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## USER'S MANUAL FOR 'RAVFAC' A RADIATION VIEW FACTOR DIGITAL COMPUTER PROGRAM

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## FOREWORD

# This document presents a portion of the work accomplished under two separate contracts: Contract NAS8-20230, Modification 2, DCN 17-75-20109, and Contract NAS8-30154. <br> The work was performed by the Thermal Environment Section of Lockheed's Huntsville Research \& Engineering Center for the Aero-Astrodynamics Laboratory of Marshall Space Flight Center, National Aeronautics and Space Administration. 

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## SUMMARY

A new technique for calculating diffuse radiation view factors was developed using contour integrals. This technique along with the finite difference (double summation) was incorporated into a Radiation View Factor Digital Computer Program (RAVFAC).

Two techniques for calculating radiation view factors were included in RAVFAC because the contour integral offers greater accuracy and the finite difference offers faster run times. A combination of the two provides accurate results and keeps the run time within reason.

A technique was also developed and incorporated into RAVFAC to account for the effects of shading by other surfaces. The calculation of shading effects is lengthy and requires a significant amount of computer time. A computer routine was developed that reduces the run time by eliminating surfaces that cannot cause shading on the areas for which the view factors are being calculated.

This technical report contains:

- Documentation of techniques and equations used to calculate the view factors and to account for the shading effects
- A description of how RAVFAC works
- A detailed input guide and definition of all input parameters, including a sample problem
- A description of the program output
- A comparison of the contour integral and finite difference techniques
- A comparison of RAVFAC with other similar programs
- Suggestions for expanding the capabilities of RAVFAC.

This report fully documents the RAVFAC computer program. It contains information for those who want to know how the program works, what equations were used and how to use the program. The report is organized in such a way that the reader can refer only to those sections of interest. For example a complete input guide is contained in Section 7.3.

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## NOMENCLATURE

| A. | area |
| :---: | :---: |
| a | first coefficient of conic surface equation |
| B | magnitude of approximate rectangular element in the $\beta$ direction |
| b | above magnitude used in the contour integration limits in the $\beta$ direction; also second coefficient of conic surface equation |
| C | coefficient used in contour integration defined by Eq. (5.31) |
| $\mathrm{C}_{\mathrm{lm}}$ | coefficient used in contour integration defined by Eq. (5.32) |
| $\mathrm{C}_{10}$ | coefficient used in contour integration defined by Eq. (5.33) |
| $\mathrm{C}_{\mathrm{ml}}$ | coefficient used in contour integration defined by Eq. (5.34) |
| $\mathrm{C}_{\mathrm{mo}}$ | coefficient used in contour integration defined by Eq. (5.35) |
| $\mathrm{C}_{\mathrm{x}}$ | coefficient used in contour integration defined by Eq. (5.29) |
| $\mathrm{C}_{\mathrm{y}}$ | coefficient used in contour integration defined by Eq. (5.30) |
| $\mathrm{Cl}_{1}$ |  |
| $\left.\begin{array}{l}\mathrm{C}_{2} \\ \mathrm{C}_{3}\end{array}\right\}$ | matrix components defined by the matrix Eq. (5.36) and also used in the contour integration |
| c | third coefficient of conic surface equation |
| c.a. | centroid of area |


d

du
$\mathrm{dF}_{\mathrm{dA}}^{\ell}{ }-\mathrm{dA} \mathrm{m}_{\mathrm{m}}$
$F_{\mathrm{dA}_{\ell}}-\mathrm{A}_{\mathrm{m}}$
$\mathrm{F}_{\mathrm{dA}}^{\ell}{ }_{l}-\mathrm{A}_{\mathrm{j}}$
${ }^{F_{\Delta A_{l}}-A_{j}}$
f

G
g
vector coefficients used in vector operation for shading check and defined by the set of Eq. (3.13)
denominator of contour integrals and defined by Eq. (5.28)
evaluation of above denominator around the four sides of the approximate rectangular element and defined by Eqs. (5.38)
distance between the origin of the test cylinder system and the origin of a sphere that encloses a whole "shade" surface, in the $X_{t}-Y_{t}$ plane, defined by Eq. (4.86)
$x, y$ and $z$ differentials used in contour integration
differential used in general type of integration given by Eq. (5.39)
differential view factor from a differential area "dA ${ }_{\ell}$ " to a differential area 'dA ${ }^{\prime}$ "
view factor from a differential area "dA $\ell$ " to a finite
elemental area "A " elemental area "A "m"
view factor from a differential area "dA ${ }_{l}$ " to a finite nodal area '"A. "'
view factor from a small finite elemental area " $\triangle \mathrm{A}_{\ell}{ }^{1 \prime}$ to a finite nodal area "A."
variable used in defining surface integral evaluating view factors from an infinitesimal area to a finite area, given by Eq. (5.17)
magnitude of approximate rectangular element in the $y$ direction
above magnitude used in the contour integration as limits in the $\gamma$ direction

| h | element counter used in defining the center of area (c.a.) of an elemental area according to the type of surface |
| :---: | :---: |
| I | refers to a surface |
| $\left.\begin{array}{c}\stackrel{\rightharpoonup}{i} \\ \stackrel{\rightharpoonup}{\mathrm{j}} \\ \stackrel{\rightharpoonup}{\mathrm{k}}\end{array}\right\}$ | unit vectors in the $X, Y$ and $Z$ directions, respectively |
| j | refers to a node |
| k | node counter used in defining the c.a. of an elemental area according to the type of surface |
| $l_{1}$ | lower limit of integration in general type of contour integration |
| $1_{2}$ | upper limit of integration in general type of contour integration |
| NB | number of elements in which a node is divided in the $\beta$ direction |
| NG | number of elements in which a node is divided in the $\gamma$ direction |
| NVB | number of nodes in which a surface is divided in the $\beta$ direction |
| NVG | number of nodes in which a surface is divided in the $\gamma$ direction |
| $\stackrel{\rightharpoonup}{\mathrm{N}}$ | unit normal of an element located at its c.a. |
| n | refers to the prime node (i.e., the node for which the view factor is being calculated) |
| $\stackrel{\rightharpoonup}{n}$ | unit vector nominal to the surface at a point $\overrightarrow{\mathrm{P}}$ and defined by Eq. (2.37) |
| $\overrightarrow{\mathrm{P}}$ | vector from the origin of the surface coordinate system to any arbitrary point on a surface |
| $\stackrel{\rightharpoonup}{P}_{b}$ | vector from the origin of the surface coordinate system to the center of the sphere that encloses the whole surface |


| $\vec{P}_{n}$ | vector from the origin of the surface coordinate system to the center of the sphere that encloses a node |
| :---: | :---: |
| $\vec{P}_{j 2 n}$ | vector from the center of the $j^{\text {th }}$ node to the center of the $n^{\text {th }}$ node |
| p | denotes components depending on the subscripts of the vectors $\vec{P}_{P}, \vec{P}_{m}, \vec{P}_{b}$ and $\vec{P}_{j 2 n}$ defined above; also an arbitrary constant used in the general type of contour integrals |
| q | an arbitrary constant used in the general type of contour integrals |
| $\mathrm{q}_{\mathrm{n} \stackrel{\mathrm{r}}{ } \mathrm{j}}$ | radiative heat transfer rate between nodes " n " and " j " |
| $\overrightarrow{\mathrm{R}}$ | vector position from the origin of the surface coordinate system to the possible point of intersection on a surface, |
| $\mathrm{R}_{1}$ |  |
| $\left.\begin{array}{l}R_{2} \\ R_{3}\end{array}\right\}$ | vector coefficients used in the vector operation for shading check and defined by Eqs. (3.12) |
| $\overrightarrow{\mathrm{R}}_{c 2 i}$ | vector from the origin of the central coordinate system to the origin of surface coordinate system |
| $r$ | vector components of the vector $r_{m \ell}$ when subscripted by $x, y$ and $z$, respectively |
| ${ }^{\text {b }}$ b | magnitude of the radius of a sphere that encloses a whole body. When subscripted with numbers it denotes distance between the c.a. of a trapezoidal surface to the four corner points |
| $\mathrm{r}_{\mathrm{i}}$ | magnitude of the radius of the c.a. of an element in the X-Y plane of the surface coordinate system |
| $r_{\ell}{ }_{1} r^{\ell_{2}}$ | in conic surfaces it is the radii of the start and end of an element in the $X-Y$ plane of surface coordinate system |


| $r_{n}$ | magnitude of the radius of sphere that encloses a whole node. Also the vector components when double subscripted with letters $x, y$ and $z$, respectively, of a vector of magnitude " $r n$ " and in the direction of vector $\vec{P}_{\mathrm{j} 2 \mathrm{n}}$. When subscripted with numbers it denotes distance between the c.a. of trapezoidal node to the four corner points |
| :---: | :---: |
| $\stackrel{\rightharpoonup}{r}$ | vector from the origin of the surface coordinates system to the c.a. of an element |
| $\vec{r}_{1 m}$ | vector from the c.a. of the $\ell^{\text {th }}$ element to the c.a. of the $m^{\text {th }}$ element |
| $S_{i}$ | surface equation in Cartesian coordinates given by Eq. (2.33) |
| T | temperature of node |
| $\left[\mathrm{T}_{\mathrm{i} 2 \mathrm{c}}\right]$ | transformation matrix from "i" system to the "c" system |
| U | quadratic equation that represents numerator of the general type of contour integrals and given by Eq.(5.13) |
| $\stackrel{\rightharpoonup}{U}_{j 2 n}$ | unit vector from the c.a. of $j^{\text {th }}$ node to the c.a. of the $n^{\text {th }}$ node |
| u | vector components of the above vector $\bar{U}_{j 2 n}$; independent variable of integration of general type of contour integrals |
| X | denotes Cartesian coordinate axis |
| x | denotes distance measured in the x direction of any given system |
| $\mathrm{x}_{\mathrm{b}}$ | X component of distance from the c.a. of a trapezoidal surface to the four corner points according to the double subscript |
| $\mathrm{x}_{\mathrm{n}}$ | X component of distance from the c.a. of a trapezoidal node to the four corner points according to the double subscript |
| Y | denotes Cartesian coordinate axis |
| y | denotes distance measured in the $y$ direction of any given system |


| $Y_{b}$ | $Y$ component of distance from the c.a. of a trapezoidal <br> surface to the four corner points according to the <br> double subscript |
| :--- | :--- |
| $y_{n} \quad$$Y$ component of distance from the c.a. of a trapezoidal <br> node to the four corner points according to the double <br> subscript |  |
| $Z$ | denotes Cartesian coordinate axis |
| $z$ | denotes distance measured in the $z$ direction of any <br> given system |

## Greek

$\alpha$
$\beta$
$\beta$
max
$\beta$
min
$\gamma$
$\gamma_{\text {max }}$
$\gamma_{\text {min }}$
$\triangle \mathrm{A}$
$\Delta \beta$
$\Delta Y$
$\delta$
scalar quantity used to describe a type of surface
beta coordinate of a point in the surface coordinate system
maximum beta dimension of a surface
minimum beta dimension of a surface
gamma coordinate of a point in the surface coordinate system
maximum gamma dimension of a surface
minimum gamma dimension of a surface
finite differential area
finite increment in the beta direction
finite increment in the gamma direction
scalar quantity in the vector Eq. (3.3) that determines points of intersection on a surface
variable used in the shading check defined by Eq. (3.9)
variable used in the shading check defined by Eq. (3.10)
angle between the unit normal of a differential area and the vector joining the two differential areas for which the view factor has been defined


| $\ell$ | denotes $\ell^{\text {th }}$ element reference to its surface coordinate system |
| :---: | :---: |
| m | denotes $m^{\text {th }}$ element reference to its surface coordinate system |
| n | denotes either an $n^{\text {th }}$ surface or an $n^{\text {th }}$ node referenced to its surface coordinate system |
| $\max$ | maximum value |
| min | minimum value |
| t | denotes test cylinder coordinate system |
| $x$ |  |
| $y\}$ | denotes components in Cartesian coordinate system |
| $z$ |  |
| $\beta$ | denotes beta direction |
| $\gamma$ | denotes gamma direction |
| Operations |  |
| $\partial / \partial \mathrm{x}$ | partial derivative |
| $\nabla$ | gradient |
| $\sum$ | summation |
| $\int$ | integration |
| 5 | contour integration |
| 11 | absolute value |
| $\\|\\|$ | maximum value of arguments inside brackets |
| $\rightarrow$ | vector |
| [] | matrix |
| ( ) | minimum value of an argument |
| ()$^{+}$ | maximum value of an argument |
| ' | denotes two extreme points of sphere that enclose a whole surface |

## Section 1 INTRODUCTION

Lockheed's Huntsville Research \& Engineering Center, as a subtask under Modification 2 of Contract NAS8-20230, and on a task under Contract NAS8-30154, conducted studies to develop techniques for determining the diffuse radiation view factors including the effects of shading.

During these studies, a finite difference (or double summation) technique and a contour integral technique were developed, each of which offers advantages and disadvantages. Selecting the better of the two techniques is dependent on the problem. A Radiation View Factor (RAVFAC) computer program, in which both of these techniques were used, was developed and written for use with the Univac 1108 computer. The program is easy to use and offers a variety of surface shapes. The program also accounts for surface shading effects on the view factors.

This report presents the equations and techniques used in the RAVFAC program, and details of how the program can be used to obtain meaningful solutions to radiation problems. View factor calculations, shading techniques and the preshade routine that is used to increase the efficiency of the program are presented and discussed. How the program works, how to set up an input data deck, and the program output are described. Also included as Appendixes are a sample problem, comparisons of the finite difference and contour integral techniques, and a comparison of RAVFAC and other view factor programs. The objective of this report is to provide the user with an easy input guide, documentation of the equations used, and information of interest to a programmer.

The size of this report may discourage the potential user of RAVFAC from becoming familiar with the capabilities and use of the computer program. The report does contain information not normally found in a user's manual; however, it is organized in such a way that the user can refer only to those
sections of interest. Some of the major interest areas are:

- Development of programmed equations (Sections 1 through 5)
- How the program works (Section 6.1)
- Definition of input parameters (Section 6.2)
- Input guide (Section 7.3)
- Sample problem (Appendix A)
- Comparison of RAVFAC results (Appendixes B and C)

Supplementary information can be found in other sections, as noted in the report, or from the Contents.

# Section 2 <br> DESCRIPTION OF SURFACE TYPES 

### 2.1 TYPES OF SURFACES

The RAVFAC program can be used to calculate diffuse radiation view factors, including the effects of shading, for the following types of surfaces:

- Rectangular plates
- Circular plates
- Trapezoidal plates
- Cylinders
- Cone or cone frustums
- Spheres
- Circular paraboloids

These surface types are shown in Fig. 1 in which the five quantities $\alpha, \beta$ min , $\beta_{\max }, \gamma_{\min }$ and $\gamma_{\max }$ define the dimensions of the surface relative to its surface coordinate system (SCS). As shown in Fig. 1, each surface type is rigorously positioned with respect to its SCS. Depending on the type of surface, they may represent either distances (in arbitrary dimensions), or angles in degrees. In the case of distances, all dimensions must be in the same units.

Each surface is divided into smaller areas (nodes), and the nodes are divided into even smaller areas (elements) as shown in Fig. 2. A subsequent discussion will explain how the view factors are calculated between elements and then summed for each node (i.e., the program output is view factors between nodes). The surface is divided into NVB nodes in the $\beta$ direction times NVG nodes in the $\gamma$ direction in equal increments of $\beta$ and $\gamma$, respectively. Every node is subdivided further into $N B$ elements in the $\beta$ direction

Surface Type $\pm 1$ Recternle


$$
\begin{aligned}
& \beta_{\min }<\beta_{\max } \\
& \gamma_{\min }<\gamma_{\text {max }}
\end{aligned}
$$

Surface Coordinate System ( $\mathrm{X}^{\prime}, \mathrm{Y}^{\prime}, \mathrm{Z}^{\prime}$ )


$$
0 \leq \beta_{\min }<\beta_{\max }
$$

$$
\left\{\begin{aligned}
0^{\circ}<\gamma_{\min }<\gamma_{\max } \leq+360^{\circ} \\
\gamma_{\text {max }} \leq \gamma_{\min }+360^{\circ}
\end{aligned}\right.
$$



Fig. 1 - Input Surface Types (Sheet 1)

Surface rype $\pm 4$ Cylinder


Fig. 1-Input Surface Types (Sheet 2)


Fig. 1 - Input Surface Types (Sheet 3)


Fig. 2-Example of Node and Element Distribution and Numbering System
times NG elements in the $\gamma$ direction. The nodal increments are therefore given in the $\beta$ and $\gamma$ directions

$$
\begin{align*}
& \Delta \beta_{n}=\frac{\beta_{\max }-\beta_{\min }}{N V B}  \tag{2.1}\\
& \Delta y_{n}=\frac{\gamma_{\max }-\gamma_{\min }}{N V G} \tag{2.2}
\end{align*}
$$

and the elemental increments are given in the $\beta$ and $\gamma$ directions, respectively, by

$$
\begin{align*}
\Delta \beta_{\ell} & =\frac{\Delta \beta_{n}}{N B}  \tag{2.3}\\
\Delta \gamma_{\ell} & =\frac{\Delta \gamma_{n}}{N G} \tag{2.4}
\end{align*}
$$

The value of the surface area $A$ of the element $\ell$ and its centroid $X_{\ell} Y^{\prime}{ }^{\prime}$ $Z_{\ell}$ depend on the type of surface as follows:

1. Rectangular Plate

$$
\begin{align*}
& A_{\ell}=\Delta \beta_{\ell} \Delta \gamma_{\ell}  \tag{2.5}\\
& x_{\ell}=\left(\gamma_{n}\right)^{-}+\left(k_{\gamma}-\frac{1}{2}\right) \Delta \gamma_{n}-\left[\frac{N G+1}{2}-h_{\gamma}\right] \Delta \gamma_{\ell} \\
& y_{\ell}=\left(\beta_{n}\right)^{-}+\left(k_{\beta}-\frac{1}{2}\right) \Delta \beta_{n}-\left[\frac{N B+1}{2}-h_{\beta}\right] \Delta \beta_{\ell}  \tag{2.6}\\
& z_{\ell}=\alpha
\end{align*}
$$

2. Circular Plate

$$
\begin{align*}
& \mathrm{A}_{\ell}=\mathrm{y}_{\ell} \Delta \beta_{\ell} \Delta \gamma_{\ell}  \tag{2.7}\\
& \mathbf{x}_{\ell}=\left(\gamma_{n}\right)^{-}+\left(k_{\gamma}-\frac{1}{2}\right) \Delta \gamma_{\mathrm{n}}-\left[\frac{\mathrm{NG}+1}{2}-h_{\gamma}\right] \Delta \gamma_{\ell} \\
& \mathrm{y}_{\ell}=\left(\beta_{\mathrm{n}}\right)^{-}+\left(k_{\beta}-\frac{1}{2}\right) \Delta \beta_{\mathrm{n}}-\left[\frac{N B+1}{2}-h_{\beta}\right] \Delta \beta_{\ell}  \tag{2.8}\\
& \mathrm{z}_{\ell}=\alpha
\end{align*}
$$

3. Trapezoidal Plate

$$
\begin{equation*}
A_{l}=\frac{\sin \left(\Delta \gamma_{\ell}\right) \Delta \beta_{\ell} \beta_{\ell}}{\cos \left(\gamma_{\ell}-\frac{\Delta \gamma_{\ell}}{2}\right) \cos \left(\gamma_{\ell}+\frac{\Delta \gamma_{\ell}}{2}\right)} \tag{2.9}
\end{equation*}
$$

where

$$
\left.\begin{array}{l}
\gamma_{\ell}=\left(\gamma_{n}\right)^{-}+\left(k_{\gamma}-\frac{1}{2}\right) \Delta \gamma_{n}-\left[\frac{N G+1}{2}-h_{\gamma}\right] \Delta \gamma_{\ell} \\
\beta_{\gamma}=\left(\beta_{n}\right)^{-}+\left(k_{\beta}-\frac{1}{2}\right) \Delta \beta_{n}-\left[\frac{N B+1}{2}-h_{\beta}\right] \Delta \beta_{\ell}
\end{array}\right\}
$$

4. Cylinder

$$
\begin{equation*}
A_{\ell}=r_{\ell} \Delta \beta_{\ell} \Delta \gamma_{\ell} \tag{2.12}
\end{equation*}
$$

where

$$
\begin{equation*}
r_{\ell}=\alpha \tag{2.13}
\end{equation*}
$$

5. Cone or Cone Frustum

$$
\begin{equation*}
A_{\ell}=\Delta \gamma_{\ell} r_{\ell} \Delta \beta_{\ell} \sec (\alpha) \tag{2.14}
\end{equation*}
$$

where

$$
\begin{equation*}
r_{\ell}=\tan (\alpha) \beta_{\ell} \tag{2.15}
\end{equation*}
$$

6. Sphere

$$
\begin{equation*}
A_{\ell}=2 \alpha \Delta \gamma_{\ell} r_{\ell} \sin \left(\frac{\Delta \beta_{\ell}}{2}\right) \tag{2.16}
\end{equation*}
$$

where

$$
\begin{equation*}
r_{\ell}=\alpha \sin \left(\beta_{\ell}\right) \tag{2.17}
\end{equation*}
$$

7. Circular Paraboloid

$$
\begin{equation*}
A_{\ell}=\frac{2}{3} \Delta \gamma_{\ell} \frac{r_{\ell}}{\left(\beta_{\ell}\right)^{1 / 2}}\left[\left(\beta_{\ell}+\frac{\Delta \beta_{\ell}}{2}+\alpha\right)^{3 / 2}-\left(\beta_{\ell}-\frac{\Delta \beta_{\ell}}{2}+\alpha\right)^{3 / 2}\right] \tag{2.18}
\end{equation*}
$$

where

$$
\begin{equation*}
r_{\ell}=2\left(\alpha \beta_{\ell}\right)^{1 / 2} \tag{2.19}
\end{equation*}
$$

The centroid of the elements of the cylinder, cone or cone frustum, sphere and circular paraboloid can be all given by the same relations:

$$
\left.\begin{array}{l}
x_{\ell}=r_{\ell} \sin \left(\gamma_{\ell}\right)  \tag{2.20}\\
y_{\ell}=r_{\ell} \cos \left(\gamma_{\ell}\right) \\
z_{\ell}=\beta_{\ell}
\end{array}\right\}
$$

except for the sphere in which case $z_{\ell}$ becomes

$$
\begin{equation*}
z_{\ell}=\alpha \cos \left(\beta_{\ell}\right) \tag{2.21}
\end{equation*}
$$

In Eqs. (2.20) and (2.21) $\beta_{\ell}$ and $\gamma_{\ell}$, are defined by Eqs. (2.10).

