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STUDY OF INCOMPLETE MODELS OF DYNAMIC STRUCTURES

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Prepared by

Kaman Aerospace Corporation
Bloomfield, Connecticut

for

Goddard Space Flight Center
Greenbelt, Maryland

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SUMMARY

This report presents the theoretical development and a qualitative evaluation of a new concept in the mathematical modeling of dynamic structures. The method has several unique features. It uses both test data and analytical approximations to identify the parameters of what is termed an "incomplete model". The technique makes one of the first substantial applications of the matrix pseudo inverse to structural dynamics. The "incomplete model" itself is unusual in that, while the mass matrix and damping coefficient are physically meaningful, the stiffness matrix is, in a sense, not physically related to the true (but unknown) values. The model does, however, have the capability of predicting the response of the points of interest on the structure over the frequency range of interest and can be used to predict the changes in natural frequencies and normal modes due to structural changes. It is the ability to correctly predict the effects of structural changes which have been tested in the work presented in this report.

The theory was tested by running simulated tests on a relatively simple structure, identifying the parameters of the incomplete model, and using this model to predict

the effects on frequency and mode shapes of several mass and stiffness changes. The conditions of the tests were varied by selecting different numbers of points of measurement, varying the frequency range, and by including assumed measurement error. Several analytical approximations were used to formulate the incomplete model.

Based on the qualitative evaluation made, the following conclusions are indicated.

1. The method presented appears to be an excellent tool for the prediction of changes in normal modes and natural frequencies due to structural changes.

2. The predicted effects are quite insensitive to reasonable variations in the analytical mass distribution.

3. The procedure is relatively insensitive to measurement error.

4. Attempts to make the model more complete by using a greater frequency range for testing will not always be successful unless the number of points of measurement are also increased or off-diagonal masses are added. There is a point in frequency beyond which the model will tend to deteriorate.

Because of the success of this preliminary evaluation, it is recommended that the theoretical development be continued and that applications to more complex structures be carried out in order to develop a better understanding of the limitations and capabilities of the method. A successful, more definitive evaluation, could lead to immediate practical applications.

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INTRODUCTION

The need for mathematical models of aerospace structures is obvious. No attempt to determine a structure's suitability for its intended mission could be made without a means of predicting its response to the expected loads. No rational means of improvement or optimization would be available without some ability to predict the effects of structural changes on the response of the structure.

Purely analytical modeling is necessary in the preliminary design stage of any structure. The evaluation of alternative basic designs, the selection of materials, the arrangement of components are some of the problems which must be solved prior to construction of a test specimen. The science of structural analysis has progressed to the point where such problems may be treated with reasonable confidence.

Prior to usage, any critical structure must be submitted to actual testing, since the analysis, after all, is only based on a hypothetical model of the system. An analytical model is as important at this stage as it was in the preliminary design. The actual mission force environment cannot economically be duplicated in test and the effects of possible changes cannot all be tested.

It would be reassuring if the results of dynamic testing could be used to give an analytical model of the actual structure. To date, however, there is no generally accepted method applicable to the dynamics of structures. In addition, it appears that the unique identification of an analytical model from test data alone must be limited to rather special conditions.

The number of degrees of freedom of a linear model of a dynamic system is equal to the number of independent spatial coordinates and also to the number of normal modes of the system. The response of any point on the structure is the superposition of the responses of each of the normal modes at that point. It is recognized that in linear systems, measurable excitation of each normal mode is only achieved by forces having frequencies near or above the natural frequency of the normal mode in question. Thus, in order that test data contain the information necessary for the unique identification of a linear model having a specified number of degrees of freedom, it is necessary that the applied forces must have had significant components at frequencies up through the same number of natural frequencies. Data obtained from smaller ranges of frequencies of excitation would not contain sufficient significant information for such a unique identification.

There are conditions where it is possible to perform a unique identification. One is where the number of points of interest on the structure is small and it is feasible to excite the structure over the appropriate frequency range. Such a situation is treated in Reference 1. Another condition is where the major portion of the motions are due to rigid body responses and there is in effect only six physical degrees of freedom. This is the primary area of past applications of identification techniques to such problems as determining aerodynamic stability derivatives.

The problem considered in this report is concerned with what is probably the more common situation when the application is made to elastic structures. It is taken to be impossible, uneconomical, or generally undesirable to excite the structure and measure responses over a sufficiently wide frequency range so as to provide the information necessary for the identification of a unique linear analytical model.

In this report, the concept of an "incomplete model" is developed. This is an analytical model which has fewer normal modes than coordinates and is valid only over a limited frequency range. It is shown how the parameters may be determined using test data together with analytical approximations. The attributes of the model are discussed and certain of these are tested using computer simulated test data. It is shown that it is possible to determine a rational mass matrix and structural damping coefficient and that it is possible to use the incomplete model to predict the changes in frequencies and normal modes due to mass and stiffness changes. It is further shown that the procedures are relatively insensitive to measurement error.

This research project has demonstrated a new and useful analytical tool. It is suggested that further development of the theory and methods of application would result in procedures which will lead to better understanding of structures and increased capability to improve and optimize the design of aerospace vehicles.

I. DESCRIPTION OF THE PROBLEM

An aspect of the following problem is considered in this report. Consider a structure which is to be subjected to dynamic testing. There are certain deflections and rotations of points on the structure which are of interest. It is desired to be able to predict each of these motions under various loading conditions for the structure actually tested and for modified versions of the structure.

It is assumed that an analytical model consisting of P lumped masses interconnected by linear springs with scalar structural damping will be adequate to represent the deformation of the structure under consideration (the more general mass matrix with off-diagonal terms is not excluded). The motion of each of the P masses is considered to represent the motion of P points on the structure including the points of interest. It is also assumed that the loading conditions of interest will contain primarily force components at frequencies below some finite value including less than P natural frequencies of the model. It is required that the analytical model faithfully represent the dynamics of the "points of interest" over the "frequency range of interest". It is further required that the model have the capability of predicting the changes in response due to structural changes including mass, stiffness and support changes.

The work reported here is limited in scope as follows. The analysis assumes knowledge of the normal mode shapes at each of P points and natural frequencies of the structure through the frequency range of interest (Knowledge of the modal masses is not required). It is considered that this data has been obtained from testing. The P points are distributed over the structure so as to represent the characteristics of the measured modes and include all the points of interest. In the rest of this report, all these points will be considered to be the "points of interest".

In the computer experiments described below, the dissipative component of the resonant response was taken as a good approximation to the normal mode. However, other more sophisticated procedures such as those given in References 1 or 2 might be justified, for example, if the resonances were not well separated. In addition, it is assumed that a "reasonable" analytical model of the mass matrix of the structure has been derived through analytical or intuitive means. Using these assumed data, a procedure is developed for identifying the parameters in the equations of motion such that the model has the capability of predicting the effects of changes in mass and stiffness on natural frequencies and modes.

II. BASIC RELATIONSHIPS

The material presented in this section is not new but is derived here in the form in which it will be referenced in following sections of this report. Definitions of all the symbols used are summarized in the Glossary of Symbols.

The matrix equation of the spring-mass-structural damping model discussed in the previous section may be written (see, for example, Reference 3)

$$M\ddot{y}(t) + (1 + ig)Ky(t) = f(t) \tag{1}$$

$\ddot{y}$, y, f are column matrices (vectors) representing the acceleration, displacement (or slope), applied force (or moment) as a function of time at each of the P points of interest. M and K are PxP symmetric matrices representing the mass and stiffness coefficients of the model. i is $\sqrt{-1}$ and g is the structural damping coefficient. The imaginary term is used here to indicate that the damping is in phase with the velocity but proportional to the displacement. Under steady state sinusoidal oscillation at a frequency, ω, the equation becomes

$$\{-\omega^2 M + (1 + ig)K\}y = f \tag{2}$$

where f may be thought of as the amplitude of the applied forces and y as the amplitude of the displacements. y will, in general, be complex because of the phase between the force and displacement.

Writing this equation in terms of velocity ($\dot{y} = i\omega y$)

$$\{\frac{g}{\omega}K + i(\omega M - \frac{1}{\omega}K)\}\dot{y} = f \tag{3}$$

the (velocity) impedance matrix is given by

$$Z = \frac{g}{\omega}K + i(\omega M - \frac{1}{\omega}K) \tag{4}$$

and the mobility matrix by

$$Y = Z^{-1} = \{\frac{g}{\omega}K + i(\omega M - \frac{1}{\omega}K)\}^{-1} \tag{5}$$

It is interesting to note that the quantity directly obtained through analysis is the impedance, Z, while the quantity measured in testing is the mobility, Y.

Consider now, the eigenvalue problem corresponding to Equation (2).

$$\{K - \Omega_i^2 M\}\phi_i = 0 \qquad i = 1,2,\ldots P \tag{6}$$

where Ω_i are the natural frequencies and ϕ_i are the normal modes of the system. This equation may be written in two ways:

$$M^{-1}K\phi_i = \Omega_i^2\phi_i \qquad i = 1,2,\ldots P \tag{7}$$

or

$$CM\phi_i = \frac{1}{\Omega_i^2}\phi_i \qquad i = 1,2,\ldots P \tag{8}$$

where C, the influence coefficient matrix, equals K^{-1}. The orthogonality relationship is given by

$$\begin{aligned} \phi_i^T M \phi_j &= 0 \qquad j \neq i \\ &= m_i \qquad j = i \end{aligned} \tag{9}$$

It is important to note that the normal modes (eigenvectors) of $M^{-1}K$ and its inverse, CM, are the same and that the respective eigenvalues are reciprocals. The dominant mode of $M^{-1}K$ is the one having the highest frequency and the dominant mode of CM is the one having the lowest frequency.

It is convenient to write (7), (8), (9) using the Φ matrix where

$$\Phi = [\phi_1 \quad \phi_2 \quad \ldots \phi_p] \tag{10}$$

is a square PxP matrix. Then

$$M^{-1}K\Phi = \Phi\lceil\Omega_i^2\rfloor \tag{11}$$

$$CM\Phi = \Phi\left\lceil\frac{1}{\Omega_i^2}\right\rfloor \tag{12}$$

and

$$\Phi^T M \Phi = \lceil m_i \rfloor \tag{13}$$

From each of the above three equations, one can write

$$K = M\Phi \left[\diagdown \Omega_i^2 \diagdown\right] \Phi^{-1}$$

$$C = \Phi \left[\diagdown \frac{1}{\Omega_i^2} \diagdown\right] \Phi^{-1} M^{-1}$$

and

$$\Phi^{-1} = \left[\diagdown \frac{1}{m_i} \diagdown\right] \Phi^T M$$

Substituting the third equation into the first two

$$K = M\Phi \left[\diagdown \frac{\Omega_i^2}{m_i} \diagdown\right] \Phi^T M \tag{14}$$

$$C = \Phi \left[\diagdown \frac{1}{\Omega_i^2 m_i} \diagdown\right] \Phi^T \tag{15}$$

These two equations may be written in the equivalent form

$$K = \sum_{i=1}^{p} \frac{\Omega_i^2}{m_i} M\phi_i \phi_i^T M \tag{16}$$

$$C = \sum_{i=1}^{p} \frac{1}{\Omega_i^2 m_i} \phi_i \phi_i^T \tag{17}$$

In connection with the discussion in Section VI, these may be written in terms of principal idempotents (See Appendix I) as follows

$$M^{-1}K = \sum_{i=1}^{p} \Omega_i^2 \left(\frac{1}{m_i}\phi_i \phi_i^T M\right) \tag{18}$$

$$CM = \sum_{i=1}^{P} \frac{1}{\Omega_i^2}\left(\frac{1}{m_i}\phi_i\phi_i^T M\right) \tag{19}$$

Note that the product $\phi_i\phi_i^T$ appearing in Equations (16) - (19) are square matrices of order P but rank 1. When P of the matrices of this type are summed as indicated, there result PxP matrices of rank P which are thus nonsingular.

The impedances and the mobility may also be expressed in terms of the normal modes. Write Equation (4) as

$$Z = \frac{1}{\omega}(g - i)K + i\omega M$$

Use Equation (14) for K and factor M out on both sides and the equation becomes

$$Z = M\{\frac{1}{\omega}(g - i)\Phi \left[\frac{\Omega_i^2}{m_i}\right]\Phi^T + i\omega M^{-1}\}M$$

From (13), one can write

$$M = \Phi^{-T}\lceil m_i \rfloor \Phi^{-1}$$

or

$$M^{-1} = \Phi\left[\frac{1}{m_i}\right]\Phi^T$$

Substituting into the above equation for Z and factoring the Φ and Φ^T matrices, there results

$$Z = M\Phi\{\frac{1}{\omega}(g - i)\left[\frac{\Omega_i^2}{m_i}\right] + i \left[\frac{1}{m_i}\right]\}\Phi^T M$$

or

$$Z = \frac{1}{\omega} M\Phi \left\lceil \frac{g\Omega_i^2 + i(\omega^2 - \Omega_i^2)}{m_i} \right\rfloor \Phi^T M \tag{20}$$

and in summation form

$$Z = \frac{1}{\omega} \sum_{i=1}^{p} \frac{\Omega_i^2 \{ g + i [(\frac{\omega}{\Omega_i})^2 - 1] \}}{m_i} M\phi_i \phi_i^T M \tag{21}$$

The mobility can be written as the inverse of the impedance, from Equation (20):

$$Y = Z^{-1} = \omega \{ M\Phi \left\lceil \frac{g\Omega_i^2 + i(\omega^2 - \Omega_i^2)}{m_i} \right\rfloor \Phi^T M \}^{-1}$$

$$= \omega (\Phi^T M)^{-1} \left\lceil \frac{m_i}{g\Omega_i^2 + i(\omega^2 - \Omega_i^2)} \right\rfloor (M\Phi)^{-1}$$

but from (13)

$$(\Phi^T M)^{-1} = \{ \lceil m_i \rfloor \Phi^{-1} \}^{-1} = \Phi \lceil \frac{1}{m_i} \rfloor$$

and

$$(M\Phi)^{-1} = \{ \Phi^{-T} \lceil m_i \rfloor \}^{-1} = \lceil \frac{1}{m_i} \rfloor \Phi^T$$

then

$$Y = \omega \Phi \left\lceil \frac{1}{m_i [g\Omega_i^2 + i(\omega^2 - \Omega_i^2)]} \right\rfloor \Phi^T \tag{22}$$

or in summation form

or

$$Y = \omega \sum_{i=1}^{p} \frac{g - i\left[\left(\frac{\omega}{\Omega_i}\right)^2 - 1\right]}{\left[\left(\frac{\omega}{\Omega_i}\right)^2 - 1\right]^2 + g^2} \frac{1}{\Omega_i^2 m_i} \phi_i \phi_i^T \qquad (23)$$

III. THE INCOMPLETE MODEL

The structure under consideration is represented analytically by g, M, K (see Equation (1)). Consider, for the time being, that g, M are known. It was seen that K and its inverse, C, could each be written as a sum of matrices containing the eigenvectors (Equations (16), (17)).

$$K = \sum_{i=1}^{P} \frac{\Omega_i^2}{m_i} M\phi_i\phi_i^T M \tag{24}$$

$$C = K^{-1} = \sum_{i=1}^{P} \frac{1}{\Omega_i^2 m_i} \phi_i\phi_i^T \tag{25}$$

and the impedance and mobility could be written in similar forms (Equations (21), (23))

$$Z = \frac{1}{\omega} \sum_{i=1}^{P} \frac{\Omega_i^2}{m_i}\{g + i[(\frac{\omega}{\Omega_i})^2 - 1]\}M\phi_i\phi_i^T M \tag{26}$$

$$Y = Z^{-1} = \omega \sum_{i=1}^{P} \frac{1}{\Omega_i^2 m_i} \frac{g - i[\left(\frac{\omega}{\Omega_i}\right)^2 - 1]}{[\left(\frac{\omega}{\Omega_i}\right)^2 - 1]^2 + g^2} \phi_i\phi_i^T \tag{27}$$

All the square matrices (K, C, Z, Y, M) are of order P, the number of degrees of freedom, and the ϕ vectors have P elements. The square matrices $\phi_i\phi_i^T$ are each PxP but are of rank 1 and are thus individually singular. Since the ϕ_i's are linearly independent, a linear combination of P of these simple products will be of rank P and thus the summations will be nonsingular (See Reference 4, p.6).

If g, M and all the normal modes of the system were known, the behavior of the system could be predicted by forming the complete equations of motion. The question posed is this: If incomplete information is available, i.e. only the first N normal modes, is it possible to generate a mathematical model that will give useful information about the behavior of the system and modifications of it?

The following postulate is advanced: <u>The K, C, Z, Y matrices obtained by using less than P terms can be used to predict the behavior of the structure and certain modifications of it</u>. Part of the purpose of this report is to test aspects of the above postulate. The analytical model described by the incomplete summations will be called an <u>incomplete model</u>. These matrices will be written

$$K_{inc} = \sum_{i=1}^{N} \frac{\Omega_i^2}{m_i} M\phi_i \phi_i^T M \tag{28}$$

$$C_{inc} = \sum_{i=1}^{N} \frac{1}{\Omega_i^2 m_i} \phi_i \phi_i^T \tag{29}$$

$$Z_{inc} = \frac{1}{\omega} \sum_{i=1}^{N} \frac{\Omega_i^2}{m_i} \{g + i[(\frac{\omega}{\Omega_i})^2 - 1]\} M\phi_i \phi_i^T M \qquad (30)$$

$$Y_{inc} = \omega \sum_{i=1}^{N} \frac{1}{\Omega_i^2 m_i} \frac{g - i[(\frac{\omega}{\Omega_i})^2 - 1]}{[(\frac{\omega}{\Omega_i})^2 - 1]^2 + g^2} \phi_i \phi_i^T \qquad (31)$$

Certain characteristics of the incomplete model are apparent:

(1) Since the terms containing the higher values of Ω_i are not included, the dominant terms of K and Z will be missing and thus K_{inc} and Z_{inc} will not resemble the true K and Z matrices.

(2) Conversely, the dominant terms of C and Y are included in C_{inc} and Y_{inc}. These are the matrices which represent the responses due to applied forces and for the model to have validity, it is necessary that they approach the true values for $\omega < \Omega_N$.

(3) The four matrices are of order P (and represent the P points of interest) but are of rank N. Thus, they are all singular and they must be all formed separately and not by inversion.

(4) The eigenvalue equation from (28) or (29) can be seen to be

$$M^{-1}K_{inc}\phi_j = \sum_{i=1}^{N} \frac{\Omega_i^2}{m_i} \phi_i\phi_i^T M\phi_j = \Omega_j^2\,\phi_j \qquad j = 1,2,\ldots N$$
$$= 0 \qquad j > N$$

and similarly for CM. Thus it may be said that the incomplete model contains only the first N modes of the corresponding complete model.

IV. IDENTIFICATION OF THE MASS MATRIX

The previous discussion has assumed knowledge of the normal modes. It is apparent that knowledge of the mass matrix is also required. Before proceeding further with the discussion of the incomplete model, a method will be described for identifying the mass matrix. As above, it is assumed that the first N natural frequencies and normal modes have been determined through testing. Each of these modes contains P elements representing the relative motion of all the points of interest.

The normal modes are orthogonal with respect to the mass as given in Equation (9).

$$\phi_i^T M \phi_j = 0 \qquad j \neq i \tag{32}$$

Writing out the indicated multiplications in terms of the individual elements, this equation becomes

$$\sum_{k=1}^{p} \phi_{ki}\phi_{kj}m_{kk} + \sum_{k=1}^{p-1}\sum_{n=k+1}^{p} (\phi_{ki}\phi_{nj} + \phi_{ni}\phi_{kj})m_{kn} = 0$$

$$i = 1,2\ldots N-1 \qquad j = i+1\ldots N \tag{33}$$

where ϕ_{ki} indicates the kth element of ϕ_i and m_{kn} is an element of the M matrix. The mass matrix has been assumed symmetrical but not necessarily diagonal. Equation (33) is, in reality, N(N-1)/2 linear equations having the mass elements as unknowns with products of the elements of the known normal modes as coefficients.

It is possible that certain mass elements may be known to be zero or to have some definite value. If they are to be zero, the corresponding terms are dropped from the equation. If they are to be restricted to a particular value, the corresponding terms are placed on the right-hand side of the equation.

In addition, if any of the generalized modal masses, m_i, are known, Equation (9) for j=i can be used.

$$\phi_i^T M \phi_i = m_i \tag{34}$$

or

$$\sum_{k=1}^{p} \phi_{ki}^2 m_{kk} + 2\sum_{k=1}^{p-1} \sum_{n=k+1}^{p} \phi_{ki}\phi_{ni} m_{kn} = m_i \tag{35}$$

There is another possible known condition. The total of the diagonal elements may be considered to be known (the total mass of the structure, for example). This leads to the equation

$$\sum_{k=1}^{p} m_{kk} = m_T \tag{36}$$

Equations (33), (35) and (36) may be written as

$$A\bar{m} = R \tag{37}$$

where $\bar{m}$ is a column matrix made up of the unknown elements of M, A is a matrix formed by the coefficients of these unknowns from Equation (33), and, if used, from (35) and (36). R is a column matrix made up of the right-hand side terms corresponding to known masses, if any, known generalized masses, if any, and possibly the known total mass.

There are, then, at least, $N(N-1)/2$ equations and possibly as many as $N(N+1)/2+1$ if all the possible equations are used. A typical situation would use Equations (33) and (36) resulting in $N(N-1)/2+1$ equations. If none of the mass elements are considered to be known (other than zero), there are at least P unknown diagonal masses and as many as $P(P+1)/2$ if the matrix is taken to be completely filled. In many situations, it will be desired to obtain a purely diagonal mass matrix. In a typical situation, there will be, say, $N(N-1)/2+1$ equations in P unknowns.

When the number of equations is less than the number of unknowns, there are an infinite number of solutions. When the reverse is true, there will ordinarily be no solution.

This treatment will be limited to the first situation where there are an infinite number of solutions to the equations. This is not a severe restriction since the class of problems considered includes a relatively large number of points of interest and a relatively small number of known normal modes. If these were nearly equal, it may have been possible with a small amount of additional testing to have completed the information requirements for other techniques (Reference 1). A typical situation might include 4 normal modes plus the total mass giving 7 equations and thus requiring more than seven unknown masses. Similarly 6 normal modes would give 16 equations and 10 normal modes would give 46 equations. As more normal modes become known for the same number of points of interest, it may be necessary to introduce off-diagonal masses as unknowns.

Return to the consideration of Equation (37) where A is an $n_e \times n_v$ matrix (n_e being the number of equations and n_v the number of variables) and $n_e < n_v$. $\bar{m}$ is $n_v \times 1$ and R is $n_e \times 1$. The equations have an infinite number of solutions, that is, there are an infinite number of mass distributions which will cause the modes to be orthogonal. In fact, it is quite possible for there to be an infinite number of mass distributions which will also give the same generalized masses, m_i, and thus result in the same C_{inc} and Y_{inc} (see Equations (29),(31)). In other words, it is quite possible that different valid mass distributions used with the measured normal modes will predict identical responses of the system to sinusoidal forcing.

If, however, it is desired to use the model for making predictions under other conditions, especially to predict the effects of changes in parameters, then it is apparent that the masses used in the model should be as near to the "true" values as possible. The best information available as to what the "true" values are, is the approximation arrived at by the analyst. These analytical values will not, in general, satisfy the orthogonality condition of the normal modes, i.e. Equation (37).

The pseudo-inverse (see Appendix II) is an elegant mathematical tool which can be used to obtain the solution to Equation (37) which is the closest (in a least squares sense) to any specified analytical approximation. Another way of saying this is that the smallest possible changes in the approximation can be found so as to satisfy the conditions of orthogonality.

