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A LITERATURE SURVEY OF CLEAR AIR TURBULENCE

Don R. Veazey

Texas A & M University
College Station, Texas

March 1970

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Abstract

The subject matter concerned with turbulence in clear air is summarized. Principal subjects covered include observational techniques, properties of CAT, correlations between CAT and synoptic parameters, forecasting of CAT, theories of CAT, and the remote detection of CAT A brief summary of the method of perturbations also is given

The literature pertaining to the major subject divisions is cited extensively. References in addition to those cited also are listed.

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1. INTRODUCTION

The problems concerned with clear-air turbulence (CAT) were not revealed until the latter stages of World War II. Until that time aviators as well as meteorologists felt that the air would be increasingly "smooth" as one went higher into the troposphere. Above the tropopause, in the stratosphere, it was generally conceded that the airflow would be laminar and that powered flight in this region would be comfortable, safe, and smooth.

However, high-flying combat aircraft often encountered bumpiness and irregular flight at altitudes above the tropopause. Pilots reported that the bumpiness was similar in nature to that encountered at lower altitudes, with one important difference. That difference was the fact that the turbulence was encountered in clear air and could not be attributed to convective activity or any other obvious cause.

As aircraft began flying higher and faster, the reports of turbulence in a clear sky became more numerous. More importantly, the potential danger to aircraft flying through these regions was recognized. With this recognition, CAT became the problem of aircraft engineers, meteorologists (both theoretical and practical), military leaders, commercial airlines personnel, and many others involved with flight through the atmosphere.

CAT is important to the communications engineer since it scatters signals transmitted by electromagnetic waves (Tatarski, 1961). As pointed out by Reiter (1963), CAT is responsible also for the scintillation of stars, and hence the astronomer becomes concerned with the problem. Apart from the problem of forecasting the occurrence of CAT for flight operations, the meteorologist also must determine the effect of turbulence on the energy budget of the atmosphere and any possible influence on global weather phenomena.

Even though CAT is an important problem, the definition of clear-air turbulence is not agreed upon universally. Reiter (1962c) suggested that "bumpiness in flight through clear air" might be a suitable definition. Others refer to the phenomenon as critical air turbulence (Dutton 1967 Monson, *et al.*, 1969), or in special instances, as undulance (Hildreth *et al.*, 1963). In 1966, the National Committee for Clear Air Turbulence (U.S. Dept. of Commerce, 1966) adopted the following definition of CAT

> "...all turbulence in the free atmosphere of interest in aerospace operations that is not in, or adjacent to, visible convective activity. This includes turbulence found in cirrus clouds not in or adjacent to visible convective activity."

In addition, most studies of CAT are restricted to altitudes greater than 10,000 ft (Anderson, 1956; Clodman and Ball, 1959; Colson and Panofsky, 1965; etc.) with no absolute upper limit. Any upper limit would realistically be imposed by the specific problem or area of application with which one is concerned.

2. TECHNIQUES OF OBSERVING CAT

Numerous techniques exist by which we may determine the structure, characteristics, and scale of CAT. In this section we shall be concerned primarily with those techniques which provide insight into the problem and data from which we might deduce the physical processes which lead to the formation of CAT. This is opposed to those techniques that exist as a means of providing a forewarning or probable existence of CAT. These latter types will be included in the section on detection of CAT.

2.1 Primary Observational Tools

2.11 Observations by Aircraft

Many types of aircraft have contributed data to the CAT problem. In many instances, the data consisted of nothing more than the pilot's report on the location of the turbulence and his own personal reaction to it. Much of the earliest data came from VGH (velocity, acceleration, and altitude) instruments which represent time-history measurements of the acceleration of the aircraft's center of gravity, air speed, and pressure altitude (Hislop, 1951; Pinus 1957; Zbrozek, 1965). The data obtained from VGH measurements depend heavily on aircraft type, assumptions involved in computations of aircraft gust-loading, pilot response, and to some degree the analyst or the method used to analyze the data.

In order to obtain gust velocities from VGH measurements, the dynamic response of the aircraft to the gust must be removed (Pratt and Walker, 1954). As pointed out by Zbrozek (1965), it is impossible to account for all the dynamic properties of the aircraft and therefore some simplifying assumptions must be made.

The assumptions used to derive the maximum airplane acceleration are (Crooks et al., 1968a):

a.) the airplane is rigid,

b.) the airplane is free to rise but not pitch;

c.) prior to entry into a gust, the airplane is in steady and level flight; and

d.) the gust velocity profile is a one-minus-cosine shape with a length of 25 wing chords.

This approach to the problem of gust loads on aircraft is known as the discrete-gust technique. With the above assumptions, measured acceleration increments Δn can be converted into derived gust velocities by the equation

$$U_{de} = \frac{2W\Delta n_{max}}{C_{L\alpha}\rho_0 S V_e K_g}, \qquad 2.11(1)$$

where:

W is aircraft weight (lb),

$C_{L\alpha}$ is wing lift curve slope, (rad^{-1}),

ρ_0 is air density at sea level (lb sec^2 ft^{-4}),

S is wing area (ft^2),

V_e is equivalent airspeed (ft sec^{-1}),

K_g is gust alleviation factor, and

U_{de} is the derived equivalent gust velocity (ft sec^{-1}), positive upward.