In Eqs. (2.6), 2.8), (2.10) and (2.11) the coefficients $k$ and $h$ are defined as

$$
\begin{aligned}
k_{\gamma} & =1, \ldots, N V G \\
h_{\gamma} & =1, \ldots, N G \\
k_{\beta} & =1, \ldots, N V B \\
h_{\beta} & =1, \ldots, N B
\end{aligned}
$$

### 2.2 COORDINATE SYSTEMS

Complex vehicles and vehicle clusters are resolved into a series of surfaces consisting of available surface types. Three different coordinate systems are used by the program which are:

- Surface Coordinate System (SCS)
- Central Coordinate System (CCS)
- Intermediate Coordinate System (ICS).

Each surface is defined in its SCS as discussed in Section 2.1 which is fixed. The CCS is conveniently located by the user and all SCS are referenced to the CCS. In some cases it is easier to reference a group of surfaces to an ICS and then reference the ICS to the CCS which requires only one additional data card. Another use of the ICS is when a group of surfaces are to be moved from one location to another. The entire group can be moved by redefining the location of the ICS.

Relationships of the different coordinate systems can best be illustrated by Fig. 3. In this example the problem view factors are to be calculated with

Fig. 3 - Coordinate Systems

Module 1 (surfaces 3, 4, and 5) located at ports 1 and 3. The CCS was arbitrarily located at the front end of the main body (surface 1). Surface 2, which will be located in the same position in both problems, is referenced directly to the CCS. Surface 5 , which will be located in a different position, is referenced to an ICS which is in turn referenced to the CCS. When Module 1 is moved to port 3 the ICS is relocated with respect to the CCS. This will make Module 1 (i.e., surfaces 3, 4, and 5) to be located at port 3.

The relation between the surface coordinate and the central coordinate systems is given by the following transformation matrix and translational vector defined as

$$
\begin{gather*}
{\left[\mathrm{T}_{\mathrm{i} 2 \mathrm{c}}\right]=\left[\begin{array}{ccc}
\lambda_{\mathrm{xi}} & \mu_{\mathrm{xi}} & \nu_{\mathrm{xi}} \\
\lambda_{\mathrm{yi}} & \mu_{\mathrm{yi}} & \nu_{\mathrm{yi}} \\
\lambda_{\mathrm{zi}} & \mu_{\mathrm{zi}} & \nu_{\mathrm{zi}}
\end{array}\right]}  \tag{2.22}\\
\overrightarrow{\mathrm{R}}_{\mathrm{c} 2 \mathrm{i}} \equiv \mathrm{R}_{\mathrm{xi}} \overrightarrow{\mathrm{i}}_{\mathrm{c}}+\mathrm{R}_{\mathrm{yi}} \overrightarrow{\mathrm{y}}_{\mathrm{c}}+\mathrm{R}_{2 \mathrm{i}} \overrightarrow{\mathrm{k}}_{\mathrm{c}} \tag{2.23}
\end{gather*}
$$

where

$$
\begin{align*}
& \lambda_{\mathrm{xi}}=\cos \phi \cos \psi \\
& \lambda_{\mathrm{yi}}=-\sin \phi \cos \psi  \tag{array}\\
& \lambda_{\mathrm{zi}}=\sin \psi \\
& \mu_{\mathrm{xi}}=\sin \phi \cos \omega-\cos \phi \sin \psi \sin \omega \\
& \mu_{\mathrm{yi}}=\cos \phi \cos \omega+\sin \phi \sin \psi \sin \omega  \tag{2.25}\\
& \mu_{\mathrm{zi}}=\cos \psi \sin \omega \\
& \nu_{\mathrm{xi}}=-\sin \phi \sin \omega-\cos \phi \sin \psi \cos \omega \\
& \nu_{\mathrm{yi}}=-\cos \phi \sin \omega+\sin \psi \sin \phi \cos \omega  \tag{2.26}\\
& \nu_{\mathrm{zi}}=\cos \psi \cos \omega
\end{align*}
$$

and where $R_{x i}, R_{y i}, R_{z i}, \phi, \psi$ and $\omega$ are given in Fig. 4.
$X_{c}, Y_{c}, Z_{c}$ - Central Coordinate System (CCS)
$X_{3}, Y_{3}, Z_{3}$ - Surface Coordinate System (SCS)
$X_{c}^{\prime}, Y_{c}^{\prime} Z_{c}^{\prime}$ - CCS translated to the SCS to show
the rotational angles


Fig. 4-Relative Position of Surface Coordinate System with Respect to Central Coordinate System

A similar transformation matrix and translational vector, as defined above, exists between the surface and the intermediate coordinate system. The matrix multiplication of the transformation matrices and the vector addition of the translational vectors yields the direct transformation and translation between the surface and central coordinate system for those surfaces which were referenced to the intermediate coordinate system.

The orientation of each surface coordinate system is specified by three angles $-\phi, \psi$, and $\omega$. These angles define three coordinate rotations that-rotate the central coordinates ( $X_{c}, Y_{c}, Z_{c}$ ) into the surface coordinates $\left(X_{3}, Y_{3}, Z_{3}\right)$. The rotations are in the direction $X_{c}, Y_{c}, Z_{c} \rightarrow X_{3}, Y_{3}, Z_{3}$ and must be carried out in the strict order $\phi$, then $\psi$, and $\omega$, as follows:
$\phi$ rotation: Rotate $Y_{c}$ toward $X_{c}$ about $Z_{c}$ obtaining $X_{1}, Y_{1}, Z_{1}$ system $\psi$ rotation: Rotate $X_{1}$ toward $Z_{1}$ about $Y_{1}$ obtaining $X_{2}, Y_{2}, Z_{2}$ system $\omega$ rotation: Rotate $Y_{2}$ toward $Z_{2}$ about $X_{2}$ obtaining $X_{3}, Y_{3}, Z_{3}$ system.

Similar orientations apply between the surface and the intermediate and central coordinate system.

### 2.3 FLAT SURFACES

The three flat surfaces, rectangular plate, circular plate and trapezoidal plate are described by a unit vector normal to the surface in the surface coordinate system and by the $\beta_{\min }, \beta_{\max }$, and $\gamma_{\min }, \gamma_{\max }$ limitations described in Fig. 1.

The normal to the surface is given by

$$
\begin{equation*}
\vec{n}_{i}= \pm \vec{k}_{i}= \pm\left\{\vec{i}_{i} \times \vec{j}_{i}\right\} \tag{2.27}
\end{equation*}
$$

as shown in Fig. 5.


Fig. 5 - Unit Normal - Flat Surface


Fig. 6 - Unit Normal-Conic Surface

The unit normal is positive when the plate is input as a positive surface, that is, when the upper surface of the plate is taken into consideration. The unit normal is negative when the lower surface of the plate is taken into consideration.

A point, $\overrightarrow{\mathrm{P}}$, on the surface given in the surface coordinate system, is trans formed to the central coordinate system by the following relation

$$
\vec{P}=\left[\begin{array}{l}
x_{c}  \tag{2.28}\\
y_{c} \\
z_{c}
\end{array}\right]=\left[\begin{array}{l}
T_{i 2 c}
\end{array}\right]\left[\begin{array}{c}
x_{i} \\
y_{i} \\
0
\end{array}\right]+\left[\begin{array}{l}
R_{x i} \\
R_{y i} \\
R_{z i}
\end{array}\right]
$$

### 2.4 CONIC SURFACES

The conic surfaces are given by the four remaining surfaces, the cylinder, the cone or cone frustum, the sphere and the circular paraboloid. Any point, $\vec{P}$, in the surface coordinate system is described by the vector equation

$$
\begin{equation*}
\vec{P}=x_{i} \vec{i}_{i}+y_{i} \vec{j}_{i}+z_{i} \vec{k}_{i} \tag{2.29}
\end{equation*}
$$

where

$$
\begin{align*}
& r_{i}=\left(x_{i}^{2}+y_{i}^{2}\right)^{1 / 2}  \tag{2.30}\\
& x_{i}=r_{i} \sin \phi_{o}  \tag{2.31}\\
& y_{i}=r_{i} \cos \phi_{o} \tag{2.32}
\end{align*}
$$

as shown in Fig. 6.

A point constrained to lie on the surface will satisfy the relation

$$
\begin{equation*}
S_{i}\left(x_{i}, y_{i}, z_{i}\right)=0 \tag{2.33}
\end{equation*}
$$

All conic surfaces can be expressed mathematically by

$$
\begin{equation*}
x_{i}-\left(a+b z_{i}+c z_{i}^{2}\right)^{1 / 2}=0 \tag{2.34}
\end{equation*}
$$

values of " $Z_{i}$ " for the cylinder, cone or cone frustum and circular paraboloid are given by

$$
\begin{equation*}
\beta_{\min } \leq z_{i} \leq \beta_{\max } \tag{2.35}
\end{equation*}
$$

and for the sphere are given by

$$
\begin{equation*}
\alpha \cos \left(\beta_{\min }\right) \leq z_{i} \leq \alpha \cos \left(\beta_{\max }\right) \tag{2.36}
\end{equation*}
$$

The constants $a, b$ and $c$ take on the following values depending on the sype of surface:

$$
\begin{gathered}
\text { Cylinder }\left\{\begin{array}{l}
a=\alpha^{2} \\
b=0 \\
c=0
\end{array}\right. \\
\text { Cone or Cone Frustum }\left\{\begin{array}{l}
a=0 \\
b=0 \\
c=\tan ^{2} \alpha
\end{array}\right. \\
\text { Sphere }\left\{\begin{array}{l}
a=\alpha^{2} \\
b=0 \\
c=-1
\end{array}\right. \\
\text { Circular Paraboloid }\left\{\begin{array}{l}
a=0 \\
b=4 \alpha \\
c=0
\end{array} .\right.
\end{gathered}
$$

A unit vector normal to the surface at a point, $P$, is defined by

$$
\begin{equation*}
\vec{n}_{i} \equiv \frac{\nabla S_{i}}{\left|\nabla S_{i}\right|}=\left(\frac{\partial}{\partial x_{i}} S_{i} \vec{i}_{i}+\frac{\partial}{\partial y_{i}} S_{i} \vec{j}_{i}+\frac{\partial}{\partial z_{i}} S_{i} \vec{k}_{i}\right) \div\left|\nabla S_{i}\right| \tag{2.37}
\end{equation*}
$$

where

$$
\begin{equation*}
\left|\nabla S_{i}\right| \equiv\left[\left(\frac{\partial S_{i}}{\partial x_{i}}\right)^{2}+\left(\frac{\partial S_{i}}{\partial y_{i}}\right)^{2}+\left(\frac{\partial S_{i}}{\partial z_{i}}\right)^{2 \cdot}\right]^{1 / 2} \tag{2.38}
\end{equation*}
$$

which for the conic equation reduces to

$$
\begin{equation*}
\vec{n}_{i}=\frac{\sin \phi_{o} \vec{i}_{i}+\cos \phi_{o} \vec{j}_{i}-\left[\frac{b+2 c z}{2 r_{i}}\right] \vec{k}_{i}}{\left|\nabla S_{i}\right|} \tag{2.39}
\end{equation*}
$$

where

$$
\begin{equation*}
\left|\nabla s_{i}\right|=\sqrt{\left(\frac{b+2 c z_{i}}{2 r_{i}}\right)^{2}+1} \tag{2.40}
\end{equation*}
$$

The unit normal is positive when the conic is input as a positive surface, that is, when the outer surface of the conic is taken into consideration. The unit normal is negative (toward the body axis) when the inner surface of the conic is taken into consideration.

Similar to the flat surfaces, a point, $\overrightarrow{\mathrm{P}}$, can be given in the central coordinate system as

$$
\vec{P}=\left[\begin{array}{l}
x_{c}  \tag{2.41}\\
y_{c} \\
z_{c}
\end{array}\right]=\left[T_{i 2 c}\right]\left[\begin{array}{l}
x_{i} \\
y_{i} \\
z_{i}
\end{array}\right]+\left[\begin{array}{l}
R_{x i} \\
R_{y i} \\
R_{z i}
\end{array}\right]
$$

### 2.5 METHOD OF INTEGRATION

The method of integration of each node, the smaller divisions of a surface, depends on the numerical technique used for calculating the view factors (Section 5).

If the finte difference is used, the integration of each node is accomplished as follows:

- The node is divided into an arbitrary number of elemental areas.
* The centroid of the elemental area in the surface coordinate system is determined (Section 2.1).
- The surface normal at this point, assuming the absolute value of the elemental area to be concentrated at the center of area, is evaluated (Section 2.4).
- The view factor between the elemental areas are evaluated, accounting for any shading effects.
- The elemental view factors are summed over the two nodes in question for which the view factors are being calculated.

If the contour integration is used, the process of integration is as follows:

- The node is divided into an arbitrary number of elemental areas.
- The centroid of the elemental area in the surface coordinate system is determined.
- The surface nommal at this point is evaluated and considered to be the positive $Z$ axis of an element coordinate system, where the $X$ and $Y$ axes are constructed arbitrarily according to the lype of surface, so that they satisfy a "righthand system" rule.
- The elemental area is approximated by a rectangular element perpendicular to the $Z$ axis of the element and with its sides parallel to the $X$ and $Y$ element axes, respectively, and in the plane of the origin. The magnitude of the rectangular element will depend on the type of surface.
- A contour integration is performed over the elements of one node while a summation of elements is performed over the area of the other node in question.

In both cases the integration of the conic surfaces takes place along the $Z_{i}$ direction between the limits of $\beta_{\ell}$ and $\beta_{\ell}+\Delta \beta_{\ell}$ and around the body between the limit angles $\gamma_{\ell}$ and $\gamma_{\ell}+\Delta \gamma_{\ell}$ in arbitrary sizes. In the case of a rectangular flat plate, integration takes place in rectangular strips, while in the case of a circular plate, integration takes place in circular strips and for a trapezoidal plate, the integration is performed by summing trapezoidal elements. The integration is performed between the limits of $\beta_{\ell}$ and $\beta_{\ell}+\Delta \beta_{\ell}$, and $\gamma_{\ell}$ and $\gamma_{\ell}+\Delta \gamma_{\ell}$.

The magnitude of the sides of the approximate rectangular element are given according to the type of surface as:

- Rectangular plate

$$
\begin{aligned}
& G=\Delta \gamma_{\ell} \\
& B=\Delta \beta_{\ell}
\end{aligned}
$$

- Circular plate

$$
\begin{aligned}
& G=\beta_{\ell} \Delta \gamma_{\ell} \\
& B=\Delta \beta_{\ell}
\end{aligned}
$$

- Trapezoidal plate

$$
\begin{aligned}
& G=\beta_{\ell}\left[\tan \left(\gamma_{\ell}+\frac{\Delta \gamma_{\ell}}{2}\right)-\tan \left(\gamma_{\ell}-\frac{\Delta \gamma_{\ell}}{2}\right)\right] \\
& B=\Delta \beta_{\ell}
\end{aligned}
$$

- Cylinder

$$
\begin{aligned}
\mathrm{G} & =2 \alpha \sin \left(\frac{\Delta \gamma_{\ell}}{2}\right) \\
\mathrm{B} & =\Delta \beta_{\ell}
\end{aligned}
$$

- Cone or cone frustum

$$
\begin{aligned}
& G=2 \beta_{\ell} \tan (\alpha) \sin \left(\frac{\Delta \gamma_{\ell}}{2}\right) \\
& B=\Delta \beta_{\ell}
\end{aligned}
$$

- Sphere

$$
\begin{aligned}
& G=2 \alpha \sin \left(\beta_{\ell}\right) \sin \left(\frac{\Delta \gamma_{\ell}}{2}\right) \\
& B=2 \alpha \sin \left(\frac{\Delta \beta_{\ell}}{2}\right)
\end{aligned}
$$

- Circular Paraboloid

$$
\begin{aligned}
G & =4 \sqrt{\alpha \beta_{\ell}} \sin \left(\frac{\Delta \gamma_{\ell}}{2}\right) \\
B & =\frac{1}{4 \alpha}\left\{r_{\ell_{2}}\left[4 \alpha^{2}+r_{\ell_{2}}^{2}\right]^{1 / 2}-r_{\ell}\left[4 \alpha^{2}+r_{\ell_{1}}^{2}\right]^{1 / 2}\right. \\
& \left.+4 \alpha^{2}\left(\ell n\left[r_{\ell}+\left(4 \alpha^{2}+r_{\ell}^{2}\right)^{1 / 2}\right]-\ln \left[r_{\ell}+\left(4 \alpha^{2}+r_{\ell}^{2}\right)^{1 / 2}\right]\right)\right\}
\end{aligned}
$$

where

$$
\begin{aligned}
& r_{\ell_{1}}=\left[4 \alpha\left(\beta_{\ell}-\frac{\Delta \beta_{\ell}}{2}\right)\right]^{1 / 2} \\
& r_{\ell_{2}}=\left[4 \alpha\left(\beta_{\ell}+\frac{\Delta \beta_{\ell}}{2}\right)\right]^{1 / 2}
\end{aligned}
$$

$G$ and $B$ are the magnitudes in the $X$ and $Y$ directions, respectively.

The arbitrary integration step size is left to the discretion of the user. The smaller the elemental areas, the more accurate the integration process and the longer the computer run time.

## Section 3

SHADING

To calculate the view factor between a pair of given elements, whether the elemental areas present themselves to each other, or whether any other portion of the body or any other surface intersects the line of view between their corresponding centers of area must be determined.

Shading between a pair of elemental areas can occur in two ways:

- The elemental areas do not present themselves to each other
- The elemental areas present themselves to each other but one or more surfaces obstruct the vector joining their centers of area.

Shading between the first areas is easily checked, since both the normals to the elemental areas and the vectors from the origin of the surface coordinate system to the centroid of the elemental areas have been calculated. These vectors can be easily translated and rotated to the central coordinate system. A dot product is sufficient to determine if two elemental areas present themselves to each other as shown in Fig. 7. The elemental areas present themselves to each other, if and only if,

$$
\begin{align*}
& \left(\left\{\left[T_{i 2 c}\right] \stackrel{\rightharpoonup}{r}_{\ell}+\stackrel{\rightharpoonup}{R}_{c 2 i}\right\}-\left\{\left[T_{j 2 c}\right] \stackrel{\rightharpoonup}{r}_{m}+\vec{R}_{c 2 j}\right\}\right) \cdot \stackrel{\rightharpoonup}{n}_{\ell} \leq 0  \tag{3.1}\\
& \left(\left\{\left[T_{i 2 c}\right] \stackrel{\rightharpoonup}{r}_{\ell}+\vec{R}_{c 2 i}\right\}-\left\{\left[T_{j 2 c}\right] \vec{r}_{m}+\vec{R}_{c 2 j}\right\}\right) \cdot \stackrel{\rightharpoonup}{n}_{m} \geq 0 \tag{3.2}
\end{align*}
$$

That is, two conditions must be met if the two elements $m$ and $\ell$ present themselves to each other. The first condition is that the angle between the normal vector of the $\mathrm{m}^{\text {th }}$ element and the vector joining the centroids of the


Fig. 7 - Surface Shading - Dot Product
$\ell^{\text {th }}$ and $\mathrm{m}^{\text {th }}$ elements is between +90 and -90 deg . The second condition is that the angle between the normal vector of the $\ell^{\text {th }}$ element and the vector joining the centroids of the $\ell^{\text {th }}$ and $\mathrm{m}^{\text {th }}$ elements is either between +90 and +180 deg or between -90 and -180 deg . If either of these two conditions is not met, the elements cannot "see" each other.

If the above tests show that the elements do not "see" each other, no further test is necessary and the view factors can be calculated accordingly (i.e., if the elements do not see each other the elemental view factor is set equal to zero). If these tests show that the elements could possibly see each other, additional checks must be made to determine if the elements are shaded by another surface. The test for an intervening surface is somewhat more complex than the above analysis; however, it is still a straightforward vector operation. The method is explained below.

In Fig. 8 an $\ell^{\text {th }}$ elemental area in the $n$ surface coordinate system (SCS) and an $\mathrm{m}^{\text {th }}$ elemental area in the $\mathrm{j} \operatorname{SCS}$ is set up by surface integration routines. A check must be made to determine whether the line of view between the $\ell^{\text {th }}$ and $\mathrm{m}^{\text {th }}$ element is shaded by some surface, i. From Fig. 8 it can be seen that the vector, $\vec{R}$, is given by

$$
\begin{equation*}
\stackrel{\rightharpoonup}{\mathrm{R}}=\left[\mathrm{T}_{\mathrm{c} 2 \mathrm{i}}\right]\left\{\stackrel{\rightharpoonup}{\mathrm{R}}_{\mathrm{c} 2 \mathrm{j}}+\left[\mathrm{T}_{\mathrm{j} 2 \mathrm{c}}\right] \overrightarrow{\mathrm{r}}_{\mathrm{m}}+\delta \stackrel{\rightharpoonup}{\mathrm{r}}_{\mathrm{m} \mathrm{\ell}}-\overrightarrow{\mathrm{R}}_{\mathrm{c} 2 \mathrm{i}}\right\} \tag{3.3}
\end{equation*}
$$

where

$$
\begin{equation*}
\vec{r}_{m \ell}=\left[T_{n 2 c}\right] \vec{r}_{\ell}+\vec{R}_{c 2 n}-\left[T_{j 2 c} \mid \vec{r}_{m}-\vec{R}_{c 2 j}\right. \tag{3.4}
\end{equation*}
$$

$$
\left[\begin{array}{ll}
\mathrm{T}_{\mathrm{c}} 2 \mathrm{i}
\end{array}\right]=\left[\begin{array}{lll}
\lambda_{\mathrm{xi}} & \lambda_{\mathrm{yi}} & \lambda_{\mathrm{zi}}  \tag{3.5}\\
\mu_{\mathrm{xi}} & \mu_{\mathrm{yi}} & \mu_{\mathrm{zi}} \\
\nu_{\mathrm{xi}} & \nu_{y i} & \nu_{z i}
\end{array}\right]
$$



Fig. 8 - Surface Shading - Vector Operation Diagram
the vectors $\vec{r}_{\mathrm{ml}}$ and $\overrightarrow{\mathrm{R}}_{\mathrm{c} 2 \mathrm{i}}$ can be written as

$$
\begin{align*}
& \vec{r}_{m \ell}=r_{x} \vec{i}_{c}+r_{y} \vec{j}_{c}+r_{z} \vec{k}_{c}  \tag{3.6}\\
& \vec{R}_{c Z i}=R_{x i} \vec{i}_{c}+R_{y i} \vec{j}_{c}+R_{z i} \vec{k}_{c} \tag{3.7}
\end{align*}
$$

and where $\delta$ is a scale factor, which is the variable to be determined. The transformation matrices $\left\{\mathrm{T}_{\mathrm{n} 2 \mathrm{c}}\right\}$ and $\left\{\mathrm{T}_{\mathrm{j} 2 \mathrm{c}}\right\}$ and translational vectors $\overrightarrow{\mathrm{R}}_{\mathrm{c} 2 \mathrm{n}}$ and $\vec{R}_{c 2 j}$ are defined as in Section 2 .

It can then be said that if there exists a real value of $\delta$ which will produce a point in the $i^{\text {th }}$ (SCS) which satisfies the boundaries of surface $i$ then the line of view between the elemental areas is shaded by the surface. If no real value exists, then the elemental areas are not shaded by the $i^{\text {th }}$ surface.

