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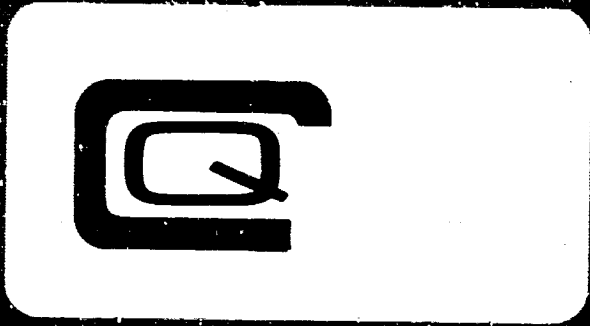
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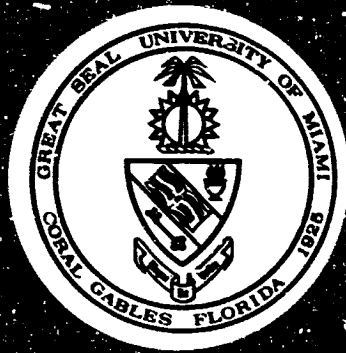
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TOLERANCE SPACES AND BEHAVIOR[*]

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I. The concept of tolerance is made central in studying certain behavioral models. A precise mathematical definition of tolerance will be used in order to interpret psychological evidence in terms of these models. The models will be phenomenological in nature and are therefore of limited value in interpreting complex human behavior. However, they exhibit certain interesting features worth study. The following exposé is not considered as the endpoint of our study but as starting point in investigating similar models which may hopefully become more and more adequate to describe human behavior.

II. Human behavior is complex largely because the human brain is complex. Each of its 10^9 neurons forms on the average about a hundred synaptic junctions by its telodendria and as many interconnections by its dentrites. However, the human nervous system is not entirely pre-wired, but is capable of growth and reorganization. Learning, for example, imposes again and again additional organization on the higher levels of the nervous system. It seems unlikely that the genes would carry the information to specify each of the 10^{11} interconnections of brain cells. The brain grows and matures for many years. However, when enough junctions in a circuit are established, many concepts of systems theory, as e.g., the concept of feedback, become debatable. One approach to the study of brain function involves the investigation of relatively small and simple neuron networks and the search for evidence in human behavior which may be the outcome of their activity. Instabilities in neural circuits of the brain, for example, are related by some investigators to various kinds of disorders of thought. But still we will not be able to describe such activity adequately, e.g., in conventional systems theory alone. For example, fundamental concepts

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of neural systems such as the facilitation and timing of their growth, their plasticity, maturation and adaptability, fundamental properties of the organization of brain matter, are outside its scope. Such underlying brain properties may result in behavioral events described in psychological terms and are far from being understood. It is shown by psychological experiments [Asratian, 1965] that, e.g., functional plasticity of the human nervous system is responsible for certain traits of personality. Memory may be related to neuron growth and strengthening of the synaptic junctions and so on.

However, we may also pose the question in the reverse direction. That is, we may ask what implications for the functioning of specific neural systems of the brain derive from observed patterns of behavior such as the rise of a conflict, its repression, the ego-id control or reality testing? The hope is to find by such an approach features of brain matter unknown in conventional systems theory but relevant to the maturation of human personality.

An example would be to proceed from the study of conflict situations to their implications on control circuits of neurons, or we may try to find implications for the reactivity and strength of neuron interconnections derived from the observed inverse relationship between the dimension of a personality called "strength of its nervous system" and its reactivity. Another well-known example from physiological psychology is the assumption of reinforcement processes in certain brain parts which lead to pleasure seeking drives.

III. The complexity of human behavior forces us at once to study a behavioral model which is simple enough to be manageable.

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Thus, we will make few specific assumptions rather than overload the behavioral model with complexity. However, such a model should still exhibit sufficiently complex features relevant for human behavior.

The model's behavior is supposed to consist entirely of adaptation and defense acts. The "internal" behavior of the model is schematized by states which are completely described by the model's adaptabilities and reactivities with regard to external (physical or mental) stimuli. That is, the internal states are described by the model's ability to change its responses to environmental stimuli. These abilities have to be functions of time (t) if the model is able to "mature". This explicit time dependence of states will be distinguished from the time dependence arising from the time dependence of the external stimuli. In addition, the internal states are described by the sensitivity of the model to changes in external stimuli, i.e., by its reactivities. Such reactivities lead to defense acts of the model.

