NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Memorandum 33-431

Volume I

FEDGE—A General-Purpose Computer Program for Finite Element Data Generation

User's Manual

F. A. Akyuz

JET PROPULSION LABORATORY

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Preface

The work described in this report was performed by the Engineering Mechanics Division of the Jet Propulsion Laboratory.

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FEDGE—A General-Purpose Computer Program for Finite Element Data Generation

User's Manual

I. Introduction

A general-purpose computer program—for the generation of natural coordinate systems (Ref. 1) and the preparation of input data for finite-element analysis—is described in two volumes.

Volume I contains preliminary definitions and step-by-step instructions for the use of the FEDGE program. Although they have many basic, distinct definitions and properties as far as input procedure is concerned, the two- and three-dimensional problems are presented in a unified form. This form of presentation not only facilitated the composition of this report, but offers for the users integrity of the definitions for two- and three-dimensional problems. Although the general input procedure has been presented by choosing two specific examples from the practice, the users, during their examination of the examples, must try to recognize the potential extension and versatility that FEDGE offers for geometrical configurations of any degree of complexity. Volume I is a tool for the user whose concern is solely the generation of data for his problem.

Volume II contains information concerning the structure of the program (with summarized flow charts) and a complete listing of the program.

Volumes I and II, together with Ref. 1, comprise a complete set of information for those who wish to adopt or modify the existing program or prepare their own program for their specific field of application. The application area ranges from the finite-element analysis of structures to finite-difference solution of any physical problem, or any professional field wherein the description of a domain with a known boundary is required by the use of a natural coordinate system concept. The need for repeated use of such a description increases the efficiency of the concept and of the program itself. The physical program FEDGE can be obtained from COSMIC,* a NASA agency for the distribution of the computer programs.

*Computer Software Management and Information Center, Computer Center, University of Georgia, Athens, Georgia 30601; telephone (404) 452-3265.

II. Definition of the Problem

A. Topological Definitions and Classifications

The finite-element method is one of the most versatile tools for solving engineering analysis problems; it eliminates the difficulties arising from the complex geometrical definitions of the solution domain in one-, two-, or three-dimensional space. The only restriction to the geometrical form of the physical model to be treated by the finite-element method is that it must be representable by simple, open, bounded, smooth lines and surfaces (and combinations of them). Topologically, this class of figures is defined as developable in plane into squares or circles, and in three-dimensional space into cubes or spheres by continuous deformations (Ref. 2). Furthermore, a two- or three-dimensional closed domain might be either concave or convex. The use of the FEDGE program requires the definition of a nearly convex domain; that is, a domain that resembles a convex domain but might have practically insignificant concave portions on its boundary (Fig. 1). Any concave domain can be subdivided into a number of nearly convex domains, and represented as a collection of them.

With respect to its use in the finite-element analysis and the FEDGE program, a line, surface, or volume can be classified as:

(1) Real when it represents a physical material, such as a line representing a frame element or surface representing a shell.

(2) A boundary when it separates two regions, such as the boundary of solution domain and the outer space.

(3) Fictitious when it does not represent anything, as described in (1) and (2), but is used to define the connectivity between real or boundary lines, surfaces, and volumes.

In this report, solid lines and solid, crosshatched surfaces will be used consistently to illustrate real material; thin lines and light, crosshatched surfaces will represent boundary; and dotted lines and light, simple, hatched surfaces will show fictitious lines and surfaces.

B. Model Representation of a Physical Problem

Figure 2 illustrates the axisymmetrical representation of a spherical, solid-propellant rocket motor. The circular line *ABCD* represents the external spherical shell; therefore, it is a *real* line, and is represented by a thick, solid line. The grain represented by the crosshatched

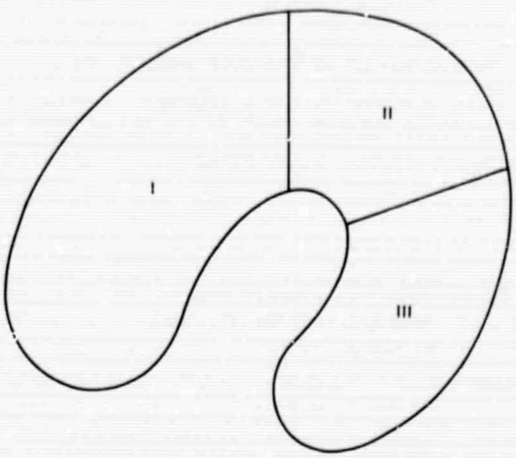


Fig. 1. Concave domain subdivided into three nearly convex domains

region is bounded by the lines *BE* and *EC* and the shell *BC*. This is a real surface, and lines *BEC* and *BC* are boundary lines of the grain. The location of the symmetry axis and the overall coordinate system, as well as the equation of the lines, are sufficient for complete description of the model.

A model representation of a cylindrical shell is illustrated in Fig. 3. The shell is supported and open at both ends D_1 and D_3 and divided into two subdomains at the middle by D_2. The overall coordinate system is chosen as shown, and the geometry of the model shell is defined by the equation of the cylinder defining the shell and the plane defining the support condition in the overall coordinate system. The planes D_1 D_2 D_3 are fictitious; the shell itself is a real surface; the volumes bounded by D_1 D_2 D_3 and the shell are fictitious domains.

The objective of the FEDGE program is to generate the meshes that would represent the shell *ABCD* and the grain *BEC* in Fig. 2, and the mesh and the boundary conditions for the cylindrical shell of Fig. 3, which could be used for finite-element analysis of these or other models. It should be noted that, by suitably changing intrinsic properties corresponding to the problem, one can obtain the input information for either stress-analysis or heat-transfer problems. It is assumed that the real lines and boundary lines are completely defined in the overall coordinate system x_1, x_2, x_3. Furthermore, the interest of the analyst might be focused on some specific point, which is called a *point of interest*. Such points of interest are hypothetically chosen in the example considered herein. Around this point, some degree of refinement of the mesh will improve the convergence characteristics

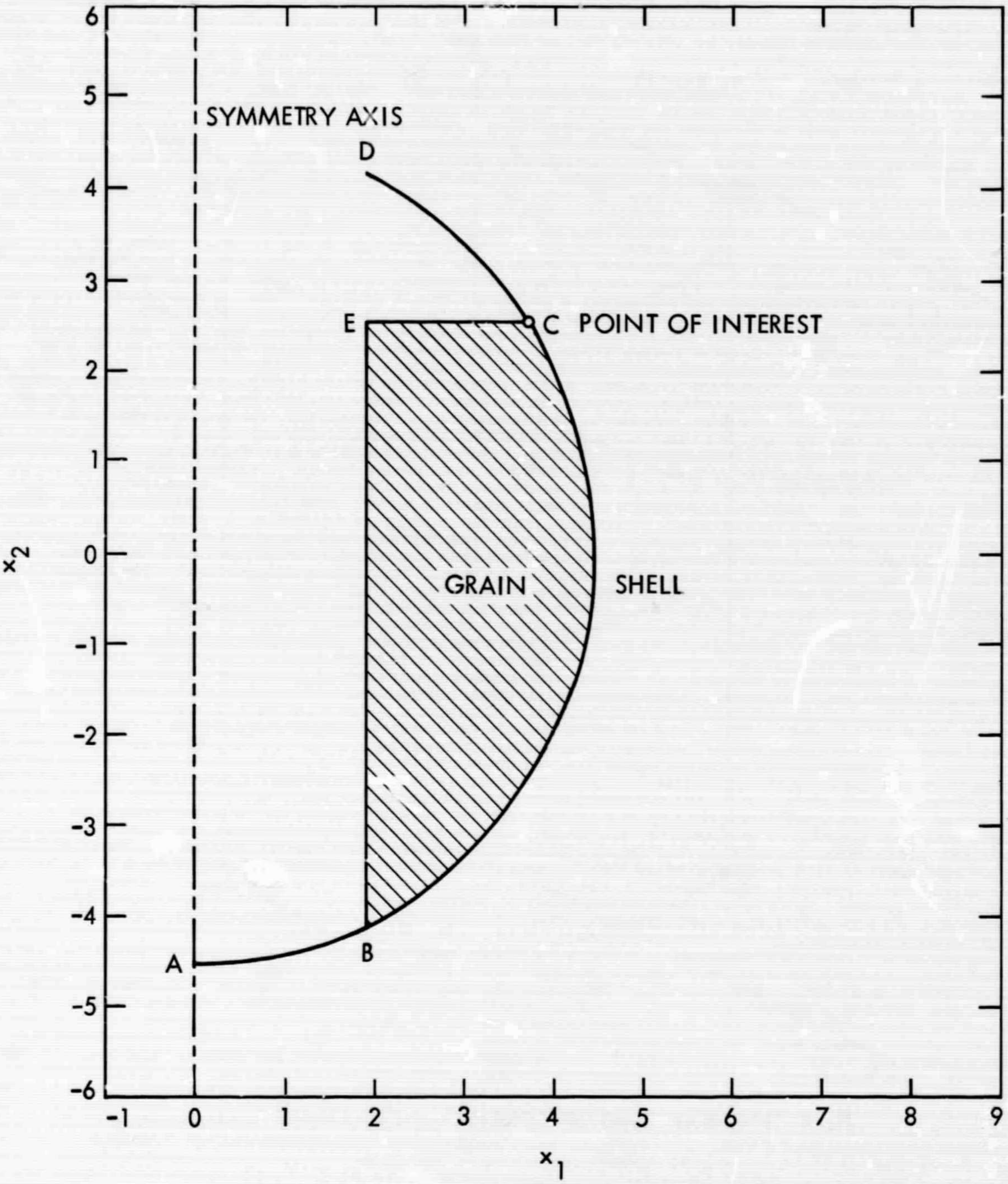


Fig. 2. Axisymmetrical model of spherical solid-propellant rocket motor

of the finite-element method; therefore, the accuracy of the results will also be improved.

Although the information as described above for Fig. 2 or 3 is completely sufficient for the definition of the problem, some additional work is required to translate the topological characteristics of the model into a form acceptable by FEDGE. Once the model representation is set up, the preparation of the input for FEDGE follows the rules described below, separately, for two- and three-dimensional examples. As a definite rule, the collection of closed, bounded subdomains, interconnected with each other in two- and three-dimensional space, is the only possible form of model setup for input to

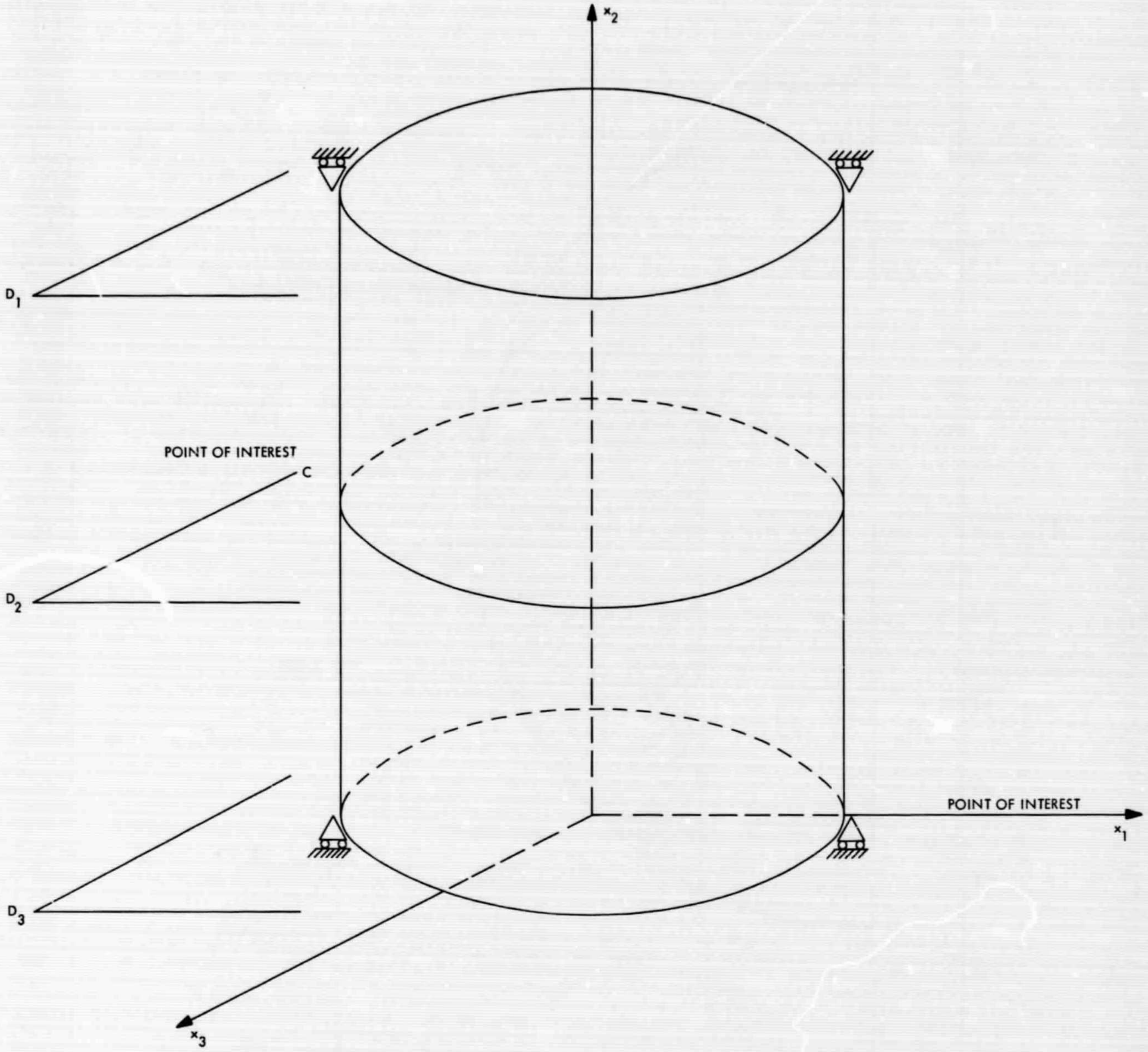


Fig. 3. Three-dimensional cylindrical open-end shell model representation

FEDGE. A real line in two-dimensional space, or real surface in three-dimensional space, must be a portion of a subdomain. (Besides those lines and surfaces, additional lines and surfaces might be used as additional information.) Subdomains that constitute a suitable input tool for scattered lines or surfaces are distinguished from the real subdomains by additional information. Furthermore, for each subdomain, the boundary must be divided into regions. In this work, these regions are called *faces* in analogy with the faces of regular geometrical figures, such as squares and cubes, in two- and three-dimensional space. From the above consideration, it follows that any model has its equivalent collection of squares and cubes in two- and three-dimensional space.

Partition of the boundary into faces is, in general, arbitrary and can be used judiciously to obtain versatile mesh forms, whereas the regular analog, composed of a collection of squares or cubes, might (in general) be unique.

C. Setup for FEDGE Input for Two-Dimensional Problems

Figures 4a and 4b illustrate the topological analog model and the setup itself, respectively, for input preparation of the two-dimensional model illustrated in Fig. 2. A few additional explanations will clarify the logic steps required between the model representation and the setup for input. After careful investigation of both Figs. 4a and 4b, it can easily be concluded that, as far as the topological properties of the components of the two figures and labelling systems are concerned, they are completely equivalent; and the following can easily be observed:

(1) The real lines *ABFCD*.

(2) Boundary lines *BE* and *EC*.

(3) Fictitious lines *AG, GE, EH,* and *HD*.

(4) Fictitious domains II and III.

(5) The *I* and *J* coordinate lines (Ref. 1).

(6) Direction of positive normal to the lines, and the labels in both Figs. 4a and 4b.

The lines 10 at point *F* and 1 at point *D* are introduced to define the corner of the domain on a continuous line in Fig. 4b.

From this simple example, the degree of geometrical complexity that can be reached and input into FEDGE can be imagined easily by adding more and more real or fictitious domains (squares in Fig. 4a or any geometrical closed form as shown in Fig. 4b). The compatibility of faces between the faces of squares must be observed after the *I,J* coordinate system has been determined. The origin (in this example, the point *E*) is the only point whose coordinates must be known for input purposes if all other lines, fictitious or real, are defined by their equations. The additional arrows and signs of the line labels are explained later, together with labelling rules of the lines and surfaces, following the brief exposition of the logic and structure of the automation of the problem.

