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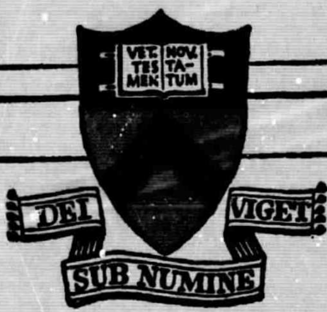

PRINCETON UNIVERSITY

OBSERVATIONS OF INTERSTELLAR LYMAN-α ABSORPTION*

by

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ABSTRACT

Absorption at the Lyman-α transition from interstellar neutral hydrogen has been observed in the ultraviolet spectra of 18 nearby O and B stars. Radiation damping is the dominant cause of line broadening, which makes the derived line-of-sight column densities proportional to the square of the observed equivalent widths. An average hydrogen density on the order of 0.1 atom cm^{-3} has been found for most of the stars observed so far. This is in contrast to the findings from surveys of 21-cm radio emission, which suggest 0.7 atom cm^{-3} exists in the local region of the Galaxy. Several effects which might introduce uncertainties into the Lyman-α measurements are considered, but none seem to be able to produce enough error to explain the disagreement with the 21-cm data. The possibility that small-scale irregularities in the interstellar gas could give significantly lower values at Lyman-α is explored. However, a quantitative treatment of the factor of ten discrepancy in Orion indicates the only reasonable explanation requires the 21-cm flux to come primarily from small, dense, hot clouds which are well separated from each other. The existence of such clouds, however, poses serious theoretical difficulties.

1. Introduction

Stellar spectroscopy in the far ultraviolet has the potential of substantially increasing our knowledge on the distribution and composition of interstellar matter. A rich variety of resonance lines for various elements are observable between the Lyman limit at 912 Å and wavelengths where conventional ground-based equipment can operate (Spitzer and Zabriskie 1959), and this prospect provides a strong incentive for further development of ultraviolet astronomical instrumentation. As of now, however, resolutions ranging from 1 to 10 Å have limited us to collecting significant data only for the strongest transition of the most abundant element: the 1216 Å Lyman-α line of atomic hydrogen. In interstellar space the absorption from this transition is strong enough to produce line widths on the order of 10 Å over typical distances to the brightest and most easily observed O and B stars. A measurement of the absorption's equivalent width allows a direct determination of the neutral hydrogen abundance along the line of sight.

Presently, the total number of observations of the Lyman-α absorption in various directions is small, and the coverage of the sky is still rather spotty (see Fig. 1). We cannot yet really class these measurements in the usual sense as a "general survey" of HI near the sun, and even in the near future we cannot hope to measure up to the overall scope and detail of information on the distribution of hydrogen provided by the 21-cm radio observations. In view of this, the ultraviolet spectroscopic data would therefore seemingly represent a paltry and unimportant contribution to our understanding of interstellar hydrogen. Nonetheless, it is important to realize that the Lyman-α observations represent a truly independent means of collecting neutral hydrogen data, and of particular interest is the fact that many of the measurements available so far have strongly disagreed with the 21-cm results.

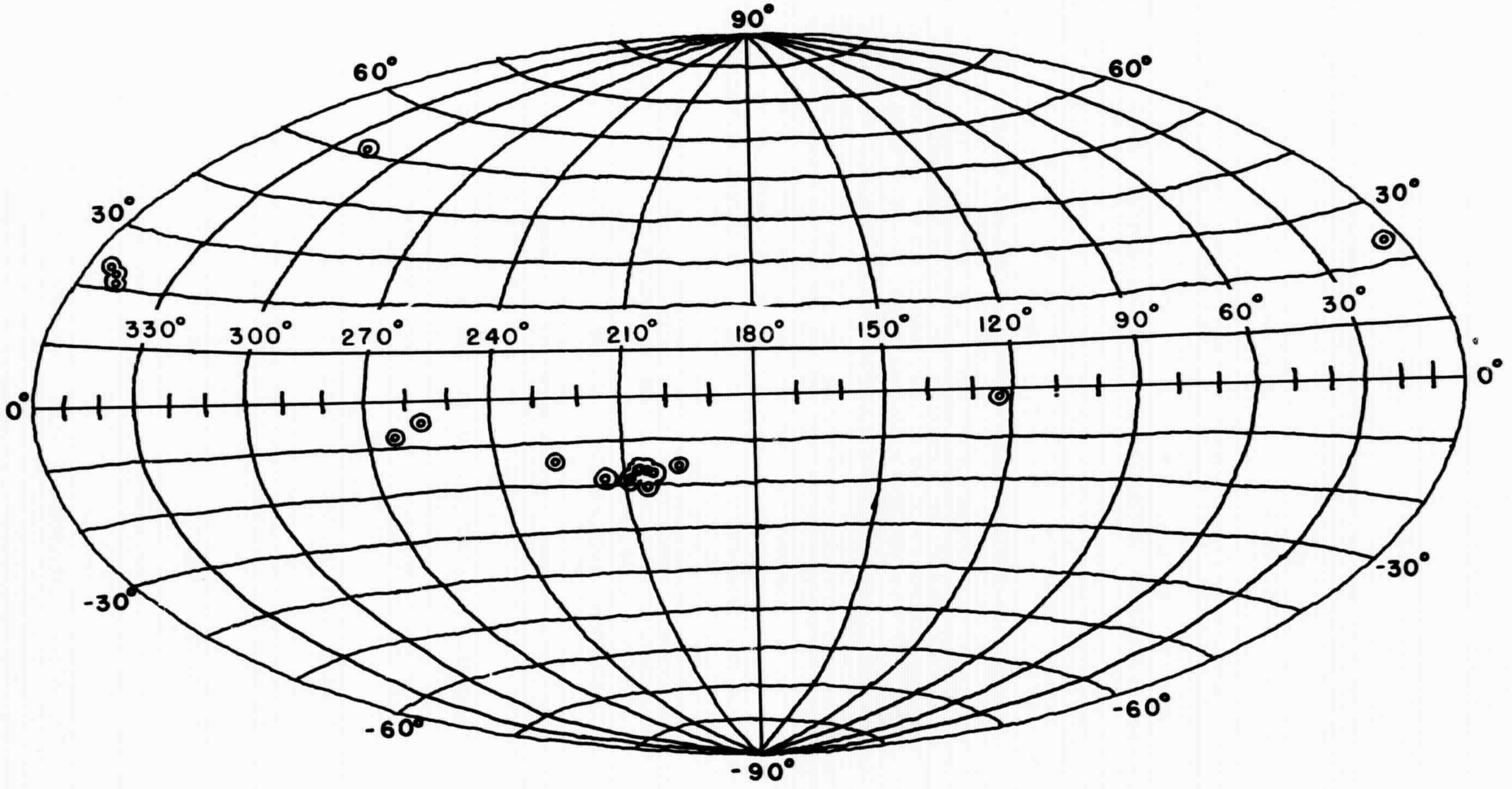

FIG. 1. The distribution in l^{II} and b^{II} of stars whose Lyman-α lines have been observed to date (see Table 1). Stars near the galactic anticenter in the middle of the diagram show the deficiency of HI, while π, δ, and ρ^1 Sco (far left) and ζ Oph (far right) are in better agreement with the 21-cm data.

