NASA
LACRE G91C\#61-30

COMPUTER PROGRAMS NAAUAL

## ANALYTICAL STUDY OF CATALYTIC REACTOKS FOR HYDRAZINE DECOMPOSITION

## ONE-AND TYO-DIMENSIONAL STEADY-STATE PROERAMS

by
E.S. Stinh , D. B. SMITH und A.S. RESTEN
prepred for
MATIGHRL AEROMAUTICS AHD SPACE ADNEASTAATIOH


United Aircraft Research Laboratories


## COMPUTER PROGRAMS MANUAL

# ANALYTICAI STUDY OF CATALYTIC REACTORS FOR HYDRAZINE DECOMPOSITION 

ONE-AND TWO-DIMENSIONAL STEADY-STATE PROGRAMS

by<br>E.J. SMITH, D.B. SMITH and A.S. KESTEN<br>эīepared for<br>NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

AUGUST, 1968

## CONTRACT NAS $7-458$

United Aircraft Research Laboratories

# AHALYMICAL STZDY OR CADALYIIC RXEACTORS FOR HYDRAZIER DECOMPOSITITON 

## Connater Program 3anual

One- and Tro-Dimensionsl Steady-Stete Iodels
by
G. J. Smith, D. B. Smith and A. S. Kesten

## TAGLE O 00 RHWIS

Fage
ABSTRACP ..... i.
FGREMORD ..... ii
 ..... 1
INTMODUCTEON ..... 2
DESRATPITON OR MELSESS ..... 3
Onemimencional Steady-State Model ..... 4
2wo-Mmensional stenuy-state kadel ..... 10
 ..... 13
One-Dimension Steadymstate Model ..... 13
Ingut leer mption ..... 13
Table - Imput Format ..... 18
Fig. 1: Coding of a Bample Data Cuge ..... 21
Fig. 2: Lesting of Input Dats Punch Cards: Sample Case ..... 22
Gatput Dascristion ..... 23
Figs. 30-32: Liating of Output for Smple Data Csse ..... 28
Conmon Operationai Problenze ..... 34

## Page

Twn-Dimensional Steady-Stice Model ..... 35
Input Description ..... 35
Table II: Input Format ..... 1
Figs. 4a-4b: Coding of a Sample Data Case ..... 44
Figs = 5a-5b: Listins cf Input Jata Punch Cards: Sanpie Case ..... 16
Output Descripticn ..... -
Figs. Ea-6i: Iisting cf Output for Scmple iata Case ..... 51
Conmon Operational Froblens ..... $6:$
RIESERAKCES ..... 61
LIST OP SYMPOJS ..... 62
APPERDIX I ..... 65
Description of Subroutines ..... 65
One-Dimensionel Steady-State Model ..... 65
Two-Dianensionai Steady-State Hodel ..... 6
Flow Diagrams ..... ?
Figs. I-I through I-5: One-Dimensional Steady-State frodeI ..... 74
Figs. I-6 through I-8: Two-Dimensional Steady-State woder. ..... 79
ADPENTIX II ..... $8=$
Listing of Conluputer Programs ..... 83
One-Dimensional Steady-State Model ..... 83
Two-Dimensionai Steady-State Model ..... ilo


#### Abstract

         formbien.      


## FOREHORD

This work was performed by United Aircraft Fesearch Laboratortes for the Hetional Aeronautics axid Space Adminietration under Contract NAS $7-458$ initiated April 15, 1966.

Included among those who cocperated in performance of the work imder NAS T-459 were Dr. A. S. Kesten, Erogrsm Manager, Dr. W. G. Burwell, Chiax, Finetics and Thermel Sciences Section, isc. D. B. Smith, Project Analyst, and Mis. E. J. Smith, Applied Whthewatician.

This work was conducted under mograp managenent of the TASA Chief, Liquid Propulaion Experimental Bagiaeerduy Systens, NisA Headquarters, Waslington, D. C.; and the Techaical Manager Wras Mr, M. W. Vrice, Jet Propulsion Laboratory, Pasadena, Californif.

Heport G910461-30

# Analytical Study of Catalytic Reactors 

for Hydrazine Deccaposition

Computer Pragrams Manuai
One- and Two-Dimensional Steady-State Kodels

Contract NAS 7-458

## sTMARM

A description is contained herein of two anchine computitional programs deveicped under Contract IRAS $7-458$ with the National Aeronautics and Space Admindatrestion, These programs represent one- and two-dimensional steady-state modela of catolyzed hydrazine decomposition reaction chambers. Both of these models consider both thermal and catalytic decomposition of reactants, along with simultaneous heat and mass transfer between the free-gas phase and the gas within the pores of the catalyst pellets. The one-dimensional model of the reactor sjatem describes the behavior of reactors having radially uniforn injection profiles and catalyst bed configurations, while the two dimensional model permits consideration of nonuniform radial injection and of catalyst bed configurations exhibiting both radial and axial nonuniformities.

A genersl description of the ans and two-dimensional models and a discussion of the machine programs developed from these models are contained in this manual. A description of input and output for both the one- and twodimensional steady-state programs are included in the discussion together with examples of typical data cases. Also included is a description of severai operational problems wich oight be encountered while using the programs along with appropriate neans for solviry these problems. In addition, a short writeup of the subroutines contained in eseh deck is inciuded along with generai flow charts of the major routines.

## INYTRODUCIION

Under Contract NAS T-458, the Research Laboratories of United Aircraft Corporation are perfoming analytical studies of the behavior of distributedfeed catalytic reactors for hydrazine decomposition. The specific objectives of this program are (a) to develop computer programs for predicting the temperature and concentration distributions in monopropellant hydrazine catalytic reactors in which hydrazine can be injected at arbitrary incations in the reaction chamber and (b) to perform calculations using these computer programs to demonstrate the effects of various system parameters on the performance of the reactor.

Progress previously reported in the first annual report (Ref. 1) included the developant of a computer program which describes the steady-state behavior of a continhous fiow type reactor system in which complete radial mixing in the free-gas (or liquid) phase was assumed. Progress previously reported in the second annual report (Ref. 2) included an extension of the steady-state program to include radial as well as axial variations in temperature and concentrations in order to permit an analysis of various injection schemes and catalyst bed configurations which exhibit radial nonuniformities. These programs had been used to calculate temperature and reactant concentration distributions as functions of feed temperature, chamber pressure, mass flow rate distribution, catalyst size distribution, and embedded injector locations. As part of the third year of contract effort attention has been directed toward preparing a manual describing to potential users the operation of these computer programs. The manual includes a general description of the one- and two-dimeasional models as well as a detailed discussion of the machine programs representing these models.

## DECCRIPPION OF ANALYSEG


#### Abstract

The anz?ysis of a hydrazine engine reaction system carried out to aate pertains to a reaction chamber of arbitrary cross section paried with uataly particles inio which liquid hydrazine is injected at arbitrasily selected locations. Catalyst particles are represented as "equivalent" spheres wj.'ra aia. meter taken as a function of the particle size and shape. Both inermal ann catalytic vapor phase decomposition of hydrazine ard anmonia are considered in developing equations describing the concentration distributions of these reattants. Diffusion of reactants from the free-gas phase to the outside surface of the catalyst pellets is taken into account. Since the catalyst maletiai s.j impregnated on the interior and exterior surfaces of porous particies, the diffusion of reactants into the porous structure must also be considered. In addition, the conduction of heat within the porous particles must ve taken into account since the decomposition reactions are accompanied by the evolution or absorption of heat.


## One-Dimensional Steady-State Mcdel

In developing the one-dimensional ateady-state model, the temperature and reactant concentrations in the interstitial phase (i.e.s the free.mfluid phase as aistinguished from the ges phase within the porous particles) are assumed to vary only with axial distance along the bed. In the entrance region of the reaction chamber, where the temperature is low enough to permit the existence of liquid hydrazine, vaporization of liquid is essumed to ocsur as a result of decomposition af vapor hydrazine within the pores of the catalyst particies. IThat is, catelytic reaction is assumed to be fast enough to keep liquid hydrazine frem wetting the pores of the particles; the hydrazine concentration at the surface of the catalyst particles at any axial location in the entrance region is then computed from the vapor pressure of liquid hydrazine in the interstitial phase at the same axial location, Neglecting axial diffusion of heat or mass, the change in enthalpy of the interstitial phase in the region where liquid hyararine is present (i.e., where $h_{i} \leq h_{j}$ ) is related to the con. centration gradient at the surface of the porous catalyst particles by

$$
\begin{gather*}
G \frac{d h}{d z}+N^{N_{2} H_{3}} D_{P} A_{P}\left(\frac{\left.d C_{p}^{N_{2} H_{4}}\right)}{d h_{8}}\right)+F\left(h_{i}-h_{F}\right)=0  \tag{1}\\
\text { for } h_{i} \leq h_{i}^{V}
\end{gather*}
$$

The variation of mass flow rate, $G$, with axial distance is easily computed from the rate of feed of liquid hydrazine from the distributed injectors into the system. In the region where liquid hydrazine exists at temperatures nelow the vaporization temperature, the temperature may be obtained from

$$
\begin{equation*}
T_{i}=T_{F}+\frac{h_{i}-h_{F}}{C_{F}} \quad \text { for } h_{i}<h_{i}^{L} \tag{2}
\end{equation*}
$$

In the two-phase region, where $T_{i}=T_{\text {vap }}$, the weight-fraction of vapor may be crmputied from

$$
\begin{equation*}
\text { WEIGHT - FRACTION VAPOR }=\frac{h_{i}-h_{i}^{L}}{h_{i}^{V}-h_{i}^{L}} \text { for } h_{i}^{L} \leq h_{i} \leq h_{i}^{v} \tag{3}
\end{equation*}
$$

At the axial position at which the enthalpy of the intexstitial phase is just equal to the enthalpy of vapor hydrazine at the boiling point ( $h_{1}=h_{i}{ }^{V}$ ), the
fraction of hydrazine injected upstream of that point which has been decomposeri is easily calculated from an ovexall heat balance. The associated amounts of ammonia, nitrogen, and hydrazine formed from decomposition of hydrazine can then be calculated taking the decomposition reaction as

$$
2 \mathrm{~N}_{2} \mathrm{H}_{4}-2 \mathrm{NH}_{3}+\mathrm{N}_{2}+\mathrm{H}_{2}
$$

$\therefore$ should be noted that this is the overall reaction scheme determined exper. imentally for both homogeneous decomposition of hydrazine (Refs. 3, 4,5) and low pressure heterogeneous decomposition of hydrazine on platinum surfeces (Ref. 6).*

In the remainder of the reaction $s$ mber, where $h_{i}>h_{i}{ }^{v}$, heat is being supplied to the system by homogeneous as well as heterogeneous decompositior of hydrazine. In addition, at sufficiently high temperature, heat is removed from the system by the endothermic decomposition of anmonia. For $h_{i}>h_{i}{ }^{v}$ then, the change in enthalpy with axial distance is related to the reactant concentrations in the interstitial phese and at the surface of the porous catalyst particles by

$$
\begin{equation*}
\frac{d h_{i}}{d z}=-\frac{1}{G}\left\{F\left(h_{i}-h_{F}\right)+A_{p} h c\left[T_{i}-\left(T_{p}\right)_{5}\right]+H^{H_{2} H_{4}} r_{\text {hom }}^{N_{2} H_{4}} \delta\right\} \tag{4}
\end{equation*}
$$

The changes in reactant weight fractions in the interstitial phase with axial distance are related to the reactant concentrations in the interstitial phase and at the surface of the porous catalyst particles by

$$
\begin{equation*}
\frac{d W_{i}^{N_{2} H_{4}}}{d Z}=\frac{i}{G}\left\{F-r_{h o m}^{\mu_{2} H_{4}} \delta-A_{p}\left(k_{C} C_{i}\right)^{N_{2} \Gamma_{4}}-F\left(\frac{C_{i}}{\rho_{1}}\right)^{N_{2} H_{4}}\right\} \tag{5}
\end{equation*}
$$

[^0]$$
\text { (f) two-step }=3\left(f^{\prime}\right) \text { present }+1
$$
process $\qquad$
\[

$$
\begin{align*}
& \left.-A_{P}\left(k_{C}\left[c_{i}-\left(C_{P}\right)_{S}\right]\right)^{N H_{3}}-F\left(\frac{C_{1}}{\rho_{i}}\right)^{\mathrm{NH}_{3}}\right\}  \tag{ó}\\
& \frac{d_{w}^{H_{2}}}{d Z}=\frac{1}{G}\left\{\frac{1}{2} r_{m}^{H_{m} H_{4}} \delta \frac{M^{H_{2}}}{M^{N_{2} H_{4}}}+\frac{A_{p}}{2}\left(k_{c} C_{1}\right)^{N_{2} H_{4}} \frac{A^{N_{2}}}{M_{2}^{H_{2} H_{2}}}\right. \tag{7}
\end{align*}
$$
\]

wher: the film coefficients, $h_{c}$ and $k_{c}$, way be estimated fromi (heri. 7)

$$
\begin{equation*}
A_{C}=0.74\left(\frac{G}{A_{p} \mu}\right)^{-04}\left(\delta_{F} G\right) \tag{9}
\end{equation*}
$$

and

$$
\begin{equation*}
k_{c}^{J}=\left(\frac{C G 1 G}{\rho_{1}}\right)\left(\frac{\mu}{\rho_{i} D_{i}^{J}}\right)^{-0.667}\left(\frac{G}{A_{p} \mu}\right)^{-041} \tag{10}
\end{equation*}
$$

The changes in reactant concentrations with axid $3=$ tax ${ }^{2}$ are then giver by

$$
\begin{equation*}
\frac{d c_{j}^{\prime}}{d z}=\rho_{i} \frac{d w}{d z} \quad \therefore \frac{1}{d z} \tag{II}
\end{equation*}
$$

6910461-30
where

$$
\frac{d \rho_{i}}{d z}=\rho_{1} \quad\left[\frac{1}{M} \frac{d \bar{M}}{d Z}-\frac{1}{T_{1}} \frac{d T_{1}}{d Z}+\frac{1}{P} \frac{d \rho}{d z}\right]
$$

and

$$
\begin{equation*}
\frac{1}{\bar{M}} \frac{d \bar{M}}{d z}=-\frac{1}{\sum_{j} \frac{w_{i}}{M^{J}}} \sum \frac{1}{M^{j}} \frac{d_{w_{i} J}}{d z} \tag{13}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{d p}{d z}=-\left(\frac{1-\delta}{\delta^{3}}\right)\left(175+\frac{150(1-\delta)}{20 G / \mu}\right)\left(\frac{G^{2}}{20 \rho_{i} g_{C}}\right) \tag{14}
\end{equation*}
$$

The temperature of the interstitial phase in this region is related to the enthalpy by

$$
\begin{equation*}
h_{1}-h_{i}^{v}=\int_{T_{v o p}}^{T_{1}} c_{F} d T_{i} \tag{15}
\end{equation*}
$$

It should be noted that the hydrazine concentration at the surface of a catalyst partizle in the vapor region, $\left(c_{p}\right)_{S}{ }_{2} \mathrm{H}_{4}$, is taken as zero. This refiects the fact that the catalytic reaction is so fast that the rate of decomposition is controlled by the rate of diffusion of hydrazine from the bulk vapor, through a stagnant gas film surrounding the catalyst particles, to the outside surface of the particles. In the case of ammonia, film diffusion is rapid relative tc the rate of dissociation of ammonia within the particles. The conceatration of ammonia at the surface of the catalyst particles, $\left(c_{p}\right)_{s}{ }^{N H} 3$, is therefore fairly close to the ammonia coicentrations in the bulk vapor phase, $\varepsilon_{i}{ }^{N H} 3$. The surface concentration can be cacculated, along with the concentration profile in the porous particles, at any axial Ioction by solving simultaneously the equations representing film and pore diffusion of heat and mass. In descriving the diffusion of mass within a porous pellet, it is assumed that changes in tas mass density of fluid within the particle are negligible relative to changes in concentration of the reacting species. In addition, pressure changes within the particle resulting from nonequimolar diffusion are neglected, as is heat transporced by pore diffusion of mass. Assuming constant diffusion coefficients, $D_{p}$, and thermal conductivities, $K_{p}$, the equations describing heat and mass transfer within a catalyst particle may be written as

$$
\begin{equation*}
D_{p}^{\mathrm{NH}_{3}} \nabla^{2} C_{p}^{\mathrm{NH}_{3}}-r_{\text {het }}^{\mathrm{NH}_{3}}=0 \tag{16}
\end{equation*}
$$

$$
\begin{equation*}
\mathrm{K}_{\mathrm{P}} \nabla^{2} \mathrm{~T}_{\mathrm{P}}-\mathrm{H}^{\mathrm{NH}_{3}{ }_{\mathrm{h}} \mathrm{NH}_{3}}=0 \tag{17}
\end{equation*}
$$

The boundary conditions which consider diffusion of heat and mass through a fin.. surrounding a spherical particle are

$$
\begin{equation*}
D_{p}^{N H_{3}}\left(\frac{d C_{p}}{d n}\right)_{s}^{N H_{3}}=k_{c}^{N H_{3}}\left[C_{i}^{N H_{3}}-\left(C_{p}\right)_{s}^{N H_{3}}\right] \tag{18}
\end{equation*}
$$

and

$$
\left(H H_{c} c_{i}\right)^{\mathrm{N}_{2} \mathrm{H}_{4}}+H^{\mathrm{NH}_{3} \mathrm{O}_{p} \mathrm{HH}_{3}}\left(\frac{\partial \mathrm{C}_{p}}{d x}\right)_{S}^{\mathrm{NH}}=h_{c}\left[T_{i}-\left(T_{P} \mathrm{l}_{5}\right]\right.
$$

Using Eqs. (16) and (17), Prater (Ref: 8) has pointed out that temperature and concentration are related quite simply by

$$
\begin{equation*}
T_{p}-\left(T_{p}\right)_{s}=\frac{H D_{p}}{K_{p}}\left[\left(c_{p}\right)_{s}-c_{p}\right] \tag{20}
\end{equation*}
$$

The use of this relationship enebles the reaction rate: $\mathrm{r}_{\text {het }} \mathrm{NH}_{3}$, tc be written as a function of concertration alone instead of concentration and temperature. In this case, however, the reaction rate is a function of two parameters, $\left(\mathrm{T}_{\mathrm{p}}\right)_{\mathrm{s}}$ and $\left(c_{\mathrm{p}}\right)_{\mathrm{s}}{ }^{\mathrm{Ni}} 3$, which are yet to be determined. Equation 16 can be solved for the concentration at any point in the porous particle in terns of the reaction rate, $\mathrm{r}_{\text {het }}{ }^{\mathrm{NH}} 3$, and the intarstitial concentration, $\mathrm{c}_{1} \mathrm{NH}_{3}$. The solution is derived in Refs. 2 and 9 as an impiscit integral equation given by

$$
\begin{align*}
c_{p}(x)^{N H_{3}}= & c_{i}^{N H_{3}}-\left[\frac{1}{x}-\frac{a k_{c}^{N H_{3}}-D_{p}^{N H_{3}}}{a^{2} k_{c}^{\mathrm{NH}}}\right] \int_{0}^{x} \xi^{2} \frac{r_{\text {het }}^{\mathrm{NH}_{3}}\left(C_{p}\right)}{D_{p}^{N H_{3}}} d \xi  \tag{21}\\
& -\int_{x}^{a}\left[\frac{1}{\xi}-\frac{a k_{c}^{N H_{3}}-D_{p}^{N H_{3}}}{0^{2} k_{c}^{N H_{3}}}\right] \xi^{2} \frac{\left.r_{\text {hel }}^{N H_{p}} C_{p}\right)}{D_{p}^{N H_{3}}} d \xi
\end{align*}
$$

6.20461-30

In order to determane the particle amonia concentration profile directiy ins terms of the interstitial temperiture and reactant concentrainans it is neces. sary to solve Eqs. 18, 19 ana 21 sim.itaneousiy.