Thus, we have a set of states induced by external stimuli. They are described by time dependent coordinates, the reactivities and adaptabilities. We then have a map of the set of stimuli X into the state space describing the internal reaction of the model to the environment and its changes, and a map of X into Y, the set of possible responses of the model.

In order to illustrate this, let us give some examples.

If, quite generally, the operational definition of our models can be given in terms of relations, S_i, between a sequence of inputs $(x_1 \ldots \ldots x_n)$ and a sequence of outputs $(Y_1 \ldots \ldots y_m)$ i.e., if

$$y_i = S_i(x_1(t), \ldots x_n(t), t), i = 1, \ldots, m,$$

then reactivities, r_{ij}, and adaptabilities, a_i, may be defined according to G. Houghton [2,3] by the derivatives*

$$r_{ij} \equiv \frac{\partial}{\partial x_j} S_i (x_1 \ldots x_n, t) \text{ and } a_i \equiv \frac{\partial}{\partial t}\Big|_{ex} S_i (x_1 \ldots x_n, t), \quad j=1 \ldots m.$$

The additional time parameter gives the explicit time dependence of the model and $\partial/\partial t|_{ex}$ denotes the derivative with regard to explicit time. For example, within limits the discharge frequency ω of a single nerve fits the logarithmic response law:

$$\omega(x,t) = S(x,t) = a_i e^{-b(t-t_o)} + C_i \log x/x_o + \omega_o \ ,$$

$$\omega(x \leq x_o, t) = \omega(x, t \leq t_o) = O$$

where x is the stimulus amplitude at time t_o, and x_o is the minimum amount of stimulus which triggers the fiber to fire with frequency ω_o. The time dependence in $e^{-b(t-t_o)}$ describes the adaptation of the nerve fiber (a,b,C, are constants of dimension $[1/t]$).

Then the internal states of the nerve fiber are described by the adaptability

$$|a| = b \ \frac{\omega(x,t) - \omega_o}{C \ \exp[b(t-t_o)]+1}$$

and the reactivity in regard to changes of x

$$r = ax^{-1}[e^{-b(t-t_o)} + C] \ , \quad (x \geq x_o, \ t \geq t_o) \ .$$

The nerve fiber exhibits no adaptability for the minimum amount of

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*Here $|a_i|$ should be called "adaptability" rather than a_i.

stimulus. Its adaptability increases if the stimulus amplitude increases, but decreases with time; $|a|$ is proportional to the decay constant b. The reactivity in regard to changes in x decreases with time as well as with increasing stimulus amplitude.

The counterpart of this example in physiological psychology would be a generalization of the Weber-Fechner law or Stevens law [Teitelbaum 1967] and the corresponding adaptability and reactivity.

IV. An invaluable characteristic of the brain is its invulnerability. Malfunction in one of its (numerous) local working parts does not shut down the entire system. Its basic elements function for many years despite the fact that each day several thousand neurons die and are never replaced. A superfluous number of functional units in the brain provides a latent reserve so that they can take over functions from other damaged units. This allows the brain to circumvent unreliability arising from malfunction or death of single localized units. Furthermore, such a safety factor may have the consequence that a specific function of the brain is not inherent in any particular local neuron system but is distributed over several brain parts. Indeed, attempts to localize complex mental functions in specific parts of the brain often fail [Luria, 1963].

In order to simulate such behavior we assume that each time the model is exposed to external stimulus 1) the adaptabilities and reactivities of functional units change and 2) in general more than one unit responds. We introduce such a multiplexity so that the reliability of the entire model can be greater than the reliability of its components. This is characteristic of human behavior.

Now we will give a more specific structure to our model:

<u>Definition (i)</u> u-cluster (denoted by C_u): set of functional units $(u_\alpha, \alpha = 1,2....)$ whose internal states depend at most on the stimulus variables on which the states of unit u depend. The responses of these units (in a given state) are identical and furthermore the intersection of the set of their states, W_{u_α}, with the set of states of u is not empty. We denote by W_{C_u} the set $\bigcup_{u_\alpha \in C_u} W_{u_\alpha}$. A unit may belong to several clusters, $u \in C_u$ of course.

