NASA CONTRACTOR
REPORT

NASA CR-61324

A STATISTICAL SAMPLING PROCEDURE FOR JIMSPHERE WINDS ALOFT POPULATION

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February 2, 1970

Prepared for

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER
Marshall Space Flight Center, Alabama 35812

TECHNICAL REPORT STANDARD TITLE PAGE

1. REPORT NO.	2. GOVERNMENT ACCESSION NO.	3. RECIPIENT'S CATALOG NO.
NASA CR-61324		

4. TITLE AND SUBTITLE

A STATISTICAL SAMPLING PROCEDURE FOR JIMSPHERE WINDS ALOFT POPULATION

5. REPORT DATE

February 2, 1970

6. PERFORMING ORGANIZATION CODE

7. AUTHOR(S)

Charles H. Carlisle

8. PERFORMING ORGANIZATION REPORT #

D5-15800

9. PERFORMING ORGANIZATION NAME AND ADDRESS

The Boeing Company
Aerospace Group, Southeast Division
Huntsville, Alabama

10. WORK UNIT NO.

11. CONTRACT OR GRANT NO.

NAS8-5608

12. SPONSORING AGENCY NAME AND ADDRESS

NASA-Marshall Space Flight Center
Aero-Astrodynamics Laboratory
Marshall Space Flight Center, Alabama, 35812

13. TYPE OF REPORT & PERIOD COVERED

CONTRACTOR REPORT

14. SPONSORING AGENCY CODE

15. SUPPLEMENTARY NOTES

Contract Monitor: J. W. Kaufman, Aero-Astrodynamics Lab, Aerospace Environment Div.

16. ABSTRACT

The large amount of available Jimsphere wind data necessitates the reduction of this information to a relatively small set of analytical probabilistic structures. This task has been accomplished for the time average power spectral density (PSD) of Jimsphere scalar wind profiles. These Jimsphere wind profile data were acquired at the Kennedy Space Center, Florida. It is shown that the distribution of PSD at a fixed wave length is well fitted by a gamma distribution (Pearson type III) and that the mean and square root of the variance of this distribution are essentially exponential functions of wave number. Sampling distributions on the mean as well as confidence intervals are obtained and the rate of convergence to normal sampling distributions is discussed. It is concluded that the time average PSD has a simple tractable probabilistic structure at all wave lengths between 100 m and 4000 m.

17. KEY WORDS

Statistics, Gamma Distribution
Power Spectral Density, Sampling Distribution, Confidence Intervals, Jimsphere Scalar Wind Profile

18. DISTRIBUTION STATEMENT

PUBLIC RELEASE

E. D. GEISSLER
Director, Aero-Astrodynamics Laboratory
MSFC

19. SECURITY CLASSIF. (of this report)	20. SECURITY CLASSIF. (of this page)	21. NO. OF PAGES	22. PRICE
U	U	52	

MSFC - Form 3292 (May 1969)

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PREFACE

This document has been prepared by the Postflight Trajectories Group of The Boeing Company to satisfy Contract NAS8-5608, Schedule II, Section A, Task 8.1.7 (see Exhibit CC, DRL 049, Line Item 171). The task performed was in support of the Aerospace Environment Division, Atmospheric Dynamics Branch (S&E-AERO-YE), Aero-Astrodynamics Laboratory, Marshall Space Flight Center, National Aeronautics and Space Administration. J. W. Kaufman, Chief of the Atmospheric Dynamics Branch, was the technical coordinator.

The author wishes to express his sincere appreciation to W. W. Vaughan, Chief, Aerospace Environment Division, S&E-AERO-Y, Marshall Space Flight Center-NASA, for the initial stimulating discussions defining the problem reported on herein and the need for a solution. The author is very grateful to J. W. Kaufman and D. Camp both of MSFC for their many hours spent in guiding the problem to a fruitful end, both technically and editorially and for their complete support of the author's various, sometimes needless, data requests. Dr. G. H. Fichtl, also of MSFC, has gracefully given much of his time to developing the author's understanding of past PSD calculations on scalar wind and the present state of available information on scalar wind. The author wishes to thank Dr. Fichtl for those many exciting conversations.

The author is very indebted to other members of The Boeing Company. In particular, he wishes to thank J. Williams, R. Simmons, and G. Engels for coding and implementing the large number of computer programs necessary to complete this document. To R. D. McCurdy, supervisor of the Postflight Trajectories, G. Engels, and R. Wheeler, the author is indebted for many hours spent in technical discussions and editorial suggestions. Finally, the author wishes to thank V. V. Moore, supervisor of Operational Flight Analysis Unit, for his valuable editorial comments.

SECTION 1

INTRODUCTION

1.0 GENERAL

An understanding of the statistical structure of local wind contributes to both environmental science and the design, development, and launch support of aerospace vehicles. The compilation of empirical statistics as well as the explanation of observed data by applied dynamic meteorology has produced many useful results. However, the analysis of the analytical structure of observed statistics with a view toward obtaining a deeper insight into the probabilistic structure of the wind has in some cases tended to lag behind these other efforts. The purpose of this report is to aid in filling this gap.

This report concentrates on one of the many statistical variables of meteorological interest; namely, the power spectra of the scalar wind (where the scalar wind is viewed as a function of height instead of time). To generate the statistics, scalar wind profiles obtained by use of the FPS-16 Radar/Jimsphere balloon system were analyzed for their power spectral content. This profile data was collected at Kennedy Space Center (KSC), Florida. At each frequency of interest, the PSD values from all profiles were assembled to form a time average statistic. This statistic exhibits a simple probabilistic structure. The wind has no preferred frequencies, the PSD probability distribution at each frequency has the same functional form, and the parameters of the probability distributions at different frequencies are related by a simple exponential function.

1.1 GENERAL DESCRIPTION OF ANALYSIS

The statistical analysis of power spectra was intended to produce both the time average population statistics and the sampling distributions associated with random sampling from the complete set of wind profiles. It was assumed that the population was completely defined by approximately two and one-half years of Jimsphere observations. No effort was made to compensate for time trends or seasonal and diurnal oscillations. It seems plausible, however, to contend that the functional form of the population statistics obtained in this report will be correct for a larger more carefully screened population. If this proves to be the case, only the estimation of those parameters specified in the function need be recalculated from the longer data record. Methods for performing these calculations are given below.

For comprehensive spectral statistics, a multivariate PSD population analysis would be desirable. For example, the power spectral densities used in this report were calculated at each of 256 wave numbers (cycles/4000 meters) for each scalar wind profile. The distribution of power density among the different wave numbers is necessary for an exhaustive analysis. A complete multivariate analysis would require a 256 variate distribution function. However, the labor involved in analyzing such a distribution is extremely prohibitive. For these reasons the sampling distributions are constructed independently at each wave number. Conse-

1.1 (Continued)

quently, a single sample which yields "good statistics" (as represented by the population statistics specified below) at one wave number may give poor statistics at another wave number.

It should also be mentioned that the estimation of the parent population probability density function severely affects each sampling density function. For example, one method for estimating the parent population may yield a good sampling distribution on the mean and a biased sampling distribution on the median, whereas a second method for estimating the parent population density function may do just the reverse. Caution must then be exercised in employing estimated parent parameters in sampling distributions.

In this report efforts are concentrated on the sampling distribution of the mean since this is more fully developed in the existing literature. Due to its frequent utility, a discussion of sampling distributions on the medians and other quantiles is presented.

Confidence intervals for the resulting sampling distributions were taken to be tolerance intervals centrally located about the mean value of the sampling distribution. For sufficiently high confidence, half the length of a central confidence interval will exceed the value of the mean. Since PSD is a scalar quantity and thus positive, the left end point of a high confidence interval centered at the mean will lie to the left of zero. Consequently, the left end point will not represent an obtainable PSD value and central confidence intervals on PSD are not defined for sufficiently high confidence. However, central confidence intervals are easy to compare with confidence intervals used in connection with a normal distribution having the same mean and variance as the sample distribution. The limiting formulas for large samples may then be employed with a known error.

SECTION 2

POPULATION AND SAMPLING STATISTICS FOR JIMSPHERE WIND PROFILE POWER SPECTRA

2.0 GENERAL

The purpose of this section is to describe and discuss the data and methods used to obtain probability inferences from compiled statistics. The first paragraph gives a brief description of the data and its condition. The second paragraph describes how the data were processed and how the statistics were compiled. This is followed by several paragraphs discussing the methods used to make statistical inferences.

2.1 DATA DESCRIPTION AND CONDITIONS

This investigation was based upon 1,196 FPS-16 radar/Jimsphere scalar wind profiles covering approximately two and one-half years of data from December 1964 to May 1967. Of the 1,196 profiles, 861 had complete data records of wind speed between the altitudes of 4 and 16 kilometers in 25-meter increments. Those profiles with complete data spanned approximately the same time period with no obvious gaps. These 861 profiles were taken as the population sample and all analysis was confined to the 4 to 16 kilometer altitude range.

Scalar wind profiles were obtained from raw FPS-16 data collected every 0.1 second by a preprocessing technique described in detail in References 1 and 2. The preprocessing essentially consisted of a tolerance filter to eliminate spurious points, an RMS interpolation to find the balloon position at every 25-meter height interval, and a finite differencing technique to obtain the velocity. Other authors have commented (Reference 3) that the processing attenuates power in wave numbers above 40 (where the unit of measure is cycles/4000 meters). This effect did not interfere with the statistical analysis developed below since a statistical analysis for wave numbers above 30 was not attempted.

Approximately 65 percent of the sample consisted of Jimsphere profiles taken at 0100 and 1300 GMT. The daily distribution of the data over the two and one-half year period is represented in Figure 2-1. The hourly and monthly distribution of the data are represented in Figures 2-2 and 2-3, respectively. Clearly, the early months of the year were more heavily weighted than the later months. No adjustment has been made to compensate for seasonal or diurnal bias in the data.

2.2 DATA PROCESSING

The power spectra for each of these profiles was constructed by using the fast Fourier transform method (References 4 and 5). In this type of spectral analysis, height replaces time as the independent variable. The fast Fourier transform method generated the spectra for some 861 profiles in less than two hours of computer time.

2.2 (Continued)

In the past, MSFC has generated power spectra after first prewhitening the signal with a Scoggins filter. The resulting statistics were compared with the statistics of the raw spectra obtained without prewhitening. No significant change in the statistical structure could be noted. A 41-weight Martin Graham filter was also tested as a prewhitening filter. Here again, there was apparently no significant change in statistics.

Of course, the basic purpose of prewhitening is to produce a white noise spectra so that a symmetric smoothing filter may be employed to estimate the true spectra. No spectral smoothing was employed due to the difficulties involved in analyzing the statistical bias of such a filter. A discussion of the indiscriminate use of symmetrically weighted smoothing filters is given in Section 3.

After the 861 power spectra were obtained, the spectral density values were grouped according to frequency. This resulted in 861 values at each of 256 wave numbers ranging from 0.3125 to 80 cycles per 4000 meters. At each wave number the values were ordered according to magnitude. From these ordered statistics the cumulative distributions on power spectral density were obtained. Ordered statistics were selected in preference to assembling density functions. These statistics allow the application of the distribution free "goodness of fit" tests such as the Kolmogorov test.

The population distributions were assumed to be defined by theoretical distributions fitting the ordered statistics. Sampling distributions for the mean and quantiles were then calculated from these theoretical population distributions. The sampling distributions were compared with equivalent numerical sampling distributions obtained from a machine oriented random sampling program (Reference 6), sampling on the 861 PSD values at a selected set of 6 frequencies. The numerical and theoretical sampling distributions agree relatively well. However, the numerical sampling distributions on quantiles are poorly defined with only 861 data values.

Central sampling confidence intervals for the mean for sample sizes ranging from 2 to 40 were calculated. Central intervals were chosen so that the rate of convergence of the sampling distributions to a normal distribution could be presented.

2.3 PEARSON TYPE III DISTRIBUTIONS

This section is offered for those readers who lack a familiarity with the normalized incomplete gamma function and its statistical relevance. Reference 7 contains most of the arguments presented in this section. Information on this distribution is generally available in the literature on statistics.

2.3 (Continued)

The incomplete gamma function may be defined by the integral formula

$$\gamma(y) = \int_0^y z^{\lambda-1} e^{-z}\, dz \tag{2.1}$$

where λ is a parameter which must be greater than zero.

The normalized function may be defined as

$$F(y) = \frac{\gamma(y)}{\Gamma(\lambda)}$$

where

$$\Gamma(\lambda) = \int_0^\infty z^{\lambda-1} e^{-z} dz \tag{2.2}$$

Since $\lim_{y \to \infty} F(y) = 1$

$F(0) = 0$, and F is monotonic increasing, the function $F(y)$ is a cumulative distribution function on the domain $0 < y < \infty$. The parameter λ shapes the function F. The density function associated with this distribution is given by

$$\frac{dF}{dy} = f(y) = \frac{1}{\Gamma(\lambda)} y^{\lambda-1} e^{-y} \tag{2.3}$$

In fitting such a function to data, one usually extends the definition of the function to include some transformation on the independent variable, such as a linear transformation. In our case, only a scaling factor is required since the minimal value of PSD is zero and a translation on y in the function $F(y)$ would destroy the property $F(0) = 0$. Thus, we require only

$$x = z/\beta \text{ or } z = \beta x$$

where β is a scaling parameter. Employing the transformation for z on equation (2.2) and noting that $\Gamma(\lambda)$ is a constant, the distribution function becomes

$$F(y) = \int_0^{(y/\beta)} \frac{1}{\Gamma(\lambda)} (\beta x)^{\lambda-1} e^{-\beta x} (\beta dx) \equiv F(x) \tag{2.4}$$

with density function

$$\frac{dF(x)}{dx} = f(x) = \frac{\beta}{\Gamma(\lambda)} (\beta x)^{\lambda-1} e^{-\beta x} \tag{2.5}$$

2.3 (Continued)

It is now necessary to fit two parameters to the data: a scaling parameter, β, and shaping parameter λ. To simplify further discussion, define $f(x)=0$ if $x < 0$.

The moments about the mean of this distribution (mean, variance, etc.) are relatively easy to calculate by employing characteristic functions. Let μ denote the mean value and let

$$\phi(t) = \int_{-\infty}^{\infty} e^{it(x-\mu)} f(x)dx \qquad (2.6)$$

denote the characteristic function. Notice that the nth derivative of $\phi(t)$ is given by

$$\frac{d^n\phi}{dt^n} = \int_{-\infty}^{\infty} [i(x-\mu)]^n e^{it(x-\mu)} f(x)dx \qquad (2.7)$$

If the nth derivative of $\phi(t)$ is evaluated at $t = 0$, one obtains

$$\left.\frac{d^n\phi}{dt^n}\right|_{t=0} = i^n \int_{-\infty}^{\infty} (x-\mu)^n f(x)dx \qquad (2.8)$$

The right-hand side of equation (2.8) is just the definition of the nth moment about the mean times i^n.

One may obtain an explicit form for $\phi(t)$ as follows:

$$\phi(t) = \int_{-\infty}^{\infty} e^{it(x-\mu)} \frac{\beta}{\Gamma(\lambda)}(\beta x)^{\lambda-1} e^{-\beta x}dx$$

$$= e^{-it\mu}\frac{\beta^\lambda}{\Gamma(\lambda)} \int_{-\infty}^{\infty} e^{x(it-\beta)}x^{\lambda-1}dx \qquad (2.9)$$

Perform the variable substitution

$$w = -(it - \beta)x$$

in the integral in (2.9) and obtain

$$\phi(t) = \frac{\beta^\lambda e^{-it\mu}}{\Gamma(\lambda)} \int_{-\infty}^{\infty} e^{-w}\left(\frac{-w}{it-\beta}\right)^{\lambda-1}\left(\frac{-1}{(it-\beta)}\right)dw$$

2.3 (Continued)

$$= \frac{\beta^\lambda e^{-it\mu}}{\Gamma(\lambda)(\beta-it)^\lambda} \int_{-\infty}^{\infty} e^{-w} w^{\lambda-1} dw$$

Since $e^{-w} w^{\lambda-1} = 0$ when $w < 0$ the above equation may be written as

$$\frac{\beta^\lambda e^{-it\mu}}{\Gamma(\lambda)(\beta-it)^\lambda} \int_{0}^{\infty} e^{-w} w^{\lambda-1} dw$$

$$= \frac{\beta^\lambda e^{-it\mu}}{\Gamma(\lambda)(\beta-it)^\lambda} \Gamma(\lambda)$$

$$= e^{-it\mu}(1 - \frac{it}{\beta})^{-\lambda} \qquad (2.10)$$

The first derivative of ϕ evaluated at zero is

$$-i(\mu-\lambda/\beta) \qquad (2.11)$$

Since equation (2.11) is $(-i)^1$ (see eq. 2.8) times the first moment about the mean, and since the mean, μ, is the first moment about the mean, one obtains

$$\mu - \lambda/\beta = 0$$

$$\mu = \lambda/\beta \qquad (2.12)$$

Substituting in equation (2.10) for μ, from (2.12), the second derivative evaluated at $t = 0$ is

$$(-i)^2 \ \lambda/\beta^2$$

Thus, the variance, denoted by μ_2, is

$$\mu_2 = \lambda/\beta^2 \qquad (2.13)$$

The higher moments may be obtained by a similar process but the first two moments are sufficient to determine the distribution. Solving equations (2.12) and (2.13) for the parameters λ and β one obtains

$$\lambda = \frac{\mu^2}{\mu_2}$$

$$\beta = \frac{\mu}{\mu_2}$$

2.4 PARAMETER ESTIMATION

In estimating λ and β from data several techniques were employed. The first technique discussed is called the maximum likelihood method. In this method a function called the likelihood function is maximized with respect to the parameters λ and β and the resulting formulas are called maximum likelihood estimators for λ and β.

The likelihood function L for the Pearson distribution is the finite product

$$L = \prod_{i=1}^{n} \left[\frac{\beta}{\Gamma(\lambda)}(\beta x_i)^{\lambda-1} e^{-\beta x_i}\right]$$

$$= \left(\frac{\beta}{\Gamma(\lambda)}\right)^n \left[\prod_{i=1}^{n}(\beta x_i)^{\lambda-1} e^{-\beta x_i}\right] \quad (2.14)$$

This function may be maximized with respect to β by maximizing the ln L with respect to β.

$$\ln L = n(\ln\beta - \ln\Gamma(\lambda)) + \sum_{i=1}^{n}[(\lambda-1)(\ln\beta + \ln x_i) - \beta x_i] \quad (2.15)$$

$$\frac{\partial \ln L}{\partial\beta} = n/\beta + \sum_{i=1}^{n}[(\lambda-1)/\beta - x_i] \quad (2.16)$$

Set equation (2.16) to zero and obtain

$$\bar{x} \equiv \frac{1}{n}\sum_{i=1}^{n} x_i = 1/\beta + (\lambda-1)/\beta = \lambda/\beta \quad (2.17)$$

Since λ/β is the true mean (equation 2.12) of the distribution, equation (2.17) demonstrates that the maximum likelihood estimator of the mean is given by the usual formula for estimating the mean.