Define a column matrix, m_A, which is the approximation to $\bar{m}$ and subtract Am_A from both sides of (37) giving

$$A(\bar{m} - m_A) = R - Am_A \tag{38}$$

At this point, a weighting function is introduced in the form of a diagonal matrix, W. Each element is a measure of the analysists confidence in the corresponding approximation. The result will be that masses having higher values of weighting functions will tend to vary least. Inserting the identity $W^{-1}W$ into the above equation results in

$$(AW^{-1})\{W(\bar{m} - m_A)\} = R - Am_A \tag{39}$$

Defining $(AW^{-1})^+$ as the pseudo-inverse of AW^{-1}, the solution given by

$$W(\bar{m} - m_A) = (AW^{-1})^+\{R - Am_A\} \tag{40}$$

is the one of the infinite number possible having the smallest weighted sum of squares of the differences of $\bar{m}$ and m_A. Using ordinary matrix algebra from this point, there results

$$\bar{m} - m_A = W^{-1}(AW^{-1})^+\{R - Am_A\}$$

$$\bar{m} = W^{-1}(AW^{-1})^+R + \{I - W^{-1}(AW^{-1})^+A\}m_A$$

It is shown in Appendix II that

$$(AW^{-1})^{+} = W^{-1}A^{T}\{A(W^{-1})^{2}A^{T}\}^{-1}$$

Then defining

$$B_R = W^{-1}(AW^{-1})^{+} = (W^{-1})^{2}A^{T}\{A(W^{-1})^{2}A^{T}\}^{-1}$$

$$B_m = I - B_R A$$

the equation for the mass elements is

$$\bar{m} = B_R R + B_m m_A \qquad (41)$$

V. MASS CHANGES

One of the criteria for evaluating the usefulness of an analytical model is its ability to predict the effects of changes. In this section, the use of an incomplete model to predict the effects of mass changes on the natural frequencies and normal modes is discussed.

The stiffness matrix and the influence coefficient matrix are independent of the mass of the system. The expression derived in terms of the normal modes do contain the mass, however. (See Equations (16), (17)). Thus, it must be concluded that when the mass is changed, the normal modes and frequencies must change in such a way that the summations remain invariant. In other words

$$K = \sum_{i=1}^{p} \frac{\Omega_i^2}{m_i} M\phi_i\phi_i^T M = \sum_{i=1}^{p} \frac{(\Omega_i+\Delta\Omega_i)^2}{(m_i+\Delta m_i)} (M+\Delta M)(\phi_i+\Delta\phi_i)(\phi_i^T+\Delta\phi_i^T)(M+\Delta M) \tag{42}$$

$$C = \sum_{i=1}^{p} \frac{1}{\Omega_i^2 m_i} \phi_i\phi_i^T = \sum_{i=1}^{p} \frac{1}{(\Omega_i+\Delta\Omega_i)^2(m_i+\Delta m_i)}(\phi_i+\Delta\phi_i)(\phi_i^T+\Delta\phi_i^T) \tag{43}$$

where the Δ's indicate the changes due to the change in mass, ΔM.

While the above expressions must be true when summed over all the modes, they will not be exact for incomplete summations, i.e. for the incomplete model. Of the two (K and C), it is to be expected that C_{inc} will be less sensitive to mass changes. The reason is that the dominant terms are included in C_{inc} and omitted in K_{inc}, thus, C_{inc} is much closer to the invariant matrix C than K_{inc} is to the invariant K.

This hypothesis has been tested by calculating the frequencies and modes of a modified system using matrix iteration on $C_{inc}(M + \Delta M)$. The changes predicted were in excellent agreement with the true values. These results are given in Section XI.

VI. STIFFNESS CHANGES

The effect of a change in the stiffness matrix cannot be handled as directly as the mass change discussed in the previous section. Both the K and C matrices must change when the stiffness is changed. Since the dominant terms of K are missing in K_{inc} it does not appear to be reasonable to hypothesize that $(K + \Delta K)_{inc} = K_{inc} + \Delta K$ since even small ΔK's can easily be greater by orders of magnitude than the elements of K_{inc}.

As discussed in Appendix I (and also derived previously, Equation (16)), it is possible to write K in the following form

$$K = \sum_{i=1}^{p} \frac{\Omega_i^2}{m_i} M\phi_i \phi_i^T M \tag{44}$$

if and only if the ϕ_i's are eigenvectors of $M^{-1}K$. Otherwise there must be coupling terms of the form $a_{ij} M\phi_i \phi_j^T M$. Thus, if $K + \Delta K$ is expressed in terms of the eigenvectors of $M^{-1}K$, the equation must be of the form

$$K + \Delta K = \sum_{i=1}^{p} \sum_{j=1}^{p} a_{ij} M\phi_i \phi_j^T M \tag{45}$$

Now, pre- and post-multiply this equation by ϕ_k^T, ϕ_n

$$\phi_k^T (K + \Delta K)\phi_n = \sum_{i=1}^{p} \sum_{j=1}^{p} a_{ij} (\phi_k^T M\phi_i)(\phi_j^T M\phi_n) = a_{kn} m_k m_n \tag{46}$$

because of the orthogonality of the ϕ's. And since $K\phi_n = \Omega_n^2 M\phi_n$

$$\phi_k^T(K + \Delta K)\phi_n \equiv \phi_k^T K\phi_n + \phi_k^T \Delta K\phi_n \equiv \Omega_n^2 \phi_k^T M\phi_n + \phi_k^T \Delta K\phi_n = a_{kn} m_k m_n$$

or

$$a_{ij} = \frac{\Omega_j^2 \phi_i^T M\phi_j + \phi_i^T \Delta K\phi_j}{m_i m_j} = \frac{\Omega_i^2}{m_i} + \frac{\phi_i^T \Delta K\phi_i}{m_i^2} \qquad j = i$$

$$= \frac{\phi_i^T \Delta K\phi_j}{m_i m_j} \qquad j \neq i \qquad (47)$$

Thus, substituting into (45), the expression for $K + \Delta K$ can be written

$$K + \Delta K = \sum_{i=1}^{p} \frac{\Omega_i^2}{m_i} M\phi_i\phi_i^T M + \sum_{i=1}^{p} \sum_{j=1}^{p} \left(\frac{\phi_i^T \Delta K\phi_j}{m_i m_j}\right) M\phi_i\phi_j^T M \qquad (48)$$

This expression when summed over all P modes is exact. Note that the first summation is equal to K.

Now, truncating the series at the last known mode, the expression can be written

$$(K + \Delta K)_{inc} = K_{inc} + \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{\phi_i^T \Delta K\phi_j}{m_i m_j} M\phi_i\phi_j^T M \qquad (49)$$

This expression can now be evaluated and it is hypothesized that $M^{-1}(K + \Delta K)_{inc}$ can be used to obtain good approximations to the new natural frequencies and normal modes. This hypothesis has been tested and the results given later appear quite satisfactory.

VII. IDENTIFICATION OF DAMPING COEFFICIENT

If the identified model is required to predict responses, in addition to predicting the effects of changes on the natural frequencies, it is necessary to identify the damping coefficient. The responses to sinusoidal loading is, in effect, the mobility (See Equation (23)). The real component of the velocity response at the driving point is given by:

$$\dot{y}_R = \omega \sum_{i=1}^{p} \frac{g}{\left[\left(\frac{\omega}{\Omega_i}\right)^2 - 1\right]^2 + g^2} \frac{1}{\Omega_i^2 m_i} \tag{50}$$

where the modes are normalized at this point. When the modes are reasonably well separated, $\dot{y}_R$ at a resonance is nearly completely dependent on the term due to the one mode. Thus

$$\dot{y}_R(\Omega_i) = \frac{1}{g\Omega_i m_i} \tag{51}$$

can be used to compute g when $\dot{y}_R(\Omega_k)$ has been measured.

If the modes are not separated, it is possible to use the incomplete version of (50) which also contains only the one unknown, g.

$$\dot{y}_R(\Omega_j) = \Omega_j \sum_{i=1}^{N} \frac{g}{\left[\left(\frac{\Omega_j}{\Omega_i}\right)^2 - 1\right]^2 + g^2} \frac{1}{\Omega_i^2 m_i} \qquad (52)$$

Using this procedure, g's may be obtained at each resonance. Experiments discussed in following sections have shown fine consistency among these values and good agreement with the exact value.

Having obtained g and knowing the mass matrix, the natural frequencies, Ω_i, and the normal modes, ϕ_i, for i = 1, N the dominant portion of the mobility matrix may be constructed at any frequency up to the vicinity of the N^{th} natural frequency. It is expected that this information allows the prediction of steady-state responses although this has not specifically been tested in the work reported in this report.

VIII. DESCRIPTION OF COMPUTER PROGRAM

In order to subject some of the hypotheses presented to a qualitative evaluation, a computer program has been developed. The program performs three functions: (1) it simulates a test; (2) it identifies the mass; (3) it finds the modes and frequencies after mass and stiffness changes. The program logic is outlined in Figure 1 and the program listing is given in Appendix III. Below is a general description of the program.

Part I - Simulated Test

In order to simulate a test, the program first accepts a complete description of the "actual system" consisting of a mass matrix, M, a stiffness matrix, K, (or optionally, an influence coefficient matrix, C) and a scalar damping coefficient, g. The "actual" number of degrees of freedom (the order of the matrices) is limited to 20.

The test to be simulated consists of measurements of the resonant response at P points on the structure due to sinusoidal excitation at one point. Thus, the data required for the simulation of the tests consists of the driving point, the points at which measurements are taken, the frequencies of excitation, and the measurement error.

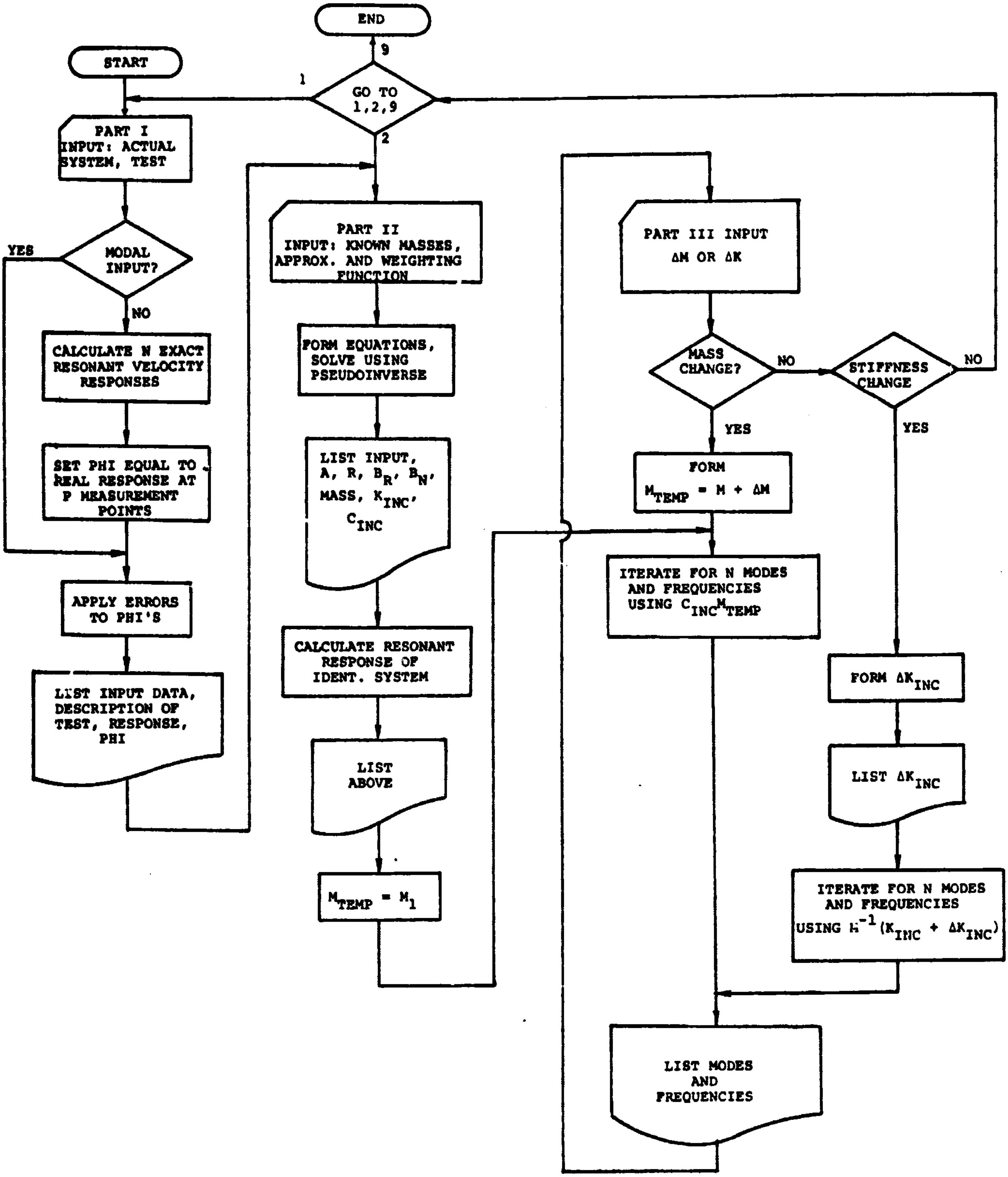


FIGURE 1. LOGIC DIAGRAM OF COMPUTER PROGRAM.

The frequencies of excitation, which should be the N resonant frequencies, are assumed to have been computed outside this program. The measurement error is taken to be a fixed bias error plus a uniform random distribution, each expressed as a percentage of the exact measurement. The procedure, then, is as follows: (1) calculate the impedance, $\frac{g}{\omega} K + i(\omega M - \frac{1}{\omega} K)$, of the "actual" system at each of the N resonant frequencies; (2) invert these complex matrices to obtain the exact mobilities; (3) store the elements corresponding to the P measurement points of the column corresponding to the driving point; (4) apply the specified measurement errors to these values.

This data will be a good simulation of data which would be taken in an actual test. The test data required in the following parts of the program are the normal modes of the system. It is known that a column of the real component of the mobility near resonance will be a good approximation to the normal mode when the natural frequencies are separated. This can be seen from Equation (23). It is this information, then, which is passed on to Part II of the program as the N measured normal modes, each consisting of P points.

As an option to the above procedure, the program will accept as input the normal modes themselves. This procedure would simulate the use of a more sophisticated method of computing them. These modes are then polluted with errors as before, prior to being transferred to Part II of the program.

Part II - Mass Identification

The procedures given in Section IV of this report are carried out numerically. The input consists of any known generalized masses or mass elements and the masses which are to be treated as unknowns. For each unknown mass, an approximation and a weighting function is supplied. In addition, one may specify that the total of the diagonal masses must remain constant. Based on the input, the A matrix is formed as are Am_A and W. The solution for the "best" mass distribution is obtained as given in Equation (41).

At this point, the identified mass matrix is formed, the generalized mass matrix ($\Phi^T M \Phi$) is computed as are the incomplete stiffness and influence coefficient matrices (Equations (28), (29)). As a check, the resonant response and the natural frequencies and normal modes of the identified incomplete model are computed.

Part III - Mass or Stiffness Changes

There now is sufficient data to compute the changes in frequencies and modes due to mass or stiffness changes. The program will accept changes to either the mass matrix or the stiffness matrix. For mass changes the matrix $M + \Delta M$ is formed, where M is the identified mass matrix. The first N frequencies and modes are obtained by iteration on $C_{inc}(M + \Delta M)$ as discussed in Section V.

For stiffness changes, $(K + \Delta K)_{inc}$ is formed as given in Equation (37). The N frequencies and modes are computed by iteration from $M^{-1}(K + \Delta K)_{inc}$.

The program is written so that changes can be made on the identified model in sequence without rerunning the first two parts of the program.

IX. THE SIMULATED TESTS

In order to test the hypotheses presented regarding the characteristics of the incomplete model, it was necessary to select a structure on which to perform the simulated testing. It was decided not to select so simple a system that success would give no confidence that the methods would work in practice. On the other hand, it was not desirable to make the first tests using such a complex system that one would not have a good intuitive understanding of the system and modifications of it.

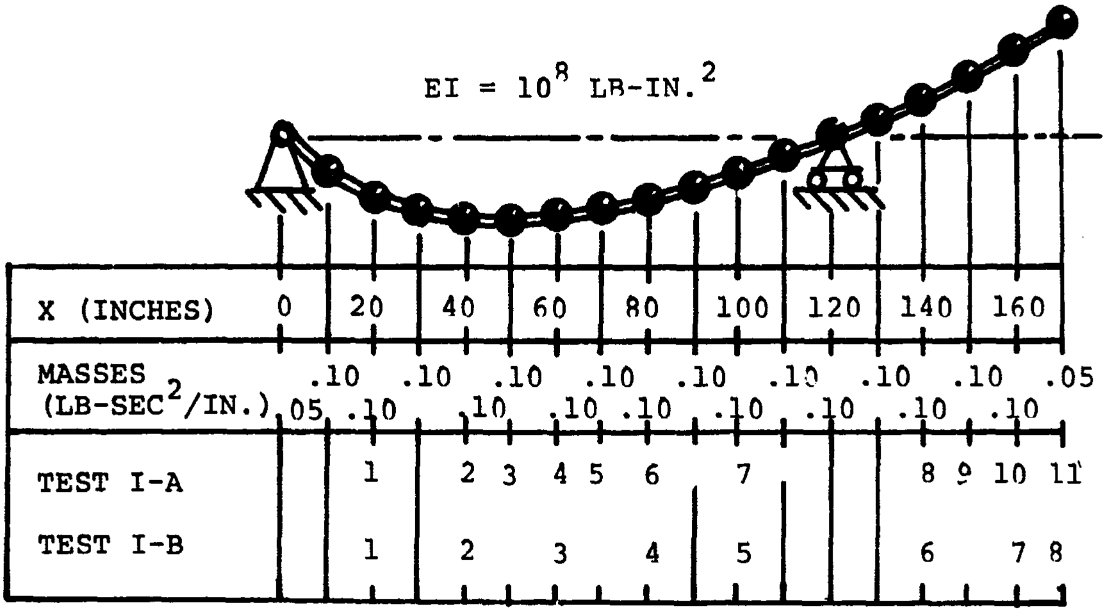
As a compromise, a structure was selected whose behavior is quite well understood yet is not so simple as to yield trivial results. The structure selected is a simple, thin, beam of constant EI having 18 lumped masses arranged so as to approximate a uniform mass distribution. The beam is allowed to deflect transversely only. The deflection is constrained at one end and at a point approximately 70 percent of the length of the beam, leaving an overhang of approximately 30 percent. A structural damping coefficient of .02 was used.

Most of the simulated testing was performed using this system. This system is designated Specimen I. Some tests were also carried out on a structure identical with the above except that two rotary inertias were added, one at the second support and one at the free tip of the beam. This system is called Specimen II. These beams are illustrated schematically in Figure 2. The natural frequencies and mode shapes of the two beams were computed using a standard computer program. These results are illustrated in Figure 3.

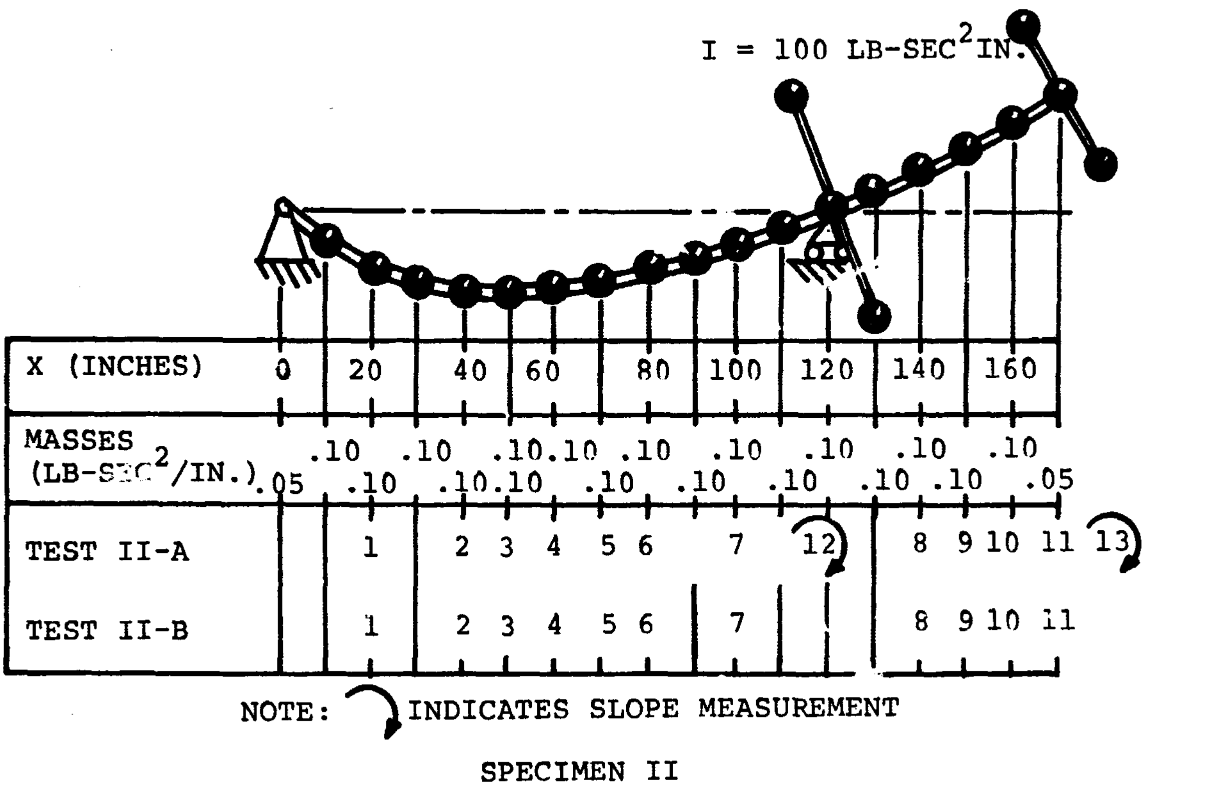
For each of the two specimens, two arrangements of test measurements were selected, referred to as tests A and B. These are also illustrated in Figure 2. In each test, the system was considered to be driven at station 60.

Associated with each specimen and each test is an approximate mass distribution. For Specimen I, test A, there are three approximate mass distributions (I, II, III). Each of these is somewhat reasonable. Table I lists the various mass approximations used. These are discussed further in Section X.

The simulated tests which were run are summarized and described in Table II.



SPECIMEN I



SPECIMEN II

Figure 2. Schematic Representation of the Simulated Test Specimens.

