5804-01 1   +01 6951-01-3625-01 2209-01-1559-01
-1251-01 2698-01-8906-01 1017+00-3816-01-1503-01 1   +01 8333-01-4317-01 2916-01
 8858-02-1909-01 6385-01-7114-01 2634-01-3696-03-4613-01 1   +01 9012-01-4682-01
-0.9652   -0.988    -0.63     0.0       -0.8773   -0.9929   -0.8      0.0
 1.0       0.96      0.53     0.0       -0.9983   -0.8202   -0.2344   0.0
-0.8089   -0.9652   -0.988    -0.63     -0.6378   -0.8773   -0.9929   -0.8
 0.7339    1.0       0.96      0.58     -0.92     -0.9983   -0.8202   -0.2344
-0.74     -1.0      -0.7       0.0      -0.5071   -0.8284   -0.95      0.0
 0.9       1.0       0.63      0.0      -1.0      -0.98     -0.2898    0.0
-0.1967   -0.74     -1.0      -0.75     -0.1186   -0.5071   -0.8284   -1.0
 0.2222    0.9       1.0       0.66     -0.3      -1.0      -0.98     -0.2898
-0.9677   -0.9991   -0.72     -0.3502    0.9902    0.9894    0.66      0.2294
 0.7673    0.9361    0.94      0.3001   -0.9862   -0.9915   -0.66     -0.2276
-0.4264   -0.6037   -0.87     -0.9566    0.4935    0.69      0.94      0.9849
 0.1316    0.2295    0.5079    0.7375   -0.4751   -0.6686   -0.92     -0.9796
-1.2214    1.2786    6.7786    9.7786   -1.3892    0.8608    3.3608    8.8608
-0.18      3.57      13.57     17.07    -3.68     -0.18      3.57      13.57
-3.6197    0.13      12.38     15.88    -3.73      0.02      6.02      9.77
        20        20        21        22        28        28
  0.5       0.2       0.75      0.25      0.6       0.4
 0.93105  -0.19023   0.089381  0.91715  -0.24938   0.1116    1.133    -0.24506
 0.12149   0.85781  -0.39294   0.21894   1.1765   -0.2354    0.12069   1.2356
-0.3057    0.15845
 658953+02 141898+03 218344+03 288758+03 349232+03 398517+03 43591 +03 460371+03
 47081 +03 466415+03 446915+03 412695+03 364752+03 304518+03 23363 +03 157396+03
 927739+02 432502+02 106483+02 828505-05
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245042+01 529698+01 825329+01 110622+02 135234+02 155011+02 16919 +02 17745 +02
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264904+01 868492+00 327218-01 294694-05
424226+03 866266+03 126976+04 161906+04 190812+04 213262+04 228939+04 237614+04
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29562 +03 102769+03 17328 +02-103933-01 486374-03
140977+02 293034+02 447096+02 594357+02 727003+02 838783+02 925297+02 983975+02
101375+03 10147 +03 987661+02 934213+02 85673 +02 758617+02 644455+02 520009+02
392098+02 268438+02 15757 +02 689365+01 131242+01 376999-05
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Section 5
EXAMPLE

Vibrational characteristics of Saturn V first stage flight are studied in this section. The vehicle is modeled as a series of four beams rigidly connected to each other (see Fig. 1c). Two cases of one slosh mode per tank and three slosh modes per tank are investigated. The longitudinal acceleration of Saturn V is shown in Fig. 8 and the first three slosh frequencies of the tanks of the vehicle are shown in Figs. 9 through 11. Most of the slosh modes, it was found, came before the vehicle bending modes. In the latter portion of the flight, however, some of the slosh modes (mainly higher modes) will mingle with the vehicle bending modes. Further observations will be made in the second phase of the present contract. It is to use the system bending modes obtained from this program to develop a hybrid simulation program for Saturn V atmospheric flight.

The first 25 modes of the case having three slosh modes per tank at flight time $t = 0$ are shown in Figs. 12 through 36. The corresponding frequency and generalized mass of each mode are given on the top of each plot. The first three vehicle bending modes of this case are also indicated in these figures. Similarly, the first 13 modes of the case having only one slosh mode per tank are presented in Figs. 37 through 49. Mass and stiffness properties of the vehicle for the above cases are provided in Figs. 50 through 53. For comparison, the mass distribution and vehicle bending modes of the vehicle at flight time $t = 40$ are shown in Figs. 54 through 57.

The lateral force distribution coefficients due to the liquid propellant of Saturn V are computed. It is found that the influence of higher slosh modes to the coefficients ${}_mA_\alpha$ and ${}_mB_\psi$ is small and diminishes rapidly for layers away from the propellant-free surface. These coefficients which take the first three slosh modes into account and the coefficients $({}_mC_\xi)_i$, $i = 1, 2, 3$, are shown in Figs. 58 through 67.

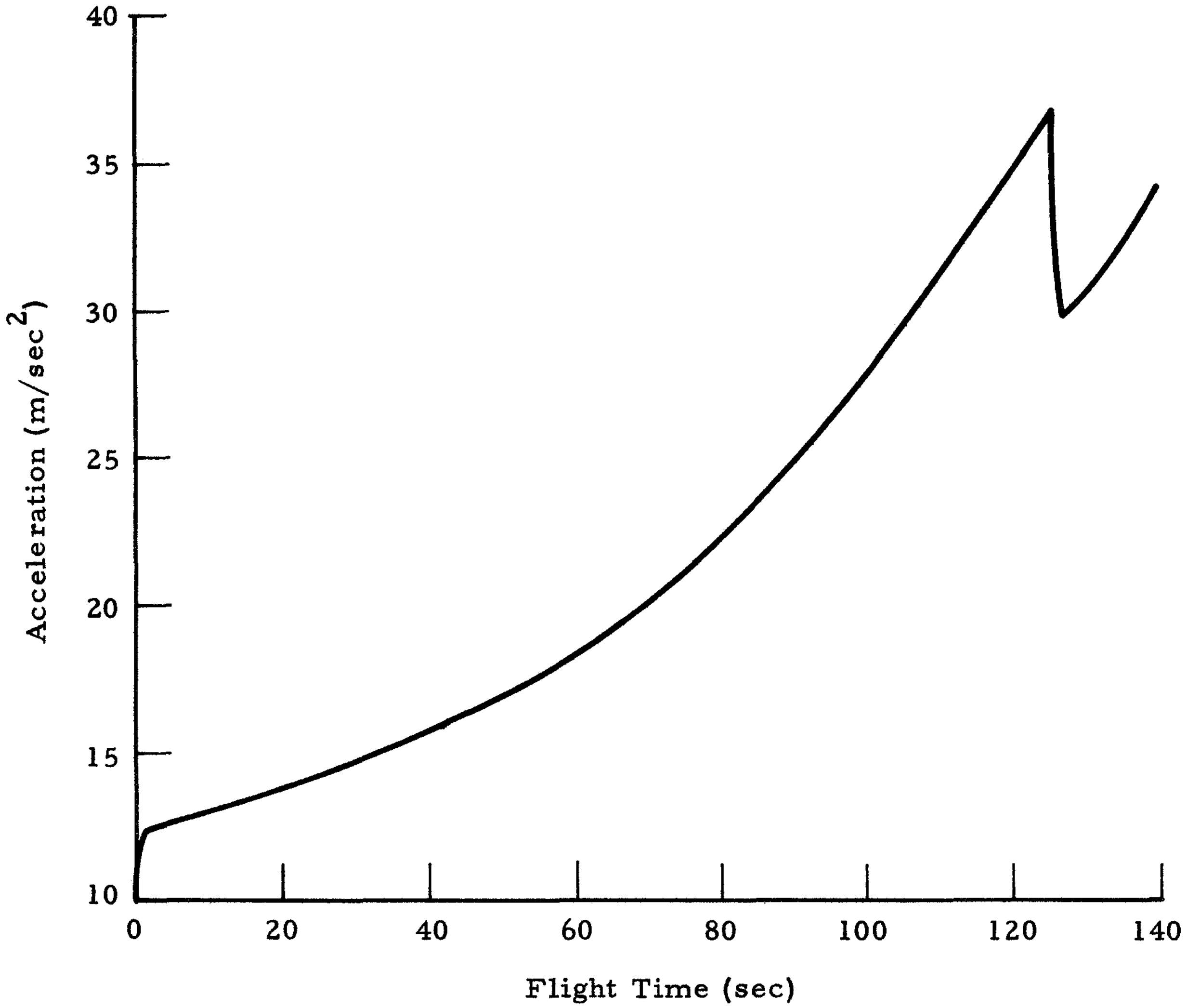


Fig. 8 - Longitudinal Acceleration of Saturn V

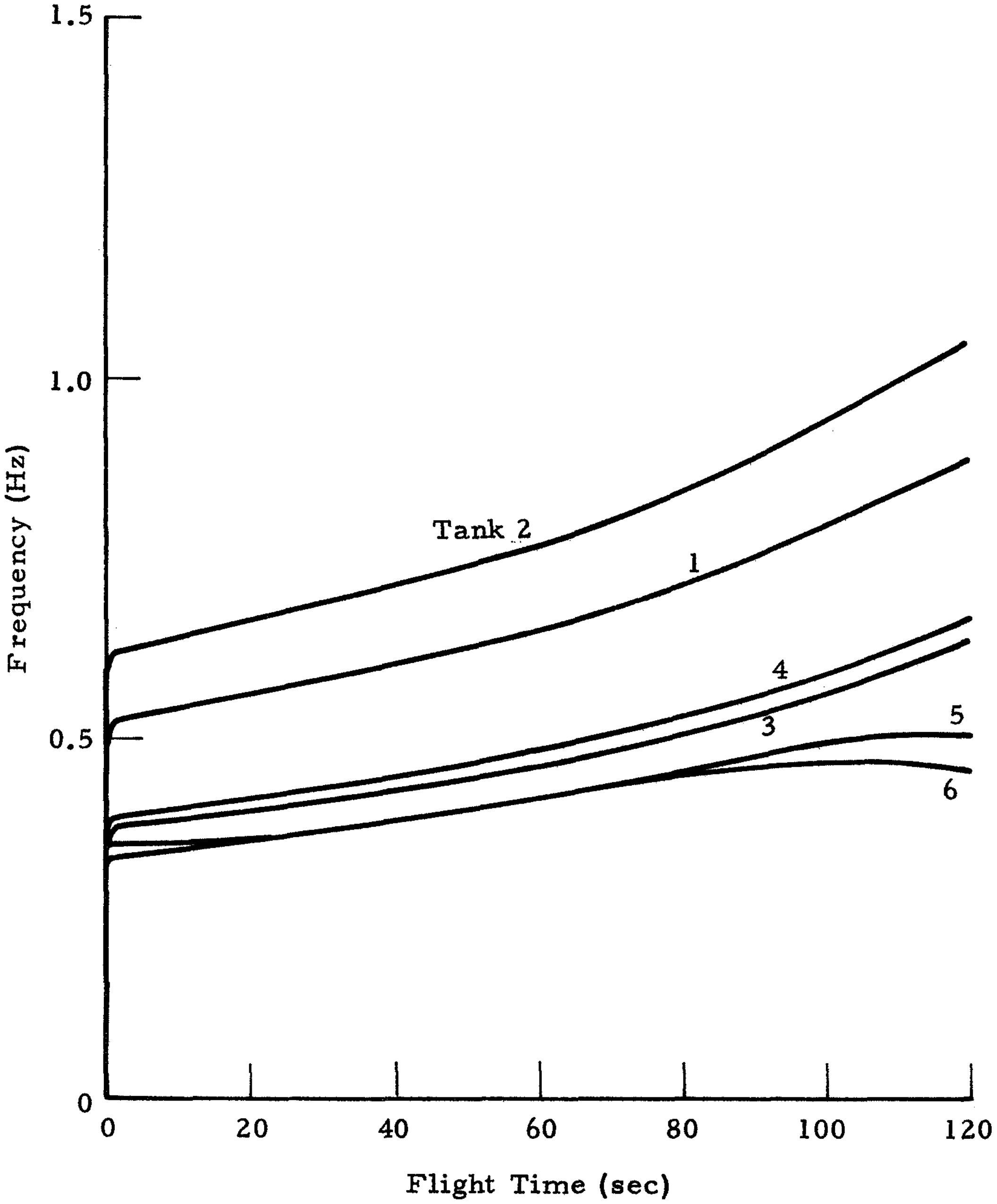


Fig. 9 - First Mode Slosh Frequencies of the Tanks of Saturn V

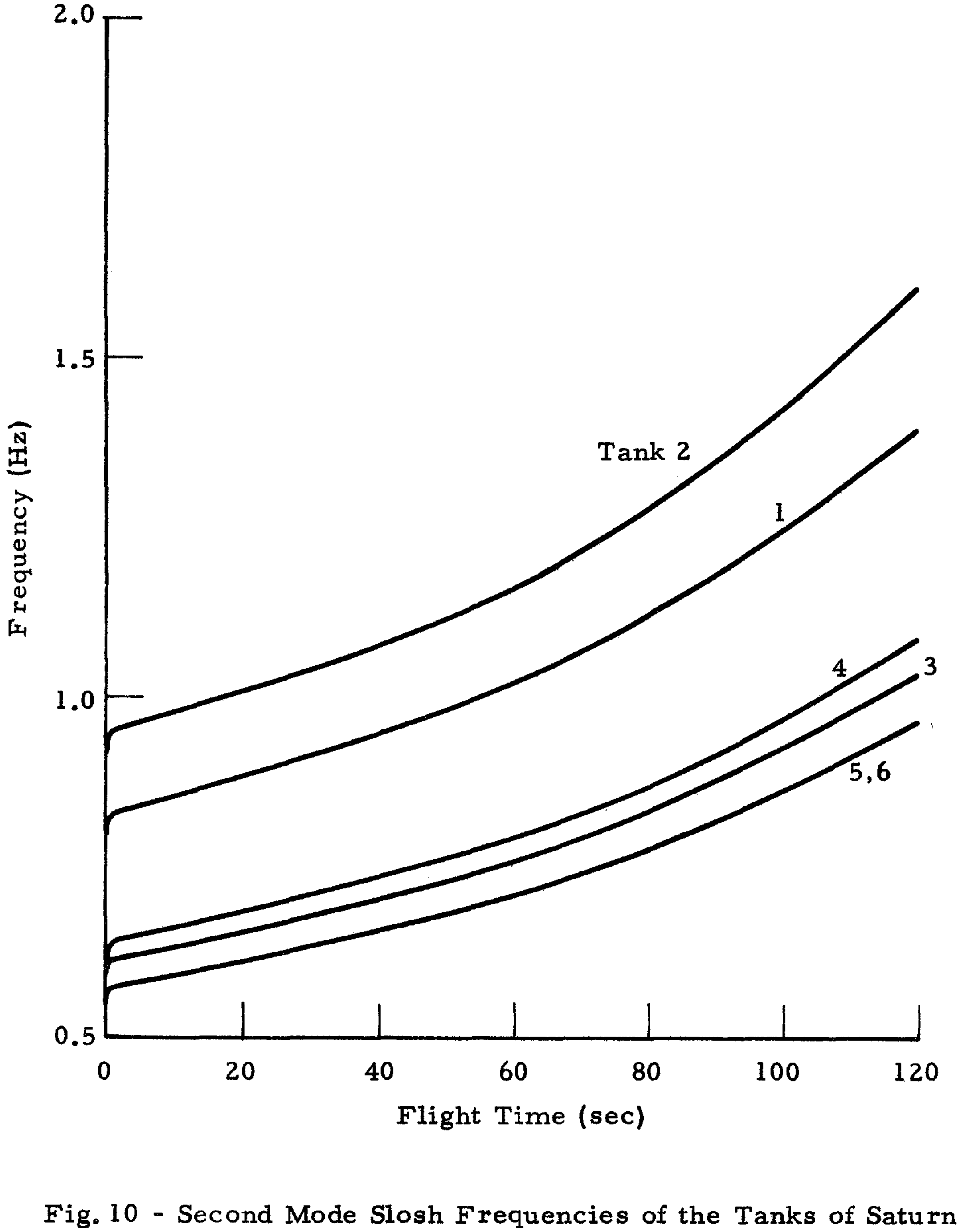


Fig. 10 - Second Mode Slosh Frequencies of the Tanks of Saturn V

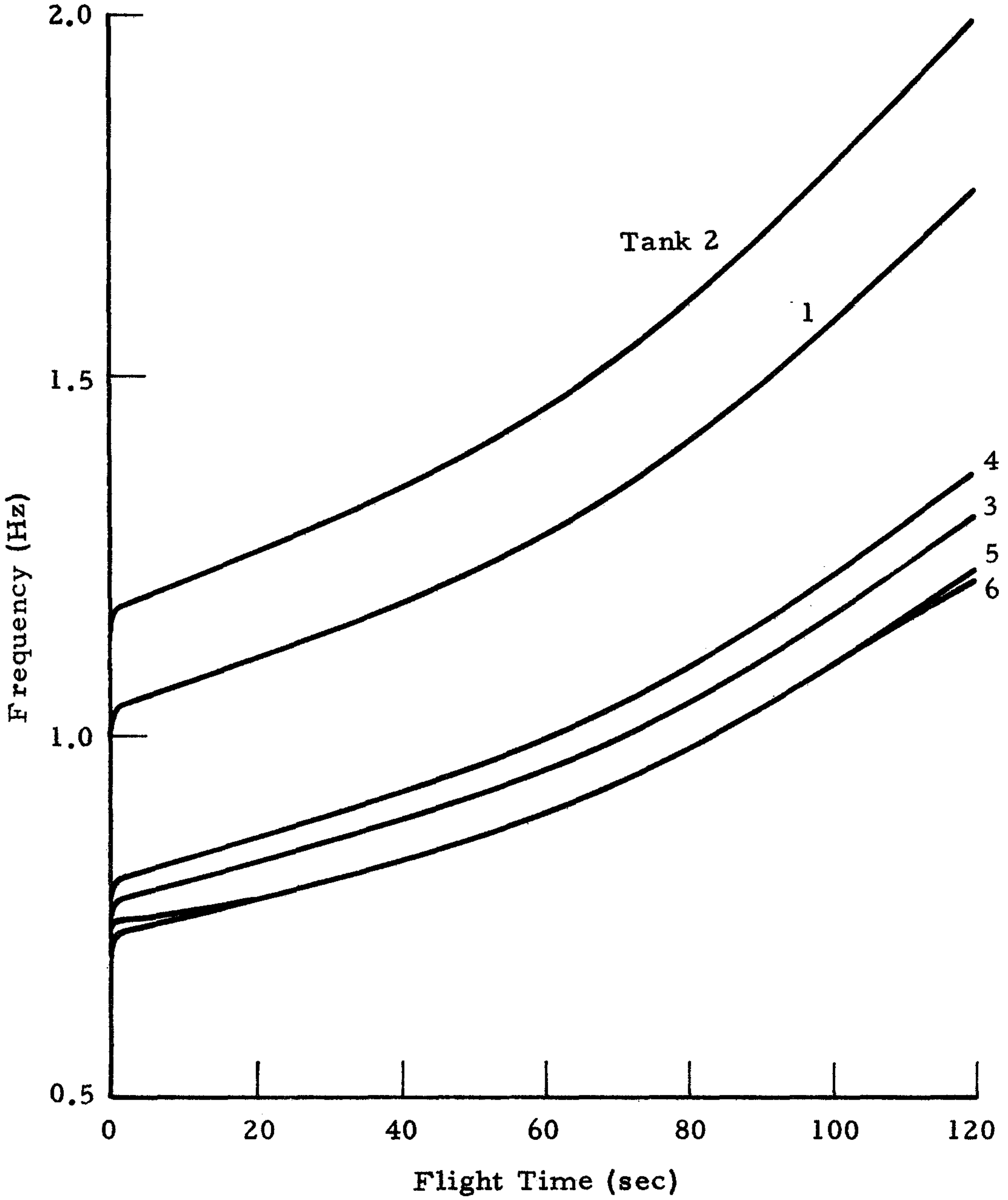


Fig. 11 - Third Mode Slosh Frequencies of the Tanks of Saturn V

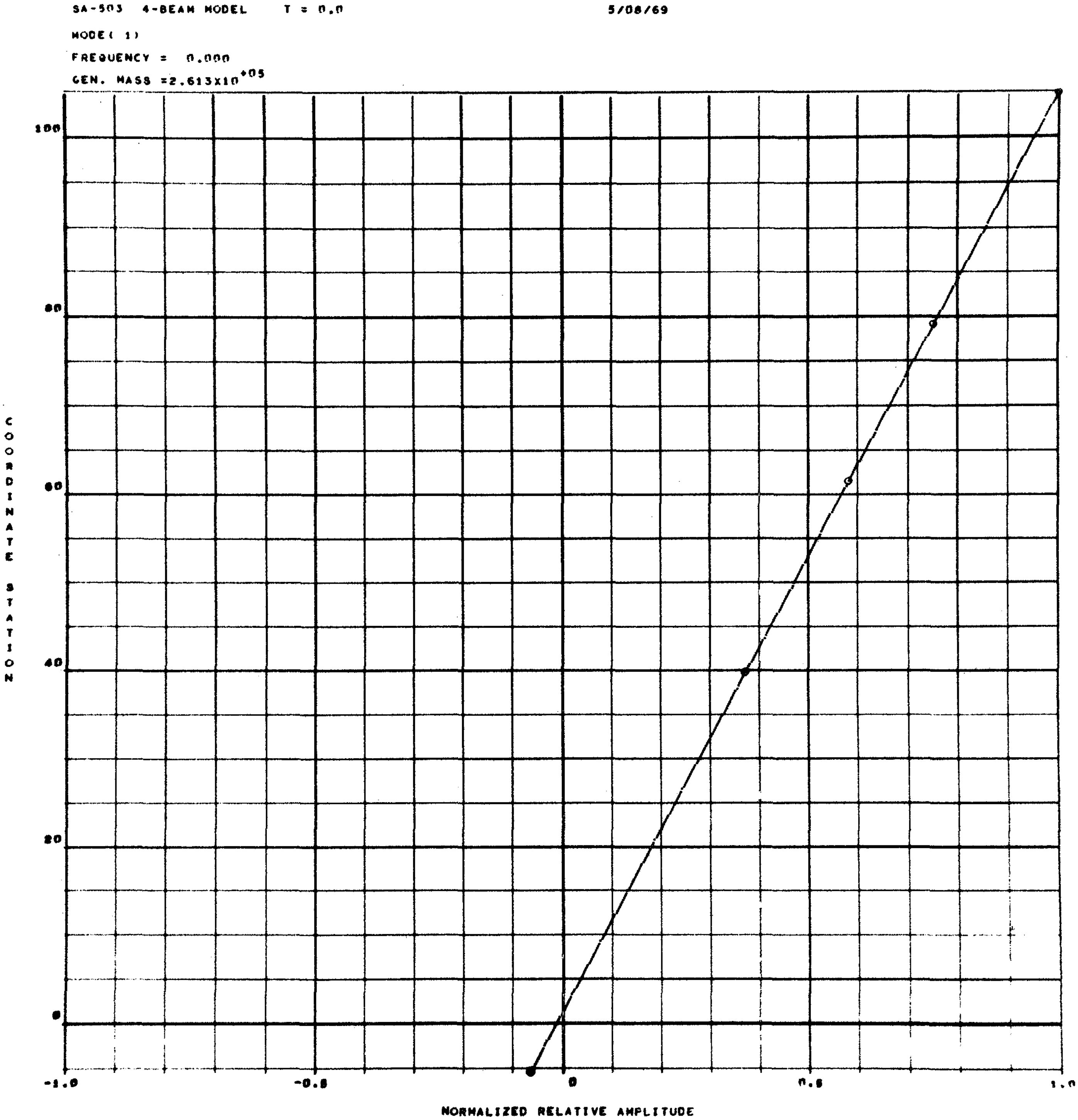


Fig. 12 - 1st Mode Shape (three slosh modes per tank)

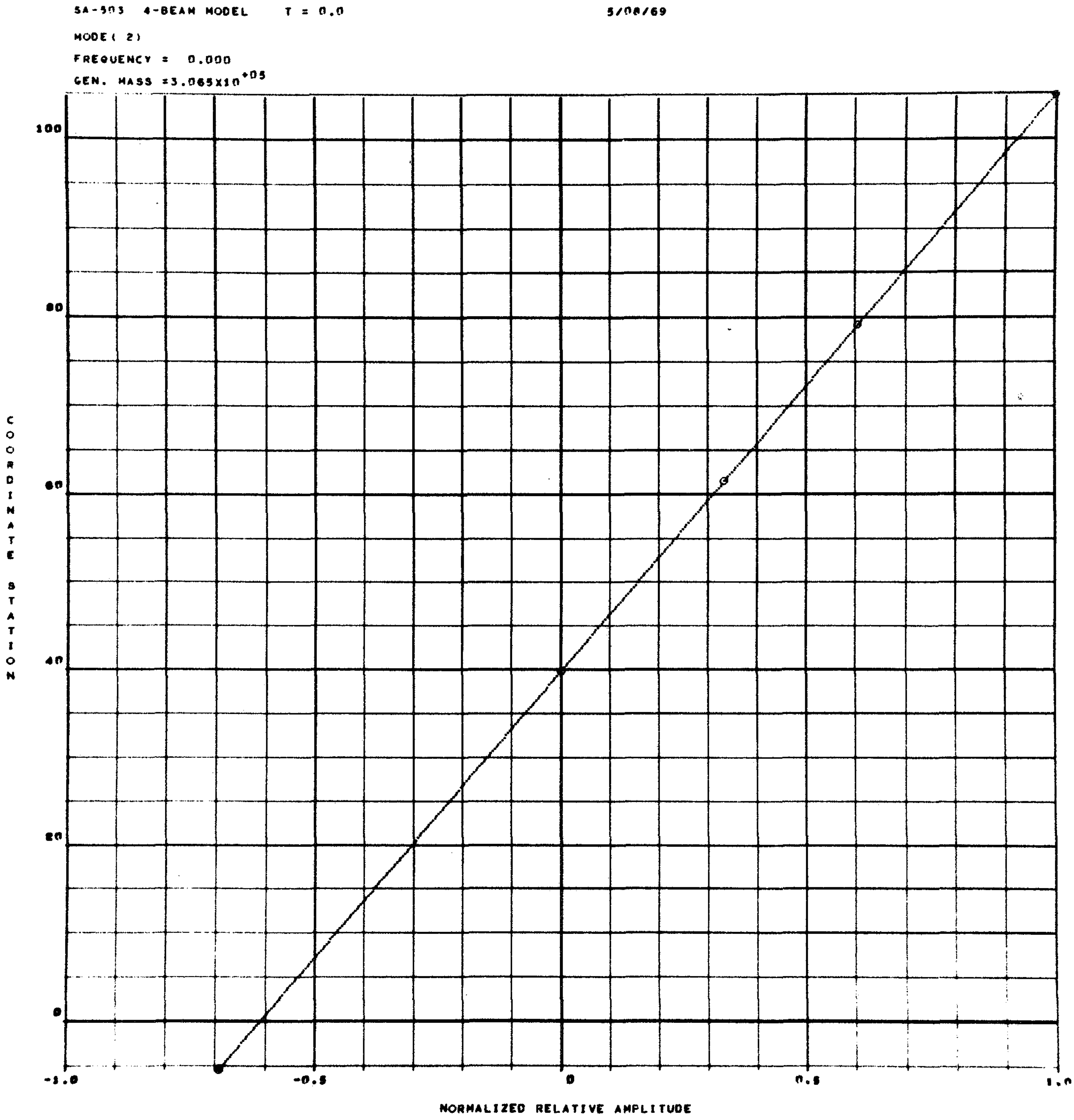


Fig. 13 - 2nd Mode Shape (three slosh modes per tank)

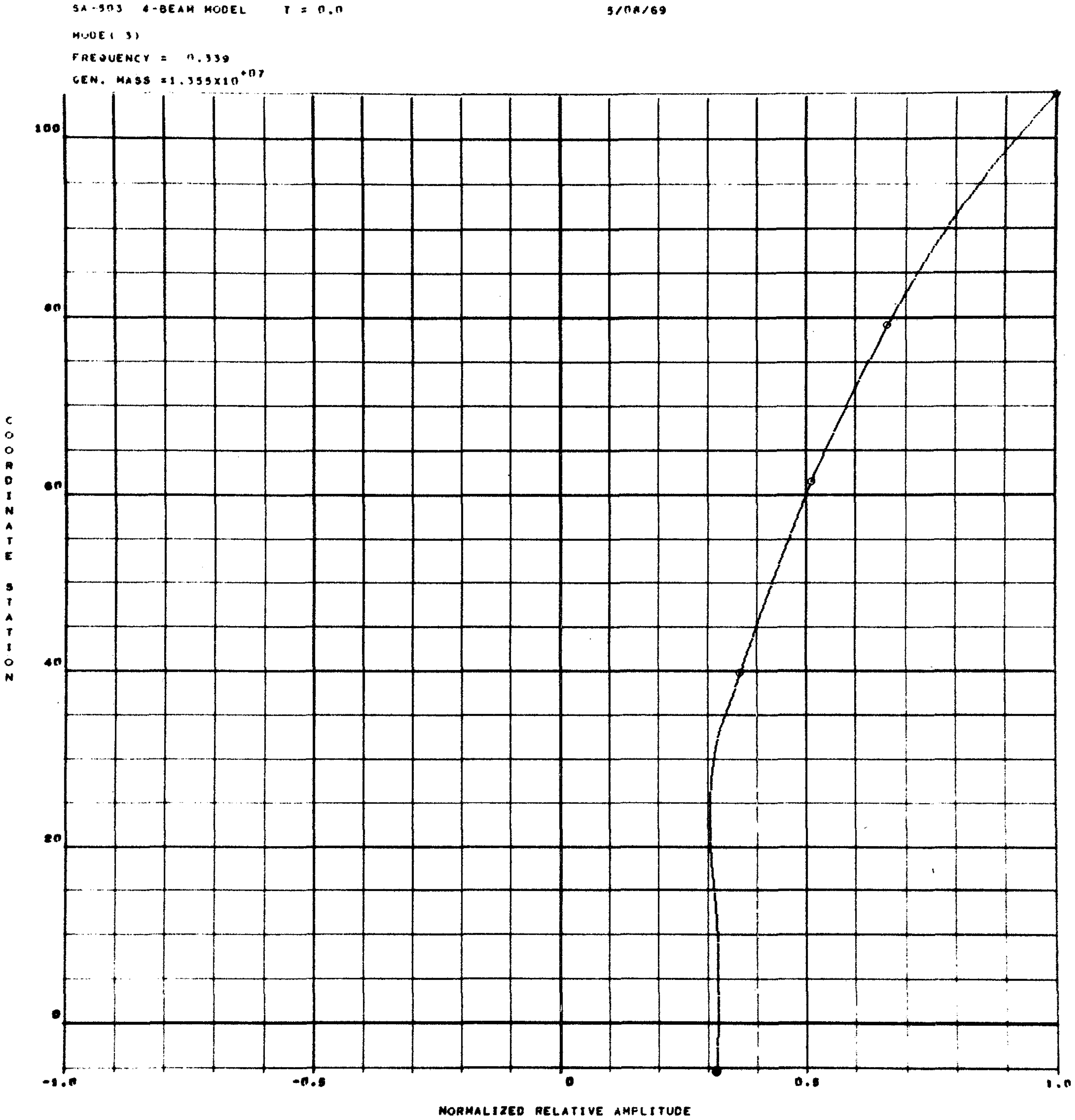


Fig. 14 - 3rd Mode Shape (three slosh modes per tank)

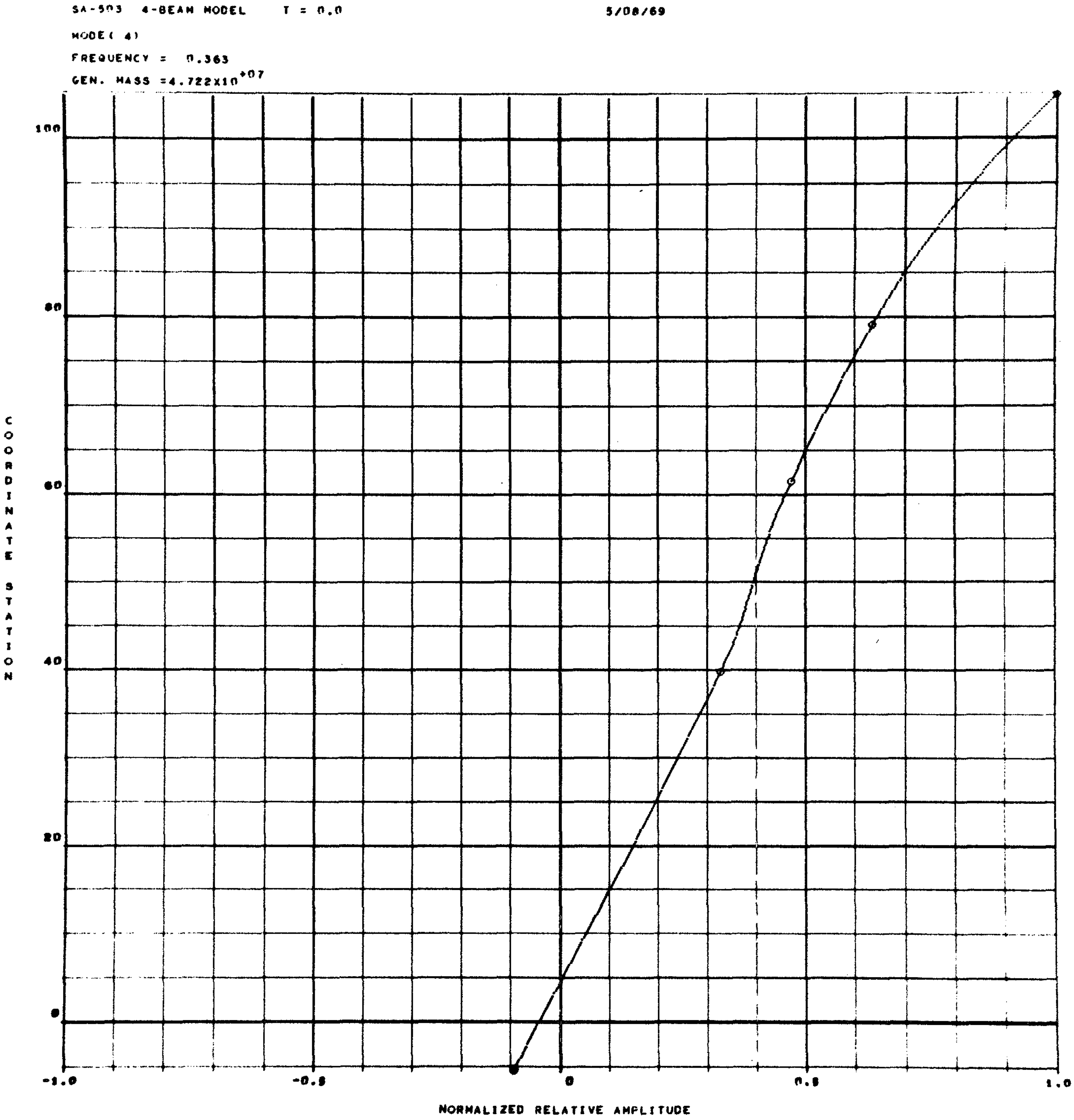


Fig. 15 - 4th Mode Shape (three slosh modes per tank)

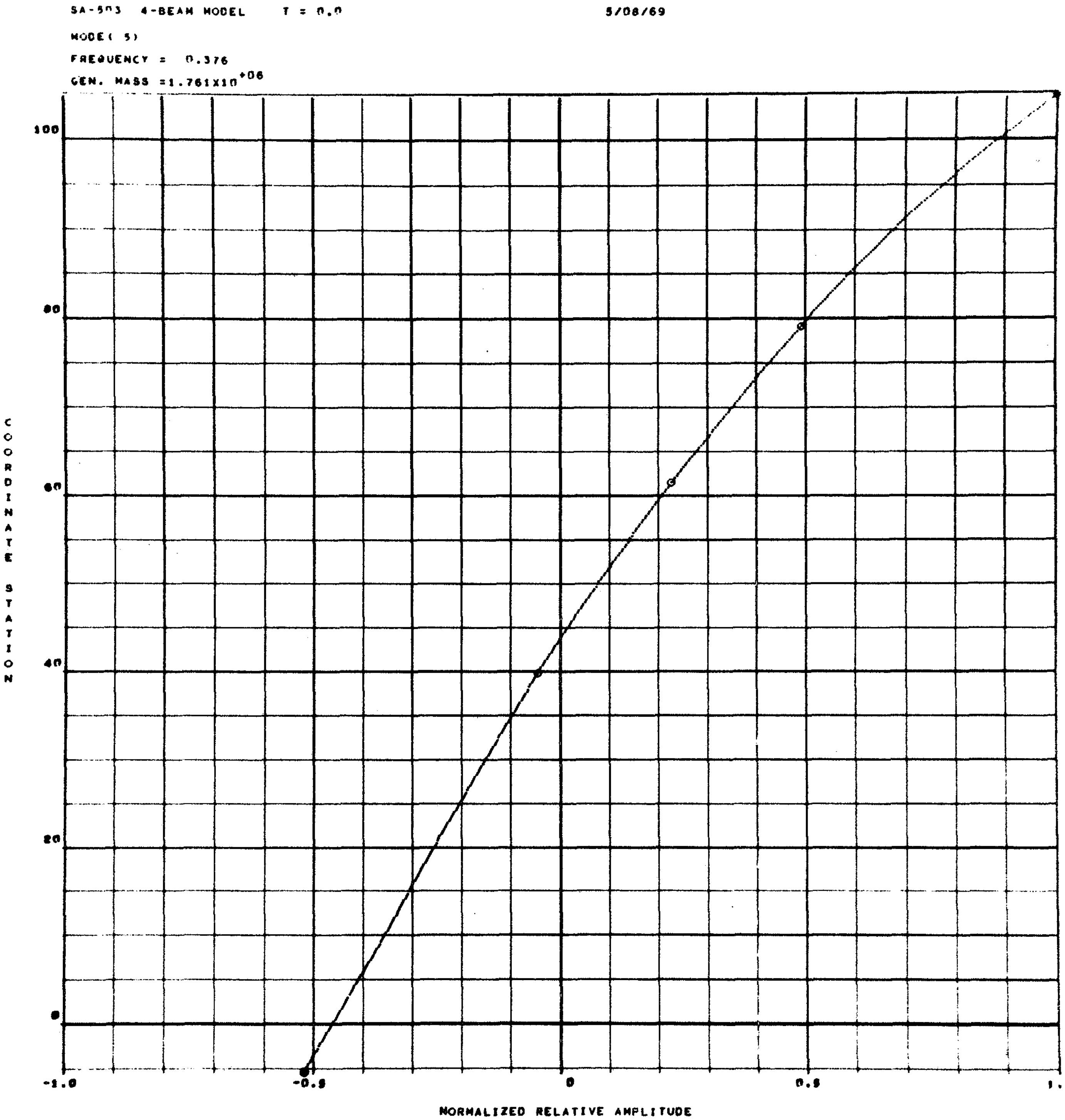


Fig. 16 - 5th Mode Shape (three slosh modes per tank)

