A STUDY OF TWO-BODY STRONG INTERACTIONS OF ELEMENTARY PARTICLES

by

J. Edwin Rush, Jr.

Final Technical Report

This research work was supported by
National Aeronautics and Space Administration
Grant NGR 01-002-063

Research Institute
The University of Alabama in Huntsville
P. O. Box 1247
Huntsville, Alabama 35807

May 1970

A Study of Two-Body Strong Interactions of Elementary Particles

By J. Edwin Rush, Jr.

This report summarizes work done under NASA grant NGR 01-002-063 during the period 27 January 1969 to 1 May 1970.

The Process $\pi^- p \rightarrow \Lambda K^o$:

Work on Λ - K production has been published previously.[1,2] It concerned the calculation of cross-section and polarization data by determining the coupling parameters in a pole resonance model, using a computer program to vary parameters and fit the data. In the former work[1] the fit was accomplished in a simple manner by direct calculation and manual variation of parameters. In the latter[2], fitting was done using a subroutine SEARCH obtained by the author from a colleague at Vanderbilt University. The results of this paper have recently been incorporated in the compilation of Rosenfeld, et al.[3], who quote partial widths for decay into $\Lambda - K^o$ of the resonant states: $D_{15}(1670)$, $F_{15}(1688)$, $S_{11}(1700)$, $P_{11}(1780)$, and $P_{13}(1860)$. These are the most reliable and, for the S_{11}, P_{11}, and P_{13}, the only values available.

During the reporting period the principal investigator used two more-elaborate computer programs for performing fits by "chi-square" minimization. One minimizes the function χ^2 given by

$$\chi^2 = \sum_i (D_i^2 - E_i^2)/\Delta_i^2$$

where D_i is a calculated cross-section or polarization value, E_i is a corresponding experimental data point, and Δ_i is the experimental error. One program, MINFUN[4], is capable of performing a routine search of the parameter space and converging on each minimum in turn. The other, MINUIT[5], performs a Monte Carlo search of the parameter space before minimizing. Both programs were modified to run on the UNIVAC 1108 computer and were checked with the original $\Lambda - K^o$ calculation used in Ref. 2.

The results of Ref. 2 involved the use of form factors on pole terms to restrict their contributions at energies near the upper limit of the calculated range, which was 1613 MeV (threshold) to 1840 MeV total center-of-mass energy. It was interesting to see if an acceptable solution could be obtained without form factors but with the NΛK coupling constant no longer held constant at a value of 13.5 (since there is considerable dispute over this value). MINUIT produced minima with relatively low values of χ^2 for both fixed and variable NΛK coupling. The highest value of the varied coupling constant was $\sim$ 7.5. There were, of course, some variations in the resonance coupling parameters from the previous results (Ref. 2). Unfortunately, these results for these minima showed a strong increase in the total cross-section just beyond the calculated energy range, due to the strong contribution of the pole terms with no form factors. These results are, as yet, unreported.

Additional work on $\Lambda - K$ production was stimulated by a preprint of a paper by Van Dyck, et al., which was later published.[6] These authors reported a sharp peak in the total $\Lambda - K^o$ cross-section near the Σ - K threshold, and suggested the possibility that the peak might be a manifestation of a cusp effect due to the opening of the Σ - K channel. The principal investigator wrote a computer program to calculate partial wave amplitudes for $\pi^- p \rightarrow \Lambda K^o$ using parameters discussed previously. The S-wave amplitude was then studied to see whether the expected cusp could reasonably produce an increase of ~30% in the total cross-section, in agreement with Ref. 6. It was concluded that such an increase would require a quite unusual behavior of the K-matrix amplitude, which are expected to vary smoothly in the cusp region. These results have been recently reported. Further work on the peak phenomenon will require very accurate angular-distribution and polarization data, which are as yet unavailable.

Form Factors in the Peripheral Model:

Another recent paper[8] on the use of form factors to compensate for Kronecker-δ terms in the peripheral model recalled some results of pole term calculations in Λ - K^o production by the principal investigator. These showed that Feynman techniques were incompatible with dispersion-relation results for higher spin particles (spin -1 and above). The dispersion-relation results do not contain unwanted Kronecker-δ contributions, which have required form factors for their suppression. This fact was then pointed out in a subsequent paper.

