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THE RELATION OF SOME GEODETIC DATUMS TO A GLOBAL GEOCENTRIC REFERENCE SYSTEM

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1. — Introduction

In this paper the relationships between three geodetic datums and the new Smithsonian Astrophysical Observatory's Standard Earth (determined by Gaposchkin and Lambeck [1]) have been established through comparisons of coordinates of stations tied to both the global geocentric system and the respective datum. The datums considered are the North American Datum of 1927 (NAD), the European Datum of 1950 (EUR), and the Provisional South American Datum of 1956 (SAD). For the NAD 12 stations were used, for EUR 7 stations, and for SAD 3 stations. For several other datums one or two stations are available for determining at least the translation of the datum, but it has been found that these elements are not representative of the entire datum. These datums are not discussed further here.

2. — The Method

The geodetic datum is usually defined by the coordinates adopted at the origin, and in such a definition the minor axis of the reference ellipsoid will generally be considered parallel to the earth's rotation axis if Laplace's condition at the origin and elsewhere has been satisfied.

A more practical definition of the geodetic coordinate system is by the coordinate values of the physical points in the geodetic net. If the computations have been carried out rigorously, the geoidal heights are known at all points, and all observations have been reduced to a consistent pole; these two definitions are essentially the same, but these conditions are not always satisfied. In particular, a knowledge of the propagation of systematic or model errors through the net is often unavailable. Thus, the distortions introduced may manifest themselves as rotations of the geodetic system with respect to the earth's rotation axis, and rotation elements should be introduced into the transformation between the two systems.

If the geocentric coordinates of a point P_j are denoted by $X_j Y_j Z_j$; the geodetic cartesian coordinates of the same point by $x_j y_j z_j$; the translation between the two systems by ΔX, ΔY, ΔZ; the rotations by the Euler angles by ω, ψ, ϵ (where ω, ψ, ϵ are positive when rotations from the geodetic to the geocentric systems are anticlockwise); and the scale difference between the two systems by ΔL — then

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_j = \begin{pmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{pmatrix} + (1 + \Delta L) \begin{pmatrix} 1 & \omega & -\psi \\ -\omega & 1 & \epsilon \\ \psi & -\epsilon & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}_j . \qquad (1)$$

Each station with coordinates in both systems provides such a set of equations, and the seven transformation elements are estimated from the combined sets.

Both sets of coordinates are stochastic quantities, so that equation (1) must be written as

$$\begin{pmatrix} 1 & 0 & 0 & x & +y & -z & 0 \\ 0 & 1 & 0 & y & -x & 0 & +z \\ 0 & 0 & 1 & z & 0 & +x & -y \end{pmatrix}_j \begin{pmatrix} \underline{\Delta X} \\ \underline{\Delta Y} \\ \underline{\Delta Z} \\ \underline{\Delta L} \\ \underline{\omega} \\ \underline{\psi} \\ \underline{\epsilon} \end{pmatrix} + \begin{pmatrix} -1 & 0 & 0 & +1 & 0 & 0 \\ 0 & -1 & 0 & 0 & +1 & 0 \\ 0 & 0 & -1 & 0 & 0 & +1 \end{pmatrix} \begin{pmatrix} \underline{\epsilon}_X \\ \underline{\epsilon}_Y \\ \underline{\epsilon}_Z \\ \underline{\epsilon}_x \\ \underline{\epsilon}_y \\ \underline{\epsilon}_z \end{pmatrix}_j = \begin{pmatrix} \underline{X} - \underline{x} \\ \underline{Y} - \underline{y} \\ \underline{Z} - \underline{z} \end{pmatrix}_j ,$$

where $\underline{\epsilon}_X$, $\underline{\epsilon}_x$ are corrections to the functions of observed quantities $\underline{X}$, $\underline{x}$. If G is the covariance matrix of the X and g is the covariance matrix of the x, the covariance matrix used in the adjustment of the above expression is $\begin{pmatrix} G & 0 \\ 0 & g \end{pmatrix}$.

The latitudes, longitudes, and heights of stations in the global cartesian reference system can be computed for a defined geocentric reference ellipsoid. Thus, the coordinates of the origin of a geodetic datum (ϕ_D λ_D H_D) when transformed into the global system by using equation (1) and converted to ellipsoidal coordinates (ϕ_G λ_G H_G) give the corrections to the deflections of the vertical ($\delta\zeta = \phi_G - \phi_D$), $\delta\eta = \cos\phi\,(\lambda_G - \lambda_D)$, $\delta H = (H_G - H_D)$ adopted at the datum origin. In this paper, the geocentric ellipsoid of reference has a flattening of 1/298.25 and a semimajor axis of 6,378,155 m.

In the subsequent analyses, the relationships between various datums and the recent Smithsonian Astrophysical Observatory's global reference system (Gaposchkin and Lambeck [1]) have been established by use of the above transformation formula. This global solution is a combination of results obtained by the dynamic and the geometric methods of satellite geodesy, data obtained from an analysis of deep-space probes, and surface-gravity measurements. The parameters solved for include the geocentric coordinates of 39 stations with an accuracy generally better than 10 m. The scale of the global solution is defined by $GM = 3.986013 \times 10^{20}\ cm^3\ sec^{-2}$, and the orientation by the mean pole of 1903-1905, the UT-1 of Bureau International de l'Heure, and the polar-motion data of the International Polar Motion Service. The GM value corresponds to a velocity of light of $2.997925 \times 10^{10}\ cm\ sec^{-1}$.

3.— North American Datum (1927)

A total of 12 stations tied to the North American Datum have been used. Table 1 lists the stations and their coordinates in the two coordinate systems. All geoid heights have been estimated from the astrogeoid map of Fischer *et al.* [2].

The covariance matrix derived from the combination solution is used for accuracy estimates of $\underline{X}_j$, $\underline{Y}_j$, $\underline{Z}_j$. Fig. 1 gives the error ellipses for the station coordinates, although the full matrix is used in the adjustment.

Accuracy estimates for the 1927 adjustment of North America are given by Simmons [3] in the form of loop closures of the sections forming the loops after the junction positions have been adopted. These have been used to derive the covariance matrix (Table 2) for the horizontal coordinates. The accuracy at any one station has been assumed equal in all directions (Fig. 2). The accuracies of the distances between pairs of stations derived from this matrix are essentially in agreement with the empirical formulation of Simmons; that is, the proportional accuracy is of the order of 1 in 20,000 $M^{1/3}$, where M is the distance in miles.

Accuracy estimates of the heights of the stations above the ellipsoid (Table 2) are based on the analyses of the 35th parallel geoid section and the central United States sections by Rice [4,5], as well as on recent results obtained by the U. S. Coast and Geodetic Survey (unpublished). No correlation between height and horizontal positions has been assumed.

