NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Report 32-1465

JPL Development Ephemeris Number 69

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Preface

The work described in this report was performed by the Mission Analysis Division of the Jet Propulsion Laboratory.

Acknowledgment

The system of computer programs used in this research effort were formulated by Dr. Charles C. Lawson of JPL. The system analysis and coding was carried out by the Federal Systems Division West of International Business Machines Corporation.

In particular, two individuals Rex E. Reed and Dan Dannenfeldt of IBM are singled out for their interest and dedication in the successful implementation of the system of programs to improve the knowledge of the locations of planets in the solar system.

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Abstract

The third issue of JPL Ephemeris Tapes is described, and is designated JPL Development Ephemeris No. 69 (DE 69). It is a special-purpose ephemeris that covers a short time span and does not replace DE 19 (Ref. 1) as the JPL export ephemeris. These tapes carry the positions and velocities of the planets and of the moon, nutations and nutation rates in longitude and obliquity, and second and fourth modified differences of all these quantities for the interval from October 28, 1961 to January 23, 1976. The description includes discussions of the improvements in the Lunar Ephemeris and the planetary ephemerides made subsequently to the second issue of the JPL Ephemeris Tapes (DE 19). These tapes will be distributed through the NASA Computer Software Management and Information Center (COSMIC).

JPL Development Ephemeris Number 69

I. Introduction

The JPL Development Ephemeris 69 described in this report is the third release from the JPL Ephemeris Tape System. It is a special purpose ephemeris that covers a short time span and will not replace DE 19 (Ref. 1) as the JPL Export Ephemeris. For users who need planetary positions before 1962 and after 1976, the DE 19 ephemeris is still available.

Tape	Julian date (Calendar date)	to	Julian date (Calendar date)
DE 69	243 7600.5		244 2800.5
	(1961 Oct. 28.0)		(1976 Jan. 23.0)

This ephemeris is the first gravitationally consistent ephemeris computed and exported from JPL. The computations are carried out by a system of programs referred to as the Solar System Data Processing System (SSDPS). The numerical integration, the observational data set, and comparison of these data with the simultaneous integrations of the nine planets are discussed.

The lunar data in DE 69 were not produced by the SSDPS integration, but are nonetheless quite different from those of DE 19. These data are the result of a composite process that included a long-span numerical integration of the moon only. The construction process is discussed in some detail in the following section.

Planetary data are heliocentric and are expressed in astronomical units (AU) and AU/day, and lunar data are geocentric and expressed in fictitious units called "earth radii" (R_{em}) and "earth radii"/day. Translation between the geocenter and the earth-moon barycenter is accomplished using the earth/moon mass ratio μ^{-1}. The values of these parameters currently recommended for most satisfactory use of DE 69 are (Ref. 2) as follows:

$$AU = 149{,}597{,}893.0 \text{ km}$$

$$R_{em} = 6378.1492 \text{ km}$$

$$\mu^{-1} = 81.301$$

Master copies of DE 19 and DE 69 have been supplied to COSMIC, University of Georgia, which will serve as the primary distribution point for these data.

II. Lunar Ephemeris

The requirements of high-precision analysis of spacecraft data and the accurate determination of the coordinates of DSN tracking stations have been major motivating factors in the persistent efforts to improve the quality of the JPL lunar ephemerides. Residual characteristics have sometimes suggested problem areas and potential improvement techniques. It is just such a situation that led to the development of JPL Lunar Ephemeris Number 16.

A. Background

It has been noted for some time that the tracking station locations derived from analyses of planetary spacecraft data differ systematically from the locations of the same stations based on data from lunar missions, the differences being on the order of tens of meters. This circumstance has been difficult to understand. If it were attributed to errors in the Lunar Theory, or in its fitting to observations, the size and nature of the error would make it easily observable. On the other hand, the proliferation of coordinate systems in astronomy presents a more subtle pitfall. Van Flandern (Ref. 3) pointed out that, for the fitting of the Lunar Theory to observations, E. W. Brown determined the equinox from his own reference stars. Thus, the reference direction of the theory is unique to that theory; it is apparently very near to Newcomb's equinox. To be consistent with the planetary ephemerides based on modern observations, the lunar ephemeris should be referred to the FK4 coordinate system (Ref. 4). It seems safe to assume that virtually all users of the JPL Ephemeris Tape System have, from its inception, tacitly assumed that this was the case. This must be regarded as an error in the precepts for application of the ephemeris rather than in the ephemeral data themselves. Nonetheless, the transformation to the FK4 model should be done in the Ephemeris Tape System, if the necessary parameters are known.

Van Flandern has, in fact, undertaken to solve for the transformation between Brown's equinox and that of the FK4, as a part of a more general study to correct the lunar elements. A very preliminary discussion of this effort is given in Ref. 3, and a complete solution is presented in Ref. 5. These latter results were not the final results of the work; some of the values have changed subsequently, but the equinox shift has remained fairly stable throughout the work. However, the coupling between parameters is such that the equinox shift so derived should be applied as a part of the overall system.

B. Transforming the JPL Ephemeris

This entire question will vanish when the lunar ephemeris is based on a numerical integration fit to real observations referred to the FK4 frame. This work is underway, but may not be expected to produce operationally useful results for some time yet. Nonetheless, previously reported work (e.g., Ref. 6) has indicated the urgent need for an integrated lunar ephemeris for operational use, and it seems desirable that it be on the FK4 system if possible.

The problems involved in producing long-interval integrations of the lunar motion were discussed in Ref. 7, where the effects not modelled in a PLOD integration (Ref. 8) were described, as was a theoretical ephemeris (LE 13) which had these effects removed analytically. In attempting to place such an ephemeris on the FK4 system, no alternative currently exists but the application of the results in Ref. 5. The corresponding expressions to be applied to a theoretical ephemeris were supplied by Van Flandern and these were applied to LE 12. The effect of the equinox shift was to change the mean longitude by an amount

$$\Delta L = -0''.8746 + 0''.732T$$

T in centuries from 1950. It may be presumed that this ephemeris (LE 14) is on the FK4 system, contains none of the effects known to be unmodelled in PLOD, and suffers the gravitational defects of the Brown theory. This was used as a source theory to which a PLOD numerical integration was fit over the interval 1950–1970. The statistics of the fit are given in Table 1, where the ephemeris is designated LE 15. This ephemeris does not have the gravitational defects of the Brown theory, but cannot be used directly because of the effects unmodelled in the integration, the effects that were removed analytically in the construction of LE 12. Thus, it was necessary to replace them; this was done by analytic modifications applied to the coordinates of LE 15. This

Table 1. Statistics of the fit of LE 15 to LE 14

Coordinate	δr	$r \cos \beta \, \delta\lambda$	$r \, \delta\beta$
Mean deviation	+64.2 m	−0.2 m	−4.8 m
Standard deviation	223.8	211.4	249.5
Extremal deviation	+851.4	+757.7	−758.8

last step in the sequence was designated LE 16[1], which should fit Van Flandern's version of the Brown theory in the same way that LE 15 fits LE 14. A sample comparison of LE 16 with LE 4 (Ref. 9), which is in DE 19, is given in Fig 1.

Because of the interest in the station location problem, and of the insensitivity of the *Mariner* trajectory to such small changes in the lunar ephemeris, LE 16 was incorporated into DE 69, the operational ephemeris for *Mariner* Mars 1969 pre-encounter activities. It is not yet clear how much of an improvement the equinox shift has produced, but station location analyses by other JPL investigators (Refs. 10 and 11) indicate that spacecraft trajectory residuals are reduced by use of LE 16. Although consistency between lunar and planetary mission results has not yet been achieved unambiguously, the disparities are reduced in most cases. The initial tests of the application of real observations of the moon have also been encouraging. The lunar starting conditions for a ten-body integration were taken from LE 16 and, while the integration and comparison with observations covered only 100 days, an equinox error would appear as a roughly constant bias in right ascension. None was observed as large as a few tenths of an arc second.

[1]This discussion presents some insight into the proliferation of LE and DE numbers in the ephemeris development effort. Each distinct ephemeris receives a number, to minimize possible confusion. In this case, LE 12, 14 and 15 will never be used operationally, but they were necessary steps in the construction of LE 16.

III. Planetary Ephemeris

A. Numerical Integration

The current JPL Export Ephemeris, DE 19 (Ref. 1), consists of nine successive single-body integrations. The relativistic differential equations of motion used in this integration were based upon the Schwarzschild line element which is given in Brouwer and Clemence (Ref. 12).

The SSDPS was completed late in 1967. It is a series of programs that is used to integrate numerically the motions of the nine planets simultaneously, compare

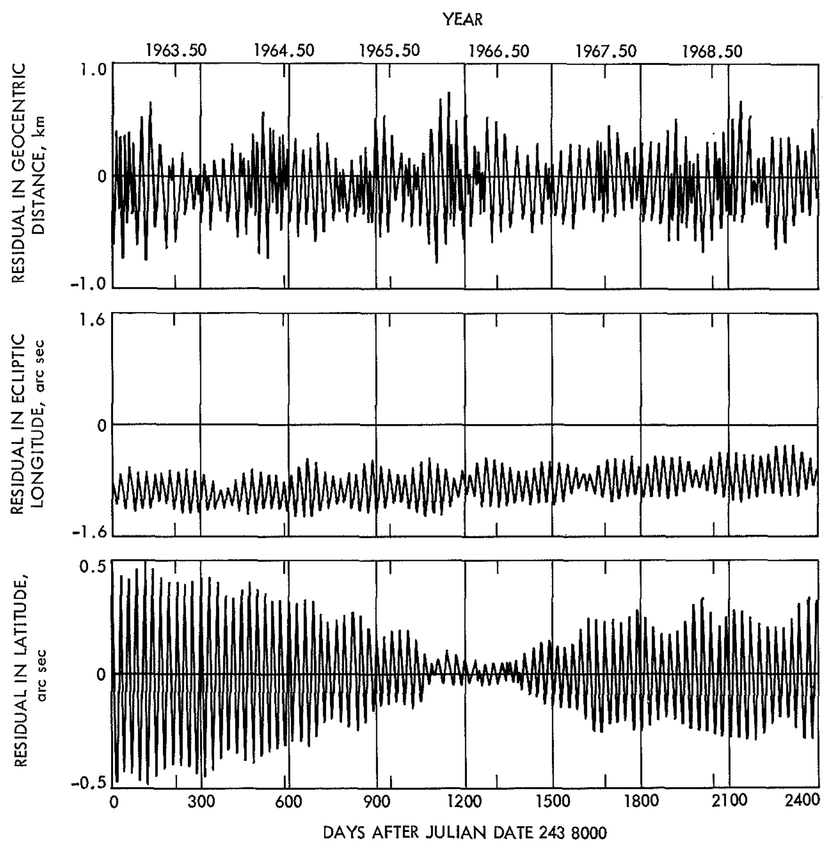

Fig. 1. Effects of TVF corrections—LE 16—LE 4.

with optical, radar, and spacecraft observations, form partial derivatives with respect to the orbital elements and additional parameters, and provide corrections to the initial set of osculating elements and parameters.

The heliocentric equations of motion are:

$$\ddot{\underline{r}}_j = -(w_o + w_j)\,\underline{u}_j - \sum_{\substack{i=1,\,10\\ i\neq j}} w_i(\underline{v}_{ij} - \underline{u}_i) + \underline{R}\,(\underline{r}_j, \dot{\underline{r}}_j, m_j)$$

where

$$\underline{s}_{ij} = \underline{r}_j - \underline{r}_i$$

$$\underline{u}_j = r_j^{-3}\underline{r}_j$$

$$\underline{v}_{ij} = s_{ij}^{-3}\underline{s}_{ij}$$

$$w_j = Gm_j \quad \text{where } G = (0.01720\,20989\,5)^2$$

$$w_o = G$$

The function $\underline{R}$ is used to introduce the effects associated with general relativity.

In order to be compatible with the form of the relativity metric used in the double-precision orbit determination program (DPODP), the SSDPS used the "Robertson" form of the metric (Ref. 13). The Robertson parameters β and γ are now input quantities with the nominal values of unity. With these values, the equations reduce to the isotropic form of the metric.

The vector transformation, relating standard form coordinates r to isotropic coordinates ρ, is

$$r = \rho\left(1 + \frac{m}{2\rho}\right)^2.$$

This transformation, together with its first time derivative, must be used to convert the new isotropic coordinates and velocity components in DE 69 to the standard Schwarzschild metric used in DE 19. Here, m is the Schwarzschild radius.

$$m = 1.47 \text{ km}$$

The values of the Sun/planet mass ratios have been altered from those appropriate for DE 19. In DE 19 the planetary masses were identical with the internationally adopted set [(IAU), 1964] and are given in the first column of Table 2. The planetary masses used in the 60-year integration reflect more recent determinations (Ref. 2).

Table 2. Reciprocal planetary masses

Planet	IAU (1964)	JPL (1969)
Mercury	6 000 000	5 983 000
Venus	408 000	408 522
Earth-moon	329 390	328 900.1
Mars	3 093 500	3 098 700
Jupiter	1 047.355	1 047.3908
Saturn	3 501.6	3 499.2
Uranus	22 869	22 930
Neptune	19 314	19 260
Pluto	360 000	1 812 000
Sun = 1.0.		