But to obtain a formula for the pair of values of λ and β which maximize the likelihood function the value of

$$\frac{\partial \ln L}{\partial\lambda}$$

must be calculated. Thus, differentiating equation (2.15)

$$\frac{\partial \ln L}{\partial\lambda} = n\frac{\partial(-\ln\Gamma(\lambda))}{\partial\lambda} + \sum_{i=1}^{n}(\ln\beta + \ln x_i)$$

2.4 (Continued)

$$= -n\left(\frac{\Gamma'(\lambda)}{\Gamma(\lambda)}\right) + n \ln \beta + \sum_{i=1}^{n} \ln x_i \qquad (2.18)$$

where $\Gamma'(\lambda)$ is the derivative of Γ. Setting equation (2.18) equal to zero, one obtains

$$n\left(\frac{-\Gamma'(\lambda)}{\Gamma(\lambda)} + \ln \beta\right) = -\sum_{i=1}^{n} \ln x_i = \ln \left(\prod_{i=1}^{n} x_i\right) \qquad (2.19)$$

Now employ the constraint given by equation (2.17) to remove β, and obtain

$$n\left(\frac{\Gamma'(\lambda)}{\Gamma(\lambda)} - \ln \frac{\lambda}{\bar{x}}\right) = \ln \left(\prod_{i=1}^{n} x_i\right) \qquad (2.20)$$

where $\bar{x}$ is again given by

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

Rewriting (2.20)

$$n\left(\frac{\Gamma'(\lambda)}{\Gamma(\lambda)} - \ln\lambda\right) = \ln\left(\prod_{i=1}^{n} \frac{x_i}{\bar{x}}\right)) \qquad (2.21)$$

This equation is easy to solve numerically for λ.

These formulas were used to fit the functions to the data. In Section 3 the results will be compared with other methods of fit. Although it is not obvious from equation (2.21), the results of Section 3 show that the formula for estimating the variance of a normal distribution, namely

$$\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

is not an adequate estimator of the variance. The values of the variance given by this estimator were calculated from the data for comparison purposes and were used with the values of the mean, given by equation (2.17), to calculate the parameters λ and β. Using these values of λ and β the gamma function had data fit errors in excess of 10 percent.

The second method of estimation employed was a least squares curve fit. The parameters λ and β are estimated by minimizing the sum of the squared deviations of the Pearson distribution from the empirical distribution.

2.4 (Continued)

This technique yields an excellent fit to the empirical distribution of data. The resulting parameter estimates were employed in building analytical sampling distributions and central confidence intervals. Of course, these estimates are not necessarily unbiased although they are consistent.

The third method of fit employed was a constrained least squares method. In constructing a fit which would yield an unbiased sampling distribution on the mean, the ratio λ/β, was held constant at the true population mean μ. The mean square deviation was minimized under this constraint.

An additional method of fit was also studied but was not used in this study. This method is much more promising for future work because it employs more information than is generally available from the distribution functions. An assumption is made that the percentile levels are given as functions of wave number by a function of the form

$$P = P_0 \, k^{-s}$$

where P is the PSD value for a given percentile level, at wave number k; P_0 and s are parameters. This assumption is well supported by References 3 and 4 and the results given below.

Assuming that the quantiles are reasonably close to normal distributions (with a sufficiently large sample of wind profiles this is quite reasonable) a root mean square estimate of P_0 and s may be made by fitting linearly

$$\ln P = \ln P_0 - s \ln k \; .$$

If this estimate is made at each wave number k, the functional form of $s(k)$ and $P_0(k)$ may be recovered.

In using the data in this report, the noise level in the high frequency range restricts the usable values of k. In fact, the approach just described should definitely be supported by careful data conditioning and spectral smoothing. See section 3 for a further discussion on this subject.

2.5 SAMPLING DISTRIBUTIONS

After the distribution functions on the PSD population have been fitted, the sampling distributions on the mean, variance and quantiles can be constructed. For simple random sampling with replacement, the sampling density function for n-random samples is given by the finite product.

$$\prod_{j=1}^{n} \left[\frac{\beta}{\Gamma(\lambda)} (\beta x_j)^{\lambda-1} e^{-\beta x_j}\right] \equiv L \qquad (2.22)$$

2.5.1 Sampling Distributions on the Mean

The sampling distribution on the mean, m, is then given by

$$F(m) = \underbrace{\int\int\cdots\int}_{n\ \text{times}} L\, dx_1\, dx_2 \ldots dx_n \tag{2.23}$$

integrated over the region specified by the set of all x_j's such that

$$\frac{1}{n}\sum_{j=1}^{n} x_j \leq m \tag{2.24}$$

The moments of the distribution may be found by the use of equation (2.10) which indicates that the characteristic function of

$$\frac{\beta}{\Gamma(\lambda)}(\beta x_j)^{\lambda-1}\ e^{-\beta x_j}$$

about zero is given by

$$\phi_j(t) = (1 - it/\beta)^{-\lambda} \tag{2.25}$$

Now the characteristic function, $\Phi(t)$, of $F(nm)$ is given by

$$\Phi(t) = \underbrace{\int_0^\infty \int_0^\infty \cdots \int_0^\infty}_{n\ \text{times}} e^{itm}\, L\, dx_1\, dx_2 \ldots dx_n$$

$$= \prod_{j=1}^{n} \left[\int_0^\infty e^{itx_j} \frac{\beta}{\Gamma(\lambda)}(\beta x_j)^{\lambda-1}\ e^{-\beta x_j}\, dx_j \right] = \prod_{j=1}^{n} \phi_j \tag{2.26}$$

Using the result of (2.25) in the final expression of (2.26) one obtains

$$\Phi(t) = (1 - it/\beta)^{-n\lambda} \tag{2.27}$$

According to equations (2.10) and (2.25) equation (2.27) is the characteristic function of the density function

$$\frac{\beta}{\Gamma(n\lambda)}(\beta x)^{n\lambda-1}\quad e^{-\beta x} \tag{2.28}$$

another gamma density function. Thus

$$F(nm) = \int_0^{nm} \frac{\beta}{\Gamma(n\lambda)}(\beta x)^{n\lambda-1}\ e^{\beta x}\ dx \tag{2.29}$$

2.5.1 (Continued)

or by the variable substitution $x = nx$

$$F(m) = \int_0^m \frac{n\beta}{\Gamma(n\lambda)} (n\beta x)^{n\lambda-1} e^{-n\beta x} dx \tag{2.30}$$

In this example, λ is replaced by $n\lambda$ and β is replaced by $n\beta$. According to equations (2.12) and (2.13), the mean and variance of the sampling distribution on the mean are then given by

$$\text{mean} = \mu_s = \frac{n\lambda}{n\beta} = \frac{\lambda}{\beta} = \mu \tag{2.31}$$

$$\text{var.} = \mu_{2s} = \frac{n\lambda}{(n\beta)^2} = \frac{\lambda}{n\beta^2} = \frac{1}{n}\mu_2 \tag{2.32}$$

where μ and μ_2 are the mean and variance from the parent population.

Comparison of equation (2.32) for the variance of the sampling distribution with the formula for sampling from a normal distribution

$$\text{var} = \frac{1}{n} \sigma^2 \tag{2.33}$$

shows that the true variance, equation (2.32), is identical to the usual formula employed for sampling from a normal distribution. The rate of convergence of the gamma function sampling distribution to the normal sampling distribution centered at the mean, μ, with standard deviation

$$\sigma \equiv \sqrt{\frac{\mu_2}{n}} \tag{2.34}$$

as a function of n is presented in Section 3.

2.5.2 Sampling Distribution on the Variance

The sampling distribution on the variance, v, is given by

$$F(v) = \underbrace{\int\int \ldots \int}_{n \text{ times}} L dx_1 \ldots dx_n \tag{2.35}$$

integrated over the values of the x_i's satisfying

$$\frac{1}{n} \sum (x_i - \mu)^2 \leq m, \tag{2.36}$$

Once again, μ denotes the population mean. If the reader wishes to pursue the structure of this function, Reference 7 offers several usable techniques.

2.5.3 Sampling Distribution on the Quantiles

The sampling distributions on the quantiles (percentage levels) are of some utility due to the fact that the power spectra are usually presented in the form of exceedance envelopes. In order to treat the theory easily a simplifying assumption will be made. If the sample size is n, only those percentage levels of the form

$$\frac{r-1}{n-1}$$

where r is a positive integer, will be discussed. Thus, if the sample size is 10, the sampling distribution for the 50 percent level cannot be obtained exactly (under the above assumption) since there is no integer r such that (r-1)/9 - 0.5.

Let F denote the population distribution (in our case F is given by equation 2.4) and f the density function of the variable in question. Then the sampling density function on the $\frac{r-1}{n-1}$ percentile value is (Reference 7):

$$f_r(x) = \frac{n!}{(r-1)!(n-r)!} (F^{r-1}(x))(1-F(x))^{n-r} f(x) \tag{2.37}$$

The distribution function associated with $f_r(x)$ is easily obtained by expanding the right hand side of (2.37) and integrating term by term.

$$\int_0^y f_r(x) = \sum_{j=0}^{n-r} (-1)^j \frac{n!}{(r-1)!(n-r)!} \binom{n-r}{j} \left(\frac{1}{r+j-1}\right) F^{r+j}(y) \tag{2.38}$$

Confidence intervals may be easily calculated numerically since

$$\int_{y_1}^{y_2} f_r(x)dx = \sum_{j=0}^{n-r} (-1)^j \frac{n!}{(r-1)!(n-r)!} \binom{n-r}{j} \left(\frac{1}{r+j-1}\right) (F^{r+j}(y_2) - F^{r+j}(y_1)) \tag{2.39}$$

Appendix B contains a further discussion of quantile sampling.

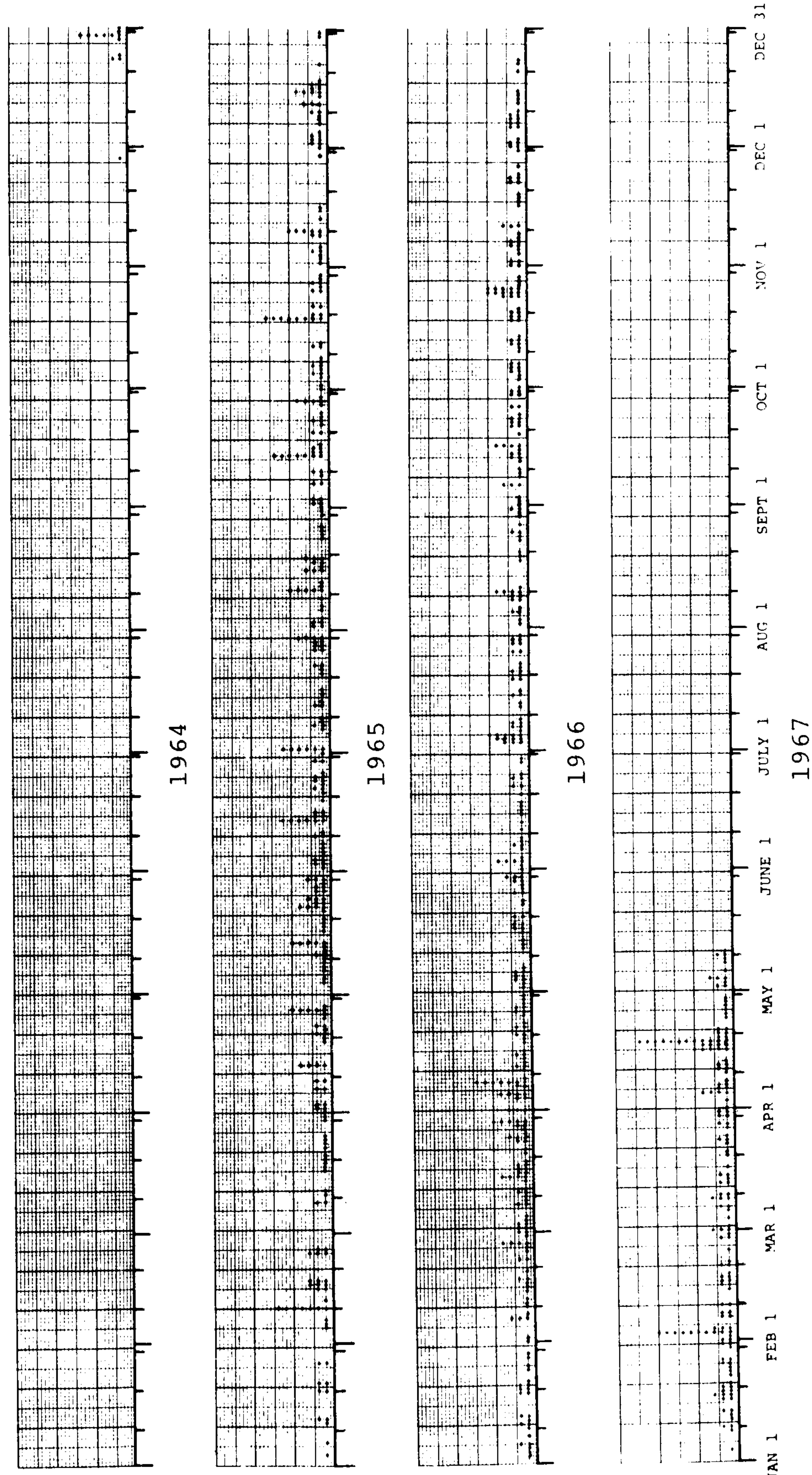

FIGURE 2-1. DAILY DISTRIBUTION OF JIMSPHERE RELEASES

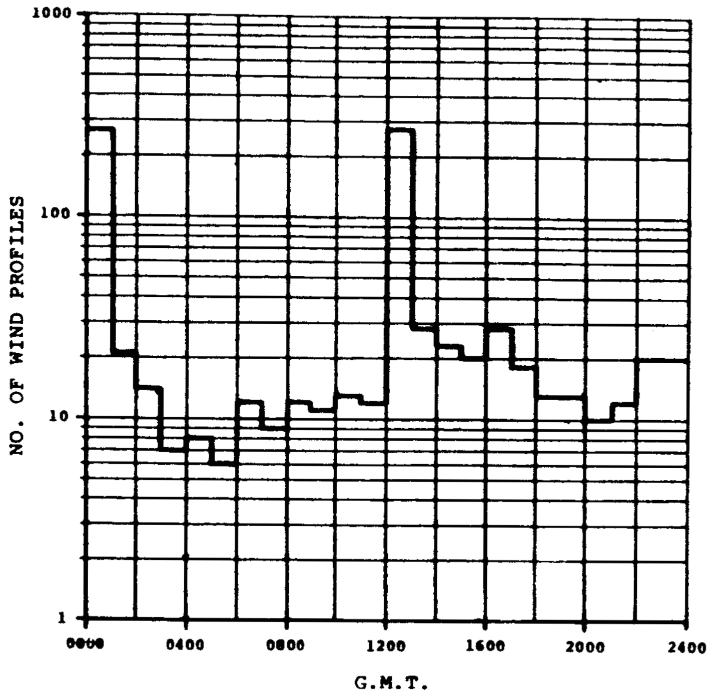

FIGURE 2-2. HOURLY DISTRIBUTION OF JIMSPHERE RELEASES

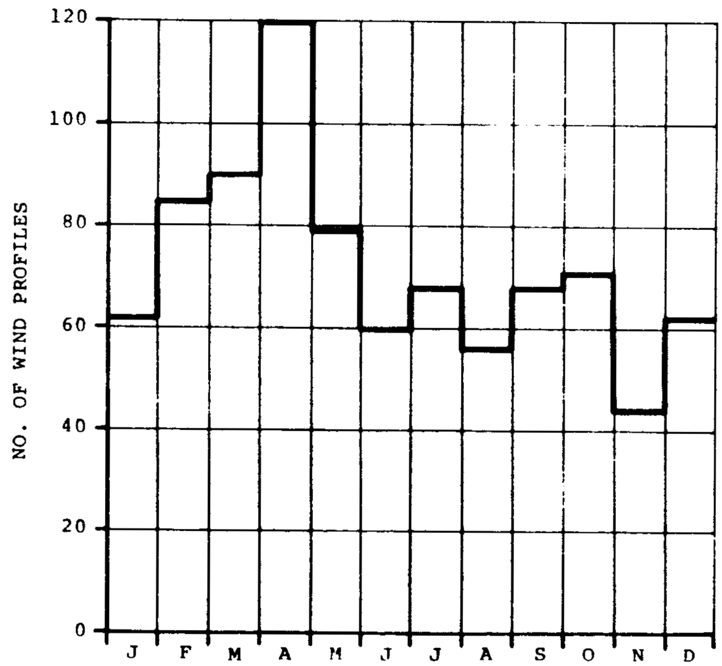

FIGURE 2-3. MONTHLY DISTRIBUTION OF JIMSPHERE RELEASES

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SECTION 3

RESULTS AND RECOMMENDATIONS

3.0 GENERAL

In this section the results of applying the fitting techniques of Section 2 and the subsequent implications are discussed. Recommendations for future work on PSD are also presented.

3.1 RESULTS

The cumulative distributions illustrated in Figures 3-1 through 3-6 are easily fitted by the gamma distribution. Each of the three methods of fit considered, least squares, constrained least squares, and maximum likelihood was designed to meet some specific criteria.

Consider first the least squares fit. Using this method of fit alone, it was possible to conclude that the empirical distributions illustrated in Figures 3-1 through 3-6 were almost certainly gamma distributions. The maximum deviation of the fitted function from the empirical distribution was always less than 0.025 with an RMS deviation of less than 0.015.

Now the Kolmogorov test (Reference 7) places confidence intervals on distribution functions and this least squares fit falls well within the 95 percent confidence level on a distribution function constructed from 861 values for most cases and never falls outside the 99% level. If the test was applicable, one could almost certainly conclude that the true PSD distribution is a gamma distribution with parameters given by the least squares fit. The difficulty comes from attempting to demonstrate that the test is applicable. First, it assumes that the ordered statistical distribution is constructed from 861 randomly selected samples which in this case may not be true. Secondly, the test places symmetric confidence intervals about the distribution function ordinates which almost never occur in practice but do happen to be closely approximated by the least squares fits obtained in this report. In any case, the test is not conclusive for the methods of fit employed in this report.