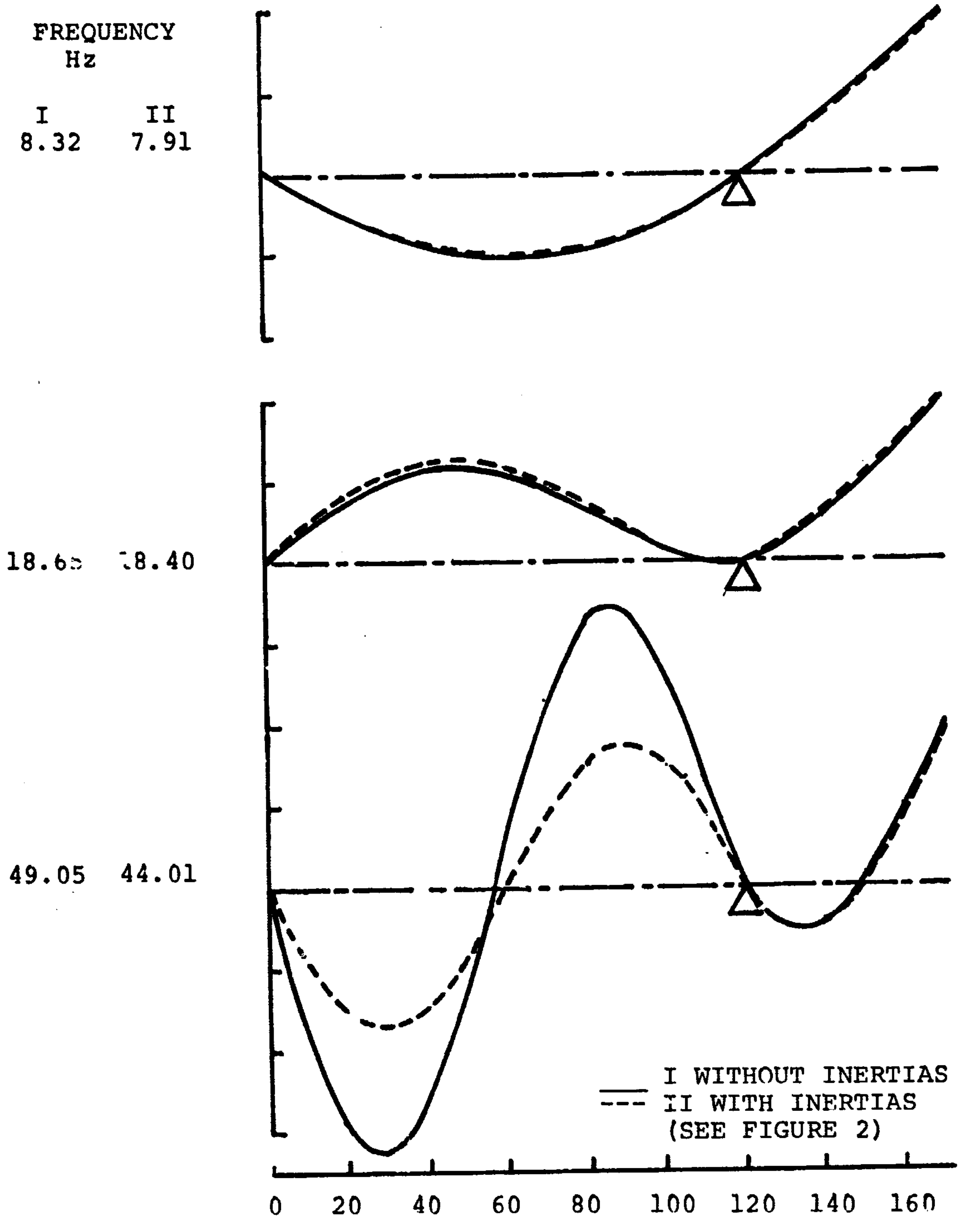


Figure 3. Exact Normal Modes of Test Specimens.

TABLE I. APPROXIMATE MASS DISTRIBUTIONS

	Specimen I						Specimen II		
Sta. In.	True Mass	Test A I	Test A II	Test A III	Test A W*	Test B	True Mass	Test A	Test B
0.Δ	.05						.05		
10.	.10						.10		
20.	.10	.30	.25	.25	0.1	.25	.10	.30	.30
30.	.10						.10		
40.	.10	.15	.15	.13	1.0	.20	.10	.15	.15
50.	.10	.10	.10	.12	0.5		.10	.10	.10
60.	.10	.10	.10	.10	1.0	.20	.10	.10	.10
70.	.10	.10	.10	.12	0.5		.10	.10	.10
80.	.10	.15	.15	.13	1.0	.20	.10	.15	.15
90.	.10						.10		
100.	.10	.30	.25	.25	0.2	.25	.10	.30	.30
110.	.10						.10		
120.Δ	.10						.10		
130.	.10						.10		
140.	.10	.25	.20	.15	0.5	.25	.10	.25	.25
150.	.10	.10	.10	.13	1.0		.10	.10	.10
160.	.10	.10	.10	.12	1.0	.15	.10	.10	.10
170.	.05	.05	.05	.05	1.0	.05	.05	.05	.05
I@120.	0						100.	100. (W=.01*)	0
I@170.	0						10.	10. (W=0.1*)	0

Δ Support points

* W = 1.0 except for test 6 when it was used as shown and for tests 18-20 when it was used to minimize the dominance of the large magnitude of the inertias.

TABLE II. DESCRIPTION OF SIMULATED TESTS

Test No.	Specimen	Test (See Fig. 2)	No. of Resonances or Modes	Mass Approx. (Table I)	Remarks*
1	I	A	4R**	I	
2	I	A	4M	I	
3	I	A	3R	I	
4	I	A	3M	I	
5	I	A	3R	I	Variable mass
6	I	A	3R	I	Weighting fun. (See Table I)
7	I	A	3R	II	
8	I	A	4R	III	
9	I	A	3R	III	Variable mass
10	I	A	3R	III	
11	I	A	3R	III	5% Bias and
12	I	A	3R	III	±5% Random
13	I	A	3R	III	Measurement
14	I	A	3R	III	Error
15	I	A	3R	III	
16	I	B	3R		
17	I	B	3R		Variable mass
18	II	A	4R		Inertias
19	II	A	3R		weighted
20	II	A	2R		(See Table I)
21	II	B	4R		
22	II	B	3R		

* Unless so indicated, total mass held constant, no weighting, and no errors assumed.

**R indicates resonant data used, M indicates actual mode shapes used.

X. THE IDENTIFIED MASSES

The masses identified by the program based on the simulated tests are given in Table III. The data is listed by test number. Refer to Table II for a description of the tests. The masses are given in the table to two significant figures for ease of reading except for tests 10-15 in order that the scatter may be observed. These are all identical except that tests 11-15 contain test measurement errors of 5 percent bias and ±5 percent random on amplitude. In addition to the masses themselves, the table indicates the changes from the approximation in terms of the rms of the variance.

Certain characteristics of this data can be observed. It appears that "better" approximations have smaller variances associated with them. This is what would be expected since the better the approximation, the smaller the change required to make it satisfy the equations. In the cases of Specimen I, test A, approximations II and III are intuitively better than I since I includes the masses at the supports. It is apparent, by comparing similar conditions from tests 1-6 with 7-15 that the variances are significantly lower for approximations II and III. Comparison of 16, 17 with 9, 10 which use effectively the

TABLE III. IDENTIFIED MASSES

Test+ No.	Point of Measurement (See Figure 1) 1	2	3	4	5	6	7	8	9	10	11	Var. RMS
	Specimen I (See Figure 1)											
AI*	.30	.15	.10	.10	.10	.15	.30	.25	.10	.10	.05	
1	.28	.16	.12	.11	.12	.17	.26	.23	.088	.093	.074	.019
2	.25	.20	.12	.08	.11	.22	.21	.25	.11	.078	.092	.043
3	.29	.15	.11	.12	.12	.15	.29	.23	.077	.092	.076	.016
4	.29	.15	.11	.12	.12	.15	.29	.23	.077	.092	.075	.016
5	.29	.15	.12	.12	.12	.16	.29	.24	.081	.095	.077	.016
6	.27	.15	.13	.11	.13	.15	.25	.24	.097	.10	.066	.023
AII*	.25	.15	.10	.10	.10	.15	.25	.20	.10	.10	.05	
7	.25	.15	.11	.11	.11	.15	.24	.19	.091	.097	.061	.0065
AIII*	.25	.13	.12	.10	.12	.13	.25	.15	.13	.12	.05	
8	.25	.15	.12	.085	.12	.15	.26	.15	.11	.099	.059	.014
9	.25	.13	.13	.11	.12	.13	.25	.15	.12	.12	.047	.0040
10	.250	.134	.125	.106	.124	.131	.248	.146	.124	.115	.047	.0040
11	.256	.142	.130	.104	.117	.123	.245	.147	.125	.115	.044	.0066
12	.255	.138	.125	.101	.118	.126	.249	.150	.129	.116	.043	.0044
13	.251	.138	.130	.109	.125	.130	.244	.144	.120	.111	.048	.0069
14	.252	.138	.130	.109	.124	.126	.243	.143	.120	.113	.052	.0070
15	.257	.142	.127	.100	.114	.122	.247	.150	.129	.117	.044	.0059
B*	.25	.20	.20	.20	.25	.25	.15	.05				
16	.24	.20	.22	.21	.24	.24	.14	.055				.010
17	.24	.20	.22	.21	.24	.24	.14	.055				.010
	Specimen II (See Figure 1)											
A**	.30	.15	.10	.10	.10	.15	.30	.25	.10	.10	.05	
18	.24	.16	.14	.12	.089	.14	.35	.26	.10	.071	.080	.034
19	.30	.15	.11	.11	.11	.16	.30	.24	.090	.098	.067	.015
20	.30	.15	.10	.10	.10	.15	.30	.25	.10	.10	.051	.0005
B*	.30	.15	.10	.10	.10	.15	.30	.25	.10	.10	.05	
21	.73	-.36	-.08	.41	.27	-.33	.27	1.11	-.31	-.19	.18	.409
22	.29	.12	.068	.082	.10	.17	.33	.28	.13	.11	.027	.023

* Mass approximation used for tests following (See Table 1)
** Inertias not tabulated. Identified inertias were virtually exact.
\+ See Table II for description of test.

same distribution suggests that the smaller the number of points, the greater the variance will be. One reason for this is that the approximation having the greater number of masses is an intuitively better representation of the system, everything else being equal.

There is another, more mathematical reason for this effect. There are fewer equations than there are unknown masses (See Section IV). If these numbers were the same, the mass would be unique and have no necessary relation to the approximation. As the number of unknowns is increased, the set of solutions expands and the probability of finding one of these solutions closer to a given approximation increases. The same effect is true when the number of unknowns stays the same but the number of equations is decreased, as when the number of modes used is reduced. Notice tests 18, 19, 20, for example, where the variances associated with 4, 3, 2 modes are .034, .015, .0005 respectively.

Thus, it could be reasoned that, the more nearly complete the model (for a constant number of masses), the greater will be the variances in the identified masses from the approximation. The data presented here tends to bear out this argument.

The effect of the constraint on the sum of the masses has little effect as can be seen by comparing tests 3 and 5, 9 and 10, 16 and 17.

The use of exact modes instead of the resonant responses, in these examples, shows no significant improvement in terms of the variances. It is interesting to note that test 1 using four resonant responses has a variance of .019 while test 2 which was identical except that true normal modes were used has a variance of .043. In tests 3, 4, which are identical to 1,2 except that 3 modes were used, the results are virtually identical to each other.

For Specimen II, test B, an intentionally poor test and approximation was made by ignoring the large inertias and omitting the measurements of slope at these points. Test 21 using 4 resonances results in very poor masses including several negative values. However, when only three resonances were used, the identified masses show general agreement with the approximation. In actual testing, results such as from test 21 could point up the omission of a significant parameter in the analysis.

The effects of the measurement errors is rather small. In five simulated identical tests (11-15), errors of 5 percent bias and 5 percent random were applied to the response amplitudes. This is identical to a uniform random distribution between 0 and +10 percent. In each case, the variances are small. None of the individual mass elements vary by more than 8 percent over the five tests.

In general, the masses identified appear to be acceptable approximations (except in test 21). The test as to whether they are satisfactory or not will come when they are used to predict the effects of structural changes.

XI. THE EFFECTS OF MASS CHANGES

Two different lumped masses were considered to be added to the structure and the new frequencies and modes were calculated for the tests described in Table II. The method used has been described in Section V and its implementation in Section VIII.

The lumped masses were added at a point midway between the supports (change "A") and at the free end of the beam (change "B"). These masses were 1.0 lb-sec^2/in. This represents an increase in total mass of the beam of almost 60 percent.

Table IV summarizes the results of the frequency calculations. The table shows the frequencies at which the simulated testing was conducted (the frequencies of the original beam), the exact frequencies of the modified beams, and the predicted results for each of the simulated tests.

In general, the results are surprisingly good and relatively insensitive to the quantities that were varied. For Specimen I, change B (the more extreme of the effects), the first mode changes frequency by 54 percent and the poorest prediction is within 8 percent of the correct value.

TABLE IV. COMPUTED FREQUENCIES DUE TO MASS CHANGE

Mode	1		2		3		4	
f*	8.32		18.65		49.06		96.12	
+	A	B	A	B	A	B	A	B
f**	6.05	3.82	15.46	14.77	48.37	47.06	79.53	78.81
1	6.30	4.09	15.67	14.85	48.46	47.31	84.65	85.17
2	6.33	4.12	15.69	14.85	48.49	47.31	83.81	84.43
3	6.29	4.08	15.69	14.87	48.52	47.48		
4	6.29	4.08	15.68	14.87	48.55	47.48		
5	6.33	4.12	15.71	14.88	48.53	47.48		
6	6.32	4.11	15.70	14.88	48.50	47.41		
7	6.19	3.97	15.60	14.84	48.50	47.43		
8	6.20	3.98	15.59	14.82	48.44	47.28	83.48	84.22
9	6.21	3.99	15.62	14.84	48.48	47.37		
10	6.21	3.99	15.62	14.84	48.48	47.37		
11	6.23	3.83	15.51	14.96	48.47	47.46		
12	6.10	3.89	15.81	14.69	48.48	47.34		
13	6.26	4.03	15.54	14.73	48.46	47.42		
14	6.25	4.05	15.76	14.99	48.49	47.35		
15	6.12	3.91	15.71	15.00	48.45	47.29		
16	6.17	3.95	15.59	14.84	48.53	47.50		
17	6.18	3.95	15.60	14.84	48.53	47.50		
f*	7.91		18.40		44.01		65.94	
f**	5.91	3.78	14.92	14.58	44.00	40.23	60.33	62.72
18	6.14	4.02	15.15	14.68	44.00	40.40	61.42	62.82
19	6.19	4.01	15.15	14.69	44.00	40.69		
20	6.06	3.94	15.07	14.72				
21	5.01	2.92	14.30	14.37	43.99	40.36	63.46	64.48
22	5.88	3.73	14.91	14.56	43.99	40.01		

* Frequency before changes, i.e. frequency tested.
** Exact frequency after changes.
\+ Addition of 1.0 $lb\text{-}sec^2/in.$ at Sta 60 ("A") or Sta 170 ("B").

For a more typical situation, Specimen I, change A, where the second mode changes frequency by about 17 percent, the poorest prediction is within about 2.5 percent.

The data bears out the expectation that the better mass approximations will result in better frequency predictions. The results of tests 7-10 (the better mass distributions) are seen to be slightly better than tests 1-6.

A better illustration of this effect is seen in test 21 which, as discussed in Section X, was given an intentionally bad mass distribution. Even though this model would predict the resonant responses and also the sinusoidal responses over the frequency range of interest (this has not been shown but is assumed to be true from theoretical considerations), its ability to predict the changes in frequency due to mass changes has been adversely affected by the poor approximation.

Test 22 which is the same as 21 except that only 3 resonances are used seems to contradict the above conclusions since the predicted frequencies are excellent. The fact that these predictions are better than those corresponding to better mass approximations is probably a coincidence. It is not surprising, however, that the 3 resonance data gives better results than the 4 resonance data. The effect is discussed in Section X in connection with the mass identification. It may be surmised from this limited data that as the number of equations approaches the number of unknowns the solutions corresponding to poorer mass approximations will tend to deteriorate more rapidly.

The effect of measurement errors is seen to be slight from tests 11-15. The maximum scatter in predicted frequencies is about 2.5 percent even though the measurement error was randomly distributed between 0 to 10 percent.

The predicted normal modes behave in a manner similar to the predicted natural frequencies in that they are generally quite acceptable. Figure 4 illustrates the effect of change B on Specimen I for the first three modes. The figures show the original modes and the exact new modes. They are both normalized to be equal at station 60 which was the driving point in the simulated test. For modes 1 and 2, the results of tests 1-9 are all very close to the exact curve. The scatter obtained in tests 11-15 containing error compared to the exact curve and to test 10 (same conditions but without error) are quite satisfactory.

The results for the third mode show an interesting effect. The results based on data containing only 3 modes does not predict the proper shape as well as the tests containing 4 mode data. There is a slight tendency in this direction in the frequency data also (see tests 1, 2, 8, 18). The reason for this effect may be surmised. Assuming that the new modes contain components of the same and adjacment old modes, then one may expect that the predictions of the highest mode measured will be missing the significant effects of the next higher old mode.

Figure 4d illustrates the third mode of Specimen II, change B for the three tests: 18, 21, 22. Test 18 is the "good" approximation to Specimen II using 4 modes and shows good agreement with the exact mode. Tests 21, 22 represent the "bad" approximation for 4 and 3 modes, respectively. While both of these predict the frequency of this mode nearly equally well, the four mode data produces a considerably better third mode than the three mode data. This is in agreement with the effect previously noted.

While only sample modes are illustrated, these are quite typical of those obtained in all the computations made. The fourth modes are not shown because of the small amount of data available.

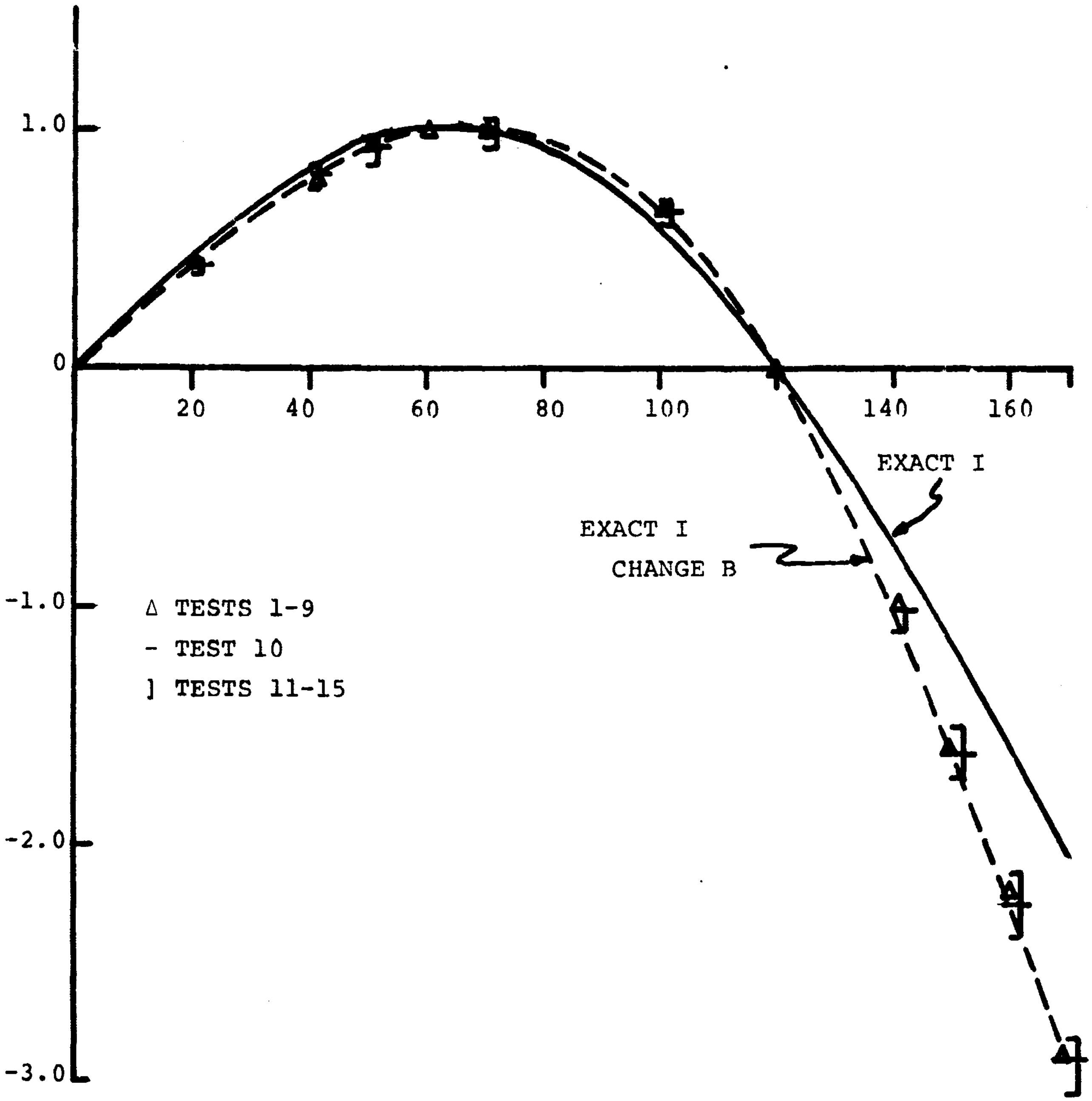


Figure 4a. Computed Mode Changes Due to Mass Change First Mode, Specimen I, Change B.

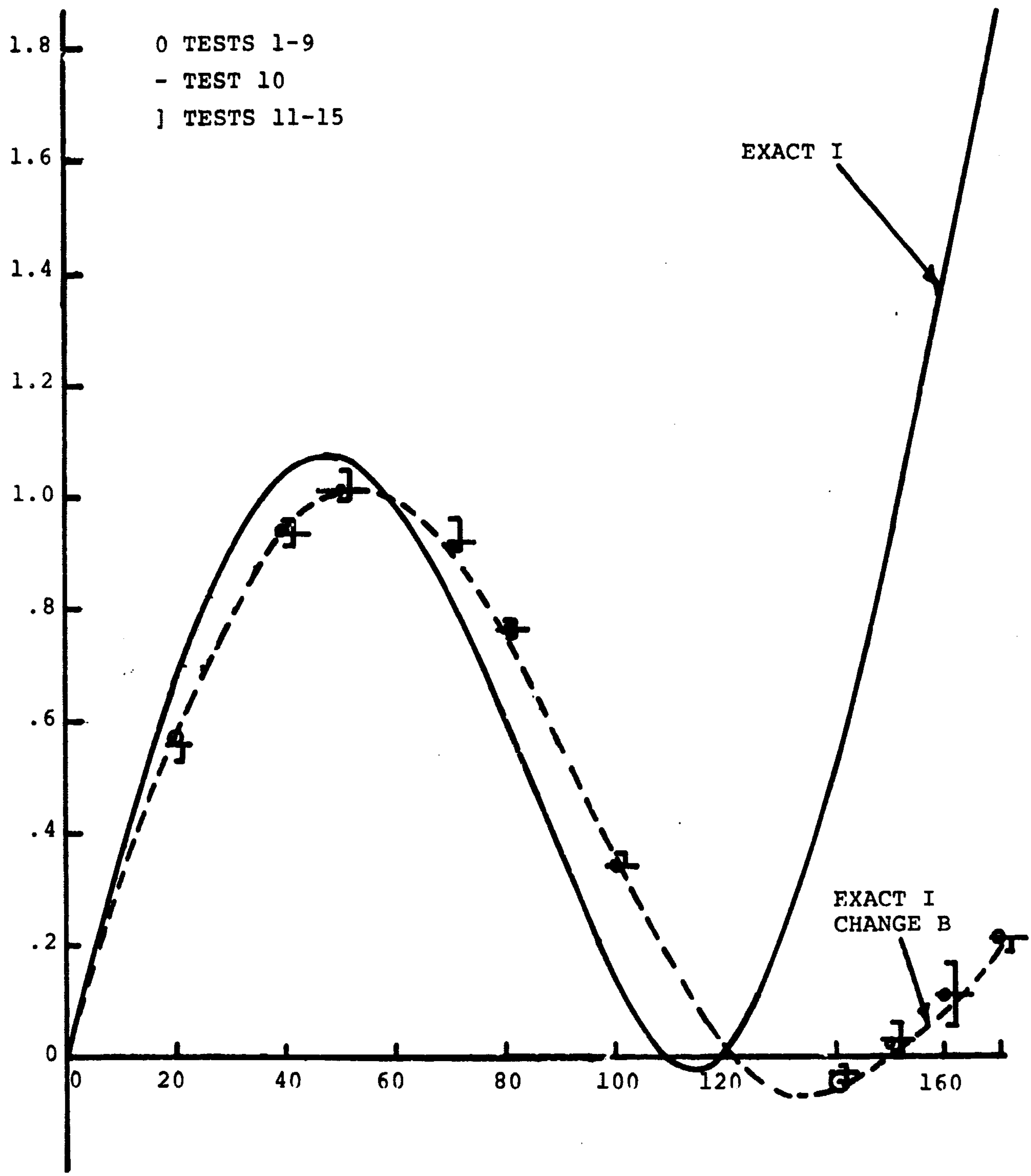


Figure 4b. Computed Mode Changes Due to Mass Change Second Mode, Specimen I, Change B.

