OPTIMIZATION OF A LINEAR INDUCTIVE MHD MACHINE CONSIDERING THE LONGITUDINAL END EFFECT

ОПТИМИЗАЦИЯ ЛИНЕЙНОЙ ИНДУКЦИОННОЙ МГД МАШИНЫ С УЧЕТОМ ПРОДОЛЬНОГО КРАЕВОГО ЭФФЕКТА

Optimizatsiya Lineynoy Induktsionnoy MGD Mashiny s uchetom Prodol'nogo Krayevogo Effekta

by
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OPTIMIZATION OF A LINEAR INDUCTIVE MHD MACHINE CONSIDERING THE LONGITUDINAL END EFFECT

By means of utilizing the variation method, a distribution of the current load in a linear induction MHD-machine has been found in which the efficiency has a maximum value. Presented are the dependencies of the optimal efficiency on the relative length of the MHD-machine. The investigation was conducted both for the pumping and the generator regimes (operating conditions).

The theory of the longitudinal end effect allows for the possibility of creating this or that form of a magnetic field in the work (operating) space of the MHD stator by varying the current loads (Refs. 1 and 2). Usually, preference is given to the field in the form of a free-playing wave which is created by utilizing an analogous current load with various compensating devices. But, there has not been enough exact research verifying either one or another field distribution for operating conditions. It is well known that additional loses appear under such conditions that are related to current diffusion beyond the limits of the active part of the stator. These loses are especially essential for generator conditions of operation where, in agreement with Ref. 3, short MHD devices are more advantageous (in terms of polar division). For such generators it would be desireable to reduce the additional loses, conditioned by the longitudinal end effect, to a minimum. With this goal in mind in Ref. 3, we propose to utilize the additional compensating poles and to divide the operating channel, outside of the active zone, by isolating partitions in the longitudinal direction. In Ref. 3 we also indicate some experimental results that corroborate the effectiveness of the improvements. However, there are no sufficient theoretical bases for showing that these methods are the most effective. That is why this question needs to be further explained.

Let us look at a model of the MHD machine shown in Fig. 1 where 2L is the length of the active part of the device, V is the velocity of movement of the conductive body. Here are the assumed simplifications: the model is limitless laterally; the operating body is a reliable conductor and fills up the entire space; the magnetic field forms a complete closed circuit within the limit of the stator's active part ($|Z|<L$); the field is horizontally parallel (not depending on x). We presuppose the density of the longi-

tudinal current load ($j_0=I_0/\delta$) in the area $|Z|<L$, whose distribution we shall determine from maximum conditions of efficiency. The latter is determined for the pumping conditions by the expression

$$\eta_H = -\frac{\operatorname{Re}\left\{\int_{-L}^{L} jB^* v\,dz\right\}}{\operatorname{Re}\left\{\int_{-L}^{L} jE^* dz\right\}}. \tag{1}$$

For generator operating condition effeciency we obtain from

$$\eta_\Gamma = 1/\eta_H \tag{1'}$$

after the formal calculation (1). All values in (1) are scalar inso-far as they differ from zero by the components $j_y=j$, $B_x=B$, $E_y=E$, *and* $v_z=v$. The quantities in the numerator and denominator (1) are temporarily neutralized (dependency is on time j, B and $E\sim e^{i\omega t}$). Since j, B, and E depend on the longitudinal current load j_0, the dependency η_H and η_Γ from j_0 is functional. In order to determine j_0, leading to maximum efficiency, it is necessary to find variant η_H and η_Γ and to approximate zero. But a strong dependancy of j, B, and E on j_0 must be known to do this. We assume that $|j|<<|j_0|$, then we get the following by integrating Maxwell's equation

$$H = \int_{-L}^{z} j_0(z_1)\,dz_1 + C_H,\quad E = i\omega\mu_0 \int_{-L}^{z} H(z_1)\,dz_1 + C_E, \tag{2}$$

$$j = \sigma(E + vB).$$

C_H is determined by condition $\int_{-L}^{L} H dz = 0$:

$$C_H = -\frac{1}{2L}\int_{-L}^{L}\int_{-L}^{z_1} j_0(z_2)\,dz_2\,dz_1. \tag{3}$$