The alleviation factor, K_g, is equivalent to

$$\frac{0.88\ U_g}{5.3 + U_g}.$$

where

$$U_g = \frac{2W}{C_{L\alpha} C g S} \quad \text{(Pratt and Walker, 1954)}$$

and C is the mean aerodynamic chord, g is the acceleration of gravity (ft sec^{-2}), and ρ is air density at flight level (lb sec^2 ft^{-4}). K_g allows for some of the dynamic properties of the aircraft including unsteady lift and is determined empirically. *It must be stressed that U_{de} is a* derived gust velocity and is not the same as might be measured by an anemometer. Also, $U_{de} = U_d(\rho/\rho_0)^{\frac{1}{2}}$, so that at sea level, U_{de} is equal to U_d. A more detailed explanation of Eq. 2.11(1) may be found in Pratt and Walker (1954), Houbolt et al. (1964), Zbrozek (1965), and Crooks et al. (1968a).

The use of the discrete-gust concept is a convenient and simple way of relating the accelerations experienced by one airplane to those which are likely to be experienced by another. The concept further implies that the relative loads for single, isolated gusts are a measure of the relative loads in a sequence of gusts (Houbolt et al., 1964). The overall description of atmospheric turbulence in terms of discrete gusts is usually given as a frequency of gusts of given magnitude per mile for a range of altitudes (Ferri et al., 1962).

As aircraft configuration and response characteristics began to change, as airframes became larger and more flexible, and as their mass distributions became different, an approach more general than the discrete-gust concept was needed. The approach adopted was that of power spectral techniques or generalized harmonic analysis. A complete treatment of this subject can be found in Press (1957), Press and Houbolt (1955), Houbolt et al. (1964) and others. A simplified summary of this concept can be found in Zbrozek (1965) and Dutton (1967).

The power spectral approach has many advantages over the discrete-gust method. According to Hildreth et al. (1963), the main assets are:

1. A more realistic representation of the continuous nature of atmospheric turbulence;
2. A more realistic accounting for aircraft configuration and response characteristics; and
3. Allowance for design and operational changes.

There are many methods in existence for obtaining continuous or at least high-frequency measurements of the wind field relative to a flying aircraft. If some desired quantity cannot be measured directly, another quantity is measured that can be related to the one desired. The instrumentation used in Project HICAT (Crooks et al., 1968a), for example, provided measurements of dynamic pressure, static pressure, temperature, aircraft attitude (pitch, roll, and yaw), angular rates, components of linear acceleration, etc. The angle of attack and angle of sideslip are measured from flow vanes or differential pressure probes (Houbolt et al., 1964; Burns and Rider, 1965). These latter measurements are the basis for the description of atmospheric turbulence. Methods for treating the data and means of correcting the measurements can be found in the works of Burns and Rider (1965), Vinnichenko (1966), Crooks et al. (1968a), and others.

Dutton (1967) shows that the vertical component of the gust velocity can be obtained from aircraft measurements by using the equation

$$w_g = V\alpha_v - V\theta + \int_0^t a_z dt + l_x\dot{\theta} . \qquad 2.11(2)$$

Houbolt et al. (1964) give the lateral and longitudinal components, respectively, as

$$v_g = -V\beta_v - V\psi + l_x\dot{\psi} + \int_0^t (a_y + g\phi)dt + l_z\dot{\phi} , \qquad 2.11(3)$$

and

$$u_g = (V - \bar{V}) - \int_0^t (a_x - g\theta)dt, \qquad 2.11(4)$$

where:

a_x, a_y, a_z are longitudinal, lateral, and vertical accelerations, respectively;
g is acceleration due to gravity;
l_x, l_z are longitudinal and vertical distances from accelerometer to flow vane;
u_g, v_g, w_g are longitudinal, lateral, and vertical gust velocities;
V is airplane speed;
α_v is vertical vane angle;
β_v is side vane angle;
θ is pitch angle;
ϕ is roll angle; and
ψ is yaw angle.

Equations 2.11(2) to 2.11(4) contain the assumptions (Houbolt et al., 1964) that boom flexure is negligible, that disturbances are small enough that the angle in radians can be used in place of the sine of the angle, and that the effects of variation in upwash on vane indications may be allowed for or are negligible.

Using the time histories of u_g, v_g, and w_g obtained from Eqs. 2.11(2) to 2.11(4), one can employ standard techniques to obtain the spectral characteristics of atmospheric gusts. A standard analysis generally consists of prewhitening the data, estimating values of the auto-correlation function, obtaining raw estimates of power, smoothing the estimates of power, and then postdarkening the smoothed estimates to obtain the final estimates of the power spectrum (Blackman and Tukey, 1959; Houbolt et al., (Appendix D), 1964).

An alternate method may be used to find the spectrum of atmospheric gusts. If one knows the response function of the data-gathering aircraft, then the frequency spectrum of atmospheric gusts is characterized by (Reiter, 1962e)

$$\Phi_0(\omega) = [T(\omega)]^2 \Phi_i(\omega), \qquad 2.11(5)$$

where $\Phi_i(\omega)$ and $\Phi_0(\omega)$ are the frequency spectra of input and output, respectively, and $T(\omega)$ is the response function of the aircraft. Figure 1, taken from a report by Houbolt and Kordes (1954), illustrates Eq. 2.11(5).