D. Setup for FEDGE Input for Three-Dimensional Problems

Figures 5a and 5b illustrate the topological analog model and the setup, respectively, for input preparation of the three-dimensional cylindrical shell problem

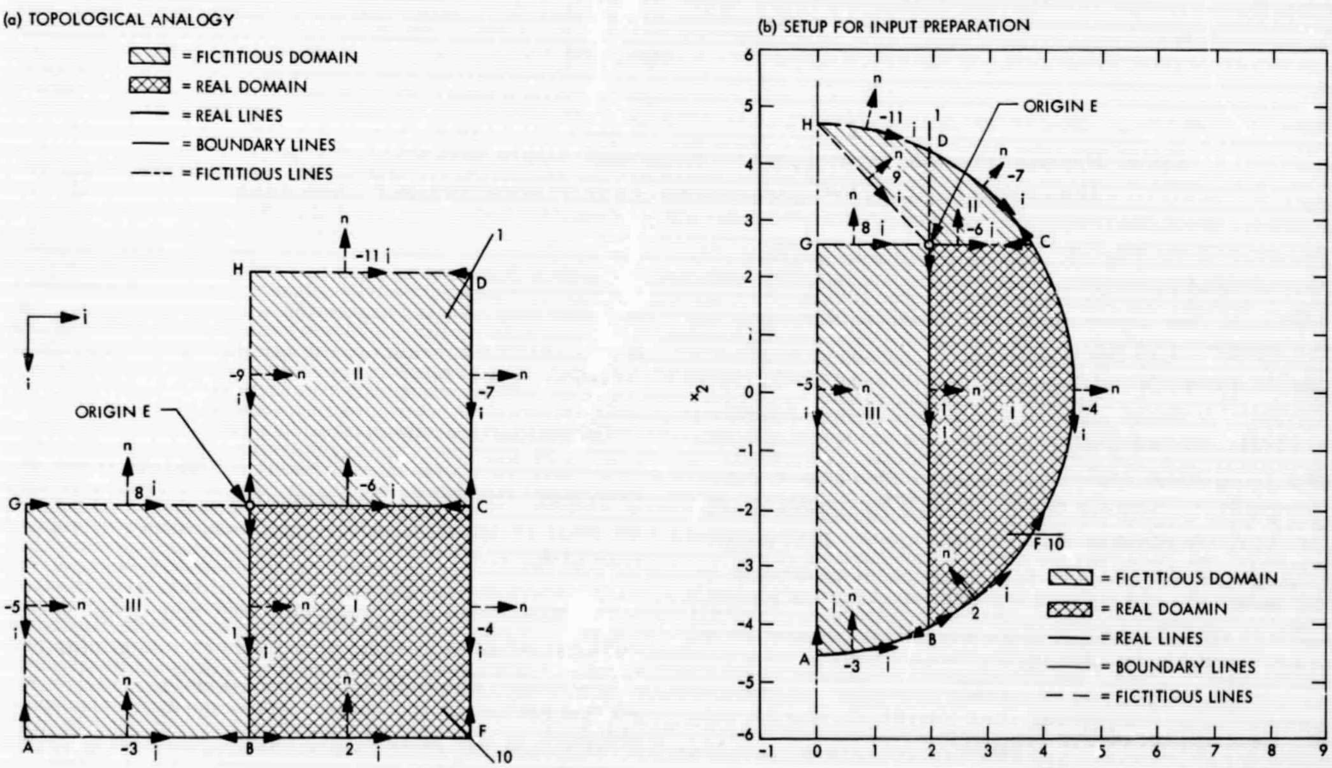


Fig. 4. General schematic input model for two-dimensional problem with application to example of Fig. 2: (a) topological analogy; (b) setup for input preparation

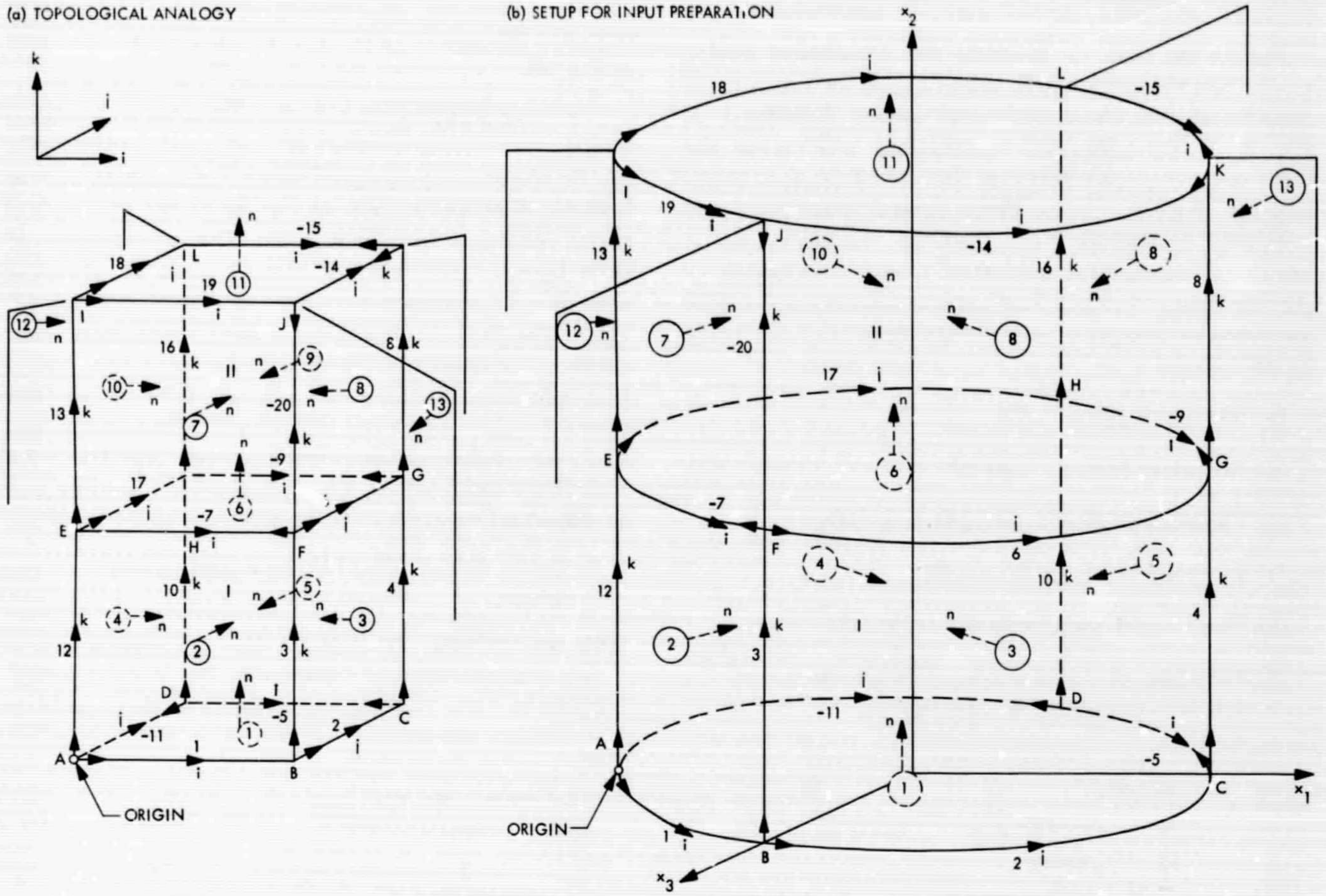


Fig. 5. General schematic input model for three-dimensional shell problem with application to example of Fig. 3: (a) topological analogy; (b) setup for input preparation

illustrated in Fig. 3. The topological analogy is obvious, and one can easily follow the numbers and the letters on the cubes and cylindrical subdomains, on the surfaces and lines, and on the corners, which explicitly show the analogy. As in the case of the two-dimensional model, cubes can be added in three directions to obtain a topologically simple analogy between complex geometrical configurations and collections of cubes. Compatibility between the faces must be observed after the I,J,K coordinates have been chosen. The positive direction of the surfaces can be selected arbitrarily by the users. The origin and the equations of all labelled surfaces must be defined in the overall coordinate system x_1,x_2,x_3 as shown in Fig. 5b.

III. Description of the Program

A. Method of Solution

The solution starts by integration along the lines in two- and three-dimensional problems (see Ref. 1). The origin (the point at which the integration starts) must be defined as input. The length of each line face is measured, and the natural coordinate system is determined along each face. In this program, the number of divisions along each line face is fixed equal to 16 (Fig. 6).

The natural coordinate system or the surface faces are generated by using the division established along the line faces (see Ref. 1). The mesh on any surface face will be 16×16 quadrilaterals, which constitutes a special case of the natural-coordinate-system concept.

In three-dimensional problems, the generation of the mesh on the faces is done on a plane, which is obtained by defining a plane normal to the component of the all-normal vectors of the surface along its face. The mesh obtained on this plane is projected to the surface. This procedure violates the application of the natural-coordinate system on the surface faces, but it is a very close approximation for shallow surfaces, and provides an extremely fast automation scheme.

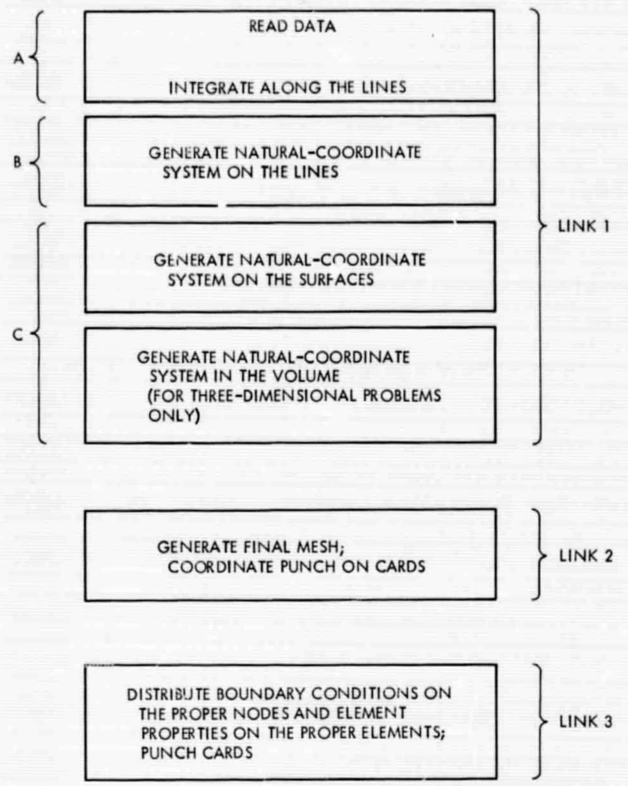


Fig. 6. General flow diagram for computer program

In three-dimensional subdomains, the natural-coordinate system is obtained by an interpolation scheme that generates each mesh nodal point through a systematic interpolation procedure between the corresponding points on the opposite faces.

All procedures described above are automatically done by the program after the necessary information concerning the lines, surfaces, and volumes, as well as their topological relationships, have been provided by the user. In two-dimensional space, a line can be defined either by its equation, which must be linear or quadratic, or point by point. In three-dimensional space, the lines are defined by the intersection of two planes or two quadratic surfaces or by a plane and quadratic surface. A line face in two-dimensional space can be defined by more than one type of line, and a surface face in three dimensions can be defined by more than one type of surface.

In the second link (see Fig. 6), the mesh is generated and the nodal points are generated in the sequence K, J, I (i.e., K varies first, then J, then I). The equivalent FORTRAN input/output (I/O) statement can be written as

$$\{[(K = 1, NZ), J = 1, NY], I = 1, NX\} \tag{1}$$

For the form of the mesh, the users have three options:

(1) The number of divisions in the three directions of the natural-coordinate system for the first subdomain (the number of divisions in the other subdomains is determined by the program, with the resulting overall mesh depending upon the next two additional options of the users).

(2) The location of a special point in the first subdomain around which the users might want a refined mesh.

(3) The degree of refinement, which is defined by the ratio of the two successive division intervals, with the intervals being measured in the sense of the number of natural-coordinate-system divisions in the interval.

The labelling of the nodes starts from the first subdomain. Fictitious lines, surfaces, and volumes are not labelled, and do not show in the sequence. The points are labelled the first time they appear in a subdomain, and are processed in the program sequentially as they appear in the input. The user either judiciously labels the subdomains and chooses natural-coordinate systems to obtain the minimum bandwidth in the complex configurations, or uses the relabelling scheme of Ref. 3 to reduce the bandwidth for any complex configuration. The resulting coordinates of the mesh nodal points are punched on the cards.

In link 3, the boundary conditions and element data are generated. In two- or three-dimensional space, the boundary conditions are assigned by the users along the complete line face or surface face, respectively. Only linear relations between various degrees of freedom at a point can be taken into account. The boundary conditions are distributed to the faces in the sequence as the corresponding subdomains and associated faces appear in the input. The boundary information is punched in the form ready for use in the program of Ref. 4.

The element-data information is distributed to all real elements. The subdomains are treated sequentially as they are assigned in the input, and the sequence number of the elements follows rules similar to the sequencing of the nodal point numbers. The face elements are treated first (if they are real) in a subdomain in the sequence, as they appear in the corresponding connectivity information of the input, related to the faces of the subdomain. The elements in the subdomain are labelled if the subdomain is real.

The typical numbers corresponding to the intrinsic properties of the faces or subdomains in which the elements lie are punched ready for use in the program of Ref. 4.

B. Capabilities and Restrictions of FEDGE Program

The FEDGE program permits users to generate input data for their finite-element analysis problem, where the geometrical configuration arising from the model representation of the physical object offers any degree of complexity. The objectives of the program are to provide the users with complete sets of coordinates and mesh topology and boundary condition information by automatically generating a mesh configuration as it is required in the finite-element analysis. The lines and information types associated with various regions are the only necessary input information for FEDGE. In two-dimensional space, the lines can either be straight or quadratic lines, or can be prescribed point by point. In three-dimensional space, the faces are planes or quadratic surfaces. No practically defined model exists that cannot be handled by the program (see Refs. 1 and 2 for exceptional geometrical configurations). The computer time and engineers' hand labor increase with increasing complexity of the geometrical configurations. Typical numbers corresponding to the limited capacity of the program are listed in Table 1. These numbers are based upon the storage capacity of 32K 36-bit word-capacity computers and upon the practical ranges encountered in engineering applications. With its actual form, the program can accommodate any finite-element analysis problem of continuous complex structures of any size within practically feasible, computer-time expenditures. However, if it is desirable to handle problems beyond the capability of this program (e.g., by the application of larger and faster machines), the entire program should be reviewed for minor changes.

Table 1. Typical maximum and minimum numbers of items that can be handled by FEDGE

Description	Maximum or minimum number
Subdomains in two- or three-dimensional space (maximum number)	50
Surface units in three-dimensional space (maximum number)	100
Line units in two-dimensional space (maximum number)	200
Boundary condition types (maximum number)	50
Boundary condition units that appear in a linear relation in a point (maximum number)	8
Nodal points (maximum number)	8000
Elements (maximum number)	8000
Number of nodal points in an input unit in two-dimensional space (maximum)	5
Number of surface units in a surface face in three dimensions[a]	2

[a]The surface interfaces must cross two opposite line faces.

C. Programming Language and Operational System

The computer program has been developed for the 32K IBM 7094/7044 direct-coupled system; however, FEDGE may be used in other systems that have the FORTRAN II compiler and FAP assembler. The program uses the following tape units during execution:

(1) FORTRAN Unit 1, system.

(2) FORTRAN Unit 2, chain.

(3) FORTRAN Unit 3, scratch.

(4) FORTRAN Unit 4, scratch.

(5) FORTRAN Unit 5, input.

(6) FORTRAN Unit 6, output.

(7) FORTRAN Unit 7, punch.

(8) FORTRAN Unit 8, scratch.

(9) FORTRAN Unit 9, scratch.

(10) FORTRAN Unit 10, scratch.

D. Description of Deck Arrangement

The FEDGE program consists of three chain links. The deck arrangement is shown in Fig. 7. For each model problem, there must be one complete data set and any number of additional data sets for the same model problem if various types of meshes are requested. In the generation of the additional meshes, the users take advantage of the fact that, *once the natural coordinate systems are stored* (the generation of which takes the major portion of the machine time), *the computer time for the generation of the additional meshes is very short.*

The source deck consists of approximately 5000 cards, and the binary deck contains approximately 1000 cards.

