TOWARDS "GENERALIZED CONTROL THEORY"*

by

Y. C. Ho

A favorite guessing game popular among symposia, meetings, after-dinner conversations in control-theoretic circles in recent days seems to be the question "What Next?". This is appropriate in view of the growing mathematical respectibility of the discipline on one hand and the various "success stories" in practice on the other. In this note, an attempt will be made in establishing a framework within which such a question might be asked and answered.† The discussion will be informal and speculative.

It seems to the author that there are three main ingredients in an optimization problem, whether static or dynamic. They are: the criterion function, the controller, and the information available to the controller. For traditional control theory, all the ingredients are singular in the sense that there is only one criterion, one central controller coordinating all control actions, and one information set available to the controller. Within this setup, some very general and deep statements can and have been made for the problem. On the other hand, one can also argue that this viewpoint is unnecessarily narrow. Surely, we can all visualize situations or problems in which there are more than one performance measure, more than one intelligent controller operating with or without coordination from others, and

---

*The research reported in this note was made possible through support extended by the U. S. Office of Naval Research under the Joint Services Electronics Program by Contract N00014-67-A-0298-0006 and by the National Aeronautics and Space Administration under Grant NGR 22-007-068.

†I have deliberately avoided the inclusion here of related but nevertheless important subjects such as, mathematical system theory, stability theory, model theory, etc. where optimization is not the main concern.

To appear as Letter to the Editor, IEEE Transactions on Automatic Control, December 1969.

finally all the controllers may or may not have the same information set available to them. As a start, we can divide each of the ingredients into three subcategories. For the criterion J, we have (i) one J (ii) two J's with $J_1 = -J_2$ and (iii) multiple J's with J_i $i=1,\ldots,n$. For the controller C, we have (i) one C, (ii) two C's and (iii) multiple C's. Finally for the information set, we have the cases (i) one perfect information set (ii) one noisy (or imperfect) information set and (iii) multiple information set. With these subclassifications, the following table of organization is self-explanatory.

	Controller			Criterion			Information Set			
							One			
	One	Two	Multiple	One	Two $J_1 = -J_2$	Multiple	Perfect	Imperfect	Multiple	References
(1) Deterministic Optimal Control	✓			✓			✓			
(2) Stochastic Optimal Control	✓			✓				✓		
(3) Zero-Sum Differential Game		✓			✓		✓			[1]
(4) Stochastic Zero-Sum Differential Game		✓			✓				✓	[2,3,4]
(5) Vector Value Optimization Problem (Negotiation pb.)	✓					✓	✓			[5,6]
(6) Nonzero-Sum Differential Games			✓			✓	✓			[7,8,9]
(7) Team Theory			✓	✓					✓	[10, 11, 12]
(8) MC/MJ/MI pb.			✓			✓			✓	[13]

With the exception of the first two categories, I have given some references containing representative works where introductory materials, results, and examples of application may be found. One also notes that in this framework, the names attached to these categories appear inadequate but entrenched. For example, a more descriptive title for (3) is "2C/2J/1I problem in generalized control theory." However, one can hardly expect such a terminology to take hold at this stage. Except for the first three categories, most results for the other categories are available only for the static optimization case. Even there many basic concepts are subtle and difficult, e.g. the coalition problems in (6) and the definition of equilibrium solution of (8). Suffice is to paraphrase a famous remark: "they will not be solved in the first thousand days . . . but let us begin".

Y. C. Ho
Harvard University
Cambridge, Massachusetts

References:

1. Rufus Isaacs, Differential Games, John Wiley, 1965.

2. R. D. Behn and Y. C. Ho, "On a Class of Linear Stochastic Differential Games," IEEE Transactions on Automatic Control, AC-13, No. 3, June 1968, pp. 227-239.

3. I. B. Rhodes and D. G. Luenberger, "Differential Games with Imperfect State Information," IEEE Transactions on Automatic Control, AC-14, No. 1, February 1969, pp. 29-39.

4. W. W. Willman, "Formal Solutions for a Class of Stochastic Pursuit-Evasion Games," IEEE Transactions on Automatic Control, AC-14, No. 5, October 1969, to appear.

5. L. A. Zadeh, "Optimality and Non-Scalar Valued Performance Criteria," IEEE Transactions on Automatic Control, Vol. AC-8, 1963, pp. 59-60.

6. N. O. da Cunha and E. Polak, "Constrained Minimization under Vector-Valued Criteria in Linear Topological Spaces," Memorandum ERL-M191, November 1966, University of California, Berkeley.

7. J. H. Case, "Equilibrium Points of N-Person Differential Games," University of Michigan, Dept. of Ind. Eng. Tech. Report 1967-1.

8. A. W. Starr and Y. C. Ho, "Nonzero-Sum Differential Games," Journal of Optimization Theory and Applications, 3, No. 3, March 1969.

9. A. W. Starr and Y. C. Ho, "Further Properties of Nonzero-Sum Differential Games," Journal of Optimization Theory and Applications, 3, No. 4, April 1969.

10. R. Radner, "Team Decision Problems," Annal of Math. Stat. Sept. 1962. pp. 857-881.

11. R. Radner, "The Evaluation of Information in Organizations," Proc. of Fourth Berkely Symp. on Math. Stat. 1961, pp. 491-533.

12. H. Witsenhausen, "A Counterexample in Stochastic Optimal Control," SIAM Journal on Control 6, No. 1, 1968, pp. 131-147.

13. J. C. Harsanyi, "Games with Incomplete Information Played By "Bayesian" Players," I-III, Management Science, Vol. 14, No. 3, November 1967, pp. 159-182, No. 5, January 1968, pp. 320-334, No. 7, March 1968, pp. 486-502.