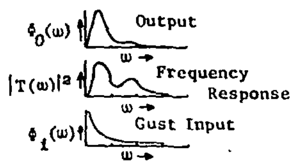


Fig. 1 Illustration of Eq. 2.11(5) (after Houbolt and Kordes, 1954)

2.12 Observations by Balloons

Balloons have been employed as wind sensors for many years. In most instances, the balloon is tracked through space by either radar or theodolites, and the balloon may or may not carry instrumentation. Wind profile data are obtained from the positional records of the balloon, but as shown by Scoggins (1963c), most of the small-scale motions are averaged out. An excellent discussion of the errors associated with standard wind measurements by balloons can be found in Reiter (1963). Balloons may also carry accelerometers and thus provide data on atmospheric gusts. This latter system is often referred to as a gust-sonde (Anderson, 1956). Reiter (1962c) suggests that constant-level balloons might carry microbarographs and produce more reliable data than gust-sondes. In the past few years, superpressure balloons and highly accurate tracking systems have been employed to determine the fine structure of detailed vertical wind profiles (Leviton, 1962, Johnston, 1962; Scoggins, 1963b).

Experimental results of flow around spheres may be found in many books on fluid dynamics (e.g., Goldstein, 1938; Schlichting, 1960). A brief summary of the theoretical considerations of balloon dynamics may be found in Scoggins (1964). A more detailed treatment is given by Eckstrom (1965) and Scoggins (1967).

The chief difficulty in using balloons to measure atmospheric turbulence is that smooth-skinned balloons experience spurious, self-induced oscillations even when rising in calm air (Scoggins, 1964; Murrow and Henry, 1965). Hence, it is impossible to measure true wind motions on a small scale.

This difficulty has been overcome largely by using a superpressure, rough-skinned balloon known as the Jimsphere (Eckstrom, 1965; Scoggins, 1967). When coupled with a highly accurate tracking radar such as the FPS-16, detailed vertical wind profiles may be obtained. Some of these profiles and possible explanations for the exhibited structure of the profiles may be found in Scoggins (1964, 1965), Scoggins and Vaughan (1965), Weinstein et al. (1966), DeMandel and Scoggins (1967), and others.

2.13 Cloud Observations

Perturbations in the upper-level winds may be deduced by detailed analysis of cloud photographs. This technique has been used by Ludlam (1952), Reiter (1962b), Reiter and Hayman (1962), and Reiter and Nania (1964). The photographs allow one to determine (or at least infer) the wave lengths and orientation of perturbations at the cirrus level. An excellent photograph of turbulence indicated by cirrus clouds was taken by Regula (1968) while flying near Newfoundland in 1967. According to Colson (1962), the presence of high clouds appears to increase the probability of the occurrence of turbulence. Kadlec (1963) has found evidence of a strong relation between CAT and the thickness of cirrus clouds. Other workers have attempted to use satellite pictures of cloud patterns in order to study CAT relationships (Weigman, 1965; Cooley and Ball, 1969), but little progress has been made in this area. The primary reason for the failure of satellites to detect cloud patterns associated with CAT is that the resolution threshold of weather satellite data is too large.

2.14 Observations by Artificial Tracers

Various methods using artificial tracers have been employed to measure the effects of upper-level turbulence or to measure this turbulence directly. Clodman (1958a) studied the effects of turbulence on the behavior of aircraft contrails, and smoke trails have been studied by Durst (1948). Henry et al. (1961) have shown how detailed vertical wind profiles may be obtained from smoke trails released from rockets. Reiter (1964a) used smoke as a tracer to follow CAT "patches" when they were found in research flights over Australia. Other techniques use aluminum foil or some other radio-reflective material released at high levels and these materials are then tracked by radar. For high-altitude study, i.e., 80-150 km, sodium vapor released from rockets is used to determine the structure of the wind and properties of diffusion (Manring, 1961).

Reiter (1962c) lists two basic difficulties in the application of the above techniques to the study of CAT. They are:

a.) The aircraft or rocket may disturb the airflow to a degree so as to produce serious imperfections in the data, and

b.) Contrails or smoke trails will spread out and dissipate due to small-scale turbulence and diffusion processes.

Motions on a scale corresponding to CAT may therefore be extremely difficult to separate from these superimposed small-scale effects.

2.2 Supporting Observational Tools

The techniques which are deemed to be of a support nature are those which either have been used sparingly or do not lend themselves to widespread use by the scientific community. Included in this group are direct observations of CAT by radar (Atlas and Naito, 1966; Atlas *et al*., 1968), lidar observations (Collis, 1964; Bourquin and Shigemoto, 1965; Collis *et al*., 1969), acoustical sensing techniques (Bates, 1965), and astronomical observations such as the scintillation of starlight (Protheroe, 1961; Battan, 1962) and the trembling in the Sun's image (Okoshi, 1967).