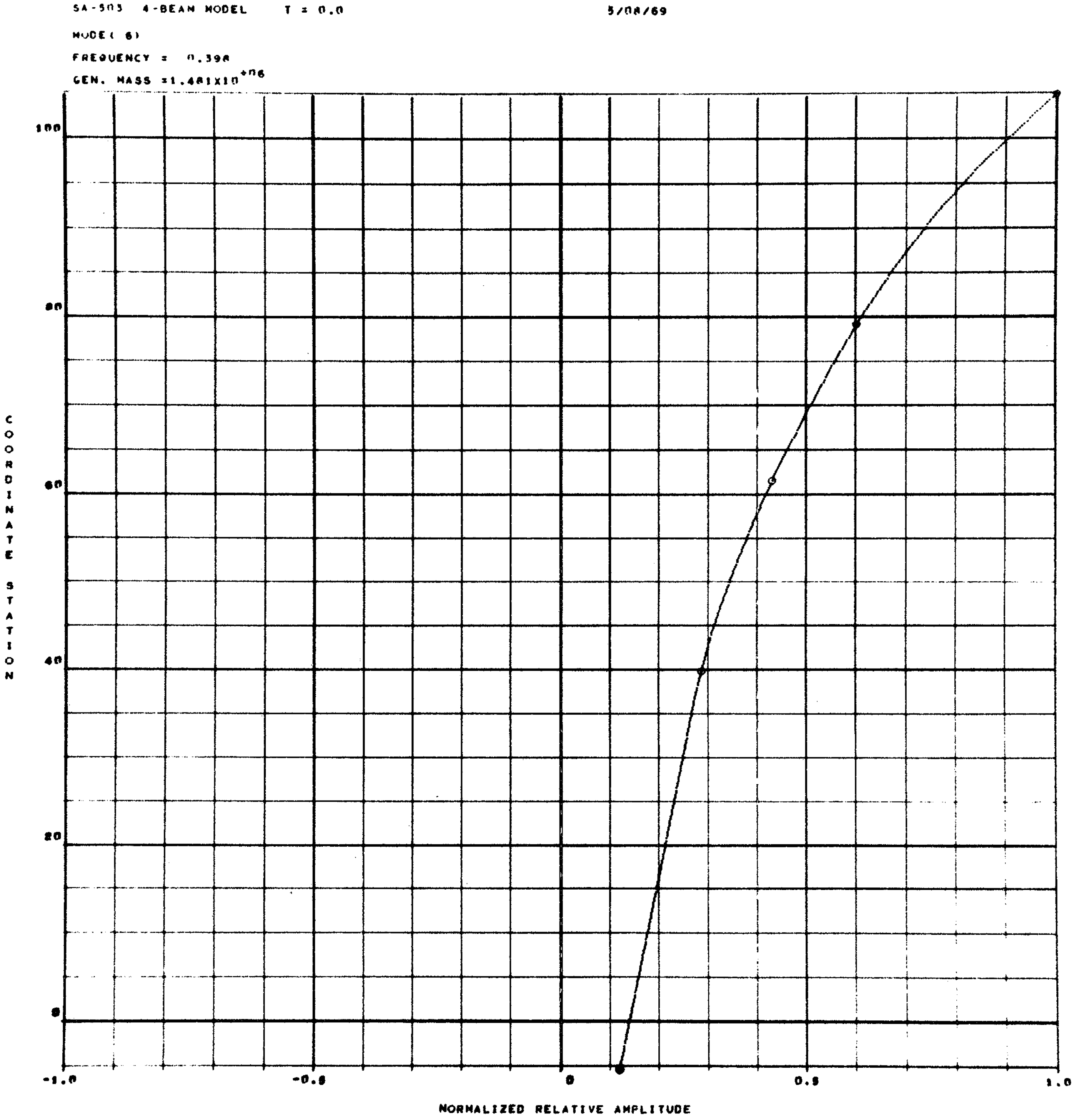


Fig. 17 - 6th Mode Shape (three slosh modes per tank)

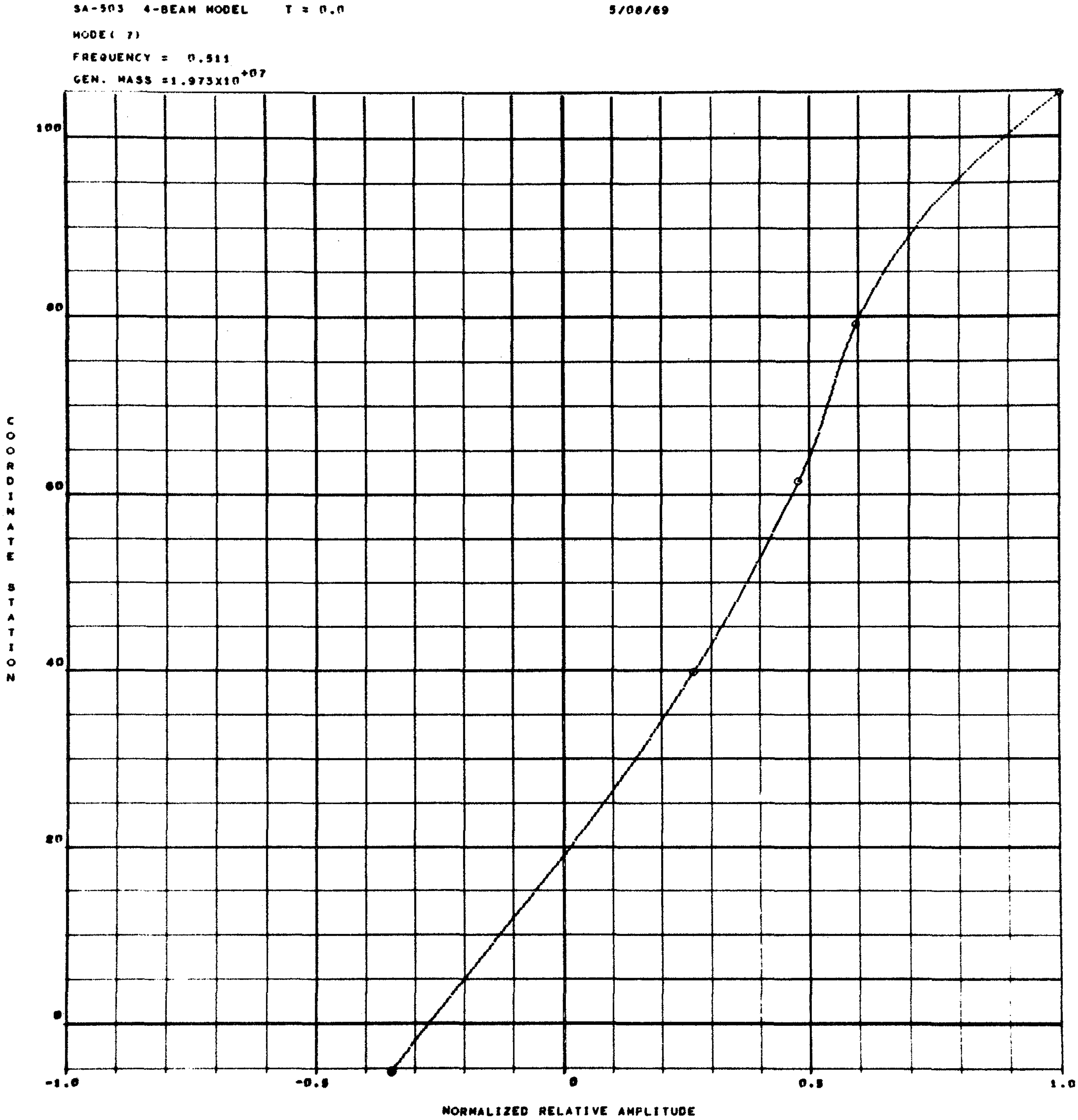


Fig. 18 - 7th Mode Shape (three slosh modes per tank)

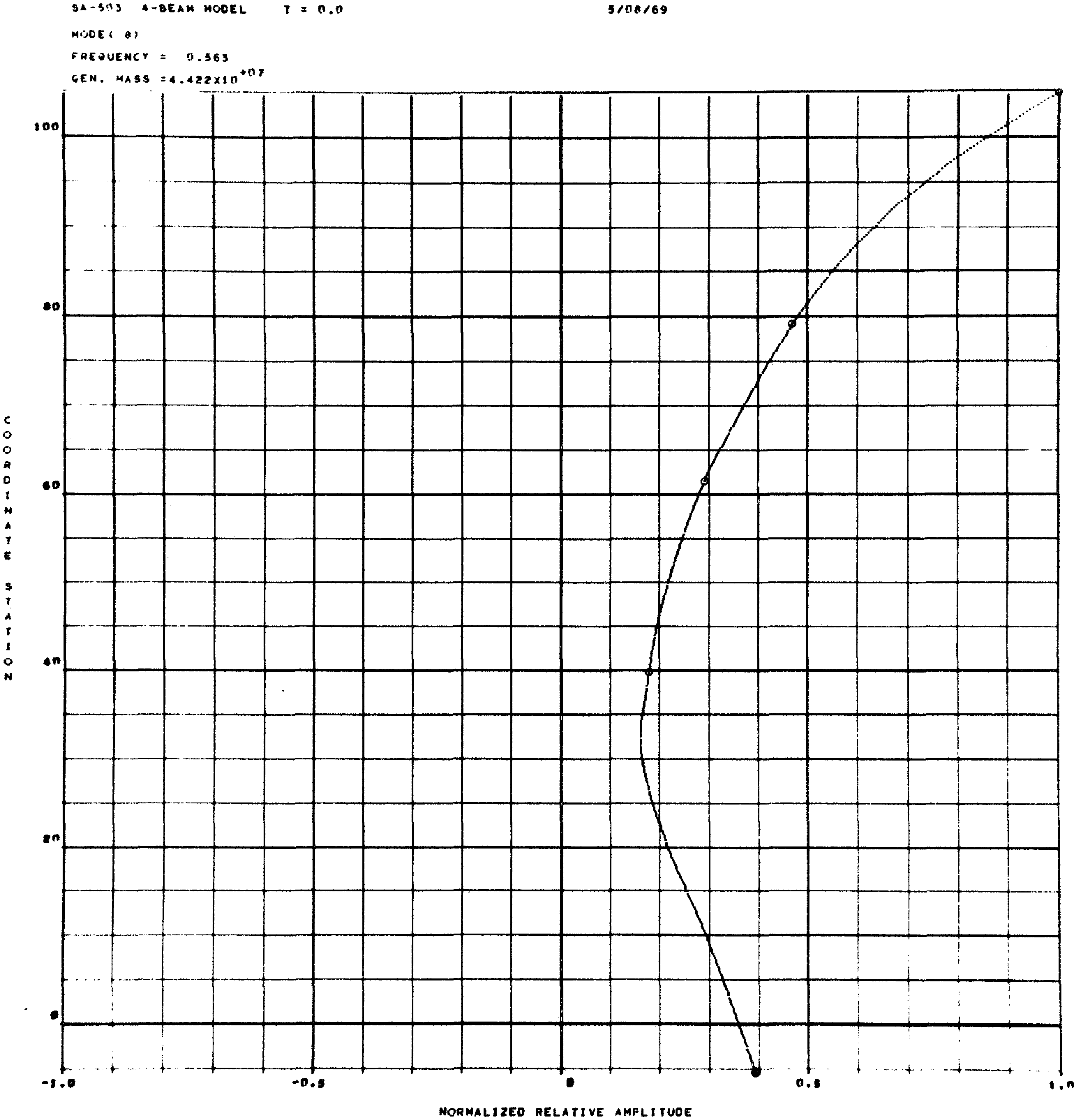


Fig. 19 - 8th Mode Shape (three slosh modes per tank)

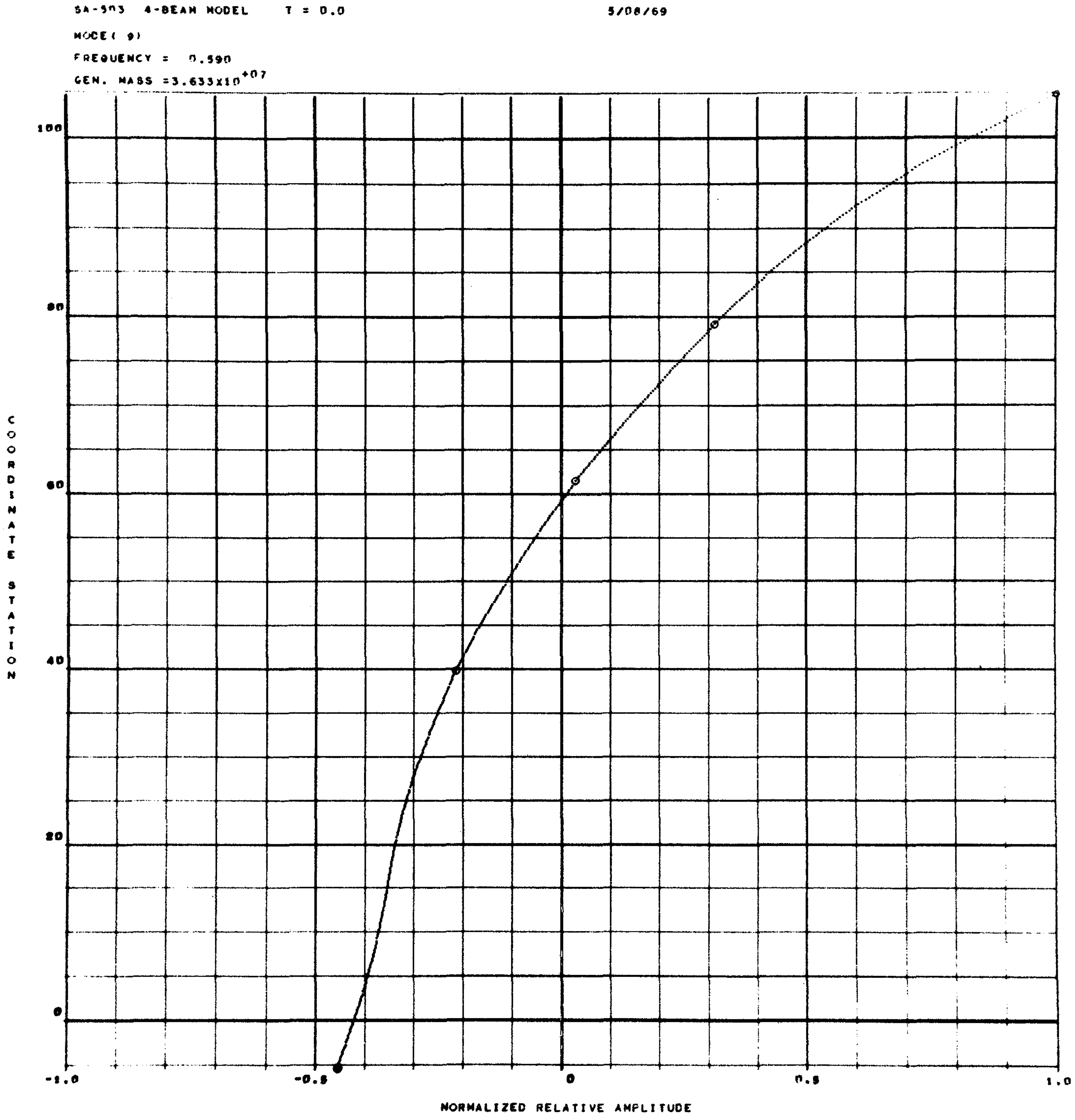


Fig. 20 - 9th Mode Shape (three slosh modes per tank)

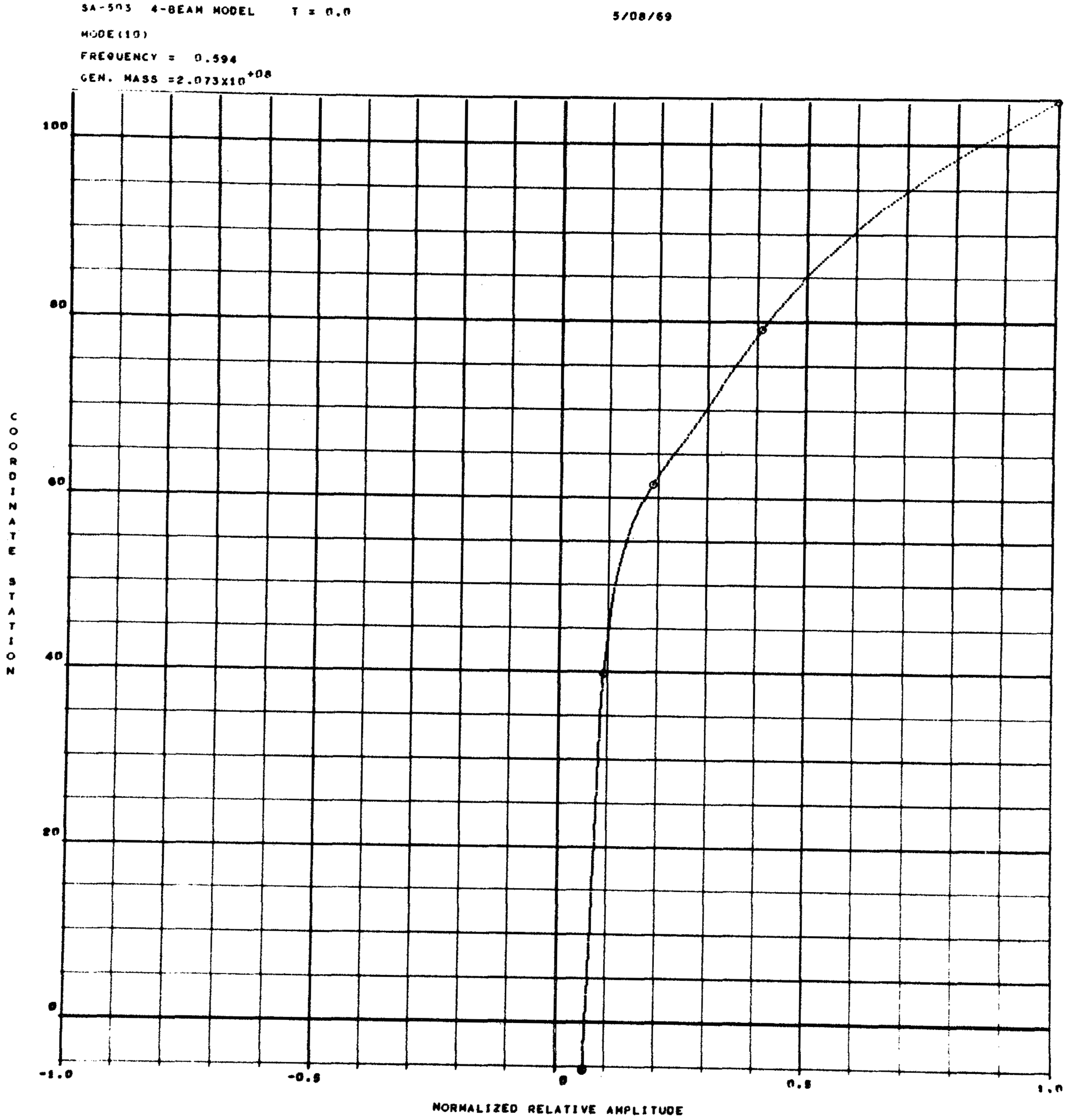


Fig. 21 - 10th Mode Shape (three slosh modes per tank)

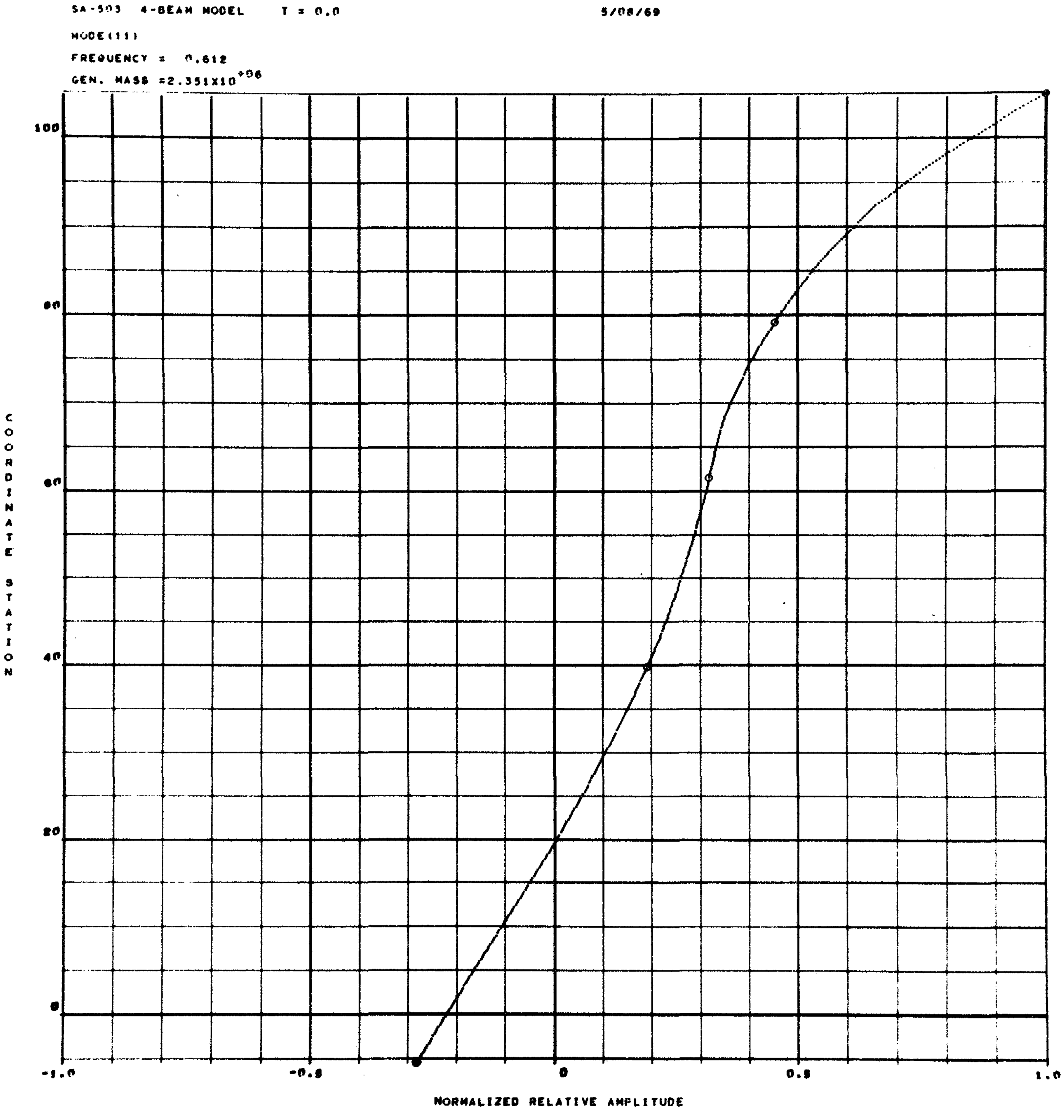


Fig. 22 - 11th Mode Shape (three slosh modes per tank)

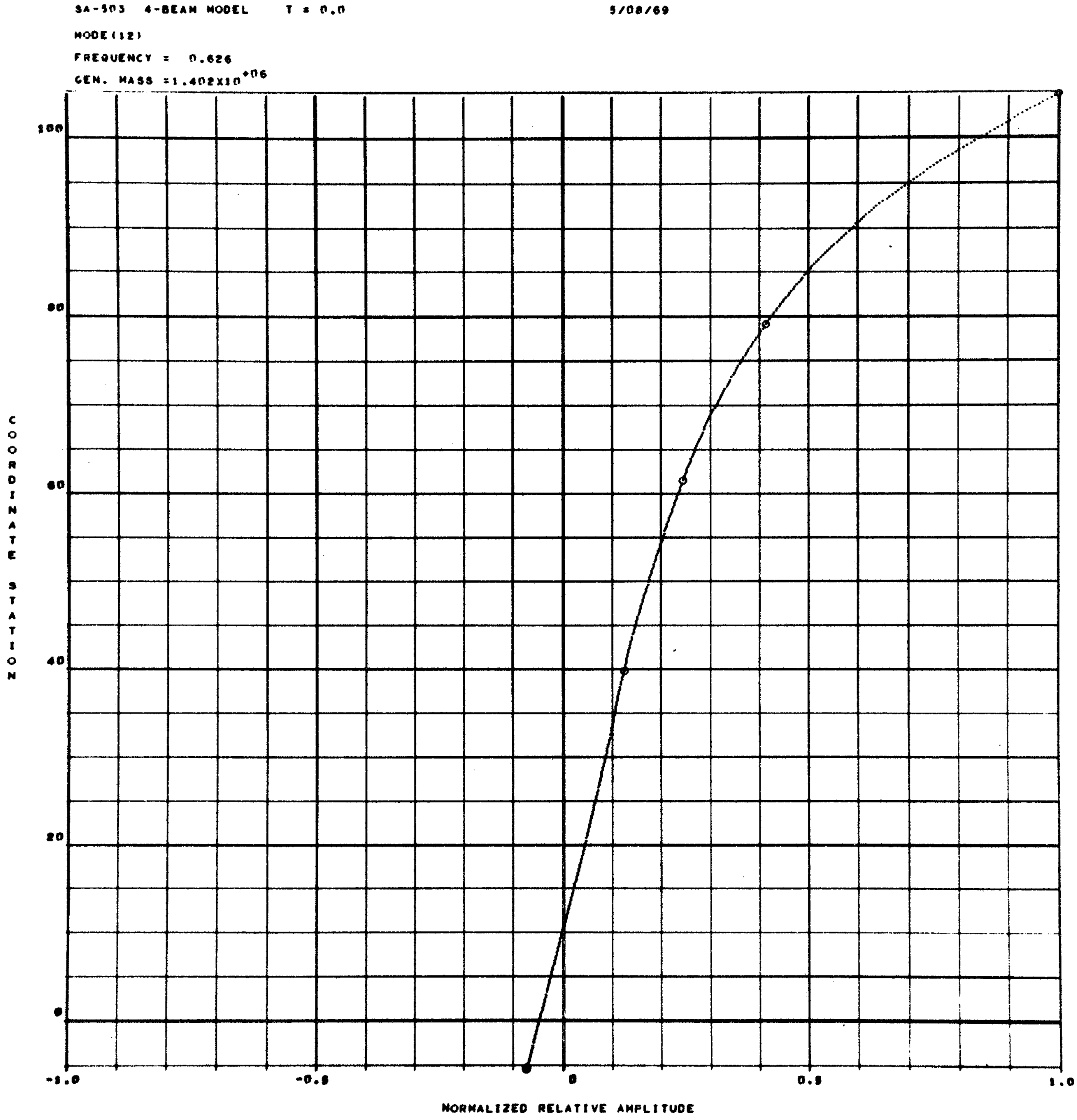


Fig. 23 - 12th Mode Shape (three slosh modes per tank)

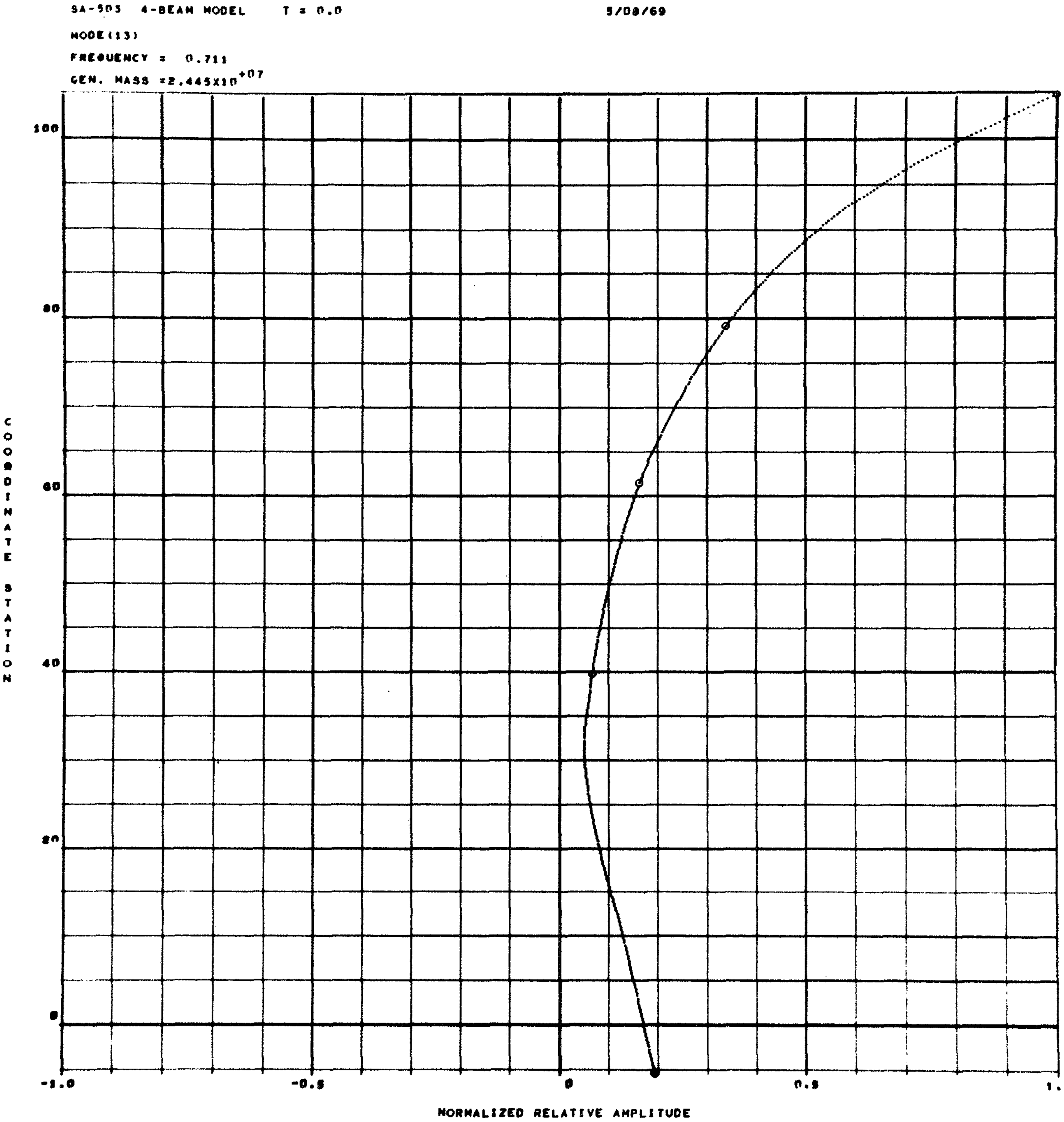


Fig. 24 - 13th Mode Shape (three slosh modes per tank)

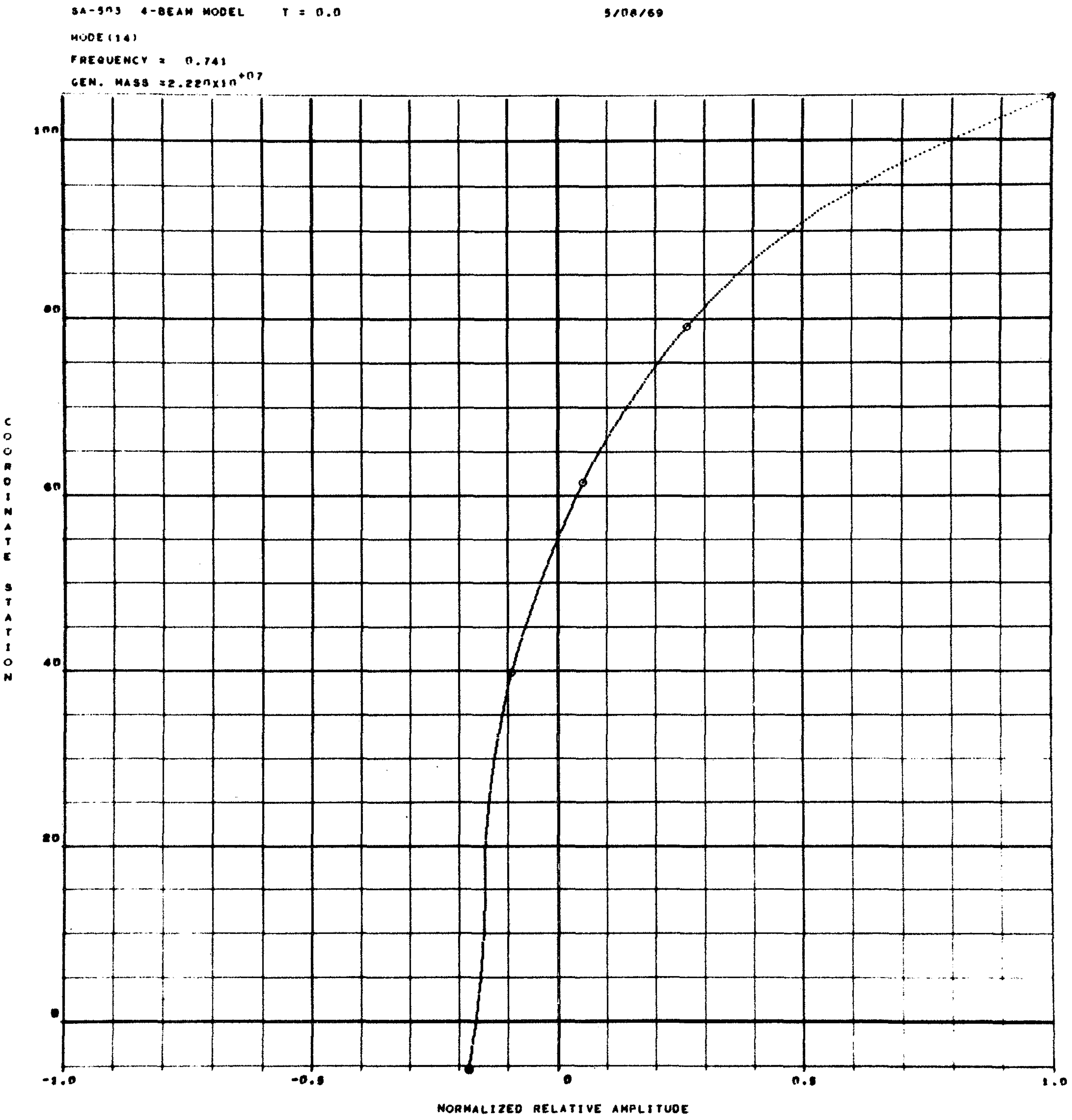


Fig. 25 - 14th Mode Shape (three slosh modes per tank)

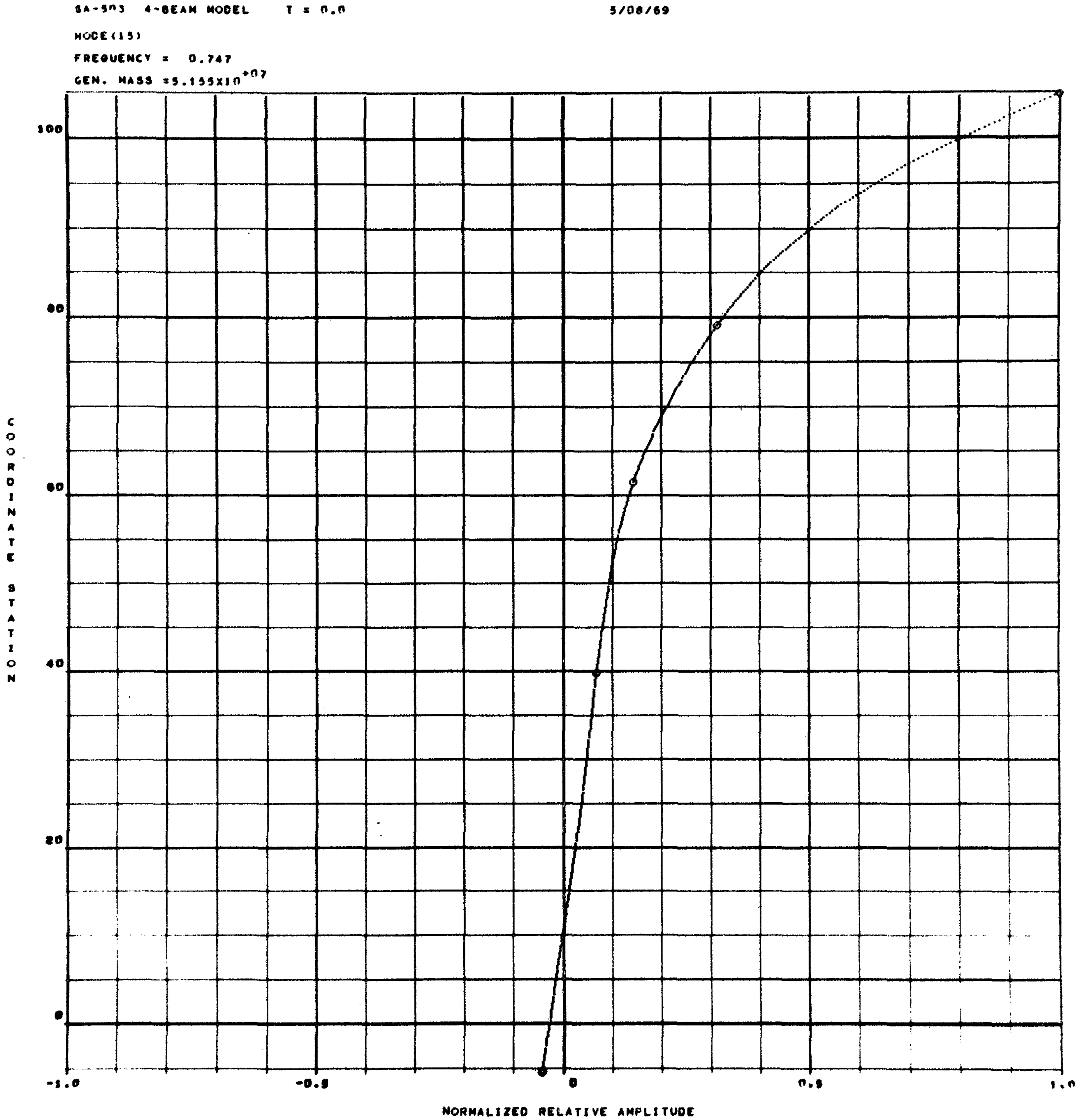


Fig. 26 - 15th Mode Shape (three slosh modes per tank)

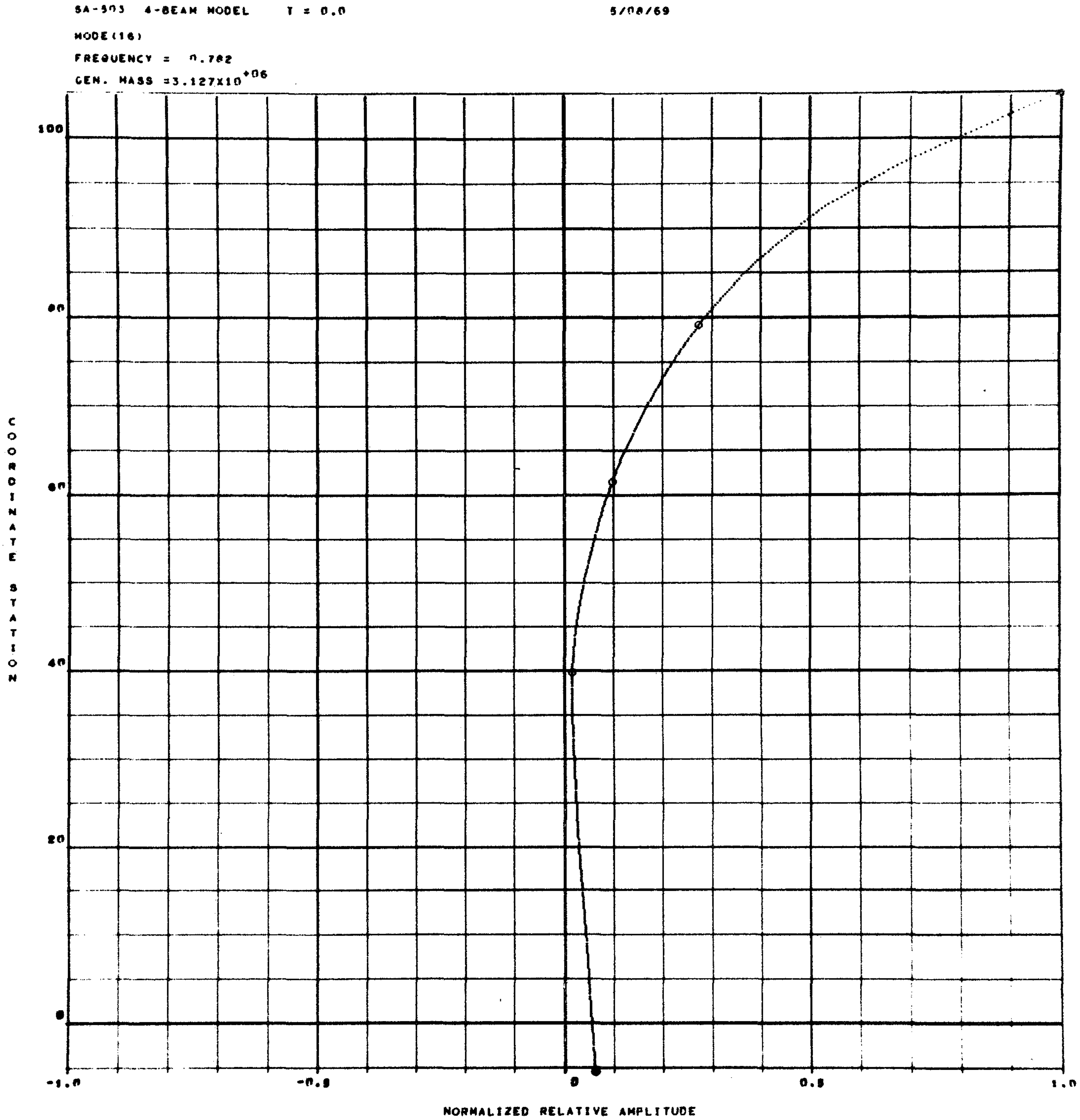


Fig. 27 - 16th Mode Shape (three slosh modes per tank)