The Processes $\pi N \rightarrow \Sigma K$:

The major portion of the work under NGR 01-002-063 was concerned with the processes $\pi^+ p \rightarrow \Sigma^+ K^+$, $\pi^- p \rightarrow \Sigma^o K^o$, and $\pi^- p \rightarrow \Sigma^- K^+$. These processes were studied jointly, in a manner similar to that used for

$\pi^- p \rightarrow \Lambda K^o$. Charge independence (isospin conservation) was used to relate the three processes by two independent isospin states.

All available data below 2045 MeV center-of-mass energy were surveyed, and 451 data points (differential cross-section or polarization) at about 20 different energies were selected for the final calculations. (The energy limit was chosen to be one half-width above the mass of the F_{37}(1940) resonance.) Data which were omitted involved too few events to be meaningful, were unnormalized (no total cross-section), or were not unambiguously defined. The data selected[10-29] are summarized in Table I.

Resonances used in the model are given in Table II. They were chosen in accord with results of phase-shift analyses of π - N scattering, as given in Ref. 3 and previous versions of the same report. Masses and widths of resonances were taken from the 1969 average values.[30] The nearest poles in each channel were included as background. These are the nucleon pole in the S channel, the Λ and Σ poles in the u channel, and the K* pole in the t channel. The most obvious omission is that of the K** (spin 2) whose inclusion would have added 5 additional parameters to the 15 parameter model (the other parameters are resonance and pole couplings). It is reasonable to expect that the addition of these 5 parameters would not significantly improve the fit, and that if one does not take the K* coupling constants too seriously, then the K* gives a reasonable representation of t-channel singularities. The form of resonance and pole contributions follows Ref. 2. The convention on coupling constants is in accord with Sakurai.[31]

A subroutine to be used in MINUIT was written and modified for MINFUN. It calculates differential cross-sections and polarizations for all three processes, incorporating charge independence, and gives the function χ^2 as output. Monte Carlo calculation with arbitrary input was performed by MINUIT, producing the 15 best values of χ^2 and of the parameters x_i. A minimum was then found for each of the 15 inputs. For comparison, several minimum regions were also found with MINFUN and their minima obtained. There resulted 6 distinct minima with χ^2 values as shown in Table II. If one defines N = (no. data points) - (no. adjustable parameters), so that N = 436, then the value of $\sqrt{\chi^2/N}$ obtained from Minimum A is 1.7.

It is necessary to know decay asymmetry parameters for $\Lambda \to p\pi^-$ (from $\Sigma^o \to \Lambda\gamma$) and for $\Sigma^+ \to \pi^o p$, since the polarizations are determined by the decay asymmetry in these processes. These parameters were taken to be, respectively, - 0.647 and 1.0, in accord with Ref. 30. The Σ^- polarization must be measured by scattering, since the asymmetry parameter for Σ^- decay is almost zero. The Breit-Wigner formula for the partial-wave resonances contains an interaction radius R which we chose as 1 Fermi.[2] The resonance parameters x_i are roots of products of reduced widths multiplied by R and are also given in Table II, as are the pole parameters, including both vector and tensor K* contributions.

Various resonance parameters, including those taken from Ref. 30 and the results of Minimum A are shown in Table III. The most remarkable result in Table III is the extremely low value of the reduced width for the $F_{37}(1940)$. This resonance was found to have a dominant effect in simpler models [32-34] for $\pi^+ p \to \Sigma^+ K^+$. Recent work by Feuerbacher and Holladay[35], incorporating the possible $P_{33}(1688)$ and $D_{35}(1954)$ and the well-known $P_{33}(1236)$ in addition to the 5 T = 3/2 resonances used in this model, gave $\gamma_{\Sigma K} = 0.004$ for the $F_{37}(1940)$. Of all the minima in the present model, however, the largest value obtained is 1.4×10^{-3} (for Minimum F).