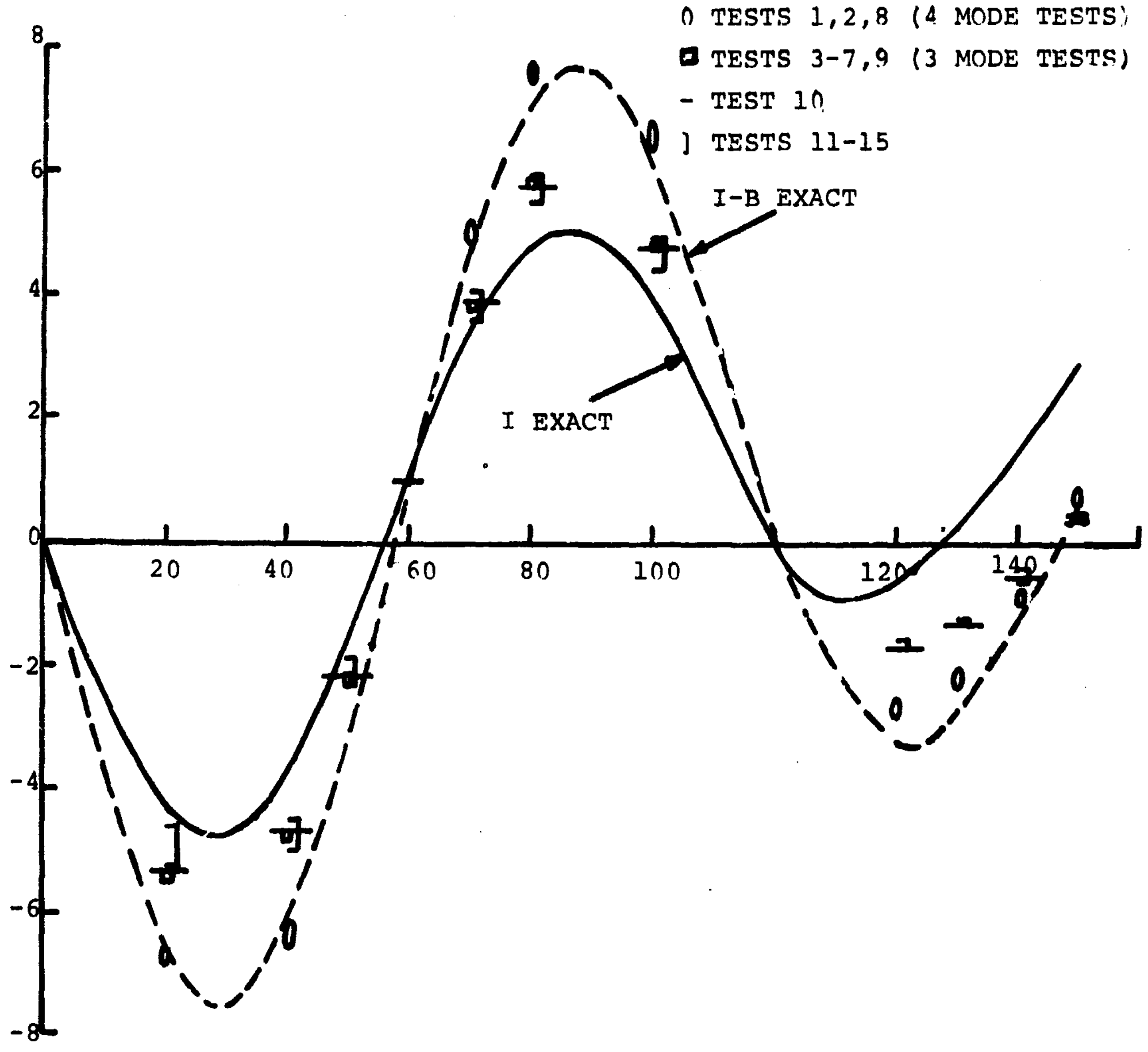


Figure 4c. Computed Mode Changes Due to Mass Change Third Mode, Specimen I, Change B.

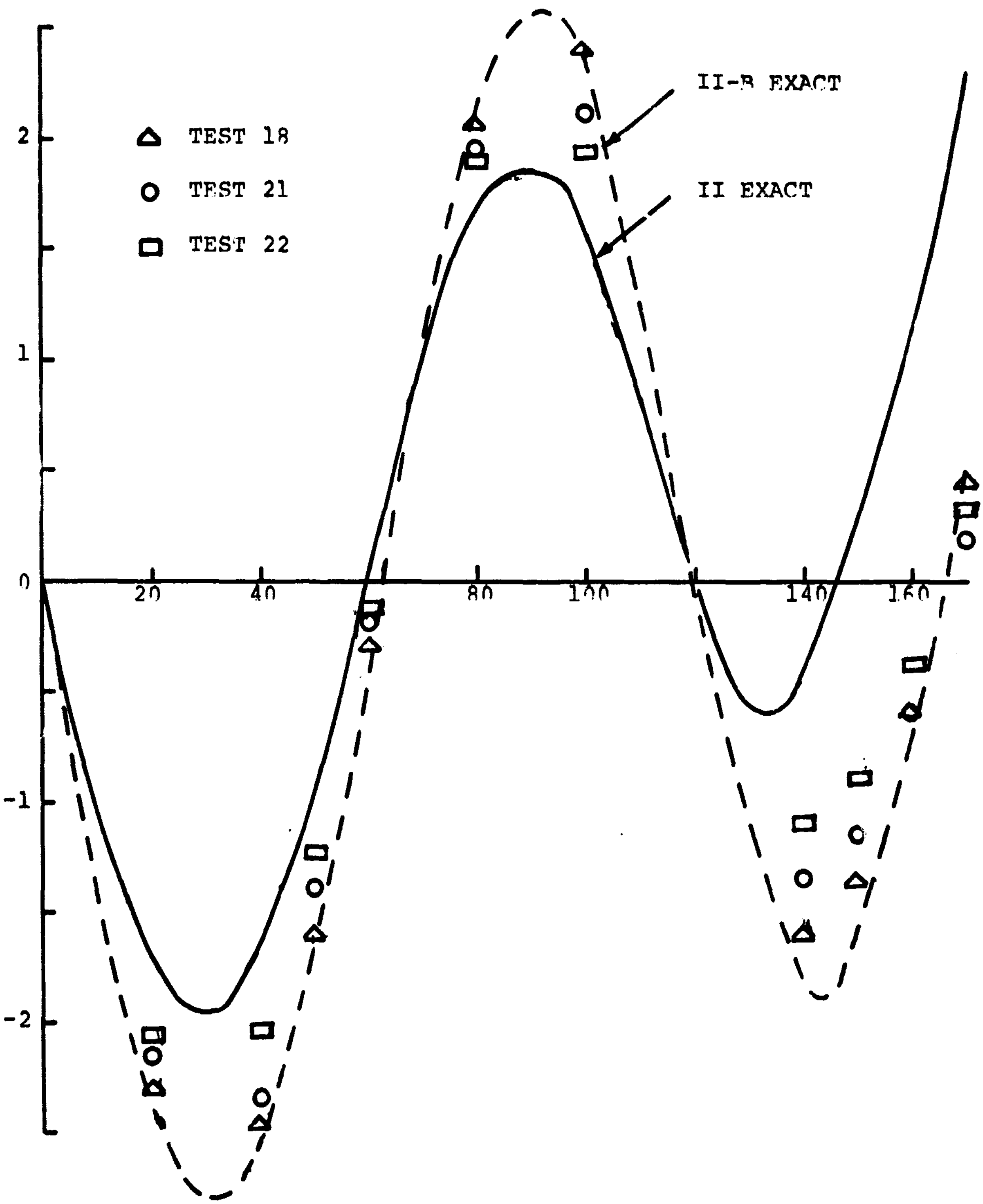


Figure 4d. Computed Mode Changes Due to Mass Change
Third Mode, Specimen II, Change B.

XII. THE EFFECTS OF STIFFNESS CHANGES

In general, the same conclusions may be drawn regarding stiffness changes as mass changes. The only possible exception being the conclusions regarding the prediction of the change in the highest mode shape measured. This is in doubt only because the stiffness changes made had very little effect on the third mode and no conclusions can be drawn.

Two kinds of stiffness changes were considered. The first type consisted of adding a spring to ground at each of the same points where the masses were added. These stations were the mid-point between the supports (change "K") and the tip of the beam (change "L"). The spring constants were 1000 lb/in. ΔK consisted of a change of the corresponding diagonal element of the K matrix (See Figure 6). As discussed in Section VI, the full $(K + \Delta K)_{inc}$ matrix was computed and the frequencies and modes were obtained by iteration on $M^{-1}(K + \Delta K)_{inc}$. The results of the frequency computations are given in Table IV and sample modes are illustrated in Figure 5.

The other stiffness change consisted of the addition of a uniform beam as illustrated in Figure 6, resulting in a ΔK matrix consisting of nine terms. The results are given in Table V(b).

All the stiffness changes resulted in quite adequate predictions of the changes in frequency and normal mode with the exception of test 2] (as anticipated).

TABLE V(a). COMPUTED FREQUENCIES DUE TO STIFFNESS CHANGE

Mode	1		2		3		4	
f*	8.32		18.65		49.06		96.12	
+	K	L	K	L	K	L	K	L
f**	9.34	11.25	19.32	21.23	49.07	49.20	96.26	96.49
1	9.20	10.97	19.21	20.80	49.07	49.19	96.22	96.39
2	9.19	10.93	19.20	20.76		49.18	96.22	96.40
3	9.21	11.00	19.22	20.84		49.19		
4	9.21	11.00	19.22	20.85		49.19		
5	9.19	10.96	19.21	20.78		49.18		
6	9.20	10.97	19.21	20.80		49.19		
7	9.27	11.13	19.26	21.03	49.07	49.20		
8	9.26	11.10	19.26	20.98	49.07	49.20	96.24	96.44
9	9.26	11.10	19.26	20.99	49.07	49.20		
10	9.26	11.11	19.26	20.99	49.07	49.20		
11	9.24	11.34	19.28	21.13		49.21		
12	9.34	11.16	19.23	21.25		49.22		
13	9.22	11.02	19.26	21.02		49.20		
14	9.24	11.08	19.21	20.80		49.20		
15	9.32	11.26	19.25	20.99		49.22		
16	9.28	11.14	19.27	21.05	49.07	49.20		
17	9.28	11.14	19.27	21.05	49.07	49.19		
f*	7.91		18.40		44.01		65.94	
f**	8.86	10.80	19.11	20.66	44.01	44.41	66.03	66.15
18	8.73	10.53	19.00	20.29	44.01	44.36	66.02	66.12
19	8.75	10.57	19.01	20.34	44.01	44.37		
20	8.78	10.68	19.05	20.51				
21	9.47	11.89	19.75	22.77	44.01	44.82	66.01	66.11
22	8.89	10.87	19.14	20.77	44.01	44.52		

* Frequency before changes, i.e. frequency tested.
** Exact frequency after changes.
\+ Addition of 1000 lb/in. spring to ground at Sta 60 ("K") or Sta 170 ("L"). See Figure 6.

TABLE V(b). COMPUTED FREQUENCIES DUE TO STIFFNESS CHANGE 'M'

Mode	1	2	3	4
f*	8.32	18.65	49.06	96.12
f**	8.92	19.32	49.13	99.86
Test No.				
1	8.83	19.26	49.18	99.86
3	8.87	19.33	49.19	
6	8.86	19.32	49.20	

* Frequency before changes.
** Exact frequency after change M. See Figure 6.

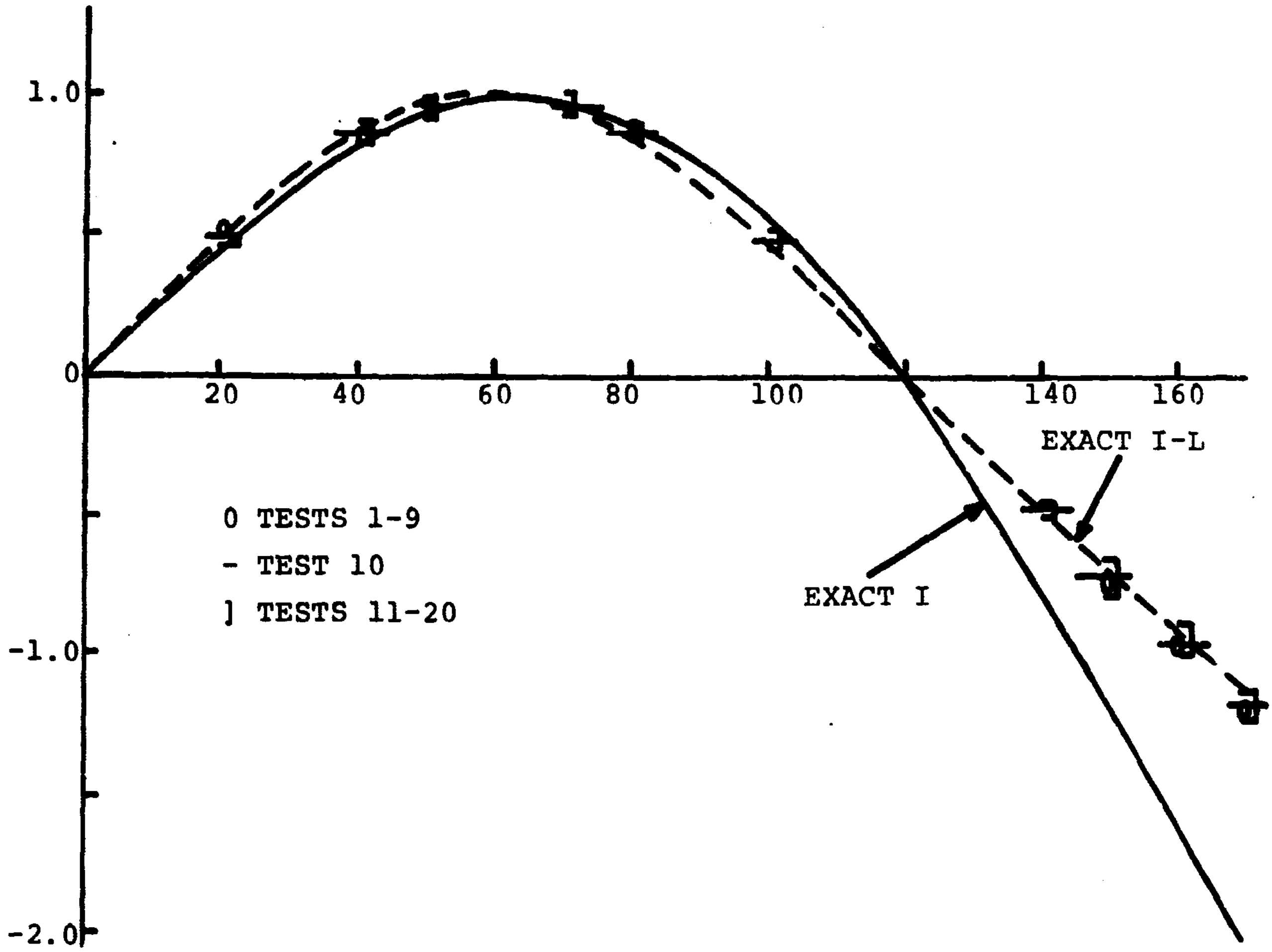


Figure 5a. Computed Mode Changes Due to Stiffness Change First Mode, Specimen I, Change L.

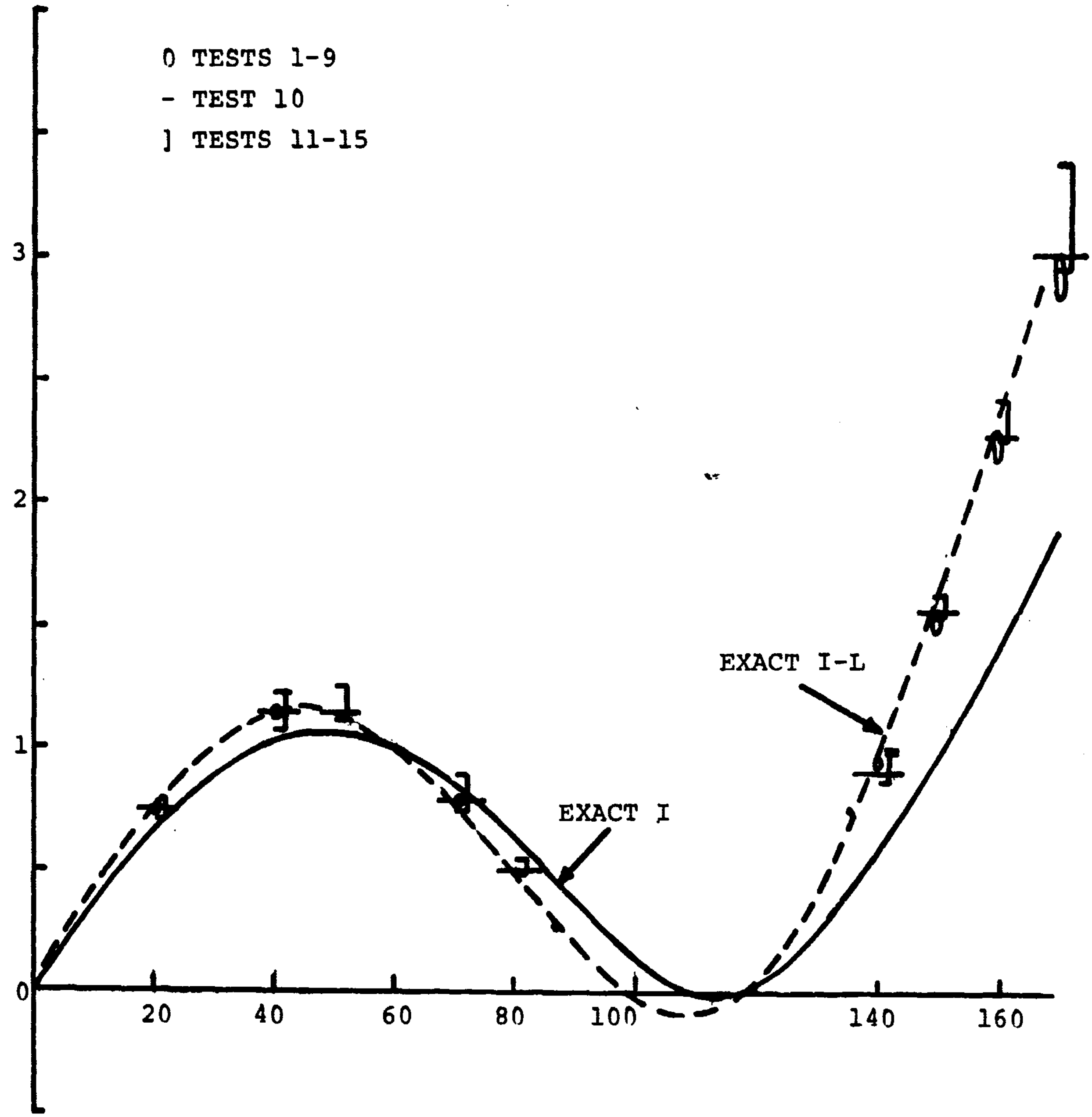


Figure 5b. Computed Mode Changes Due to Stiffness Change
Second Mode, Specimen I, Change L.

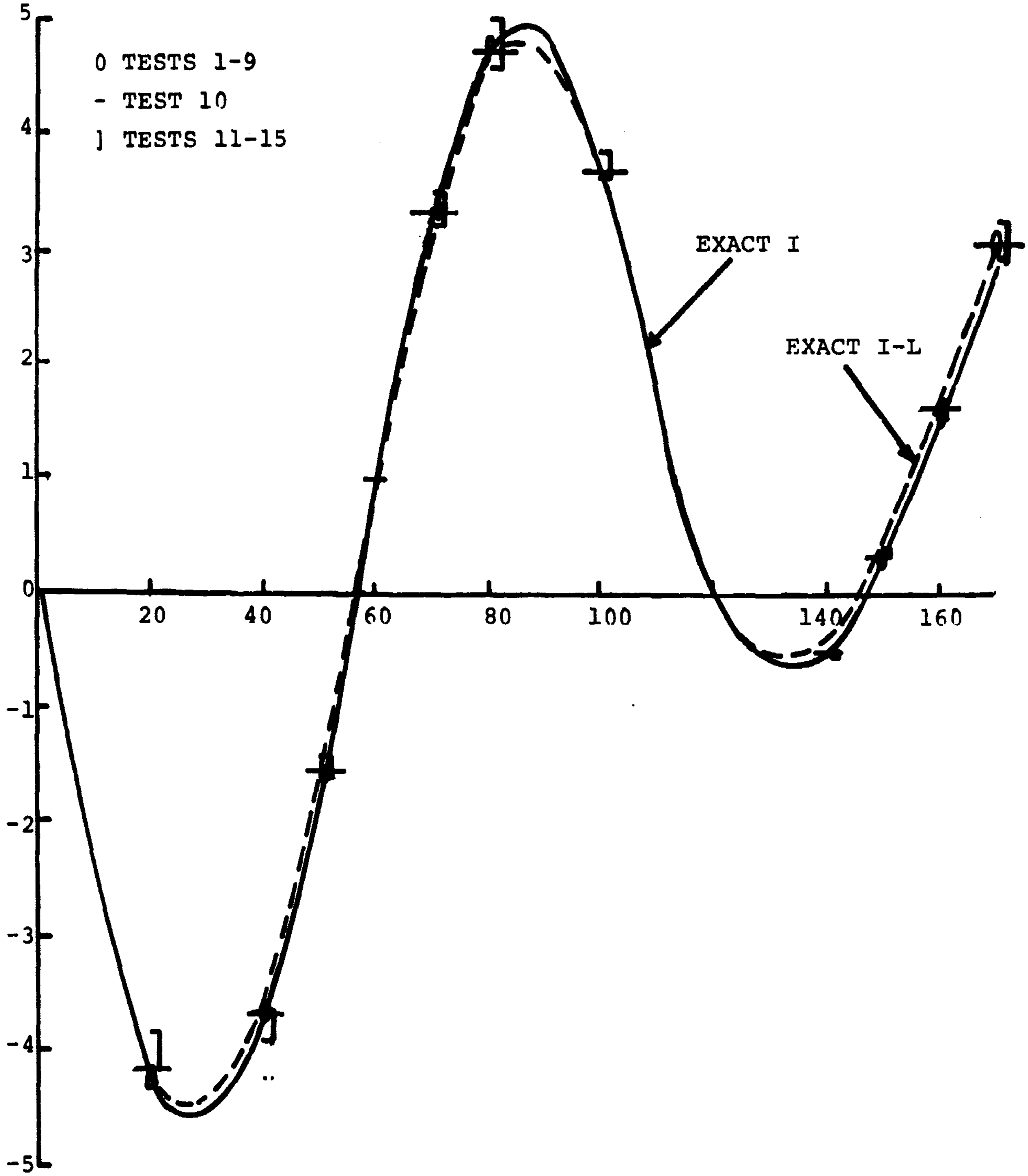


Figure 5c. Computed Mode Changes Due to Stiffness Change Third Mode, Specimen I, Change L.