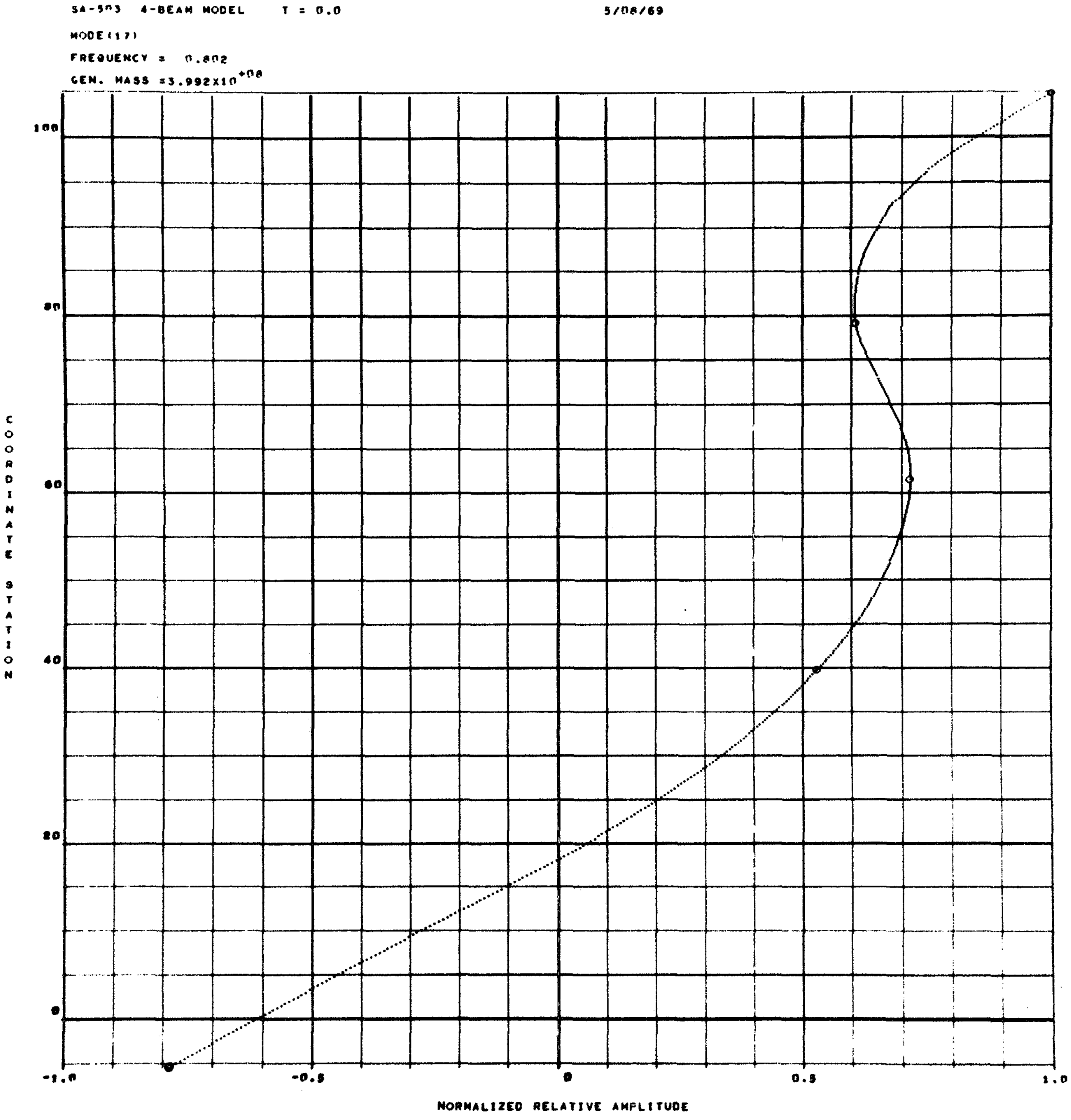


Fig. 28 - 17th Mode Shape (three slosh modes per tank)

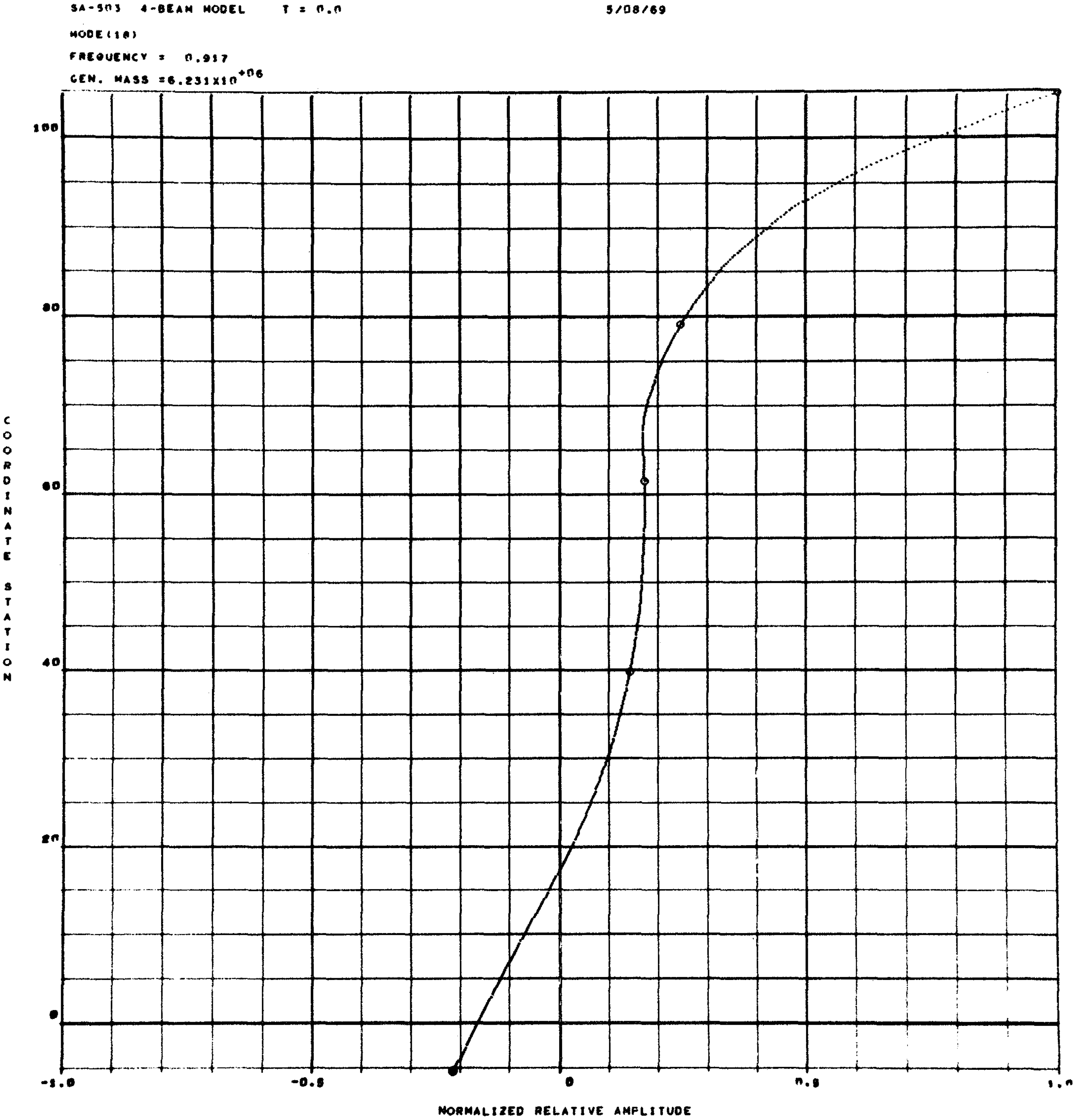


Fig. 29 - 18th Mode Shape (three slosh modes per tank)

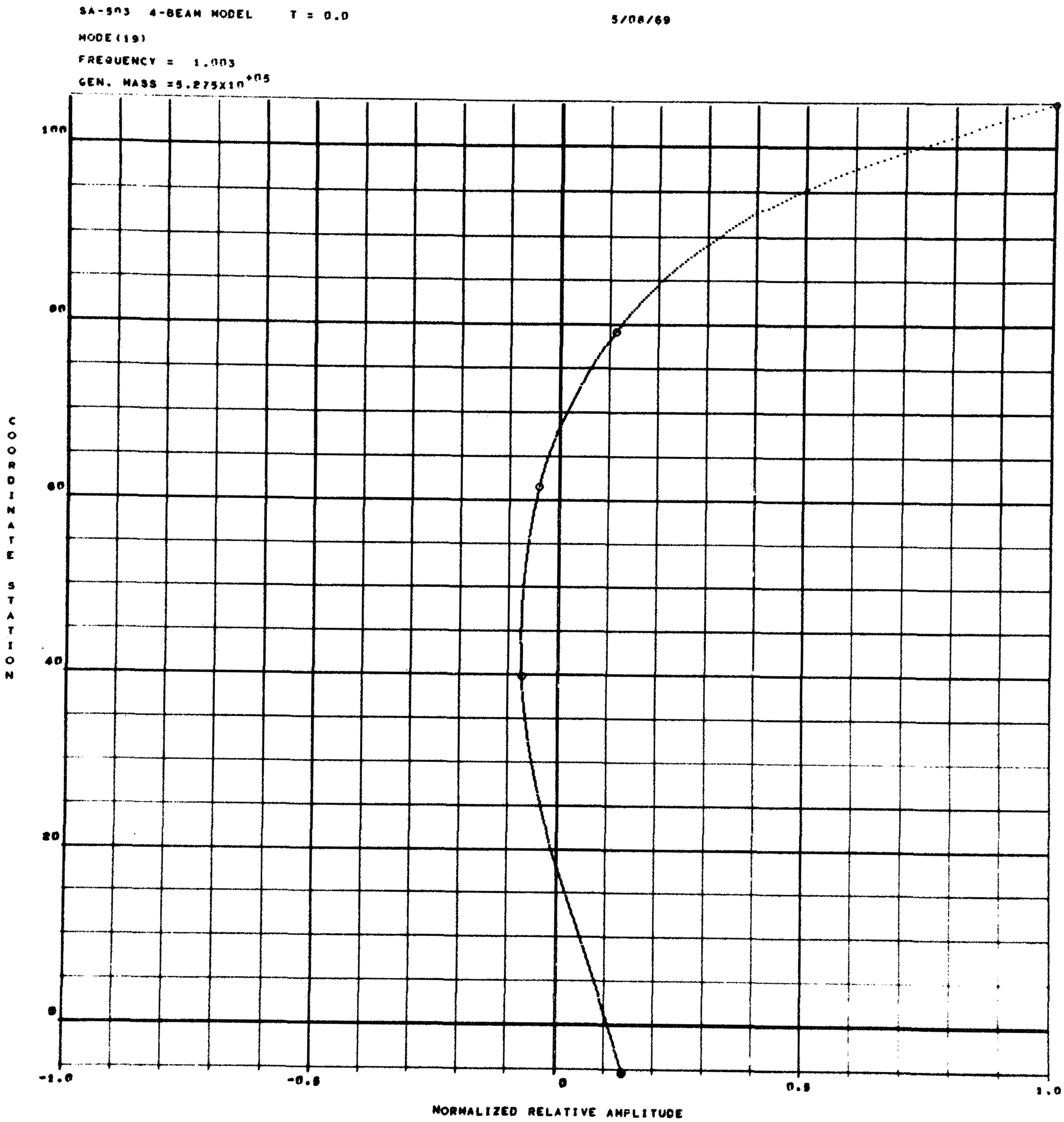
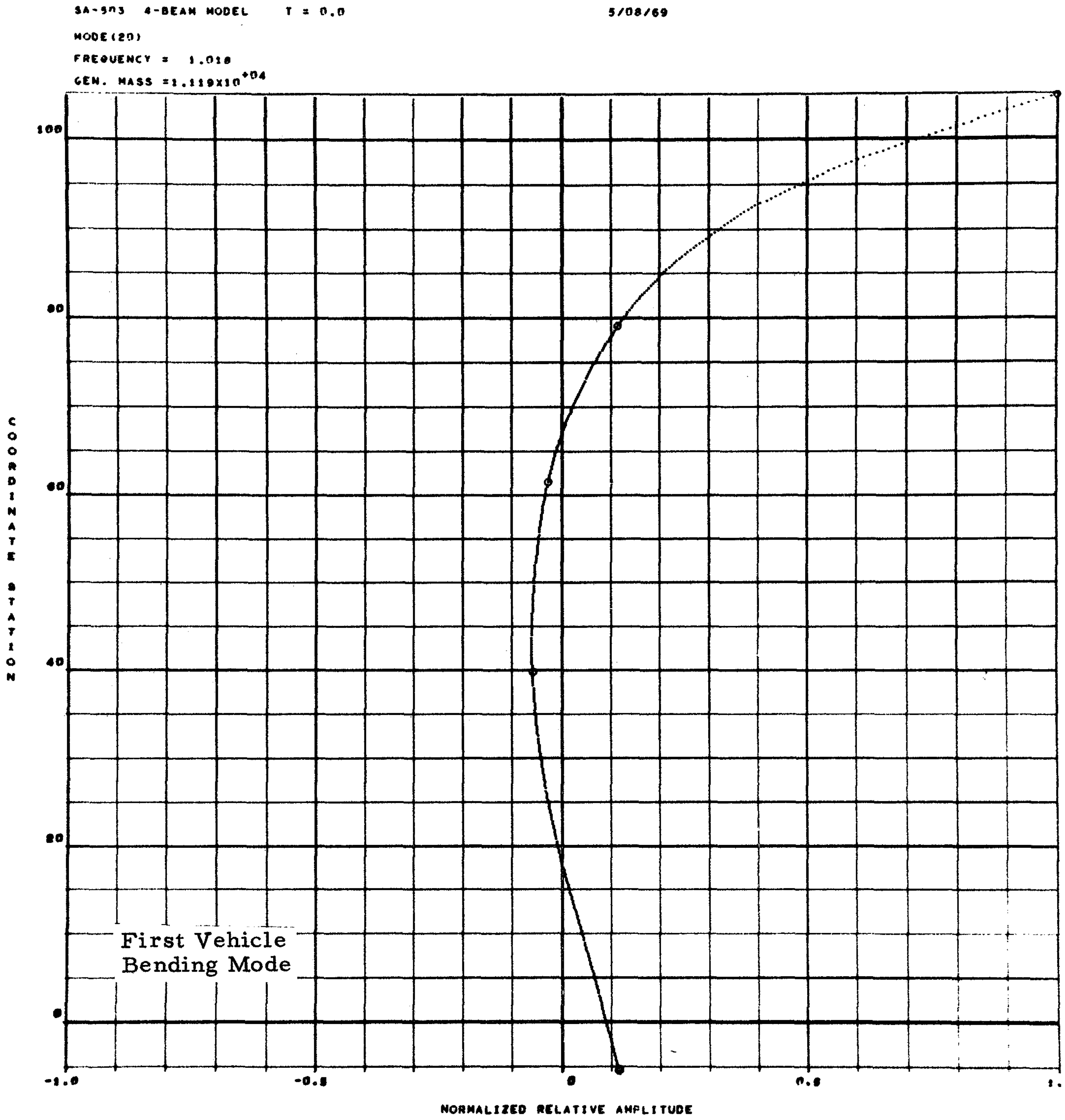


Fig. 30 - 19th Mode Shape (three slosh modes per tank)



First Vehicle
Bending Mode

Fig. 31 - 20th Mode Shape (three slosh modes per tank)

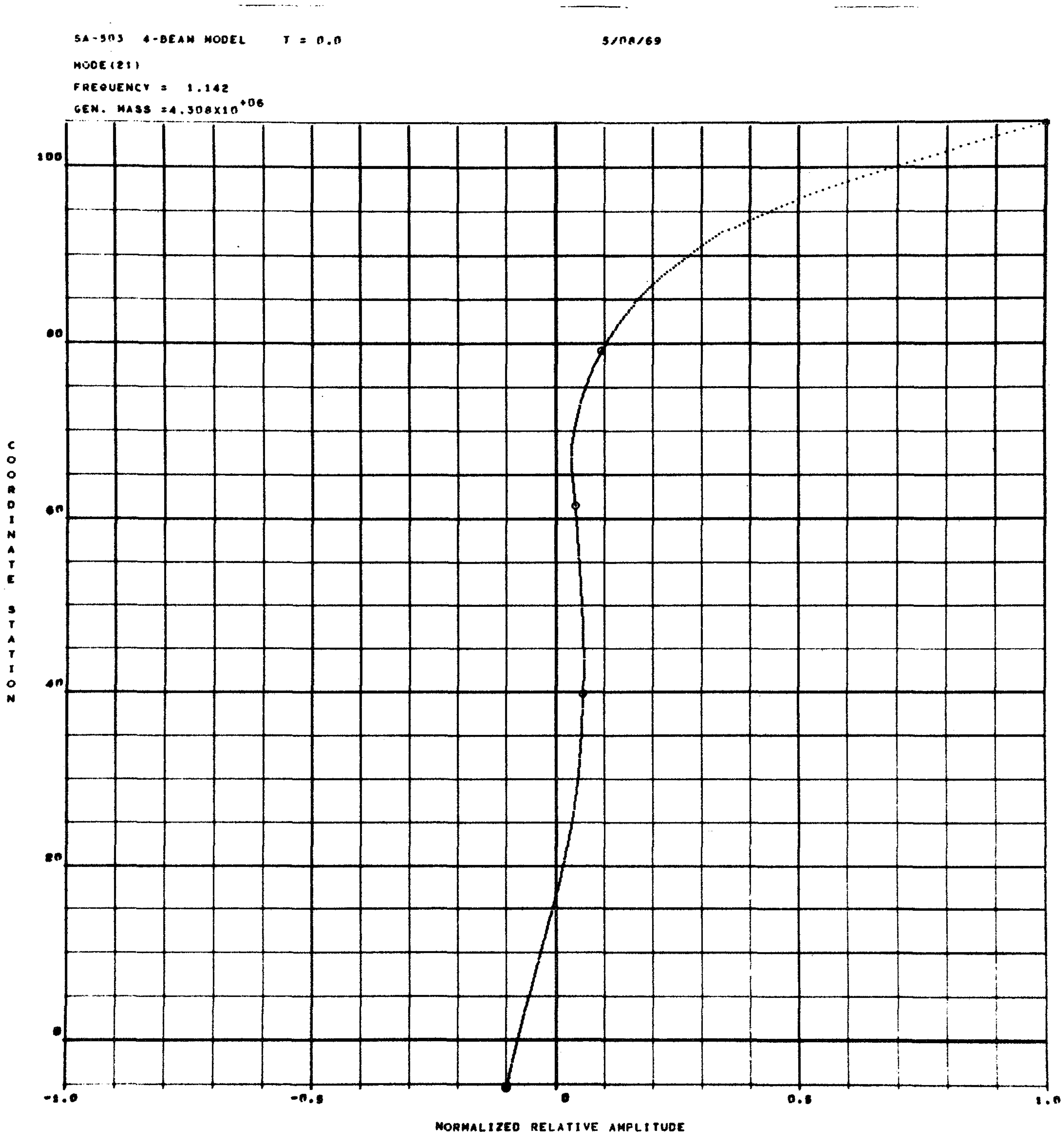


Fig. 32 - 21st Mode Shape (three slosh modes per tank)

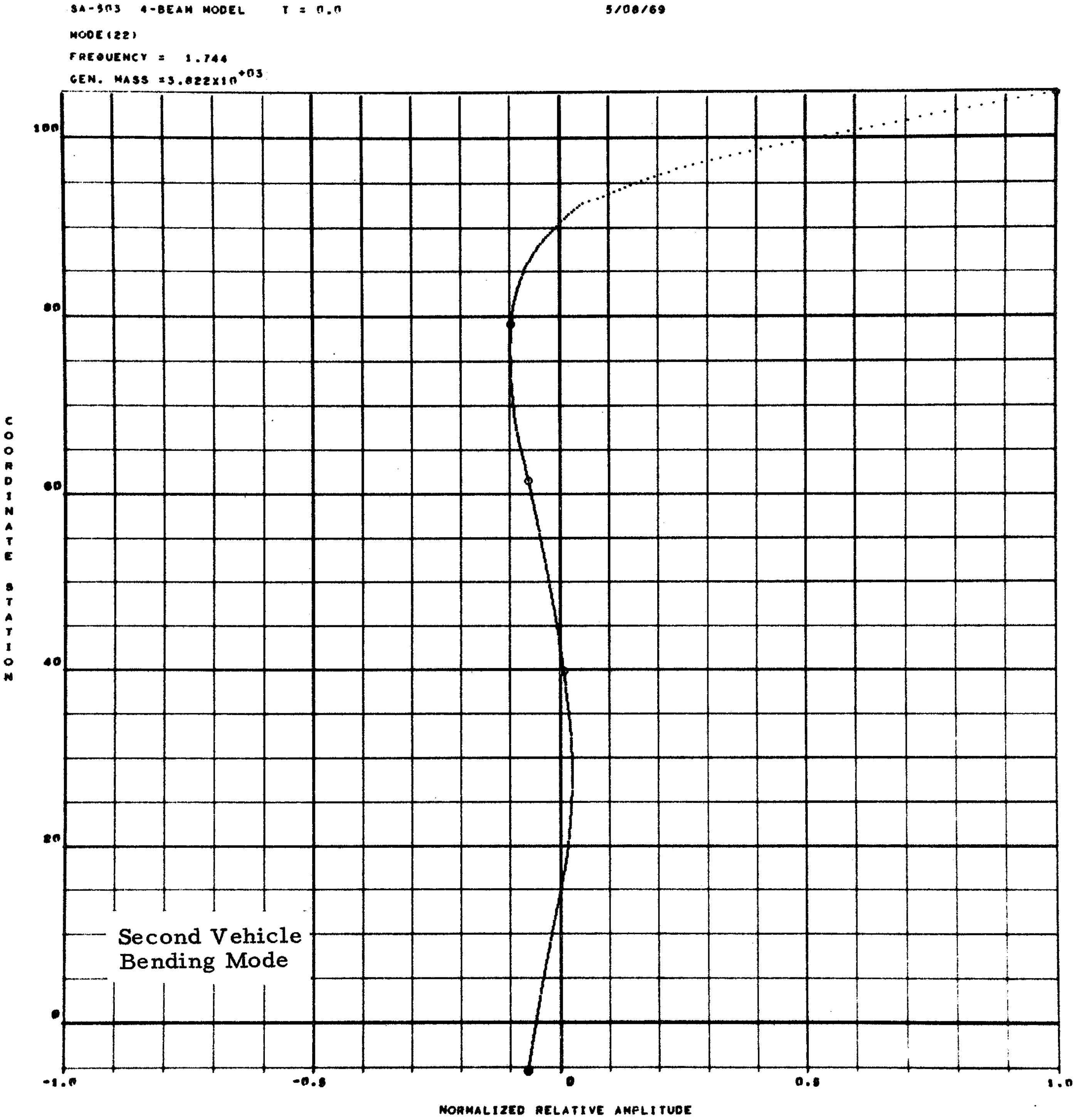


Fig. 33 - 22nd Mode Shape (three slosh modes per tank)

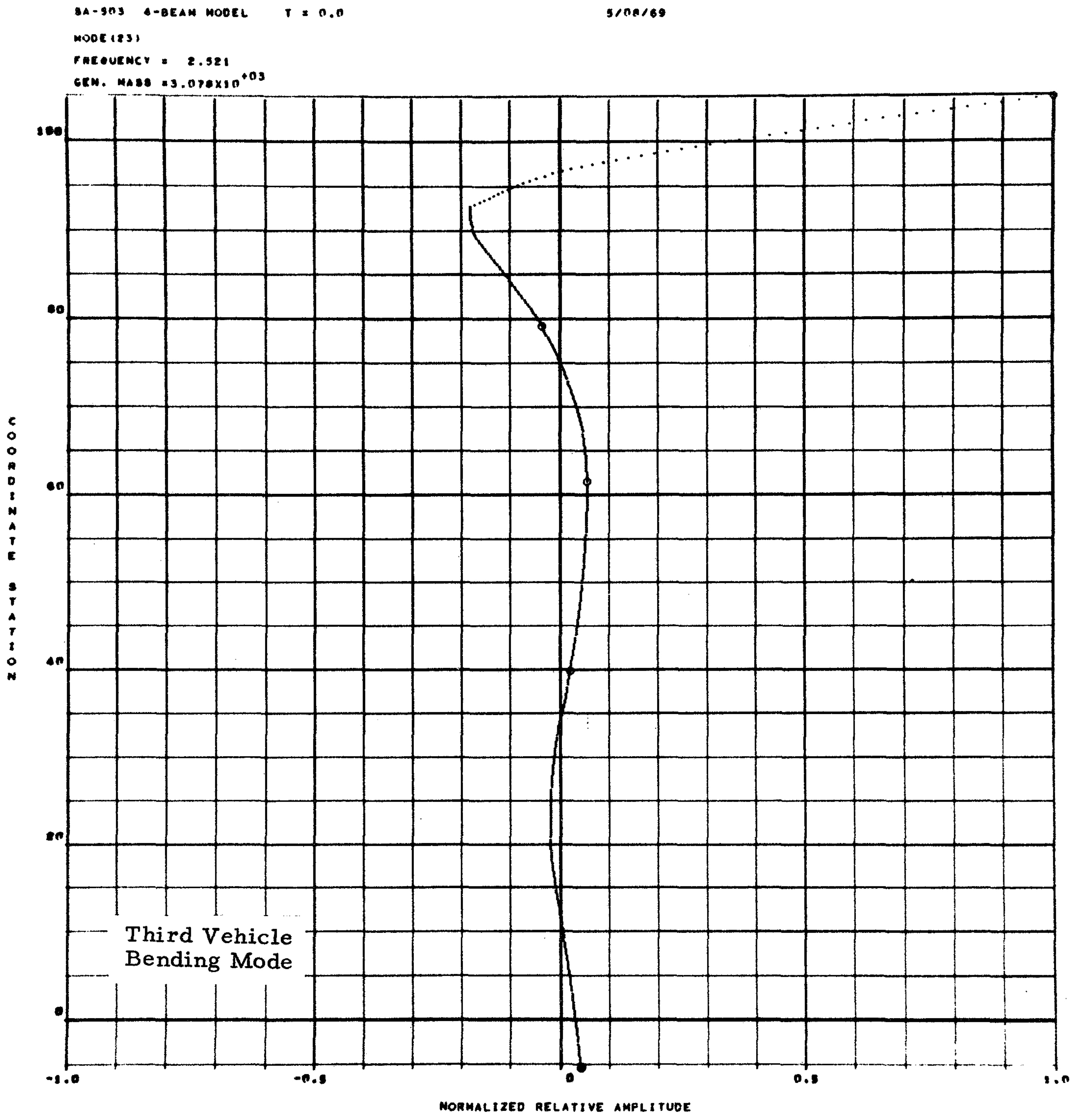


Fig. 34 - 23rd Mode Shape (three slosh modes per tank)

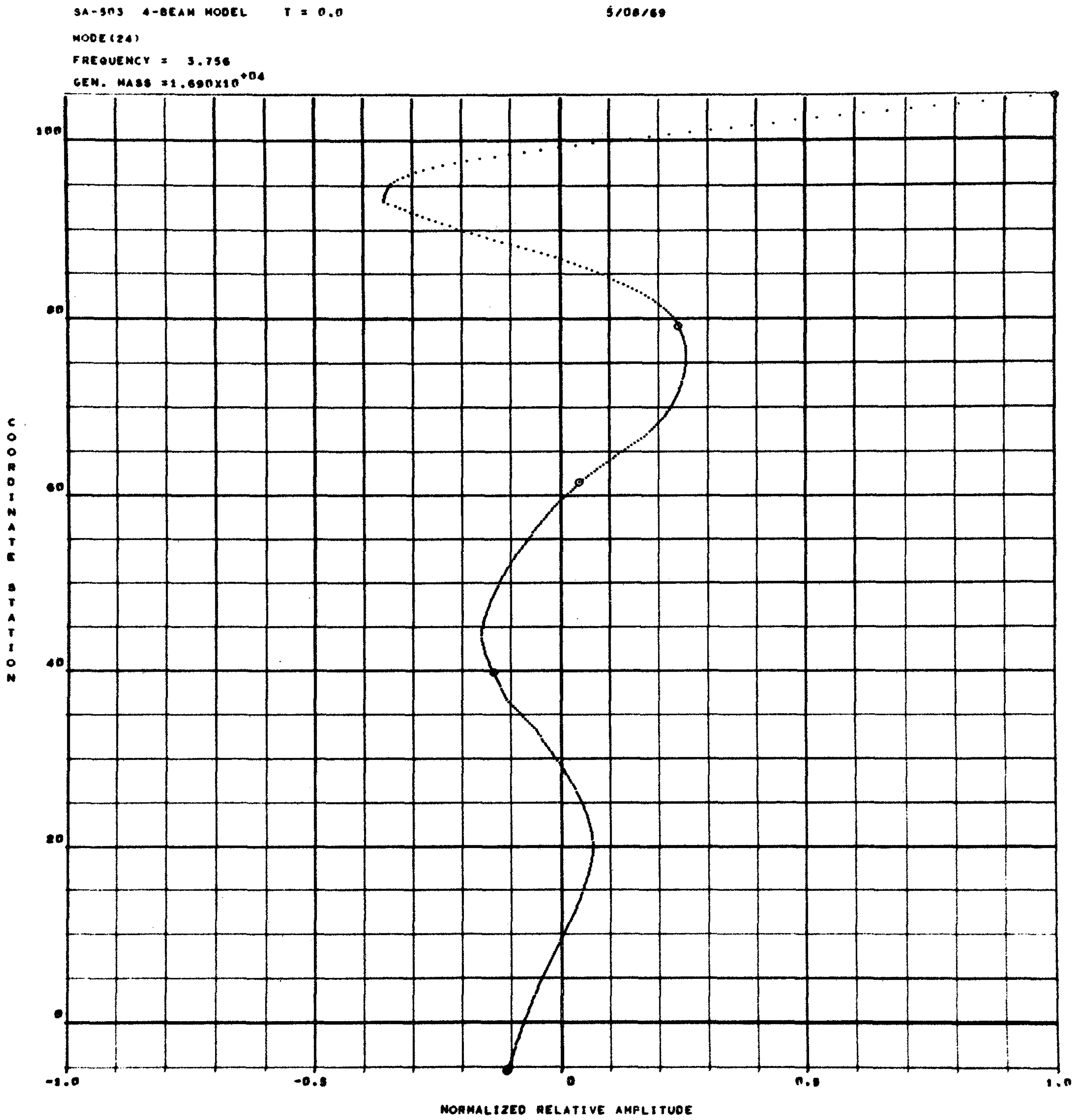


Fig. 35 - 24th Mode Shape (three slosh modes per tank)

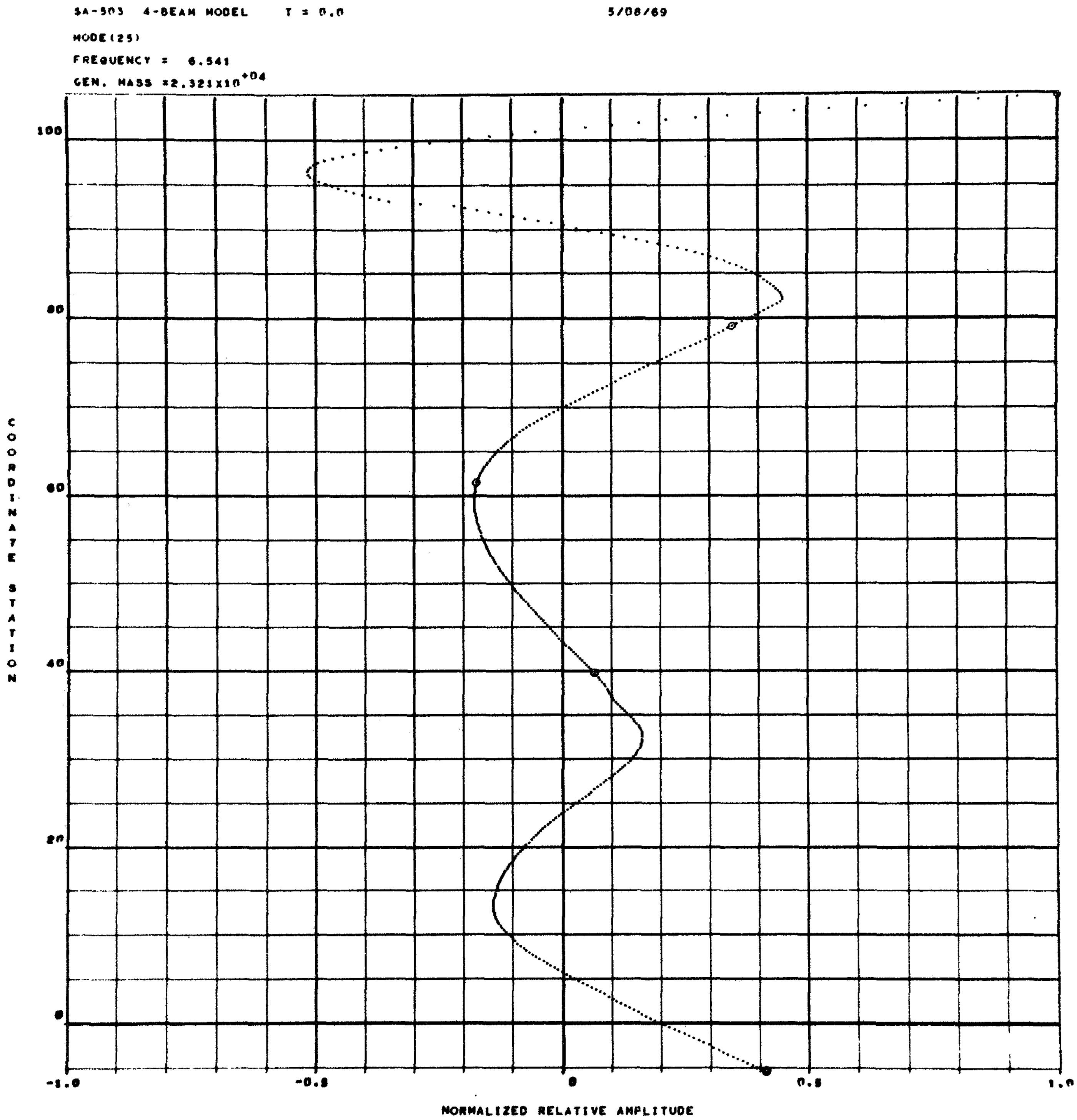


Fig. 36 - 25th Mode Shape (three slosh modes per tank)

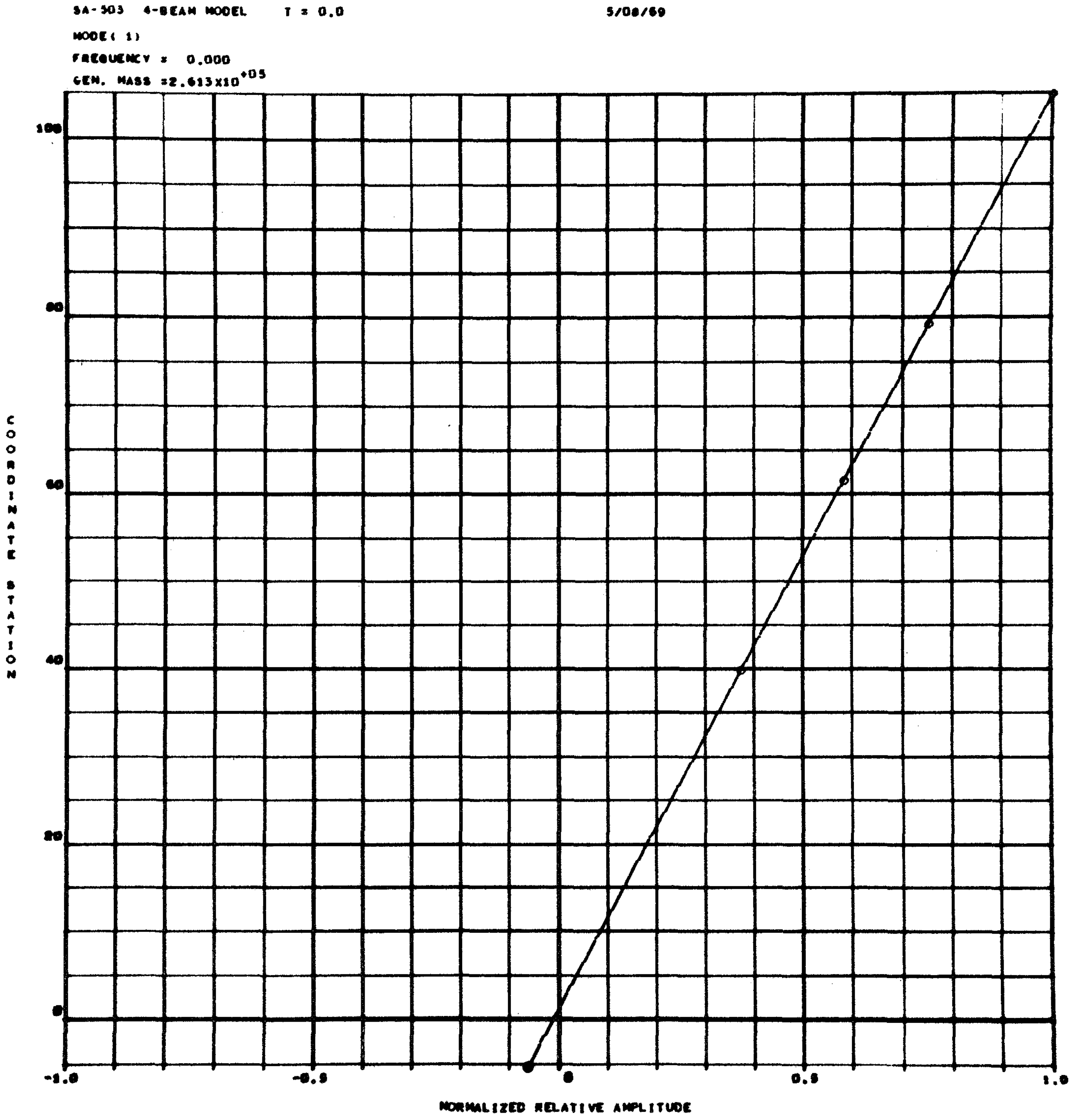


Fig. 37 - 1st Mode Shape (one slosh mode per tank)

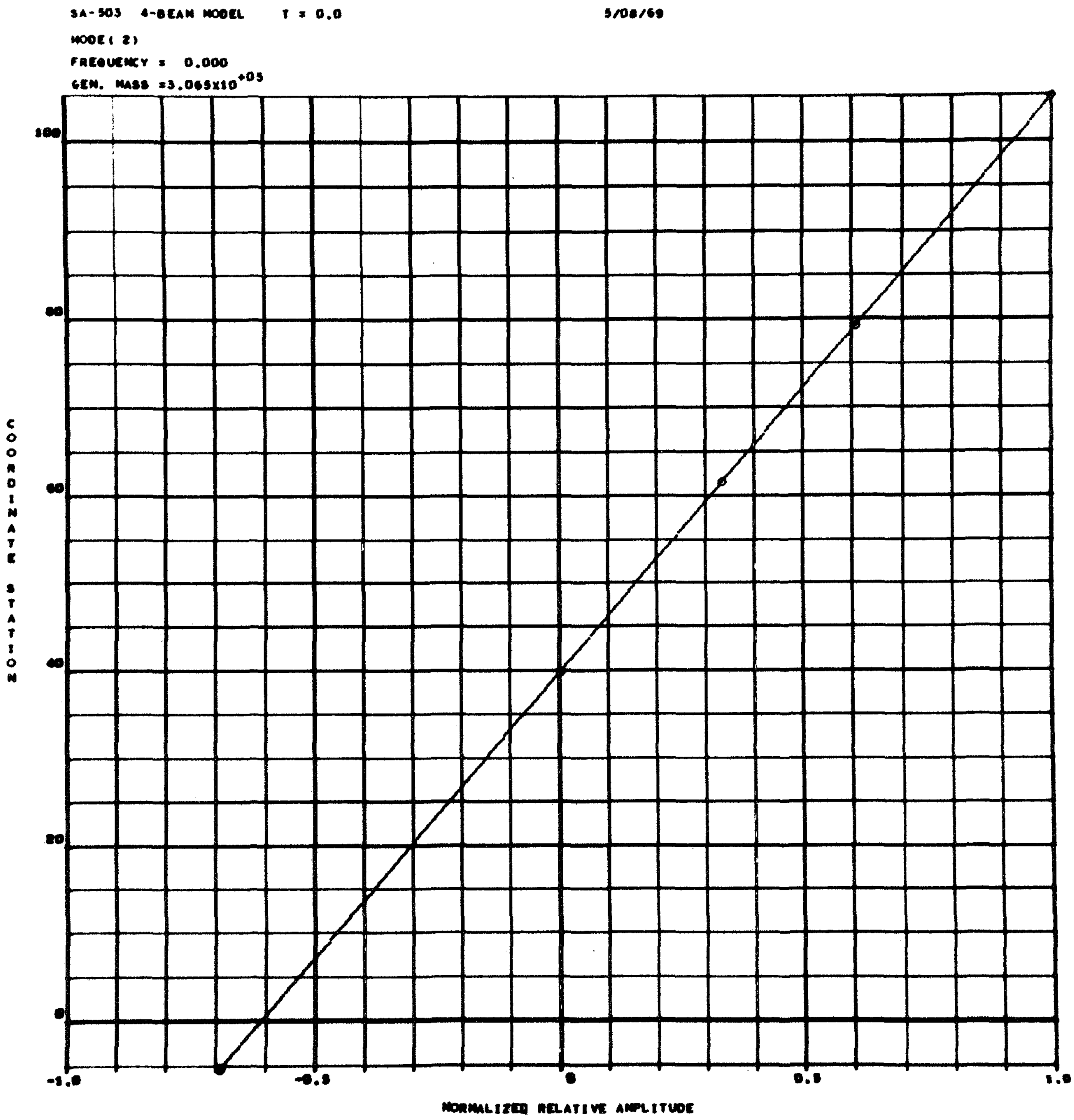


Fig. 38 - 2nd Mode Shape (one slosh mode per tank)

