NASA CR 109628

MEMORANDUM #2

RELATIONSHIP OF CHANGES IN THE HEAT INPUT TO THE POWER OUTPUT OF A THERMOELECTRIC DEVICE

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RELATIONSHIP OF CHANGES IN THE HEAT INPUT TO THE POWER OUTPUT OF A THERMOELECTRIC DEVICE

July 30, 1969

Prepared For:

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JPL CONT. GP-505281

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This work was performed for the Jet Propulsion Laboratory, California Institute of Technology, sponsored by the National Aeronautics and Space Administration under Contract NAS7-100.

INTRODUCTION

It is well known that the electrical power output of a thermoelectric device is proportional to the square of the temperature difference across the thermoelements. It is sometimes assumed that the heat traversing the device is proportional to the first power of the temperature difference across the thermoelements and therefore that the electrical power output is doubly changed for a given change in the heat input. This assumption is only valid in the limit of a vanishing current inasmuch as it neglects all current dependent effects. Other equally unacceptable assumptions are frequently made with the result that considerable confusion exists on the heat input dependence of the power output of a thermoelectric device. The purpose of the present memorandum is to investigate this problem in somewhat more detail than is done usually and to derive a relationship that correctly relates the heat input to the power output of a thermoelectric device.

TYPICAL ANALYSES

Before proceeding with the analysis that correctly relates the heat input to the power output of a thermoelectric device, it may be worthwhile to consider two typical, albeit incorrect, approaches to the problem. For convenience, all calculations will be performed on the basis of single thermocouples. The generalization of the results to multi-thermocouple devices is straightforward.

Probably the most common method used for relating the heat input to the power output of a thermoelectric device assumes that all of the heat traversing the thermocouples is transported by conduction. Thus, it is assumed that

$$Q = K\Delta T, \qquad (1)$$

where Q is the total heat input to the thermocouples, K is the thermal conductance of the thermocouples and ΔT is the temperature difference between the hot and cold junctions of the thermocouples. The power output P of a thermocouple is given in terms of current I and load resistance R_L as

$$P = I^2 R_L . \tag{2}$$

The current I may be defined as

$$I = \frac{S\Delta T}{R(1+m)} , \tag{3}$$

where S is the combined average Seebeck coefficient of the n- and p-type thermoelements, R is the total internal electrical resistance of the thermocouple and $m = R_L/R$ is the ratio of load to internal electrical resistance. The combination of Eqs. (1), (2) and (3) and their differentials results in the following relationship between changes in the power output and heat input to a thermocouple:

$$\frac{dP}{P} = 2\frac{dQ}{Q} \tag{4}$$

Equation (4) shows that changes in the heat input result in a doubly changed power output of a thermocouple. This result of course is incorrect because the heat balance, Eq. (1), neglects all current dependent effects such as Peltier, Joule and Thomson heating. Only in the limit of vanishing current, infinite load resistance, does it apply.

Another analysis that yields an incorrect relationship between the heat input and power output of a thermoelectric device makes use of the efficiency expression as the starting point. The efficiency η of a thermoelectric device may be defined as

$$\eta = \frac{P}{Q} . \tag{5}$$

Differentiation of Eq. (5) and division by η shows that

$$\frac{dP}{P} = \frac{d\eta}{\eta} + \frac{dQ}{Q} \quad . \tag{6}$$

It is now assumed that efficiency may be expressed as the product of a Carnot efficiency η_C and a material efficiency η_M, with the latter independent of temperature. Although not rigorous, this assumption is reasonable and is based on the traditional analysis of the fixed operating temperature performance of a thermoelectric device (see, for example, A.F. Ioffe, Semiconductor Thermoelements and Thermoelectric Cooling, Infosearch, London 1957). As a result of this assumption $d\eta/\eta$ may now be written

$$\frac{d\eta}{\eta} = \frac{d\eta_C}{\eta_C} = \frac{1}{T_H - T_C}\left[\frac{T_C}{T_H} dT_H - dT_C\right] , \tag{7}$$

where T_H and T_C represent thermocouple hot and cold junction temperatures respectively and use has been made of the definition of Carnot efficiency, $\eta_C = \left(T_H - T_C\right)/T_H$. Assuming that the cold junction temperature is constant and rewriting $T_C/T_H = 1-\eta_C$, Eq. (7) becomes