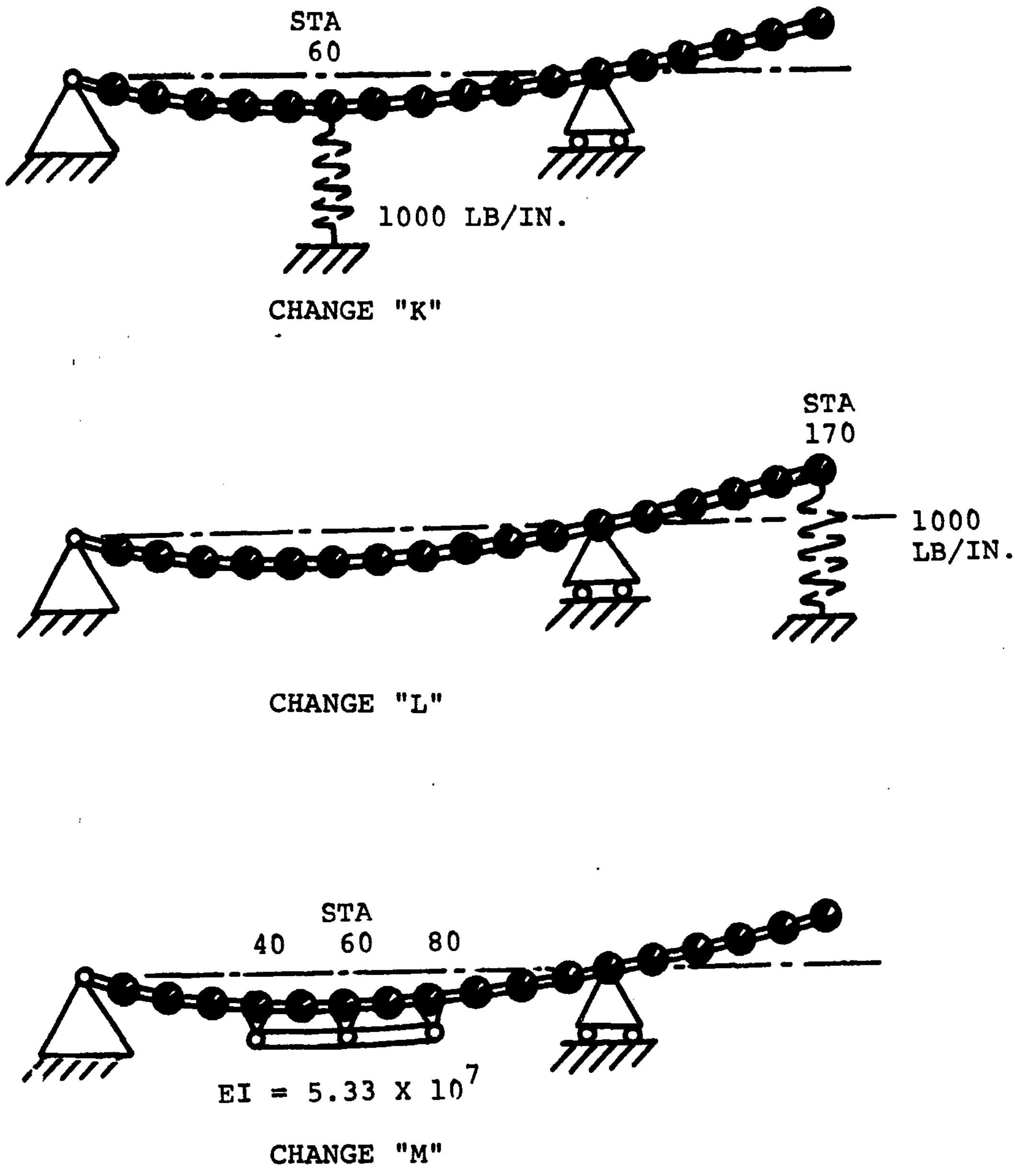


Figure 6. Stiffness Changes.

XIII. THE IDENTIFIED DAMPING COEFFICIENTS

The damping coefficient was obtained using Equation (51). This depends on the natural frequency, the real resonant velocity response, and the generalized mass. Thus, the damping coefficient may vary with the mode and with the identified mass. Table VI gives all the identified coefficients and it is seen that almost all the identifications fall within 15 percent. Tests 21, 22 which started with bad mass distributions behave as they did before. Test 21 gives very poor results while 22 gives quite good results for the first two modes. The poor results for the third mode of tests 18, 19 are not explained. It is noted, however, that the response is very small.

In the tests with error (11-15), the simulated erroneous response was used. The total scatter is of the order of 15 percent.

TABLE VI. IDENTIFIED GENERALIZED MASSES AND DAMPING COEFFICIENTS

Mode	1		2		3		4	
Ω_i	8.016		18.65		49.06		96.12	
$\dot{y}_R$	.7487		.3985		.01097		.08263	
	m_1	g	m_2	g	m_3	g	m_4	g
1	1.512	.0175	1.268	.0169	17.32	.0170	1.338	.0150
2								
3	1.496	.0176	1.254	.0170	17.44	.0166		
4								
5	1.533	.0172	1.285	.0167	17.78	.0176		
6	1.521	.0174	1.247	.0172	16.83	.0188		
7	1.391	.0189	1.166	.0184	15.77	.0183		
8	1.404	.0188	1.178	.0182	16.18	.0192	1.134	.0176
9	1.411	.0187	1.182	.0181	15.40	.0192		
10	1.410	.0187	1.181	.0181	15.39	.0192		
11	1.436	.0178	1.139	.0188	15.04	.0196		
12	1.291	.0183	1.235	.0171	15.36	.0185		
13	1.473	.0170	1.168	.0173	14.95	.0187		
14	1.443	.0173	1.278	.0166	16.32	.0181		
15	1.317	.0181	1.189	.0175	14.35	.0189		
16	1.376	.0186	1.155	.0185	16.48	.0180		
17	1.379	.0186	1.157	.0185	16.51	.0179		
Ω_i	7.909		18.40		44.01		65.94	
$\dot{y}_R$	.6938		.4301		.000178		.0510	
	m_1	g	m_2	g	m_3	g	m_4	g
18	1.705	.0170	1.108	.0170	587.4	.0345	2.585	.0183
19	1.674	.0173	1.159	.0173	587.5	.0346		
20	1.594	.0182	1.093	.0184				
21	.784	.0370	.542	.0371	277.8	.0732	2.894	.0164
22	1.401	.0207	.959	.0210	418.4	.0486		

XIV. CONCLUSIONS

It should be recognized that the following general conclusions are based on a small amount of simulated data on a relatively simple structure. While these conclusions are not proven, they are strongly suggested by the data obtained.

1. The concept of an incomplete model of a dynamic structure is valid and useful, at least, for predicting the effects of structural changes on the normal modes and frequencies.

2. The parameters of an incomplete model may be determined from measured modal data and a "reasonable" approximation to the mass matrix.

3. The identified mass matrix will be approximately equal to the assumed values when the assumption is reasonably valid. The identified structural damping coefficient will also be a reasonable approximation under this condition.

4. The identified stiffness matrix, while not approximating the true values, may be used to predict the effects of stiffness changes.

5. The identified influence coefficient matrix will be an approximation to the true values.

6. The effects of mass and stiffness changes on the normal modes and natural frequencies can be estimated using the techniques that have grown out of the incomplete model concept. These predictions are not overly-sensitive to the mass approximation and to errors of measurement.

7. The procedures may tend to deteriorate when fewer points or more modes are used, reducing the degeneracy of the equations. It is expected that this condition can be simply corrected by including off-diagonal masses as unknowns.

In general, the results of the qualitative study are quite encouraging. It is recommended that a more definitive evaluation of the concepts be carried out both from a theoretical viewpoint and by more detailed computer experimentation with more complex structures.

NEW TECHNOLOGY

This entire report is considered to be a "reportable item" in accordance with the New Technology Clause (NASA Form 1162).

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GLOSSARY OF SYMBOLS

A	Coefficient matrix in mass equation (See Section IV)
B_R	Matrix multiplying R in mass identification (See Section IV)
B_m	Matrix multiplying m_A in mass identification (See Section IV)
C	Influence coefficient matrix
f	Force vector
g	Structural damping coefficient
K	Stiffness matrix
M	Mass matrix
m_i	Generalized mass of i-th mode
m_{ij}	Element of M
m_A	Approximation to $\bar{m}$
m_T	Total of diagonal masses
$\bar{m}$	Vector consisting of unknown elements of M
N	Number of modes
n_e	Number of equations
n_v	Number of variables
P	Number of points of interest
R	Right-Hand side of mass equation (See Section IV)
W	Weighting matrix referring to confidence in m_A (diagonal)
Y	Mobility matrix
y	Displacement vector

GLOSSARY OF SYMBOLS (Continued)

$\dot{y}_R$	Real component of velocity vector
Z	Impedance
Δ	Any change (as a prefix)
Φ	Matrix of modes
ϕ_i	Modal vector (i-th mode)
Ω_i	Natural frequency of i-th mode
ω	Forcing frequency
+	Superscript - pseudo inverse
$\lceil \, \rfloor$	Indicates diagonal matrix

APPENDIX I

PRINCIPAL IDEMPOTENTS

The expression of positive definite matrices in terms of their principal idempotents has been discussed in Reference 5. It has been useful in the development of the incomplete model concept and some of the associated techniques. A brief discussion is given below.

Define ϕ_i as an arbitrary set of orthogonal vectors (P vectors each having P elements) and A as an arbitrary positive definite PxP matrix.

It is possible to express A as follows:

$$A = \sum_{i=1}^{P} \sum_{j=1}^{P} C_{ij} \phi_i \phi_j^T \qquad \text{(I-1)}$$

since the ϕ's are independent and there are as many C_{ij}'s as there are elements in A. Note that $\phi_i \phi_j^T$ is a non-zero square matrix for all i, j combinations but $\phi_j^T \phi_i$ is a scalar and equal to 0 unless i=j since the vectors are orthogonal.

Now pre- and post-multiply the equations by ϕ_n^T and ϕ_k

$$\phi_n^T A \phi_k = \sum_{i=1}^{p} \sum_{j=1}^{p} C_{ij} (\phi_n^T \phi_i)(\phi_j^T \phi_k) = C_{nk} (\phi_n^T \phi_n)(\phi_k^T \phi_k)$$

since each term in the summation is zero except when i=n and j=k. Thus (note that $\phi_n^T A \phi_k$ is a scalar)

$$C_{ij} = \frac{\phi_i^T A \phi_j}{(\phi_i^T \phi_i)(\phi_j^T \phi_j)} \qquad \text{(I-2)}$$

Thus for any set of orthogonal vectors it is possible to express the matrix A as in Equation (I-1) by evaluating the coefficients as in Equation (I-2).

However, if the ϕ's are the eigenvectors of A, $A\phi_j = \lambda_j \phi_j$ and

$$C_{ij} = \frac{\lambda_j \phi_i^T \phi_j}{(\phi_i^T \phi_i)(\phi_j^T \phi_j)} = 0 \text{ for } i \neq j$$

since now $\phi_i^T \phi_j = 0$ for i = j, and

$$C_{ii} = \frac{\lambda_i}{\phi_i^T \phi_i}$$

Thus, the cross terms in Equation (I-1) become zero and

$$A = \sum_{i=1}^{p} \lambda_i \left(\frac{\phi_i \phi_i^T}{\phi_i^T \phi_i} \right) \qquad \text{(I-3)}$$

if and only if the ϕ's are eigenvectors of A. Note that (for any orthogonal ϕ's), if $V_i = \frac{\phi_i \phi_i^T}{\phi_i^T \phi_i}$, $V_i^2 = \frac{\phi_i (\phi_i^T \phi_i) \phi_i^T}{(\phi_i^T \phi_i)^2}$ $= V_i$ thus V_i is idempotent. When the ϕ_i's are the eigenvectors of A, V_i is called a principal idempotent.

By similar procedures it can be shown that the influence coefficient matrix and the stiffness matrix can be written in terms of their principal idempotents as follows (using the notation of the body of this report).

$$CM = \sum_{i=1}^{p} \frac{1}{\Omega_i^2} \left(\frac{\phi_i \phi_i^T M}{m_i} \right) \qquad \text{(I-4)}$$

$$M^{-1}K = \sum_{i=1}^{p} \Omega_i^2 \left(\frac{\phi_i \phi_i^T M}{m_i} \right) \qquad \text{(I-5)}$$

Where the terms in brackets are the principal idempotents of CM and $M^{-1}K$. It follows from the above two equations that

$$C = \sum_{i=1}^{p} \frac{1}{\Omega_i^2 m_i} \phi_i \phi_i^T \qquad \text{(I-6)}$$

$$K = \sum_{i=1}^{p} \frac{\Omega_i^2}{m_i} M \phi_i \phi_i^T M \qquad \text{(I-7)}$$

and that expressions of this form are only possible when the ϕ's are the eigenvectors of CM or $M^{-1}K$.

APPENDIX II

PSEUDO INVERSES

The pseudo inverse, sometimes called the generalized inverse of a matrix is discussed in numerous publications (e.g. Reference 6). In this appendix, only the application made to the problem at hand will be discussed.

Consider the set of independent linear equations

$$Ax = b \qquad \text{(II-1)}$$

where there are more equations than unknowns. A, then, is a rectangular matrix, $n_e \times n_v$ where $n_e < n_v$. x then is $1 \times n_v$ and b is $1 \times n_e$. There are then an infinite number of solutions to these equations.

The pseudo inverse of A, written A^+, is defined for this problem to be

$$A^+ = A^T(AA^T)^{-1} \qquad \text{(II-2)}$$

where the ordinary inverse of AA^T is used. This matrix, AA^T, is of order $n_e \times n_e$ and can be shown to be non-singular when the original equations are independent.

Now

$$\bar{x} = A^{+}b$$

is a solution to Equation (II-1) as can be seen from

$$A\bar{x} \equiv AA^{+}b \equiv AA^{T}(AA^{T})^{-1}b \equiv IB = b$$

It is shown in the literature that this solution (of the infinity of solutions) is the one having the minimum sum of squares of the individual elements, i.e. the "smallest" in a least squares sense.

Using Equation (II-2) to obtain the pseudo inverse of AW^{-1} from Equation (39) in the main text (where W^{-1} is a diagonal matrix),

$$(AW^{-1})^{+} = (AW^{-1})^{T}\{(AW^{-1})(AW^{-1})^{T}\}^{-1}$$

$$= W^{-1}A^{T}\{AW^{-1}W^{-1}A^{T}\}^{-1}$$

APPENDIX III

PROGRAM LISTING AND SAMPLE COMPUTATION

This appendix contains the Fortran listing of the computer program discussed in Section VIII. Included are the operating instructions and a sample computation of test 16.