We note that Feuerbacher and Holladay obtained a reduced width for the $P_{33}(1688)$ consistent with zero, in agreement with its omission from the present model. They find values of 1.04 and 1.99 for the $P_{33}(1236)$ and the $D_{35}(1954)$, however. The latter resonance is suspect[3] while the former probably gives a much smaller contribution, because the Breit-Wigner resonance formula has been found to give resonance tails which are much too large.[36] It was for the latter reason that the $P_{11}(1470)$, the $D_{13}(1520)$, the $S_{11}(1550)$, and the $P_{33}(1236)$ were omitted from the present model. Indeed, one can question whether the $S_{31}(1630)$, the $D_{15}(1675)$, and the $F_{15}(1690)$ ought not to be suppressed somehow at the higher energies. Deans[37] has repeated the studies of $\pi^- p \to n\eta$[38] and $\pi^- p \to \Lambda K^o$[2] with resonant amplitudes damped by exponentials beyond one half-width above the resonant mass. He found that, for those resonances actually in the energy region studied, the fits were better but the reduced widths were essentially unchanged. This makes one less dubious about the treatment of the S_{31}, D_{15}, and F_{15} in the present model, but leaves considerable doubt about the interpretation of the P-wave parameters in Ref. 35, as well as serious question about the influence of the D_{35}, which may be spurious. If one continues the comparison of reduced widths, however, one finds reasonable agreement in the case of the S_{31} and the P_{31}, but the reduced widths obtained in Ref. 35 for the D_{33} and F_{35} are much smaller than those of the present model.

There have been no significant studies of the resonance region of $\pi^- p \rightarrow \Sigma^- K^+$ of the type represented by Ref. 32-35. A phase-shift analysis[38] of $\pi^+ p \rightarrow \Sigma^+ K^+$ and two analyses of[14,26] the three processes jointly are inconclusive. After further analysis of the present results, it is intended that they be submitted for publication in the Physical Review.

Footnotes

1. J. E. Rush and W. G. Holladay, "Application of π - N Scattering Results to Λ - K^o Production," Phys. Rev. 148, 1444 (1966).
2. J. E. Rush, "Parameters of Low-Energy Λ - K^o Production," Phys. Rev. 173, 1776 (1968).
3. Particle Data Group, "Review of Particle Properties," Rev. Mod. Phys. 42, 87 (1970).
4. Bealls, Computer Program MINFUN, (Lawrence Radiation Laboratory, Berkeley, 1962).
5. F. James and M. Roos, Computer Program MINUIT (CERN, Geneva, 1967).
6. C. Van Dyck, R. Blumenthal, S. Frankel, V. Highland, J. Nagy, T. Sloan, M. Takats, W. Wales, and N. Werbeck, "Cross-Section for $\pi^- p \rightarrow \Lambda K^o$: Structure Near Σ Threshold," Phys. Rev. Letters 23, 50 (1969).
7. J. E. Rush, "Cusp Phenomena in Associated Production of Elementary Particles," presented at Annual Meeting of the Alabama Academy of Science, 10 April 1970.
8. P. K. Williams, "Form Factors, Kronecker- δ Terms, and the Absorptive Peripheral Model," Phys. Rev. 181, 1963 (1969).
9. J. E. Rush, "Kronecker- δ Terms in the Peripheral Model and Dispersion Relations," Phys. Rev. 188, 2535 (1969).
10. F. Grard and G. A. Smith, Phys. Rev. 127, 607 (1962).
11. N. L. Carayannopoulos, G. W. Tautfest, and R. B. Willmann, Phys. Rev. 138, B433 (1965).
12. F. S. Crawford, Jr., F. Grard, and G. A. Smith, Phys. Rev. 128, 368 (1962).
13. C. Baltay, H. Courant, W. J. Fickinger, E. C. Fowler, H. L. Kraybill, J. Sandweiss, J. R. Sanford, D. L. Stonehill, and H. D. Taft, Rev. Mod. Phys. 33, 374 (1961).
14. G. Borreani and C. E. Kalmus, Lawrence Radiation Laboratory Report, UCRL-18350 (unpublished).
15. H. W. J. Foelsche, A. Lopez-Cepero, C.-Y. Chien, and H. L. Kraybill (unpublished).
16. E. C. Fowler, L. R. Fortney, J. W. Chapman, P. L. Connolly, E. L. Hart, P. V. C. Hough, and R. C. Strand, Bull. Am. Phys. Soc. 9, 420 (1963).

