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TECHNICAL
MEMORANDUM

MULTIPLE SCATTERING CALCULATION
FOR PLANETARY ATMOSPHERES

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ABSTRACT

An iterative technique for calculating the multiple
scattering effect in planetary atmospheres has been developed.
Various studies have been conducted to insure the accuracy of
the technique and of the computer programs which carry out the
calculations.

The technique, while intended for a spherical geometry,
can be adapted to a planar geometry, for which exact calculations
are available through the method developed by Chandrasekhar.
Comparisons of the apparent brightness were made for a haze
model which had been developed previously to explain the photo-
metric results of Mariner IV. For the planar geometry, the
iterative technique produces results which agree with the exact
calculations to better than 1/2%.

For the spherical geometry, it was found that the
effect of multiple scattering was to increase the apparent
brightness by about 20% when the line of sight intersected the
surface, and by up to a factor of two when the line of sight lies
above the limb of the planet.

It is concluded that the iterative technique provides
an accurate method of calculating multiple scattering effects
in planetary atmospheres.

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TECHNICAL MEMORANDUM

1.0   INTRODUCTION

The purpose of this memorandum is to report on efforts to carry out multiple scattering calculations for planetary atmospheres. The specific problem for which the calculations have been made is the scattering of light in the Martian haze that has been suggested[1] to explain the data from Mariner IV.

The properties of such a Martian haze have already been studied in considerable detail on the basis of first-order calculations.[2] However, it was recognized that multiple scattering processes would cause a major effect, and would have to be included in the calculations in order to obtain useful accuracy. A method to carry out the multiple scattering calculations has been developed. In this memorandum, some preliminary calculations which were used to test the validity and accuracy of the method are reported.

The Martian haze was evoked to explain the observations made by Mariner IV. However, similar phenomena were not observed in the recent Mariner VI and VII missions. This has given additional credence to the suspicion that the data from Mariner IV were affected by a malfunction in the camera or spacecraft system,[1][3] although this opinion does not have universal acceptance.

Our purpose here is not to argue the reality of the haze, but to verify the accuracy of a procedure to perform multiple scattering calculations. The haze is a convenient model to use because of the previous work with which the calculations can be compared.

In particular, exact multiple scattering calculations have been carried out[2] using the classic methods developed by Chandrasekhar.[4] The exact calculations of Chandrasekhar are applicable only to planar atmospheres, while it is necessary, in our case, to consider the spherical geometry of the planet. For example, the brightness above the limb of a planet has no parallel in planar atmospheres.

The procedure that has been developed can be adapted to either spherical or planar geometry. The results of the planar case, compared with the exact calculations, give an indication of the overall accuracy of the technique. In addition, results for the spherical geometry have been included to illustrate the effect of multiple scattering in that case.

It is worth noting that the procedures developed for multiple scattering calculations are applicable to problems other than Martian haze. At the present time, the area of greatest interest is Rayleigh scattering in the gaseous atmosphere of Mars. Preliminary calculations[5] have been carried out to illustrate how brightness measurements above the limb of a planet can be used to deduce the density profile of the atmosphere. Multiple scattering effects must be included in the atmospheric problem in order to achieve high accuracy.

In some of the following discussions, the comments do not depend on the particular properties of the material surrounding the planet. In such cases the term "atmosphere" has been used in a general sense to refer to any medium surrounding a planet.

2.0 METHOD

The brightness seen by an observer looking through a planetary atmosphere or a haze obeys the relation

$$\frac{d\omega}{d\chi} = -\sigma(\chi)\omega(\chi,0,0) + \beta(\chi,\theta_s)\pi F e^{-\tau_s} + \int \beta(\chi,\theta)\ \omega(\chi,\theta,\phi)d\Omega \qquad (1)$$

where $\omega(\chi,\theta,\phi)$ is the luminous intensity at the point χ incident on a volume element from a direction whose polar coordinates are θ,ϕ (see Figure 1). The direction $\theta=0$ corresponds to light moving toward the observer. πF is the flux in the incident solar radiation, θ_s is the polar angle for the direction of the incident radiation, and τ_s is the optical thickness between the source of the incident radiation and the volume element at the point χ. The distance χ is measured along the line of sight, increasing toward the observer.

$\beta(\chi,\theta)$ is the fraction of the incident light in a unit solid angle that is scattered into a unit solid angle centered at an angle θ to the initial direction, per unit length travelled by the light. The function β describes completely the scattering properties of the atmosphere or haze. The function $\sigma(\chi)$ represents the extinction coefficient for the atmosphere. The dependence on χ for both $\beta(\chi,0)$ and $\sigma(\chi)$ arises from variations of the density of the atmosphere.

In the case of isotropic scattering, with which we will be exclusively concerned in this memorandum, β does not depend on θ.

We may define the scattering coefficient b through the relation

$$b(\chi) = \int \beta \, (\chi,\theta) \, d\Omega = 4\pi\beta \tag{2}$$

and Equation(1) may be written

$$\frac{d\omega}{d\chi} = - \sigma(\chi) \; \omega(\chi,0,0) \; + \; \frac{b(\chi)}{4\pi} \; \pi Fe^{-\tau}s + \; \frac{b}{4\pi} \int \omega(\chi,\theta,\phi)d\Omega \tag{3}$$

In Equation (1), the first term on the right hand side accounts for the attenuation of the light already travelling toward the observer, the second term corresponds to scattering of the incident solar radiation, and the third term comes from the scattering of the diffuse light incident on the volume element at χ from the surrounding haze and the surface of the planet. It is the third term in Equation(1), the integral, that accounts for the multiple scattering, since it represents the scattering of light which has already been scattered at least once. In the first order calculations, where only a single scattering is considered, the integral is neglected. A more detailed discussion of Equation (1) has been given previously[2].

The integro-differential equation (1) can be solved exactly for planar atmospheres with a plane parallel beam of radiation incident on the atmosphere[4]. However, for a spherical atmosphere with a plane parallel incident beam, no such solution is known. An accurate approximate solution can be obtained by numerical methods, as follows:[6]

Suppose we have an estimate of the brightness at each point in the atmosphere, for all directions. We call such an estimate a data base. The data base consists of a list of brightness values for certain directions at a selected set of points throughout the atmosphere. The geometry is shown in Figure 2. A point in the atmosphere is defined by λ, the sun angle, and h, the height above the surface. Cylindrical symmetry is assumed to exist about the direction of solar radiation. The angles θ and ϕ define a local polar coordinate system in which the direction of the incident radiation is measured.

The atmospheric brightness data is stored for a finite set of points distributed throughout the atmosphere. These local points are defined by the intersection of lines of constant height,h_j and lines of constant sun angle, λ_i, where the values h_j, λ_i are chosen to meet the requirements of the problem. At each local point, the brightness is stored for each of a set of directions defined by the angles θ_k, ϕ_k. This set of angles is the same for each local point.

At each local point, there is reflection symmetry in the plane defined by the direction of the incident radiation and the radius vector from the center of the planet. This symmetry is a direct result of the overall cylindrical symmetry of the problem. Thus the set of angles θ_k, ϕ_k may be chosen to cover only a hemisphere, or more specifically

$$0 \leq \phi_k \leq \pi$$

The brightness value for an arbitrary point in the atmosphere and for an arbitrary direction can be obtained through interpolation among values in the data base. In this way, the value of the integral in Equation (1) may be evaluated for any point in the atmosphere, and the equation can be solved by standard numerical techniques. It is clear that the accuracy of the numerical techniques will depend on the selection of matrix points for the data base, and that increasing the density of points will lead to improved accuracy.

The data base itself is constructed by an iterative procedure. In the zeroth order data base, all of the brightness values for the atmosphere are set equal to zero. This is equivalent to setting the integral in Equation (1) to zero. The next higher order data base is obtained by solving Equation (1) for the complete set of direction angles for all local points; the solutions are obtained using the zeroth order data base to evaluate the integral in Equation (1). Successive iterations lead to a data base of sufficient accuracy for the required solution.

All of the computational work is done on a Univac 1108 computer. A group of programs has been written to manipulate the data base. There are programs to move the data base between tape storage and random access bulk storage, set up the zeroth order data base, perform an iteration on the data base, compute the change in the data produced by an iteration, or solve Equation (1) in a specified geometry using the best available data base.

3.0 PLANAR ATMOSPHERE

The light scattered by a planar haze can be calculated exactly by a method developed by Chandrasekhar. These calculations, which we will refer to as the "exact calculations", can serve as a convenient reference against which to compare the results of the iterative procedure developed for use in the spherical case.

For the iterative calculations, the change from the spherical to the planar case requires only a few minor changes in the geometrical calculations involved in the program. The main features of the program are identical in the two cases. Because of this, agreement between the iterative and the exact calculations serves to indicate not only the validity and accuracy of the approach, but also the correctness of the computer program written to carry out the calculations. This is extremely important because in computer programs as long and as complicated as those involved here it is very easy for pro-gramming errors to go undetected.

The haze model that is the basis of the computations is the same as that described previously[2]. Throughout this report, a cutoff altitude of 122.3035 km has been used. (The cutoff altitude is the altitude below which no haze exists). The value used differs slightly from previous values in order that the optical thickness of the haze be almost 0.4, an essential feature to simplify the exact calculations[2]. Table I lists the parameters which describe the haze.

A major question for the iterative calculations is the rapidity with which they converge to a solution of acceptable accuracy. The convergence properties are shown in Figure 3 for a sun angle of 0° and in Figure 4 for a sun angle of 60°. It should be noted that,because of the symmetry inherent in isotropic scattering in a planar atmosphere, there is no dependence on the azimuthal angle between the sun direction and the viewing direction.

All the brightness values shown in the curves for this report are given relative to the brightness of an ideal Lambert surface normal to the incident solar radiation (and without any haze). For convenience, we use the term millilamb to indi-cate one one-thousandth of the brightness of such a reference surface.

In both Figure 3 and Figure 4, it is apparent that after just four iterations, the iterative solution closely approximates the exact values. Further iterations, which are not illustrated in the figures, bring the iterative and exact calculations into agreement of better than 1/2%. It should be noted that the accuracy of the exact calculations is itself limited to 1/2%; this uncertainty arises from the evaluation of the functions described by Chandrasekhar and in the numerical integrations required to obtain the actual numbers.

For the remainder of this section, the data base used for the calculations was created through 12 iterations. During the last iteration, the maximum change that occurred for any point in the data base was 0.1%.

In Figures 3 and 4, the effect of the multiple scattering is to increase the brightness about 30% above the first estimate (NIT = 0). Because the multiple scattering represents such a small portion of the final result, the agreement shown in these figures is not a sensitive indicator of the accuracy of the iterative technique.

A more sensitive indicator is a measurement of the multiply scattered light itself. The first two terms in Equation (1) present no computational problem since they can be calculated with high accuracy for any point in the atmosphere. The difficulty is to obtain the brightness values for the integrand of the third term in the equation, which, as we noted before, gives rise to multiple scattering. The integrand is just the diffuse light in the atmosphere (the term diffuse is used to provide a distinction between it and the plane parallel solar radiation). In the zeroth approximation the diffuse light is zero. The accuracy with which it is calculated in the final approximation serves as a sensitive indication of the overall accuracy of the multiple scattering procedure.

The diffuse light incident on the surface is shown in Figures 5 and 6, for sun angles of 0° and 30°, respectively, for both the iterative and the exact calculations. It is readily apparent from these graphs that there is extremely good agreement between the two calculations. This is a significant indication that the iterative technique converges correctly to the desired solution.

Similarly, Figure 7 shows the surface illumination as a function of sun angle. This curve indicates good agreement is achieved by the iterative calculations for the full range of sun angles.

Thus it may be concluded that the iterative technique provides accurate results for a planar atmosphere. The accuracy is at least as good as 1/2%. Part of this observed discrepancy may arise in the numerical integrations used to obtain the exact results.

The success of the iterative technique indicates, in addition, that the computer program is error free, to a high degree of certainty.

4.0 SPHERICAL GEOMETRY

The most obvious difference between the spherical and planar calculations is in the geometrical calculations. While the differences are significant, they represent straight-forward calculations and cause no conceptual or programming difficulty.

A more subtle difference between the spherical and planar cases arises because of various discontinuities in brightness which are a part of the spherical problem. Discontinuities are the bane of numerical integrations. Either care must be exercised to account for the discontinuities in the integration procedure, or they will represent a source of error in the computations.

In developing the iterative technique, care was exercised to take into account discontinuities where they could have a significant impact on the results. Figure 8 shows the discontinuities along a typical line of sight through the atmos-phere. Discontinuities arise where the line of sight enters or leaves the limited region where the haze exists, and where the line of sight enters or leaves the shadow zone. Entering or leaving the haze region gives rise to a discontinuity in the density of the haze, and hence in the extinction and scattering coefficients in Equation (3). The boundary of the shadow zone represents a discontinuous change in the incident solar radiation.

Integration of Equation (1) along the line of sight is accomplished by the rectangle method; the value of the derivative is calculated at a point and multiplied by the length of a short interval surrounding the point. The points were chosen so that all discontinuities lie on the border of two adjacent intervals.

Another discontinuity, which was not illustrated in Figure 8, occurs when the line of sight intersects the sur-face of the planet. This discontinuity, and some of those corresponding to entering and leaving the region of haze, represent just the initial and final points of the range of integration. All of the discontinuities occurring along the line of sight have been accounted for exactly, and hence do not affect the accuracy of the calculations in the spherical geometry.