By vector manipulation, the following vector equation in the unknown $\delta$ is obtained:

$$
\begin{equation*}
\vec{R}=\left[R_{1}+\delta c_{1}\right] \vec{i}_{i}+\left[R_{2}+\delta c_{2}\right] \vec{j}_{i}+\left[R_{3}+\delta c_{3}\right] \vec{k}_{i} \tag{3.8}
\end{equation*}
$$

if we define

$$
\begin{align*}
& \zeta \equiv R_{x j}-R_{x i}+\lambda_{x j} x_{j}+\mu_{x j} y_{j}+\nu_{x j} z_{j}  \tag{3.9}\\
& \eta \equiv R_{y j}-R_{y i}+\lambda_{y j} x_{j}+\mu_{y j} y_{j}+\nu_{y j} z_{j}  \tag{3.10}\\
& \rho \equiv R_{z j}-R_{z i}+\lambda_{z j}+\mu_{z j} y_{j}+\nu_{z j} z_{j} \tag{3.11}
\end{align*}
$$

then the constants $R_{1}, R_{2}, R_{3}, c_{1}, c_{2}, c_{3}$ can be represented as

$$
\left.\begin{array}{l}
\mathrm{R}_{1}=\lambda_{\mathrm{xi}} \zeta+\lambda_{\mathrm{yi}} \eta+\lambda_{\mathrm{zi}} \rho \\
\mathrm{R}_{2}=\mu_{\mathrm{xi}} \zeta+\mu_{\mathrm{yi}} \eta+\mu_{\mathrm{zi}} \rho \\
\mathrm{R}_{3}=\nu_{\mathrm{xi}} \zeta+\nu_{\mathrm{yi}} \eta+\nu_{\mathrm{zi}} \rho
\end{array}\right\}
$$

The solution of the vector equation for the unknown $\delta$ is elementary when the intersecting surface is a flat plate.

If the vector is to intersect a flat surface, then the following equation must be satisfied

$$
R_{3}+\delta c_{3}=\alpha
$$

from which $\delta$ can be solved as

$$
\begin{equation*}
\delta=\frac{\left(\alpha-R_{3}\right)}{c_{3}} \tag{3.15}
\end{equation*}
$$

With this value of $\delta$ the $i$ and $j$ components of $\vec{R}$ can be evaluated and tested to determine if they lie within the boundaries of the plate, i.e., for a rectangular plate

$$
\begin{aligned}
& \gamma_{\min } \leq R_{1}+\delta c_{1} \leq \gamma_{\max } \\
& \beta_{\min } \leq R_{2}+\delta c_{2} \leq \beta_{\min }
\end{aligned}
$$

must be satisfied in order for the surface to intersect vector $\vec{R}$ and cause shading on the element of the surface being tested.

If the intersecting surface is a conic, the solution of the vector equation for the unknown $\delta$ is obtained by substituting the $i, j$ and $k$ components of $\vec{R}$ into the general conic equation

$$
\begin{equation*}
r_{i}=\left(x_{i}^{2}+y_{i}^{2}\right)^{1 / 2}=\left(a+b z_{i}+c z_{i}^{2}\right)^{1 / 2} \tag{3.16}
\end{equation*}
$$

This substitution will yield a quadratic equation in $\delta$ and the solution is given by

$$
\begin{align*}
\delta= & -\left(2 R_{1} c_{1}+2 R_{2} c_{2}-2 \mathrm{cR}_{3} c_{3}-b c_{3}\right) \pm\left\{\left(2 R_{1} c_{1}+2 R_{2} c_{2}-2 \mathrm{cR}_{3} c_{3}-b c_{3}\right)^{2}\right. \\
& \left.-4\left(c_{1}^{2}+c_{2}^{2}-c c_{3}^{2}\right)\left[R_{1}^{2}+R_{2}^{2}-a-\left(b+c R_{3}\right) R_{3}\right)\right\} 1 / 2 \\
& -2\left(c_{1}^{2}+c_{2}^{2}-c c_{3}^{2}\right) \tag{3.17}
\end{align*}
$$

If the values of $\delta$ are real and between zero and one ( $0 \leq \delta \leq 1$ ) there is a possibility that there are two points of intersection. If the roots are real and equal and between the values of zero and one ( $0 \leq \delta \leq 1$ ) there is a possibility of one point of intersection. If the roots are either real and negative or greater than one, or imaginary, or if the denominator

$$
\begin{equation*}
c_{1}^{2}+c_{2}^{2}-c c_{3}^{2}=0 \tag{3.18}
\end{equation*}
$$

no intersection is possible. If the intersection is possible, then the $i, j$ and k components of $\overrightarrow{\mathrm{R}}$ are evaluated with the $\delta$ values obtained and are tested to see whether they lie within the boundaries of the conic surface.

The shading checks discussed in this section are made between elements, for which the view factor is being calculated, to determine if the two elements can see each other. If they are shaded, their contribution to view factor between the two nodes on which they are located is zero. These checks require much computer time, especially where there are many elements. In the program input, there are two shading flags for each surface. Each flag applies to each node and element on that surface. These flags are

KSH - can the surface cause shading on any other surface?
KSBH - can the surface be shaded by any other surface?

These flags, as discussed subsequently in Section 6, can be used to exclude a surface from all checks to determine if the surface can cause shading, or to exclude a surface from the checks to determine if it is shaded, or both. When the user is sure that a surface cannot cause shading or be shaded, these flags should be input accordingly to reduce the required run time.

## Section 4

PR ESHADE

As discussed in Section 3, input flags are used to indicate if a given surface should be checked for possible shading or if the surface can cause shading between other surfaces. Proper use of these flags can reduce the required computer time when several surfaces can be flagged as a "cannot be shaded" or "cannot cause shading" surface. The user will find that most of the actual problems, however, are such that surfaces cannot be classified in either of the two categories mentioned above. Also, unless the user is careful, errors can result when a surface is incorrectly flagged.

Use of the shading flags has one disadvantage in that they apply to all surfaces (i.e., if a surface is flagged as a "no shade" surface then the surface does not cause shading on any other surface in the problem). Usually the user can flag a surface except for one or two surfaces which can cause shading or be shaded by the surface in question. Therefore, the user must decide if he knows enough about the problem to use the shading flags effectively.

Subroutine PRESHD was developed for RAVFAC to reduce the problem of obtaining valid view factors, using only a minimum amount of computer time. The use of this subroutine does not require extra effort by the user. The use of the shading flags is still available and should always be used for those surfaces which meet the requirements.

Before preshade is discussed, a review of the shading checks that were discussed in Section 3 would be helpful. As the view factor between elements is being calculated, a group of checks are made to determine if any other surface intersects a line joining the center of areas of the two elements. The first check is to determine if the elements could "see" each other if no other
surface caused shading. Next the shading flags are checked to determine if shading checks should be made. Shading checks are made between the two elements to determine if any of the surfaces cause shading. If no surface causes shading, the view factor between elements is calculated. If shading is detected, the view factor between elements is set equal to zero. These shading checks must be made many times as shown below:

$$
\text { No. of checks } \approx \frac{(\text { No. of Elements })^{2}}{2} \times \text { (No. of Surfaces). }
$$

The function of preshade is to eliminate from the shading checks all surfaces that cannot cause shading on any portion of the two nodes for which the view factor is being calculated. Several advantages to this approach which result in a reduction of computer time, are:

- Since preshade is used on a node-to-node basis it need not consider the problem as a whole, as does the user when he selects the shading flags.
- Those surfaces that preshade determines to be outside the shading region of the two nodes in question need not be checked to determine if they can cause shading between the elements on those nodes.
- Use of preshade reduces the burden on the user to try to determine those surfaces he should flag as "cannot shade" or "cannot be shaded." Although preshade will set the proper flags, computer time is needed to make the necessary checks. Therefore, the use of shading flags should not be abandoned since they result in the largest reduction of computer time.

In the following discussion of preshade the reader should keep in mind that if:

- shading is not considered, or
- preshade is to be by passed, or
- the node, for which the view factor is being calculated, is flagged as "cannot be shaded," or
- if the surface to be checked for causing shading is flagged as a "cannot cause shading" surface
then the preshade checks are not made.

The preshade routine consists of constructing a sphere around each of the two nodes for which the view factor is being calculated. The radii of the spheres enclose cach node. A vector, $\vec{P}_{n}$, is constructed between the origin of the surface coordinate system and the center of each sphere. A test cylinder is then constructed, with a radius equal to the larger radii of the two spheres, and its axial coordinate along the vector $\overrightarrow{\mathrm{P}}_{\mathrm{nj}}$ joining the centers of the spheres. The length of the test cylinder has a magnitude equal to the vector $\vec{P}_{n j}$ plus the magnitude radii of the spheres. This test cylinder has its coordinate system defined such that the origin is at the $n$ node end of the test cylinder. The $n$ node end of the test cylinder is set, at its origin, perpendicular to the axial direction (in the $X_{t}-Y_{t}$ plane). The $j$ node end of the test cylinder lies perpendicular to the axial coordinate at the end of the length of the cylinder. The test cylinder is shown in Fig. 9. Next, a series of spheres is constructed around the surfaces that have been flagged as "can shade," with a radius that encloses the whole surface. The spheres of the surfaces are transformed to the cylinder coordinate system and are tested to determine whether or not they intersect any part of the test cylinder (as shown in Fig. 9). In the example shown in Fig. 9, preshade would eliminate surface $i_{2}$ from all shading ckecks between the elements on nodes $n$ and $j$.

The radius of the test cylinder and the radii of the spheres depend on the types of surfaces that they enclose. The radii of the spheres that enclose the nodes are given according to the type of surface:

1. Rectangular Plate

$$
\begin{equation*}
r_{n}=\frac{\left(\Delta \beta_{i}^{2}+\Delta \gamma_{i}^{2}\right)^{1 / 2}}{2} \tag{4.1}
\end{equation*}
$$

2. Circular Plate

$$
\left.\begin{array}{ll}
r_{n}=\left(\beta_{i}+\frac{\Delta \beta_{i}}{2}\right) \sin \left(\frac{\Delta \gamma_{i}}{2}\right) & \Delta \gamma_{i} \leq 180 \mathrm{deg} \\
r_{n}=\beta_{i}+\frac{\Delta \beta_{i}}{2} & \Delta \gamma_{i}>180 \mathrm{deg} \tag{4.2}
\end{array}\right\}
$$


Fig. 9-Preshade Test Cylinder
3. Trapezoidal Plate

$$
\begin{equation*}
r_{n}=\max \left\|r_{n_{1}}, r_{n_{2}}, r_{n_{3}}, r_{n_{4}}\right\| \tag{4.3}
\end{equation*}
$$

where

$$
r_{n_{k}}=\left[\left(x_{n_{k}}-p_{n_{x}}\right)^{2}+\left(y_{n_{k}}-p_{n_{y}}\right)^{2}\right]^{1 / 2}
$$

for $k=1,2,3$ and 4.
and where

$$
\left.\begin{array}{rl}
x_{n_{1,2}} & =y_{n_{1,2}} \tan \left(\gamma_{i}-\frac{\Delta \gamma_{i}}{2}\right) \\
x_{n_{3,4}} & =y_{n_{3,4}} \tan \left(\gamma_{i}+\frac{\Delta \gamma_{i}}{2}\right) \\
y_{n_{1,4}} & =\beta_{i}-\frac{\Delta \beta_{i}}{2} \\
y_{n_{2,3}} & =\beta_{i}+\frac{\Delta \beta_{i}}{2} \\
p_{n_{x}}=\frac{1}{3}\left(\frac{3}{2} \beta_{i}+\frac{\Delta \beta_{i}}{8 \beta_{i}}\right)\left[\tan \left(\gamma_{i}+\frac{\Delta \gamma_{i}}{2}\right)+\tan \left(\gamma_{i}-\frac{\Delta \gamma_{i}}{2}\right)\right] \\
p_{n_{Y}} & =\frac{2}{3}\left(\frac{3}{2} \beta_{i}+\frac{\Delta \beta_{i}^{2}}{8 \beta_{i}}\right) \tag{4.6}
\end{array}\right\}
$$

4. Cylinder

$$
\begin{array}{ll}
r_{n}=\left\{\left[2 \alpha \sin \left(\frac{\Delta \gamma_{i}}{2}\right)\right]^{2}+\Delta \beta_{i}^{2}\right\}^{1 / 2} \div 2 & \Delta \gamma_{i} \leq 180 \mathrm{deg}  \tag{4.7}\\
r_{n}=\left\{(2 \alpha)^{2}+\Delta \beta_{i}^{2}\right\}^{1 / 2} \div 2 & \Delta \gamma_{i}>180 \mathrm{deg}
\end{array}
$$

5. Cone or Cone Frustum

$$
\begin{equation*}
r_{n}=\left\{\frac{\beta_{i}}{\cos ^{2}(\alpha)}\left[\frac{\beta_{i}}{\cos ^{2}(\alpha)}-2\left(\beta_{i}-\frac{\Delta \beta_{i}}{2}\right)\right]+\left(\frac{\beta_{i}-\frac{\Delta \beta_{i}}{2}}{\cos (\alpha)}\right)^{2}\right\}^{1 / 2} \tag{4.8}
\end{equation*}
$$

6. Sphere

$$
\left.\begin{array}{l}
r_{n}=\left\{\left[2 \alpha \sin \left(\frac{\Delta \beta_{i}}{2}\right)\right]^{2}+\left[2 \alpha \sin \left(\frac{\Delta \gamma_{i}}{2}\right)\right]^{2}\right\}^{1 / 2} \div 2 \Delta \gamma_{i} \leq 180 \mathrm{deg}  \tag{4.9}\\
r_{n}=\alpha \quad \Delta \gamma_{i}>180 \mathrm{deg}
\end{array}\right\}
$$

## 7. Circular Paraboloid

$$
\begin{align*}
& r_{n}=\left\{p_{n_{z}}^{2}+4 \alpha\left(\beta_{i}-\frac{\Delta \beta_{i}}{2}\right)+\left(\beta_{i}-\frac{\Delta \beta_{i}}{2}\right)^{2}-2 p_{n_{z}}\right. \\
& {\left[4 \alpha\left(\beta_{i}-\frac{\Delta \beta_{i}}{2}\right)+\left(\beta_{i}-\frac{\Delta \beta_{i}}{2}\right)^{2}\right]^{1 / 2}\left[\frac{\Delta \beta_{i}}{\left.\left\{\Delta \beta_{i}^{2}+4 \alpha\left[\left(\beta_{i}+\frac{\Delta \beta_{i}}{2}\right)^{1 / 2}-\left(\beta_{i}-\frac{\Delta \beta_{i}}{2}\right)^{1 / 2}\right]^{2}\right\}^{1 / 2}\right]^{1 / 2}}\right\}^{14}} \tag{4.10}
\end{align*}
$$

where

$$
\begin{equation*}
p_{n_{z}}=\frac{\left(2 \alpha+\beta_{i}\right)\left\{\Delta \beta_{i}^{2}+4 \alpha\left[\left(\beta_{i}+\frac{\Delta \beta_{i}}{2}\right)^{1 / 2}-\left(\beta_{i}-\frac{\Delta \beta_{i}}{2}\right)^{1 / 2}\right]^{2}\right\}^{1 / 2}}{\left[4 \alpha\left(\beta_{i}+\frac{\Delta \beta_{i}}{2}\right)+\left(\beta_{i}+\frac{\Delta \beta_{i}}{2}\right)^{2}\right]^{1 / 2}-\left[4 \alpha\left(\beta_{i}-\frac{\Delta \beta_{i}}{2}\right)+\left(\beta_{i}-\frac{\Delta \beta_{i}}{2}\right)^{2}\right]^{1 / 2}} \tag{4.11}
\end{equation*}
$$

Vector $\vec{P}_{n}$, from the origin of the surface coordinate system to the center of the node, is defined by

$$
\begin{equation*}
\overrightarrow{\mathrm{p}}_{\mathrm{n}} \equiv \mathrm{p}_{\mathrm{n}_{\mathrm{x}}} \overrightarrow{\mathrm{i}}_{\mathrm{i}}+\mathrm{p}_{\mathrm{n}} \overrightarrow{\mathrm{j}}_{\mathrm{i}}+\mathrm{p}_{\mathrm{n}} \overrightarrow{\mathrm{k}}_{\mathrm{i}} \tag{4.12}
\end{equation*}
$$

where the vector components $p_{n_{x}}, p_{n_{y}}$, and $p_{n_{z}}$ are given according to the
type of surface:

1. Rectangular Plate

$$
\begin{align*}
& p_{n_{x}}=\gamma_{i}  \tag{4.13}\\
& p_{n_{y}}=\beta_{i}  \tag{4.14}\\
& p_{n_{z}}=\alpha_{i} \tag{4.15}
\end{align*}
$$

2. Circular Plate

$$
\left.\begin{array}{ll}
\mathrm{p}_{\mathrm{n}_{\mathrm{x}}}=\left(\beta_{\mathrm{i}}+\frac{\Delta \beta_{\mathrm{i}}}{2}\right) \cos \left(\frac{\Delta \gamma_{\mathrm{i}}}{2}\right) \sin \left(\gamma_{\mathrm{i}}\right) & \Delta \gamma_{\mathrm{i}} \leq 180 \mathrm{deg} \\
\mathrm{p}_{\mathrm{n}_{\mathrm{x}}}=0 & \Delta \gamma_{\mathrm{i}}>180 \mathrm{deg}
\end{array}\right\}
$$

3. Trapezoidal Plate

$$
\begin{align*}
& \mathrm{p}_{\mathrm{n}_{\mathrm{x}}}=\frac{1}{3}\left(\frac{3}{2} \beta_{\mathrm{i}}+\frac{\Delta \beta_{\mathrm{i}}^{2}}{8 \beta_{\mathrm{i}}}\right)\left[\tan \left(\gamma_{\mathrm{i}}+\frac{\Delta \gamma_{\mathrm{i}}}{2}\right)+\tan \left(\gamma_{\mathrm{i}}-\frac{\Delta \gamma_{\mathrm{i}}}{2}\right)\right]  \tag{4.19}\\
& \mathrm{p}_{\mathrm{n}_{\mathrm{y}}}=\frac{2}{3}\left(\frac{3}{2} \beta_{\mathrm{i}}+\frac{\Delta \beta_{i}^{2}}{8 \beta_{\mathrm{i}}}\right)  \tag{4.20}\\
& \mathrm{p}_{n_{z}}=\alpha \tag{4.21}
\end{align*}
$$

4. Cylinder

$$
\left.\begin{array}{ll}
p_{n_{x}}=\alpha \cos \left(\frac{\Delta \gamma_{i}}{2}\right) \sin \left(\gamma_{i}\right) & \Delta \gamma_{i} \leq 180 \mathrm{deg} \\
p_{n_{x}}=0 & \Delta \gamma_{i}>180 \mathrm{deg}
\end{array}\right\} .
$$

5. Cone or Cone Frustum

$$
\left.\begin{array}{ll}
p_{n_{x}}=\left(\beta_{i}+\frac{\Delta \beta_{i}}{2}\right) \tan (\alpha) \cos \left(\frac{\Delta \gamma_{i}}{2}\right) \sin \left(\gamma_{i}\right) & \Delta \gamma_{i} \leq 180 \mathrm{deg} \\
p_{n_{x}}=0 & \Delta \gamma_{i}>180 \mathrm{deg}
\end{array}\right\}
$$

6. Sphere

$$
\left.\begin{array}{ll}
\mathrm{p}_{\mathrm{n}_{\mathrm{x}}}=\alpha \sin \left(\beta_{\mathrm{i}}\right) \sin \left(\gamma_{\mathrm{i}}\right) & \Delta \gamma_{\mathrm{i}} \leq 180 \mathrm{deg} \\
\mathrm{p}_{\mathrm{n}_{\mathrm{x}}}=0 & \Delta \gamma_{\mathrm{i}}>180 \mathrm{deg} \tag{4.28}
\end{array}\right\}
$$

$$
\left.\begin{array}{ll}
\mathrm{p}_{\mathrm{n}_{\mathrm{y}}}=\alpha \sin \left(\beta_{\mathrm{i}}\right) \cos \left(\gamma_{\mathrm{i}}\right) & \Delta \gamma_{\mathrm{i}} \leq 180 \mathrm{deg} \\
\mathrm{p}_{\mathrm{n}_{\mathrm{y}}}=0 & \Delta \gamma_{\mathrm{i}}>180 \mathrm{deg}
\end{array}\right\}
$$

## 7. Circular Paraboloid

$$
\begin{align*}
& p_{n_{x}}=2\left[\left(\beta_{i}+\frac{\Delta \beta_{i}}{2}\right) \alpha\right]^{1 / 2} \cos \left(\frac{\Delta \gamma_{i}}{2}\right) \sin \left(\gamma_{i}\right)  \tag{4.31}\\
& p_{n_{x}}=0  \tag{4.32}\\
& p_{n_{y}}=2\left[\left(\beta_{i}+\frac{\Delta \beta_{i}}{2}\right) \alpha\right]^{1 / 2} \cos \left(\frac{\Delta \gamma_{i}}{2}\right) \cos \left(\gamma_{i}\right)  \tag{4.33}\\
& p_{n_{y}}=0 \\
& p_{n_{z}}=\frac{\Delta \gamma_{i}>180 \mathrm{deg}}{}=\frac{\Delta \gamma_{i} \leq 180 \mathrm{deg}}{\left(2 \alpha+\beta_{i}\right)}\left\{\Delta \beta_{i}^{2}+4 \alpha\left[\left(\beta_{i}+\frac{\Delta \beta_{i}}{2}\right)^{1 / 2}-\left(\beta_{i}-\frac{\Delta \beta_{i}}{2}\right)^{1 / 2}\right]^{2}\right\}^{1 / 2} \\
& {\left[4 \alpha\left(\beta_{i}+\frac{\Delta \beta_{i}}{2}\right)+\left(\beta_{i}+\frac{\Delta \beta_{i}}{2}\right)^{2}\right]^{1 / 2}-\left[4 \alpha\left(\beta_{i}-\frac{\Delta \beta_{i}}{2}\right)+\left(\beta_{i}-\frac{\Delta \beta_{i}}{2}\right)^{2}\right]^{1 / 2}}
\end{align*}
$$

Radius $r_{b}$ of the sphere that incloses the whole surface is also given according to the type of surface:

1. Rectangular Plate

$$
\begin{equation*}
r_{b}=\left[\left(\beta_{\max }-\beta_{\min }\right)^{2}+\left(\gamma_{\max }-\gamma_{\min }\right)^{2}\right]^{1 / 2} \div 2 \tag{4.34}
\end{equation*}
$$

2. Circular Plate

$$
\left.\begin{array}{ll}
r_{b}=\beta_{\max } \sin \left(\frac{\gamma_{\max }-\gamma_{\min }}{2}\right) & \gamma_{\max }-\gamma_{\min } \leq 180 \mathrm{deg} \\
\mathrm{r}_{\mathrm{b}}=\beta_{\max } & \gamma_{\max }-\gamma_{\min }>180 \mathrm{deg}
\end{array}\right\}
$$