The mass of Mercury is derived from analysis of radar observations of Venus. The mass values used for Venus, Earth-moon and Mars are based on radio tracking data from the Mariner, Ranger and Surveyor spacecraft series. The masses of Jupiter, Saturn, and Uranus result from the rediscussion by Mulholland (Ref. 14). The masses of Neptune and Pluto result from more recent work by Gill and Gault (Ref. 15) and Duncombe, et al., respectively (Ref. 16). These masses are given in column 2 of Table 2.

The epoch conditions for DE 69 are referred to JD 244 0800.5, 0^hET, August 2, 1970. This date has been chosen as the standard 400-day date in 1970 which will be used by the ephemeris group of representatives from NASA, the U. S. Naval Weapons Laboratory (NWL), the U. S. Naval Observatory (USNO), and JPL.

B. Observation Set

Optical observations from 1910–1968 from the Six and Nine Inch Transit Circles of the USNO have been collected and placed in a uniform format on punched cards (Ref. 17). The collection of the data over the period 1911–1949 was made by C. Oesterwinter (NWL), Dahlgren, Virginia. These observations were initially compared with DE 28 (Ref. 18). The mispunched cards and obvious printing errors were found and corrected. The final optical data set of over 34,000 observations covering the 60-year period is given in Table 3.

Radar range data from various radar sites have been collected. The collection of these data has been a joint effort by MIT-Lincoln Laboratory and JPL.

Table 3. Transit circle observations

Planet	USNO transit circle	Period	Number of observations	
			Right ascension	Declination
Mercury	6 in.	241 9937.2–243 9654.3 (1913–1967)	1756	1695
	9	242 1867.2–243 1174.2 (1918–1944)	550	532
			2306	2227
Venus	6	242 0391.3–243 9679.3 (1914–1967)	2761	2582
	9	242 2113.3–243 1129.1 (1919–1944)	451	436
			3212	3018
Mars	6	242 4793.8–243 9658.5 (1926–1967)	549	528
	9	242 0105.8–243 1164.5 (1913–1944)	122	120
			671	648
Jupiter	6	242 4311.8–243 9548.6 (1965–1967)	656	624
	9	242 0330.8–243 1165.6 (1914–1944)	260	257
			916	881
Saturn	6	242 4607.8–243 9433.6 (1926–1966)	660	622
	9	242 0085.8–243 1122.6 (1914–1944)	280	280
			940	902
Uranus	6	243 4380.8–243 9595.6 (1925–1967)	639	628
	9	242 0321.8–243 1061.7 (1914–1943)	247	245
			886	873
Neptune	6	242 4531.8–243 9662.6 1925–1967)	618	606
	9	242 0129.8–243 1214.6 (1913–1944)	285	283
			903	889
Sun	6	241 9174.2–243 9682.2 (1911–1967)	5973	5695
	9	242 1867.2–243 1444.2 (1918–1944)	1696	1668
			7669	7363

The current ephemeris can predict the position of Venus to better than 20 μs in range. It has been found that the inclusion of Arecibo Ionospheric Observatory (AIO) data degrades this ephemeris. A systematic bias of approximately 30 μs appears in the residuals. Consequently, most AIO data have been removed from the data set pending re-examination of these data which is currently under way at AIO. The data set used in this development ephemeris is given in Table 4. This radar datum is also placed in a uniform format on punched cards (Ref. 17).

The first use of spacecraft data in ephemeris development is under way with the 60-year ephemerides. A total of 214 planetary range points covering the period June 21 to November 12, 1967 from the R&D planetary ranging system on *Mariner V* were used. These points are of 0.1 μs accuracy. Through the perturbations by Venus on the orbit of *Mariner V* during the encounter phase, the center of gravity of Venus can be determined at the time of encounter by *Mariner V* to 100 m in geocentric range and 5 m/day in geocentric range rate[2].

C. Ephemeris Computations

In the spring of 1968, the first simultaneous integration of all the planets was made at JPL. The initial conditions were close to those found in DE 19. These DE 19 initial conditions resulted from individual numerical integrations of all the planets using the relativistic differential equations fit to either the source theories from the USNO, or the Newcomb theories as programmed by N. Block in 1963. The initial conditions of Venus and the earth-moon barycenter differ from DE 19. These improved initial conditions are from DE 26.[3] These

[2]This assumes that the speed of light is given exactly by the IAU value of 299792.5 km/s.

[3]Lawson, C. L., "Announcement of JPL Development Ephemeris No. 26," Internal Document, Jet Propulsion Laboratory, Pasadena, Calif., June 7, 1967.

improved Venus and Earth-moon ephemerides resulted from comparison of the theories with optical observations from the 6-in. transit circle of the USNO between 1950 and 1965 and planetary radar from JPL, MIT, and AIO between 1961 and early 1967. A set of masses unlike those used in DE 19 or DE 69 were used (Ref. 14). The epoch conditions at JD 244 0400.5 were altered to reflect this change in planetary masses by constraining the osculating mean motions while adjusting the values for the semi-major axis. The expression

$$n^2a^3 = k^2(1+m+\delta m)$$

was used. A forward integration was made to the chosen "standard epoch" 2440800.5. At this point, the integrated epoch conditions for 1970 were used to begin a 20-year backward integration.

The initial comparison with the combined set of planetary range data of AIO, the Millstone Hill and Haystack sites of MIT, and JPL's, Venus site showed variations which were quite large (see Figs. 2, 3, and 4) as follows:

(1) $+4000$ to -2000 μs for Mercury.

(2) $+1500$ to approximately 0 μs for Venus with a positive offset.

(3) $+3000$ to -1000 μs for Mars.

These radar residuals along with a set of USNO 6-Inch Transit Circle Observations over the period 1950–1967 were used to correct the orbital elements of all the planets except Pluto, along with the radii and the astronomical unit.

A 56-parameter solution was made using these data. A rank 52 solution of an eigenvalue–eigenvector analysis was applied to the original osculating elements and a new integration performed[4]. The magnitudes of the radar range residuals were reduced by two orders of magnitude.

This first gravitationally consistent ephemeris computed at JPL was used from the Spring of 1968 until February 1969. At the completion of this initial effort, it was known that the outer planets were improved over the currently available ephemerides, but also that there was a need to fit over a much longer arc in order to obtain definitive ephemerides. Further, a longer arc of optical observations was needed to better determine the orientation of the ecliptic and the mean longitude of the earth-moon barycenter. Consequently, a 60-year numerical integration of the motion of the planets was made.

Table 4. Radar range observations

Data source	Period	Number of observations
Mercury		
AIO	243 8493.2–243 9363.2 (1964–1966)	119
Haystack (MIT)	243 9425.3–244 0064.2 (1966–1968)	88
	Summary 1964–1968	207
Venus		
Haystack (MIT)	243 9161.3–244 0063.2 (1967–1968)	63
JPL (Venus DSS)	243 8541.2–243 9707.6 (1964–1967)	284
Millstone Hill (MIT)	243 8447.0–243 9725.2 (1964–1967)	101
	Summary 1964–1968	448
Mars		
AIO	243 8719.0–243 8915.0 (1964–1965)	39
Haystack (MIT)	243 9587.7–243 9643.5 (1967)	10
	Summary 1964–1967	49

After a coordinate transformation to place the epoch conditions originally on the Schwarzschild metric into an isotropic form, a 60-year ephemeris was integrated. The interval of integration was from 1970 to 1910, and was designated DE 61.

A simultaneous solution of 63 parameters of the solar system was obtained reflecting the comparison of DE 61 with all the data discussed previously. The unknowns which are considered are the elements of the eight planets except Pluto, the right ascension and declination limb bias for Mercury and Venus, the radii of Mercury, Venus, and Mars, the six elements of *Mariner* V, the mass of Venus, and the astronomical unit. After some consideration, a rank 55 solution from the eigenvalue–eigenvector analysis was selected for re-integration.

D. Comparison with Theory

The Mercury radar range residuals are shown in Fig. 5. These residuals are the result of the solution just discussed. The mean error for the fit of data is 57 μs.

[4]Lawson, C. L., "Eigenvalue-Eigenvector Analysis for SSDPS," Internal Report, Jet Propulsion Laboratory, Pasadena, Calif., Jan. 17, 1968.

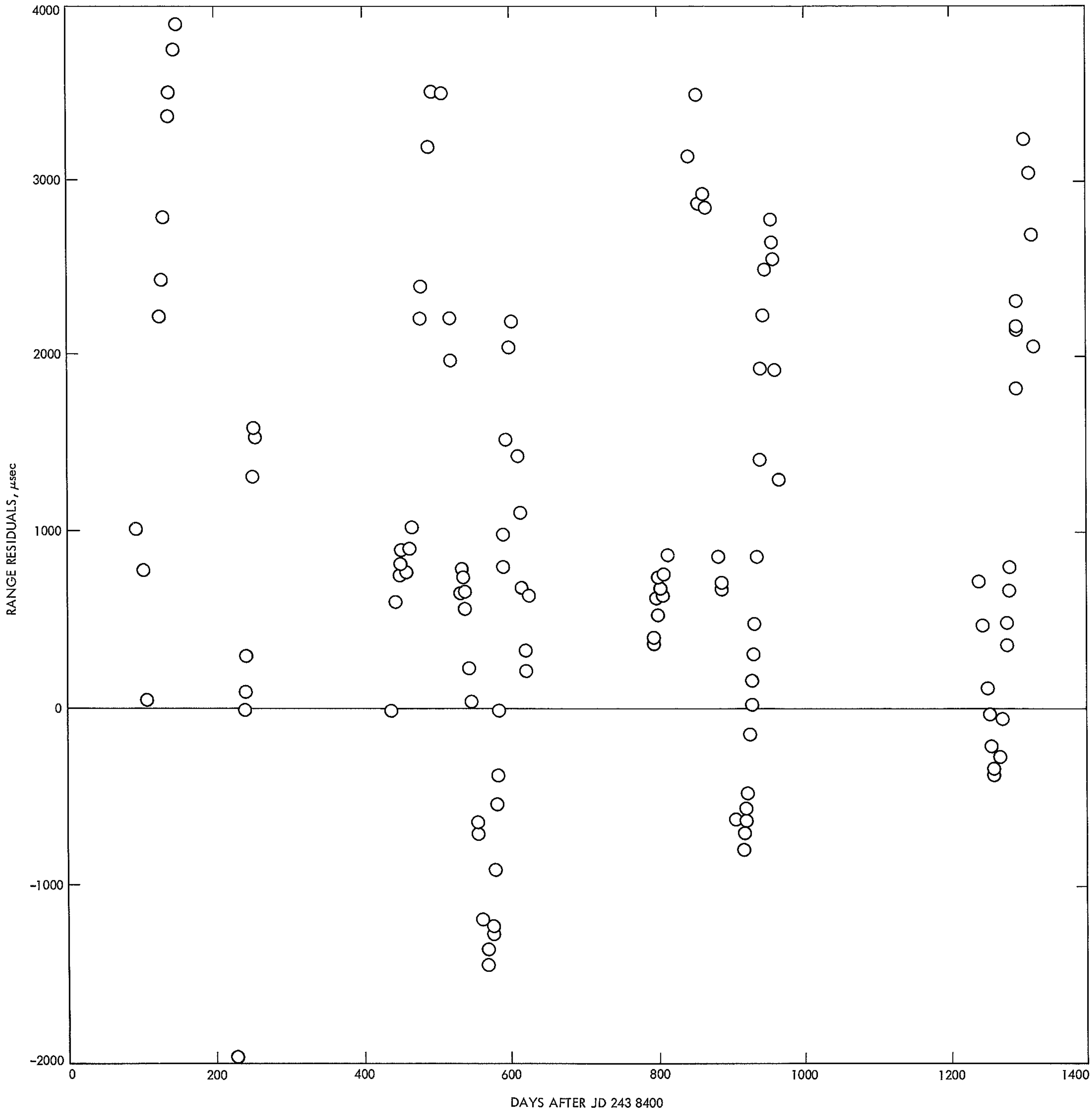

Fig. 2. Mercury radar range residuals with DE 35.