However, discontinuities may affect the evaluation of the integral in Equation (1). This integral represents the integration of light incident at the reference point from all directions. There is a discontinuity in the incident radiation corresponding to the horizon; in most circumstances the surface area below the horizon is markedly brighter than the atmosphere above the horizon.

        The integral is evaluated by summing the products of the brightness in specific directions (the selected directions in the data base) and the solid angle contained in an area surrounding the direction. The discontinuity will cause an error if the horizon does not occur on the border of these areas. If a border does correspond to the horizon, an accurate evaluation will result since brightness representative of the surface (or the atmosphere) will be used for a solid angle corresponding to that actually subtended by the surface (or atmosphere).

        The angle between the horizon and the local vertical depends on the altitude above the surface. It is 90° at the surface, 105° at the low altitude cutoff (122.3 km) and 109° at the high altitude limit of the atmosphere.

        The directions for which brightness values are stored in the data base are given in Table II. Data is stored for polar angles of 90° and 120°, so that the border between them lies at 105°, close to the horizon. In the worst case, when the horizon is at 109° below local vertical, 3.6% of the solid angle would be misplaced, that is, calculated as if it has surface brightness when in fact it corresponds to atmosphere. Moreover, the greatest error occurs where the haze is thinnest; consequently the error has a minimal effect on the observed brightness.

        Although the horizon problem is not critical for the haze calculations, provided the polar directions for the data base are carefully chosen, the problem will be significant in cases where the scattering medium extends down to the surface. Specifically, for calculations of the brightness of the gaseous atmosphere of Mars, additional consideration must be given to this problem.

        Discontinuities can also cause computational errors in another context. At regular intervals along the line of sight, points are selected for the evaluation of Equation (1). In general, these points do not correspond to points contained in the data base. In order to evaluate the integral in Equation (1), a linear interpolation is performed among the nearest points in the data base. In certain circumstances, adjacent data base points have different geometrical properties (such as shadowing). The interpolation procedure does not take into account the geometry of the data base points relative to the point at which the evaluation is being made, and where there are such differences among the data base points the interpolation does not give an accurate value. This does not cause a serious error to the overall calculation

because it occurs in a limited geometrical region and because
the values are small in the regions, near the terminator,
where it does occur.

Interpolation problems can also arise in areas where
the brightness changes rapidly. Such a region is the lower part
of the haze, where the absolute value of the density changes
most quickly. If the local points were spaced equally in height,
major changes in brightness would occur between adjacent height
points, and an inaccurate interpolation would result. Instead,
the height points were chosen to maintain an equal vertical
optical thickness between points. However, the rule was not
used for the upper portion of the atmosphere, where it would
make the distance between points excessive. In the upper regions,
uniform height spacing was used. This is shown in Table III,
which gives the matrix of the local points.

Problems similar to those described above do not occur
for the planar case. There are no shadows in the planar case.
The discontinuities at the top and bottom of the haze are treated
in the same way as for the spherical case, and so are accounted
for exactly. The horizon is at 90° regardless of the height
above the surface, and the polar angles for the planar case
were chosen so that there was a border at 90°. Neither does the
interpolation procedure cause a problem in the planar case be-
cause, for a given calculation, all points have the same sun
angle and essentially the same geometry.

Convergence properties for the spherical geometry are
shown in Figure 9 for a sun angle of 0°. It is clear that the
convergence is extremely rapid. The observed change between
NIT = 4 and NIT = 10 was less than 0.1% for 0° sun angle.

However, certain points in the data base continue to
change even for NIT = 7. This is shown in Figure 10, which
shows in histogram form the number of points in the data base
which have undergone changes of various amounts. For NIT = 7,
a small portion of the points in the data base show a modest
increase up to about 1/2%. These points in general have low
absolute value and lie primarily in regions near the terminator.
Hence they do not affect significantly those points having a
sun angle of 0°.

Figure 10 also shows a histogram for NIT = 10. At
this point, no significant changes occur for any point in the
data base.

Typical results of the multiple scattering calculations are shown in Figures 11 through 14. The first three of these figures allow a comparison to be made among the various types of calculations: multiple scattering for a spherical geometry, multiple scattering for a planar geometry, and first order calculations (NIT = 0) for a spherical geometry. Comparison of the first order and multiple scattering results for the spherical case indicate that the multiple scattering produces an increase of about 20% in the apparent brightness. Note, however, that the line of sight in these figures intersects the surface, and a major portion of the apparent brightness arises from light scattered from the surface. This is shown in Figure 11, where the lowest curve gives the apparent brightness that would be observed if the light scattered by the haze is not included; attenuation by the haze has been included. From a comparison of the increased brightness due to first order calculations with that due to multiple scattering, it is apparent that multiple scattering approximately doubles the effect of the haze in increasing the brightness.

The surface contribution is also shown in Figure 13 for a sun angle of 60°. The effect of multiple scattering, compared to the first order calculation, is not as great as for a sun angle of 0°; nevertheless it still represents a substantial contribution to the apparent brightness.

In the spherical geometry, the apparent brightness depends on the azimuthal angle between the direction of the incident solar radiation and the viewing direction (the geometry is shown in Figure 2). The range of values of the apparent brightness as the azimuthal angle is varied from 0° to 180° is indicated by cross hatching in Figures 12 through 14. For a sun angle of 0° (Figure 11), symmetry precludes any azimuthal dependence.

For sun angle of 80° or less, and for any look angle, the brightness decreases monotonically as the phase angle goes from 0° to 180°. Thus the upper boundary line of the cross-hatched area represents 0° azimuthal angle, and the lower boundary line an azimuth of 180°.

This relationship does not hold at a sun angle of 90°. In Figure 15, the apparent brightness for different azimuthal angles are shown for a sun angle of 90°. It is quite apparent that for look angles near 90° and for azimuthal angles near 180°, there is considerable structure in the brightness profiles. The structure arises from the interaction of the shadow zone and the haze-free region below the cutoff. That is to say that the structure depends on the detailed geometry of the haze, and even slight changes in the haze model would cause significant variation in the structure.

Conversely, detailed observations of the brightness in the region near the terminator (sun angle = 90°) can be used to infer the structure of the haze. This point has been discussed previously[2].

There are no exact calculations against which the results of the spherical geometry may be compared. A crude estimate of the validity may be obtained by a comparison with the results from the planar geometry, however. The relationship between the geometry from the spherical and planar cases is shown in Figure 16. When both the look angle and the sun angle are near 0°, the planar geometry is an approximation to that of the spherical case. As either the sun or the look angle changes from 0°, the quality of the approximation is degraded.

A comparison of the calculations for the flat and the spherical geometry in Figures 11 through 14 indicates that the behavior of the apparent brightness is consistent with the quality of the approximation. It is not possible to make the comparison quantitative since there are systematic differences between the flat and spherical cases even for 0°.

It is also of major importance to study the brightness above the limb of the planet. The geometry for this viewing is shown in Figure 17. The brightness profile above the limb is itself shown for two different sun orientations in Figures 18 and 19; Figure 20 displays the specific orientations used.

Figure 18 gives the brightness profile for the Mariner IV geometry; that is, the profile corresponds to the region above the limb which was photographed in the first Mariner IV picture. The first order calculations are shown for comparison. It should be noted that the brightness above the limb is essentially constant up to an altitude of about 150 km. Beyond that, the brightness falls off rapidly. This constancy of brightness occurs despite the structure of the haze. It is necessary for the brightness profile to have this characteristic in order to account for the Mariner IV data.

Figure 19 displays the brightness profile for the case where the line of sight passes above the sub-solar point. It is clear from Figure 19 that the multiple scattering effect can change the apparent brightness by a factor of two above the first order calculation. The actual effect, of course, depends on the precise geometry, and the effect is smaller when the sun angle (σ) nears 90°. The important point , however, is that multiple scattering does change the brightness above the limb by a substantial amount.

5.0 CONCLUSIONS

        The accuracy of the iterative technique for performing
multiple scattering calculations has been illustrated for a
planar atmosphere by showing that the calculations agree with
exact results computed by the method of Chandrasekhar. Further-
more, it has been shown that the iterative calculations for
the spherical geometry behave as expected. Hence it is concluded
that the iterative technique provides an accurate method of
performing multiple scattering calculations for planetary atmos-
pheres. A subsidiary conclusion is that the computer programs
developed to carry out the calculations are likewise accurate.
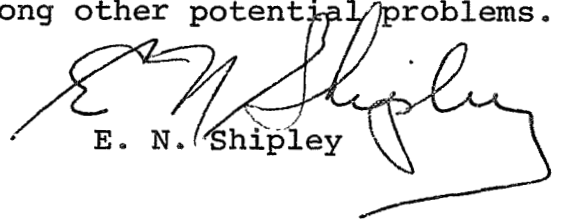
        Results were presented for the apparent brightness for
Mars under certain geometric conditions with the assumption that
a specific haze model described the optical properties above the
surface of the planet. The haze model was developed through the
use of first order calculations, and the parameters of the model
were chosen to give good agreement between the first order cal-
culations and the Mariner IV data[2]. Because multiple scattering
causes substantial changes in the brightness compared to first
order estimates, the multiple scattering results are not suitable
for comparison with Mariner IV data. It will be necessary to
adjust the parameters of the haze model to bring the new results
into agreement with the observed data.

        Because the multiple scattering affects the apparent
brightness above the limb to a much greater degree than the
apparent brightness of the surface, it is believed that a major
variation of the parameters, at least, will be required, and it
may be necessary to alter significantly the structure of the haze.

        The computational technique itself can be improved to
eliminate the remaining problems with the discontinuities that
were described earlier. A new technique for handling the horizon
discontinuity in the integral of Equation (1) is necessary in
order to permit the calculations to be used for atmospheres
which extend down to the surface (a suitable method has already
been developed), and a better interpolation procedure is desirable
in order to increase confidence in the accuracy of the spherical
calculations.

However, the important point is that a technique now exists to carry out multiple scattering calculations in a spherical geometry with incident solar radiation. The technique can be applied both to the possible Martian haze and to scattering in the gaseous atmosphere of Mars, among other potential problems.

E. N. Shipley

1014-ENS-bmn

Attachments
   References
   Tables I-III
   Figures 1-20

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6. Reference 2, Appendix A.

TABLE I

PARAMETERS OF THE HAZE MODEL

Scale height, h_o	10 km
Low altitude cutoff, h_{co}	122.3 km
High altitude cutoff, h_{max}	200 km
Surface normal albedo, ρ	0.5
Albedo for single particle scattering, ρ_s	0.5
Extinction coefficient extrapolated to the surface, σ_o	8200 km^{-1}

Scattering coefficient $b = \rho_s \sigma$, where

$$\text{Extinction coefficient } \sigma = \sigma_o e^{-h/h_o} \qquad h_{co} < h < h_{max}$$

$$\sigma = 0 \qquad h < h_{co}$$

$$\sigma = 0 \qquad h > h_{max}$$

TABLE II

DIRECTIONS IN WHICH BRIGHTNESS

DATA ARE STORED IN THE DATA BASE

k	θ_k (degrees)	ϕ_k (degrees)
1	0	0
2	30	30
3	30	90
4	30	150
5	60	0
6	60	45
7	60	90
8	60	135
9	60	180
10	90	15
11	90	45
12	90	75
13	90	105
14	90	135
15	90	165
16	120	0
17	120	45
18	120	90
19	120	135
20	120	180
21	150	30
22	150	90
23	150	150
24	180	0

The coordinate system is shown in Figure 2. θ is the polar angle from the local vertical, and ϕ is the azimuthal angle measured from the plane containing the sun and the local vertical.

TABLE III

LOCAL POINTS USED IN THE SPHERICAL CALCULATION

A. SUN ANGLE (λ_i)

0°, 2°, 4°, ... 120°

For $\lambda \geq 120°$, all brightness values are set to zero.