Microbarograph traces may be used to deduce the presence of gravity waves which may be associated with CAT. Kuettner (1952b) discusses this technique, and Gossard (1960) presents spectra on pressure variations associated with gravity waves over California. Hodge and Voltz (1968) have studied the possible relationship between mesoscale surface pressure waves and CAT.

3. CAT STUDIES

A great deal of time, money, and effort has been expended to gather data and determine the relevant properties of CAT. Because of the immense scope of the problem, most studies of CAT have been conducted or sponsored by governmental agencies from all parts of the world.

One of the first CAT-oriented programs was conducted by the Royal Aircraft Establishment in the United Kingdom during the period 1946-1948 (Met. Office Dis., 1958). From 1948 to 1950, the Ministry of Supply supported a flight program to study CAT over the air routes of British European Airways (Hislop, 1951). Measurements of free-atmosphere turbulence were conducted by the National Advisory Committee for Aeronautics (NACA) during the period 1951-52 with the use of gust-sondes (Anderson, 1956). The U.S. Air Force sponsored Project Jetstream during the winter months of 1953-54 and 1954-55, and again from 1956-1958 (Brundidge, 1958; Reiter, 1963; etc.). Project Cloud Trail was established in the U.S. Air Force Air Defense Command and ran for one year, December 1954 to December 1955 (Clem, 1957).

Another CAT measurement program, Project TOPCAT, was conducted over South Australia during 1963 (Radok and Reiter, 1964; Burns and Rider, 1965). A massive program to locate and measure atmospheric gusts (and other parameters as well) is currently being sponsored by the U.S. Air Force under the code name ALLCAT (Dutton, 1967; Atnip, 1969; Loving, 1969). Project ALLCAT is designed to measure atmospheric properties from the surface to 200,000 ft (61 km). A comprehensive measurement program has been and continues to be conducted in the U.S.S.R. (Vinnichenko, 1969). Other specialized but less comprehensive programs have been conducted by the Air Force Cambridge Research Laboratories (Penn and Pisinski, 1967), as well as by commercial airlines in the United States (Harrison, 1961; Kadlec, 1964).

Nearly all of the above studies, plus many others, were or are conducted on the basis of obtaining accurate meteorological data. Numerous other data have been compiled using CAT reports such as might be received via PIREPS, etc. Studies using data of this type have been reported by Bannon (1951), Turner (1959), Colson and Rustenbeck (1961), Kroll and Rustenbeck (1961), Briggs (1961a, 1961b, 1963), Colson (1961, 1962, 1963, 1969), Sorenson (1964), Colquhoun and Bourke (1967a), Endlich and Mancuso (1968), and many others.

4. PROPERTIES OF CAT

The one dominant fact that has emerged from studies such as the ones listed above is that CAT can and does occur at any altitude, at any location, and at any time. This means, of course, that all present-day aircraft must face the possibility of encountering some degree of turbulence while in flight. This is true also for spacecraft that are ascending or descending through Earth's atmosphere as well as for the proposed SST which will be operational in the near future (Reiter, 1964a; Kordes and Love, 1967; Ehernberger, 1968b; Burnham, 1968). Thus a knowledge of the properties of CAT is mandatory so that aerospace operations may be conducted in a reasonable and safe manner.

4.1 Seasonal Distribution

Nearly all statistical studies of CAT have shown that aircraft encounter CAT most frequently during the winter months. Clem (1957) showed that CAT occurrence for all sectors of the U.S. was greatest on a percentage basis during the winter. Table 1 is a summary of Clem's findings.

Table 1. Seasonal occurrence of CAT in the United States (after Clem, 1957).

	NW US	SW US	NE US
Winter	28%	33%	44%
Spring	24%	19%	32%
Summer	23%	14%	28%
Fall	22%	16%	27%

Briggs (1963) showed that the seasonal variation of turbulence at heights of 20,000 to 40,000 ft over Europe ranges from a summer minimum of about 6700 n mi per incident of turbulence to an autumn maximum of about 3500 n mi per incident. This is an increase of nearly 46 per cent from summer to autumn. He further shows that the variation in the eastern Mediterranean is from a summer minimum of 7570 n mi per incident to a winter maximum of 2910 n mi per incident. These latter figures represent a seasonal change of 38 per cent in the encounter of turbulence by aircraft. Other studies in the N. Hemisphere, such as those by Colson (1963), Rao and Sadogopan (1968), and others, show results of a similar nature. An exception to this general fact was reported by

Berenger and Heissat (1959), who found more turbulence over France during the summer than during the winter.

Recent measurements in the S. Hemisphere show that the occurrence of CAT on a percentage basis is greatest during the fall and winter. Colquhoun (1967c) found that the greatest percentage of CAT occurred in March (autumn) and that 52 per cent of all aircraft flights in the measuring program encountered some degree of turbulence in this time period. In summarizing the results of the Australian phase of the U.S. Air Force's HICAT program, Spillane (1967) reports that the greatest frequency of occurrence of CAT is in the winter and early spring.

4.2 Geographical Distribution

Among the studies which attempt to define the geographical distribution of CAT, there is no universal agreement. Several facts must be kept in mind when comparing one study with another. First, the method of quoting results is not uniform (Clodman et al., 1961). Furthermore, different methods of collecting the data might give different results for the same location. Terrain effects differ from one location to another so that strict comparisons become very difficult. Finally, many studies are based on data collected from scheduled airline flights so that a statistical bias might be introduced. These latter flights also use turbulence-avoidance techniques so that an additional bias is possible.