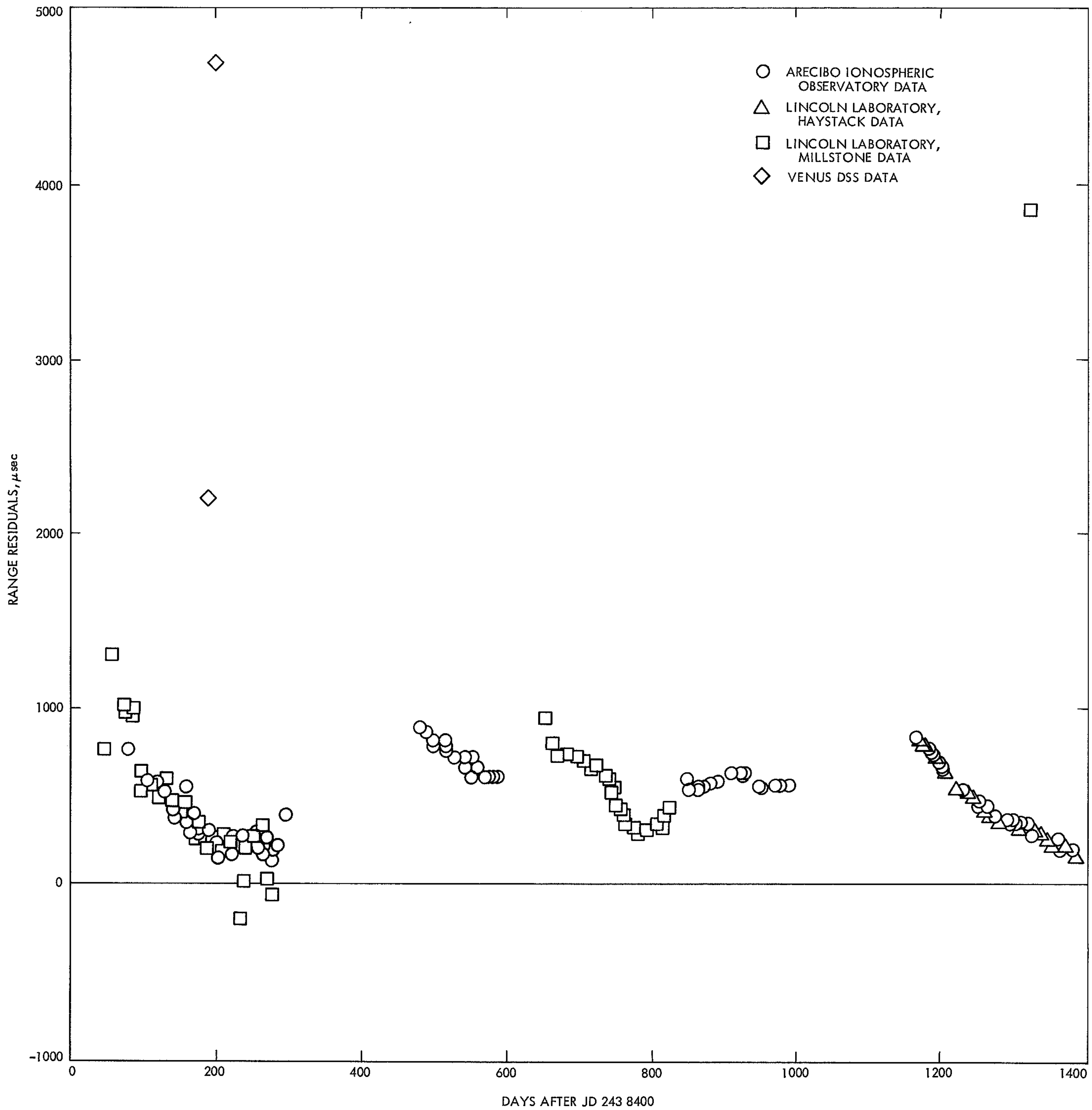

Fig. 3. Venus radar residuals with DE 35 (before fit).

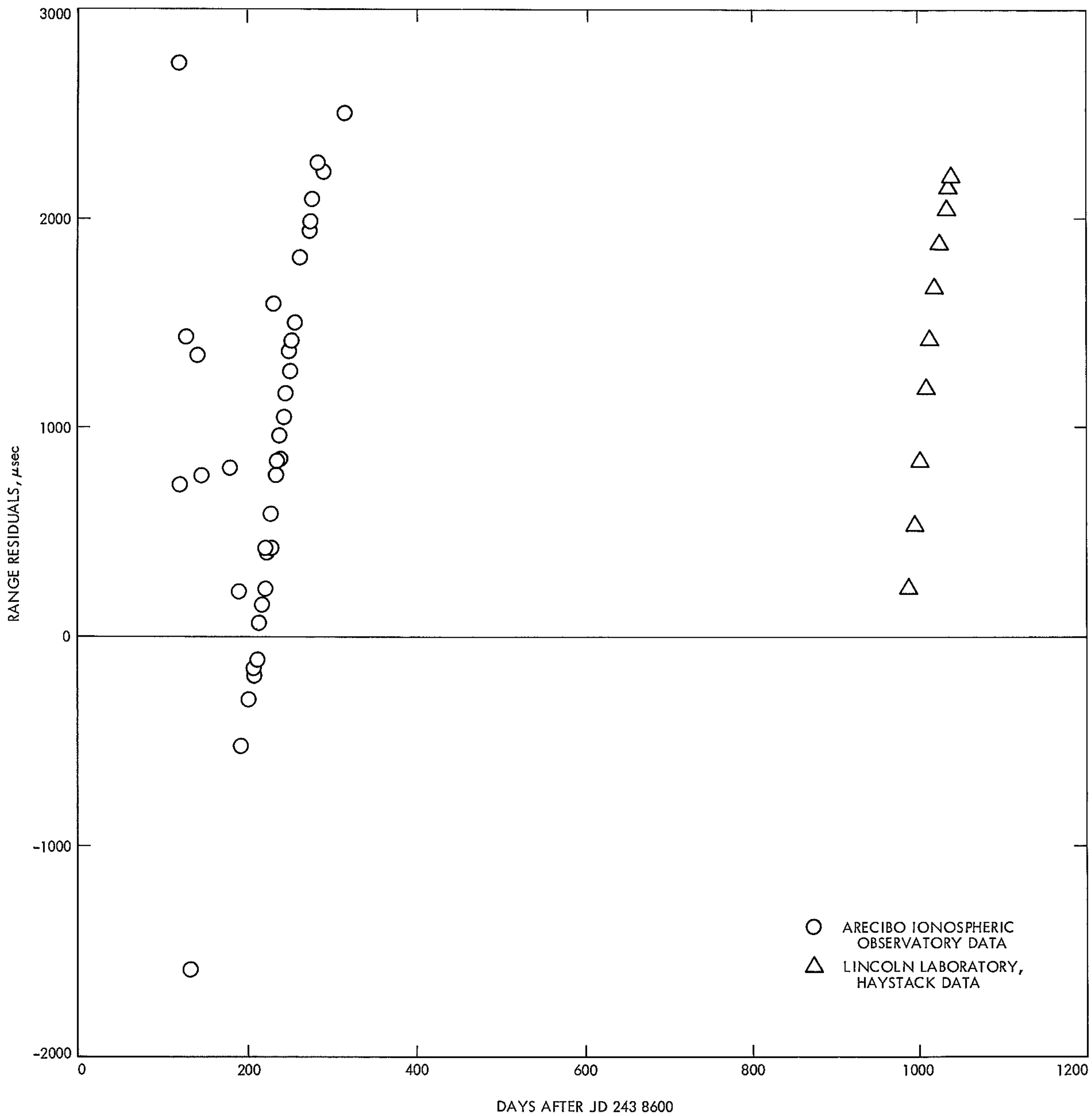

Fig. 4. Mars residuals compared with DE 35 (before fit).

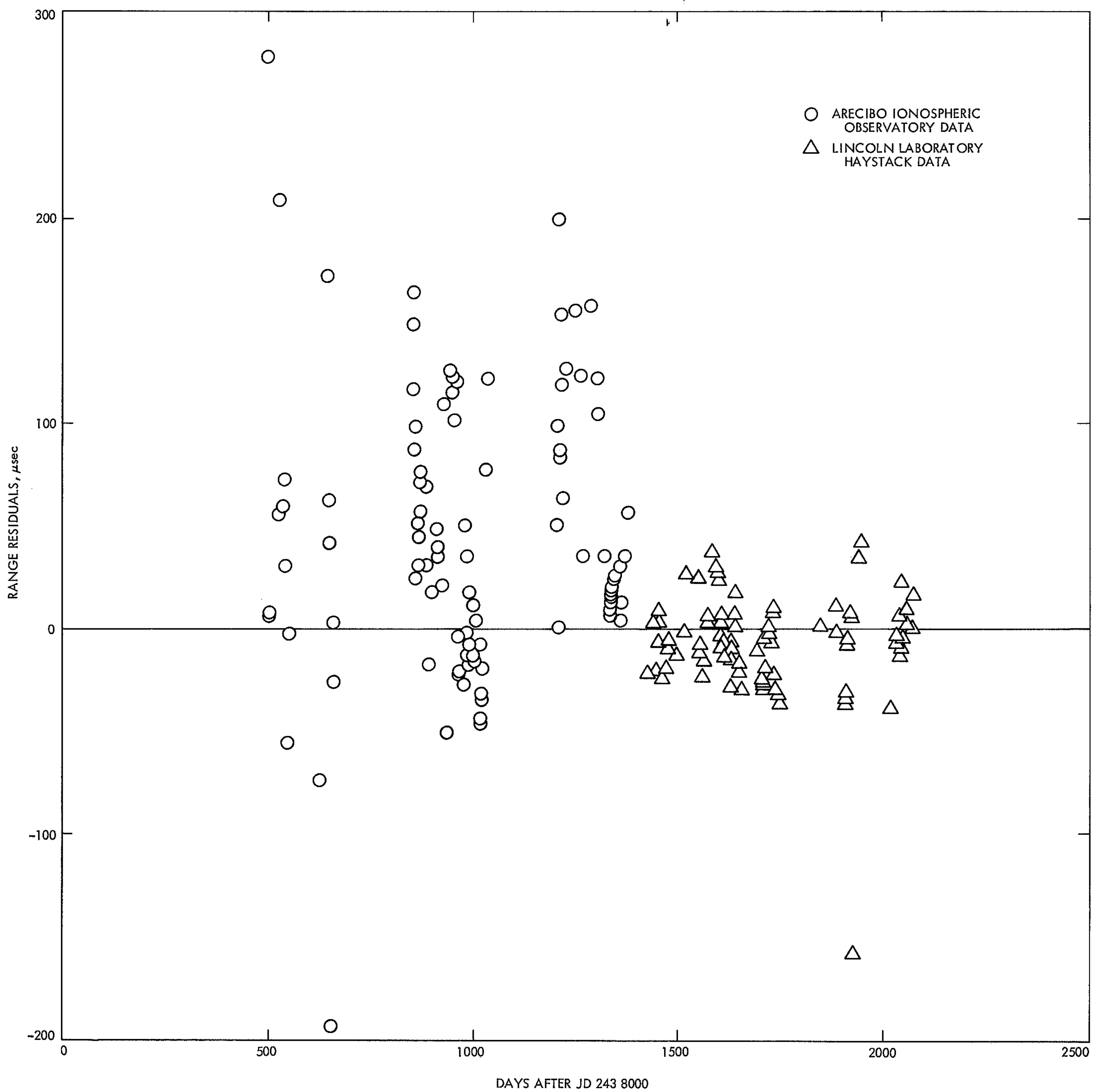

Fig. 5. Mercury radar range residuals—DE 69.

The increased radar detectability by the decreasing limits of radar residuals, as the data become more current, should also be noted. The standard deviations of the residuals on Mercury optical data are 1″.0 in right ascension and 0″.9 in declination.

Venus radar range data are from Millstone Hill, Haystack and JPL–Goldstone, the AIO data having been removed as a result of the previous discussions. The residuals based on DE 69 are shown in Fig. 6. The standard deviation for the fit of this data is 50 μsec.

The structure to be seen in the 1965/1966 Venus radar range residuals must be regarded as an anomaly in the modeling of the masses. If one considers the masses better known from spacecraft tracking, and therefore fixes the values, the reduction of degrees-of-freedom will cause the "feature" to appear in the Venus residuals. By altering the mass of Mercury, the feature and the overall sum of squared residuals are diminished.

The structure in the 1967 Venus-residuals has been removed. It was caused by mis-identification of the time base of the measurements.

Figure 7 shows the *Mariner V* residuals with respect to the 60-year integration. The precision of this new data type is seen by the scale which is in tenths of microseconds.

The optical residuals of Venus in right ascension have a standard deviation of 1″.2 and in declination 0″.9.

The 39 radar observations of Mars taken in 1964 at AIO, and the 10 high-precision-compressed points taken at Haystack in 1967, are shown in Fig. 8 compared to DE 69. The standard deviation, excluding the few early points, is 50 μs. The radar observations of Mars taken at JPL during the Spring and Summer of 1969 have not been included in DE 69.

The Sun and the planets, Jupiter, Saturn, Uranus, and Neptune, were fit to optical observations only. The residuals of the outer planets except Pluto are shown in Figs. 9 through 16. The periodic character of these residuals is indicative that further work is necessary on the planetary masses. Table 5 summarizes the statistics on the fit of the observations to DE 69.

In Appendix C are given all of the values to be used with DE 69. These reflect the solution just discussed.

Table 5. Statistics for fit of optical observations

Planet	Right ascension		Declination	
	Number of observations	Root mean square from mean	Number of observations	Root mean square from mean
1939–1967				
Mercury	913	.94	864	.93
Venus	1653	1.22	1504	.93
Mars	379	.52	360	.54
Jupiter	524	.45	498	.48
Saturn	509	.48	484	.53
Uranus	475	.33	465	.43
Neptune	519	.85	506	.60
Sun	3526	.76	3304	.60[a]
1910–1939				
Mercury	1394	1.04	1363	.85
Venus	1559	1.22	1514	.92
Mars	292	.72	288	.62
Jupiter	392	.54	383	.56
Saturn	431	.50[a]	416	.50[a]
Uranus	411	.48	408	.51
Neptune	384	.56	383	.46
Sun	4143	.80[a]	4059	.83

[a]These are estimated errors.