Let V be a subset of external stimuli $(V \subset X)$.

<u>Definition (ii)</u> We call the set of states occupied by the clusters which are sensitive to V the V-configuration of the model.

<u>Assumptions:</u> (a) The model consists of clusters as defined above. The units of a cluster are indistinguishable insofar as a configuration determines a behavioral act of the model, regardless which of the units gives a certain response.

Furthermore we suppose that

(b) the interconnections between the elements of a cluster are such that they can be represented by directed lines having the following meaning:

<u>Definition (iii)</u> A directed line $\underset{w}{o} \rightarrow \underset{w_j}{o}$ (j $\in$ I, I set of indices) means: If at given time state w is occupied by a unit, u say, then $w_j \in W_{C_u}$ and w_j is occupied too. (Occupied by a unit which of course belongs to the u-cluster.)

If two units are in the same state we neglect the corresponding line since they give the same response in any case.

(c) It is assumed that the set $U = \{w_j \mid$ line $w\, w_j$ exists$\}$ is finite for a finite set V and that the internal state w also belongs to such a finite set.

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Then each cluster of functional units is represented for fixed
time t and given V, by a (not necessarily connected) digraph [Harary,
1965]. We denote the reachable set of w by R(w) and its antecedent
set by Q(w). R_p(w) denotes the set of states reachable from w by
paths with length $\leq$ p (p a natural number). In the usual way we have
a classification of states as receivers, transmitters, carriers or
isolates.

Two examples given in appendix A may help to clarify the fore-
going. There we use the method described above to construct internal
states. In appendix B we give all possible structurally distinct di-
graphs if the number (n) of states is n = 2 or 3 for given V. This
shows that the number of possible models is considerably restricted
for finite state spaces. The appendix also shows that our assumptions
and definitions are consistent since they can be illustrated by ex-
amples.

V. A digraph reflects detailed internal structure of the models.
Yet we are mainly interested in correlations between general features
of digraphs and the way these models respond to external stimuli.

Given a certain mechanism for reaction to the environment there
is little remarkable about its activities. However, we may sometimes
wonder why in nature complex organisms do not react to certain stimuli.
What additional features enable such "action mechanisms" to restrain
from action? Actually there are many varieties of inaction in nature
such as death, rest, inhibition, or repression.

On the other hand the action of a realistic model should not ex-
hibit a one to one relation between its internal "configurations" and
its responses. For example, two states of a brain which occur at

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different times and which are represented by the reactivities and adaptabilities of its many functional units, composed of about 10^9 neurons, may be associated with the same thought even if the brain lost a few thousand neurons by aging. A realistic model should be able to mature this is why we have included explicit time. Furthermore, we do not exclude the possibility that the model may tend to respond to a certain stimulus variable in different ways - different units of a cluster may be in different states giving opposed responses. The model may then be in a "conflict situation".

Therefore, in order to provide precise definitions for such terms, borrowed from psychology, and in order to be able to describe the above mentioned features of a realistic model to some extent, we will make set W_{C_u} into a "tolerance space". Behavioral acts of the model will be determined by its internal configuration as well as by a tolerance arising from the interconnections between the states of this configuration as shown in the corresponding digraphs.

<u>Assumption:</u> (d) The set W_{C_u} is a tolerance space (T-space).

Later on we will give concrete realizations of tolerances on W_{C_u}. First we give some basic definitions concerning tolerance spaces. Tolerance spaces were first used by E.C. Zeeman to describe certain features of visual perception and memory.

<u>Definition (iv)</u> A tolerance ξ on a set W is a subset of the cartesian product $W \times W$ with the properties:

      a) $(w,w) \in \xi$ for all $w \in W$

      b)   if $(w_\alpha , w_\beta) \in \xi$ then $(w_\beta , w_\alpha) \in \xi$

A tolerance space, (W, ξ), is the set W together with a tolerance ξ. The points w_α and w_β are said to be within tolerance, $(w_\alpha \sim w_\beta)$.

A tolerance like a basis of neighborhoods of a topological space imprints a structure on the set W. Many biological examples of tolerance spaces such as the visual field have been given by E.C. Zeeman.