$$\frac{d\eta}{\eta} = \left(1-\eta_C\right) \frac{dT_H}{\Delta T} , \tag{8}$$

where the abbreviated notation $\Delta T = T_H - T_C$ has been used. With the assumption of a fixed cold junction temperature, an inspection of Eq. (1) shows that $dT_H/\Delta T$ may be replaced by dQ/Q. Making this replacement in Eq. (8) and substituting the resultant equation in Eq. (6), it is possible to write the final result as

$$\frac{dP}{P} = \left(2-\eta_C\right) \frac{dQ}{Q} \quad . \tag{9}$$

It should be noted that although Eq. (9) in some cases yields more accurate answers than the corresponding Eq. (4), it itself, in making use of Eq. (1), neglects all current dependent effects in the thermocouple and therefore is subject to the same shortcomings as Eq. (4). Moreover, the assumption of a fixed cold junction temperature is highly questionable and therefore, if anything, the second derivation is even more unacceptable than the first. In any case, both derivations are grossly inaccurate and their results should consequently never be used in serious engineering work in thermoelectrics.

CORRECT ANALYSIS

The main problem with the typical analyses of the relationship between the heat input and the power output of a thermocouple as given above is the over-simplification of the heat input equation, Eq. (1). As pointed out, the neglect of all current dependent effects in Eq. (1) is unjustified and unacceptable. A more meaningful representation of the heat input to a thermocouple is given by (see V. Raag, Eng. Conv. 8, 173 (1968))

$$Q = K\Delta T + IS_H T_H - \{1/2\} I^2 R - \{1/2\} I \mu \Delta T, \quad (10)$$

where S_H pertains to the combined Seebeck coefficient of the n- and p-type thermoelements at the hot junction temperature of the thermocouple and μ is the average combined Thomson coefficient of the n- and p-type thermoelements. The rest of the symbols have previously been defined. The use of Eq. (3) in Eq. (10) enables the elimination of the ΔT's, and the resultant quadratic equation in current may be solved to yield

$$I = A \left\{ 1 - \left[1 - \frac{Q}{A^2 B} \right]^{1/2} \right\}, \quad (11)$$

where

$$A = \frac{KR\,(1+m)+SS_H T_H}{R\left[S+\mu\,(1+m)\right]}\,, \qquad (12)$$

$$B = \frac{R}{2S}\left[S+\mu\,(1+m)\right]\;.$$

The substitution of Eq. (11) in Eq. (2), differentiation and simplification enables the relationship between changes in the heat input and the power output of a thermocouple to be expressed as

$$\frac{dP}{P} = 2\left\{\left[1-\frac{2Q}{\alpha A^2 B}\right]\frac{Q}{A}\,\frac{dA}{dQ} + \frac{Q}{\alpha A^2 B}\right\}\frac{dQ}{Q}\,, \qquad (13)$$

where α is defined as

$$\alpha = 2\left(1-\frac{Q}{A^2 B}\right)^{1/2}\left[1-\left(1-\frac{Q}{A^2 B}\right)^{1/2}\right]. \qquad (14)$$

The derivative of A with respect to heat input to the thermocouple may be written

$$\frac{dA}{dQ} = \frac{SS_H}{R\left[S+\mu\,(1+m)\right]}\,\frac{dT_H}{dQ}\,, \qquad (15)$$

where it has implicitly been assumed that the thermoelectric properties are independent of changes in the heat input and consequently independent of changes in the operating temperatures. This same assumption underlies Eq. (13) and is valid for reasonably small changes in heat input and operating temperatures. If temperature independent thermoelectric properties were to be used in the analysis, then of course Eqs. (13) and (15) would be exact. In this case, however, the Thomson coefficient would vanish and S and S_H would become identical. In the treatment as given, it is assumed that all thermoelectric properties represent temperature integrated averages.

The use of the expression for the power output of the thermocouple enables the derivative of the hot junction temperature of the thermocouple with respect to the heat input to be expressed as

$$\frac{dT_H}{dQ} = \frac{T_C}{Q}\left[\frac{\Delta T}{2T_C}\ \frac{dP}{P} + \frac{dT_C}{T_C}\right]\frac{Q}{dQ} \quad . \tag{16}$$