Table 1

Interstellar hydrogen measurements

Star	MK	l^{II}	b^{II}	E(B-V)	N_H (10^{20} atom cm^{-2})	d (pc)	n_H (atom cm^{-3})	Investigators
ζ Oph	09.5V	6°	+24°	0.32	4.2	170	0.8	4
γ Cas	B0IVe	124	-3	0.08	1.2	200	0.2	10
γ Ori	B2III	197	-16	0.02	$0.39^{+0.19}_{-0.16}$	95	0.135	3
δ Ori	09.5II	204	-18	0.08	$1.25^{+0.33}_{-0.28}$	460	0.088	1,2
η Ori	B0.5V	205	-20	0.11	$2.7^{+3.3}_{-2.0}$	380	0.23	2
ε Ori	B0Ia	205	-17	0.09	$1.34^{+0.34}_{-0.29}$	460	0.094	2
ζ Ori	09.5Ib	206	-17	0.09	$1.30^{+0.21}_{-0.19}$	460	0.092	1,2,3,8
σ Ori	09.5V	207	-17	0.06	$2.7^{+1.5}_{-1.2}$	460	0.19	2
θ^1Ori C	06p	209	-19	0.31	$4.8^{+2.7}_{-2.1}$	500	0.31	8
ι Ori	09III	210	-20	0.07	$0.99^{+0.26}_{-0.24}$	500	0.064	2,3,8
κ Ori	B0.5Ia	215	-19	0.04	$0.89^{+0.38}_{-0.28}$	460	0.071	3
β CMa	B1II-III	226	-14	0.01	$0.57^{+0.48}_{-0.34}$	210	0.088	3
ζ Pup	05f	256	-5	0.05	$0.61^{+0.12}_{-0.11}$	390*	0.051	3,6,9
γ Vel	WC7+07	263	-8	0.03-0.06	$0.31^{+0.21}_{-0.16}$	390*	0.026	3,6
α Vir	B1V	316	+51	0.03	<0.13	100	<0.042	9
τ Sco	B1V	347	+20	0.06	7.5^{+8}_{-4}	170	1.4	7
δ Sco	B0V	350	+23	0.18	12^{+6}_{-3}	170	2.4	7
β^1Sco	B0.5V	353	+24	0.20	16^{+16}_{-8}	170	3.0	7

*Estimated distance to the near edge of the Gum Nebula, which probably surrounds both stars.

(continued)

Key to Investigators (Table 1 continued)

1. Morton (1967a) (Not included in average since no errors were quoted).
2. Jenkins and Morton (1967).
3. Carruthers (1968).
4. Stecher (1968).
5. Morton, Jenkins, and Bohlin (1968).
6. Morton, Jenkins, and Brooks (1969).
7. Jenkins, Morton, and Matilsky (1969).
8. Carruthers (1969).
9. Smith (1969).
10. Morton, Jenkins, and Bohlin (1969).

With the exception of the results for ζ Oph and three stars in Scorpius, the strength of the Lyman-α absorptions have indicated the presence of roughly one-tenth the column densities found by 21-cm emission measurements in the same direction. Or, from a slightly different viewpoint, the average volume densities to the stars (all less than 0.5 kpc away) are on the order of 0.1 atom cm^{-3}, in contrast to a description by Kerr and Westerhout (1965) of 21-cm data which indicates the density is more near 0.7 atom cm^{-3} in the solar neighborhood.

Table 1 is a compilation of data presently available from the Lyman-α measurements for 18 stars, all of which have been observed on rocket flights. The stars are listed in ascending order of l^{II} and are grouped according to mutual proximity in the sky. For stars which have been observed more than once, the column density N_H is computed from a weighted average of the separate measurements of the equivalent width W_λ , with the weights being proportional to the inverse squares of the respective error estimates σ_i for each observation. The quoted error, when specified by an author, is used to compute the uncertainty in N_H, and multiple observations are assigned a net uncertainty in W_λ equal to $(\sum_i \sigma_i^{-2})^{-\frac{1}{2}}$. The estimated distance d to each star is used for deriving the average volume density n_H along the line of sight. The value shown for α Vir represents an upper limit since a good fraction of the Lyman-α absorption is probably stellar.

The far greater capability of satellite telescopes for providing a large volume of measurements should soon allow us to contemplate a much longer list of stars. The tabulation is already on the verge of being out of date as spectroscopic data will shortly be available from the Wisconsin Experiment Package aboard the first Orbiting Astronomical Observatory.

2. Profile Structure

Before exploring the implications of the Lyman-α results in more detail, it would be appropriate to review the theory which applies to the observed profiles and follow with a discussion of the various effects which could lead to errors in the measurements. The large abundance of interstellar hydrogen and the strength of the transition dictate that to even the nearest stars the absorption is very strongly saturated. An optical depth at the line center is typically about 10^7, which is sufficient to place an equivalent width measurement far up on the damping portion of the curve of growth, and at interstellar densities radiation damping is the dominant cause of broadening. Any turbulent or thermal doppler broadening is completely insignificant. We are fortunate to have the other forms of broadening of no importance here, since in the end we can obtain a very straightforward relation for the column density N_H in terms of the equivalent width W_λ without having to worry about any assumptions regarding temperatures, densities, or velocities.

The basic equation for a transition's optical depth τ as a function of frequency ν

$$\tau(\nu) = \frac{e^2}{mc} f_{12} \frac{\gamma}{4\pi} \frac{N_H}{(\nu-\nu_0)^2 + (\gamma/4\pi)^2} \tag{1}$$

may be immediately simplified by dropping the $(\gamma/4\pi)^2$ term in the denominator since $|\nu-\nu_0| >> \gamma/4\pi$ at any point where τ is not unreasonably large. After substituting in the expression

$$\gamma = \frac{g_1}{g_2} f_{12} \frac{8\pi^2 e^2 \nu^2}{mc^3} \tag{2}$$

and re-expressing the equation in terms of wavelength λ, we obtain a more convenient representation

$$\tau(\lambda) = 2\pi N_H \left(\frac{e^2}{mc^2}\right)^2 \lambda_0^2 \left(f^2_{12,1/2} \frac{g_1}{g_{2,1/2}} + f^2_{12,3/2} \frac{g_1}{g_{2,3/2}}\right) (\lambda-\lambda_0)^{-2} \tag{3}$$

In this formula the f and g values for the two terms in the doublet are shown separately. The integration of 1 - exp $\left[-\tau(\lambda)\right]$ is straightforward and we obtain

$$N_H = \left(1.865 \times 10^{18} \text{ atom cm}^{-2} \text{ Å}^{-2}\right) W_\lambda^2 \tag{4}$$

after substituting in the numerical values for the various constants which are all well known.

3. Possible Misinterpretations from the Lyman-α Data

From equation 4 it is evident that the factor of ten discrepancy in density results from equivalent width measurements which are typically a factor of three too low for agreement with the radio data. This amount seems to be well outside any reasonable experimental errors in the actual Lyman-α measurements, and one can be confident that the observed 21-cm brightness temperatures should be even more accurate. Allowances for some of the complications which commonly arise in optical interstellar line investigations tend only to aggravate the discrepancy. For instance, we assume that stars earlier than about B3 should have their own Lyman-α absorption line well within the saturated region of the interstellar absorption. If the stellar component were wide enough to be of consequence, we would be forced to conclude the actual interstellar absorption is narrower than the observed line which is already too narrow. An excessively large velocity for some very small fraction of the interstellar hydrogen would similarly worsen the disagreement with the radio results. On the whole, the existence of a large number of weak absorptions on top of the stellar continuum should neither increase nor decrease the observed equivalent width, unless these absorptions were very oddly distributed near the Lyman-α wavelength.

In the category of possible errors in the actual processing of the recorded data, one should include incorrect conversions of densities recorded on film into relative intensities, through the application of a badly determined H and D curve. Although this could lead to some inaccuracies in the equivalent widths, from the author's experience any reasonable variations in the shape of the H and D curve do not seem to have an appreciable effect on the derived values. Another point to consider is that since the wings of the theoretical profile extend fairly well beyond the main core, have the measurements really included all of the absorption. We might question whether the published equivalent widths were based on an assumed continuum level which had been drawn in too low. This might be a possibility if a star's continuum were rather confusing, and it could be a particularly tricky question when the resolution of a recording is comparable to W_λ and the profile is badly washed out.