Results

The solution for the seven parameters gives the following results:

$$\Delta X = -31.8 \pm 8.0 \text{ m} \qquad \omega = -0.''04 \pm 0.''27$$

$$\Delta Y = 178.0 \pm 12.2 \text{ m} \qquad \psi = -0.''32 \pm 0.''27$$

$$\Delta Z = 177.6 \pm 11.9 \text{ m} \qquad \epsilon = -0.''53 \pm 0.''35 \quad .$$

$$\Delta L = (3.4 \pm 1.8) \times 10^{-6}$$

The variance of unit weight σ_0^2 is given as 1.07, and with 29 degrees of freedom, $F_{0.95, 29, \infty} = 1.46$.

The corrections to the coordinates are given in Fig. 1, and the corrections to the geodetic coordinates in Fig. 2. The covariance matrix of the adjusted station coordinates, referred to the geocentric reference system, shows little correlation between the components of the various stations.

The angle $\delta\lambda$ between the XZ and xz planes is related to the orientation elements by (Bursa [6])

$$\delta\lambda = (\omega^2 + \epsilon^2)^{1/2} = -0.''53 + 0.''50 \quad ,$$

the axis being to the east of the X axis. Similarly, the angle $\delta\beta$ between the two rotation axes is given by

$$\delta\beta = (\epsilon^2 + \psi^2)^{1/2} = 0.''62 \pm 0.''50$$

in a direction about 35° west of Greenwich (Fig. 3).

Relating the Meades Ranch origin on the Clarke ellipsoid to the earth-centered ellipsoid by use of the above transformation elements gives the following corrections to the deflections of the vertical adopted for the datum origin:

$$\zeta = -0.''02 \pm 0.''10 \qquad \eta = -1.''60 \pm 0.''11 \qquad H = -42.2 \text{ m} \pm 3.3 \quad .$$

The value for ΔL expresses the scale difference between the two systems and indicates that the NAD 1927 is smaller than the global solution by 3.4 in 10^6.

The corrections to the NAD coordinates (Fig. 2) indicate a possible systematic error in latitude for stations in the northeastern part of the U.S. of about 5 to 7 m.

Because of the smallness of the three rotation elements, a second adjustment has been made on the assumption that the two coordinate systems are parallel. The results are

$$\Delta X = -25.8 \pm 3.5 \text{ m} \qquad \Delta L = (3.1 \pm 1.9) \times 10^{-6}$$

$$\Delta Y = 168.1 \pm 9.7 \text{ m} \qquad \sigma_0^2 = 1.08 \quad .$$

$$\Delta Z = 167.3 \pm 7.4 \text{ m}$$

The difference between the two sets of transformation elements is marginal, and they yield almost identical results when used to transform coordinates from NAD to the geocentric system or vice versa, indicating that, within the limits of the accuracies discussed here, there are no serious distortions in NAD 1927. In view of the way the NAD was computed — first by adjusting the western half and then by tying the other parts to it while keeping this block fixed — this conclusion is somewhat surprising.

Bursa [7, 8] and Veis [9 , 10] have both given solutions for the relationship between the NAD and from satellite-determined reference systems. Their results are generally inconclusive because of the few stations available. Bursa [8], for example, used observations from stations 9001, 9007, 9009, and 9010 as well as some GIMRADA data to estimate rotation elements, but two of these stations, 9007 and 9009, are in South America and only loosely tied to NAD 1927. A recent solution by Lambeck [11] gives results similar to the present solution using coordinates derived from an earlier iteration of the global reference system.

4. — European Datum 1950

Altogether nine stations (or groups of stations) are available for relating the European Datum of 1950 to the global reference system. Table 1 lists the appropriate coordinates. The geoid heights have been estimated from

Fischer [12]. The average accuracies of the station coordinates, relative to the Potsdam origin, have been estimated from the results summarized by Whitten [13] as 3×10^{-6} times the distance to the origin. This may appear overoptimistic, but because of the seven-parameter transformation, systematic distortions over the entire datum need not be considered in assessing these accuracy estimates. Station heights above the reference ellipsoid have been assumed to be accurate to ±3 m, and no correlation between horizontal and vertical positions has been assumed.

From initial comparisons between the two solutions, two results become apparent immediately: there is a large-scale difference between the global solution and EUR 1950, and stations 9074 (Riga) and 9115 (Oslo) give poor agreement. Fig. 4 illustrates these results in the form of differences in distances computed from the two solutions. Unfortunately, we do not have enough information to assess the accuracy with which the geodetic coordinates of station 9074 are known, while for station 9115 we consider the global solution to be suspect because of the limited data available and because of systematic timing errors that have been known to exist in satellite observations from this station. For these reasons, these two stations have been rejected from the datum adjustment.

Results

The solution for the seven transformation elements is

$$\Delta X = -64.5 \pm 19.0 \text{ m} \qquad \omega = -0''.2 \pm 0''.4$$

$$\Delta Y = -154.8 \pm 11.0 \text{ m} \qquad \psi = -1''.7 \pm 0''.7$$

$$\Delta Z = -46.2 \pm 17.5 \text{ m} \qquad \epsilon = 1''.4 \pm 0''.4$$

$$\Delta L = (-12.4 \pm 2.6) \times 10^{-6} \qquad \sigma_0^2 = 0.72 \quad .$$

The corrections to the geocentric coordinates are given in Fig. 5, and the corrections to the datum coordinates in Fig. 6. In most cases, these corrections are smaller than the assumed accuracy estimates.

The angle between the xy and XZ planes is $\delta\lambda = +1''.4 \pm 0''.6$, the x axis being to the east of the X axis, and the angle $\delta\beta$ between the two rotation axes is $2''.2 \pm 0''.7$, in a direction 219° longitude (Fig. 7).

The scale difference indicates that the European Datum is too large by 12.4 in 10^6 when compared with the scale based on GM.

Relating the Potsdam origin on EUR 1950 to the earth-centered ellipsoid gives the following components of the deflection of the vertical to be added to the values initially adopted for the origin:

$$\delta\zeta = -3''.16 \qquad \delta\eta = -5''.79 \qquad \delta H = 0.5 \text{ m} \quad .$$

Solving only for the scale and translation elements yields the following results:

$$\Delta X = -26.8 \pm 16.8 \text{ m} \qquad \Delta L = (-13.7 \pm 3.5) \times 10^{-6}$$

$$\Delta Y = -121.0 \pm 14.0 \text{ m}$$

$$\Delta Z = -82.9 \pm 16.1 \text{ m} \qquad \sigma_0^2 = 1.45 \quad .$$

This solution is statistically still acceptable as $F_{0.95, 14, \infty} = 1.67$. However, the corrections to the heights again show systematic tendencies (Fig. 6) so that the seven-parameter solution appears to be the better one.