B. HEIGHT (h_j)

j	h_j	Optical Thickness Interval
1	0.0	
		.00
2	122.3035	
		.0277
3	123.0	
		.0278
4	123.8	
		.0278
5	124.6	
		.0278
6	125.6	
		.0278
7	126.6	
		.0278
8	127.7	
		.0278
9	129.0	
		.0278
10	130.4	
		.0278
11	132.1	
		.0278
12	134.2	
		.0278
13	136.7	
		.0278
14	140.2	
		.0278
15	145.6	
		.0210
16	153.4	
		.0097
17	161.2	
		.0044
18	168.9	
		.0020
19	176.7	
		.0009
20	184.5	
		.0005
21	192.2	
		.0002
22	200.0	

LIGHT INCIDENT
ON SCATTERING
ELEMENT

SMALL
SCATTERING
ELEMENT

LINE OF SIGHT ——————————————————→ Z ⊅ OBSERVER

θ

φ

X

Y

(DOWN)
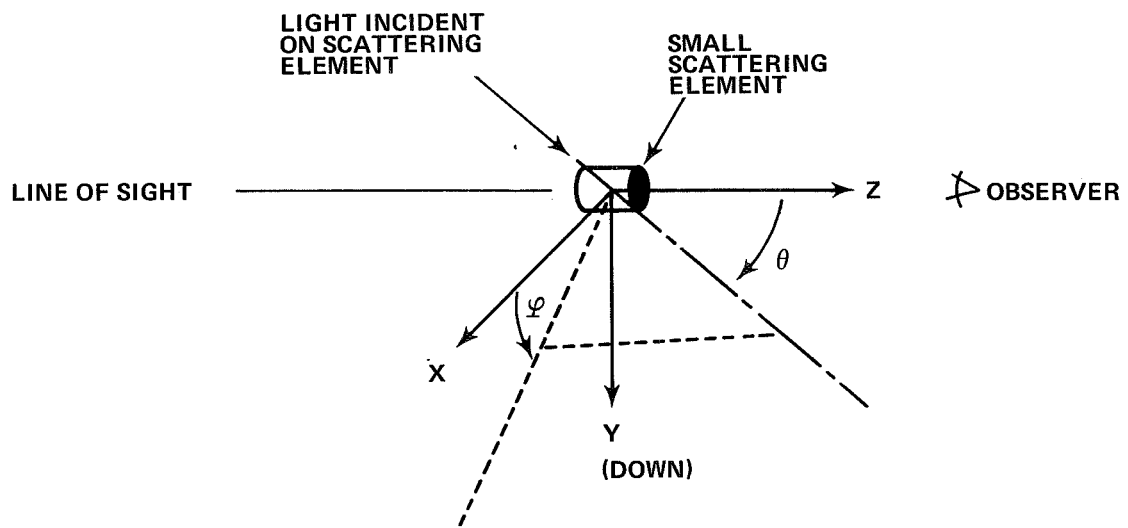
FIGURE 1 - GEOMETRY FOR SCATTERING CALCULATION
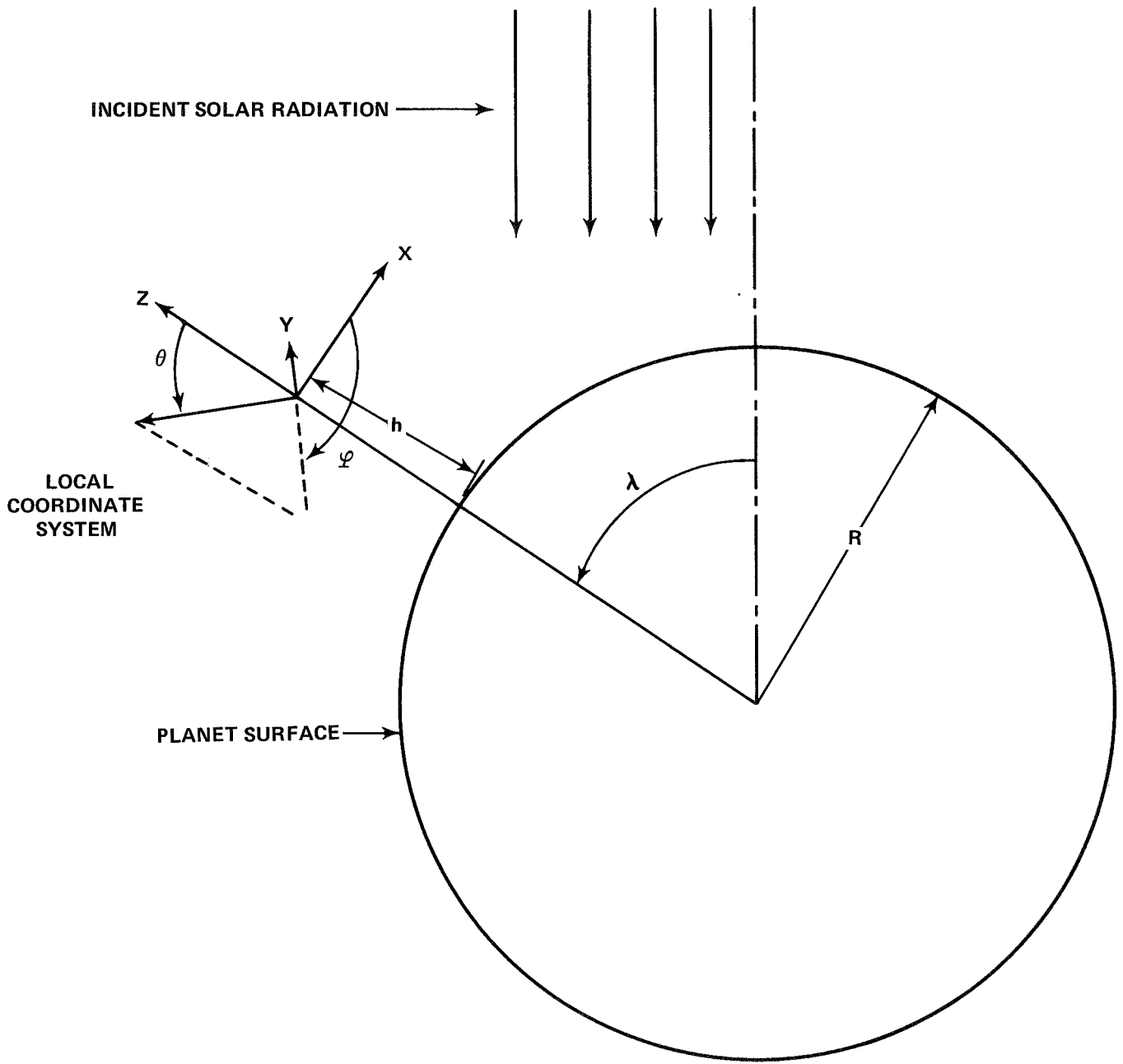
FIGURE 2 - COORDINATE SYSTEM FOR THE DATA BASE
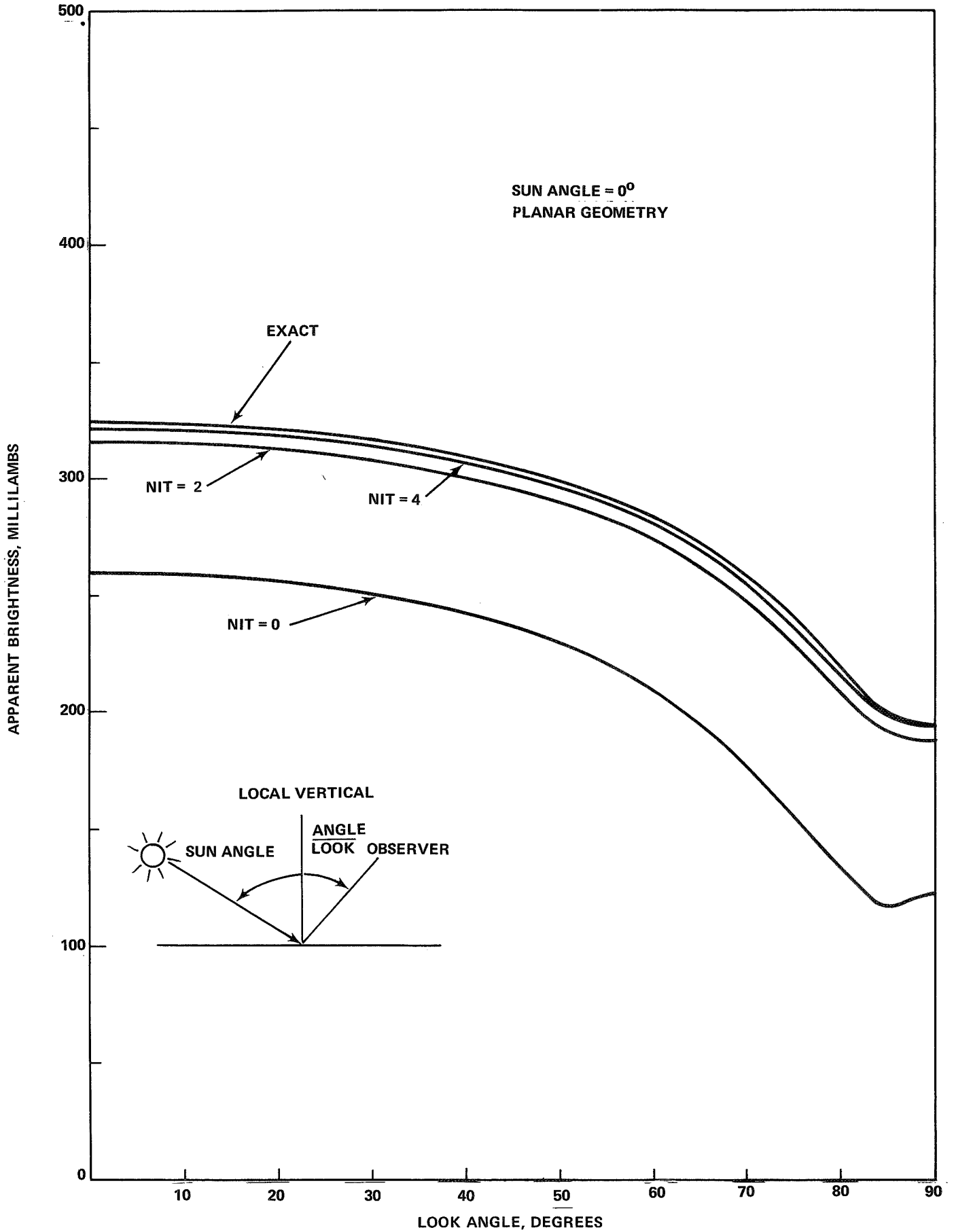
FIGURE 3 - ILLUSTRATION OF THE CONVERGENCE OF THE ITERATIVE CALCULATIONS FOR THE PLANAR HAZE. THE PARAMETER NIT IS THE NUMBER OF ITERATIONS.