Some insight on the validity of a few of the W_λ derivations is available from spectra having a resolution of wavelength intervals considerably smaller than W_λ. In such cases the actual shape of the profiles may be discerned, and in a consistency check we can make use of this extra recorded detail plus our _a priori_ knowledge of the theoretical line shape. The bottom tracings in Figure 2 are a sampling of such spectra obtained by Princeton investigators. In each case the observations were recorded on film by objective spectrographs in a rocket, and the resolutions were on the order of 1 Å. To gain a perspective on the acceptability of various values of N_p we may consider the shape of the original tracings after they have been multiplied by $e^{+\tau(\lambda)}$. These profiles are basically reconstructions (except near the core) of how the stellar output should have appeared in the absence of interstellar hydrogen. The profiles shown above the original data use $\tau(\lambda)$ functions based on one, two, and four times the

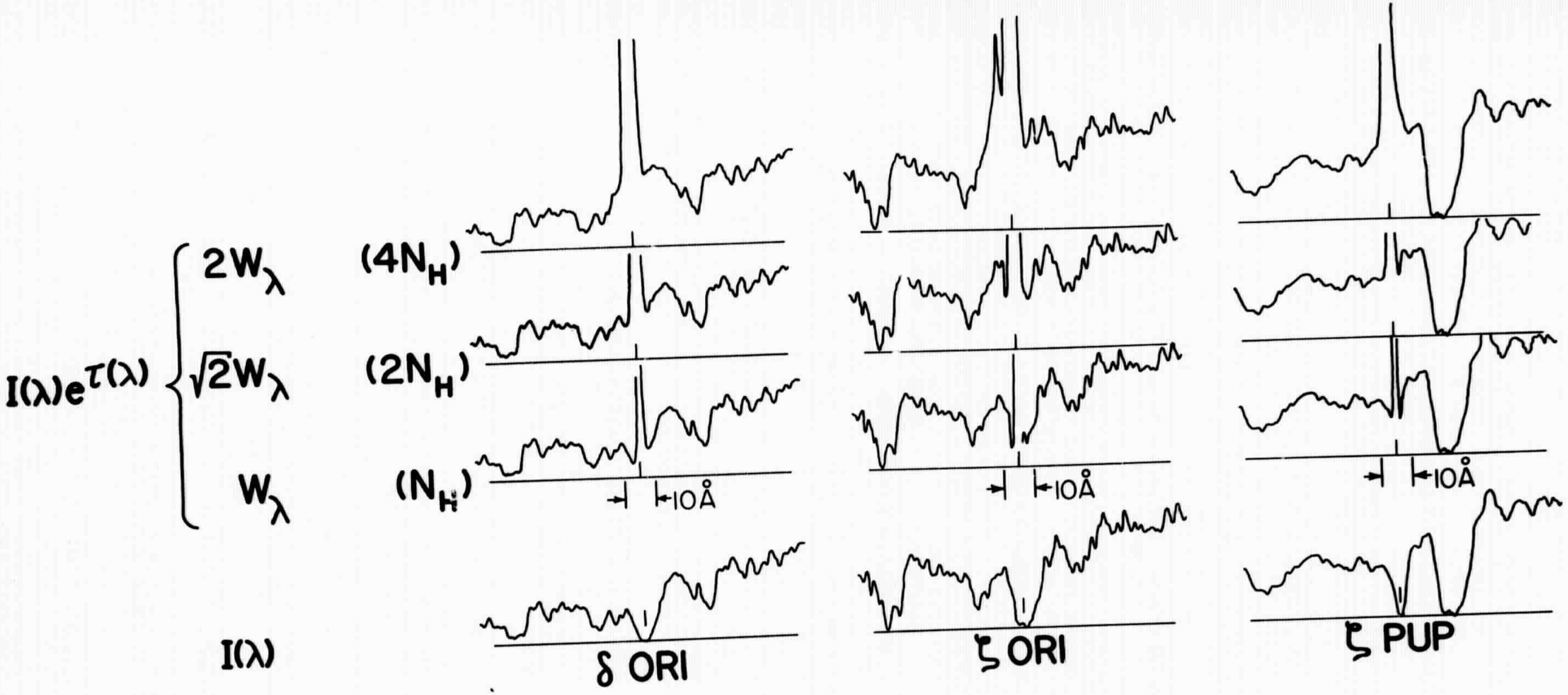

FIG. 2. A plausibility test for different interstellar hydrogen densities. Above the originally recorded spectra of δ Ori, ζ Ori and ζ Pup are three reconstructions of how the stellar output would have appeared in the absence of interstellar Lyman-α absorption. The examples show the original tracing multiplied by $e^{+\tau(\lambda)}$, where $\tau(\lambda)$ is derived from equation 3 in the text, assuming 1, 2 and 4 times the column densities quoted by Jenkins and Morton (1967) for the Orion stars and Morton, Jenkins and Brooks (1969) for ζ Pup. These cases respectively correspond to 1, $\sqrt{2}$ and 2 times the measured equivalent widths.

column densities derived from the W_λ estimates. Whether or not the larger values of N_H are plausible should depend upon our judgement on how reasonably the reconstructed stellar flux behaves near 1216 Å. The drawings seem to reveal that four times the originally quoted densities are out of the question, and even where only twice these densities are assumed, the tracing turns upward reasonably far from the line center, i.e. at a wavelength where a measurable intensity had been recorded. With the actual assumed densities (N_H times one), however, it appears that the flux falls below the continuum at some distance from the line core, although the upturn is nicely confined to wavelengths where the recorded signal loses significance. The fact that the theoretical absorption profile is not reproduced perfectly by the data is evidenced by our inability to obtain a fairly smooth continuum near both the core and the wings, but the fidelity seems to be as good as one could expect from these observations. Generally speaking, for spectra of moderately good resolution, a reasonably placed continuum level should not result in a quoted column density being more than a factor of two lower than the real value.

After looking at the top curves in Figure 2, we might be tempted to ask whether material in the immediate vicinity of the star (or for that matter, the entire HII region) is not actually emitting a strong and quite broad Lyman-α line which could pull up the wings of the observed profile and reduce the apparent W_λ. For this effect to be important, it would be necessary to have an emission strength comparable to the stellar continuum extending at least several Å from the line center. Ordinary thermal and turbulent doppler broadening from a 10^4 °K gas having an rms bulk velocity on the order of 10 km s^{-1} would lead to a Gaussian profile with a dispersion $\sigma \approx 0.06$ Å. Since this profile approaches zero very rapidly as $\Delta\lambda$ exceeds several σ, random velocities cannot be responsible for pushing any of the Lyman-α emission into a wavelength region where the interstellar absorption is not overwhelming.

On the other hand, the more gradual fall-off of the Lorentz profile might render broadening from radiation damping an important consideration if the total emission flux were strong enough. For each stellar photon more energetic than the Lyman limit we would expect the eventual production of one Lyman-α photon from either within or not far beyond the edge of the surrounding HII region. If we neglect the other forms of broadening, the emission profile $N_{\alpha e}(\nu)$ will equal the number of photons N_L having $\lambda < 912$ Å times the expression in Equation (1) normalized in frequency,

$$N_{\alpha e}(\nu) = N_L \frac{\gamma/4\pi^2}{\Delta\nu^2 + (\gamma/4\pi)^2} \tag{5}$$

where $\Delta\nu = \nu - \nu_0$. If we now solve for the value of $\Delta\nu/\nu_0$ which gives an emission flux $N_{\alpha e}(\nu)$ equal to the stars continuum radiation $N_{\alpha c}(\nu)$ near 1216 Å, we obtain

$$\frac{\Delta\nu}{\nu_0} = (1.615 \times 10^{-12}\, s^{1/2}) \left(\frac{N_L}{N_{\alpha c}(\nu)}\right)^{1/2} \tag{6}$$

(Again we have ignored the $(\gamma/4\pi)^2$ term.) For representative values of N_L and $N_{\alpha c}(\nu)$ we consider as a worst case the output from the hottest type star observed and adopt the fluxes $N_{\alpha c}(\nu) = 2.4 \times 10^9$ phot cm^{-2} s^{-1} Hz^{-1} and $N_L = 3.5 \times 10^{24}$ phot cm^{-2} s^{-1} given by Hickok and Morton (1968) and Morton (1969a) for a model O5V star ($T_e = 37\ 450$°K). Substituting these values into Equation (6) gives $\Delta\nu/\nu_0 = 6.2 \times 10^{-5}$ which corresponds to 0.075 Å from the line's center. Or to put it differently, at 1 Å away from the center the emission would be only 0.0057 times the stellar continuum flux. Thus our rough calculation appears to demonstrate that, unless some violent nonequilibrium processes are important, the total Lyman-α emission is insufficient to generate enough photons at wavelengths which are relevant to the interstellar hydrogen measurements.