Definition (v) The induced tolerance on the power set of W, L_W, is defined as follows:

Given A, $A' \subset L_W$; $A \overset{\xi}{\sim} A'$ (A and A' are in tolerance) iff $A' \subset \xi A$ and $A \subset \xi A'$, where $\xi A \equiv \{w \mid w \in W, w_\alpha \in A \text{ and } (w_\alpha, w) \in \xi\}$ and similarly for $\xi A'$.

Let f be a map between two sets X, Y and let (X, ξ) be a tolerance space. Then the induced tolerance, $\eta \equiv \xi f$, on Y is given by $y_1 = f(x_1) \sim y_2 = f(x_2)$ iff $x_1 \sim x_2$.

Definition (vi) We call the set $T_\xi(w_\alpha) \equiv \{w \mid w, w_\alpha \in W \text{ and } (w, w_\alpha) \in \xi\}$ the tolerance base of w_α (T-base of w_α).

Thus, the behavioral model is realized 1) by the relation structures: $M_u = \{W_{C_u}; \xi_u, 1\}$ where ξ_u is a given tolerance on W_{C_u} and $1(w,w')$ the relation: "line w'w exists", and 2) by maps

$$g_{u_\alpha} : X \rightarrow W_{C_u}$$
$$f : W_{C_u} \rightarrow Y$$

performed by the functional units, $u_\alpha \in C_u$, $\alpha \in I$.[*]

VI. Let V and t be given, where V is chosen such that the model contains at least one functional unit, u say, whose states at time t depend on V and not only on a subset of V. We will consider clusters of such units and the associated digraphs.

[*] u lables the units of the model.

An intrinsic tolerance on W_{C_u}, having a plausible interpretation, is realized as follows:

$\xi_u \subset W_{C_u} \times W_{C_u}$ with $(w,w) \epsilon \xi_u$ and $(w_\alpha, w_\beta) \epsilon \xi_u$ iff w_α and w_β are 3-joint by paths of length one in D_u, where $w_\alpha, w_\beta \epsilon W_{C_u}$ and D_u is the digraph describing C_u (for V at time t).[*]

Obviously ξ_u is a symmetric and reflexive relation on W_{C_u}. In general it is not transitive.

States in tolerance are always occupied simultaneously. Elements of C_u can (and do) exchange their states before the model "notices" any difference - that is, before it changes its responses - if these states are within tolerance. (To verify this note that because of assumption (a) the units generate the same sequence of responses before and after the change). Therefore we can regard two states in tolerance as equivalent to some extent even if they correspond to different responses of the model. Vertices of complete symmetric (asymmetric) digraphs are all within (outside) tolerance. The tolerance ξ_u is time dependent and takes part in the maturation process.

We can generalize the tolerance given above including in ξ_u pairs of points which are 3-joint by paths of length $\leq \ell$ where ℓ is a natural number. Models may be characterized by the strength of their tolerance (ℓ).

In what follows we consider a given tolerance ξ_u on W_{C_u} with given strength ℓ and suppose that the model has developed a tolerance (in a psychological sense) as a necessity for its reliability and its stability in regard to certain environmental changes. That is, if

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[*] We choose this rather (unnecessarily) complicated description of ξ_u because it is generalizable.

$\tilde{\eta} = \xi_f$ is the induced tolerance on the set of possible responses Y, then responses as well as adaptabilities and reactivities within (mathematical) tolerance, $\tilde{\eta}$ and ξ_u respectively, have no opposed tendencies and lead to one well-defined reaction of the model.

The induced tolerance $\tilde{\eta}$ is not necessarily the maximum tolerance on Y; instead $\tilde{\eta} \subset \eta$ where η is the set of all pairs of responses mutually compatible and leading to one well-defined reaction.

For example, the reacting of the model may not depend on (small) quantitative differences in the responses of the units as long as these responses are within tolerance.

On the other hand, given a tolerance on W, the set of environmental stimulus variables is also a T-space according to the following definition:

$$x_i \sim x_i \text{ and } x_i \sim x_j \, (i \neq j) \text{ iff } R_1(w_i^u) \overset{\xi_u}{\sim} \{w_j^u\} \text{ and } R_1(w_j^u) \overset{\xi_u}{\sim} \{w_i^u\}$$

where

$$w_k^u = g_u(x_k)$$

This tolerance corresponds to a "just noticeable difference" in physiological psychology [Zeemann, 1968] which is a symmetric, reflexive and nontransitive relation between stimuli. Two stimuli must be out of this tolerance if the model is to distinguish between them. Again this tolerance, i.e. the J.n.d., depends on the "maturity" of the model.