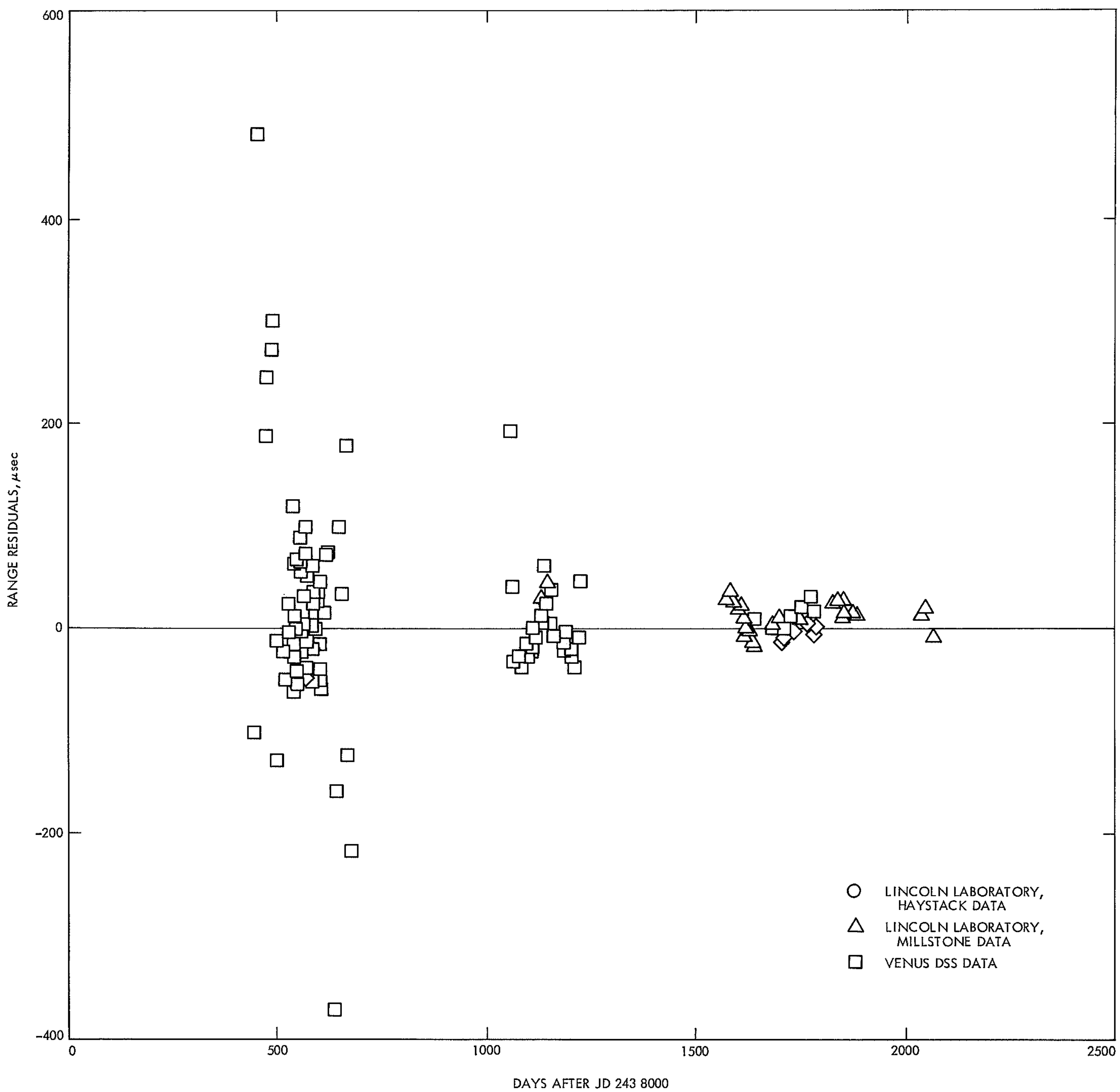

Fig. 6. Venus radar range residuals—DE 69.

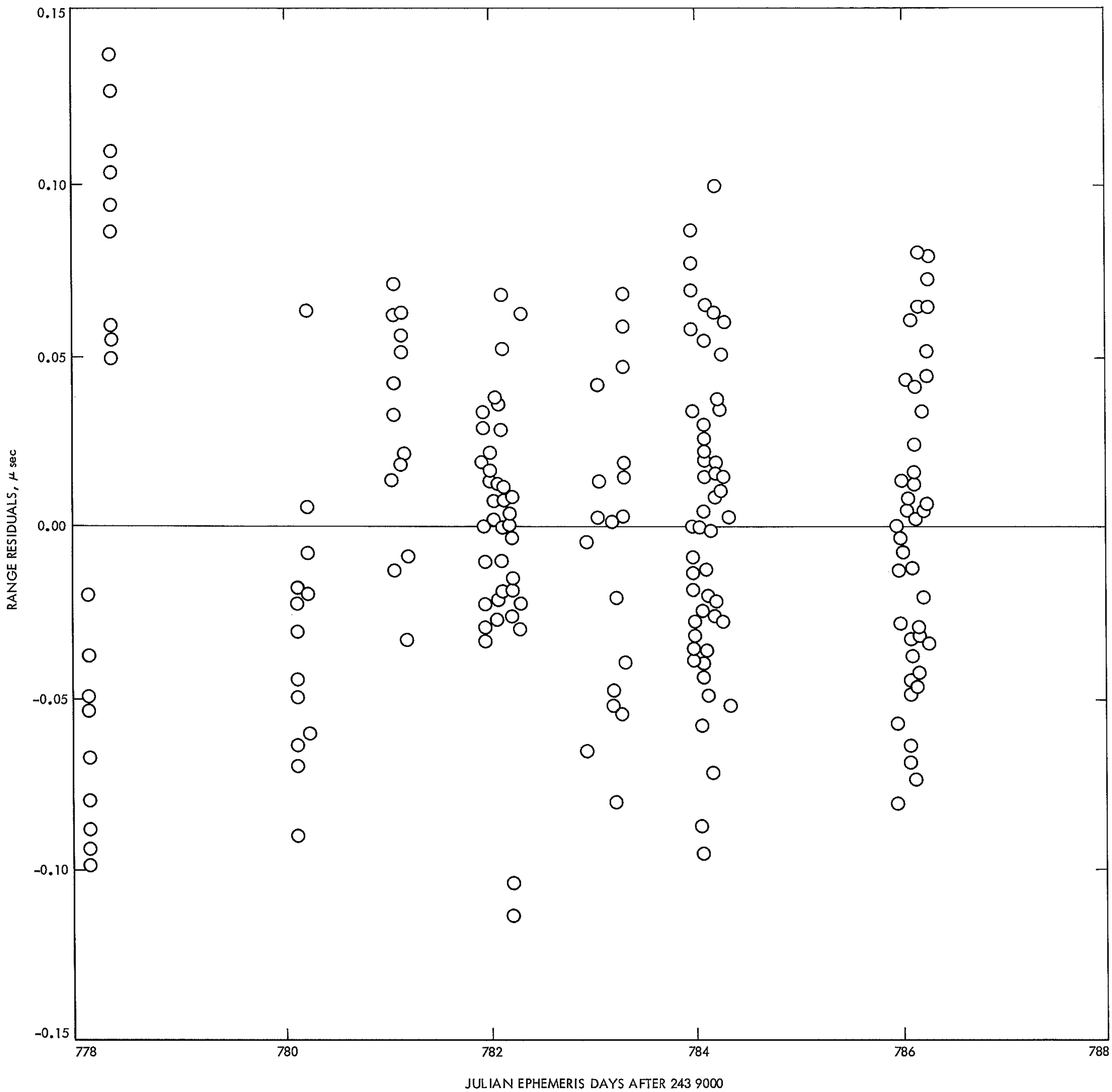

Fig. 7. ***Mariner V*** **residuals.**

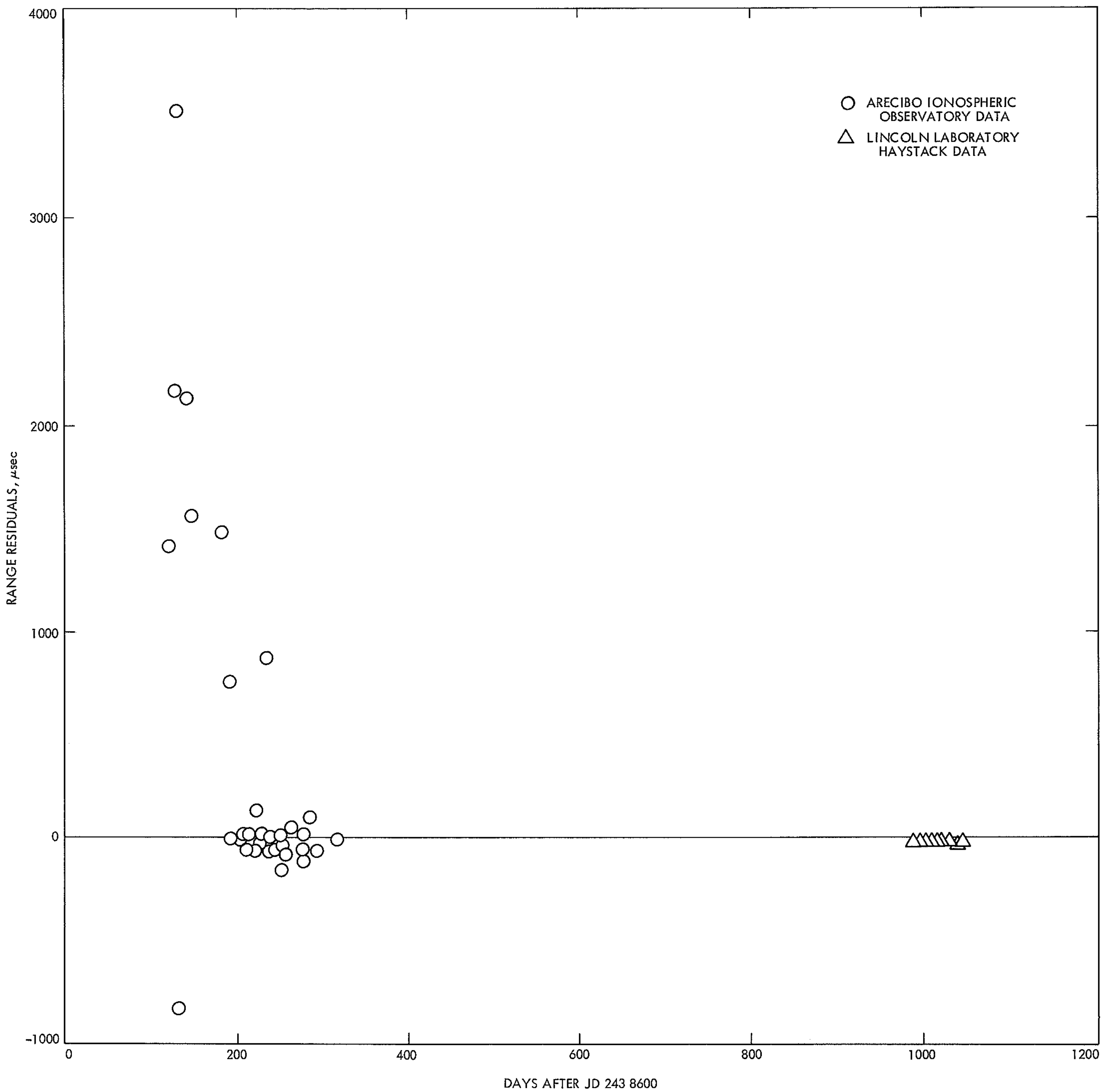

Fig. 8. Mars radar range residuals—DE 69.