The program was compiled and run on an IBM 360, Model 40, having 128K bytes of storage under DOS, release 17.

```
C************************************ INCMOD ************************************
C
C        INCOMPLETE MODEL THEORY - TEST PROGRAM - USES RESONANT RESP AS MODE
C
C  INPUT - PART  I - SIMULATED TEST
C        1.  HEAD1  COL 1 = IC,  COL 2-80 HEADING
C                IC = 0, NORMAL    (INPUT 2,(3),4,5,7,8FF)
C                     1, SAME AS 0, BUT C INPUT INSTEAD OF K (2,(3),4,6,7,8FF)
C                     2, NEW ACTUAL MASS AND FREC ONLY (INPUT 4,7,8FF)
C                     3, NEW FREQUENCIES ONLY  (INPUT 7,8FF)
C                     4. SAME ACTUAL SYSTEM, NEW TEST (INPUT 2A,(3),7,8FF)
C                     5. NO ANALYTICAL MODEL, INPUT PHI'S DIRECTLY (2A,7,7A,8)
C
C        2.  CONTROL CARD (PT. I)
C           COL  1,5  ND,        NO OF ACTUAL DEG OF FREEDOM           I5    20 MAX
C                6,10 NU,        NO OF MEASUREMENT  POINTS (USED)      I5    20 MAX
C                             IF NU .LT. ND, READS 3.
C               11,20 NROW,      DRIVING POINT INDEX                   I10
C                             MUST BE A USED POINT
C               21,30 G          SCALAR STRUCT DAMPING COEF            F10
C               31,40 PCT        RANDOM ERROR ON AMPLITUDE, ERROR      F10
C                             UNIFORMLY DISTRIBUTED BETWEEN -/+
C                             PCT*RESPONSE
C               41,50 PCTB       BIAS ERROR ON AMP = PCTB*RESP         F10
C               71,80 IZ         SEED FOR RANDOM NO GENERATOR          I10
C
C        2A. SAME AS 2. EXCEPT DOES NOT READ ND
C
C        3.  USED STA. CARD
C                    USED(I),I=1,NU      INDICES OF STATIONS
C                 TO BE USED   , IN NUMERICAL SEQUENCE            19I4    19 MAX
C
C        4.  ACTUAL MASS CARD(S)
C           FIRST CARD(S)  M(I,I),I=1,ND  DIAGONAL MASSES IN
C                 SEQUENCE, AS MANY  CARDS AS NECESSARY           8F10
C           NEXT CARD  COL 1,10  NOD,  NO OF OFF-DIAGONAL MASSES  I10
C           NEXT CARDS (IF NOD .NE. 0) I,J,MASS                 8(2I2,F6)
C                 LOWER TRIANGLE ONLY (SYMMETRY ASSUMED)
C                 8 PER CARD, AS MANY CARDS AS NECESSARY
C
C        5.  ACTUAL K CARDS  K(I,J)   I=1,ND ,J=1,I
C           STIFFNESS MATRIX, LOWER TRIANGLE INPUT ONLY, MATRIX
C                 WILL BE SYMMETRIZED
C                 START EACH ROW ON NEW CARD, END ON DIAGONAL
C                 ELEMENT                                         8F10
C
C        6.  ACTUAL C CARDS  C(I,J)
C           INFLUENCE COEF MATRIX, SAME FORM AS K
C
C        7.  ACTUAL NATURAL FREQ  (USED IN SIMULATED TEST)
C           FIRST CARD     NFREQ, NUMBER OF FREQUENCIES            I10    10 MAX
C           NEXT CARD(S)    FREQ(I),I=1,NFREC                     8F10
C
C        7A. ACTUAL PHI'S  (PHI(I,J),J=1,NFREC),I=1,NU
```

```
C
C  INPUT - PART II - IDENTIFICATION
C        8.  HEAD2  COL 1 = IC2,  COL 2-80 HEADING
C                   IC2 = 0, NORMAL (INPUT 9,10,11,12FF)
C                         1, NEW APPROX MASS - OTHER DATA UNCHANGED (11,12FF)
C
C        9.  GENERALIZED MASS - KNOWN VALUES ONLY - IF NONE USE 1 BLANK CARD
C                   CD1  COL 1-10  NG, NO OF VALUES                     I10  10 MAX
C                   FOLLOWING CARD(S)  I,GM(I)                          8(I2,F8)
C
C       10.  KNOWN MASSES - IF NONE USE 1 BLANK CARD
C                   CD1  COL 1-10  NK, NO OF VALUES                     I10  20 MAX
C                   FOLLOWING CARD(S) I,J,KM(I,J)  (SEE *NOTE)
C                   ENTER LOWER TRIANGLE VALUES ONLY (I .GE. J)    8(2I2,F6)
C                   SYMMETRY ASSUMED
C                   ONLY NON ZERO MASSES
C
C       11.  APPROX MASS CARDS - DEFINE UNKNOWNS
C                   CD1  COL 1-10  NV, NO OF UNKNOWN MASSES             I10  20 MAX
C                        COL 20    ICT = 0  NO CONTROL ON TOTAL MASS
C                                        1  TOTAL UNKNOWN DIAGONAL MASSES CONSTANT
C                   FOLLOWING CARDS, ONE PER MASS - LOWER TRIANGLE ONLY
C                   COL 1-5    I      (SEE *NOTE)
C                       6-10   J      (SEE *NOTE)
C                      11-20   AM, APPROX MASS (MAY BE ZERO IF OFF DIAG)
C                      21-30   W,  WEIGHTING FACTOR                 2I5,2F10
C                   HIGHER W INDICATES GREATER CONFIDENCE IN ESTIMATE
C                   DO NOT USE W = 0
C
C            NOTE, ALL MASSES NOT DEFINED IN 10 OR 11 WILL BE ZERO
C            SEE SUBROUTINE IDMASS FOR ERROR MESSAGES
C
C  INPUT - PART III - EFFECTS OF VARYING MASS OR  STIFFNESS
C
C       12.  HEAD3  COL 1= IC3,  COL 2-80 HEADING
C                   IC3 = 0, MASS CHANGE (INPUT 13,12FF)
C                       = 1, RETURN TO PART I (1)
C                       = 2, RETURN TO PART II (8)
C                       = 3, STIFFNESS (14,12FF)
C                       = 9, END OF RUN
C
C       13.  CHANGED MASS CARDS
C                    FIRST CARD    COL 1-10  NC, NO OF CHANGES          I10
C                    NEXT CARDS(S)  I,J,MASS CHANGE  (SEE *NOTE)  8(2I2,F6)
C                         LOWER TRIANGLE ONLY, SYMMETRY ASSUMED
C
C       14.  CHANGED STIFFNESS CARDS
C                    SAME FORM AS 13. EXCEPT                            4(2I2,F16)
C
C
C            NOTE, NEXT CARD READ IS 12.   THESE MASS CHANGES ARE ONLY
C            TEMPORARILY STORED.  VARIOUS CHANGES CAN BE RUN IN SEQUENCE.
C
C
C           *NOTE  I,J ARE INDICES OF TEST POINTS, NOT ACTUAL SYSTEM
```

```
C
C
C
C      SUBROUTINES SYM,INVRS,MOUT2,MCB,CINV,MMRY,RANCU,ICMASS,PSEUDO,MITER,GEN
C
      INTEGER HEAD(20),USED(20),HEAD2(20)
      INTEGER HEAD3(20),ITN(10)
      REAL M(20,21),K(20,21),C(20,21),FREC(10),ZR(20,21),ZI(20,21),
     1  YR(20,21) ,YI(20,21),RESR(10,20),RESI(10,20),GPP(10),PHI(20,10),
     2  GPPF(10),MINC(20,20),CINC(20,20),KINC(20,20)
      REAL MTEMP(20,21),MODE(20),PHIC(20,10),FRECC(10),CTEMP(20,20)
      REAL KTEMP(20,20),MINV(20,21),MODM(20),DK(20,20),GMASS(10),
     1 DCOF(20,10)
      COMMON M,K,C,ZR,ZI,YR,YI,MINC,CINC,KINC,MTEMP,CTEMP
      COMMON KTEMP,MINV,DK,DCOF
C                                     PART I - SIMULATED TEST
    1 READ (1,1000) IC,HEAD
 1000 FORMAT(I1,A3,19A4)
      WRITE (3,1001) IC,HEAD
 1001 FORMAT ('1'//T5,'INCOMPLETE MODEL THEORY - SIMULATED TEST'//
     1  T5 ,15('*'),I2,3X,A3,19A4,5X,15('*')///)
      ICC=IC+1
      GO TO (10,10,20,70,11,11      ),ICC
   10 READ (1,1002) ND, NU,      NROW,G,PCT,PCTB,IZ
 1002 FORMAT (2I5,I10,3F10.0,20X,I10)
      GO TO 13
   11 READ (1,2002) NU,NROW,G,PCT,PCTB,IZ
 2002 FORMAT (5X,I5,I10,3F10.0,20X,I10)
   13 IX=IZ*2+1
      GO TO (14,14,14,14,14,18),ICC
   14 IF(NU-ND)15,18,18
   15 READ (1,1003) (USED(I),I=1,NU)
 1003 FORMAT (19I4)
      DO 16 I=1,NU
      IF(USED(I)-NROW)16,17,16
   16 CONTINUE
      NROW=USED(1)
      NROWN=1
      WRITE (3,999) NROW
  999 FORMAT (T5,'***  FORCING POINT INVALID, SET TO' I3///)
      GO TO 20
   17 NROWN=I
      GO TO 20
   18 NROWN=NROW
      DO 19 I=1,NU
   19 USED(I)=I
   20 GO TO (25,25,25,25,70,70),ICC
   25 DO 30 I=1,ND
      DO 30 J=1,ND
   30 M(I,J)=0
      READ (1,1004) (M(I,I),I=1,ND)
 1004 FORMAT (8F10.0)
      READ(1,1005) NOD
 1005 FORMAT (I10)
      IF (NOD) 40,40,35
```

```
   35 READ (1,1006) (I,J,M(I,J),L=1,NDD)
 1006 FORMAT (8(2I2,F6.0))
      CALL SYM (M,ND)
   40 GO TO (50,60,70,70,70,70),ICC
   50 DO  55 I=1,ND
   55 READ (1,1004) (K(I,J),J=1,I)
      CALL SYM(K,ND)
      CALL INVRS (K,ND,C)
      GO TO 70
   60 DO 65 I=1,ND
   65 READ (1,1004) (C(I,J),J=1,I)
      CALL SYM (C,ND)
      CALL INVRS (C,ND,K)
   70 READ (1,1005) NFREQ
      READ (1,1004) (FREQ(I),I=1,NFREQ)
      GO TO (71,71,71,71,71,72),ICC
   71 WRITE (3,1007) ND,G
 1007 FORMAT (T10,'ACTUAL SYSTEM PARAMETERS'I10,' DEG OF FREEDOM, STRUCT
     1  DAMPING COEF ='F6.3/T10,24('-')//T50,'MASS MATRIX'/)
      GO TO 74
   72 WRITE (3,2007) NFREQ,G
 2007 FORMAT (T10,'ACTUAL MODES'I10,' MODES, STRUCTURAL DAMPING COEF ='
     1 F6.3/T10,12('-')//)
      GO TO 90
   74 CALL MOUT2 (M,ND,ND)
      GO TO (75,75,80,80,80,80),ICC
   75 WRITE (3,1008)
 1008 FORMAT ('1',T50,'STIFFNESS MATRIX'//)
      CALL MOUT2 (K,ND,ND)
      WRITE (3,1009)
 1009 FORMAT ('1'T50,'INFLUENCE COEFFICIENT MATRIX'//)
      CALL MOUT2 (C,ND,ND)
      GO TO 90
   80 WRITE (3,1010)
 1010 FORMAT (//T10,'ACTUAL K AND C SAME AS PREVICUS CASE')
   90 WRITE (3,1011) NROW,NU
 1011 FORMAT ('1'//T10,'DESCRIPTION OF SIMULATED TEST'/T10,29('-')//
     1  T20,'POINT AT WHICH SYSTEM IS FORCED'I4/T20,'NO OF POINTS OF MEA
     2SUREMENT'I8)
      GO TO (94,94,94,94,94,96),ICC
   94 IF(ND-NU) 100,100,95
   95 WRITE (3,1012) (USED(I),I=1,NU)
 1012 FORMAT (T20,'POINTS AT WHICH MEASUREMENTS ARE TAKEN'/T30,20I4)
  100 WRITE (3,1013)(FREQ(I),I=1,NFREQ)
 1013 FORMAT (T20,'FREQUENCIES OF EXCITATICN - HZ'/(T30,1P5F15.4))
   96 WRITE (3,1014) PCT,PCTB,IZ
 1014 FORMAT(/T20,'AVE RANDOM AMPLITUDE ERROR'F10.3/T20,'BIAS AMPLITUDE
     1ERROR'F16.3//T20,'SEED FOR RAND NO GENERATOR'I12//)
      GO TO (109,109,109,109,109,101),ICC
  101 DO 102 J=1,NU
  102 READ (1,1004)   (RESR(I,J),I=1,NFREQ)
      GO TO 111
C                                CALCULATE RESCNANT RESPCNSES
  109 DO 110 I=1,NFREQ
      OM=FREQ(I)
```

```
      CALL MOB (M,K,G,ND,CM,ZR,ZI,YR,YI)
      DO 110 J=1,NU
      JJ=USED(J)
C                                  COMPLEX FORM
      RESR(I,J)=YR(NROW ,JJ)
  110 RESI(I,J)=YI(NROW ,JJ)
C                                  CALC ASSUMED PHI'S = REAL RESPONSE WITH ERROR
  111 DO 120 I=1,NU
      DO 120 J=1,NFREQ
      CALL RANDU (IX,IY,YFL)
      IX=IY
  120 PHI(I,J)=RESR(J,I)*(1.0+2.0*PCT*(YFL-0.5)+PCTR)
C                                  NORMALIZE ON DRIVING POINT
      DO 130 I=1,NFREQ
      GPP(I)=RESR(I,NROWN)
      GPPE(I)=PHI(NROWN,I)
      DO 130 J=1,NU
      PHI(J,I)=PHI(J,I)/GPPE(I)
      IF(IC-5) 129,130,130
  129 RESI(I,J)=RESI(I,J)/GPP(I)
  130 RESR(I,J)=RESR(I,J)/GPP(I)
      GO TO (135,135,135,135,135,131),ICC
  131 WRITE (3,2015) (FREQ(I),I=1,NFREQ)
 2015 FORMAT (//T10,'EXACT MODES OF ACTUAL SYSTEM, NORMALIZED ON DRIVING
     1 POINT'//T3,'FREQ    '1P10E12.4)
      GO TO 139
  135 WRITE (3,1015) (FREQ(I),I=1,NFREQ)
 1015 FORMAT(//T10,'EXACT RESPONSE OF ACTUAL SYSTEM,IN/SEC/POUND, NORMAL
     1IZED ON REAL DRIVING POINT RESPONSE'//T30,'REAL PART'//T3,'FREQ
     2'1P10E12.4)
      WRITE (3,1016) (GPP(I),I=1,NFREQ)
 1016 FORMAT(/T3,'D.P.'/T4,'RESP   '1P10E12.4)
  139 WRITE (3,1004)
      DO 140 I=1,NU
  140 WRITE (3,1017) USED(I),(RESR(J,I),J=1,NFREQ)
 1017 FORMAT (I6,3X,10F12.5)
      GO TO (142,142,142,142,142,141),ICC
  141 WRITE (3,2019)
 2019 FORMAT ('1'//T10,'SIMULATED MEASURED MODES, ACTUAL PHI''S WITH ERR
     1OR'/)
      GO TO 154
  142 WRITE (3,1018)
 1018 FORMAT ('1'//T30,'IMAGINARY PART'/)
      DO 145 I=1,NU
  145 WRITE (3,1017) USED(I),(RESI(J,I),J=1,NFREQ)
      WRITE (3,1019)
 1019 FORMAT ('1'//T10,'SIMULATED TEST RESPONSE,  REAL MOBILITY WITH ERR
     1ORS USED AS PHI''S'/)
      WRITE (3,1016) (GPPE(I),I=1,NFREQ)
      WRITE (3,1004)
  154 CALL MOUT2 (PHI,NU,NFREQ)
C                                  PART II - IDENTIFY MASSES
  155 READ (1,1000) IC2,HEAD2
      CALL IDMASS (IC2,HEAD2,FREQ,PHI,NU,NFREQ,MINC,CINC,KINC,GMASS)
C                         CALUATE RESONANT RESPONCE  FROM KINC,MINC
```

```
      DO 160 I=1,NFREQ
      OM=FREQ(I)
      CALL MOB (MINC,KINC,G,NU,OM,ZR,ZI,YR,YI)
      DO 160 J=1,NU
      RESR(I,J)=YR(J,NROWN)
  160 RESI(I,J)=YI(J,NROWN)
      DO 170 I=1,NFREQ
      GPP(I)=RESR(I,NROWN)
      DO 170 J=1,NU
      RESI(I,J)=RESI(I,J)/GPP(I)
  170 RESR(I,J)=RESR(I,J)/GPP(I)
      WRITE (3,1020) (FREQ(I),I=1,NFREQ)
 1020 FORMAT ('1'//T10,'RESONANT RESPONSE OF IDENTIFIED SYSTEM'//
     1 T30,'REAL PART'//
     2 T3,'FREQ     '1P10F12.4)
      WRITE (3,1016) (GPP(I),I=1,NFREQ)
      WRITE (3,1004)
      DO 180 I=1,NU
  180 WRITE (3,1017) USED(I),(RESR(J,I),J=1,NFREQ)
      WRITE (3,1018)
      DO 190 I=1,NU
  190 WRITE (3,1017) USED(I),(RESI(J,I),J=1,NFREQ)
      IFIRST =0
      GO TO 255
C                                    PART III - CHANGED MASSES
  200 READ (1,1000)  IC3,HEAD3
      IF(IC3-9)220,210,210
  210 CALL EXIT
  220 ICC=IC3+1
      GO TO (250,1,155,250 ),ICC
  250 WRITE (3,1030) IC3,HEAD3
 1030 FORMAT ('1'//T5,'PART III - CHANGED MASSES OR  STIFFNESSES'//T5,
     1 15('*'),I2,3X,A3,19A4,5X,15('*')///)
      READ (1,1005) NC
  255 DO 260 I=1,NU
      DO 260 J=1,NU
      KTEMP(I,J)=0
  260 MTEMP(I,J)=0
      IF(IFIRST)261,269,261
  261 IF(IC3)265,265,300
  265 READ (1,1006) (I,J,MTEMP(I,J),L=1,NC)
      CALL SYM (MTEMP,NU)
      WRITE (3,1031)
 1031 FORMAT (T50,'MASS MATRIX CHANGES'/)
      CALL MOUT2 (MTEMP,NU,NU)
  269 DO 270 I=1,NU
      DO 270 J=1,NU
      CTEMP(I,J)=CINC(I,J)
  270 MTEMP(I,J)=MTEMP(I,J)+MINC(I,J)
C                                    ITERATE USING CINC * MTEMP
      DO 280 I =1,NFREQ
      CALL MITER(CTEMP,MTEMP,NU,.00001,25,MODE,OMEG,IT)
      CON=MODE(NROWN)
      DO 275 J=1,NU
      MODE(J)=MODE(J)/CON
```

```
  275 PHIC(J,I)=MODE(J)
      ITN(I)=IT
      FREQC(I)=1.0/SQRT(OMEG)/6.2832
      GPPE(I)=GEN(MODE,MTEMP,NU)
      CON=OMEG/GPPE(I)
      DO 280 L=1,NU
      DO 280 J=1,NU
  280 CTEMP(L,J)=CTEMP(L,J)-CON*MODE(L)*MODE(J)
      IF(IFIRST)285,281,285
  281 WRITE(3,2032) (FREQ(I),I=1,NFREQ)
 2032 FORMAT ('1'//T10,'FREQUENCIES AND MODES OF IDENTIFIED SYSTEM'//
     1 T3,'FREQ    '1P10F12.4)
      GO TO 289
  285 WRITE (3,1032) (FREQC(I),I=1,NFREQ)
 1032 FORMAT ('1'//T10,'NEW FREQUENCIES AND MODES OF MODIFIED SYSTEM'//
     1 T3,'FREQ    '1P10F12.4)
  289 WRITE (3,1033) (GPPE(I),I=1,NFREQ)
 1033 FORMAT (/T2,'GEN MASS'1P10F12.4)
      WRITE (3,1034) (ITN(I),I=1,NFREQ)
 1034 FORMAT (/T3,'ITER'10I12)
      WRITE (3,1004)
      DO 290 I=1,NU
  290 WRITE (3,1017) USED(I),(PHIC(I,J),J=1,NFREQ)
      IFIRST=IFIRST+1
      GO TO 200
  300 READ (1,3031) (I,J,KTEMP(I,J),L=1,NC)
 3031 FORMAT (4(2I2,E16.0))
      CALL SYM (KTEMP,NU)
      WRITE (3,2031)
 2031 FORMAT (T50,'K MATRIX CHANGES')
      CALL MOUT2 (KTEMP,NU,NU)
C                                  CALCULATE DELTA K INCOMPLETE
      CALL MMPY (KTEMP,PHI,NU,NU,NFREQ,DK)
      DO 310 I=1,NU
      DO 310 J=1,NFREQ
  310 MINV(J,I)=PHI(I,J)
      CALL MMPY (MINV,DK,NFREQ,NU,NFREQ,DCOF)
      DO 320 I=1,NU
      DO 320 J=1,NU
  320 DK(I,J)=0
      DO 330 IJ=1,NFREQ
      DO 330 JI=1,NFREQ
      DO 330 I=1,NU
      DO 330 J=1,NU
  330 DK(I,J)=DK(I,J)+PHI(I,IJ)*PHI(J,JI)*DCOF(IJ,JI)/GMASS(IJ)/
     1 GMASS(JI)
      CALL MMPY (MINC ,DK,NU,NU,NU,KTEMP)
      CALL MMPY (KTEMP,MINC ,NU,NU,NU,DK)
      WRITE (3,1040)
 1040 FORMAT ('1'T50,'DELTA K INCOMPLETE')
      CALL MOUT2 (DK,NU,NU)
      DO 340 I=1,NU
      DO 340 J=1,NU
  340 KTEMP(I,J)=KINC(I,J)+DK(I,J)
      CALL INVRS (MINC ,NU,MINV)
```

```
C                                 ITERATE USING MINV*KTEMP
      DO 380 II=1,NFREQ
      I=NFREQ-II+1
      CALL MITER(MINV  ,KTEMP,NU,.00001,25,MODE,OMEG,IT)
      CON=MODE(NROWN)
      DO 375 J=1,NU
      MODE(J)=MODE(J)/CON
  375 PHIC(J,I)=MODE(J)
      ITN(I)=IT
      CALL MMPY (MINC  ,MODE,NU,NU,1,MODM)
      FREQC(I)=     SQRT(OMEG)/6.2832
      GPPE(I)=GEN(MODE,MINC  ,NU)
      CON=OMEG/GPPE(I)
      DO 380 L=1,NU
      DO 380 J=1,NU
  380 KTEMP(L,J)=KTEMP(L,J)-CON*MODM(L)*MODM(J)
      GO TO 285
      END
```

```
      SUBROUTINE MOUT2 (A,M,N)
      REAL A(20,21)
      ID=MIN0(N,10)
      WRITE (3,1000) (I,I=1,ID)
 1000 FORMAT (/T5,10I12)
      WRITE (3,1000)
      DO 10 I=1,M
   10 WRITE (3,1001) I,(A(I,J),J=1,ID)
 1001 FORMAT (I5,5X,1P10F12.4)
      IF (ID-N) 20,50,50
   20 WRITE (3,1000) (I,I=11,N)
      WRITE (3,1000)
      DO 30 I=1,M
   30 WRITE (3,1001) I,(A(I,J),J=11,N)
   50 RETURN
      END
```

```
      SUBROUTINE CINV (A,B,N,C,D)
C
C                                 C+I*D = INVERSE OF A+I*B       I=SQRT(-1)
C
C                                 B ASSUMED NON SINGULAR
C
      REAL A(20,21),B(20,21),C(20,21),D(20,21),E(20,21)
      CALL INVRS(B,N,C)
      CALL MMPY(C,A,N,N,N,E)
      CALL MMPY(A,E,N,N,N,C)
      DO 10 I=1,N
      DO 10 J=1,N
   10 C(I,J)=C(I,J)+B(I,J)
      CALL INVRS(C,N,D)
      CALL MMPY(E,D,N,N,N,C)
      DO 20 I=1,N
      DO 20 J=1,N
   20 D(I,J)=-D(I,J)
      RETURN
      END
```

```
      SUBROUTINE SYM (A,N)
C               FORMS SYMMETRIC MATRIX FROM LOWER TRIANGLE
      REAL A(20,21)
      N1=N-1
      DO 10 I=1,N1
      I1=I+1
      DO 10 J=I1,N
   10 A(I,J)=A(J,I)
      RETURN
      END
```

```
      SUBROUTINE IDMASS (IC,HEAD,FR,PHI,P,A,MASS,CINC,KINC,GMASS)
C                 USES PSEUDO INVERSE TO OBTAIN UNKNOWN MASSES AND
C                 INCOMPLETE STIFFNESS AND INFLUENCE COEFFICIENT MATRICES
C                 SEE MAIN PROGRAM FOR INPUT DESCRIPTION
C
C   INPUT ERROR MESSAGES
C                       MASS 'I' INDEX TOO LARGE
C                       MASS 'I'  I LESS THAN J
C                       MASSES 'I' AND 'J' HAVE DUPLICATE INDICES
C                       WEIGHTING FUNCTION 'I' =0
C                       APPROX MASS 'I' AND KNOWN MASS 'J' HAVE SAME INDICES
C
      INTEGER HEAD(20),P,IGM(10),IKM(20),JKM(20),IAM(20),JAM(20)
      REAL PHI(20,10),GM(10),KM(20),AM(20),W(20),A(20,20),R(20),
     1 AW(20,20),BR(20,21),BM(20,20),MMIN(20),MPAR(20),DEL(20),
     2 FR(10),MASS(20,20),CCOFF(10),KCOFF(10),CINC(20,20),KINC(20,20)
      REAL GMASS(10)
C                                INPUT
   10 WRITE (3,1001) IC,HEAD
 1001 FORMAT ('1'//T10,'PART II - IDENTIFICATION'//T5,15('*'),I2,3X,
     1  A3,19A4,5X,15('*')///)
      IFR=0
      IERR=0
      ICC =IC+1
 1002 FORMAT (2I10)
      GO TO (41,42),ICC
   41 READ (1,1002) NG
   42 IF(NG) 50,50,60
   50 WRITE (3,1007)
 1007 FORMAT (//T20,'ALL GENERALIZED MASSES UNKNOWN'/)
      GO TO 70
   60 GO TO (61,62),ICC
   61 READ (1,1008) (IGM(I),GM(I),I=1,NG)
 1008 FORMAT (8(I2,F8.0))
   62 WRITE (3,1009) (IGM(I),GM(I),I=1,NG)
 1009 FORMAT (//T20,'KNOWN GENERALIZED MASSES'//(T5,  'M('I2,') = '
     1 1PE10.4,T29,I2,') = 'E10.4,T27,'M('T51,I2,') = 'E10.4,T49,'M('
     2 T73,I2,') = 'E10.4,T71,'M('T95,I2,') = 'E10.4,T93,'M('))
   70 GO TO (71,72),ICC
   71 READ (1,1002) NK
   72 IF (NK) 80,80,90
   80 WRITE (3,1010)
 1010 FORMAT (//T20,'ALL MASSES UNKNOWN'/)
      GO TO 100
   90 GO TO (91,92),ICC
   91 READ (1,1011) (IKM(I),JKM(I),KM(I),I=1,NK)
 1011 FORMAT (8(2I2,F6.0))
   92 WRITE (3,1012) (IKM(I),JKM(I),KM(I),I=1,NK)
 1012 FORMAT (//T20,'KNOWN MASSES'//(T4   ,'M('I2,',',I2,') = '1PE11.4,
     1T31,I2,',',I2,') ='F12.4,T29,'M('T57 ,I2,',',I2,') ='F12.4,T55 ,'M('
     2T81,I2,',',I2,') ='F12.4,T79,'M('T106,I2,',',I2,') ='E12.4,T104,
     3'M('))
C                                INPUT ERROR TEST
      DO 920 I=1,NK
```

```
      L=IKM(I)
      K=JKM(I)
      IF(L-P) 901,901,902
  901 IF(K-P) 905,905,902
  902 IF(IER) 903,903,904
  903 WRITE (3,2000)
 2000 FORMAT (/T5,'INPUT ERRORS, KNOWN MASSES'/)
  904 IER=IER+1
      WRITE (3,2001) I
 2001 FORMAT (T10,'MASS'I4,' INDEX TOO LARGE')
  905 IF(L-K) 906,910,910
  906 IF(IER)907,907,908
  907 WRITE (3,2000)
  908 IER =IER+1
      WRITE (3,2002) I
 2002 FORMAT (T10,'MASS'I4,', I LESS THAN J')
  910 IF(I-NK) 911,920,920
  911 J1=I+1
      DO 920 J=J1,NK
      IF(L-IKM(J)) 920,912,920
  912 IF(K-JKM(J)) 920,913,920
  913 IF(IER) 914,914,915
  914 WRITE (3,2000)
  915 IER=IER+1
      WRITE (3,2003) I,J
 2003 FORMAT (T10,'MASSES'I4,' AND'I4,' HAVE DUPLICATE INDICES')
  920 CONTINUE
  100 READ (1,1013) NV,ICT,(IAM(I),JAM(I),AM(I),W(I),I=1,NV)
 1013 FORMAT (I10,9X,I1/(2I5,2F10.0))
C                                   INPUT ERROR TEST
      DO 950 I=1,NV
      L=IAM(I)
      K=JAM(I)
      IF(L-P) 921,921,922
  921 IF(K-P) 925,925,922
  922 IF(IERR) 923,923,924
  923 WRITE (3,2004)
 2004 FORMAT (/T5,'INPUT ERRORS, APPROX MASSES'/)
  924 IERR=IERR+1
      WRITE (3,2001) I
  925 IF(L-K) 926,929,929
  926 IF(IERR) 927,927,928
  927 WRITE (3,2004)
  928 IERR=IERR+1
      WRITE (3,2002) I
  929 IF(W(I)) 935,930,935
  930 IF(IERR) 931,931,932
  931 WRITE (3,2004)
  932 IERR=IERR+1
      WRITE (3,2005) I
 2005 FORMAT (T10,'WEIGHTING FUNCTION'I4,' = 0')
  935 IF(I-NV)936,942,942
  936 J1=I+1
      DO 941 J=J1,NV
      IF(L-IAM(J)) 941,937,941
```

```
  937 IF(K-JAM(J)) 941,938,941
  938 IF(IERR)939,939,940
  939 WRITE (3,2004)
  940 IERR=IERR+1
      WRITE (3,2003) I,J
  941 CONTINUE
  942 IF(NK) 950,950,943
  943 DO 950 J=1,NK
      IF(L-IKM(J)) 950,944,950
  944 IF(K-JKM(J))950,945,950
  945 IF(IERR) 946,946,947
  946 WRITE (3,2004)
  947 IERR=IERR+1
      WRITE (3,2006) I,J
 2006 FORMAT (T10,'APPROX MASS'I4,' AND KNOWN MASS'I4,'HAVE SAME INDICES
     1')
  950 CONTINUE
      IF(IER )951,951,500
  951 IF(IERR)101,101,500
  101 NZE = N*(N-1)/2
      NE = NZE+NG+MINO(ICT,1)
      NR=NV-NE
      WRITE (3,1014) NE,NV,NR
 1014 FORMAT (//T20,'PROBLEM DEFINITION   NO OF EQUATIONS = 'I2/
     1 T41,'NO OF VARIABLES = 'I2/T37,'ORDER OF DEGENERACY = 'I2)
      IF (NR) 110,120,149
  110 WRITE (3,1015)
 1015 FORMAT ('+'T65,'NOTE - NO EXACT SOLUTION POSSIBLE')
      GO TO 149
  120 WRITE (3,1016)
 1016 FORMAT ('+'T65,'NOTE - ONLY ONE SOLUTION POSSIBLE, APPROX MASSES N
     1OT USED')
  149 IF(ICT)130,151,130
  130 WRITE(3,2016)
 2016 FORMAT (/T41,'TOTAL OF DIAGONAL MASSES CONSTANT')
  151 IF(NE-20) 159,159,152
  152 WRITE (3,3016)
 3016 FORMAT (//T10,'*** TOO MANY EQUATIONS, LIMIT IS 20 ***')
      GO TO 500
  159 GO TO (150,250),ICC
C                                FORM A MATRIX (NE X NV), R VECTOR (NE)
  150 DO 160 I=1,NE
  160 R(I)=0
C                                OFF-DIAGONAL EQUATIONS
      IE=0
      N1=N-1
      DO 200 I=1,N1
      J1=I+1
      DO 200 J=J1,N
      IE=IE+1
      DO 180 IV=1,NV
      L=IAM(IV)
      K=JAM(IV)
      IF(K-L) 170,165,170
  165 A(IE,IV)=PHI(L,I)*PHI(L,J)
```

```
      GO TO 180
  170 A(IE,IV)=PHI(L,I)*PHI(K,J)+PHI(K,I)*PHI(L,J)
  180 CONTINUE
      IF(NK)200,200,185
  185 DO 199 IK=1,NK
      L=IKM(IK)
      K=JKM(IK)
      IF(K-L) 195,190,195
  190 R(IE)=R(IE)-KM(IK)*PHI(L,I)*PHI(L,J)
      GO TO 199
  195 R(IE)=R(IE)-KM(IK)*(PHI(L,I)*PHI(K,J)+PHI(K,I)*PHI(L,J))
  199 CONTINUE
  200 CONTINUE
      IF(NG) 250,250,210
C                                 EQS CORRESPONDING TO DIAG TERMS
  210 DO 240 II=1,NG
      I=IGM(II)
      IE=IE+1
      DO 230 IV=1,NV
      L=IAM(IV)
      K=JAM(IV)
      IF (K-L) 225,220,225
  220 A(IE,IV)=PHI(L,I)*PHI(L,I)
      GO TO 230
  225 A(IE,IV)=2.0*PHI(L,I)*PHI(K,I)
  230 CONTINUE
      R(IE)=GM(I)
      IF(NK) 240,240,231
  231 DO 239 IK=1,NK
      L=IKM(IK)
      K=JKM(IK)
      IF(K-L) 235,232,235
  232 R(IE)=R(IE)-KM(IK)*PHI(L,I)*PHI(L,I)
      GO TO 239
  235 R(IE)=R(IE)-2.0*KM(IK)*PHI(L,I)*PHI(K,I)
  239 CONTINUE
  240 CONTINUE
  250 IF(ICT)251,260,251
C                                 EQUATION FOR SUM OF DIAGONAL MASSES
  251 IE=IE+1
      R(IE)=0
      DO 259 IV=1,NV
      L=IAM(IV)
      K=JAM(IV)
      IF(L-K) 258,252,258
  252 A(IE,IV)=1.0
      R(IE)=R(IE)+AM(IV)
      GO TO 259
  258 A(IE,IV)=0
  259 CONTINUE
C                                 A MATRIX OUTPUT
  260 WRITE (3,1017) NE,NV
 1017 FORMAT ('1'//T20,'A MATRIX'I4,' X'I3//)
      CALL MOUT2  (A,NE,NV)
      WRITE (3,1019)
```

```
 1019 FORMAT (///T20,'F VECTOR'//)
      WRITE (3,1020)(P(I),I=1,NF)
 1020 FORMAT (/(T6,1P10E12.4))
C                                   FORM AW = A/W
      DO 270 J=1,NV
      W1=1.0/W(J)
      DO 270 I=1,NF
  270 AW(I,J)=A(I,J)*W1
C                                   FORM BR
      CALL PSEUDO (AW,NE,NV,BR)
      DO 280 I=1,NV
      W1=1.0/W(I)
      DO 280 J=1,NE
  280 BR(I,J)=BR(I,J)*W1
C                                   FORM BM
      CALL MMPY (BR,A,NV,NF,NV,BM)
      DO 290 I=1,NV
      DO 290 J=1,NV
      BM(I,J)=-BM(I,J)
      IF(I-J) 290,285,290
  285 BM(I,J)=BM(I,J)+1.0
  290 CONTINUE
      WRITE (3,1021) NV,NE
 1021 FORMAT ('1'//T20,'BR MATRIX'I4,' X'I3//)
      CALL MOUT2 (BR,NV,NE)
      WRITE (3,1022) NV,NV
 1022 FORMAT ('1'//T20,'BM MATRIX'I4,' X'I3//)
      CALL MOUT2 (BM,NV,NV)
C                                   FORM SOLUTION
      CALL MMPY (BR,R,NV,NE,1,MMIN)
      CALL MMPY (BM,AM,NV,NV,1,MBAR)
      TM =0
      TMB=0
      DELS=0
      DO 310 I=1,NV
      MBAR(I)=MBAR(I)+MMIN(I)
      TM=TM+AM(I)
      TMB=TMB+MBAR(I)
      DEL(I)=MBAR(I)-AM(I)
  310 DELS=DELS+DEL(I)*DEL(I)
      DELS=SQRT(DELS/NV)
C                                   MASS OUTPUT
      WRITE (3,1023)
 1023 FORMAT ('1'//T30,'BEST MASS SOLUTION'//T5,'I    J     APPROX MASS   W
     1GT FACT      MIN MASS        BEST MASS          CHANGE'//)
      WRITE (3,1024) (IAM(I),JAM(I),AM(I),W(I),MMIN(I),MBAR(I),DEL(I),
     1 I=1,NV)
 1024 FORMAT (I5,I4,1PE15.4,0PF10.4,1P3E15.4)
      WRITE (3,1025) TM,TMB,DELS
 1025 FORMAT (//T4,'TOTALS'1PE15.4,T50,E15.4//T4,'RMS OF CHANGES'F12.4)
C                                   FORM COMPLETE MASS MATRIX
      DO 320 I=1,P
      DO 320 J=1,P
  320 MASS(I,J)=0
      DO 330 I=1,NV
```

```
      L=IAM(I)
      K=JAM(I)
      MASS(L,K)=MBAR(I)
  330 MASS(K,L)=MBAR(I)
      IF(NK) 355,355,340
  340 DO 350 I=1,NK
      L=IKM(I)
      K=JKM(I)
      MASS(L,K)=KM(I)
  350 MASS(K,L)=KM(I)
  355 WRITE (3,1026)
 1026 FORMAT ('1'//T30,     'FULL MASS MATRIX'//)
      CALL MOUT2 (MASS,P,P)
C                                   FORM GENERALIZED MASS MATRIX
C                                   (BM,BR DESTROYED)
      DO 370 I=1,P
      DO 370 J=1,N
  370 BM(J,I)=PHI(I,J)
      CALL MMPY (BM,MASS,N,P,P,BR)
      CALL MMPY (BR,PHI,N,P,N,BM)
      WRITE (3,1027)
 1027 FORMAT ('1'//T30,'GENERALIZED MASS MATRIX'//)
      CALL MOUT2 (BM,N,N)
      DO 375 I=1,N
  375 GMASS(I)=BM(I,I)
C                                   FORM C, K MATRICES
      DO 390 I=1,N
      OM=FR(I)*FR(I)*39.4784
      CCOEF(I)=1.0/(OM*BM(I,I))
  390 KCOEF(I)=OM/BM(I,I)
      DO 395 I=1,P
      DO 395 J=1,P
      CINC(I,J)=0
  395 KINC(I,J)=0
      DO 400 IN=1,N
      DO 400 I=1,P
      DO 400 J=1,P
      CINC(I,J)=CINC(I,J)+CCOEF(IN)*PHI(I,IN)*PHI(J,IN)
  400 KINC(I,J)=KINC(I,J)+KCOEF(IN)*BR(IN,I)*BR(IN,J)
      WRITE (3,1029) (KCOEF(I),I=1,N)
 1029 FORMAT ('1'//T30,'INCOMPLETE STIFFNESS COEF'//T10,'MODAL COEFFICIE
     1NTS'/T6,1P10F12.4)
      CALL MOUT2 (KINC,P,P)
      WRITE (3,1028) (CCOEF(I),I=1,N)
 1028 FORMAT ('1'//T30,'INCOMPLETE INFLUENCE COEF'//T10,'MODAL COEFFICIE
     1NTS'/T6,1P10F12.4)
      CALL MOUT2 (CINC,P,P)
  500 RETURN
      END
```

```
      SUBROUTINE MITER (A,B,N,TOL,ITMAX,FUN,VAL,IT)
C
C                 ITERATES ON A*B FOR DOMINENT EIGENFUNCTION (FUN)
C                    AND EIGENVALUE (VAL).
C                 N IS ORDER
C                 TOL IS DECIMAL (.01 PERCENT) TOLERANCE ON VAL.
C                 ITMAX IS MAX NO OF ITERATIONS.
C                 IT IS NUMBER OF ITERATIONS PERFORMED.
C
C                 A,B ARE SQUARE OF ORDER N (DIMENSIONED (20,21) ).
C
C           USES MMPY (A,B,N1,N2,N3,C)
      REAL A(20,21),B(20,21),C(20,21),DUM(20),FUN(20)
      CALL MMPY (A,B,N,N,N,C)
      VALO=100.
      IT=1
      DO 10 I=1,N
   10  FUN(I)=1.0
   11 CALL MMPY (C,FUN,N,N,1,DUM)
      VAL=DUM(1)
      DO 20 I=2,N
      IF(ABS(VAL)-ABS(DUM(I)))15,20,20
   15 VAL=DUM(I)
   20 CONTINUE
      DO 30 I=1,N
   30 FUN(I)=DUM(I)/VAL
      IF(ABS(VAL/VALO-1.0)-TOL) 50,50,40
   40 IT=IT+1
      VALO=VAL
      IF(IT-ITMAX) 11,11,50
   50 RETURN
      END
```

```
      FUNCTION GEN (FUN,A,N)
C
C                                 GEN = FUN(TRANS) * A * FUN
C
      DIMENSION A(20,21),FUN(20)
      GEN=0
      DO 20 I=1,N
      DUM=0
      DO 10 J=1,N
   10 DUM=DUM+A(I,J)*FUN(J)
   20 GEN=GEN+DUM*FUN(I)
      RETURN
      END
```

```
      SUBROUTINE PSEUDO (A,NR,NC,C)
C
C             C = PSEUDOINVERSE OF A     A UNDISTURBED
C             A IS A RECTANGULAR MATRIX OF MAXIMAL RANK  (NR X NC)
C             NR .GT. OR .LT. NC
C
C                           -1                          -1
C             C =   (A'A)   A'    OR    A'(AA')
C
C             NR,NC MAY NOT EXCEED 20
C
       REAL A(20,20),B(20,21),C(20,21)
C                                                 B = A'
      DO 10 I=1,NR
      DO 10 J=1,NC
   10 B(J,I)=A(I,J)
      IF(NR-NC)20,20,30
C                                     NR .LE. NC
C                                     C = AA'
   20 CALL MMPY (A,B,NR,NC,NR,C)
C                                     A = INV OF C
      CALL INVRS (C,NR,A)
C                                     C = PSEUDOINVERSE OF A (NC X NR)
      CALL MMPY (B,A,NC,NR,NR,C)
      GO TO 40
C                                     NC .LT. NR
C                                     C = A'A
   30 CALL MMPY (B,A,NC,NR,NC,C)
C                                     A = INV OF C
      CALL INVRS (C,NC,A)
C                                     C = PSEUDOINVERSE OF A  (NC X NR)
      CALL MMPY (A,B,NC,NC,NR,C)
C                                     RESTORE A
   40 DO 50 I=1,NR
      DO 50 J=1,NC
   50 A(I,J)=B(J,I)
      RETURN
      END
```

```
      SUBROUTINE INVRS (B,N,A)
C     A = INVERSE OF B          B UNDISTURBED
      DIMENSION A(20,21),D(20,21),IROW(21),ICOL(21),B(20,21)
      DO 1 I=1,N
      DO 1 J=1,N
    1 A(I,J)=B(I,J)
      M=N+1
      DO 7  I=1,N
      IROW(I)=I
    7 ICOL(I)=I
      DO 20 K=1,N
      AMAX= A(K,K)
      DO 10 I=K,N
      DO 10 J=K,N
      IF(ABS( A(I,J))-ABS(AMAX))10,9,9
    9 AMAX= A(I,J)
      IC=I
      JC=J
   10 CONTINUE
      KI=ICOL(K)
      ICOL(K)=ICOL(IC)
      ICOL(IC)=KI
      KI=IROW(K)
      IROW(K)=IROW(JC)
      IROW(JC)=KI
      IF(AMAX) 11,12,11
   12 WRITE (3,13)
   13 FORMAT(' SOLUTION OF EXISTING MATRIX NOT POSSIBLE')
      GO TO 100
   11 DO 14 J=1,N
      E=A(K,J)
      A(K,J)=A(IC,J)
   14 A(IC,J)=E
      DO 15 I=1,N
      E=A(I,K)
      A(I,K)=A(I,JC)
   15 A(I,JC)=E
      DO 16 I=1,N
      IF(I-K) 18,17,18
   17 A(I,M)=1.
      GO TO 16
   18 A(I,M)=0.
   16 CONTINUE
      PVT=A(K,K)
      DO 8 J=1,M
    8 A(K,J)=A(K,J)/PVT
      DO 19 I=1,N
      IF(I-K)21,19,21
   21 AMULT=A(I,K)
      DO 22 J=1,M
   22 A(I,J)=A(I,J)-AMULT*A(K,J)
   19 CONTINUE
      DO 20 I=1,N
   20  A(I,K)=A(I,M)
```

```
      DO 25 I=1,N
      DO 24 L=1,N
      IF(IROW(I)-L)24,23,24
   24 CONTINUE
   23 DO 25 J=1,N
   25 D(L,J)=A(I,J)
      DO 26 J=1,N
      DO 28 L=1,N
      IF(ICOL(J)-L) 28,29,28
   28 CONTINUE
   29 DO 26 I=1,N
   26 A(I,L)=D(I,J)
  100 RETURN
      END
```

```
      SUBROUTINE MMPY (A,B,N1,N2,N3,C)
C
C                                C = A * B
C                                A (N1 X N2)    B (N2 X N3)    C (N1 X N3)
C
      REAL A(20,21),B(20,21),C(20,21)
      DO 20 I=1,N1
      DO 20 J=1,N3
      C(I,J)=0.
      DO 20 K=1,N2
   20 C(I,J)=C(I,J)+A(I,K)*B(K,J)
      RETURN
      END
```

```
      SUBROUTINE RANDU (IX,IY,YFL)
C                   THIS SUBROUTINE IS FROM  SSP VERS. II
      IY=IX*65539
      IF(IY)5,6,6
    5 IY=IY+2147483647+1
    6 YFL=IY
      YFL=YFL*.4656613E-9
      RETURN
      END
```

```
      SUBROUTINE MOB (M,K,G,N,OM,ZR,ZI,YR,YI)
C
C               CALCULATES COMPLEX IMPEDANCE AND MOBILITY
C               M IS SQUARE MASS MATRIX
C               K IS SQUARE STIFFNESS MATRIX
C               G IS SCALAR STRUCTURAL DAMPING
C               OM IS FREQUENCY IN HERTZ
C               N IS ORDER
C
C               IMPEDANCE IS ZR + I*ZI     (I = SQRT(-1))
C               MOBILITY  = YR + I*YI
C
C               ALL SQUARE MATRICES ARE DIMENSIONED (20,21)
C
C        USES CINV, INVRS, MMPY
C
      REAL M(20,21),K(20,21),ZR(20,21),ZI(20,21),YR(20,21),YI(20,21)
      OMR=OM*6.283185
      CON=G/OMR
      DO 10 I=1,N
      DO 10 J=1,N
      ZR(I,J)=CON*K(I,J)
   10 ZI(I,J)=OMR*M(I,J)-K(I,J)/OMR
      CALL CINV (ZR,ZI,N,YR,YI)
      RETURN
      END
```