In the future we will differentiate two cases: a) with $|z|>L$ the channel is separated by isolating partitions in the longitudinal direction; b) the partitions are missing. In the first instance

$$\int_{-L}^{L} j\,dz = 0,\quad C_E = -\frac{i\omega\mu_0}{2L}\int_{-L}^{L}\int_{-L}^{z_1} H(z_2)\,dz_2\,dz_1, \tag{4a}$$

in the second

$$C_E = 0. \tag{4б}$$

We can rewrit[illegible] those elements entering the actual parts from the product of complex function in the following way:

$$\operatorname{Re}\left\{\int_{-L}^{L} jB^* v\,dz\right\} = \frac{1}{2}\int_{-L}^{L} v(jB^* + Bj^*)\,dz \tag{5}$$

and we can variate j_0 & j^*_0 independently. When this happens the equations for j^*_0 & j_0 become complexly linked. We get an equation for j_0, by varying j^*_0:

$$2v^2B + vE(1+\eta_H) + i\omega\int_{-L}^{z} [2\eta_H E(z_1) + vB(z_1)(1+\eta_H)]\,dz_1 = \text{const.} \tag{6}$$

The forms of equation (6) for (4a) & (4b) coincide. However, they contain quantities B & E; the latter is expresses through (2) which contains the constant C_E, differing for (4a) & (4b). From (6) it is easier to go to the differential equation for j_0. Utilizing $H'=j_0$ & $E'=i\omega B$, we must differentiate equation (6) according to z three times.

$$2v^2B'+i\omega v(1+\eta_H)B+i\omega[2\eta_H E+v(1+\eta_H)B]=0, \qquad (7)$$

$$2v^2j'_0+i\omega v(1+\eta_H)j_0+i\omega[2i\omega\eta_H H+v(1+\eta_H)H']=0, \qquad (8)$$

$$j''_0+\frac{i\omega}{v}(1+\eta_H)j'_0-\frac{\omega^2}{v^2}\eta_H j_0=0. \qquad (9)$$

The primes in (7), (8), & (9) refer to the differential process according to z. j_0 is determined from (9) while (6), (7), & (8) are needed to determine constant integration, - they act as limiting factors. Since E is differentiated only by a constant for (4a) & (4b) & (8) & (9) will serve as common in both cases.

The general solution (9)

$$j_0=C_1e^{-i\omega/v\,z}+C_2e^{-i\omega/v\,\eta_H z} \qquad (10)$$

combines two flat waves, moving in the direction of transference of the conducting body. The velocity of the movement of the first wave equals the velocity of the conductor's movement, while the velocity of the second one exceeds it by $(\eta_H\leqslant 1)$.).

To determine C_1, C_2 & η_H let us utilize condition (7), (8), & (1). For (4a) we get the following from (7) & (8)

$$\eta_H C_1\sin\Omega+C_2\sin\Omega\eta_H=0,$$
$$\eta_H^2C_1\sin\Omega+C_2\sin\Omega\eta_H=0, \qquad (11)$$

where

$$\Omega=\omega/vL. \qquad (12)$$

System (11) has solutions distinguished from zero given

$$\eta_H{}^n=\frac{\pi n}{\Omega},\quad C_1=0, \qquad (13)$$

$$j_0{}^n=C_2{}^ne^{-\frac{i\pi nz}{L}}, \qquad (14)$$

where n - is a whole positive quantity $(n\neq 0)$. In each solution (14) conditions (1) are executed identically.

The dependency η_H from parameter Ω is illustrated in Fig. 2 by a set of curves (1). Those parts of the curve, which correspond to the absolute optimum, are represented by a solid line. The other parts of the curves (1) correspond only to relative optimums & are indicated by a broken curve. We can observe a certain periodicity in the dependency of the optimum Ω from η_H, whose amplitude diminishes with the growth of Ω. When $\Omega=n\pi,\eta_H$ reaches its maximum value. In area $\Omega<\pi$ pumping conditions are impossible. In Fig. 2 there is a definite solution that corresponds to each part of curve 1. (14) i.e., there is a definite winding coefficient with a whole quantity of waves for length 2L.