In the special case of negligiblf film resistance to heat and mess trans. fer (i.e. $\left(T_{p}\right)_{s}=T_{i}$ and $\left.\left(c_{p}\right)_{s}=c_{i}\right)$, Eq. (2l) can be written, for any neseting species, as

$$
\begin{align*}
c_{p}(x)= & c_{i}-\left[\frac{1}{x}-\frac{1}{a}\right] \int_{0}^{x} \xi^{2} \frac{r_{h p t}\left(c_{p}\right)}{D_{p}} o \hat{\}} \\
& -\int_{x}^{0}\left[\frac{1}{\xi}-\frac{1}{a}\right] \xi^{2} \frac{r_{h e t}\left(C_{p}\right)}{D_{p}} d \xi \tag{2?}
\end{align*}
$$

It is $\mathrm{Eq} .(\underset{)}{(?)}$ which is used tc describ: tie hydrazine concentration proiiles within the catalyst pariicies located in the liquid region of the reaction chamber. In this liquid region it is assumed that liquid hydrazine wets the outside surface of he catalyst particles so that $\left(c_{p}\right)_{s}{ }^{\mathrm{N}_{2} \mathrm{H}_{4}}=\mathrm{c}_{i} \mathrm{~N}_{2} \mathrm{H}_{4}$, where $\mathrm{c}_{\mathrm{i}} \mathrm{N}_{2} \mathrm{H}_{4}$ is the vapor concentration in equilibrium with liquid hydrazine ai temperature $\mathbb{T}_{i}$. In the liquid-vapor region the situation is somewhat more complicated since it is difficu?t to predict whether liquid or a combination of liquid and vapor wets the outsile surface of the catalyst particles. Boin of these options are presently ta the computer program representing the steadystate model. In the case in which both the liquid and vapor are taken to weit the particle surface, it is assumed that, at a given axial location, the fraction of ine surface covered by vapor is equal to the weight-fraction of vapor present. Decomposition rates, computed assuming pure liquia surface coverage anu then pure vapor coverage, are weighted accordingly. Fortuitously, for the system conside ed here, the liquid-rapor region is so namow that the choice of either of these options has negligible effect on the resulting temperature distributions (Ref. 1).

Finite difference methods have be zn used to program Por digital computation the ordinary difierential equations describing the changes in enthalpy and reactant concentrations in the interctitial phase. No iteration is necessury to solve these equations numertcally when the incremental exial distances are sufficiently small. The size of a succeeding increment is calculated at each axis.i position as a function of the rates of change of temperature and fractionel ammonia dissociation with axial distance. However, Eqs. (2l) and (22), wh.ch must be solved simultaneowsly with the difierential equations, are implicat intesral equations which require iterative procedures ior solution. Hand calculations have indicated that convergence to solutions for $c_{p}(x)$ are difficult to achieve unless the initial estimates of the concentration distributions are fairly accurate. Methods have been developed for generating these estimates and ite. tive procedures have been devised which exfect rapid convei-
gence over a fsix?y wide range ar condizions, These procedures are present?y used as aucroutines in the main progian representing the steady-state modei.

Sm-Dinensional Stesiy-State Model
I. aeveloping the two-dimensional sieady-state moiel of a hydrazine reactor systers the temperature and reactant concentrations in the bulk fluic phase ere permitted to vary with radial and axial position in the reaction chanber. In the entrance region of the reactor, where ths teaperature is low enough to permit the existence of liquid hydrezine, radial mixing between sijacent layers of lizquid is neglected. the equations representing the change in liquid enthslpy and temperature sith axial distance at any radial position are the sane as thase developed for the one-dimensional model descrited previously. As in the one-disprisional nodel, cacaiytic reaction is assumed to be fast enough to keep liquid hydazine from wetting the pores of the particles; the hydrazine concentrution at the surface of the catalyst particles at any location in the entrance region is then coupated fros the vapor gressure of liquid hydrazine in the interstitial phese at the seme locstion.

In the vapor regions of the reaction chamber, turbulent diffusion of heat and mass is considered as a mecha. am for radial mixing. Radial heat and mass fluxen are computed as functione of temperature and reactant concentration gradjents. Heat is being suppiled to the system by homogeneous as weil as heterogeneous decomposition of hydrazine, and is being removed from the system by the catalytic decomposition of enmonis. The change in enthalpy with exial distance at any redial location is related to the reactant concentrations in the interstitial phase and at the surface of the porous catalyst particles by

$$
\begin{align*}
\frac{\partial h_{i}}{\partial z}= & -\frac{1}{G}\left\{F\left(h_{i}-h_{F}\right)+A_{p} h_{C}\left[T_{1}-\left(T_{P}\right)_{S}\right]+H^{N_{2} H_{a}} r_{r_{0 m}}^{H_{2} H_{4}} \delta\right. \\
& \left.+\frac{\partial q_{r}}{\partial r} \delta+\frac{q_{r}}{r} \delta+\frac{\partial T_{1}}{\partial r} \delta \sum_{3} N_{r}^{J} C_{F}^{J}\right\} \tag{23}
\end{align*}
$$

The changes in reactant weight fractions in the interstitial phase with axial distance at any radial location are related to the reactant concentrations in the interstitial phase and at the surface of the pcrous catalyrt particles by

[^1]\[

$$
\begin{align*}
& \frac{\partial W_{i} \mathrm{~N}_{2} \mathrm{H}_{4}}{\partial z}=\frac{1}{G}\left\{F-\mathrm{r}_{\text {hom }}^{\mathrm{N}_{2} \mathrm{H}_{4}} \delta-A_{p}\left(K_{C} C_{i}\right)^{\mathrm{N}_{2} \mathrm{H}_{4}}\right. \\
& \left.-\frac{\partial N_{r}^{\mathrm{N}_{2} \mathrm{H}_{4}}}{\partial r} \delta-\frac{N_{r}^{N_{2} \mathrm{H}_{4}}}{r} \delta-F\left(\frac{C_{i}}{\rho_{i}}\right)^{\mathrm{N}_{2} \mathrm{H}_{4}}\right\} \tag{24}
\end{align*}
$$
\]

$$
\begin{align*}
& -A_{F}\left(K_{C}\left[C_{i}-\left(C_{P} H_{5}\right]\right)^{\mathrm{NH}_{3}}-\frac{\partial N_{r} \mathrm{NH}_{3}}{\partial r} \delta-\frac{N_{r}{ }^{N H_{3}}}{r} \delta-F\left(\frac{C_{i}}{\rho_{i}}\right)^{\mathrm{NH}_{3}}\right\} \tag{25}
\end{align*}
$$

$$
\begin{align*}
& \left.+\frac{A_{p}}{2}\left(x_{C}\left[C_{i}-\left(C_{P}\right)_{S}\right]\right)^{N H_{3}} \frac{H^{N_{2}}}{M^{N H_{3}}}-\frac{\partial N_{i}^{H_{2}}}{\partial r} \delta-\frac{N_{i}^{N_{2}}}{r} \delta-F\left(\frac{C_{i}}{p_{i}}\right)^{N_{2}}\right\} \tag{26}
\end{align*}
$$

$$
\begin{align*}
& +\frac{3 A_{p}}{2}\left(k_{C}\left[C_{i}-\left(C_{P)_{S}}\right]\right)^{N H_{3}} \frac{M^{H_{2}}}{N^{N H_{3}}}-\frac{\partial N_{r}^{H_{2}}}{\partial r} 8-\frac{N_{r}^{H_{2}}}{r} 8-=\left(\frac{C_{1}}{\rho_{i}}\right)^{H_{2}}\right\} \tag{27}
\end{align*}
$$

where

$$
\begin{equation*}
q_{r}=-\lambda\left(\partial r_{i} / \partial r\right) \tag{28}
\end{equation*}
$$

$$
\begin{equation*}
N_{r}^{J}=-\left(\partial c_{i}^{J} / \partial r\right)_{r} \tag{29}
\end{equation*}
$$

$$
\begin{align*}
& h_{C}=0.74\left[\frac{G}{A_{P} \mu}\right]^{-0.41}\left[C_{F} G\right]  \tag{30}\\
& k_{C}^{J}=\left[\frac{0.61 G}{\rho_{i}}\right]\left[\frac{\mu}{A O_{i}^{J}}\right]^{-0.657}\left[\frac{G}{A_{p} \mu}\right]^{-0.41} \tag{31}
\end{align*}
$$

The eddy conductivity and diffucirity may be estinatod fran (Kef. 11)

$$
\begin{equation*}
\lambda=\frac{a \bar{C}_{F} G}{5 \delta} \quad \text { and } \quad \leqslant=\frac{a G}{\$ p_{i}} \tag{32}
\end{equation*}
$$

The changes in reactant concentrations with axial Aistance are then giver by

$$
\begin{equation*}
\frac{\partial c_{i}^{J}}{\partial z}=\rho_{i} \frac{\partial w_{i}^{J}}{\partial z}+\frac{c_{i}^{d}}{\rho_{i}} \frac{\partial \rho_{i}}{v_{z}} \tag{33}
\end{equation*}
$$

where

$$
\begin{align*}
& \frac{\partial \rho_{i}}{\partial z}=\rho_{i}\left[\frac{1}{\bar{M}} \frac{\partial \bar{B}}{\partial z}-\frac{1}{T_{i}} \frac{\partial T_{i}}{\partial z}+\frac{1}{P} \frac{d P}{d z}\right]  \tag{34}\\
& \frac{1}{M} \frac{\partial \bar{M}}{\partial z}=-\frac{1}{\sum_{J}\left(w_{i} J / M^{J}\right)} \sum_{J} \frac{1}{M^{J}} \frac{\partial w_{i}^{J}}{\partial z} \tag{35}
\end{align*}
$$

and the pressure drop may be estimated from the Ergun equation (R=P. 7) as

$$
\begin{equation*}
\frac{d p}{d z}=-\left(\frac{1-\delta}{\delta^{3}}\right)\left(1.75+\frac{150(1-\delta)}{20 G / \mu}\right)\left(\frac{G^{2}}{2 a \rho_{i} g_{c}}\right) \tag{36}
\end{equation*}
$$

2he mass flow rate, $G$, is computed as a function of the rate of feed of Ilquia hydrazine from the distributed injectors into the system. Bulk radiai flow, caused by particle-fluid viscous interaction, is neglected. It is assumed, therefore, that downstream of the injectors the mass flow rate profile remains unchanged.

## DISCUSSTON OF ONE: RAD TWO-DIIERNSTONAL SITRADY-STATE COMPUIER PROGRAMS

The equations representing the onem and two-dimensional steady-state models have been programmed for the UNIVAC 1108 digitai computer. These comfuter programs are discussed below. Included in this discussion are input and output descriptions and descriptions of common operstional problems associated with the programs.

One-Dimensional steady-State Model

## Input Description

The following is a listing of the necessary input for the one-dimensional steady-state computer program. The input format is given in Table I. The coding of a sample data case is shown in Fig. I and a listing of the input data punch cards corresponaing to this sample data case is shown in Fig. 2. The card numbers in the text below correspond to the card numbers (first column) of table I. For each run there will be only one card number one. Cards 2 through 16 should be repeated for each data case to be run.

1. The first card contains the number NCASE. This number indicates the number of data cases with each run. l $\leq$ NCASE $\leq 999$.
2. The second cerd is the title card used for individual data case identification. The title may be any alphe numeric information desired.
3. The third card contains the indicators $\emptyset \mathrm{PPC} \phi_{\mathrm{N}}$ and PRTNT and the number N $\varnothing \mathrm{FZ}$. ${ }^{\mathrm{P} M T \phi \mathrm{~N}}$ is used to indicate which method of analyzing the liquid-vapor region is desired. If $\oint \mathrm{PTI} \phi \mathrm{N}=2$, the program will use the methoi in subroutine LQV2. If $\varnothing \mathrm{PII} \phi \mathrm{N} \neq 2$, the program will use the method in subroutine LQVP. These two methods are described in Appendix $Y$. FRINT is used to indicate which type of printout is desired. If PRINM $=0$ or is blank, the "standard output" described in the section (. output is printed. If PRINI $=1$, both $e$ "etandard" and "nonstandard output" are printed. "Nonstandara outpu'" is also described in the section on output. IV $\phi \mathrm{F} Z$ is the number of axial stations ( Z 's) to be used in the three tables input on cards 8 through 16.
4. The fourth card contains the eight constants $\mathrm{ZO}, \mathrm{GO}, \mathrm{FC}, \mathrm{ALPHA}, \mathrm{HF}$, $\mathrm{E}_{2}$ WM4, and WM3.
zo is the axial distance to the end of a buried injector in ft. (Ref. 1).

GO is the inlet mass flow rate in $1 \mathrm{~b} / \mathrm{It}^{2}-\mathrm{sec}$. It must be greater than zero.

FC is the rate of feed of hydrazine from buried injectoris (hef. 1) into the system in $1 \mathrm{~b} / \mathrm{ft}^{3}-\mathrm{sec}$.

ALPFAA3 is the preexponential factor in the rate equation for the thermal dacomposition of hydrazine (See Ref. 1). It equals $2 . \therefore 4 \times 10^{10} \mathrm{sec}^{-1}$.

HF is the enthalpy of liquid hydrazine ertering the bed in Btu/lb.
R is a gas consiant. It equals 10.73 (psia-ft $\left.\mathrm{t}^{3}\right) /(\mathrm{lb}$ mole-deg $R$ ).
WM4 is the molecular weight of hydrazine. It equals $32.048 \mathrm{1b} /$ If mole.

WM? is the molecular weight of amonia. It equals $17.032 \mathrm{1b} / \mathrm{Ib}$ mole.
5. The fifth card contairs the eight constants WMR, WMI, ALPYAAI, ATPHAP, AGM, BGM, KP, and CCM.

WM2 is the molecular weight of nitrogen. It equals 28.016 Ib/Ib mole.

WMI is the molecular weight of hydrogen. It equals $2.016 \mathrm{lb} / \mathrm{lb}$ mole.
$A L P H A D$ is the preexponential factor in the rate equation for the catalytic decomposition of hydrazine (See Ref. 1). For the Shell 405 catalyst it equals $10^{10} \mathrm{sec}^{-1}$.

ALPHAR is the preexponential factor in the rate equatior for the cataiytic decomposition of anmonia (See Ref, l). For the Shell 405 catalyst it equals $10^{11}\left(1 \mathrm{~b} / \mathrm{ft}^{3}\right)^{1.6}(\mathrm{sec})^{-1}$.

AGM is the activation energy for the catalytic decomposition of hydrazine, divided by the gas constunt. For the Shell 405 catalyst it equals 2500 deg R.

BGM is the activation energy for the catalytic decomposition of amonia, divided by the gas constant. For the Shell 405 catalyst it equals $50,000 \mathrm{deg} \mathrm{R}$.

KP is the themal conductivity of the porous catalygt particle. Far the Snell 405 catalyes it equals $0.4 \times 10^{-4}$ Btu/ftrsec-deg R.

CGM is the activetion energy for the thermal decomposition of hydiszine, dirided by the gas constant. It equals 33,000 $\operatorname{deg} \mathrm{R}$.
6. The sixth card contains the seven constants TT, WFL, ENMXI, ENMEN, ENMX3, DIF3, DIF4, and the inlet value of PRES.

TF is the temperature of isquid hydrazine entering the bed in aer R.

CFL is the specific heat of liquid hydrazine. It equals 0.7332 Btu/1b-zeg $R$.

ENMXI is the constant used to determine the size of axial station increments in the liquid region. It equals 200. Increas. ing this number would result in a deccease in size of axial station increments (and an increase in computer zun time).

ENMX2 is the constant used to determine the size of axial station increments in the liquid-vapor region. It equals 40. Increasing this number would result in a decrease in size of axial. station increments (and an increase in computer run time).

ENMXX is the constant used to determine the size of axial station increments in the vapor region. It equals 80. Increasing this number would result in a decrease in size of axial staticn increments (and an increase in computer run time).

DIF3 is the diffusion coefficient of ammonia in the gas phase at STP. It equals $0.17 \times 10^{-3} \mathrm{ft}^{2} / \mathrm{sec}$.

DIF4 is the diffusion coefficient of hydrazine in the gas phase at STP. It equals $0.95 \times 10^{-4} \mathrm{ft}^{2} / \mathrm{sec}$.

PRES is the inlet chamber pressure in peia.
7. The seventh card contains the four constants ZEND, END, FnD, and EN3.

TEND is the catalytic bed length in ft.
is the order of hydrazine catalytic decomposition reaction with respect to hydrazine. For the Shell 405 catalyst is equals 1.0.
is the order of ammonia catalytic decomposition reacticr with respect to ammonia. For the Shell 405 catalyst it equals 1.0 .
is the order of ammonia catalytic decomposition reaction with respect to hydrogen. For the Shell 405 catalyst it equals -1.6.
8. Cards 8 through 10 contain $\operatorname{ETBLA}(I)$, the interpolation table used to obtain the catalyst particle radius at any point along the reactor bed. Subroutine UNBAR, an interpolation routine developed at the ünited Aircraft Research Laboratories, is used to obtain an appropriate particle radius, A, for a given axial station, $Z(I)$, along the bed. For this table there should be a total of (NめI'Z) z 's and (NDFZ) ${ }_{h}$ 's. The table is set up as follows.