In addition to the stations used in the above transformations, EUR 1950 coordinates of the following stations are available: 9002 — South Africa, 9006 — India, 9008 — Iran, 9028 — Ethiopia, and 7818 — Algeria. These stations have not been used in the datum adjustment because it is felt that they are too far from the origin to give transformation elements that are representative of EUR 1950 in Europe. With the above seven transformation elements, the satellite-determined coordinates of these stations can be transformed into EUR 1950 and compared with the datum values (Fig. 8 and Table 3). Geoid heights were estimated from Fischer [12]. These comparisons indicate that, even for such distant stations as South Africa, the datum coordinates are consistent to about 1 in 10^5.

The similarity in the coordinate differences obtained for stations 9008 and 9006 suggests that the connection through Turkey and Iran may be in error by about 30 m. Similarly, the coordinate differences for stations 9002 and 9028 suggest errors of about 40 m in the connection of the arc datum to EUR 1950.

Bursa [7, 8] and Veis [9, 10] have also given solutions for the European Datum relative to the global system but did not attempt to solve for the scale factor, which, as indicated in the present solution, is significant. Their analysis was again limited by the few stations available, and Bursa, for example, used stations 9008 in Iran and 9006 in India, which, as this analysis has shown, are not connected reliably to the EUR Datum.

5.— South American Datum

There are only three stations in South America whose coordinates in the South American Datum 1956 (La Canoa origin) are available. These three stations lie on the 13,000-km-long loop of triangulation around the northern part of South America; its closing error appears to be less than 40 m (Fischer and Slutsky [14]). In view of the large geoid-ellipsoid separation in parts of South America, serious distortions, particularly in scale, may be expected to exist. Table 1 gives the appropriate station coordinates. The accuracy of the geodetic coordinates relative to the origin has been taken as 40 parts in 13×10^6, or 3 in 10^6. The geoid heights are taken from Fischer [15].

Results

Solving for the seven parameters gives

$$\Delta X = -355.3 \pm 31.0 \text{ m} \qquad \omega = -1''.4 \pm 0''.7$$

$$\Delta Y = 194.3 \pm 17.6 \text{ m} \qquad \psi = 1''.2 \pm 0''.6$$

$$\Delta Z = -382.1 \pm 18.4 \text{ m} \qquad \epsilon = -0''.9 \pm 0''.6$$

$$\Delta L = (16.6 \pm 2.8) \times 10^{-6} \qquad \sigma_0^2 = 0.27 \quad .$$

The corrections to both sets of coordinates are always less than a few meters, as would be expected since there are only two degrees of freedom in the adjustment.

Solving only for the translation and scale parameters gives the following results:

$$\Delta X = -320.2 \pm 12.1 \text{ m} \qquad \Delta L = (15.8 \pm 4.1) \times 10^{-6}$$

$$\Delta Y = 203.8 \pm 24.1 \text{ m}$$

$$\Delta Z = -391.7 \pm 7.5 \text{ m} \qquad \sigma_0^2 = 2.4 \quad .$$

The corrections to the coordinates in this case are considerably larger than before (see Fig. 9), particularly for station 9029.

In view of the limited data available, the four-parameter solution is considered to be the better one. For this solution, the additional deflections to the La Canoa origin become

$$\delta\zeta = -13.3 \qquad \delta\eta = -6.4 \qquad \delta H = -70 \text{ m} \quad .$$

6.— Conclusion

The comparisons between geodetic datums and the geocentric system have indicated some large systematic "discrepancies" between the corresponding coordinates, discrepancies that could be reduced by the introduction of scale and rotation parameters. These differences appear to be the result of incorrect reductions of length measurements to the reference ellipsoid or of the unknown character of the propagation of model errors rather than the result of erroneous definitions of scale and orientation in establishing the datum. Once these systematic tendencies have been removed, surface triangulation appears to be reliable to about 2 or 3 in 10^6 over areas of continental size and to about 1 in 10^5 over such large distances as Europe to India and Europe to South Africa. Whether the transformation elements are representative of the entire datum depends on the station distribution. For example, in Europe the transformation elements may not be reliable for stations in Scandanavia and the discrepancies found at Oslo and Riga have to be resolved. Important new data now collected from satellite-tracking cameras in Helsinki and Uppsala will help to resolve this problem. For the SAD the transformation elements can be expected to be reliable only for points in or close to the northern loop and, because of the steep astro-geoid slopes, are certainly not expected to be representative for areas such as Chile or Argentina. When the SAD 1956 coordinates of SAO stations in Argentina become available, a new evaluation of this datum should give improved results.

Future improvements for datum positions and adjustments can be expected if more satellite-tracking stations participate. Bursa [7] gives some requirements in this respect to determine the orientations of datums. Improvements, particularly in the scale determination, will come from simultaneous optical direction and laser range observations. Some work for NAD has been done by SAO between its stations in Florida and New Mexico (Lambeck [16]), and by Centre Nationale d'Etudes Spatiales and Institute Géographique Nationale (Lefebvre [17]) in Europe.

REFERENCES

[1] K. LAMBECK and E. M. GAPOSCHKIN: Geodetic parameters for a Standard Earth, Journ. Geophys. Res. and Smithsonian Astrophys. Obs. Spec. Rep. (in press).

[2] I. FISCHER, M. SLUTSKY, R. SHIRLEY, and P. WYATT: Geoid charts of North and Central America, Army Map Service Tech. Rept. 62, 1967.

[3] L. G. SIMMONS: How accurate is first-order triangulation, The Journal, Coast and Geodetic Survey, 3, 53-56, 1950.

[4] D. A. RICE: A geoidal section in the United States, Bull. Géod., 65, 243-251, 1962.

[5] D. A. RICE: The development of geoidal sections in the Central United States, in Aus der Geodätischen Lehr en Forschung, Festschrift-Walter Grossman, Stuttgart, 28-35, 1967.

[6] M. BURSA: The theory for the determination of the nonparallelism of the minor axis of the reference ellipsoid and the inertial polar axis of the earth, and of the planes of the initial astronomic and geodetic meridians from observations of artificial earth satellites, Studia Geophys. et Geod., 6, 209-214, 1962.

[7] M. BURSA: On the possibility of determining the rotating elements of geodetic reference systems on the basis of satellite observations, Studia Geophys. et Geod., 11, 390-396, 1967.

[8] M. BURSA: On the mutual position of Clarke's and Hayford's ellipsoids of the NAD-27 and the EUR reference systems, Studia Geophys. et Geod., 12, 122-135, 1968.

[9] G. VEIS: Geodetic interpretation, in Geodetic Parameters for a 1966 Smithsonian Institution Standard Earth, Vol. 3, edited by C. A. Lundquist and G. Veis, Smithsonian Astrophys. Obs. Spec. Rep. 200, 1-14, 1966.

[10] G. VEIS: The determination of the radius of the earth and other geodetic parameters as derived from optical satellite data, Bull. Géod., 89, 253-275, 1968.

[11] K. LAMBECK: New estimates for the relation of the North American Datum to a geocentric satellite reference system, Studia Geophys. et Geod., 13, 482-485, 1969.