Taking the formal correlation $\eta_r=1/\eta_H$, we may investigate the generator conditions of operation by an analogous method. In this way we get a set of straight lines rather than a set of hyperboles (13).

$$\eta_r=\frac{\Omega}{\pi n}. \qquad (15)$$

It is obvious that the generator conditions of operation are possible with all Ω; the same winding coefficient with a whole quantity of polar paris lead to the optimum.

For coefficients C_1, & C_2 we get the following equation for case (4b)

$$\eta_H^2 C_1 \sin\Omega + C_2 \sin\Omega\eta_H = 0,$$
$$\eta_H^2 C_1 \cos\Omega + C_2 \cos\Omega\eta_H = 0. \qquad (16)$$

For j_0 we have a set of solutions (17) where

$$j_0^n = C^n\left[e^{-i\omega/v\, z} - (-1)^n \eta_H^2 e^{-i\omega/v\, \eta_H^n z}\right], \qquad (17)$$

$$\eta_H^n = 1-\frac{\pi n}{\Omega}. \qquad (18)$$

The condition (1) for (17) becomes an identity because of the orthogonal character of the 1st and 2nd term in the interval $|z|\leqslant L$. The optimal η_H is attributed to the curve with the least n, i.e., when $n=1$ (curve 3 Fig. 2). In the entire area Ω the optimum η_H is less than in case (4a) (only at points $\Omega=2\pi, 3\pi$, . . . the curves 1 & 3 cross each other).

For generator conditions of operation

$$\eta_r^n=\frac{\Omega}{\Omega+\pi n}, \qquad (19)$$

while the optimum η_r is expressed by (19) when $n=1$ (curve 4 fig. 2).

When clearly running fields manifest as B, & E. in figure 2, η is greater both for generator and pumping conditions of operation. This is only possible when there are isolating dividers present (4a), when $|z|>L$ then $\mu=0$ & $\sigma=0$, and both B & E can take on any value at the end of the stator. Conditions $\int Bdz=0$ lead to the necessity of a whole quantity of waves along length 2L. In case (4b) it is necessary to satisfy conditions $E=0$ when $|z|=L$; otherwise, in the reverse situation the current will spill out in area $|z|>L$ at that the leakage will be considerable. Such conditions cannot be achieved by means of a freely running current. An altered current (the superimposition if two waves) presents a solution and leads to a reduction of efficiency. When Ω are great then for both situations we have an efficiency that approximates one.

In the above instance we assume $|j| \ll |j_0|$; however, it is not difficult to generalize for various instances. For inference (2) we must not use

$$\text{rot}\, H = j_0 , \tag{20}$$

but

$$\text{rot}\, H = j + j_0 \tag{20'}$$

or in (2) j_0 must be considered a summary density of current $j_0 = j + \tilde{j}_0$ and $\tilde{j}_0$ taken as the current density of the magnetic field.

Let us take the following to examine case (4a)

$$B = B_0 e^{-i\omega/v\, \eta_H z} . \tag{21}$$

From equation $\text{rot}\, E = -i\omega B$ we find

$$E_0 = -\frac{vB_0}{\eta_H} \tag{22}$$

and

$$j = -\sigma v \mu_0 \frac{(1-\eta_H)}{\eta_H} H_0 . \tag{23}$$

Now, (20) leads to

$$\tilde{j}_0 = \left[-i \frac{\omega}{v} \eta_H + \sigma v \mu_0 \frac{(1-\eta_H)}{\eta_H} \right] H_0 . \tag{24}$$

Analogous to (4b) we obtain

$$\tilde{j}_0 = -i \frac{\omega}{v} H_0 \left\{ e^{-i\omega/v\, z} + \left[(1-\pi/\Omega)^2 + \frac{i\varepsilon\pi}{\Omega} \right] e^{-i\omega/v(1-\pi/\Omega)z} \right\}, \tag{25}$$

where

$$\varepsilon = \frac{\omega \mu_0 \sigma \tau^2}{\pi^2} , \tag{26}$$

but, τ is the polar division determined by the first member (25) (i.e., $\omega/v = \pi/\tau$). If for case (4à) the distribution of the charge remained the same (only the amplitude and phase changed), for case (4b) the relationship between the 1st and 2nd time in (25) and (17) changes and with it the distribution of $\tilde{j}_0$.