The substitution of Eqs. (16) and (15) in Eq. (13) permits the relationship between changes in the heat input and the power output of a thermocouple to be finally written as

$$\frac{dP}{P} = 2\left[\frac{\beta + \Gamma Q\ \frac{dT_C}{dQ}}{1-\Gamma\Delta T}\right]\frac{dQ}{Q} \ , \tag{17}$$

where β and Γ are defined by

$$\beta = \frac{Q}{\alpha A^2 B} \ , \tag{18}$$

$$\Gamma = \left[1-2\beta\right]\left[\frac{SS_H}{KR(1+m) + SS_H T_H}\right] .$$

It is noted that the terms entering Eq. (17) are functions of thermocouple operating temperatures, dimensions, thermoelectric properties, heat input and load characteristics. The use of Eq. (17) thus enables the determination of the effect of changes in heat input on the power output of a thermocouple. The derivative of the cold junction temperature with respect to heat input in Eq. (17) depends on system heat rejection characteristics and thus in a general treatment of the present type cannot be precisely specified. In case of heat rejection by radiation, such as in space thermoelectric systems, the term may be written as

$$\frac{dT_C}{dQ} = \frac{T_C}{4Q} \ , \tag{19}$$

where the sink temperature has been assumed to be negligible in comparison to the thermocouple cold junction temperature.

Attention is finally called to the fact that the present treatment is not only farily rigorous within its presently intended scope, but also reasonably represents the behavior of actual thermoelectric devices when typically experienced thermal and electrical losses are included in the thermal conductance K and the electrical resistance R of the thermocouple. Thermoelectric generator thermal end losses can approximately be taken into account by including them in the total heat input.

ILLUSTRATION

A silicon-germanium Air-Vac thermocouple* has been selected for the purpose of numerically illustrating the results of the preceding discussion. The thermocouple component dimensions are given as follows:

Thermoelement length	2.54 cm
Cross-sectional area	
n-type thermoelement	0.2101 cm^2
p-type thermoelement	0.1168 cm^2
Hot shoe area	4.00 cm^2
Radiator area per couple	15.00 cm^2

The heat input into the thermocouple is 5.40 watts, this resulting in the approximate operating temperatures of 1000° C and 260° C for the hot and cold junctions respectively at the maximum power output point. Using thermoelectric property data that pertain to those extant initially in silicon-germanium alloys, the performance of the thermocouple as a function of load current may be shown to be given by the solid curve in Figure 1. If the heat input into the thermocouple is decreased to 4.93 watts, the operating temperatures of the thermocouple decrease and the performance is given by the dashed curve in Figure 1. The reduction of heat input to the thermocouple from 5.40 watts to 4.93 watts represents the decay experienced with a Pu-238 heat source after

* The same thermocouple configuration was derived and discussed in Memorandum #1, prepared by V. Raag, RESALAB SCIENTIFIC, for JPL, April 29, 1969.

some 11.4 years of operation. It is noted from Figure 1 that for the given decrease in heat input, the decrease in power output is load dependent, as predicted by Eq. (17) of the preceding section. Using Eq. (17), it may be calculated that for the thermocouple in question, the dependence of power output on heat input as a function of the ratio of load to internal electrical resistance m is given as follows:

m	0	0.5	1.0	1.5	3.0	10.0	∞
$\frac{dP}{P}/\frac{dQ}{Q}$	1.58	1.67	1.72	1.74	1.83	1.93	2.00