Consider secondly the constrained least squares fit. In contrast to the least squares fit, the constrained least squares fit maintains the expected value of PSD computed from the gamma functions at the mean value computed from the data. The resulting fit to the empirical distributions illustrated in Figures 3-1 through 3-6 is relatively poor. It roughly has a maximum error of +0.06 and a minimum error of -0.04. Since these deviations are not symmetric, the fit cannot be easily rejected by using the Kolmogorov test. Of course, this type of fit was employed to guarantee that the sampling distributions on the mean would be unbiased when compared with the data. The error in the maximum likelihood method of fit is approximately equal to the error from the constrained least squares method of fit.

These errors in fit are reflected in the mean value sampling density function presented in Figures 3-7 through 3-15. From a cursory examination of these curves, it appears that the least squares fit is, so to say, "too

3.1 (Continued)

good"; i.e., errors in the data values are worked into the fitting process and these errors appear in the sampling distribution. On the other hand, the maximum likelihood fit relies too heavily on the data distribution so that errors, such as an incorrect variance, strongly affect its shape. Consequently, it is a more realistic fit in the sense of being unbiased but is somewhat poorly shaped. The only solution to this inconsistency is to iterate towards the correct answer. A re-examination of the construction of the statistics is required.

In figure 3-16, selected percentile levels are plotted as a function of wave number. The peculiar rise in the higher percentage levels towards the high frequency range is due to balloon tracking noise. Elimination or filtering of those profiles containing tracking noise would increase the number of wave numbers for which distributions could be obtained. The functional dependence of the parameters on wave number might then be investigated. For example, a preliminary investigation indicates that the mean and square root of the variance have the same functional form as the median. Removal of the noise will also improve the estimation of the quantile levels. The principle question of interest here is whether or not these quantile lines are parallel. The drop in these curves around wave number 80 is probably due to power attenuation by data preprocessing. This could be resolved by reanalyzing the raw FPS-16 data but is probably not necessary.

A significant refinement of the distribution shape might occur by employing a more accurate spectral estimate than that obtained from the raw power spectra. This is usually a delicate operation, but in the case of wind spectra, it appears that there are many saving graces.

Most methods of spectral estimation, such as filter smoothing, reduce the resolution. However, all of the evidence indicates that there are no preferred frequencies (Reference 3) in the spectra of wind profiles. Thus, with nothing to resolve, resolution is not too important and a significant trade of resolution for accurate estimation could be made.

A second error usually employed in power spectral smoothing could also be avoided. Often times a symmetrically weighted narrow band filter is used as a smoothing filter. This is a correct procedure to apply to a white noise signal; i.e., one where the power viewed as a function of wave length is constant. Since an a priori knowledge of the rough structure of the spectra is necessary to properly prewhiten the signal, application of symmetrically weighted filters is often times incorrectly used.

In the case of wind spectra, an a priori knowledge of the general shape of the spectra is available (Reference 3) and a properly whitened signal could be constructed. Alternatively, if the direct transform method is preferred, a more correct nonsymmetrically weighted smoothing filter can be employed. Thus, it should be possible to obtain an excellent estimate of the true spectra. The statistics which result may indeed clarify the discrepancies in methods of distribution

3.1 (Continued)

function fit and allow one to be much more conclusive about the probabilistic structure of the wind.

Finally, some change in the distributions might be obtained by deleting the more obvious biases in the time distribution of the data. But the effect of such an effort is not at all clear and may indeed be negligible.

The values of the parameters λ and β obtained by the different methods of fit at six selected wave numbers are presented and compared in Table 3-I. In summary, the population statistics for PSD values at all wave numbers between 5 cy/4km and 25 cy/4km are tentatively given by Pearson Type III distributions with shaping parameter, λ, equal to approximately 9/10. The parameter β is not constant but the logarithm of β is a linear function of the logarithm of the wave number, k. Since λ is approximately constant, the log of the mean λ/β, and the log of the square root of the variance $\sqrt{\lambda/\beta^2}$, are also linear functions of the logarithm of k.

The resulting mean value sampling distributions together with the machine oriented random sampling distributions are shown in Figures 3-7 through 3-15 for three selected wave numbers and three selected sample sizes. The tables in the Appendix show the tolerance limits on central confidence intervals together with a comparison of the normal approximation. The central limit theorem is well illustrated by the rate of convergence of these tolerance levels as sample size increases. Clearly, good approximations may be obtained by using the Gaussian normal confidence intervals.

As a final speculation, it is very attractive to consider the possibility that λ is actually 1 instead of 9/10, since the distribution function then reduces to a χ^2 function for two degrees of freedom. Such a situation could possibly be explained by considering the Fourier coefficients a_k and b_k in the Fourier expansion of the wind profile as normally distributed variables with zero mean and equal variance. Since the power at wave number k is given by

$$P_k = \frac{1}{2}(a_k^2 + b_k^2)$$

normality in a_k and b_k would imply the χ^2 with two degrees of freedom as the appropriate distribution for power. Attempts to fit the χ^2 function to the PSD distributions were unsuccessful. But, to this author at least, there is no apparent reason why the bivariate distribution of the pairs (a_k, b_k) should not have zero mean. Otherwise there is some preferred pairing of sine and cosines in the expansion. Furthermore, it is not obvious why the variance on a_k should be different from the variance on b_k or why the correlation coefficient should not be zero.

In fact, it seems much more likely that the bivariate distribution is not normal, either because of the structure of the wind or because of contamination from sources not presently understood. In any case, it is clear that the bivariate statistics of the Fourier coefficients is more fundamental

3.1 (Continued)

than the PSD statistics. For obviously the wind profile may be recovered from a knowledge of the expansion coefficients where a knowledge of the PSD values do not allow for the recovery of this information.

3.2 RECOMMENDATIONS

If the probabilistic structure of power spectra is to be pursued further, the data should be screened to eliminate noisy profiles. An accurate filter should be developed and applied and the method of fit given at the end of paragraph 2.4 should be attempted. Of course, the data set should be expanded if feasible and some study of the effect of daily data bias undertaken.

As regards fitting the gamma function in future work, the most convenient method of estimating these parameters for enlarged data sets is the maximum likelihood method. These formulas are convenient since they do not require the construction of a distribution function. Also, a continual update of the parameter estimates is easily obtained. The sampling distributions on the mean which result from the maximum likelihood method are unbiased.

As mentioned at the end of the last subsection, the bivariate distribution of the Fourier coefficients is more closely related to the wind profile. Since these coefficients are available when PSD are obtained with the direct transform method, their statistics should be compiled and analyzed. An increase in the data set would be necessary to obtain accurate results.

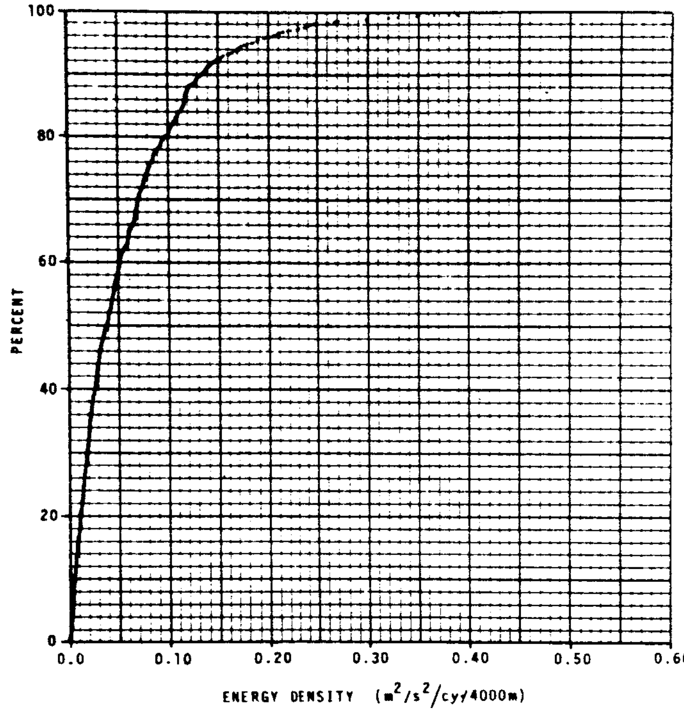

FIGURE 3-1. EMPIRICAL CUMULATIVE PSD DISTRIBUTIONS AT WAVE NUMBER 5

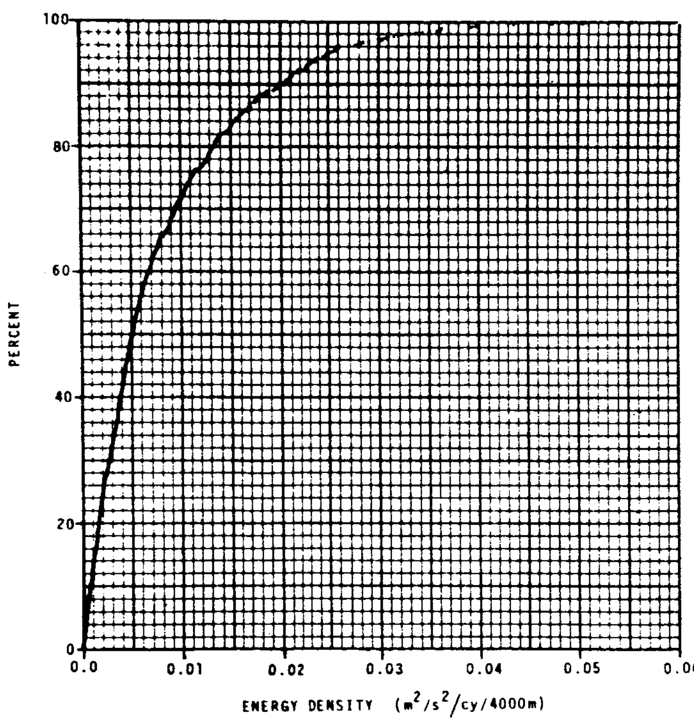

FIGURE 3-2. EMPIRICAL CUMULATIVE PSD DISTRIBUTIONS AT WAVE NUMBER 10

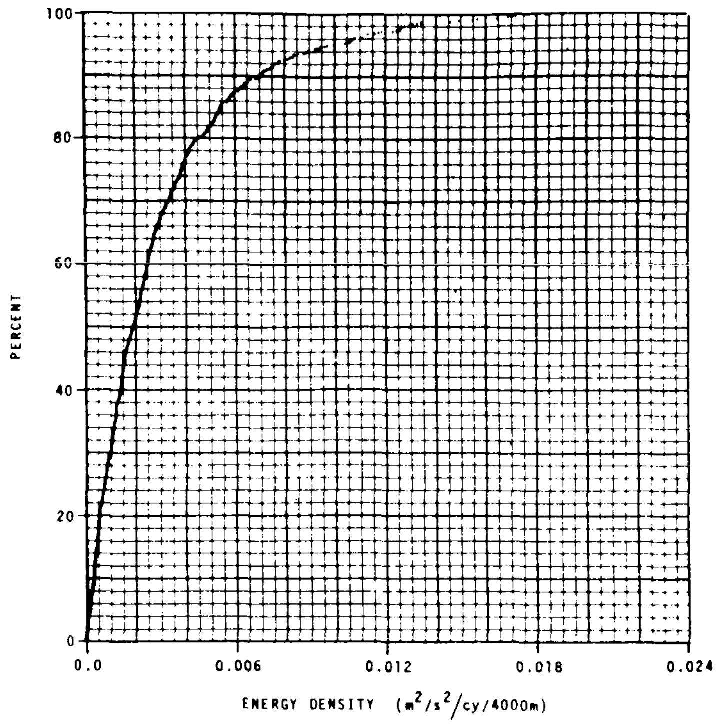

FIGURE 3-3. EMPIRICAL CUMULATIVE PSD DISTRIBUTIONS AT WAVE NUMBER 15

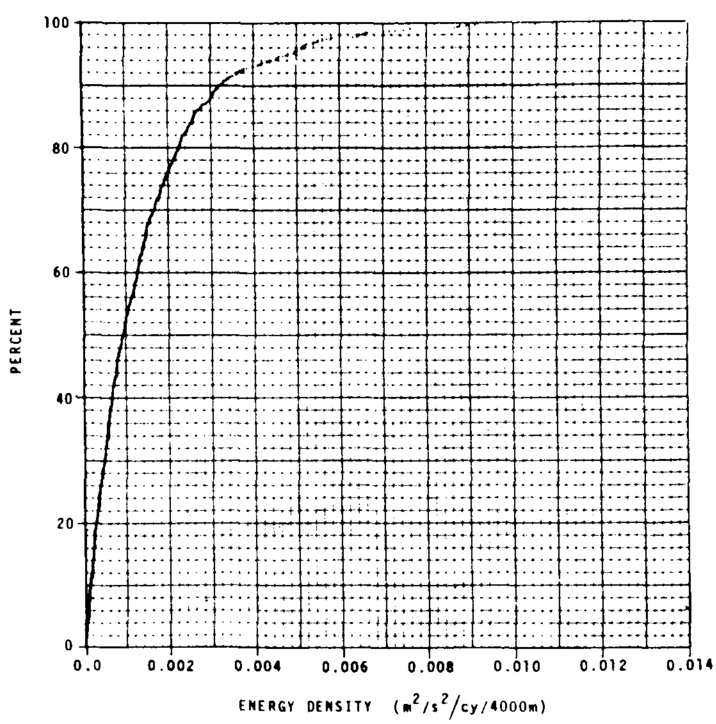

FIGURE 3-4. EMPIRICAL CUMULATIVE PSD DISTRIBUTIONS AT WAVE NUMBER 20

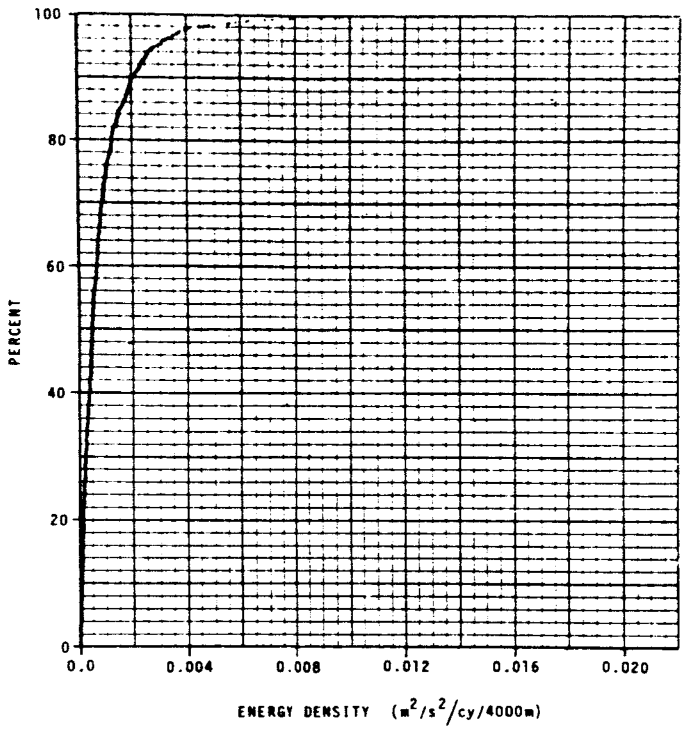

FIGURE 3-5. EMPIRICAL CUMULATIVE PSD DISTRIBUTIONS AT WAVE NUMBER 25

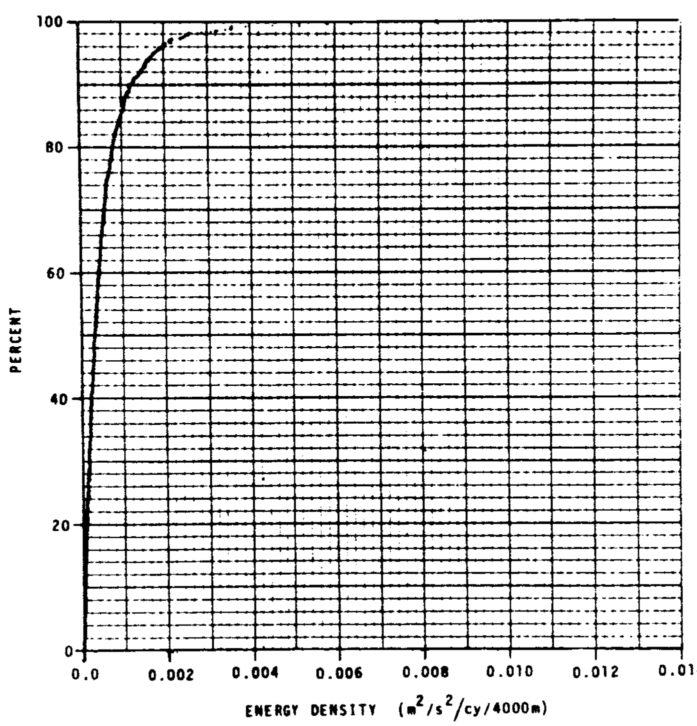

FIGURE 3-6. EMPIRICAL CUMULATIVE PSD DISTRIBUTIONS AT WAVE NUMBER 30

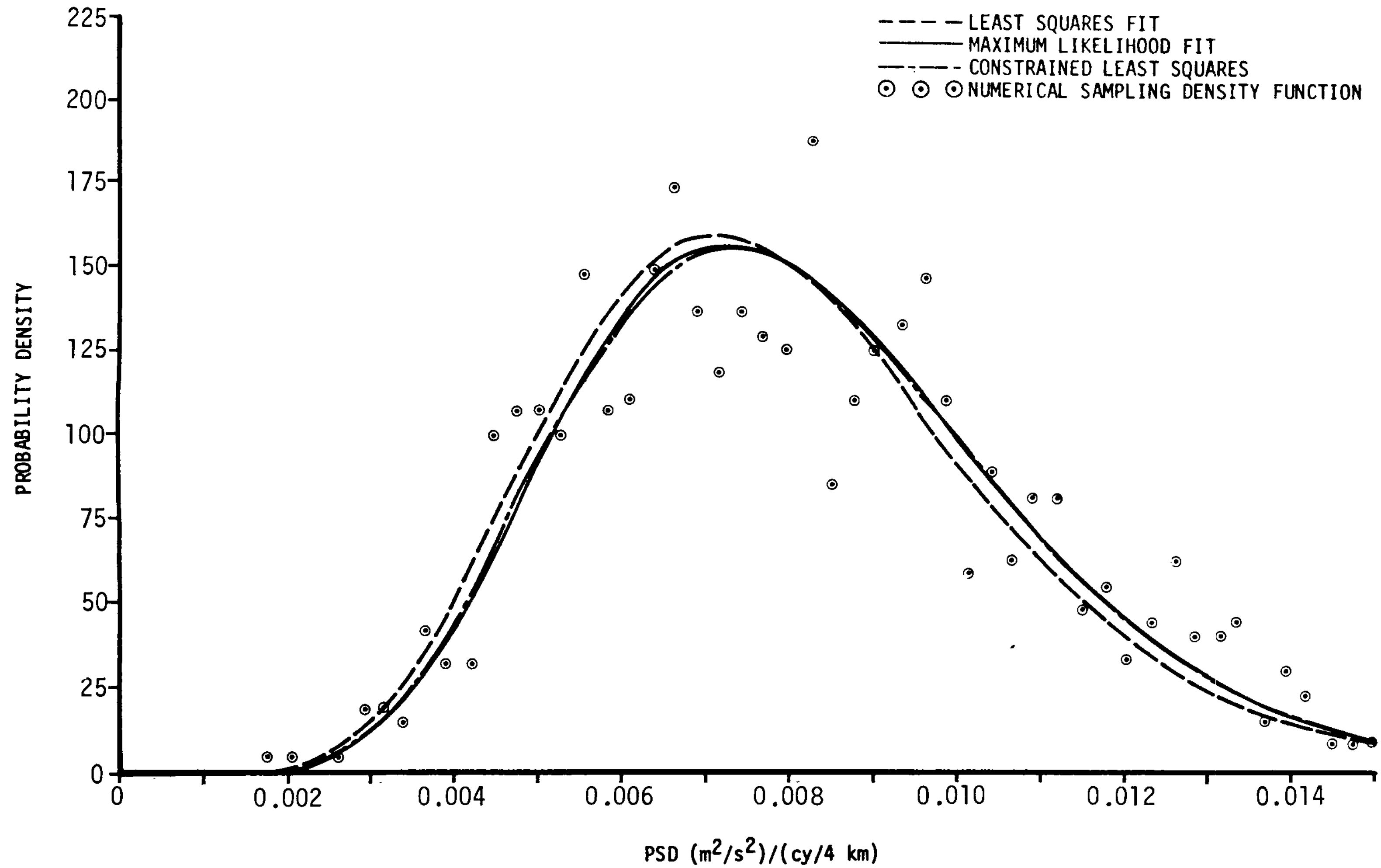

FIGURE 3-7. COMPARISON OF NUMERICAL AND THEORETICAL SAMPLING DISTRIBUTIONS ON THE MEAN AT WAVE NUMBER 10 FOR SAMPLE SIZE 10

