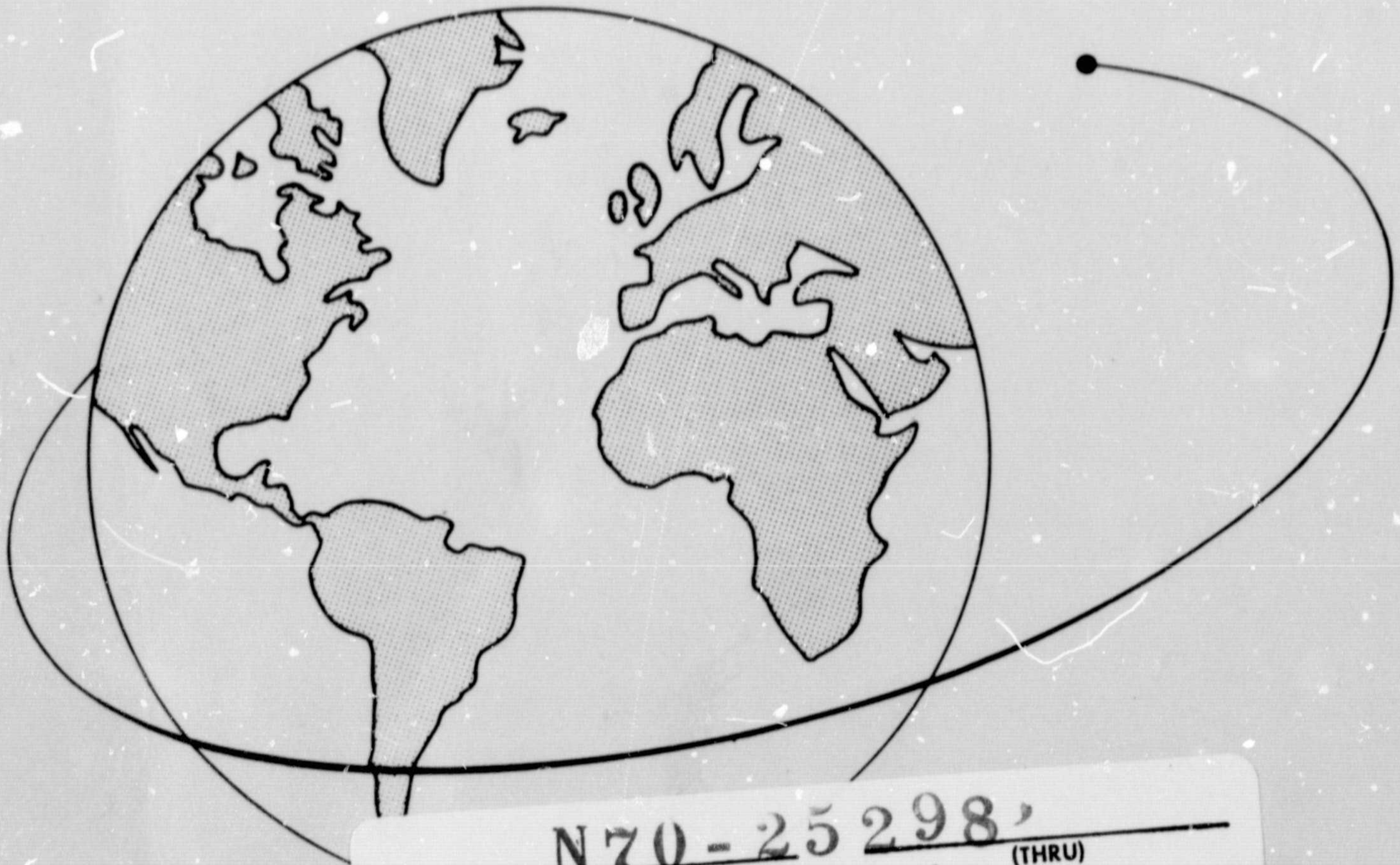
NGR-09-015-103
Smithsonian Institution

SEASONAL VARIATIONS OF THE GEOPOTENTIAL

Y. KOZAI



Smithsonian Astrophysical Observatory
SPECIAL REPORT 312

Research in Space Science
SAO Special Report No. 312

SEASONAL VARIATIONS OF THE GEOPOTENTIAL INFERRED FROM SATELLITE OBSERVATIONS

Yoshihide Kozai

April 2, 1970

Smithsonian Institution
Astrophysical Observatory
Cambridge, Massachusetts 02138

TABLE OF CONTENTS

ABSTRACT

Annual variation of J_2, the coefficient of the dominant term in the geopotential, is identified from the variations in the longitudes of the ascending nodes for two satellites. It is concluded that about half the seasonal variation of the length of day is due to a change in the principal moment of inertia.