The study by Clem (1957) of Project Cloud Trail data showed that the highest incidence of CAT between 25,000 and 45,000 ft was in the northeastern United States. Balzer and Harrison (1959) report that there is reason to believe that the exposure to high-level CAT over the United States is fairly uniform. Colson (1963), in studying over 12,000 CAT reports covering the period September 1960 to August 1962, found a maximum of CAT occurrence over southeast Colorado with a secondary maximum over southern New England.

Variations of CAT occurrence over Western Europe have been found by Hislop (1951), Berenger and Heissat (1959), Briggs (1963), and others. Colquhoun (1967a, 1967b, 1967c) and Colquhoun and Bourke (1967a, 1967b) show that CAT occurrences in the South Pacific have maximum values over southern Australia with another area of high occurrence in the vicinity of Borneo.

Coleman and Steiner (1960) and Steiner (1966) have analyzed data from U-2 flights made over the U.S., Japan, Turkey, England, and Germany. Their results show that the frequency of exceeding given values of gust velocity per mile of

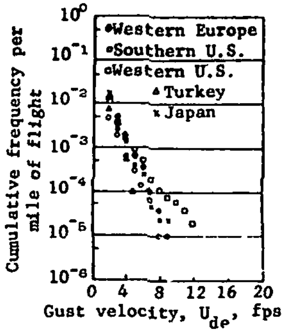


Fig. 2. Frequency of exceeding given values of gust velocity per mile of flight for five areas (after Coleman and Steiner, 1960).

flight differs very little among these locations (see Fig. 2). Figure 3 shows values of average turbulence frequency vs height for the United States. These values were obtained by Endlich and Mancuso (1968) in their attempt to estimate a turbulence climatology between 20,000 and 45,000 ft.

A study by Clodman et al. (1961) shows that the frequency of occurrence of CAT over the oceans is about one-tenth of that over the continents. In an experiment to relate CAT to terrain, Turner (1959) found that over the British Isles, CAT was four times as frequent over the coastlines and more than twice as frequent over land as over the sea. He noted also that the difference was less marked during the summer.

To explain the variability of CAT occurrences over land and over the ocean, one is led naturally to consider the effects of terrain on high-level turbulence. Ludlam (1952) suggests that cirrus clouds can be caused by hills 1000 ft high. According to Jenkins (1952), some obstacles can affect the air flow up to 25 times their height. Wave patterns that are orographically induced have been detected in the ozonosphere (Paetzold and Zschorner, 1955). Terrain effects of CAT occurrence are reported for areas over Australia (Colquhoun, 1967c), the Middle East (Rao and Sadogopan, 1968), and the United States (Clodman and Ball, 1959; Endlich and Mancuso, 1965a; Foltz, 1967). The importance of terrain in the production of CAT will be discussed in Section 7.

4.3 Meso-scale Properties of CAT

4.31 Horizontal Dimensions

CAT encountered by aircraft is normally described as being of a patchy nature. The occurrence ratio of turbulent patches to smooth patches within a given turbulent area varies from the order of 1:1 to about 1:10 (Clem, 1957). Clodman et al. (1961) conclude that, on the average, about 3 per cent of the distance flown by

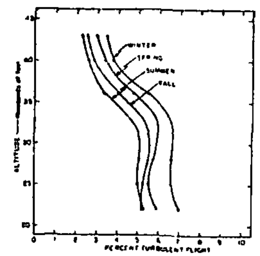


Fig. 3. Curves of average turbulence frequency vs height for the United States (after Endlich and Mancuso, 1968).

an aircraft in the range of 20,000 to 45,000 ft over land will be turbulent. Hislop (1951) found that the mean horizontal extent of turbulent areas was 75 to 100 km. Clodman (1953) gives this figure at about 90 km with an extreme of approximately 450 km. Press et al. (1953) report horizontal extents of 15 to 60 km, while Clem (1957) reports that 50 per cent of the cases have horizontal dimensions less than 80 km. Clem also found that CAT patches are elongated in the direction of the wind and that they are frequently several times longer than they are wide. Cunningham (1958), using the data from Project Jet Stream (Fetner, 1956), gives the frequency distribution of horizontal extent shown in Table 2. These data pertain to the continental United States.

Table 2. Horizontal extent of turbulent regions (after Fetner, 1956)

Extent (mi)	10.0	10-19.9	20-29.9	30-39.9	40-49.9
% Occurrence	31.5	21.4	9.5	10.1	6.5
Extent (mi)	50-59.9	60-69.9	70-79.9	80-89.9	90-99.9
% Occurrence	4.8	2.4	2.4	0.6	1.2
Extent (mi)	100-149.9	150-199.9	200.0		
% Occurrence	6.5	2.4	0.6		

Steiner (1966) found that in the flights of the U-2 aircraft, 50 per cent of the turbulent areas exceeded 10 mi in length.