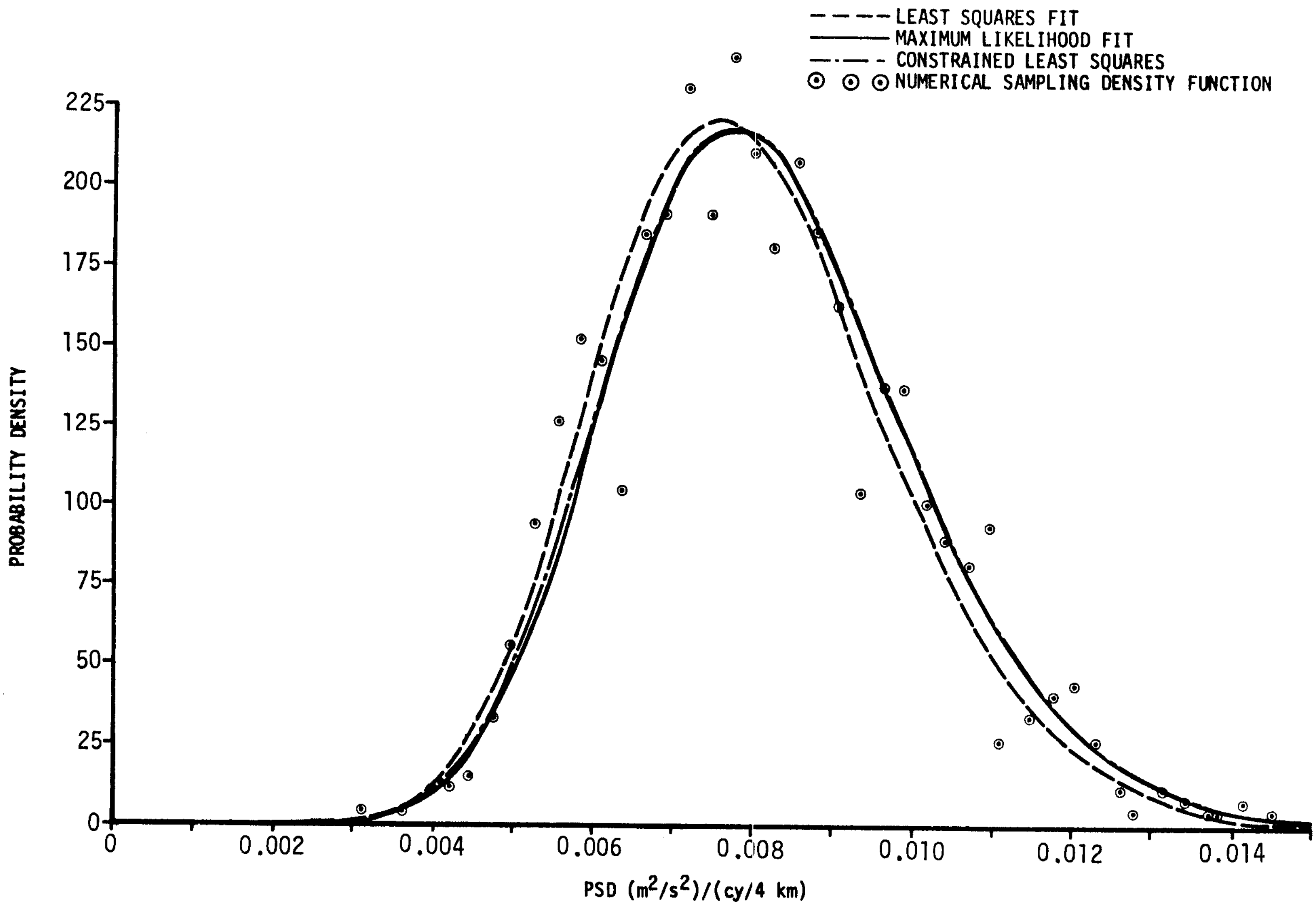

FIGURE 3-8. COMPARISON OF NUMERICAL AND THEORETICAL SAMPLING DISTRIBUTIONS ON THE MEAN AT WAVE NUMBER 10 FOR SAMPLE SIZE 20

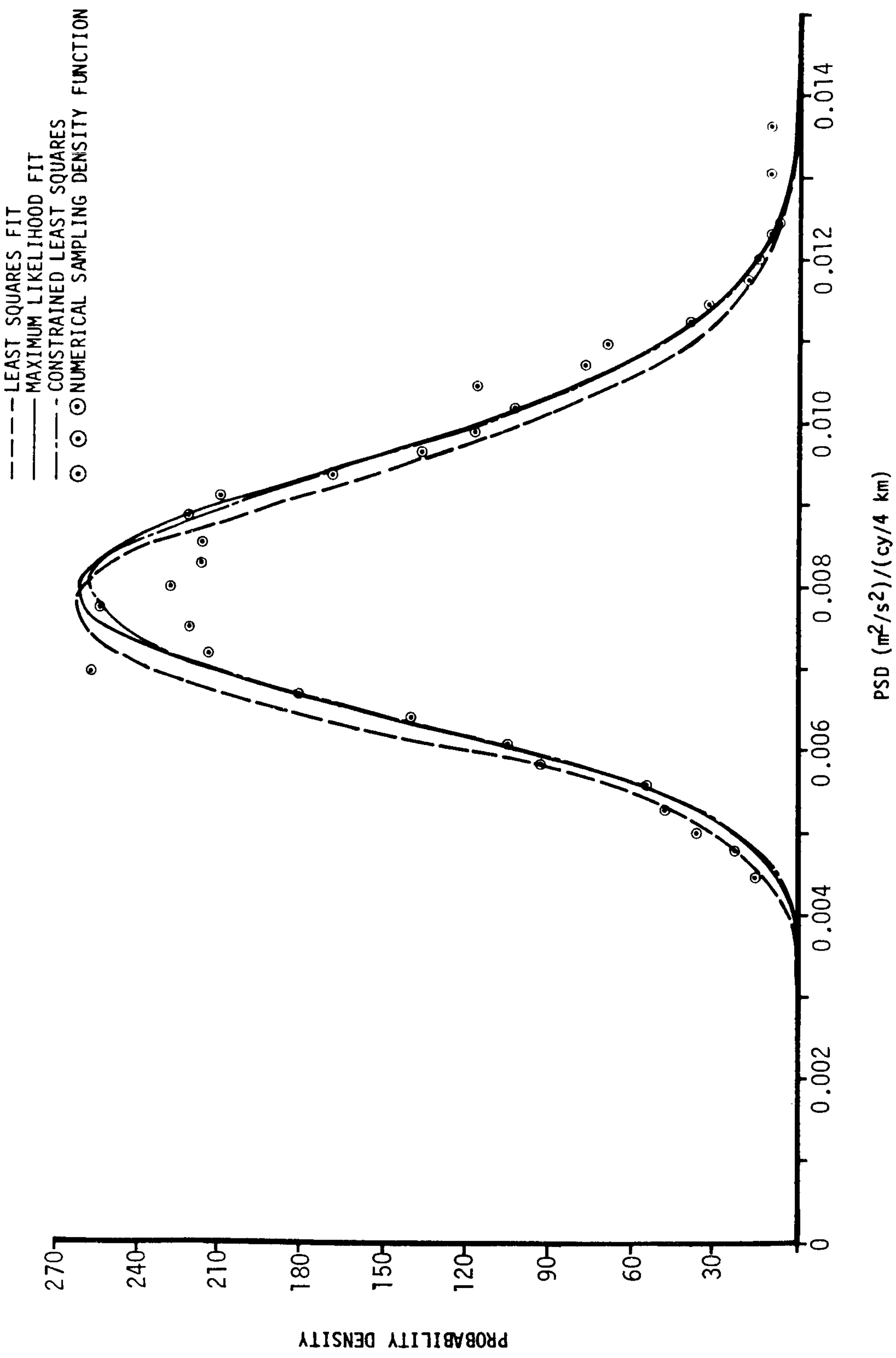

FIGURE 3-9. COMPARISON OF NUMERICAL AND THEORETICAL SAMPLING DISTRIBUTIONS ON THE MEAN AT WAVE NUMBER 10 FOR SAMPLE SIZE 30

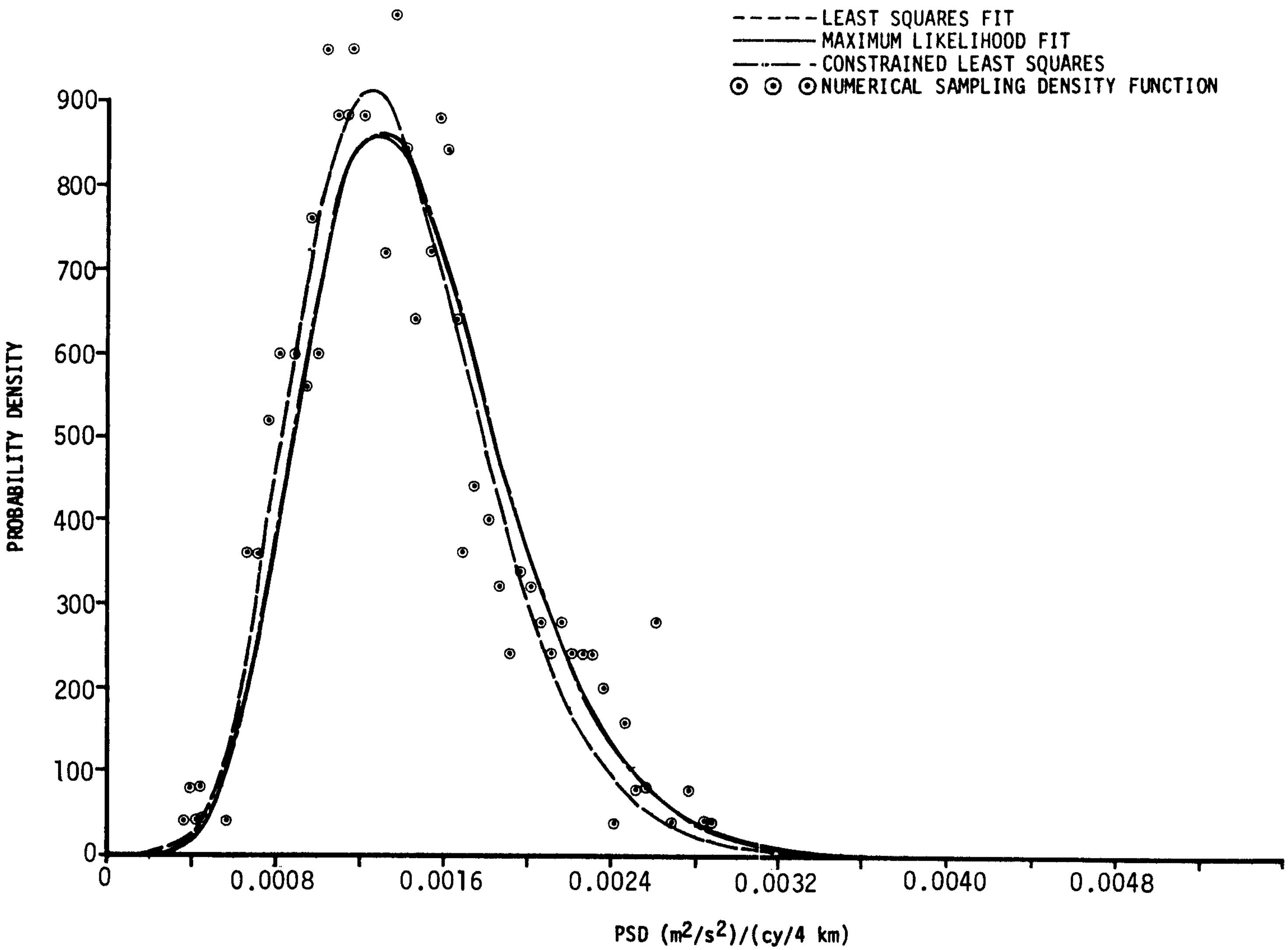

FIGURE 3-10. COMPARISON OF NUMERICAL AND THEORETICAL SAMPLING DISTRIBUTIONS ON THE MEAN AT WAVE NUMBER 20 FOR SAMPLE SIZE 10

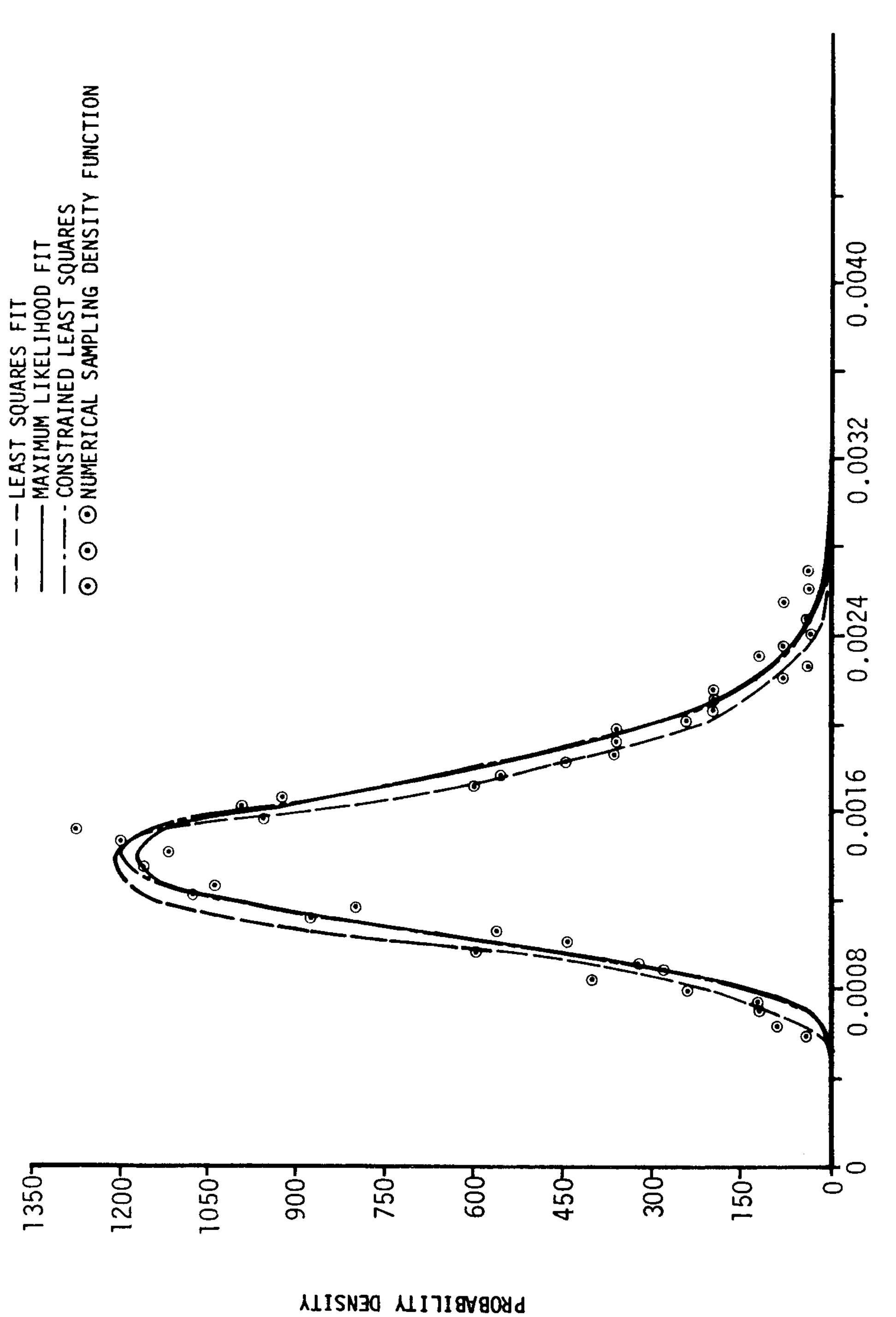

FIGURE 3-11. COMPARISON OF NUMERICAL AND THEORETICAL SAMPLING DISTRIBUTIONS ON THE MEAN AT WAVE NUMBER 20 FOR SAMPLE SIZE 20