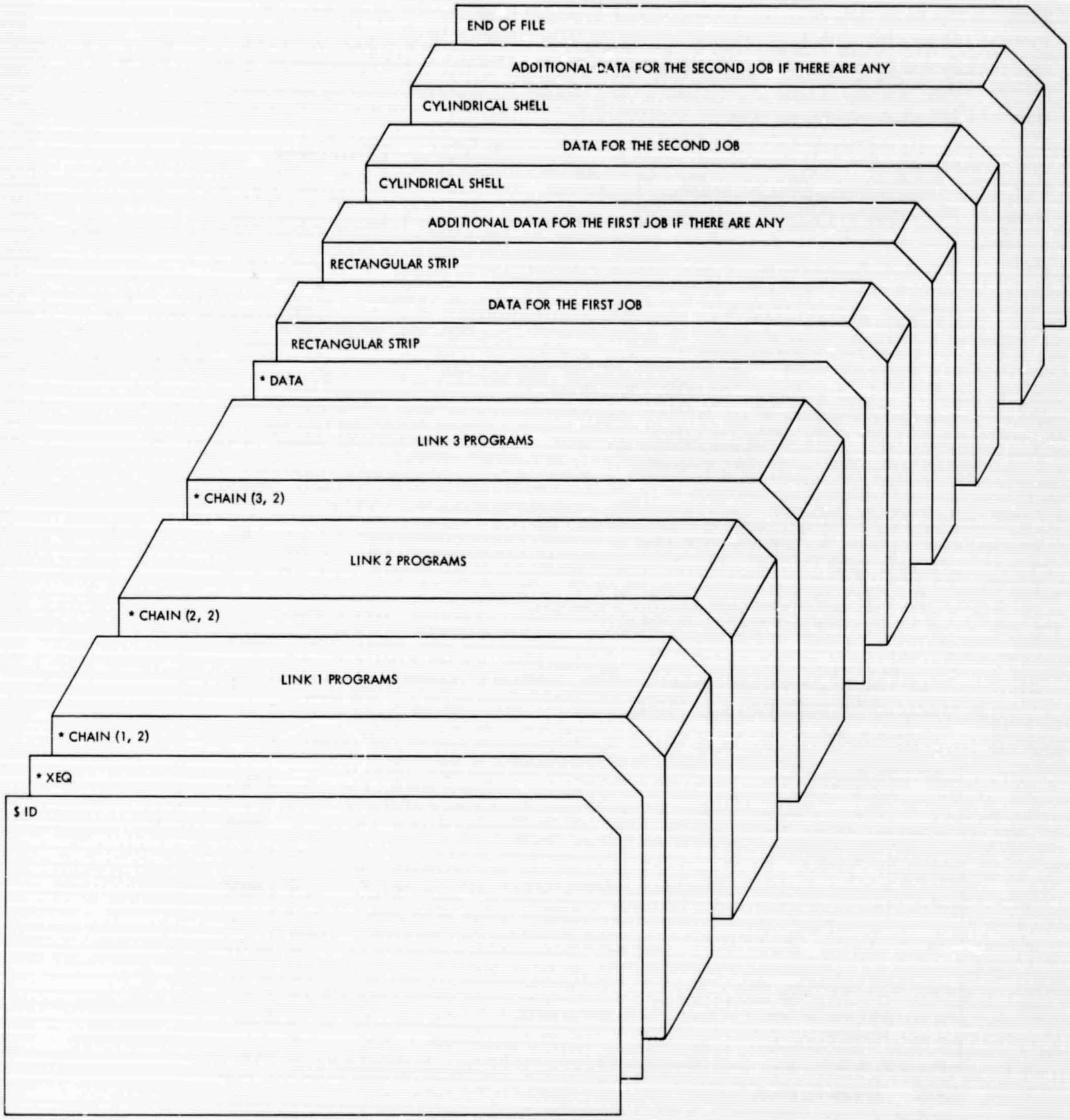


Fig. 7. Physical deck arrangement of FEDGE program

IV. Preparation of Input

A. General Information

Regardless of whether or not it is a three-dimensional problem, the input data deck can be considered in three groups: (1) the title and control cards, (2) topological connectivity and properties information deck, and (3) numerical values associated with the surface or line equations and boundary conditions (Fig. 8).

The first group of data is prepared in a similar way in both three- and two-dimensional problems. In the presensation that follows (Tables 2–8), each table describes a card, or the deck of cards, with a sequential instruction that refers to the column number on the card or deck of cards in question. Additional or unusual properties of input parameters are explained whenever they appear. The name of the parameter, as it appears in the program, is also included in the last column, following the description. The arrays are distinguished from the single-valued constants by a dummy index.

B. Title and Control Cards

The input forms for title and control cards are presented in Tables 2 and 3, respectively.

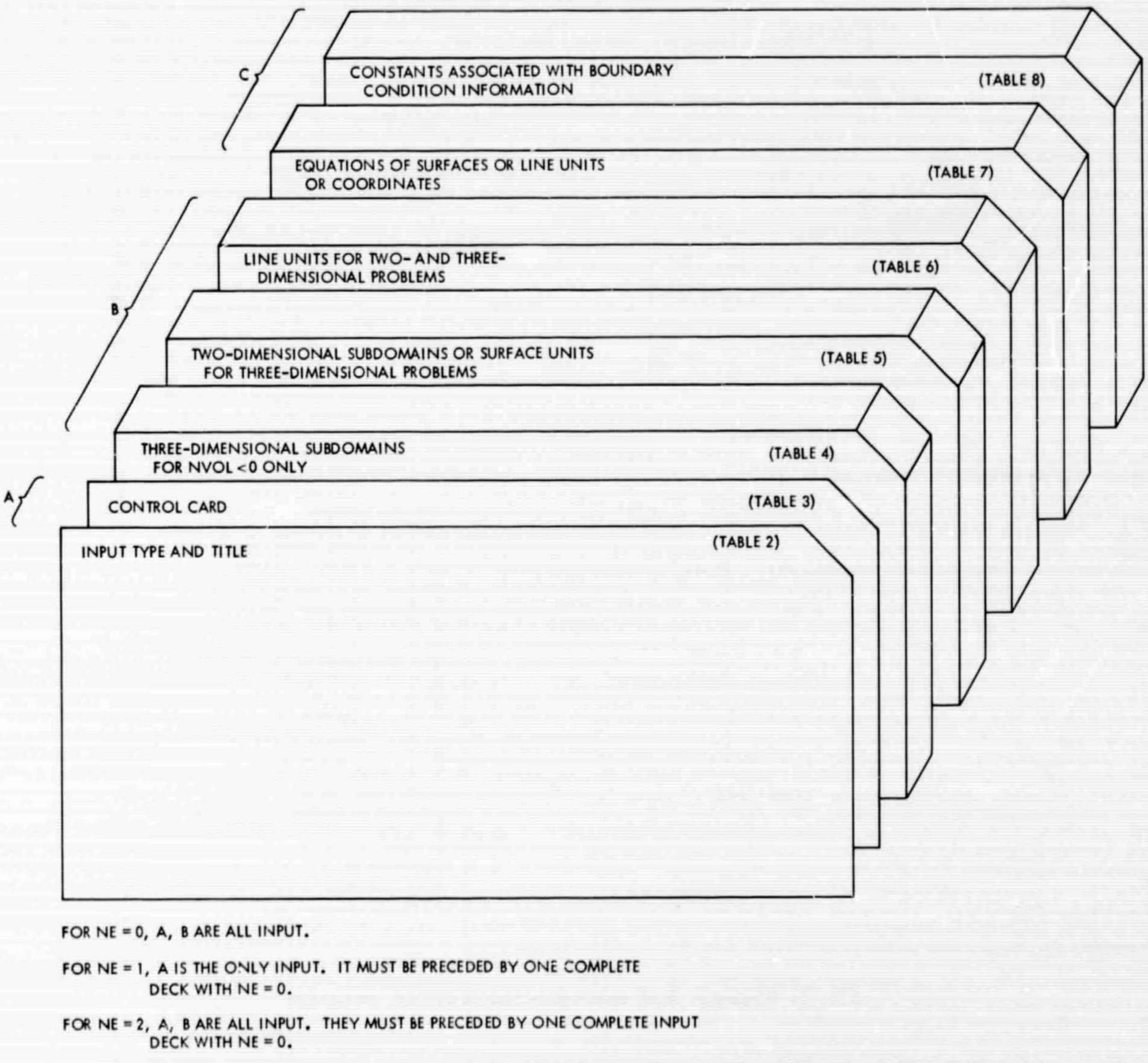


Fig. 8. Physical deck arrangement of data for FEDGE program

Table 2. Title card for input

Column	Description of input item	Name
1–2	A constant that defines the input type $NE = 0$ The complete deck of data is required $NE = 1$ The title card and control card are required. The point of interest, the refinement constant, and the number of final mesh divisions in three directions can be altered. The remaining values are assumed unchanged from the previous complete data information $NE = 2$ The complete deck of data is required. Complete input data deck with $NE = 0$ must precede this deck. In this deck, the element properties and boundary conditions can be altered. The first link is skipped during execution $NE \leq -3$ The complete data deck is required in two-dimensional problems. In addition to the standard output deck, mesh topology information for plotting (element by element) is punched out	NE
3–80	Any title information to identify the problem	—

Table 3. Control card for input—format (I2, 2I3, 4I2, E4.0, 12E, 5.0)

Column	Description of input item	Name
1–2	Number of volume subdomains; zero for two-dimensional problem only	NVOL
3–5	Number of surface units in three dimensions or number of surface subdomains in two dimensions	NSUR
6–8	Number of line units in three and two dimensions	NLIN
9–10	Label of subdomain at which the special point lies (in this program, always $IO = 1$)	IO
11–12	Number of divisions in the direction I in the first subdomain	NX(IO)
13–14	Number of divisions in the direction J in the first subdomain	NY(IO)
15–16	Number of divisions in the direction K in the first subdomain; nonzero only for three-dimensional problems	NZ(IO)
17–20	Factor of refinement $CF > 1$, and if $1 < CF < 1.01$, it is automatically assumed to be practically 1 in the program	CF
21–25	The x_1 coordinate of the special point in the overall coordinate system; it must be within or on the boundary of the first subdomain	XO(IO)
26–30	The x_2 coordinate of the special point in the overall coordinate system	YO(IO)
31–35	The x_3 coordinate of the special point in the overall coordinate system for three-dimensional problems only	ZO(IO)
36–40	The x_1 coordinate of the origin in the overall coordinate system; the origin must be on the starting point of the line labelled 1; the first line must be the I line	XB(1)
41–45	The x_2 coordinate of the origin in the overall coordinate system	XB(1)
46–50	The x_3 coordinate of the origin in the overall coordinate system for three-dimensional problems only	ZB(1)
51–55	Minimum x_1 coordinate in the overall coordinate system	XMIN
56–60	Maximum x_1 coordinate in the overall coordinate system	XMAX
61–65	Minimum x_2 coordinate in the overall coordinate system	YMIN
66–70	Maximum x_2 coordinate in the overall coordinate system	YMAX
71–75	Minimum x_3 coordinate in the overall coordinate system for three-dimensional problems only	ZMIN
76–80	Maximum x_3 coordinate in the overall coordinate system for three-dimensional problems only	ZMAX

C. Topological Connectivity and Properties Information Deck

If the number of three-dimensional subdomains NVOL > 0, the program will expect the information associated with the three-dimensional subdomains according to Table 4. For two-dimensional problems, the deck associated with Table 4 is completely excluded from the input deck.

Surface information associated with the face of each subdomain is input according to Table 5. The labelling of faces must be sequential; otherwise, the labelling is

Table 4. Information associated with three-dimensional subdomains—two subdomains per card—input format [2(8I4,8X)]

Column	Description of input item	Name[a]
1–4 (41–44)	Label of the sequential subdomain	MDM_i
5–6 (45–46)	Type of material in the *i*th subdomain	$IMAT = NMATE_i/100$
7–8 (47–48)	Type of temperature in the *i*th subdomain (if the volume subdomain is fictitious, columns 5–8 must be blank)	$ITEM = NMATE_i - 100 * IMAT$
9–12 (49–52)	Label of the first face that is conventionally defined as the *JK* face for which $I = 1$ in the natural coordinate system	$NFL_{i,1}$
13–16 (53–56)	Label of the second face that is conventionally defined as the *JK* face for which $I = 17$ in the natural coordinate system	$NFL_{i,2}$
17–20 (57–60)	Label of the third face that is conventionally defined as the *KI* face for which $J = 1$ in the natural coordinate system	$NFL_{i,3}$
21–24 (61–64)	Label of the fourth face that is conventionally defined as the *KI* face for which $J = 17$ in the natural coordinate system	$NFL_{i,4}$
25–28 (65–68)	Label of the fifth face that is conventionally defined as the *IJ* face for which $K = 1$ in the natural coordinate system	$NFL_{i,5}$
29–32 (69–72)	Label of the sixth face that is conventionally defined as the *IJ* face for which $K = 17$ in the natural coordinate system	$NFL_{i,6}$

[a]Computations shown in this column are in the FORTRAN integer sense.

Table 5. Information associated with faces and surface units of two- and three-dimensional subdomains—two surface units per card—input format [2(9I4,4X)]

Column	Description of input item	Name[a]
1–4 (41–44)	Label of the sequential surface faces; if the face is defined by more than one surface, the sequence number of the face is repeated for each surface	MDM_i
5–6 (45–46)	Element type number in the sense it is defined in Ref. 4	$IELT = MELMA_i/100$
7–8 (47–48)	Material type number; in the case of fictitious surfaces, columns 5–8 must be blank	$IMAT = MELMA_i - 100 * IELT$
9–10 (49–50)	Pressure type	$IPRS = MPRTI_i/100$
11–12 (51–52)	Thickness type	$ITIC = MPRTI_i - 100 * IPRS$
13–14 (53–54)	Temperature change type	$ITEM = MTETG_i/100$
15–16 (55–56)	Temperature gradient along the thickness type	$ITGY = MTETG_i - 100 * ITEM$
17–18 (57–58)	Boundary condition type	$MBON = MBOVR_i/100$
19 (59)	Convexity index *MCV*: $MCV = 0$ for a perfectly convex surface domain; $MCV = 1$ for a nearly convex surface domain	$MCV = MBOVR_i/10 - 10 * MBON_i$
20 (60)	Blank	—
21–24 (61–64)	Label of the first line face that is conventionally defined as the first line of the second coordinate label of a surface defined by *IJ* or *JK* or *KI* coupled labels	$ML_{i,1}$

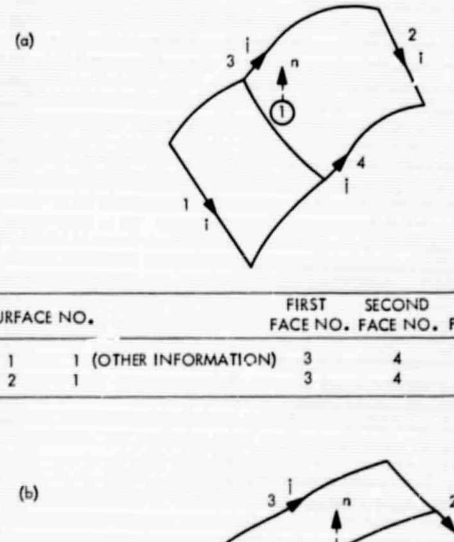
[a]Computations shown in this column are in the FORTRAN integer sense.

Table 5 (contd)

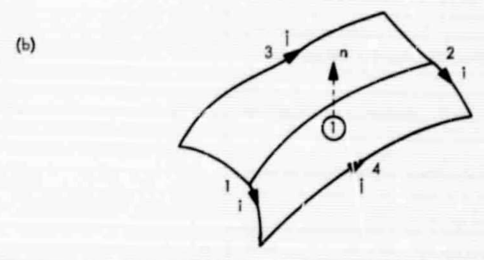
Column	Description of input item	Name*
25–28 (65–68)	Label of the second line face that is conventionally defined as the seventeenth line of the second coordinate label of the surface defined by *IJ* or *JK* or *KI* coupled labels	$ML_{i,2}$
29–32 (69–72)	Label of the third line face that is conventionally defined as the first line of the first coordinate label of the surface defined by *IJ* or *JK* or *KI* coupled labels	$ML_{i,3}$
33–36 (73–76)	Label of the fourth line face that is conventionally defined as the first line of the first coordinate label of the surface defined by *IJ* or *JK* or *KI* coupled labels	$ML_{i,4}$
	In the case of a face defined by more than one surface, the label corresponding to the interface of surfaces will be zero	
	In two-dimensional problems, only the *IJ* surface is in question; $ML_{i,j} = 0$ for all fictitious faces	

*Computations shown in this column are in the FORTRAN integer sense.

arbitrary. In three-dimensional problems, one face can be defined up to two surface units, the label of which must appear in the surface information list as the label of the face repeated for each of the units (Fig. 9a).



SURFACE NO.			FIRST FACE NO.	SECOND FACE NO.	THIRD FACE NO.	FOURTH FACE NO.
1	1	(OTHER INFORMATION)	3	4	1	0
2	1		3	4	0	2

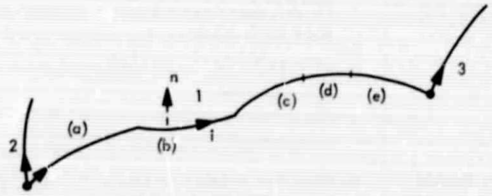


SURFACE NO.			FIRST FACE NO.	SECOND FACE NO.	THIRD FACE NO.	FOURTH FACE NO.
1	1	(OTHER INFORMATION)	3	0	1	2
2	1		0	4	1	2

Fig. 9. A face labelled (1) defined by two surfaces and the corresponding form of sequence label and line information (two cases)

In two-dimensional problems, the surface is in the x_1,x_2 plane, and all information associated with x_3 is null. Furthermore, each subdomain is defined by a single surface; i.e., $x_3 = 0$. One face can be defined up to any practical number of line units, the label of which must appear in the line information list as the label of the face repeated for each of the units (Fig. 10).