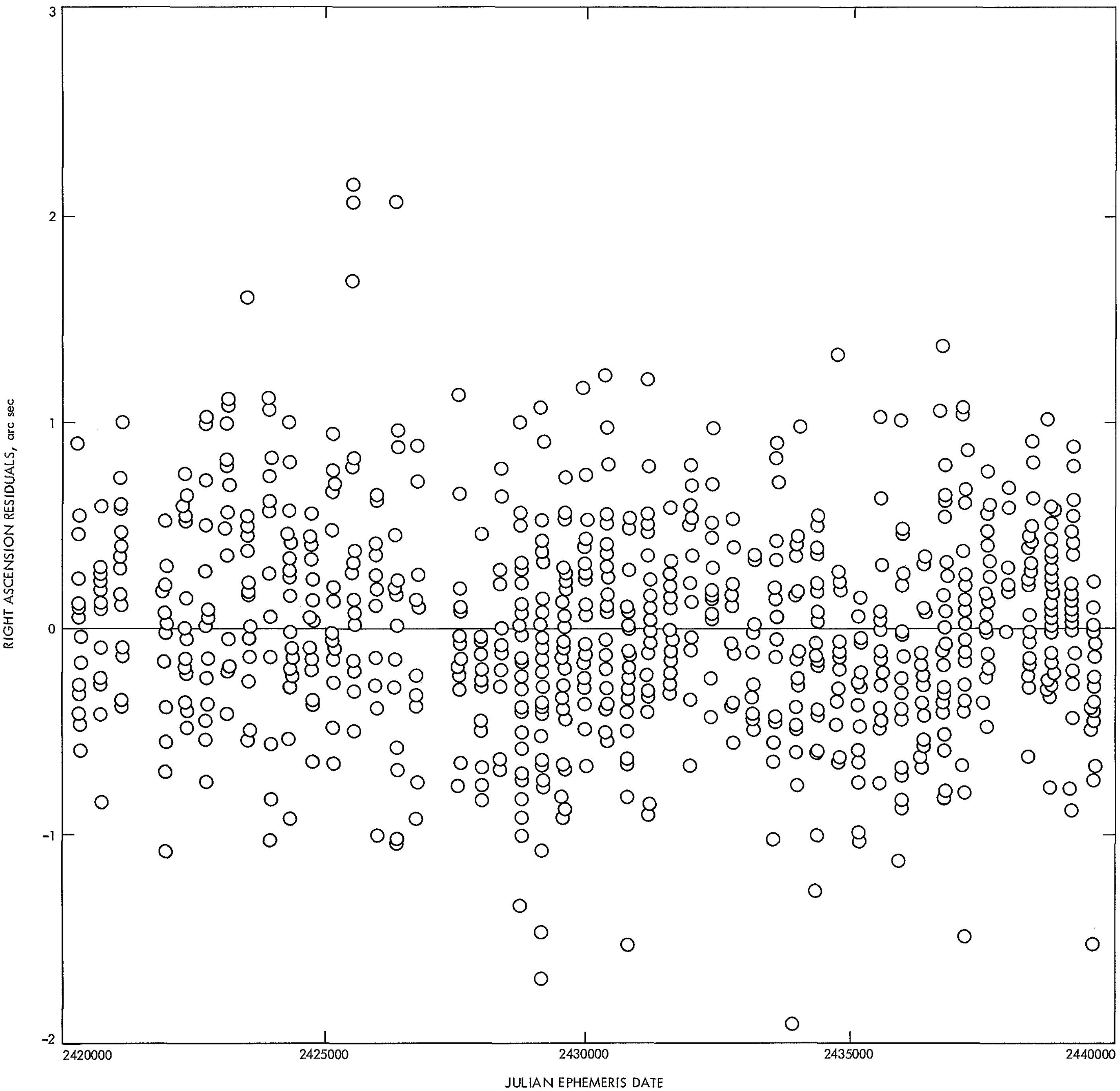

Fig. 9. Right ascension residuals—Jupiter.

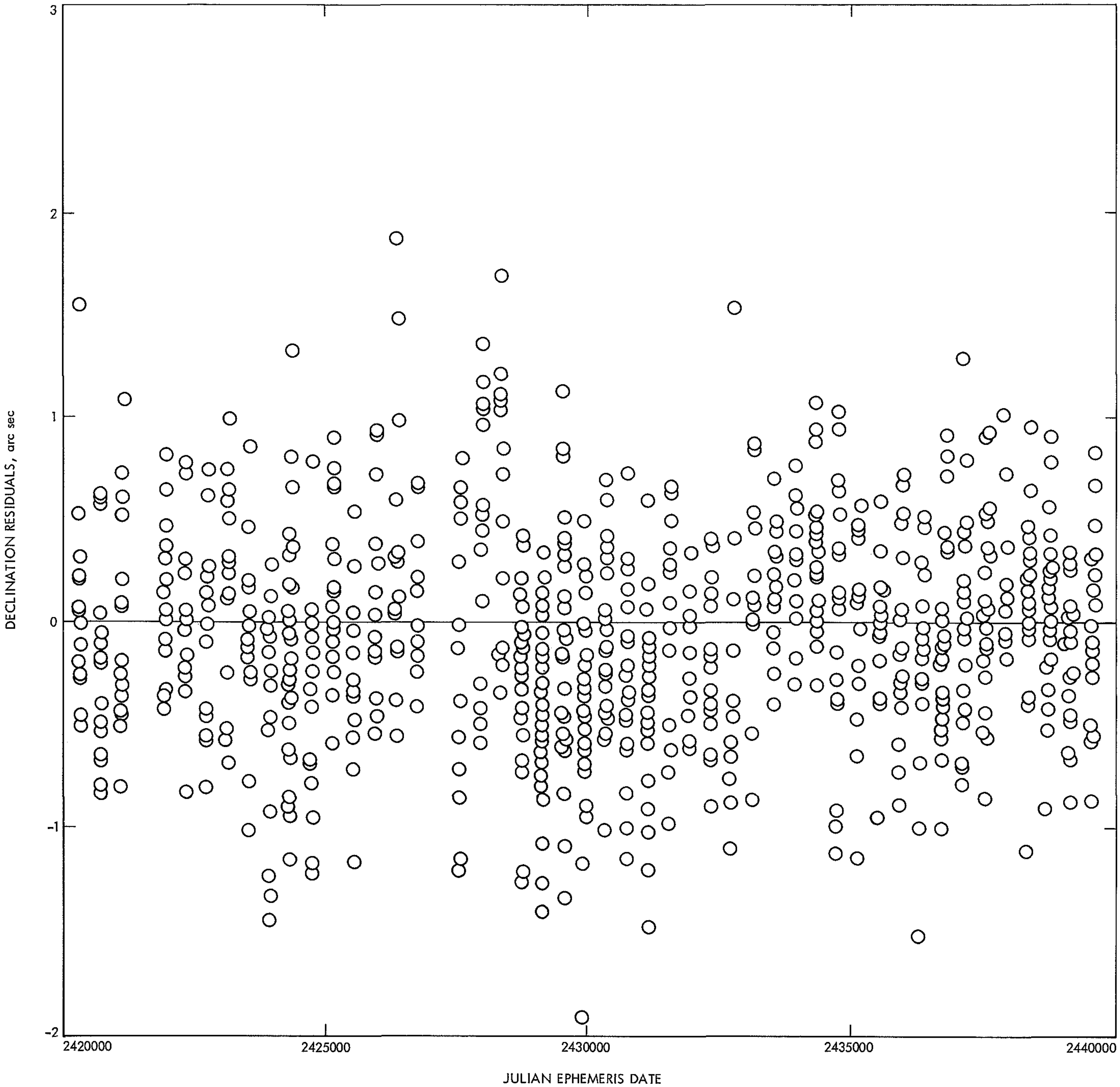

Fig. 10. Declination residuals—Jupiter.

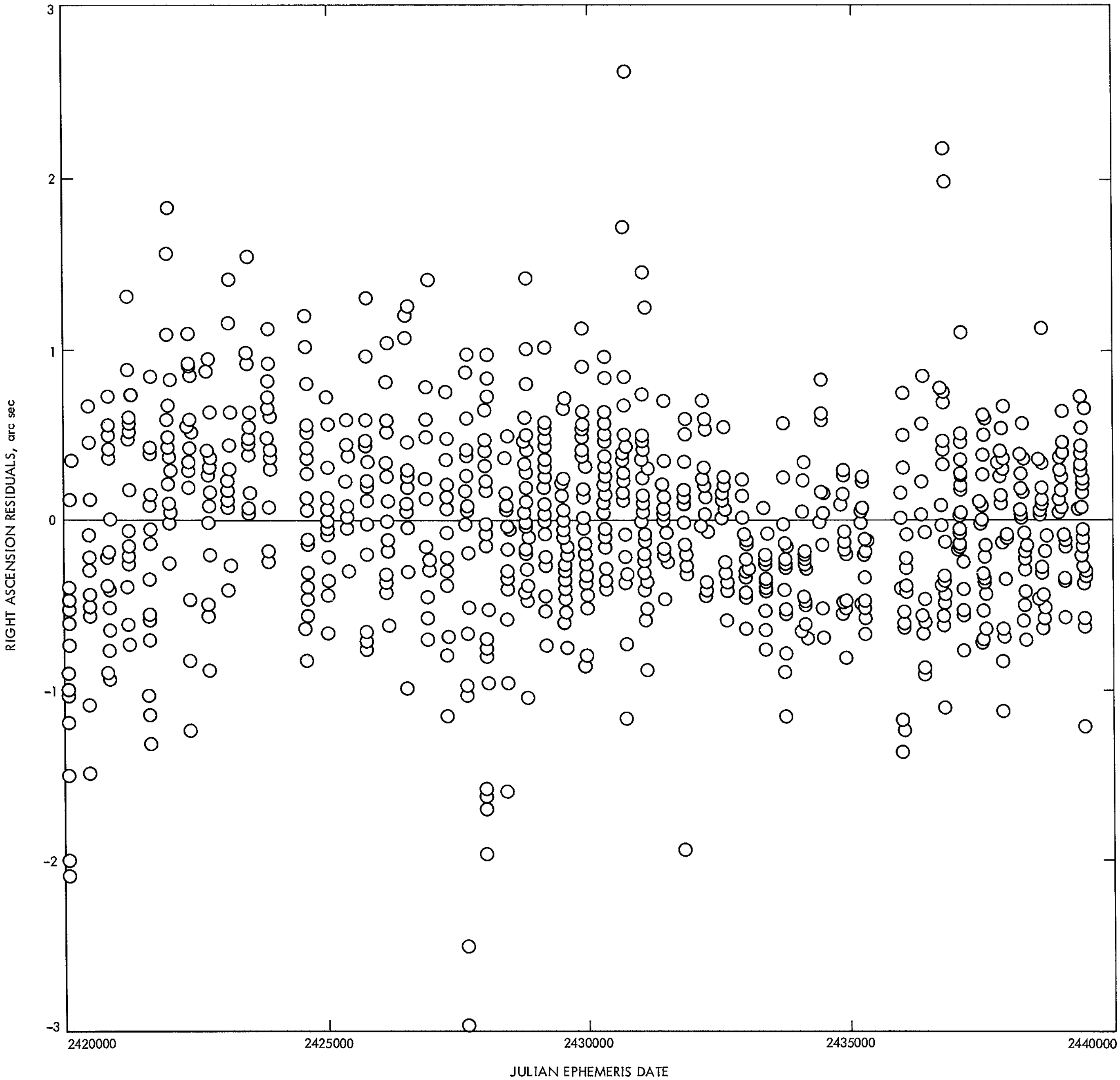

Fig. 11. Right ascension residuals—Saturn.

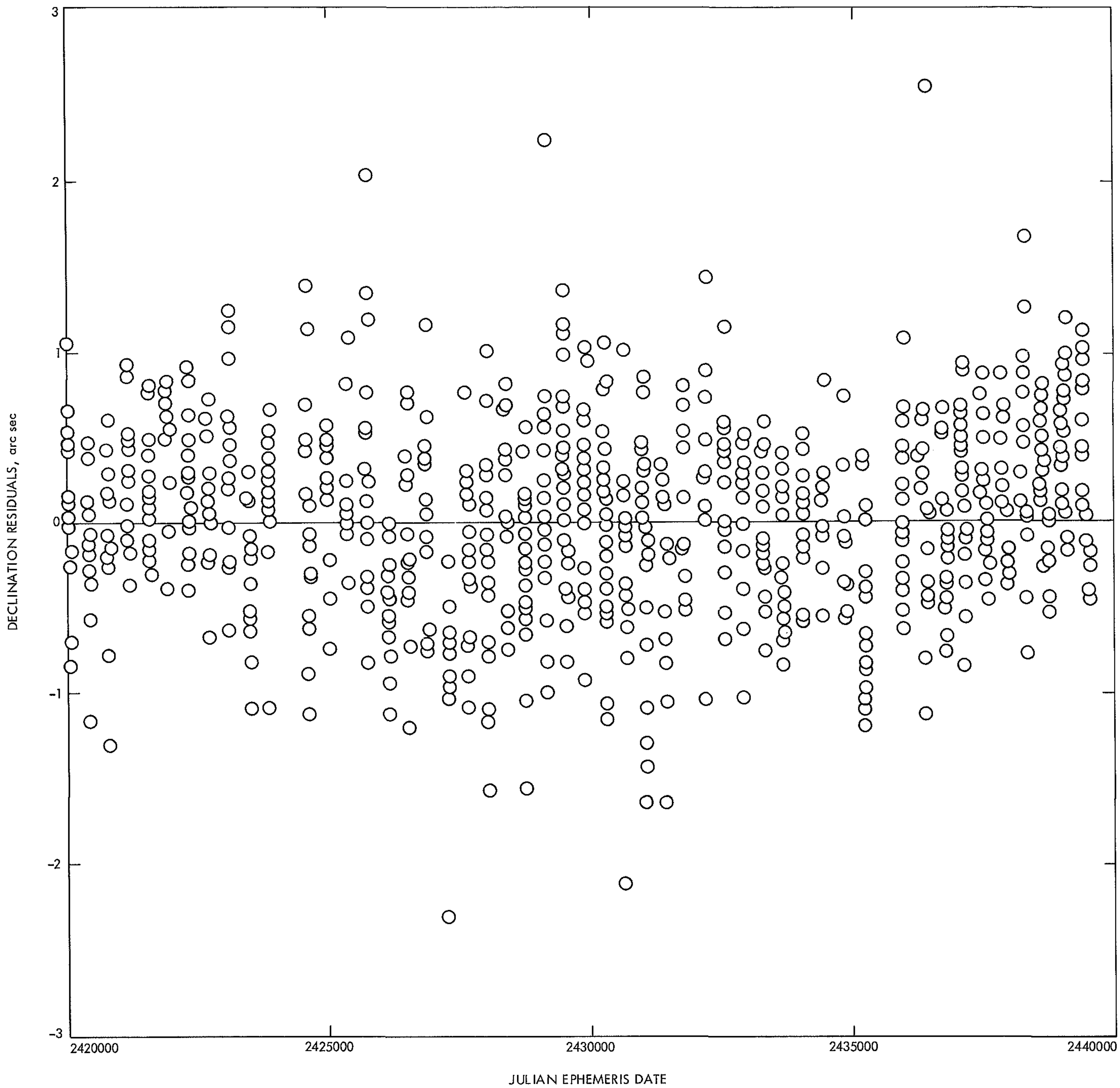

Fig. 12. Declination residuals—Saturn.