CARD NO.
8 Tis card contain the four table descriptcrs used by UNBAR. The first descriptor signifies the table number. For this program it equals 0.0 . The second descriptor tells at what location in the array the table starts; the tables in this program axe read in such that this number equals 1.0. The thind deseriptor is the number of independent variables in the table (in this case, the number of $Z$ 's). This number squals NWEZ. The fourth descriptor for a univariate table such as this one should equal 0.0 .

9 These cards contain the monatonically increasing 2 values. Enough cards should be used to contain N NFFZ values of $Z$ at the rate of ten per card. For example, if $\mathrm{N} \phi \mathrm{F} Z=12,12$ velues or z should be input using two cards with ten values on the first card and the 2 remaining values $n n$ the second card.

10 These cards contain the A's which correspond to the Z's listed on cards 9. Enough cards should be used to contain $\mathrm{N} \phi \mathrm{FZ}$ values of A at the rate of ten per card.
9. Cards 11 through 13 contain $\operatorname{ZTBLAP}(\mathrm{I})$, the interpolation table used to obtain the total extemal catalyst particle surface area per unit volume of bed (AP). These AP yalues are obtained from UNBAR as func-
ticas of exial distance ( $Z$ ) as in the ZIT A sable discussed above. For this table there should be a total of (NoFZ) z's and (NXFZ) AP's. The table is set up as followe:

CARD NO.
11 This card is exactly the same as card 8.

1'd These cards are exactly the same as cards 9 ,
13 These cards contain the AP values which correspond to the Z's listed on cards 12. shough cards should be used to contain 30 F values of AP at the rate of ten per card.
10. Cards 14 through 16 contain $\operatorname{ZTBLD}(I)$, the intexpolaticn table used to obtain the interparticle void fraction (DELA). These DELA values are obtained from UNBAR as functions of axial distance ( $Z$ ) as in the ZIMBEA table discussed above. For this table there should be a total of (NめFZ) Z's and (NDFZ) DETA's. The table is set up as follows:

CARD HO.

14 This card is exactly the game as card 8.
15 These cards are exactly the same as sards 9 .
16 These cards contain the DELA values which correspond to the Z's listed on cards 15. Enough cards should be used to coritain N $\varnothing \mathrm{HZ}$ values of DELA at the rate of ten per card.
6910461-30
Ons-Dimensional Steadymstate Computer Program Input Format,

| CARD NUMBEF | PUMEER OF CAROS | $\begin{aligned} & \text { FORTPAN } \mathbf{Z ~} \\ & \text { FORMAT } \end{aligned}$ | $\begin{aligned} & \text { COLWMans } \\ & \text { USED } \end{aligned}$ | SYMEOL , H2 OESCRIPTION | COARESPONDING SYMBOL USED IN ECNMTIONS | NOAENCLATURE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | I3** | 1-3 | NCASE | - | No. Of casen |
| 2 | 1 | 14 A 6 | 1-80 | T2tre | $\cdots$ | - |
| 3 | 1 | 212, I3\% | 1-2 | фFII $\phi$ N PRIMT NOFZ | " | Tiq.-Vap. Indicator <br> Prirt Indicator <br> No. of 71 s in Trput Iracies |
| 4 | 1 | 8 E 10.5 | $\begin{aligned} & 1 .-1.0 \\ & 11-20 \\ & 21-30 \\ & 31-40 \\ & 1.1-50 \\ & 51-60 \\ & 6.2-70 \\ & 71-80 \end{aligned}$ | 20 <br> 60 <br> TC <br> ALPHA3 <br> ET <br> R <br> WM 4 <br> WM3 | $\%$ <br> Qo <br> F <br> $\alpha_{\text {hom }}$ <br> hr <br> R <br> $\mathrm{M}_{2} \mathrm{H}_{4}$ <br> $\mathrm{M}^{\mathrm{NH}} 3$ | Axial aistance to injector eal <br> Inlot mass flow rato: <br> Distributed t'ecd inte <br> Freexponential factor <br> Enthat py of dued <br> Gess constran': <br> Molecular wh. of $\mathrm{N}_{\mathrm{i}} \mathrm{H} / \mathrm{H}_{1}$ <br> Molecuitar vit. of Nilld |
| 5 | 1 | 2E10.5 | $\begin{aligned} & 1-10 \\ & 1.1-20 \\ & 21 .-30 \\ & 3.2040 \\ & 41-50 \\ & 5.1-60 \\ & 61-70 \\ & 71-30 \end{aligned}$ | WM2 <br> AMI. <br> ALI'HAL <br> ALITAR <br> AGM <br> EGM <br> KP <br> CGM | $M^{N+2}$ <br> $\mathrm{MH}_{2}$ <br> $\mathrm{a}^{\mathrm{N}} \mathrm{N}^{3} \mathrm{H}_{4}$ <br> $\alpha^{\mathrm{NH}_{3}}{ }^{\mathrm{N}_{2} \mathrm{H}_{4} / \mathrm{R}}$ <br> phet <br> Quentin! $3 / R$ <br>  | Molccinlar wt. of $\mathrm{N}_{\mathrm{f}}$, Molecular wt. of $\mathrm{H}_{2}$ Preexponential factor Preexponential finctor Acisivation energy, deg $R$ Activation energy, herr $R$ Thermol zonuatitivi".; <br>  |
| 6 | 1 | 8 El 10.5 | $\begin{aligned} & 1-10 \\ & 11-20 \\ & 21-30 \\ & 31-40 \\ & 41-50 \\ & 51-60 \\ & 61-70 \\ & 71-80 \end{aligned}$ | TT <br> CFL <br> IMMXI. <br> EnMX2 <br> IRNMX <br> DTF'3 <br> DIF't <br> PRES | $\begin{aligned} & \mathrm{DF} \\ & \mathrm{C}_{4} \\ & - \\ & - \\ & m \\ & D_{0} \mathrm{NH}_{2} 3 \\ & D_{0} \mathrm{NO}_{2} \mathrm{H}_{4} \\ & \mathrm{D} \end{aligned}$ | Feed tempnieture <br>  <br> Determines ater ais. is. 11 ?uid rerion <br> Determdnes step shad in dq-vapor ren <br> Determinces step bise in mior rewion <br> Difrusion cost. of int. <br> Distiusion cool' of NeH2 <br> Inlet chanber presure |

*A11 I format numbers shnuld be right adjusted

| CARD NUABER | NUMBER OF CARES | FORTRAN I FCRMAT | $\begin{aligned} & \text { COLUMNS } \\ & \text { USED } \end{aligned}$ | SYMBO: OR | CORRESPONUING SYMBOL USED in equations | NOMENCLA URTE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ? | 1 | $8 \mathrm{FL10.5}$ | $\begin{aligned} & 1-10 \\ & 11-20 \\ & 21-30 \\ & 31-40 \end{aligned}$ | $\begin{aligned} & \text { ZEND } \\ & \text { ENL } \\ & \text { EN2 } \\ & \text { ENO } \end{aligned}$ | $\begin{aligned} & \bar{n} \\ & n \\ & n \\ & n \end{aligned}$ | Bed length <br> order of decomposition reaction Order of decomposition reaction order of decomposition resetion |
| 8 | $1$ | $4 \mathrm{~F}, 8.5$ | $\begin{gathered} 1-8 \\ 9-16 \\ 17-24 \\ 25-32 \end{gathered}$ | 0. <br> 1. <br> NXIFZ. <br> 0. |  | Table deseriptor <br> table deacriftor <br> Table descriftor <br> Table descriptor |
| 9* |  | $2088.5$ | $\begin{array}{r} 1-18 \\ 9-16 \\ 17-24 \\ \left.\right\|_{73-80} \end{array}$ | $Z(I)$ | 2 | Axisl station |
| $10 * *$ | ** | $\text { 20m8. } 5$ | $\begin{gathered} 1-8 \\ 9-16 \\ 17-24 \end{gathered}$ | $A(\tau)$ | © | Catalyst Particle Radius |
| 21 | $1$ | $4 E 8.5$ | $\begin{gathered} 1-8 \\ 9-16 \\ 17-24 \\ 25-32 \end{gathered}$ | 0. 1. N $\varnothing$ FZ. 0. |  | Table deseriptor Table descriptor Table deswriptor Table descriptor |


TABLE I
(Continued)

| CARD mumber | $\begin{aligned} & \text { NUMBER } \\ & \text { OF CARDS } \end{aligned}$ | FORTRAN I FORMAT | COUMGNS USED | SYMBOL OR DESERIPTION | CORMESPONOING SYMEOL USED in EqUATIONS | NOMENCL.ATURE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 12* | * | 10 E 8.5 | 1.8 $9-16$ $17-24$ $73-80$ | $Z(I)$ | $z$ | Axial station |
| 13** | ** | 1058. 5 | $\begin{gathered} 1-8 \\ 9-16 \\ 17-24 \\ \vdots \\ 73-80 \end{gathered}$ | $\mathrm{AP}(\mathrm{I})$ | ${ }^{\prime \prime} A_{p}$ | Total external :atal.yst parti le surface area per unit vol. of beri |
| 14 | 1 | 4 ES .5 | $\begin{gathered} 3-8 \\ 9-16 \\ 17-24 \\ 25-32 \end{gathered}$ | $\begin{aligned} & 0 . \\ & 1 . \\ & \mathrm{N} \not \mathrm{FZ} . \\ & 0 . \end{aligned}$ |  | Table deszriptor <br> Table descriplor' <br> Tab?e deseriptor <br> Thble descriftor |
| 15* | * | 10E8.5 |  | $\because(I)$ | $z$ | Axial .'tatlor |
| 16** | ** | 1058.5 | $\begin{gathered} 1-8 \\ 9-16 \\ 1 \\ 73-80 \end{gathered}$ | DELA (I) | $\delta$ | Interpartiale vola fra tion |



営

Cara No.



G910451-30

## OLtput Description

Düsput from the one-dimensional st ady-state program is entirely in printout form. Standerd output, which is printed out when input option FRINT = 0 , inciudes all printing nozmally done during execution of any representative data case, three messages which pertain to calculations which do not fallow the normal pattern in a typical run, and one error message which is followed by program termination. Non-standard output is printed in addition to the standard output when PRDTP $=1$. This non-standard output includes additional calculated values and comments which pertain to intermediate calcalainons. The print statements associated with each routine in which output is generated are described below.

Standiand Output
MAIN program

1. A complete listing of all program input including Fofiran variable titles for all input variables.
2. Axial position, ( Z ), temperature, (TrMP), enthalpy, (H), and rate of change of enthalpy with axial distance, ( $D H D Z$ ), for each axial position in the liquid region.

Subroutine IQVP or LQV2
$=\mathrm{Fm}=\mathrm{m}=\mathrm{m}=\mathrm{m}=\boldsymbol{=}=$

1. Axial position, (2), temperature, (TENP), entha?py, (H), and weight fraction of yapor, (WFV), for eack axial position in the liquid-vapor region.

Dubroutine VAPめR

1. Axial position: (Z), temperature, (TEMP), pressure (PRES), enthalpy, (H), and coacentrations of hydrogen, (Cl), nitrogen, (C2), ammenia, (C3), and hydrazine, (C4), at each axial position in the vapor region.
2. Mole fractions of hydrogen, (MFRACl), nitrogen, (MFRAC2), ammonia, (MFRAC3), and hydrazine, (MFRAC4), and the fractional aissaciation of dmonia; (FRAC3D), at each exial position in the vapor region.
3. All axial positions, ( $Z$ values), in the vapor region listed consecutively and MBAR and $r$ values at the end of the reactor for use in preparing input, to the transient model computer program.

4．＂KOUNT $=\mathrm{XX}$－－－THIS INTERVAL HAS BEEN REDIVIDED XXXX TTMES＂
For all cases invjlving a non－zero embedded injector feed rate，a check is made on the $Z$ step size after each calculation．If the increment proves too large to yield satisfactory results，it is halved and re－ checked．The procedure continues until a satisfactory interval size is found，and the above message is then printed．

5．＂THERE IS A PUDDLE $\phi$ C COLD HIDRAZINE AT THE LIQUID－VAPOR／VAPOR INIEREACE．－－－TRY USING A Larger value F $\phi$ R co＂

When using a buried injector scheme it is possible to＂flood＂the region surrounding the injector tip with cold，liquid hydrazine．A sudden drop in axial．temperatures at the li．quid－vapor／vapor interface indicates that this nas occurred，and in such cases the above message is printed and no further calculations are made．

## Subroctine SGRAD

1．＂WE HAVE CALCULATED A NEGATIVE KO DURING ITERATIめN Nめ．XX．SET $\mathrm{XO}=0$ ，CALCULATE TPS $=. \mathrm{XXXXX} \pm \mathrm{XX}$ ，AND CODNTINUE ${ }^{1 \prime}$

XO represents an approximation of the radial distance to which hydra－ zine penetrates the catalyst particle before being dissipated．It is determined through an iterative procedure，and in some instances initial guesses do not yield astisfactory results．In this case，corrective measures to yield a better approximation to $X 0$ are instituted and the procedure repeated．This message indicates only that corrective cal－ culations to improve on the accuracy of $X 0$ are being initiated．

2．＂UNABLE TX CONVERGE $\emptyset$ N CPS IN 50 TRIES $-\cdots C P(X / A)=. X X X X X \pm X X "$
If subroutine SGRAD cannot calculate a＂converged＂value for CPS after 50 iterations，the final value for de at the particle surface is used to approximate CPS．This is a gooa approximation to CPS，however，and program calculations continue with the above message being printed．

3．＂UNABLE T $\varnothing$ FIND SUITABLE XO AFTIER F $\phi$ UR TRIES $\phi \mathrm{F} 25$ ITERATIめNS EACH $-\cdots$ PR\＆GRAM STめP F $\varnothing L L \phi W S "$

If aiter four corrective attempts to approximate $X 0$ the procedure still does not yield satisfactory results，this message along with all unaccept－ able values for $X O$ is printed and further calculations are stopped．An octal dump of core accompanies the program stop．

## Non-Standard_Output

## Subroutine SLDPE

1. "INITTAL CH $\varnothing$ ICE THEøUGH $\emptyset$ RIGIN IS T $\varnothing \varnothing \varnothing$ Large"

When iterating to find a satisfactory approximation to the radial depth of penetration of hydrazine in a catalyst particle (Xo calculation), an initial guess is the particle radius itself. If this proves to be an unsatisfactory choice, the above message is printed and a different initial guess is used.
2. "SATISFACThRY STARTING CURVE FhUND AFTER XX TRAILS. THE VALUE $\phi F$ B (XO) IS . $\mathrm{XXXXXX} \pm X X^{\prime \prime}$

This message indicates that a satisfactory approximation to the radial depth of penetration of hydrazine in a catalyst particle has been found, and appears frequently in calculations involving the liquid region of the reactor.
3. "INITIAL CH $\phi$ ICE THRU $\emptyset R I G I N$ SEEMINGLY $\emptyset K$, BUT RESULIS R $\varnothing$ TIEN AFIER 99 ITTRATTONS --- SEP X0 $=.000001 *$ A AND USE MdRE REFINED TECHNIQUE"

When calculating a concentration ve radial position profile within the catalyst particle, an initial guess at the profile is used assuming a linear profile from the center of the particle to the surface. It can happen that this appears to be a satisfactory first guess, but ultimately yields unsatisfactory resilts for the final "converged" values of CPA. In such instances the above message is printed and the iteration procedure is repeated using a new initial guess.
4. ${ }^{\text {ITIERAITI }} \mathrm{N}=\mathrm{XX}$

| x/A | CPA | $\mathrm{x} / \mathrm{A}$ | CPA | X/A | CPA | x/A | CPA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| xxxx | xxxx | xxxx | xaxx | x xxx | XXXX | xXXX | xXXX |
| xxXX | XXXX | xxOx | XXXX | XXXX | XXXX | XXXX | XxXX |
| : | : | : | : | : | : | : | : |
| xxxX | xxxx | x $\mathrm{x} \times \mathrm{X}$ | x xXXX | x xXx | x $\times X X$ | xxxx | x xxx |

THE SL/PE C CDNVERGES TX 。XXXXXXX $\pm X X "$
When a converged value for the slope of the concentration profile curve at the catalyst particle surface has been calculated, the above "concentration profile" will be printed. The word "ITERATI $\varnothing \mathrm{N}$ " refers to the
iteration count at the time of convexgences. $X / A$ is the nommined distance from the center of the catalyst particie of radius $A$ to the surfece. CPA is the concentration of hydrazine within the particle at the corresponding nomslized radial distance. The finai message indicates the final converged value of the slope. This block will be printed for each axiax stathon of the liquid region.

This messpge indientes thet the fterative procedure has achieved convergenee on value of tae hyurazine coatentration gradient at the catalys particlevingree, and appearemequently in calculations involving the liquia region of the reactor.

Subroutine , schad.
$====x^{x}=$

1. A listing of converged, reactant concentration (CP(X/A)) versus normalized ardial distance within the catalyst partıcle ( $X / A$ ) at each axial position in the vapor zegion.

(b) "OP(X) AT PARTICLE SURFACW $=. X X X X X ~+X X "$

(a) "HC* (T-TPS) $=X X X X X+X X^{18}$

Print message (a) indicates the number of iterations that were needed to find a converged value for the concentration gradient.

Print message (b) gives the converged value for the concentration at the particle surface $\left(c_{p}\right)_{s}$.

Print messages (c) and (d) give calculated values where KC3 is the mass transfer coefficient for amonia, CI3 is the interstitial concentration of amonia at the catalyst surface, HC is the heat transfer coefficient, $T$ is the interstitial temperature, and TPS is the temperature at the surface of the catalyst.

Print mesaages (a), (b), (c), and (d) appear at each axial position in the vapor region.
3. "SATISFACTORY XO FIUND AFTER XXX TRIES, $X O=. X X X X X \pm X X "$

When calculating an aumonia concentration radial profile within a catalyst particle it is necessary to determine the radial depth of penetration of ammonia. The approximate radial position of "zero" concentration is referred to as XO in subroutine SGRAD, and wher "he iterative procedure employed has successfully determined a value of XO, the above message, with iteration count, is printed.

A sample listing of the output for a typical one-dimensional steady-state data case is shown in Figs. 3a through 3f.

LISTING OF OUTPUT FOR SAMPLE DATA CASE
ONE－DIMENSIONAL STEADY－STATE
N


$983000-01$




C
 OU～Cuzinsで
IJーUUUE86＊


（1002
ciG. 3b


FIG. 3c


FIG. 3d


FIG. 3 e
.26443258-0n

.13845610 AmOO



FIG. 3 f


## Common Operational Froblems

Kany different data cases have been run with the one-dimensional steadystate compiter program. During these zuns, most of the problems which have developed heve been eliminated through program modification. Hovever, two problems which may still occur are noted below, together with appropriate techniquee Por solving them.