In fig. 3a we have the instant values j_0 (curve 1) and $\tilde{j}_0$ (curve 2) when $\Omega = 2\pi$, $\varepsilon = 1$ & $\omega t = 0$,; fig.3 b shows the same given $\omega t = \pi/2$. Fig. 3a also indicates the distribution of the effective value of j_0 along the stators length (curve 3). Evidently, the distribution of a linear current load must possess a certain assymmetry with great current concentration at the inlet. This assymmetry is conditioned by the fact that the field is carried along by the moving conductor, while the summary current must be symmetrical.

To clarify the expediency in utilizing a winding coefficient of type (17), the following claculation was made for the rate of comparison. We assumed that the 2nd time in (17) was absent and $\varepsilon = 1$ (for the quantity of polar pairs $p = 1$ ---curve 5, but $p = 2$ ---curve 6 - Fig. 2). The corresponding curves are indicated by the dotted line when $\varepsilon \ll 1$, or when $|j| \ll |j_0|$.

The last case was first explored in Ref. 5, and, evidently, the maximal one does not exceed 18% independent from the quantity of polar pairs efficiency. If $\varepsilon \sim 1$ we can achieve greater efficiency, but they would be far from the optimal given Ω. Besides, with utilization of (17), we investigated the dependence $\eta(\varepsilon)$ for fixated Ω (Eq. 25 was not considered). Fig. 4 shows dependence $\eta(\varepsilon)$ when $\Omega=2\pi$ (curve 1) and $\Omega=3\pi$ (curve 2). The dotted line shows the optimum value, which is given by η winding coefficients (25) and does not depend on ε. As we can see from these curves, the calculation (25) improves the efficiency significantly when the end effect is ε.

Even though above we only considered joule-effect loses in the working body and calculation of other loses (loses in the sheath, wall, etc.) could change the obtained correspondences, nevertheless, the results can be applied in the construction of various types of MHD devices.

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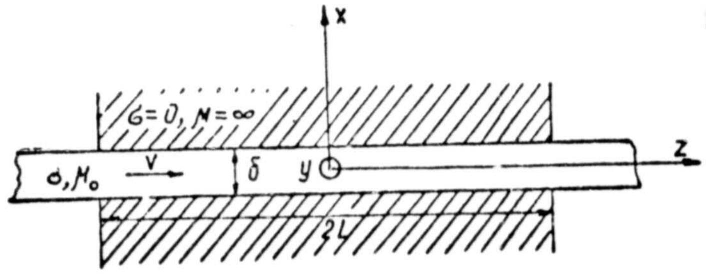
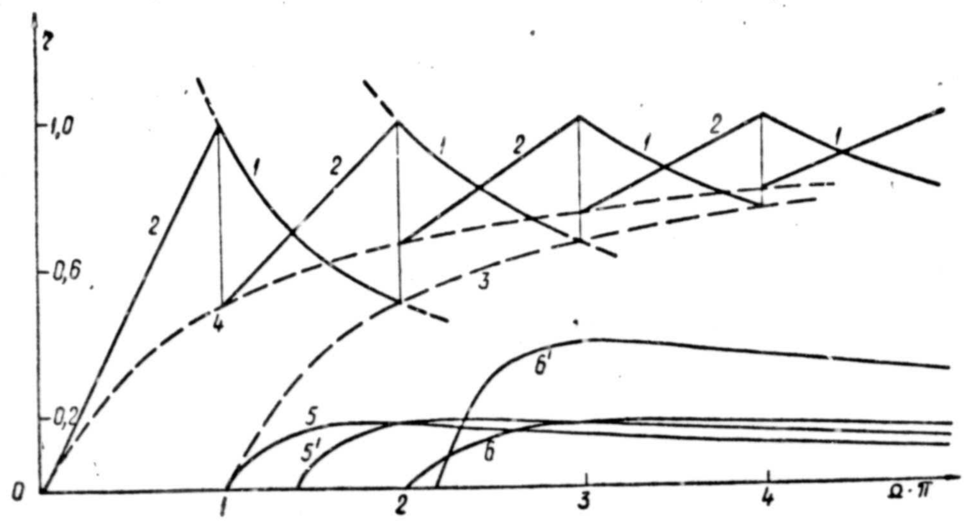


Рис. 1.