[12] I. FISCHER: New pieces in the astro geodetic geoid map of the world. Presented at the XIV General Assembly IUGG, Lucerne, October 1967.

[13] C. A. WHITTEN: Adjustment of European triangulation, Bull. Géod., 24, 187-206, 1952.

[14] I. FISCHER and M. SLUTSKY: A study of the geoid in South America, Presented to 10th Consultation on Cartography of the Pan-American Institute of Geography and History, Guatamala, 1965.

[15] I. FISCHER: The geoid in South America referred to various reference systems, Presented at the Fall Meeting of the American Geophysical Union, San Francisco, 1969.

[16] K. LAMBECK: Scaling a satellite triangulation net with laser range measurements, Studia Geophys. et Geod., 12, 339-349, 1968.

[17] M. LEFEBVRE: Etude comparative des différents résultats obtenus en géodesie spatiale sur le réseau Européen, presented at Symposium sur la Triangulation Spatiale Européen, Paris, February 1969.

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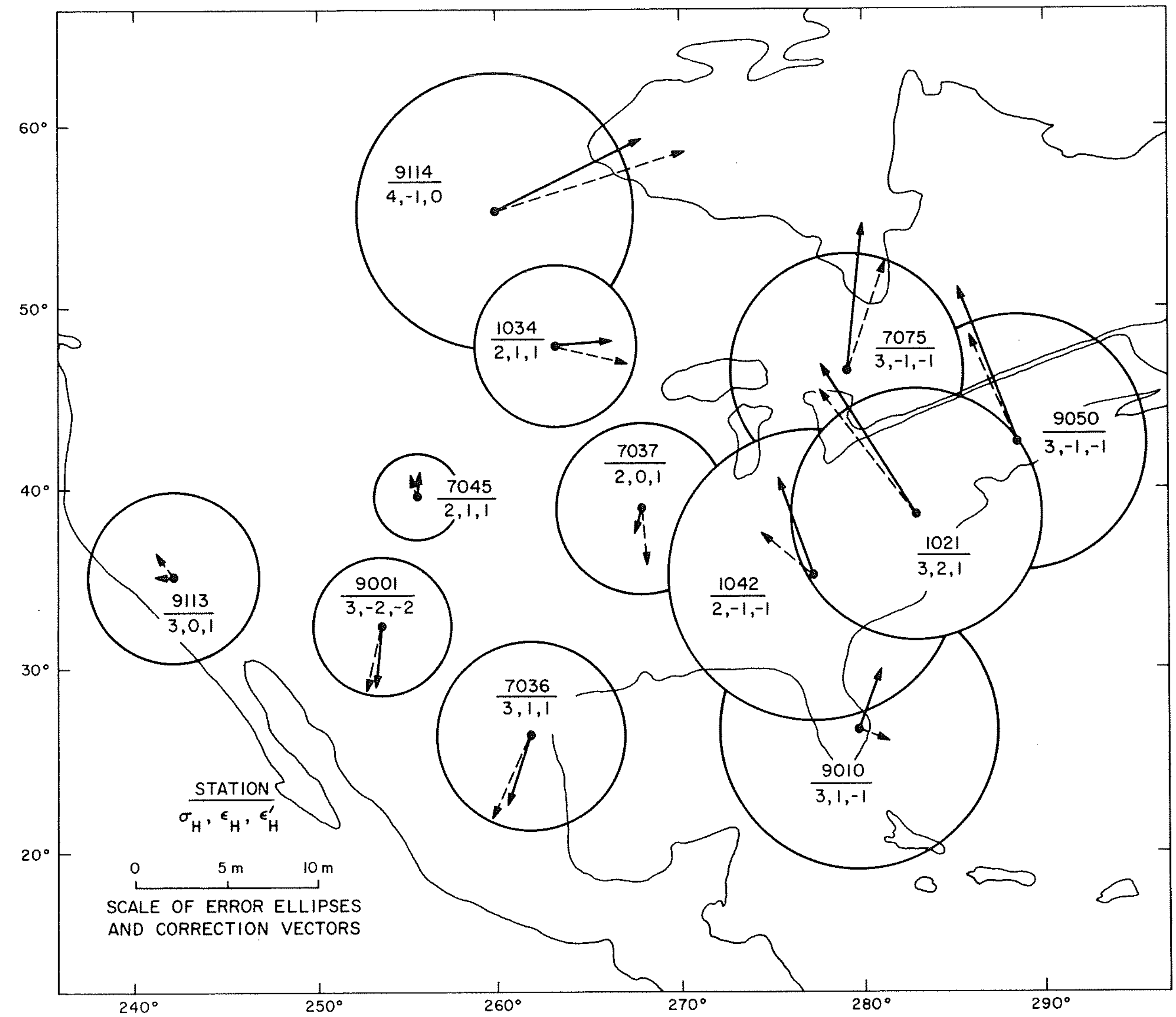

Figure 1 — Accuracy estimates of station positions in NAD 1927 relative to the origin at Meades Ranch. The vectors (solid lines, seven-parameter solution; dashed lines, four-parameter solution) represent the corrections to the geodetic coordinates resulting from combination with the global solution. ϵ_H and ϵ'_H are height corrections (ϵ_H from the seven-parameter solution and ϵ'_H from the four-parameter solution).