3. Trapezoidal Plate

$$
\begin{equation*}
r_{b}=\max \left\|r_{b_{1}}, r_{b_{2}}, r_{b_{3}}, r_{b_{4}}\right\| \tag{4.36}
\end{equation*}
$$

where

$$
\begin{equation*}
r_{b_{k}}=\left[\left(x_{b_{k}}-p_{b_{x}}\right)^{2}+\left(y_{b_{k}}-p_{b_{y}}\right)^{2}\right]^{1 / 2} \tag{4.37}
\end{equation*}
$$

as $k=1,2,3,4$
and

$$
\left.\begin{array}{c}
x_{b_{1}}=y_{b_{1}} \tan \left(\gamma_{\min }\right) \\
x_{b_{2}}=y_{b_{2}} \tan \left(\gamma_{\min }\right) \\
x_{b_{3}}=y_{b_{3}} \tan \left(\gamma_{\max }\right) \\
x_{b_{4}}=y_{b_{4}} \tan \left(\gamma_{\max }\right) \tag{4.39}
\end{array}\right\}
$$

$$
\begin{gathered}
\mathrm{p}_{\mathrm{b}}=\frac{1}{3}\left(\frac{\beta_{\max }^{3}-\beta_{\min }{ }^{3}}{\beta_{\max }^{2}-\beta_{\min }^{2}}\right)\left[\tan \left(\gamma_{\min }\right)+\tan \left(\gamma_{\max }\right)\right] \\
\mathrm{p}_{\mathrm{b}_{\mathrm{Y}}}=\frac{2}{3}\left(\frac{\beta_{\max }{ }^{3}-\beta_{\min }{ }^{3}}{\beta_{\max }^{2}-\beta_{\min }^{2}}\right)
\end{gathered}
$$

4. Cylinder

$$
\left.\begin{array}{l}
r_{b}=\frac{\left\{\left[2 \alpha \sin \left(\frac{\gamma_{\max }-\gamma_{\min }}{2}\right)\right]^{2}+\left[\beta_{\max }-\beta_{\min }\right]^{2}\right\}^{1 / 2}}{2} \gamma_{\max }-\gamma_{\min } \leq 180 \mathrm{deg}  \tag{4.41}\\
r_{b}=\frac{\left\{[2 \alpha]^{2}+\left[\beta_{\max }-\beta_{\min }\right]^{2}\right\}^{1 / 2}}{2} \\
\gamma_{\max }-\gamma_{\min }>180 \mathrm{deg}
\end{array}\right\}
$$

5. Cone or Cone Frustum

$$
\begin{equation*}
r_{b}=\left\{\frac{\beta_{\max }+\beta_{\min }}{2 \cos ^{2}(\alpha)}\left[\frac{\beta_{\text {max }}+\beta_{\text {min }}}{2 \cos ^{2}(\alpha)}-\frac{2 \beta_{\min }}{\cos (\alpha)}\right]+\left[\frac{\beta_{\text {min }}}{\cos (\alpha)}\right]^{2}\right\}^{1 / 2} \tag{4.42}
\end{equation*}
$$

6. Sphere

$$
\begin{align*}
& r_{b}=\frac{\left\{\left[2 \alpha \sin \left(\frac{\beta_{\max }-\beta_{\min }}{2}\right)\right]^{2}+\left[2 \alpha \sin \left(\frac{\left.\gamma_{\max }-\gamma_{\min }\right)}{2}\right)\right\}^{2}\right\}^{1 / 2}}{2} \gamma_{\max }-\gamma_{\min } \leq 180 \mathrm{deg}  \tag{4.43}\\
& r_{b}=\alpha
\end{align*}
$$

7. Circular Paraboloid

$$
\begin{gather*}
r_{b}=\left[p_{b_{z}\left\{p_{b}-\frac{\left[\beta_{\min }\left(4 \alpha+\beta_{\min }\right)\right]^{1 / 2}\left[\beta_{\max }-\beta_{\min }\right]}{\left\{\left[\beta_{\max }-\beta_{\min }\right]^{2}+4 \alpha\left[\left(\beta_{\max }\right)^{1 / 2}-\left(\beta_{\min }\right)^{1 / 2}\right]^{2}\right\}^{1 / 2}}\right\}} \begin{array}{c}
\left.\beta_{\min }\left(4 \alpha+\beta_{\min }\right)\right]^{1 / 2}
\end{array},\right.
\end{gather*}
$$

$$
\begin{equation*}
p_{b_{z}}=\frac{\left(4 \alpha+\beta_{\max }+\beta_{\min }\right)\left\{\left(\beta_{\max }-\beta_{\min }\right)^{2}+4 \alpha\left[\left(\beta_{\max }\right)^{1 / 2}-\left(\beta_{\min }\right)^{1 / 2}\right]^{2}\right\}^{1 / 2}}{2\left\{\left[4 \alpha \beta_{\max }\left(1+\beta_{\max }\right]^{1 / 2}-\left[\beta_{\min }\left(4 \alpha+\beta_{\min }\right)\right]^{1 / 2}\right\}\right.} \tag{4.45}
\end{equation*}
$$

Vector $\vec{P}_{b}$, from the origin of the surface coordinate system to the center of the sphere of the surface, is defined by

$$
\begin{equation*}
\overrightarrow{\mathrm{P}}_{\mathrm{b}} \equiv \mathrm{p}_{\mathrm{b}_{\mathrm{x}}} \overrightarrow{\mathrm{i}}_{\mathrm{i}}+\mathrm{p}_{\mathrm{b}} \overrightarrow{\mathrm{j}}_{\mathrm{i}}+\mathrm{p}_{\mathrm{b}_{\mathrm{z}}} \stackrel{\rightharpoonup}{k}_{\mathrm{i}} \tag{4.46}
\end{equation*}
$$

Where the vector components $\mathrm{p}_{\mathrm{b}_{\mathrm{x}}}, \mathrm{p}_{\mathrm{b}_{\mathrm{y}}}$ and $\mathrm{p}_{\mathrm{b}_{\mathrm{z}}}$ are given according to the type of surface:

1. Rectangular Plate

$$
\begin{align*}
& \mathrm{p}_{\mathrm{b}_{\mathrm{x}}}=\frac{\gamma_{\min }+\gamma_{\max }}{2}  \tag{4.47}\\
& \mathrm{p}_{\mathrm{b}_{\mathrm{y}}}=\frac{\beta_{\min }+\beta_{\max }}{2}  \tag{4.48}\\
& \mathrm{p}_{\mathrm{b}_{\mathrm{z}}}=\alpha \tag{4.49}
\end{align*}
$$

2. Circular Plate

$$
\left.\begin{array}{ll}
\mathrm{p}_{\mathrm{b}_{\mathrm{x}}}=\beta_{\max } \cos \left(\frac{\gamma_{\max }-\gamma_{\min }}{2}\right) \sin \left(\frac{\gamma_{\min }+\gamma_{\max }}{2}\right) & \gamma_{\max }-\gamma_{\min } \leq 180 \mathrm{deg} \\
\mathrm{p}_{\mathrm{b}_{\mathrm{x}}}=0 & \gamma_{\max }-\gamma_{\min }>180 \mathrm{deg} \tag{4.50}
\end{array}\right\}
$$

$$
\left.\mathrm{p}_{\mathrm{b}}=\beta_{\max } \cos \left(\frac{\gamma_{\max }-\gamma_{\min }}{2}\right) \cos \left(\frac{\gamma_{\min }+\gamma_{\max }}{2}\right) \gamma_{\max }-\gamma_{\min } \leq 180 \mathrm{deg}\right)
$$

$$
\begin{equation*}
\left.\mathrm{p}_{\mathrm{b}}=0 \quad \quad \gamma_{\max }-\gamma_{\min }>180 \mathrm{deg}\right\} \tag{4.51}
\end{equation*}
$$

$$
\begin{equation*}
p_{b_{z}}=\alpha \tag{4.52}
\end{equation*}
$$

3. Trapezoidal Plate

$$
\begin{align*}
& \mathrm{p}_{\mathrm{b}_{\mathrm{x}}}=\frac{1}{3}\left(\frac{\beta_{\max }{ }^{3}-\beta_{\min }{ }^{3}}{\beta_{\max }{ }^{2}-\beta_{\min }{ }^{2}}\right)\left[\tan \left(\gamma_{\min }\right)+\tan \left(\gamma_{\max }\right)\right]  \tag{4.53}\\
& \mathrm{p}_{\mathrm{b}_{\mathrm{Y}}}=\frac{2}{3}\left(\frac{\beta_{\max }{ }^{3}-\beta_{\min }{ }^{3}}{\beta_{\max }{ }^{2}-\beta_{\min }{ }^{2}}\right)  \tag{4.54}\\
& \mathrm{p}_{\mathrm{b}_{\mathrm{z}}}=\alpha \tag{4.55}
\end{align*}
$$

4. Cylinder

$$
\left.\begin{array}{ll}
\mathrm{p}_{\mathrm{b}_{\mathrm{x}}}=\alpha \cos \left(\frac{\gamma_{\max }-\gamma_{\min }}{2}\right) \sin \left(\frac{\gamma_{\min }+\gamma_{\max }}{2}\right) & \gamma_{\max }-\gamma_{\min } \leq 180 \mathrm{deg} \\
\mathrm{p}_{\mathrm{b}_{\mathrm{x}}}=0 & \gamma_{\max }-\gamma_{\min }>180 \mathrm{deg}
\end{array}\right\}
$$

## 5. Cone or Cone Frustum

$$
\left.\begin{array}{l}
p_{b_{x}}=\beta_{\max } \tan (\alpha) \cos \left(\frac{\gamma_{\max }-\gamma_{\min }}{2}\right) \sin \left(\frac{\gamma_{\min }+\gamma_{\max }}{2}\right) \\
p_{b_{x}}=0 \\
\gamma_{\max }-\gamma_{\min } \leq 180 \mathrm{deg}  \tag{4.60}\\
p_{b_{y}}=\beta_{\max }-\gamma_{\min }>180 \mathrm{deg}
\end{array}\right\}(
$$

$$
\begin{align*}
& p_{b_{y}}=0  \tag{4.61}\\
& p_{b_{z}}=\frac{\beta_{\min }+\beta_{\max }}{2 \cos ^{2}(\alpha)}
\end{align*}
$$

## 6. Sphere

$$
\left.\begin{array}{ll}
\mathrm{p}_{\mathrm{b}_{\mathrm{x}}}=\alpha \sin \left(\frac{\beta_{\min }+\beta_{\max }}{2}\right) \sin \left(\frac{\gamma_{\min }+\gamma_{\max }}{2}\right) & \gamma_{\max }-\gamma_{\min } \leq 180 \mathrm{deg} \\
\mathrm{p}_{\mathrm{b}_{\mathrm{x}}}=0 & \gamma_{\max }-\gamma_{\min }>180 \mathrm{deg}
\end{array}\right\}
$$

## 7. Circular Paraboloid

$$
\begin{align*}
& \left.\mathrm{p}_{\mathrm{b}_{\mathrm{x}}}=2\left(\alpha \beta_{\max }\right)^{1 / 2} \cos \left(\frac{\gamma_{\max }-\gamma_{\min }}{2}\right) \sin \left(\frac{\gamma_{\min }+\gamma_{\max }}{2}\right) \gamma_{\max }-\gamma_{\min } \leq 180 \mathrm{deg}\right)  \tag{4.65}\\
& \mathrm{p}_{\mathrm{b}_{\mathrm{x}}}=0 \quad \gamma_{\max }-\gamma_{\min }>180 \mathrm{deg} \\
& \left.\begin{array}{ll}
p_{b}=2\left(\alpha \beta_{\max }\right)^{1 / 2} \cos \left(\frac{\gamma_{\max }-\gamma_{\min }}{2}\right) \cos \left(\frac{\gamma_{\min }+\gamma_{\max }}{2}\right) & \gamma_{\max }-\gamma_{\min } \leq 180 \mathrm{deg} \\
p_{b_{y}}=0 & \gamma_{\max }-\gamma_{\min }>180 \mathrm{deg}
\end{array}\right\}  \tag{4.66}\\
& p_{b_{z}}=\frac{\left(4 \alpha+\beta_{\min }+\beta_{\max }\right)\left\{\left(\beta_{\max }-\beta_{\min }\right)+4 \alpha\left[\left(\beta_{\max }\right)^{1 / 2}-\left(\beta_{\min }\right)^{1 / 2}\right]^{2}\right\}}{2\left\{\left[4 \alpha \beta_{\max }\left(1+\beta_{\max }\right)\right]^{1 / 2}-\left[\beta_{\min }\left(4 \alpha+\beta_{\min }\right)\right]^{1 / 2}\right\}} \tag{4.67}
\end{align*}
$$

The vector to the center nodes is transformed to the central coordinate system by the following relation

$$
\left[\begin{array}{c}
p_{n x_{j}}  \tag{4.68}\\
p_{n y_{j}} \\
p_{n z}
\end{array}\right]_{c}=\left[\begin{array}{l}
T_{i 2 c}
\end{array}\right]\left[\begin{array}{c}
p_{n x_{j}} \\
p_{n y_{j}} \\
p_{n z_{j}}
\end{array}\right]_{i}+\left[\begin{array}{l}
R_{x_{j}} \\
R_{y_{j}} \\
R_{z_{j}}
\end{array}\right]_{c}
$$

The vector joining the two center nodes in the central coordinate system is then given by the difference between the two

$$
\vec{P}_{j 2 n} \equiv\left[\begin{array}{c}
p_{j n_{x}}  \tag{4.69}\\
p_{j n_{y}} \\
p_{j n_{z}}
\end{array}\right]_{c}=\left[\begin{array}{l}
p_{n x_{n}} \\
p_{n y_{n}} \\
p_{n z_{n}}
\end{array}\right]_{c}-\left[\begin{array}{c}
p_{n x_{j}} \\
p_{n y_{j}} \\
p_{n x_{j}}
\end{array}\right]_{c}
$$

and the unit vector between the $j^{\text {th }}$ and $n^{\text {th }}$ node is given by

$$
\vec{U}_{j 2 n} \equiv\left[\begin{array}{c}
u_{j n_{x}}  \tag{4.70}\\
u_{j n_{y}} \\
u_{j n_{z}}
\end{array}\right]_{c}=\left[\begin{array}{c}
p_{j n_{x}} /\left|\vec{P}_{j 2 n}\right| \\
p_{j n_{y}} /\left|\vec{P}_{j 2 n}\right| \\
p_{j n_{z}} /\left|\vec{P}_{j 2 n}\right|
\end{array}\right]
$$

where

$$
\begin{equation*}
\left|\vec{P}_{j 2 n}\right|=\left[\left(p_{j n_{x}}\right)^{2}+\left(p_{j n_{y}}\right)^{2}+\left(p_{j n_{z}}\right)^{2}\right]^{1 / 2} \tag{4.71}
\end{equation*}
$$

Since the above unit vector represents the axial direction of the test cylinder, the transformation matrices between the test cylinder coordinate system and the central coordinate system is given by the following direction cosines. ${ }^{*}$.

[^0]
\[

\left.$$
\begin{array}{l}
\lambda_{x t}=\left(1-u_{j n_{x}}^{2}\right)^{1 / 2} \\
\lambda_{y t}=-u_{j n_{x}} u_{j n_{y}} \div\left(1-u_{j n_{x}}^{2}\right)^{1 / 2} \\
\lambda_{z t}=-u_{j n_{x}} u_{j n_{z}} \div\left(1-u_{j n_{x}}^{2}\right)^{1 / 2} \\
\mu_{x t}=0  \tag{4.74}\\
\mu_{y t}=-u_{j n_{z}} \div\left(1-u_{j n_{x}}^{2}\right)^{1 / 2} \\
\mu_{z t}=u_{j n_{y}} \div\left(1-u_{j n_{x}}^{2}\right)^{1 / 2} \\
\nu_{x t}=-u_{j n_{x}} \\
\nu_{y t}=-u_{j n_{y}} \\
\nu_{z t}=-\dot{u}_{j n_{z}}
\end{array}
$$\right\}
\]

All the above direction cosines are valid if $u_{j n_{x}} \neq 1$.
If $u_{j n_{x}} \cong 1$ then the direction cosines are given by

$$
\left.\begin{array}{l}
\begin{array}{l}
\lambda_{x t}=u_{j n_{y}} \div\left(1-u_{j n_{z}}^{2}\right)^{1 / 2} \\
\lambda_{y t}=-u_{j n_{x}} \div\left(1-u_{j n_{z}}^{2}\right)^{1 / 2} \\
\lambda_{z t}=0
\end{array} \\
\begin{array}{l}
\mu_{x t}=0 \\
\mu_{y t}=0
\end{array} \\
\begin{array}{ll}
\mu_{z t}=1 & u_{j n_{z}} \geq 0 \\
\mu_{z t}=-1 & u_{j n_{z}}<0
\end{array}
\end{array}\right\}
$$

$$
\left.\begin{array}{l}
\nu_{x t}=-u_{j n_{x}} \div\left(1-u_{j n_{z}}^{2}\right)^{1 / 2} \\
\nu_{x t}=u_{j n_{x}} \div\left(1-u_{j n_{z}}^{2}\right)^{1 / 2}  \tag{4.77}\\
\nu_{y t}=-u_{j n_{y}} \div\left(1-u_{j n_{z}}^{2}\right)^{1 / 2} \\
\nu_{y t}=u_{j n_{y}} \div\left(1-u_{j n_{z}}^{2}\right)^{1 / 2} \\
\nu_{z t}=0
\end{array}\right\}
$$

Vector $\overrightarrow{\mathrm{P}}_{\mathrm{b}}$, defined previously in this section, is transformed to the central system by the following transformation

$$
\vec{P}_{b} \equiv\left[\begin{array}{l}
p_{b_{x}}  \tag{4.78}\\
p_{b_{y}} \\
p_{b_{z}}
\end{array}\right]_{c}=\left[T_{i 2 c}\right]\left[\begin{array}{l}
p_{b_{x}} \\
p_{b_{y}} \\
p_{b_{z}}
\end{array}\right]_{i}+\left[\begin{array}{l}
R_{x_{i}} \\
R_{y_{i}} \\
R_{z_{i}}
\end{array}\right]_{c}
$$

The radius of the first node ( ${ }^{\text {th }}$ node) is transformed also into a vector in the central coordinate system that runs along the axis of the test cylinder by the following operation

$$
\left[\begin{array}{c}
r_{n_{x}}  \tag{4.79}\\
r_{n} \\
r_{n_{z}}
\end{array}\right]_{c}=r_{n}\left[\begin{array}{c}
u_{j n_{x}} \\
u_{j n} \\
u_{j n_{z}}
\end{array}\right]_{c}
$$

A vector from the origin of the test cylinder copordinate system to the center of the sphere constructed around a "can shade" surface is given, in the central coordinate system, by

$$
\left[\begin{array}{c}
x_{0}  \tag{4.80}\\
y_{o} \\
z_{o}
\end{array}\right]_{c}=\left[\begin{array}{c}
p_{b_{x}} \\
p_{b_{y}} \\
p_{b_{z}}
\end{array}\right]_{c}-\left[\begin{array}{c}
p_{n x_{n}} \\
p_{n y} \\
p_{n z_{n}}
\end{array}\right]_{c}-\left[\begin{array}{c}
r_{n_{x}} \\
r_{n_{y}} \\
r_{n_{z}}
\end{array}\right]_{c}
$$

The two extremes of the sphere in the axial direction of the test cylinder system can be given by

$$
\left[\begin{array}{c}
\mathrm{x}_{\mathrm{o}}^{\prime}  \tag{4.81}\\
\mathrm{y}_{\mathrm{o}}^{\prime} \\
\mathrm{z}_{\mathrm{o}}^{\prime}
\end{array}\right]_{\mathrm{c}}=\left[\begin{array}{c}
\mathrm{x}_{\mathrm{o}} \\
\mathrm{y}_{\mathrm{o}} \\
\mathrm{z}_{\mathrm{o}}
\end{array}\right]_{\mathrm{c}}+\left[\begin{array}{c}
\mathrm{r}_{\mathrm{b}_{\mathrm{x}}} \\
\mathrm{r}_{\mathrm{b}_{\mathrm{y}}} \\
\mathrm{r}_{\mathrm{b}_{\mathrm{z}}}
\end{array}\right]_{\mathrm{c}}
$$

and

$$
\left[\begin{array}{c}
\mathrm{x}_{\mathrm{o}}^{\prime \prime}  \tag{4.82}\\
\mathrm{y}_{\mathrm{o}}^{\prime \prime} \\
\mathrm{z}_{\mathrm{o}}^{\prime \prime}
\end{array}\right]_{\mathrm{c}}=\left[\begin{array}{c}
\mathrm{x}_{\mathrm{o}} \\
\mathrm{y}_{\mathrm{o}} \\
\mathrm{z}_{\mathrm{o}}
\end{array}\right]_{\mathrm{c}}-\left[\begin{array}{c}
\mathrm{r}_{\mathrm{b}_{\mathrm{x}}} \\
\mathrm{r}_{\mathrm{b}_{\mathrm{y}}} \\
\mathrm{r}_{\mathrm{b}_{\mathrm{z}}}
\end{array}\right]_{\mathrm{c}}
$$

where

$$
\left[\begin{array}{c}
r_{b_{x}}  \tag{4.83}\\
r_{b} \\
r_{y} \\
b_{z}
\end{array}\right]_{c}=r_{b}\left[\begin{array}{c}
u_{j n_{x}} \\
u_{j n} \\
u_{j n_{z}}
\end{array}\right]_{c}
$$

Since the extreme points are given in the central coordinate system, a trans formation from the central coordinate system to the test cylinder coordinate system must be performed in order to test for the intersection of the sphere with the test cylinder. The axial components of the two extreme points on the sphere are given, in the test cylinder coordinate system, by

$$
\begin{align*}
& z_{t}^{\prime}=\nu_{x t} x_{o}^{\prime}+\nu_{y t} y_{o}^{\prime}+\nu_{z t} z_{o}^{\prime}  \tag{4.8+}\\
& z_{t}^{\prime \prime}=\nu_{x t} x_{o}^{\prime \prime}+\nu_{y t} y_{o}^{\prime \prime}+\nu_{z t} z_{o}^{\prime \prime} \tag{4.85}
\end{align*}
$$

The axial components of the extreme points are tested to see whether they lie in the positive axial direction of the test cylinder between the origin and the end of the test cylinder. If either of them do lie in the positive axial direction, and the sum of the radius of the test cylinder and the radius of the sphere around the surface that can shade is greater than the distance between the origins of the test cylinder and the sphere in the plane perpendicular to the axial direction (Fig. 11), then the surface is said to have a possibility of causing shading between the pair of nodes for which the view factor is being evaluated.