SUN ANGLE = 60°
PLANAR GEOMETRY

APPARENT BRIGHTNESS, MILLILAMBS

EXACT

NIT = 2

NIT = 4

NIT = 0

LOOK ANGLE, DEGREES
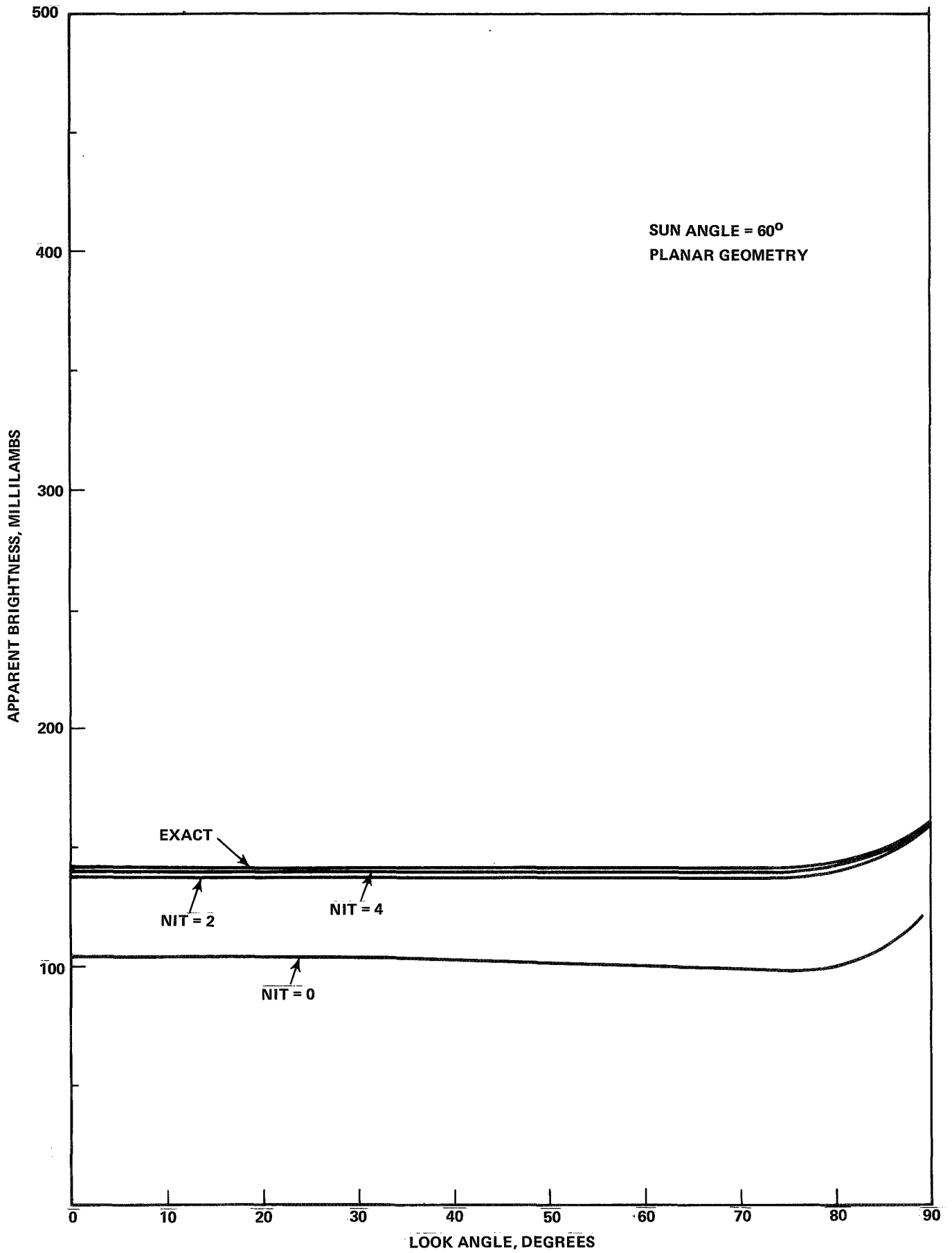
FIGURE 4 - ILLUSTRATION OF THE CONVERGENCE OF THE ITERATIVE CALCULATIONS
FOR THE PLANAR HAZE. THE PARAMETER NIT IS THE NUMBER OF ITERATIONS.
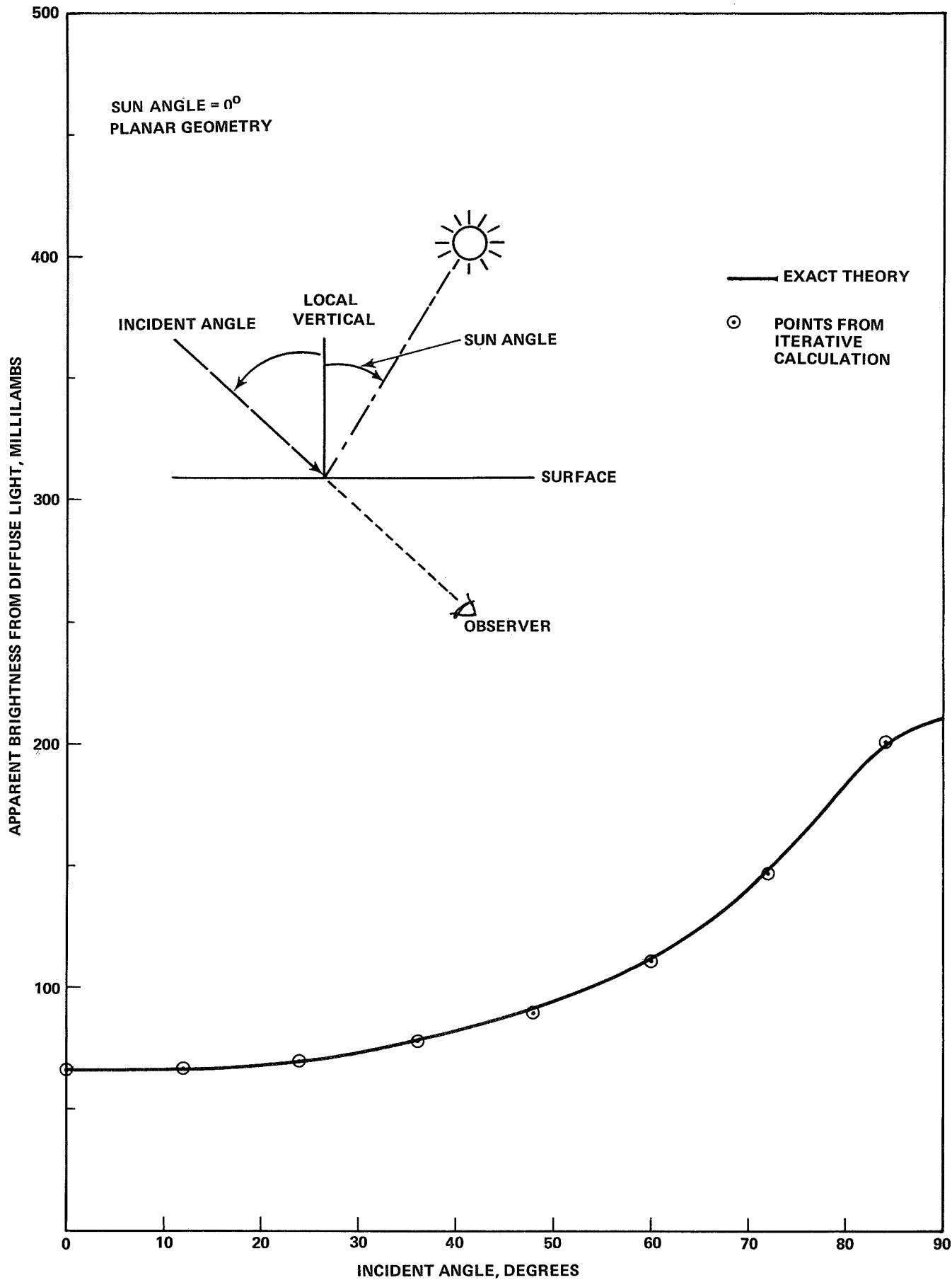
FIGURE 5 - THE INTENSITY OF THE INCIDENT DIFFUSE LIGHT AT THE SURFACE,
AS A FUNCTION OF INCIDENCE ANGLE.
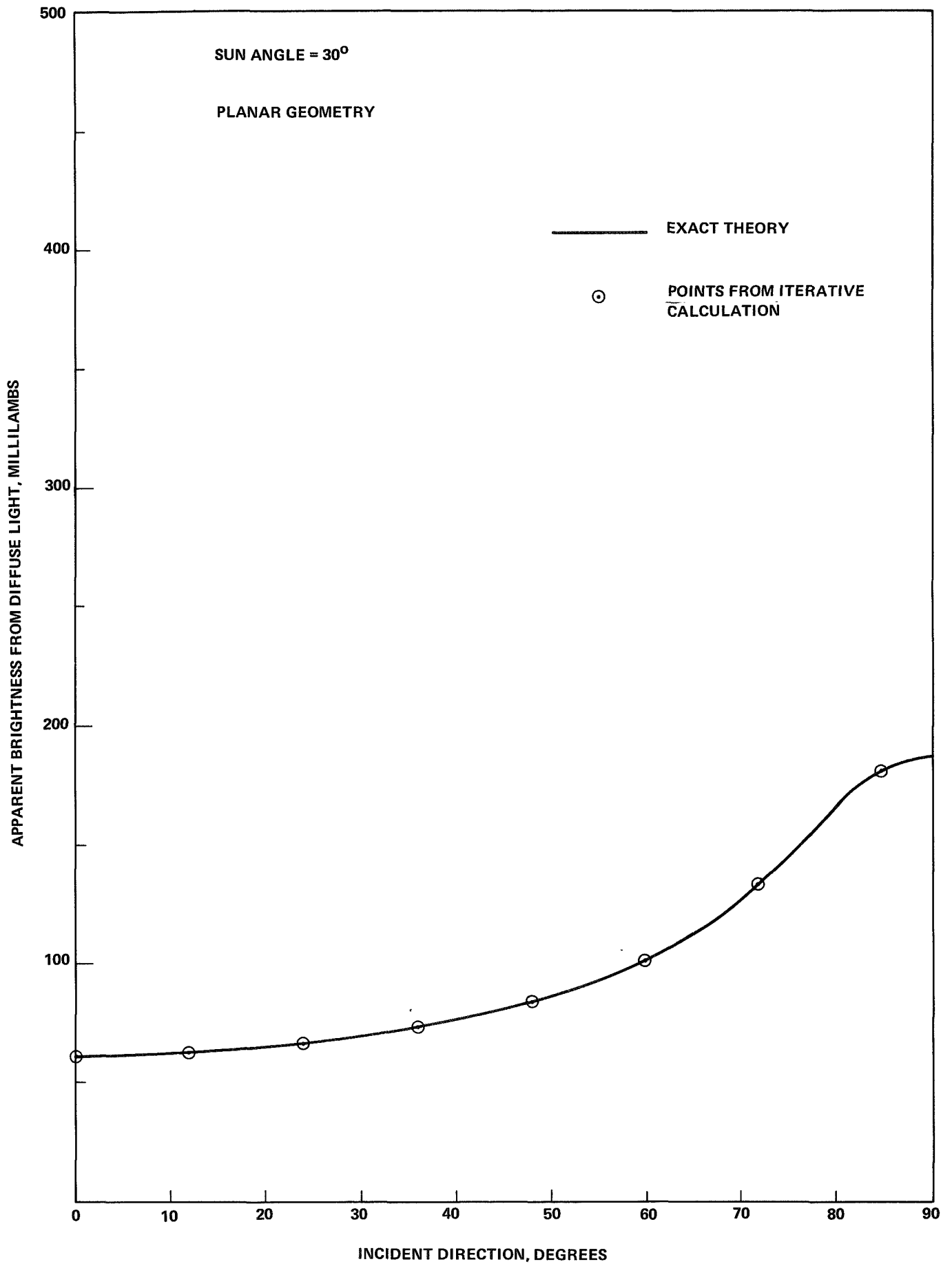
FIGURE 6 - THE INTENSITY OF THE INCIDENT DIFFUSE LIGHT AT THE SURFACE,
AS A FUNCTION OF THE INCIDENCE ANGLE.
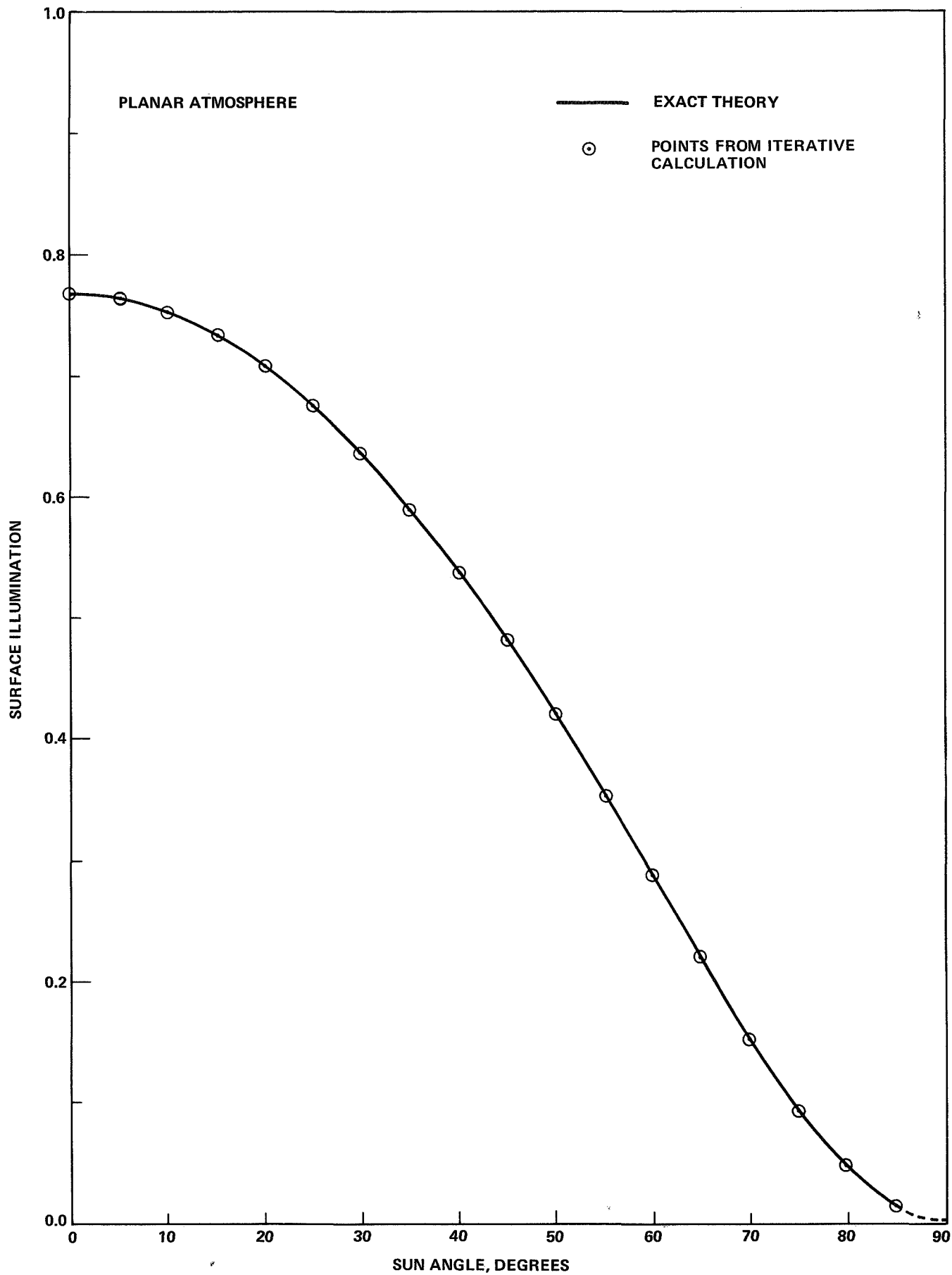
FIGURE 7 - SURFACE ILLUMINATION AS A FUNCTION OF SUN ANGLE.
A VALUE OF UNITY REPRESENTS THE ILLUMINATION OF A
SURFACE NORMAL TO THE DIRECT SOLAR RADIATION
(WITH NO HAZE).