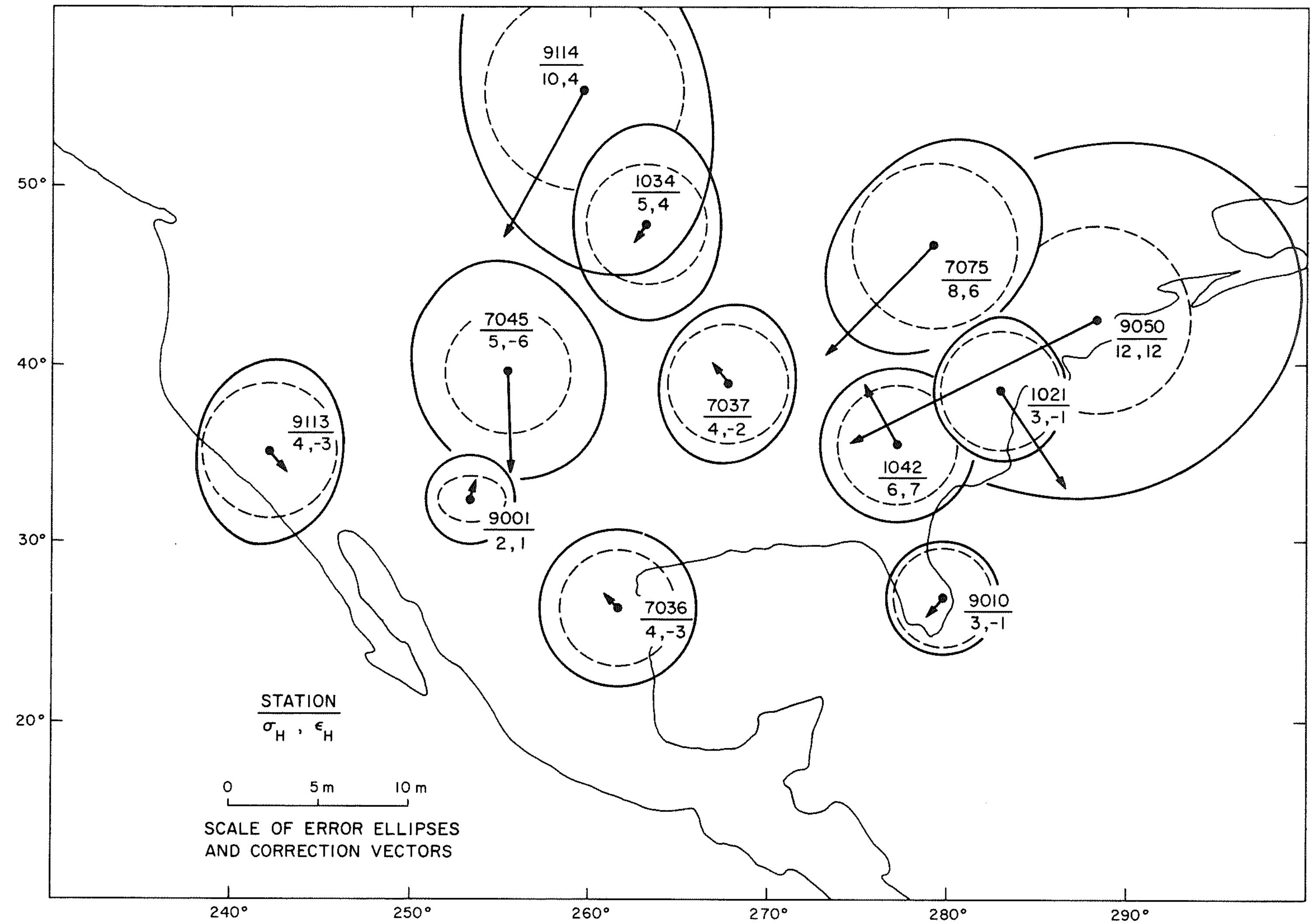

Figure 2 — Accuracy estimates of the station positions derived from the global solution (solid-line ellipses) and from the combination of this solution with geodetic survey data (broken-line ellipses). The vectors represent the corrections to the geocentric coordinates resulting from this combination.

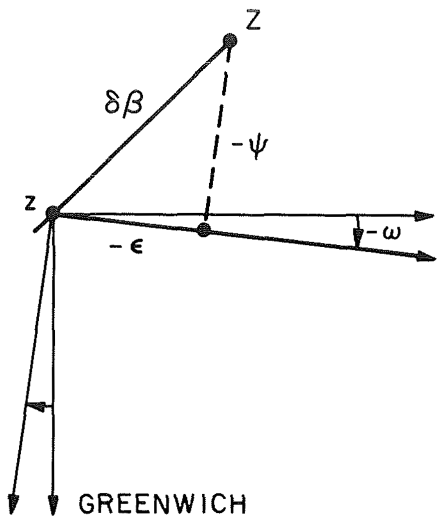

Figure 3 — Position of the pole of NAD 1927 (z) relative to the pole of the global solution (Z).

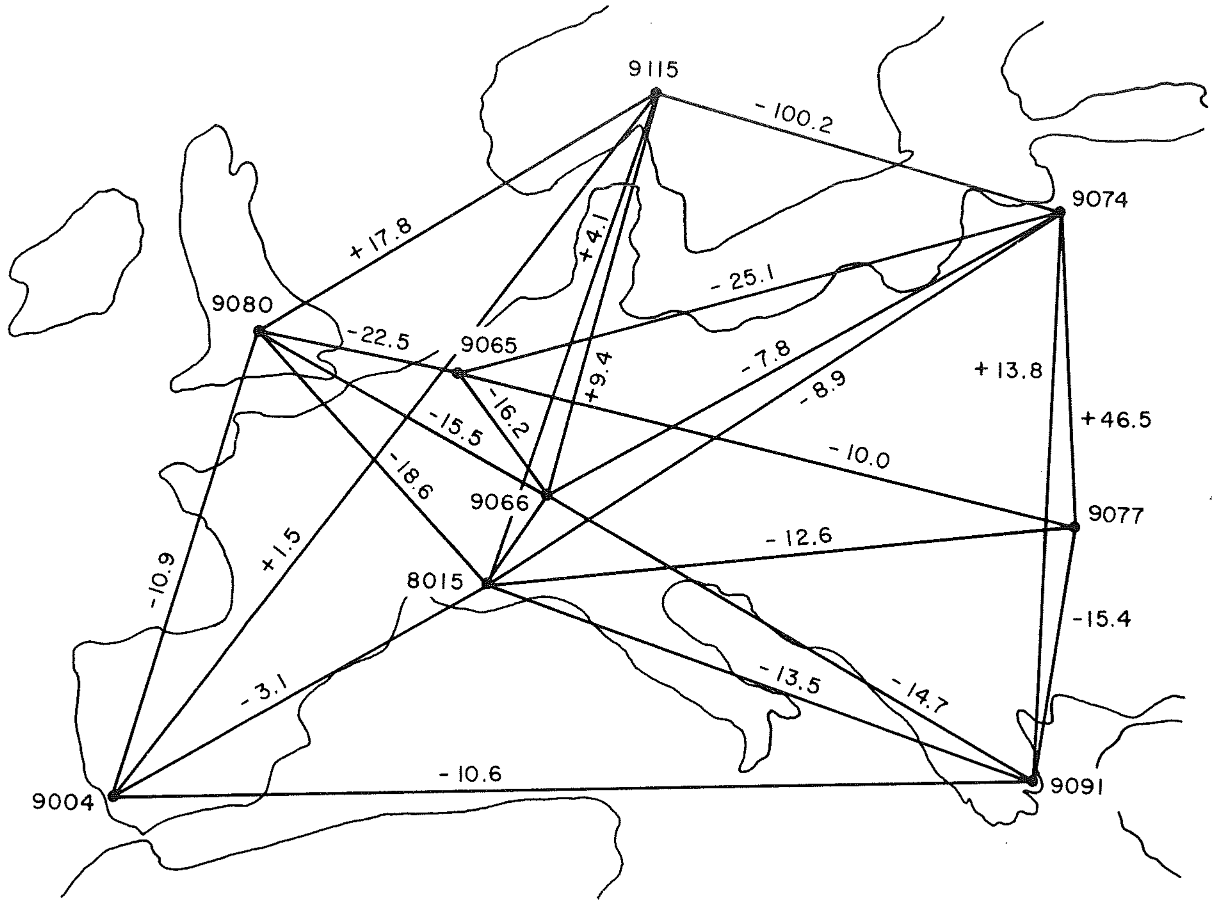

Figure 4 — Differences (in parts in 10^6) in interstation distances computed from the global solution and EUR 1950.