RÉSUMÉ

La variation annuelle de J_2, le coefficient du terme principal dans le géopotentiel, est déterminée à partir des variations des longitudes des noeuds ascendants pour deux satellites. Nous en avons conclu qu'environ la moitié de la variation saisonnière de la longueur du jour est dûe à un changement du principal moment d'inertie.

КОНСПЕКТ

Годовое изменение J_2, коэффициента доминирующего члена геопотенциала, определено по изменению расстояний между восходящими точками пересечения орбит 2-х спутников. Было заключено, что сезонные изменения продолжительности дня наполовину объясняются изменением основного момента инерции.

SEASONAL VARIATIONS OF THE GEOPOTENTIAL INFERRED FROM SATELLITE OBSERVATIONS

Yoshihide Kozai

For the past decade, coefficients in the expressions of the geopotential have been derived by many investigators analyzing satellite observations under the assumption that the geopotential is time independent. Only combinations of observations made for different periods for different satellites provided sufficient information to determine the geopotential. There were too many unknowns to be determined, and not enough observations with sufficient accuracy and well distributed both in time and space to allow a search for time dependence in the geopotential.

The geopotential will vary, however, whenever mass displacements occur on or inside the earth because of tides, atmospheric motions, earthquakes, and other mass motions. It has already been possible from satellite observations to identify tidal effects, which are the perturbations in the orbits due to the tidal deformations of the earth (Newton, 1968; Kozai, 1968).

In this paper, efforts are made to identify the effects due to seasonal variation of J_2 from Baker-Nunn observations of two satellites, 1960 ι2 and 1962 $\beta\mu$1. Approximate values of the orbital elements, secular motions of the nodes, and periods of observations used for the two satellites are given in Table 1.

This work was supported in part by grant NGR 09-015-103 from the National Aeronautics and Space Administration.

Table 1. Orbital elements.

Satellite	n(rev/day)	i	e	$\dot{\Omega}$ (per day)	Period
1960 ι2	12.197	47°.23	0.0114	-3°.101	September 1960-December 1964
1962 βμ1	13.345	50°.14	0.0070	-3°.609	November 1962-December 1964

For the periods mentioned in Table 1, SAO has determined the mean orbital elements of the two satellites every 2 days by use of 4-day-interval observations. At each epoch, perturbations not included in the derivation of the mean orbital elements by the differential-orbit improvement program are evaluated and subtracted from the elements. For two angular elements, the argument of perigee and the longitude of the ascending node, the secular motions are also evaluated from the mean orbital elements, and (O-C) are derived for each epoch by assuming that (O-C) are zero at the initial epoch. Since the instantaneous values of the mean motion and the eccentricity are used to compute the secular motions, the air-drag effects in the two angular elements are eliminated.

From values of (O-C), corrections to the secular motions and the amplitudes of long-periodic terms due to zonal harmonics in the geopotential are derived. Then the residual computed for each element and for each epoch can be used to identify periodic perturbations due to other sources. In fact, the tidal perturbations could be identified in this way.

Autocorrelation functions computed from the residuals of the longitudes of the nodes for the two satellites show periodic terms of 1-yr period in the residuals; the following terms are derived:

$$\delta\Omega = \underset{\pm 4}{0^\circ.22} \times 10^{-3} \sin (2\pi t + \underset{\pm 10}{350^\circ}), \quad \text{for 1960 } \iota 2,\ T_0 = 1963.0$$

$$\delta\Omega = \underset{\pm 4}{0^\circ.30} \times 10^{-3} \sin (2\pi t + \underset{\pm 10}{330^\circ}), \quad \text{for 1962 } \beta\mu 1,\ T_0 = 1964.0 \quad . \tag{1}$$

The longitude of the ascending node is expressed approximately as

$$\Omega = - \int \frac{J_2}{p^2} n \cdot \cos i \cdot dt \quad . \tag{2}$$

If it can be assumed that there is an annual variation in J_2, that is, if J_2 in (2) can be replaced by

$$J_2 + \delta J_2 = J_2 + A \cos (2\pi t + \beta) \quad , \tag{3}$$

the following expression is derived:

$$\Omega = \dot{\Omega} \left[t + \frac{A}{2\pi J_2} \sin (2\pi t + \beta) \right] \quad , \tag{4}$$

where

$$\dot{\Omega} = - \left(\frac{J_2}{p^2} \right) n \cdot \cos i \quad . \tag{5}$$