1. "UNABLE TW FITD SUITABLE XO AFTHER FUUR TRIES \$F 25 ITERATIONS EACH ... PROGRAK STDP FDIUOWS"

If a satisfactory value for XO camot be found after four attempts, this uressage is printed and program execution is terminated. An appropriate solution to this problem would be to try different values for $\mathrm{f}_{\mathrm{i}}$ [: zq . (I-11) in discussion of. SGRAD, Appendix I]. These yalues cald be greater than 0.95 . To make this change, subroutine SGRAD would have to be recomplied using the new values of $f_{i}$.
2. "THEFE IS A PLDDLE OF COLD MYDRAZINE AT THE LIQUID-VAF $/$ R/VAPGR


When uring a buriea injector acheme it is possible to "elood" the region surrounding the injector tip with cold, liquid hydrazine, A sudden drop in axial temperatures at the liquid-vapor/vapor interface indicates that this has occurreds and in such cases the above mesagge in printed and no further calculations are made. ini appropriate solution to this problem would be to try a larger input value for so and rerun the program with the revised input,

## Tro-Dimensional Steady-State Model

## Irput Description

The following is a description of the necessary input for the two-dimensional steady-state corputer progran. The input format is given in Table II. The coding of the sample data case for this program is shown in Figs. Ha and 4b, and a iisting of the input data punch cards corresponding to thie sample data case is shown in Figs. $5 a$ and 50 . The statement numbers in the text below refer to the cird numbers (first column) of Table II. For each run there will be only one card number one. Cards $t w$ through twenty-one shouid be included for each data case to be run.

1. The first card contains the number NCASs. This number indicates the number of data cases with each run. $1 \leq$ NCASE $\leq 999$.
2. The second card is the title card used for individusil data case idera $=$ tification. The title may be any alpha numeric information desired.
3. The third card contains the indicators NRINGS and N⿰亻 FZ. KRINGS indicates the number of evenly spaced radial stations at which calculations are to be made where radial station number one is that one nearest the center of the reactor snd radial station number (NRINGS) is that station nearest the reactor sall. For typical runs, NRTNGS $=10$ was found adequate to insure . good results. Increasing this number would allow more detailed radial analysis, but it would also insrease conputer run time. NofZ is the number of axial stations (2's) to be used in the three tables input on cards 10 through 21.
4. Caris four contain the values of $F(I)$, (the raves of feed of hydrazine from buried injectors (Ref. 1) into the systen in lb/ft ${ }^{3}-\sec$ ). One value of F for each radial station (total number of radial stations = NRINGS) should be input. Ien numbers ere allowed to a card. For the suggested NRINGS of 10 , there would be one card with ten values of $F$.
5. Cerde five contain the values of $\operatorname{co}(x)$, (the inlet mass flow rates in $\mathrm{lb} / \mathrm{p} \mathrm{t}^{2}$-sec $)$ for each radiai station. Ten numbers are allowed to" a card. Fow the suggested NRINGS of 10 . there would be one card with ten values of GO. All values of $G O$ must be greater than zero.
6. Cards sif contain the values of $\mathrm{ZO}(\mathrm{I})$, (the sxial aistance to the end of a buined injector in $f t$ ) for each radial station. Ten numbers are allowed to a sard. For the suggested NRINGS of 10 , there would be one card with ten values of 20 .
7. The seventh card contains the eight constants ALFHA3, HF, R, Marait, MNH3, MNE, MH2, and ALMHAL.

ALPHA3 is the preexponential factor in the rate equation for the thermal decomposition of hydrazine. It eqiels $2.14 \times 10^{10} \mathrm{sec}^{-1}$.

HF is the enthalpy of liquid hyarrasine entering the bed in deg $R$.
$R$ is the gas constant. It eçuals 10.73 (psia-ft $\left.{ }^{3}\right) /(1 b-m o l e-$ deg R ).

MNOH4 Is the molecular weight of nydrazine tat equals 32.048 1b/1b mole.

Mnis 3 is the molecular weight of ammonia. Jt equals 17.03 j Io/1b mole.

MN2 is the molecular weight of nitrogen. It equals $28.016 \mathrm{lb} / \mathrm{lb}$ mole.

MH2 : is the molecular weight of hydrogen. It equals $2.016 \mathrm{lb} / \mathrm{Ib}$ mole.

ALPHAS is the preexponential factor in the rate equation for the catalytic decomposition of hydrazine. For the Shell 405 catalyst it equais $10^{10} \mathrm{sec}^{-1}$.
8. The eighth card contains the eight constants AJPHAD, AGM, BCM, KP, TH, GF, MMAXI, and MMAXP.

ALPFAR is the preexponential factor in the rate equation for the catalytic decomposition of amonia. For the Shell 405 cataIyst it equals $10^{11} \mathrm{sec}^{-1}$.

AGM Is the activetion enerey for the catalytic decomposition of hydrazine, divided by the gas constant, For the Shell 405 catalyst it equals $2,500 \mathrm{deg} R$.

BGM is the activation energy for the catalytic decomposition of anmonia; divided by the gas constant. For the shell 405 catalyst it equals 50,000 deg $R$.
$K P$ Is the effective thamal conductivity of the porous catalyst particle. For the shell 405 catalyst it equals $0.4 \times 10^{-4}$ Btu/ft-sec-deg R.
is the temperature of liquid hydramine entering the bed in deg $R$

CF is the specific heat of liquid hydrazine. It equals 0.1332 Btu/lb-deg R.

MMAXI is the constant used to determine the size of axial station increments in the liquid region. It equals 200. Increasing this number would result in a decrease in size of axial station increments (and an increase in computer run time).

MMAX2 is the constantu used to determine the size of axial station increments in the Iiquid-vapor region. It equals 40. Increasing this number would result in a decrease in size of axial stetion increments (and an increase in computer run time).
9. The ninth card conteins the inlet value of $P$ and five constants ZEND, DONER4, DONH3, CGM, and RADIUS.

P is the inlet chamber pressure in psia.
ZAND is the catalyst bed length in feet.
DON2H4 is the diftusion coefficient of hydrazine in the gas phase at SIPP. It equals $0.95 \times 10^{-4} \mathrm{Pt}^{2} / \mathrm{sec}$.

DONH3 is the diffusion coefficient of amonia in the gas phase at STP. It equals $0.17 \times 10^{-3} \mathrm{ft}^{2} / \mathrm{sec}$.

CGM is the activation energy for the thermal decomposition of hydrazine, divided by the gas constant. It equals 33,000 deg R.

RADIUS is the radius of the catalyst bed in feet.
10. Cards ten through thirteen contain AVSZ(I), the bivariate interpolation teble used to obtain the catalyst particle racius, $A(x, r) *$. These $A$ values are obtained from aubroutine UNBAR, an interpolation routine developed at the United Aircraft Research Laboratories, as functions of axial distance, $Z$, and radial distance, RAD. For this table there should be a totel of (NめFC) Z's, (NRINGS) RAD's and (N $\quad$ (FZ $x$ NRINGS) A's. The tatole is set up as follows:
*Tis vaxibile is not subscripted in the program. This notation is used to show that the variable in punction of both axial distance and radial diam tance and to clardisy the way the table is set up.

10 This card contains the four table descriptors used by UNBAR. The first descriptor signifies the table number. For this program it equals 0.0. The second descriptor signifies the location in the array at which the table starts; the tables in this program are read in such that this number equals i. 0. The third descriptor for a bivariate table such as this one is the nuriber of elements in the first set of independent variables in the table (in this case, the number of $Z^{\prime} s$ ). inis number equals NめFZ. The fourth descriptor is the number of elements in the second set of independent variables in the table (in this case, the number of RAD's). This nu.ber equals NRINGS.

11 Thuse cards contain the monatonically increasing $Z$ values. Fnough cards should be used to contain n $\% \mathrm{FZ}$ values of $Z$ at the rate of ten per card. For example, if $N \phi F=12,12$ values of $Z$ should be input using 2 cards with ten values on the first card and the 2 remaining values on the second card.

12 These cards contain the monatonically increasing RAD's. Enough cards should be used to contain NRINGS values of RAD at the rate of ten per card.

13 These cards contain the values for $\mathrm{A}(\mathrm{z}, \mathrm{r})$. The A values are input at each $Z$ value for all KAD's (i.e., (NRINGS) val. ues of A for each $Z$ ) at the rate of ten per card.

Example 1: if N $\varnothing F=10$ and NRINGS $=5$, the first card would contain five A values corresponding to the five RAD's on card 12 for $Z(1)$; the second card would contain the five A values corresponding to the five RAD's for $Z(2)$; ... etc. ....; the loth card would contain the five $A$ values corresponding to the five RAD's for $Z(10)$.

Example 2: if NめFZ $=10$ and NRINGS $=12$, the first card would contain 10 A values corresponding to the first ten KAD 's (on card 12a) for $Z(1)$; the second card would contain the two remeining $A$ 's corresponding to the last two RAD'e (on card libb) for $Z(1)$; the third card would contain the $10 A^{\prime}$ 's corresponding to the first ten RAD's for $Z(2)$; the fourth card would contain the 2 remaining $A^{\prime}$ s for $Z(2) ; \ldots$ etc. ....; the 19th card would contain the ten A's corresponding to the first ten RAD's at $Z(10)$; the $20 t h$ card would contain the two A's corresponding to the 1est two RAD's at $Z(10)$.
11. Cards 14 through 17 contain APVSZ (I), the bivariate interpolation table used to obtain the catalyst particle surface area, $A P(z, r) *$. These $A P$ values are obtained from UNBAR ad functions of axial distance, 7 , and radial distance, $R A D$, as in the AVSZ table discussed above. For this
 $x$ NRINGS) AP's. The taile is set up on follows

CARD NO.
14 This card is exactly the ame as card 10.
15 These cards are exactly the sume as cards 11.
16 These cards are exactly the same as cards 12.
17 These cards contain the values for $\operatorname{AP}(z, r)$. These values are input at each $Z$ value for all RAD's at the rate of ten values per card. (See examples in the discussion of the AVSZ table as the table setup is the same.)
17. ORats 18 through 21 contain DELVSZ(I), the bivariate interpolation table used to obtain the interparticle void fraction, DELTA ( $z, r$ )*. fisege DRLIP values are obtained from UNBAR as functions of axial dis-
 dievar ror this table there should be a total of (NめFZ) $Z \%$, (NRINGS)
 CARD 10

18 This card is exsctiy the same as cari 10.
19. These cards are exactly the same as cards 11.

20 These cards are exactly the same as cards 12 .

21 These cards contain the values for DELIA ( $z, r$ ). These values are input at each $Z$ value for all RAD's at the rate of ten values per card. (See examples in the discusaion of the AVSZ table as the table setup is the same).

NOIE: The values for the ordere of the decomposition reactions (called ENl, EN2, and FN3 in the one-dimenslomal model) are included in the equations in the two-dimensional model and therefore are not input.
*This variable is not subscripted in the program. This notation is used to show that the variable is a function of both axial distance and radial dis= tance and to clarify the way the table is set up.
pable IT
Two-Dixensional Computer Program: Thput Format

| cacen | of caros | ${ }_{\text {Forpran }}$ | COUMANS | Snopor or |  | MOSAEMCLATURE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 13* | 2-3 | ITCASE |  | Number of data eases |
| 2 | 1 | 14 A 6 | $1-80$ | Title |  | - . |
| 3 | 1 | 213* | $\begin{aligned} & 1-3 \\ & 4-6 \end{aligned}$ | $\begin{aligned} & \text { NRTNGS } \\ & \text { N } \phi \text { FZ } \end{aligned}$ | - | Humber of radial stations Number of $z$ " $s$ in input tables |
| 4 | ** | 1088.4 | $\begin{gathered} 1-8 \\ 9-16 \\ \vdots \\ 73-80 \end{gathered}$ | F(I) | F | Distributed Feed Rate |
| 5 | ** | 10 E .4 | $\begin{gathered} 1-8 \\ 9-16 \\ 73-80 \end{gathered}$ | Co(I) | $G_{0}$ | Inlet mass flow rate |
| 6 | ** | 3058.4 | $\begin{gathered} 1-8 \\ 9-16 \\ f 3-80 \end{gathered}$ | ZO(I) | $z_{0}$ | Axial distance to inje:tor end |

** Enough cards should be used to contain (NRINGS) values of $F(I), G O(I)$ or $Z O(I)$ at the rate of ten per card.

| CARD FHMABER | NUABER <br> OF CARDS | $\begin{aligned} & \text { FORTRAN I } \\ & \text { FORMAT } \end{aligned}$ | COUWBANS USED | SYMBOL OR DESCPIPTION | CORRESPONDING SYMBOL USED IN EQUATIONS | NOMENCLATURE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 8 ETO 5 | $\begin{aligned} & 1-10 \\ & 11-20 \\ & 21-30 \\ & 31-40 \\ & 41-50 \\ & 51-60 \\ & 61-70 \\ & 71-80 \end{aligned}$ | ALPHA3 <br> HF <br> R <br> MN2H4 <br> MNH3 <br> MN2 <br> MH2 <br> ALETHAI | ${ }^{\alpha} \mathrm{hoh}$ <br> $h_{F}$ <br> K <br> $\mathrm{M}^{\mathrm{N}} \mathrm{N}^{2} \mathrm{~F}_{4}$ <br> $\mathrm{M}^{\mathrm{NH}} 3$ <br> $\mathrm{M}^{\mathrm{N}} 2$ <br> $\mathrm{MH}_{2}$ <br> $\alpha^{\mathrm{N}_{2} \mathrm{H}_{4}}$ | Constant in rate equation <br> Enthalpy of feed <br> Gas constant <br> Molecular weight of $\mathrm{N}_{2} \mathrm{H}_{4}$ <br> Moleculaz weight of $\mathrm{NH}_{3}$ <br> Mole var weight of. $\mathrm{N}_{2}$ <br> Molec ar weignt of $\mathrm{H}_{2}$ <br> Preexponential factor |
| . 8 | 1 | $8 \mathrm{E} 10.5$ | $\begin{array}{r} 1-10 \\ 11-20 \\ 21-30 \\ 31-40 \\ 41-50 \\ 51-60 \\ 61-70 \\ 71-80 \end{array}$ | ALPHA2 AGM BGM KP ITF CF MMAXI NMAXZ |  | Preexponential factor <br> Activation energy, deg $R$ <br> Activation energy, deg $R$ <br> Thermal conductivity <br> Feed temperature <br> Specific heat of liquid $\mathrm{N}_{2} \mathrm{H}_{4}$ <br> Determ. axial step size (liq. reg.) <br> Determ. axial step size(liq.vap.reg. |
| 9 | $1$ | 8 ElO 5 | $\begin{array}{r} 1-10 \\ 11-20 \\ 21-30 \\ 31-40 \\ 41-50 \\ 51-60 \end{array}$ | P. <br> ZEND <br> DON2 4 <br> DONH3 <br> CGM <br> RADTUS | $\begin{aligned} & \mathrm{P} \\ & -\mathrm{N}_{2} \mathrm{H}_{4} \\ & \mathrm{D}_{0} \\ & \mathrm{D}_{\mathrm{NH}} \mathrm{NH}_{3} \\ & \varepsilon_{\text {hom }} \mathrm{N}_{2} \mathrm{H}_{4} / \mathrm{R} \end{aligned}$ | Inlet chamber pressure <br> Bed Iength <br> Diffusion coefficient of $\mathrm{N}_{2} \mathrm{H}_{4}$ <br> Diffusion coefficient of $\mathrm{NH}_{3}$ <br> Activation energy, deg $k$ <br> Bed radius |
| 10! | $1$ | $4 \mathrm{E} 8.4$ | $\begin{gathered} 1-8 \\ 9-16 \\ 17-24 \\ 25-32 \end{gathered}$ | 0. <br> 1. NøFZ. NRINGS. | 菖 | Table descriptor Table descriptor ráble descriptor Table descriptor |
| 11 |  | $10 E 8.4$ | $\begin{gathered} 3-8 \\ 9-16 \\ +73-80 \\ \hline \end{gathered}$ | $Z(I)$ | 2 | Axial station |

Honoug rards should be used to contain (NDFZ) values of $z$ at the rate of ter. rur ary.
691046:-30
PSABIE II (Cont.)

| FAMD | $\begin{aligned} & \text { NLKaTER } \\ & \text { NOCASAS } \end{aligned}$ | FORTRAM y FOPMAT | $\begin{aligned} & \text { coumans } \\ & \text { USED } \end{aligned}$ | $\begin{aligned} & \text { SWMEO OR } \\ & \text { DESMRHTION } \end{aligned}$ | COnRESPOHDIMG SHMEOL USED IN ECUATIONS | MCANENCLATURE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ir | \% | $1078.4$ |  | RAD (I) | z' | Radyal station |
| $\left\lvert\, \begin{array}{r}13 \\ \end{array}\right.$ | ** * | $1058.4$ | $\begin{gathered} 1.8 \\ 9.16 \\ 73080 \end{gathered}$ | $A(z ; r)$ | ¢ | cata,iyst rseticle racius |
| $\cdots$ | 1 | 4 EB .4 | $\begin{gathered} 3-8 \\ 9-16 \\ 17-24 \\ 25-32 \end{gathered}$ | 0 : <br> 1. NOFZ. NRINGS. |  | table deseriptor Irable men ariptor Taiole deseriptor freble dezeriptor |
| i5 |  | 10 EP .4 | $\begin{gathered} 1.8 \\ 9.16 \\ 73-80 \end{gathered}$ | $Z(I)$ | $2{ }^{\prime \prime}$ | Axisal station |
| 36 | ** | 1088.4. | $\begin{gathered} 1-8 \\ 9-16 \\ 73-80 \end{gathered}$ | $\operatorname{RAN}(I)$ | \% | Fadial suatjor |
| 17 | *** | 9.OEB. 4 | $\begin{gathered} 1-8 \\ 9.16 \\ 73 \times 30 \end{gathered}$ | $A P(z, x)$ | $A_{p}$ | Toter exterrai satalyst zartius ewface area per init, volume of buri |

 exumple in text).

| Cango | Wumperf of capdos | FORTran E Frmaty | $\begin{aligned} & \text { COLUMAS } \\ & \text { USEM? } \end{aligned}$ | SYMBOL OR DESCRIPTIOA | CORTESPMMDING STMBOL USED IN EOUATIONS | MOAF MCLATURE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 18 | 1 | 4E8.4 | $\begin{gathered} 2-8 \\ 2-16 \\ 17-24 \\ 25-32 \end{gathered}$ | 0. <br> 1. NOWH. JTRINGS. | 4 | Table descriptor Thble descriptor Table descriptor Table descriptor |
| 19 | * | 10m8. 4 | $\begin{gathered} 1.8 \\ 9-16 \\ 1 \\ \hline 7.80 \end{gathered}$ | $2(I)$ | z | Axial station |
| 20 | ** | 10E8. 4 | $\begin{gathered} 1-8 \\ 9-16 \\ 13-80 \end{gathered}$ | $\operatorname{RAD}(\mathrm{T})$ | $\mathbf{r}$ | Radial station |
| 21 | *** | 1058. 4 | $\begin{gathered} 1-8 \\ 9 \times 10 \\ 4 \\ 73-80 \end{gathered}$ | $D E L D A(z, z)$ | $\delta$ | Interparticle void fraction |




(CONT.:




FiG. $5 \mathbf{b}$

INPUT DATA PUNCH CARDS: SAMPLE CASE (CONTINUED)
TWO - DIMENSIONAL STEADY - STATE MODEL.
LISTING OF


## Output Description

Output from the two-dimensional steadymstate program is entirely in printout form. There is no print option as in the onemimensional program; therefore, all printing deacribed below is "atandard" and could possibly occur with each data case run. The print statements generated from each routine are described below; they include all printing normally done during execution of any reprem sentative data case, error messages, and ceriain comments pertaining to caleulations which do not follow the normel pattern in a typical run.