In view of the evidence for mass loss from the supergiant stars observed by Morton (1967a), Carruthers (1968), Morton, Jenkins and Bohlin (1968), and Morton, Jenkins and Brooks (1969), we should explore yet another possibility for a significant effect from Lyman-α emission. The observations have shown that prominent stellar absorption lines exhibit negative velocity shifts corresponding to about 10 Å. We therefore must recognize a possible source of Lyman-α photons which unquestionably could be shifted well out of the region of heavy interstellar absorption. The ejected material from the star is highly ionized, and we should explore the possibility that hydrogen recombinations could generate enough Lyman-α photons to be of concern to us. Morton (1967b) has developed a model for the mass loss from some Orion supergiant stars, and in spite of some simplifying assumptions in his analysis, the model should be sufficiently close to reality for us to determine whether or not the emission would be troublesome.

In Morton's model, the velocity of the gas did not change with increasing distance from the star's surface, and hence the electron density n_e followed the relationship

$$n_e = n_{e*} \frac{r_*^2}{r^2} \tag{7}$$

where r_* was the stellar radius and n_{e*} was the density at the surface. The total Lyman-α emission $N_{\alpha e}$ is the volume integral of the number of recombinations $n_e^2 \alpha_2$ above the star's surface,

$$N_{\alpha e} = \int_{r_*}^{\infty} \alpha_2 \frac{n_{e*}^2 r_*^4}{r^4} (4\pi r^2 dr) = 4\pi \alpha_2 n_{e*}^2 r_*^3 \tag{8}$$

The recombination coefficient α_2 is assumed not to vary over the volume, which is in effect saying that $T = 10^4$ °K everywhere. Next, we again wish to compare the derived $N_{\alpha e}$ with the continuum flux from the star, and in this case we want the total flux $4\pi r_*^2 N_{\alpha c}(\lambda)$. For a BOI-II star, it is appropriate to use Hickok and Morton's (1968) BOV model flux reduced by a factor of 2.7 (Morton 1969b), which gives $N_{\alpha c}(\lambda) = 8.7 \times 10^{20}$ phot s^{-1} cm^{-2} $Å^{-1}$. Assuming $n_{e*} = 5 \times 10^9$ elec. cm^{-3} and $r_* = 2 \times 10^{12}$ cm, we find the quotient $N_{\alpha e}/N_{\alpha c}(\lambda)$ equals 0.017 Å, which is to say that the Lyman-α recombination emission is equivalent to the stellar continuum flux within a 0.017 Å passband and hence is insignificant.

As a final consideration, we might briefly review the consequences of having extremely small scale irregularities existing in the distribution of interstellar hydrogen. If these fluctuations were sufficiently strong and small to cause substantial variations in N_H across the apparent disk of the star, then the observed W_λ would be too narrow for the corresponding average N_H over the area. This prospect is untenable, however, because a supergiant's radius is about 2×10^{12} cm, and we would not expect to find the scale length for the density gradients of interstellar hydrogen to be much less than the mean free path, which is on the order of 10^{16} cm.

4. Comparison with 21-cm Data

As the introductory discussion has indicated, there is a need to resolve an apparent disagreement between the available Lyman-α data and the findings from surveys of 21-cm emission in the galaxy. A really precise comparison of the two is difficult to achieve since there are several distinct differences in the methods of sampling. These factors which set the Lyman-α and 21-cm results apart, however, should be a key to the explanation of the apparent differences in density, and in turn we may gain an insight on some otherwise obscure attributes of the interstellar hydrogen.

To some extent, we are unsure of how much of the observed 21-cm emission came from beyond the stars, which generally are not too distant since they must have a relatively large apparent brightness to have been observed so far. For stars which are reasonably far off the galactic plane, little of the radio energy is likely to have come from beyond, and these stars offer the most secure comparisons with the 21-cm data in the same direction. Conversely, at small galactic latitudes it is difficult to tell how much of the radiation is background from the galaxy, and the stars are not far enough away to make velocity ranging from galactic rotation a useful means for separating foreground and background 21-cm components. Unfortunately, a majority of the bright O and B stars have a small b^{II}, and hence for many observations we must fall back on a more indirect comparison with an overall estimate of the local density from the 21-cm data.

Together with the Orion association, which will be treated in some detail in a later section, the stars listed in Table 2 have a high enough galactic latitude to permit meaningful comparisons with the radio data. The results of McGee, et al. (1966) are listed for the 21-cm readings which were made closest to each star. If we refer to Table 1, it should be evident that both β CMa and α Vir show considerably lower column densities from the Lyman-α equivalent widths than from the strength of the 21-cm emission. Near $l^{II} = 0$, on the other hand, the Lyman-α data for the Scorpius stars and ζ Oph indicate N_H values slightly greater than those listed in Table 2.

In addition to the emission measurements, 21-cm absorption has been observed in the radiation from two different continuum sources, both of which are bright H II regions not far from stars seen at Lyman-α. The Orion Nebula, which surrounds θ^1 Ori C, has been observed by Clark (1965), who obtained 16 km s^{-1} for

Table 2

21-cm column densities

Star	N_H* (10^{20} atom cm^{-2})
ζ Oph	9.5
ρ CMa	5.3
α Vir	6.9
π Sco	5.9
δ Sco	10.3
ρ^1 Sco	8.8

*Derived from the $\int T_b dv$ measurements by McGee, et al. (1966) using Equation (9) in the text.

the integral of the 21-cm optical depth τ over velocity. Earlier measurements by Muller (1959) and Clark, Radhakrishnan and Wilson (1962) have ranged from 8.3 to 17 km s^{-1}, but we shall adopt Clark's (1965) value for the purposes of discussion. Similarly, Menon (1969) has observed 4.9 km s^{-1} for NGC 2024, which is very near ζ Ori. For the absorption studies, the well defined end-point in the path eliminates the uncertainty on how much of a contribution may come from beyond, but since the optical depth is proportional to the column density N_H divided by the hydrogen's spin temperature T_s, we now are troubled by an uncertainty of a different nature, namely, the actual value of T_s. If we adopt the value of 125°K (Schmidt 1957), which has been a popular estimate in the past, the resulting column densities of 3.7×10^{21} atom cm^{-2} for the Orion Nebula and 1.1×10^{21} atom cm^{-2} for NGC 2024 are higher than the Lyman-α measurements for θ^1 Ori C and ζ Ori by factors of 7.7 and 8.5, respectively. Although we could again say that the radio data shows the presence of considerably more hydrogen than the Lyman-α measurements, we could equally well propose that T_s is more on the order of 15°K. In view of the observed brightness of 21-cm radiation over the sky, a 15°K T_s may seem unusually low, but close comparisons of 21-cm emission and absorption in certain directions have demonstrated that T_s may vary over a wide range (Clark 1965, Radhakrishnan and Murray 1969). It may not be too unreasonable to find lower than usual temperatures selectively located near the radio sources, which might be associated with dense HI clouds where strong cooling may occur. For the Orion Nebula we should be cautious about our conclusions from the comparison, since Clark (1965) has shown that the 21-cm absorption is produced by a small, opaque cloud located in front of the relatively large region of continuum emission, instead of a uniform, weaker absorption distributed evenly over the apparent area of the source. Thus it is conceivable that the line of sight to

θ^1 Ori C could be missing the main absorbing clump.

Returning to the 21-cm emission measurements, it should be clear that we cannot reconcile the disagreement with the Lyman-α data by raising the spin temperature, since the formula customarily used for deriving the column density,

$$N = 1.83 \times 10^{18} \text{ atom cm}^{-2}\ {}^\circ\text{K}^{-1}\ (\text{km s}^{-1}) \int T_b \, dv \qquad (9)$$

already assumes that $\tau \ll 1$, which is to say that the observed $T_b \ll T_s$. It follows that Equation (9) really represents a lower limit for the amount of hydrogen, regardless of the actual spin temperature. This assertion, however, does not take into account the possible existence of negative spin temperatures, which could be responsible for a strong maser emission at 21 cm. In a treatment of the evidence for high velocity clouds in the galactic corona, Shklovsky (1967) proposed that blue-shifted Lyman-α radiation from H II regions could, according to the theoretical criteria of Varshalovich (1967), lead to an optical pumping of the hydrogen. The consequent population inversion of the hyperfine levels could produce a 21-cm signal which was overrepresentative of the hydrogen actually present (using Equation 9). Fischel and Stecher (1967) have recognized this possibility might also apply to the emission from hydrogen near the stars in Orion, which could explain the difference between the Lyman-α and 21-cm data. The analyses of Van Bueren and Oort (1968) and Storer and Sciama (1968), on the other hand, point out that the Lyman-α radiation becomes rapidly thermalized to the local velocity field as it scatters through many optical depths, and the required frequency gradient in the radiation is lost for virtually all of the neutral hydrogen which is not immediately outside the boundary of the H II region. It therefore appears difficult for us to reason that a significant 21-cm contribution could come from a maser amplification in the interstellar medium.