INCOMPLETE MODEL THEORY - SIMULATED TEST

*************** 1 (1,C2) - R(3R) - E(C) TEST 16 ***************

ACTUAL SYSTEM PARAMETERS 18 DEG OF FREEDOM, STRUCT. DAMPING COEF = 0.020

MASS MATRIX

	1	2	3	4	5	6	7	8	9	10
1	5.0000E-02	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	1.0000E-01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	1.0000E-01	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	1.0000E-01	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	1.0000E-01	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	1.0000E-01	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	1.0000E-01	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0000E-01	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0000E-01	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0000E-01
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

	11	12	13	14	15	16	17	18
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	1.0000E-01	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	1.0000E-01	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	1.0000E-01	0.0	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	1.0000E-01	0.0	0.0	0.0	0.0
15	0.0	0.0	0.0	0.0	1.0000E-01	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	1.0000E-01	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0	1.0000E-01	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.0000E-02

STIFFNESS MATRIX

	1	2	3	4	5	6	7	8	9	10
1	1.1609E 06	-3.6482E 05	2.5824E 05	-6.8662E 04	1.8239E 04	-5.5125E 03	3.0578E 03	-4.8841E 03	1.0710E 04	-1.5171E 04
2	-3.6483E 05	9.8858E 05	-9.5075E 05	4.1430E 05	-1.1078E 05	3.2076E 04	-1.3473E 04	1.4256E 04	-2.9029E 04	4.2161E 04
3	2.5825E 05	-9.5077E 05	1.4019E 06	-1.0604E 06	4.4588E 05	-1.2378E 05	3.8338E 04	-1.8724E 04	2.8521E 04	-4.4296E 04
4	-6.8673E 04	4.1434E 05	-1.0604E 06	1.4340E 06	-1.0753E 06	4.5381E 05	-1.2184E 05	3.0626E 04	-1.6065E 04	2.6407E 04
5	1.8246E 04	-1.1081E 05	4.4593E 05	-1.0754E 06	1.4442E 06	-1.0756E 06	4.4196E 05	-1.0715E 05	2.5729E 04	-1.8699E 04
6	-5.5138E 03	3.2086E 04	-1.2381E 05	4.5384E 05	-1.0756E 06	1.4330E 06	-1.0583E 06	4.3051E 05	-1.0992E 05	3.7033E 04
7	3.0546E 03	-1.3416E 04	3.8338E 04	-1.2185E 05	4.4196E 05	-1.0583E 06	1.4205E 06	-1.0590E 06	4.4455E 05	-1.2676E 05
8	-4.8811E 03	1.4246E 04	-1.8713E 04	3.0623E 04	-1.0715E 05	4.3051E 05	-1.0590E 06	1.4351E 06	-1.0908E 06	4.6117E 05
9	1.0711E 04	-2.9030E 04	2.8525E 04	-1.6073E 04	2.5739E 04	-1.0993E 05	4.4457E 05	-1.0809E 06	1.4563E 06	-1.0917E 06
10	-1.5173E 04	4.2171E 04	-4.4314E 04	2.6426E 04	-1.8714E 04	3.7051E 04	-1.2679E 05	4.6121E 05	-1.0919E 06	1.4570E 06
11	1.2206E 04	-3.5196E 04	4.0561E 04	-2.9742E 04	2.2118E 04	-2.2963E 04	4.2690E 04	-1.2946E 05	4.5909E 05	-1.0832E 06
12	-6.0173E 03	1.8119E 04	-2.2854E 04	1.9700E 04	-1.6067E 04	1.3475E 04	-1.5627E 04	3.5827E 04	-1.2264E 05	4.5067E 05
13	3.4689E 03	-9.9953E 03	1.1940E 04	-1.0029E 04	8.0799E 03	-6.1097E 03	5.4219E 03	-9.6593E 03	3.1954E 04	-1.1935E 05
14	-4.8616E 03	1.3180E 04	-1.3139E 04	8.8194E 03	-7.2049E 03	6.3200E 03	-5.4684E 03	5.1721E 03	-9.1440E 03	3.1219E 04
15	6.9699E 03	-2.0459E 04	2.1389E 04	-1.5025E 04	1.2695E 04	-1.2311E 04	1.1564E 04	-8.9016E 03	6.4079E 03	-1.1119E 04
16	-6.7368E 03	2.2827E 04	-2.8414E 04	2.4238E 04	-1.9425E 04	1.6086E 04	-1.5127E 04	1.2789E 04	-8.9256E 03	9.8168E 03
17	3.5331E 03	-1.4867E 04	2.2263E 04	-2.2328E 04	1.7175E 04	-1.1511E 04	1.0286E 04	-9.8256E 03	7.3914E 03	-7.2288E 03
18	-7.3075E 02	4.1139E 03	-7.1322E 03	7.9279E 03	-5.9874E 03	3.3915E 03	-2.8271E 03	3.0017E 03	-2.3305E 03	2.0784E 03

	11	12	13	14	15	16	17	18
1	1.2206E 04	-6.0192E 03	3.4726E 03	-4.8691E 03	6.9790E 03	-6.7398E 03	3.5297E 03	-7.2839E 02
2	-3.5192E 04	1.8121E 04	-1.0004E 04	1.3200E 04	-2.0487E 04	2.2829E 04	-1.4850E 04	4.1042E 03
3	4.0548E 04	-2.2848E 04	1.1944E 04	-1.3157E 04	2.1404E 04	-2.8397E 04	2.2227E 04	-7.1152E 03
4	-2.9726E 04	1.9690E 04	-1.0021E 04	8.8205E 03	-1.5013E 04	2.4196E 04	-2.2282E 04	7.9105E 03
5	2.2103E 04	-1.6052E 04	8.0700E 03	-7.1933E 03	1.2659E 04	-1.9363E 04	1.7126E 04	-5.9720E 03
6	-2.2540E 04	1.3451E 04	-6.0955E 03	6.3038E 03	-1.2263E 04	1.6012E 04	-1.1458E 04	3.3765E 03
7	4.2652E 04	-1.5599E 04	5.4145E 03	-5.4619E 03	1.1526E 04	-1.5066E 04	1.0742E 04	-2.8151E 03
8	-1.2942E 05	3.5805E 04	-9.6632E 03	5.1761E 03	-8.8802E 03	1.2750E 04	-9.7980E 03	2.9943E 03
9	4.5905E 05	-1.2262E 05	3.1954E 04	-9.1361E 03	6.3819E 03	-8.8911E 03	7.3660E 03	-2.3305E 03
10	-1.0832E 06	4.5046E 05	-1.1935E 05	3.1200E 04	-1.1088E 04	9.7782E 03	-7.1935E 03	2.0648E 03
11	1.4430E 06	-1.0724E 06	4.4690E 05	-1.1930E 05	3.6708E 04	-1.8882E 04	1.0828E 04	-2.5700E 03
12	-1.0724E 06	1.4354E 06	-1.0704E 06	4.4909E 05	-1.2717E 05	4.5544E 04	-1.8996E 04	4.6229E 03
13	4.4689E 05	-1.0704E 06	2.4360E 06	-1.0748E 06	4.5830E 05	-1.3587E 05	4.2766E 04	-8.9398E 03
14	-1.1930E 05	4.4908E 05	-1.0748E 06	1.4431E 06	-1.0824E 06	4.5971E 05	-1.2125E 05	2.1445E 04
15	3.6217E 04	-1.2715E 05	4.5827E 05	-1.0824E 06	1.4399E 06	-1.0583E 06	4.0736E 05	-6.5873E 04
16	-1.8905E 04	4.5533E 04	-1.3579E 05	4.5952E 05	-1.0581E 06	1.3690E 06	-9.1220E 05	2.4403E 05
17	1.0861E 04	-1.8996E 04	4.2685E 04	-1.2104E 05	4.0709E 05	-9.1205E 05	9.4247E 05	-3.4784E 05
18	-2.8845E 03	4.6258E 03	-8.9110E 03	2.1365E 04	-6.5767E 04	2.4396E 05	-3.4782E 05	1.5429E 05