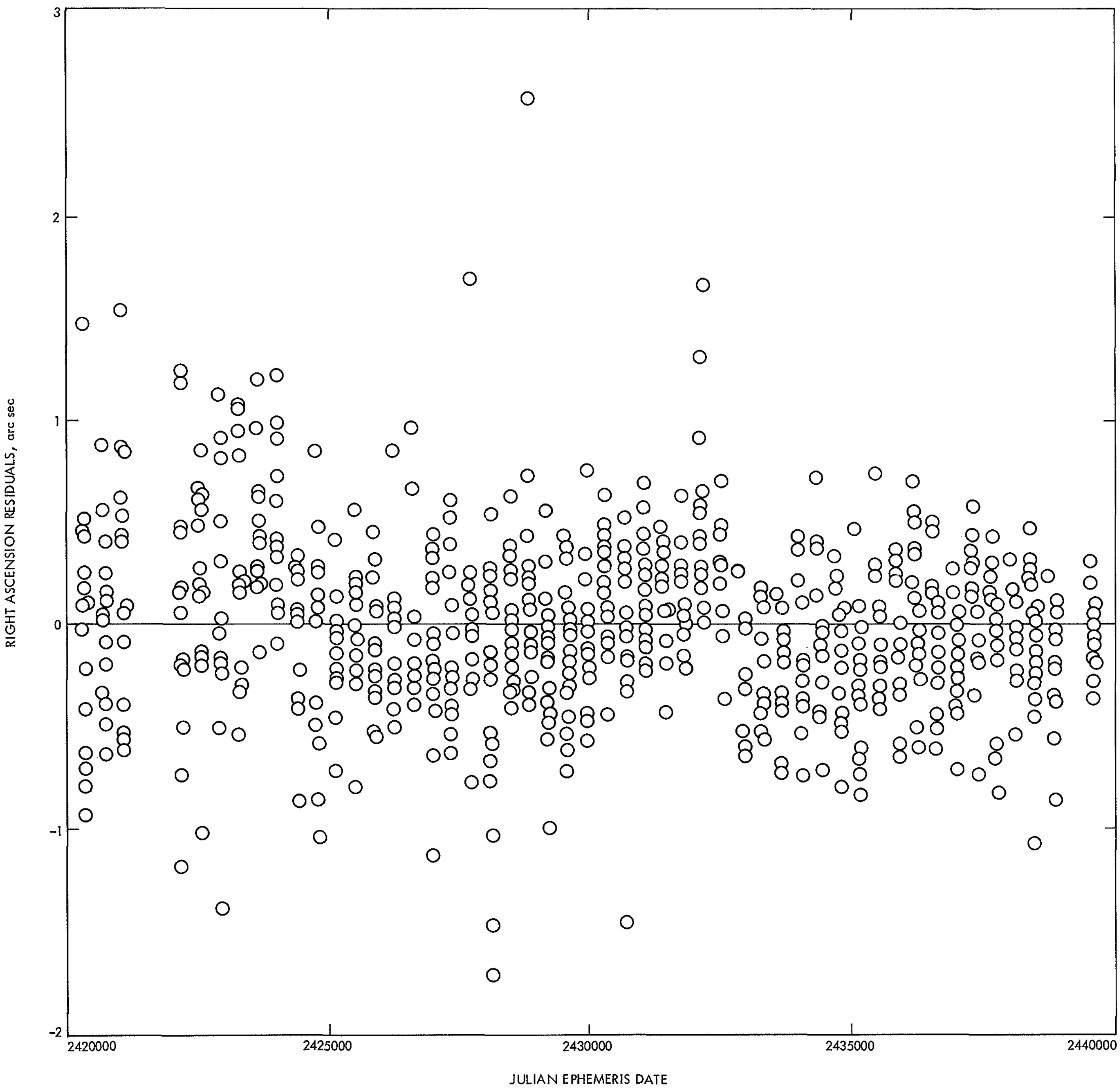

Fig. 13. Right ascension residuals—Uranus.

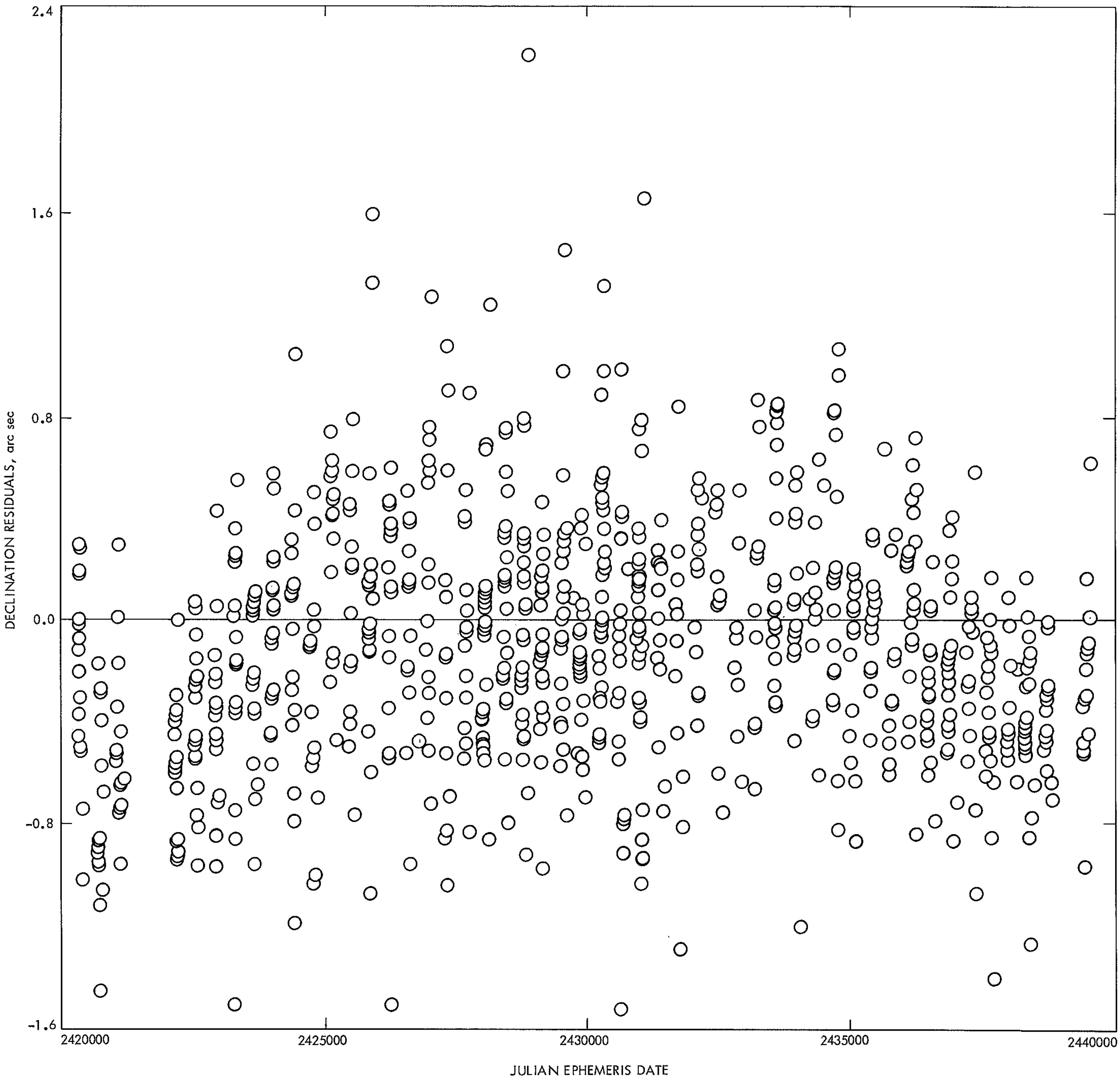

Fig. 14. Declination residuals—Uranus.

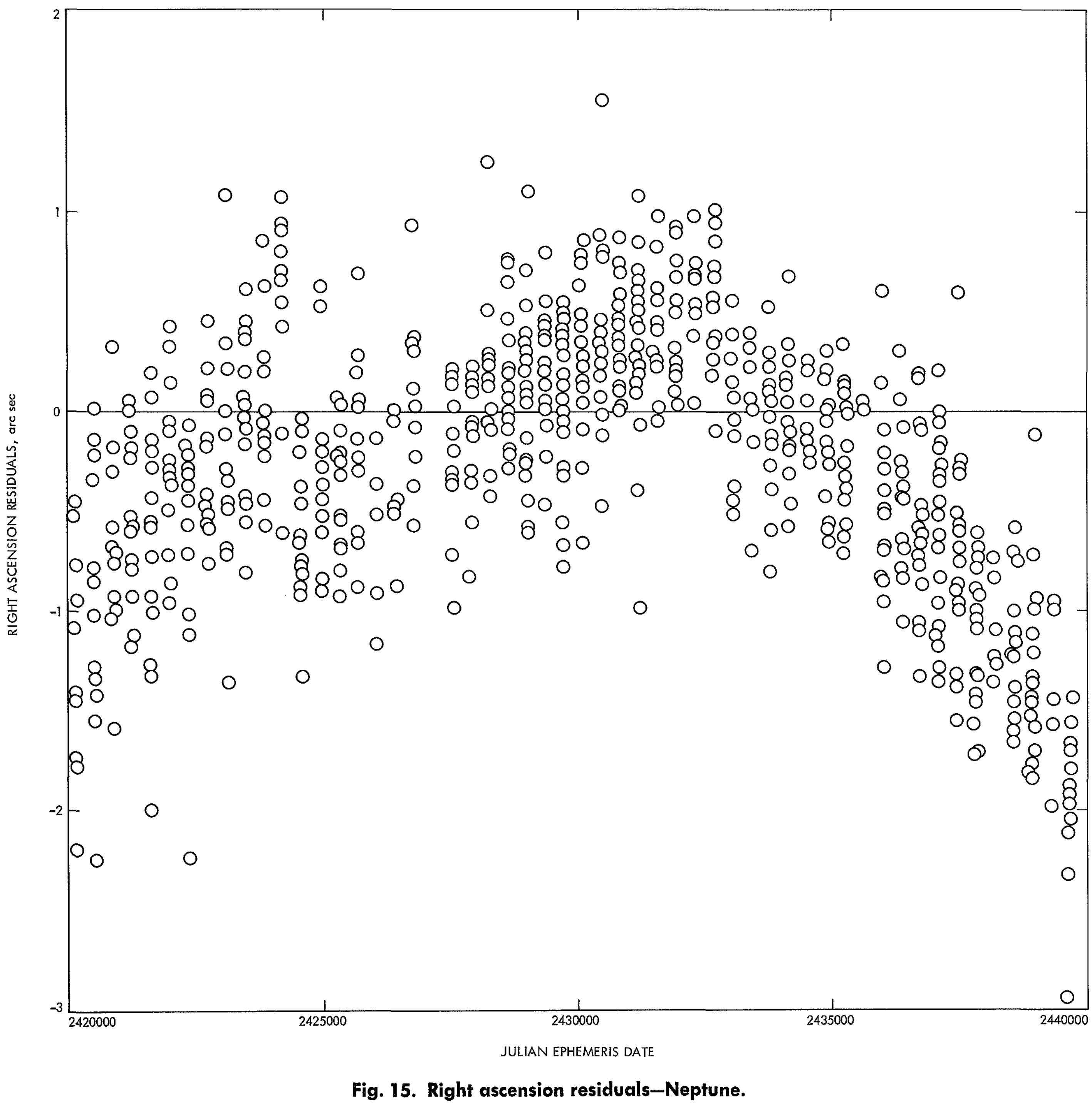

Fig. 15. Right ascension residuals—Neptune.