5. Interstellar Gas Clumps

We have not yet discussed one particularly significant difference in the sampling geometry of the 21-cm and Lyman-α measurements: The radio antenna beam averages the brightness distribution over a finite solid angle, usually on the order of 1° in diameter, whereas the Lyman-α measurements subtend a virtually infinitesimal sampling area. One could expect therefore to find some differences in the two results if there were any substantial irregularity or bunching in the distribution of hydrogen over a scale much smaller than the radio beam size.

The possibility that this concept may be of importance is illustrated by some sample comparisons in Figure 3 of the velocity profiles for interstellar matter observed both in 21-cm emission and optical absorption in the visible. In each case we have a pair of observations which are closely analogous to the direct comparisons of 21-cm and Lyman-α results. The dashed lines show 21-cm emission as seen in a radio beam 22' in diameter centered on a star (Goldstein 1968). The solid lines depict the velocity profiles for each star's interstellar sodium D absorption line (Hobbs 1969) and in some cases data were available from Takakubo's (1963) study of the Ca II K line absorptions (dotted lines). Except for the K line observations, the velocity resolutions of the visual and radio determinations are comparable. Since it is difficult to assign a multiplying factor to normalize one observation to the other, the 21-cm T_b and the visual line τ values have been plotted logarithmically with an arbitrary vertical placement of one curve with respect to the other. The greater detail in the velocity structure of the D line absorption would suggest that the interstellar gas is not homogeneous over an apparent angular length corresponding to

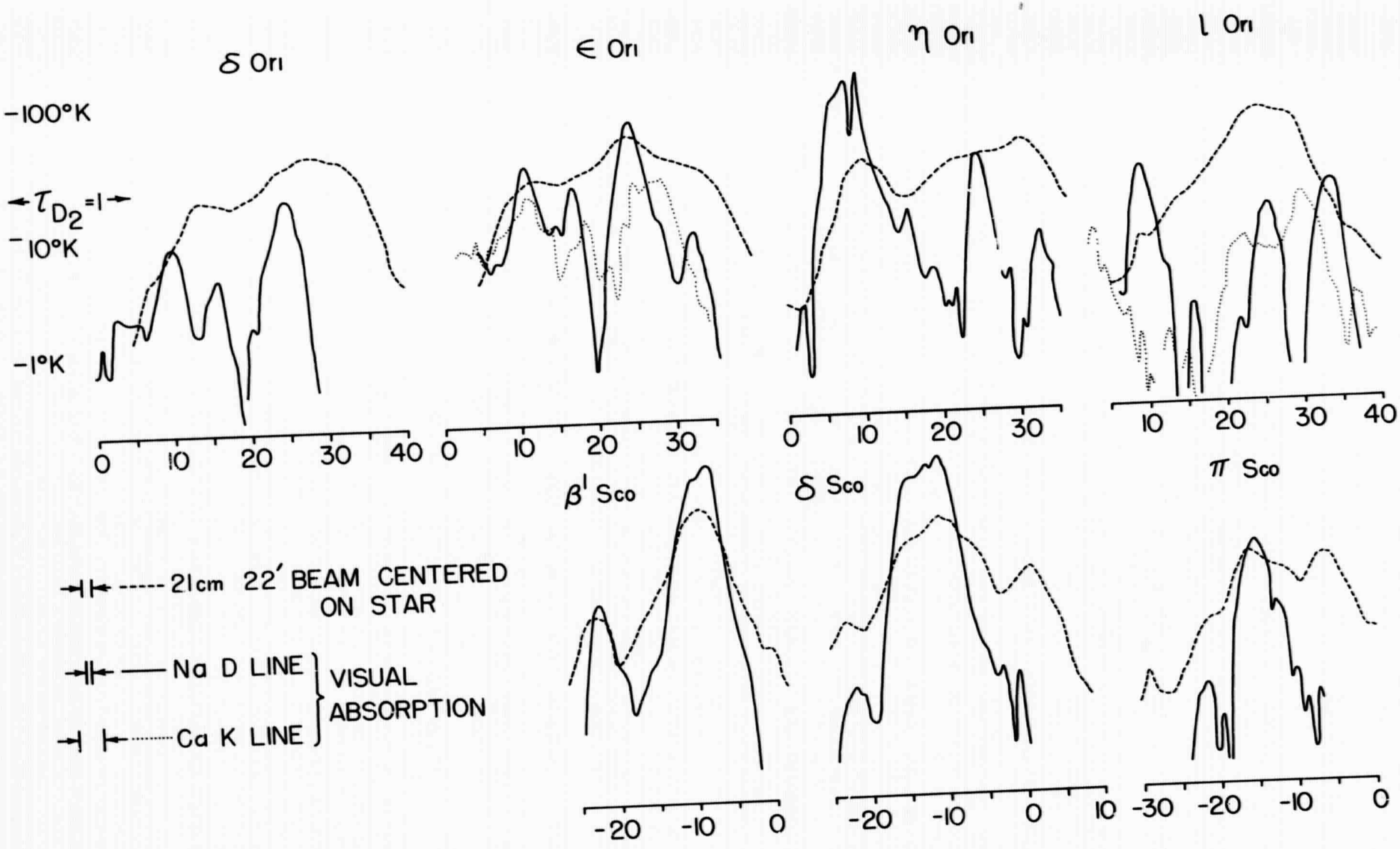

FIG. 3. A comparison of velocity profiles for optical line absorptions and 21-cm emission. Logarithms of the absorption optical depths and the 21-cm brightness temperatures are plotted against heliocentric velocities. The 21-cm, Na I D and Ca II K line observations are from the results of Goldstein (1968), Hobbs (1969) and Takakubo (1963), respectively, and their velocity resolutions are shown in the lower left corner.

the beam size. It is not difficult to imagine that the optical absorption path could cut through only a few small clouds which have narrow, well-defined velocities, while the 21-cm emission is observed to come from a large collection of neighboring clumps which together exhibit a blended spectrum of velocities. To be sure, this comparison may be further complicated by possibly large fluctuations in the relative sodium abundances or the ionization equilibria at various places. Without question, however, the contrast between these optical and radio observations demonstrates that it is still difficult to say with much conviction that one is seeing truly equivalent samples of interstellar gas in the two cases.

It may also be interesting to note that a simple interpretation of the observed galactic absorption of the soft, intergalactic X-ray background shows a deficiency of hydrogen similar to the Lyman-α results. From this, Bowyer and Field (1969) have on their own concluded that the material is unevenly distributed and that each clump is optically thick for the X-rays. The overall attenuation of X-rays would be therefore less than if the gas distribution were uniform.

With this encouragement, we might now ask whether we can invoke the existence of small-scale irregularities to explain the apparent deficiency at Lyman-α. What picture could we construct for an interstellar gas distribution which could satisfy both observations and yet remain a statistically reasonable explanation? The basic approach will be to presume that the voids between the concentrations occupy a large enough area in the sky to make all but a very few of the Lyman-α observations show an abundance significantly lower than the overall average hydrogen density. We could then suppose that up to now no observations have penetrated any of the very dense clumps which are responsible for most of the radio emission. There should be little doubt that the conclusions derived from the arguments do not constitute a sound observational premise on the actual properties of the medium, since far too few observations are at hand. It is better to say that the spirit of the analysis will be to gain a perspective on the minimum quantitative extremes which are necessary to make the explanation viable.