17. P. Daronian, A. Daudin, M. A. Jabiol, C. Lewin, C. Kochowski, B. Ghidini, S. Mongelli, and V. Picciarelli, Nuovo Cimento 41, 503 (1966).

18. J. A. Anderson, F. S. Crawford, Jr., and J. C. Doyle, Phys. Rev. 152, 1139 (1966).

19. T. O. Binford, M. L. Good, V. G. Lind, D. Stern, R. Krauss, and E. Dettman, Phys. Rev. 183, 1134 (1969).

20. Y. S. Kim, G. R. Burleson, P. I. P. Kalmus, A. Roberts, C. L. Sandler, and T. A. Romanowski, Phys. Rev. 151, 1090 (1966).

21. R. L. Crolius, V. Cook, B. Cork, D. Keefe, L. T. Kerth, W. M. Layson, and W. A. Wenzel, Phys. Rev. 155, 1455 (1967).

22. L. L. Yoder, C. T. Coffin, D. I. Meyer, and K. M. Terwilliger, Phys. Rev. 132, 1778 (1963).

23. Y. S. Kim, G. R. Burleson, P. I. P. Kalmus, A. Roberts, and T. A. Romanowski, Phys. Rev. 143, 1028 (1966).

24. O. Goussu, M. Sene, B. Ghidini, S. Mongelli, A. Romano, P. Waloschek, and V. Alles-Borelli, Nuovo Cimento 42, 606 (1966).

25. O. I. Dahl, L. M. Hardy, R. I. Hess, J. Kirz, D. H. Miller, and J. A. Schwartz, Phys. Rev. 163, 1430 (1967).

26. J. J. Edgington, V. J. Howard, M. C. Miller, R. J. Ott, P. J. Duke, R. E. Hill, W. R. Holley, D. P. Jones, and J. J. Thresher, Phys. Rev. 177, 2103 (1969).

27. W. Weldon, H. Steiner, G. Shapiro, C. Schultz, C. H. Johnson, Jr., L. Holloway, M. Hansroul, P. D. Grannis, B. Dieterle, O. Chamberlain, and J. Arens, Phys. Rev. 167, 1199 (1968).

28. M. L. Good and R. R. Kofler, Phys. Rev. 183, 1142 (1969).

29. J. C. Doyle, F. S. Crawford, Jr., and J. A. Anderson, Phys. Rev. 165, 1483 (1968).

30. Particle Data Group, "Review of Particle Properties," Rev. Mod. Phys. 41, 109 (1969).

31. J. J. Sakurai, Invariance Principles and Elementary Particles (Princeton University Press, 1964).

32. L. E. Evans and J. M. Knight, "Resonance Model for $\Sigma^+ - K^+$ Production," Phys. Rev. 137, B1232 (1965).

33. W. G. Holladay, "Poles and Resonances in the Process $\pi^+ + p \rightarrow \Sigma^+ + K^+$," Phys. Rev. 139, B1348 (1965).

34. J. L. Feuerbacher and W. G. Holladay, "The Parity of N(1925, 7/2) in a Model for the Process $\pi^+ + p \rightarrow \Sigma^+ + K^+$," Phys. Letters 19, 599 (1965).

35. J. L. Feuerbacher and W. G. Holladay, "Partial Widths for the Decay of T = 3/2 Nucleon States into Σ^+K^+" (unpublished).

36. See, e.g., A. Donnachie, Rapporteur's Talk in 14th International Conference on High-Energy Physics (CERN, Geneva, 1968), p. 146.

37. S. R. Deans (private communication).

38. C. Lovelace, F. Wagner, and J. Iliopoulos, "Phase-shift Analysis of $\pi^- p \rightarrow \Lambda K^0$ and $\pi^+ p \rightarrow \Sigma^+ K^+$," CERN preprint, quoted in Ref. 36 (unpublished).

Table I. Data used in the study of $\pi N \to \Sigma K$. (+, 0, -) refer to $\pi^+ p \to \Sigma^+ K^+$, $\pi^- p \to \Sigma^0 K^0$, and $\pi^- p \to \Sigma^- K^+$, respectively.