## Standard Output

## MAIN Program

1. A complete listing of the punch card input with appropriate keadings and F $\varnothing$ EIPRAN variable titles for all input variables.
2. Axial positions, (2), and temperatures, (T), in each annular region for both liquid and liquia-vapor regions.
3. Axial position, radial position and temperature at the liquid/liquidvapor interface for each annular region.
4. Axisl position, radial position and temperature at the liquid-vapor/ vapor interface for each annular region.

## Subroutine vazon

든

1. Concentratiens of hydrazine, (C4), ammonia, (C3), nitrogen; (C2), and hydrogen, "CJ. and mole fractions of hydragine, (MFRAC4), ammonia, (MSRAC3), $x^{-3}$ "igen, (MFRAC2), and hydrogen, (MFRACl), at the liquidvapor/vapor miterfaces (these values will be indentical for each ring).
2. Axial position, (Z), temperature, (JNMP), and concentrations of hydramine, (N2H4), amonia, (NH3), nitrogen, (N2), and hydrogen, (H2), for each annulay region at every axial increment along the reactor.
3. Assumed uniform preasure (calculated by averaging the pressure drop calculated for each ring over the reactor cross-section) In the reactor at each axial increment.
4. Wole fractions of hydrazine, (MFRG4), smranise, (MFRAC3), nitrogen, (MrAC2), and hydrogen, (MFRAC1), ant the equivalent fractional ammonia discociation, (EQUTVALENM FRAC3D), for each annular ring at every axial increment.
 FACE - - - WRy USING A LaRger value F $\phi$ R go"
When using a buried injector scheme it is possible to "flood" the region surrounding the injector tip with cold, liquid hydrazine. A sudden drop in axial temperatures at the liquid-vapor/vapor interface in any annular ring indicates that this has occurred, and in such cases the above messise printed and no further calculations are made.
5. "IHE PRØGRAM HAS CALCUALIEED A NEGATIVE PRESSURE -..- REIURN AND TERMiNATG"

If a negative pressure is calculated at some axial station, further calculationa for the current dats case are stopped and this message is printed out.
7. "the pedgram has calculated a nigative tmaperature in ring XX -um rexurn AND terminate"

If a negative temperature is ca?culated in any annular region at any axial station, further calculations are stopped and this message, including the current annuler ring, is printed.

Subroutine SGRAD


1. "he havs calculatid a negative xo durcig ingration N $\quad$. $\mathrm{XX} . \operatorname{set} X 0=0$,


XO represents an approximation of the radial diatance to which hydrazine penetrates the catalybt particle before being disbipated. It is determined through an iterative procedure, and in some instances iritial guesses do not yield aatiafactory results. In this cqse, corrective méssures to yield a better approximetion to XO are instituted and thie procedure repeated. This message indjeates only that corrective calculations to improve on the accuracy of $X 0$ are being initiated.
2. "UNABIE TX FIND SUTTABLE XD AFTER FXUR TRIES OF' 25 ITtERATIONS EACH $-\cdots$ PRDGRAM STOP F

If after Pour corrective attempts to approximate $X 0$ the procedure still does not yield batisfactory reauits, this message aiong with all unaceap$\therefore$ le values fer XO is printed and further calculations are atopped. An oc.und dump of core accompanien, the programi stop.

```
9910461-30
```

3. "UTABLE MX CONVERGE $\emptyset$ N CPS IN 50 TRIES $-. .-\mathrm{CP}(\mathrm{X} / \mathrm{A})=. \mathrm{XXXXX} \pm \mathrm{XX}$ "

If sukroutine SGRAD cannot calculate a "converged" value for CPS after 50 iterstions, the final value for $C P$ at the particle surface is used to approximate CPS. : This is a good approximation to CPS, however, and program calculations continue with the above message being printed.

A listing of typical output for the two-dimensional sample data case is shown in Fige. 6e through 61.
C910461-30
LISTING OF OUTPUT FOR SAMPLE DATA GASE $\mid$ TWO-DIMENSHONAL STEADY-STATE
76:60



F16: 6e


| －1v2ck－ur？ | ．659144＋10 | ．14290－02 | ．67191＋03 | ．10343－0＜ | ． $684777+03$ | ． $10388-107$ | ． $6976: 9+03$ | ．10426－02 | ． $71047+03$ | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ．10453－68 | ．723s2＋03 | ． 10474 －02 | ．73615＋03 | ．10491－02 | ． $74899+0.3$ | ．10505－09 | －76182＋n3 | ．10515－02 | ． $77465+03$ | $\stackrel{0}{0}$ |
| ． $105<0-18 \mathrm{C}$ | ． $78748+015$ | ．10530－02 | ．80031＋03 | ．10536－02 | ． $81313+03$ | ．105．34－0？ | －82（901）＋n3 | ．10557－02 | ． $82000+03$ |  |
| ． | －23200＋43 | ．14602－02 | ． $82000+03$ | ．10624－92 | － $2100+4{ }^{\text {d }}$ | ．10647－07 | －82004＋a3－ | － 10670 －42 | － $82000+03$ |  |
| －10゙ロyローUミ | ．820100＋43 | ．10716－02 | ． $82000+03$ | ．10738－02 | ．82．0u0＋03 | ．10761－32 | －32000 0 ＋03 | ．10777－02 | ． $82000+03$ | $\stackrel{\sim}{6}$ |
|  |  |  |  | RI | 7 |  |  |  |  |  |
| 004\％ | couder | ${ }_{-41948-43}^{2}$ | $.54294+03$ | $06489-03$ | －5568643 | $-78786-03$ | $-50972+03$ | $\begin{gathered} 2 \\ \sim 7096 m 3 \end{gathered}$ | ${ }^{\top}-5440+03$ |  |
| －91770－03 | ． $59 \times 00+0.5$ | ．95120－03 | $.60750+03$ | ． 98024 －03 | ．62040＋03 | ．99989－03 | ． $63328+03$ | ．10119－02 | ． $64616+03$ |  |
|  | ．6．39u4＋us | ．10290－42 | ．67191＋63 | ． $10345-02$ | ．68477＋03 | ．10388－02 | ． $69762+n 3$ | ．10426－02 | ． $71047+03$ |  |
| －1154．53－02 | ． $72332+4 \mathrm{~S}$ | ．10474－02 | ． 73615 ＋03 | ．10491－02 | ． $744999+0.3$ | －10505－02 | ． $76182+03$ | ．10515－02 | ． $77465+03$ |  |
| －105cs－02 | ． 7874 ＋+63 | ．10550－02 | ． $30031+03$ | ．10536－02 | ．81313＋03 | －10534－03 | －82003＋03 | －10557－02 | － $82000+03$ |  |
| －1U5アy－0゙2 | ． $82000+03$ | ．1v002－02 | $.82000+03$ | ．10625－02 | ． $82000+03$ | ．10647－02 | $.82000+03$ | ．10670－02 | ． $82000+03$ |  |
| －200\％ 2 －42 | －924 | －1476－42 | －8000－93 | －1073－42 | －82000＋05 | ．10761－82 | －2009＋03 | 10777－02 | ＋32000403 |  |
|  |  |  |  | RI | 8 |  |  |  |  |  |
| $<$ | 1 | 2 | $\uparrow$ | 2 | $T$ | 2 | T | 2 | ${ }^{\top}{ }^{\top}$ |  |
| －visucu | ． $53000+62$ | ．83036－04 | ． $54294+03$ | ．13358－03 | ． $55586+03$ | ．15758－03 | －56873＋03 | ．17418－03 | －58169＋03 |  |
| －10590－03 | ． 590 | ．14495－03 | .64750 .03 | ．19606－03 | －62440＋03－ | －19999－03 | －63320＋03 | －20239－03 | ． $64696+03$ |  |
| －20440－us | ．65904＋3د | ．20582－03 | ．67191＋03 | ．20688－03 | ． $68477+03$ | ．20778－1；3 | ． $69762+n 3$ | ．2n854－03 | ． $71047+03$ |  |
| －209uy－03 | ． $72532+65$ | ．24961－03 | ． $73615+03$ | ．20984－03 | $.74899+03$ | ．21011－03 | － $76182+03$ | ．21032－03 | ．77465＋03 |  |
| ．21040－03 | ． $78748+03$ | ．21062－03 | $.80031+03$ | ．21074－03 | ． $81313+03$ | ．21070－（13 | －8с103＋n3 | ．211：5－03 | ． $82500+03$ |  |
| ．21101－03 | ． $02000+03$ | ．21206－43 | ． $82000+03$ | ．21251－03 | ． $82000+03$ | ．21297－03 | －82000＋03 | ．21．342－03 | ． $82000+03$ |  |
| ．21380－03 | ．82000＋0S | ．21．433－03 | ． $82000+03$ | ，21479－03 | －82000＋03 | ．21524－0．3 | － $82000+n 3$ | ．21556－03 | $.92000+03$ | 0 |
| 1 |  |  |  |  | 9 |  |  |  |  |  |
| 2 | $T$ | $\angle$ | T | 2 | T | $z$ | ${ }^{\text {r }}$ | 2 | $T$ |  |
| －nouse | ． $53000+00$ | ． $83836-04$ | ．54294＋03 | ．13353－03 | ．55586＋03 | ．15758－03 | ． $56878+03$ | ． 27418 mm | ． $58169+03$ |  |
| ．18350003 | ． $59400+03$ | ．19025－03 | ．6リ750＋03 | ．19606－03 | $.6204(1)+03$ | ．19999－03 | $.63328+03$ | ．202x9－03 | ．64616＋03 |  |
| － $20440^{2}-43$ | － $659844+03$ | －265ec－j3 | － $67191+03$ | －20684－03 | －68477＋03 | 20778－03－ | ． $69762+03$ | －20854－03 | ． $71047+03$ |  |
| －20909－03 | ． $72332+03$ | ．20951－03 | ． $73615+03$ | ．20984－03 | ． $7481899+03$ | ．21011－03 | －76182＋03 | ．21032－03 | ．77485＋03 |  |
| － 21040003 | ． 73743843 | ．21162－－3 | ．80031＋03 | ．21074－03 | ．81313＋03 | ，21070－03 | ． $82000+03$ | ．21115－03 | ． $82000+03$ |  |
| －21101－03 | ． $02000+03$ | ．21206－03 | ． $82000+03$ | ．21251－03 | －8200n＋03 | ．21297－03 | －82000＋03 | ．21342－03 | ．82000＋03 |  |
| －くi306m43 | －82040＋03 | ．21433－03 | ． $82000+03$ | ．21479－03 | ． $82000+03$ | ．21524－03 | －82000＋03 | ．21556－03 | $.82000+03$ |  |
|  |  |  |  |  | 10 |  |  |  |  |  |
| $<$ | $\boldsymbol{r}$ | Z | $\uparrow$ | $z$ |  | 2 | T | 2 | $T$ |  |
| ． 000 us | ．53000＋03 | ．83836－04 | ． $54294+03$ | ．13358－03 | ．555d6＋03 | ．15758－0．3 | －56878＋n3 | ．17418－03 | ．58169＋03 |  |
| ．18356－03 | ． $59460+03$ | ．19025－03 | $.60750+03$ | ．19606－03 | ． $62040+03$ | ．19999－03 | ． 6332 cion | ．20239－03 | ． $64616+03$ |  |
| ．20443－03 | ． $65904+03$ | ．20582－03 | ．67191＋03 | ．20688－03 | ．68477＋03 | ．20778－03 | ． $69762+03$ | ．20854－03 | ． $71047+03$ |  |
| － | －72332＋03 | －20904－43 | ． $73615+03$ | －20984－03 | －74499－03 | －21011－03 | －764 42.483 | －21032－03 | ． $77465+03$ |  |
| ． $21040-03$ | ． $78748+03$ | ．21062－03 | ． $80031+03$ | ．21074－03 | ． $81513+03$ | ．21070－03 | ．82000＋03 | ．21115－03 | ． $82000+03$ |  |
| －21101－03 | ． $82.200+05$ | ．21206－03 | ． $82000+03$ | ．21251－03 | ． $82060+03$ | ．21297－03 | －82000＋03 | ．21342－03 | ． $82000+03$ |  |
| ．21380－03 | ． $82004+03$ | ．21433－03 | ． $82000+03$ | ．21479－03 | ． $82300+03$ | ．21524－03 | －32000＋03 | ．21556－03 | ． $82000+03$ |  |

FIG． $6 d$


FIG. 6e


FIG. 6 f



FIG. 69


**井丰** OPERATIONS COMPLETE ******

FIG. 6 i

## Common Operational Problems

The two-dimensional steady-sta'te computer program has been run with a large varlety of data cases. During these runs, most of the problems which developed were eliminated by modifying the program. However, a few problems may still remain; these problems are outlined below together with appropriate techniques for solving them.

1. "IHE PRめGRAM HAS CALCULATED A NEGATIVE PRESSURE---REIURN AND TERMINATE"

If a negative pressure is calculated at some axial station in the vapor region, further calculations for the current data case are stopped and this message is printed out. This diagnostic statement indicates that a physical limitation of the reactor system has been exceeded. Therefore, this particular case cannot be run. A lower mass flow rate or a higher feed pressure should work.
 RETURN AND TERMINATE"

If a negative temperature is calcualted in any annular region at any axial station in the vapor region, further calculations are stopped and this message, including the current annular ring, is printed. An appropriate solution to this problem would be to increase the number of radial regions into which the reactor is divided.
 PRめGRAM ST $\varnothing$ P F

If a satisfactory value for $X 0$ cannot be round after four attempts, this message is printed and program execution is terminated. An appropriate solution to this problem would be to try different values for $f_{i}$ [Eq. (I-11) in discussion of SGRAD, Appendix I ]. These values could be greater than 0.95 . To make this change, subroutine SGRAD would have to be recompiled using the new values of $f_{1}$.
4. "THERE IS A PUDDIE $\varnothing$ F C $\varnothing \mathrm{LD}$ HYDRAZINE AT THE LIQUID-VAP $\varnothing R / V A P \phi R$ INTERFACE --- TRY USING A LARGER VALUE F $\varnothing$ R GO"

When using a buried " $\urcorner$ jector scheme it is possible to "flood" the region surrounding the injector tip with cold, liquid hydrazine. A sudden drop in axial temperatures at the liquid-vapor/vapor interface in any annular ring indicates that this has occurred, and in such cases the above message is printed and no further calculations are made. An appropriate solution to this problem would be to try a langer input value for $G O$, and rerun the program with the revised input.

## RWFERHNCTSS

1. Kesten, A. S.: Analytical Study of Catalytic Reactors for Hydrazine Decomposition. United Aircraft Research Laboratories Report F910461-12, First Annual Progress Report, Contract INAS 7w 458, May 1967.
2. Kesten, A. S.: Analytical Study of Catalytic Reactors for Hydrazine Decomposition. United Alrcraft Research Laboratories Report G910461-24, Second Annual Progress Report, Contract NAS 7-458, May 1968.
3. Eberstein, I. J., and I. Glassman: The Gas-Phase Decomposition of Hydrazine and Its Methyl Derivations. Tenth Symposium (International) on Combustion, Pittsburgh, The Combustion Institute, 1965, pp. 365-374.
4. McHale, E. T., B. E. Knox, and H. B. Palmer: Determination of the Decomposition Kinetics of Hydrazine Using a Single-Pulse Shock Tube. Tenth Symposium (International) on Combustion, The Combustion Institute, Pittsburgh, 1965, pp. 341-351.
5. Michel, K. W., and H. GG. Wagner: The Pyrolysis and Oxidation of Hydrazine Behind Shock Waves. Tenth Symposium (International) on Combustion, The Combustion Institute, Pittsburgh, 1965, pp. 353-364.
6. Askey, P. J.: The Thermal Decomposition of Hydrazine. J. Am. Chem. Soc., Vol. 52, 1930, pp. 970-974.
7. Bird, R. B., W. E. Stewart, and E. N. Lightfoot: Transport Phenomena. John Wiley \& Sons, Inc., New York, 1960.
8. Prater, D. C.: The Temperature Produced by Heat of Reaction in the Interior of Porous Particles. Chemical Engineering Science, Vol. 8, 1958, pp. 84-286.
9. Kesten, A. S.: Analytical Study of Catalytic Reactors for Hydrazine Decomposition - Part I:, Steady-State Behavior of Hydrazine Reactors. Proceedings of the Hydrazine Monopropellant Technology Symposium, The Johns Hopkins University Applied Physics Laboratory, Silver Spring, Maryland, November 1967.
10. Argo, W. B., and J. M. Smith: Heat Transfer in Packed Beds, Chemical Engineering Progress, 49, 1953, pp. 443-451.
a Radius of spherical particle, ft
$A_{p} \quad$ Total external surface of catalyst particle per unit volume of bed, $\mathrm{ft}^{-1}$
$c_{i}$
Reactant concentration in interstitial fluid, $1 \mathrm{lb} / \mathrm{f}^{3} \mathrm{t}^{3}$ Reactant concentration in gas phase within the porous particle, $1 \mathrm{lb} / \mathrm{ft}^{3}$ Specific heat of fluid in the interstitial phase, Btu/lb - deg $R$ Average specific heat of fluid in the interstitial phase, Btu/lb deg $R$
$D_{i} \quad$ Diffusion coefficient of reactant gas in the interstitial fluid, $\mathrm{ft}^{2} / \mathrm{sec}$
$D_{0} \quad$ Diffusion coefficient of reactant gas in the interstitial fluid at STP, $f t^{2} / \mathrm{sec}$
$D_{p} \quad$ Diffusion coefficient of reactant gas in the porous particle, $\mathrm{ft}^{2} / \mathrm{sec}$
$f_{i} \quad$ Weighting factor in Eq. (I-11)
F Rate of feed of hydrazine from buried injectors into the system (Ref. I), 1b/ft ${ }^{3}$-sec