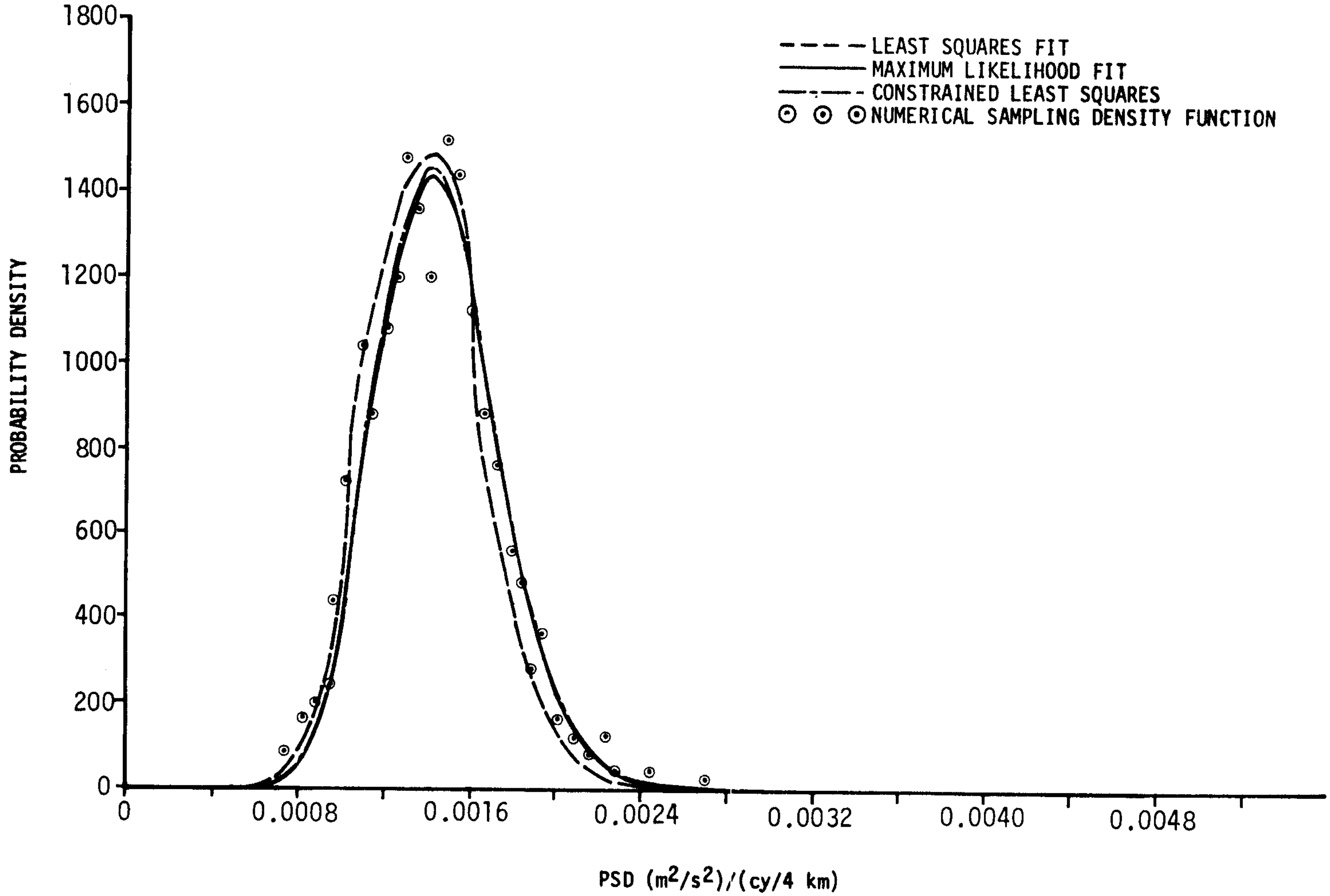

FIGURE 3-12. COMPARISON OF NUMERICAL AND THEORETICAL SAMPLING DISTRIBUTIONS ON THE MEAN AT WAVE NUMBER 20 FOR SAMPLE SIZE 30

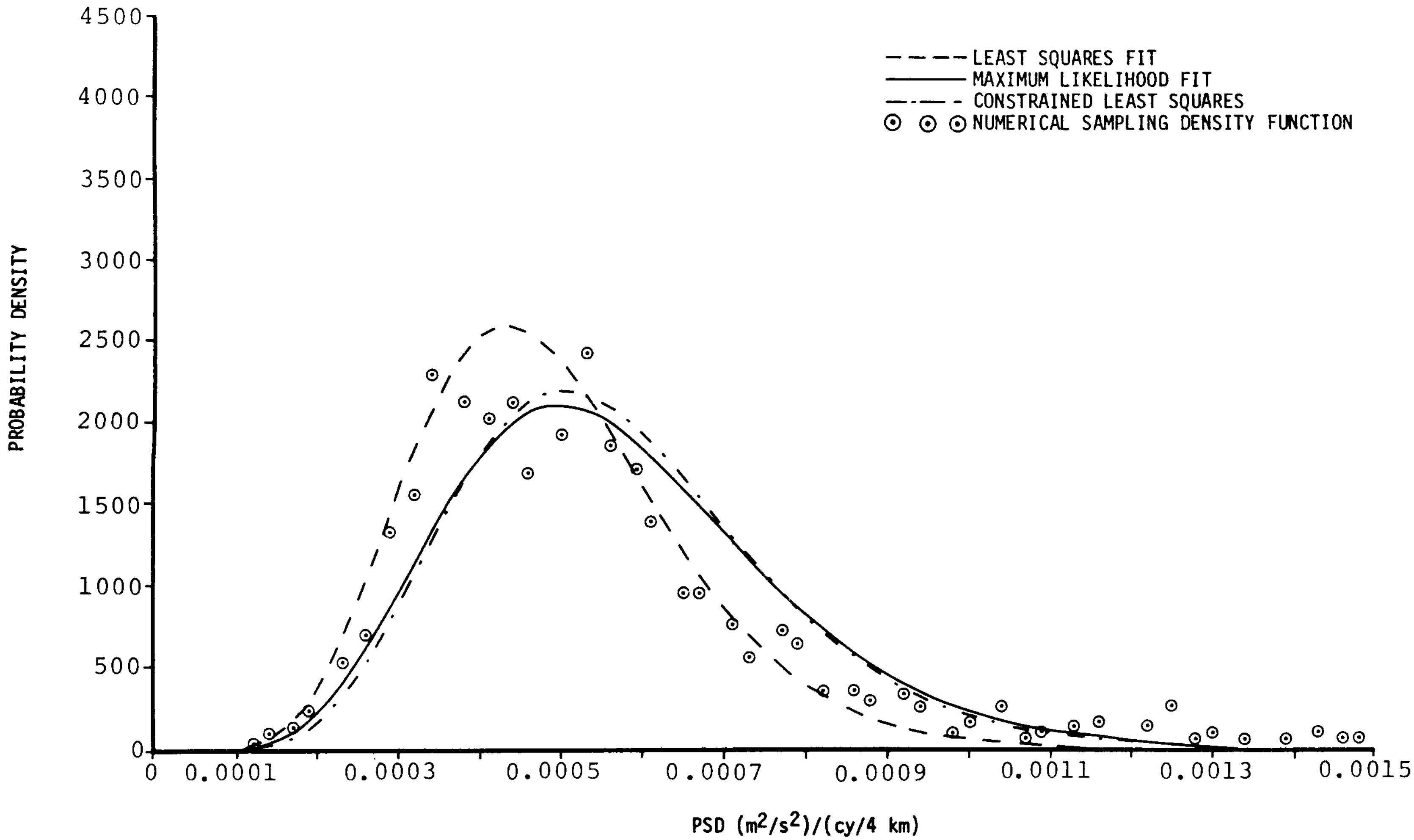

FIGURE 3-13. COMPARISON OF NUMERICAL AND THEORETICAL SAMPLING DISTRIBUTIONS ON THE MEAN AT WAVE NUMBER 30 FOR SAMPLE SIZE 10

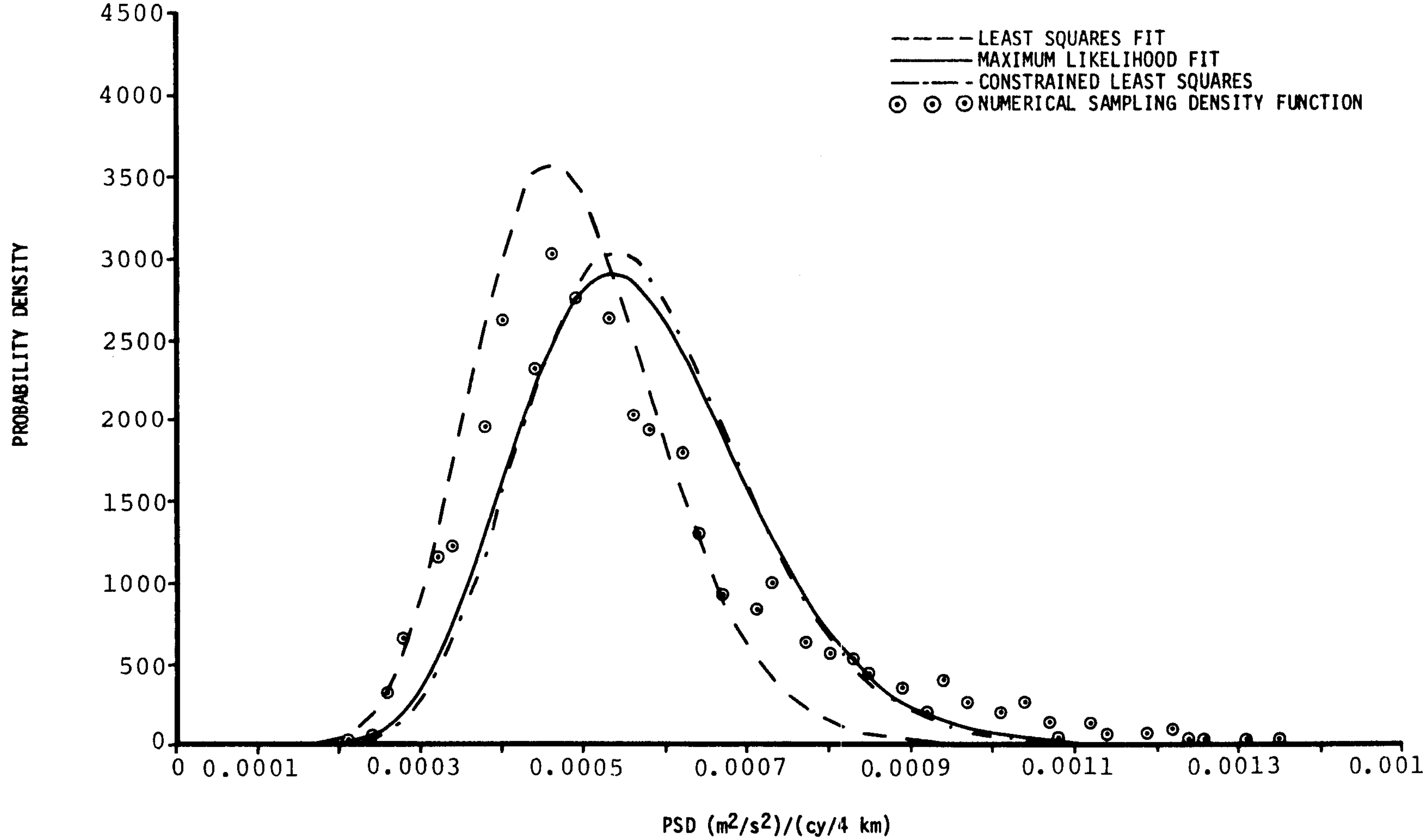

FIGURE 3-14. COMPARISON OF NUMERICAL AND THEORETICAL SAMPLING DISTRIBUTIONS ON THE MEAN AT WAVE NUMBER 30 FOR SAMPLE SIZE 20

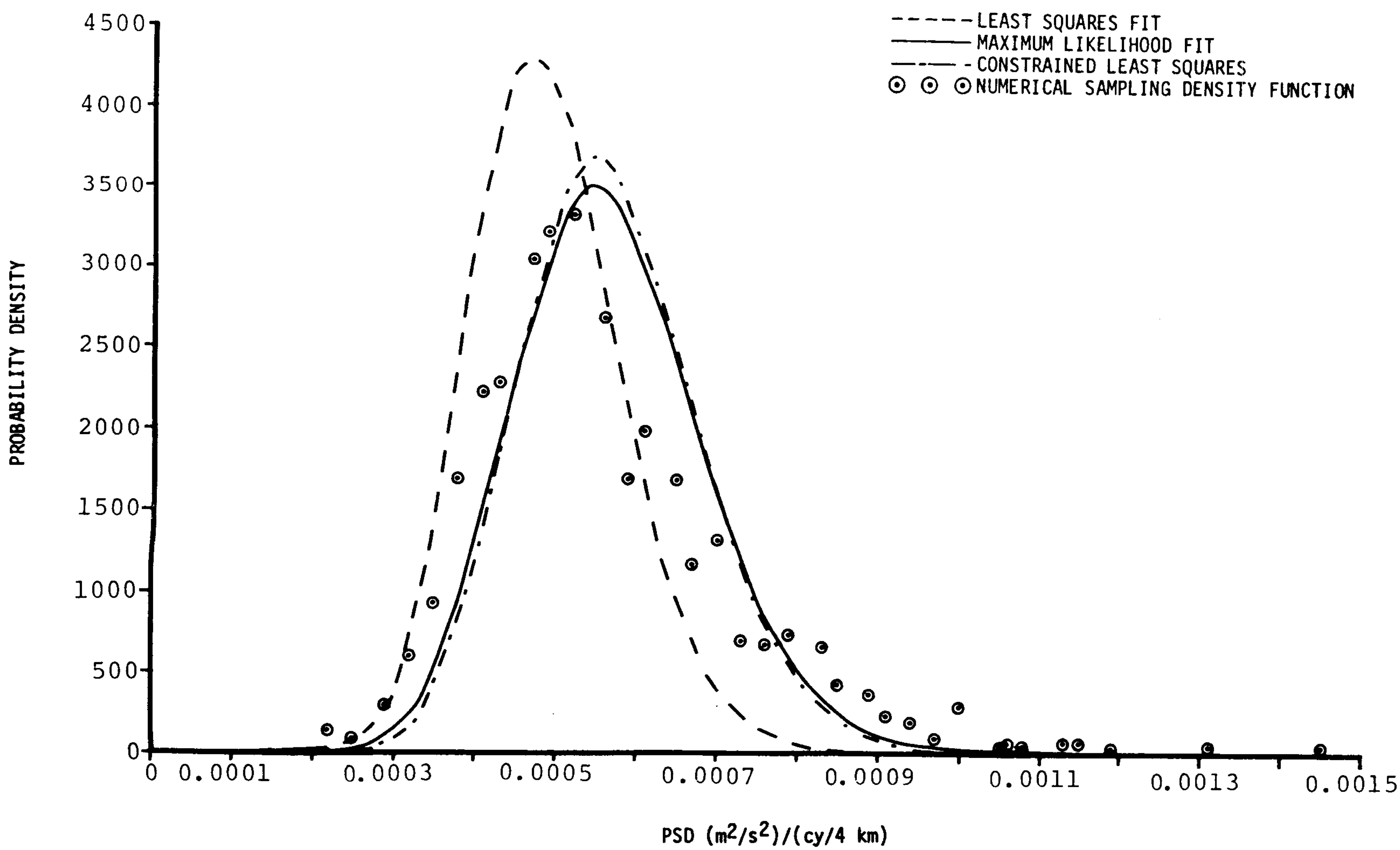

FIGURE 3-15. COMPARISON OF NUMERICAL AND THEORETICAL SAMPLING DISTRIBUTIONS ON THE MEAN AT WAVE NUMBER 30 FOR SAMPLE SIZE 30