When various intensities of CAT are categorized, the statistical distribution is found to change. Clodman (1953), for example, states that areas with strong turbulence usually are larger and thicker. This fact was noted also by Press et al. (1953). Briggs and Roach (1963) report that in 22 flights through jet streams, the following was found:

Turbulence Intensity	Duration (min)	% of Total Flying Time
slight	165	5.4
moderate	122	4.0
severe	5	0.2

These data seem to show that the more severe CAT areas are smaller, or just the opposite from what was found by Clodman and Press et al., mentioned above. A summary of some recent flight data is shown in Fig. 4 (after Burnham, 1968).

The horizontal dimensions of CAT over the Atlantic Ocean have been reported by Morgan (Clodman et al., 1961) to be about 100 n mi, on the average. His figures, when broken down into intensity categories, show that CAT areas of moderate, severe, and violent have horizontal dimensions of 93.4, 124.6, and 87.0 n mi, respectively.

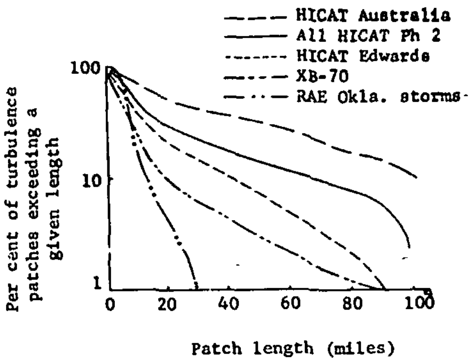


Fig. 4. Percentage of turbulence exceeding a given length encountered between 40,(and 70,000 ft during a number of research flights (after Burnham, 1968)

4.32 Vertical Dimensions

CAT areas are generally quite thin in the vertical with 2000 ft being the average thickness (Hislop, 1951; Clem, 1957; Farthing, 1959; Clodman et al., 1961; etc.). Press et al. (1953) report that 50 per cent of the cases they studied were less than 2000 ft thick. The extreme thickness values which have been reported range from 100 to 20,000 ft (Clodman, 1953; Berenger and Heissat, 1959; Farthing, 1959). Turbulent patch often occur in multiple layers which may have several thousand feet of relati ly smooth air separating them (Anderson, 1956; Clem, 1957).

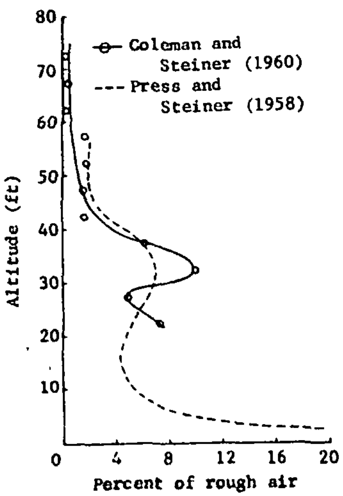


Figure 5 is a compilation of the variat with altitude of the percentage of time spent in rough air by aircraft (Coleman and Steiner, 1960). Also shown in the figure, for comparison purposes, are th results obtained by Press and Steiner (1958). This figure indicates that CAT encountered 7 to 10 per cent of the tim at flight levels between 30,000 and 40, ft, and less than about 2 per cent of t time at altitudes greater than 50,000 f Anderson's (1956) gust-sonde analysis showed the following altitudes where t

Fig. 5. Variation in percentage of rou air with altitude (after Colem and Steiner, 1960).

bulence encounters were maxima and minima:

Minimum Exposure	Maximum Exposure
6.2 km	7.4 km
9.5 km	11.0 km
12.8 km	14.4 km

His data showed a rapid decrease in turbulence encounters above 15,000 m. A recent study by Spillane (1967) showed that the expectation of encountering CAT per 100 flights is greatest in the height range of 50,000 to 55,000 ft. His results are summarized in Table 3.

Table 3. Expectation of encountering CAT per 100 flights (after Spillane, 1967)

Height (x 10^3 ft)	%	Height (x 10^3 ft)	%
10-15	20	40-45	11
15-20	19	45-50	15
20-25	11	50-55	43
25-30	20	55-60	35
30-35	37	60-65	17
35-40	13		

4.33 Temporal Distribution

CAT is often called a "random" phenomenon (Reiter, 1963) and this would certainly be in keeping with the statistical approach to the theory of CAT. This idea is not unfounded since oftentimes an aircraft will report turbulence while another aircraft passing through the same area a short time later will not (Hislop, 1951). However, many observations tend to show that CAT, although transient, is persistent. Endlich (1963) has reported that moderate or severe CAT may persist for many hours, while Colson (1963) has shown that CAT areas may persist for periods up to a day. Some detailed vertical wind profiles have shown persistent fine structure for periods up to several hours (Weinstein et al., 1966; DeMandel and Scoggins, 1967). Reiter (1964a) has written of a turbulent patch over South Australia that was marked by smoke. The research aircraft was able to follow this patch of turbulence for about 45 minutes. The distance traversed by the patch was approximately 100 mi. Reiter also states that the intensity level of CAT as well as the size of the patch stayed about the same during the entire period.

4.34 Intensity of CAT

The intensity of CAT is of extreme importance to the aircraft engineer, for this is one of the parameters that determine the structural loads which the aircraft must be built to withstand. Aircraft controllability and pilot/passenger comfort also are dependent on this factor.