SOLAR RADIATION

LOWER HAZE LIMIT (h_{co})

LINE OF SIGHT

PLANET

DISCONTINUOUS POINTS

UPPER HAZE LIMIT (h_{max})

SHADOW ZONE
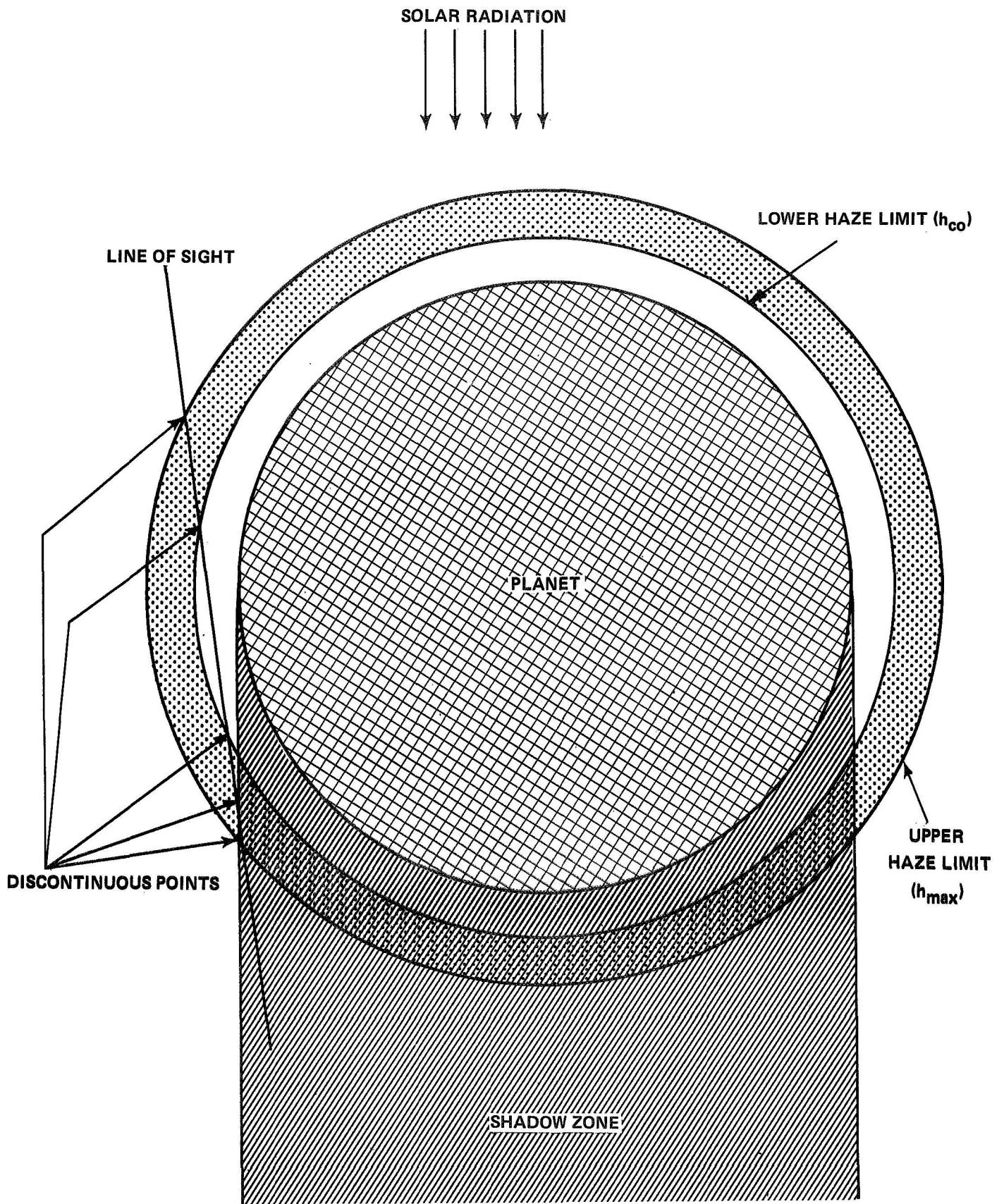
FIGURE 8 - DISCONTINUITIES FOR A TYPICAL LINE OF SIGHT THROUGH THE HAZE
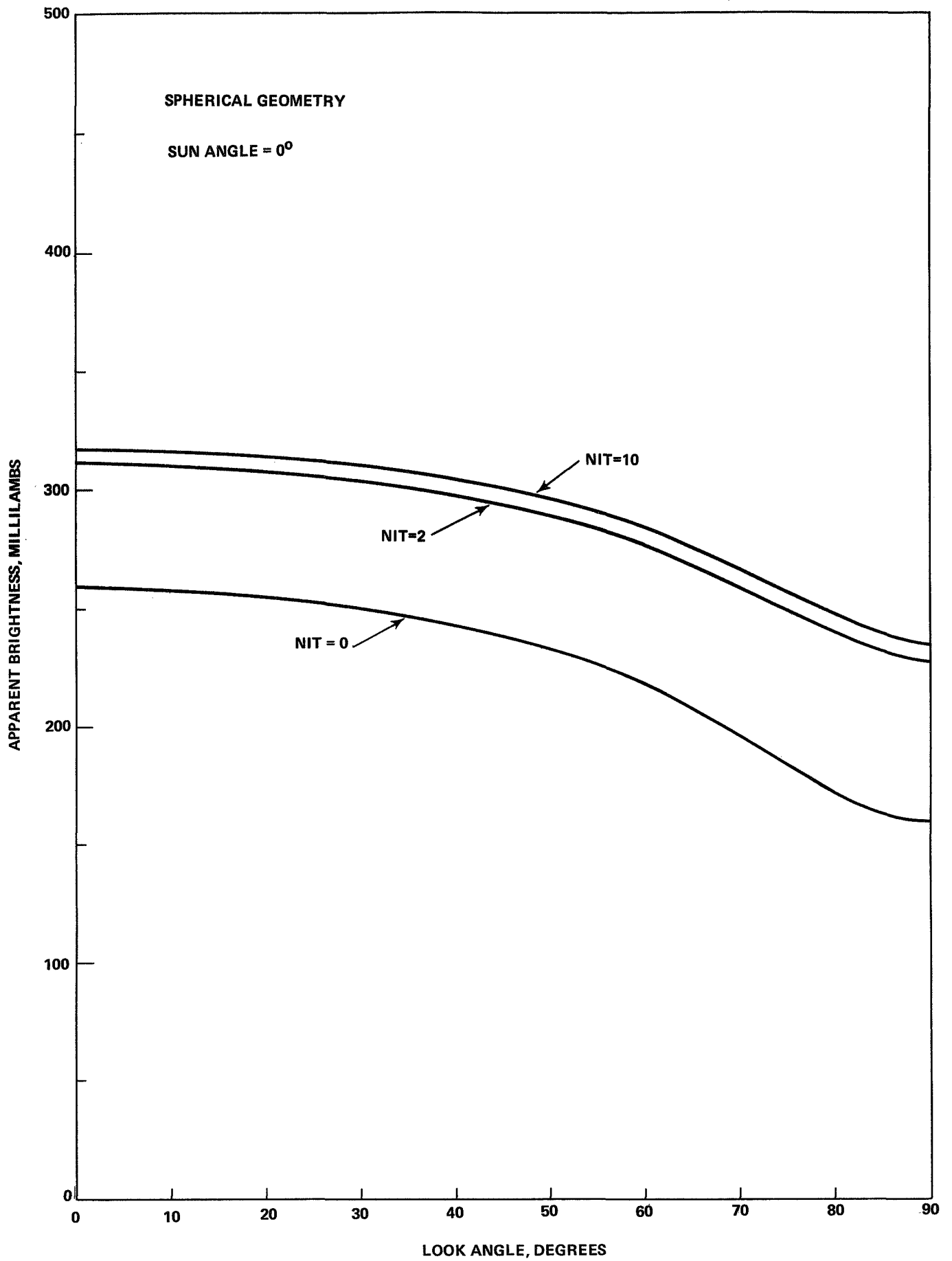
FIGURE 9 - ILLUSTRATION OF THE CONVERGENCE FOR THE SPHERICAL
GEOMETRY. NIT INDICATES THE NUMBER OF ITERATIONS.
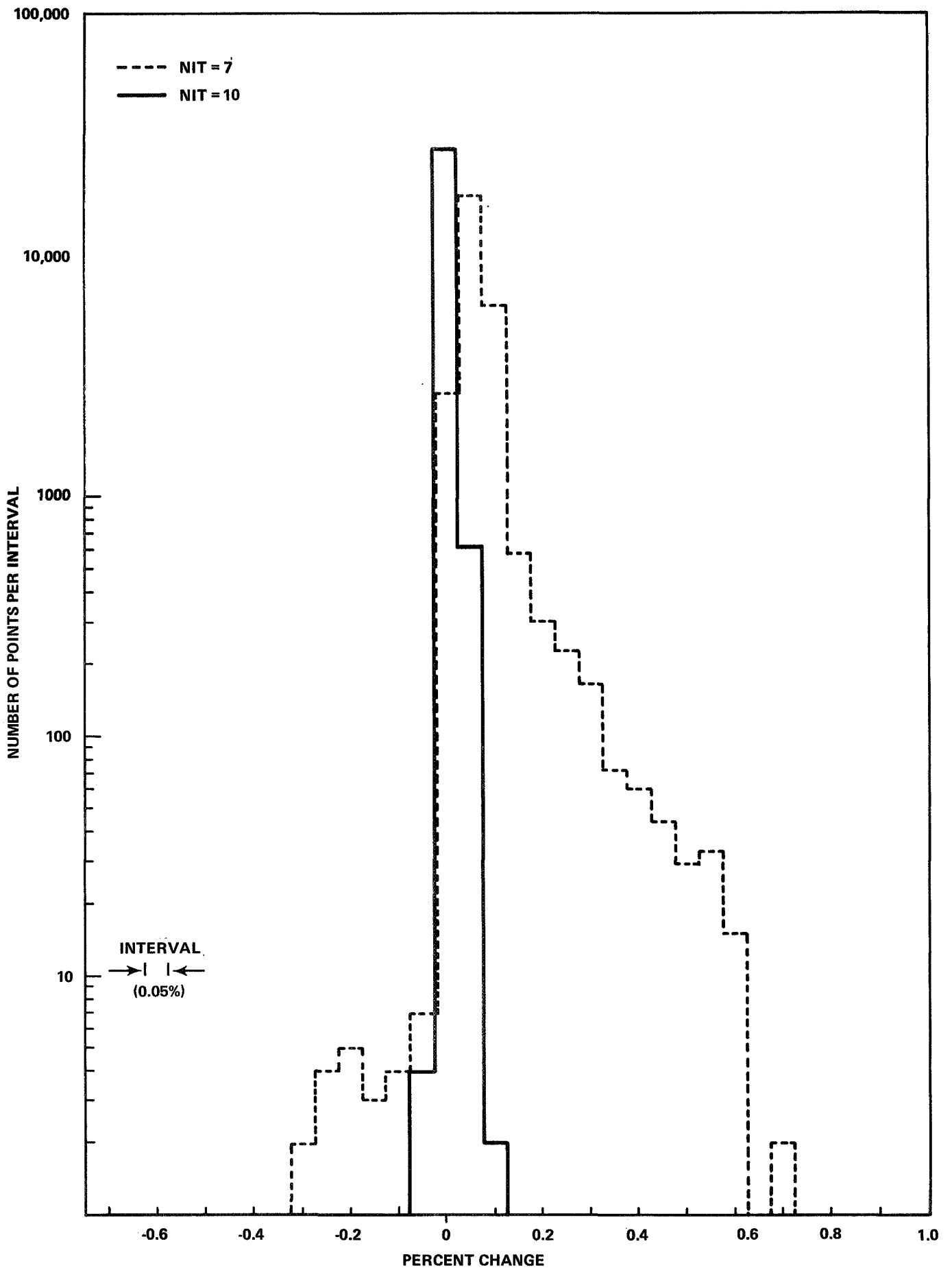
FIGURE 10 - HISTOGRAM SHOWING THE PERCENTAGE CHANGE IN THE POINTS
OF THE DATA BASE DUE TO A SINGLE ITERATION. NOTE THE
LOGRITHMIC VERTICAL SCALE.
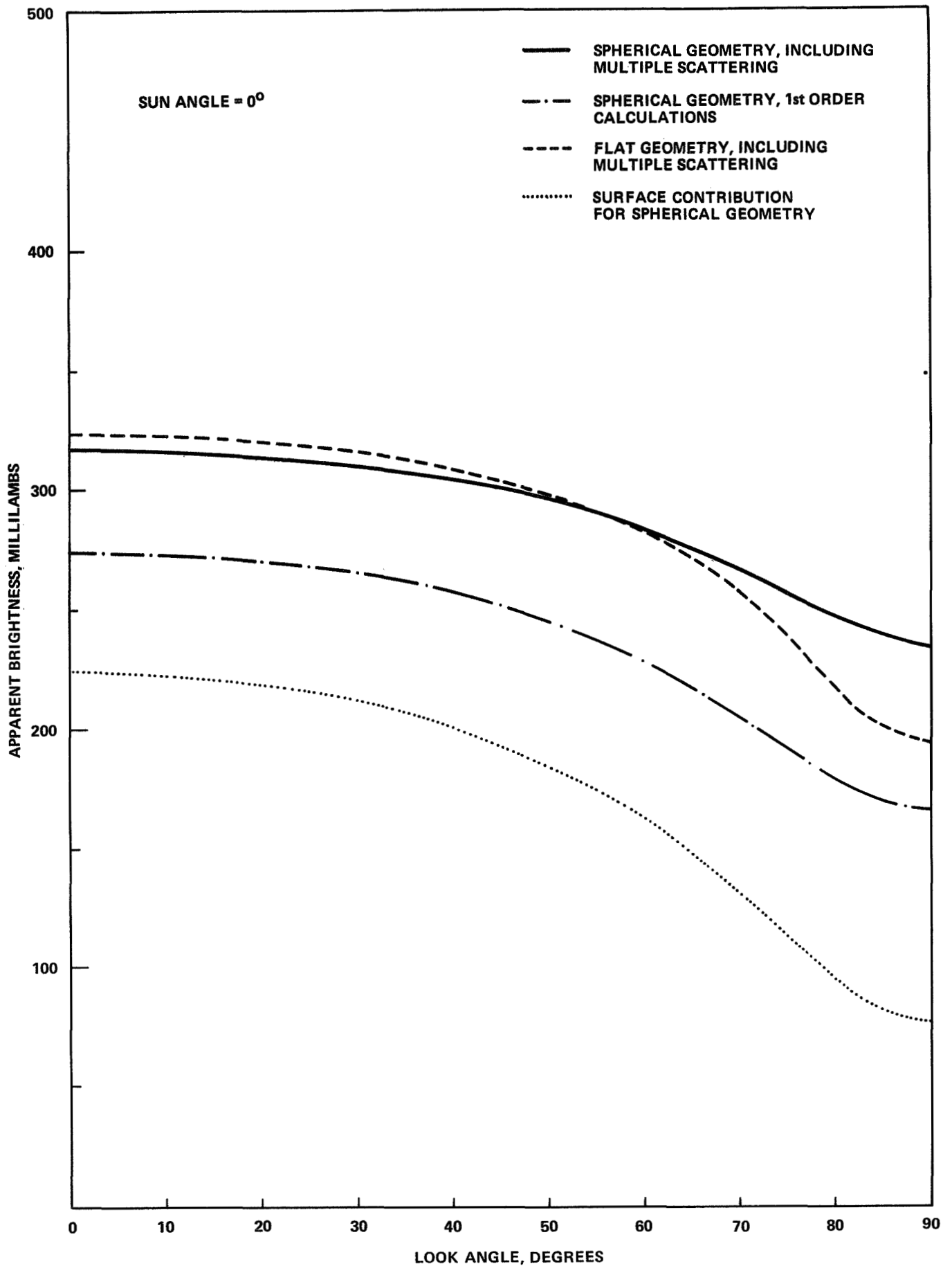
FIGURE 11 - APPARENT BRIGHTNESS, FOR A SUN ANGLE OF 0°, CALCULATED BY SEVERAL DIFFERENT TECHNIQUES.
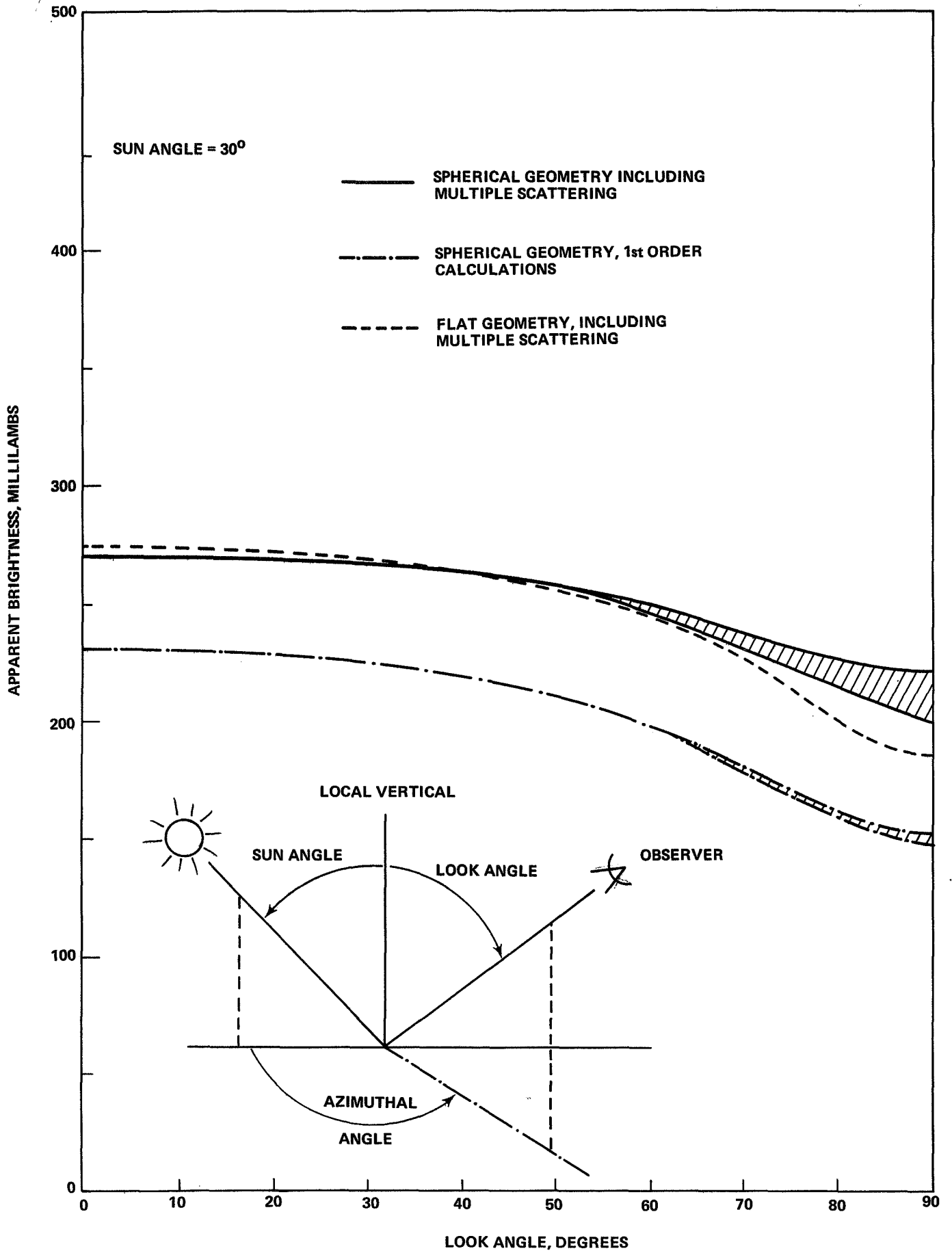
FIGURE 12 - APPARENT BRIGHTNESS FOR A SUN ANGLE OF 30°, CALCULATED BY SEVERAL
DIFFERENT TECHNIQUES. THE CROSS-HATCHED AREAS INDICATE THE RANGE OF
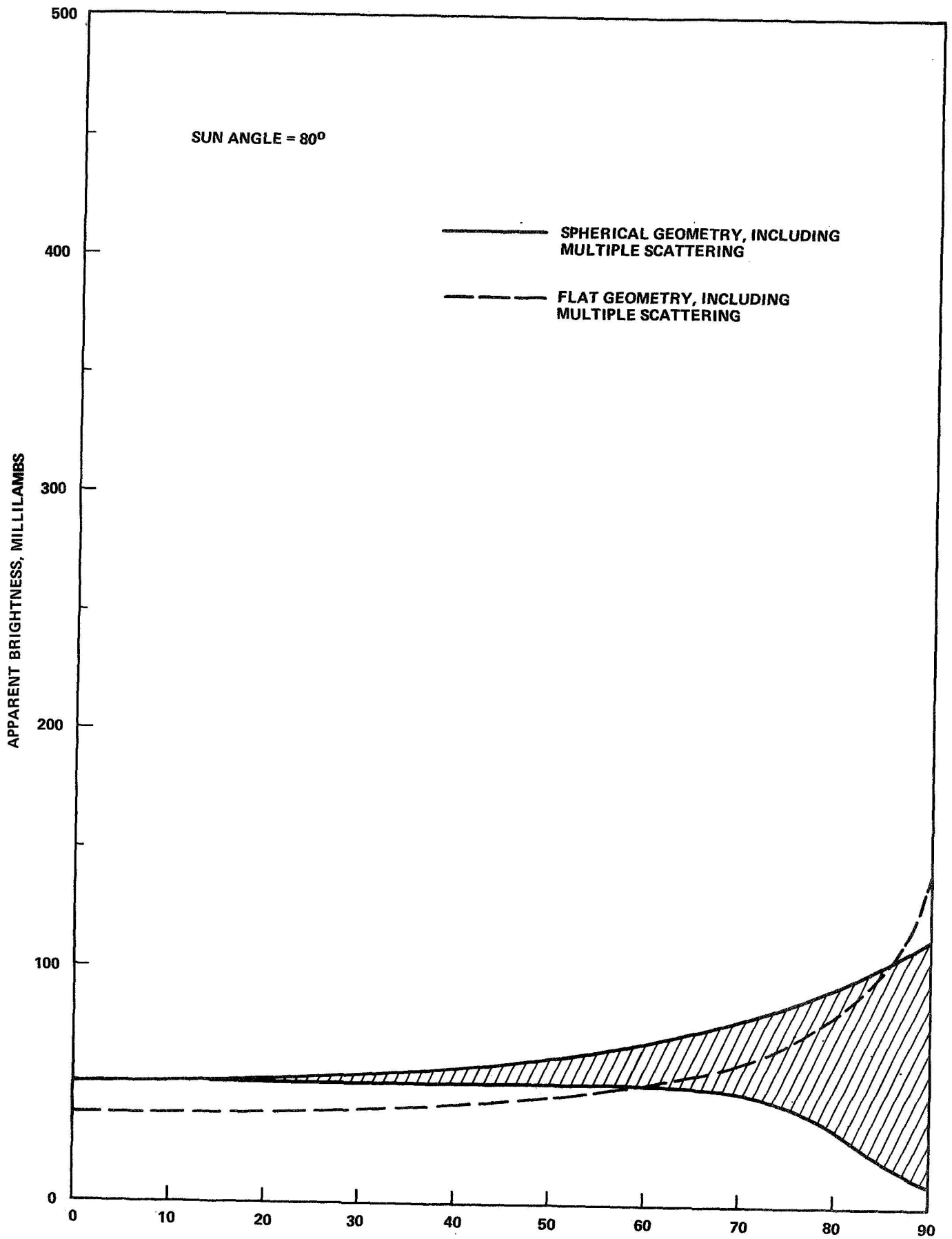VARIATION IN THE BRIGHTNESS DUE TO CHANGE IN THE AZIMUTHAL ANGLE.
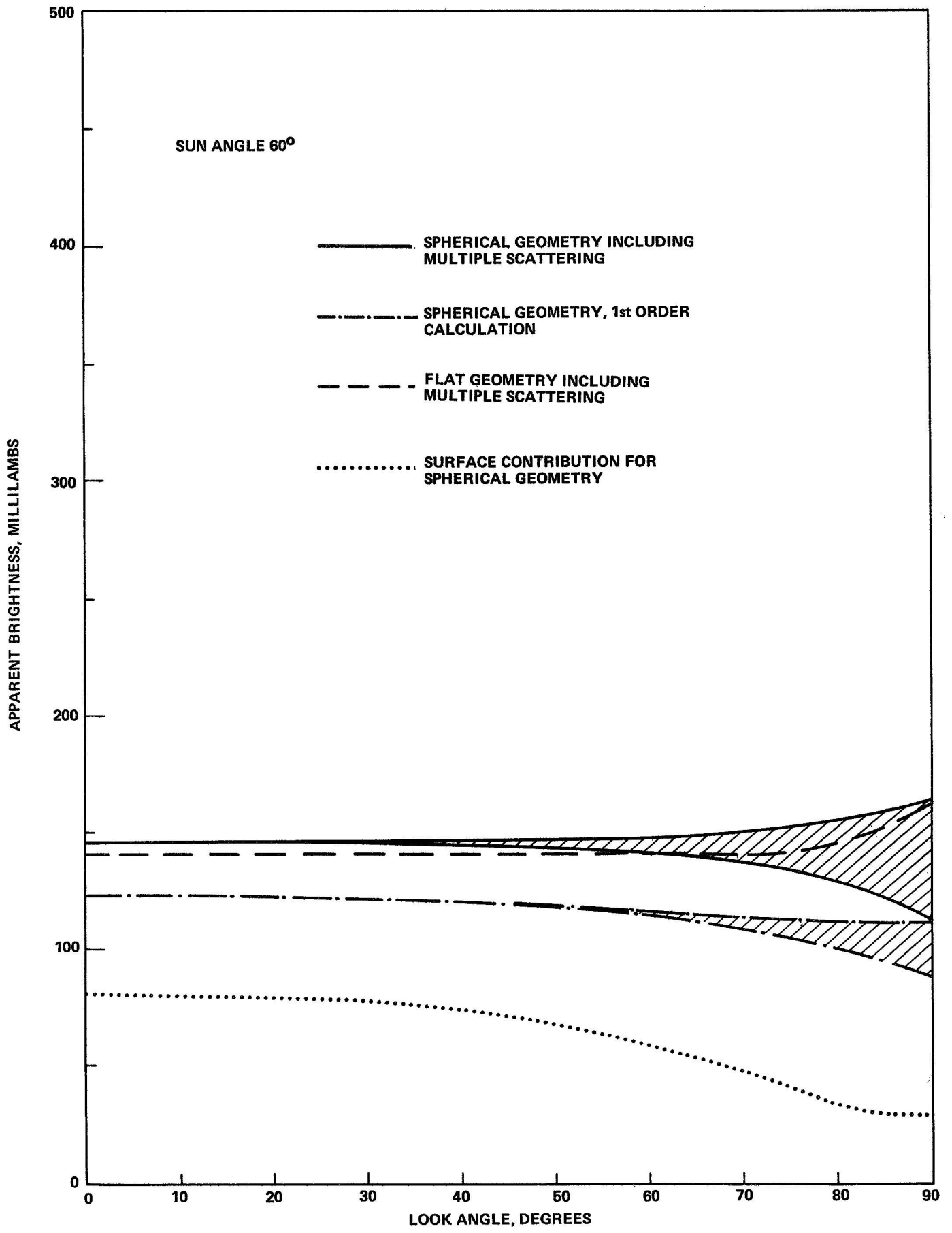
FIGURE 13 - APPARENT BRIGHTNESS FOR A SUN ANGLE OF 60°, CALCULATED BY SEVERAL DIFFERENT TECHNIQUES. THE CROSS-HATCHED AREAS INDICATE THE RANGE OF VARIATION IN THE BRIGHTNESS DUE TO CHANGE IN THE AZIMUTHAL ANGLE.