G. Houghton [2] interpretes the units of his model and their

functioning by Freudian concepts such as the id, ego, superego, their conscious and unconscious parts. He simulates, for example, the ego-id control mechanism by a feedback circuit of linear units by which he describes the ego and the id assuming that nonlinearities can be neglected in a first approximation. Nonlinearities may arise from interactions between the conscious and unconscious levels of activity of the ego and id units. However, in our picture their states are represented in first approximation by isolates in which case a tolerance is impossible. A tolerance of finite strength can only be developed in the case of interactions of consciousness and unconsiousness.

We conclude this section with some examples from the behavioral repertoire of our models.

Conflicting action tendencies can and do struggle against each other within human beings. We expect that a theoretical formulation of conflict can help us to understand such behavior.

Consider the conflict behavior of the models as competition between incompatible response tendencies. The models may then exhibit two different kinds of conflict behavior if $R_1(w_u) \not\subseteq T_{\xi_u}(w_u)$. (For example $R_1(w_u)$ may contain receivers or transmitters.) An unconscious conflict occurs if f maps $R_1(w_u)$ into $T_\eta(y_u)$ (where $f(w_u) = y_u$); an open conflict occurs if $f(R_1(w_u)) \not\subseteq T_\eta(y_u)$. In the first case the responses have no opposite tendencies. However, the internal states, occupied by cluster C_u, may have opposite reactivities or adaptabilities in regard to changes of the environment. The formation of conflict depends on the strength of the tolerance. In some cases a conflict can be resolved by developing a stronger tolerance.

Similarly, the model exhibits different forms of inactivity. As

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mentioned, in any complete account of behavior we must be able to explain the organism's inaction as well as its action - and there are many varieties of inaction in nature. Let y_o represent "no response". If for some i, $y_i \, \varepsilon \, T_\eta(y_o)$, we say the responses y_i are aroused rather then performed since they may lie under the behavioral threshold. If

$$R_1(w_u^o) \subset T_{\xi_u}(w_u^o) \quad , \quad f(w_u^o) = y_o$$

and f maps $R_1(w_u^o)$ into $T_\eta(y_o)$, we may have the following inaction. Either $T_\xi(y_o) = \{y_o\}$ (rest) or $T_\eta(y_o) \cap \{y_o\} \neq y_o$ (inhibition), or we may have the corresponding situation with underlying conflict if

$$R_1(w_u^o) \not\subset T_{\xi_u}(w_u^o).$$

Recall that the model acquired several means of responding to the same stimuli. Essentially this is possible because of the pool of superfluous functional units which serve as latent reserve. This pool confers great reliability on the model. We saw that this assumption has important implications. We might add, as an additional example, that states which remain occupied for a period of time, even by different units, may act as memory.

VII. As previously remarked we intended in this paper to operate with a few specific assumptions so as to avoid overloading the models with complexity. However, in order to study detailed behavior we need to impose more empirical structure on the models. A possible

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further step would be to turn the digraphs into signed digraphs (or nets) where the weights determine the internal dynamics of the models. In any case the different elements of the behavioral repertoire of these models provide foundation stones with which we can build more sophisticated models.

ACKNOWLEDGEMENTS

APPENDIX A: We use the method of page 4 to construct internal states
(denoted by circles). The input - output relation of functional
units is given by $y = u(x,z,t)$.

In both examples we choose three units with $n = 9$ for a certain
V. $D_{C_{u_\alpha}}$ denotes the digraph associated with the u_α-cluster, where
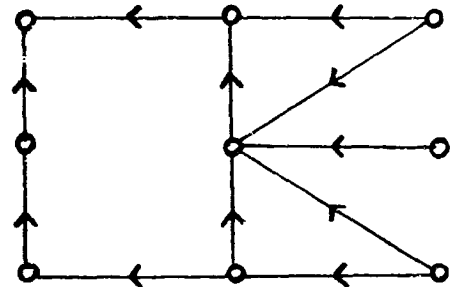first points of a line are internal states of u_α.