Various classifications have been used to describe the intensity of CAT. One subjective classification uses the categories mild, light, moderate, severe, and violent (Hyde, 1954b). The meaning of these categories is as follows:

- Mild - "Cobblestone", 6-8 cycles sec^{-1}, discernable to passengers;
- Light - "Cobblestone", control displacement, passenger discomfort;
- Moderate - uncomfortable control, unstable all axes, seat belts required;
- Severe - Cb-type turbulence, extremely uncomfortable for passengers; and
- Violent - intense bumps, high "g", completely unacceptable, threshold of structural failure.

Other classifications use incremental vertical acceleration as a basis for catagorizing intensity. Crooks et al. (1968a) offer the following:

CAT Description	Peak "g" Increment
very light	± 0.05 to ± 0.10
light	± 0.10 to ± 0.25
moderate	± 0.25 to ± 0.50
severe	± 0.50 to ± 0.75
extreme	± 0.75 or greater

For a basis of comparison, Burnham (1968) states that acceleration increments of ± 0.06 g would be of some inconvenience to a passenger who was trying to drink. Figure 6 shows the human reaction to a range of aircraft accelerations.

Since structural damage is possible for an aircraft encountering intense turbulence (Reiter, 1962a), it is of interest to note some of the extreme values which have been found by various workers. Hislop (1951) reported that the "Mosquito" aircraft flying over Europe experienced incremental accelerations of 1.5 g or 35 ft $sec^{-1}U_{de}$. Other values which have been reported are Bindon (1951), 3 g; Hyde (1954a), 2.5 to 4.0 g; Farthing (1959), 36 ft $sec^{-1}U_{de}$;

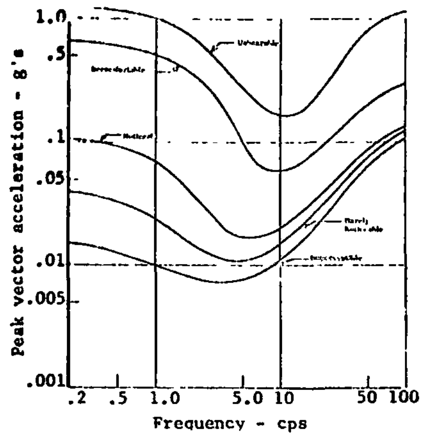


Fig. 6. Vibration threshold levels of human feeling (after Hildreth et al., 1963).

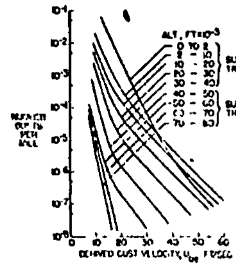


Fig. 7. Estimated overall gust distributions for operations at various altitudes (after Rhyne and Steiner, 1962).

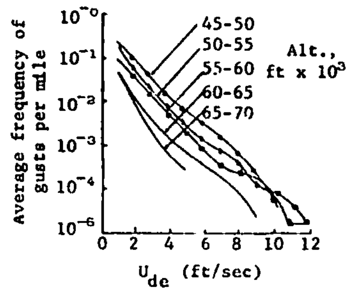


Fig. 8. Average frequencies at which given values of derived gust velocity were exceeded per mile of flight during the phase 2 HICAT program (after Burnham, 1968).

Steiner (1966), 58 ft $sec^{-1}U_{de}$; and Crooks et al. (1968a), 1.1 g or 22 ft $sec^{-1}U_{de}$. Some of the most severe encounters with CAT have occurred in mountainous regions. In a spectacular accident in which a B-52 lost 85 per cent of its vertical stabilizer, lateral gusts were estimated to exceed 100 ft sec^{-1} (Dutton, 1967). Later measurements using F-106 fighter aircraft in the Sangre de Cristos Mountains, Colorado, revealed more than 25 gusts with amplitudes exceeding 100 ft sec^{-1}. The maximum gust recorded during the tests was 175 ft sec^{-1} (Dutton, 1967).

In the interest of aircraft design, data of the above nature have been subjected to various statistical treatments. Hislop (1951) offers the following analysis: Operating a fleet of 20 Comet aircraft at an annual utilization of 3,000 hr, one would expect that one of the airplanes would encounter a 36 ft sec^{-1} gust on the average of every 2 wk and a 50 ft sec^{-1} gust once every 4 yr. Other examples of studies of this type are those made by Rhyne and Steiner (1962), Coy (1967), Burnham (1968), and Crooks et al. (1968a). Figures 7 and 8 illustrate several of the estimates for overall gust distributions in the vertical.

4.35 Lateral vs Vertical Gustiness

Horizontal atmospheric gusts may cause aerospace vehicles to experience vertical accelerations due to the change of lift induced across the wings or other lifting elements. Clodman (1957, 1958b) indicates that for speeds of high flying aircraft, it would take horizontal gusts of about 30 ft sec^{-1} (on the average) to produce moderate to heavy bumpiness. Gusts of this magnitude are about four times as great as the vertical velocities needed to produce equivalent aircraft accelerations (Hislop, 1951). In the light of the measurements made in the Sangre de Cristos which were mentioned earlier, it seems that, under certain conditions and in certain locations, horizontal gusts become very important.