The line information associated with the line units of the two- and three-dimensional subdomains are input according to Table 6. Labelling of the lines must be sequential. The values of the lines that define the two opposite boundaries of a face defined by more than one surface are repeated as described in the tabulation (Fig. 9b). Labelling of the lines is not completely arbitrary. Users must follow imaginary integration procedures along each line element. The lines connected at the end of the lines must assume a higher label than that of the



LINE NO.			FACE NO.		
1	1		1	0	0
2	1		1	0	0
3	1	(OTHER INFORMATION)	1	0	0
4	1		1	0	0
5	1		1	0	3

Fig. 10. A face in a two-dimensional problem labelled (1) defined by five different line units and the corresponding form of sequence label and line information

Table 6. Information associated with line units of two- and three-dimensional subdomains—two line units per card—input format [2(7I4,4I3)]

Column	Description of input item	Name[a]
1–4 (41–44)	Label of the sequential line units; if the surface face is defined by more than one surface in the direction of those line faces, the sequence number of the line is repeated for each surface unit; in two-dimensional problems, the face number is repeated for each line unit associated with the face if the face is defined by more than one line unit; the line label must be negative if the integration is opposite the positive direction of the natural coordinate base vector along the line	IDM_i
5–6 (45–46)	Element type number in the sense defined in Ref. 4	$IELT = IELMA_i/100$
7–8 (47–48)	Material type number; in the case of a fictitious or boundary line, columns 5–8 must be blank	$IMAT = IELMA_i - 100 * IELT$
9–10 (49–50)	Pressure type	$IPRS = IPRTE_i/100$
11–12 (51–52)	Temperature type	$ITEM = IPRTE_i - 100 * IPRS$
13–14 (53–54)	Temperature gradient type in the local *y*-axis direction	$ITGY = IGYGZ_i/100$
15–16 (55–56)	Temperature gradient type in the local *z*-axis direction	$ITGZ = IGYGZ_i - 100 * ITGY$
17–18 (57–58)	Area type	$IARE = IARMX_i/100$
19–20 (59–60)	Moment of inertia around local *x* axis of the member type; the local *x* axis is along the member and in the direction of the base vector of the natural-coordinate system	$IMMX = IARMX_i - 100 * IARE$
21–22 (61–62)	Moment of inertia around the local *y* axis of the member	$IMMY = IMYMZ_i/100$
23–24 (63–64)	Moment of inertia around the local *z* axis of the member[b]	$IMMZ = IMYMZ_i - 100 * IMMY$
25–26 (65–66)	Angle type that defines the direction of the local *y* axis[b]	$IMFI = IMFBO_i/100$
27–28 (67–68)	Boundary-condition type along the line in question (for two-dimensional problems only)	$JBON = IMFBO_i - 100 * IMFI$
29–30 (69–70)	Indicator for the type of expression that defines the line (in two-dimensional problems only): $NDT = 3$ for straight-line input $NDT = 6$ for quadratic-line input $2 \leq NDT \leq 10$, the number of coordinates input when the line is defined point by point (maximum 5 points)	$NDT = IDTNR_i/10$
31 (71)	Indicator for the type of line input for two-dimensional problems: $INR = 0$–the line is defined point by point; the number of points is defined by $NDT/2$ (Fig. 11) $INR = 1$–the integration along the line stops the first time the end surface or line is reached $INR = 2$–the integration along the line stops the second time the end surface or line is reached	$INR = IDTNR_i - 10 * NDT$
32–34 (72–74)	In three-dimensional problems, the label of the *first* face whose interface with the *second* face defines the line in question; the normal of the *first* face, the normal of the *second* face, and the positive base vector of the natural-coordinate system on the line form a right-hand coordinate system in this sequence In two-dimensional problems, the label of one of the line faces connected to the end point of the line along which the integration is performed	$IFL_{i,1}$
35–37 (75–77)	In three-dimensional problems, the label of the *second* face whose interface with the *first* face defines the line in question (the sequence is defined above for $IFL_{i,1}$) In two-dimensional problems, the labels of one of the line faces connected to the end point of the line along which the integration is performed	$IFL_{i,2}$
38–40 (78–80)	In three-dimensional problems, the label of the end surface where the integration continues; if one of the surfaces that define the line is fictitious, the lines connected at the end of the line are chosen automatically as the two lines of the assumed end surface of the subdomain associated with the line information are available; not more than one fictitious surface is allowed to define a line In two-dimensional problems, the label of the end line where the integration continues—zero if the next line unit defines the same line face	$IFL_{i,3}$

[a]Computations shown in this column are in the FORTRAN integer sense.
[b]This is dummy information. The generation of mesh in three-dimensional frame elements is not possible.

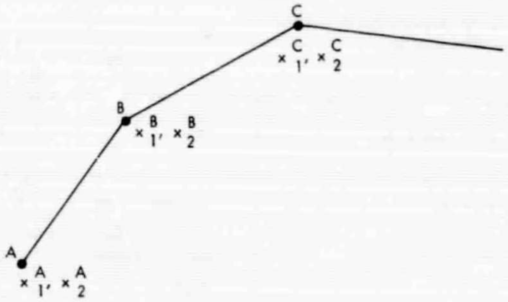


Fig. 11. The line defined by the coordinates of points

previous lines. The starting point at each line is determined through the integration procedure of the previous line (and only once—the first time they are reached during the integration process). The direction of the integration thus obtained in a given line might or might not be in the direction of the base vector of the natural-coordinate system. The label is then positive if these two directions coincide; it is negative if they are opposite.

D. Numerical Values Associated With Surface or Line Equations and Boundary Conditions

For three-dimensional problems for each labelled surface, as described in Table 5, even if the equations corresponding to various input units are identical, an equation of the quadratic surface must be read by the program, as shown in Table 7. The form of the equation will be as given by Eq. (2), and the sign of the equation will be determined by the positive direction of the normal to the surface.

$$a_1 + a_2x_3 + a_3x_2 + a_4x_1 + a_5x_1x_3 + a_6x_3x_2 + a_7x_2x_1 + a_8x_3^2 + a_9x_2^2 + a_{10}x_1^2 = 0 \qquad (2)$$

For two-dimensional problems for each labelled line, as described in Table 6, even if the equations corresponding to various input units are identical, a set of coordinates x_1, x_2, or a straight-line equation with three constants, or an equation of a quadratic line must be read depending upon the value of the corresponding indicator constant, as defined in Table 6. Equations (3), (4), and (5), and Fig. 11 clarify these input items.

$$x_1^A, x_2^A, x_1^B, x_2^B, x_1^C, x_2^C, x_1^D, x_2^D, x_1^E, x_2^E \qquad (3)$$

$$a_1 + a_2x_2 + a_3x_1 = 0 \qquad (4)$$

$$a_1 + a_2x_2 + a_3x_1 + a_4x_1x_2 + a_5x_2^2 + a_6x_1^2 = 0 \qquad (5)$$

The constants of the equation of one input unit or ten coordinates (five points) are read per card, according to the form presented in Table 7.

Two integer numbers and one constant constitute a set of information for the linear relation between various degrees of freedom on a given point. Up to eight sets of information can be read for a given type of boundary condition along a surface face in three-dimensional problems, or along a line face in two-dimensional problems.

Each card, therefore, will contain eight sets of information, as shown in Table 8, and each card will correspond to one type of boundary condition cited in columns 17–18 (57–58) of the connectivity information card, as described in Table 5, or in columns 27–28 (67–68) of the connectivity information card in Table 6. The number of cards for boundary conditions or the maximum number of boundary condition types is the maximum number that appears in the corresponding column of Table 5 or 6.

Table 7. Input of constants associated with equations of surfaces, line units, or coordinates—input format (10F8.0)

Column	Description of input item	Name[a]	Column	Description of input item	Name[a]
1–8	a_1 of Eqs. (2), (4), (5), or x_1^A of Eq. (3)	$AN_{10(n-1)+1}$	41–48	a_6 of Eqs. (2), (5), or x_2^C of Eq. (3)	$AN_{10(n-1)+6}$
9–16	a_2 of Eqs. (2), (4), (5), or x_2^A of Eq. (3)	$AN_{10(n-1)+2}$	49–56	a_7 of Eq. (2) or x_1^D of Eq. (3)	$AN_{10(n-1)+7}$
17–24	a_3 of Eqs. (2), (4), (5), or x_1^B of Eq. (3)	$AN_{10(n-1)+3}$	57–64	a_8 of Eq. (2) or x_2^D of Eq. (3)	$AN_{10(n-1)+8}$
25–32	a_4 of Eqs. (2), (5), or x_2^B of Eq. (3)	$AN_{10(n-1)+4}$	65–72	a_9 of Eq. (2) or x_1^E of Eq. (3)	$AN_{10(n-1)+9}$
33–40	a_5 of Eqs. (2), (5), or x_1^C of Eq. (3)	$AN_{10(n-1)+5}$	73–80	a_{10} of Eq. (2) or x_2^E of Eq. (3)	$AN_{10(n-1)+10}$

[a]The n refers either to the sequence number of input items or to the sequence number of cards from which the information is read.

Table 8. Input of constants associated with boundary condition information—input format (8I2,8E8.0)

Column	Description of input item[a]	Name
(1–2), (3–4), ···, (15–16)	Number of degrees of freedom associated with the nodal points on the face considered; each pair of numbers represents two direction labels on a point in connection with the prescribed boundary condition	$IBON_{i,j}$
(17–24), (25–32), ···, (73–80)	Constant that relates the degrees of freedom defined by the integers in columns 1–16. Each set is complete by consecutive pairs of integers in columns 1–16 and a corresponding constant in columns 17–80. Integers in columns 1–2 correspond to constant in columns 17–24. Integers in columns 3–4 correspond to constant in columns 25–32, etc.	$BORC_{i,j}$

[a]The actual input format in the program is different, but this can be used as is. It is more explicit than the actual format, which is (4I4,8E8.0).

V. Error Messages and Diagnostics

A. General Information

The occurrence of an error that causes a message generally has a detrimental effect on the results. The program stops when the continuation of the execution does not yield any additional information about the status of the input. When error messages occur, the user must check carefully that portion of the input designated by the error message. The error messages are as self-explanatory as possible, and are listed below in the order of probable appearance during execution of the program. The meaning of the printed information (italics in parentheses), the name of the subroutine in which the error is detected, and the probable cause of the error message are also given.

B. Error Messages in Link 1

VOLUME SUBDOMAIN NUMBERS ARE NOT IN SEQUENCE (*sequence number of volume subdomain and user's label*)

Occurs in DORB when the user's sequence label for the volume subdomain is not correct.

ERROR IN THE FACE LABELLING OF SUBDOMAIN (*number of subdomains involved*) OR (*number of adjacent subdomains*)

Occurs in DORB when the face numbers are not correct in the subdomains involved with the label printed out in the message.

TOTAL NUMBER OF FACES MAY NOT BE ZERO

Occurs in DORB. The zero or negative number of surfaces and lines is not acceptable by the program.

INPUT ERROR PROBABLY IN CONNECTION WITH INPUT UNIT (*number of the input unit involved*)

Occurs in COPY when an error exists in the label of lines or surfaces corresponding to the input unit indicated in the error message. Execution continues.

ERROR IN THE INPUT UNIT (*number of input units*) THE FIRST FOUR CONSTANTS FOLLOW (*the coordinates x_A, y_A, x_B, y_B of the segment AB*)

Occurs in DSAN when the boundary is defined by straight line segments and the coordinates of the end points of the segments are not correct. Execution continues.

AFTER (*number of steps*) STEPS OF INTEGRATION STARTING POINT IS REACHED, COORDINATES AND COEFFICIENTS OF SURFACES FOLLOW (*x, y, z coordinates and the coefficients of the surface equations for left, right end surfaces sequentially*)

Occurs in INTR. The coefficients involved are probably not correct.

AFTER (*number of steps*) STEPS OF INTEGRATION END SURFACE IS NOT REACHED, COORDINATES AND COEFFICIENTS OF SURFACES FOLLOW (*x, y, z coordinates and the coefficients of the surface equations for left, right, end surfaces sequentially*)

Occurs in INTR. The coefficients involved are probably not correct.

ERROR IN THE INPUT UNIT (*number of the unit*) COORDINATES AND COEFFICIENTS FOLLOW (*x, y, z coordinates and the coefficients of the surface equations for left, right, end surfaces sequentially)*

Occurs in LENG when the maximum or minimum coordinates assigned by the user are reached. The maximum and minimum coordinates, the coefficients of the equations involved, or the direction of the integration may be incorrect.

ERROR IN THE COEFFICIENTS OF QUADRATIC. COORDINATES, COMPUTED CONSTANTS, COEFFICIENTS FOLLOW [*The integers and the constants in the common map from 1–150* (see Table 2 of Vol. II) *and the coefficients of all equations up to the one in question are printed out.*]

Occurs in ROOT when the root of the quadratic cannot be found in the step considered. The coefficients of the quadratic may be incorrect.

INPUT ERROR DETECTED DURING THE INTEGRATION OF (*label of line along which the integration is performed*) TH LINE, RELATED INFORMATIONS FOLLOW (*the label of the line, the surface informations related to the line, user's input and program selection, and the label of lines at the end of the integration*)

Occurs in BACO when an error exists in the connectivity information of three surfaces constituting three faces around a nodal point in three dimensions. Line labels corresponding to the surfaces and the surface labels corresponding to the lines must be reexamined.

INPUT ERROR IN THE SURFACE LINE LABEL INFORMATION, CHECK WITH THE FORMAT SPECIFICATION (*the surface number and corresponding line labels are printed out for the surface in question and the consecutive one*)

Occurs in BOLI when a face is defined by more than one surface and the labels of the boundary lines are not written properly.

END POINTS OF THE (*number of lines in question*) TH LINE DO NOT MATCH END POINTS OF THE PREVIOUS LINE (*number of previous lines*), COORDINATES FOLLOW (*coordinates of the two ends of the previous line, the coordinate of the first point of the line in question*)

Occurs in TEST when an error exists in the line label or the computation of coordinates on the lines.

THE ERROR IS (*error*) ON THE FACE (*face number*) ERROR TOLERANCE (*error tolerance*) IS EXCEEDED, COEFFICIENTS FOLLOW (*coefficients of the surface equation*)

Occurs in NOCO when the surface equations associated with the face in question are not satisfied by the substitution of the coordinates computed.

ERROR IN THE COEFFICIENTS OF THE SURFACE EQUATION, RELATED INFORMATIONS FOLLOW [*The integers and the constants in the common map from 1–150* (see Table 2 of Vol. II), *and the coefficients of all surface equations up to the one in question are printed out.*]

Occurs in ROTZ when the roots of the quadratic equations do not satisfy the surface equations involved. The coefficients of the surface equations may be incorrect.

C. Error Messages in Link 2

IN LINK 2 ERROR DETECTED IN CONNECTIVITY INFORMATION (*the number of subdomains, surface in two dimensions, volume in three dimensions*)

Occurs in ENFI. Usually this type of error is detected in link 1.

D. Error Messages in Link 3

ERROR PROBABLY IN THE CONNECTIVITY INFORMATIONS OF (*the number of the subdomain*) TH SUBDOMAIN

Occurs in ARBU. It is unlikely that any error of this kind would be detected at this stage. If an error occurs, the related input should be checked carefully. Deeper study involving the structure of the program may be required.

VI. Description of Output

A. General Information

The output of FEDGE can be considered in two basic groups:

(1) The output in the printed form, which is basically for documentation, but also allows the user to follow some of the procedural aspects of the program and the information concerning his input data.

(2) The output in the punched-card form, which is basically for use as input data in a finite-element analysis program (specifically in the actual form for Ref. 4); in some cases, the user may optionally request additional punched cards for plotting purposes for two-dimensional problems.

B. Output in Printed Form

1. First page information. The first page starts with the information in the title card. Under the title, the dimension of the problem is printed out. The information in the control card follows, line by line, in the same sequence as it appears in the control card (see Table 3), and the description of each item is given as explicitly as possible.

2. Volume, surface, and lines connectivity information. The next page starts with the information that is read in as described in Section IV-C. The content of Table 4 appears in the input deck and in the output list for three-dimensional problems only.

3. Coefficients of surfaces, lines, or coordinates. The following surface and line information is printed out with explicit headings, in the sequence they are read in, and in as explicit a form as possible. The numerical values associated with the surface equations in three-dimensional problems appear under the heading: *Coefficients of Surface Equations.* The numerical values associated with the line equations or coordinates of the given points appear under the heading: *Coefficients of Line Equations or Coordinates.* The coordinates appear in two lines: the first line for the x_1, the second for the x_2.