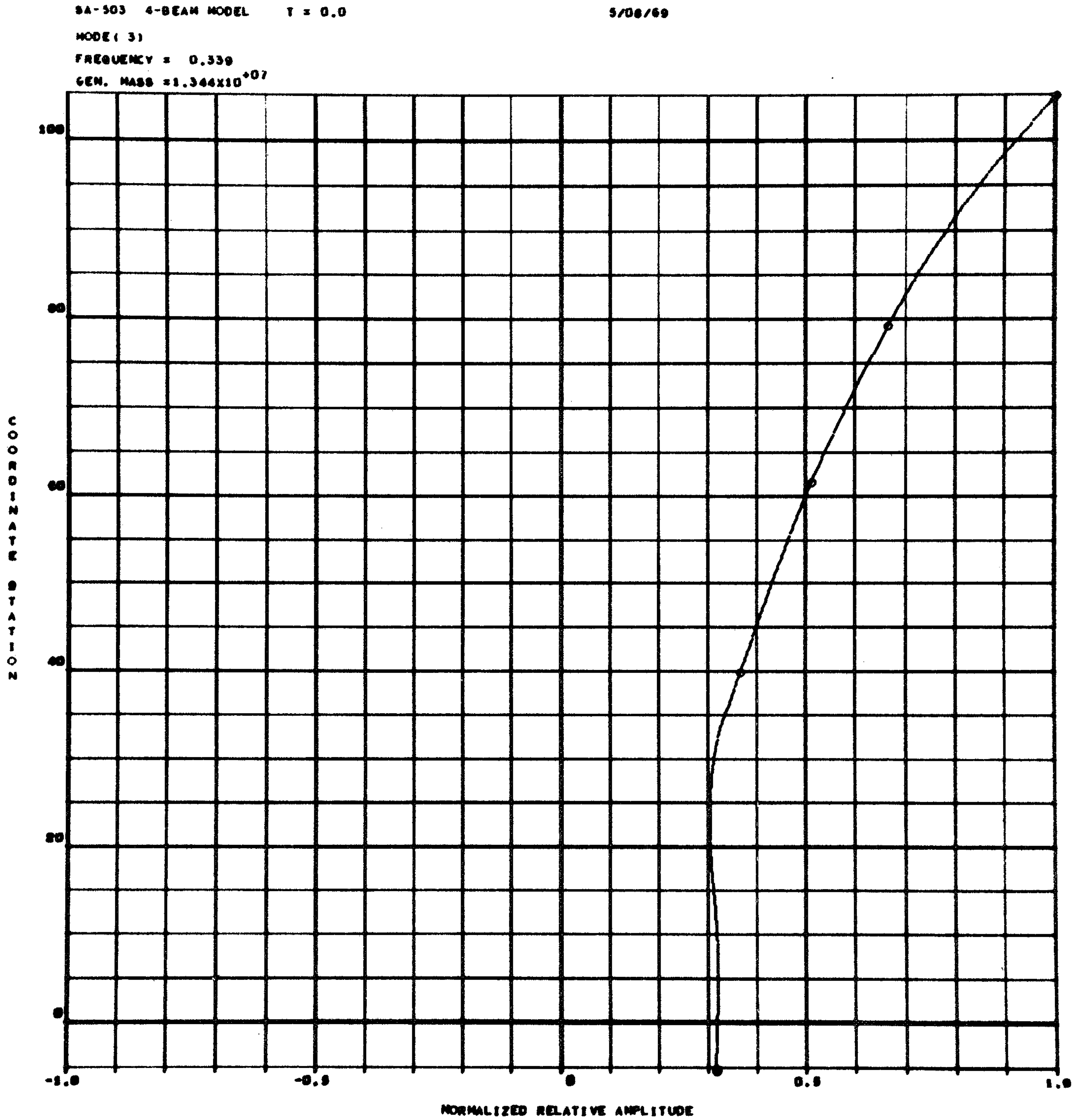


Fig. 39 - 3rd Mode Shape (one slosh mode per tank)

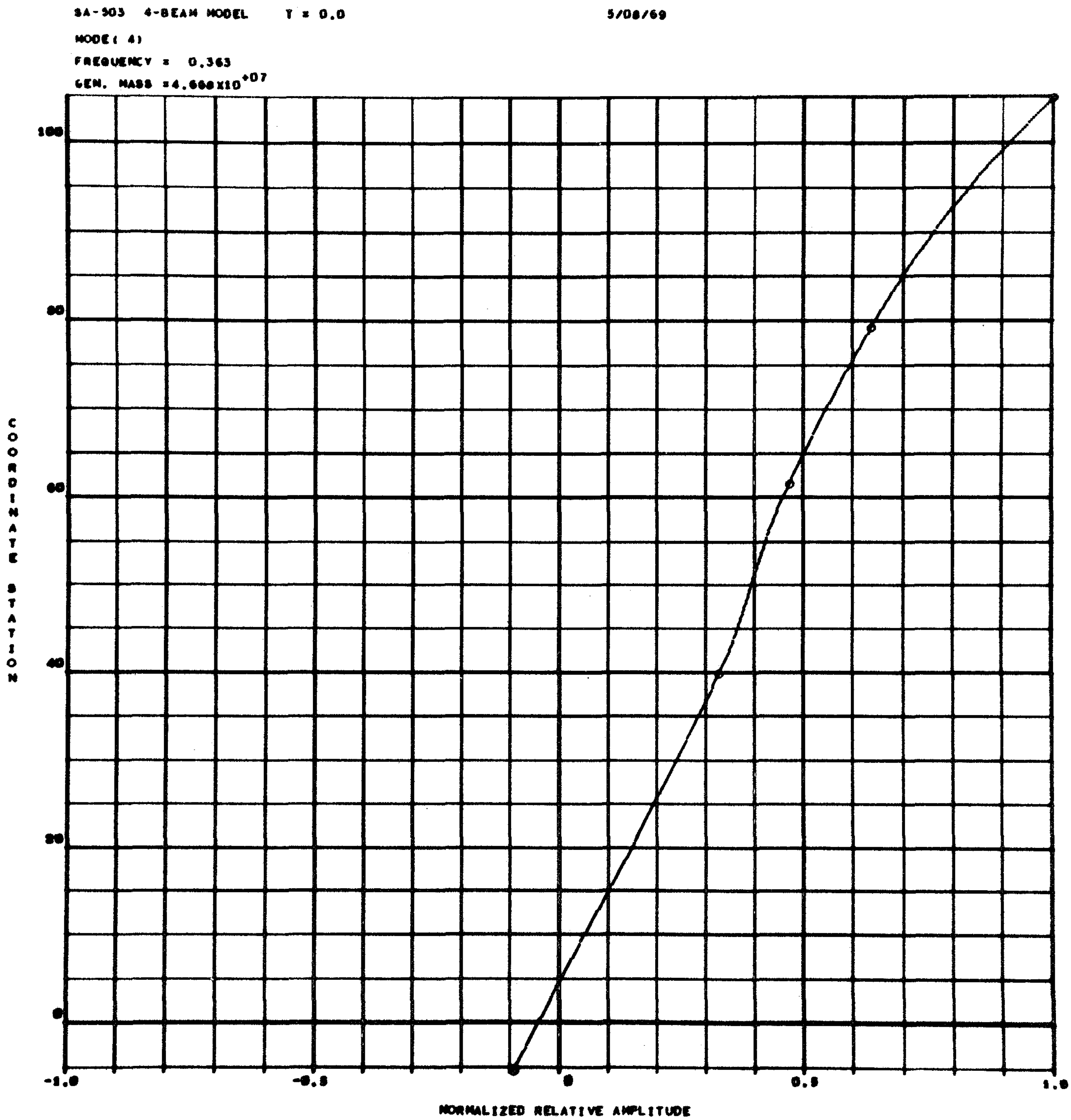


Fig. 40 - 4th Mode Shape (one slosh mode per tank)

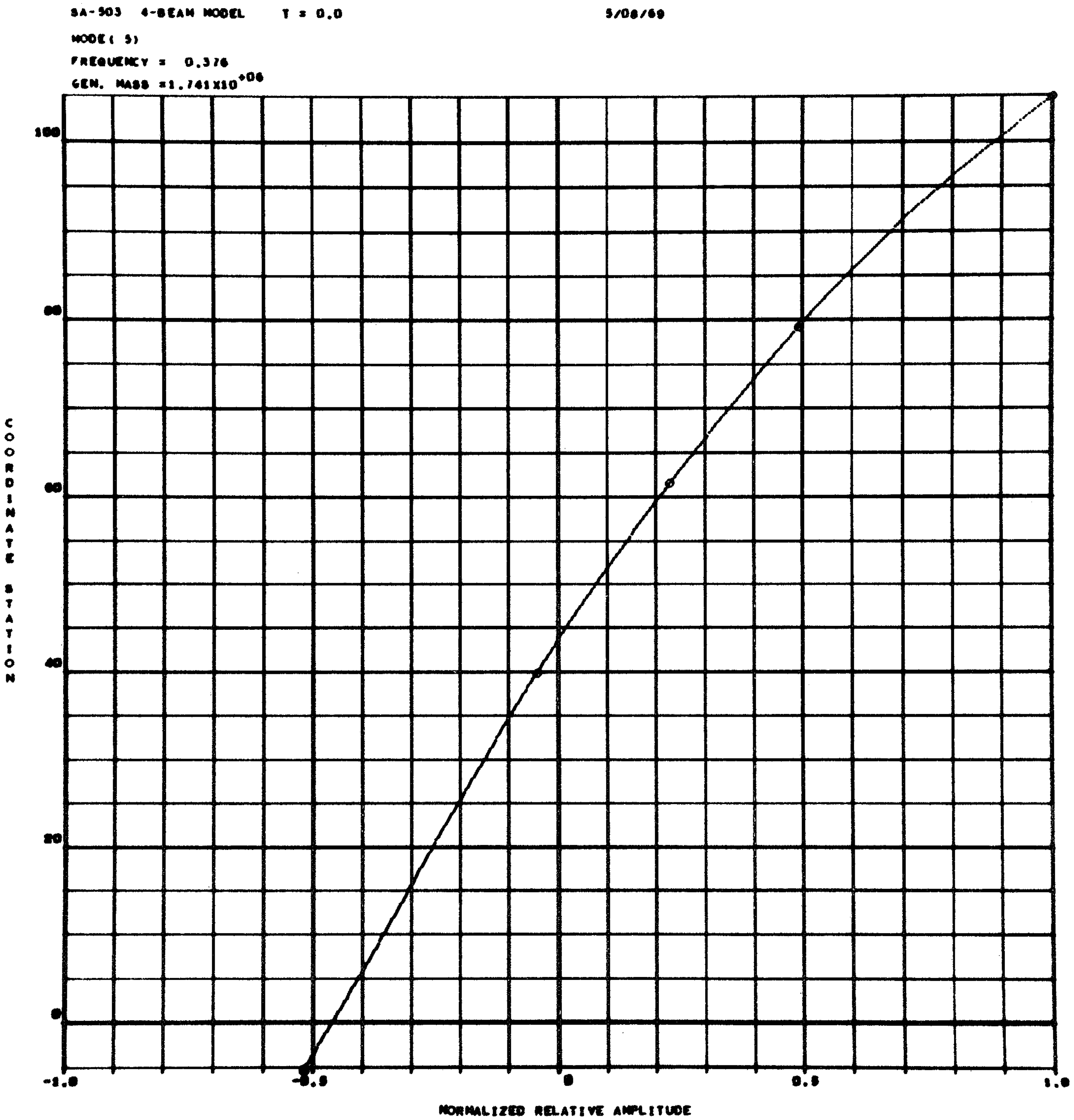


Fig. 41 - 5th Mode Shape (one slosh mode per tank)

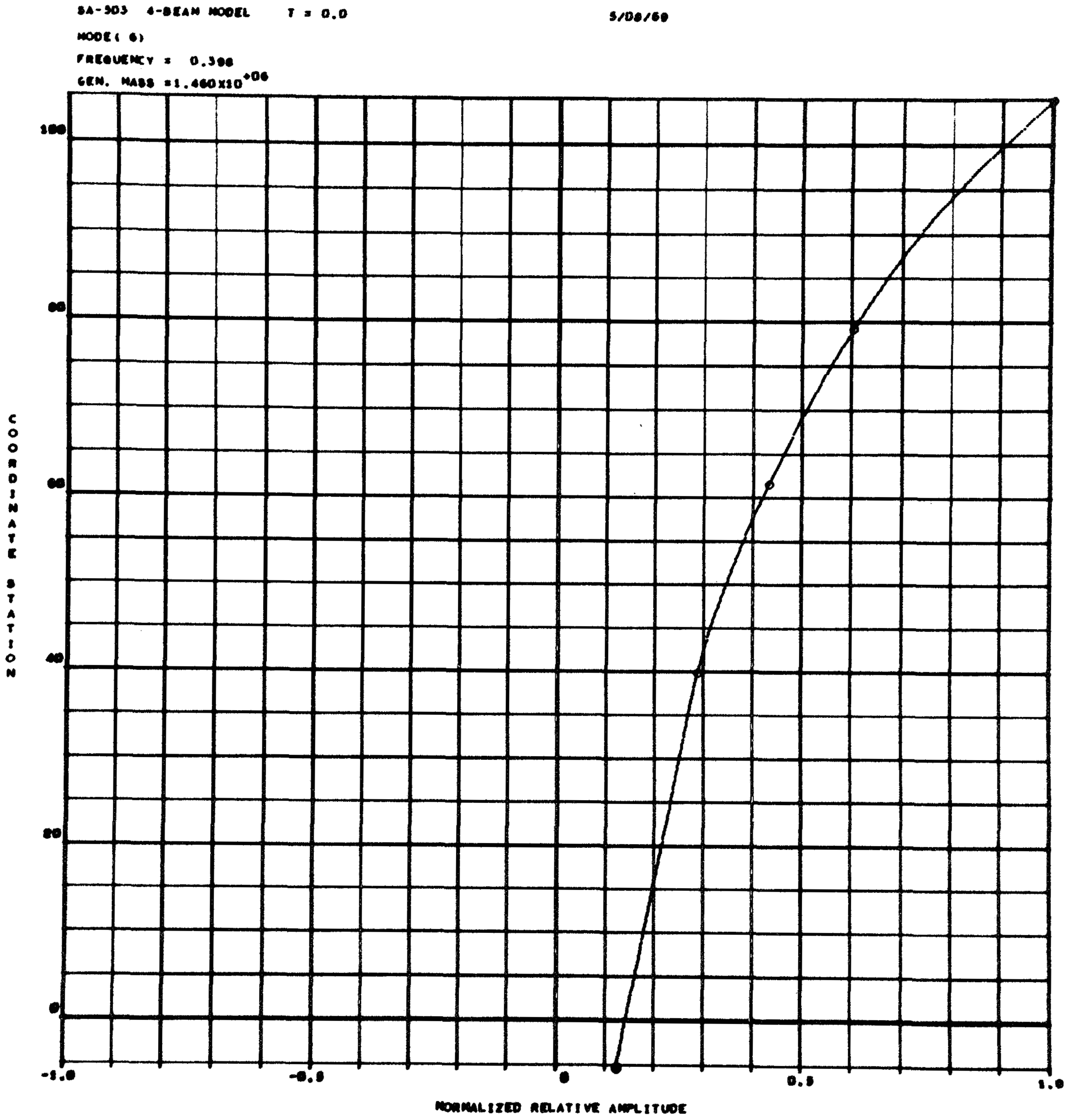


Fig. 42 - 6th Mode Shape (one slosh mode per tank)

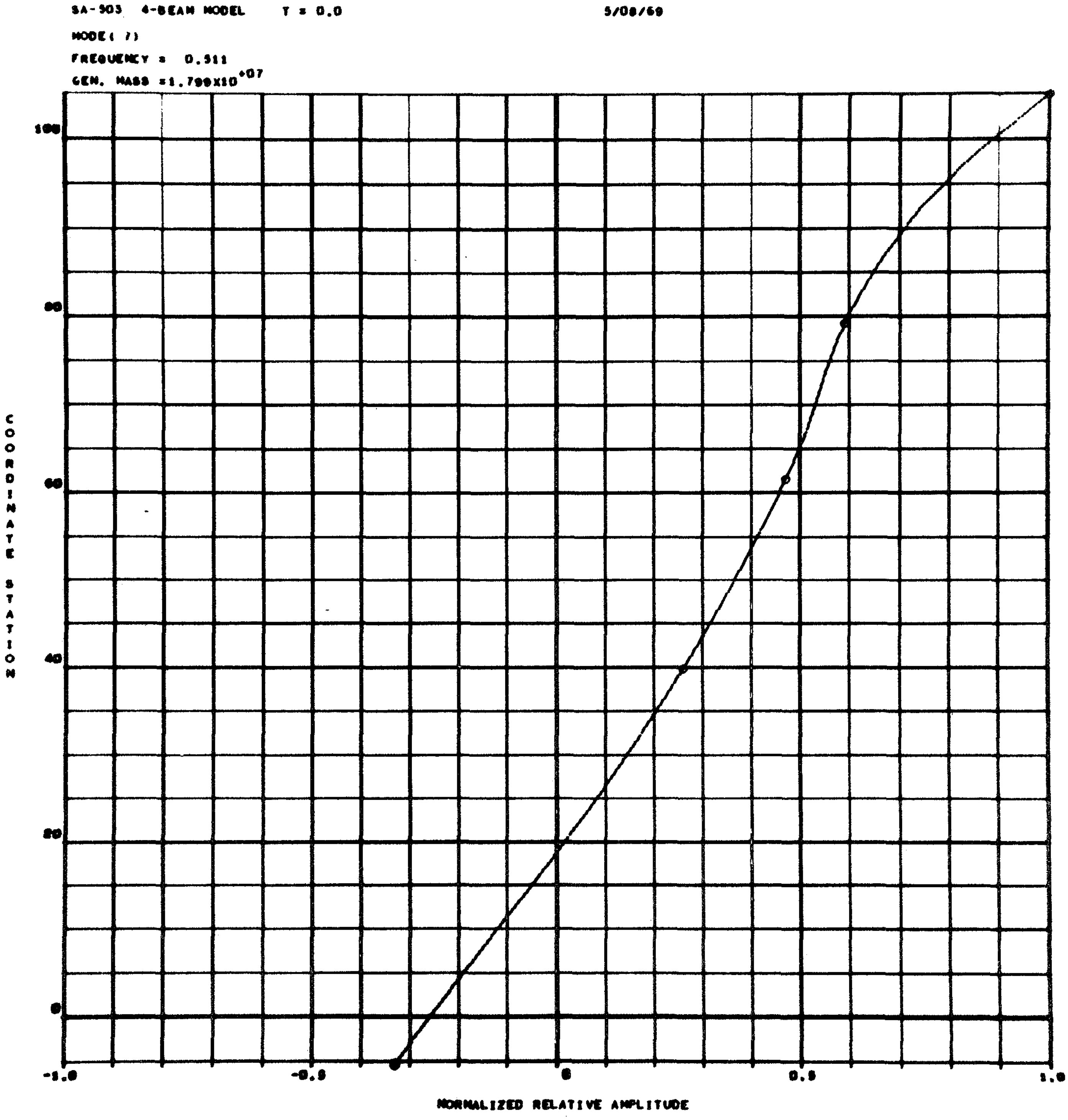


Fig. 43 - 7th Mode Shape (one slosh mode per tank)

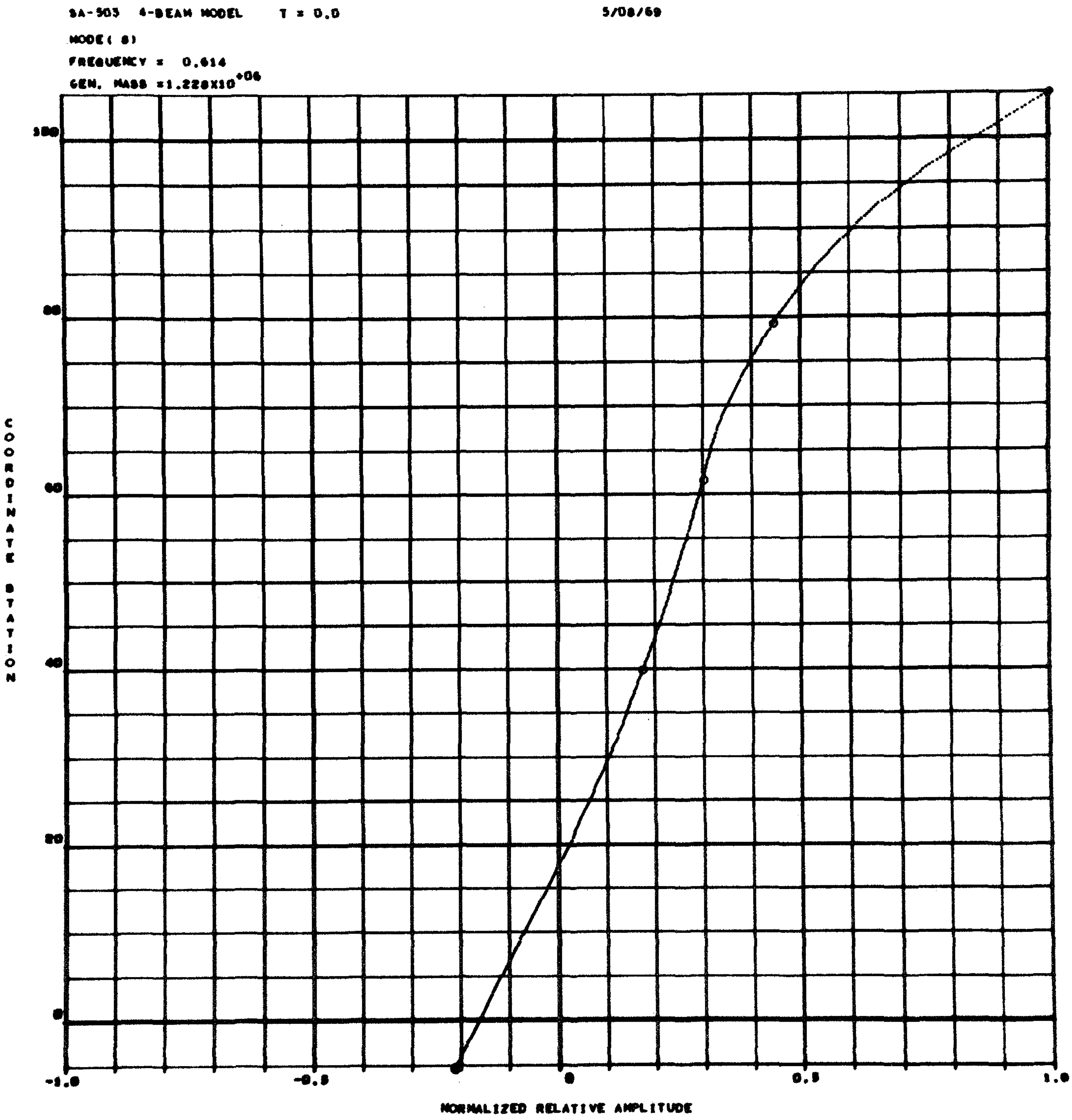


Fig. 44 - 8th Mode Shape (one slosh mode per tank)

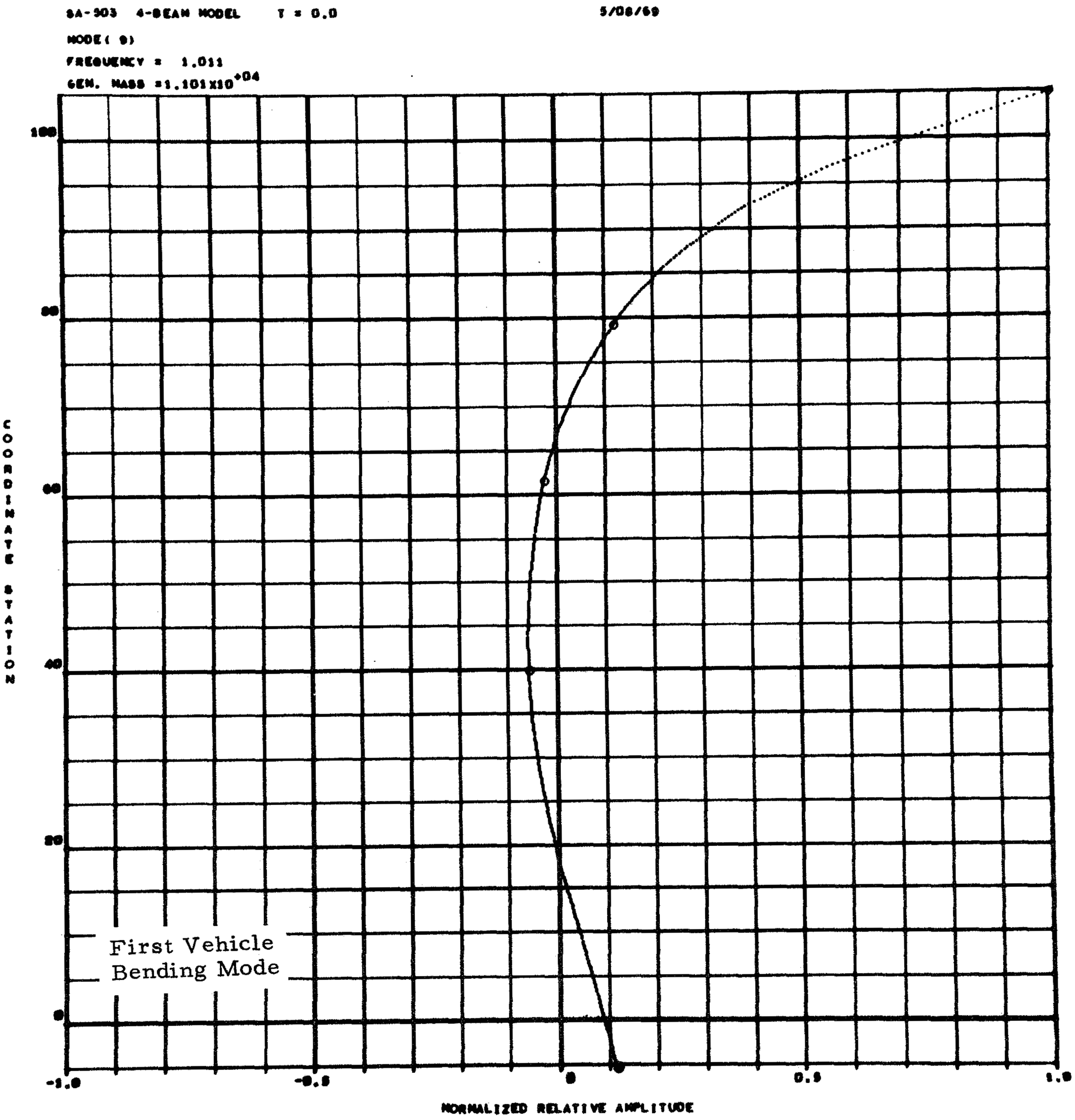
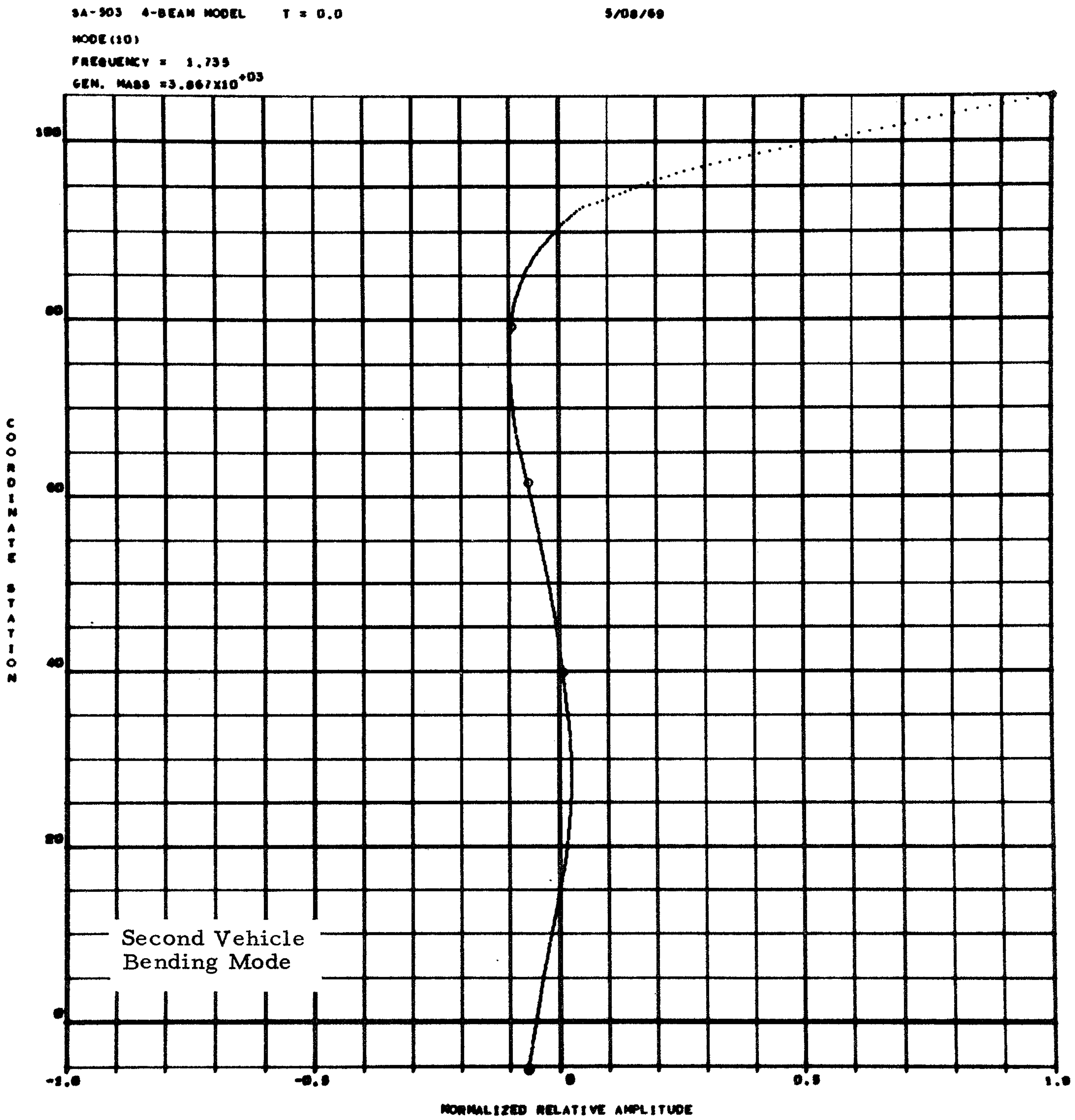


Fig. 45 - 9th Mode Shape (one slosh mode per tank)



Second Vehicle Bending Mode

Fig. 46 - 10th Mode Shape (one slosh mode per tank)

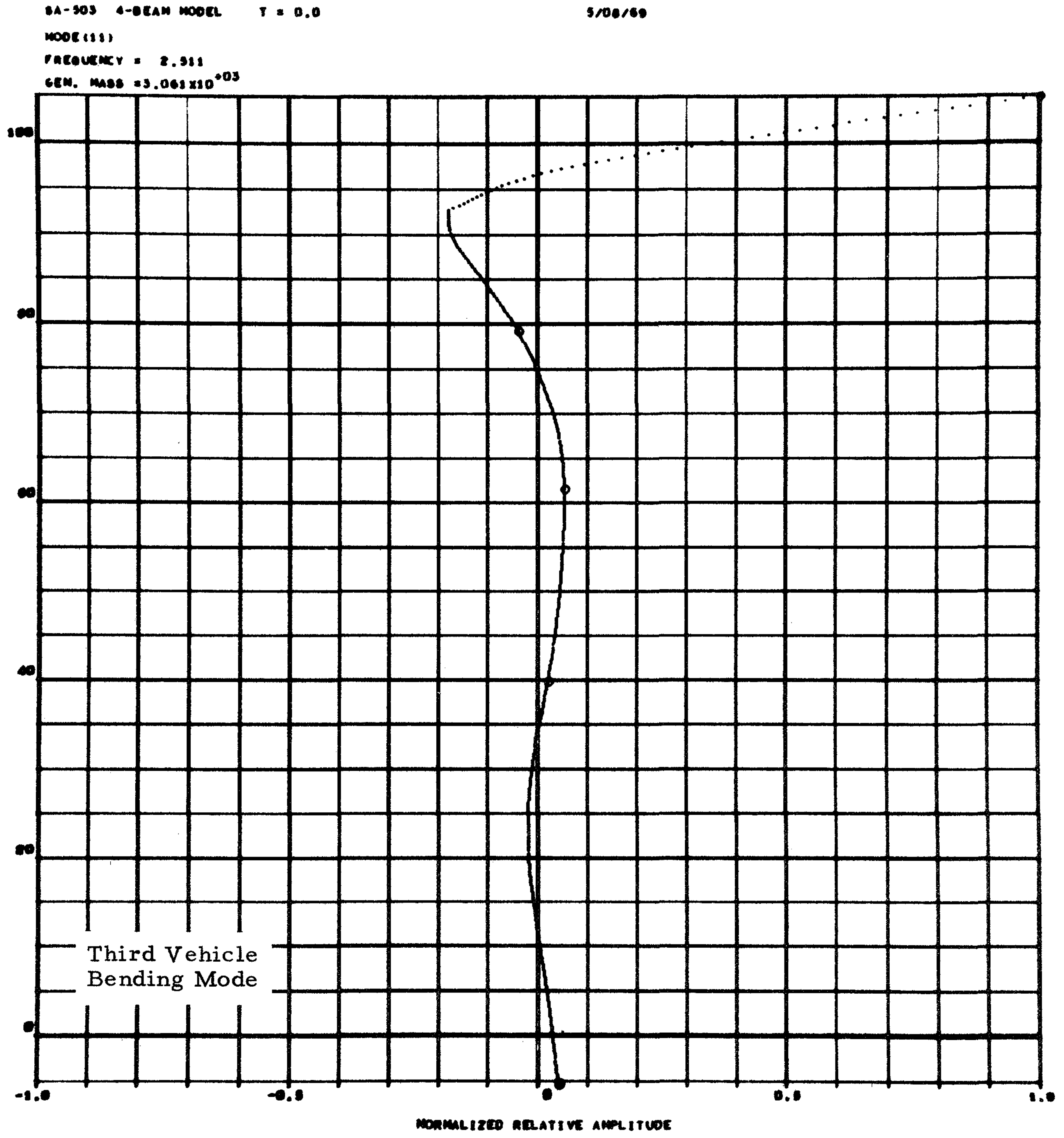


Fig. 47 - 11th Mode Shape (one slosh mode per tank)

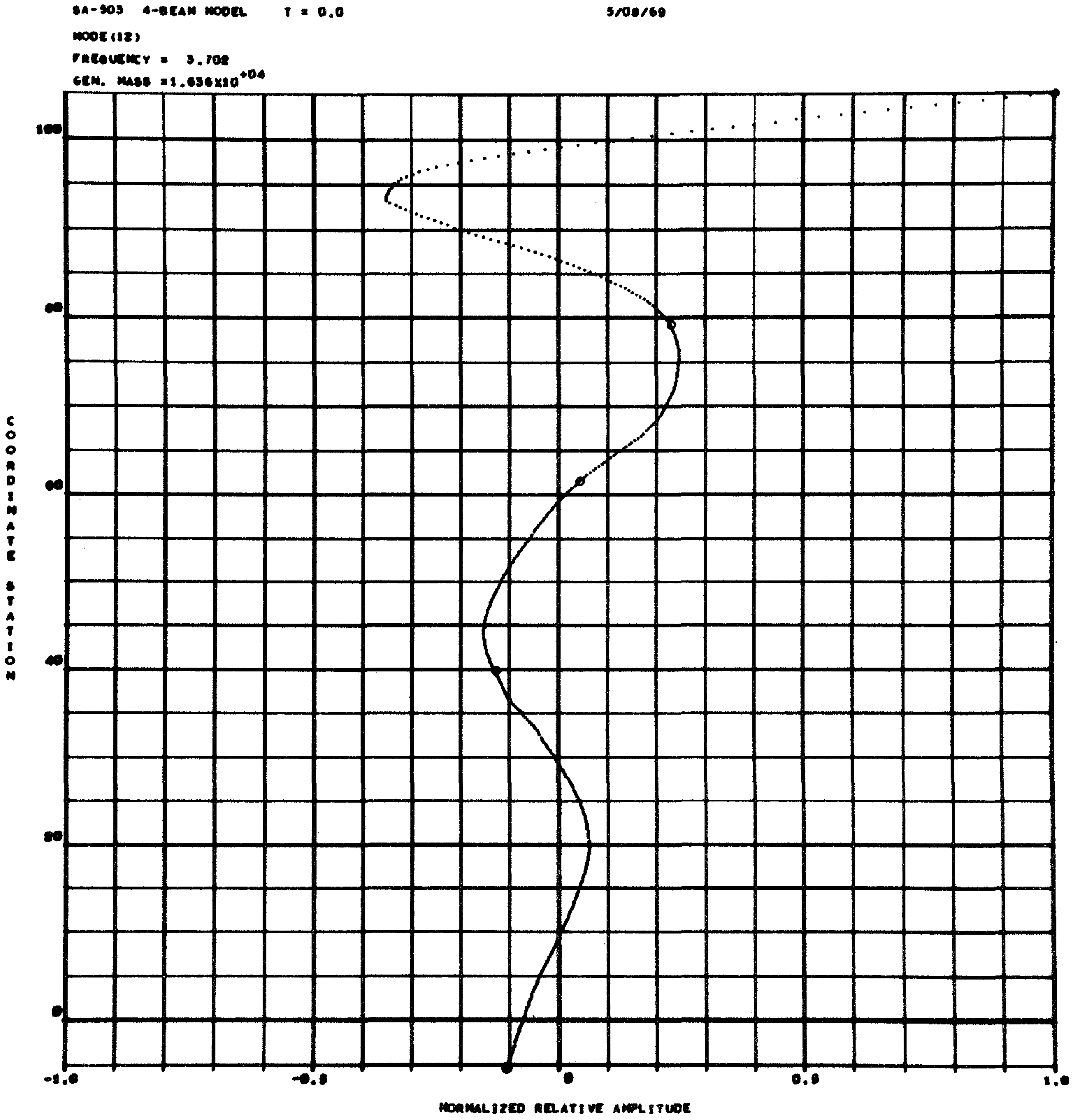


Fig. 48 - 12th Mode Shape (one slosh mode per tank)