INFLUENCE COEFFICIENT MATRIX

	1	2	3	4	5	6	7	8	9	10
1	1.0000E-06	9.1654E-07	8.3327E-07	7.4948E-07	6.6615E-07	5.8281E-07	4.9948E-07	4.1615E-07	3.3281E-07	2.4948E-07
2	9.1654E-07	3.4471E-05	6.0525E-05	7.8244E-05	8.8462E-05	9.2011E-05	8.9724E-05	8.2434E-05	7.0973E-05	5.6147E-05
3	8.3327E-07	6.0525E-05	1.1188E-04	1.4823E-04	1.6958E-04	1.7759E-04	1.7393E-04	1.6027E-04	1.3827E-04	1.0959E-04
4	7.4948E-07	7.8244E-05	1.4823E-04	2.0321E-04	2.3735E-04	2.5149E-04	2.4813E-04	2.2976E-04	1.9888E-04	1.5799E-04
5	6.6615E-07	8.8462E-05	1.6958E-04	2.3735E-04	2.8511E-04	3.0788E-04	3.0731E-04	2.8673E-04	2.4948E-04	1.9988E-04
6	5.8281E-07	9.2011E-05	1.7759E-04	2.5149E-04	3.0788E-04	3.4093E-04	3.4648E-04	3.2703E-04	2.8673E-04	2.2976E-04
7	4.9948E-07	8.9724E-05	1.7393E-04	2.4813E-04	3.0731E-04	3.4648E-04	3.6065E-04	3.4648E-04	3.0731E-04	2.4813E-04
8	4.1615E-07	8.2434E-05	1.6027E-04	2.2976E-04	2.8673E-04	3.2703E-04	3.4648E-04	3.4093E-04	3.0788E-04	2.5149E-04
9	3.3281E-07	7.0973E-05	1.3827E-04	1.9888E-04	2.4948E-04	2.8673E-04	3.0731E-04	3.0788E-04	2.8511E-04	2.3735E-04
10	2.4948E-07	5.6147E-05	1.0959E-04	1.5799E-04	1.9988E-04	2.2976E-04	2.4813E-04	2.5149E-04	2.3735E-04	2.0320E-04
11	1.6615E-07	3.8871E-05	7.5899E-05	1.0959E-04	1.3826E-04	1.6027E-04	1.7393E-04	1.7759E-04	1.6958E-04	1.4823E-04
12	8.2975E-08	1.9896E-05	3.8871E-05	5.6173E-05	7.0971E-05	8.2432E-05	8.9722E-05	9.2009E-05	8.8460E-05	7.8243E-05
13	2.5798E-11	8.2401E-08	1.6503E-07	2.4788E-07	3.3093E-07	4.1417E-07	4.9756E-07	5.8109E-07	6.6474E-07	7.4847E-07
14	-8.2845E-08	-1.9875E-05	-3.8826E-05	-5.6105E-05	-7.0877E-05	-8.2310E-05	-8.9573E-05	-9.1830E-05	-8.8252E-05	-7.8004E-05
15	-1.6615E-07	-3.9814E-05	-7.7799E-05	-1.1244E-04	-1.4207E-04	-1.6502E-04	-1.7963E-04	-1.8423E-04	-1.7716E-04	-1.5675E-04
16	-2.5052E-07	-5.9776E-05	-1.1677E-04	-1.6876E-04	-2.1324E-04	-2.4771E-04	-2.6966E-04	-2.7661E-04	-2.6605E-04	-2.3548E-04
17	-3.3437E-07	-7.9696E-05	-1.5571E-04	-2.2503E-04	-2.8436E-04	-3.3034E-04	-3.5964E-04	-3.6893E-04	-3.5489E-04	-3.1417E-04
18	-4.2031E-07	-9.9596E-05	-1.9460E-04	-2.8126E-04	-3.5540E-04	-4.1289E-04	-4.4955E-04	-4.6120E-04	-4.4368E-04	-3.9282E-04

	11	12	13	14	15	16	17	18
1	1.6615E-07	8.2975E-08	2.5798E-11	-8.2845E-08	-1.6615E-07	-2.5052E-07	-3.3437E-07	-4.2031E-07
2	3.8871E-05	1.9896E-05	8.2401E-08	-1.9875E-05	-3.9814E-05	-5.9776E-05	-7.9696E-05	-9.9596E-05
3	7.5899E-05	3.8871E-05	1.6503E-07	-3.8826E-05	-7.7799E-05	-1.1677E-04	-1.5571E-04	-1.9460E-04
4	1.0959E-04	5.6173E-05	2.4788E-07	-5.6105E-05	-1.1244E-04	-1.6876E-04	-2.2503E-04	-2.8126E-04
5	1.3826E-04	7.0971E-05	3.3093E-07	-7.0877E-05	-1.4207E-04	-2.1324E-04	-2.8436E-04	-3.5540E-04
6	1.6027E-04	8.2432E-05	4.1417E-07	-8.2310E-05	-1.6502E-04	-2.4771E-04	-3.3034E-04	-4.1289E-04
7	1.7393E-04	8.9722E-05	4.9756E-07	-8.9573E-05	-1.7963E-04	-2.6966E-04	-3.5964E-04	-4.4955E-04
8	1.7759E-04	9.2009E-05	5.8109E-07	-9.1830E-05	-1.8423E-04	-2.7661E-04	-3.6893E-04	-4.6120E-04
9	1.6958E-04	8.8460E-05	6.6474E-07	-8.8252E-05	-1.7716E-04	-2.6605E-04	-3.5489E-04	-4.4368E-04
10	1.4823E-04	7.8243E-05	7.4847E-07	-7.8004E-05	-1.5675E-04	-2.3548E-04	-3.1417E-04	-3.9282E-04
11	1.1188E-04	6.0526E-05	8.3228E-07	-6.0253E-05	-1.2133E-04	-1.8240E-04	-2.4345E-04	-3.0447E-04
12	6.0526E-05	3.4471E-05	9.1613E-07	-3.4169E-05	-6.9251E-05	-1.0433E-04	-1.3939E-04	-1.7444E-04
13	8.3228E-07	9.1613E-07	1.0000E-06	1.0832E-06	1.1665E-06	1.2498E-06	1.3331E-06	1.4164E-06
14	-6.0253E-05	-3.4169E-05	1.0832E-06	4.4530E-05	8.9640E-05	1.3474E-04	1.7984E-04	2.2491E-04
15	-1.2133E-04	-6.9251E-05	1.1665E-06	8.9640E-05	1.8811E-04	2.8824E-04	3.8834E-04	4.8841E-04
16	-1.8240E-04	-1.0433E-04	1.2498E-06	1.3474E-04	2.8824E-04	4.5173E-04	6.1684E-04	7.8191E-04
17	-2.4345E-04	-1.3939E-04	1.3331E-06	1.7984E-04	3.8834E-04	6.1684E-04	8.5535E-04	1.0954E-03
18	-3.0447E-04	-1.7444E-04	1.4164E-06	2.2491E-04	4.8841E-04	7.8191E-04	1.0954E-03	1.4189E-03

DESCRIPTION OF SIMULATED TEST

POINT AT WHICH SYSTEM IS FORCED 7
NO OF POINTS OF MEASUREMENT 8
POINTS AT WHICH MEASUREMENTS ARE TAKEN
3 5 7 9 11 15 17 18
FREQUENCIES OF EXCITATION - HZ
8.3160E 00 1.8654E 01 4.9055E 01

AVE RANDOM AMPLITUDE ERROR 0.0
BIAS AMPLITUDE ERROR 0.0

SEED FOR RAND NO GENERATOR 0

EXACT RESPONSE OF ACTUAL SYSTEM,IN/SEC/POUND, NORMALIZED ON REAL DRIVING POINT RESPONSE

REAL PART

FREQ	8.3160E 00	1.8654E 01	4.9055E 01
D.P. RESP	7.4873E-01	3.9852E-01	1.0967E-02
3	0.47153	0.66158	-4.26195
5	0.83368	1.04122	-3.72077
7	1.00000	1.00000	1.00000
9	0.91745	0.60815	4.77066
11	0.57490	0.14263	3.75490
15	-0.75043	0.51650	-0.58588
17	-1.58366	1.39374	1.48294
18	-2.00865	1.87239	2.88484

SIMULATED TEST RESPONSE, REAL MOBILITY WITH ERRORS USED AS PHI'S

D.P.
RESP 7.4873E-01 3.9852E-01 1.0967E-02

	1	2	3
1	4.7153E-01	6.6158E-01	-4.2619E 00
2	8.3368E-01	1.0412E 00	-3.7208E 00
3	1.0000E 00	1.0000E 00	1.0000E 00
4	9.1745E-01	6.0815E-01	4.7707E 00
5	5.7490E-01	1.4263E-01	3.7549E 00
6	-7.5043E-01	5.1650E-01	-5.8588E-01
7	-1.5837E 00	1.3937E 00	1.4829E 00
8	-2.0087E 00	1.8724E 00	2.8848E 00

PART II - IDENTIFICATION

*************** 0   T(.02) - R(3R) - T(C)   TEST 16

ALL GENERALIZED MASSES UNKNOWN

ALL MASSES UNKNOWN

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PROBLEM DEFINITION   NO OF EQUATIONS =  4
                     NO OF VARIABLES =  8
                  ORDER OF DEGENERACY =  4

                     TOTAL OF DIAGONAL MASSES CONSTANT
```

A MATRIX 4 X 8

	1	2	3	4	5	6	7	8
1	3.1195E-01	8.6805E-01	1.0000E 00	5.5794E-01	8.1998E-02	-3.8760E-01	-2.2072E 00	-3.7610E 00
2	-2.0096E 00	-3.1019E 00	1.0000E 00	4.3768E 00	2.1587E 00	4.3966E-01	-2.3485E 00	-5.7946E 00
3	-2.8196E 00	-3.8741E 00	1.0000E 00	2.9013E 00	5.3556E-01	-3.0261E-01	2.0668E 00	5.4016E 00
4	1.0000E 00	1.0000E 00	1.0000E 00	1.0000E 00	1.0000E 00	1.0000E 00	1.0000E 00	1.0000E 00

B VECTOR

0.0 0.0 0.0 1.5500E 00

BR MATRIX 8 X 4

	1	2	3	4
1	-4.3193E-02	-4.2510E-03	-7.0886E-02	1.4660E-01
2	1.7264E-01	-9.0493E-02	-5.7130E-03	1.4509E-01
3	4.3394E-01	-1.2120E-01	1.7112E-01	1.3181E-01
4	8.4392E-02	4.0135E-02	6.8805E-02	1.4657E-01
5	-1.8300E-01	9.8495E-02	-7.0437E-02	1.5234E-01
6	-2.7448E-01	1.0553E-01	-1.1863E-01	1.4612E-01
7	-2.7324E-01	6.7839E-02	-8.0967E-02	9.8685E-02
8	8.2929E-02	-9.6052E-02	1.0671E-01	3.2784E-02

BM MATRIX 8 X 8

	1	2	3	4	5	6	7	8
1	6.5846E-01	-3.9691E-01	-2.8270E-02	1.0176E-01	-9.5918E-02	-1.8292E-01	-1.0541E-01	4.9210E-02
2	-3.9691E-01	4.0221E-01	-2.2153E-01	1.7123E-01	3.9159E-02	-4.0117E-02	3.5257E-02	1.0706E-02
3	-2.8270E-02	-2.2153E-01	3.8433E-01	-3.3990E-01	2.6029E-03	1.4145E-01	1.8768E-01	-1.2637E-01
4	1.0176E-01	1.7123E-01	-3.3990E-01	4.3106E-01	-2.7697E-01	-1.1068E-01	-8.2490E-03	3.1745E-02
5	-9.5918E-02	3.9159E-02	2.6034E-03	-2.7697E-01	6.8777E-01	-2.8789E-01	-1.7936E-01	1.1061E-01
6	-1.8292E-01	-4.0117E-02	1.4146E-01	-1.1068E-01	-2.8789E-01	6.6519E-01	-2.5892E-01	7.3882E-02
7	-1.0541E-01	3.5256E-02	1.8768E-01	-8.2482E-03	-1.7936E-01	-2.5892E-01	6.2489E-01	-2.9588E-01
8	4.9209E-02	1.0705E-02	-1.2637E-01	3.1745E-02	1.1061E-01	7.3882E-02	-2.9588E-01	1.4610E-01

BEST MASS SOLUTION

I	J	APPROX MASS	WGT FACT	MIN MASS	BEST MASS	CHANGE
1	1	2.5000E-01	1.0000	2.2723E-01	2.4410E-01	-5.9003E-03
2	2	2.0000E-01	1.0000	2.2489E-01	2.0163E-01	1.6285E-03
3	3	2.0000E-01	1.0000	2.0431E-01	2.1967E-01	1.9667E-02
4	4	2.0000E-01	1.0000	2.2718E-01	2.0853E-01	8.5347E-03
5	5	2.5000E-01	1.0000	2.3612E-01	2.4370E-01	-6.3010E-03
6	6	2.5000E-01	1.0000	2.2649E-01	2.3807E-01	-1.1926E-02
7	7	1.5000E-01	1.0000	1.5296E-01	1.3892E-01	-1.1085E-02
8	8	5.0000E-02	1.0000	5.0815E-02	5.5379E-02	5.3789E-03
TOTALS		1.5500E 00			1.5500E 00	

RMS OF CHANGES 1.0191E-02

FULL MASS MATRIX

	1	2	3	4	5	6	7	8
1	2.4410E-01	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	2.0163E-01	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	2.1967E-01	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	2.0853E-01	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	2.4370E-01	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	2.3807E-01	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	1.3892E-01	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.5379E-02

GENFRALIZED MASS MATRIX

	1	2	3
1	1.3760E 00	-5.9605F-08	2.563CE-06
2	5.9605E-08	1.1547F 00	-1.1921F-07
3	1.8477E-06	-5.9605F-08	1.6475F 01

INCOMPLETE STIFFNESS COEF

MODAL COEFFICIENTS
1.9841E 03 1.1897E 04 5.7663E 03

	1	2	3	4	5	6	7	8
1	6.5775E 03	4.9422E 03	-8.4557E 02	-5.6807E 03	-5.3906E 03	1.0322E 03	-9.1406E 02	-7.8457E 02
2	4.9422E 03	3.8258E 03	-3.2836E 02	-3.9231E 03	-3.8250E 03	8.5094E 02	-4.8096E 02	-4.6923E 02
3	-8.4537E 02	-3.2836E 02	9.4806E 02	1.6750E 03	1.3110E 03	6.6814E 01	6.7104E 02	4.2487E 02
4	-5.6807E 03	-3.9231E 03	1.6750E 03	5.9710E 03	5.3550E 03	-6.8244E 02	1.3904E 03	1.0307E 03
5	-5.3906E 03	-3.8250E 03	1.3110E 03	5.3550E 03	4.8817E 03	-7.3480E 02	1.1059E 03	8.5494E 02
6	1.0322E 03	8.5094E 02	6.6814E 01	-6.8244E 02	-7.3480E 02	3.5540E 02	1.9553E 02	6.2629E 01
7	-9.1406E 02	-4.8096E 02	6.7104E 02	1.3904E 03	1.1059E 03	1.9553E 02	7.8669E 02	4.7717E 02
8	-7.8457E 02	-4.6923E 02	4.2487E 02	1.0307E 03	8.5494E 02	6.2629E 01	4.7717E 02	2.9964E 02

INCOMPLETE INFLUENCE COEF

MODAL COEFFICIENTS
2.6618E-04 6.3042E-05 6.3892E-07

	1	2	3	4	5	6	7	8
1	9.8380E-05	1.5820E-04	1.6450E-04	1.2752E-04	6.7881E-05	-7.1050E-05	-1.4468E-04	-1.9187E-04
2	1.5820E-04	2.6720E-04	2.8517E-04	2.3217E-04	1.2801E-04	-1.3123E-04	-2.6347E-04	-3.2969E-04
3	1.6450E-04	2.8517E-04	3.2986E-04	2.8559E-04	1.6442E-04	-1.6756E-04	-3.3273E-04	-4.1478E-04
4	1.2752E-04	2.3217E-04	2.8559E-04	2.6190E-04	1.5731E-04	-1.6524E-04	-3.2878E-04	-4.0995E-04
5	6.7881E-05	1.2801E-04	1.6442E-04	1.5731E-04	9.8267E-05	-1.1160E-04	-2.2625E-04	-2.8362E-04
6	-7.1050E-05	-1.3123E-04	-1.6756E-04	-1.6524E-04	-1.1160E-04	1.6694E-04	3.6116E-04	4.6112E-04
7	-1.4468E-04	-2.6347E-04	-3.3273E-04	-3.2878E-04	-2.2625E-04	3.6116E-04	7.9144E-04	1.0140E-03
8	-1.8187E-04	-3.2969E-04	-4.1478E-04	-4.0995E-04	-2.8362E-04	4.6112E-04	1.0140E-03	1.3003E-03

RESONANT RESPONSE OF IDENTIFIED SYSTEM

REAL PART

FREQ	8.3160E 00	1.8654E 01	4.7055E 01
D.P. RESP	6.9333E-01	3.6940E-01	9.8470E-03
3	0.47377	0.66153	-4.26025
5	0.83608	1.04131	-3.71752
7	1.00000	1.00000	1.00000
9	0.92231	0.60820	4.77137
11	0.57813	0.14266	3.75450
15	-0.75294	0.51638	-0.58527
17	-1.58554	1.39374	1.48293
18	-2.01776	1.87224	2.88604

FREQUENCIES AND MODES OF IDENTIFIED SYSTEM

FREQ	8.3160E 00	1.8654E 01	4.9055E 01
GEN MASS	1.3760E 00	1.1547E 00	1.6475E 01
ITER	10	7	3
3	0.47153	0.66158	-4.26195
5	0.83368	1.04122	-3.72076
7	1.00000	1.00000	1.00000
9	0.91744	0.60815	4.77061
11	0.57490	0.14263	3.75489
15	-0.75043	0.51650	-0.58589
17	-1.58366	1.39374	1.48291
18	-2.00865	1.87240	2.88485

PART III - CHANGED MASSES OR STIFFNESSES

*************** 0 I(.02) - R(3P) - I(C) - A TEST 16 ***************

MASS MATRIX CHANGES

	1	2	3	4	5	6	7	8
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	1.0000E 00	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

NEW FREQUENCIES AND MODES OF MODIFIED SYSTEM

FREQ	6.1740E 00	1.5593E 01	4.8525E 01
GEN MASS	2.1092E 00	7.0792E 00	1.2993E 02
ITER	9	6	4
3	0.48715	0.82100	-12.91041
5	0.85134	1.21949	-11.97098
7	1.00000	1.00000	1.00000
9	0.88426	0.32243	11.88652
11	0.53196	-0.27173	9.80743
15	-0.61529	1.79564	-1.58307
17	-1.26345	4.41810	3.86989
18	-1.59039	5.82078	7.57423

PART III - CHANGED MASSES OR STIFFNESSES

*************** 0 I(.02) - B(30) - I(C) - 8 TEST 16 ***************

MASS MATRIX CHANGES

	1	2	3	4	5	6	7	8
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0000E 00

NEW FREQUENCIES AND MODES OF MODIFIED SYSTEM

FREQ	3.9499E 00	1.4835E 01	4.7504E 01
GEN MASS	1.0368E 01	6.6600E-01	2.4805E 01
ITER	6	5	3
3	0.44550	0.56205	-5.29938
5	0.80324	0.93473	-4.74020
7	1.00000	1.00000	1.00000
9	0.97245	0.75637	5.74816
11	0.65901	0.34192	4.71396
15	-1.02809	-0.03372	-1.64567
17	-2.24498	0.11025	-0.51286
18	-2.87363	0.20256	0.44880

PART III - CHANGED MASSES OR STIFFNESSES

*************** 3 I(.02) - B(3R) - I(C) - K TEST 16 ***************

K MATRIX CHANGES

	1	2	3	4	5	6	7	8
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	1.00000E 03	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

DELTA K INCOMPLETE

	1	2	3	4	5	6	7	8
1	2.4714E 01	4.1442E 01	5.8242E 01	4.9590E 01	3.0060E 01	-5.1006E 00	3.2559E 00	2.9921E 00
2	4.1442E 01	6.6789E 01	9.3866E 01	7.9922E 01	4.8446E 01	-8.2203E 00	5.2474E 00	4.8222E 00
3	5.8242E 01	9.3866E 01	1.3192E 02	1.1232E 02	6.8087E 01	-1.1553E 01	7.3748E 00	6.7772E 00
4	4.9590E 01	7.9922E 01	1.1232E 02	9.5636E 01	5.7972E 01	-9.8366E 00	6.2772E 00	5.7704E 00
5	3.0060E 01	4.8446E 01	6.8087E 01	5.7972E 01	3.5141E 01	-5.9627E 00	3.8063E 00	3.4979E 00
6	-5.1006E 00	-8.2203E 00	-1.1553E 01	-9.8366E 00	-5.9627E 00	1.0117E 00	-6.4586E-01	-5.9350E-01
7	3.2559E 00	5.2474E 00	7.3748E 00	6.2792E 00	3.8063E 00	-6.4586E-01	4.1228E-01	3.7887E-01
8	2.9921E 00	4.8222E 00	6.7772E 00	5.7704E 00	3.4979E 00	-5.9350E-01	3.7887E-01	3.4816E-01

NEW FREQUENCIES AND MODES OF MODIFIED SYSTEM

FREQ	9.2806E 00	1.9269E 01	4.9071E 01
GEN MASS	1.6264E 00	1.0250E 00	1.5871E 01
ITER	4	9	9
3	0.45874	0.65373	-4.17224
5	0.81931	1.03218	-3.63420
7	1.00000	1.00000	1.00000
9	0.94080	0.62384	4.69599
11	0.60901	0.16623	3.69139
15	-0.85668	0.44015	-0.57543
17	-1.83513	1.21228	1.45786
18	-2.33705	1.63518	2.83554

PART III - CHANGED MASSES OR STIFFNESSES

*************** 3   I(,02) - R(3R) - I(C) - L   TEST 16   ***************

K MATRIX CHANGES

	1	2	3	4	5	6	7	8
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0000E 03

DELTA K INCOMPLETE

	1	2	3	4	5	6	7	8
1	7.7993E 00	3.2064E 00	-6.5364E 00	-8.8821E 00	-1.0671E 00	-3.8484E 01	-5.9272E 01	-3.1660E 01
2	3.2064E 00	1.3182E 00	-2.6872E 00	-3.6516E 00	-4.3869E-01	-1.5821E 01	-2.4368E 01	-1.3016E 01
3	-6.5365E 00	-2.6872E 00	5.4781E 00	7.4439E 00	8.9430E-01	3.2253E 01	4.9679E 01	2.6533E 01
4	-8.8821E 00	-3.6516E 00	7.4439E 00	1.0115E 01	1.2153E 00	4.3827E 01	6.7501E 01	3.6055E 01
5	-1.0671E 00	-4.3868E-01	8.9433E-01	1.2152E 00	1.4605E-01	5.2653E 00	8.1094E 00	4.3317E 00
6	-3.8484E 01	-1.5821E 01	3.2253E 01	4.3827E 01	5.2652E 00	1.8989E 02	2.9247E 02	1.5622E 02
7	-5.9272E 01	-2.4368E 01	4.9675E 01	6.7501E 01	8.1094E 00	2.9247E 02	4.5045E 02	2.4060E 02
8	-3.1660E 01	-1.3016E 01	2.6533E 01	3.6055E 01	4.3316E 00	1.5622E 02	2.4060E 02	1.2852E 02

NEW FREQUENCIES AND MODES OF MODIFIED SYSTEM

FREQ	1.1144E 01	2.1051E 01	4.9195E 01
GEN MASS	9.9799E-01	2.0937E 00	1.6072E 01
ITER	3	10	9
3	0.50149	0.75213	-4.19111
5	0.86779	1.13590	-3.65111
7	1.00000	1.00000	1.00000
9	0.85995	0.48712	4.70362
11	0.48955	-0.01144	3.68883
15	-0.47878	0.90316	-0.51111
17	-0.93919	2.28498	1.62507
18	-1.16655	3.02693	3.05987