Conversion factor, $\left(1 b_{m} / 1 b_{f}\right) \mathrm{ft} / \mathrm{sec}^{2}$
G
Mass flow rate, $\mathrm{lb} / \mathrm{ft}^{2}-\mathrm{sec}$
Enthalpy, Btu/Ib
hc Heat transfer coefficient, Btu/ft ${ }^{2}$-sec-deg R
$\mathrm{H} \quad$ Heat of reaction (negative for exothermic reaction), Btu/lo
$k_{c} \quad$ Mass transfer coefficient, ft/sec
$k_{0} \quad$ Reaction rate constant, equals $\alpha e^{-\gamma}$
$K_{p} \quad$ Thermal conductivity of the porous catalyst particle, Btu/ft-sec-deg $R$

G910461-30

M
$\bar{M}$
n
$\mathrm{N}_{\mathrm{r}}$

P Chamber pressure, psia
$q_{r}$
Qhet Activation energy for (heterogeneous) chemical reaction on the catalyst surfaces, $\mathrm{Btu} / \mathrm{lb}$ mole

Qhom Activation energy for (homogeneous) chemical reaction in the interstitial phase, Btu/Ib mole
$r$ Radial distance from the center of the cylindrical reaction chamber, fit
Thet Rate of (heterogeneous) chemical reaction on the catalyst surfaces, 1b/ft ${ }^{3}$-sec
$r_{\text {hom }} \quad$ Frite of (homogeneous) chemical reaction in the interstitial phase, lb/ft3-sec
$R \quad$ Gas constant, equals $10.73 \mathrm{psia}-\mathrm{ft}^{3} / \mathrm{lb}$ mole - deg $R$, or, Radius of reactor
$T$
$T_{\text {vap }} \quad$ Vaporization temperature, deg $R$
$w_{i} \quad$ Weight fraction of reactant in interstitial phase
x
$X_{0} \quad$ Defined in Appendix I (Discussion of Subroutine SGRAD)
$z \quad$ Axial distance, ft
$z_{0} \quad$ Axial distance to the end of buried injectors, ft
a Preexponential factor in rate equation

G910461-30
$\beta \quad$ Equals $\left[-\left(C_{p}\right)_{s} \mathrm{HD}_{\mathrm{p}}\right] /\left[\mathrm{K}_{\mathrm{p}}\left(\mathrm{T}_{\mathrm{p}}\right)_{\mathrm{s}}\right]$
$\gamma \quad$ Equals $Q_{\text {het }} / R\left(T_{p}\right)_{s}$
$\delta$ Interparticle roid fraction
є Eddy diffusivity, $\mathrm{ft}^{2} / \mathrm{sec}$
$\lambda \quad$ Eddy conductivity, Btu/ft-sec-deg R
$\mu \quad$ Viscosity of interstitial fluid, $\mathrm{lb} / \mathrm{ft}$ - sec
$\rho_{i} \quad$ Density of interstitial fluid, $\mathrm{lb} / \mathrm{ft}^{3}$
Subscripts
F Refers to feed
i Refers to interstitial phase
$\mathrm{p} \quad$ Refers to gas within the porous catalyst particle
s Refers to surface of catalyst particle
Superscripts
$J$ Refers to chemical species
I RePers to liquid at vaporization temperature
V Fef'ers to vapor at vaporization temperature

## APPENDIX I

## Description of Subroutines

The following is a list and brief description of the subroutines which comprise the UNIVAC 1108 computer programs describing the one- and twodimensional steady-state models of a hydrazine catalytic reactor. Subroutine SGRAD, since it is the key subroutine in each program is described in detail. The flow charts for the main programs and major subroutines are included immediately after this list in Figs. I..l tinrough I-8. The number outside of and next to any block on the flow charts indicates the approximate statement number in that routine at which that perticular operation orcurs.

One-Dinensional Model

| MAIN (Fig. I-1) | Controls input and calcuiates concentrations and temperstures in the liquid regicn of the reactor. |
| :---: | :---: |
| SIOPE (Fig. I-1) | Calculates concentration axd temperature profiles within the catalyst particles for the liguid and liquid vapor regions of the reactor. This subroutine is similar to SGRAD which is described in detail later in this section. |
| IQVP (Fig. I-2) | Calculates enthalpy during the liquid vapor region of the reactor (concentration of $\mathrm{N}_{2} \mathrm{H}_{4}$ and temperature remain constant). |
| LQV2 (Fig. I-2) | Calculates hydrazine concentration, enthalpy and temperatures during the liquid-liquid vapor region of the reactor (concentration of hydrazine varies). |
| VAPdR (Figs. I-3 \& I-4) | Calculates concentrations, temperatures and pressures in the vapor region of the reactor. |
| PARAM (Fig. I-5) | Cafculates psrameters needed for calculations done in subroutine SLDPE. |
| C¢NC (Fig. I-5) | Calculates reactant concentrations at the liquid vaporvapor interface of the reactor. |
| UnPBAR | Interpolation routine used to obtain values from a table. |
| BLDCK DATA TABIES | Tables of: |

(1) temperature vs. viscosity
'a) temperature vs. vapor pressure
(3) temperature vs. heats of reaction
(4) temperature vs. specific heat
(5) vapor pressure vs. temperature
(6) enthalpy vs. temperature

This routine is the same as it is in the two-dimensional model. For a detailed description, see the section describing two-dimensional subroutines.

Two-Dimensional Model

MAIN (Fig. I-6) Controls input and calculates concentrations and temperatures in the liquid region of the reactor for all annular regions.

Calculates concentration and temperature profiles within the catalyst particles for the liquid and liquid vapor regions of the reactor for all annular regions. This subroutine is similar to SGRAD which is described in detail later in this section.

Calculates enthalpy during the liquid vapor region of the reactor for all annular regions (concentration of $\mathrm{N}_{2} \mathrm{H}_{4}$ and temperature remain constant).

Calcuiates concentrations, temperatures and pressures in the vapor region of the reactor for all annualr regions.

Calculates axial increments for the vapor region.

Arranges an array of numbers in ascending order
Interpolation routine used to cbtain values from a table.

Tables of:
(1) temperature vs. viscosity
(2) temperature vs. vapor pressure
(3) temperature vs. heats of reaction
(4) temperature vs. specific heat
(5) vapor pressure vs. temperature
(6) enthalpy vs. temperature

SGRAD (Fig. I-8) Detailed description follows:

G910461-30

SGRAD (FIg. I-8)

The purpose of subroutine SGRAD is to solve the implicit integral equations describing reactant concentration and temperature profiles in the porous catalyst particles and to calculate the slope of the reactant concentration gradient at the surface of the catalyst particles. This routine is used for calculations in the vapor region of the reactor only. In the hydrazine catalytic reactor system, amonia concentration profiles are calculated but the subroutine is very general and can be used for many other reactants. The key equation to be solved is an implicit integral equation of the form (Refs. 2 and 9):

$$
\begin{align*}
& C_{p}{ }^{N H_{3}}(x ; a)=c_{i}{ }^{N H_{3}}-a^{2}\left[\frac{1}{x / a}-\frac{a k_{c}^{N H_{3}} D_{p}^{N H_{3}}}{a k_{c}^{N H_{3}}}\right] \int_{x_{0} / a}^{x / a} \xi^{2} \frac{{ }_{h}{ }_{h H_{3}}\left[C_{p}{ }^{N H_{3}}(x / a)\right]}{D_{p}{ }^{N H_{3}}} d \xi \tag{I-1}
\end{align*}
$$

where $c_{p}{ }^{\mathrm{NH}_{3}}(x)$ is the reactant (ammonia) concentration as a function of $x$ (the radial position within the catalyst particle), $c_{i}{ }^{\mathrm{NH}} 3$ is the interstitial reactant concentration and $a$ is the radius of the spherical catalyst particle. To solve this equation, a two-phase iterative scheme is used. First, an initial estimate for $c_{p}{ }^{\mathrm{NH}} 3(x)$ is found through an iterative method of calculating successively better approximations. Second, using the good initial estimate found in the first phase, a similar iterative method is used to arrive at converged values of the actual $c_{p}{ }^{1 / 3}(x)$ distribution.

## Phase I

It was found through hand calculation that solutions of Eq. (I-I) were very likely to diverge if the initial estimate wai not a very good estimate. Therefore, in the first phase of this subroutine the iterative scheme is used to find this good first estimate. A linear function of the type shown in Fig. I-9 was found to be a fairly close approximation to the actual concentration distribution. The point at which the reactant concentration profile changes slope is referred to as $X_{0}$.


Fig. (I-9)
The final solution to Phase $I$ is a distribution of this type,

Iterative Procedure: Phase I

1. First a guess is made at a value for the reactant concentration at the surface of the catalyst particle: $\left(c_{p}\right)_{s}{ }^{N H} 3=c_{i}{ }^{\mathrm{NH} 3} / 2$.
2. Using this v . 1 ue , a value is found for the slope of the concentration profile at the surface, $\left[\mathrm{dc}_{\mathrm{p}} \mathrm{NH}_{3} / \mathrm{dx}\right]_{\mathrm{x}=\boldsymbol{a}^{\circ}}$

$$
\begin{equation*}
\left[d c_{p}^{N H_{3}} / d x\right]_{x=a}=\frac{k_{c}^{N H_{3}}}{D_{p}{ }^{N H_{3}}}\left[c_{i}-\left(c_{p}\right)_{s}\right]^{N H_{3}} \tag{I-2}
\end{equation*}
$$

where $\mathrm{k}_{\mathrm{c}}{ }^{\mathrm{NH}_{3}}$ is calculated from an equation given in Ref. 1 and $\mathrm{D}_{\mathrm{p}} \mathrm{NH}_{3}$ is calculated from Eq. (I-3).

$$
\begin{equation*}
D_{P}{ }^{N H_{3}}=D_{0}{ }^{\mathrm{NH}_{3}}\left\{\left(\frac{\left(T_{P}\right)_{\mathrm{s}}}{492}\right)^{1.823} \cdot\left(\frac{14.7}{P}\right) \cdot\left[1-e^{-0.0672(P / 14.7)(492 /(T P) s)}\right]\right\} \tag{I-3}
\end{equation*}
$$

3. The temperature at the particle surface, $\left(T_{p}\right)_{s}$, is calculated from

$$
\begin{equation*}
\left(T_{p}\right)_{s}=T_{i}-\frac{1}{h_{c}}\left[\left(H \cdot k_{c} \cdot c_{i}\right)^{N_{2} H_{4}}+\left(H \cdot D_{p} \cdot\left[d c_{p} / d x\right]_{x=0}\right)^{N H_{3}}\right] \tag{I-4}
\end{equation*}
$$

where $\mathrm{T}_{\mathrm{i}}$ and $\mathrm{c}_{\mathrm{i}} \mathrm{N}_{2} \mathrm{H}_{4}$ are input to the subroutine, $\mathrm{H}^{\mathrm{N}_{2} \mathrm{H}_{4}}$ and $\mathrm{H}^{\mathrm{NH}_{3}}$ are taken from tables in the computer: program, and $h_{c}$ and $k_{c}{ }^{N_{2} H_{4}}$ are calculated according to the equations in Ref. 1.
4. Using the point $\left[a,\left(c_{p}\right)_{s} \mathrm{NH}_{3}\right]$ and the slope $\left[\mathrm{ac}_{\mathrm{p}} \mathrm{NH}_{3} / \mathrm{dx}\right]_{\mathrm{x}=\mathrm{a}}$, a line is established and extrapolated to the $c_{p}{ }^{\mathrm{NH}_{3}}=\delta_{a x i s}{ }_{a}$ line, intersecting the axis line at $X_{0}$ (as in Fig. I-9).
5. The value for $X_{0}$ is calculated from

$$
\begin{equation*}
x_{0}=a-\left\{\left(\theta_{p}\right)_{s} /\left[d c_{p} / d x\right]_{x=0}\right\}^{N H_{3}} \tag{I-5}
\end{equation*}
$$

Since the region of pimary interest is the particle surface, it is at this point that convergence on a value for $c_{p}{ }^{\mathrm{NH}_{3}}(x)$ is tested. To test for convergence, a new ( $c_{p}$ ) $\mathrm{NH}_{3}$ is calculated and compared to the previous ( $c_{p}$ ) $\mathrm{NH}_{3}$. The new value for $\left(\mathrm{p}_{\mathrm{p}}\right)_{\mathrm{s}} \mathrm{NH}_{3}$ can be calculated from Eq. (I-I) by noting that, at the catalyst particle surface, where $\mathbf{x}=\mathbf{a}$, the second integral term in Eq. (I-I) drops out leaving

$$
\begin{equation*}
\left(c_{p}\right)_{s}^{N H_{3}}=c_{i}^{N_{3}}-\left[\frac{1}{x}-\frac{a k_{c}-D_{p}^{N H_{3}}}{a^{2} k_{c}{ }^{N H_{3}}}\right] \int_{0}^{a} \xi^{2} \frac{r_{\text {het }}\left[c_{p}{ }^{N H_{3}}(x)\right]}{D_{p}{ }^{N H_{3}}} d \xi \tag{I-6}
\end{equation*}
$$

As can be seen in Fig. (I-9) in distributions of this type all values of $c_{p} \mathrm{NH}_{3}(x)$ between 0 and $X_{0}$ are zero. Therefore, in evaluating the integrals, all points between 0 and $X_{0}$ can be ignored. If this is done and if $x$ is normalized by dividing by 5 , Eq. (I-6) reduces to
$\left(c_{p}\right)_{s}^{N H_{2}}=c_{1}^{N H_{3}}-a^{2}\left[1-\frac{a k_{c}^{N H_{3}}-D_{p}^{N H_{3}}}{a k_{c}^{N H_{3}}}\right] \int_{x_{0} / 0^{2}}^{1} \frac{\xi_{\text {het }}\left[c_{p}^{N H_{3}}(x)\right]}{D_{p}^{N H_{3}}} d \xi$
where all tirne have been previously determined except rhet which is calculated from

$$
\begin{gather*}
r_{\text {het }}{ }^{N H_{3}}=k_{0}\left(c_{i}{ }^{N H_{3}}\right)^{1-n} \cdot\left[C_{p}^{N H_{3}}(x)\right]^{n} \exp \left\{r \beta\left(1-C_{p} N_{3}(x) / c_{i}^{N H_{3}}\right)\right. \\
\left.1 /\left[1+\beta\left(1-c_{p}{ }^{N H_{3}}(x) / c_{i}^{N H_{3}}\right)\right]\right\} \tag{I-8}
\end{gather*}
$$

where $n, k_{0}, \gamma$, and $\beta$ are defined in the List of Symbols.
6. A new value for $\left(c_{p}\right)_{s}{ }^{\mathrm{NH}} 3$ is calculated using Eq. (I-7) where the integral is evaluated numerically using the trapezoidal method.
$\begin{array}{l}\text { 7. A new value for } \\ \text { newly calculated } \\ \left(c_{p}\right)_{s} \mathrm{NH}_{3}\end{array} \mathrm{NH}_{3}$. $]_{x=a}$ is calculated from Eq. (I-3) using the
8. New values are calculated for $\left(T_{p}\right)_{\mathrm{S}}, \mathrm{D}_{\mathrm{p}}^{\mathrm{NH}} 3, \gamma, \beta, \mathrm{k}_{\mathrm{O}}$.
9. The following convergence tests are made:

$$
\left|\frac{\left[T_{i}-\left(T_{p}\right)_{S}\right]_{O \perp O}-\left[T_{i}-\left(T_{p}\right)_{S}\right]_{N E Y}}{\left[T_{i}-\left(T_{P}\right)_{S}\right]_{N E W}}\right| \begin{align*}
& ?  \tag{I-9}\\
& \stackrel{y}{\leq} 0.05
\end{align*}
$$

end

$$
\begin{equation*}
\left|\frac{\left[c_{i}-\left(c_{p}^{\prime}\right)_{S}\right]_{O L D}^{N H_{3}}-\left[c_{i}-\left(c_{p}\right)\right]_{S E W}^{N H_{3}}}{\left[c_{i}-\left(c_{p}\right)_{S}\right]_{N E W}^{N H_{3}}}\right| ? \frac{?}{\leq} 0.05 \tag{I-10}
\end{equation*}
$$

If these tests are both satisfied, the value of $X_{0}$ calculated in Eq. (I-5) is saved and the program moves on to Phase II.

Tf both tests are not satisfied, an averaged value of $\left(c_{p}\right)_{s} \mathrm{NH}_{3}$ is calculated using as many as three averaging techniques to insure rapid convergence.

Using this new value of $\left(c_{p}\right)_{s}{ }^{\mathrm{NH}} 3$, steps 2 through 9 are repeated up to a maximum of twenty-five times. If no convergence is reached after twentyfive iterations, a "weighted" estimate of $X_{0}$ is tried:

$$
\begin{equation*}
X_{0}=f_{i} \cdot\left(X_{0}\right)_{\underset{c}{\text { previously }} \text { calculated }}+\left(1-f_{i}\right) \cdot\left(X_{0}\right) \underset{\text { calculated }}{\text { cast }} \tag{I-11}
\end{equation*}
$$

Steps 1 through 9 are repeated up to twenty-five times. Succeeding values $f_{i}=$ $0.80,0.85,0.90$, and 0.95 are tried until convergence is reached. If convergence still is not reached and therefore a satisfactory $X_{0}$ is not found, a program termination with an appropriate error message follows.

Phase II

Using as an initial approximation the straight line determined by the convergent $X_{0}$ and $\left[d c_{p} N / 3 / d x\right] \quad x=a$ found in Phase $I$, an iterative scheme similar to that in Phase I is now employed to find convergent values for the entire $c_{p} \mathrm{NH}_{3}(x)$ distribution within the catalyst particle. It was found through hand calculations that the convergent values of $c_{p} \mathrm{NH}_{3}(x)$ near the surface were not changed by more than 5 percent when the values of $c_{p}{ }^{N} 3(x)$ between 0 and $X_{0}$ were not considered in the iterative procedure. Therefore, the points in this range are ignored.