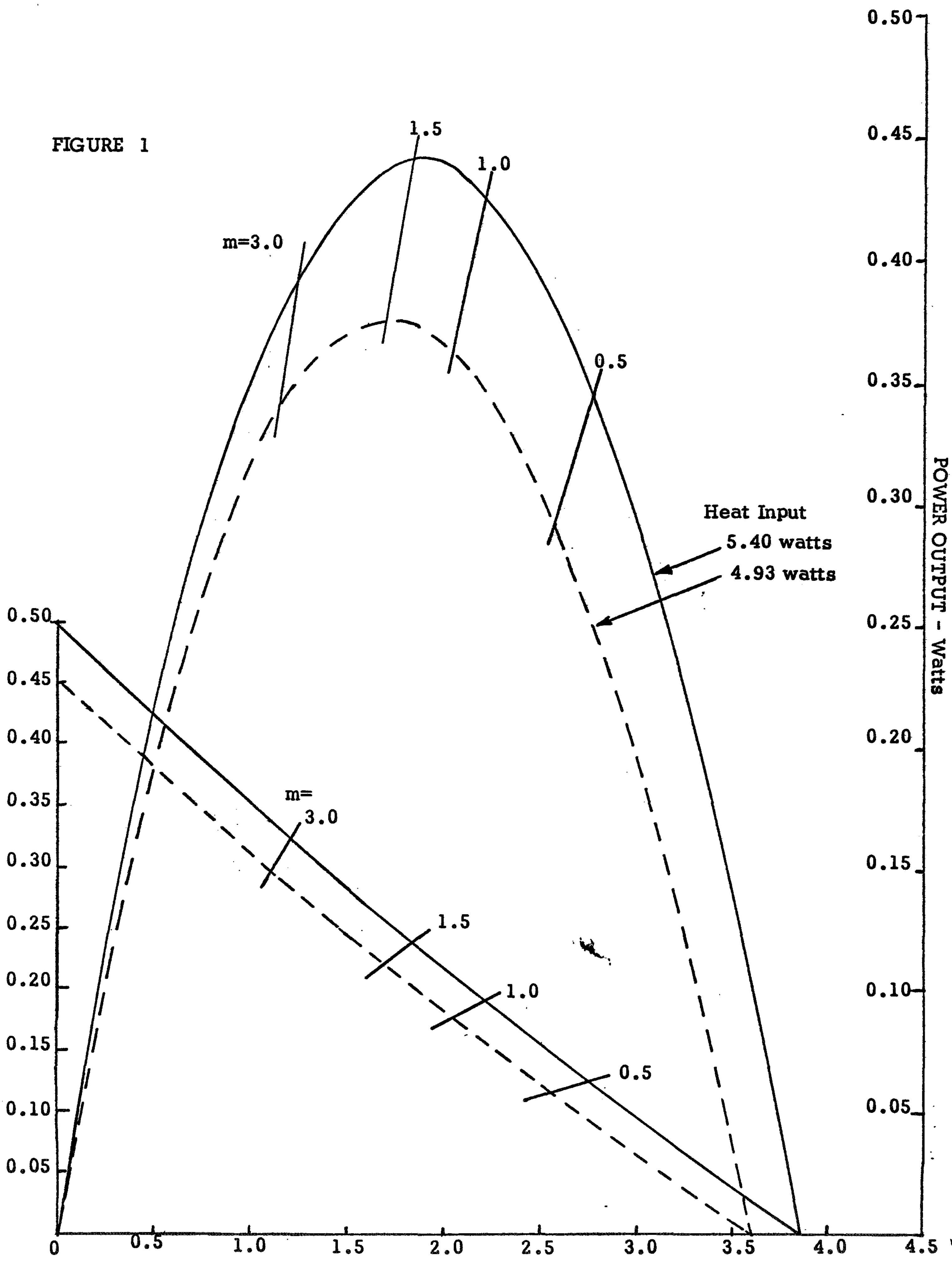
The agreement of the calculated values of (dP/P)/(dQ/Q) with those obtainable from Figure 1 is good. It should be noted that the internal resistance of the thermocouple in this example is 85.9 milliohms. From the above table it is seen that dependence of changes in power output on heat input near the maximum power point is less than a factor of two, being of the order of 1.70 to 1.75. Only when m becomes very large, the limit being the open circuit operation of the thermocouple, does the power output have a square dependence on heat input. The reason for this is that under open circuit operating conditions, the heat balance, as given by Eq. (10), only possesses the conduction term and therefore changes in the temperature differential across the thermoelements are directly proportional to changes in heat input. Power output, however, changes as the second power of the temperature differential across the thermoelements and therefore also as the second power of heat input.

A comparison of the results obtained with the present analysis and those calculable by means of more approximate methods such as those discussed in the Section entitled "Typical Analyses", shows the inaccuracy of the results commonly obtained. For example, in the illustration just discussed, it is found that the present analysis yields a value of 1.73 for (dP/P)/(dQ/Q) at the maximum power output point (m = 1.3). The two common methods of analysis discussed under "Typical Analyses" give the respective values of 2.00 and 1.42 for the same case. The fallacy is obvious.

SUMMARY

A new analysis has been developed for relating changes in the heat input to the power output of a thermoelectric device because most existing treatments of this problem are completely inadequate. A numerical example has been used to illustrate the results obtainable with the new method and also to indicate the gross difference between the present results and those begotten by the methods previously used.

FIGURE 1



0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
m=3.0
1.5
1.0
0.5
m=
3.0
1.5
1.0
0.5
Heat Input
5.40 watts
4.93 watts
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
POWER OUTPUT - Watts