4. Boundary conditions. Boundary conditions are printed out, if there are any, and the type is assigned as described in Sections IV-C and IV-D, under the heading: *Number of Directions Related to Each Other and the Relation Constants.* Eight possible combinations are grouped, and each group is printed out as zero (even if they are not assigned by the user) if one among them has been assigned.

The output related to the direct input terminates at this point. If users fail to obtain this part of the output, they should check carefully with the number of input cards and the corresponding numbers previously assigned by them.

5. Results of computations. The computations start and continue with: (a) an error message, as described in Section V, or (b) a message at the end of the integration of each line face that is printed out: LABELS OF SURFACES AND LINES INVOLVED IN THE (i)TH LINE INTEGRATION ARE l, m, n, p, q, and r, where, for three-dimensional problems, l and m are two surfaces that define the line and n is the end surface for the line. Line p is the line along which the integration is performed, and q and r are labels of the two lines that are connected to the end of line i. It follows that $p = i$, and, in complex geometrical configurations, this information might help users to adjust their input information as described in Tables 4, 5, and 6 if any related error message is printed out. In two-dimensional problems, only p and q are meaningful integers that represent the labels of the lines connected to the end of the ith line; l, m, and n are zero.

6. Time messages. Three time messages are printed out that indicate the computation time for parts A, B, and C of link 1, respectively (see Fig. 6):

(1) INTEGRATION ALONG THE LINE SEGMENTS TOOK ... SECONDS.

(2) DIVISION ALONG THE LINE SEGMENTS TOOK ... SECONDS.

(3) GENERATION OF NATURAL COORDINATES TOOK ... SECONDS.

7. Coordinate listing. The list of coordinates of the mesh that is generated follows the last time message in the format [2(I4,3F12.6)]:

$$(i) \quad x_1^{\,i}\, x_2^{\,i}\, x_3^{\,i} \qquad (i+1) \quad x_1^{\,i+1}\, x_2^{\,i+1}\, x_3^{\,i+1}$$

where i is the nodal point number and x_j^i are the associated three coordinates in the overall coordinate system.

8. Computation time. At the end of the list, a time message is printed out:

GENERATION OF FINAL MESH COORDINATES

TOOK . . . SECONDS

which denotes the computation time for link 2 (see Fig. 6).

9. Boundary conditions. The boundary conditions (if any) are printed out in the format [8(I4,I1,I4,I1,F6.3)]:

$$(i)(k)(i)(l)C_m,\ (j)(p)(j)(q)C_n,\ \cdots$$

where $i, j, \cdots$ are the labels of the nodal points for which kth (or pth) degrees of freedom on ith (or jth) points are related to the lth (or qth) degrees of freedom by a constant C_m (or C_n) typically expressed as the mth (nth) boundary-condition type in the input information. The linear relation can be expressed as

$$\alpha_{ik} = C^0_m + C^1_m\, \alpha_{il} + \cdots$$

In the input format and output listing, C^0_m is distinguished from C^1_m by $k = l$ and α_{ik}, α_{il} are the deflections of the ith point in the kth and lth directions (see Ref. 4).

10. Element-data listing. The list for element data is printed out in the format (20I4), wherein the content of each integer number directly corresponds to the corresponding input item in Ref. 4. The sequence of the labels of the element vertices is determined, assuming always the positive normal of the surface inward the subdomain containing the corresponding face and that has the smallest sequence number.

11. End card. The number of nodes, boundary conditions, and elements are printed out in the end card:

NO. OF NODES (IN) NO. OF BC (IB)

NO. OF ELTS (IE) END

where IN, IB, and IE are computed during the generation of nodal coordinates, boundary conditions, and element data, respectively.

12. Plotting information. For NE ≤ -3 (for two-dimensional problems in addition to the standard output deck), a mesh topology information for element-by-element plotting is printed out in the format (20I4). The nodal-point numbers associated with the elements are selected in such a manner that the common faces of the elements are not plotted twice. Each group of five integers represents the nodal-point labels of an element in sequence in the ccw direction. The nodal numbers corresponding to common faces are not printed out, and the remaining ones appear in the listing or in the cards as left-justified in each group of five integers. The plotting sequence follows the rule explained in Section III-A. Only labelled elements are accounted for.

13. Time message. The time message for the generation of elements is printed out as:

GENERATION OF ELEMENT DATA TOOK

. . . SECONDS

which denotes the computation time for link 3 (see Fig. 8).

C. Output in Punched-Card Form

For NE ≤ 2, the output in printed form—as explained in B7, B9, B10, and B11—is punched in the same format and sequence, with each line corresponding to one card. This complete deck is ready for use with the program described in Ref. 4 (with a few additional information cards).

For NE ≤ -3 (for two-dimensional problems in addition to the standard output deck described above), mesh-topology information for element-by-element plotting is punched out as described in B_{12}. This information, together with the punched information corresponding to B_7 coordinates of the nodal points, can be used for plotting purposes.

VII. Examples

A. Axisymmetrical Model for Spherical Motor, Cylindrical Inlet

The input listed in Table A-1 of the appendix can be followed in the light of the corresponding instructions of Sections II–IV, and with reference to Fig. 4. The users must follow the following logical steps:

1. Determination of subdomains. Subdomain I is determined by the grain material. Subdomains II and III are selected as simply as possible (otherwise, arbitrarily), and connected to subdomain I, to be able to include portions CD and AB of the outer shell in the analysis. The partition of the boundary of each subdomain into faces is a result of a very natural choice, as in the case of subdomain III, or else depends upon the form of the final mesh as requested by the users. In subdomain II, the partition is a result of the selected form of the subdomain. In subdomain I, point F is selected to provide a quasirectangular form. It should be noted that each face has homogeneous properties; that is, EC is a boundary of the grain, BF is a shell, AG is fictitious, etc.

2. ***Definition of origin.*** The origin can be any defined point on the common joint of two faces. The E is chosen because it is known without any computation. The H, G, or A could have been chosen as well because they are defined in an equally simple way.

3. ***Determination of coordinate lines.*** In two-dimensional problems, the first line determines the IJ natural coordinate system; i.e., EB is selected as the first line, and therefore determines the I direction. The other coordinate lines are the natural result of this selection. The positive normal of each line is on the left side of an observer, heading toward the direction of the base vectors for the I and J lines. The integration directions and the labels of the lines are determined at the same time, the integration directions being denoted by small arrows at each corner. The label of face 2 is positive because the small integration arrow at B is in the same direction as the J base vector. By the same token, the label of face AB is -3, the label of FC is -4, the label of AG is -5, etc. Face BF can be labelled 2 because the origin is determined from the integration along line 1; face AB can be labelled -3 for the same reason; face FC can be labelled with the next sequence number, -4, because the origin has been determined from the integration along line 2, etc.

4. ***Input deck preparation.*** Once Fig. 4 is set up properly, the preparation of input decks according to the instructions of Tables 2, 3, 5, 6, and 7 is a simple exercise.

5. ***Output listing.*** In Table A-2 of the appendix, the output is listed in the form in which it appears at the end of the successful execution of the program. Parts corresponding to Sections VI-B-5 and VI-B-6 do not appear when $NE = 1$ or $NE = 2$. Parts corresponding to Sections VI-B-7 through VI-B-11 and VI-B-13 are always printed and punched in the same sequence and in the same format. The part corresponding to Section VI-B-12 is punched only when $NE \leq -3$.

6. ***Generation of natural coordinate system and final mesh.*** The natural coordinate system and final mesh are illustrated in Figs. 12 and 13, respectively.

B. Cylindrical Shell Problem

The input listed in Table A-3 of the appendix can be followed in the light of the corresponding instructions of Sections II–IV, and with reference to Fig. 5. The users must follow the logical steps described below.

1. ***Determination of subdomains.*** Volumes of subdomains I and II (which are fictitious) are distinguished by the D_2 plane of Fig. 3. The partition of the boundary of each subdomain is realized by two planes: $x_1 = 0$ and $x_3 = 0$; that is, the six faces of subdomain I are (1) the $x_2 = 0$ plane, (2) the portion of the shell between the $-x_1$ *and* $+x_3$ axes, (3) the portion between the $+x_3$ and $+x_1$ axes, etc. With reference to Figs. 5a and 5b, one can see the fictitious planes 12 and 13 defining the faces of the subdomains; the base of the cylinder by 1, which is a fictitious plane; and the four faces 2, 3, 4, and 5, which constitute real surfaces and represent the lower portion of the shell. The boundary conditions are prescribed at the planes D_3 and D_1 defined by $x_2 = 0$ and $x_3 = 96$, respectively, which are fictitious surfaces and labelled 6, 11. The positive direction of the surfaces is assumed inward for the shell; the positive direction of overall coordinate axes is assumed for the other fictitious planes 1, 6, 11, 12, and 13.

2. ***Definition of origin.*** The origin has been chosen arbitrarily at A ($x_1 = -153$, $x_2 = 0$, $x_3 = 0$).

3. ***Determination of coordinate lines.*** The first line of integration is AB. The resulting simple, natural coordinate system is shown on Figs. 5a and 5b, where the arrows along the lines are the base vectors for the first and last lines of the subdomain. The sequence label of the lines follows the rules described in Table 6; that is, the integration along line 1 yields the coordinates and integration directions for lines BC and BF, which are labelled $+2$ and $+3$, respectively (according to this information, sequentially positive), because these directions are the same as the direction of base vectors along these lines. Lines CG and CD are labelled $+4$ and -5 by the same token, etc.

4. ***Input deck preparation.*** Tables 2 and 3 are self-explanatory. Columns 5–8 (45–48) in Table 4 are blank because the volume is void. In Table 5, columns 5–8 (45–48) contain the element-type numbers in the sense in which they are described in Ref. 4, and material-type numbers of faces 2–5 and 7–10 are left blank for faces 1, 6, 11, 12, and 13. The connectivity information in columns 21–36 (61–76) is blank for faces 12 and 13. The

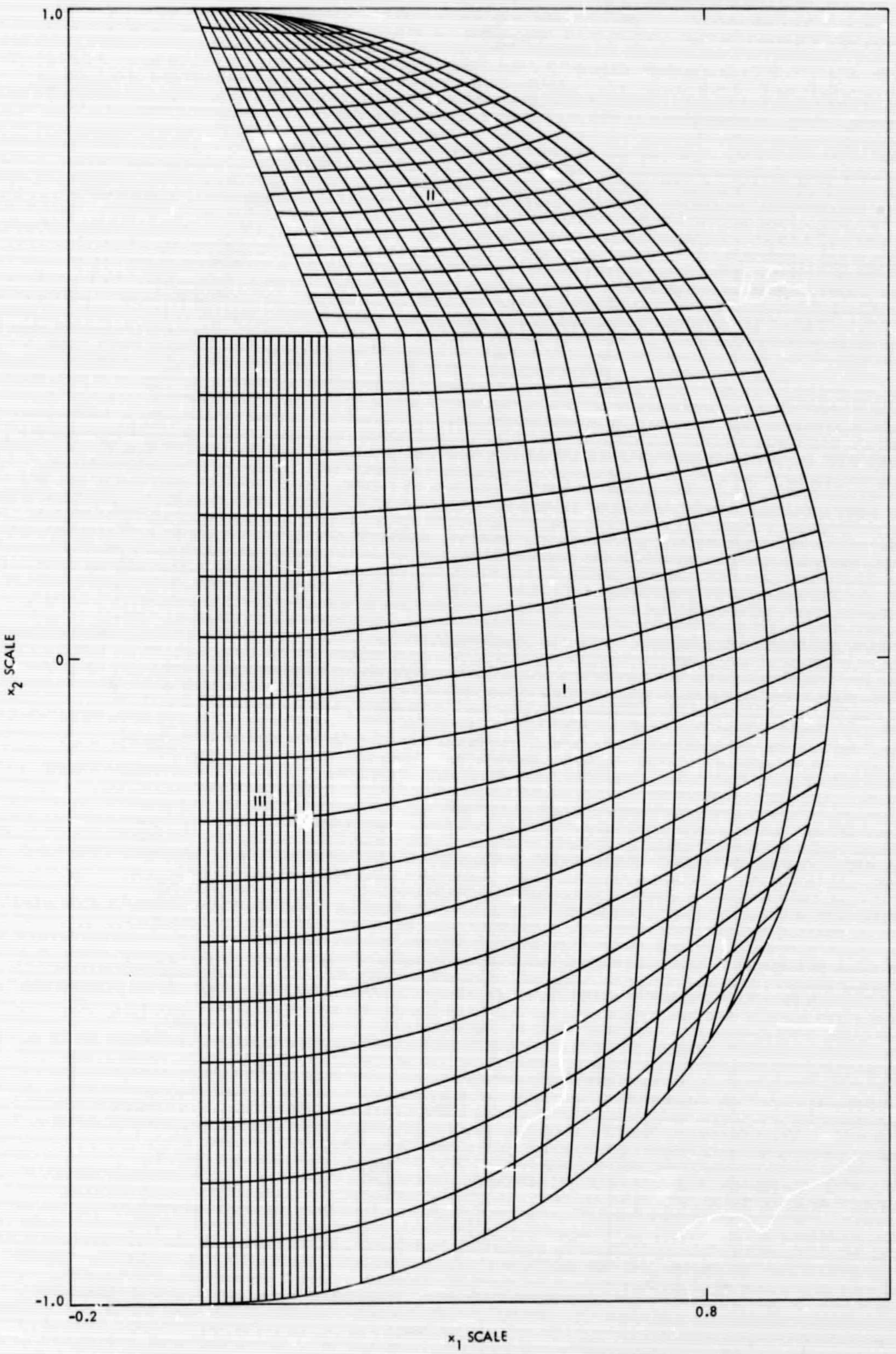


Fig. 12. Natural coordinate system for axisymmetrical model

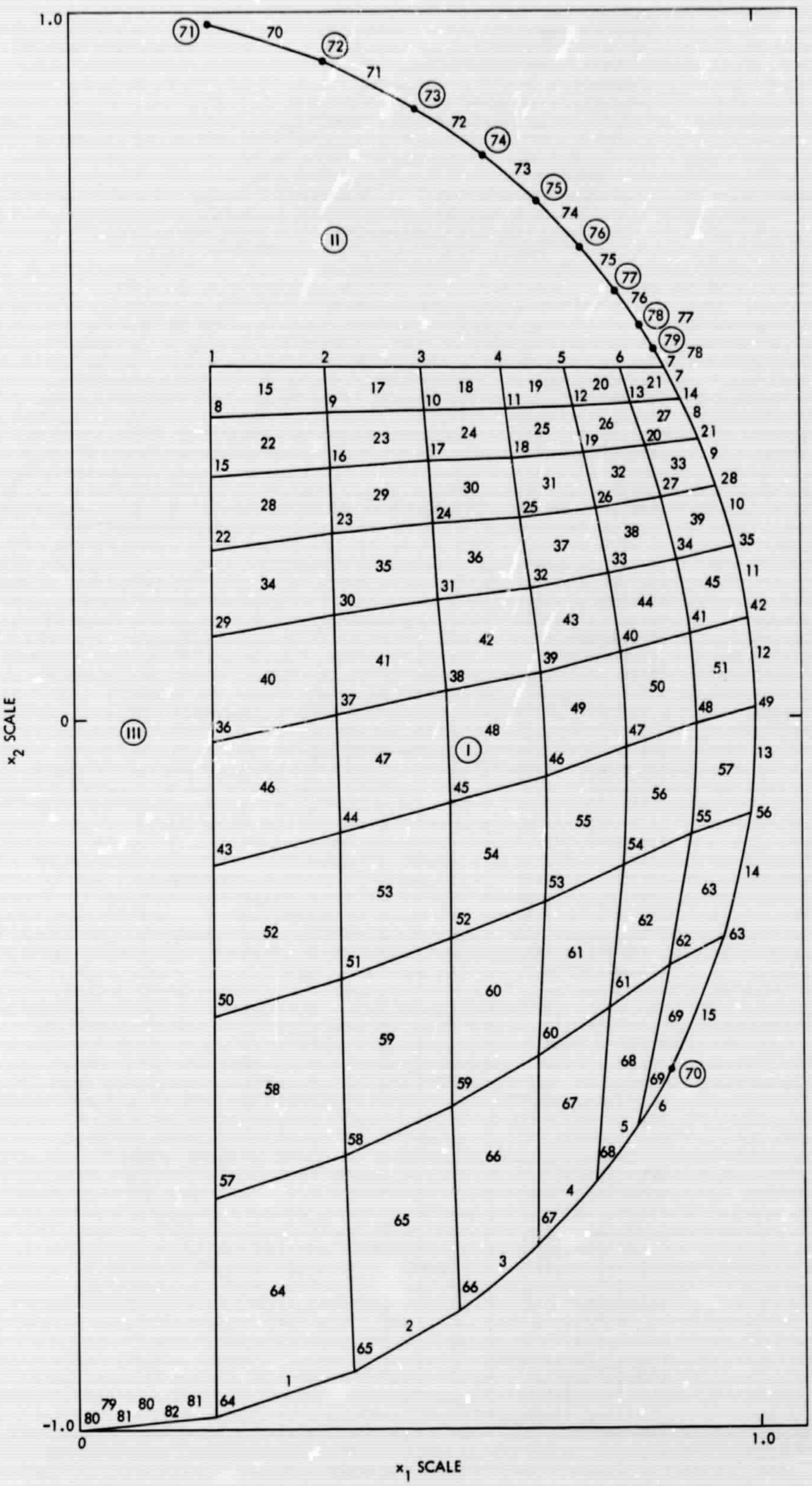


Fig. 13. Final mesh for axisymmetrical model

list that follows contains line information according to Table 6. Next, the list of constants associated with the surface equation is given according to Table 7. Each line corresponds to one surface input unit of Table 5, sequentially.