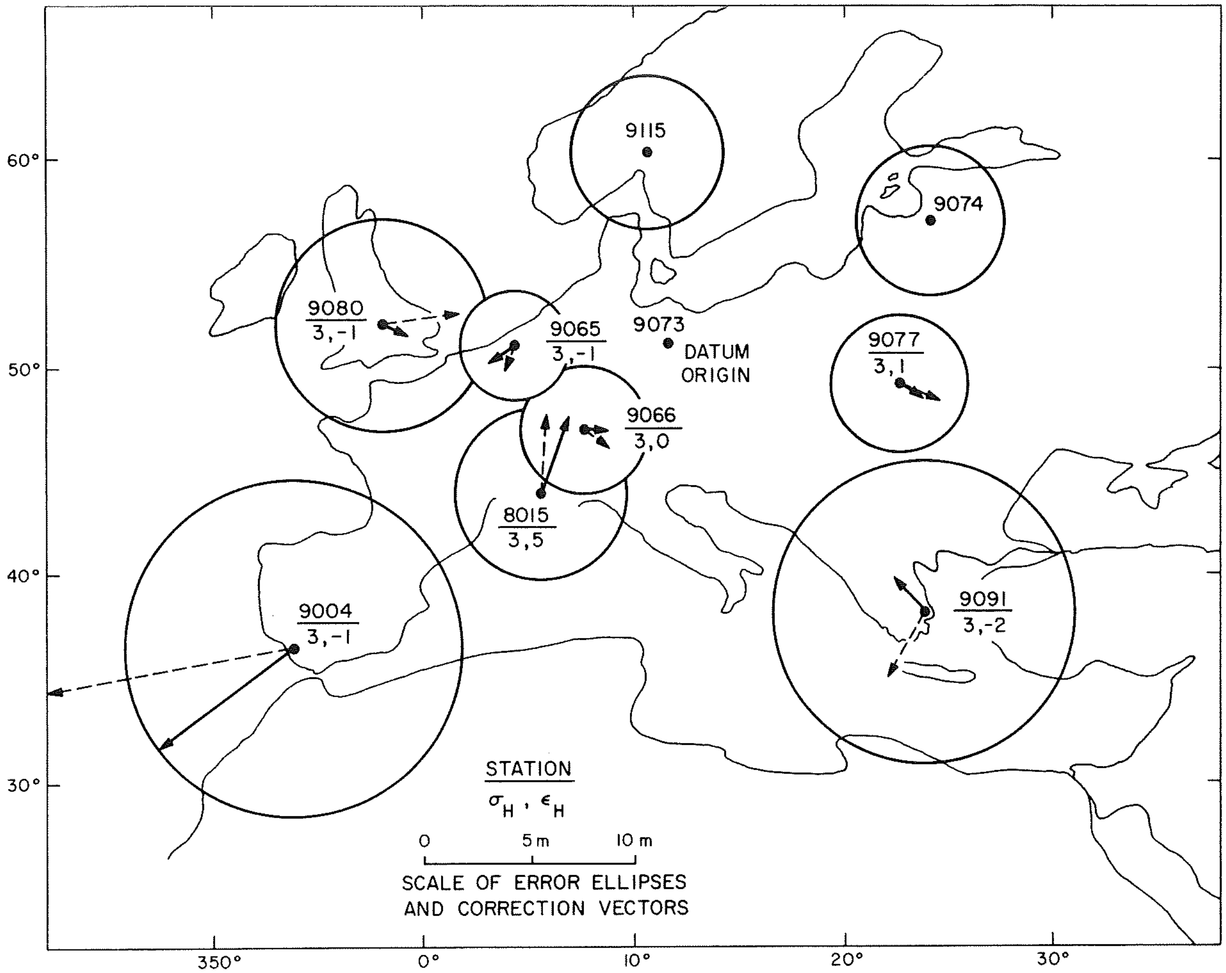

Figure 5 — Accuracy estimates of stations in EUR 1950 relative to the Potsdam origin. Solid vectors refer to the seven-parameter solution, and broken vectors to the four-parameter solution.