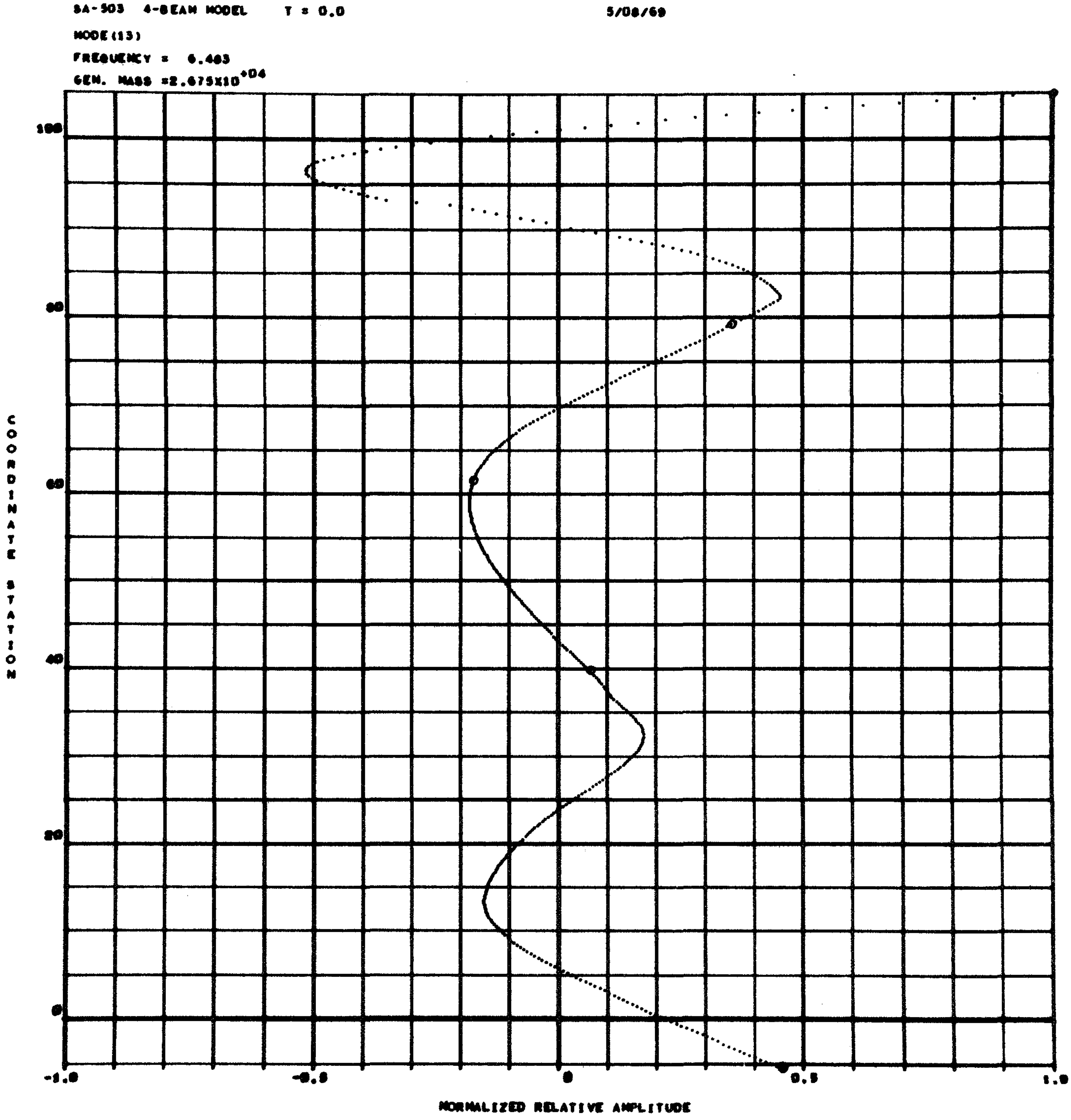


Fig. 49 - 13th Mode Shape (one slosh mode per tank)

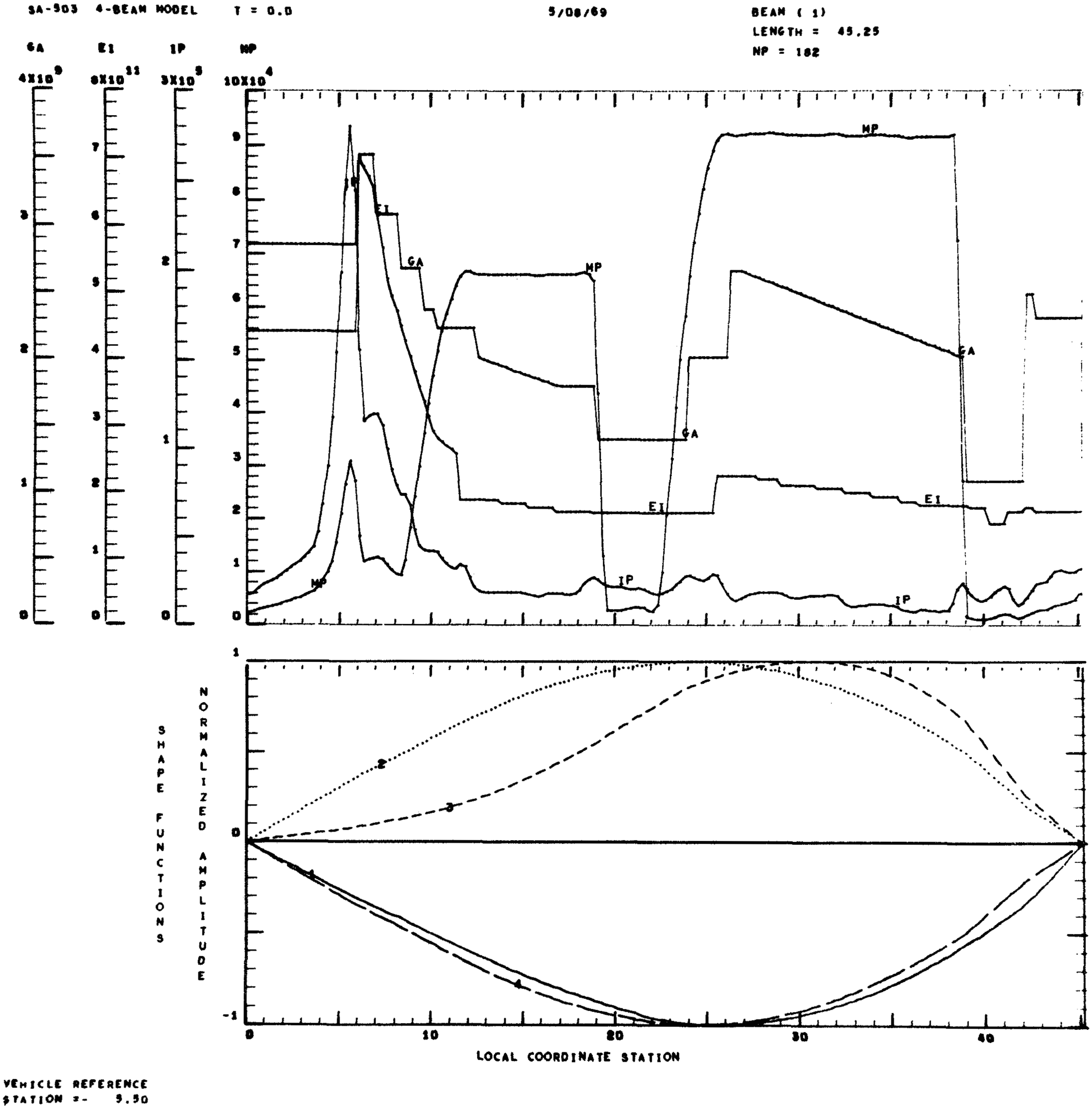


Fig. 50 - Fundamental Deflection Functions, Mass and Stiffness Properties of Beam 1

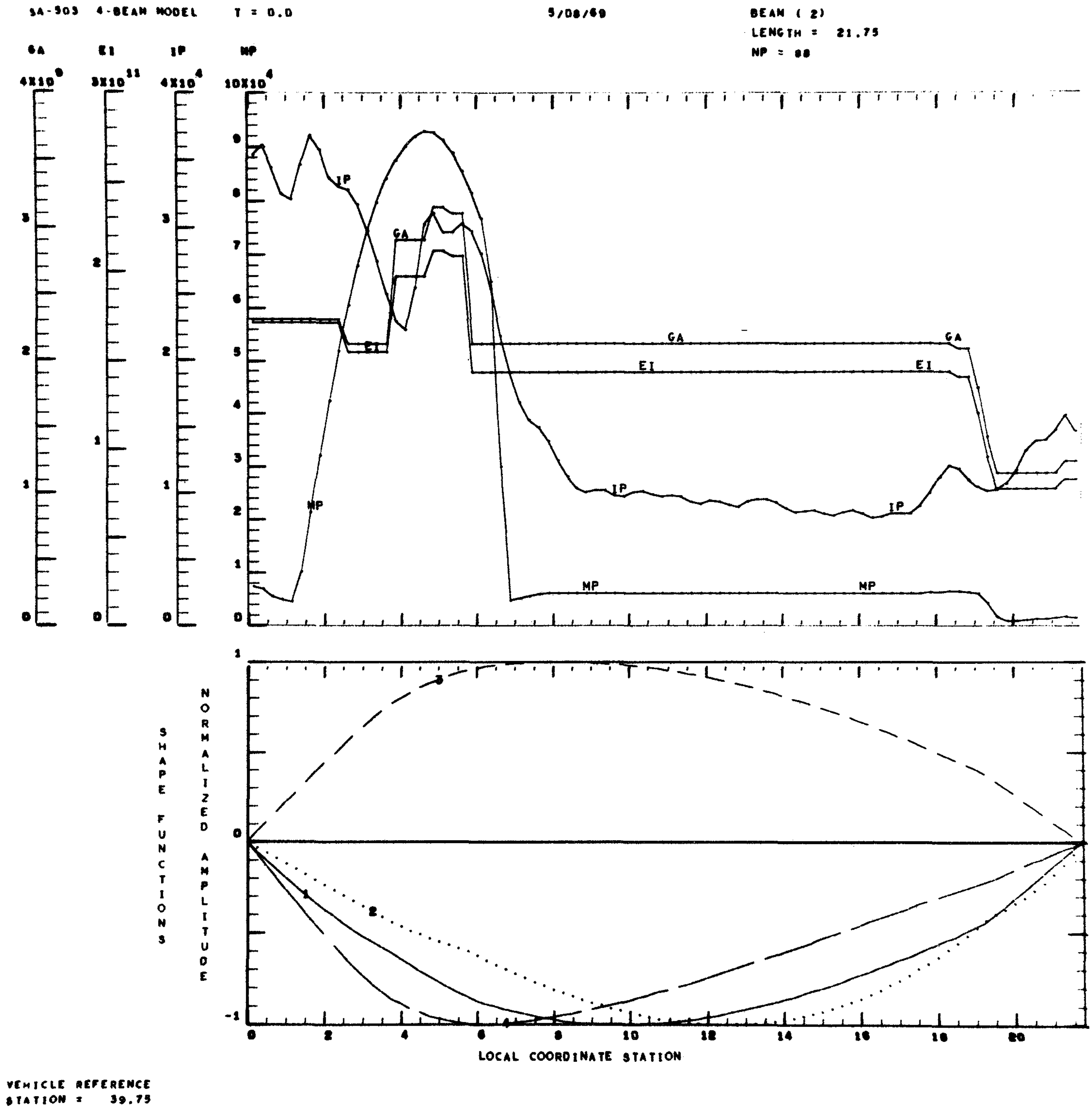


Fig. 51 - Fundamental Deflection Functions, Mass and Stiffness Properties of Beam 2

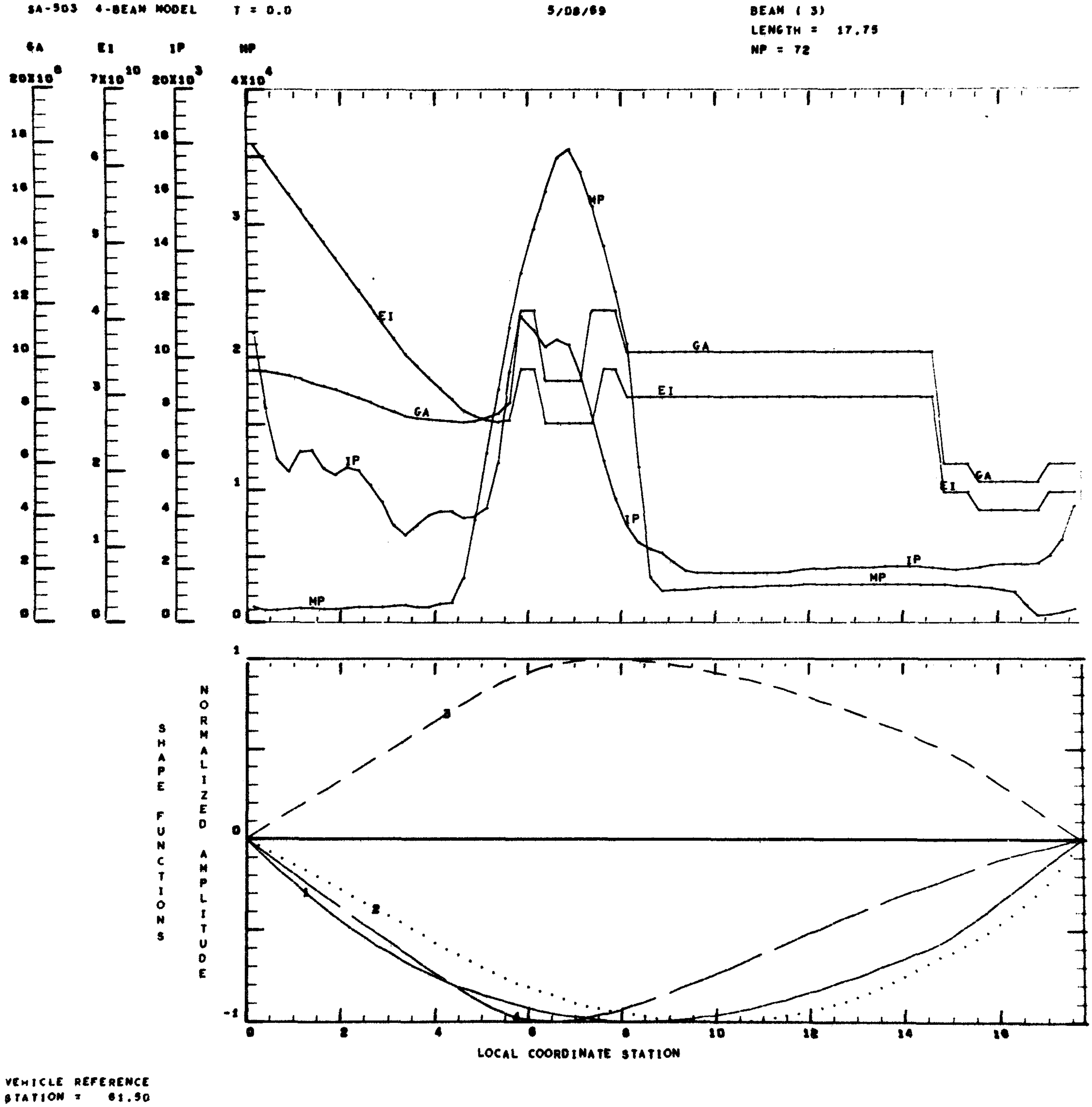


Fig. 52 - Fundamental Deflection Functions, Mass and Stiffness Properties of Beam 3

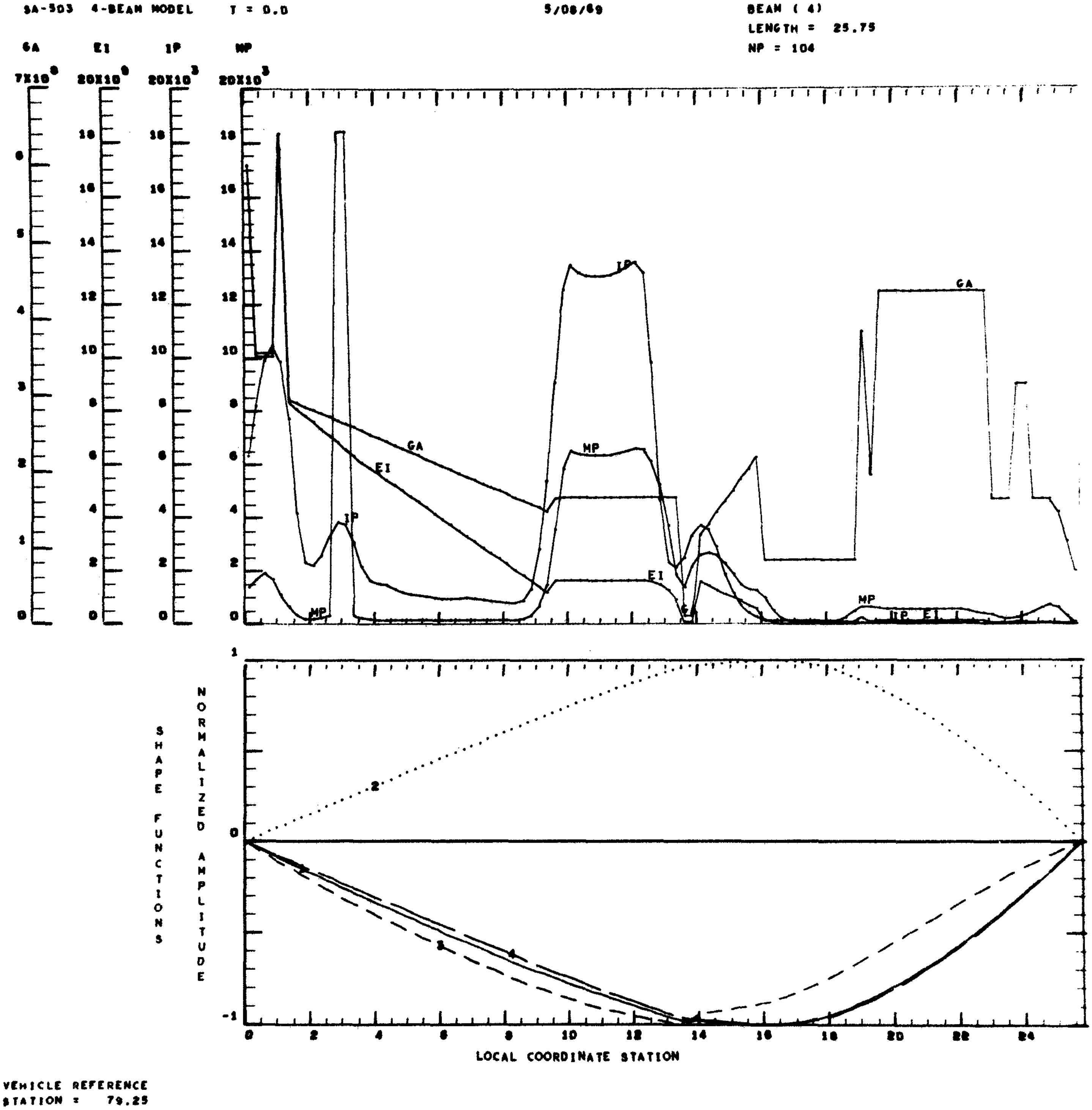
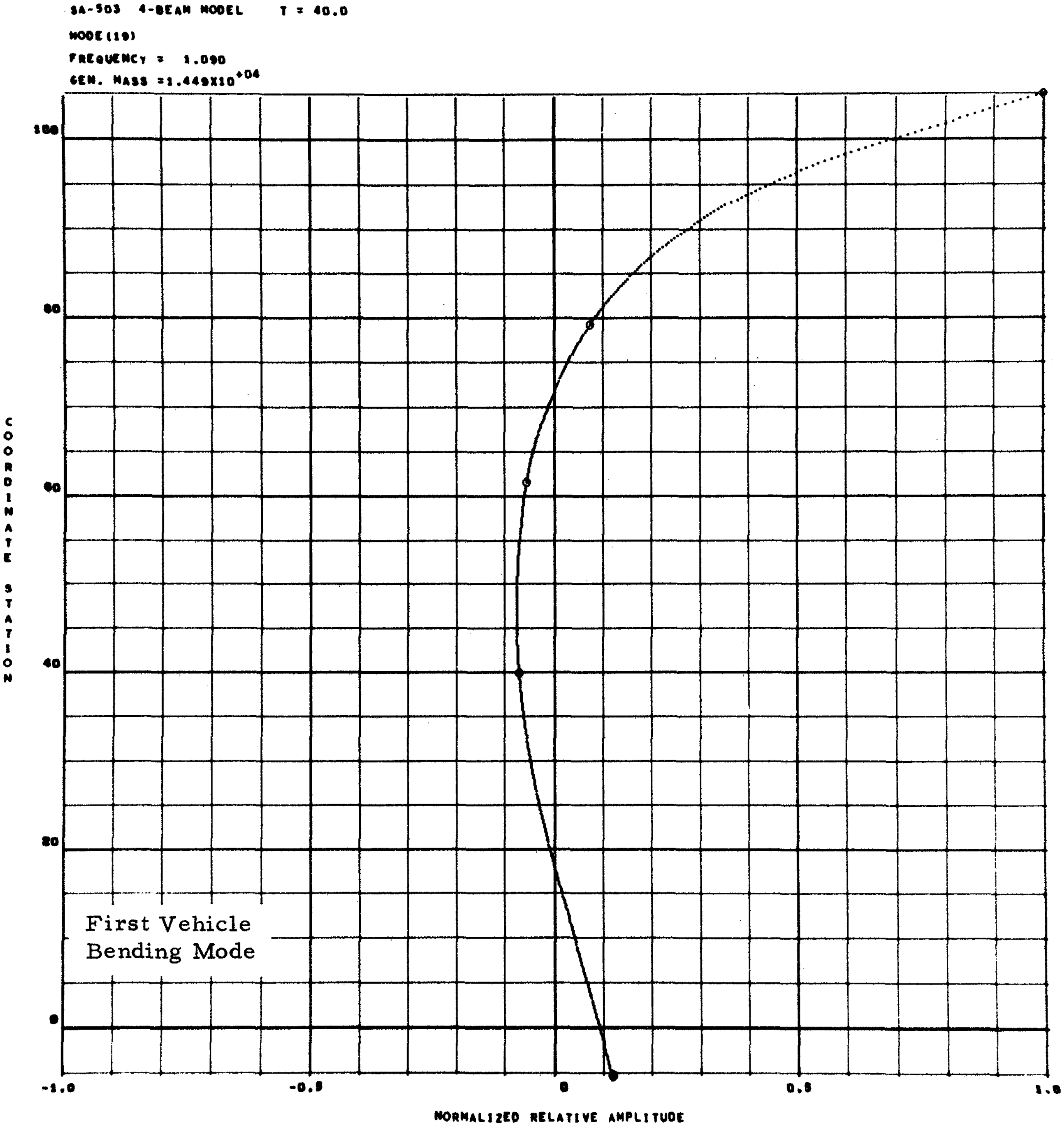


Fig. 53 - Fundamental Deflection Functions, Mass and Stiffness Properties of Beam 4



First Vehicle Bending Mode

Fig. 54 - 19th Mode Shape (three slosh modes per tank)

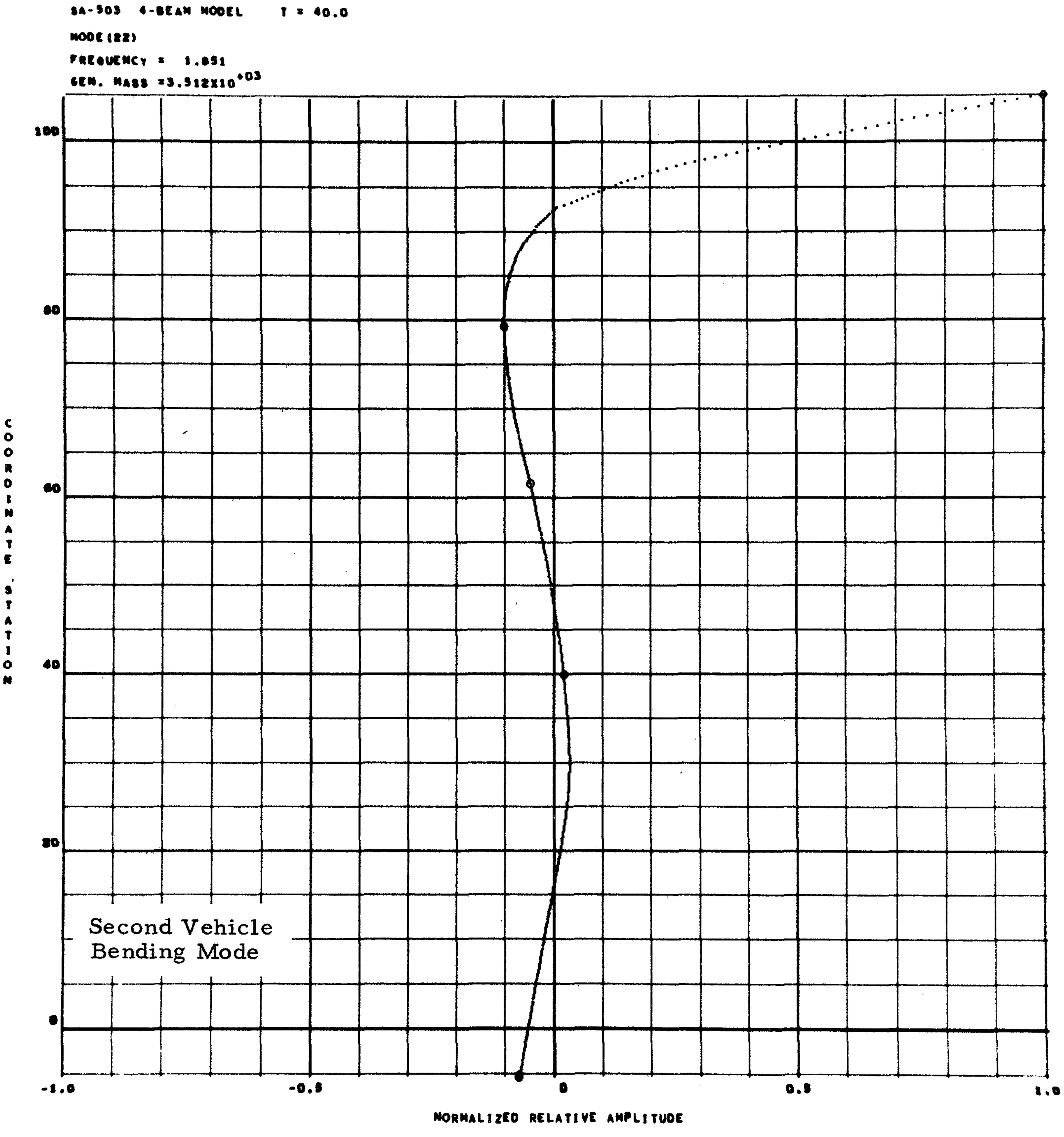


Fig. 55 - 22nd Mode Shape (three slosh modes per tank)

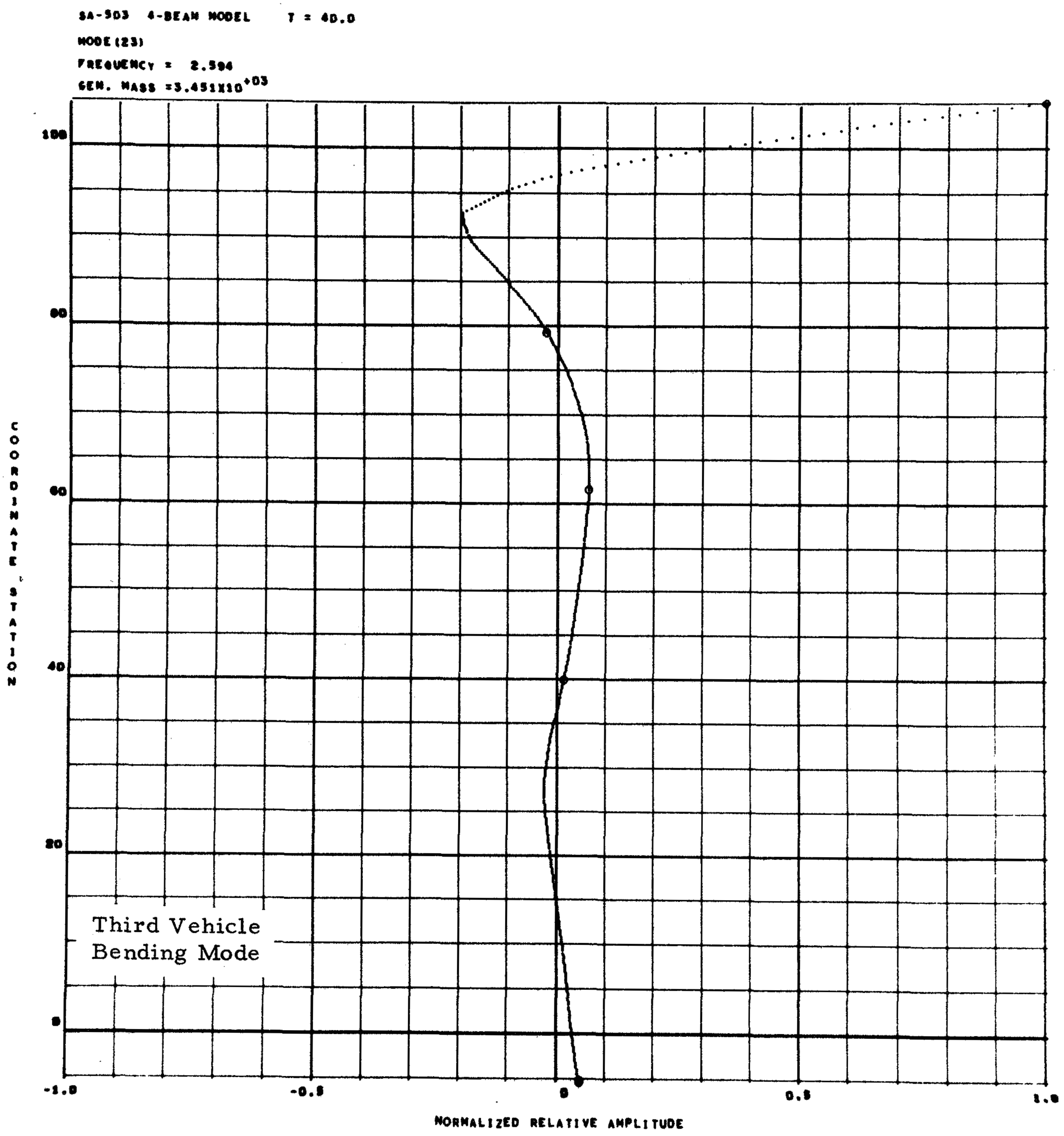


Fig. 56 - 23rd Mode Shape (three slosh modes per tank)

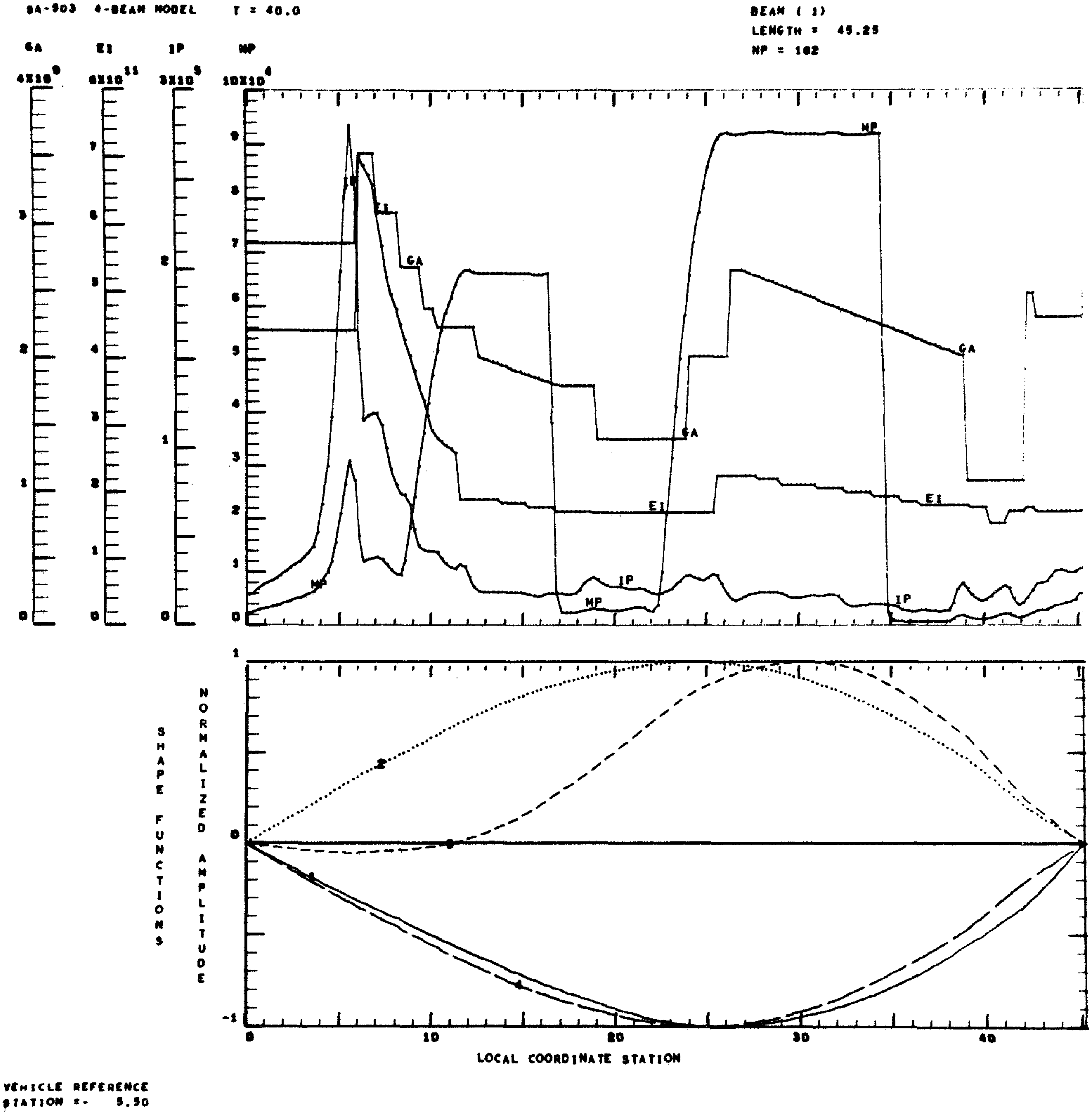


Fig. 57 - Fundamental Deflection Functions, Mass and Stiffness Properties

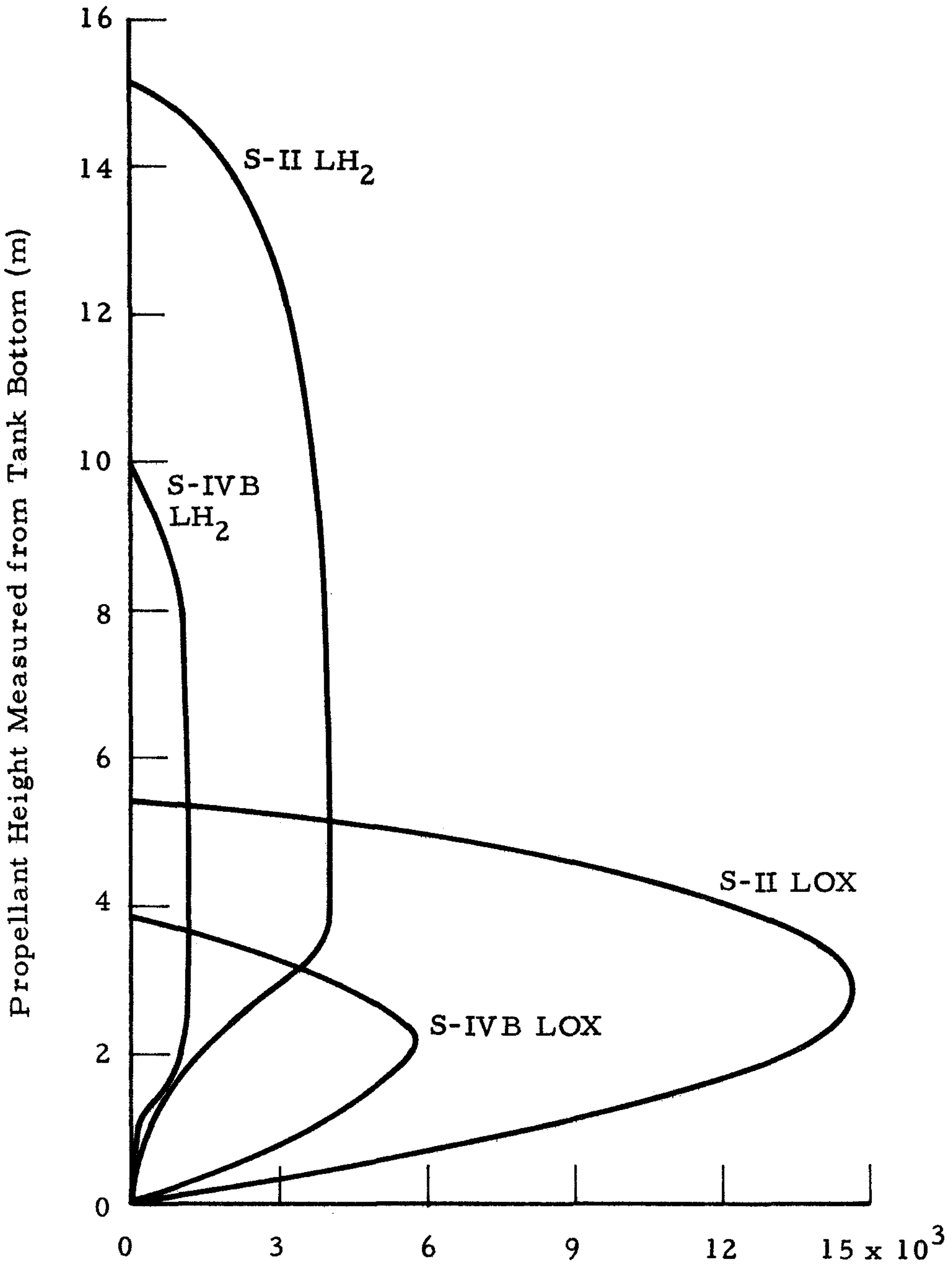


Fig. 58 - Lateral Force Distribution Coefficients ${}_mA_\alpha$ (kg) for Second- and Third-Stage Tanks

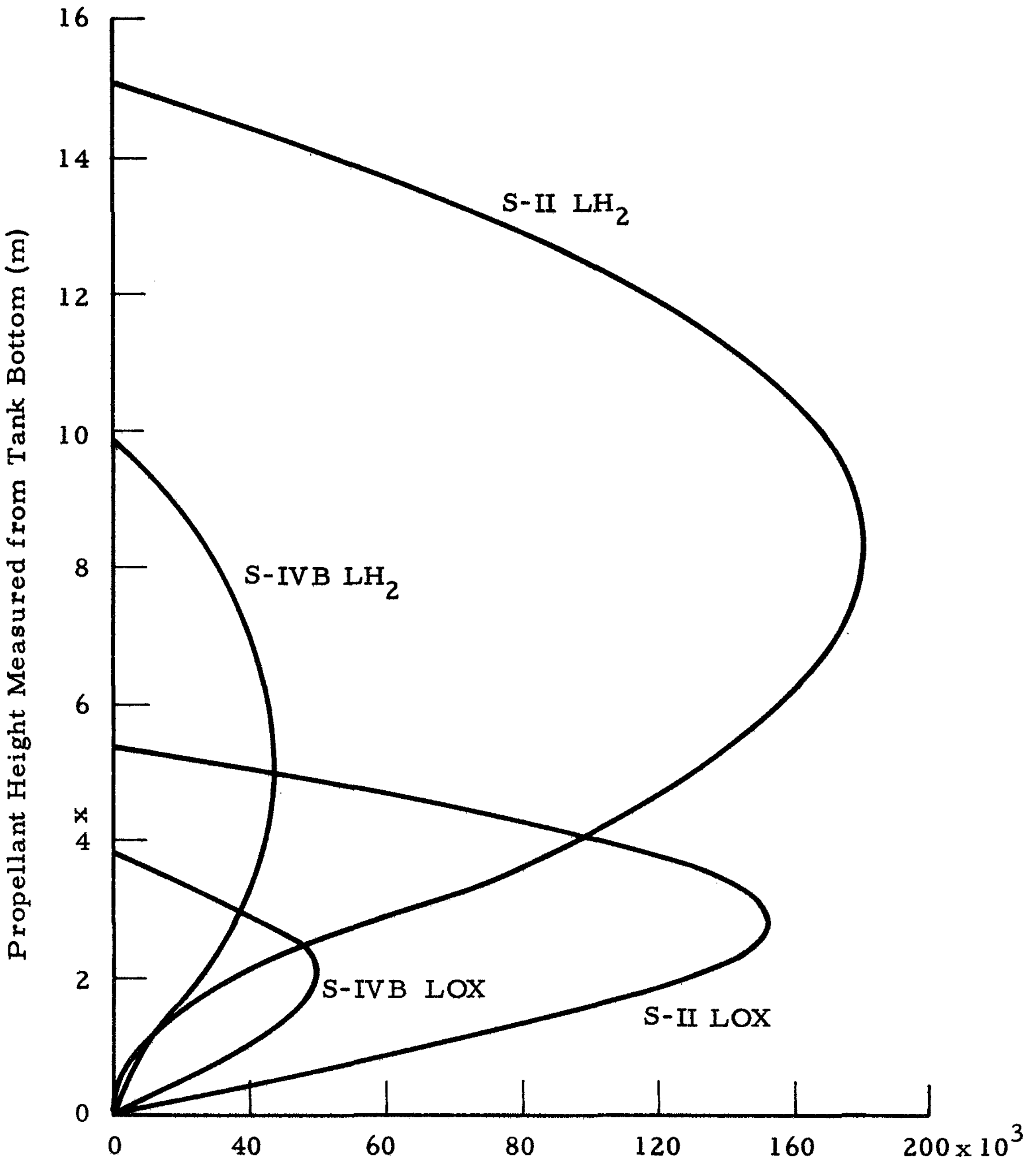


Fig. 59 - Lateral Force Distribution Coefficients $_m B_\psi$ (kg/m) for Second- and Third-Stage Tanks