The boundary conditions are given according to Table 8. Three directions along which the points that are in the 1st and 11th surfaces are restrained are 11, 22, and 33; and complete restraint is expressed by zero at corresponding columns 17–24. The type number at columns 17–18 (or 57–58) corresponding to the line of the 1st and 11th faces expresses the fact that the boundary condition, as defined by the last card, will be assigned to all points on faces 6, 1, and 11.

5. Output listing. In Table A-4 of the appendix, the output is listed after successful execution of the program. In the part corresponding to Section VI-B-5, a message is printed out for each line-integration process. This part would not appear when $NE = 1$ or $NE = 2$. Parts corresponding to Sections VI-B-7 through VI-B-11 and VI-B-13 are always printed and punched in the same sequence and in the same format.

6. Generation of natural coordinate system and final mesh. The final mesh is illustrated in Fig. 14. For cylindrical shells with the same geometry, the input and the output for the generation of another mesh configuration are illustrated in Tables A-5 and A-6 of the appendix, respectively. It should be noted that the type of shell is changed to membrane and bending by the change of element-type numbers from 14 to 12 in the third column of the surface information table. The boundary conditions are also changed by fixing completely both end points of the shell; that is, 6 deg of freedom, as shown in the table for boundary conditions (see Section VI-B-4).

This output was obtained with $NE = 2$ from the natural coordinate system, which has been generated and stored in the previous case, and takes only 23 s compared with approximately 7 min for the first case, which includes the generation of natural coordinate systems in two- and three-dimensional subdomains.

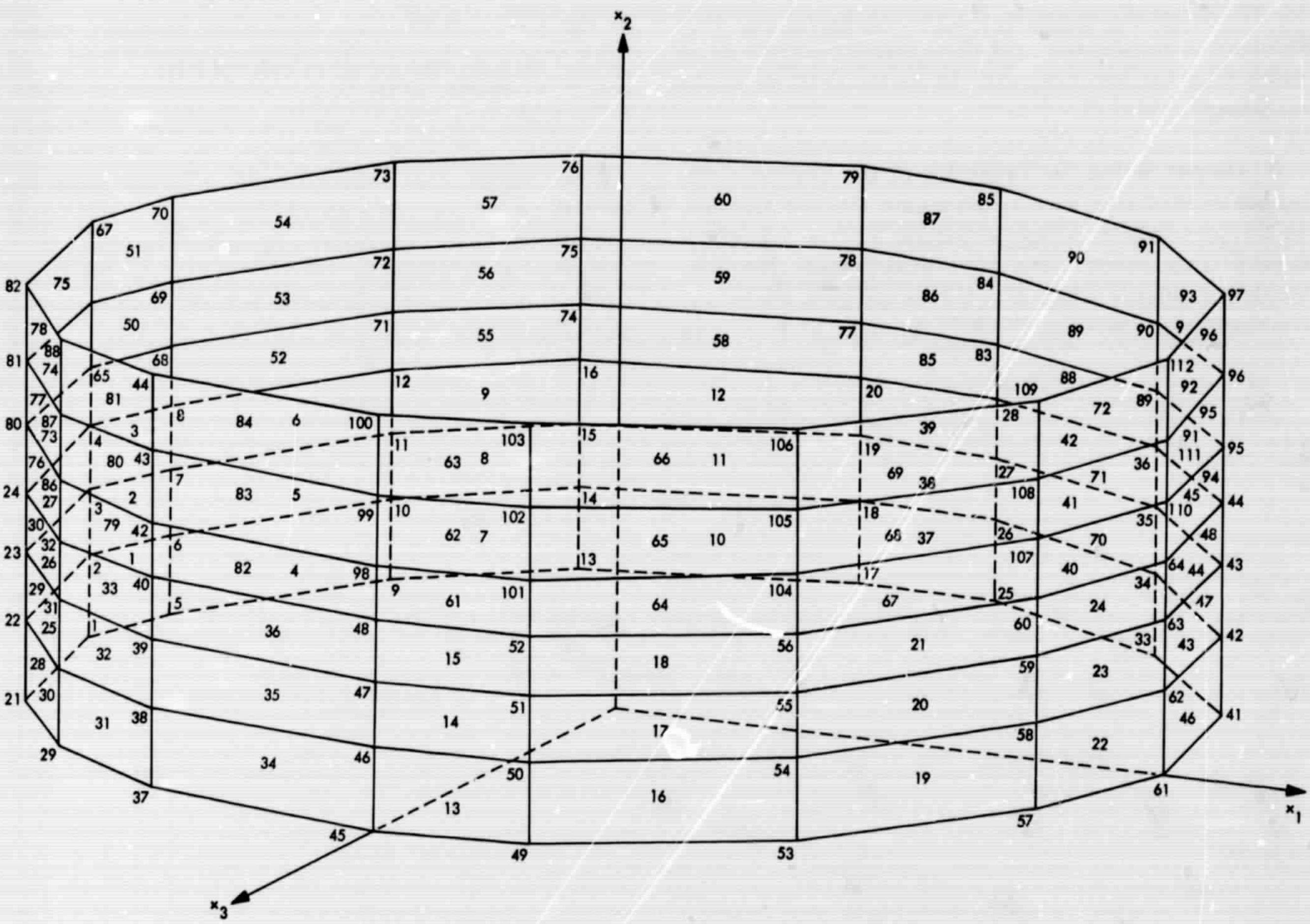


Fig. 14. Final mesh for cylindrical shell problem

Appendix

Computer Results for the Examples

This appendix contains a listing of input data, together with the respective output data obtained after successful execution, for the following:

(1) Axisymmetrical model for spherical motor, cylindrical inlet.

(2) Cylindrical shell problem for the membrane case.

(3) Cylindrical shell problem for the membrane and bending case.

Cited in the appendix are sectional references, which the user will find helpful in following the applicable procedural aspects of the FEDGE program.

Table A-1. List of input data for axisymmetrical model for spherical motor, cylindrical inlet

AXISYMMETRICAL MODEL FOR SPHERICAL MOTOR, CYLINDIRICAL INLET   INPUT TYPE NO. -3

DATA FOR TWO DIMENSIONAL PROBLEM

NUMBER OF THREE DIMENSIONAL SUBDOMAINS	NVOL	-0
NUMBER OF FACES OR TWO DIM. SUBDOMAINS	NSUR	3
NUMBER OF LINEAR FACES OR ONE DIM. ELEMENTS	NLIN	11
NO. OF SUBDIVISION IN WHICH SPECIAL POINT LIES	ID	1
NUMBER OF DIVISION IN THREE DIRECTIONS	NX(ID)	9
	NY(ID)	6
	NZ(ID)	-0
FACTOR OF REFINEMENT	CF	0.120000E 01
COORDINATES OF SPECIAL POINT	XO(ID)	0.866000E 00
	YO(ID)	0.500000E 00
	ZO(ID)	-0.
COORDINATES OF THE ORIGIN	XB(1)	0.200000E-00
	YB(1)	0.500000E 00
	ZB(1)	-0.
MINIMUM AND MAXIMUM BOUNDARY COORDINATES	XMIN	-0.200000E-00
	XMAX	0.120000E 01
	YMIN	-0.120000E 01
	YMAX	0.120000E 01
	ZMIN	-0.
	ZMAX	-0.

SURFACES INFORMATION

SURF	NO	ELEMNT	MATERL	PRESSR	THICNS	TEM CH	TEM GR	BND CN	CV IDX	INP TP	1ST FC	2ND FC	3RD FC	4TH FC
1	1	16	1	-0	0	-0	0	-0	-0	0	6	2	1	4
2	2	-0	0	-0	0	-0	0	-0	-0	0	11	6	9	7
3	3	-0	0	-0	0	-0	0	-0	-0	0	8	3	5	1

LINES INFORMATION

LINE	NO	ELEMNT	MATERL	PRESSR	TEM CH	TEM GY	TEM GZ	S AREA	TOR CS	MOIN Y	MOIN Z	ANG FI	BND CN	NDTINR	FACE NO			
1	1	-0	0	1	-0	-0	0	-0	0	-0	0	-0	0	3	1	-0	2	3
2	2	17	2	-0	0	-0	0	-0	0	-0	0	-0	0	6	1	-0	4	10
3	-3	17	2	1	-0	-0	0	-0	0	-0	0	-0	0	6	1	-0	-0	5
4	-4	17	2	-0	0	-0	0	-0	0	-0	0	-0	0	6	1	-0	7	6
5	-5	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	3	1	-0	-0	8
6	-6	-0	0	1	-0	-0	0	-0	0	-0	0	-0	0	3	1	-0	-0	1
7	-7	17	2	1	-0	-0	0	-0	0	-0	0	-0	0	6	1	-0	11	1
8	8	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	3	1	-0	9	1
9	-9	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	3	1	-0	-0	11
10	-10	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	3	1	-0	-0	-0
11	-11	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	6	1	-0	-0	5

L UN	NO	COEFFICIENTS OF LINE EQUATIONS OR COORDINATES					
1	1	-0.2000	-0.	1.0000			
2	2	1.0000	-0.	-0.	-0.	-1.0000	-1.0000
3	-3	1.0000	-0.	-0.	-0.	-1.0000	-1.0000
4	-4	-1.0000	-0.	-0.	-0.	1.0000	1.0000
5	-5	-0.	-0.	1.0000			
6	-6	-0.5000	1.0000	-0.			
7	-7	-1.0000	-0.	-0.	-0.	1.0000	1.0000
8	8	-0.5000	1.0000	-0.			
9	-9	-1.0000	1.0000	2.5000			
10	-10	0.5000	1.0000	-0.			
11	-11	-1.0000	-0.	-0.	-0.	1.0000	1.0000

Table A-2. List of output data after successful execution for axisymmetrical model

```
LABELS OF SURFACES AND LINES INVOLVED IN THE     1 TH LINE INTEGRATION ARE    0    0    0          2    -3    0
LABELS OF SURFACES AND LINES INVOLVED IN THE     2 TH LINE INTEGRATION ARE    0    0    0         -4   -10    0
LABELS OF SURFACES AND LINES INVOLVED IN THE     3 TH LINE INTEGRATION ARE    0    0    0         -5     0    0
LABELS OF SURFACES AND LINES INVOLVED IN THE     4 TH LINE INTEGRATION ARE    0    0    0         -7    -6    0
LABELS OF SURFACES AND LINES INVOLVED IN THE     5 TH LINE INTEGRATION ARE    0    0    0          8     0    0
LABELS OF SURFACES AND LINES INVOLVED IN THE     6 TH LINE INTEGRATION ARE    0    0    0          1     0    0
LABELS OF SURFACES AND LINES INVOLVED IN THE     7 TH LINE INTEGRATION ARE    0    0    0        -11     1    0
LABELS OF SURFACES AND LINES INVOLVED IN THE     8 TH LINE INTEGRATION ARE    0    0    0         -9     1    0
LABELS OF SURFACES AND LINES INVOLVED IN THE     9 TH LINE INTEGRATION ARE    0    0    0        -11     0    0
LABELS OF SURFACES AND LINES INVOLVED IN THE    11 TH LINE INTEGRATION ARE    0    0    0         -5     0    0

INTEGRATION ALONG THE LINE SEGMENTS TOOK     10.78 SECONDS.

DIVISION ALONG THE LINE SEGMENTS TOOK        10.05 SECONDS.


GENERATION OF NATURAL COORDINATES TOOK       81.30 SECONDS.

   1    0.199999    0.500000    0.          2    0.366897    0.500000    0.
   3    0.565979    0.500000    0.          4    0.621881    0.500000    0.
   5    0.718466    0.500000   -0.          6    0.798953    0.500000    0.
   7    0.866026    0.500000    0.          8    0.200000    0.428852    0.
   9    0.370838    0.436009    0.         10    0.511542    0.439811    0.
  11    0.629645    0.441486    0.         12    0.730437    0.445233   -0.
  13    0.816499    0.449593    0.         14    0.889755    0.455604    0.
  15    0.200000    0.343475    0.         16    0.373930    0.358477    0.
  17    0.517125    0.367108    0.         18    0.640162    0.371499   -0.
  19    0.745670    0.380031    0.         20    0.837125    0.389337    0.
  21    0.915572    0.401018    0.         22    0.200000    0.241023    0.
  23    0.375915    0.264307    0.         24    0.523060    0.279348    0.
  25    0.652483    0.288028   -0.         26    0.764604    0.302830    0.
  27    0.860574    0.317309    0.         28    0.942323    0.333678    0.
  29    0.200000    0.118380    0.         30    0.377584    0.149738    0.
  31    0.530163    0.173039    0.         32    0.665823    0.189008   -0.
  33    0.782226    0.210394    0.         34    0.882399    0.230279    0.
  35    0.967849    0.250569    0.         36    0.200000   -0.029452    0.
  37    0.362415    0.011026    0.         38    0.542645    0.044396    0.
  39    0.682555    0.070226   -0.         40    0.800870    0.100136    0.
  41    0.902266    0.126428    0.         42    0.988509    0.148306    0.
  43    0.200000   -0.206490    0.         44    0.385949   -0.158008   -0.
  45    0.548485   -0.113285   -0.         46    0.689247   -0.075670   -0.
  47    0.808246   -0.034836   -0.         48    0.910779   -0.000643    0.
  49    0.999226    0.023623    0.         50    0.200000   -0.418936    0.
  51    0.386609   -0.363403    0.         52    0.548606   -0.306217    0.
  53    0.687517   -0.254049   -0.         54    0.804595   -0.200099    0.
  55    0.904191   -0.155312    0.         56    0.991856   -0.126348    0.
  57    0.200000   -0.673872    0.         58    0.389848   -0.612410    0.
  59    0.546027   -0.541915    0.         60    0.674714   -0.472193   -0.
  61    0.780376   -0.403029    0.         62    0.870503   -0.342612    0.
  63    0.952775   -0.302170    0.         64    0.200000   -0.979795    0.
  65    0.401642   -0.915782    0.         66    0.556138   -0.830707   -0.
  67    0.672061   -0.740275   -0.         68    0.757449   -0.652364    0.
  69    0.820073   -0.571578    0.         70    0.866026   -0.500000    0.
  71    0.200000    0.979796    0.         72    0.367298    0.929783    0.
  73    0.498520    0.866781    0.         74    0.599703    0.799819    0.
  75    0.677463    0.735177    0.         76    0.737070    0.675583    0.
  77    0.782633    0.622124    0.         78    0.817642    0.575209    0.
  79    0.844770    0.534565    0.         80   -0.000001   -1.000000    0.
  81    0.079572   -0.996811    0.         82    0.145518   -0.989336   -0.

GENERATION OF FINAL MESH COORDINATES TOOK     6.58 SECONDS.
   -11702    0    0   64   65   -21702    0    0   65   66   -31702    0    0   66   67   -41702
    0    0   67   68   -51702    0    0   68   69   -61702    0    0   69   70   -71702    0    0
    7   14   -81702    0    0   14   21   -91702    0    0   21   28 -101702    0    0   28   35
 -111702    0    0   35   42 -121702    0    0   42   49 -131702    0    0   49   56 -141702
    0    0   56   63 -151702    0    0   63   70 -161601    0    0    1    8    9    2 -171601
    0    0    2    9   10    3 -181601    0    0    3   10   11    4 -191601    0    0    4   11
   12    5 -201601    0    0    5   12   13    6 -211601    0    0    6   13   14    7 -221601
    0    0    8   15   16    9 -231601    0    0    9   16   17   10 -241601    0    0   10   17
   18   11 -251601    0    0   11   18   19   12 -261601    0    0   12   19   20   13 -271601
    0    0   13   20   21   14 -281601    0    0   15   22   23   16 -291601    0    0   16   23
   24   17 -301601    0    0   17   24   25   18 -311601    0    0   18   25   26   19 -321601
    0    0   19   26   27   20 -331601    0    0   20   27   28   21 -341601    0    0   22   29
   30   23 -351601    0    0   23   30   31   24 -361601    0    0   24   31   32   25 -371601
    0    0   25   32   33   26 -381601    0    0   26   33   34   27 -391601    0    0   27   34
   35   28 -401601    0    0   29   36   37   30 -411601    0    0   30   37   38   31 -421601
    0    0   31   38   39   32 -431601    0    0   32   39   40   33 -441601    0    0   33   40
   41   34 -451601    0    0   34   41   42   35 -461601    0    0   36   43   44   37 -471601
    0    0   37   44   45   38 -481601    0    0   38   45   46   39 -491601    0    0   39   46
   47   40 -501601    0    0   40   47   48   41 -511601    0    0   41   48   49   42 -521601
    0    0   43   50   51   44 -531601    0    0   44   51   52   45 -541601    0    0   45   52
   53   46 -551601    0    0   46   53   54   47 -561601    0    0   47   54   55   48 -571601
    0    0   48   55   56   49 -581601    0    0   50   57   58   51 -591601    0    0   51   58
```