## Iterative Procedure: Phase II

The values of $c_{p}{ }^{\mathrm{NH}} 3(x),\left(\mathrm{T}_{p}\right)_{5}, \mathrm{k}_{0} \mathrm{NH}_{3}, \beta^{\mathrm{NH}_{3}}, \gamma^{\mathrm{NH}} 3$, etc. found in the last iteration in Phase I are the initial input to the following iteration.

1. A new $c_{p}{ }^{\mathrm{NH}} 3(x)$ profile is calculated irom Eq. (I-12). $C_{p}{ }^{N H_{3}}(x / \sigma)=c_{i}{ }^{N H_{3}}-a^{2}\left[\frac{1}{x / a}-\frac{a k_{c}^{N H_{3}}-D_{p}^{N H_{3}}}{a k_{c}^{N H_{3}}}\right] \int_{x_{0} / a}^{x / a} \xi^{2} \frac{r \text { het }^{N H_{3}}\left[C_{P}^{N H_{3}}(x / a)\right]}{D_{P}{ }^{N H_{3}}} d \xi$

As before, the limits of the integral have been normalized by dividing by a. The integrals are evaluated numerically using the finite sum approximation described below.

To evaluate the integral terms in Eq. (I-12) the following procedure, using a finite sum approximation, is used:
(a) the interval $X_{0} / a \leq x / a \leq 1$ is divided into 24 equally spaced subdivisins, and an average value for rhet $\left[c_{p}{ }^{N H}(x / a)\right]$ is calculated for each of these divisions.
(b) treating $r_{\text {hat }}\left[c_{p}{ }^{\mathrm{NH}} 3(\mathrm{x} / \mathrm{a})\right]$ as constant over each of these subdivisions, Eq. (I-12) can be approximated by

$$
\begin{align*}
& C_{p}{ }^{N H_{3}}(x / \sigma)=c_{i}{ }^{N H_{3}}-\frac{a}{D_{P}^{N H_{3}}}\left[\frac{1}{x / \sigma}-\frac{a k_{f}^{N H_{3}}-D_{p}^{N H_{3}}}{a k_{c} N_{3}}\right]\left\{r_{\text {set }}^{1} \int_{x_{0} / a}^{x_{0} / a+\Delta x / a} \quad \xi d \xi\right. \\
& \left.+r_{\text {ht }}{ }^{2} \int_{x_{0} / a+\Delta x / a}^{x_{0} / a+2 \Delta x / a} \xi d \xi+\cdots+r_{h_{01}} 24 \int_{x_{0} / a+(k-1) \Delta x / a}^{x_{0} / a+k \Delta x / a} \quad \xi d \xi\right\} \\
& -\frac{a^{2}}{D_{p}^{N H_{3}}}\left\{r_{\text {ht }}{ }^{\prime} \int_{x_{0} / a+k \Delta x / a}^{x_{0} / a+(k+1) \Delta x / a}\left[\frac{1}{\xi}-\frac{a k_{c}{ }^{N H_{2}}-D_{p}{ }^{N H_{3}}}{a k_{c}^{N H_{3}}}\right] \xi^{2} d \xi\right.  \tag{I-13}\\
& +r_{\text {set }}{ }^{2} \int_{x_{0} / a+(k+1) \Delta x / a}^{x_{0} / a+(k+2) \Delta x / a}\left[\frac{1}{\xi}-\frac{a k_{c}^{{ }^{N H_{3}}-D_{p}^{N H_{3}}}}{a k_{c}{ }^{N H_{3}}}\right] \xi^{2} d \xi+\cdots \\
& +r_{\text {net }} 24 \int_{x_{0} / a+23 \Delta x / a}^{x_{0} / a+24 \Delta x / a}\left[\frac{1}{\xi}-\frac{a k_{c}^{N H_{3}}-D_{p}^{N H_{3}}}{a k_{c}^{N H_{3}}}\right] \xi^{2} d \xi
\end{align*}
$$

where $k=1,2, \ldots, 24$
(c) the integrals in Eq. (I-13) can now be evaluated directly

$$
\begin{aligned}
& \operatorname{viz} \int_{a}^{b} \xi d \xi=\left.\frac{\xi^{2}}{2}\right|_{a} ^{b}=\frac{\frac{b}{}^{2}}{2}-\frac{\frac{a}{2}^{2}}{2} \\
& \int_{a}^{b} \operatorname{constant} \cdot \xi^{2} d \xi=\left.\operatorname{constant} \cdot \frac{\xi^{3}}{3}\right|_{a} ^{b}=\operatorname{constant}\left(\frac{b^{3}}{3}-\frac{a^{3}}{3}\right)
\end{aligned}
$$

(d) rearranging and integrating term by term in Eq. (I-13) yields the finite sum approximation for $c_{p}{ }^{N H} 3(x / a)$ at each subdivision of the interval from $X_{0} / a$ to 1 :

$$
\begin{align*}
& C_{P}^{N H_{3}}(x / a)_{k+1}=c_{i}^{N H_{3}}-\frac{a^{2}}{D_{p}{ }^{N H_{3}}}\left\{\left(\frac{1}{X_{k} / \sigma}-\frac{a v+1}{a v}\right) .\right. \\
& \sum_{j=1}^{k} \frac{\text { het }^{j}}{3}\left[\left(\frac{x_{j}}{a}\right)^{3}-\left(\frac{x_{j}-1}{a}\right)^{3}\right]+\sum_{j=k}^{24} \frac{\text { inet }^{j+1}}{2}\left[\left(\frac{x_{j+1}}{a}\right)^{2}-\left(\frac{x_{j}}{a}\right)^{2}\right]  \tag{I-14}\\
& \left.-\left(\frac{\rho v+1}{a v}\right) \cdot \sum_{j=k}^{24} \frac{r_{\text {hef }}^{j+1}}{3}\left[\left(\frac{x_{j}+1}{a}\right)^{3}-\left(\frac{x_{j}}{a}\right)^{3}\right]\right\}
\end{align*}
$$

where $v=\left(a k_{c}-D_{P}\right)^{N H_{3}} / a k_{c} \mathrm{NH}_{3}$ and $K=1,2, \ldots, 24$.
(e) the values for $\left.c_{p}{ }^{N_{3}} 3(x / a)\right|_{x=X_{0}}$ and $\left.c_{p}{ }^{N H H} 3(x / a)\right|_{x=a}$ are special cases where one or the other of the integral terms in Eq. (I-12) vanishes. Evaluation follows from a simple reduction of Eq. (I-14).
2. A new value for $\left[\begin{array}{l}\left.d c_{p} \mathrm{NH}_{3} / d x\right] \\ \text { newly calculated } \\ \left(c_{p}\right)_{s}\end{array} \mathrm{NH}_{3}\right.$. $\mathrm{x}=\mathrm{a}$ is calculated from Eq. (I-3) using the
3. A new value for $\left(T_{p}\right)_{s}$ is calculated from Eq. (I-4).
4. Convergence tests are made (as they were in Phase I) using Eqs. (I-9) and (I-10):
(a; If the convergence tests are both satisfied, the quantities GRAD and TGRAD are calculated according to Eqs. (I-15) and (I-16), and the program returns to the point from which the subroutine was called.

$$
\begin{equation*}
G R A D=\left[d c_{p}^{N H_{3}} / d x\right]_{x=0} \cdot D_{p}^{N H_{3}} \tag{I-15}
\end{equation*}
$$

$$
\begin{equation*}
\text { TGRAD }=h_{c}\left[T_{i}-\left(T_{p}\right)_{S}\right] \tag{I-16}
\end{equation*}
$$

(b) If the tests are not both satisfied, a new $c_{p}{ }^{N H 3}(x)$ distribution is calculated using one of various averaging techniques. Corresponding $\operatorname{dc}_{p}{ }^{\mathrm{NH}} 3 /\left.\mathrm{dx}\right|_{\mathrm{x}=\mathrm{a}}\left(\mathrm{T}_{\mathrm{p}}\right)_{\mathrm{s}}, \mathrm{k}_{0}, \gamma, \beta$, etc. are also calculated. Then steps 1 through 4 are repeated up to a maximum of 50 times. If convergence criteria are not met after 50 iterations, approximations to acceptable values of GRAD and TGRAD are made using the results of the Phase I iterative procedure, an appropriate message is printed, and the program returns to the point from which the subroutine was called.

Distributions of the type show in Fig. (I-10) ere typical of those found in this iterative procedure.
(1) converged linear approximation from
Phase I
(2) curve calculated from curve (1) using Eq. (I-11)
(Phase II, step 1)
(3) averaged curve calculated from curves (1) and (2)
(Phase II, step 4b)


Fig. (I-10)

ONE-DIMENSIONAL STEADY-STATE COMPUTER PROGRAM main program and subrøutine sløpe: Flow Diagrams




# ${ }^{\text {G910461-30 }}$ ONE-DIMENSIONAL STEADY-STATE COMPUTER PROGRAM SUBRDUTINE VAPDR (cont.) 




TWO-DIMENSIONAL STEADY-STATE COMPUTER PROGRAM MAIN PROGRAM FLOW DIAGRAM


TWO-DIMENSIONAL STEACY-STATE COMPUTER PROGRAM SUBRØUTINE VAP@R: Flow Diagram


TWO-DIMENSIONAL STEADY-STATE COMPUTER PROGRAM SUBRQUTINES DELTAZ, SGRAD, and ØRDER

Flow Diagrams


G910461-30
APPENDIX II

## LISTING OF COMPUTER PROGRAMS

## ONE-DIMENSIONAL STEADY-STATE MODEL





SUsicOUTi,NE LQVP (H, LLV, Q,JJoQ1,TEMP) ..... 0
HEAK Kr ..... 10SAGEEER PARINT20











G910461－30
3 1F！1K．EG．1）GOTO 322 ..... 2150
321 けWLUKUEKIF ..... 2160
1F（KJ．E甘． 1 ）G0 TO 19\％ ..... 2170
1F（1K．tは．1）G0 TO 2bu ..... 2180
322 RAIL＝DER1F ..... 2100
 ..... 2200
2210
4 IF（ 1 －－HL ）777，777．5 ..... 22ア0
 ..... ‘2230
CALL U：VUAK（VISVST（1），1，HEIAP，0．．，VIS．KK） ..... $2 \dot{2} 40$
H H ..... 2050
AKC゙ニ（．Gl＊G）／（RHO）＊（（VIS／（RHO＊DIFN））＊＊－．667）＊（（G／（AP＊VIS））＊＊－．41） ..... ＂くら0
KAT＝AKC＊CG（i）／LPA（I） ..... 2270
1F（KんTE゙ーKAT）776．776．0 ..... 2280
MFLACIS ..... く290
RAIt．$=R A T$ ..... 2310
 ..... ？ 310
778 MFLAG＝n ..... ？ 320
777 1F（15．ST．．9．33＊A（1））60 TU 8BK ..... 2330
IF（r）RluT．ciu．l）WR1Tt（u）182）IM ..... 2340
102 FUMTAAT（11H ITLAATION＝，I3） ..... 2350
IF（HRSil．EU．1）WKITE（0．336） ..... 2360

1 ヘ／A ..... 2330
IF（PRINT．E．X．1）WRITE（0．183）（XOA（1），CPA（I），I＝1，MMAX，4） ..... 2390
 ..... 2440
12．7．（X，ヒ12．7） ..... 2410
838 IF（PRLNT．EU．1）WKITL（U．323）RATE ..... 2420
 ..... $243 n$
327 KLIURV ..... 2440
32 $\mathrm{HERC=}$（abs（JtRIF－HOLD）／OERIF）＊100． ..... 2450
ALL＝5． ..... 24650
GU T0 3Z2
GU T0 3Z2 ..... 2470 ..... 2470
C SAMPLIFIEC ..... 2480
701 B二．חi）nnol＊A（I） ..... 2490
700 FOISHAT（／B4H INITIAL CHOICE THRU ORIGIN SEEMINGLY OK，HUT RESULTS ..... 2500
$\times$ HUITEN AFTER 99 ITCRATIONS ．．．／4BH SET XO＝．000001＊A ANO USE MORE ..... 2510
 ..... 2520
HREHAAGIV ..... 2530
BUK＝（A（I）－b）／BUIV ..... 2540
MMAX $=1+1 \mathrm{r}+\mathrm{JIHT}+1$ ..... $255{ }^{\circ}$
MLHT $=I \cdot W I T+1$ ..... 2560
MAX $=$ MAX -1 ..... 2570
$1 \times=0$ ..... 2580
00 TU 2.37 ..... 2590
Eivi ..... 2610
SUBKOUTLIVL SGRAL（GRAD，TGRAD） ..... 0
REAL KIIMPOKC3，KC4，MU ..... 10
INTLIGKK PRIKI ..... 20
COMINOIV／FTZ／TBLVP（70），TBLH4（42），TBLH3（42），SHTBL1（34），SHTBL2（34）． ..... 30
1 SHTBL3（34），SHTBL4（34），ZTBLÜ（46），ZTBLAP（46），ZTELA（46） ..... 401NM4，WM3，W＇Y2，WM1，ALPHA3，R，TVAF，ZEND，BGM，HF，UZ，ALPHA1，ALPHAZ60






| 9910461－30 |  |  |  |
| :---: | :---: | :---: | :---: |
| － | TGKAL $=$ HC末（Y－TPS <br> IF（PRINT．EQ． 11 WRITE（6，83）GRAD，TGRAD |  | 2720 |
|  |  |  | 2730 |
| － 83 |  | 54X．${ }^{\text {He＊}}$（T－TPS $\left.)=0 . E 12.5\right)$ | 2740 |
| 28 | KETUKN |  | 2750 |
|  | E゙NO |  | 2750 |

SUlakuUTINE TKAF＇P（U，V．NPART，RIESUM） ..... 0
REAL KOOKCS ..... 10
COMMUN AEBBG／OPS，A，KC3．KU。XOA，CPS，CIJ．GAMMA，BETA ..... 20
C UEFIINE RIHET FOR VARIABLE CP，CPS，TP
HILTF（A，B，C，O，EFN）$=E * A * *(1-N) * B * * N \neq E X P(C * D *(1 .-B / A) /(1 .+D *(1 .-$ ..... 30
$X$ B／A）！ ..... 40
－C－FUNETIGNG GEFTNING INFEORANOS
FOXII（X，R）＝X＊＊2＊R ..... 50
FUACTIUN DEFINING CP（X）FOR RIHET FUNCTION$C$ CH（X）IS ASSUMED TO VARY LINEARLY WITH $X$
CPXF（X，Y，Z）＝（X－Y）f（1，－Y）＊Z ..... 60
$N=\operatorname{IVHART}-1$ ..... 70
Patranimity ..... 80
$H=(V-U)$ i YAIRT ..... 90
UPH $=U+$ H ..... 100
SUM＝0． ..... 110
CPAI＝CHXF（U．XOA：CPS） ..... 120
CP×2＝CPXF（V，XOA，CPS） ..... 130
WHETI＝HHETFHEIJVEPKITOAMMAVBEFAOKOTH ..... 140
RHET T2＝RHETF（CI3，CPX2，GAMMA， $\mathrm{BETA}, \mathrm{KO}, 1)$ ..... 150
C－GALCULATE FIIRSTGLASF TERMS OF RIEMANN SUM FIRST
TRMI＝F0XII（U．RHEY1）／2． ..... 160
THM2＝FOXI： $1 \forall$ OHETEI／2。 ..... 170
$60081=1, N$ ..... 180
EPx＝EPKFHWPHOA日ATEPST ..... 190
RHET＝RHETF（CI3，CPX，GAMMA，BETA，KO，1） ..... 200
Sthin＝SUM＋fOXH（UPHVRHET） ..... 210
UPri $=$ UPH＋H ..... 220
COIVFINUE ..... 230
RIESUM 3 H ${ }^{\left(T R N_{1}\right.} 1+$ SUM + TRM2） ..... 240
ENU ..... 250




```
3910461-30
E - GLOCK \AFA FUK SPEEIFIC HEAT VS TEMPERATURE
            COMMON IUDU/CFTEL4(34),CFTUL3(34),CFTBL2(34),CFTBL1(34)}1
            UAIA (CFTALT(I)FI=1.34)/0., 1.: 15.0 0..% 20
    C TEMPEKATURES
```



```
        x 1960. . 2160. . 2340. . 2520. . 2700. . 2880. . 3060. . 40
    C SHECLSIC HEEATS
        X 3.4194, 3.4596. 3.4685, 3.4765, 3.4899, 3.5151, 3.5454, 3.5006, 50
        X 3.f2n&, 3.6654, 3.7150, 3.7696. 3.8291, 3.8802. 3.9288 / 60
        UAIA (CFTBL2(I),I=1.34) / 0., 1.. 15.1 .. }7
```



```
    x 1980. . 2160. . 2340. . 2520. . 2700. . 2880. . 3060. . 90
    C SHECIFIC HEATS
        X.246.5 . .2495 . .2524 . .2569 . . 2624 . . 2682 . . 2738 . . 2790 . 
        X.2836 , .2878 , .2914 . . 2946 . . 2974 . . 2998 . . 3n19 / 110
```



```
    C TEMPLKATUHESS
```



```
        x 1900. . 2160. . 2340. . 2520. . 2700. . 2880. . 3060. . 140
    C SPECIFIC HEATS
        X.5005 . .5424 . . 5891 . .6344 . .6773 . . 7176 . .7553 . .7905 . 150
```



```
            LATA (CFTBL4(I).,I=1.34)/0., 1., 15.. 0.. 170
    C TEMPLKATUKES
        \lambda b40. . 720. . 900. . 1080. . 1260., 1440., 1020., 1800., 180
        x 1980. , غj6t%. , 2340. . 2520. . 2700. . 2880. . 3060.. 190
    C SPECIFIC HEATS
```



```
            LNU -. 220
```



G910461-30

$\begin{array}{lr}\text { MLUCK JATA } & 0 \\ \text { COMMO H TDAVIBL/VPTUL(44) }\end{array}$
isluck lata lable of vapor pressure vs temp (USEd to fino tvap)
 30 50. 50. . 550. . 600. . 650. . 700. . 750. . 80.. an