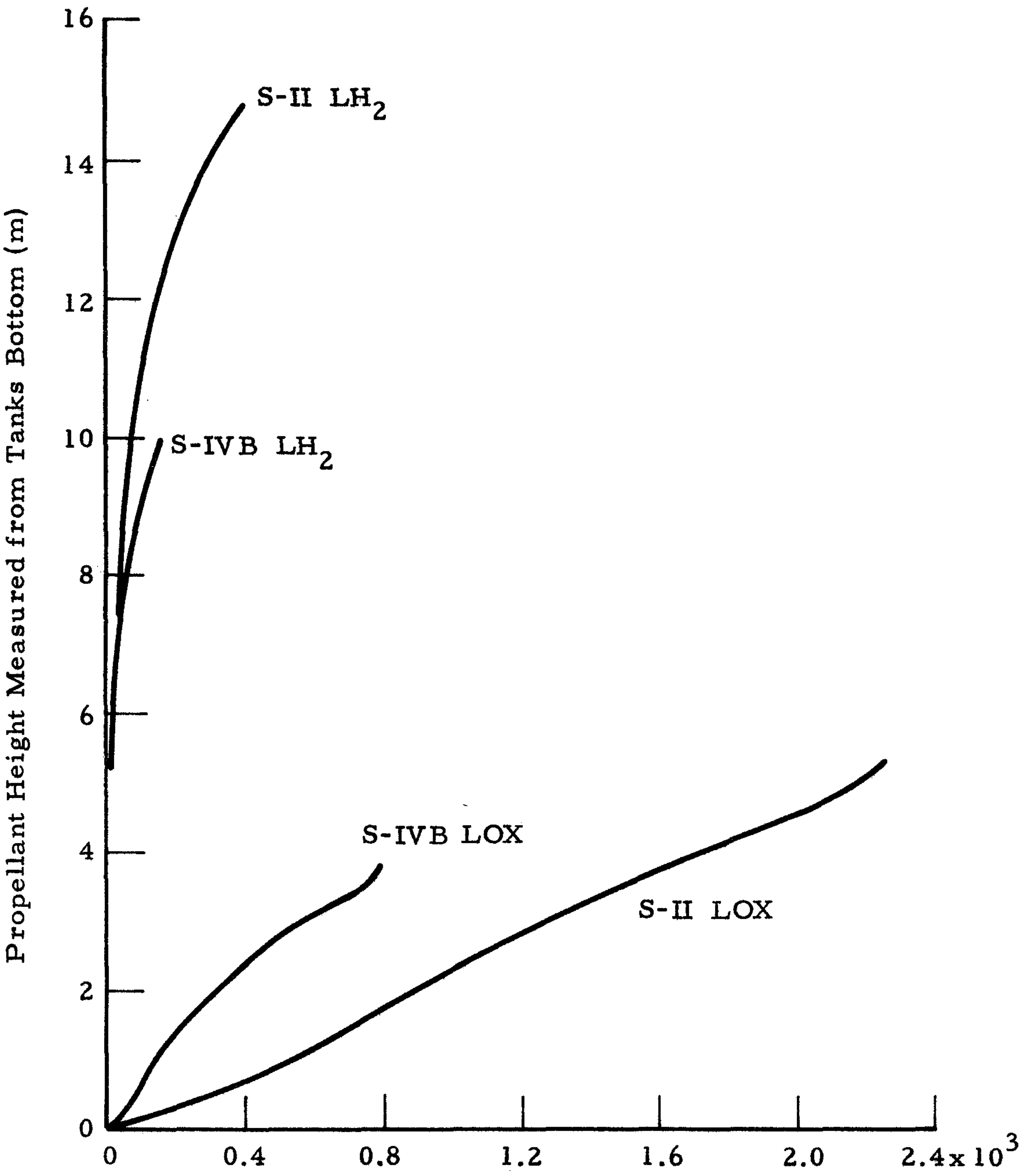


Fig. 60 - Lateral Force Distribution Coefficients $({}_{m}C_{\xi})_1/\alpha_3$(kg/m) for Second- and Third-Stage Tanks

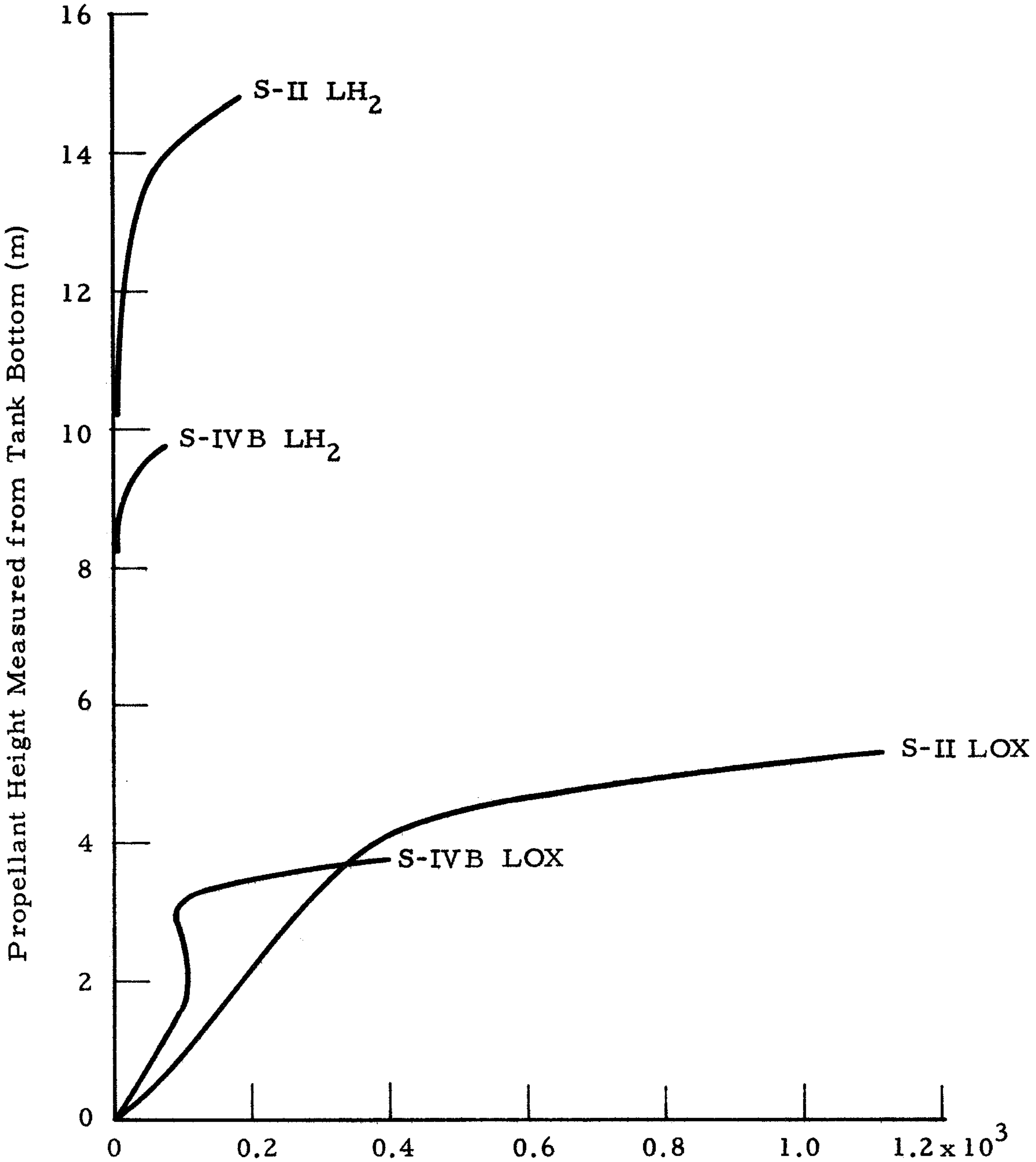


Fig. 61 - Lateral Force Distribution Coefficients $({}_{m}C_{\xi})_2/\alpha_3$ (kg/m) for Second- and Third-Stage Tanks

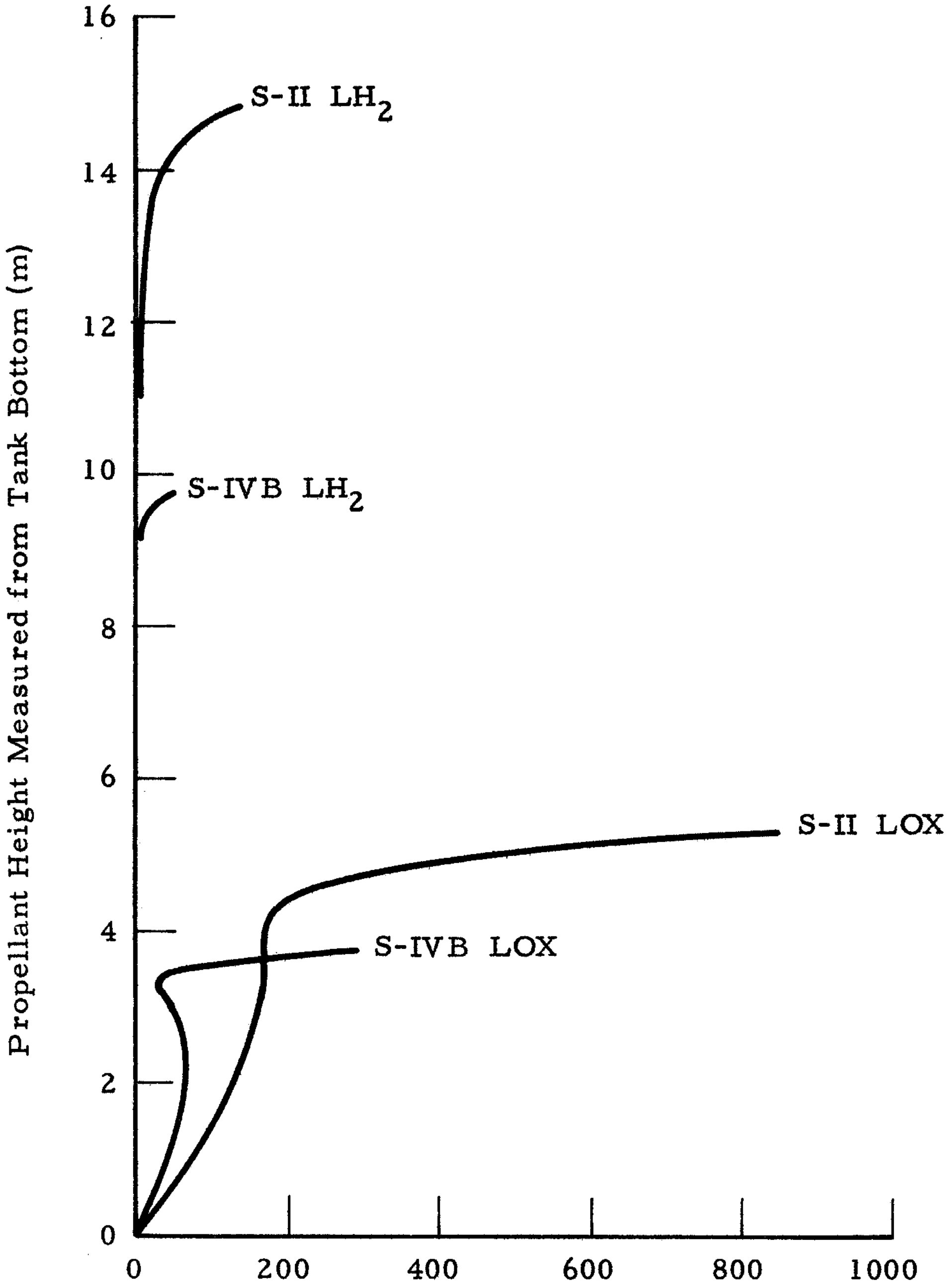


Fig. 62 - Lateral Force Distribution Coefficients $({}_mC_\xi)_3/\alpha_3$ (kg/m) for Second- and Third-Stage Tanks

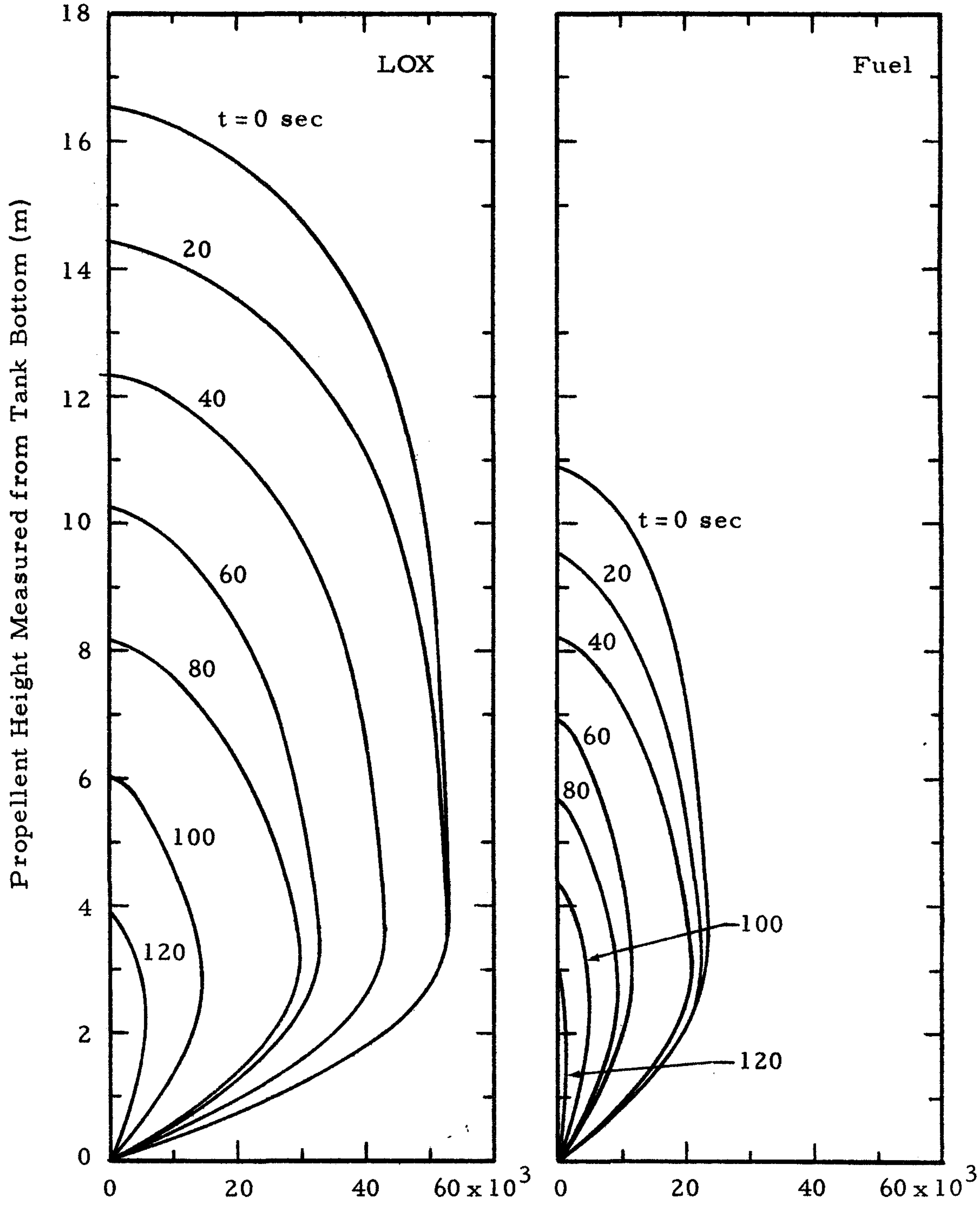


Fig. 63 - Lateral Force Distribution Coefficients ${}_{m}A_{\alpha}$ (kg) for S-IC Tanks

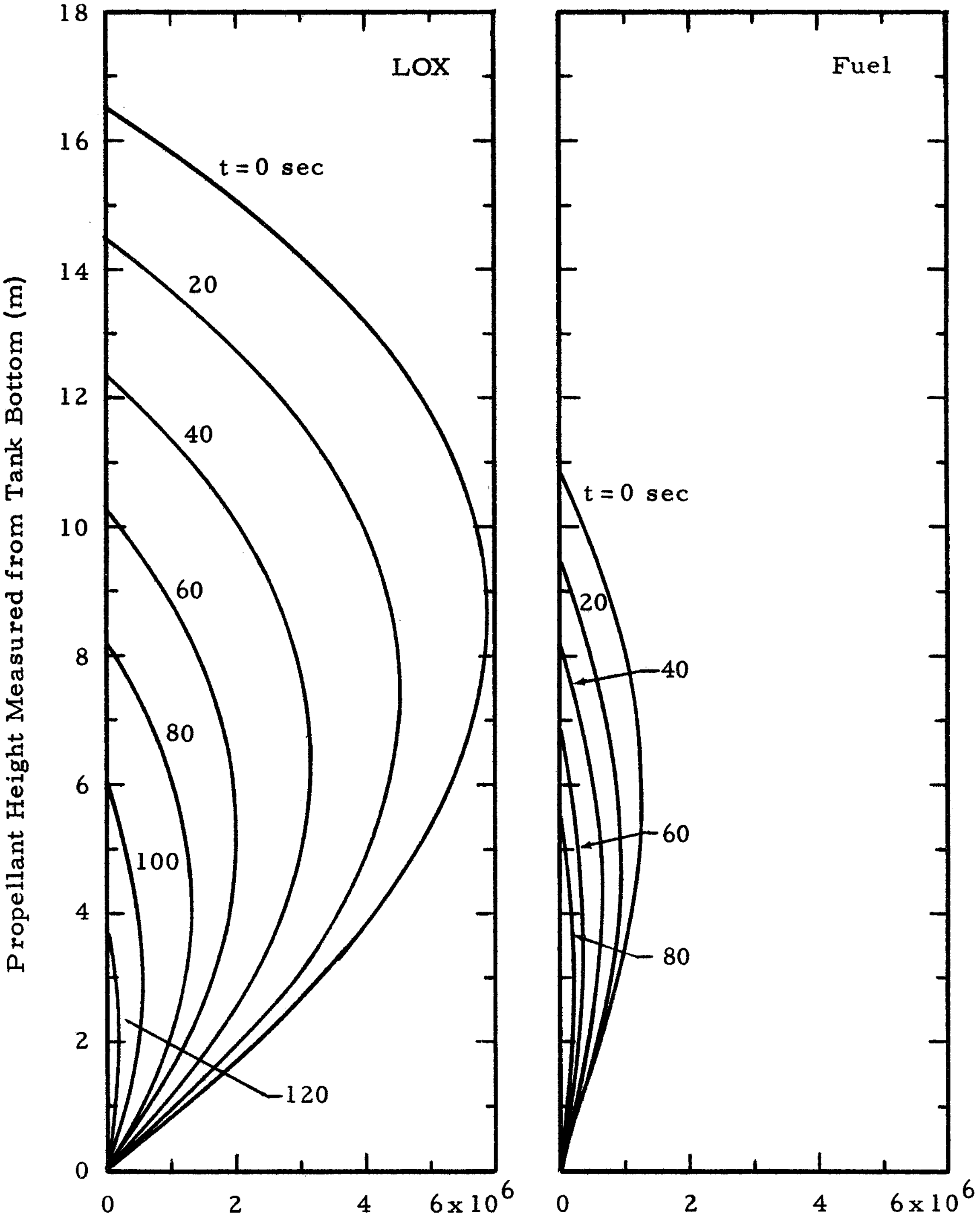


Fig. 64 - Lateral Force Distribution Coefficients ${}_{m}B_{\psi}$ (kg/m) for S-IC Tanks

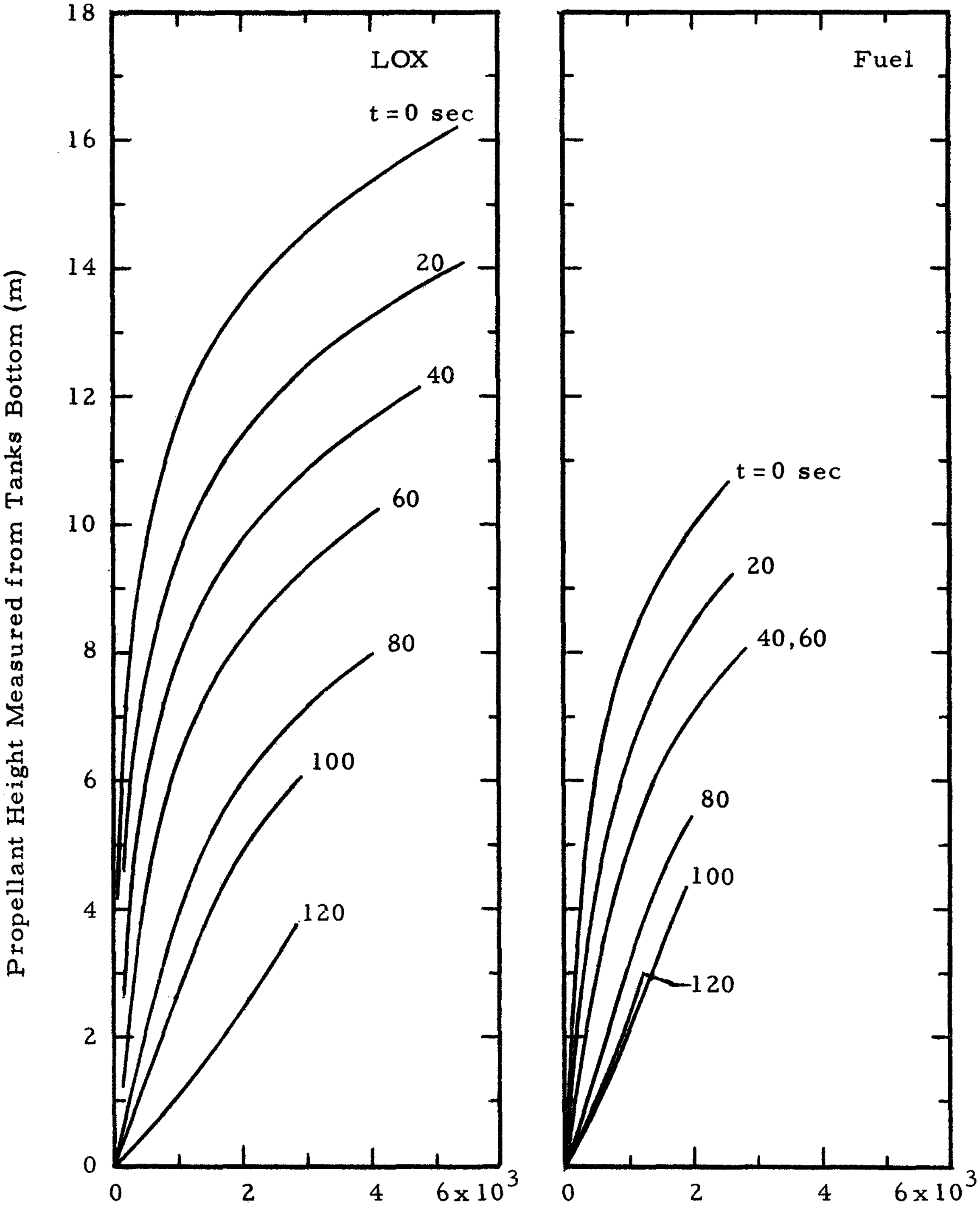


Fig. 65 - Lateral Force Distribution Coefficients $({}_{m}C_{\xi})_1/\alpha_3$ (kg/m) for S-IC Tanks

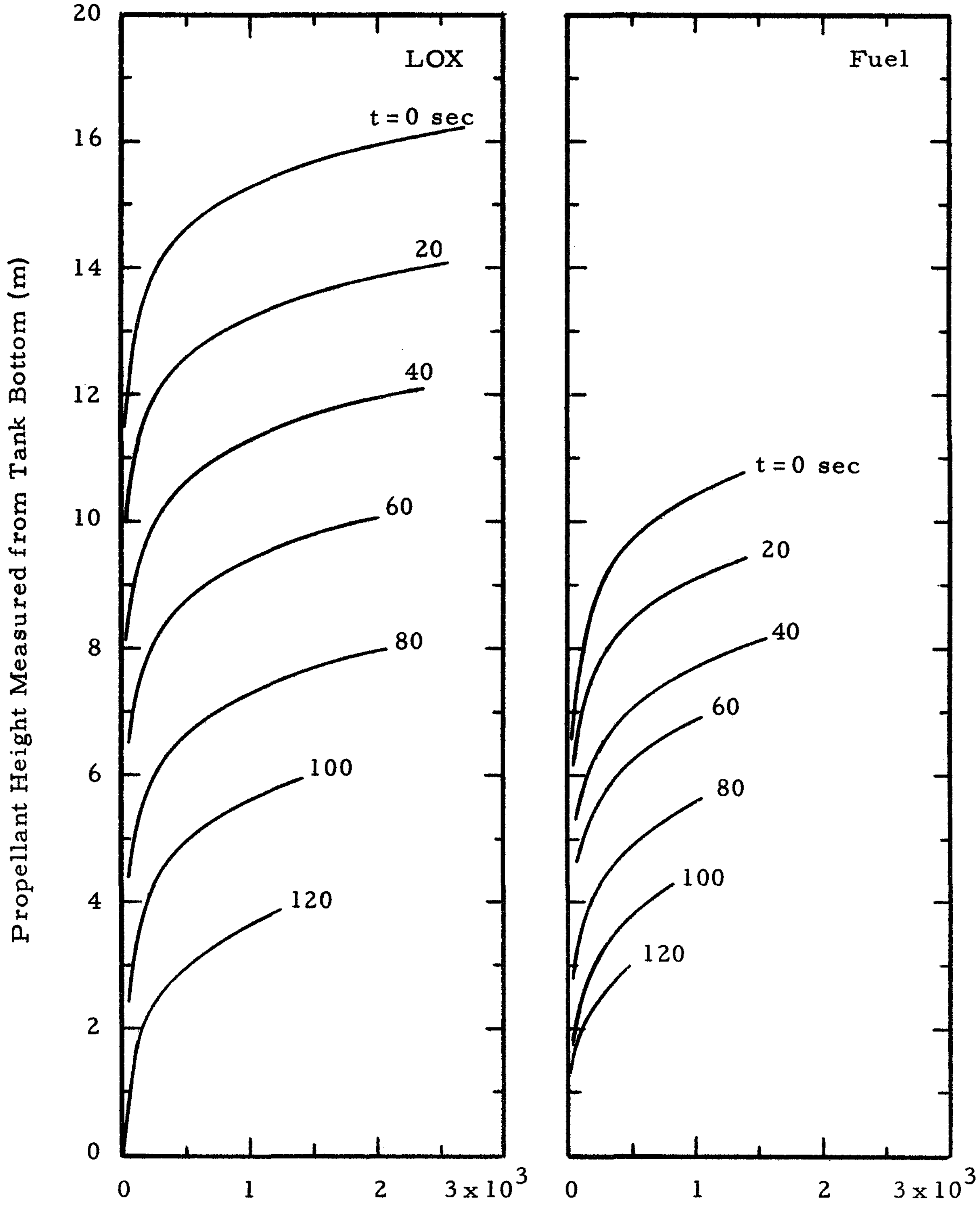


Fig. 66 - Lateral Force Distribution Coefficients $({}_{m}C_{\xi})_{2}/\alpha_{3}$ (kg/m) for S-IC Tanks

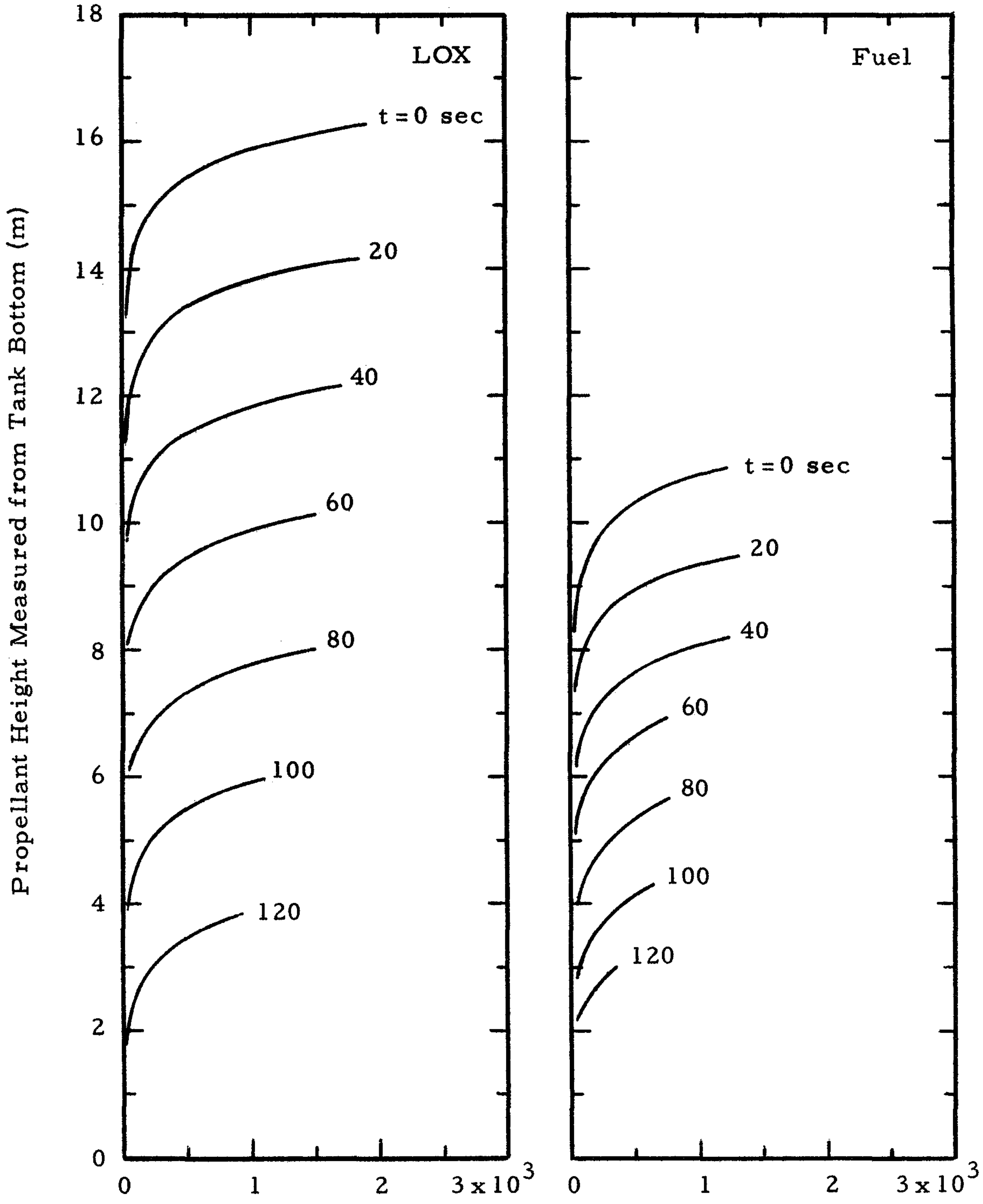


Fig. 67 - Lateral Force Distribution Coefficients $({}_mC_\xi)_3/\alpha_3$ (kg/m) for S-IC Tanks

Section 6
CONCLUSION AND RECOMMENDATION

A new method was derived to study the vibrational characteristics of a coupled elastic and fluid system idealized from a liquid-propellant space vehicle. This method presents a consistent formulation for the physical problem and has less restrictions in application than the conventional mechanical model approach. In addition, the method provides a possibility that just a few system modes which are influencing the vehicle dynamics will be needed to define a mathematical model for flight simulation of a space vehicle. Consequently, it will not only lead to a simple and reliable model but also meet the limitations of a computer. For instance, the limited capacity of a hybrid computer and excessive computation time required on a digital computer are the problems which are commonly encountered in flight simulation.

Due to the complexity of the computer program, certain possible improvements of the program are not able to be made in this contract. Specific areas are simplification of the input data deck, possible savings of computer time and to evaluate the integrals of Eq. (A.4) in a better manner. However, the method was demonstrated in this preliminary study that it is a logical approach to solve the coupled bending and sloshing problem of a large-liquid propellant space vehicle.

For an axisymmetric tank, the velocity field of the fluid can be expressed by Eqs. (3.2), (3.3) and (3.4). The eigenvectors c_{km}^{n} and slosh frequencies λ_{kn} can be obtained from Ref. 4. In case of an arbitrary tank which does not possess a nice geometric symmetry, seeking an analytic expression of the velocity potential of the fluid is almost impossible. Perhaps, to define an empirical equation based on experiment is the only solution to the problem. Once the velocity potential of a fluid system is given in an explicit form, the kinetic and potential energy terms associated with a tank can be readily computed. Hence, the presented method may be used to study non-beamlike vehicles whose propellant tanks do not have symmetric properties. Furthermore, utilization of the current capability of the developed program, the Saturn V vehicle may be modeled more accurately than the present four-beam model.

REFERENCES

1. Whetstone, W. D., and M. L. Pearson, "Vibrational Characteristics of Large Complex Space Vehicles," LMSC/HREC A783589, Lockheed Missiles & Space Company, Huntsville, Ala., December 1966.

2. Abramson, H. N., Editor, "The Dynamic Behavior of Liquids in Moving Containers," NASA SP-106, National Aeronautics and Space Administration, Washington, D. C., 1966.

3. Ryan, R. S., and A. W. King, "The Influential Aspects of Atmospheric Disturbances on Space Vehicle Design Using Statistical Approaches for Analysis," NASA TN D-4963, National Aeronautics & Space Administration, Washington, D. C., January 1969.

4. Lomen, D. O., "Analysis of Fluid Sloshing," GDC-DDE66-018, General Dynamics/Convair Division, San Diego, Calif., June 1966.

5. Abramowitz, M., and I. A. Stegun, Editors, Handbook of Mathematical Functions, National Bureau of Standards, Washington, D. C., August 1966.

6. Lowan, A. N., N. Davids and A. Levenson, "Table of the Zeros of the Legendre Polynomials of Order 1-16 and the Weight Coefficients for Gauss' Mechanical Quadrature Formula," Bull. Am. Math. Soc., Vol. 48, No. 10, pp. 739-743, October 1942.