For a case study, we shall simplify the situation by focusing our attention on just one region of the sky. The Orion region is a particularly appropriate choice since, relatively speaking, it has been intensively studied by both the radio and rocket astronomers. There are a total of 8 stars in close proximity to each other which have been observed at Lyman-α . All of the stars have shown substantially less hydrogen than one would expect from the 21-cm emission in the same general direction observed by Menon (1958) and more extensively by Schwartz and Van Woerden (1967). Both of the 21-cm surveys agree that the column densities range from 1.5 to 2.5 x 10^{21} atom cm^{-2} which is about ten times the Lyman-α N_H. Again, we should consider how much of the hydrogen could be beyond the stars, but with $b^{II} \approx 18^{\circ}$ and d $\approx$ 460 pc (see Table 1), the stars should be approximately 140 pc off the plane of the galaxy. Thus it seems difficult to propose that roughly nine-tenths of the hydrogen is beyond the stars, since in our neighborhood of the galaxy the total thickness between the half-density points is about 220 pc (Schmidt 1957).

In all fairness, however, we should be aware that significant interstellar reddening does occur beyond the Orion association. The Orion stars have a B-V color excess on the order of 0.1, while out to 1 kpc in the same direction an additional reddening of around 0.1 and 0.2 has been observed (Fitzgerald 1968).

If the gas to dust ratio were constant, it might be reasonable to suppose that as much as one-half to two-thirds of the gas may lie beyond the stars. Shane and Wirtanen's (1967) galaxy counts also indicate that the Orion stars lie within a spur of absorption projecting out of the main plane of galactic obscuration. On the other hand, γ Ori at 95 pc is much closer than the other Orion stars, and yet it shows an average density n_H not much different from the others. This would suggest the hydrogen seen at Lyman-α is somewhat evenly distributed between here and the more distant stars and is not just concentrated around the association. In the analysis that follows, we shall assume that one-half of the 21-cm emission comes from beyond the stars. Even with this fairly liberal allowance, a substantial disagreement with the Lyman-α column densities is still evident.

Starting with a uniform distribution of hydrogen, we can envision the successively greater degrees of condensation which are shown in the sequence in Figure 4. Whether these condensations are actually filamentary or globular is of minor importance for this study; for simplicity in the analysis here, the hydrogen is assumed to be contained entirely within small spheres of uniform density n_c and radius $\underline{a}$ (this is a common practice in discussions on interstellar clouds). If we let the parameter x represent the average number of clouds penetrated along various random lines of sight, the probability of missing a cloud in one observation is e^{-x}. To have missed the clouds in all 8 observations, we obviously need to have x very small since the joint probability equals e^{-8x}. In a portrayal of the sizes and distributions of clouds, a specification for x is basically an expression of how extreme the clumping must be to satisfy the condition that there is now still an inadequate number of Lyman-α observations to give a representative sample of what hydrogen is really present.

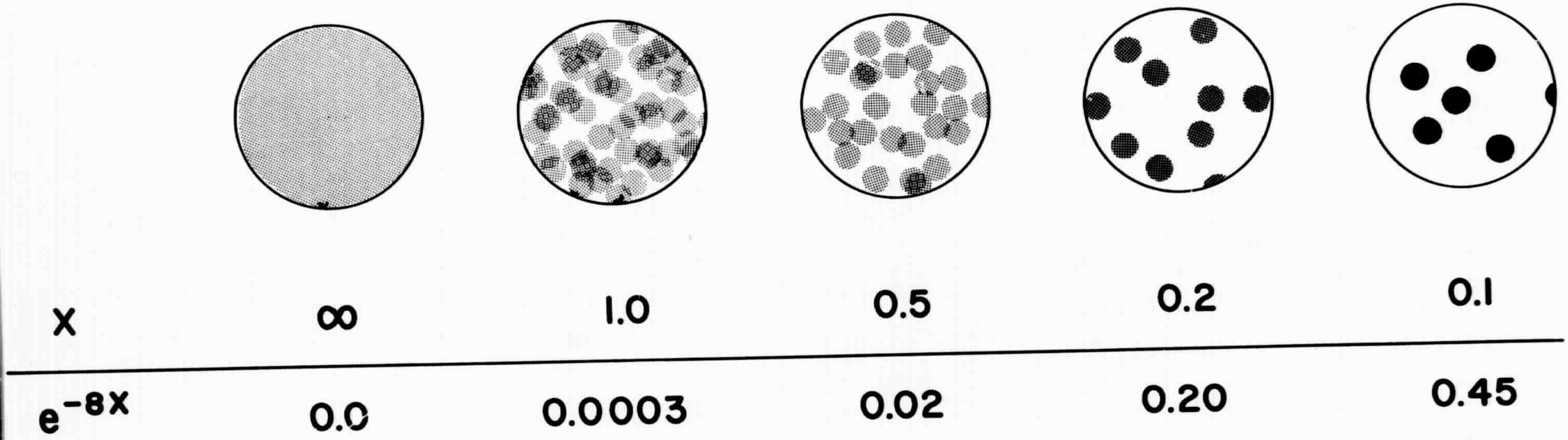

FIG. 4. An illustration of the appearances at 21-cm for hypothetical, small clouds within the solid angle of a radio beam. Each example represents a different value for x, but all cases would give the same brightness temperature.

We should next consider the constraints on the problem imposed by the radio observations and require that the clouds radiate a sufficient amount of energy within the radio beam to maintain the observed 21-cm flux. As the clouds become fewer, their 21-cm surface brightness must correspondingly increase to compensate for the absence of emission in the voids. It would normally follow that the average radiation would remain the same if the overall density of hydrogen did not change as the condensation into well-separated, dense clouds took place. However, the clouds soon become optically deep to 21-cm radiation if $T_s \approx 100^\circ K$, and naturally, self absorption prevents the surface brightness of a cloud from ever exceeding T_s. It should be clear that as x decreases, we must substantially raise T_s.

A rectangle 50°K high and 20 km s^{-1} wide is a simple but adequate approximation for the shape of the observed 21-cm radiation profile in the Orion region. A reduction of both the height T_b and width Δv by a factor of $\sqrt{2}$ allows for our estimate that one-half of the emission may come from beyond, and we obtain the profile shown in the corner of Figure 5, which will be adopted in the analysis that follows.

Within any of the radio beams illustrated in Figure 4, the expression

$$T_b = \sum_{I=0}^{\infty} P_x(I)\, T_s \left[1 - \exp\left(-I\sigma_c / k\, \Delta v\, T_s\right)\right] \tag{10}$$

is a reasonably accurate prediction for the average brightness temperature, where the Poisson distribution term $P_x(I)$ represents the fraction the beam's area covered by I clouds in line. Each cloud has a projected mean surface density specified by σ_c (which equals $4an_c/3$), and k is the constant shown in Equation (9). A straightforward evaluation of the sum gives

$$T_b = T_s\left(1 - \exp\left\{-x\left[1 - \exp\left(-\sigma_c / k\, \Delta v\, T_s\right)\right]\right\}\right) \tag{11}$$