By assuming that the annual variations in Ω are due to those in J_2, we get

$$\delta J_2 = \underset{\pm 2}{1.2} \times 10^{-9} \cos (2\pi t + \underset{\pm 10}{170°}), \qquad \text{for } 1960\ \iota 2,$$

$$\delta J_2 = \underset{\pm 2}{1.4} \times 10^{-9} \cos (2\pi t + \underset{\pm 10}{150°}), \qquad \text{for } 1962\ \beta\mu 1 \quad . \tag{6}$$

The results derived from the two satellites agree with each other within the errors, and if their mean is taken, the annual variation of J_2 is expressed as

$$\delta J_2 = \underset{\pm 2}{1.3} \times 10^{-9} \cos (2\pi t + \underset{\pm 10}{160°}) \quad , \tag{7}$$

where the epoch is at the beginning of the year. Because of this term, the geoid oscillates around the mean position by 8 mm at most with 1-yr period and J_2 takes the maximum value around July 20 and the minimum around January 20.

It is known that the rotation speed of the earth changes annually and that the expression of UT2-UT1 has the following terms:

$$0^s.025 \sin (2\pi t + 331°) + 0^s.009 \sin (4\pi t + 130°) \quad . \tag{8}$$

Therefore, the angular velocity ω of the earth changes as

$$\frac{\delta\omega}{\omega} = 0.50 \times 10^{-8} \cos (2\pi t + 331°) + 0.36 \times 10^{-8} \cos (4\pi t + 130°) \quad . \tag{9}$$

If we assume that the angular momentum $C\omega$ is conserved for the earth, we have the following expression:

$$\frac{\delta C}{C} + \frac{\delta\omega}{\omega} = 0 \quad . \tag{10}$$

Since the expression of J_2 is

$$J_2 = \frac{C - (A + B)/2}{M a_e^2} \quad , \tag{11}$$

by assuming that the mass M and the equatorial radius a_e are constant and $\delta A + \delta B + \delta C = 0$, we get

$$\delta J_2 = \frac{1}{2} \frac{\delta C}{C} = - \frac{1}{2} \frac{\delta\omega}{\omega} \quad , \tag{12}$$

since $C = M a_e^2/3$. Therefore, under the assumption that the angular momentum $C\omega$ is conserved, the anticipated seasonal variation of J_2 is computed as

$$\delta J_2 \times 10^9 = 2.5 \cos (2\pi t + 151°) + 1.8 \cos (4\pi t + 310°) \quad . \tag{13}$$

By comparing expression (13) with (7) derived from satellite observations, we see that the angular momentum is not conserved. However, since the amplitude of the annual term in (13) is twice as large as that in (7) and the phase angles are almost identical, about half the annual variation of the rotation speed may be due to the mass displacement expressed by (7), that is, the seasonal variation in the principal moment of inertia; the other half as well as semiannual variation is presumably due to wind in the atmosphere, since the semiannual variation cannot be found in J_2 from the satellite observations.

There is also a possibility that J_3 changes in a similar way. If J_3 can be replaced by

$$J_3 + \delta J_3 = J_3 + B \cos (2\pi t + \gamma) \quad , \tag{14}$$

long-periodic perturbations with $2\pi t \pm \omega + \gamma$ as arguments appear in the orbital elements. Efforts have been made to identify such perturbations from the residuals in the orbital elements, but no definitive results have yet been obtained. This would suggest that B is less than 10^{-9} even if it exists.

When much more accurate observations become available and the perturbations caused by solar-radiation pressure can be computed with the same accuracy, secular and irregular variations of J_2 as well as variations in nonzonal terms in the geopotential can be identified from satellite observations.

REFERENCES

KOZAI, Y.

1968. Love's number of the earth derived from satellite observations. Publ. Astron. Soc. Japan, vol. 20, pp. 24-26.

NEWTON, R. R.

1968. A satellite determination of tidal parameters and earth deceleration. Geophys. Journ. Roy. Astron. Soc., vol. 14, pp. 505-539.

BIOGRAPHICAL NOTE

YOSHIHIDE KOZAI received the M.S. and D.S. degrees from Tokyo University in 1951 and 1958, respectively. He has been associated with the Tokyo Astronomical Observatory since 1952 and has held concurrent positions as staff astronomer with that observatory and consultant to SAO since 1958.

Dr. Kozai specializes in celestial mechanics, his research at SAO being primarily in the determination of zonal harmonics coefficients in the earth's gravitational potential by using precisely reduced Baker-Nunn observations. He is also interested in the seasonal variability of the earth's potential. Using satellite-tracking data in studies of the earth's geopotential, he has derived new values for the coefficients of the zonal harmonics of the earth's gravitational field.