FIGURE 14 - APPARENT BRIGHTNESS, FOR A SUN ANGLE OF 80°, CALCULATED BY TWO DIFFERENT TECHNIQUES. THE CROSS-HATCHED AREA INDICATES THE RANGE OF VARIATION IN THE BRIGHTNESS DUE TO CHANGE IN THE AZIMUTHAL ANGLE.
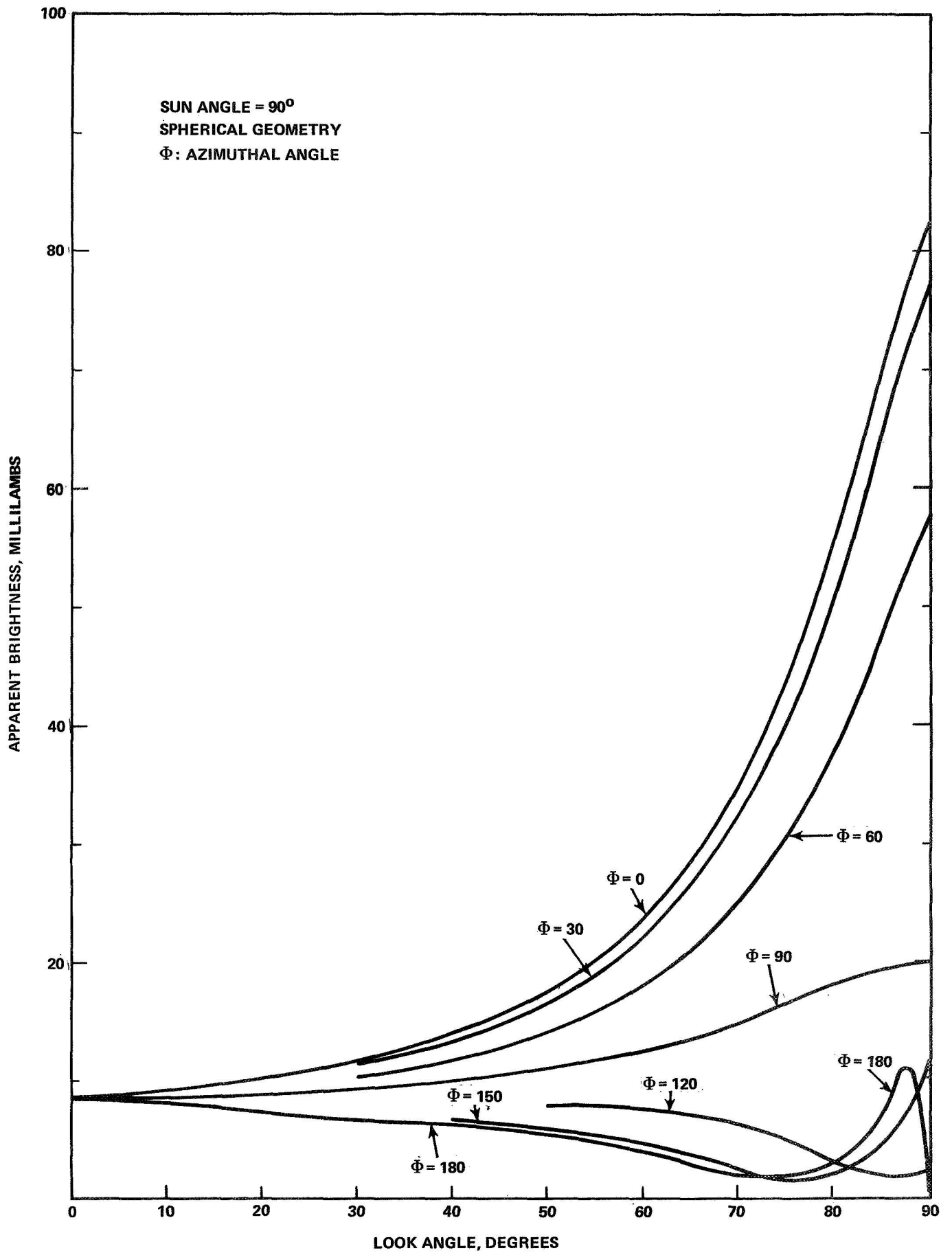
FIGURE 15 - APPARENT BRIGHTNESS FOR SEVERAL VALUES OF THE AZIMUTHAL
ANGLE Φ. THE VERTICAL SCALE HAS BEEN MAGNIFIED OVER THAT USED
IN PREVIOUS FIGURES.