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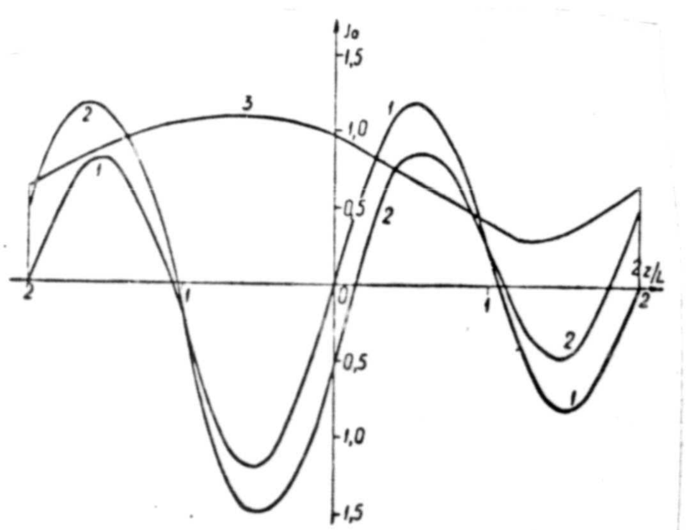

Рис. 3а.

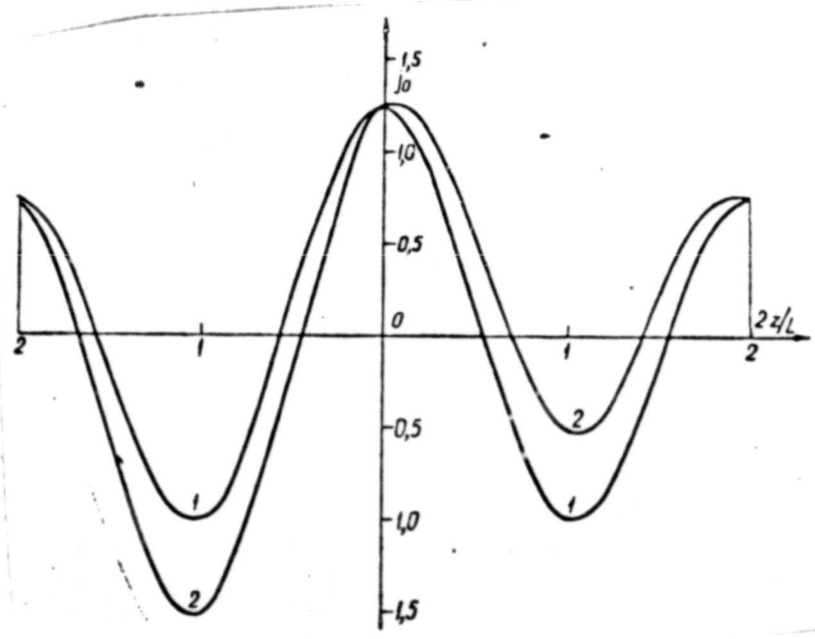

Рис. 3б.

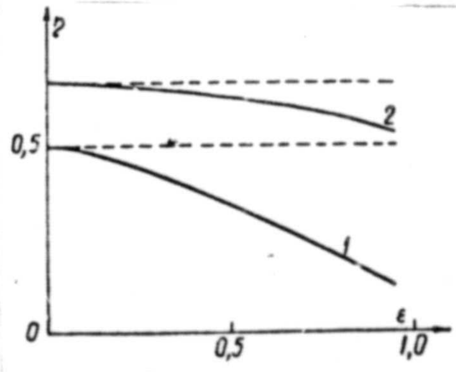

Рис. 4.