From Fig. 11, the distance between the origins of the test cylinder and the sphere in the plane perpendicular to the axial direction is given by

$$
\begin{equation*}
d=\left(x_{t}^{2}+y_{t}^{2}\right)^{1 / 2} \tag{4.86}
\end{equation*}
$$

where

$$
\begin{align*}
& x_{t}=\lambda_{x t} x_{o}+\lambda_{y t} y_{o}+\lambda_{z t} z_{o}  \tag{4.87}\\
& y_{t}=\mu_{x t} x_{o}+\mu_{y t} y_{o}+\mu_{z t} z_{o} \tag{4.88}
\end{align*}
$$

In Fig. 11, the relation between the sum of the radii of the test cylinder, $r_{n}$, and the sphere, $r_{b}$, and the distance, $d$, between the origin of the test cylinder and the sphere, is as follows:

$$
\begin{aligned}
& r_{n}+r_{b}<d \text { circles are outside and have no point in common } \\
& r_{n}+r_{b}=d \text { circles are outside and have one point in common } \\
& r_{n}+r_{b}>d \text { circles are either partially or totally inside. }
\end{aligned}
$$



Fig. 11 - Preshade Test Cylinder and Sphere (Cross Section Parallel to the $X_{t}, Y_{t}$ Plane)

The preshade subroutine was added to the RAVFAC program as an option, which should be bypassed when the number of surfaces and nodes is small or when shading does not occur. The subroutine should be used when the number of surfaces and nodes increases and shading occurs frequently, since the use of this subroutine will save computer run time in proportion to the magnitude of the problem.

## Section 5

## VIEW FACTOR CALCULATION

The diffuse radiation view factor (also referred to as the angle factor, shape factor, form factor, geometric factor or configuration factor) can be defined by considering two gray ${ }^{*}$ surfaces, of areas of $A_{n}$ and $A_{j}$, separated by a nonabsorbing, nonscattering medium. The net interchange of thermal radiant energy between the two surfaces at temperature $T_{n}$ and $T_{j}$ is

$$
\begin{equation*}
q_{n \rightleftharpoons j}=A_{n} F_{A_{n} \rightarrow A_{j}} \sigma\left(T_{n}^{4}-T_{j}^{4}\right) \tag{5.1}
\end{equation*}
$$

The view factor $F_{A_{n}} \rightarrow A_{j}$ is defined as the fraction of radiant energy leaving surface $A_{j}$, in all directions, that strikes surface $A_{n}$.

The view factor between two surfaces is considered to be a function of geometry only. When the effects of emissivity, absorptivity, transmissivity and multiple reflections are taken into consideration, the view factor F is usually written as $\mathscr{F}$ and is beyond the scope of this effort. If these effects are considered, however, the view factor as discussed in this section will be required and plans are being made to include these effects into RAVFAC (Section 9).

### 5.1 GENERAL VIEW FACTOR EQUATION

To evaluate the view factor, consider the radiant energy exchange between two small gray surface elements, $\mathrm{dA}_{\ell}$ and $\mathrm{dA}_{\mathrm{m}}$, which are located on $A_{n}$ and $A_{j}$, respectively, as shown in Fig. 12.

The subscripts $n, j, \ell$ and $m$ were chosen because they are the subscripts used in RAVFAC. The node (viewing) in question is $n$, and the node being viewed is $j$. The incremental area on node $n$ is $\ell$ and the incremental area

[^1]

Fig. 12 - View Factor Notation


Fig. 13 - Finite Difference Notations
on node j is m . The view factor $\mathrm{dF}_{\ell-\mathrm{m}}$ between $\mathrm{dA}_{\ell}$ and $\mathrm{dA}_{\mathrm{m}}$ is derived in most heat transfer text books (such as Ref. 1) and assumes that Lambert's cosine distribution has the form of

$$
\begin{equation*}
\mathrm{dF}_{\mathrm{dA}}^{\ell} \text { - } \mathrm{dA} \mathrm{~A}_{\mathrm{m}}=\frac{\cos \left(\theta_{\ell}\right) \cos \left(\theta_{\mathrm{m}}\right)}{\pi \mathrm{r}_{\ell \mathrm{m}}^{2}} \mathrm{dA} \mathrm{~A}_{\mathrm{m}} \tag{5.2}
\end{equation*}
$$

The view factor between $A_{m}$ and $A_{j}$ can be determined by integrating Eq. (5.2) over both surfaces and can be written as

$$
\begin{equation*}
F_{A_{n}}-A_{j}=\frac{1}{A_{n}} \int_{A_{n}} \int_{A_{j}} \frac{\cos \left(\theta_{\ell}\right) \cos \left(\theta_{m}\right)}{\pi r_{l m}^{2}} d A_{\ell} d A_{m} \tag{5.3}
\end{equation*}
$$

The reciprocity theorem states that

$$
\begin{equation*}
A_{n} F_{A_{n}}-A_{j}=A_{j} F_{A_{j}}-A_{n} \tag{5.4}
\end{equation*}
$$

A closed form solution to Eq. (5.3) is impossible except for a few simple shapes uniquely oriented. When surface shading must be considered, the number of obtainable solutions is further reduced. Several analytical techniques have been developed for evaluating Eq. (5.3), such as the unit sphere, contour integral and finite difference. After these techniques were reviewed and the effects of surface shading considered, the conclusion was reached that the finite difference technique requires less computer time and that the contour integral technique produces a more accurate solution. Therefore, both of these techniques were included in RAVFAC. The choice of the technique is left to the user or he can let the program select the most efficient one, as discussed in Section 6.

The development of solutions to Eq. (5.3) using the finite difference and the contour integral techniques are discussed in the remainder of this section. Both techniques utilize the incremental area approach in which each surface
area (node) is subdivided into incremental areas (elements). View factors are calculated between elements which are summed over the entire area to yield the view factor between nodes. The shading effects are taken into account by checking to determine if the element centers can "see" each other. If they can, the incremental view factor is calculated. If the element centers cannot "see" each other, the entire element is considered to be shaded and the incremental view factor is zero. The techniques used to determine whether or not an element is shaded were discussed in Section 3 .

### 5.2 FINITE DIFFERENCE TECHNIQUE

The finite difference technique, also referred to as the double summation, consists of approximating Eq. (5.3) by the following finite difference equation:

$$
\begin{equation*}
F_{A_{n}}-A_{j}=\frac{1}{A_{n}} \sum_{A_{n}} \sum_{A_{j}} \frac{\cos \left(\theta_{\ell}\right) \cos \left(\theta_{m}\right)}{\pi r_{\ell m}^{2}} A_{\ell} A_{m} \tag{5.5}
\end{equation*}
$$

The use of Eq. (5.5) assumes that the incremental elements ( $\ell$ and $m$ ) are small enough so that the view factor can be considered as a constant over the areas of $A_{\ell}$ and $A_{m}$. An additional requirement of Eq. (5.5) is that the distance between the two elements being considered be large as compared to the incremental areas and that:

$$
\begin{equation*}
\frac{\mathrm{A}_{\ell}}{\left(\mathrm{r}_{\ell \mathrm{m}}\right)^{2}} \leq 0.01 \tag{5.6}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{A_{\mathrm{m}}}{\left(\mathrm{r}_{\mathrm{lm}}\right)^{2}} \leq 0.01 \tag{5.7}
\end{equation*}
$$

The solution to Eq. (5.5) can be simplified by using vectors. One must also keep in mind that each surface has its own coordinate system ( $n$ or j ) which is related to a central coordinate system (c). Equation (5.5) can be readily evaluated with the aid of Fig. 13.

Using Fig. 13, the cosine terms can be written as:

$$
\begin{align*}
& \cos \left(\theta_{\ell}\right)=-\frac{\stackrel{\rightharpoonup}{N}_{\ell} \cdot \vec{r}_{\mathrm{m} \ell}}{\left|\overrightarrow{\mathrm{r}}_{\mathrm{m} \ell}\right|}  \tag{5.8}\\
& \cos \left(\theta_{\mathrm{m}}\right)=\frac{\stackrel{\rightharpoonup}{\mathrm{N}}_{\mathrm{m}} \cdot \overrightarrow{\mathrm{r}}_{\mathrm{m} \ell}}{\left|\overrightarrow{\mathrm{r}}_{\mathrm{m} \ell}\right|} \tag{5.9}
\end{align*}
$$

since

$$
\begin{equation*}
\left|\stackrel{\rightharpoonup}{\mathrm{N}}_{\ell}\right| \equiv\left|\overrightarrow{\mathrm{N}}_{\mathrm{m}}\right|=1 \tag{5.10}
\end{equation*}
$$

Equation (5.5) can be written to include the coordinate transformations and the cosine terms as:

$$
\begin{equation*}
F_{A_{n}}-A_{j}=\frac{1}{A_{n}} \sum_{A_{n}} \sum_{A_{j}} \frac{\left(-\left[T_{n 2 c}\right\rfloor \vec{N}_{\ell} \cdot \stackrel{\rightharpoonup}{r}_{m \ell}\right)\left(\left[T_{j 2 c}\right] \stackrel{\rightharpoonup}{N}_{m} \cdot \stackrel{\rightharpoonup}{r}_{m \ell}\right)}{\pi\left|\vec{r}_{m \ell}\right|^{4}} A_{m} A_{\ell} \tag{5.11}
\end{equation*}
$$

where

$$
\begin{equation*}
\stackrel{\rightharpoonup}{r}_{m \ell}=\left[T_{j 2 c}\right] \stackrel{\rightharpoonup}{r}_{m}-\left[T_{n 2 c}\right] \vec{r}_{\ell}-R_{c 2 n}+R_{c 2 j} \tag{5.12}
\end{equation*}
$$

and

$$
\begin{equation*}
\left|\stackrel{\rightharpoonup}{r}_{m \ell}\right|=\left\{\left(x_{n}-x_{j}\right)^{2}+\left(y_{n}-y_{j}\right)^{2}+\left(z_{n}-z_{j}\right)^{2}\right\}^{1 / 2} \tag{5.13}
\end{equation*}
$$

The transformation matrices $[T]$, the unit normal $\vec{N}$, and the vectors $\vec{r}_{n}, \vec{r}_{j}$, $\vec{R}_{c 2 n}, \vec{R}_{c 2 j}$ and the incremental area terms $A_{\ell}$ and $A_{m}$ are defined in Section 2.

### 5.3 CONTOUR INTEGRAL TECHNIQUE

The contour integral technique employs Stokes' theorem to reduce the double area integrals of Eq. (5.3) to contour or line integrals. This technique is discussed by Sparrow in Ref. 2 for the case of unshaded surfaces.

For two flat surfaces, regardless of relative position (translation and rotation), the first integration can be performed analytically and the second can be approximated by a numerical summation. This technique produces view factors which are more accurate than those produced by the finite difference technique; and as the distance between the surfaces becomes smaller the accuracy increases (i.e., the restrictions of Eqs. (5.6) and (5.7) do not apply). The contour integral technique, however, requires more computer time than does the finite difference technique.

Contour integrals can be written for calculating the view factor between two surfaces without dividing the surfaces into incremental areas. This is not practical when shading is considered, however, because there is no easy way to describe the contour around the shaded portion of the surface. Therefore, the surfaces are divided into incremental areas in the same manner as they were for the finite difference technique. Each incremental area is approximated by a rectangular element with equivalent area and unit normal vector. During this study, contour equations were developed for surface types other than for a flat plate, but it was beyond the present scope of work to include these equations in RAVFAC.

Development of the contour integral technique is based on Eq. (5.2) written for a differential area $\mathrm{dA}_{\ell}$ and a finite area $A_{m}$.

$$
\begin{equation*}
\mathrm{F}_{\mathrm{d} A_{\ell}}-\mathrm{A}_{\mathrm{m}}=\int_{\mathrm{A}_{\mathrm{m}}} \frac{\cos \left(\theta_{\ell}\right) \cos \left(\theta_{\mathrm{m}}\right)}{\pi \mathrm{r}_{\ell \mathrm{m}}^{2}} \mathrm{~d} A_{\mathrm{m}} \tag{5.14}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathrm{r}_{\ell \mathrm{m}}^{2}=\left(\mathrm{x}_{\mathrm{m}}-\mathrm{x}_{\ell}\right)^{2}+\left(\mathrm{y}_{\mathrm{m}}-\mathrm{y}_{\ell}\right)^{2}+\left(\mathrm{z}_{\mathrm{m}}-\mathrm{z}_{\ell}\right)^{2} \tag{5.15}
\end{equation*}
$$

$$
\begin{array}{ll}
x_{\ell}, y_{\ell}, z_{\ell} & \text { denotes position of surface } \mathrm{dA} \\
\ell \\
\mathrm{x}_{\mathrm{m}}, \mathrm{y}_{\mathrm{m}}, \mathrm{z}_{\mathrm{m}} & \text { denotes position on surface } \mathrm{dA}
\end{array}
$$

Equation (5.14) can be expanded and rewritten as

$$
\begin{equation*}
F_{d A_{\ell}}-A_{m}=\int_{A_{m}}\left\{\lambda_{m}\left(x_{\ell}-x_{m}\right) f+\mu_{m}\left(y_{\ell}-y_{m}\right) f+\nu_{m}\left(z_{i}-z_{m}\right) f\right\} d A_{m} \tag{5.16}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathrm{f}=\left[\lambda_{\ell}\left(\mathrm{x}_{\mathrm{m}}-\mathrm{x}_{\ell}\right)+\mu_{\ell}\left(\mathrm{y}_{\mathrm{m}}-\mathrm{y}_{\ell}\right)+\nu_{\ell}\left(\mathrm{z}_{\mathrm{m}}-\mathrm{z}_{\ell}\right)\right] \div \pi \mathrm{r}_{\ell \mathrm{m}}^{4} \tag{5.17}
\end{equation*}
$$

$$
\begin{aligned}
& \lambda_{\ell}, \mu_{\ell}, \nu_{\ell} \text { denotes directional cosines of the normal of surface } \mathrm{dA}_{\ell} \\
& \lambda_{\mathrm{m}}, \mu_{\mathrm{m}}, \nu_{\mathrm{m}} \text { denotes directional cosines of the normal of surface } \mathrm{dA} \mathrm{~m}_{\mathrm{m}}
\end{aligned}
$$

The area integral Eq. (5.16) can be reduced to a contour integral by applying Sotkes theorem, yielding:

$$
\begin{align*}
& F_{d A_{\ell}}-A_{m}=\lambda_{\ell} \oint_{c_{m}} \frac{\left(z_{m}-z_{\ell}\right) d y_{m}-\left(y_{m}-y_{\ell}\right) d z_{m}}{2 \pi r_{\ell m}^{2}} \\
& +\mu_{\ell} \oint_{c_{m}} \frac{\left(x_{m}-x_{\ell}\right) d z_{m}-\left(z_{m}-z_{\ell}\right) d x_{m}}{2 \pi r_{\ell m}^{2}} \\
& +v_{\ell} \oint_{c_{m}} \frac{\left(y_{m}-\dot{y}_{\ell}\right) d x_{m}-\left(x_{m}-x_{\ell}\right) d y_{m}}{2 \pi r_{\ell m}^{2}} \tag{5.18}
\end{align*}
$$

Each surface is defined in its own coordinate system, referred to as the surface coordinate system, to simplify the input requirements for RAVFAC. All of the individual coordinate systems must be referenced to a common coordinate system, referred to as the central coordinate system. RAVFAC allows the use of a third coordinate system, referred to as the intermediate coordinate system, to provide additional flexibility. The transformation from the intermediate to the central is not discussed in this section because it is handled in the same manner as the transformation from the individual to the central coordinate system.

As already mentioned, each incremental area ( $A_{\ell}$ or $A_{m}$ ) is approximated by an equivalent rectangular element. A surface is approximated by a number of rectangular elemental areas. Each element has a coordinate system with its origin at the centroid of the element. The positive $z$ axis is along the unit normal of the element as shown in Fig. 14.

To integrate Eq. (5.18) around the defined rectangular elements, the following transformations must be performed:

$$
\begin{align*}
& {\left[\begin{array}{c}
\lambda_{\ell} \\
\mu_{\ell} \\
\nu_{\ell}
\end{array}\right]_{\mathrm{c}}=\left[\mathrm{T}_{\mathrm{n} 2 \mathrm{c}}\right]\left[\begin{array}{c}
\lambda_{\ell} \\
\mu_{\ell} \\
\nu_{\ell}
\end{array}\right]_{\mathrm{n}}}  \tag{5.19}\\
& {\left[\begin{array}{l}
x_{m} \\
y_{m} \\
z_{m}
\end{array}\right]_{c}=\left[T_{m 2 c}\right]\left[\begin{array}{l}
x_{m} \\
y_{m} \\
z_{m}
\end{array}\right]_{m}+\left[T_{j 2 c}\right]\left[\begin{array}{l}
x_{j} \\
y_{j} \\
z_{j}
\end{array}\right]_{j}+\left[\begin{array}{l}
R_{x_{j}} \\
R_{y_{j}} \\
R_{j} \\
z_{i}
\end{array}\right]_{c}}  \tag{5.20}\\
& {\left[\begin{array}{l}
x_{\ell} \\
y_{\ell} \\
z_{\ell}
\end{array}\right]_{c}=\left[T_{n 2 c}\right] \cdot\left[\begin{array}{c}
x_{\ell} \\
y_{\ell} \\
z_{\ell}
\end{array}\right]_{n}+\left[\begin{array}{c}
R_{x_{n}} \\
R_{y_{n}} \\
R_{z_{n}}
\end{array}\right]_{c}} \tag{5.21}
\end{align*}
$$



Fig. 14 - General Rectangular Element Area Coordinate System

at I: $x_{m}=-g / 2, d x_{m}=0$
at II: $\mathrm{y}_{\mathrm{m}}--\mathrm{b} / 2, \mathrm{dy} \mathrm{y}_{\mathrm{m}}=0$
at III: $x_{m}=+\mathrm{g} / 2, d x_{m}=0$
at IV: $y_{m}=+b / 2, d y_{m}=0$

Fig. 15 - Contour Integration Around Rectangular Element

Where the transformation matrices $\left[T_{n 2 c}\right]$ and $\left[T_{j 2 c}\right]$ are given in Section 2 and where the matrix $\left|T_{m 2 d}\right|$ is given by

$$
\left[T_{m 2 c}\right] \equiv\left[T_{j 2 c}\right]\left[T_{m 2 j}\right] \equiv\left[\begin{array}{lll}
\lambda_{x} & \mu_{x} & \nu_{x}  \tag{5.22}\\
\lambda_{y} & \mu_{y} & \nu_{y} \\
\lambda_{z} & \mu_{z} & \nu_{x}
\end{array}\right]
$$

where

$$
\left.\begin{array}{l}
\lambda_{x}=\lambda_{x_{j}} \lambda_{x_{m}}+\mu_{x_{j}} \lambda_{y_{m}}+\nu_{x_{j}} \lambda_{z_{m}} \\
\mu_{x}=\lambda_{x_{j}} \mu_{x_{m}}+\mu_{x_{j}} \mu_{y_{m}}+\nu_{x_{j}} \mu_{z_{m}} \\
\nu_{x}=\lambda_{x_{j}} \nu_{x_{m}}+\mu_{x_{j}} \nu_{y_{m}}+\nu_{x_{j}} \nu_{z_{m}}
\end{array}\right\}
$$

The differentialable variables are also transformed by the following solution

$$
d\left\{\left[\begin{array}{l}
x_{m} \\
y_{m} \\
z_{m}
\end{array}\right]_{c}\right\}=d\left\{\left[T_{m 2 c}\right]\left[\begin{array}{l}
x_{m} \\
y_{m} \\
z_{m}
\end{array}\right]_{m}+\left[T_{j 2 c}\right]\left[\begin{array}{l}
x_{j} \\
y_{j} \\
z_{j}
\end{array}\right]_{j}+\left[\begin{array}{l}
R_{x_{j}} \\
R_{y_{j}} \\
R_{z_{j}}
\end{array}\right]\right\}
$$

which simplifies to

$$
\left[\begin{array}{l}
d x_{m}  \tag{5.26}\\
d y_{m} \\
d z_{m}
\end{array}\right]_{c}=\left[\mathrm{T}_{\mathrm{m} 2 \mathrm{c}}\right]\left[\begin{array}{c}
\mathrm{d} \mathrm{x}_{\mathrm{m}} \\
\mathrm{~d} \mathrm{y}_{\mathrm{m}} \\
\mathrm{~d} z_{m}
\end{array}\right]_{\mathrm{m}},
$$

since the transformations and translations are invariants with respect to the operator "d." Substituting the transformations (Eqs. (5.19) through (5.26)) in Eq. (5.18) and noting that for a flat surface in the $X_{m}-Y_{m}$ plane

$$
z_{m}=0 \Rightarrow d z_{m}=0
$$

Eq.(5.18) reduces to

$$
\begin{equation*}
F_{d A_{\ell}}-A_{m}=\oint_{C_{m}} \frac{C_{\ell m} y_{m}+C_{\ell o}}{2 \pi D} d x_{m}+\oint_{C_{m}} \frac{C_{m \ell} x_{m}+C_{m o}}{2 \pi D} d y_{m} \tag{5.27}
\end{equation*}
$$

where

$$
\begin{align*}
D & \equiv x_{m}^{2}+y_{m}^{2}+2 C_{x} x_{m}+2 C_{y} y_{m}+C  \tag{5.28}\\
C_{x} & \equiv \lambda_{x} C_{1}+\lambda_{y} C_{2}+\lambda_{z} C_{3}  \tag{5.29}\\
C_{y} & \equiv \mu_{x} C_{1}+\mu_{y} C_{2}+\mu_{z} C_{3}  \tag{5.30}\\
C & \equiv C_{1}^{2}+C_{2}^{2}+C_{3}^{2} \tag{5.31}
\end{align*}
$$

$$
\begin{align*}
& C_{\ell m} \equiv \lambda_{\mathrm{x}}\left(\nu_{\ell} \mu_{\mathrm{y}}-\mu_{\ell} \mu_{\mathrm{z}}\right)+\lambda_{\mathrm{y}}\left(\lambda_{\ell} \mu_{\mathrm{z}}-\nu_{\ell} \mu_{\mathrm{x}}\right)+\lambda_{\mathrm{z}}\left(\mu_{\ell} \mu_{\mathrm{x}}-\lambda_{\ell} \mu_{\mathrm{y}}\right)  \tag{5.32}\\
& \mathrm{C}_{\ell 0} \equiv \lambda_{\mathrm{x}}\left(\nu_{\ell} \mathrm{C}_{2}-\mu_{\ell} \mathrm{C}_{3}\right)+\lambda_{\mathrm{y}}\left(\lambda_{\ell} \mathrm{C}_{3}-\nu_{\ell} \mathrm{C}_{1}\right)+\lambda_{\mathrm{z}}\left(\mu_{\ell} \mathrm{C}_{1}-\lambda_{\ell} \mathrm{C}_{2}\right)  \tag{5.33}\\
& \mathrm{C}_{\mathrm{m} \ell} \equiv \mu_{\mathrm{x}}\left(\nu_{\ell} \lambda_{\mathrm{y}}-\mu_{\ell} \lambda_{\mathrm{z}}\right)+\mu_{\mathrm{y}}\left(\lambda_{\ell} \lambda_{\mathrm{z}}-\nu_{\ell} \lambda_{\mathrm{x}}\right)+\mu_{\mathrm{z}}\left(\mu_{\ell} \lambda_{\mathrm{x}}-\lambda_{\ell} \lambda_{\mathrm{y}}\right)  \tag{5.34}\\
& \mathrm{C}_{\mathrm{mo}} \equiv \mu_{\mathrm{x}}\left(\nu_{\ell} \mathrm{C}_{2}-\mu_{\ell} \mathrm{C}_{3}\right)+\mu_{\mathrm{y}}\left(\lambda_{\ell} \mathrm{C}_{3}-\nu_{\ell} \mathrm{C}_{1}\right)+\mu_{\mathrm{z}}\left(\mu_{\ell} \mathrm{C}_{1}-\lambda_{\ell} \mathrm{C}_{2}\right) \tag{5.35}
\end{align*}
$$

and where

$$
\left[\begin{array}{l}
C_{1}  \tag{5.36}\\
C_{2} \\
C_{3}
\end{array}\right] \equiv\left[T_{j 2 c}\right]\left[\begin{array}{c}
x_{j} \\
y_{j} \\
z_{j}
\end{array}\right]_{j}+\left[\begin{array}{l}
R_{x_{j}} \\
R_{y_{j}} \\
R_{z_{j}}
\end{array}\right]_{c}-\left[T_{n 2 c}\right]\left[\begin{array}{c}
x_{\ell} \\
y_{\ell} \\
z_{\ell}
\end{array}\right]_{n}-\left[\begin{array}{l}
R_{x_{n}} \\
R_{y_{n}} \\
R_{z_{n}}
\end{array}\right]_{c} .
$$

Let the dimensions of the rectangular element be " $g$ " units in the $X_{m}$ direction and " $b$ " units in the $Y_{m}$ direction (Fig. 15), where " $g$ " and " $b$ " will depend on the type of surface.