Fig. 1

$y_I = u_I(x,z) = \frac{1}{2}xz(\text{sgn } xz + 1)$

$y_{II} = u_{II}(x) = \frac{1}{2}x(\text{sgn } x + 1)$

$y_{III} = u_{III}(z) = \frac{1}{2}z(\text{sgn } z + 1)$

$V = \{x = 0, \pm 1; z = 0, \pm 1, xz > 0\}$

$D_{C_{u_{II}}}$ and $D_{C_{u_{III}}}$ are totally disconnected.

Fig. 2 (see next page)

$y_1 = u_1(x,t) = xt(\text{sgn } xt + 1)$

$y_2 = u_2(x,t) = xt \text{ sgn } xt$

$y_3 = u_3(x,t) = -xt(\text{sgn}(-xt) + 1)$

$V = \{x_o, 0, -x_o\}$

Fig. 2
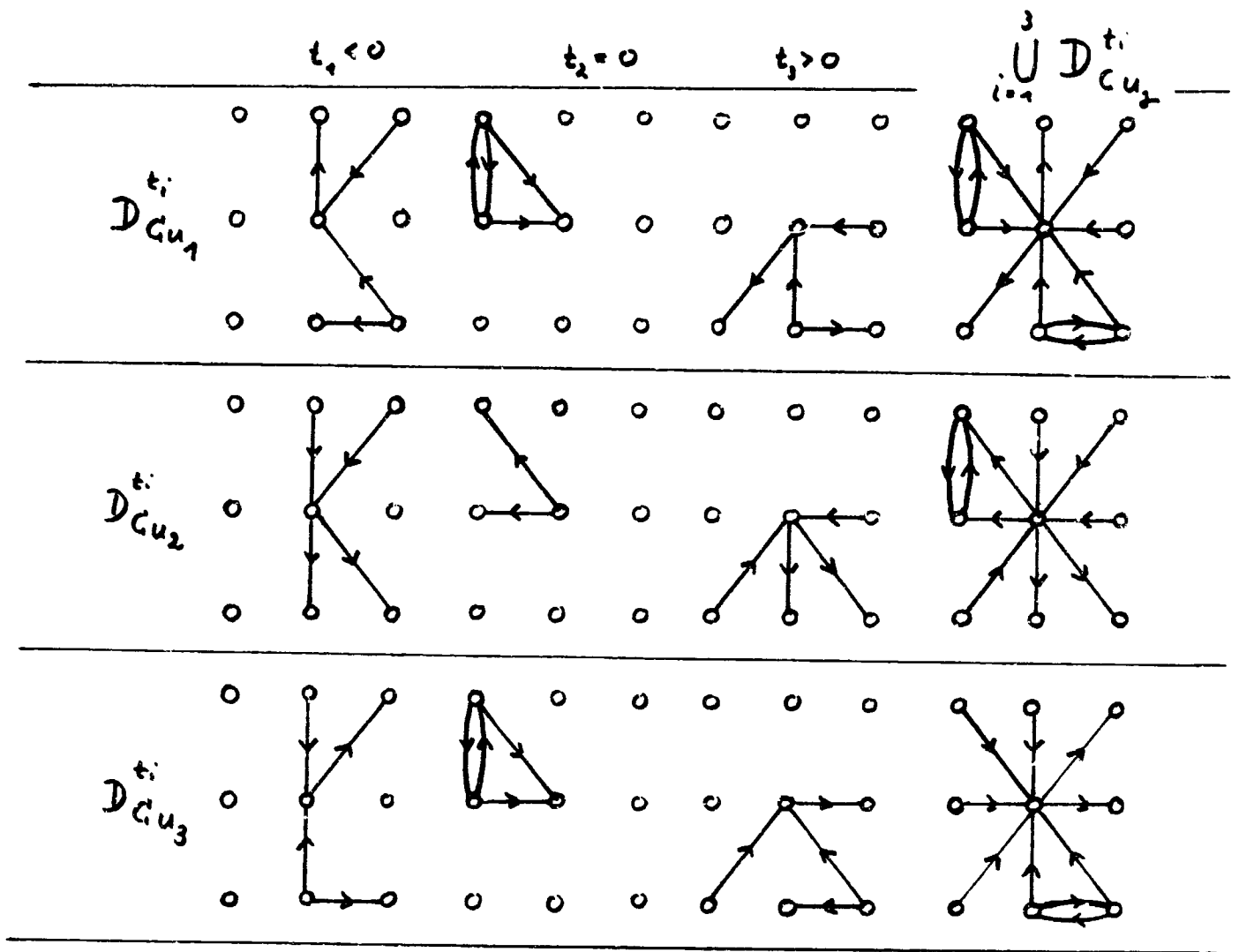
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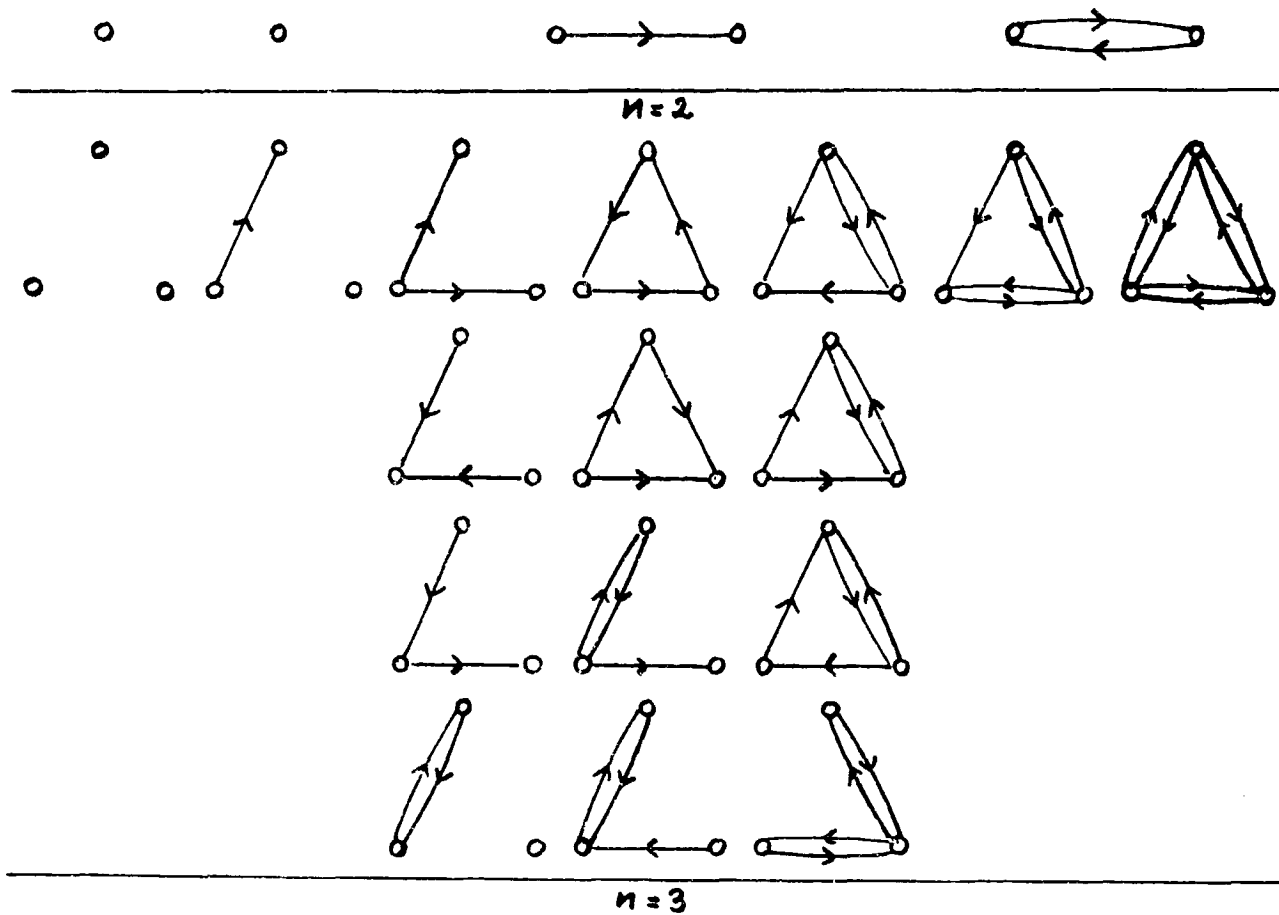Fig. 3: All structurally distinct digraphs for n=2 and 3.

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