610
$\qquad$

G910461-30
TWO-DIMENSIONAL STEADY-STATE MODEL




|  | $\cdots$ |
| :---: | :---: |
| 6910461-30 |  |
| 41 | READ (5.41) (AVSZ(I).I=5,NOF24) |
|  | FOKMAT (1ILEB.4) |
|  | REAO (5,41) (AVSZ(I), I=NOFZ5,NOF26) |
|  | $11=$ NOF26+1 |
|  | $\pm 2=$ NOF-26+HHINOS |
|  | U0 $42 \mathrm{~J}=1$ - NOFZ |
|  | READ (5.41) (AVSZ(I),I=I1:52) |
|  | $11=12+1$ |
|  | $12=1+$ NK 1 NGS -1 |
| 42 | Colntidue: |
|  | REA |
|  | READ (5,41) (APVSZ(I), I=5,NOF24) |
|  | KEAU (5041) (APVSZ(1), I=NOF $25, N O F 26$ ) |
|  | $11=\mathrm{NOF} 26+1$ |
|  | $12=$ NUFZ6+NKINGS |
|  | HC $43 \mathrm{~J}=1$, NOFZ |
| 43 |  |
|  | $11=1 \gamma+1$. |
|  | $\underline{L}=21+\mathrm{HRINGS}-1$ |
|  | CONTIINUE |
|  | REAU (5.40) (OLLVS $2(1), I=1,4)$ |
|  |  |
|  |  |
| $I 1=\text { NUF } 26+1$ |  |
|  |  |
|  |  |
|  | KEAO (5.41) (0ELVSZ(1),I=11.12) |
| $11=12+1$ |  |
| 44 |  |
|  |  |
|  | HRITE (0,604) |
| 604 | FOKMAT (///59X.1314 A VS 2 TAitle) |
|  | WRITE (6,45t-fAVSZ(i),i=1,4) |
|  | WIRIE ( 6.50 ) |
|  |  |
|  |  |  |
| 43 | WKITE (6T50) |
|  | WKITE (6,46) (AVSZ(1),I=NOF25,NOFZ6) |
|  | WRITE (6,50) |
|  | FUKMAT (39x.4E13.6) |
| 46 |  |
| $11=10+26+1$ |  |
| $12=10 F 26+1$ NRINGS |  |
| N0 $47 \mathrm{~J}=1$ •110FZ |  |
| WKITE (0,46t (AVSZ(I),I=I1,12)LI $=12+1$ |  |
|  |  |  |
| 47 CONTINUE |  |
|  |  |  |
| 606 | WRITE (6,606) |
|  | FORMAA (///59x. 13 HAP VS 2 TA(iLE) |
|  | White (6,45) (APVSZ(I), $1=1,4$ ) |
|  | WRITE (0,50) |
|  |  |
|  |  |  |
|  | WRITE (6,4Gt (APVSZ(I),IENUFZ5,NOFZ6) |
| WRITE (0,50) . |  |
| 11 = HUF $26+1$ |  |
| $12=$ NOF $26+$ NKIIVGS |  |
| WRITE (6,46) (APVSZ(1), J=11.12) |  |
|  |  |  |




```
G910461-30
C PHOCEEO ON TO \forallAFOR REGION
        CALL VAPOR (ALPHA3,ZLV,TLV) 2160
        KUUIST = KOUNT+1 217C
        IF (KOUNT.LE.NCASE) GO TO 705 218S
```



```
        107 FORRIAT (////48X,35H****** OPERATIONS COMPLETE ****** ) 22U(
        STUP' 2210
        ENu 22as
            SUBIROUTINE LGVP (I,J,DERIV,T,G,Z,ZZ,FT,DZI
    C THIS KOUTINE HANDLES REACTOR CALCULATIONS FOR LIQUID VAPOR REGIONS
    C OF LACH ANNItLAR RING
            REAL INMAX2 10
```



```
            COMMON /ULOK2/A,AP,DELTA,DK.DPA,OON2H4,DONH3,R,CGM,RAUIUS,NMAX1& 3G
        X NMAX2,ALPHA1,ALPHA2,GETA,AGM,BGM,P,ZEND,MH2,IAN2,MNH3,MN2114 4C
            COMMON/BLOK3/K,KO,KP,KC3,KC4,HF,HL,HV,TF,CF,CFBAR,GAMMA,C4,NRINGS SC
            COMMON /BVTULS/A\forallSZ(234),APVSZ(234),DELVSZ(234) 6C
            COMMON/CCC/H4TBL(4n).H3TRL(40)}70
```



```
    C ASSUME TEMPERATURE CONSTANT IN LIQUIU VAPOR REGION
            CALL UNGAF (H4TBE,1FT,O.,H4.KK) 9C
    1甘2 CALL UNGAR (APVSZ,1,ZOKAD(I),AP,KK) 1UC
            DPA = ONN2H4*(F/492.)**1.832*14.7/P**1.-EXP{-.067*P*492./ 110
            X (14.7*T))) 120
```



```
            UZ =-H4/(NNMAXZ*DHDZ) 14C
            Z=z+0Z . 150
            J=J+1 16C
-G - STGNE CUHHEAT Z IA OUTPUT BLUCK
            22(1.J)= Z
                17C
```



```
    C CHECK IF WE HAVE REACHED THE INJECTOR TIP FOR THIS RING
            IF (Z-L0{It} 34,34,35 196.
    34G=GO(I)+F(I)*Z 200
            GO TO-7%-... 21C
    35G = GATZO(I) 220
```



```
    78 H(I)=H(I)+OHDZ*DZ 24C
            IF (H+IH-HV) 182,195,184 25C
    C WE HAVE EXCEEDED LIQUID VAPOK-VAFOR INTERFACE FOR THIS RING ---
    c. GACKSTEP TO t\forall-V GGUNJJARY
    184 OL = (HV-H(I))/CHUZ 26C
```



```
            ZZ(I.J)= 2 280
            H(E)=HV 290
    195 REIURN 30C
            ENO - 31C
```

    C ANNULAK RING
    
X
MN2HI -MBAR






G910461-30


```
    G910461-30
        UN()Z(J)=-(2.-DI:LFA)/OELFA**3**1.75+75.*VIS*(1.-DELTA)/(A*G(J)))* 3050
    x G(J)**2/(64,4*A*RHO(J))
        SUMFF= SUMP+OPGZ(J)*RAD(J)
    75 CONTINUE
```



```
        WFITE (6,599)
```



```
    A SX,'N2',10X,'H2' ) 3120
        WRITE (Ó,600) (J,ZZ(J),TT(J),C5(J),CJ(J),CZ(J),C1(J),J=1,NRINGG)
    GN\mp@code{ FURMAT (2GX,12,5X,GE12.5)}
    97 FORNAT (//54X,'PRESSURE =',E12.5) 3160
        WKITE (O, 36) (F1(J),F2(J),F3(J),F4(J),F3D(J),J=1,NRINGS)}317
    3n FORMAT (//3t,'EQUIVALENT'/ 40X,'MFRAC1',6X,OMFRAC2'.6X,'MFRAC3', 3180
        X 6X."MFRAC4*'0X, 'FRAC30'/ (35x,5E12.5))}319
        WRITE (6,006) 3200
```




```
        UO i00 J=1,NRINGGS 3230
        It (TT(J).LT.TLV(U)) GO TO 106 3240
    1OB CONTINUE 3éい?
    C CHECK IF WE HAVE REACHEO THE END OF THE REACTOR 
    C CHECK IF WE HAVE REACIUED ALL-VAPOR REGION
    C I7 IF (2-2\forallO(INRINGS)) 8.2:,21
    21 CALL DELITAL (CII,C12,CI3,CI4,T,G,RHO,RHOM,CK4,GR,TGR,Z,DZ) 328O
        VAPOK = . 3290
```



```
        IF (VP.GT.3) VF=3 3.31%
        GOTO & 33?0
        WRITE (6.95) 3.33C
        FOKMAT 1//30X:*THE PKOGRAM HAS CALCULATED A NEGATIVE PRESSURE --- }334
        XI<CTURN ANE TERMMINATE' )
        60)70-9% 3360
        94 WR1TE (0,103) J 337U
        103 FOKMAT 1//23X,'THE PKOGRAM HAS CALCULATED A NEGATIVE TEMPERATURE I 3380
        XIN KIIIG',I3,'m-- RETURII ANU TERMINATE' ) 339|
            GO TO 8 3400
    106 wNITE. (6,107) J 3410
```



```
    XAPOK/VAPOR INTERFACE IN RING',I3.' --- TRY USING A LARGER VALUE FO 34.30
    XH GO') }344
    99 RETURN 3450
        EHYG 3460
        SUUROUIINE URDER (<V,ZVO) O
    c IHIS HOUIIANE PLACES THE NUMGERS OF THE FIRST ARGUMENT ARRAY IN
    C ASCENDING ORDER AND STORES THEM IN THE SECOND ARGUMENT ARRAY
```



```
    UIMENSION ZV(100),ZVO(10U)
        20
        LAST = NKINGS-1 30
        UO 10 I=1,NRINCS 40
        ZVO(I) = ZV(I) 50
        1n CONTINUL
        60
            CONTINUL 60
```




G910451－30

| ALLUW $=0.05$ | 160 |
| :--- | :--- |
| FiKAC＝．99 | 170 |

Fiरal＝．99 ..... 170
$12 \quad B=0.11$ ..... 180
702 I＝1 ..... 190
NOT＝サ ..... 200
ADIV＝100． ..... 210
UULV＝100． ..... 220
TULE． 01 ..... 230
STUitt $=1.0$ ..... 240
KJ＝？ ..... 250
HULU日 ..... 260
$N 1=0$ ..... $<70$
13 M1二0 ..... 280
IL＝0 ..... 290
iK＝0 ..... 340
MNN＝0 ..... 310
320
$F=0.5$ ..... 330
20 MMA＝MM +1 ..... 340
A $\mathrm{CR}=\mathrm{L} / \mathrm{ALLIV}$ ..... 350
BUK＝（AtI）－BI／BUIV ..... 360
JIVT＝tsuIV ..... 370
 ..... उत0
IF（MM．E゙U．1） 60 TO 15 ..... 390
GU 1016 ..... 400
15 MMAK＝JIINT＋1 ..... 410
MAX＝NMAX－1 ..... 420
MINT＝0 ..... 430
60－10－17 ..... 440
16 MMA $=$ INI $T+J I N T T+1$ ..... 450
MIVT $=$ IHIT＋I ..... 460
MAX $=V_{1} \| A \lambda-1$ ..... 470
$17 \cdots x=0 . \theta$ ..... 480
DO 411 I $J=1$, MHAX ..... 490
 ..... $-540$
$X=F L O A T(1 U-1) * A D R$ ..... 510
60 T6 23 ..... 520
$49 \mathrm{KMJ}=\mathrm{IJ}$－MIIVT ..... 530
$x=\forall+F L U A T(K M J)$ WBQR ..... 540
$23 \mathrm{CP}=\mathrm{CG}(\mathrm{I}) * X /(\mathrm{A}(\mathrm{I})-\mathrm{B})-\{\mathrm{B} /(\mathrm{A}(\mathrm{I})-\mathrm{B})) * \mathrm{C}(\mathrm{C}(1)$ ..... 550
 ..... 560
C THIS IS THE GENERAL EQUATION FOR A LINE WITH NEGATIVE Y－INTERCEPT 24 FACFIEKfI）＊（CG\｛1） $2 * *(1 .-E N 12) *(C P) * * E N 12$ ..... 570
$\operatorname{EFACI}=\operatorname{GMMA}(I) * \operatorname{BETA}(I) *(1,-\operatorname{CP} / C G(1)) /(1 .+\operatorname{BETA}(1) *(1 .-C P / C G(I))$ ..... 580
IF（EFACF．GT， 88, IEFACI $=88$ ． ..... 590
FACT2＝EXP（EFACT） ..... 610
HF $(x)-37-40-37$ ..... 610
37 RCウニト ACT1＊FACT？ ..... 620
35 SfiJt＝（1．fX－1．fA（I））＊X＊i＊RCP ..... 630
40 CONT INUE ..... 640
50 SUMEA．A ..... 650
SUMA $=0.0$ ..... 660
00 H－Hzeromax ..... 670
IF（1L．GT．NINT）GO TO 61 ..... 480
SUM $=$ SUMT－S（IL） ..... 690
GO TU ho ..... 700
G1 SUMA＝SUNA＋S（IL） ..... 710
60 CUNTINUE ..... 720
SH1＝ ..... 730
SGMA $=(5(1)+2 . * S U M) *(A D R /(2 . * D P A(I)))+(5(M M A X)+2 . * S U M A) *(B D R /(2$. ..... 740







```
*
$10461-30
    K2 = . N-N... 1550
    PS1 = n. 1570
    HS2 =A% - -\cdots................... 1580
    ULLXUA = (1.-XOA)/FLOAT(NX) 1590
    EALEULATE-PAOFILEL-CHNUES FOR-INTEORANO-PUNCTIONS
        XA = XOA 16,10
        UO 770 I=10NXI - ... - . - 1610
        CP(X/A) IS A LINEAR PROFILE DURING FIRST APPROXIMATION
        IF (LPP.GT.1) 60 TO 664 1620
        CPX(1) = (XA-XOA)/(1,-XOA)*CPS 1630
```



```
            x (1.+BETA*(1.-CPX(1)/CI3))) 1650
        UX(1) = XA -- .- 1660
        XA = XA+UELXOA 1670
70 CONTIHUE 1680
    TAKE INTERVAL FUNCT'N MIDPTS AS CONSTANT VALUE FOR CP(X/A) AND RHET
    #*74-1ご相
    CP\times(I) = (CPX(I)+CPX(I+1))/2. 1700
    RHLT(I) = (RHET(I)+RHET(I+1))/2. & 17:0
71 COISTINUE 1720
    XA = XOA OELXOA 1
    CTr,Mi = (A*VINU+1.)/(A*VINU) 1740
    INHEONAL EOUATION-POLTOWS
    CPOX(1) IS SPECIAL CASE:... }x=x
        UXL = XOA
        1750
        DXU = UXL+UELXOA 1
    KK1 = 0.
        1770
    DO 377 I=1,NX 1780
```



```
    EXL = UXU 18U0
    OX゙U = UXUCUELXOA 1810
77 CUIVTIINUL 1620
    CHGX(1) =C13-A*A/OP3*RR1 1830
    It (CPUX(1).LT.U.) CPOX(1)=0. 1840
```



```
    R1 = HIt+RHET(I)*EVALI(XOA,XA) 1660
    XOA = XA 
    XA = XA+HELXOA 1880
    CONTIHUE 1890
```




```
    XA = XA-DF.LXOA 1920
    WO 773 1=1NT1,NXM1 1930
    FS1 = PSI+RHET(I+1)*EVAL2(XA,XAD) 1940
    PS? = PS2+RHET(1+1)*EVAL1\XA,XAD) 1950
```



```
    XAL = XAD+UELXOA 1970
    CONIINUE 1980
    K2 = PS.L-CTRM*PS? 1990
    INT: = INT1+1. 2000
    CHUX(L) := CI3-A*A/UP3*(R1+R2) 2010
```



```
    XOA =XO/A 2030
    XA = XOA+DELXOA 2040
    L =L+1 2050
    K1 = n.
    R2 = 0.
    2060
    2070
    usf. = n.
2000
```




| G910461-30 |  |
| :---: | :---: |
| $705 \mathrm{~N}=1$ | 80 |
| GU TO 707 | 90 |
| $706 \mathrm{~m}=2$ | 100 |
| 707 N2= 1 | 110 |
| -701 $\mathrm{H}=11+1$ | 120 |
| $702 \mathrm{NL}=\mathrm{N}+1$ | 130 |
| $0030 \mathrm{~L}=\mathrm{IIOII}$ | 140 |
| IF ( T(L) + 0. ) 00,60,51 | 150 |
| $60 \quad k k=-1$ | 160 |
| $1 . c=0$. | 170 |
| 60 70-9999 | 140 |
| 51 NX $=$ T(L) | 190 |
| IF (T(L+1) + n. ) 60. 52.50 | 200 |
| $52 \mathrm{NY}=0 \mathrm{n}$ | 210 |
| -50 1053 | 220 |
| 50 NY $=T(L+1)$ | 230 |
| - E3-CONTINUE | 240 |
| KK $=0$ | 250 |
| $K Y=0$ | 260 |
| $x X=X 1 y$ | 270 |
| $Y Y=Y i N$ | 280 |
| $J 1=11+2$ | 290 |
|  | 300 |
| IF (XX-T (J1) $301 ; 306,4 \mathrm{C})$ | 310 |
| 400 Do 302 JこJiod2 | 320 |
| IF ( XX -TiJ) $304,304,302$ | 330 |
| 302 CUnTIHHE | 340 |
| $309 \mathrm{KK}=2$ | 350: |
|  | 360 |
|  | 370 |
|  | 380 |
| $301 \mathrm{KK}=1$ | 390 |
| $\cdots \mathrm{XA}_{\boldsymbol{\lambda}}=\mathrm{F}(\mathrm{JI})$ | 400 |
| $306 \mathrm{JX1}=\mathrm{J1}$ | 410 |
| 304 IF (J-J1-1) 301,306,307 | 420 |
|  | 430 |
| 307 IF (J-J2) 303,308,309 | 440 |
| 303 J 1 = J-HE | 450 |
| --305 CONTINUE | 460 |
| XINT $=X X$ | 470 |
| 1500 00 1599 L=1,N1 | $480{ }^{-}$ |
|  | 490 |
| $\cdots \quad-x(t) \equiv$ FtJxit | 500 |
| $L Y=J X 1+N X$ | 510 |
| 1599 $\quad \begin{array}{ll}\text { Y(L) } & =T(E Y)\end{array}$ | 520 |
| 1599 JX1 $=$ JX1+1 | 530 |
| $\underline{L}=1$ |  |
| - $3000 \mathrm{JI}=\mathrm{di}+\mathrm{NX}$ - | 560 |
| $J 2=J 2+N Y$ | 570 |
| IF 1 YY-Tfditi311,316,401 | 580 |
| 401 DO $312 \mathrm{~J}=\mathrm{J} 1, \mathrm{~J} 2$ | 590 |
| 312 CONTINUE (KY-TH) 314.3140312 | 609 |
|  | 610 |
| - $319 \mathrm{KY}=-6$ | 620 |
| $Y Y=T(J 2)$ | 630 |
| 318 UY1 = J2-N | 640 |
| GO 10315 | 650 |
| $Y Y=T(J 1)$ | 660 670 |






[^0]:    *It is more commonly assumed, without benesit of experimen'al evidence, that the decomposition reaction is $3 \mathrm{~N}_{2} \mathrm{H}_{4} \rightarrow 4 \mathrm{NH}_{3}+\mathrm{N}_{2}$, followed by dissociation of one of the four ammonia molecules to nitrogen and hydrogen. This two-step process leads to the same overal. reaction cited above but assumes that a minimum of 25 percent of the amonia produced by hydrazine decomposition also decomposes. The fractional ammonia dissociation, $f$, calculated assuming the validity of the two-step process is 1 elated by the fractional ammonia dissociation calculated in the present report by

[^1]:    *Equations of this type are presented in somewht dipferent form in Ref. 7. The last term on the right-hand side of the equation reflects the neat trans. ferred by the radial diffuston of nass.