Table A-2 (contd)

```
 59  52 -601601    0    0   52   59   60   53 -611601    0    0   53   60   61   54 -621601
  0   0   54   61   62   55 -631601    0    0   55   62   63   56 -641601    0    0   57   64
 65  58 -651601    0    0   58   65   66   59 -661601    0    0   59   66   67   60 -671601
  0   0   60   67   68   61 -631601    0    0   61   68   69   62 -691601    0    0   62   69
 70  63 -701702    0    1   71   72 -711702    0    1   72   73 -721702    0    1   73   74
-731702    0    1   74   75 -741702    0    1   75   76 -751702    0    1   76   77 -761702
  0   1   77   78 -771702    0    1   78   79 -781702    0    1   79    7 -791702    0    1
 80  81 -801702    0    1   81   82 -811702    0    1   82   64

NO. OF NODES   82  NO. OF BC    0  NO. OF ELTS   81                                END

 64  65   0   0   0  65  66   0   0   0  66  67   0   0   0  67  68   0   0   0
 68  69   0   0   0  69  70   0   0   0   7  14   0   0   0  14  21   0   0   0
 21  28   0   0   0  28  35   0   0   0  35  42   0   0   0  42  49   0   0   0
 49  56   0   0   0  56  63   0   0   0  63  70   0   0   0   1   8   9   2   1
  9  10   3   2   0  10  11   4   3   0  11  12   5   4   0  12  13   6   5   0
 13  14   7   6   0   8  15  16   9   0  16  17  10   0   0  17  18  11   0   0
 18  19  12   0   0  19  20  13   0   0  20  21  14   0   0  15  22  23  16   0
 23  24  17   0   0  24  25  18   0   0  25  26  19   0   0  26  27  20   0   0
 27  28  21   0   0  22  29  30  23   0  30  31  24   0   0  31  32  25   0   0
 32  33  26   0   0  33  34  27   0   0  34  35  28   0   0  29  36  37  30   0
 37  38  31   0   0  38  39  32   0   0  39  40  33   0   0  40  41  34   0   0
 41  42  35   0   0  36  43  44  37   0  44  45  38   0   0  45  46  39   0   0
 46  47  40   0   0  47  48  41   0   0  48  49  42   0   0  43  50  51  44   0
 51  52  45   0   0  52  53  46   0   0  53  54  47   0   0  54  55  48   0   0
 55  56  49   0   0  50  57  58  51   0  58  59  52   0   0  59  60  53   0   0
 60  61  54   0   0  61  62  55   0   0  62  63  56   0   0  57  64  65  58   0
 65  66  59   0   0  66  67  60   0   0  67  68  61   0   0  68  69  62   0   0
 69  70  63   0   0  71  72   0   0   0  72  73   0   0   0  73  74   0   0   0
 74  75   0   0   0  75  76   0   0   0  76  77   0   0   0  77  78   0   0   0
 78  79   0   0   0  79   7   0   0   0  80  81   0   0   0  81  82   0   0   0
 82  64   0   0   0

GENERATION OF ELEMENT DATA TOOK          11.20 SECONDS.
```

Table A-3. List of input data for the shell cylindrical problem for membrane case

```
CYLINDIRICAL SHELL PROBLEM                                              INPUT TYPE NO. -0


DATA FOR THREE DIMENSIONAL PROBLEM


NUMBER OF THREE DIMENSIONAL SUBDOMAINS                 NVOL              2
NUMBER OF FACES OR TWO DIM. SUBDOMAINS                 NSUR             13
NUMBER OF LINEAR FACES OR ONE DIM. ELEMENTS            NLIN             20
NO. OF SUBDIVISION IN WHICH SPECIAL POINT LIES           ID              1
NUMBER OF DIVISION IN THREE DIRECTIONS               NX(ID)              4
                                                     NY(ID)              4
                                                     NZ(ID)              3
FACTOR OF REFINEMENT                                     CF   0.100010E 01
COORDINATES OF SPECIAL POINT                         XO(ID)  -0.153000E 03
                                                     YO(ID)   0.480000E 02
                                                     ZO(ID)  -0.
COORDINATES OF THE ORIGIN                             XB(1)  -0.153000E 03
                                                      YB(1)  -0.
                                                      ZB(1)  -0.
MINIMUM AND MAXIMUM BOUNDARY COORDINATES               XMIN  -0.154000E 03
                                                       XMAX   0.154000E 03
                                                       YMIN  -0.100000E 01
                                                       YMAX   0.970000E 02
                                                       ZMIN  -0.154000E 03
                                                       ZMAX   0.154000E 03

VOLUME INFORMATION

SUBD NO    MATERIAL  TEMPERATURE  1ST FACE NO  2ND FACE NO  3RD FACE NO  4TH FACE NO  5TH FACE NO  6TH FACE NO

    1          -0          0            5            3            2            4            1            6
    2          -0          0           10            8            7            9            6           11
```

Table A-3 (contd)

SURFACES INFORMATION

SURF	NO	ELEMNT	MATERL	PRESSR	THICNS	TEM CH	TEK GR	BND CN	CV IDX	INP TP	1ST FC	2ND FC	3RD FC	4TH FC
1	1	-0	0	-0	0	-0	0	1	0	0	11	2	1	5
2	2	14	1	1	1	-0	0	-0	-0	0	1	7	12	3
3	3	14	1	1	1	-0	0	-0	-0	0	3	4	2	6
4	4	14	1	1	1	-0	0	-0	-0	0	5	9	10	4
5	5	14	1	1	1	-0	0	-0	-0	0	12	10	11	17
6	6	-0	0	-0	0	-0	0	-0	-0	0	17	6	7	9
7	7	14	1	1	1	-0	0	-0	-0	0	7	19	13	20
8	8	14	1	1	1	-0	0	-0	-0	0	20	8	6	14
9	9	14	1	1	1	-0	0	-0	-0	0	9	15	16	8
10	10	14	1	1	1	-0	0	-0	-0	0	13	16	17	18
11	11	-0	0	-0	0	-0	0	1	0	0	18	14	19	15
12	12	-0	0	-0	0	-0	0	-0	-0	0	-0	-0	-0	-0
13	13	-0	0	-0	0	-0	0	-0	-0	0	-0	-0	-0	-0

LINES INFORMATION

LINE	NO	ELEMNT	MATERL	PRESSR	TEM CH	TEM GY	TEM GZ	S AREA	TOR CS	MOIN Y	MOIN Z	ANG FI	BND CN	NUTINR		FACE NO		
1	1	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1	2	1	12
2	2	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1	3	1	13
3	3	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1	12	2	6
4	4	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1	3	13	6
5	-5	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1	1	4	12
6	6	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1	8	6	13
7	-7	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1	7	6	13
8	8	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1	8	13	11
9	-9	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1	6	9	12
10	10	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1	4	12	6
11	-11	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1	1	5	13
12	12	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1	13	5	6
13	13	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1	13	10	11
14	-14	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1	8	11	12
15	-15	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1	11	9	12
16	16	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1	9	12	11
17	17	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1	6	10	12
18	18	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1	11	10	12
19	19	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1	7	11	12
20	-20	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1	12	7	6

S UN	NO	COEFFICIENTS OF SURFACE EQUATIONS									
1	1	-0.	-0.	1.0000	-0.	-0.	-0.	-0.	-0.	-0.	-0.
2	2	23409.0000	-0.	-0.	-0.	-0.	-0.	-0.	-1.0000	-0.	-1.0000
3	3	23409.0000	-0.	-0.	-0.	-0.	-0.	-0.	-1.0000	-0.	-1.0000
4	4	23409.0000	-0.	-0.	-0.	-0.	-0.	-0.	-1.0000	-0.	-1.0000
5	5	23409.0000	-0.	-0.	-0.	-0.	-0.	-0.	-1.0000	-0.	-1.0000
6	6	-48.0000	-0.	1.0000	-0.	-0.	-0.	-0.	-0.	-0.	-0.
7	7	23409.0000	-0.	-0.	-0.	-0.	-0.	-0.	-1.0000	-0.	-1.0000
8	8	23409.0000	-0.	-0.	-0.	-0.	-0.	-0.	-1.0000	-0.	-1.0000
9	9	23409.0000	-0.	-0.	-0.	-0.	-0.	-0.	-1.0000	-0.	-1.0000
10	10	23409.0000	-0.	-0.	-0.	-0.	-0.	-0.	-1.0000	-0.	-1.0000
11	11	-96.0000	-0.	1.0000	-0.	-0.	-0.	-0.	-0.	-0.	-0.
12	12	-0.	-0.	-0.	1.0000	-0.	-0.	-0.	-0.	-0.	-0.
13	13	-0.	1.0000	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.

3 UN NO NO OF DIRECTIONS RELATED TO EACH OTHER AND THE RELATION CONSTANTS

1 2 2 -0.   0 0 -0.   -0-0 -0.   -0 0 -0.   -0-0 -0.   -0 0 -0.   -0-0 -0.   -0 0 -0.

Table A-4. List of output data after successful execution for membrane case

LABELS OF SURFACES AND LINES INVOLVED IN THE	1 TH LINE INTEGRATION ARE	2	1	3	1	3	2
LABELS OF SURFACES AND LINES INVOLVED IN THE	2 TH LINE INTEGRATION ARE	3	1	4	2	4	5
LABELS OF SURFACES AND LINES INVOLVED IN THE	3 TH LINE INTEGRATION ARE	3	2	6	3	6	7
LABELS OF SURFACES AND LINES INVOLVED IN THE	4 TH LINE INTEGRATION ARE	3	4	6	4	6	9
LABELS OF SURFACES AND LINES INVOLVED IN THE	5 TH LINE INTEGRATION ARE	1	4	5	5	11	10
LABELS OF SURFACES AND LINES INVOLVED IN THE	6 TH LINE INTEGRATION ARE	8	6	9	6	8	9
LABELS OF SURFACES AND LINES INVOLVED IN THE	7 TH LINE INTEGRATION ARE	7	6	10	7	13	17
LABELS OF SURFACES AND LINES INVOLVED IN THE	8 TH LINE INTEGRATION ARE	8	9	11	8	14	15
LABELS OF SURFACES AND LINES INVOLVED IN THE	9 TH LINE INTEGRATION ARE	6	9	10	9	17	16
LABELS OF SURFACES AND LINES INVOLVED IN THE	10 TH LINE INTEGRATION ARE	4	5	6	10	9	17
LABELS OF SURFACES AND LINES INVOLVED IN THE	11 TH LINE INTEGRATION ARE	1	5	2	11	1	12
LABELS OF SURFACES AND LINES INVOLVED IN THE	12 TH LINE INTEGRATION ARE	2	5	6	12	7	17
LABELS OF SURFACES AND LINES INVOLVED IN THE	13 TH LINE INTEGRATION ARE	7	10	11	13	19	18
LABELS OF SURFACES AND LINES INVOLVED IN THE	14 TH LINE INTEGRATION ARE	8	11	7	14	20	19
LABELS OF SURFACES AND LINES INVOLVED IN THE	15 TH LINE INTEGRATION ARE	11	9	10	15	18	16
LABELS OF SURFACES AND LINES INVOLVED IN THE	16 TH LINE INTEGRATION ARE	9	10	11	15	15	18
LABELS OF SURFACES AND LINES INVOLVED IN THE	17 TH LINE INTEGRATION ARE	6	10	9	17	9	16
LABELS OF SURFACES AND LINES INVOLVED IN THE	18 TH LINE INTEGRATION ARE	11	10	9	18	15	16
LABELS OF SURFACES AND LINES INVOLVED IN THE	19 TH LINE INTEGRATION ARE	7	11	8	19	20	14
LABELS OF SURFACES AND LINES INVOLVED IN THE	20 TH LINE INTEGRATION ARE	8	7	6	20	6	7

INTEGRATION ALONG THE LINE SEGMENTS TOOK 49.03 SECONDS.

DIVISION ALONG THE LINE SEGMENTS TOOK 47.87 SECONDS.

GENERATION OF NATURAL COORDINATES TOOK 276.05 SECONDS.

1	-152.995995	0.	-0.004241	2	-153.000000	16.015967	-0.000257
3	-153.000000	32.030346	-0.000257	4	-152.998035	48.000000	-0.001542
5	-141.345861	0.	-58.558903	6	-139.711567	16.015972	-62.366685
7	-139.713243	32.030349	-62.362927	8	-141.353840	48.000000	-58.543404
9	-108.168578	0.	-108.200986	10	-108.174532	16.015974	-108.199963
11	-108.180648	32.030351	-108.193646	12	-108.191693	48.000000	-108.179247
13	-58.511314	0.	-141.366039	14	-62.328458	16.015972	-139.728619
15	-62.336586	32.030350	-139.724993	16	-58.553762	48.000000	-141.349056
17	0.046825	0.	-152.997025	18	0.049793	16.015967	-152.999989
19	0.049793	32.030346	-152.999989	20	-0.012536	48.000000	-152.996964
21	-141.356544	0.	58.542020	22	-139.719490	16.015972	62.348930
23	-139.719635	32.030349	62.348608	24	-141.355469	48.000000	58.540884
25	58.585550	0.	-141.335882	26	62.394121	16.015972	-139.699316
27	62.393792	32.030349	-139.699463	28	58.585550	48.000000	-141.335882
29	-108.195269	0.	108.178452	30	-108.195547	16.015974	108.178947
31	-108.195554	32.030351	108.178941	32	-108.194603	48.000000	108.177729
33	108.211785	0.	-108.159132	34	108.213788	16.015974	-108.160702
35	108.213788	32.030351	-108.160702	36	108.211785	48.000000	-108.159132
37	-58.558090	0.	141.349415	38	-62.365395	16.015972	139.712135
39	-62.365078	32.030348	139.712276	40	-58.557750	48.000000	141.348957
41	141.366999	0.	-58.511584	42	139.732157	16.015972	-62.320529
43	139.732302	32.030348	-62.320200	44	141.366999	48.000000	-58.511584
45	-0.017166	0.	152.999027	46	-0.018133	16.015967	152.999996
47	-0.018133	32.030346	152.999996	48	-0.017166	48.000000	152.999027
49	58.526139	0.	141.362057	50	62.334103	16.015972	139.726107
51	62.333775	32.030349	139.726254	52	58.526139	48.000000	141.362057
53	108.166166	0.	108.206160	54	108.167381	16.015974	108.207111
55	108.167381	32.030351	108.207112	56	108.166166	48.000000	108.206160
57	141.342365	0.	58.572493	58	139.705429	16.015972	62.380419
59	139.705574	32.030350	62.380093	60	141.342365	48.000000	58.572493
61	152.998013	0.	0.032784	62	152.999996	16.015967	0.030801
63	152.999996	32.030346	0.030801	64	152.998013	48.000000	0.032784
65	-152.999996	64.016764	0.000423	66	-152.999996	79.994410	0.000423
67	-152.998035	96.000000	-0.001542	68	-139.715899	64.016759	-62.356981
69	-139.715900	79.994402	-62.356974	70	-141.353840	96.000000	-58.543404
71	-108.184807	64.016760	-108.189691	72	-108.185081	79.994404	-108.169416
73	-108.191693	96.000000	-108.179247	74	-62.340665	64.016758	-139.723173
75	-62.341702	79.994402	-139.722710	76	-58.553762	96.000000	-141.349056
77	0.049793	64.016764	-152.999489	78	0.049793	79.994410	-152.999989
79	-0.012536	96.000000	-152.996964	80	-139.719604	64.016752	62.348677
81	-139.719578	79.994395	62.348732	82	-141.355019	96.000000	58.540558
83	62.393934	64.016757	-139.699400	84	62.393979	79.994402	-139.699379
85	58.585550	96.000000	-141.335882	86	-108.195961	64.016743	108.178534
87	-108.195966	79.994386	108.178529	88	-108.193878	96.000000	108.177068
89	108.213788	64.016759	-108.160702	90	108.213788	79.994402	-108.160702
91	108.211785	96.000000	-108.159132	92	-62.366426	64.016713	139.711674
93	-62.366533	79.994373	139.711628	94	-58.556608	96.000000	141.347885
95	139.732241	64.016758	-62.320341	96	139.732218	79.994399	-62.320387
97	141.366999	96.000000	-58.511584	98	-0.021734	64.016655	152.999996
99	-0.021734	79.994370	152.999996	100	-0.018752	96.000000	152.996986
101	62.334064	64.016708	139.726128	102	62.334104	79.994375	139.726107
103	58.524957	96.000000	141.360973	104	108.167439	64.016741	108.207058
105	108.167424	79.994387	108.207073	106	108.165405	96.000000	108.205496
107	139.705524	64.016754	62.380208	108	139.705498	79.994399	62.380268
109	141.341960	96.000000	58.572165	110	152.999996	64.016764	0.030801
111	152.999996	79.994410	0.030801	112	152.998013	96.000000	0.032784