Application of the contour integral Eq. (4.27) to the rectangle, as shown in Fig. 15, yields the following integral equation in the counterclockwise direction.

$$
\begin{align*}
& F_{d A_{\ell}}-A_{m}=\frac{1}{2 \pi}\left\{\left(-\frac{g}{2} C_{m \ell}+C_{m o}\right) \int_{+b / 2}^{-b / 2} \frac{d y_{m}}{D_{I}}\right. \\
& +\left(-\frac{b}{2} C_{\ell m}+C_{\ell 0}\right) \int_{-g / 2}^{+g / 2} \frac{d x_{m}}{D_{I I}} \\
& +\left(\frac{g}{2} C_{m l}+C_{m o}\right) \int_{-b / 2}^{+b / 2} \frac{d y_{m}}{D_{I I I}} \\
& \left.+\left(\frac{b}{2} C_{l m}+C_{\ell 0}\right) \int_{g / 2}^{-g / 2} \frac{d x_{m}}{D_{I V}}\right) \tag{5.37}
\end{align*}
$$

where

$$
\left.\begin{array}{rl}
D_{I} & =y_{m}^{2}+2 C_{y} y_{m}+C+\frac{g^{2}}{4}-C_{x} g \\
D_{I I} & =x_{m}^{2}+2 C_{x} x_{m}+C+\frac{b^{2}}{4}-C_{y} b \\
D_{I I I} & =y_{m}^{2}+2 C_{y} y_{m}+C+\frac{g^{2}}{4}+C_{x} g  \tag{5.38}\\
D_{I V} & =x_{m}^{2}+2 C_{x} x_{m}+C+\frac{b^{2}}{4}+C_{y} b
\end{array}\right\}
$$

The four integrals on the right hand of Eq. (5.37) are of the general type

$$
\begin{equation*}
\int_{\ell_{1}}^{\ell_{2}} \frac{d u}{U} \quad \text { where } U=u^{2}+2 p u+q \tag{5.39}
\end{equation*}
$$

$$
\begin{aligned}
\ell_{1}, \ell_{2} & \text { lower and upper limits } \\
p, q & \text { arbitrary constants. }
\end{aligned}
$$

The integral (Eq. (5.39)) has three solutions depending on the value of $p^{2}-q$

$$
\left.\begin{array}{l}
\frac{1}{\sqrt{q-p^{2}}}\left\{\tan ^{-1}\left[\frac{\ell_{2}+p}{\sqrt{q-p^{2}}}\right]-\tan ^{-1}\left[\frac{\ell_{1}+p}{\sqrt{q-p^{2}}}\right]\right\} \\
\frac{-1}{\ell_{2}+p}-\frac{-1}{\ell_{1}+p}  \tag{5.40}\\
\frac{1}{2 \sqrt{p^{2}-q}}\left\{\ln \left[\frac{p^{2}-q<0}{\ell_{2}+p+\sqrt{p^{2}-q}}\right]-\ln \left[\frac{\ell_{1}+p-\sqrt{p^{2}-q}}{\ell_{1}+p+\sqrt{p^{2}-q}}\right]\right\}
\end{array}\right\}
$$

With the solution of integral (Eq. 5.39 ) known, $F_{d A_{\ell}}-A_{m}$ can then be eval-
uated by Eq. $(5.37$ ). uated by Eq. (5.37).

Summing all the view factors from $d A_{\ell}$ to $A_{m}$ over $A_{j}$ will yield the view factor between $d A_{l}$ and $A_{j}$, that is

$$
\begin{equation*}
F_{d A_{\ell}}-A_{j}=\sum_{A_{j}} F_{d A_{\ell}}-A_{m} \tag{5.41}
\end{equation*}
$$

The view factor from a surface $A_{n}$ to a surface $A_{j}$ is given in terms of the view factor from an elemental surface $d_{l}$ to a surface $A_{j}$ as

$$
\begin{equation*}
F_{A_{n}}-A_{j}=\frac{1}{A_{n}} \int_{A_{n}} F_{d A_{\ell}}-A_{j} d A_{\ell} \tag{5.42}
\end{equation*}
$$

The integration over the surface $A_{n}$ in Eq. (5.42) can then be approximated as a summation, since a closed form solution is usually not possible, as

$$
\begin{equation*}
F_{A_{n}-A_{j}} \cong \frac{1}{A_{n}} \sum_{\Delta A_{\ell}} F_{\Delta A_{\ell}-A_{j}} \Delta A_{\ell} \tag{5.43}
\end{equation*}
$$

Section 6
DESCRIPTION OF COMPUTER PROGRAM AND INPUT PARAMETERS

### 6.1 DESCRIPTION OF THE RAVFAC COMPUTER PROGRAM

The RAdiation View FACtor (RAVFAC) program is written in Fortran V language for the Univac 1108 computer. RAVFAC can be used on any large ( 64 K core) computer with a few modifications, which would be limited to replacing or eliminating the SLEUTH ${ }^{*}$ routines used to calculate the elasped run time, and the routine (NTRAN) which is used to read the view factors from the restart tape.

At present RAVFAC has the capability to:

- Calculate the diffuse radiation view factors using:
- Finite difference technique
- Contour integral technique
- Combination of the above techniques.
- Calculate the effects of surface shading.
- Normalize the radiation view factors so that
- $\sum V F_{n}=1$
- $\sum \mathrm{VF}_{\mathrm{n}} \leq 1$
- Analyze any of seven surface types:
- Rectangular Plate
- Circular Plate
- Trapezoid
- Cylinder
- Cone
- Sphere
- Paraboloid.

[^2]- Use up to 150 different surfaces on each case.
- Divide the surfaces into nodes for which view factors are given. (Each case can consist of up to 150 nodes.)
- Use up to 2500 elements for each case to improve accuracy of view factor calculation and shading effects.
- Reduce the computer time required for calculating the effects of shading by using a "preshade routine."
- Select surfaces which are to be included in the shading effects by using "shading flags."
- Restart after computer failure or termination as a result of maximum run time or machine failure.
- Check out options including:
- Shading override.
- One element per node.
- Use of three coordinate systems.

When the program logic and coding for RAVFAC were being developed, several other capabilities were taken into consideration. Although these capabilities have not been included, some of the basic work was performed so that they can be included without major changes to RAVFAC. These additional capabilities are discussed in Section 9. The following paragraphs briefly describe how RAVFAC processes a case.

RAVFAC consists of a main program, four Fortran subroutines and several SLEUTH and system routines. The main program, called "RAVFAC," maintains control of the problem as it is being processed. All input/output is processed by the main program; as are the view factor calculations. When RAVFAC is executed all constants are initialized. The TITLE card is read in and checked to determine if the run has been completed. The CASE control card is read in and the option flags are set. Next the first set of surface data is read in and stored in arrays for later use. If any portion of the checkout option is requested, the surface data are modified accordingly. The transformation matrices are calculated and stored, assuming a surface-to-central transformation. The surface data are printed in the output. The next set of surface data is read in and the above procedure is repeated until the end-of-
surface data card is encountered. If any surface was referenced to an intermediate coordinate system (ICS) the first ICS data card is read in. All sur faces are checked to determine if the ICS was referenced. If the ICS was referenced, the transformation matrix is modified to account for the extra transformation. The next ICS data card is read in and the above procedure is repeated until the end of the ICS data cards is encountered. After all the surface and ICS data have been read in, RAVFAC calls subroutine SURF.

The function of SURF is to calculate necessary data for the nodes and elements. All the surfaces are processed in a single call to SURF. Each surface is processed as follows: the surface is divided into nodes and the position vector, area and coefficients are determined. Each node is then divided into elements. The position and unit normal vectors and areas are calculated. If the contour integral technique was requested or if the program is to select the method, additional contour parameters are calculated. These data, which will be used later, are stored. After the last surface has been processed, control is returned to RAVFAC.

RAVFAC checks to determine if its storage capacity has been exceeded and terminates if it has. If an output tape was requested, most of the data discussed above is written on tape. A summary of all the surface data is printed along with the computer time required thus far (IELT=1). As noted previously, the elapsed time is not presently available under Exec VIII.

The next major operation is to calculate the view factors. These calculations take place inside four nested "do-loops." The first loop is set up for the base node and another loop for the other node: the view factor will be between the two. Several checks are made to ensure that the two nodes can "see" each other if no other surfaces cause shading. If they cannot see each other, the VF's are set equal to zero and the next pair of nodes is checked for shading. The storage location of the data stored by subroutine SURF is detexmined, as well as the limits of the next two do-loops. If required, subroutine PRESHD is called.

The function of PRESHD is to eliminate the surfaces that cannot cause shading between the two nodes under consideration. A test cylinder is constructed between the two nodes that are being considered. Each surface is then checked to determine if any portion lies inside the test cylinder. If the surface does not lie within the test cylinder, at least in part, the surface is flagged as a "no shade" surface. PRESHD is called each time the view factor is calculated between nodes that were flagged (via input) as "can be shaded" surfaces. After all surfaces have been checked, control is returned to RAVFAC.

Next RAVFAC sets up the two innermost do-loops. There is one loop for the elements on each of the two nodes. The angles are claculated between the vector connecting the element centers and their unit normals. Checks are made to determine if the elements can see each other if they are not shaded by other surfaces. If they cannot see each other, the dVF is set equal to zero and the next element is considered. If they can see each other, subroutine SHADE is called, if shading between the two nodes under consideration is to be considered.

The function of subroutine SHADE is to determine if the two elements can see each other. Each surface which was flagged by PRESHD, as a "can shade" (or by the user if PRESHD was bypassed) is checked to determine if it intersects a line joining the centers of the two elements. When an intersection is found, the elements are flagged as being shaded. If no intersection is detected, the elements are not shaded. These checks require substantial computer time because they are lengthy and are made for each surface on an element-to-element basis. Time can be saved if the surfaces that cause shading on the most elements are input first. After all surfaces have been checked, or an intersection is detected, control is returned to RAVFAC.

If the elements cannot see each other, the dVF is set equal to zero. If they can see each other, the view factor technique requested is selected to calculate the dVF between the two elements under consideration. When the contour integral is used, subroutine TEGRAL is called to perform the
numerical integration of four terms of the contour equation. After $d V F$ is calculated, it is added to the view factor between the element on the base node and the other node under consideration (dVFS). This ends the innermost do-loop. The dVFS is then added to the view factor between the two nodes under consideration (VFS). If the time to print each $5 \%$ of the view factors is requested, a check is made to determine if it is time to print. This ends the second and third innermost do-loops. The view factors for the base node are written on a data storage drum and, if required, on the restart tape for later use. This ends the outermost loop and all the view factors have been calculated.

The large view factor array is zeroed out and the view factors are read into the array from the data storage drum. The time required to calculate all the view factors is printed as IELT=2. The view factor array is printed in the output. The time required to calculate and print the view factors is printed as IELT=3.

If normalized view factors are requested, they are calculated and printed in the output. The elapsed time from the start of the view factor calculations is printed as IELT=4. The view factor sum for each node is calculated and printed in the output along with the elapsed time as IELT=5. Control is returned to the start of RAVFAC to read in the second case.

A restart case starts the same as a non-restart case until the case control card has been processed. Control is then transferred to the restart section. Surface data and control flags are read from the restart tape, then the view factors are read from the restart tape. Subroutine NTRAN is used to read the restart tape. After the last view factor is read and stored on drum, the do-loop indices for the nodes are calculated and control is transferred to the view factor calculation section. The remainder of the restart case is the same as a non-restart case.

### 6.2 DESCRIPTION OF INPUT PARAMETERS

The objective of this section is to provide the unfamiliar reader with a complete description of what each input variable for the LMSC Radiation View Factor Computer Program (RAVFAC) represents. Once the reader is familiar with these input variables the Input Guide in Section 7 can be used to set up an input data deck.

The input data for RAVFAC consist of five general card types which are:

- Title Card
- Case Control Card
- Surface Data Cards
- Intermediate Coordinate System Card
- End Card.

The general formats of these cards are shown in Fig. 16. The decimal point precedes the X's in Fig. 16 whenever a non-integer format is used. The exact format for each card is presented in Section 7. The variable names are given as they appear in the input guide and as used elsewhere in this report if applicable.

Title Card Any comment on this card will be printed at the beginning of the problem output for identification purposes.

Case Control Card This card contains nine variables which control the manner the program will process the problem. Variables which are zero can be omitted.

NSTART - Restart control flag
Some of the problems which can be processed by RAVFAC will take several hours of computer time. During this length of time there is always the possibility of the computer rebelling or the operator tripping on the stop button, and the "remote" possibility of the user under-estimating the run time and exceeding the maximum run time. In either of these cases the run is terminated and unless the NSTART and NT options are used all is lost. When a termination occurs during the view factor calculations $*$, the run can be

[^3]GENERAL PURPOSE CODING FORM

PROGRAMMER:
PROBLEM RAVFAC Program Input Data Summary - Figure 16



迬相 T P
Tr $x$ $\mathbb{N}^{W}$ - $-\frac{x}{x}$ NW
$-14$

$$
=
$$T

continued at a later date using the restart option. The restart flag instructs the program either to read surface data from input cards and start the view factor calculations (normal case), or to read surface data and previously calculated view factors from an input tape (restart case). The input tape consists of the output tape (NT) which was made and saved on the terminated run. The program will determine where termination occurred and continue from that point. When a "restart only" is being run, the Title, Case Control and End Cards are required. The NPRT and NT options are not available on a restart case, and the remaining control flags are read from tape.

NSTART $\geq 0$ : Normal case.
 card is read, the program will read the terminated problem data from tape INSTARTI. This tape must be assigned before the program is executed.

NT - Output tape for restart
The NT option is used to generate the restart input tape. During a normal run, data will be written on tape NT as it is calculated. If a computer failure occurs or the operator terminates the run, tape NT can be used to restart the problem where termination occurred. If the run will take longer than 15 minutes the NT option should be used.
$\mathrm{NT} \leq 0$ : Restart output tape is not written.
NT $>0$ : Restart data is written on tape NT. If the problem is terminated during the view factor calculations, tape NT can be used to restart and continue the problem at a later date.

When NT is greater than zero, the following guidelines must be followed if a usable tape is to be available for a restart:

- The proper tape unit must be assigned before RAVFAC is executed
- A SAVE label must be included with the run (check local procedure)
- The operator can be instructed not to save the tape if the job gets a normal exit
- Use only those tape units (numbers) available on local system

NVFCAL - View factor calculation technique option
The user has three options in selecting the two techniques available with RAVFAC for calculating view factors. Each option has its own advantages and disadvantages as pointed out below.

$$
\begin{aligned}
\text { NVFCAL }=1: & \text { Program selects the finite difference } \\
& \text { technique. This technique requires } \\
& \text { less computer time but becomes in - } \\
& \text { accurate when: }
\end{aligned}
$$

$$
r_{m_{\ell}}^{2}>\frac{A_{\ell}}{0.1} \quad \text { (refer to Section 5.2). }
$$

NVFCAL = 2: Program selects the contour integral technique. This technique is more accurate than the finite difference and is not restricted by the distance between the elements. However, the technique requires more computer time in that additional parameters are required and the basic view factor solution also requires more computer time than does the finite difference technique.

NVFCAL $=3$ : Program selects most efficient technique. When this option is used the program makes the following check

$$
\text { Is }\left\{\frac{\mathrm{A}_{\ell}}{\mathrm{r}_{\mathrm{ml}}^{2}}>\mathrm{RMAX}\right\}
$$

If the above is true, the program selects the contour integral technique. If the check is false, the program selects the finite difference technique. This option requires more computer time for the same problem than the NVFCAL $=1$ option and less computer time than the NVFCAL $=2$ option. However, since it is not necessary to use a large number of elements on surfaces which are close together (to avoid the restrictions of the finite difference technique), fewer elements can be used. This can result in less computer time than
the NVFCAL $=1$ option to obtain the same accuracy. One should also keep in mind that the number of elements also influence the effects of shading.

NORM - View factor normalization option
Techniques used to calculate view factors will result in inaccuracies. In some cases the finite difference technique will produce a total view factor for a node (i.e., the sum of all the view factors for that node) which is greater than one. In other cases the user may have a closed system in which the total view factor for a node is equal to one. This option allows the user to adjust, or normalize, the calculated view factors. When the view factors are normalized all view factors greater than one are set equal to one. Each non-zero view factor for a given node will be adjusted according to the option selected. The zero view factors are not adjusted. A table of non-normalized view factors will be printed out for each option. If NORM $>0$, a table of normalized view factors will also be printed.

> NORM $\leq 0$ : Normal run, view factors are not normalized.

NORM = 1: View factors for a node are normalized if the view factor sum for that node is greater than one.
NORM $\geq 2$ : View factors for each node are normalized so that the view factor sum is equal to one.

RMAX - Maximum area-to-distance ratio
This parameter gives the user some control of the accuracy of the view factors to be calculated (used with NVFCAL $=3$ only). The program selects the contour integral technique when

$$
\frac{A_{\ell}}{r_{m \ell}^{2}}>\operatorname{RMAX}
$$

which will provide better accuracy. A value of 0.01 will provide as accurate results as is usually required. A value over 0.1 will probably produce view factors which are too large. If a value of RMAX is not given or if it is zero or negative, the program will set RMAX equal to 0.1 ,

$$
6-10
$$

NPRT - Immediate output control
During a normal run all view factors are calculated before they are printed out. For problems consisting of a large number of elements which can shade and be shaded, this can take hours. The objective of NPRT is to give the user some idea of the required run time if the run is not completed.

NPRT $\leq 0$ : No immediate printout.
NPRT $>0$ : The time required to compute each $5 \%$ of the view factors will be printed on the output listing.

NPRT =. 2: Each view factor will be printed on the output listing as it is calculated. Use of this option will increase the required run time and an excessive amount of output will occur (approximately ( $\mathrm{N}^{2} / 2$ lines).

NFE - Element override option
An input error is costly in a dollar sense as well as the time delay in obtaining a long computer run. A short checkout run should be made first to assure that all surfaces have been input correctly. All problems can be processed in a relatively short time if all nodes have only one element. Results of a one-element-per-node will reflect most input errors.

NFE $\leq 0$ : Normal run, number of elements taken from surface data cards.

NFE > 0: Program sets $\mathrm{NB}=\mathrm{NG}=1$ for all nodes; i.e., one element per node.

NFS - Shading override option
The shading override can also be used to reduce the run time required to check out a data deck or an actual case when shading will not be considered.

NFS $\leq 0$ : Normal run. Shading flags are set by surface data input.

NFS $>0$ and $\neq 2$ : Program sets $\mathrm{KSH}=\mathrm{KBSH}=0$; all shading is neglected.

$$
\begin{aligned}
& \text { NFS }=2: \text { Subroutine PRESHD is bypassed along with } \\
& \text { the calculation of variables required only by } \\
& \text { PRESHD. Shading flags are taken from sur- } \\
& \text { face data input and processed accordingly. } \\
& \text { Use of this option will increase the run time } \\
& \text { of a normal case except on rare cases. }
\end{aligned}
$$

NTVF - View factor output tape
The view factors can be output on a magnetic tape which can be saved for later use. Unlike the restart tape only the view factors, areas and associated node numbers are written on the tape.

NTVF = 0: A view factor tape will not be made.
NTVF $>0$ : View factors are written on tape NTVF.

## Surface Data Cards

These cards are used to describe the surfaces and to divide them into the proper number of nodes and elements. The description of each surface consists of three data cards. The first card of a dummy surface (ISP =-1) is used to terminate the reading of the surface data. (Do not input the second or third card of the dummy surface.)

Card No. 1
ISF - First node number on the surface
Each surface can be divided into several nodes. ISF is the number assigned to the first node of the surface in all output. It will normally be the number that will be assigned to the node in a latter thermal analyzer run. If there is more than one node on the surface, they will be numbered consecutively from ISF; the numbering goes from $\beta_{\min }$ to $\beta_{\max }$ for each $\gamma$ going from $\gamma_{\min }$ to $\gamma_{\max }$ (Fig. 2 ). Nodes may be assigned the same number.

NSF $<0$ : Surface description is completed.
NSF $=0$ (or blank): Identification number will be one plus the identification number of the last node on the preceding surface.

NSF $>0$ : Identification number of the first node on the surface.

KSH, KSBH - Shading flags
The KSH and KSBH are flags to indicate whether the surface can cause shading on any other surface (KSH) or if the surface can be shaded by any other surface (KSBH). The use of these flags can reduce the required computer time by eliminating flagged nodes from the shading checks.
$\mathrm{KSH}(\mathrm{KSBH})=0$ : The surface cannot cause shading (be shaded)
$K S H(K S B H)>0$ : The surface can cause shading (be shaded).

## COMM - Surface identification

Any comment can be made on this section of the card to identify the surface. The se comments will be listed along with the surface input data. It is also useful in identifying the surface data cards.

## Card No. 2

ILK - Surface type
Seven types of surface can be input:

$$
\begin{aligned}
& \text { ILK }= \pm 1: \text { Rectangle } \\
& \text { ILK }= \pm 2: \text { Disc } \\
& \text { ILK }= \pm 3: \text { Trapezoid } \\
& \text { ILK }= \pm 4: \text { Cylinder } \\
& \text { ILK }= \pm 5: \text { Cone } \\
& \text { ILK }= \pm 6: \text { Sphere } \\
& \text { ILK }= \pm 7: \text { Circular paraboloid }
\end{aligned}
$$

Figure 1 shows the basic input for each of the seven types. For the flat surfaces $( \pm 1, \pm 2, \pm 3)$ the positive value indicates the top or $+Z^{\prime}$ side of the surface is the active side, while the negative value indicates the bottom or - Z side. For the surfaces of revolution $( \pm 4, \pm 5, \pm 6, \pm 7$ ) the positive value indicates the outside of the surface is active while the negative value indicates the inside.