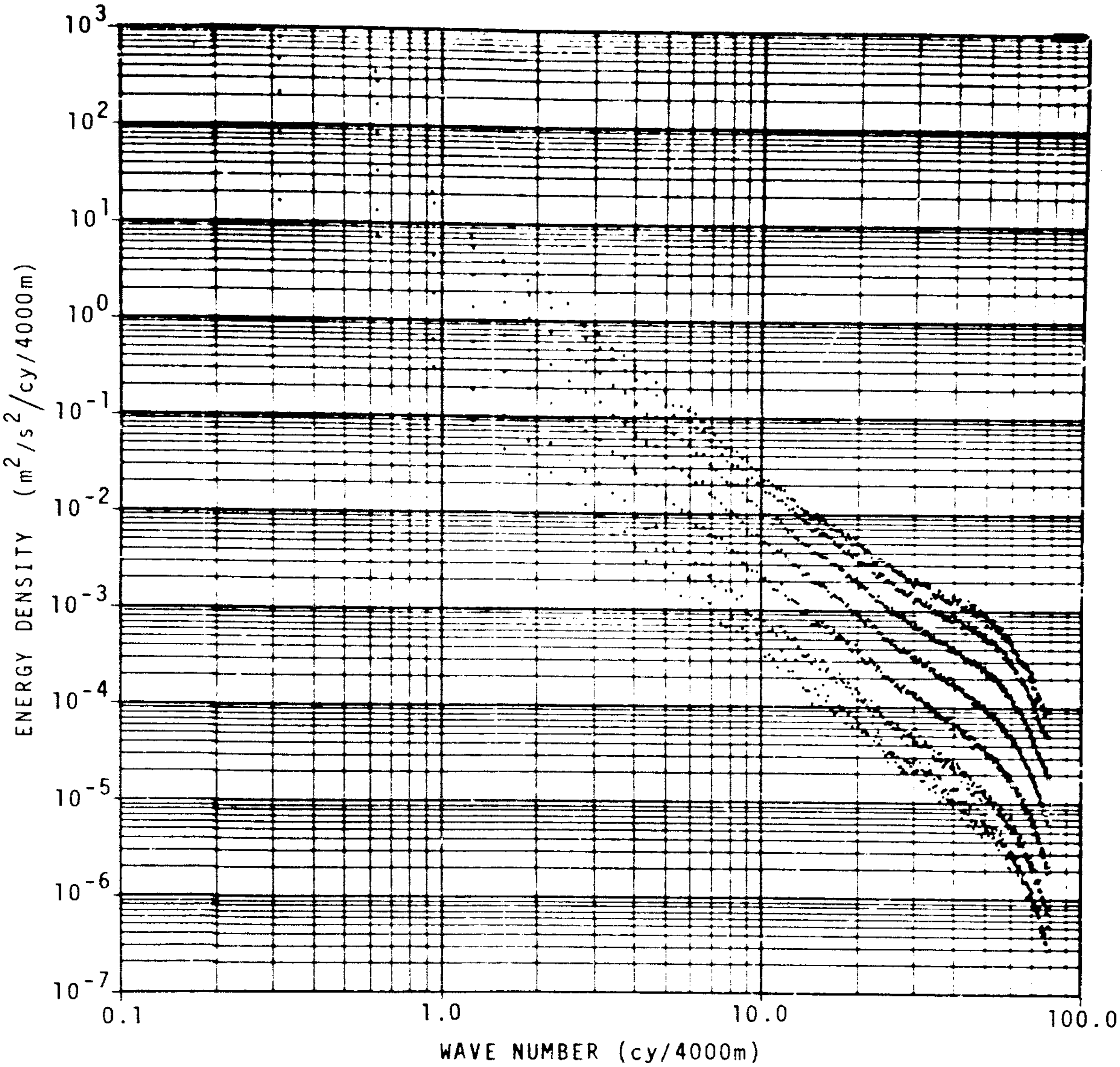

FIGURE 3-16. 5, 10, 25, 50, 75, 90, AND 95 PERCENTILE PSD VALUES PLOTTED AS A FUNCTION OF WAVE NUMBER

WAVE NO.	λ	β	λ/β	λ/β^2
		LEAST SQUARES ESTIMATORS		
5	0.865179	14.7415	0.0586905	0.00398134
10	0.927396	116.492	0.00796103	0.0000683397
15	0.929622	329.636	0.00282282	0.00000855534
20	0.916526	657.703	0.00139353	0.00000211878
25	0.902129	1162.06	0.000776321	0.000000668056
30	0.909847	1868.19	0.000487021	0.000000260691
		MAXIMUM LIKELIHOOD ESTIMATORS		
5	0.903647	15.1393	0.0596888	0.00394262
10	0.945772	115.771	0.00816933	0.0000705646
15	0.905528	306.794	0.00295158	0.00000962071
20	0.890628	611.172	0.00145725	0.00000238435
25	0.800717	880.540	0.000909348	0.00000103272
30	0.812664	1429.96	0.000568312	0.000000397432
		CONSTRAINED LEAST SQUARES ESTIMATORS		
5	0.865032	14.4930	0.05968606	0.00411825
10	0.925964	113.350	0.00816907	0.0000720694
15	0.927418	314.224	0.00295146	0.00000939284
20	0.915363	628.166	0.00145720	0.00000231977
25	0.881433	969.302	0.000909348	0.000000938147
30	0.893350	1571.94	0.000568310	0.000000361534
		NORMAL PARAMETER ESTIMATORS		
5	0.786545	13.1780	0.0596860	0.00452920
10	0.903435	110.592	0.00816909	0.0000783670
15	0.816986	276.807	0.00295146	0.0000106625
20	0.753781	517.281	0.00145720	0.00000281704
25	0.429262	472.063	0.000909331	0.00000192629
30	0.395605	696.125	0.000568296	0.000000816371

TABLE 3-I. PARAMETERS OBTAINED FROM THE DIFFERENT METHODS OF FIT

APPENDIX A

TABLES OF CONFIDENCE INTERVALS ON THE MEAN

In this appendix the tables of confidence intervals together with the errors associated with assuming a normal distribution are presented. The tolerance intervals are grouped according to confidence level. Thus the first table, Table A-I, lists the tolerance value for the 50 percent confidence intervals for sample sizes 2, 4, etc. for selected wave numbers. Referring to Table A-I, and assuming the maximum likelihood method of fit is employed, a sample of 16 wind profiles selected at random will give the true PSD mean at wave number 10 with an error of $\pm$ 1.412 x 10^{-3} $(m^2/s^2)/$(cy/4km), 50 percent of the time. Referring to the table in Figure A-V the PSD mean at wave number 10, calculated from a random sample of size 16, will be correct to within $\pm$ 5.678 x 10^{-3} (m^2/s^2)/(cy/4km) 99 percent of the time.

The difference between the several methods of fit, least squares (L.S.), maximum likelihood (M.L.) and constrained least squares (C.L.S.) is in part a measure of the relative accuracy of these methods. Disregarding the higher wave numbers, which were increasingly contaminated by balloon tracking error, the difference between the several methods of fit is negligible.

If confidence intervals at other wave numbers are desired, they may be quickly approximated by a graphical method. Select the confidence interval, sample size, and type of fit desired. Plot the tolerance value (table entry) vs. wave number on log-log graph paper. Since this is essentially a straight line, interpolation and extrapolation may be effected graphically.

For sample sizes above 40, the normal distribution can be used to obtain confidence intervals. To calculate confidence intervals, refer to a table of normal integrals where

$$P = \int_{-x}^{x} \phi(x)dx$$

in the table and ϕ is a Gaussian density function with zero mean and unit variance (σ = 1). Select the sample size, n, and confidence desired, P. Locate the value of x giving the probability P in the table. Decide on the wave number desired and obtain the variance λ/β^2 from Table 3-1. Divide the variance by n and take the square root of the result. Multiply the result by x to obtain the tolerance value δ. Thus

$$\delta = x\sqrt{\frac{\lambda}{n\beta^2}}$$

Tables A-IX and A-X list the error in confidence incurred when using the normal approximation for sample sizes 10, 20, etc. The entries in this table represent the percentages which must be added to the confidence calculated from the normal approximation to obtain the true confidence values. This error is clearly negligible when the sample size exceeds 40.

SAMPLE SIZE	WAVE NO. 5			WAVE NO. 10			WAVE NO. 15			WAVE NO. 20			WAVE NO. 25			WAVE NO. 30		
	L.S. $X10^{-3}$	M.L. $X10^{-3}$	C.L.S. $X10^{-3}$	L.S. $X10^{-4}$	M.L. $X10^{-4}$	C.L.S. $X10^{-4}$	L.S. $X10^{-4}$	M.L. $X10^{-4}$	C.L.S. $X10^{-4}$	L.S. $X10^{-5}$	M.L. $X10^{-5}$	C.L.S. $X10^{-5}$	L.S. $X10^{-5}$	M.L. $X10^{-5}$	C.L.S. $X10^{-5}$	L.S. $X10^{-5}$	M.L. $X10^{-5}$	C.L.S. $X10^{-5}$
2	29.20	29.09	29.70	38.34	38.98	39.37	13.57	14.37	14.21	67.48	71.51	70.60	37.87	33.76	44.84	23.66	29.10	27.85
4	20.99	20.90	21.34	27.52	27.98	28.26	9.739	10.32	10.20	48.46	51.38	50.70	27.20	27.71	32.22	16.99	20.95	20.01
6	17.22	17.14	17.51	22.58	22.94	23.18	7.988	8.469	8.369	39.75	42.15	41.59	22.31	24.06	26.44	13.94	17.19	16.41
8	14.95	14.88	15.20	19.59	19.91	20.12	6.933	7.350	7.264	34.50	36.59	36.10	19.37	21.55	22.95	12.10	14.93	14.25
10	13.39	13.33	13.62	17.55	17.83	18.02	6.209	6.583	6.505	30.89	32.77	32.33	17.35	19.69	20.55	10.84	13.37	12.76
12	12.23	12.18	12.44	16.03	16.29	16.46	5.672	6.014	5.943	28.23	29.94	29.54	15.85	18.25	18.78	9.901	12.22	11.66
14	11.33	11.28	11.53	14.85	15.09	15.25	5.255	5.572	5.506	26.15	27.74	27.36	14.68	17.08	17.40	9.172	11.32	10.80
16	10.61	10.56	10.79	13.90	14.12	14.27	4.917	5.214	5.152	24.47	25.96	25.60	13.74	16.11	16.28	8.583	10.59	10.11
18	10.00	9.955	10.17	13.11	13.32	13.46	4.638	4.918	4.859	23.08	24.48	24.15	12.96	15.28	15.36	8.095	9.992	9.533
20	9.492	9.447	9.654	12.44	12.64	12.77	4.401	4.667	4.611	21.90	23.23	22.92	12.30	14.58	14.57	7.682	9.482	9.046
22	9.053	9.009	9.207	11.86	12.05	12.18	4.197	4.450	4.398	20.89	22.15	21.85	11.73	13.96	13.90	7.326	9.043	8.627
24	8.669	8.627	8.817	11.36	11.54	11.67	4.019	4.262	4.211	20.00	21.22	20.93	11.23	13.41	13.31	7.015	8.660	8.261
26	8.330	8.290	8.472	10.92	11.09	11.21	3.862	4.095	4.047	19.22	20.39	20.11	1.079	12.93	12.79	6.741	8.322	7.939
28	8.028	7.990	8.165	10.52	10.69	10.80	3.723	3.947	3.901	18.52	19.65	19.38	10.40	12.49	12.32	6.497	8.020	7.651
30	7.759	7.721	7.891	10.17	10.33	10.44	3.597	3.814	3.769	17.90	18.99	18.73	10.05	12.10	11.91	6.279	7.749	7.394
32	7.513	7.477	7.641	9.844	10.00	10.11	3.483	3.693	3.650	17.33	18.39	18.14	9.733	11.74	11.53	6.080	7.506	7.160
34	7.289	7.254	7.413	9.551	9.705	9.808	3.379	3.583	3.541	16.82	17.84	17.60	9.443	11.41	11.19	5.899	7.282	6.946
36	7.084	7.050	7.205	9.282	9.432	9.532	3.284	3.483	3.441	16.34	17.34	17.10	9.177	11.11	10.88	5.733	7.078	6.751
38	6.896	6.683	7.013	9.035	9.181	9.279	3.197	3.390	3.350	15.91	16.88	16.65	8.933	10.82	10.59	5.580	6.889	6.572
40	6.722	6.689	6.836	8.807	8.949	9.044	3.116	3.304	3.265	15.51	16.45	16.23	8.707	7.102	10.32	5.439	6.715	6.405

TABLE A-I. TOLERANCE VALUES IN $(m^2/s^2)/(cy/4km)$ FOR 50 PERCENT CONFIDENCE INTERVALS

SAMPLE SIZE	WAVE NO. 5			WAVE NO. 10			WAVE NO. 15			WAVE NO. 20			WAVE NO. 25			WAVE NO. 30		
	L.S. X10^{-2}	M.L. X10^{-2}	C.L.S. X10^{-2}	L.S. X10^{-3}	M.L. X10^{-3}	C.L.S. X10^{-3}	L.S. X10^{-4}	M.L. X10^{-4}	C.L.S. X10^{-4}	L.S. X10^{-4}	M.L. X10^{-4}	C.L.S. X10^{-4}	L.S. X10^{-4}	M.L. X10^{-4}	C.L.S. X10^{-4}	L.S. X10^{-4}	M.L. X10^{-4}	C.L.S. X10^{-4}
2	4.463	4.469	4.539	5.905	6.017	6.063	20.90	22.08	21.89	10.38	10.97	10.86	5.816	7.102	6.869	3.637	4.416	4.272
4	3.393	3.386	3.450	4.466	4.544	4.585	15.80	16.73	16.56	7.857	8.319	8.221	4.407	5.433	5.214	2.755	3.374	3.240
6	2.835	2.826	2.883	3.725	3.789	3.825	13.18	13.96	13.81	6.556	6.946	6.860	3.679	4.549	4.355	2.299	2.824	2.705
8	2.483	2.474	2.525	3.260	3.316	3.348	11.54	12.22	12.09	5.739	6.082	6.004	3.221	3.988	3.814	2.012	2.475	2.369
10	2.236	2.228	2.274	2.935	2.984	3.014	10.38	11.01	10.88	5.166	5.476	5.405	2.900	3.594	3.434	1.812	2.230	2.133
12	2.050	2.042	2.085	2.690	2 735	2.763	9.519	10.09	9.974	4.736	5.021	4.955	2.658	3.296	3.149	1.661	2.046	1.955
14	1.904	1.896	1.937	2.498	2.539	2.565	8.839	9.369	9.261	4.397	4.662	4.601	2.469	3.062	2.924	1.542	1.900	1.816
16	1.785	1.778	1.816	2.342	2.380	2.405	8.286	8.783	8.682	4.123	4.371	4.314	2.314	2.872	2.742	1.446	1.782	1.702
18	1.686	1.679	1.715	2.212	2.248	2.271	7.825	8.295	8.199	3.893	4.129	4.074	2.186	2.713	2.589	1.366	1.683	1.608
20	1.602	1.595	1.629	2.101	2.135	2.157	7.434	7.880	7.789	3.699	3.922	3.870	2.076	2.578	2.460	1.297	1.599	1.527
22	1.529	1.523	1.555	2.005	2.038	2.059	7.096	7.522	7.435	3.531	3.744	3.694	1.982	2.461	2.348	1.238	1.527	1.458
24	1.466	1.459	1.491	1.922	1.953	1.973	6.800	7.209	7.125	3.383	3.588	3.540	1.900	2.359	2.250	1.187	1.463	1.397
26	1.409	1.403	1.433	1.848	1.878	1.898	6.538	6.931	6.850	3.253	3.450	3.404	1.826	2.268	2.164	1.141	1.407	1.344
28	1.359	1.353	1.382	1.782	1.811	1.830	6.304	6.684	6.606	3.137	3.327	3.282	1.761	2.187	2.087	1.100	1.357	1.296
30	1.314	1.308	1.336	1.722	1.750	1.769	6.094	6.461	6.385	3.032	3.216	3.173	1.703	2.115	2.017	1.064	1.312	1.252
32	1.273	1.267	1.295	1.669	1.696	1.713	5.904	6.259	6.186	2.938	3.116	3.074	1.649	2.049	1.954	1.030	1.271	1.213
34	1.235	1.230	1.256	1.619	1.646	1.663	5.730	6.075	6.004	2.851	3.024	2.983	1.601	1.988	1.897	1.000	1.234	1.178
36	1.201	1.196	1.222	1.574	1.600	1.617	5.571	5.906	5.837	2.772	2.940	2.900	1.556	1.933	1.844	.9723	1.199	1.145
38	1.170	1.164	1.189	1.533	1.558	1.574	5.424	5.751	5.683	2.699	2.863	2.824	1.515	1.883	1.795	.9467	1.168	1.115
40	1.140	1.135	1.160	1.495	1.519	1.535	5.288	5.607	5.541	2.631	2.791	2.753	1.477	1.836	1.751	.9230	1.139	1.087

TABLE A-II. TOLERANCE VALUES IN $(m^2/s^2)/(cy/4km)$ FOR 75 PERCENT CONFIDENCE INTERVALS

SAMPLE SIZE	WAVE NO. 5			WAVE NO. 10			WAVE NO. 15			WAVE NO. 20			WAVE NO. 25			WAVE NO. 30		
	L.S. $X10^{-2}$	M.L. $X10^{-2}$	C.L.S. $X10^{-2}$	L.S. $X10^{-3}$	M.L. $X10^{-3}$	C.L.S. $X10^{-3}$	L.S. $X10^{-4}$	M.L. $X10^{-4}$	C.L.S. $X10^{-4}$	L.S. $X10^{-4}$	M.L. $X10^{-4}$	C.L.S. $X10^{-4}$	L.S. $X10^{-4}$	M.L. $X10^{-4}$	C.L.S. $X10^{-4}$	L.S. $X10^{-4}$	M.L. $X10^{-4}$	C.L.S. $X10^{-4}$
4	4.672	4.672	4.752	6.167	6.280	6.332	21.83	23.08	22.86	10.85	11.47	11.35	6.078	7.456	7.186	3.801	4.634	4.467
6	3.957	3.950	4.025	5.210	5.302	5.349	18.44	19.52	19.31	91.66	97.04	95.90	5.141	6.337	6.083	3.214	3.935	3.780
8	3.488	3.479	3.548	4.587	4.665	4.710	16.23	17.19	17.00	8.071	8.550	8.445	4.529	5.594	5.360	2.830	3.473	3.330
10	3.153	3.143	3.207	4.142	4.212	4.253	14.66	15.53	15.35	7.290	7.725	7.627	4.091	5.062	4.843	2.556	3.142	3.008
12	2.898	2.888	2.947	3.806	3.869	3.908	13.47	14.27	14.11	6.698	7.099	7.008	3.759	4.654	4.451	2.349	2.888	2.765
14	2.696	2.686	2.742	3.539	3.599	3.634	12.52	13.27	13.12	6.230	6.603	6.518	3.497	4.332	4.141	2.185	2.689	2.572
16	2.532	2.522	2.575	3.322	3.377	3.412	11.76	12.46	12.32	5.848	6.200	6.119	3.283	4.068	3.888	2.051	2.525	2.414
18	2.393	2.384	2.434	3.140	3.192	3.224	11.11	11.78	11.64	5.528	5.860	5.784	3.103	3.848	3.675	1.939	2.388	2.282
20	2.275	2.266	2.314	2.985	3.034	3.065	10.56	11.19	11.07	5.254	5.571	5.498	2.950	3.659	3.494	1.843	2.270	2.169
22	2.173	2.164	2.210	2.850	2.897	2.927	10.09	10.69	10.57	5.018	5.321	5.251	2.817	3.495	3.337	1.760	2.169	2.072
24	2.084	2.075	2.119	2.733	2.778	2.806	9.669	10.25	10.13	4.811	5.101	5.034	2.701	3.351	3.199	1.687	2.080	1.987
26	2.004	1.996	2.038	2.629	2.673	2.700	9.303	9.862	9.748	4.629	4.907	4.843	2.599	3.224	3.077	1.624	2.001	1.911
28	1.934	1.926	1.967	2.536	2.578	2.605	8.974	9.513	9.403	4.465	4.735	4.672	2.507	3.110	2.970	1.566	1.930	1.844
30	1.870	1.862	1.902	2.452	2.493	2.518	8.677	9.199	9.092	4.318	4.579	4.518	2.424	3.009	2.872	1.514	1.867	1.783
32	1.812	1.804	1.843	2.376	2.415	2.440	8.408	8.914	8.810	4.184	4.437	4.378	2.349	2.916	2.783	1.467	1.809	1.728
34	1.759	1.752	1.789	2.307	2.345	2.369	8.163	8.654	8.553	4.062	4.307	4.250	2.280	2.831	2.701	1.425	1.757	1.677
36	1.711	1.703	1.740	2.243	2.280	2.304	7.938	8.415	8.317	3.950	4.189	4.133	2.217	2.753	2.627	1.385	1.708	1.631
38	1.666	1.659	1.695	2.185	2.220	2.243	7.730	8.195	8.099	3.846	4.079	4.025	2.159	2.681	2.558	1.349	1.664	1.588
40	1.625	1.618	1.653	2.130	2.165	2.188	7.538	7.992	7.898	3.751	3.978	3.925	2.106	2.615	2.495	1.316	1.623	1.549