Table A-4 (contd)

```
GENERATION OF FINAL MESH COORDINATES TOOK   21.70 SECONDS.
  12   12-0.      52   52-0.      92   92-0.     132  132-0.     172  172-0.
 212  212-0.     252  252-0.     292  292-0.     332  332-0.     372  372-0.
 412  412-0.     452  452-0.     492  492-0.     532  532-0.     572  572-0.
 612  612-0.     672  672-0.     702  702-0.     732  732-0.     762  762-0.
 792  792-0.     822  822-0.     852  852-0.     882  882-0.     912  912-0.
 942  942-0.     972  972-0.    1002 1002-0.    1032 1032-0.    1062 1062-0.
1092 1092-0.    1122 1122-0.
  -11401 100   1   1   5   6   2  -21401 100   1   2   6   7   3  -31401 100   1
   3   7   8   4  -41401 100   1   5   9  10   6  -51401 100   1   6  10  11   7
  -61401 100   1   7  11  12   8  -71401 100   1   9  13  14  10  -81401 100   1
  10  14  15  11  -91401 100   1  11  15  16  12 -101401 100   1  13  17  18  14
 -111401 100   1  14  18  19  15 -121401 100   1  15  19  20  16 -131401 100   1
  45  46  50  49 -141401 100   1  46  47  51  50 -151401 100   1  47  48  52  51
 -161401 100   1  49  50  54  53 -171401 100   1  50  51  55  54 -181401 100   1
  51  52  56  55 -191401 100   1  53  54  58  57 -201401 100   1  54  55  59  58
 -211401 100   1  55  56  60  59 -221401 100   1  57  58  62  61 -231401 100   1
  58  59  63  62 -241401 100   1  59  60  64  63 -251401 100   1   1   2  22  21
 -261401 100   1   2   3  23  22 -271401 100   1   3   4  24  23 -281401 100   1
  21  22  30  29 -291401 100   1  22  23  31  30 -301401 100   1  23  24  32  31
 -311401 100   1  29  30  38  37 -321401 100   1  30  31  39  38 -331401 100   1
  31  32  40  39 -341401 100   1  37  38  46  45 -351401 100   1  38  39  47  46
 -361401 100   1  39  40  48  47 -371401 100   1  17  25  26  18 -381401 100   1
  18  26  27  19 -391401 100   1  19  27  28  20 -401401 100   1  25  33  34  26
 -411401 100   1  26  34  35  27 -421401 100   1  27  35  36  28 -431401 100   1
  33  41  42  34 -441401 100   1  34  42  43  35 -451401 100   1  35  43  44  36
 -461401 100   1  41  61  62  42 -471401 100   1  42  62  63  43 -481401 100   1
  43  63  64  44 -491401 100   1   4   8  68  65 -501401 100   1  65  68  69  66
 -511401 100   1  66  69  70  67 -521401 100   1   8  12  71  68 -531401 100   1
  68  71  72  69 -541401 100   1  69  72  73  70 -551401 100   1  12  16  74  71
 -561401 100   1  71  74  75  72 -571401 100   1  72  75  76  73 -581401 100   1
  16  20  77  74 -591401 100   1  74  77  78  75 -601401 100   1  75  78  79  76
 -611401 100   1  48  98 101  52 -621401 100   1  98  99 102 101 -631401 100   1
  99 100 103 102 -641401 100   1  52 101 104  56 -651401 100   1 101 102 105 104
 -661401 100   1 102 103 106 105 -671401 100   1  56 104 107  60 -681401 100   1
 104 105 108 107 -691401 100   1 105 106 109 108 -701401 100   1  60 107 110  64
 -711401 100   1 107 108 111 110 -721401 100   1 108 109 112 111 -731401 100   1
   4  65  80  24 -741401 100   1  65  66  81  80 -751401 100   1  66  67  82  81
 -761401 100   1  24  80  86  32 -771401 100   1  80  81  87  86 -781401 100   1
  81  82  88  87 -791401 100   1  32  86  92  40 -801401 100   1  86  87  93  92
 -811401 100   1  87  88  94  93 -821401 100   1  40  92  98  48 -831401 100   1
  92  93  99  98 -841401 100   1  93  94 100  99 -851401 100   1  20  28  83  77
 -861401 100   1  77  83  84  78 -871401 100   1  78  84  85  79 -881401 100   1
  28  36  89  83 -891401 100   1  83  89  90  84 -901401 100   1  84  90  91  85
 -911401 100   1  36  44  95  89 -921401 100   1  89  95  96  90 -931401 100   1
  90  96  97  91 -941401 100   1  44  64 110  95 -951401 100   1  95 110 111  96
 -961401 100   1  96 111 112  97

NO. OF NODES  112  NO. OF BC   32  NO. OF ELTS   96                        END

GENERATION OF ELEMENT DATA TOOK            6.52 SECONDS.
```

Table A-5. List of input data for cylindrical shell problem for membrane and bending case

```
CYLINDIRICAL SHELL PROBLEM  BENDING                                    INPUT TYPE NO.  2


DATA FOR THREE DIMENSIONAL PROBLEM


NUMBER OF THREE DIMENSIONAL SUBDOMAINS          NVOL                  2
NUMBER OF FACES OR TWO DIM. SUBDOMAINS          NSUR                 13
NUMBER OF LINEAR FACES OR ONE DIM. ELEMENTS     NLIN                 20
NO. OF SUBDIVISION IN WHICH SPECIAL POINT LIES    ID                  1
NUMBER OF DIVISION IN THREE DIRECTIONS         NX(ID)                 2
                                               NY(ID)                 2
                                               NZ(ID)                 2
FACTOR OF REFINEMENT                              CF      0.100010E 01
COORDINATES OF SPECIAL POINT                   XO(ID)    -0.153000E 03
                                               YO(ID)     0.480000E 02
                                               ZO(ID)    -0.
```

Table A-5 (contd)

```
COORDINATES OF THE ORIGIN                          XB(1)   -0.153000E 03
                                                   YB(1)   -0.
                                                   ZB(1)   -0.
MINIMUM AND MAXIMUM BOUNDARY COORDINATES            XMIN   -0.154000E 03
                                                    XMAX    0.154000E 03
                                                    YMIN   -0.100000E 01
                                                    YMAX    0.970000E 02
                                                    ZMIN   -0.154000E 03
                                                    ZMAX    0.154000E 03
```

VOLUME INFORMATION

SUBD NO	MATERIAL	TEMPERATURE	1ST FACE NO	2ND FACE NO	3RD FACE NO	4TH FACE NO	5TH FACE NO	6TH FACE NO
1	-0	0	5	3	2	4	1	6
2	-0	0	10	8	7	9	6	11

SURFACES INFORMATION

SURF	NO	ELEMNT	MATERL	PRESSR	THICNS	TEM CH	TEM GR	BND CN	CV IDX	INP TP	1ST FC	2ND FC	3RD FC	4TH FC
1	1	-0	0	-0	0	-0	0	1	0	0	11	2	1	5
2	2	12	1	1	1	-0	0	-0	-0	0	1	7	12	3
3	3	12	1	1	1	-0	0	-0	-0	0	3	4	2	6
4	4	12	1	1	1	-0	0	-0	-0	0	5	9	10	4
5	5	12	1	1	1	-0	0	-0	-0	0	12	10	11	17
6	6	-0	0	-0	0	-0	0	-0	-0	0	17	6	7	9
7	7	12	1	1	1	-0	0	-0	-0	0	7	19	13	20
8	8	12	1	1	1	-0	0	-0	-0	0	20	8	6	14
9	9	12	1	1	1	-0	0	-0	-0	0	9	15	16	8
10	10	12	1	1	1	-0	0	-0	-0	0	13	16	17	18
11	11	-0	0	-0	0	-0	0	1	0	0	18	14	19	15
12	12	-0	0	-0	0	-0	0	-0	-0	0	-0	-0	-0	-0
13	13	-0	0	-0	0	-0	0	-0	-0	0	-0	-0	-0	-0

LINES INFORMATION

LINE	NO	ELEMNT	MATERL	PRESSR	TEM CH	TEM GY	TEM GZ	S AREA	TOR CS	MOIN Y	MOIN Z	ANG FI	BND CN	NDTINR	FACE NO
1	1	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1 2 1 12
2	2	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1 3 1 13
3	3	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1 12 2 6
4	4	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1 3 13 6
5	-5	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1 1 4 12
6	6	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1 8 6 13
7	-7	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1 7 6 13
8	8	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1 8 13 11
9	-9	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1 6 9 12
10	10	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1 4 12 6
11	-11	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1 1 5 13
12	12	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1 13 5 6
13	13	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1 13 10 11
14	-14	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1 8 11 12
15	-15	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1 11 9 12
16	16	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1 9 12 11
17	17	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1 6 10 12
18	18	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1 11 10 12
19	19	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1 7 11 12
20	-20	-0	0	-0	0	-0	0	-0	0	-0	0	-0	0	0	1 12 7 6

S UN NO COEFFICIENTS OF SURFACE EQUATIONS

S UN	NO										
1	1	-0.	-0.	1.0000	-0.	-0.	-0.	-0.	-0.	-0.	-0.
2	2	23409.0000	-0.	-0.	-0.	-0.	-0.	-0.	-1.0000	-0.	-1.0000
3	3	23409.0000	-0.	-0.	-0.	-0.	-0.	-0.	-1.0000	-0.	-1.0000
4	4	23409.0000	-0.	-0.	-0.	-0.	-0.	-0.	-1.0000	-0.	-1.0000
5	5	23409.0000	-0.	-0.	-0.	-0.	-0.	-0.	-1.0000	-0.	-1.0000
6	6	-48.0000	-0.	1.0000	-0.	-0.	-0.	-0.	-0.	-0.	-0.
7	7	23409.0000	-0.	-0.	-0.	-0.	-0.	-0.	-1.0000	-0.	-1.0000
8	8	23409.0000	-0.	-0.	-0.	-0.	-0.	-0.	-1.0000	-0.	-1.0000
9	9	23409.0000	-0.	-0.	-0.	-0.	-0.	-0.	-1.0000	-0.	-1.0000
10	10	23409.0000	-0.	-0.	-0.	-0.	-0.	-0.	-1.0000	-0.	-1.0000
11	11	-96.0000	-0.	1.0000	-0.	-0.	-0.	-0.	-0.	-0.	-0.
12	12	-0.	-0.	-0.	1.0000	-0.	-0.	-0.	-0.	-0.	-0.
13	13	-0.	1.0000	-0.	-0.	-0.	-0.	-0.	-0.	-0.	-0.

3 UN NO NO OF DIRECTIONS RELATED TO EACH OTHER AND THE RELATION CONSTANTS

```
1   1 1 -0.   2 2 -0.   3 3 -0.   4 4 -0.   5 5 -0.   6 6 -0.   -0-0 -0.   -0 0 -0.
```

Table A-6. List of output data after successful execution for membrane and bending case

```
 1 -152.995995   0.          -0.004241   2 -153.000000   24.022748   -0.000257
 3 -152.998035  48.000000    -0.001542   4 -108.166550   0.        -108.203171
 5 -108.175476  24.022755  -108.199190   6 -108.189716  48.000000 -108.181431
 7    0.046825   0.        -152.997025   8    0.049793  24.022748 -152.999989
 9   -0.012536  48.000000  -152.996964  10 -108.193292   0.         108.180635
11 -108.193742  24.022755   108.180927  12 -108.192625  48.000000  108.179913
13  108.213968   0.        -108.157154  14  108.215780  24.022755 -108.158879
15  108.213968  48.000000  -108.157154  16   -0.017166   0.         152.999027
17   -0.018133  24.022748   152.999996  18   -0.017166  48.000000  152.999027
19  108.168350   0.         108.204182  20  108.169363  24.022755  108.205363
21  108.168350  48.000000   108.204182  22  152.998013   0.           0.032784
23  152.999996  24.022748     0.030901  24  152.998013  48.000000    0.032784
25 -152.999996  72.005980     0.000423  26 -152.998035  96.000000   -0.001542
27 -108.162280  72.005976  -108.192390  28 -108.189716  96.000000 -108.181431
29    0.049793  72.005980  -152.999989  30   -0.012536  96.000000 -152.996964
31 -108.194199  72.005959   108.180473  32 -108.191899  96.000000  108.179251
33  108.215782  72.005978  -108.158879  34  108.213968  96.000000 -108.157154
35   -0.021734  72.005905   152.999996  36   -0.018752  96.000000  152.996986
37  108.169422  72.005953   108.205247  38  108.167589  96.000000  108.203518
39  152.999996  72.005980     0.030801  40  152.998013  96.000000    0.032784

GENERATION OF FINAL MESH COORDINATES TOOK   17.95 SECONDS.
  11   11-0.      12   12-0.      13   13-0.      14   14-0.      15   15-0.
  16   16-0.      41   41-0.      42   42-0.      43   43-0.      44   44-0.
  45   45-0.      46   46-0.      71   71-0.      72   72-0.      73   73-0.
  74   74-0.      75   75-0.      76   76-0.     101  101-0.     102  102-0.
 103  103-0.     104  104-0.     105  105-0.     106  106-0.     131  131-0.
 132  132-0.     133  133-0.     134  134-0.     135  135-0.     136  136-0.
 161  161-0.     162  162-0.     163  163-0.     164  164-0.     165  165-0.
 166  166-0.     191  191-0.     192  192-0.     193  193-0.     194  194-0.
 195  195-0.     196  196-0.     221  221-0.     222  222-0.     223  223-0.
 224  224-0.     225  225-0.     226  226-0.     261  261-0.     262  262-0.
 263  263-0.     264  264-0.     265  265-0.     266  266-0.     281  281-0.
 282  282-0.     283  283-0.     284  284-0.     285  285-0.     286  286-0.
 301  301-0.     302  302-0.     303  303-0.     304  304-0.     305  305-0.
 306  306-0.     321  321-0.     322  322-0.     323  323-0.     324  324-0.
 325  325-0.     326  326-0.     341  341-0.     342  342-0.     343  343-0.
 344  344-0.     345  345-0.     346  346-0.     361  361-0.     362  362-0.
 363  363-0.     364  364-0.     365  365-0.     366  366-0.     381  381-0.
 382  382-0.     383  383-0.     384  384-0.     385  385-0.     386  386-0.
 401  401-0.     402  402-0.     403  403-0.     404  404-0.     405  405-0.
 406  406-0.
  -11201 100    1   1   4   5   2  -21201 100   1   2   5   6   3  -31201 100   1
   4   7   8   5  -41201 100   1   5   8   9   6  -51201 100   1  16  17  20  19
  -61201 100    1  17  18  21  20  -71201 100   1  19  20  23  22  -81201 100   1
  20  21  24  23  -91201 100   1   1   2  11  10 -101201 100   1   2   3  12  11
 -111201 100    1  10  11  17  16 -121201 100   1  11  12  18  17 -131201 100   1
   7  13  14   8 -141201 100   1   8  14  15   9 -151201 100   1  13  22  23  14
 -161201 100    1  14  23  24  15 -171201 100   1   3   6  27  25 -181201 100   1
  25  27  28  26 -191201 100   1   6   9  29  27 -201201 100   1  27  29  30  28
 -211201 100    1  18  35  37  21 -221201 100   1  35  36  38  37 -231201 100   1
  21  37  39  24 -241201 100   1  37  38  40  39 -251201 100   1   3  25  31  12
 -261201 100    1  25  26  32  31 -271201 100   1  12  31  35  18 -281201 100   1
  31  32  36  35 -291201 100   1   9  15  33  29 -301201 100   1  29  33  34  30
 -311201 100    1  15  24  39  33 -321201 100   1  33  39  40  34

NO. OF NODES    40   NO. OF BC    96   NO. OF ELTS    32                    END

GENERATION OF ELEMENT DATA TOOK               4.28 SECONDS.
```

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