NVB, NVG - Number of nodes per surface
Each surface can be divided into nodes as shown in Fig. 2. The surface is divided into NVB nodes in the $\beta$ direction times NVG nodes in the $\gamma$ direction in equal increments of $\beta$ and $\gamma$, respectively. The total number of nodes on all surfaces may not exceed 150 .

$$
\begin{aligned}
& \sum_{\text {all surfaces }} \mathrm{NVB} * \mathrm{NVG} \leq 150 \\
& \text { and }
\end{aligned}
$$

The view factor between each two nodes is calculated and printed (if non zero) in the output listing.

NB, NG - Number of elements per node
Every node on a surface can, and usually should be, further divided into elements, The view factors between nodes are actually the summation of the view factors between elements. Also the shading is accounted for by checking to see if the element centers can "see" each other. If they cannot, the entire element is considered to be shaded. The accuracy of the view factors are therefore controlled by the size of the elements. NB is the number of elements in the $\beta$ direction and NG is the number of elements in the $\gamma$ direction in equal increments of $\beta$ and $\gamma$ respectively.

$$
\mathrm{NB} * \mathrm{NG}=\text { number of elements per node. }
$$

The total numbex of all elements may not exceed 2500 .

$$
\begin{aligned}
& \sum \mathrm{NVB} * \mathrm{NVG} * \mathrm{NB} * \mathrm{NG} \leq 2500 \\
& \text { all surfaces }
\end{aligned}
$$

A, BMIN, BMAX, GMIN, GMAX - Surface dimensions
The five quantities $A(\alpha), \operatorname{BMDN}\left(\beta_{\min }\right)$, $\operatorname{BMAX}\left(\beta_{\text {max }}\right)$, GMIN ( $\gamma_{\text {min }}$ ), and GMAX ( $y_{m a x}$ ) define the dimensions of the surface relative to its surface coordinate system ( $X^{\prime}$, $Y^{\prime}, Z^{\prime}$ ) as shown in Fig.1. De pending on the type of surface, they may represent either distances (any units of distance) or angles in degrees.

LLK $= \pm 1$ (Rectangles): Rectangles are, by definition, parallel to the $X^{\prime} Y^{\prime}$-plane with the side parallel to the $X^{5}$ and $Y^{8}$ axes. $\alpha$ is the distance of the rectangle above (or below) the $X^{\prime} Y^{\prime}$-plane. $\beta_{\min }$ and $\beta_{\text {max }}$ are the dimensions in the $Y^{\prime}$ direction and $\gamma_{\text {min }}$ and $\gamma_{\text {max }}$ the dimensions in the $X^{\prime}$ direction. All five may be either positive or negative; the only restrictions are that $\beta_{\text {min }}$ must be less than $\beta_{\max }$ and $\gamma_{\text {min }}$ less than $\gamma_{\text {max }}$. This means algebraically less - for example, $\left(\beta_{\mathrm{min}}=+2\right.$, $\left.\left.\beta_{\max }=+5\right), \beta_{\min }=-10, \beta_{\max }=+3\right), \beta_{\min }=-10, \beta_{\max }$ $=-4)$ ase all valid inputs; $\left(\beta_{\min }=-4, \beta_{\text {max }}=-10\right)$ and $\left(\beta_{\min }=+3, \beta_{\text {max }}=-10\right)$ are not. This basic restriction applies to all surface types.

ILK $= \pm 2$ (Discs): Discs are defined parallel to the $X^{\prime} Y^{\prime}$-plane with the centex on the $Z$ axis. $\alpha$ is the distance of the disc above (or below) the $X^{\prime} Y^{\prime}$-plane, $\beta_{\text {min }}$ and $\beta_{\text {max }}$ are the inner radius and outer radius, respectively, $\left(0 \leq \beta_{\min }<\beta_{\max }\right) . \quad \gamma \min$ and $\gamma \max$ are the angles that the radii defining the wedge required make with the $Y^{\prime}$ axis, measured about the $Z^{\prime}$ axis from the $+Y^{\prime}$ axis toward the $+X^{\prime}$ axis. Restrictions are $-360^{\circ}<$ $\gamma_{\min }<\gamma_{\max }=+360^{\circ}$ and $\gamma_{\max }-\gamma_{\min } \leq 360^{\circ}$.

ILK $=+3$ (Trapezoids): Trapezoids are, by definition, parallel to the $X^{\prime} Y^{\prime}$-plane with the apex on the $Z^{\prime}$ axis and the bases parallel to the $X^{\prime}$ axis. $\alpha$ is the distance of the trapezoid above (or below) the $X^{\prime} Y^{\prime}$-plane. $\beta_{\min }$ and $\beta_{\max }$ are the distances of the two bases along the $Y^{\prime}$ axis with the restriction that either $0 \leq \beta_{\min }<\beta_{\max }$ or $\beta_{\min }<\beta_{\max } \leq 0\left(\beta_{\min }\right.$ or $\beta_{\max }=0$ produces a triangle). $\gamma_{m i n}$ and $\gamma_{\max }$ are the angles made by the two sides with the $Y^{\prime}$ axis, measured about the $Z^{\prime}$ axis from the $+Y^{\prime}$ axis toward the $+X^{\prime}$ axis; restrictions are either $-90^{\circ}<\gamma_{\min }<\gamma_{\max }<+90^{\circ}$ (with $0 \leq \beta_{\min }<$ $\beta_{\text {max }}$ ), or $+900<\gamma_{\min }<\gamma_{\max }<+2700\left(\right.$ with $\beta_{\min }<$ $\beta_{\max } \leq 0$ ).

ILK $= \pm 4$ (Cylinde $x$ ) : Right circular cylinders are define $\bar{d}$ with the cylinder axis coincident with the $Z$ axis. $\alpha$ is the cylinder radius $(\alpha>0) . \beta_{\min }$ and $\beta_{\max }$ are the limits of the cylinder above (or below) the $X^{\prime} Y^{\prime}$-plane $\left(\beta_{\min }<\beta_{\max }\right) . \gamma \min$ and $\gamma_{\max }$ are the same as for the disc.
$I L K=+5$ (Cone): Right circular cones are defined with the apex at the origin of the $X^{\prime} Y^{\prime} Z^{\prime}$ coordinates and the cone axis along the $+Z^{\prime}$ axis. $\alpha$ is the cone half-angle $\left(0<\alpha<90^{\circ}\right)$. $\beta_{\text {min }}$ and $\beta_{m a x}$ are the limits of the cone above (not below) the $X^{\prime} Y^{\prime}$-plane measured parallel to the $Z^{\prime}$ axis $\left(0 \leq \beta_{\min }<\beta_{\max }\right) . \gamma_{\min }$ and $\gamma_{\max }$ are the same as for the disc.
$I L K= \pm 6$ (Sphere): Spheres are defined with the center of the sphere at the origin of the $X^{\prime} Y^{\prime} Z^{\prime}$ coordinates. $\alpha$ is the sphere radius $(\alpha>0) . \beta_{\min }$ and $\beta_{\max }$ are measured down from the $+Z^{\prime}$ axis $\left(0 \leq \beta_{\min }<\beta_{\max } \leq\right.$ 1800. $\gamma \min$ and $\gamma_{\max }$ are angles measured from the $+Y^{\prime}$ axis toward the $+X^{\prime}$ axis $\left(-3600<\gamma_{\min }<\gamma_{\max } \leq\right.$ $360^{\circ}$ ). If you compare the sphere with a globe of the earth, the $X^{\prime} Y^{\prime}$-plane is the equatorial plane, $+Z^{\prime}$ is the direction of the North Pole, $\beta_{\text {min }}$ and $\beta_{\text {max }}$ are lines of
co-latitude ( $90^{\circ}$ minus North latitude), and $\gamma_{\text {min }}$ and $\gamma_{\text {max }}$ are meridians of West longitude.


#### Abstract

ILK $= \pm 7$ (Circular paraboloid): Circular paraboloids are defined with the vertex (not the focus) at the origin of the $X^{\prime} Y^{\prime} Z^{\prime}$, coordinates and the axis of the paraboloid along the $+Z^{\prime}$ axis. $\alpha$ is the distance from the vertex to the focus $(\alpha>0) . \beta_{\min }$ and $\beta_{\max }$ are the distances above the $X^{\prime} Y^{\prime}$-plane, measured parallel to the $Z^{\prime}$ axis, of the part of the paraboloid required ( $0 \leq \beta_{\text {min }}<\beta_{\text {max }}$ ). $\gamma_{\text {min }}$ and $\gamma_{\text {max }}$ are the same as for the disc.


## Card No. 3

NCS - Intermediate coordinate system number.
In some cases, surface input data can be simplified by locating groups of related surfaces relative to an intermediate coordinate system (ICS) instead of the central coordinate system (CCS), and then locating the ICS relative to the CCS. The ICS is also useful when the problem must be run several times with the position of a group of surfaces being changed. When the position of a group of surfaces is to be changed, they should be input referenced to one or more ICS's. The entire group can be repositioned by changing the location of the ICS with respect to the CCS. If any surface contains a positive NCS number, the program will require the ICS data discussed later (negative or blank NCS's are ignored). There is no limit to the number of different NCS's (ICS's) or the number of surfaces that can be referenced to the same NCS number.

NCS $\leq 0$ (or blank): Position data on this card refers to the CCS.

NCS $>0$ : Position data on this card refers to ICS number NCS. The relationship between the ICS and CCS will be input later.

RX, RY, RZ - Distance between coordinate systems
Each surface is located by inputting the distance of the origin of the surface coordinate system ( $X^{\prime} Y^{\prime} Z^{\prime}$ ) from the origin of the CCS (X, Y, Z), if NCS is positive the CCS referred to here is actually the ICS, along the three ICS axes. $R X\left(R_{x}\right)$ is the distance along the $X$ axis, $R Y\left(R_{y}\right)$ along the $Y$ axis and $\mathbb{R}^{2}\left(\mathrm{R}_{\mathrm{Z}}\right)$ along the Z axis; as shown in Fig. l7a. Units must be the same as those used in defining the surfaces linear dimensions.

Coordinate System
$\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ - Central (Intermediate)


Fig. 17 - Orientation of Surface Coordinate System to Central Coordinate System

## PHI, PSI, OMEGA - Coordinate system orientation angles

The orientation of each surface with respect to the CCS (or ICS) is specified by the three angles PHI ( $\phi$ ), PSI $(\psi)$, and OMEGA $(\omega)$ which are input in degrees. These angles define three coordinate rotations that rotate the central coordinates ( $X, Y, Z$ ) into the surface coordinates ( $X^{\prime}, Y^{\prime}, Z^{\prime}$ ), as shown in Fig. 17b. If NCS is positive, these angles rotate the intermediate coordinates ( $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ ) into the surface coordinates ( $X^{\prime}, Y^{\prime}, Z^{\prime}$ ). The rotations are in the direction $X, Y, Z$ $\longrightarrow X^{\prime}, Y^{\prime}, Z^{\prime}\left(\operatorname{not} X^{\prime}, Y^{\prime}, Z^{\prime} \longrightarrow X, Y, Z\right)$ and must be carried out in the strict order of $\phi$, then $\psi$, then $\omega$.

PHI The yaw angle ( $\phi$ ) is the angle from the $X$ axis to the projection of the $X^{\prime}$ axis on the X - Y plane measured positive in the clockwise direction when viewed from the $+Z$ axis.

PSI The pitch angle $(\psi)$ is the angle from the projection of the $X^{\prime}$ axis on the $X-Y$ plane to the $X^{\prime}$ axis measured positive in the clockwise direction when viewed from the projection of the $+Y^{\prime}$ axis on the $X-Y$ plane.

OMEGA The roll angle $(\omega)$ is the angle from the projection of the $Y^{\prime}$ axis on the $X-Y$ plane to the $Y^{\prime}$ axis measured positive in counterclockwise direction when viewed from the projection of the $X^{\prime}$ axis on the $X$ - $Y$ plane.

Card No. 4
Each surface is described by inputting a set (Cards 1, 2 and 3) of surface data cards. A maximum of 150 surfaces (producing a maximum of 150 nodes) can be input. The surface description data are terminated by a card containing a negative number in column 5 (i.e., a dummy surface with a negative ISF number).

## Intermediate Coordinate System Card

The use of the intermediate coordinate system was discussed along with the NCS number. RAVFAC assumes that if a positive NCS number was given for a surface the data given on Card No. 3 related the surface coordinate system (SCS) to the intermediate coordinate system (ICS). A ICS card is input for each different NCS number included in the surface data cards to relate the ICS to the central coordinate system (CCS). If a ICS card is not given for a NCS number then the program will assume that the transformation data given in the surface data is that required to
reference the SCS to the CCS. After the last ICS card insert a card ( -1 in columns 9 and 10) to terminate the ICS data.

As stated previously the variables on the third surface data card relates the SCS to either the ICS or the CCS. The variables on the ICS card are the same as the third surface data card except they relate the ICS to the CCS. That is the RX, RY, and RZ are the distances from the origin of the CCS to the origin of the ICS along the three CCS axes. The $\phi, \psi$ and $\omega$ angles define the three coordinate rotations that relate the CCS into the ICS.

If any surface refers to an ICS, at least the termination (-1) card must be present. If no surfaces refer to an ICS, the presence of any ICS cards or of the ICS termination card will cause an error stop.

## END Card

Several cases can be analyzed on the same computer run by stacking cases one behind the other. However, if one case causes a program or computer error the remaining cases will not be processed. After the last (or only) case, insert a card containing END in columns 1, 2 and 3. This card will insure that all data tapes are rewound and give a normal run termination.

## Section 7

INPUT GUIDE

This section contains a summary of the required input parameters and the format of the data cards of the RAVFAC program. When the input parameters are fully understood, the summary can be used as an input guide without reference to other sections of this report. Each parameter is discussed in detail in Section 6, to which the reader is referred for further information concerning input parameters. Figure 16 may also be used as an input guide.

The input data for RAVFAC is, with the exception of the Case Control Card, the same as the view factor input data for the Lockheed Heat Rate Computer program (Ref. 3), although the variable names may be different in some cases. This format was chosen because it is straightforward and a number of large models have already been composed in this format. Some of the options available for the Heat Rate program have not been included in RAVFAC, but the view factor results from RAVFAC can be used as input to the Heat Rate program to calculate radiation resistor constants and orbital heat rates. The Heat Rate program uses the finite difference technique (the contour integral technique is not available) to calculate view factors and does not have the timesaving preshade option. Plans have been made to include the radiation resistor constant calculations in RAVFAC and to output onto magnetic tape the data required by the Heat Rate program to calculate the orbital heat rates.

The user is cautioned against inputting any parameters or comments in the portion of data cards which appear to be skipped. The reason is that as RAVFAC is expanded these columns will be used to read additional data. If data of the wrong formats are detected, a program error will result. The exception to this is the additional data on the Heat Rate program data cards.

RAVFAC was written for use on the Univac 1108 computer, although it could be modified for use on any computer having a 64 K core memory. The
most current production version of RAVFAC will be available for use at the NASA/MSFC Computation Laboratory. Use of the program requires computer control cards and input data cards.

### 7.1 UNIVAC 1108 COMPUTER CONTROL CARDS

A typical set of control cards required for each run is described below. In these descriptions, NNNNN is the reel number of the program tape and X is the tape unit on which the program tape is mounted. "Assignments" on the other ASG cards refer to the magnetic tape unit assignments for output from or input to RAVFAC.

RAVFAC Program Control Cards

1 (Card Column)
$\nabla R U N, T$
$\nabla A S G, T M \quad X, T$
$\nabla A S G, T \quad B, T$
$\nabla$ REWIND X 。
$\nabla \mathrm{COPIN} \quad \mathrm{X}_{\bullet}, \mathrm{TPF}$.
$\nabla$ FREE $\quad \mathrm{X}$ 。
$\nabla \mathrm{XQT}$
RAVFAC data deck - case l
RAVFAC data deck - case 2

END
$\nabla$ REWIND B.
$\nabla$ FIN

### 7.2 GENERAL FORMAT

The formats of the data cards that must be followed are of four basic types: (1) the 15 (integers); (2) the E10.5 (floating point); (3) the F5.3 (floating point without exponent); and the A6 alphameric field. The I5 consists of a

5-column field, without a decimal, and may be positive or negative (if no sign is given, the number is assumed positive). The last digit of the number must end in the last column of the field. The El0.5 consists of a 10 -column field as shown below

$$
\begin{aligned}
& 12345678910 \\
& \pm \times \mathrm{XXXXX} \pm \mathrm{XX}
\end{aligned}
$$

If the number is positive, the sign need not be punched in the first column of the field. Field columns 8,9 and 10 contain the exponent. A ( + ) in column 8 will cause the decimal, which is assumed to be between columns 2 and 3 as indicated by the .5 of E10.5, to be shifted XX places to the right and a (-) will cause it to be shifted to the left. If a decimal is punched, it will override the assumed position. The F5.3 consists of a 5-column field as shown below,

$$
\begin{aligned}
& 12345 \\
& \pm \\
& \pm \times X X X
\end{aligned}
$$

If the number is positive, the sign need not be punched in column 1. The decimal is assumed to be between columns 2 and 3 as indicated by the . 3 of F5.3. If a decimal is punched, it will override the assumed position. The A6 consist of a 6-column field which can contain any standard alphameric (A-Z, 0-9, + , l, etc.) data.
7.3 DATA INPUT GUIDE
FIELD PROGRAM
Location Variable Explanation

Title Card - Format 13A6, A2
Columns 1-80 Title
Any comment can be made on this card to identify the case.

FIELD
Location

PROCRAM
Variable

## Explanation

Case Control Card - Format 4T5, 10x, El0.5, 20x, 415
Columns $1-5$ NSTART Restart control flag
$\geq 0$ Normal case requiring surface data cards.

RO restart case using data tape |NSTART|.

Columns 6-10 NT

Columns 11-15

Columns 16-20

Columns $31=40$

NVEACL

NORM

RMAX

Output tape which can be used for a restart case in case of termination.
$\leq 0$ restart tape will not be made.
$>0$ restart data written on tape NT which must be assigned in the computer control cards.

View factor calculation technique option.
$=1$ Finite difference technique.
$=2$ Contour integral technique.
$=3$ Program selects technique to use based on
$\frac{A}{x^{2}}>$ RMAX - Contour Integral
$\frac{A}{r^{2}} \leq R M A X-F i n i t e$ Difference.
View factor normalization option.
$\leq 0$ view factors are not normalized.
$=1$ view factors are normalized if

$$
\sum V F_{n}>1
$$

$\geq 2$ view factors are normalized so
that

$$
\sum V F_{n}=1
$$

Maximum area-to-distance ratio. If a. value is not given $\mathrm{RMAX}=0.1$ will be used.

FIELD
Localion

Columns 61-65

Columns 66-70

Columns 71-75

Columns 76-80

PROGRAM
Variable

NPRT Immediate output flag.
$\leq 0$ no immediate output.
$>0^{*}$ time required to calculate each $5 \%$ of the view factors is printed.
$=2$ each view factor is printed as it is calculated.

Element override option.
$\leq 0$ no override. Number of elements taken from surface data cards.
$>0$ one element per node.
Shading override option.
$\leq 0$ shading will be considered using shading flags on surface data cards.
$>0$ and $\neq 2$ all shading is neglected.
$=2$ shading will be considered using shading flags on surface data cards, but PRESHD is bypassed.

NTVF View factor tape option.
$\leq 0$ no view factor tape written.
$>0$ view factors are written on tape NTVF.

Surface Data Cards - 3 cards per surface

The description of each surface consists of a set of three (3) cards. The complete set is input for each surface followed by a set for the next surface. After the last surface is input, a termination card (card No.4) is required. The maximum number of nodes is 150 and the maximum number of elements is 2500 .

Card No. 1 - Format I5, 2I1, 43X,5A6
Columns 1-5 ISF
First node number on the surface.
$>0$ first node number.
$<0$ termination card (see card No. 4 )
$=0$ first node number will be one plus the last node number of the preceding surface

[^4]FIELD

Location
Column 6

PROGRAM
Variable Explanation

KSH Can shadeflag. $=0$ surface cannot cause shading. $>0$ surface can cause shading.

Column 7 KSBH Can be shaded flag.
$=0$ surface cannot be shaded.
$\geqslant 0$ surface can be shaded.
Columns 51-80 COMM Any comment can be made in these columns to identify surface.

Card No. 2-Format 5X, 5I5, 5E10.5

Columns 6-10
ILK

Columns 11-15 NVB
Columns 16-20
Columns 21-25

Columns 26-30 NG

Columns 31-40
Columns 41-50
Columns 51-60
*Refer to Fig. 1.

Surface type.
$\pm 1$ Rectangle
$= \pm 2 \mathrm{Disc}$
$\pm \pm$ Trapezoid
$= \pm 4$ Cylinder
$= \pm 5$ Cone
$= \pm 6$ Sphere
$\pm 7$ Circular Paraboloid
A positive ILK indicates that the active side of the surface is in the + direction (or outside of surface). A negative ILK indicates that the active side is in the - direction (or inside of surface).

Number of nodes in the $\beta$ direction.
Number of nodes in the $\gamma$ direction.
Number of elements per node in the $\beta$ direction.

Number of elements per node in the $\gamma$ direction.
Surface dimension $\alpha^{*}$
Surface dimension $\beta_{\min }$ *
Surface dimension $\beta_{\max }$.

FIEI, D
Localion
Columns 61-70
Columns 71-80

PROGRAM
Variable
Explanation
GMIN Surface dimension $\gamma_{\min }{ }^{*}$ *
GMAX Surface dimension $\gamma_{\max }$.

[^5]Intermediate Coordinate System Card - Format 5X, 15, 10X, 6E10.5

An intermediate coordinate system (ICS) card is required for each different NCS number used in the surface description. If an ICS card is not given, the program will assume that the data given on the NCS card relate to the CCS instead of the ICS. If any NCS was input as a positive value the termination card is required.

FIELD
Location
Columns 6-10
Columns 21-30

| Columns 31-40 | RYR | Same as RXR except that it is measured along the $Y$ axis. |
| :---: | :---: | :---: |
| Columns 41-50 | RZR | Same as RXR except that it is measured along the $Z$ axis. |
| Columns 51-60 | PHIR | Coordinate rotation angle $\phi$ for rotating the CCS into the ICS; rotates $Y$ toward X about Z . |
| Columns 61-70 | PSIR | Coordinate rotation angle $\psi$ for rotating the CCS into the ICS; rotates X toward $Z$ about $Y$. |
| Columns 71-80 | OMEGAR | Coordinate rotation angle $\omega$ for rotating the CCS into the ICS; rotates $Y$ toward Z about X . |

NOTE: The rotations are in the direction of CCS $\rightarrow$ ICS and must be carried out in the strict order $\phi$, then $\psi$, $\omega$. After the last ICS has been defined the following card is required.
Columns 6-10 ICS Intermediate coordinate system data termination card.
$<0-$ ICS data are complete.
$>0$ - ICS data are not complete.