TABLE A-III. TOLERANCE VALUES IN $(m^2/s^2)/(cy/4km)$ FOR 90 PERCENT CONFIDENCE INTERVALS

SAMPLE SIZE	WAVE NO. 5			WAVE NO. 10			WAVE NO. 15			WAVE NO. 20			WAVE NO. 25			WAVE NO. 30		
	L.S. $X10^{-2}$	M.L. $X10^{-2}$	C.L.S. $X10^{-2}$	L.S. $X10^{-3}$	M.L. $X10^{-3}$	C.L.S. $X10^{-3}$	L.S. $X10^{-4}$	M.L. $X10^{-4}$	C.L.S. $X10^{-4}$	L.S. $X10^{-4}$	M.L. $X10^{-4}$	C.L.S. $X10^{-4}$	L.S. $X10^{-4}$	M.L. $X10^{-4}$	C.L.S. $X10^{-4}$	L.S. $X10^{-4}$	M.L. $X10^{-4}$	C.L.S. $X10^{-4}$
6	4.844	4.824	4.927	6.354	6.459	6.525	22.48	23.83	23.56	11.19	11.86	11.70	6.279	7.786	7.438	3.923	4.831	4.618
8	4.224	4.207	4.296	5.542	5.634	5.692	19.61	20.78	20.55	9.757	10.34	10.21	5.477	6.784	6.486	3.422	4.214	4.028
10	3.801	3.786	3.865	4.987	5.069	5.121	17.64	18.70	18.49	8.778	9.306	9.185	4.928	6.104	5.836	3.079	3.788	3.624
12	3.482	3.468	3.542	4.569	4.644	4.691	16.17	17.13	16.94	8.042	8.527	8.415	4.515	5.600	5.348	2.821	3.475	3.321
14	3.237	3.223	3.292	4.245	4.315	4.360	15.02	15.92	15.74	7.474	7.924	7.820	4.196	5.200	4.970	2.621	3.227	3.086
16	3.034	3.021	3.086	3.979	4.045	4.087	14.08	14.93	14.75	7.006	7.429	7.330	3.933	4.881	4.659	2.457	3.028	2.893
18	2.866	2.853	2.915	3.758	3.820	3.859	13.30	14.10	13.93	6.616	7.016	6.923	3.714	4.610	4.400	2.321	2.861	2.732
20	2.723	2.711	2.769	3.572	3.630	3.668	12.64	13.39	13.24	6.288	6.665	6.580	3.529	4.380	4.180	2.205	2.718	2.595
22	2.600	2.590	2.645	3.411	3.467	3.503	12.07	12.80	12.65	6.005	6.369	6.284	3.372	4.182	3.994	2.106	2.595	2.480
24	2.493	2.482	2.536	3.269	3.322	3.357	11.57	12.26	12.12	5.755	6.103	6.022	3.231	4.012	3.828	2.019	2.489	2.377
26	2.397	2.387	2.438	3.143	3.195	3.228	11.12	11.79	11.65	5.534	5.869	5.790	3.107	3.858	3.681	1.941	2.394	2.285
28	2.312	2.302	2.351	3.031	3.080	3.113	10.72	11.37	11.24	5.336	5.659	5.584	2.996	3.721	3.550	1.872	2.309	2.204
30	2.235	2.225	2.273	2.930	2.978	3.009	10.37	10.99	10.86	5.158	5.471	5.398	2.896	3.597	3.431	1.809	2.232	2.130
32	2.165	2.155	2.202	2.838	2.885	2.915	10.04	10.65	10.52	4.997	5.300	5.229	2.806	3.485	3.324	1.753	2.162	2.064
34	2.102	2.092	2.137	2.755	2.800	2.829	9.748	10.33	10.21	4.850	5.144	5.075	2.723	3.383	3.227	1.702	2.099	2.003
36	2.043	2.034	2.078	2.678	2.722	2.751	9.477	10.05	9.930	4.716	5.002	4.934	2.648	3.289	3.137	1.654	2.041	1.948
38	1.990	1.981	2.024	2.608	2.650	2.678	9.228	9.784	9.669	4.592	4.870	4.804	2.578	3.203	3.055	1.611	1.987	1.896
40	1.940	1.931	1.973	2.543	2.584	2.611	8.997	9.539	9.427	4.477	4.748	4.684	2.514	3.123	2.978	1.570	1.937	1.849

TABLE A-IV. TOLERANCE VALUES IN $(m^2/s^2)/(cy/4km)$ FOR 95 PERCENT CONFIDENCE INTERVALS

SAMPLE SIZE	WAVE NO. 5			WAVE NO. 10			WAVE NO. 15			WAVE NO. 20			WAVE NO. 25			WAVE NO. 30		
	L.S. $\times10^{-2}$	M.L. $\times10^{-2}$	C.L.S. $\times10^{-2}$	L.S. $\times10^{-3}$	M.L. $\times10^{-3}$	C.L.S. $\times10^{-3}$	L.S. $\times10^{-3}$	M.L. $\times10^{-3}$	C.L.S. $\times10^{-3}$	L.S. $\times10^{-4}$	M.L. $\times10^{-4}$	C.L.S. $\times10^{-4}$	L.S. $\times10^{-4}$	M.L. $\times10^{-4}$	C.L.S. $\times10^{-4}$	L.S. $\times10^{-4}$	M.L. $\times10^{-4}$	C.L.S. $\times10^{-4}$
10	5.616	5.567	5.711	7.318	7.419	7.515	2.589	2.750	2.713	12.89	13.70	13.47	7.248	9.093	8.592	4.525	5.640	5.335
12	5.052	5.009	5.138	6.584	6.682	6.762	2.329	2.474	2.441	11.60	12.34	12.13	6.521	8.185	7.733	4.071	5.073	4.805
14	4.624	4.589	4.703	6.033	6.123	6.196	2.134	2.267	2.237	10.63	11.30	11.10	5.975	7.490	7.080	3.730	4.642	4.398
16	4.287	4.255	4.360	5.594	5.678	5.745	1.979	2.102	2.074	9.857	10.47	10.30	5.540	6.943	6.565	3.459	4.303	4.078
18	4.015	3.986	4.083	5.241	5.320	5.382	1.854	1.969	1.943	9.233	9.810	9.648	5.189	6.494	6.148	3.240	4.025	3.819
20	3.788	3.761	3.852	4.945	5.020	5.078	1.749	1.857	1.833	8.712	9.255	9.107	4.896	6.125	5.803	3.057	3.797	3.603
22	3.594	3.572	3.655	4.698	4.769	4.825	1.662	1.765	1.742	8.276	8.791	8.647	4.651	5.811	5.508	2.904	3.602	3.423
24	3.430	3.407	3.489	4.481	4.549	4.601	1.585	1.683	1.661	7.893	8.384	8.251	4.435	5.545	5.255	2.769	3.437	3.264
26	3.284	3.266	3.340	4.295	4.361	4.411	1.520	1.613	1.592	7.566	8.035	7.906	4.251	5.307	5.034	2.655	3.290	3.129
28	3.159	3.138	3.212	4.128	4.192	4.239	1.460	1.550	1.530	7.271	7.721	7.601	4.085	5.098	4.839	2.551	3.160	3.006
30	3.044	3.026	3.096	3.979	4.042	4.090	1.408	1.495	1.475	7.013	7.442	7.328	3.939	4.916	4.665	2.460	3.048	2.898
32	2.944	2.925	2.994	3.846	3.907	3.954	1.361	1.445	1.426	6.780	7.194	7.084	3.808	4.749	4.509	2.378	2.944	2.801
34	2.851	2.833	2.900	3.729	3.788	3.829	1.319	1.399	1.382	6.566	6.968	6.862	3.688	4.600	4.368	2.303	2.853	2.713
36	2.766	2.748	2.813	3.617	3.675	3.715	1.280	1.358	1.341	6.370	6.767	6.660	3.578	4.463	4.239	2.235	2.768	2.635
38	2.688	2.673	2.734	3.518	3.574	3.613	1.245	1.321	1.304	6.196	6.575	6.475	3.480	4.337	4.121	2.174	2.690	2.560
40	2.618	2.604	2.663	3.423	3.478	3.516	1.211	1.286	1.269	6.035	6.405	6.305	3.389	4.225	4.013	2.117	2.618	2.494

TABLE A-V. TOLERANCE VALUES IN $(m^2/s^2)/(cy/4km)$ FOR 99 PERCENT CONFIDENCE INTERVALS

SAMPLE SIZE	WAVE NO. 5			WAVE NO. 10			WAVE NO. 15			WAVE NO. 20			WAVE NO. 25			WAVE NO. 30		
	L.S. $X10^{-3}$	M.L. $X10^{-3}$	C.L.S. $X10^{-3}$	L.S. $X10^{-3}$	M.L. $X10^{-3}$	C.L.S. $X10^{-3}$	L.S. $X10^{-4}$	M.L. $X10^{-4}$	C.L.S. $X10^{-4}$	L.S. $X10^{-4}$	M.L. $X10^{-4}$	C.L.S. $X10^{-4}$	L.S. $X10^{-4}$	M.L. $X10^{-4}$	C.L.S. $X10^{-4}$	L.S. $X10^{-4}$	M.L. $X10^{-4}$	C.L.S. $X10^{-4}$
2	40.22	40.21	40.91	5.308	5.405	5.451	18.79	19.87	19.68	9.336	9.874	9.767	5.233	6.420	6.186	32.72	39.90	38.46
4	30.01	29.93	30.52	3.945	4.013	4.051	13.96	14.79	14.63	6.943	7.355	7.264	3.895	4.813	4.611	24.34	29.88	28.64
6	24.92	24.84	25.35	3.273	3.328	3.361	11.58	12.27	12.13	5.761	6.105	6.028	3.233	4.003	3.828	20.20	24.85	23.78
8	21.77	21.68	22.14	2.857	2.904	2.933	10.11	10.71	10.59	5.029	5.331	5.262	2.823	3.499	3.343	17.64	21.71	20.76
10	19.57	19.49	19.90	2.567	2.609	2.636	9.083	9.627	9.517	4.519	4.791	4.728	2.537	3.146	3.005	15.85	19.52	18.66
12	17.92	17.85	18.23	2.351	2.389	2.414	8.317	8.816	8.714	4.138	4.388	4.330	2.323	2.882	2.752	14.51	17.89	17.09
14	16.63	16.56	16.91	2.181	2.217	2.240	7.717	8.180	8.086	3.840	4.072	4.018	2.156	2.675	2.554	13.47	16.60	15.86
16	15.58	15.52	15.85	2.043	2.077	2.098	7.230	7.665	7.576	3.593	3.815	3.764	2.020	2.507	2.393	12.62	15.56	14.86
18	14.71	14.65	14.96	1.929	1.961	1.981	6.825	7.236	7.152	3.396	3.602	3.554	1.907	2.368	2.259	11.91	14.69	14.03
20	13.97	13.91	14.21	1.832	1.862	1.881	6.482	6.872	6.791	3.225	3.420	3.375	1.811	2.249	2.145	11.31	13.95	13.32
22	13.33	13.27	13.56	1.748	1.777	1.795	6.185	6.558	6.481	3.078	3.264	3.220	1.728	2.146	2.047	10.79	13.32	12.71
24	12.78	12.72	12.99	1.675	1.702	1.720	5.926	6.283	6.209	2.949	3.127	3.085	1.656	2.056	1.962	10.34	12.76	12.18
26	12.28	12.23	12.49	1.610	1.636	1.653	5.697	6.040	5.969	2.835	3.007	2.966	1.592	1.977	1.886	9.943	12.27	11.71
28	11.84	11.79	12.04	1.552	1.578	1.594	5.492	5.823	5.755	2.733	2.899	2.860	1.534	1.906	1.818	9.586	11.83	11.29
30	11.45	11.39	11.64	1.500	1.525	1.541	5.308	5.628	5.562	2.641	2.802	2.764	1.483	1.843	1.757	9.265	11.43	10.91
32	11.09	11.04	11.28	1.453	1.477	1.492	5.142	5.452	5.388	2.559	2.714	2.677	1.437	1.785	1.702	8.974	11.07	10.57
34	10.76	10.71	10.94	1.410	1.433	1.448	4.990	5.291	5.228	2.483	2.634	2.598	1.394	1.732	1.652	8.709	10.75	10.26
36	10.46	10.41	10.64	1.371	1.393	1.408	4.851	5.143	5.083	2.414	2.560	2.526	1.355	1.684	1.606	8.467	10.45	9.970
38	10.18	10.14	10.36	1.335	1.356	1.371	4.723	5.007	4.948	2.350	2.493	2.459	1.320	1.640	1.563	8.243	10.17	9.706
40	9.929	9.883	10.10	1.301	1.322	1.336	4.604	4.882	4.824	2.291	2.430	2.397	1.286	1.598	1.524	8.036	9.917	9.463

TABLE A-VI. TOLERANCE VALUES IN $(m^2/s^2)/(cy/4km)$ FOR 68.72 PERCENT CONFIDENCE INTERVALS

SAMPLE SIZE	WAVE NO. 5 L.S. X10^{-2}	WAVE NO. 5 M.L. X10^{-2}	WAVE NO. 5 C.L.S. X10^{-2}	WAVE NO. 10 L.S. X10^{-3}	WAVE NO. 10 M.L. X10^{-3}	WAVE NO. 10 C.L.S. X10^{-3}	WAVE NO. 15 L.S. X10^{-4}	WAVE NO. 15 M.L. X10^{-4}	WAVE NO. 15 C.L.S. X10^{-4}	WAVE NO. 20 L.S. X10^{-4}	WAVE NO. 20 M.L. X10^{-4}	WAVE NO. 20 C.L.S. X10^{-4}	WAVE NO. 25 L.S. X10^{-4}	WAVE NO. 25 M.L. X10^{-4}	WAVE NO. 25 C.L.S. X10^{-4}	WAVE NO. 30 L.S. X10^{-4}	WAVE NO. 30 M.L. X10^{-4}	WAVE NO. 30 C.L.S. X10^{-4}
6	4.986	4.962	5.071	6.534	6.640	6.710	23.12	24.51	24.22	11.50	12.20	12.04	6.459	8.033	7.654	4.035	4.983	4.751
8	4.332	4.314	4.405	5.683	5.776	5.835	20.11	21.31	21.07	10.00	10.61	10.47	5.616	6.967	6.652	3.508	4.323	4.130
10	3.891	3.875	3.957	5.105	5.189	5.242	18.06	19.14	18.93	8.987	9.528	9.403	5.044	6.257	5.975	3.152	3.883	3.710
12	3.564	3.550	3.625	4.676	4.753	4.802	16.54	17.54	17.33	8.231	8.728	8.613	4.621	5.732	5.473	2.887	3.557	3.399
14	3.309	3.295	3.365	4.340	4.412	4.457	15.36	16.28	16.09	7.641	8.102	7.995	4.289	5.322	5.081	2.680	3.303	3.155
16	3.102	3.089	3.155	4.068	4.135	4.178	14.39	15.26	15.08	7.162	7.595	7.494	4.021	4.990	4.763	2.512	3.096	2.957
18	2.930	2.917	2.980	3.842	3.905	3.945	13.59	14.41	14.24	6.764	7.172	7.077	3.797	4.713	4.498	2.372	2.924	2.793
20	2.783	2.771	2.831	3.650	3.709	3.748	12.91	13.69	13.53	6.425	6.814	6.723	3.607	4.478	4.273	2.254	2.779	2.653
22	2.657	2.645	2.702	3.484	3.541	3.577	12.33	13.07	12.92	6.133	6.504	6.417	3.443	4.275	4.079	2.151	2.652	2.533
24	2.546	2.535	2.590	3.339	3.393	3.428	11.81	12.52	12.38	5.878	6.233	6.150	3.300	4.097	3.909	2.061	2.542	2.427
26	2.448	2.438	2.490	3.210	3.263	3.296	11.36	12.04	11.90	5.652	5.994	5.914	3.173	3.940	3.759	1.982	2.445	2.334
28	2.361	2.351	2.401	3.096	3.146	3.179	10.95	11.61	11.48	5.450	5.780	5.702	3.060	3.800	3.625	1.912	2.358	2.251
30	2.283	2.272	2.321	2.992	3.041	3.073	10.59	11.23	11.09	5.268	5.587	5.512	2.958	3.674	3.505	1.848	2.279	2.176
32	2.211	2.201	2.249	2.899	2.946	2.977	10.26	10.87	10.75	5.104	5.413	5.340	2.865	3.559	3.395	1.790	2.208	2.108
34	2.146	2.137	2.183	2.814	2.859	2.889	9.955	10.55	10.43	4.954	5.254	5.183	2.781	3.455	3.295	1.738	2.143	2.046
36	2.087	2.077	2.122	2.735	2.780	2.809	9.679	10.26	10.14	4.816	5.108	5.039	2.704	3.359	3.204	1.689	2.084	1.989
38	2.032	2.023	2.067	2.664	2.707	2.735	9.424	9.992	9.875	4.689	4.974	4.907	2.633	3.271	3.120	1.645	2.029	1.937
40	1.981	1.972	2.015	2.597	2.639	2.667	9.189	9.742	9.628	4.572	4.850	4.784	2.567	3.189	3.042	1.604	1.979	1.888