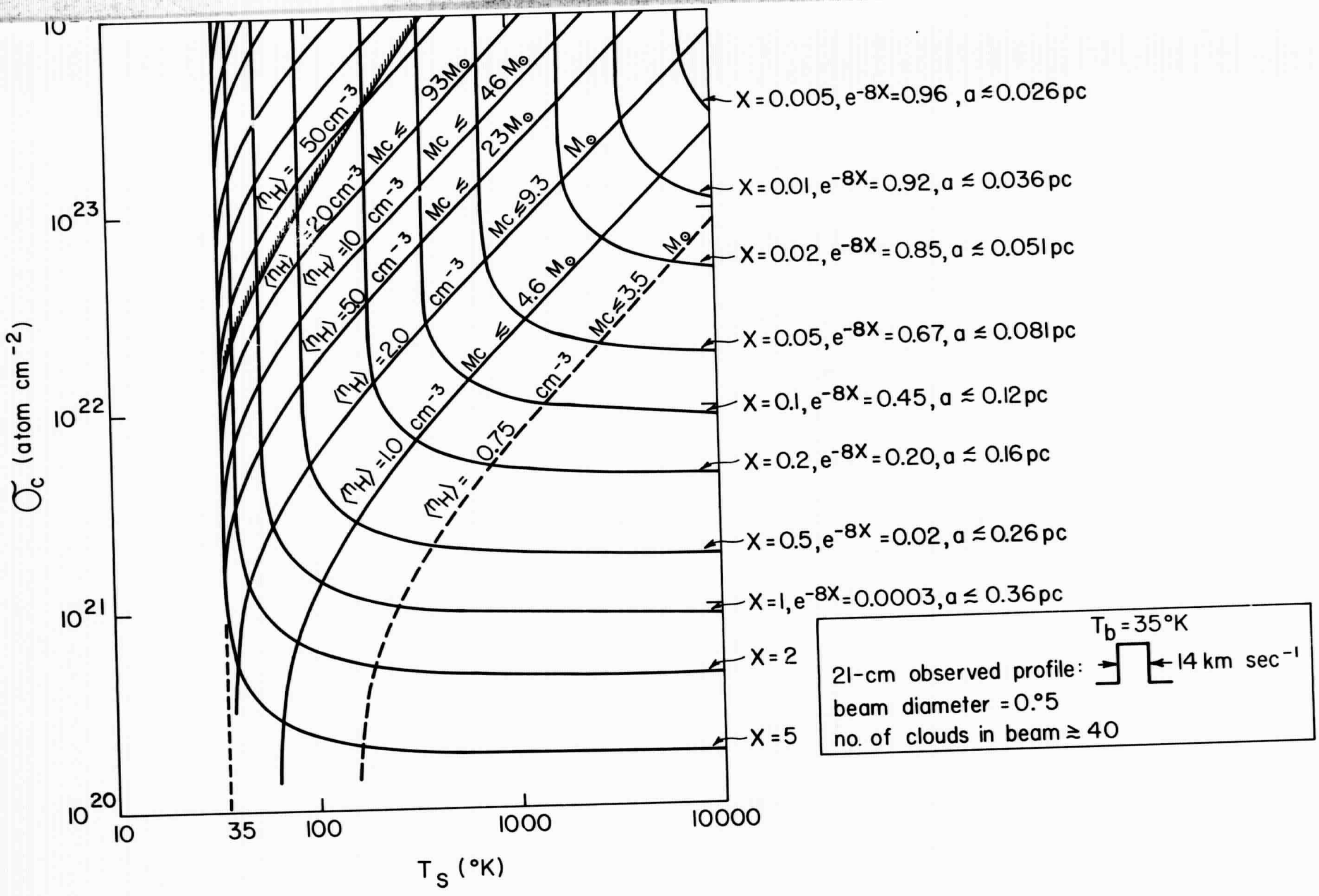

FIG. 5. Possible combinations of cloud parameters (see text) subject to the idealized 21-cm observations shown in the box. Gravitational binding for individual clouds is important only above the feathered line in the diagram.

Our main objective will be to try to arrive at a description of the clouds and their distribution which avoids any unreasonable values for the physical parameters x, σ_c and T_s, which we are free to vary. In so doing, it is helpful to graphically display these parameters and their behavior in Equation (11). If Equation (11) is rewritten in the form

$$\sigma_c = -k\,\Delta v\,T_s\,\ln\left[1 + \frac{\ln(1 - T_b/T_s)}{x}\right] \tag{12}$$

with $T_b = 35^\circ$K and $\Delta v = 14$ km s^{-1}, we obtain the family of curves for different values of x shown in the plot of log σ_c versus log T_s in Figure 5. The vertical portions of the x curves correspond to clouds which are quite opaque at 21 cm, where Equation (11) converges to

$$T_b = T_s\,(1 - e^{-x}) \tag{13}$$

as σ_c becomes much larger than k $\Delta v\ T_s$. The horizontal sections represent optically thin clouds where Equation (12) approaches

$$\sigma_c = k\,\Delta v\,T_b/x \tag{14}$$

when T_s is much greater than both T_b and T_b/x. In proposing a model, we wish to avoid unnecessarily large values for either σ_c or T_s, and hence the intermediate sections of the lines of constant x (where they curve sharply) are of most interest to us. We should, in fact, hesitate to consider values for T_s much greater than 1000°K, since in ~~this~~ the Orion region many of the observed velocity dispersions σ for the 21-cm Gaussian components are as small as 3 km s^{-1} (Van Woerden 1967).

The average volume density of hydrogen over all space (both clouds and voids) is given by

$$\langle n_H \rangle = \tfrac{4}{3}\pi a^3 R\, n_c \tag{15}$$

where R is the number of clouds per unit volume. If the stars are situated at a distance d, we may use x to eliminate R in Equation (15) and obtain

$$\langle n_H \rangle = \tfrac{4}{3}\, a x n_c / d = \sigma_c x / d \tag{16}$$

The diagonal lines in Figure 5 depict the different values for $\langle n_H \rangle$ with d = 460 pc. It is evident that over a wide range of x values, the average density does not differ much from 1 atom cm^{-3} in the region where the constant x lines have their lowest σ_c and T_s.

Even after we have decided upon a favorable location in the diagram, the quantities a and R remain undetermined. In principle we are unable to distinguish between a radio beam which sees many small clouds or just a few much larger clouds, either of which cover the same relative area in the sky. A very approximate lower limit for the number of clouds inside the beam may be found by examining how constant the 21-cm emission is for a number of adjacent, independent measurements. Within the limits $171^\circ < l^I < 179^\circ$ and $-18^\circ < b^I < -13^\circ$, 28 profiles exhibited by Schwarz and Van Woerden (1967) have an rms fluctuation in $\int T_b dv$ equal to one-ninth its mean value. This would appear to preclude our having less than about 81 clouds in the beam, since a smaller number of clouds would produce statistical fluctuations greater than the observed variation. Since we are assuming one-half of the hydrogen emission may have come from beyond the stars, there should be more than 40 clouds out to d = 460 pc within a beam whose diameter is approximately 0.01 radians. Assuming, at worst, all of the clouds were actually at the distance d, we may specify an upper limit for the clouds' radius

$$a \lesssim \frac{1}{2}\left(\frac{x}{40}\right)^{1/2} 0.01\, d = 0.36\, x^{1/2}\ \mathrm{pc} \tag{17}$$

which is tabulated alongside the x values in Figure 5. From Equations (16) and (17) we find each cloud's mass M_c should satisfy the relation

$$M_c = \frac{4\pi}{3} n_c a^3 = \pi d a^2 \langle n_H \rangle / x \lesssim (4.6\ M_\odot\ \mathrm{atom}^{-1}\,\mathrm{cm}^{3}) \langle n_H \rangle \tag{18}$$

The choice of a "most conservative" model is governed by one's willingness to tolerate the ever diminishing measures of plausibility, e^{-8x}, which are associated with increasing values of x. How large x can be, then, is to a large degree a matter of personal judgement. Let us say, however, that it would be reasonable to adopt a value of 0.1 for x, and along the curved portion of the x = 0.1 line we arrive at $\sigma_c = 1.5 \times 10^{22}$ atom cm^{-2} and $T_s = 600^\circ$K. Our desire for moderation might also favor the consideration of the a and M_c upper limits as actual equalities, which would lead us to 4.6 $M_\odot$ clouds which are 0.12 pc in diameter.

As we contemplate the existence of such clouds in interstellar space, we must face the difficult question of what holds the clouds together. An exceptionally large external pressure would be required to prevent the clouds from expanding, since $n_c T = 1.5 \times 10^7$ atom cm^{-3} $^\circ$K. Gravitational binding is relatively unimportant for regions in the log σ_c — log T_s diagram below the feathered line in the upper left corner, if we assume the kinetic temperature equals T_s throughout the cloud. We might also be hard pressed to explain what dynamical conditions might have led to this state for the interstellar matter. Unless some favorable theoretical conclusions or more compelling observational material can be presented to uphold the picture that neutral hydrogen can exist in the form we have just considered, it would appear that the main accomplishment of the foregoing analysis has been to reveal the inadequacy of the explanation that the lack of agreement between the 21-cm and Lyman-α observations in Orion is chiefly a result of the differing responses of the two sampling modes to a very inhomogeneous distribution of gas. Also, this example has illustrated how in the future one might investigate a possible repetition of the problem in Orion, when more Lyman-α data are available elsewhere in the sky.