LAMBERT SURFACE

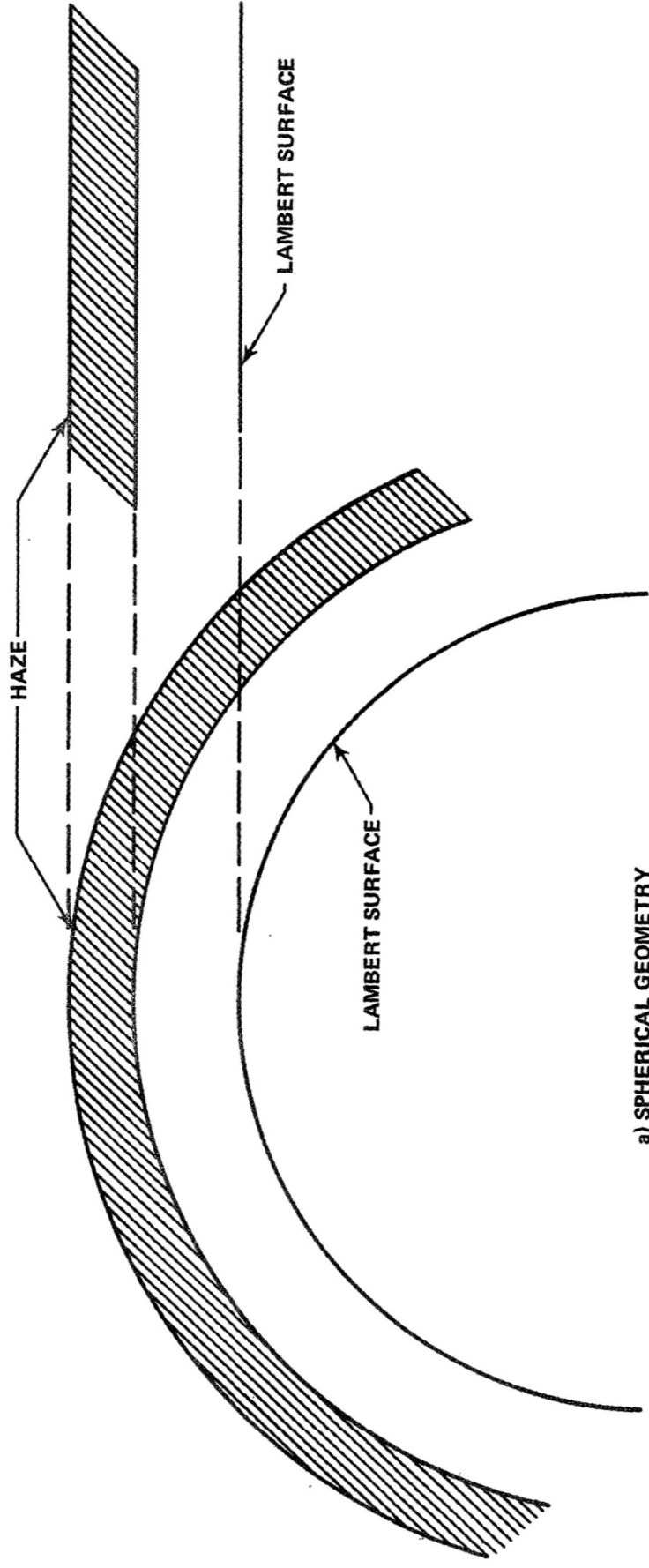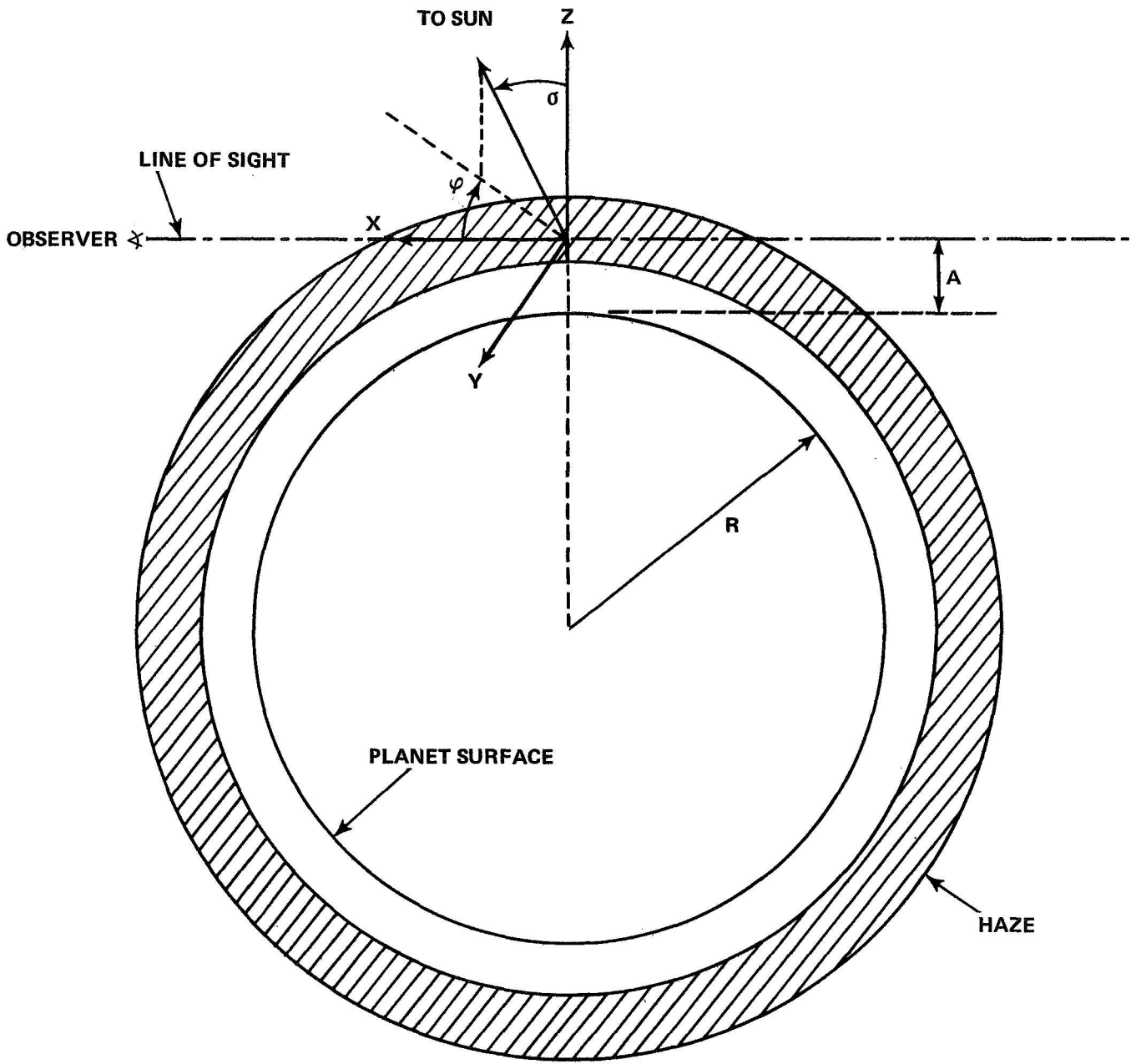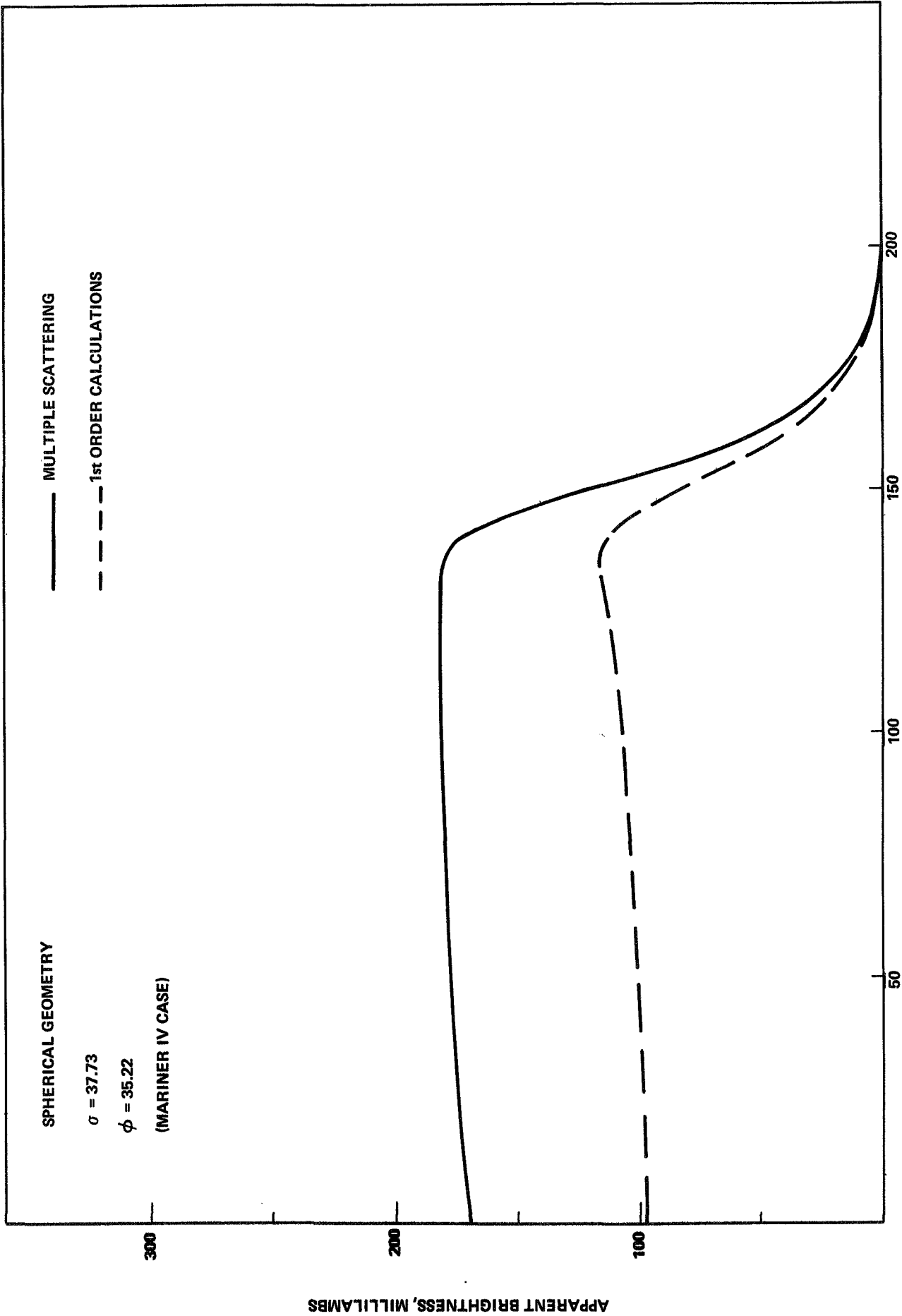b) FLAT GEOMETRY