Reference	Process	Momenta MeV/C	No. Points		No. Events
			dσ/dΩ	P(θ)	
10	+	1053	10		274
11	+	1111	10	5	259
11	+	1206	10	5	340
11	+	1265	10	5	296
12	+	1170	20	5	251
13	+	1222	10		227
13	+	1393	10		52
14	+	1350	20		256
14	+	1430	20		277
15	+	1390	10		56
15	+	1760	10		64
16	+	1490	10		
17	+	1590	10		192
18	0	1170	10	5	918
19	0	1128	10		756
19	0	1235	10		256
19	0	1277	10		314
19	0	1326	10		168
20	0	1170		7	846
20	0	1320		8	919
21	0	1225		2	
21	0	1275	9	2	44
21	0	1325	10	2	78
21	0	1365	10	2	50
22	0	1508	8		134
23	0	1500	9	3	253
24	0	1590	10		65
25	0	1500	12		
25	0	1600	12		70
25	0	1700	12		
26	-	1130		7	
27	-	1145		1	
28	-	1125	10		465
28	-	1225	10		216
28	-	1275	10		237
28	-	1325	10		156
29	-	1170	10		1338
24	-	1590	10		285
25	-	1500	10		
25	-	1600	10		
25	-	1700	10		

Table II. Pole and resonance parameters[a] for individual minima. All numbers should be multiplied by 10^{-2}.

Resonance or Pole	Minima					
	A	B	C	D	E	F
S_{31}(1630)	2.68	-2.48	1.34	-0.0900	1.82	1.34
P_{31}(1905)	4.09	-4.06	1.33	-1.53	-1.56	1.56
D_{33}(1670)	-6.10	-0.967	7.57	7.37	2.33	1.09
F_{35}(1880)	-1.38	3.15	2.36	2.65	1.53	1.30
F_{37}(1940)	0.023	-0.621	0.383	0.401	-0.101	-0.867
S_{11}(1715)	-8.58	8.94	7.89	-7.40	-8.40	3.57
P_{11}(1785)	4.54	-1.66	2.96	-3.40	-1.47	-10.13
P_{13}(1855)	-0.720	-0.446	2.87	3.04	3.17	-1.74
D_{15}(1675)	-1.89	3.50	3.71	-4.01	-2.52	0.409
F_{15}(1690)	-1.21	8.13	5.14	-4.24	-0.797	1.81
Nucleon	11.5	47.6	5.69	15.8	6.45	25.3
Sigma	-51.8	-17.7	80.8	74.5	55.4	-25.1
Lambda	61.0	-69.3	60.2	-71.6	-56.4	18.7
K*(Vector)	-14.9	9.14	12.8	16.6	-0.0850	-11.2
K*(Tensor)	21.3	1.29	3.19	-6.35	-5.75	-30.0
χ^2	1317	1409	1525	1535	1651	1712

[a] Resonance parameters are $R(\gamma_{\pi N}\,\gamma_{\Sigma K})^{\frac{1}{2}}$ where R is the interaction radius and γ's are reduced widths. Pole parameters are $g_1 g_2/4\pi$.

Table III. Resonance results for Minimum A. All units are MeV except for ratios.

Resonance	W_r	Γ	$(\gamma_{\pi N}\gamma_{\Sigma K})^{\frac{1}{2}}$	$\gamma_{\Sigma K}$	$\gamma_{\pi N}$	$\Gamma_{\Sigma K}/\Gamma$	$\Gamma_{\pi N}/\Gamma$
S_{31}	1630	160	5.29	3.50	7.99		
P_{31}	1905	300	8.07	5.82	11.2	0.065	0.25
D_{33}	1670	225	-12.03	18.63	7.77		
F_{35}	1880	250	-2.72	0.617	12.0	9.6×10^{-4}	0.18
F_{37}	1940	210	4.6×10^{-3}	1.02×10^{-6}	20.7	3.9×10^{-9}	0.42
S_{11}	1715	280	-16.9	9.26	30.8	0.046	0.66
P_{11}	1785	405	8.95	3.42	23.4	0.015	0.34
P_{13}	1855	335	-1.42	0.142	14.2	0.0012	0.27
D_{15}	1675	145	-3.73	0.837	16.6		
F_{15}	1690	125	-2.39	0.160	35.6		