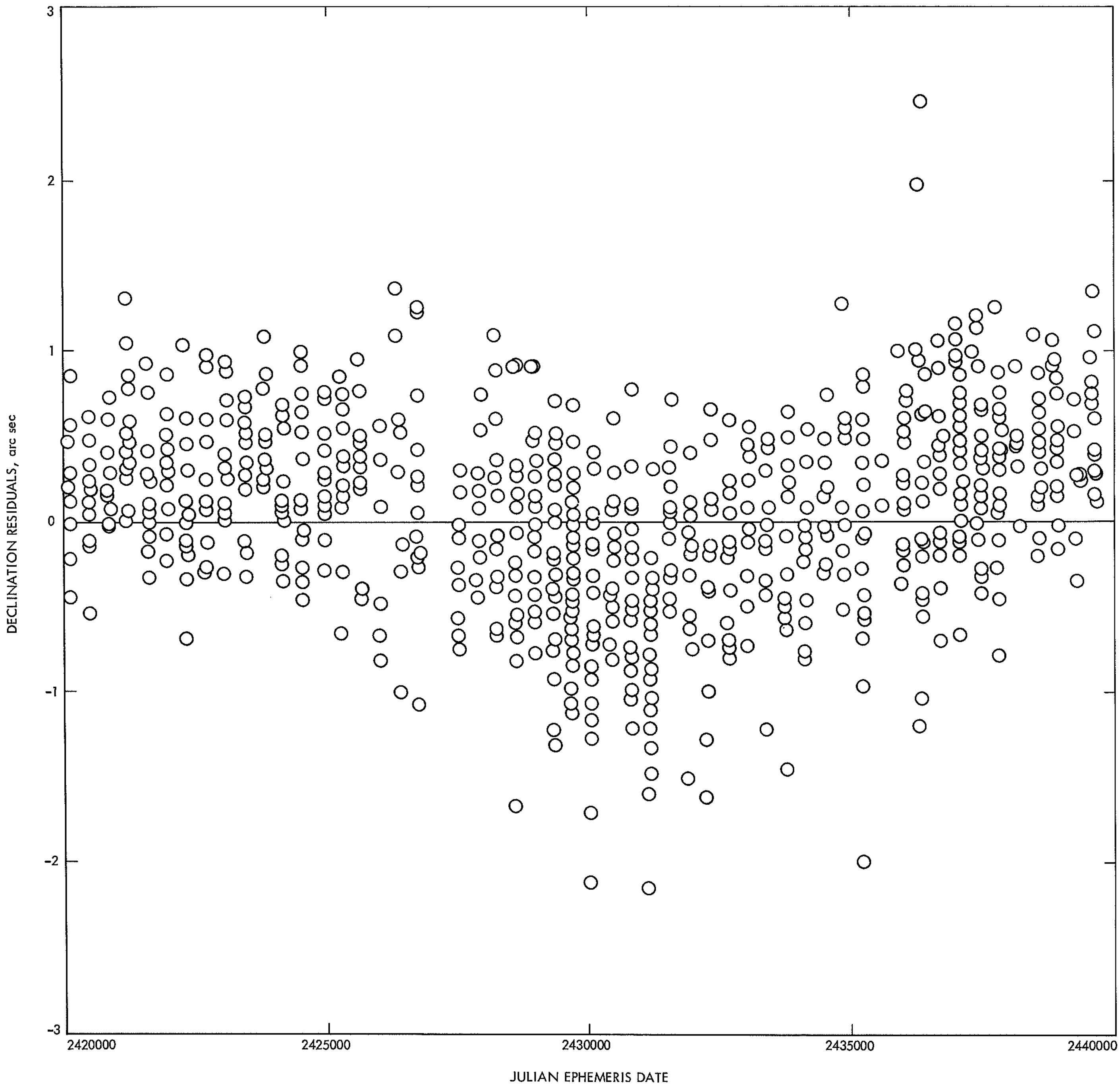

Fig. 16. Declination residuals—Neptune.

Appendix A

JPL Ephemeris Tape Format, Type 50

The JPL Ephemeris Tape Format, Type 50, contains two information records at the beginning of each tape followed by the data records. The record format may be described as follows:

(1) The first record of each tape contains 24 BCD words written in binary. These 24 words serve to describe the general nature of the information on the tape.

(2) The second record of each tape contains the following information in the order listed:

   (a) Number of bodies on tapes = 10.

   (b) A floating point 50, which denotes the Type 50 format.

   (c) Initial Julian date for which data are provided.

   (d) Final Julian date for which data are provided.

   (e) Step size of the logical data record = 8.0 days.

   (f) Ten pairs of numbers. The first number of the pair denotes the body in increasing order out from the Sun, with a zero used for lunar data. The second number of each pair is the step size of data provided for that body.

(3) The JPL Ephemeris Tapes contain data in buffered and overlapped 8-day logical records. The end points of the 8-day span are repeated as the first points of the succeeding 8-day record. This format allows ease of handling by the interpolation program.

The format for the JPL Ephemeris Tape records is listed in Table A-1. All data, with the exception of those for nutations, are double precision, so that the total record size is 1863 words. The step size for lunar data and nutations is ½ day. Mercury data are given in 2-day steps, and all other data in 4-day steps.

The Julian date is the epoch (Ephemeris Time, ET) of the start of the data record. Lunar positions and velocities are referred to the geocentric equatorial rectangular reference frame of the mean equator and equinox of 1950.0 = JD 243 3282.423. They are expressed in units called "earth radii" and "earth radii"/mean solar day. Planetary positions and velocities are referred to the heliocentric equatorial rectangular frames of 1950.0, and are expressed in units of AU and AU/mean solar day.

The conversion of position and velocity tabulations to laboratory units, such as kilometers and kilometers per

Table A-1. Ephemeris tape record format

Word in record	Date[a]
0	Julian Date
2	Mercury: $X, d^2X, d^4X, Y, d^2Y, d^4Y, Z, d^2Z, d^4Z$ Followed by four more position data points $\dot{X}, d^2\dot{X}, d^4\dot{X}, \dot{Y}, d^2\dot{Y}, d^4\dot{Y}, \dot{Z}, d^2\dot{Z}, d^4\dot{Z}$ Followed by four more velocity points
182	Venus: $X, d^2X, d^4X, Y, d^2Y, d^4Y, Z, d^2Z, d^4Z$ Followed by two more position data points $\dot{X}, d^2\dot{X}, d^4\dot{X}, \dot{Y}, d^2\dot{Y}, d^4\dot{Y}, \dot{Z}, d^2\dot{Z}, d^4\dot{Z}$ Followed by two more velocity data points
290	Earth–moon barycenter: Same as Venus
398	Mars: Same as Venus
506	Jupiter: Same as Venus
614	Saturn: Same as Venus
722	Uranus: Same as Venus
830	Neptune Same as Venus
938	Pluto: Same as Venus
1046	Moon: $X, d^2X, d^4X, Y, d^2Y, d^4Y, Z, d^2Z, d^4Z$ Followed by sixteen more position data points $\dot{X}, d^2\dot{X}, d^4\dot{X}, \dot{Y}, d^2\dot{Y}, d^4\dot{Y}, \dot{Z}, d^2\dot{Z}, d^4\dot{Z}$ Followed by sixteen more velocity data points
1658	Nutations: $\Delta\Psi, d^2\Delta\Psi, d^4\Delta\Psi, \Delta\varepsilon, d^2\Delta\varepsilon, d^4\Delta\varepsilon$ Followed by sixteen more nutation data points $\Delta\dot{\Psi}, d^2\Delta\dot{\Psi}, d^4\Delta\dot{\Psi}, \Delta\dot{\varepsilon}, d^2\Delta\dot{\varepsilon}, d^4\Delta\dot{\varepsilon}$ Followed by sixteen more nutation rate data points
1862	Check sum

[a]Where d^2 and d^4 are defined as in Eq. (A.1).

Table A-2. Maximum magnitude of *K*th difference of quantities tabulated

K	Mercury	Venus	Earth-moon barycenter	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto	Moon	Δψ	Δε
	AU	AU	AU	AU	AU	AU	AU	AU	AU	Earth radii	Rad	Rad
0	0.410E 00	0.725E 00	0.100E 01	0.165E 01	0.544E 01	0.951E 01	0.182E 02	0.280E 02	0.304E 02	0.636E 02	0.914E-04	0.481E-04
1	0.674E-01	0.811E-01	0.699E-01	0.582E-01	0.306E-01	0.235E-01	0.150E-01	0.125E-01	0.126E-01	0.745E 01	0.510E-06	0.202E-06
2	0.109E-01	0.911E-02	0.477E-02	0.245E-02	0.193E-03	0.538E-04	0.142E-04	0.497E-05	0.488E-05	0.916E 00	0.888E-07	0.384E-07
3	0.270E-02	0.103E-02	0.348E-03	0.983E-04	0.113E-05	0.156E-06	0.159E-07	0.704E-08	0.755E-08	0.124E 00	0.238E-07	0.106E-07
4	0.701E-03	0.117E-03	0.236E-04	0.560E-05	0.101E-07	0.221E-08	0.198E-08	0.199E-08	0.199E-08	0.188E-01	0.711E-08	0.304E-08
5	0.300E-03	0.138E-04	0.199E-05	0.325E-06	0.147E-08	0.142E-08	0.143E-08	0.143E-08	0.143E-08	0.359E-02	0.223E-08	0.992E-09
6	0.129E-03	0.166E-05	0.161E-06	0.306E-07	0.227E-08	0.227E-08	0.227E-08	0.227E-08	0.227E-08	0.857E-03	0.790E-09	0.371E-09
7	0.833E-04	0.225E-06	0.205E-07	0.663E-08	0.412E-08	0.412E-08	0.412E-08	0.412E-08	0.412E-08	0.262E-03	0.457E-09	0.188E-09
8	0.507E-04	0.340E-07	0.101E-07	0.803E-08	0.791E-08	0.791E-08	0.791E-08	0.791E-08	0.791E-08	0.936E-04	0.587E-09	0.260E-09
9	0.422E-04	0.869E-08	0.138E-07	0.142E-07	0.140E-07	0.140E-07	0.140E-07	0.140E-07	0.140E-07	0.425E-04	0.954E-09	0.484E-09
10	0.328E-04	0.533E-08	0.201E-07	0.275E-07	0.273E-07	0.273E-07	0.273E-07	0.273E-07	0.273E-07	0.284E-04	0.189E-08	0.966E-09
11	0.323E-04	0.692E-08	0.343E-07	0.498E-07	0.489E-07	0.489E-07	0.489E-07	0.489E-07	0.489E-07	0.487E-04	0.358E-08	0.183E-08
12	0.299E-04	0.122E-07	0.516E-07	0.964E-07	0.956E-07	0.956E-07	0.956E-07	0.956E-07	0.956E-07	0.895E-04	0.715E-08	0.360E-08
13	0.335E-04	0.181E-07	0.910E-07	0.177E-06	0.174E-06	0.174E-06	0.174E-06	0.174E-06	0.174E-06	0.174E-03	0.137E-07	0.695E-08
14	0.354E-04	0.337E-07	0.142E-06	0.343E-06	0.340E-06	0.340E-06	0.340E-06	0.340E-06	0.340E-06	0.335E-03	0.273E-07	0.136E-07
15	0.437E-04	0.534E-07	0.260E-06	0.635E-06	0.624E-06	0.624E-06	0.624E-06	0.624E-06	0.624E-06	0.658E-03	0.528E-07	0.268E-07
	AU/day	AU/day	AU/day	AU/day	AU/day	AU/day	AU/day	AU/day	AU/day	Earth radii/day	Rad/day	Rad/day
0	0.338E-01	0.203E-01	0.175E-01	0.145E-01	0.764E-02	0.588E-02	0.376E-02	0.313E-02	0.315E-02	0.149E 02	0.102E-05	0.407E-06
1	0.548E-02	0.228E-02	0.119E-02	0.612E-03	0.482E-04	0.134E-04	0.356E-05	0.124E-05	0.122E-05	0.183E 01	0.178E-06	0.776E-07
2	0.136E-02	0.258E-03	0.870E-04	0.246E-04	0.283E-06	0.391E-07	0.402E-08	0.182E-08	0.186E-08	0.247E 00	0.483E-07	0.215E-07
3	0.353E-03	0.293E-04	0.590E-05	0.140E-05	0.254E-08	0.589E-09	0.517E-09	0.519E-09	0.520E-09	0.377E-01	0.142E-07	0.618E-08
4	0.152E-03	0.344E-05	0.498E-06	0.814E-07	0.602E-09	0.604E-09	0.603E-09	0.603E-09	0.603E-09	0.717E-02	0.440E-08	0.201E-08
5	0.654E-04	0.416E-06	0.400E-07	0.784E-08	0.107E-08	0.107E-08	0.107E-08	0.107E-08	0.107E-08	0.172E-02	0.161E-08	0.743E-09
6	0.426E-04	0.564E-07	0.534E-08	0.251E-08	0.203E-08	0.203E-08	0.203E-08	0.203E-08	0.203E-08	0.527E-03	0.769E-09	0.377E-09
7	0.261E-04	0.857E-08	0.306E-08	0.361E-08	0.352E-08	0.352E-08	0.352E-08	0.352E-08	0.352E-08	0.188E-03	0.746E-09	0.395E-09
8	0.219E-04	0.210E-08	0.394E-08	0.689E-08	0.681E-08	0.681E-08	0.681E-08	0.681E-08	0.681E-08	0.833E-04	0.130E-08	0.693E-09
9	0.171E-04	0.159E-08	0.649E-08	0.123E-07	0.121E-07	0.121E-07	0.121E-07	0.121E-07	0.121E-07	0.476E-04	0.249E-08	0.128E-08
10	0.170E-04	0.204E-08	0.102E-07	0.237E-07	0.235E-07	0.235E-07	0.235E-07	0.235E-07	0.235E-07	0.506E-04	0.478E-08	0.243E-08
11	0.159E-04	0.352E-08	0.173E-07	0.431E-07	0.423E-07	0.423E-07	0.423E-07	0.423E-07	0.423E-07	0.879E-04	0.920E-08	0.467E-08
12	0.179E-04	0.539E-08	0.295E-07	0.835E-07	0.826E-07	0.826E-07	0.826E-07	0.826E-07	0.826E-07	0.164E-03	0.181E-07	0.922E-08
13	0.191E-04	0.983E-08	0.507E-07	0.153E-06	0.150E-06	0.150E-06	0.150E-06	0.150E-06	0.150E-06	0.302E-03	0.349E-07	0.178E-07
14	0.237E-04	0.163E-07	0.925E-07	0.298E-06	0.294E-06	0.294E-06	0.294E-06	0.294E-06	0.294E-06	0.575E-03	0.685E-07	0.350E-07
15	0.280E-04	0.297E-07	0.161E-06	0.550E-06	0.541E-06	0.541E-06	0.541E-06	0.541E-06	0.541E-06	0.109E-02	0.133E-06	0.681E-07

[a]Step size is 0.5 day for moon and nutations, 2 days for Mercury, and 4 days for others.

second, requires scaling by the conversion factors kilometers/AU and kilometers/"earth radius." Conversion of planetary data from a heliocentric to a geocentric frame of reference requires specification of the earth-moon mass ratio μ to locate the earth-moon barycenter in the geocentric frame. Finally, if data are required for a particular epoch in Universal Time (UT), the time correction $\Delta t_q = \mathrm{ET} - \mathrm{UT}$ must be specified.