End Card - Format A6

Several different cases can be processed on the same run by placing the data deck for one case behind another case. An END card is required after the last (or only) case.

## FIELD

Location

PROGRAM
Variable

Explanation

Columns 1-3

Columns 4-6

END
The alphameric characters END are punched on the data card.

These columns must be blank.

## Section 8 <br> DESCRIPTION OF PROGRAM OUTPUT

In this section the output from RAVFAC is discussed. The symbol is used to identify each group of data. This same symbol has been used in the Sample Problem Output (Appendix A, Fig. A-4). The reader should refer to Fig. A-4 while reading this section.
(1) An LMSC title is printed at the start of each run.
(2) * The date and time run was started.
(3) Case title.
(4) Case control card variables are printed along with the options available.
(5) Surface data.
(5a) Summary of surface data Card No.l.
(5b) Surface radiation properties - currently not used.
(5c) Summary of surface data Card No. 2 .
(5d) Summary of surface data Card No. 3 .
(6) Intermediate coordinate system data (for each ICS number).
(7) Total number of surfaces, nodes and elements used.
(8) Surface data summary.

- All transformations have been made to the central coordinate system.
- Angles are in radians.
- The first digit of KS is the "can be shaded" flag and the second digit is the "can shade" flag.
(9) Node Areas.
- The units of length are the same as input for the surface data.

[^6]Table 8-1
ELAPSED TIMES

| ELT = | Zero <br> Reference | Comment |
| :---: | :---: | :---: |
| 1 | a | Start the writing of the restart tape. |
| 2 | a | Surface data have been processed and written. |
| 3 | a | Start reading the restart tape. |
| 4 | a | Surface data have been read from the restart tape. |
| 5 | a | View factors have been read from the restart tape. |
|  | b | Number of view factor calculations completed (only printed if requested). |
| 6 | b | View factors have been calculated. |
| 7 | b | Vew factors have been written. |
| 8 | b | View factors have been normalized. |
| 9 | b | Case completed. |
| Elapsed times are referenced to: |  |  |
| a. After writing the LMSC title and before reading the case the card. |  |  |
| b. The start of the view factor calculations. |  |  |

- Nodes are listed in the order in which they are stored, based on the order in which the surfaces were input.
- The number in parenthesis is the node ID number (ISF).
(10)* Time required to process surface data: hh:mm:ss where
hh is hours
mm is minutes
ss is seconds
(11) ${ }^{*}$ Time required to calculate each $5 \%$ of the element-to-element view factors listed as N of T
where
N is the number of dVFs calculated
$T$ is the total number of dVFs to be calculated.
Usually there will not be twenty of these times because when a node is skipped its contribution to the dVF count is not added in.
Time required to calculate all view factors. The start of the view factor calculation is used as the reference time for IELTs greater than 2.
(13) View factor array $-\mathrm{N}, \mathrm{J}, \mathrm{VF} \underset{\mathrm{N} \rightarrow \mathrm{J}}{ }$

Two sets of these values are printed.
(13a) The node numbers are those used by the program (i.e., based on the order of input).
(136) The node numbers are based on the surface index number (ISF)
These arrays are printed in groups of three in the order of nodes ( $N$ ) used by the program. Each $N$ node is started on a new line. The view factor does not have the area term included. Only non-zero view factors are printed.

Time required to calculate and print view factors.
Normalized view factors (not shown in Fig. A-4). These are the view factors after normalization has occurred. Format is the same as (13).
(15) Total view factor for each node. The program index number (the user's ID number is in parenthesis) and the sum of all view factors for this node are given. If normalization is requested these will be normalized values.
Elapsed time from the start of view factor calculations to the end of the case.
(17) Case completed statement.
(18) Data and time the run was completed.

During a run RAVFAC makes checks to determine if its limitations have been exceeded or if the restart tape is being processed correctly. If an error is detected, the following message is printed:

ERROR TYPE $* * * n * * *$ ENCOUNTERED. RUN TERMINATED.
where $n$ denotes the type of error as shown in Table 8-2.

Table 8-2
ERROR MESSAGE

| Error Type (n) | Meaning |  |
| :---: | :---: | :---: |
| 1 | Maximum number of surfaces has been exceeded. |  |
| 2 | Incorrect surface type. |  |
| 3 | Output (or restart) tape unit not allowed. |  |
| 4 | Maximum number of nodes has been exceeded. Maximum number of elements has been exceeded. |  |
| 5 |  |  |
| 6-9 | Maximum number of elements has been exceeded. Not presently used. |  |
| 10 | NTRAN error - writing array | TITLE |
| 11 |  | TII |
| 12 |  | BMIN |
| 13 |  | MS 1 |
| 14 |  | MSM |
| 15 |  | UN1 |
| 16 | 1 | PN1 |
| 17 | NTRAN error - Reading array | TITLE |
| 18 |  | TIl |
| 19 |  | BMIN |
| 20 |  | MS 1 |
| 21 |  | MSM |
| 22 | , | UMI |
| 23 | $\gamma$ | PM1 |

This error message is printed to aid the user in determining what caused the problem to terminate. If $n \geq 10$ the error is associated with reading or writing the restart tape. This is usually caused by a systems error which is beyond the user's control (suggest resubmitting job).

## Section 9

## CONCLUSIONS AND RECOMMENDATIONS

The radiation view factor (RAVFAC) digital computer program is operational on a productional status. From the many check cases which were run, the conclusion was reached that the three options for calculating the view factors should allow the user to obtain accurate results economically. The following guidelines should be used.

- Problems having $A / r^{2}<0.1$ ( $A$ is based on element) - finite difference.
- Problems having some $A / r^{2}>0.1$ - let program select method.
- Problems where the most accurate answers are required contour integral.
- Flag all surfaces that cannot cause shading or be shaded.
- Input first those surfaces which cause shading on the most elements.

RAVFAC was developed as a basic view factor program. During the development of RAVFAC several areas were noted where improvements could be made and additional capabilities that were needed. Although these improvements and capabilities were beyond the present scope of work some of the preliminary work was done. Also during the logic development and coding, provisions were made so that the necessary modifications could be made without having to make major changes to the present version of the program. These improvements and extension of capabilities are discussed in the remainder of this section.

- Semi-transparent surfaces - The fully transparent surfaces can be evaluated using the "cannot shade" flag. However, most"transparent"surfaces cause some blockage of the radiant energy which passes through them. To account for the transparent effects, the view factor must be modified when a transparent surface is encountered. Although the view factors are based on diffuse radiation, the transmissivity is often largely dependent on the incident angle. This effect should be accounted for.
- Emissivity, absorptivity and reflectivity effects - Usually, the view factors are the first step in calculating the temperature distribution of a given system. After the view factors are obtained, the effects
of surface radiation properties are included. Next, the multiple reflections are incorporated and the data input into the appropriate computer program. A routine should be written for RAVFAC to include the effects of surface properties and multiple reflections. These data can then be punched into data cards or written on tape in a format suitable for the computer programs that will be used to calculate the temperature or heat rate.
- Contour integral improvement - At present, view factor calculations are based on approximating the element with a rectangular plate. Work was done on solving the contour integral equation for the various surface types. These equations can be included in RAVFAC to provide more accurate view factor equations. Another benefit of these equations is that the view factor can be calculated on a node-to-node basis, instead of element-to-element, if the two nodes are not shaded. This reduces run time.
- Additional surface types - Additional surface types will be helpful when complex configurations are modeled. Listed below are several surface types which should be included:
- Box (5 and 6 sided)
- Ellipsoid of revolution
- Hyperboloid
- Specular radiation - Most of the thermal analyses are based on assumed diffuse radiation. However, as design requirements become more demanding, more accurate view factors will be required. Little work has been accomplished in applying specular radiation and combined specular and diffuse radiation to general problems.
- Program optimization - During the development of RAVFAC much attention was given to decreasing the run time and increasing the maximum number of nodes and elements. However, an additional effort is required to optimize the total program. Results of this optimization will be reduced cost per run and increase in capability to analyze larger problems.


## REFERENCES

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2. Sparrow, E.M., "A New and Simple Formulation for Radiative Angle Factors," Paper No. 62-HT-17, ASME/AICHE Heat Transfer Conference and Exhibit, Houston, Texas, August 5-8, 1962.
3. Newby, T.S., 'Heat Rate Computer Program - User's Manual," TXA 1954, Lockheed Missiles \& Space Company, Sunnyvale, Calif., 26 October 1966.
4. Radio Corporation of America, "A Computer Program to Calculate Radiative Configuration Factors," LTM-(H)-7009-10, 4 January 1965.
5. Toups, K.A.s "A General Computer Program for the Determination of RadiantInterchange Configuration and Form Factors -Confac II," North American Aviation, Inc., SID 65-1043-2, Downey, Calif., October 1965.

Appendix A

## SAMPLE PROBLEM

## Appendix A

This Appendix presents a sample problem, the objective of which is to familiarize the reader with the steps to be taken to obtain a solution for a radiation view factor problem. Basic data are given as well as a listing of the input data. Program output is also presented.

View factors between seven different types of surfaces are to be calculated. In Fig. A-1 a cross-sectional view of the sample case is shown. A central, an intermediate and the individual coordinate systems of each surface are also shown.

The surfaces consist of the following types:

- Surface 1 Rectangular plate
- Surface 2 Sphere
- Surface 3 Cone
- Surface 4 Circular paraboloid
- Surface 5 Cylinder
- Surface 6 Trapezoidal plate
- Surface 7 Circular plate

Surface 1 is divided into two nodes in the Beta direction and four nodes in the Gamma direction. The remainder of the surfaces are divided into two nodes, for both the Beta and Gamma directions, respectively. The nodes of surface $l$ are divided into three elements in the Beta direction and five elements in the Gamma direction. The nodes of the other surfaces are divided equally into two nodes in the Beta and Gamma directions. An intermediate coordinate system is used for referring the surface coordinate system of surface 3 (cone), to the central coordinate system. The surface coordinate systems of all other surfaces are referred directly to the central coordinate system.

View factors are calculated using either the finite difference or contour integral method based on a RMAX $=0.01$. The effects of shading are being considered. The view factors are not normalized. Tape A is utilized for a possible restart, and the time required to calculate the view factors is printed.

The location and orientation of the surfaces are shown in Fig. A-1, and given with respect to the central coordinate system. Surface dimensions are given in Fig. A-2. Based on this information, the required input data can be generated. A listing of these data is shown in Fig. A-3. The sample problem output is shown in Fig. A-4.


Fig. A-1 - Sample Problem (Cross-Sectional View)


Fig. A-3 - Sample Problem Input
A. 6








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Appendix B INTEGRAL TECHNLQUES

## Appendix B

Several comparisons were made of the accuracy of the finite difference and the contour integral techniques. For the simple shapes, comparisons were based on analytical results; this was not possible for the more complex shapes since analytical techniques are not available. Results of the comparisons indicated that the contour integral technique is more accurate than the finite difference. If the area-distance ratio $\left(A / C^{2}\right)$ is less than 0.1 , however, the accuracy of the finite difference technique is sufficient for most engineering problems.

A series of runs was made in which the configuration shown on Figs. B-la, $b$, and $c$ was used. The distance (C) between the plates was changed in each run. These runs illustrate the dependency of the area/distance ( $A_{1} / C^{2}$ ) ratio as well as an accuracy comparison of the finite difference and contour integral techniques. Each surface consisted of one node and one element.

Results of these runs are presented in Table B-l and Figs. B-la, b, and $c$. When these results are reviewed, it should be kept in mind that most of these runs were made with the plates placed extremely close to each other (large $A_{1} / C^{2}$ ), and as stated in the main report, one should model so that

$$
\frac{A}{C^{2}} \leq 0.1
$$

As more elements per node are used, the accuracy of both techniques improve as shown in Figs. B-2a, b, and c. However, RAVFAC, as well as other view factor programs, is limited in the size of problems that it can process. Also the run time increases as the number of elements increases. If the problem is such that $\left(A / C^{2}\right)$ is greater than 0.1 is unavoidable, the contour integral technique should be selected (NVFCAL $=2$ or 3 ).

Table B-1
PARALLEL FLAT PLATES

| C | $\mathrm{A}_{1} / \mathrm{C}$ | $\mathrm{A}_{1} / \mathrm{C}^{2}$ | Exact ${ }^{1}$ | View Factor |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Finite Difference ${ }^{2}$ | Contour <br> Integral ${ }^{2}$ |
| 0.0152 | 66.0 | 4356.0 | . 97059 | 1386.6 | . 99925 |
| 0.0160 | 62.5 | 3906.0 | . 96899 | 1243.4 | . 99916 |
| 0.0278 | 36.0 | 1296.0 | . 94716 | 412.5 | . 99748 |
| 0.0400 | 25.0 | 625.0 | . 92525 | 198.9 | . 99479 |
| 0.0667 | 15.0 | 225.0 | . 87981 | 71.6 | . 98567 |
| 0.100 | 10.0 | 100.0 | . 82699 | 31.8 | . 96834 |
| 0.133 | 7.5 | 56.25 | . 77806 | 17.9 | . 94511 |
| 0.200 | 5.0 | 25.00 | . 69024 | 8.0 | . 88465 |
| 0.250 | 4.0 | 16.00 | . 63204 | 5.1 | . 83103 |
| 0.286 | 3.5 | 12.25 | . 59399 | 3.9 | . 79041 |
| 0.222 | 3.0 | 4.00 | . 54738 | 2.9 | . 73520 |
| 0.400 | 2.5 | 6.25 | . 48922 | 2.0 | . 65914 |
| 0.500 | 2.0 | 4.00 | . 41525 | 1.3 | . 55413 |
| 0.667 | 1.5 | 2.25 | . 32006 | 0.71620 | . 41285 |
| 1.0 | 1.0 | 1.00 | . 19982 | 0.31831 | . 23946 |
| 2.0 | 0.5 | 0.25 | . 06859 | 0.07578 | . 07348 |
| 4.0 | 0.25 | 0.063 | . 01911 | 0.01989 | . 01949 |
| 8.0 | 0.125 | 0.016 | . 00492 | 0.00497 | . 00497 |

Traken from NACA Report TN 2836, "Radiant Interchange Configuration Factors:" D.C. Hamilton and W.R. Morgan, December 1952.
${ }^{2}$ One element/node

$$
B-2
$$


Fig. B-la - View Factor vs A/C for Ratios $1 \leq A / C \leq \infty$



Fig. B-1c - View Factor vs $A / C$ for Ratios $0 \leq A / C \leq 1.0$

Fig. B-2a - View Factor vs $\mathrm{A} / \mathrm{C}$ for Ratios $1 \leq \mathrm{A} / \mathrm{C} \leq \infty$



Appendix C

RAVFAC COMPARISON

## Appendix C

In this appendix, results obtained with the RAVFAC Computer Program are compared to the results obtained with other existing computer programs. Also, the results of several sample cases are compared to data which are found in the open literature.

Two existing diffuse radiation view factor computer programs were used in this comparison study. These programs are CONFAC II (Ref. C.1) and CONFAC III (Ref. C.2). The CONFAC II Computer Program uses the geometric unit sphere method and CONFAC III uses a contour integral technique nique. The RAVFAC results were obtained by using both the double summation and contour integral techniques. One of the sample cases used consisted of two equal parallel flat plates in which both the dimensions and distance between the plates were varied, as well as the number of elements used. The results of these cases are shown in Table C-1 and Fig. C-1. The exact solutions are also shown in Table C-1, and were obtained by evaluating the closed form solution with the aid of a digital computer program. The grid size used in subdividing the flat plates were $2 \times 2$ and $24 \times 24$. The maximum grid size used with CONFAC III was 9 x 9 , since this is the maximum grid size available for that program.

Table C-l shows that the best results were obtained for this particular case by using CONFAC III, for the $2 \times 2$ grid. For the $24 \times 24$ grid the best results were obtained by using RAVFAC's contour integral technique. Note that both CONFAC II and CONFAC III were written for limited body geometries. CONFAC II is restricted such that the flux source is a plane polygon and the receiver an arbitrary polyhedron. CONFAC III is restricted to evaluate configuration factors between plane polygons only.

In Figs. $C-2, C-3$ and $C-4$ and Tables $C-2, C-3$ and $C-4$ several sample cases and their results are shown. These results were obtained both from the literature and using RAVFAC. In these cases RAVFAC view factors were obtained using the combined double summation and contour integral techniques according to the size of the elements and the distance between them (i.e., $\Delta \mathrm{A}_{\mathrm{e}} / \mathrm{r}_{\mathrm{e}}^{2} \leq .01$ ).

The sample case shown in Fig. C-2 consists of a flat plate and a cylinder. RAVFAC and CONFAC II were used to calculate the radiation view factor for this configuration. Also, view factors for this case were taken from Ref. C.3. The results are also shown in Table $\mathbb{C}-2$.

Figures C-3 and C-4 are sample cases of sphere-to-sphere and sphere-to-cone view factors. View factors for this configuration were calculated using RAVFAC, and compared to data from Ref. C. 4. The results are shown in Tables C-3 and C-4, respectively. In these last two sample cases the view factors could not be calculated with either CONFAC II or CONFAC III since both of these programs are restricted by the geometry of the surfaces. The results from the literature shown in Tables $C-2, C-3$ and $C-4$ were obtained from parametric plots of the sample cases shown in Figs. $C-2, C-3$ and $C-4$, respectively. These plots were calculated numerically since a closed form solution has not been found for these sample cases.

From this comparison study the conclusion is reached that the answers given by both CONFAC III and RAVFAC's contour integral technique were the best for the sample case shown in Fig. C-1. Since CONFAC III also uses a contour integral technique for plane polygons, it can also be concluded that the contour integration yields better results than both the double summation and the unit sphere method.

Results from the sample cases shown in Figs. $C-2, C-3$ and $C-4$ show that the view factors calculated by RAVFAC using the combined double summation and contour integral techniques are quite close to the values obtained

$$
C-2
$$

from the literature. Since the values found in the literature were not evaluated by closed form solutions but by numerical schemes with the aid of digital computers, it cannot be concluded which of the two values is closest to the exact solution.

From these comparisons two major conclusions can be drawn:

- The contour integral method yields the best results when the distance between the elements is small compared to the magnitude of the areas, while the double summation method yields accurate results when the distance between the elements is large compared to the magnitude of the areas, in a simpler and faster way. The RAVFAC Computer Program has both the contour integral and double summation methods for calculating view factors.
- The versatile RAVFAC Computer Program permits configuration factors between seven different types of surfaces and their possible combinations to be evaluated, while most of the other existing programs allow only configuration factors between plane polygons and polyhedrons to be evaluated.


## Appendix C

REFERENCES
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C. 3 Sparrow, E.M., and R.D. Cess, Radiation Heat Transfer, Brooks-Cole Publishing Company, Belmont, Calif., 1966.
C.4. Campbell, P.J., and G.D. McConnell, "Radiant Interchange Configuration Factors for Spherical and Conical Surfaces to Spheres," NASA TN D-4457, Lewis Research Center, Cleveland, Ohio, April 1968.

Fig. C-1 - Two Equal Parallel Flat Plates



1 Flat Plate
2 Cylinder
$a=2$
$b=4$
$\mathrm{c}=4$
$d=1$

Fig. C-2 - Flat Plate to Cylinder

Table C-2
RESULTS OF SAMPLE CASE SHOWN IN FIG. C- 2

| View Factor | Appendix A <br> Ref. C.3 | RAVFAC | CONFAC II |
| :---: | :---: | :---: | :--- |
| $\mathrm{F}_{1-2}$ | 0.275 | 0.30700 | 0.29950 |
| $\mathrm{~F}_{2-1}$ | 0.175 | 0.19544 | 0.15535 |

$$
C-6
$$



Fig. C-3-Sphere-to-Sphere

Table C-3
RESULTS OF SAMPLE CASE SHOWN IN FIG. C-3

| View Factor | Reference C.4 | RAVFAC |
| :---: | :---: | :---: |
| $\mathrm{F}_{1-2}$ | 0.060 | 0.05802 |
| $\mathrm{~F}_{2-1}$ | 0.060 | 0.05802 |



Fig. C-4 - Cone to Sphere

Table C-4
RESULTS OF SAMPLE CASE SHOWN IN FIG. C-4

| View Factor | Ref.C.4 | RAVFAC |
| :---: | :---: | :---: |
| $\mathrm{F}_{1-2}$ | 0.102 | 0.10167 |
| $\mathrm{~F}_{2-1}$ | 0.036 | 0.03594 |

```
DEP-T
A&IS-PAT
PM-PR-M, Mr. Goldston
A&TS-MS-H
A&TS-MS-IP
A&TS-MS-IL (8)
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    Mr. Brewer
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    Mr. Elkin (6)
    Mrs. Hightower
S&E-ASTN-PT
    Mr. Vaniman
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[^0]:    *The direction cosines were derived from trigonometric relations obtained from Fig. 10.

[^1]:    * A gray surface is defined as a surface for which the monochromatic emittance is constant with respect to wavelength and considered to be nondirectional or diffuse in character.

[^2]:    * At present the time routines are not available under Exec VIII.

[^3]:    *Almost all the computer time is taken to calculate view factors.

[^4]:    *This option is not presently available under Exec VIII.

[^5]:    *Refer to Fig. 1.
    Card No. 3 - Format 5X, 15, 10X, 6E10.5

    Columns 6-10 NCS

    Columns 21-30 RX

    Columns 31-40 RY

    Columns 41-50 RZ

    Columns 51-60 PHI

    Columns 61-70

    Columns 71-80
    OMEGA
    PSI

    Intermediate coordinate system number. $\leq 0$ data on this card relates the surface coordinate system (SCS) to the central coordinate system (CCS).
    $>0$ data on this card relate the SCS to the immediate coordinate system (NCS).
    Distance of the origin of the SCS from the origin of the CCS (or ICS) measured along the $X$ axis of the CCS (or ICS).
    Same as RX except that it is measured along the Y axis.

    Same as RX except that it is measured along the $Z$ axis.

    Coordinate rotation angle $\phi$ for rotating the CCS (or ICS) into the SCS; rotates $Y$ toward X about Z .
    Coordinate rotation angle $\psi$ for rotating the CSC (or ICS) into the SSC; rotates X toward Z about Y .

    Coordinate rotation angle $\omega$ for rotating the CSC (or ICS) into the SCS; rotates Y toward Z about X .

    NOTE: The rotations are in the direction of CCS (or ICS) $\rightarrow$ SCS and must be carried out in the strict order $\phi$, then $\psi$, then $\omega$.

    Card No. 4 - Format 15
    Columns 1-5 ISF

    Surface description input termination flag.
    < 0 surface description has been completed.
    $\geq 0$ this is a Card No. 1.

[^6]:    *These times are presently not available on the Exect VIII system. Refer to Table 8-1 for elapsed time information.