HAZE

LAMBERT SURFACE

a) SPHERICAL GEOMETRY

FIGURE 16 - COMPARISON OF THE GEOMETRIES USED FOR THE FLAT AND THE SPHERICAL BRIGHTNESS CALCULATIONS.

FIGURE 17 - VIEWING GEOMETRY ABOVE THE LIMB OF A PLANET. THE QUANTITY A IS THE HEIGHT OF THE LINE OF SIGHT ABOVE THE LIMB.

SPHERICAL GEOMETRY

$\sigma = 37.73$

$\phi = 35.22$

(MARINER IV CASE)

——————— MULTIPLE SCATTERING

– – – – – 1st ORDER CALCULATIONS

APPARENT BRIGHTNESS, MILLILAMBS

ALTITUDE ABOVE THE LIMB, km.

FIGURE 18 - APPARENT BRIGHTNESS ABOVE THE LIMB OF THE PLANET.

SPHERICAL GEOMETRY
$\sigma = 0$
$\varphi = 0$

—————— MULTIPLE SCATTERING

— — — — 1st ORDER CALCULATIONS

APPARENT BRIGHTNESS, MILLILAMBS

ALTITUDE ABOVE THE LIMB, km.
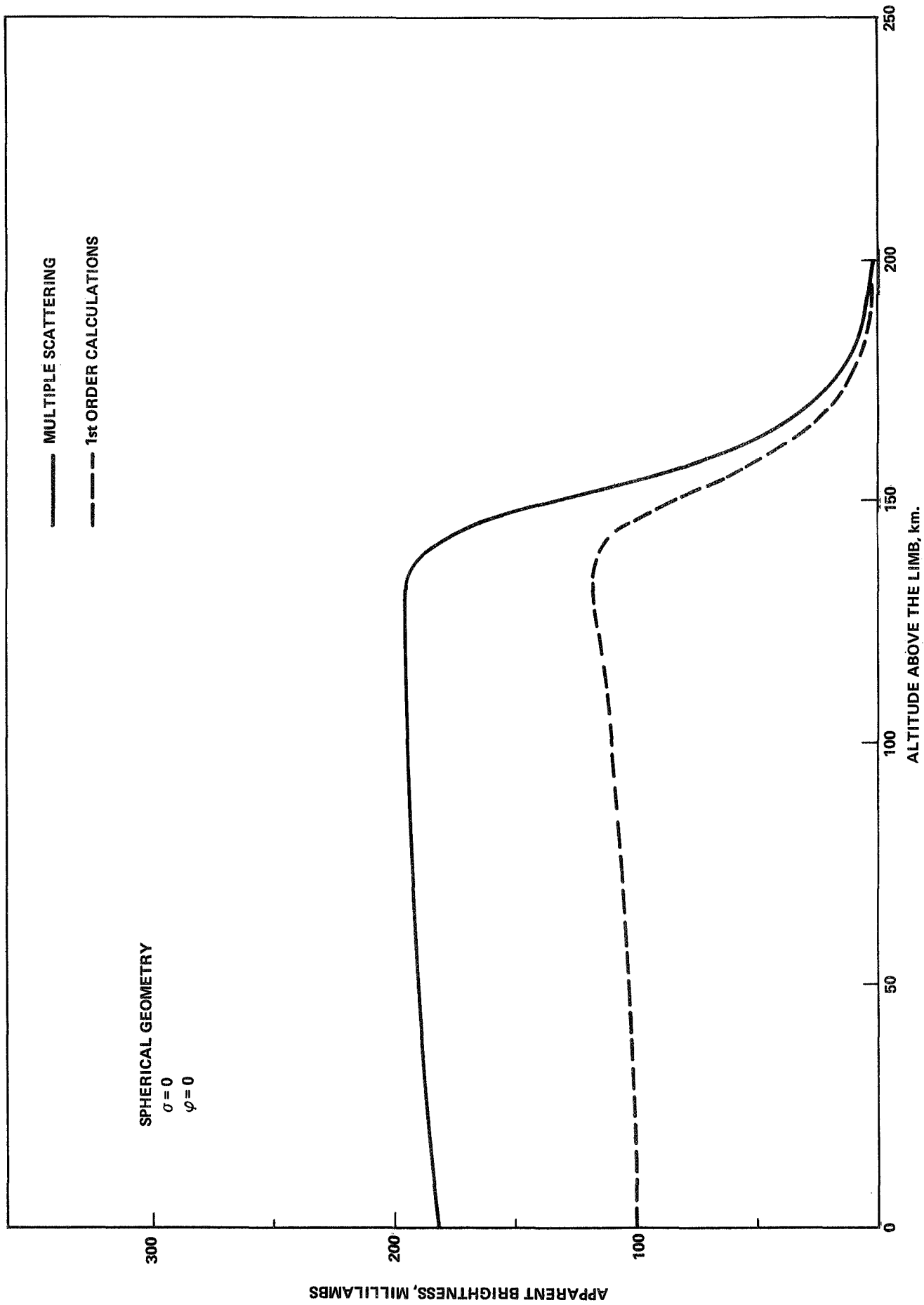
FIGURE 19 - APPARENT BRIGHTNESS ABOVE THE LIMB OF THE PLANET.

A. MARINER IV GEOMETRY

VIEWING ANGLE

90° 75° 60° 45° 30° 15°

REGION OF THE ATMOSPHERIC PROFILE

CENTER OF OBSERVED DISC

24°

SUB-SOLAR POINT

B. $\sigma = 0$ $\varphi = 0$

VIEWING ANGLE

90° 75° 60° 45° 30° 15°

REGION WHERE ATMOSPHERIC PROFILE WAS CALCULATED

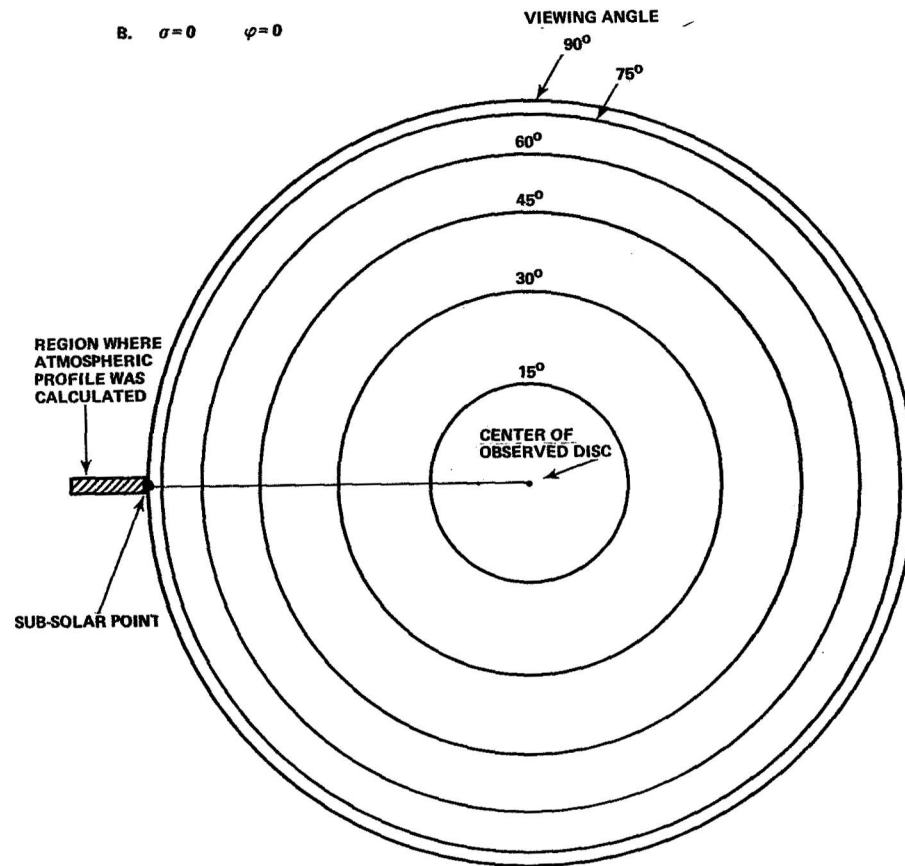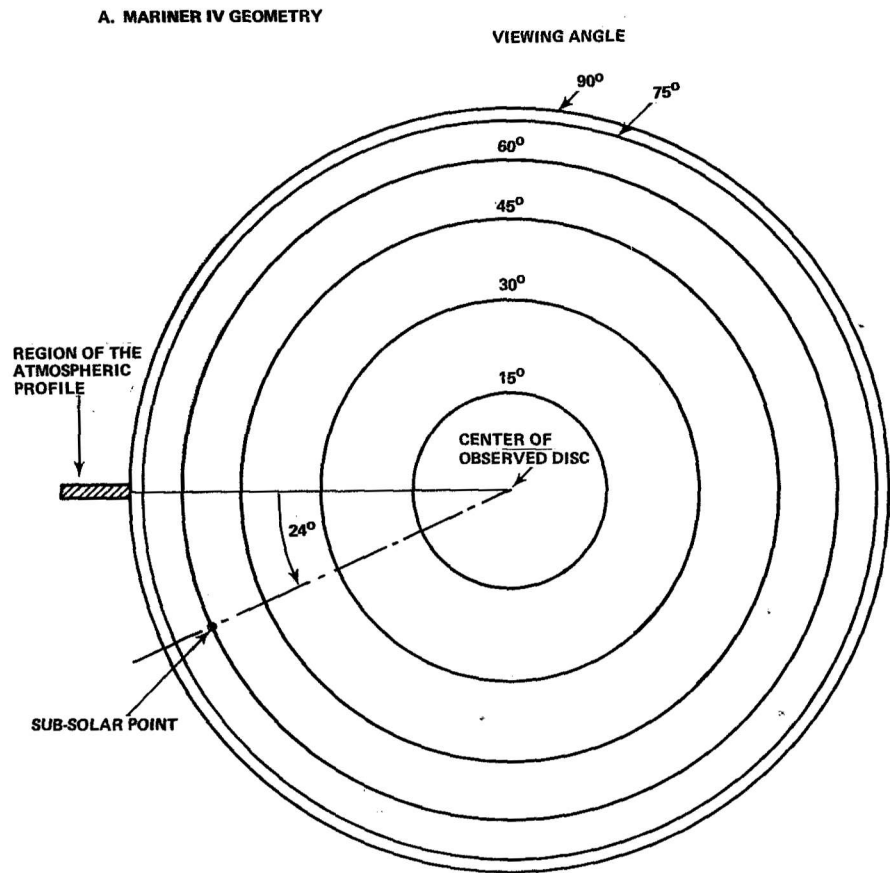CENTER OF OBSERVED DISC

SUB-SOLAR POINT

FIGURE 20 - ILLUSTRATION OF THE GEOMETRY USED IN FIGURE 18 (A) AND IN FIGURE 19 (B).