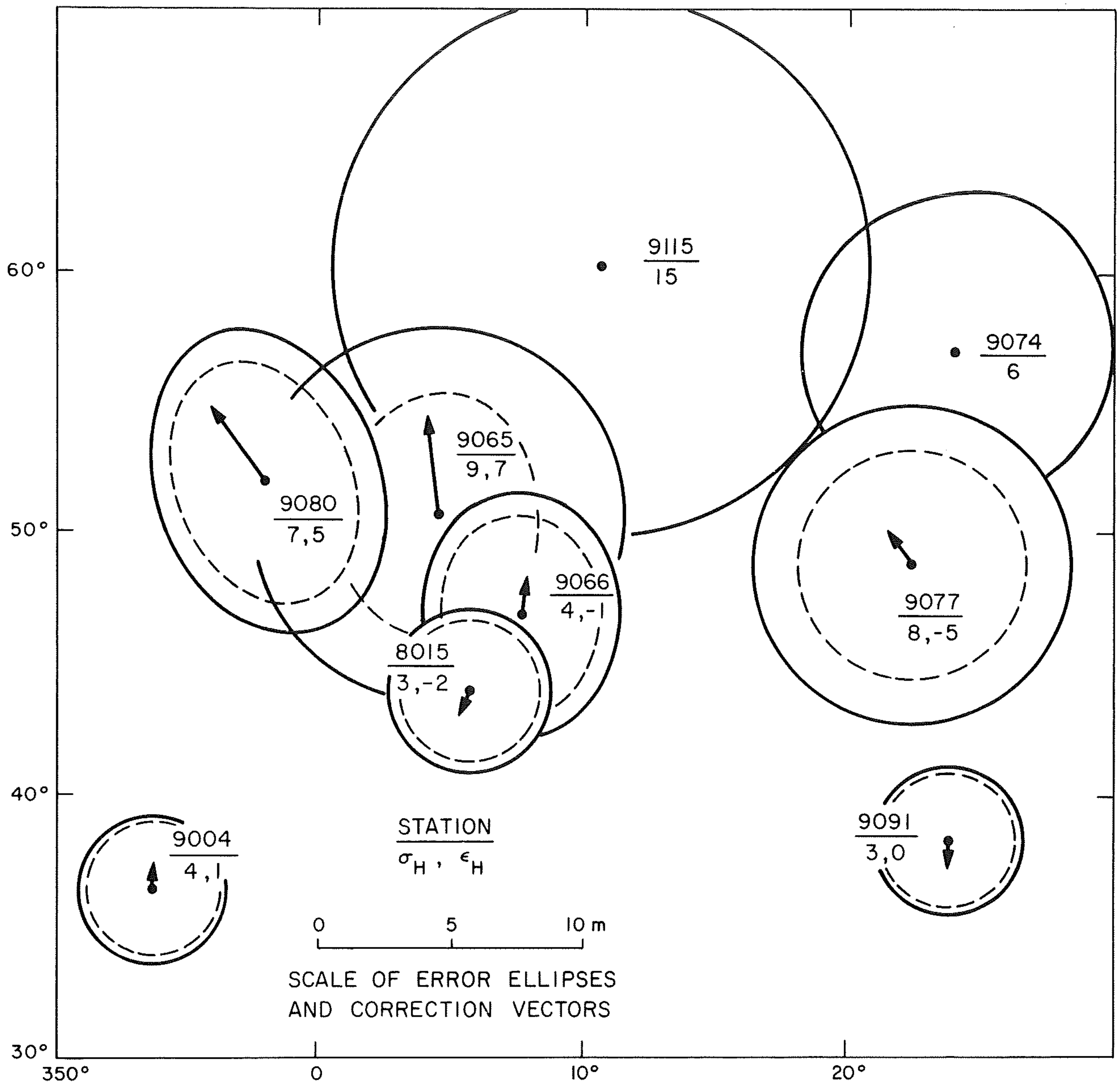

Figure 6 — Accuracy estimates of stations derived from the global solution (solid-line ellipses) and from the combination of this solution with geodetic survey data (broken-line ellipses). The vectors represent the corrections to the geocentric coordinates resulting from this combination.

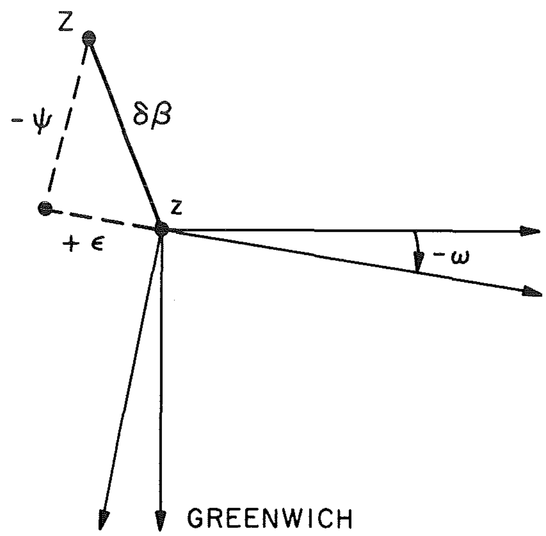

Figure 7 — Position of the pole of EUR 1950 (z) relative to the pole of the global reference system (Z).

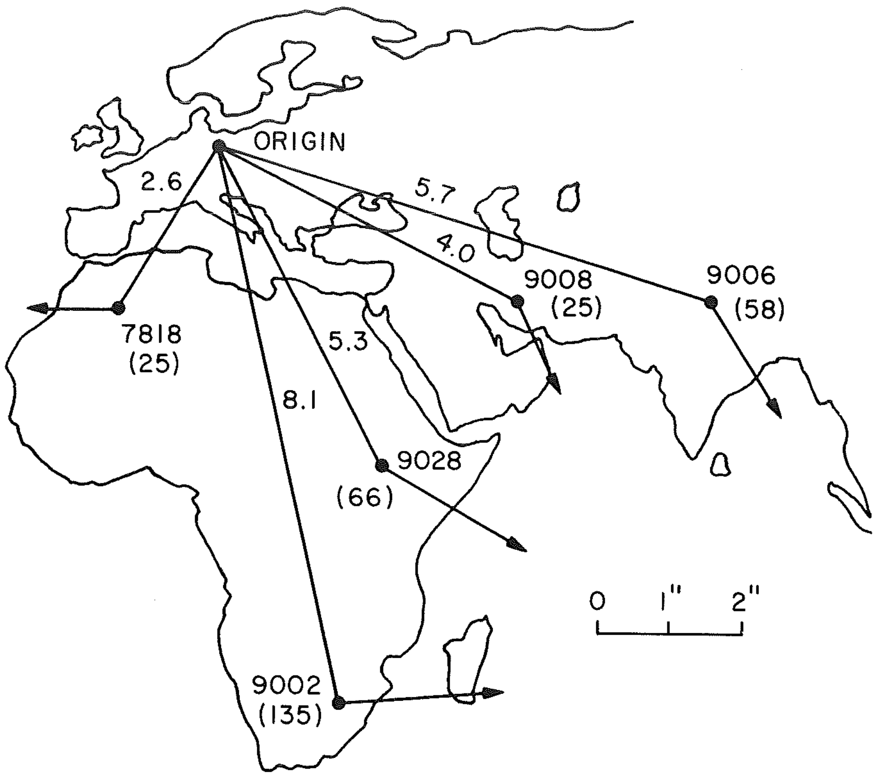

Figure 8 — Differences between satellite-solution coordinates transformed into EUR 1950 and the EUR 1950 Datum coordinates. The numbers in parentheses refer to differences in height above the reference ellipsoid. Distances of the station from the origin are in megameters.