6. Correlations of Dust and Hydrogen

Early comparisons of 21-cm fluxes with the optical absorption from dust grains have generally shown a positive correlation of the two for various regions of the sky, with a gas to dust density ratio on the order of 100 (Lilley 1955; Lambrecht and Schmidt 1957). Significant variations in the ratio have been observed, however, and the relative deficiency of 21-cm emission from regions rich in dust has been interpreted as indirect evidence that some of the hydrogen is in molecular, rather than atomic, form. (van de Hulst, et al. 1954, Heeschen 1955, Bok, et al. 1955, Varsavsky 1968). Several recent studies of specific areas having a very strong optical obscuration have shown the deficiency to be rather pronounced (Garzoli and Varsavsky 1966, Kerr and Garzoli 1968, Mészáros 1968). Such observations support the contention that a particularly favorable environment for the formation of H_2 could exist within dense dust clouds, where there is a strong attenuation of starlight which would normally heat the grains above a critical temperature for molecule formation (Knaap, et al. 1966, Solomon and Wickramasinghe 1969).

Although the correlations of the optical and radio data have shown convincing evidence for anomalies in the gas to dust ratio, they are subject to the same ambiguities arising from sampling differences that we have considered in connection with the Lyman-α and 21-cm data. On the other hand, a comparison of the Lyman-α equivalent widths and B-V color excesses for various stars could provide a more secure basis for comparison, since the path lengths and solid angles are precisely equal. In addition, it is not entirely clear how much of the 21-cm deficiency in the clouds could be attributable to lower temperatures inside, unless, of course, the 21-cm profile is measured in absorption (Gosachinskii 1966). Lyman-α measurements could provide a value for the atomic hydrogen density which is independent of temperature. Unfortunately, before probing the dense dust clouds, we must await the development of ultraviolet spectrographs which are considerably more sensitive than present day instrumentation, since absorption in the ultraviolet is far stronger than the extinction at visible wavelengths.

Figure 6 shows a plot of the color excesses and Lyman-α column densities for the stars listed in Table 1. In view of the fact that the Lyman-α data generally show less hydrogen than the 21-cm measurements, it is not surprising that the gas to dust ratio indicated by a best fit (with slope = 1) through the points in Figure 6 is lower than the previous estimates. However, the ratio here is less by only a factor of three, as opposed to the factor of seven between the Lyman-α and 21-cm local density values. This difference is probably a consequence of selecting stars which generally have a less than average reddening per unit distance. The most reddened stars observed so far, ζ Oph and θ^1 Ori C ($E_{B-V} = 0.3$), do not seem to show a significant departure from the overall trend. The other studies have shown, however, that somewhat more absorption is necessary before one can expect to notice a decrease in the ratio.

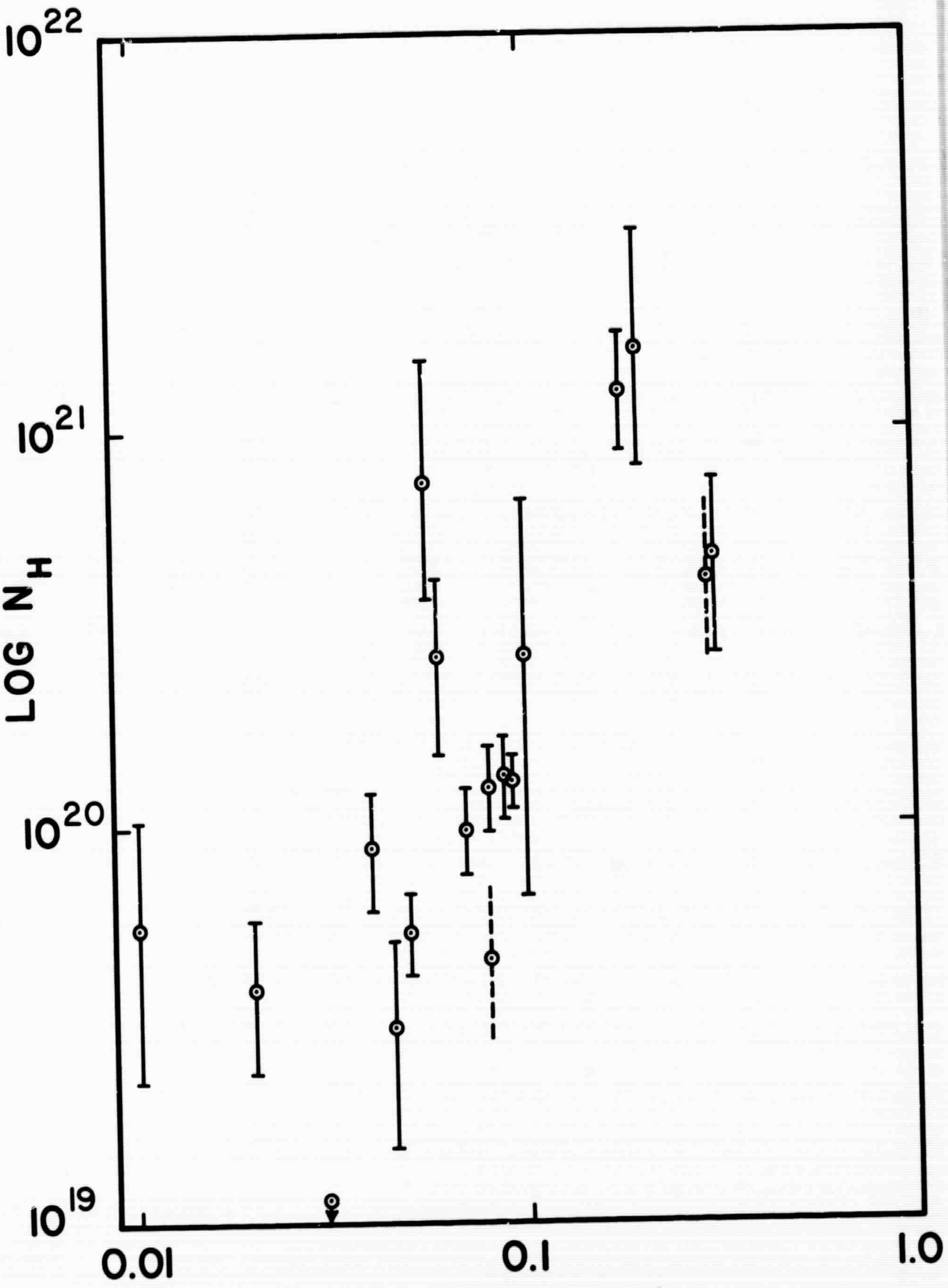

FIG. 6. A plot of hydrogen column densities (atom cm^{-2}) versus B-V color excesses for the stars listed in Table 1.

7. Conclusion

The most noteworthy aspect of the results on interstellar Lyman-α absorption is the apparent contrast with the extensive information on the distribution of hydrogen provided by surveys of line radiation at 21 cm. We have explored a number of possible effects, related either to the experimental method or to the interstellar medium, for explaining the much lower column densities deduced from the Lyman-α equivalent widths. A satisfactory explanation for the difference seems to be lacking, although we cannot deny the possibility that a combination of several of the previously considered factors may play a large enough role. It is also conceivable that the overall discrepancy may reflect the fact that the sun may be actually situated in a hole where the density of hydrogen is considerably less than normal for this region of the Galaxy. One might also argue that some unforseen selection effects play an important role when we observe hot O and B stars.

In the end, a better insight on the Lyman-α measurements will undoubtedly arise when more data are available. In some respects, a more diverse collection of observations is required before many meaningful correlations and generalizations can be made. For instance, it is not yet clear whether the four stars ζ Oph, π, δ and β^1 Sco on their own exhibit a large interstellar hydrogen density because (a) the resolutions were about 10 Å, as opposed to around 1 or 2 Å for all the other stars, (b) these stars are, on the whole, less luminous than many of the others, or (c) they are in a markedly different region of the sky. The answers to these and similar questions are not far from being realized, since a rapid accumulation of many more Lyman-α observations should be assured by our swiftly advancing technology in ultraviolet stellar spectroscopy.

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