Interpolation

The JPL ephemeris (format type 50) contains modified second and fourth differences computed as follows:

$$d_j^2 = \delta_j^2 + a_{26}\,\delta_j^6 + a_{28}\,\delta_j^8$$
$$d_j^4 = \delta_j^4 + a_{46}\,\delta_j^6 + a_{48}\,\delta_j^8 \qquad \text{(A.1)}$$

where

$$a_{26} = -0.013120 \qquad a_{28} = 0.004299$$
$$a_{46} = -0.278269 \qquad a_{48} = 0.068489$$

These modified differences are intended to facilitate the use of Everett's fifth-order interpolation formula which may be written as

$$y\,(t_j + sh) \cong P\,(s) \equiv y_j\,F_0\,(1-s) + d_j^2\,F_2\,(1-s) + d_j^4\,F_4\,(1-s) + y_{j+1}\,F_0\,(s) + d_{j+1}^2\,F_2\,(s) + d_{j+1}^4\,F_4\,(s) \qquad \text{(A.2)}$$

where

$$F_0\,(s) = s$$
$$F_2\,(s) = [(s-1)\,(s)\,(s+1)]/6$$
$$F_4\,(s) = [(s-2)\,(s-1)\,(s)\,(s+1)\,(s+2)]/120$$

Equation (A.2) is to be used only with $0 \le s \le 1$, in which case, the truncation error can be shown to be bounded as follows:

$$|y - P| \le \sum_{k=6}^{9} b_k\,M_k + b_{10}\,\widehat{M}_{10} \qquad \text{(A.3)}$$

where

$$M_k = \max\,|\delta^k|$$
$$\widehat{M}_k = h^k \max\left|\frac{d^k y}{dt^k}\right|$$
$$b_6 = 8.35 \times 10^{-7}$$
$$b_7 = 8.99 \times 10^{-6}$$
$$b_8 = 3.05 \times 10^{-7}$$
$$b_9 = 3.48 \times 10^{-6}$$
$$b_{10} = 2.40 \times 10^{-4}$$

The quantities of M_k are given in Table A-2 for $k = 0, 1, \cdots, 15$ as computed from DE19B. The quantity $\widehat{M}_{10}$ has not been computed directly because of the difficulty of computing $d^{10}\,y/dt^{10}$. However, since $h^{10}\,(d^{10}\,y/dt^{10}) = \delta^{10} - (5/12)\,\delta^{12} + \cdots$, (Ref. 19), we will use M_{10} as an approximation to $\widehat{M}_{10}$.

With $\widehat{M}_{10}$ replaced by M_{10}, Eq. (A.3) has been evaluated, using the data given in Table A-2, and the resulting interpolation error bounds are listed in Table A-3.

Table A-3. Bound for truncation error when using fifth-order Everett interpolation formula[a]

Body	Position	Velocity
Mercury	8890.00 AU	4420.00 AU/day
Venus	4.73 AU	0.62 AU/day
Earth-moon barycenter	5.19 AU	2.50 AU/day
Mars	6.74 AU	5.77 AU/day
Jupiter	6.64 AU	5.72 AU/day
Saturn	6.64 AU	5.72 AU/day
Uranus	6.64 AU	5.72 AU/day
Neptune	6.64 AU	5.72 AU/day
Pluto	6.64 AU	5.72 AU/day
Moon	10100.00 earth radii	14500.00 earth radii/day
$\Delta\psi$	0.46 rad	1.16 rad/day
$\Delta\varepsilon$	0.23 rad	0.58 rad/day

[a]All entries have been multiplied by 10^{12}; step size is 2 days for Mercury, 0.5 day for moon, $\Delta\psi$, and $\Delta\varepsilon$, and 4 days for all others.

Appendix B

Description of FORTRAN IV Ephemeris Reading Subroutine READE

It should be noted that the format of the ephemeris tapes in DE 69 is exactly the format of DE 19; hence, any subroutine used to read DE 19 may be used without modification to read DE 69. The following subroutine system has been submitted to COSMIC for secondary distribution.

I. Identification

(1) READE read, interpolate, and translate JPL Ephemeris.

(2) Program language: FORTRAN IV.

(3) Machine: IBM 7094, UNIVAC 1108.

(4) C. L. Lawson (JPL) and J. E. Ekelund (Planning Research Corp.), June 13, 1966.

II. Purpose

This subroutine is to be used to obtain ephemeris data from a JPL Ephemeris Tape. The data will be interpolated to the Julian Ephemeris Date given by JED + TSEC/86400. The position and velocity vectors may be translated to provide the position and velocity vectors of any requested set of bodies relative to any requested central body.

III. Usage Part 1 (Basic Features)

A JPL Ephemeris Tape must be mounted on FORTRAN Unit 12. The tape format must be Type 50 (see Appendix A).

The user's program must contain the following statements:

```
COMMON/CETBL1/AU, RE, TPD,EMRAT

COMMON/CETBL2/ICW,ICENT,IREQ (13)

COMMON/CETBL4/TABOUT (6, 12), NUT (4)

DOUBLE PRECISION AU, RE, TPD, EMRAT,
TABOUT, NUT, JED,TSEC

CALL READE (JED, TSEC, IERR)

IF (IERR .NE. 0) GO TO [error procedure]
```

The parameters in CETBL1 and CETBL2 and the parameters JED and TSEC must be set by the user before calling READE. READE will place its output in CETBL4 and will set the error flag IERR and may modify the flag ICW. The parameters are as follows:

Parameter	Description
AU, RE, TPD	These three parameters determine units of the output quantities. The planetary ephemerides are recorded in AU and in AU per ephemeris day. The lunar ephemeris is recorded in "earth radii" and in "earth radii" per ephemeris day. The nutation parameters are recorded in radians and radians per ephemeris day. The user must set: AU = number of output linear units in an AU RE = number of output linear units in an "earth radius" TPD = number of output time units in an ephemeris day.
EMRAT	Ratio of the mass of the Earth to the mass of the moon. This ratio is used to locate the relative position of the Earth-moon barycenter.
ICW	Flag indicating the status of arrays into which READE reads ephemeris tape records. The user must set ICW = 1 before the first CALL to READE and should generally leave ICW alone thereafter.

Parameter	Description
ICENT	Index of body to be used as central body relative to which coordinates of requested bodies will be given. The indexes are: 1 Mercury 5 Jupiter 9 Pluto 2 Venus 6 Saturn 10 Sun 3 Earth 7 Uranus 11 Moon 4 Mars 8 Neptune (Note that earth-moon barycenter is not a permissible center.)
[IREQ (J), J = 1,13]	IREQ (J) specifies the type of output desired for body J. IREQ (J) = 0 means no output = 1 means position only = 2 means position and velocity The body numbers are assigned as given above for ICENT with the addition of 12 for earth-moon barycenter and 13 for nutation parameters.
{[TABOUT(I,J), I = 1,6], J = 1,12}	Output coordinates. The first index runs through $x, y, z, \dot{x}, \dot{y}$ and $\dot{z}$ in that order. The second index is the body number.
[NUT (I), I = 1,4]	Output values of the nutation parameters: NUT (1) = $\Delta\psi$ = Δ longitude NUT (2) = $\Delta\varepsilon$ = Δ obliquity NUT (3) = time derivative of $\Delta\psi$ NUT (4) = time derivative of $\Delta\varepsilon$.
JED, TSEC	Taken together these two parameters specify the epoch of ephemeris time at which ephemeris data is desired. The epoch is ET = JED + (TSEC/86400.D0). Any combination of values of JED and TSEC such that ET falls within the time span of the ephemeris tape is permissible. Highest resolution in interpolation will be obtained if JED is an exact binary number representing a Julian date close to the epoch of interest ET.

Parameter	Description
IERR	Error flag set by READE: 0 No error 1 (JED + TSEC/86400.D0) is smaller than first date on ephemeris tape 2 (JED + TSEC/86400.D0) is greater than last date on ephemeris tape 3 IREQ (J) is not 0,1, or 2 for some J 4 ICENT is not 1,2, ···, or 11 5 ICW is not 1, 2, or 3.

IV. Usage Part 2 (Other Features)

Besides the COMMON blocks mentioned, there are other COMMON blocks used by READE and GETTAP which may be of interest to the user:

COMMON/CETBL3/TAB3(0829), NUTAT(204), CKSUM

COMMON/CETBL5/BIVECT(6,13)

COMMON/CETBL9/JD1, TDAY, JDIF, IERR1

COMMON/REC1/REC1 (24)

COMMON/REC2/TBODY, TYPE, AJD, BJD, STEP, DUM20(20)

DOUBLE PRECISION TAB3, BIVECT, JD1, TDAY, JDIF

REAL NUTAT

INTEGER CKSUM

The standard 1863 word data record from an ephemeris tape is read by GETTAP into TAB3, NUTAT, and CKSUM.

BIVECT is working space for READE. On exit from READE, BIVECT contains interpolated and scaled, but not translated, ephemeris coordinates. The body numbering is not exactly the same as in TABOUT.

CETBL9 is used for communication between READE and GETTAP.

GETTAP reads the first two identification records of an ephemeris tape into REC1 and REC2, respectively. REC1 is a 144-character BCD identification text. REC2 contains 25 single precision floating point numbers. Of these two records, only the three items AJD, BJD, and STEP are used by READE and GETTAP.

The flag ICW has three permissible values:

Value	Description
1	Means GETTAP must rewind the ephemeris tape and read records 1 and 2 and the first data record before beginning to search for the requested epoch.
2	Means GETTAP can immediately begin to search for the requested epoch since records 1 and 2 have already been read into REC1 and REC2 and the last data record read from tape is in CETBL3. GETTAP always sets ICW = 2 after reading a data record.
3	Means REC1 and REC2 have been preserved, but CETBL3 has not (possibly due to OVERLAY) and, thus, GETTAP must read one data record before beginning to search for the requested epoch.

V. Subroutines Used

The FORTRAN IV SUBROUTINE, GETTAP, is used to position the tape and read the correct data record into CETBL3. All I/O is done via standard FORTRAN IV statements.

Appendix C

DE 69 Constants

The following table lists constants for DE 69.

Velocity of light	299 752.5 km/s
Radii	
Moon	1738 km
Mercury	2441
Venus	6053
Mars	3376
Jupiter	71350
Saturn	60400
Uranus	23800
Neptune	22300
Pluto	7200
Sun	696 000

Appendix D

Nominal Value Input Cards

Table D-1 presents the nominal value input cards.

Table D-1. Nominal value input cards

Body	AU			AU/Day		
	X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Z}$
Moon	1.957201476459373D-03	−1.601313108959806D-03	9.684816460558743D-04	−4.584079593648413D-04	−2.786992359772400D-04	−1.706976322327463D-04
Earth–Moon bary-center	6.388223239692001D-01	−7.234668700684017D-01	−3.137194324577663D-01	1.308702468621249D-02	9.880583281228748D-03	4.284450687694985D-03
Mercury	−3.472828490907576D-01	−2.538198004029557D-01	−1.002522097205557D-01	1.164349789102359D-02	−1.796892566378662D-02	−1.081910535895070D-02
Venus	−2.729262059304316D-01	−6.191876072511641D-01	−2.617880857259560D-01	1.860072423467568D-02	−6.590520902680456D-03	−4.144660068370773D-03
Mars	−1.043273791584276D 00	1.149610170700971D 00	5.556918002442636D-01	−1.028576013130965D-02	−7.076373218558543D-03	−2.972113161620146D-03
Jupiter	−4.241144808148452D 00	−3.149104334834158D 00	−1.247301166271148D 00	4.626302582644397D-03	−5.059629126080921D-03	−2.283657425358879D-03
Saturn	6.459617061053484D 00	6.101128619727081D 00	2.243375059274648D 00	−4.252230932057857D-03	3.559268583257949D-03	1.655567450141698D-03
Uranus	−1.814313887576205D 01	−2.460514498777027D 00	−8.216992903049754D-01	5.261848130241593D-04	−3.735781938971764D-03	−1.644315967838234D-03
Neptune	−1.530781100256404D 01	−2.435404000527554D 01	−9.591123470954914D 00	2.693366882765193D-03	−1.429427158965292D-03	−6.534123487881678D-04
Pluto	−3.031628535965546D 01	−1.792426075828660D 00	8.620253359544401D 00	3.986115845958216D-04	−3.146654477732543D-03	−1.114235223116397D-03

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