TABLE A-VII. TOLERANCE VALUES IN $(m^2/s^2)/(cy/4km)$ FOR 95.45 PERCENT CONFIDENCE INTERVALS

SAMPLE SIZE	WAVE NO. 5			WAVE NO. 10			WAVE NO. 15			WAVE NO. 20			WAVE NO. 25			WAVE NO. 30		
	L.S. $\times 10^{-2}$	M.L. $\times 10^{-2}$	C.L.S. $\times 10^{-2}$	L.S. $\times 10^{-3}$	M.L. $\times 10^{-3}$	C.L.S. $\times 10^{-3}$	L.S. $\times 10^{-3}$	M.L. $\times 10^{-3}$	C.L.S. $\times 10^{-3}$	L.S. $\times 10^{-4}$	M.L. $\times 10^{-4}$	C.L.S. $\times 10^{-4}$	L.S. $\times 10^{-4}$	M.L. $\times 10^{-4}$	C.L.S. $\times 10^{-4}$	L.S. $\times 10^{-4}$	M.L. $\times 10^{-4}$	C.L.S. $\times 10^{-4}$
14	5.781	5.726	5.880	7.522	7.629	7.725	2.661	2.828	2.789	13.26	14.10	13.88	7.454	--	8.860	4.655	--	5.491
16	5.341	5.295	5.432	6.952	7.054	7.140	2.459	2.615	2.577	12.27	13.04	12.84	6.894	8.663	8.182	4.304	5.365	5.075
18	4.984	4.940	5.069	6.493	6.590	6.669	2.297	2.440	2.407	11.45	12.17	11.98	6.432	8.082	7.638	4.018	5.009	4.736
20	4.687	4.651	4.767	6.105	6.197	6.271	2.162	2.297	2.263	10.76	11.45	11.26	6.050	7.597	7.185	3.778	4.706	4.456
22	4.438	4.398	4.514	5.782	5.868	5.938	2.046	2.172	2.144	10.18	10.83	10.67	5.730	7.188	6.799	3.576	4.454	4.216
24	4.220	4.184	4.292	5.498	5.584	5.646	1.945	2.066	2.038	9.694	10.30	10.14	5.451	6.837	6.469	3.401	4.236	4.011
26	4.030	3.996	4.099	5.255	5.330	5.397	1.859	1.974	1.948	9.258	9.843	9.688	5.207	6.530	6.177	3.249	4.045	3.833
28	3.865	3.833	3.930	5.038	5.112	5.174	1.782	1.893	1.868	8.878	9.434	9.290	4.990	6.256	5.921	3.115	3.875	3.673
30	3.715	3.686	3.779	4.848	4.918	4.979	1.715	1.820	1.797	8.535	9.074	8.932	4.799	6.016	5.696	2.996	3.729	3.532
32	3.581	3.554	3.642	4.673	4.744	4.799	1.653	1.756	1.732	8.236	8.750	8.619	4.626	5.798	5.494	2.891	3.594	3.406
34	3.463	3.438	3.522	4.519	4.586	4.641	1.598	1.698	1.675	7.958	8.459	8.328	4.476	5.602	5.311	2.794	3.471	3.293
36	3.353	3.327	3.410	4.376	4.441	4.495	1.548	1.645	1.622	7.709	8.196	8.067	4.332	5.424	5.142	2.706	3.364	3.190
38	3.254	3.231	3.309	4.246	4.311	4.364	1.503	1.596	1.574	7.484	7.951	7.832	4.206	5.263	4.989	2.625	3.263	3.095
40	3.161	3.139	3.215	4.130	4.191	4.239	1.461	1.551	1.531	7.277	7.732	7.615	4.087	5.118	4.850	2.551	3.170	3.008

TABLE A-VIII. TOLERANCE VALUES IN $(m^2/s^2)/(cy/4km)$ FOR 99.73 PERCENT CONFIDENCE INTERVALS

SAMPLE SIZE	WAVE NO. 5 L.S.	WAVE NO. 5 M.L.	WAVE NO. 5 C.L.S.	WAVE NO. 10 L.S.	WAVE NO. 10 M.L.	WAVE NO. 10 C.L.S.	WAVE NO. 15 L.S.	WAVE NO. 15 M.L.	WAVE NO. 15 C.L.S.	WAVE NO. 20 L.S.	WAVE NO. 20 M.L.	WAVE NO. 20 C.L.S.	WAVE NO. 25 L.S.	WAVE NO. 25 M.L.	WAVE NO. 25 C.L.S.	WAVE NO. 30 L.S.	WAVE NO. 30 M.L.	WAVE NO. 30 C.L.S.
50% CONFIDENCE ERROR																		
10	0.22	0.21	0.22	0.21	0.20	0.21	0.21	0.21	0.21	0.21	0.22	0.21	0.21	0.24	0.22	0.21	0.24	0.22
20	0.11	0.10	0.11	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.11	0.10	0.10	0.12	0.11	0.10	0.12	0.11
30	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.08	0.06	0.06	0.08	0.06
40	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.05	0.04	0.04	0.05	0.04	0.05	0.05	0.05	0.05	0.05	0.05
75% CONFIDENCE ERROR																		
10	1.24	1.19	1.24	1.16	1.13	1.16	1.15	1.19	1.16	1.17	1.21	1.17	1.19	1.35	1.22	1.18	1.32	1.20
20	0.61	0.59	0.61	0.57	0.56	0.57	0.57	0.59	0.57	0.58	0.60	0.58	0.69	0.76	0.60	0.58	0.65	0.59
30	0.41	0.39	0.41	0.38	0.37	0.38	0.38	0.39	0.38	0.39	0.40	0.39	0.49	0.44	0.40	0.39	0.43	0.40
40	0.31	0.29	0.31	0.28	0.28	0.29	0.28	0.29	0.28	0.29	0.30	0.29	0.39	0.33	0.30	0.29	0.33	0.30
90% CONFIDENCE ERROR																		
10	1.41	1.35	1.41	1.31	1.29	1.32	1.31	1.34	1.31	1.33	1.37	1.33	1.35	1.52	1.38	1.34	1.50	1.36
20	0.68	0.65	0.68	0.64	0.63	0.64	0.64	0.65	0.64	0.65	0.66	0.65	0.66	0.74	0.67	0.65	0.73	0.66
30	0.45	0.43	0.45	0.42	0.41	0.42	0.42	0.43	0.42	0.42	0.44	0.42	0.43	0.49	0.44	0.43	0.48	0.44
40	0.34	0.32	0.34	0.31	0.31	0.31	0.31	0.32	0.31	0.32	0.33	0.32	0.32	0.36	0.33	0.32	0.36	0.32
95% CONFIDENCE ERROR																		
10	0.68	0.66	0.68	0.65	0.64	0.65	0.64	0.66	0.65	0.65	0.67	0.65	0.66	0.75	0.67	0.66	0.74	0.66
20	0.37	0.35	0.37	0.33	0.33	0.33	0.33	0.35	0.33	0.34	0.36	0.34	0.35	0.39	0.36	0.34	0.39	0.36
30	0.24	0.23	0.24	0.22	0.22	0.22	0.22	0.23	0.22	0.23	0.23	0.23	0.23	0.25	0.23	0.23	0.25	0.23
40	0.18	0.18	0.18	0.17	0.17	0.17	0.17	0.18	0.17	0.17	0.18	0.18	0.18	0.20	0.18	0.18	0.19	0.18
99% CONFIDENCE ERROR																		
10	-0.51	-0.49	-0.51	-0.49	-0.48	-0.49	-0.49	-0.49	-0.49	-0.49	-0.50	-0.48	-0.50	-0.53	-0.50	-0.49	-0.53	-0.50
20	-0.27	-0.26	-0.27	-0.25	-0.25	-0.25	-0.25	-0.26	-0.25	-0.26	-0.26	-0.25	-0.26	-0.30	-0.26	-0.26	-0.29	-0.26
30	-0.18	-0.17	-0.18	-0.16	-0.16	-0.17	-0.16	-0.17	-0.16	-0.17	-0.17	-0.16	-0.17	-0.19	-0.17	-0.17	-0.19	-0.17
40	-0.13	-0.13	-0.13	-0.12	-0.12	-0.12	-0.12	-0.13	-0.12	-0.13	-0.13	-0.12	-0.13	-0.15	-0.12	-0.13	-0.14	-0.13

TABLE A-IX. ERROR IN THE PERCENTAGE CONFIDENCE OBTAINED FROM USING THE NORMAL APPROXIMATION WITH EXACT CONFIDENCE INTERVALS AT PERCENTAGES 50, 75, 90, 95, AND 99

SAMPLE SIZE	WAVE NO. 5			WAVE NO. 10			WAVE NO. 15			WAVE NO. 20			WAVE NO. 25			WAVE NO. 30		
	L.S.	M.L.	C.L.S.	L.S.	M.L.	C.L.S.	L.S.	M.L.	C.L.S.	L.S.	M.L.	C.L.S.	L.S.	M.L.	C.L.S.	L.S.	M.L.	C.L.S.
	68.27% CONFIDENCE ERROR																	
10	0.95	0.91	0.95	0.88	0.87	0.88	0.88	0.90	0.88	0.89	0.92	0.89	0.91	1.02	0.93	1.00	1.01	0.92
20	0.47	0.45	0.47	0.44	0.43	0.44	0.43	0.45	0.44	0.44	0.46	0.44	0.45	0.51	0.46	0.45	0.50	0.46
30	0.31	0.30	0.31	0.29	0.29	0.29	0.29	0.30	0.29	0.29	0.30	0.30	0.30	0.34	0.31	0.30	0.33	0.30
40	0.23	0.22	0.23	0.22	0.21	0.22	0.22	0.22	0.22	0.22	0.23	0.22	0.22	0.25	0.23	0.22	0.25	0.23
	95.45% CONFIDENCE ERROR																	
10	0.57	0.55	0.57	0.54	0.53	0.54	0.53	0.55	0.54	0.54	0.55	0.54	0.55	0.60	0.56	0.54	0.60	0.55
20	0.30	0.29	0.30	0.28	0.28	0.28	0.28	0.29	0.28	0.29	0.30	0.29	0.29	0.33	0.30	0.29	0.32	0.29
30	0.20	0.20	0.20	0.19	0.19	0.19	0.19	0.20	0.19	0.19	0.20	0.19	0.20	0.22	0.20	0.19	0.22	0.20
40	0.15	0.15	0.16	0.14	0.14	0.14	0.14	0.15	0.14	0.15	0.15	0.15	0.15	0.17	0.15	0.15	0.16	0.15
	99.73% CONFIDENCE ERROR																	
20	-0.18	-0.18	-0.18	-0.17	-0.17	-0.17	-0.18	-0.18	-0.17	-0.18	-0.18	-0.17	-0.18	-0.19	-0.18	-0.18	-0.18	-0.18
30	-0.14	-0.14	-0.14	-0.14	-0.14	-0.14	-0.14	-0.14	-0.14	-0.14	-0.14	-0.14	-0.14	-0.15	-0.14	-0.14	-0.15	-0.14
40	-0.12	-0.11	-0.12	-0.11	-0.11	-0.11	-0.11	-0.11	-0.11	-0.11	-0.12	-0.11	-0.11	-0.13	-0.12	-0.13	-0.12	-0.11

TABLE A-X. ERROR IN THE PERCENTAGE CONFIDENCE OBTAINED FROM USING THE NORMAL APPROXIMATION WITH EXACT CONFIDENCE INTERVALS AT PERCENTAGES 68.27, 95.45, AND 99.73

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APPENDIX B

CALCULATION TECHNIQUES

A few guidelines to calculation techniques are listed below. These techniques are not necessarily efficient but they are generally understood by people familiar with performing numerical statistical calculations. Considerable literature exists covering their implementation (e.g., Ref. 6).

Since statistics often deal with massive amounts of data and since the formulas sometimes involve large factorials, exponentials, logarithm sums, etc., simple discretion must be employed to maintain accurate calculations. Adequate results can usually be obtained by implementing the formulas in a way which avoids the generation of large numbers.

Accurate numerical integrations can be accomplished by a three or four pass Runge-Kutta scheme and implementation is relatively easy. Also some type of numerical isolation scheme is necessary. One based on finite difference approximations to partial derivatives is adequate. A complete recalculation of the partials on each pass does not necessarily result in a serious loss of computation time and is often simpler to implement than methods based on an update scheme.

Accurate calculations of the Pearson density function may be improved by rewriting equation (2.5) as follows

$$\frac{\beta}{\Gamma(\lambda)} (\beta x)^{\lambda-1} e^{-\beta x} = e^{(\ln\beta - \ln\Gamma(\lambda) + (\lambda-1)\ln(\beta x) - \beta x)}$$

This is especially important when calculating the integrand of the sampling distribution on the mean, equation (2.30). Otherwise, the argument of the exponential may exceed the range of the exponential subroutine. Several good routines are available for calculating the gamma function $\Gamma(\lambda)$. Reference 6 lists one such subroutine. Five place accurate integration of the density function over its entire useful range may be obtained by using one hundred steps with a four pass Runge-Kutta integrator.

In employing the likelihood method (Eqs. 2.17 & 2.21) to fit the population distribution, it is necessary to use numerical isolation to obtain the value of λ satisfying (2.21). The formula can be implemented as written. Of course, the x_i's represent the data values and $\bar{x}$ the mean value of the data. Many formulas are available for calculating $\Gamma'(\lambda)/\Gamma(\lambda)$. The following series may be used but this series converges very slowly in the neighborhood of $\lambda = 1$.

$$\Gamma'(\lambda)/\Gamma(\lambda) = -\gamma - \frac{1}{\lambda} + \lambda \sum_{n=1}^{\infty} \frac{1}{n(\lambda+n)}$$

APPENDIX B (Continued)

In this formula γ is Euler's constant. Notice if $\lambda = 1$ the nth partial sum, S_n, is given by

$$S_n = -(\gamma+1)+(n/(n+1))$$

Thus, 1000 terms of the sum are required to obtain 4-place accuracy.

In calculating central confidence intervals on the quantiles, the true value of the quantile must be assumed to be given by one of the theoretical functions fitting the population distribution. Let Q denote the value of the quantile to be studied. Then one seeks the value of y satisfying the equation

$$Q = \int_0^y \frac{\beta}{\Gamma(\lambda)} (\beta x)^{\lambda-1} e^{-\beta x} dx$$

This value of y represents the true value of the population quantile. After y has been obtained, confidence intervals centered at y may be calculated numerically.

Suppose a random sample of n profiles has been selected, the power spectra has been calculated, and at some selected wave number the resulting n values of PSD have been ordered according to magnitude.

$$x_1 < x_2 < \ldots < x_r < \ldots < x_n$$

Suppose δ is a positive number such that $y - \delta > 0$. What is the probability that x_r is in the interval $y \pm \delta$? This is the probability that x_r represents the true value of the Qth percentile power density with a known error. Now if x_r is not even close to y, the probability that it represents the true value is very small. In paragraph 2.5.3, the assumption that Q was of the form $(r-1)/(n-1)$ was made to assure x_r being as close as possible to Q as the sample size increases.

For simplicity we would recommend that for a given sample size, n, confidence intervals should only be developed for quantiles of the form $(r-1)/(n-1)$ where r is a positive integer. To proceed with this calculation, select the sample size, n, and the percentile value, Q, (of the form $(r-1)/(n-1)$) and solve for the value of y. Then select the confidence desired, P, and use equation (2.38) or (2.39) to solve for that value of δ satisfying

$$P = \sum_{j=0}^{n-r} (-1)^j \frac{n!}{(r-1)!(n-r)!} \binom{n-r}{j} \left(\frac{1}{r+j-1}\right)\left(F^{r+j}(y+\delta) - F^{r+j}(y-\delta)\right)$$

APPENDIX B (Continued).

In this equation, F^{r+j} is given by

$$F^{r+j}(z) = [\int_0^z \frac{\beta}{\Gamma(\lambda)} (\beta x)^{\lambda-1} e^{-\beta x} dx]^{r+j}$$

A little experimentation with the implemented form of the equation may be necessary to obtain accurate results, especially for large sample sizes (large values of n).

Finally, the reader's attention should be brought to the distribution free methods of calculating confidence intervals on quantiles. Once again, suppose a random sample has been selected, the PSD values ordered, etc.

$$x_1 < x_2 < \ldots < x_j < \ldots < x_r < \ldots < x_n$$

The distribution free method answers the question, "What is the probability that Q lies between x_j and x_r?" This probability does not depend on the population distribution. Computations are thereby reduced to a minimum. The formula for calculating this probability, P, is

$$P = \sum_{i=j}^{n} \frac{n!}{i!(n-i)!} Q^i (1-Q)^{n-i} - \sum_{i=r}^{n} \frac{n!}{i!(n-i)!} Q^i (1-Q)^{n-i}$$

$$= \sum_{i=j}^{r-1} \frac{n!}{i!(n-i)!} Q^i (1-Q)^{n-i}$$

As an example of the application of this formula, suppose a random sample of size 15 is selected. What is the probability that the 50 percentile value of the power lies between the 7th and 9th values of the sample? Here

$Q = 0.5 = 1/2$

$r = 9$

$j = 7$

$$P = \sum_{i=7}^{8} \frac{15!}{i!(15-i)!} (\frac{1}{2})^i (\frac{1}{2})^{15-i}$$

$$\cong 0.4$$

APPENDIX B (Continued)

The interval with end points x_7 and x_9 serves as a 40 percent confidence interval for the 50 percentile value in a sample of size 15.

The distribution free techniques may also be used to measure the size of a sample necessary to include a given high percentile level of power. For the probability, P, that the percentile value Q exceeds the nth value of a sample of size n is

$$P = \frac{n!}{n!(n-n)!} Q^n (1-Q)^{n-n}$$

$$= Q^n$$

Thus if Q is selected as the 99 percentile level, the probability that the largest value in a sample of size 3 is less than Q is

$$(0.99)^3 \cong 0.97$$

A sample size large enough to include a value of the power exceeding the 99 percentile value 50 percent of the time is given by the solution of n in the equation

$$(0.99)^n = 0.5$$

Solving for n

$$n = \frac{\log(0.5)}{\log(0.99)} \cong 69$$

To exceed the 95 percentile value of power 99 percent of the time requires a sample size of 86 or more.

For

$$(0.95)^n = 0.01$$

$$n = \frac{\log(0.01)}{\log(0.95)} \cong 86$$

To exceed the 99 percentile value 99 percent of the time

$$n = \frac{\log(0.01)}{\log(0.99)} \cong 455$$

APPENDIX C

GLOSSARY OF DEFINITIONS, ABBREVIATIONS AND SYMBOLS

DEFINITIONS

Bias	-	Used in statistics to indicate that the first moment (mean values) of a distribution does not agree with the mean value of the population the distribution is supposed to represent.
Central confidence interval	-	A tolerance interval centered at some preselected value.
Quantile	-	If a set of n numbers has been ordered according to magnitude, the r^{th} quantile (where $r < n$) is the r^{th} value in the set.
Tolerance interval	-	A type of confidence interval used with sampling distributions. The confidence associated with such an interval is the probability that the desired value will be obtained within the tolerances specifying the interval.
Time average statistic	-	Used here in connection with PSD values where the time variation has been destroyed by considering data collected at different times as belonging to a single population with no time difference.

ABBREVIATIONS

GMT	-	Greenwich mean time
KSC	-	Kennedy Space Center
PSD	-	Power spectral density

SYMBOLS

β	-	A scaling parameter used in the Pearson distribution.
γ	-	Incomplete gamma function - also used as Eulers constant

APPENDIX C (Continued)

SYMBOLS (Continued)

k	-	Wave number.
ln	-	Natural logarithm.
λ	-	A shaping parameter used in the Pearson distribution.
μ	-	Mean.
μ_2	-	Variance.
μ_s	-	The mean of a sampling distribution.
μ_{2s}	-	The variance of a sampling distribution.
P	-	Power density - also used as probability.
π	-	Continued product.
ϕ	-	Characteristic function.
Φ	-	Characteristic function of the sampling distribution on the mean.