Appendix A

DETAILED DERIVATIONS OF SECTION 3

Appendix

Substituting Eqs. (3.2) and (3.3) into Eq. (3.1), the kinetic energy of the k^{th} tank can be expressed as

$$
\begin{aligned}
T_k = \frac{1}{2}\rho_k \int_V \Bigg\{ & (\dot{u}_i^\ell)^2 + 2\dot{u}_i^\ell \frac{\dot{u}_i^r - \dot{u}_i^\ell}{L_i}(d_{1k}+x_3^*) + \left(\frac{\dot{u}_i^r - \dot{u}_i^\ell}{L_i}\right)^2 (d_{1k}+x_3^*)^2 \\
& + 2\dot{u}_i^\ell \sum_{j=1}^{N_i} \dot{\zeta}_{ij} Y_{ij} + 2\frac{\dot{u}_i^r - \dot{u}_i^\ell}{L_i}(d_{1k}+x_3^*) \sum_{j=1}^{N_i} \dot{\zeta}_{ij} Y_{ij} + \sum_{j=1}^{N_i} \sum_{m=1}^{N_i} \dot{\zeta}_{ij} \dot{\zeta}_{im} Y_{ij} Y_{im} \\
& + 2\left[\dot{u}_i^\ell + \frac{\dot{u}_i^r - \dot{u}_i^\ell}{L_i}(d_{1k}+x_3^*) + \sum_{j=1}^{N_i} \dot{\zeta}_{ij} Y_{ij}\right] \Bigg[\sum_{n=1}^{N_k} \frac{1}{\lambda_{kn}} \dot{\xi}_{kn}\Big(\sin^2\theta \frac{\partial\phi_{kn}}{\partial R} \\
& + \cos^2\theta \frac{\phi_{kn}}{R}\Big)\Bigg] \\
& + \sum_{n=1}^{N_k} \sum_{m=1}^{N_k} \dot{\xi}_{kn} \dot{\xi}_{km} \Bigg[\sin^2\theta \frac{\partial\phi_{kn}}{\partial R} \frac{\partial\phi_{km}}{\partial R} + \cos^2\theta \frac{\phi_{kn}}{R} \frac{\phi_{km}}{R} \\
& + \sin^2\theta \frac{\partial\phi_{kn}}{\partial Z} \frac{\partial\phi_{km}}{\partial Z}\Bigg] \\
& + \left(\frac{\dot{u}_i^r - \dot{u}_i^\ell}{L_i} r \sin\theta\right)^2 - 2\frac{\dot{u}_i^r - \dot{u}_i^\ell}{L_i} r \sin^2\theta \sum_{n=1}^{N_k} \frac{1}{\lambda_{kn}} \dot{\xi}_{kn} \frac{\partial\phi_{kn}}{\partial Z} \Bigg\} a_k^3 R\, d\theta\, dR\, dZ
\end{aligned}
$$

$$= \pi a_k^3 \rho_k \int_S \left\{ (\dot{u}_i^{\ell})^2 \left[1 - 2 \frac{d_{1k}}{L_i} - 2 \frac{a_k}{L_i} Z^* + \left(\frac{d_{1k}}{L_i} \right)^2 + 2 \frac{d_{1k}}{L_i} \frac{a_k}{L_i} Z^* \right. \right.$$

$$\left. + \left(\frac{a_k}{L_i} Z^* \right)^2 + \frac{1}{2} \left(\frac{a_k}{L_i} R \right)^2 \right]$$

$$+ (\dot{u}_i^r)^2 \left[\left(\frac{d_{1k}}{L_i} \right)^2 + 2 \frac{d_{1k}}{L_i} \frac{a_k}{L_i} Z^* + \left(\frac{a_k}{L_i} Z^* \right)^2 + \frac{1}{2} \left(\frac{a_k}{L_i} R \right)^2 \right]$$

$$+ 2 \dot{u}_i^{\ell} \dot{u}_i^r \left[\frac{d_{1k}}{L_i} + \frac{a_k}{L_i} Z^* - \left(\frac{d_{1k}}{L_i} \right)^2 - 2 \frac{d_{1k}}{L_i} \frac{a_k}{L_i} Z^* - \left(\frac{a_k}{L_i} Z^* \right)^2 - \frac{1}{2} \left(\frac{a_k}{L_i} R \right)^2 \right]$$

$$+ 2 \dot{u}_i^{\ell} \sum_{j=1}^{N_i} \dot{\zeta}_{ij} \left[1 - \frac{d_{1k}}{L_i} - \frac{a_k}{L_i} Z^* \right] Y_{ij} + 2 \dot{u}_i^r \sum_{j=1}^{N_i} \dot{\zeta}_{ij} \left[\frac{d_{1k}}{L_i} + \frac{a_k}{L_i} Z^* \right] Y_{ij}$$

$$+ \sum_{j=1}^{N_i} \sum_{m=1}^{N_i} \dot{\zeta}_{ij} \dot{\zeta}_{im} Y_{ij} Y_{im}$$

$$+ \dot{u}_i^{\ell} \sum_{n=1}^{N_k} \frac{1}{\lambda_{kn}} \dot{\xi}_{kn} \left[\left(\frac{\partial \phi_{kn}}{\partial R} + \frac{\phi_{kn}}{R} \right) \left(1 - \frac{d_{1k}}{L_i} - \frac{a_k}{L_i} Z^* \right) + \frac{a_k}{L_i} R \frac{\partial \phi_{kn}}{\partial Z} \right]$$

$$+ \dot{u}_i^r \sum_{n=1}^{N_k} \frac{1}{\lambda_{kn}} \dot{\xi}_{kn} \left[\left(\frac{\partial \phi_{kn}}{\partial R} + \frac{\phi_{kn}}{R} \right) \left(\frac{d_{1k}}{L_i} + \frac{a_k}{L_i} Z^* \right) - \frac{a_k}{L_i} R \frac{\partial \phi_{kn}}{\partial Z} \right]$$

$$+\sum_{j=1}^{N_i}\sum_{n=1}^{N_k}\frac{1}{\lambda_{kn}}\,\dot{\zeta}_{ij}\,\dot{\xi}_{kn}\left(\frac{\partial\phi_{kn}}{\partial R}+\frac{\phi_{kn}}{R}\right)Y_{ij}$$

$$+\frac{1}{2}\sum_{n=1}^{N_k}\sum_{m=1}^{N_k}\frac{1}{\lambda_{kn}\lambda_{km}}\,\dot{\xi}_{kn}\,\dot{\xi}_{km}\Bigg[\frac{\partial\phi_{kn}}{\partial R}\,\frac{\partial\phi_{km}}{\partial R}+\frac{\phi_{kn}}{R}\,\frac{\phi_{km}}{R}$$

$$+\frac{\partial\phi_{kn}}{\partial Z}\,\frac{\partial\phi_{km}}{\partial Z}\Bigg]\Bigg]\Bigg\}\,RdRdZ \qquad \text{(A.1)}$$

where

$$Z^{*}=\frac{G_{3k}-d_{1k}}{a_k}+Z \quad \text{and} \quad dZ^{*}=dZ.$$

If Eq. (3.4) is used and integrations with respect to Z are performed (Ref. 6), Eq. (A-1) may be further reduced to the following form.

$$T_k=-\pi a_k^3\rho_k\Bigg\{V_k^{pp}(\dot{u}_i^{\ell})^2+V_k^{pq}\,\dot{u}_i^{\ell}\,\dot{u}_i^{r}+V_k^{qq}(\dot{u}_i^{r})^2$$

$$+\dot{u}_i^{\ell}\sum_{j=1}^{N_i}(ULB)_{ij}\,\dot{\zeta}_{ij}+\dot{u}_i^{r}\sum_{j=1}^{N_i}(URB)_{ij}\,\dot{\zeta}_{ij}$$

$$+\sum_{j=1}^{N_i}\sum_{m=1}^{N_i}(BB)_{ijm}\,\dot{\zeta}_{ij}\,\dot{\zeta}_{im}+\dot{u}_i^{\ell}\sum_{n=1}^{N_k}\frac{1}{\lambda_{kn}}\,S_{kn}^{p}\,\dot{\xi}_{kn}$$

$$+ \dot{u}_i^r \sum_{n=1}^{N_k} \frac{1}{\lambda_{kn}} S_{kn}^q \dot{\xi}_{kn} + \sum_{j=1}^{N_i} \sum_{n=1}^{N_k} \frac{1}{\lambda_{kn}} (BS)_{kjn} \dot{\zeta}_{ij} \dot{\xi}_{kn}$$

$$\left. + \frac{1}{2} \sum_{n=1}^{N_k} \sum_{m=1}^{N_k} \frac{1}{\lambda_{kn} \lambda_{km}} S_{kmn} \dot{\xi}_{kn} \dot{\xi}_{km} \right\} \qquad \text{(A.2)}$$

where

$$\left\{ \begin{aligned} V_k^{pp} &= V_k^{qq} + \oint \left[1 - 2 \frac{d_{1k}}{L_i} - \frac{a_k}{L_i} Z^* \right] Z^* RdR \\ V_k^{pq} &= -2 V_k^{qq} + \oint \left[2 \frac{d_{1k}}{L_i} + \frac{a_k}{L_i} Z^* \right] Z^* RdR \\ V_k^{qq} &= \oint \left[\left(\frac{d_{1k}}{L_i} \right)^2 + \frac{1}{2} \left(\frac{a_k}{L_i} R \right)^2 + \frac{d_{1k}}{L_i} \frac{a_k}{L_i} Z^* + \frac{1}{3} \left(\frac{a_k}{L_i} Z^* \right)^2 \right] Z^* RdR \end{aligned} \right. \qquad \text{(A.3)}$$

$$\left\{ \begin{aligned} (ULB)_{ij} &= -2 \int_S Y_{ij} RdRdZ - (URB)_{ij} \\ (URB)_{ij} &= -2 \int_S \left[\frac{d_{1k}}{L_i} + \frac{a_k}{L_i} Z^* \right] Y_{ij} RdRdZ \\ (BB)_{ijm} &= -\int_S Y_{ij} Y_{im} RdRdZ^* \\ (BS)_{kjn} &= -\int_S \left[\frac{\partial \phi_{kn}}{\partial R} + \frac{\phi_{kn}}{R} \right] Y_{ij} RdRdZ \end{aligned} \right. \qquad \text{(A.4)}$$

$$\left\{\begin{aligned}
S^{p}_{kn} &= - S^{q}_{kn} + \sum_{j=1}^{5} c^{n}_{kj} \left\{ 2j \oint R^{2j-1} Z^{*} dR \right\} \\
&\quad + \sum_{j=6}^{10} c^{n}_{kj} \left\{ \oint \left[(R\, J'_{1}(i_{kj}R) + \frac{1}{j_{kj}} J_{1}(j_{kj}R) \right] e^{j_{kj}(Z - \frac{\ell_k}{a_k})} dR \right\} \\
S^{q}_{kn} &= \sum_{j=1}^{5} c^{n}_{kj} \left\{ 2j \oint \left[\frac{d_{1k}}{L_i} + \frac{1}{2} \frac{a_k}{L_i} Z^{*} \right] Z^{*} R^{2j-1} dR \right\} \\
&\quad + \sum_{j=6}^{10} c^{n}_{kj} \left\{ \oint \left[\left(R\, J'_{1}(j_{kj}R) + \frac{1}{j_{kj}} J_{1}(j_{kj}R) \right) \left(\frac{d_{1k}}{L_i} - \frac{a_k}{L_i} \frac{1}{j_{kj}} + \frac{a_k}{L_i} Z^{*} \right) \right. \right. \\
&\quad \left. \left. - \frac{a_k}{L_i} R^{2} J_{1}(j_{kj}R) \right] e^{j_{kj}(Z - \frac{\ell_k}{a_k})} dR \right\}
\end{aligned}\right. \qquad (A.5)$$

and

$$\begin{aligned}
S_{knm} &= \sum_{j=1}^{5} \sum_{i=1}^{5} c^{n}_{kj} c^{n}_{ki} (4_{ji} - 2j - 2i + 2) \oint R^{2j+2i-3} Z^{*} dR \\
&\quad + \sum_{j=1}^{5} \sum_{i=6}^{10} (c^{n}_{kj} c^{m}_{ki} + c^{m}_{kj} c^{n}_{ki}) \oint \left[(2j-1)\, J'_{1}(j_{ki}R) + \frac{1}{j_{ki}R} J_{1}(j_{ki}R) \right] \\
&\quad \cdot R^{2j-1} e^{j_{ki}(Z - \frac{\ell_k}{a_k})} dR
\end{aligned}$$

$$+\sum_{j=6}^{10} \sum_{i=6}^{10} c^{n}_{kj} c^{m}_{ki} \oint \Big[j_{kj}\, j_{ki}\, R\, J_1'(i_{kj} R)\, J_1'(j_{ki} R)$$

$$+ (j_{kj}\, j_{ki}\, R + \frac{1}{R})\, J_1(i_{ki} R)\, J_1(j_{ki} R) \Big] \frac{1}{j_{kj} + j_{ki}} \cdot$$

$$\cdot\, e^{(j_{kj} + j_{ki})(Z - \frac{\ell_k}{a_k})}\, dR \qquad \text{(A.6)}$$

The potential energy of the k^{th} tank (Eq. (3.7)) can be written in the following form

$$P_k = \frac{1}{2} \rho_k \alpha_3 a_k^2 \int_{FS} \left[\sum_{n=1}^{N_k} \sum_{m=1}^{N_k} \xi_{kn} \xi_{km} \phi_{kn} \phi_{km} \right] \sin^2 \theta R dR d\theta$$

$$= \frac{\pi}{2} \rho_k \alpha_3 a_k^2 \sum_{n=1}^{N_k} \sum_{m=1}^{N_k} p_{knm}\, \xi_{kn}\, \xi_{km} \qquad \text{(A.7)}$$

where

$$p_{knm} = \int_{o}^{1} R\, \phi_{kn}\, \phi_{km}\, dR$$

Substituting Eq. (3.4) into the above equation, one finds

$$p_{knm} = \sum_{j=1}^{5} \sum_{i=1}^{5} c^{n}_{kj} c^{n}_{ki} \int_{o}^{1} R^{2j+2i-1}\, dR$$

$$+ \sum_{j=1}^{5} \sum_{i=6}^{10} (c_{kj}^{n} c_{ki}^{m} + c_{kj}^{m} c_{ki}^{n}) \int_{o}^{1} R^{2j} J_1(j_{ki} R) \, dR$$

$$+ \sum_{j=6}^{10} \sum_{i=6}^{10} c_{kj}^{n} c_{ki}^{m} \int_{o}^{1} R \, J_1(j_{kj} R) \, J_1(j_{ki} R) \, dR$$

$$= \sum_{j=1}^{5} \sum_{i=1}^{5} c_{kj}^{n} c_{ki}^{m} \frac{1}{2(i+j)}$$

$$+ \sum_{j=1}^{5} \sum_{i=6}^{10} (c_{kj}^{n} c_{ki}^{m} + c_{kj}^{m} c_{ki}^{n}) \int_{o}^{1} R^{2j} J_1(j_{ki} R) \, dR$$

$$+ \sum_{j=6}^{10} c_{kj}^{n} c_{kj}^{m} \frac{1}{2 j_{ki}^{2}} (j_{kj}^{2} - 1) \left[J_1(j_{kj}) \right]^{2} \qquad \text{(A.8)}$$