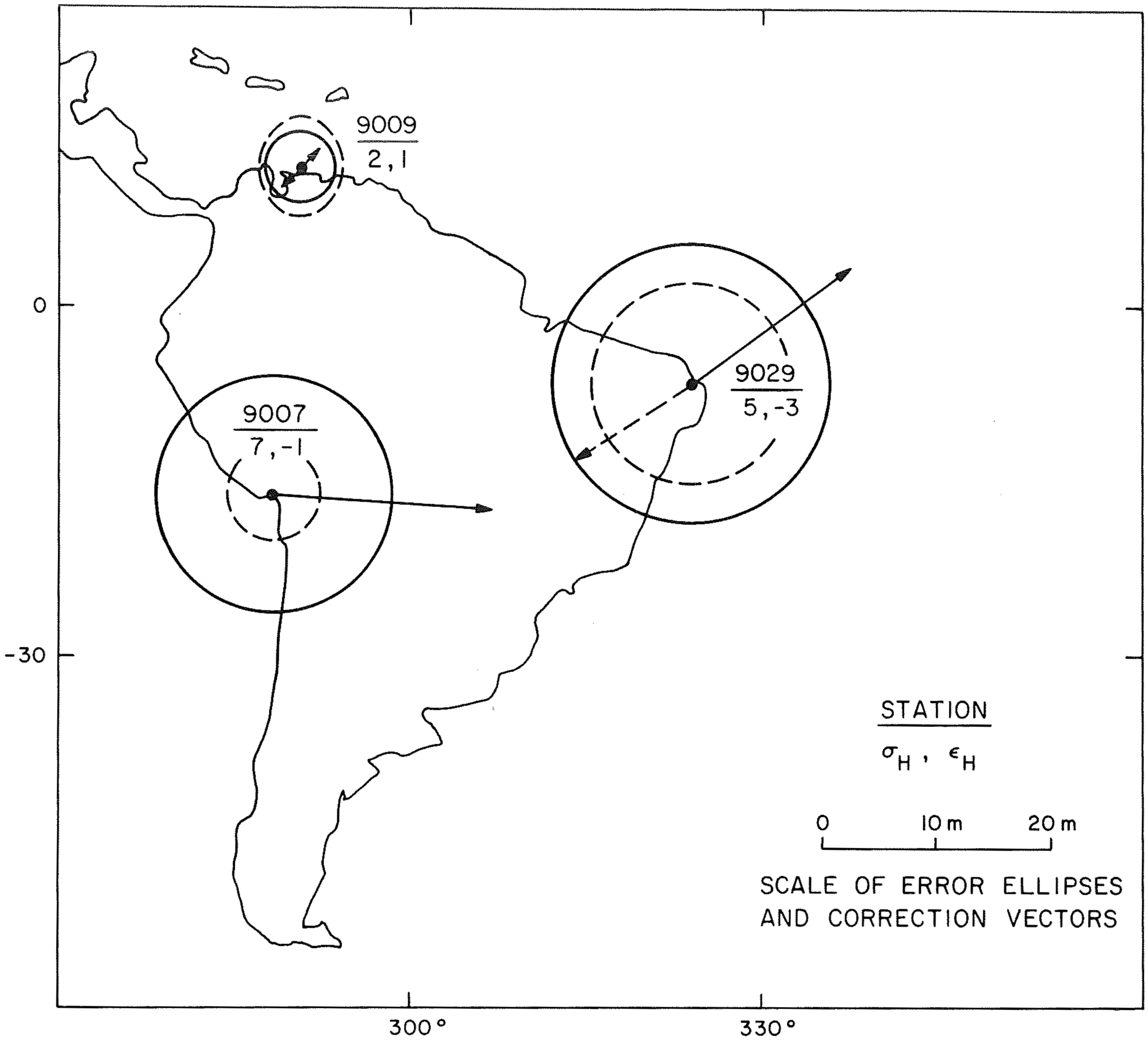

Figure 9 — Accuracy estimates for the combination of SAD 1956 with the global solution. The solid-error ellipses and correction vectors refer to the datum coordinates, and the broken-line ellipses and vectors to the global solution. σ_H and ϵ_H refer to the datum coordinates.

Table 1

Geocentric Cartesian coordinates and geodetic datum coordinates of the stations used for the adjustments. The heights are above the reference ellipsoid used.

	X (MM)	Y(MM)	Z(MM)	STATION	LATITUDE	LONGITUDE	HEIGHT
				NAD 27	6.378206	294.98	
1021	1.118029	-4.876316	3.942984	1021	38 25 49.63	282 54 48.22	7
1034	-.521702	-4.242049	4.718731	1034	48 1 21.40	262 59 21.56	255
1042	.647515	-5.177924	3.656707	1042	35 12 6.93	277 7 41.01	916
7036	-.828496	-5.657458	2.816812	7036	26 22 45.44	261 40 9.03	66
7037	-.191286	-4.967280	3.983262	7037	38 53 36.07	267 47 42.12	273
7045	-1.240479	-4.760229	4.048995	7045	39 38 48.03	255 23 41.19	1796
7075	.692628	-4.347059	4.600483	7075	46 27 20.99	279 3 10.35	280
9001	-1.535757	-5.166996	3.401042	9001	32 25 24.56	253 26 51.17	1650
9010	.976291	-5.601398	2.880240	9010	27 1 12.88	279 53 13.01	26
9050	1.489753	-4.467478	4.287304	9050	42 30 20.97	288 26 28.71	193
9113	-2.450011	-4.624421	3.635035	9113	34 57 50.74	242 5 11.58	760
9114	-1.264838	-3.466884	5.185467	9114	54 44 33.86	249 57 26.39	701
				EUR 50	6.378388	297.00	
8015	4.578328	.457966	4.403179	8015	43 56 1.14	5 42 49.28	647
9004	5.105588	-.555228	3.769667	9004	36 27 51.37	353 47 42.09	-6
9065	3.923411	.299882	5.002945	9065	52 0 9.24	4 22 21.23	23
9066	4.331310	.567511	4.633093	9066	46 52 40.32	7 27 58.24	898
9074	3.183901	1.421448	5.322772	9074	56 56 54.98	24 3 37.81	3
9077	3.907421	1.602397	4.763890	9077	48 38 4.56	22 17 57.88	189
9080	3.920178	-.134738	5.012708	9080	52 8 39.12	358 1 59.49	112
9091	4.595157	2.039425	3.912650	9091	38 4 48.24	23 56 1.61	467
9115	3.121280	.592643	5.512701	9115	60 12 40.38	10 45 8.74	591
				SAD 56	6.378388	297.00	
9007	1.942775	-5.804081	-1.796933	9007	-16 27 43.79	288 30 31.54	2322
9009	2.251829	-5.816919	1.327160	9009	12 5 36.44	291 9 51.73	19
9029	5.186461	-3.653856	-.654325	9029	-5 55 27.47	324 50 6.98	38

Table 2

Covariance matrix of the station coordinates in the NAD and referred to the origin at Meades Ranch (units are meters^2). The last column gives the variances of the station heights above the ellipsoid.

	1034	9001	9113	7036	9050	1021	7037	1042	7045	7075	9010	9114	σ_H^2
1034	19	1	1	8	8	8	7	7	1	10	7	19	4
9001		15	6	4	0	0	0	0	2	0	1	1	9
9113			20	3	0	0	0	0	4	0	0	1	4
7036				26	7	7	7	13	2	7	14	8	9
9050					50	41	14	37	0	27	16	8	9
1021						45	14	40	0	30	16	8	9
7037							21	7	0	7	14	7	4
1042								63	0	25	35	7	4
7045									4	0	0	0	4
7075										40	7	7	9
9010											57	7	9
9114												56	16

Table 3

Differences in coordinates of stations tied to the EUR 1950 Datum. ϕ_G, λ_G, H_G are the coordinates obtained in the global solution and transformed into the EUR 1950 system. ϕ_D, λ_D, H_D are the coordinates in EUR 1950 obtained from terrestrial triangulation.

				Proportional accuracy relative to origin	
Station	$\phi_G - \phi_D$	$\lambda_G - \lambda_D$	$H_G - H_D$	Horizontal position	Height
7818	-0.″03	-1.″13	25 m	1 in 90,000	1 in 105,000
9002	0.″19	2.″30	135 m	130,000	60,000
9006	-1.″54	0.″92	58 m	110,000	98,000
9008	-1.″11	0.″53	25 m	110,000	160,000
9028	-1.″15	1.″94	66 m	80,000	80,000