Appendix B

SLOSH PROGRAM LISTINGS

```
$IBFTC SLOSH   DECK
      SUBROUTINE SLOSH(AIJ,BIJ,NCOR,NTANK,NSMODE)
C
C
C  ***  THIS PROGRAM PROVIDES PRELIMINARY INFORMATION ON THE DYNAMIC ***
C          BEHAVIOR OF LIQUID PROPELLANT AND THE COEFFICIENTS OF
C          LATERAL FORCE DISTRIBUTION (SATURN-V SPACE VEHICLE)
C
C  AALPN(I,N) = COEFFICIENTS A-ALPHA-N ASSOCIATED WITH I-TH TANK
C  ACCL      = ACCELERATION LEVEL
C  ALL(I)    = LIQUID LEVEL IN I-TH TANK
C  AIJ(I,J)  = ELEMENTS OF MASS MATRIX
C  BIJ(I,J)  = ELEMENTS OF STIFFNESS MATRIX
C  BL(I)     = LENGTH OF I-TH BEAM
C  BTHEN(I,N) = COEFFICIENTS B-THETA-N ASSOCIATED WITH I-TH TANK
C  C(1,J,K)  = PRELIMINARY EIGENVECTORS ASSOCIATED WITH S-IVB LH2 TANK
C  C(2,J,K)  = PRELIMINARY EIGENVECTORS ASSOCIATED WITH S-IVB LOX TANK
C  C(3,J,K)  = PRELIMINARY EIGENVECTORS ASSOCIATED WITH S-II LH2 TANK
C  C(4,J,K)  = PRELIMINARY EIGENVECTORS ASSOCIATED WITH S-II LOX TANK
C  C(5,J,K)  = PRELIMINARY EIGENVECTORS ASSOCIATED WITH S-IC LOX TANK
C  C(6,J,K)  = PRELIMINARY EIGENVECTORS ASSOCIATED WITH S-IC FUEL TANK
C  CXIN(I,J,N) = COEFFICIENTS C-XI-N ASSOCIATED WITH THE J-TH SLOSH MODE
C                   OF I-TH TANK
C  DBTBCM(I)= DISTANCE BETWEEN TANK BOTTOM AND CENTER OF MASS ASSOCIATED
C                 WITH I-TH TANK
C  DTB(I)    = DISTANCE BETWEEN TANK COORDINATE SYSTEM AND LOWER BEAM END
C                 ASSOCIATED WITH I-TH TANK
C  ETA(I,J,K) = J-TH NORMAL MODE ASSOCIATED WITH I-TH TANK
C  FTIME     = FLIGHT TIME
C  HN(I,N)   = COEFFICIENTS HN ASSOCIATED WITH I-TANK
C  INDX1     = 0, IF MASS OF LIQUID HAS ALREADY BEEN INCLUDED IN THE
C                 BENDING PROGRAM, OTHERWISE 1
C  INDX2     = 0, NO LATERAL FORCE DISTRIBUTION COEFFICIENTS PRINT-OUT,
C                 OTHERWISE 1
C  INDX3     = 0, ZERO LENGTH INTERSTAGES, OTHERWISE 1
C  INDX4     = 0, NO NORMALIZED SLOSH NORMAL MODES PRINT-OUT, OTHERWISE 1
C  INDX5     = 0, NO INTERMEDIATE COMPUTATION PRINT-OUT, OTHERWISE 1
C  NCASE     = NUMBER OF CASES
```

```
C   NCOR      = TOTAL NUMBER OF DEGREES OF FREEDOM OF BEAM END DISPLANCE-
C                 MENTS AND BEAM DEFLECTION FUNCTIONS
C   NPT(I)    = NUMBER OF PARTITIONS OF THE I-TH TANK
C   NSFLIG    = STAGE OF FLIGHT
C   NSMODE    = NUMBER OF SLOSH MODES CONSIDERED
C   NTANK     = NUMBER OF TANKS
C   PEV(I,J) = J-TH PRELIMINARY EIGENVALUE ASSOCIATED WITH I-TH TANK
C   RHO(I)    = MASS DENSITY ASSOCIATED WITH I-TH TANK
C   TTFCB(I,N) = THIRD TERM OF COEFFICIENT B-THETA-N ASSOC WITH I-TH TANK
C   ZINC(I)   = PARTITION HEIGHT OF I-TH TANK
C
      DIMENSION C(6,3,11), ETA(6,4,24), ETAMAX(6,3), PEV(6,3)
      DIMENSION DBLSCM(6), DBTBCM(6), P(6,3,3), RHO(6), S(6,3,3),
     1  SP(6,3), SQ(6,3), PPVOL(6)
      DIMENSION YBDLF(6,4,8), ZBDLF(6,4)
      DIMENSION ELEMA(24,24), ELEMB(24,24), AIJ(60,60), BIJ(60,60)
      DIMENSION BS(6,4,24)
      DIMENSION AALPN(6,32), BN(6,3), BTHEN(6,32), CXIN(6,3,32),
     1  FASUM(3),        GAMMA(6,3), HN(6,3), NPT(6), PIRALS(6),
     2  TTFCB(6,32), ZINC(6), OMEGA(6,3)
      COMMON /CL2/DL(6), PHI(10), Z(6)
      COMMON /CL3/BL(3), DTB(6), SPU(10), SQU(10), U(10,10), V(10,10),
     1  VPP(6), VPQ(6), VQQ(6), VOL, G3(6), RADIUS
      COMMON /CL7/ NBDLF       , INDF, UF(10), INDX2, VF, VN, RALS(6),
     1              ALL(6)
      COMMON /CL8/ BB(6,4,4), BFORDS(4,3), BFSLPS(4,3), ULB(6,4),
     1  URB(6,4)
      EQUIVALENCE (ETA(1,1,1),ELEMA(1,1),AALPN(1,1)),
     1  (ELEMB(1,1),BS(1,1,1),CXIN(1,1,1)), (GAMMA(1,1),SP(1,1)),
     2  (ETAMAX(1,1),BN(1,1)), (YBDLF(1,1,1),BTHEN(1,1)),
     3  (C(1,1,1),TTFCB(1,1))
      READ(5,2) NSFLIG, NCASE, NSMODE, INDX1, INDX2, INDX3, INDX4, INDX5
    2 FORMAT(8I10)
      NBEAM = 4 - NSFLIG
      NTANK = 8 - 2*NSFLIG
      NBDLF = 4
      READ(5,6) FTIME, ACCL, (ALL(I),I=1,NTANK)
    6 FORMAT(8E10.6)
```

```
      READ(5,6) (RHO(I),I=1,NTANK)
      READ(5,6) (DBTBCM(I),I=1,NTANK)
      READ(5,6) (BL(I),I=1,NBEAM)
      READ(5,6) (DTB(I),I=1,NTANK)
      READ(5,6) ((PEV(I,J),J=1, 3    ),I=1,NTANK)
      READ(5,10) (((C(I,J,K),K=1,10),J=1, 3    ),I=1,NTANK)
      READ(5,6) (((YBDLF(K,J,I),I=1,4),J=1,NBDLF),K=1,NTANK)
      READ(5,6) ((ZBDLF(K,I),I=1,4),K=1,NTANK)
      READ(5,2)( NPT(I),I=1,NTANK)
      READ(5,6) (ZINC(I),I=1,NTANK)
      READ(5,6) ((HN(I,J),J=1, 3    ),I=1,NTANK)
   10 FORMAT(10E8.4)
C
C  COMPUTATION OF DISTANCE BETWEEN LIQUID SURFACE AND C.M. AND TANK
C   RADIUS AT LIQUID SURFACE
C
      DO 11  I=1,NTANK
      PLEVEL = ALL(I)
      CALL CONTR(PLEVEL,ORLS,I)
      RALS(I) = ORLS
   11 DBLSCM(I) = ALL(I) - DBTBCM(I)
      G3(1) = 6.8  + DBTBCM(1)
      G3(2) =  4.54 + DBTBCM(2)
      G3(3) = 4.68 + DBTBCM(3)
      G3(4) = 1.16 + DBTBCM(4)
      G3(5) = 22.6 + DBTBCM(5)
      G3(6) = 8.71 + DBTBCM(6)
      IF(INDX3 .EQ. 0) GO TO 13
      BL(1) = 36.6
      BL(3) = 12.09
      DTB(1) = 2.1814
      DTB(2) = 0.0992
      DTB(5) = 20.6197
      DTL(6) = 6.72968
      G3(1) = 1.14 + DBTBCM(1)
      G3(2) = -1.12 + DBTBCM(2)
      G3(5) = 17.1 + DBTBCM(5)
      G3(6) = 3.21 + DBTBCM(6)
```

```
13 MCASE = 0
   MCASE = MCASE + 1
   WRITE(6,14) NSFLIG, NCASE, NSMODE, NTANK
14 FORMAT(1H1, 30X, 48H PROPELLANT BEHAVIOR IN SATURN-V SPACE VEHICLE
  1  //// 19H STAGE OF FLIGHT = , I2 // 19H NUMBER OF CASES = , I2 //
  2  25H NUMBER OF SLOSH MODES = , I2 // 19H NUMBER OF TANKS = , I2)
   WRITE(6,18) MCASE, FTIME, ACCL
18 FORMAT( 1H0, 5HCASE , I2 // 15H FLIGHT TIME = , F8.2, 5H SEC //
  1  22H ACCELERATION LEVEL = , E16.8, 8H M/S**2 )
   WRITE(6,20) INDX1, INDX2, INDX3, INDX4, INDX5
20 FORMAT(1H0,  7HINDX1 =, I2, 8H INDX2 =, I2, 8H INDX3 =, I2,
  1  8H INDX4 =, I2, 8H INDX5 =, I2)
   WRITE(6,22) (ALL(I),I=1,NTANK)
22 FORMAT( 1H0, 32X, 92H     TANK 1           TANK 2          TANK 3
  1          TANK 4          TANK 5          TANK 6 //22H  PROPELLANT L
  2EVEL(M) , 11X, 6E16.8)
   WRITE(6,26) (RHO(I),I=1,NTANK)
26 FORMAT( 1H0,23H MASS DENSITY(KG/M**3) , 9X, 6E16.8)
   WRITE(6,30) (RALS(I),I=1,NTANK)
30 FORMAT( 1H0, 32H RADIUS AT LIQUID SURFACE(M)     , 6E16.8)
   WRITE(6,34) (DBTBCM(I),I=1,NTANK)
34 FORMAT( 1H0, 32H DISTANCE BETWEEN TB AND CM(M)   , 6E16.8)
   DO 43  J=1,NSMODE
   IF(J .NE. 1) GO TO 39
   WRITE(6,38) (PEV(I,J),I=1,NTANK)
38 FORMAT( 1H0, 25H PRELIMINARY EIGENVALUES , 7X, 6E16.8)
   GO TO 43
39 WRITE(6,42) (PEV(I,J),I=1,NTANK)
42 FORMAT( 1H0, 32X, 6E16.8)
43 CONTINUE
   WRITE(6,44) (DTB(I),I=1,NTANK)
44 FORMAT( 1H0, 66H DISTANCE BETWEEN LOWER BEAM END AND TANK COORD. S
  1YSTEM ORIGIN(M)  // 33X, 6E16.8)
   WRITE(6,45) (BL(I),I=1,NBEAM)
45 FORMAT( 1H0, 16H BEAM LENGTH(M) // 3E20.8)
   WRITE(6,46) ((C(1,J,K),K=1,10),J=1,NSMODE)
46 FORMAT( 1H0, 15X, 57H PRELIMINARY EIGENVECTORS ASSOCIATED WITH S-I
  1VB LH2 TANK // (10E13.4))
```

```
      WRITE(6,50) ((C(2,J,K),K=1,10),J=1,NSMODE)
   50 FORMAT( 1H0, 15X, 57H PRELIMINARY EIGENVECTORS ASSOCIATED WITH S-I
     1VB LOX TANK // (10E13.4))
      IF(NTANK .EQ. 2) GO TO 67
      WRITE(6,54) ((C(3,J,K),K=1,10),J=1,NSMODE)
   54 FORMAT( 1H0, 15X, 57H PRELIMINARY EIGENVECTORS ASSOCIATED WITH S-I
     1I LH2 TANK  // (10E13.4))
      WRITE(6,58) ((C(4,J,K),K=1,10),J=1,NSMODE)
   58 FORMAT( 1H0, 15X, 57H PRELIMINARY EIGENVECTORS ASSOCIATED WITH S-I
     1I LOX TANK  // (10E13.4))
      IF(NTANK .EQ. 4) GO TO 67
      WRITE(6,62) ((C(5,J,K),K=1,10),J=1,NSMODE)
   62 FORMAT( 1H0, 15X, 57H PRELIMINARY EIGENVECTORS ASSOCIATED WITH S-I
     1C LOX TANK  // (10E13.4))
      WRITE(6,66) ((C(6,J,K),K=1,10),J=1,NSMODE)
   66 FORMAT( 1H0, 15X, 57H PRELIMINARY EIGENVECTORS ASSOCIATED WITH S-I
     1C FUEL TANK // (10E13.4))
   67 CONTINUE
      DO 70  K=1,NTANK
      K1 = K
      WRITE(6,68) K1, (ZBDLF(K1,I),I=1,4), (ZBDLF(K1,I),I=1,4),
     1  ((YBDLF(K1,J,I),I=1,4),J=1,2)
   68 FORMAT( 1H0, 30H BENDING DEFLECTION FUNCTIONS // 6H TANK , I2, 4X,
     1  4E14.6, 4X, 4E14.6 / 12X, 4E14.6, 4X, 4E14.6)
      WRITE(6,69) (ZBDLF(K1,I),I=1,4), (ZBDLF(K1,I),I=1,4),
     1  ((YBDLF(K1,J,I),I=1,4),J=3,4)
   69 FORMAT( 1H0, 11X, 4E14.6, 4X, 4E14.6/ 12X, 4E14.6, 4X, 4E14.6)
   70 CONTINUE
      IF(INDX4 .EQ. 0) GO TO 107
C
C COMPUTATION OF SLOSH NORMAL MODES
C
      DO 81  I=1,NTANK
      DL(I) = DBLSCM(I)/RALS(I)
      Z(I) = DBLSCM(I)/RALS(I)
      DO 79  J=1,NSMODE
      R = 0.0
      DO 75  K=1,21
```

```
      CALL PSMODE(I,R,PHI)
      ETA(I,J,K) = 0.0
      DO 71  L=1,10
   71 ETA(I,J,K) = ETA(I,J,K) + C(I,J,L)*PHI(L)
   75 R = R + 0.05
   79 CONTINUE
   81 CONTINUE
      INDEX = 0
      IF(INDEX .EQ. 0) GO TO 103
      WRITE(6,82) ((ETA(1,J,K),K=1,21),J=1,NSMODE)
   82 FORMAT( 1H0, 10X, 54H PRELIMINARY SLOSH MODES(THE INCREMENT OF R I
     1S 0.05)  // 21H FOR S-IVB LH2 TANK  // (7E18.5))
   85 WRITE(6,86) ((ETA(2,J,K),K=1,21),J=1,NSMODE)
   86 FORMAT( 1H0, 21H FOR S-IVB LOX TANK  // (7E18.5))
      IF(NTANK .EQ. 2) GO TO 103
      WRITE(6,90) ((ETA(3,J,K),K=1,21),J=1,NSMODE)
   90 FORMAT( 1H0, 20H FOR S-II LH2 TANK  // (7E18.5))
      WRITE(6,94) ((ETA(4,J,K),K=1,21),J=1,NSMODE)
   94 FORMAT( 1H0, 20H FOR S-II LOX TANK  // (7E18.5))
      IF(NTANK .EQ. 4) GO TO 103
      WRITE(6,98) ((ETA(5,J,K),K=1,21),J=1,NSMODE)
   98 FORMAT( 1H0, 20H FOR S-IC LOX TANK  // (7E18.5))
      WRITE(6,102) ((ETA(6,J,K),K=1,21),J=1,NSMODE)
  102 FORMAT( 1H0, 21H FOR S-IC FUEL TANK  // (7E18.5))
  103 IF(INDEX .EQ. 1) GO TO 107
      DO 105  I = 1,NTANK
      DO 105  J=1,NSMODE
      ETAMAX(I,J) = ABS(ETA(I,J,21))
      DO 105  K=1,21
  105 ETA(I,J,K) = ETA(I,J,K)/ETAMAX(I,J)
      WRITE(6,106) ((ETA(1,J,K),K=1,21),J=1,NSMODE)
  106 FORMAT( 1H0, 10X, 68H PRELIMINARY SLOSH MODES(THE INCREMENT OF R I
     1S 0.05) -- NORMALIZED  // 21H FOR S-IVB LH2 TANK  // (7E18.5))
      INDEX = INDEX + 1
      GO TO 85
C
C COMPUTATION OF COEFFICIENTS OF KINETIC AND POTENTIAL ENERGY TERMS
C   ASSOCIATED WITH PROPELLANT SLOSHING
```

```
C
  107 INDF = 2
      PI = 3.1415927
      DO 113  K=1,NTANK
      PLEVEL = ALL(K)
      CALL CINTL(K, PLEVEL)
      PIRALS(K) = PI*RALS(K)**2
      PPVOL(K) = VOL
      DO 113  N=1,NSMODE
      SUM1 = 0.0
      SUM2 = 0.0
      OMEGA(K,N) = SQRT(ACCL*PEV(K,N)/RALS(K))
      DO 111  I=1,10
      DO 109  J=1,10
  109 SUM2 = SUM2 + C(K,N,I)*C(K,N,J)*V   (   I,J)
  111 SUM1 = SUM1 + C(K,N,I)*V   (   1,I)
      GAMMA(K,N) = PIRALS(K)*DBLSCM(K)*SUM2/PPVOL(K)
  113 BN(K,N) = PIRALS(K)*RALS(K)*SUM1/(PPVOL(K)*GAMMA(K,N))
      INDF = 0
      DO 165   K=1,NTANK
      PLEVEL = ALL(K)
      CALL CINTL(K, PLEVEL)
      DO 139  M=1,NSMODE
      DO 135  N=M,NSMODE
      SUM1 = 0.0
      SUM2 = 0.0
      DO 115  I=1,10
      DO 115  J=1,10
      SUM1 = SUM1 + C(K,M,I)*C(K,N,J)*U(I,J)
  115 SUM2 = SUM2 + C(K,M,I)*C(K,N,J)*V(I,J)
      S(K,M,N) = SUM1
  135 P(K,M,N) = SUM2
      SUM1 = 0.0
      SUM2 = 0.0
      DO 137  I=1,10
      SUM1 = SUM1 + C(K,M,I)*SPU(I)
  137 SUM2 = SUM2 + C(K,M,I)*SQU(I)
      SP(K,M) = SUM1
```

```
      SQ(K,M) = SUM2
  139 CONTINUE
      DO 141  M=2,NSMODE
      M1 = M - 1
      DO 141  N=1,M1
      S(K,M,N) = S(K,N,M)
  141 P(K,M,N) = P(K,N,M)
      IF(INDX5 .EQ. 0) GO TO 165
      WRITE(6,142) K, ((S(K,M,N),N=1,NSMODE),M=1,NSMODE)
  142 FORMAT(1H0, 96H COEFFICIENTS OF KINETIC AND POTENTIAL ENERGY TERMS
     1 ASSOCIATED WITH PROPELLANT SLOSHING OF TANK , I2 // 15X, 22H COEF
     2FICIENTS S(M,N)  // (5E24.8))
      WRITE(6,144) ((P(K,M,N),N=1,NSMODE),M=1,NSMODE)
  144 FORMAT(1H0, 15X, 22H COEFFICIENTS P(M,N)  // (5E24.8))
      WRITE(6,150) K, (SP(K,M),M=1,NSMODE)
  150 FORMAT(1H0, 30H COEFFICIENTS SP(K,M) OF TANK , I2 // 5E24.8)
      WRITE(6,154) K, (SQ(K,M),M=1,NSMODE)
  154 FORMAT(1H0, 30H COEFFICIENTS SQ(K,M) OF TANK , I2 // 5E24.8)
C
C COMPUTATION OF VOLUME AND MASS OF LIQUID PROPELLANT
C
      AMASS = RHO(K)*VOL
      WRITE(6,158) K, VOL, AMASS
  158 FORMAT(1H0, 56H VOLUME AND MASS OF LIQUID PROPELLANT CONTAINED IN
     1TANK , I2 , 5H ARE , E14.6, 10H M**3  AND, E14.6, 20H KG, RESPECTI
     2VELY.  )
  165 CONTINUE
C
C COMPUTATION OF THE ELEMENTS OF THE MASS AND STIFFNESS MATRICES
C   ASSOCIATED WITH PROPELLANT SLOSHING
C
      DO 173  K=1,NTANK
      TEMPC =  - PIRALS(K)*RHO(K)*RALS(K)
      VPP(K) = TEMPC*VPP(K)
      VPQ(K) = TEMPC*VPQ(K)
      VQQ(K) = TEMPC*VQQ(K)
      DO 169  N=1,NSMODE
      TEMPC1 = TEMPC/PEV(K,N)
```

```
    SP(K,N) = TEMPC1*SP(K,N)
169 SQ(K,N) = TEMPC1*SQ(K,N)
    TEMPC2 =  0.5*PIRALS(K)*RHO(K)*ACCL
    DO 171   M=1,NSMODE
    DO 171   N=1,NSMODE
    TEMPC1 = 0.5*TEMPC/(PEV(K,M)*PEV(K,N))
    S(K,M,N) = TEMPC1*S(K,M,N)
171 P(K,M,N) = TEMPC2*P(K,M,N)
173 CONTINUE
    IF(INDX5 .EQ. 0) GO TO 175
    WRITE(6,178) (VPP(K),K=1,NTANK)
178 FORMAT(1H0, 25H COEFFICIENTS VPP(NTANK)  // 6E20.8)
    WRITE(6,180) (VPQ(K),K=1,NTANK)
180 FORMAT(1H0, 25H COEFFICIENTS VPQ(NTANK)  // 6E20.8)
    WRITE(6,182) (VQQ(K),K=1,NTANK)
182 FORMAT(1H0, 25H COEFFICIENTS VQQ(NTANK)  // 6E20.8)
175 CONTINUE
    DO 177  I=1,24
    DO 177  J=I,24
    ELEMA(I,J) = 0.0
177 ELEMB(I,J) = 0.0
    ELEMA(1,1) = 2.0*(VPP(5) + VPP(6))
    ELEMA(1,2) = VPQ(5) + VPQ(6)
    ELEMA(2,2) = 2.0*(VQQ(5) + VQQ(6))
    ELEMA(3,3) = 2.0*(VPP(3) + VPP(4))
    ELEMA(3,4) = VPQ(3) + VPQ(4)
    ELEMA(4,4) = 2.0*(VQQ(3) + VQQ(4))
    ELEMA(5,5) = 2.0*(VPP(1) + VPP(2))
    ELEMA(5,6) = VPQ(1) + VPQ(2)
    ELEMA(6,6) = 2.0*(VQQ(1) + VQQ(2))
    K = 10 + 4*(NSMODE - 1)
    IF(INDX3 .NE. 0) GO TO 179
    ELEMA(2,2) = ELEMA(2,2) + ELEMA(3,3)
    ELEMA(2,3) = ELEMA(3,4)
    ELEMA(3,3) = ELEMA(4,4) + ELEMA(5,5)
    ELEMA(3,4) = ELEMA(5,6)
    ELEMA(4,4) = ELEMA(6,6)
    ELEMA(5,5) = 0.0
```

```
      ELEMA(5,6) = 0.0
      ELEMA(6,6) = 0.0
      K = 8 + 4*(NSMODE - 1)
  179 I6 = NTANK - 1
      DO 185  I=1,I6,2
      N = I
      L = 6 - I
      IF(I .EQ. 3) K = 8 + 2*(NSMODE - 1)
      IF(I .EQ. 5) K=6
      IF(INDX3 .NE. 0) GO TO 183
      IF(I .EQ. 1) GO TO 183
      K = 6 + 2*(NSMODE - 1)
      N = I - 1
      IF(I .EQ. 3) GO TO 183
      K = 4
      N = I - 2
  183 N1 = N + 1
      DO 185  M=1,2
      DO 184  J=1,NSMODE
      K = K + 1
      ELEMA(N,K) = SP(L,J)
  184 ELEMA(N1,K) = SQ(L,J)
  185 L = L + 1
      NRBD = NBEAM + 1
      IF(INDX3 .NE. 0) NRBD = 2*NBEAM
      NTKSM = NTANK*NSMODE
      I = 0
      L = 0
      K5 = 5
      IF(INDX3 .NE. 0 ) K5=7
      K6 = NTKSM + NRBD
      DO 193  K=K5,K6
      IF(I .EQ. 0) L = L + 1
      I = I + 1
      GO TO (186,187,188), I
  186 K2 = NSMODE
      GO TO 189
  187 K2 = NSMODE - 1
```

```
      GO TO 189
  188 K2 = 1
  189 J = I - 1
      N = 0
      DO 191  K1=1,K2
      M = K + N
      N = N + 1
      J = J + 1
      ELEMA(K,M) = 2.0*S(L,I,J)
  191 ELEMB(K,M) = 2.0*P(L,I,J)
      IF(I .EQ. NSMODE) I = 0
  193 CONTINUE
      DO 197  I=2,K6
      I1 = I - 1
      DO 197  J=1,I1
      ELEMA(I,J) = ELEMA(J,I)
  197 ELEMB(I,J) = ELEMB(J,I)
      I1 = NCOR + 1
      I2 = NCOR + NTKSM
      M1 = 1
      IF(NBEAM .EQ. 3) GO TO 200
      GO TO (198,199), NBEAM
  198 M1 = 3
      IF(INDX3 .NE. 0) M1=5
      GO TO 200
  199 M1 = 2
      IF(INDX3 .NE. 0) M1=3
  200 DO 201  I=1,I2
      DO 201  J=1,I2
      AIJ(I,J) = 0.0
  201 BIJ(I,J) = 0.0
      IF(INDX1 .EQ. 0) GO TO 207
      N1 = M1
      DO 205  I=1,NRBD
      N2 = M1
      DO 203  J=1,NRBD
      AIJ(I,J) = ELEMA(N1,N2)
  203 N2 = N2 + 1
```

```
  205 N1 = N1 + 1
  207 N1 = M1
      DO 211  I=1,NRBD
      K1 = 5
      IF(INDX3 .NE. 0) K1=7
      DO 209  J=I1,I2
      AIJ(I,J) = ELEMA(N1,K1)
      AIJ(J,I) = AIJ(I,J)
  209 K1 = K1 + 1
  211 N1 = N1 + 1
      K1 = 5
      IF(INDX3 .NE. 0) K1=7
      DO 215  I=I1,I2
      K2 = 5
      IF(INDX3 .NE. 0) K2=7
      DO 213  J=I1,I2
      AIJ(I,J) = ELEMA(K1,K2)
      BIJ(I,J) = ELEMB(K1,K2)
  213 K2 = K2 + 1
  215 K1 = K1 + 1
C
C  COMPUTATION OF BENDING-BENDING, BENDING-RIGID AND BENDING-SLOSH
C     TERM
C
      INDF = 11
      DO 603  K=1,NTANK
      DO 603  I=1,4
  603 ZBDLF(K,I) = ZBDLF(K,I)/RALS(K)
      DO 641  K=1,NTANK
      PLEVEL = ALL(K)
      DO 611  J=1,NBDLF
      DO 611  I=1,3
      L = I + 1
      BFSLPS(J,I) = (ZBDLF(K,I) - ZBDLF(K,L))/(YBDLF(K,J,I) -
     1  YBDLF(K,J,L))
  611 BFORDS(J,I) = ZBDLF(K,I) - BFSLPS(J,I)*YBDLF(K,J,I)
      CALL CINTL(K, PLEVEL)
      DO 631  J=1,NBDLF
```

```
      DO 631  N=1,NSMODE
      SUM1 = 0.0
      DO 621  I=1,10
  621 SUM1 = SUM1 + C(K,N,I)*U(J,I)
  631 BS(K,J,N) = SUM1
  641 CONTINUE
      DO 671  K=1,NTANK
      PIA3RH = PI*RHO(K)*RALS(K)**3
      DO 671  J=1,NBDLF
      ULB(K,J) = PIA3RH*ULB(K,J)
      URB(K,J) = PIA3RH*URB(K,J)
      DO 651  I=1,NBDLF
  651 BB(K,J,I) = PIA3RH*BB(K,J,I)
      DO 661  N=1,NSMODE
  661 BS(K,J,N) = PIA3RH*BS(K,J,N)/PEV(K,N)
  671 CONTINUE
      K1 = 6
      K2 = 5
      IF(INDX3 .NE. 0) GO TO 701
      J = NBEAM + 3
      DO 691  I=1,NBEAM
      I1 = I + 1
      DO 681  L=1,NBDLF
      AIJ(I,J) = ULB(K1,L) + ULB(K2,L)
      AIJ(J,I) = AIJ(I,J)
      AIJ(I1,J) = URB(K1,L) + URB(K2,L)
      AIJ(J,I1) = AIJ(I1,J)
  681 J = J + 1
      K1 = K1 - 2
  691 K2 = K2 - 2
      GO TO 731
  701 J = 2*NBEAM + 3
      DO 721  I=1,NBEAM
      I1 = 2*I - 1
      I2 = 2*I
      DO 711  L=1,NBDLF
      AIJ(I1,J) = ULB(K1,L) + ULB(K2,L)
      AIJ(J,I1) = AIJ(I1,J)
```

```
      AIJ(I2,J) = URB(K1,L) + URB(K2,L)
      AIJ(J,I2) = AIJ(I2,J)
  711 J = J + 1
      K1 = K1 - 2
  721 K2 = K2 - 2
  731 I1 = NCOR - 15
      K1 = 6
      K2 = 5
      DO 761  K=1,NBEAM
      DO 751  L=1,NBDLF
      J = I1
      DO 741  M=L,NBDLF
      AIJ(I1,J) = 2.0*(BB(K1,L,M) + BB(K2,L,M))
      IF(I1 .NE. J) AIJ(J,I1) = AIJ(I1,J)
  741 J = J + 1
  751 I1 = I1 + 1
      K1 = K1 - 2
  761 K2 = K2 - 2
      I1 = NCOR - 7
      J1 = NCOR + 1
      DO 791  K=1,NTANK
      DO 781  L=1,NSMODE
      I2 = I1
      DO 771  M=1,NBDLF
      AIJ(I2,J1) = BS(K,M,L)
      AIJ(J1,I2) = AIJ(I2,J1)
  771 I2 = I2 + 1
  781 J1 = J1 + 1
      IF(K .EQ. 2) I1 = I1 - NBDLF
      IF(K .EQ. 4) I1 = I1 - NBDLF
  791 CONTINUE
      IF(INDX5 .EQ. 0) GO TO 239
      I2 = NCOR + NTKSM
      WRITE(6,220) ((AIJ(I,J),J=1,I2), I=1,I2)
  220 FORMAT(1H0, 44H ELEMENTS OF A-MATRIX ASSOCIATED WITH SLOSH  //
     1  (10E13.4))
      WRITE(6,222) ((BIJ(I,J),J=1,I2), I=1,I2)
  222 FORMAT(1H0, 44H ELEMENTS OF B-MATRIX ASSOCIATED WITH SLOSH  //
```

```
     1  (10E13.4))
C
C COMPUTATION OF LATERAL FORCE DISTRIBUTION COEFFICIENTS
C
  239 IF(INDX2 .EQ. 0) GO TO 281
      INDF = 1
      NPTMAX = NPT(1)
      DO 253  K=2,NTANK
      IF(NPT(K) .GT. NPTMAX) NPTMAX=NPT(K)
  253 CONTINUE
      IF(NPTMAX .GT. 50) NPTMAX = 50
      DO 257  K=1,NTANK
      DO 257  I=1,NPTMAX
      AALPN(K,I) = 0.0
      BTHEN(K,I) = 0.0
      DO 257  J=1,NSMODE
  257 CXIN(K,J,I) = 0.0
      DO 279  KDK=1,NTANK
      K = KDK
  261 PLEVEL = ALL(K)
      KK = NPT(K)
      IF(KK .GT. 50) KK = NPTMAX
      DO 275  I=1,KK
      CALL CONTR(PLEVEL,ORLS,K)
      RADIUS = ORLS
      IF(I .EQ. 1) RALS(K) = ORLS
      CALL CINTL(K, PLEVEL)
      PLEVEL =  PLEVEL - ZINC(K)
      DO 267  N=1,NSMODE
      SUM3 = 0.0
      DO 263  J=1,10
  263 SUM3 = SUM3 + C(K,N,J)*UF(J)
      CXIN(K,N,I) = PIRALS(K)*RHO(K)*SUM3
  267 FASUM(N) = SUM3
      SUM1 = 0.0
      SUM2 = 0.0
      DO 271  N=1,NSMODE
      SUM1 = SUM1 + BN(K,N)*FASUM(N)
```

```
271 SUM2 = SUM2 + (G3(K)*BN(K,N) - DBLSCM(K)*(BN(K,N) - HN(K,N)))*
   1  FASUM(N)
    AALPN(K,I) = -RHO(K)*VN + PIRALS(K)*RHO(K)*DBLSCM(K)*SUM1
275 BTHEN(K,I) = - RHO(K)*VF*PIRALS(K)**2/PI - G3(K)*RHO(K)*VN
   1  + PIRALS(K)*RHO(K)*DBLSCM(K)*SUM2
279 CONTINUE
281 CONTINUE
    DO 9  I=1,NTANK
    II = NPT(I)
    IF(II .GT. 50) II = 50
  9 READ(5,6) (TTFCB(I,J),J=1,II)
    IF(INDX2 .EQ. 0) RETURN
    DO 283  K=1,NTANK
    I1 = NPTMAX - 1
    DO 283  I=1,I1
    II = I + 1
    AALPN(K,I) = AALPN(K,I) - AALPN(K,II)
    BTHEN(K,I) = BTHEN(K,I) - BTHEN(K,II) - RHO(K)*TTFCB(K,I)
    DO 283  J=1,NSMODE
283 CXIN(K,J,I) = CXIN(K,J,I) - CXIN(K,J,II)
    WRITE(6,288) (NPT(I),I=1,NTANK)
288 FORMAT(1H1, 15X, 45H COEFFICIENTS OF LATERAL FORCE DISTRIBUTION
   1  /// 32X, 92H     TANK 1            TANK 2            TANK 3
   2 TANK 4            TANK 5            TANK 6 // 25H NUMBER OF PARTITION
   3S    , 6I16)
    WRITE(6,290) (ZINC(I),I=1,NTANK)
290 FORMAT(1H0, 21H PARTITION HEIGHT(M) , 11X, 6E16.8)
    DO 297  J=1,NSMODE
    IF(J .NE. 1) GO TO 295
    WRITE(6,292) (HN(I,J),I=1,NTANK)
292 FORMAT(1H0, 26H COEFFICIENTS HN(DIM-LESS), 6X, 6E16.8)
    GO TO 297
295 WRITE(6,296) (HN(I,J),I=1,NTANK)
296 FORMAT(1H0, 32X,6E16.8)
297 CONTINUE
    DO 303  J=1,NPTMAX
    IF(J .NE. 1) GO TO 301
    WRITE(6,300) (TTFCB(I,J),I=1,NTANK)
```

```
  300 FORMAT(1H0, 47H THIRD TERM OF FORCE COEFFICIENT B-THETA(KG-M) //
     1  32X, 6E16.8)
      GO TO 303
  301 WRITE(6,302) (TTFCB(I,J),I=1,NTANK)
  302 FORMAT(32X, 6E16.8)
  303 CONTINUE
      DO 307  J=1,NSMODE
      IF(J .NE. 1) GO TO 305
      WRITE(6,304) (OMEGA(I,J),I=1,NTANK)
  304 FORMAT(1H0, 32H NATURAL FREQUENCIES(RAD/SEC)    , 6E16.8)
      GO TO 307
  305 WRITE(6,302) (OMEGA(I,J),I=1,NTANK)
  307 CONTINUE
      DO 311  I=1,NPTMAX
      IF(I .NE. 1) GO TO 309
      WRITE(6,308) (AALPN(K,I),K=1,NTANK)
  308 FORMAT(1H0, 32H COEFFICIENTS A-ALPHA-N(KG)      , 6E16.8)
      GO TO 311
  309 WRITE(6,302) (AALPN(K,I),K=1,NTANK)
  311 CONTINUE
      DO 317  I=1,NPTMAX
      IF(I .NE. 1) GO TO 315
      WRITE(6,314) (BTHEN(K,I),K=1,NTANK)
  314 FORMAT(1H0, 32H COEFFICIENTS B-THETA-N(KG-M)    , 6E16.8)
      GO TO 317
  315 WRITE(6,302) (BTHEN(K,I),K=1,NTANK)
  317 CONTINUE
      DO 323  J=1,NSMODE
      DO 323  I=1,NPTMAX
      IF(I .NE. 1) GO TO 321
      WRITE(6,320) J, (CXIN(K,J,I),K=1,NTANK)
  320 FORMAT(1H0, 54H COEFFICIENTS C-XI-N ASSOCIATED WITH SLOSH MODE(KG/
     1M) , I2 // 32X, 6E16.8)
      GO TO 323
  321 WRITE(6,302) (CXIN(K,J,I),K=1,NTANK)
  323 CONTINUE
 5991 CONTINUE
      RETURN
```

```
          END
$IBFTC SUB1     DECK
          BLOCK DATA
          COMMON /BLK1/A(7),B(7),O(7)
          COMMON /BLK2/G(16),X(16)
          COMMON /BLK3/RJ1P(5)
          DATA (A(M),M=1,7)/0.5,-0.56249985,0.21093573,-0.03954289,
         1 0.00443319,-0.00031761,0.00001109/, (B(M),M=1,7)/0.79788456,
         2 0.00000156,0.01659667,0.00017105,-0.00249511,0.00113653,
         3 -0.00020033/, (O(M),M=1,7)/-2.3561945,0.12499612,0.00005650,
         4 -0.00637879,0.00074348,0.00079824,-0.00029166/
          DATA (G(I),I=1,16)/0.18945061,0.18260342,0.16915652,0.14959599,
         1 0.12462897,0.09515851,0.06225352,0.02715246,0.02715246,
         2 0.06225352,0.09515851,0.12462897,0.14959599,0.16915652,
         3 0.18260342,0.18945061/, (X(I),I=1,16)/0.09501251,0.28160355,
         4 0.45801678,0.61787624,0.75540441,0.86563120,0.94457502,
         5 0.98940094,-0.98940094,-0.94457502,-0.86563120,-0.75540441,
         6 -0.61787624,-0.45801678,-0.28160355,-0.09501251/
          DATA (RJ1P(M),M=1,5)/1.8411837,5.3314427,8.5363163,11.706004,
         1 14.863588/
          END
$IBFTC SUB2     DECK
          SUBROUTINE PSMODE(I,R,PHI)
C
C THIS SUBROUTINE PROVIDES THE EIGENFUNCTIONS FOR THE COMPUTATION OF
C SLOSH NORMAL MODES
C
          COMMON /BLK1/A(7),B(7),O(7)
          COMMON /BLK3/RJ1P(5)
          COMMON /CL2/DL(6), PHI(10), Z(6)
          DO 6001  L=1,5
 6001     PHI(L) = R**(2*L - 1)
          DO 6017  L=1,5
          M = L + 5
          AJNR = RJ1P(L)*R
          IF(AJNR .GT. 3.0) GO TO 6009
          T = AJNR/3.0
          SUM = 0.5
```

```
      DO 6005  N=2,7
 6005 SUM = SUM + A(N)*T**(2*N - 2)
      BJ1 = AJNR*SUM
      PHI(M) = BJ1/EXP(RJ1P(L)*ABS(DL(I) - Z(I)))
      GO TO 6017
 6009 T = 3.0/AJNR
      SUM1 = 0.0
      SUM2 = AJNR
      DO 6013  N=1,7
      SUM1 = SUM1 + B(N)*T**(N - 1)
 6013 SUM2 = SUM2 + O(N)*T**(N - 1)
      BJ1 = (1.0/AJNR**0.5)*SUM1*COS(SUM2)
      PHI(M) = BJ1/EXP(RJ1P(L)*ABS(DL(I) - Z(I)))
 6017 CONTINUE
      RETURN
      END
$IBFTC SUB3    DECK
      SUBROUTINE CINTL(K, PLEVEL)
C
C GAUSSIAN QUADATURE FORMULA IS USED IN THIS SUBROUTINE TO EVALUATE LINE
C INTEGRALS WITH ARBITRARY LIMITS (P,Q).
C
      COMMON /BLK2/G(16),X(16)
      COMMON /BLK3/RJ1P(5)
      COMMON /CL3/BL(3), DTB(6), SPU(10), SQU(10), U(10,10), V(10,10),
     1  VPP(6), VPQ(6), VQQ(6), VOL, G3(6), RADIUS
      COMMON /CL4/BJ1(4,5,16), BJ1A(5,21), BJ1P(4,5,16), EPS
      COMMON /CL5/PL(4), QL(4), QMP(4), VZ(4,16)
      COMMON /CL6/VR(4,16)
      COMMON /CL7/ NBDLF       , INDF, UF(10), INDX2, VF, VN, RALS(6),
     1             ALL(6)
      COMMON /CL8/ BB(6,4,4), BFORDS(4,3), BFSLPS(4,3), ULB(6,4),
     1  URB(6,4)
      COMMON /CL9/ IDXB, RORALS
      COMMON /TEMP/BJ1PA(5,21), BJ0A(5,21)
      BOA = 0.70698979
      AORS = (4.9784/RALS(K))**2
      RIOVRP = 1.0
      IF(INDF .EQ. 1) RIOVRP = RADIUS/RALS(K)
```

```
      PL(1) = 0.0
      PL(2) = RIOVRP
      QL(2) = 0.0
      PL(3) = RIOVRP
      QL(3) = 0.0
      EPS = 0.0
      IF(INDF .EQ. 1) GO TO 6097
      GO TO (6085,6085,6089,6089,6093,6093), K
 6085 DTBBL = DTB(K)/BL(3)
      RALSBL = RALS(K)/BL(3)
      GO TO 6097
 6089 DTBBL = DTB(K)/BL(2)
      RALSBL = RALS(K)/BL(2)
      GO TO 6097
 6093 DTBBL = DTB(K)/BL(1)
      RALSBL = RALS(K)/BL(1)
 6097 IF(INDF .EQ. 11) GO TO 9061
 6099 GO TO (6101,6141,6147,6155,6161,6161), K
 6101 IF(PLEVEL .GT. 2.2606) GO TO 6105
      PL(1) = SQRT(10.903204 - (PLEVEL + 1.0414)**2)/RALS(K)
      QL(1) = 3.1334786/RALS(K)
      PL(2) = QL(1)
      QL(2) = RIOVRP
      QL(3) = PL(1)
      EPS = PL(1)
      GO TO 6109
 6105 IF(PLEVEL .GT. 7.8486) GO TO 6107
      QL(1) = 3.1334786/RALS(K)
      PL(2) = QL(1)
      QL(2) = RIOVRP
      GO TO 6109
 6107 QL(1) = 3.1334786/RALS(K)
      PL(2) = QL(1)
      QL(2) = 3.302/RALS(K)
      PL(3) = QL(2)
      QL(3) = RIOVRP
      PL(4) = RIOVRP
      QL(4) = 0.0
```

```
      NII = 4
      GO TO 6111
 6109 NII = 3
 6111 CALL TKCONF(NII,PL,QL,QMP,VR,VZ,CMH,BOA,AORS,PLEVEL,K)
      CALL BESSEL(VR,BJ1,BJ1A,BJ1P,NII)
      IF(INDF .EQ. 1) GO TO 6177
      IF(INDF .EQ. 2) GO TO 6140
      DO 6117  I=1,5
      DO 6114  J=I,5
      SUM1 = 0.0
      DO 6113  L=1,NII
      DO 6113  LL=1,16
 6113 SUM1 = SUM1 + QMP(L)*G(LL)*VZ(L,LL)*VR(L,LL)**(2*I + 2*J - 3)
      U(I,J) = FLOAT(4*I*J - 2*I - 2*J + 2)*SUM1
      V(I,J) = 0.5/FLOAT(I + J)
      IF(J .EQ. I) GO TO 6114
      U(J,I) = U(I,J)
      V(J,I) = V(I,J)
 6114 CONTINUE
      SUM2 = 0.0
      SUM3 = 0.0
      DO 6115  L=1,NII
      DO 6115  LL=1,16
      SUM2 = SUM2 + QMP(L)*G(LL)*(DTBBL*VZ(L,LL) + 0.5*RALSBL*VZ(L,LL)**
     1  2)*VR(L,LL)**(2*I-1)
 6115 SUM3 = SUM3 + QMP(L)*G(LL)*VZ(L,LL)*VR(L,LL)**(2*I-1)
      SQU(I) = FLOAT(2*I)*SUM2
 6117 SPU(I) = FLOAT(2*I)*SUM3 - SQU(I)
      DO 6125  I=1,5
      DO 6125  J=6,10
      JJ = J - 5
      SUM1 = 0.0
      SUM2 = 0.5*BJ1A(JJ,21)
      DO 6119  L=1,NII
      DO 6119  LL=1,16
 6119 SUM1 = SUM1 + QMP(L)*G(LL)*(FLOAT(2*I-1)*BJ1P(L,JJ,LL)*VR(L,LL)**(
     1  2*I-1) + BJ1(L,JJ,LL)*VR(L,LL)**(2*I-2)/RJ1P(JJ))*EXP(RJ1P(JJ)*
     2  (VZ(L,LL) + CMH))
      R =0.0
```

```
      DELR = (1.0 - EPS)/20.0
      DO 6123  L=1,20
      SUM2 = SUM2 + BJ1A(JJ,L)*R**(2*I)
 6123 R = R + DELR
      U(I,J) = SUM1
      U(J,I) = U(I,J)
      V(I,J) = SUM2*DELR
 6125 V(J,I) = V(I,J)
      DO 6137  I=6,10
      II = I - 5
      DO 6133  J=I,10
      JJ = J - 5
      SUM1 = 0.0
      DO 6129  L=1,NII
      DO 6129  LL=1,16
 6129 SUM1 = SUM1 + QMP(L)*G(LL)*(RJ1P(II)*RJ1P(JJ)*VR(L,LL)*BJ1P(L,II,
     1 LL)*BJ1P(L,JJ,LL) + (1.0/VR(L,LL) + RJ1P(II)*RJ1P(JJ)*VR(L,LL))*
     2 BJ1(L,II,LL)*BJ1(L,JJ,LL))*(1.0/(RJ1P(II) + RJ1P(JJ)))*EXP((RJ1P
     3 (II) + RJ1P(JJ))*(VZ(L,LL) + CMH))
      U(I,J) = SUM1
      IF(J .EQ. I) GO TO 6133
      U(J,I) = U(I,J)
      V(I,J) = 0.0
      V(J,I) = 0.0
 6133 CONTINUE
      SUM2 = 0.0
      SUM3 = 0.0
      DO 6135  L=1,NII
      DO 6135  LL=1,16
      SUM2 = SUM2 + QMP(L)*G(LL)*(DTBBL*(VR(L,LL)*BJ1P(L,II,LL) + BJ1(L,
     1 II,LL)/RJ1P(II)) - RALSBL*BJ1(L,II,LL)*VR(L,LL)**2 + RALSBL*(
     2 VZ(L,LL) - 1.0/RJ1P(II))*(VR(L,LL)*BJ1P(L,II,LL) + BJ1(L,II,LL)/
     3 RJ1P(II)))*EXP(RJ1P(II)*(VZ(L,LL) + CMH))
 6135 SUM3 = SUM3 + QMP(L)*G(LL)*(VR(L,LL)*BJ1P(L,II,LL) + BJ1(L,II,LL)/
     1 RJ1P(II))*EXP(RJ1P(II)*(VZ(L,LL) + CMH))
      SQU(I) = SUM2
      SPU(I) = SUM3 - SQU(I)
 6137 V(I,I) = 0.5*(1.0 - 1.0/RJ1P(II)**2)*BJ1A(II,21)**2
```

```
      SUM1 = 0.0
      SUM2 = 0.0
      SUM3 = 0.0
      SUM4 = 0.0
      DO 6139  L=1,NII
      DO 6139  LL=1,16
      SUM1 = SUM1 + QMP(L)*G(LL)*VR(L,LL)*VZ(L,LL)*RALS(K)**3
      SUM2 = SUM2 + QMP(L)*G(LL)*(VZ(L,LL)*DTBBL**2 + RALSBL*DTBBL*VZ(L,
     1  LL)**2 + (RALSBL**2)*(0.33333333*VZ(L,LL)**3 + 0.5*VZ(L,LL)*VR(
     2  L,LL)**2))*VR(L,LL)
      SUM3 = SUM3 + QMP(L)*G(LL)*((1.0 - 2.0*DTBBL)*VZ(L,LL) - RALSBL*
     1  VZ(L,LL)**2)*VR(L,LL)
 6139 SUM4 = SUM4 + QMP(L)*G(LL)*(2.0*DTBBL*VZ(L,LL)
     1  + RALSBL*VZ(L,LL)**2)*VR(L,LL)
      VQQ(K) = SUM2
      VPP(K) = SUM3 + VQQ(K)
      VPQ(K) = SUM4 - 2.0*VQQ(K)
      VOL = -6.2831853*SUM1
      RETURN
C
C COMPUTATION OF COEFFICIENTS BMN(1,M,N)
C
 6140 CONTINUE
      DO 6175  I=1,10
      DO 6175  J=I,10
      IF(I .GT. 5) GO TO 6171
      IF(J .GT. 5) GO TO 6167
      V  (   I,J) = (1.0 - EPS**(2*(I+J)))/FLOAT(2*(I+J))
      GO TO 6173
 6167 JJ = J - 5
      IF(I .NE. 1) GO TO 6169
      V  (   I,J) = (BJ1A(JJ,21) - EPS*BJ1A(JJ,1) + RJ1P(JJ)*BJ1PA(JJ,1)
     1  *EPS**2)/RJ1P(JJ)**2
      GO TO 6173
 6169 II = I - 1
      V  (   I,J) = (FLOAT(2*I-1)*BJ1A(JJ,21) - (FLOAT(2*I-1)*BJ1A(JJ,1)
     1  - RJ1P(JJ)*EPS*BJ1PA(JJ,1))*EPS**(2*I-1) - FLOAT(4*I*(I-1))*
     2    V(   II,J))/RJ1P(JJ)**2
      GO TO 6173
```

```
6171 II = I - 5
     JJ = J - 5
     IF(J .GT. I) GO TO 6172
     V  (  I,J) = 0.5*((RJ1P(JJ)**2 - 1.0)*BJ1A(JJ,21)**2 - (RJ1P(JJ)
    1  *EPS*BJ1PA(JJ,1))**2 - ((RJ1P(JJ)*EPS)**2 - 1.0)*BJ1A(JJ,1)**2)
    2  /RJ1P(JJ)**2
     GO TO 6175
6172 V  (  I,J) = (RJ1P(II)*EPS*BJ1A(JJ,1)*BJ1PA(II,1) - RJ1P(JJ)*EPS
    1  *BJ1A(II,1)*BJ1PA(JJ,1))/(RJ1P(II)**2 - RJ1P(JJ)**2)
6173 IF(J .EQ. I) GO TO 6175
     V(J,I) = V(I,J)
6175 CONTINUE
     SUM1 = 0.0
     DO 6176  L=1,NII
     DO 6176  LL=1,16
6176 SUM1 = SUM1 + QMP(L)*G(LL)*VR(L,LL)*VZ(L,LL)*RALS(K)**3
     VOL = -6.2831853*SUM1
     RETURN
6177 DO 6185  I=1,10
     SUM1 = 0.0
     IF(I .GT. 5) GO TO 6181
     DO 6179  L=1,NII
     DO 6179  LL=1,16
6179 SUM1 = SUM1 + QMP(L)*G(LL)*VZ(L,LL)*VR(L,LL)**(2*I - 1)
     UF(I) = -FLOAT(2*I)*SUM1
     GO TO 6185
6181 II = I - 5
     DO 6183  L=1,NII
     DO 6183  LL=1,16
6183 SUM1 = SUM1 + QMP(L)*G(LL)*(VR(L,LL)*BJ1P(L,II,LL) + BJ1(L,II,LL)
    1  /RJ1P(II))*EXP(RJ1P(II)*(VZ(L,LL) + CMH))
     UF(I) = - SUM1
6185 CONTINUE
     SUM1 = 0.0
     SUM2 = 0.0
     VF = 0.0
     VFC = 2.0*(G3(K) - DTB(K))/RALS(K)
     DO 6187  L=1,NII
```

```
      DO 6187  LL=1.16
      VF = VF + QMP(L)*G(LL)*VR(L.LL)*(VZ(L.LL)**2 - VFC*VZ(L.LL))
      SUM2 = SUM2 + QMP(L)*G(LL)*VR(L.LL)*VZ(L.LL)**2
 6187 SUM1 = SUM1 + QMP(L)*G(LL)*VR(L.LL)*VZ(L.LL)
      VN = -6.2831853*SUM1*RALS(K)**3
C
C COMPUTATION OF C.M. MEASURED FROM THE ORIGIN OF THE TANK
C   COORDINATE SYSTEM
C
      ZBAR = 0.5*RALS(K)*SUM2/SUM1
      RETURN
 6141 IF(PLEVEL .GT. 2.2606) GO TO 6143
      QL(1) = RIOVRP
      NII = 2
      GO TO 6111
 6143 QL(1) = 3.1334786/RALS(K)
      PL(2) = QL(1)
      QL(2) = RIOVRP
      GO TO 6109
 6147 IF(PLEVEL .GT. 3.519678) GO TO 6149
      PL(1) = 1.4144475*SQRT(12.388133 - PLEVEL**2)/RALS(K)
      QL(1) = RIOVRP
      QL(2) = PL(1)
      NII = 2
      EPS = PL(1)
      GO TO 6111
 6149 IF(PLEVEL .GT. 13.6906) GO TO 6151
      QL(1) = RIOVRP
      NII = 2
      GO TO 6111
 6151 QL(1) = 4.9784/RALS(K)
      PL(2) = QL(1)
      QL(2) = RIOVRP
      GO TO 6109
 6155 IF(PLEVEL .GT. 3.519678) GO TO 6157
      QL(1) = RIOVRP
      NII = 2
      GO TO 6111
 6157 GL(1) = 4.9784/RALS(K)
```

```
      PL(2) = QL(1)
      QL(2) = RIOVRP
      GO TO 6109
 6161 IF(PLEVEL .GT. 15.940278) GO TO 6163
      IF(K .EQ. 6 .AND. PLEVEL .GT. 9.539478) GO TO 6163
      QL(1) = RIOVRP
      NII = 2
      GO TO 6111
 6163 QL(1) = 4.9784/RALS(K)
      PL(2) = QL(1)
      QL(2) = RIOVRP
      GO TO 6109
 9061 CONTINUE
      ROROLD = 0.1
      NII = 1
      IDXB = INDF
      DO 9067  J=1,NBDLF
      URB(K,J) = 0.0
      ULB(K,J) = 0.0
      DO 9063  I=1,NBDLF
 9063 BB(K,J,I) = 0.0
      DO 9065  I=1,10
 9065 U(J,I) = 0.0
 9067 CONTINUE
      DELPLV = 0.02*PLEVEL
      DELZ = DELPLV/RALS(K)
      PLEVEL = PLEVEL - 0.5*DELPLV
      GO TO (9071,9073,9075,9077,9077,9077), K
 9071 CMH = (1.0414 - PLEVEL)/RALS(K)
      GO TO 9081
 9073 CMH = (1.2192 - PLEVEL)/RALS(K)
      GO TO 9081
 9075 CMH = -PLEVEL/RALS(K)
      GO TO 9081
 9077 CMH = (3.519678 - PLEVEL)/RALS(K)
 9081 CONTINUE
      CALL CONTR(PLEVEL,ORLS,K)
      RORALS = ORLS/RALS(K)
```

```
     1  + 1.0414)**2)/RALS(K)
      IF(K .EQ. 3 .AND. PLEVEL .LE. 3.519678) EPS=1.4144475*SQRT(12.3881
     1  - PLEVEL**2)/RALS(K)
      IF(ABS(RORALS - ROROLD) .GT. 0.0001) CALL BESSEL(VR,BJ1,BJ1A,BJ1P,
     1  NII)
      ROROLD = RORALS
      ISEG = 1
      GO TO (9111,9111,9131,9131,9151,9151), K
 9111 Z = (PLEVEL -1.0414)/RALS(K)
      IF(K .EQ. 2) Z=(PLEVEL - 1.2192)/RALS(K)
      IF(PLEVEL .LE. 2.2606) GO TO 9181
      ISEG = 2
      IF(PLEVEL .LE. 7.8486) GO TO 9181
      ISEG = 3
      GO TO 9181
 9131 Z = PLEVEL/RALS(K)
      IF(K .EQ. 4) Z=(PLEVEL - 3.519678)/RALS(K)
      IF(PLEVEL .LE. 3.519678) GO TO 9181
      ISEG = 2
      IF(PLEVEL .LE. 13.6906) GO TO 9181
      ISEG = 3
      GO TO 9181
 9151 Z = (PLEVEL - 3.519678)/RALS(K)
      IF(PLEVEL .LE. 3.519678) GO TO 9181
      ISEG = 2
      IF(K .EQ. 6 .AND. PLEVEL .LE. 9.539478) GO TO 9181
      IF(K .EQ. 5 .AND. PLEVEL .LE. 15.940278) GO TO 9181
      ISEG = 3
 9181 CONTINUE
      R = EPS
      DELR = (RORALS - EPS)/20.0
      RINC = R + DELR
      RSUM1 = 0.0
      DO 9201  L=1,20
      RSUM1 = RSUM1 + R + RINC
      R = R + DELR
 9201 RINC = RINC + DELR
```

```
      DO 9221  J=1,NBDLF
      YJ = (Z - BFORDS(J,ISEG))/BFSLPS(J,ISEG)
      URB(K,J) = URB(K,J) + DELR*RSUM1*(DTBBL + RALSBL*Z)*YJ*DELZ
      ULB(K,J) = ULB(K,J) + DELR*RSUM1*YJ*DELZ
      DO 9211  I=J,NBDLF
      YI = (Z - BFORDS(I,ISEG))/BFSLPS(I,ISEG)
      BB(K,J,I) = BB(K,J,I) + 0.5*DELR*RSUM1*YJ*YI*DELZ
 9211 IF(I .NE. J) BB(K,I,J) = BB(K,J,I)
 9221 CONTINUE
      DO 9261  I=1,10
      R = EPS
      RINC = R + DELR
      SPU(I) = 0.0
      IF(I .GE. 6) GO TO 9241
      DO 9231  L=1,20
      SPU(I) = SPU(I) + R**(2*I - 1) + RINC**(2*I - 1)
      R = R + DELR
 9231 RINC = RINC + DELR
      GO TO 9261
 9241 II = I - 5
      DO 9251  L=1,20
      N = L + 1
      SPU(I) = SPU(I) + RJ1P(II)*R*BJ1PA(II,L) + BJ1A(II,L)
     1  + RJ1P(II)*RINC*BJ1PA(II,N) + BJ1A(II,N)
      R = R + DELR
 9251 RINC = RINC + DELR
 9261 CONTINUE
      DO 9291  J=1,NBDLF
      YJ = (Z - BFORDS(J,ISEG))/BFSLPS(J,ISEG)
      DO 9281  I=1,10
      IF(I .GE. 6) GO TO 9271
      U(J,I) = U(J,I) + FLOAT(I)*DELR*SPU( I)*YJ*DELZ
      GO TO 9281
 9271 II = I - 5
      U(J,I) = U(J,I) + 0.5*DELR*SPU(I)*YJ*DELZ*EXP(RJ1P(II)*(Z + CMH))
 9281 CONTINUE
 9291 CONTINUE
      PLEVEL = PLEVEL - DELPLV
```

```
      IF (PLEVEL .LE. 0.0) GO TO 9295
      GO TO 9081
 9295 CONTINUE
      DO 9299  J=1,NBDLF
 9299 ULB(K,J) = ULB(K,J) - URB(K,J)
      RETURN
      END
$IBFTC SUB4    DECK
      SUBROUTINE TKCONF(NII,PL,QL,QMP,VR,VZ,CMH,BOA,AORS,PLEVEL,K)
C
C THIS SUBROUTINE DESCRIBES THE CONTOURS OF SATURN V TANKS IN THE THETA-
C PLANE.
C
      DIMENSION QPP(4)
      COMMON /BLK2/G(16),X(16)
      COMMON /CL5/PL(4), QL(4), QMP(4), VZ(4,16)
      COMMON /CL6/VR(4,16)
      COMMON /CL7/ NBDLF       , INDF, UF(10), INDX2, VF, VN, RALS(6),
     1             ALL(6)
      DO 6201  I=1,NII
      QMP(I) = 0.5*(QL(I) - PL(I))
 6201 QPP(I) = 0.5*(QL(I) + PL(I))
      DO 6205  I=1,NII
      DO 6205  J=1,16
 6205 VR(I,J) = QMP(I)*X(J) + QPP(I)
      GO TO (6209,6217,6225,6233,6241,6241), K
 6209 DO 6213  I=1,16
      VZ(1,I) = - 2.0828/RALS(K) + SQRT((3.302/RALS(K))**2 - VR(1,I)**2)
      VZ(2,I) = - SQRT((3.302/RALS(K))**2 - VR(2,I)**2)
      IF(NII .EQ. 3) VZ(3,I) = (PLEVEL - 1.0414)/RALS(K)
      IF(NII .EQ. 3) GO TO 6213
      VZ(3,I) = 6.8072/RALS(K) + SQRT((3.302/RALS(K))**2 - VR(3,I)**2)
      VZ(4,I) = (PLEVEL - 1.0414)/RALS(K)
 6213 CONTINUE
      CMH = (1.0414 - ALL(K))/RALS(K)
      RETURN
 6217 DO 6221  I=1,16
      VZ(1,I) = 2.0828/RALS(K) - SQRT((3.302/RALS(K))**2 - VR(1,I)**2)
```

```
      IF(NII .EQ. 2) GO TO 6221
      VZ(2,I) = SQRT((3.302/RALS(K))**2 - VR(2,I)**2)
      VZ(3,I) = (PLEVEL - 1.2192)/RALS(K)
 6221 CONTINUE
      CMH = (1.2192 - ALL(K))/RALS(K)
      RETURN
 6225 DO 6229  I=1,16
      VZ(1,I) = BOA*SQRT(AORS - VR(1,I)**2)
      IF(NII .EQ. 2) VZ(2,I) = PLEVEL/RALS(K)
      IF(NII .EQ. 2) GO TO 6229
      VZ(2,I) = 13.6906/RALS(K) + BOA*SQRT(AORS - VR(2,I)**2)
      VZ(3,I) = PLEVEL/RALS(K)
 6229 CONTINUE
      CMH = - ALL(K)/RALS(K)
      RETURN
 6233 DO 6237  I=1,16
      VZ(1,I) = - BOA*SQRT(AORS - VR(1,I)**2)
      IF(NII .EQ. 2) VZ(2,I) = (PLEVEL - 3.519678)/RALS(K)
      IF(NII .EQ. 2) GO TO 6237
      VZ(2,I) = BOA*SQRT(AORS - VR(2,I)**2)
      VZ(3,I) = (PLEVEL - 3.519678)/RALS(K)
 6237 CONTINUE
      CMH = (3.519678 - ALL(K))/RALS(K)
      RETURN
 6241 DO 6249  I=1,16
      VZ(1,I) = - BOA*SQRT(AORS - VR(1,I)**2)
      IF(NII .EQ. 2) VZ(2,I) = (PLEVEL - 3.519678)/RALS(K)
      IF(NII .EQ. 2) GO TO 6249
      IF(K .EQ. 5) GO TO 6243
      VZ(2,I) = 6.0198/RALS(K) + BOA*SQRT(AORS - VR(2,I)**2)
      GO TO 6245
 6243 VZ(2,I) = 12.4206/RALS(K) + BOA*SQRT(AORS - VR(2,I)**2)
 6245 VZ(3,I) = (PLEVEL - 3.519678)/RALS(K)
 6249 CONTINUE
      CMH = (3.519678 - ALL(K))/RALS(K)
      RETURN
      END
```

```
$IBFTC SUBB     DECK
      SUBROUTINE BESSEL(VR,BJ1,BJ1A,BJ1P,NII)
C
C THIS SUBROUTINE EVALUATES ALL OF THE BESSEL FUNCTIONS WHICH ARE NEEDED
C FOR SUBSEQUENT COMPUTATION
C
      DIMENSION D(7), E(7), F(7)
      COMMON /BLK1/A(7),B(7),O(7)
      COMMON /BLK3/RJ1P(5)
      COMMON /CL4/BJ1(4,5,16), BJ1A(5,21), BJ1P(4,5,16), EPS
      COMMON /CL6/VR(4,16)
      COMMON /CL9/ IDXB, RORALS
      COMMON /TEMP/BJ1PA(5,21), BJ0A(5,21)
      DATA (D(I),I=1,7)/1.0,-2.2499997,1.2656208,-0.3163866,0.0444479,
     1  -0.0039444,0.0002100/, (E(I),I=1,7)/0.79788456,-0.00000077,
     2  -0.0055274,-0.00009512,0.00137237,-0.00072805,0.00014476/,
     3  (F(I),I=1,7)/-0.78539816,-0.04166397,-0.00003954,0.00262573,
     4  -0.00054125,-0.00029333,0.00013558/
      DO 6329  I=1,NII
      DO 6329  J=1,5
      LB = 16
      IF(IDXB .EQ. 11) LB = 1
      DO 6329  L=1,LB
      IF(IDXB .EQ. 11) GO TO 6313
      AJNR = RJ1P(J)*VR(I,L)
      IF(AJNR .GT. 3.0) GO TO 6305
      T = AJNR/3.0
      SUM1 = 0.0
      SUM2 = 0.0
      DO 6301  LL=1,7
      SUM1 = SUM1 + A(LL)*T**(2*LL - 2)
 6301 SUM2 = SUM2 + D(LL)*T**(2*LL - 2)
      BJ1(I,J,L) = AJNR*SUM1
      BJ1P(I,J,L) = SUM2 - SUM1
      GO TO 6313
 6305 T = 3.0/AJNR
      SUM1 = 0.0
      SUM2 = AJNR
```

```
      SUM3 = 0.0
      SUM4 = AJNR
      DO 6309  LL=1,7
      SUM1 = SUM1 + B(LL)*T**(LL - 1)
      SUM2 = SUM2 + O(LL)*T**(LL - 1)
      SUM3 = SUM3 + E(LL)*T**(LL - 1)
 6309 SUM4 = SUM4 + F(LL)*T**(LL - 1)
      CAJNR = 1.0/AJNR**0.5
      BJ1(I,J,L) = CAJNR*SUM1*COS(SUM2)
      BJ1P(I,J,L) = CAJNR*SUM3*COS(SUM4) - BJ1(I,J,L)/AJNR
 6313 IF(.NOT.(I .EQ. 1 .AND. L .EQ. 1)) GO TO 6329
      R = EPS
      RBORA = 1.0
      IF(IDXB .EQ. 11) RBORA = RORALS
      DELR = (RBORA - EPS)/20.0
      DO 6327  M=1,21
      AJNR = RJ1P(J)*R
      IF(AJNR .GT. 3.0) GO TO 6321
      T = AJNR/3.0
      SUM1 = 0.5
      SUM2 = 1.0
      DO 6317  LL=2,7
      SUM2 = SUM2 + D(LL)*T**(2*LL - 2)
 6317 SUM1 = SUM1 + A(LL)*T**(2*LL - 2)
      BJ1A(J,M) = AJNR*SUM1
      BJ1PA(J,M) = SUM2 - SUM1
      BJOA(J,M)  = SUM2
      GO TO 6326
 6321 T = 3.0/AJNR
      SUM1 = 0.0
      SUM2 = AJNR
      SUM3 = 0.0
      SUM4 = AJNR
      DO 6325  LL=1,7
      SUM1 = SUM1 + B(LL)*T**(LL - 1)
      SUM3 = SUM3 + E(LL)*T**(LL - 1)
      SUM4 = SUM4 + F(LL)*T**(LL - 1)
 6325 SUM2 = SUM2 + O(LL)*T**(LL - 1)
```

```
      BJ1A(J,M) = (1.0/AJNR**0.5)*SUM1*COS(SUM2)
      BJ1PA(J,M) = (1.0/AJNR**0.5)*SUM3*COS(SUM4) - BJ1A(J,M)/AJNR
      BJ0A(J,M)  = (1.0/AJNR**0.5)*SUM3*COS(SUM4)
 6326 R = R + DELR
 6327 CONTINUE
 6329 CONTINUE
      RETURN
      END
$IBFTC SUB6    DECK
      SUBROUTINE CONTR(PLEVEL,ORLS,K)
C
C THIS SUBROUTINE PROVIDES THE RADIUS OF THE LIQUID SURFACE
C
      C1 = 10.903204
      C2 = 4.9784
      C3 = 3.519678
      GO TO (6411,6421,6431,6445,6451,6461), K
 6411 IF(PLEVEL .GT. 7.8486) KI=1
      IF(PLEVEL .LE. 7.8486 .AND. PLEVEL .GE. 1.0414) KI=2
      IF(PLEVEL .LT. 1.0414) KI=3
      GO TO (6413,6415,6417), KI
 6413 ORLS = SQRT(C1 - (PLEVEL - 7.8486)**2)
      RETURN
 6415 ORLS = 3.302
      RETURN
 6417 ORLS = SQRT(C1 - (1.0414 - PLEVEL)**2)
      RETURN
 6421 IF(PLEVEL .GE. 2.2606) KI=1
      IF(PLEVEL .LT. 2.2606) KI=2
      GO TO (6423,6425), KI
 6423 ORLS = SQRT(C1 - (PLEVEL - 1.2192)**2)
      RETURN
 6425 ORLS = SQRT(C1 - (3.302 - PLEVEL)**2)
      RETURN
 6431 IF(PLEVEL .GT. 13.6906) KI=1
      IF(PLEVEL .LE. 13.6906) KI=2
      GO TO (6433,6435), KI
 6433 ORLS = C2*SQRT(1.0 - ((PLEVEL - 13.6906)/C3)**2)
```

```
      RETURN
 6435 ORLS = C2
      RETURN
 6445 ORLS = C2*SQRT(1.0 - (1.0 - PLEVEL/C3)**2)
      RETURN
 6451 IF(PLEVEL .GT. 15.940278) KI=1
      IF(PLEVEL .LE. 15.940278 .AND. PLEVEL .GE. C3) KI=2
      IF(PLEVEL .LT. C3) KI=3
      GO TO (6453,6435,6445), KI
 6453 ORLS = C2*SQRT(1.0 - ((PLEVEL - 15.940278)/C3)**2)
      RETURN
 6461 IF(PLEVEL .GT. 9.539478) KI=1
      IF(PLEVEL .LE. 9.539478 .AND. PLEVEL .GE. C3) KI=2
      IF(PLEVEL .LT. C3) KI=3
      GO TO (6463,6435,6445), KI
 6463 ORLS = C2*SQRT(1.0 - ((PLEVEL - 9.539478)/C3)**2)
      RETURN
      END
```