In an experiment to determine the relative magnitude of vertical and horizontal gusts in the atmosphere, Donely (1940) found that eddies of the sizes which affect airplanes are largely isotropic. His results pertained to the low- and mid-troposphere.

In a stable stratification, vertical motions will be suppressed because of the large amount of work which would be required to overcome the force of gravity. Theoretically, then, horizontal gusts should prevail in the stratosphere (Reiter, 1963). This is verified somewhat by the findings of Clodman and Ball (1959). Their statistical investigation based on 25 flights in the vicinity of the jet stream show that horizontal gustiness prevails in the stratosphere while vertical gustiness is prevalent in the troposphere. Reiter and Burns (1966) found that long wave perturbations tend to be anisotropic when found in stable layers of the stratosphere and the upper troposphere; but perturbations with wave lengths below a certain critical value (see Section 7) break down into isotropic turbulence. Findings of this nature suggest that the atmospheric modes which produce either isotropic or anisotropic turbulence must be considered when forecasting high-level turbulence.

4.4 Distribution of CAT with Respect to Atmospheric Phenomena

4.41 Jet Streams

Theoretical considerations made by Richardson (1920), Arakawa (1951), and others suggest that vertical and horizontal wind shears are important mechanisms for the formation of turbulence. It follows then that the jet stream should be a region with a high incidence of CAT. This fact has been borne out by the studies of Bannon (1952), Clem (1957), Endlich and McLean (1957), Sasaki (1958), Balzer and Harrison (1959), Briggs and Roach (1963), Kao and Sizoo (1966), and

Penn and Pisinski (1967), just to name a few. Studies in the S. Hemisphere (Spillane, 1965, 1967; Colquhoun and Bourke, 1967a, 1967b; etc.) also show the jet stream to be a preferred region for the occurrence of CAT. The now-classic study made by Bannon (1952) showed that CAT appeared to be grouped in particular zones relative to the jet axis. He found also that 75 per cent of the cases studied occurred on the low-pressure side of the stream. Bannon's results have been verified by Sasaki (1958) in his analysis of nine Project Jet Stream flights. Sasaki's results are shown in Fig. 9. The figure is expressed in percentage of turbulent flying time of the total flying time. This figure clearly shows that

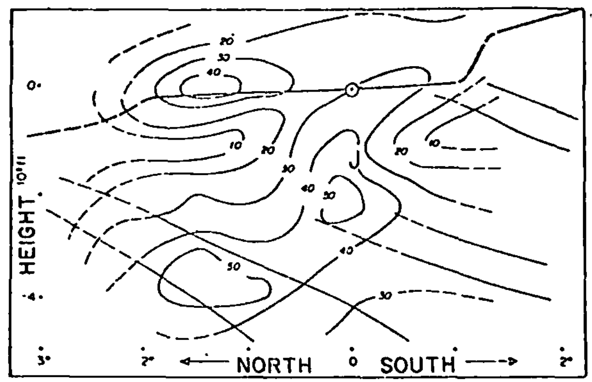


Fig. 9. Mean distribution of turbulent flying time (CAT) in per cent of total flying time for nine Project Jet Stream flights 1956-1957 (after Sasaki, 1958).

the turbulence frequency on the cyclonic (low-pressure) side of the jet exceeds that on the anticyclonic side. Contrary to this, turbulence over the ocean seems to favor the anticyclonic side of the jet stream above the axis (Clodman et al., 1961). As pointed out by Reiter (1963), the positions of the individual turbulence maxima coincide with the locations of stable baroclinic zones. This corresponds quite well with Endlich's and McLean's (1957) jet-stream "front" (a stable layer characterized by extreme vertical wind shears). This furthermore agrees with numerous findings of turbulence near the tropopause (see Danielsen, 1959) in the vicinity of the jet stream (Clem, 1957; Briggs and Roach, 1963; Penn and Pisinski, 1967; Panofsky et al., 1968).

The distribution of <u>severe</u> CAT in relation to the jet stream has been investigated by Harrison (1959), Reiter (1962c), and Endlich (1963). Harrison's study discriminated between CAT occurrence versus various jets and positions of jets; viz., occurrence relative to polar jets, occurrence between two jets, and occurrence relative to subtropical jets. He found that severe CAT occurred primarily below and poleward of the polar jet axis. His study of severe CAT occurrences between two jets revealed that nearly all cases occurred below and equatorward of the northernmost jet. He could reach no conclusion concerning the relation between severe CAT and the subtropical jet because of a very small sample size.

Other examples of the distribution of CAT relative to the jet stream are shown in Figs. 10 to 13. Figure 14 shows the location of reported occurrences

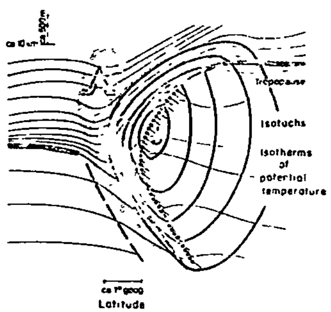


Fig. 10. Schematic cross-section through the jet stream (J): potential temperatures (thin lines), and isotachs (semi-heavy lines). The boundaries of the stable baroclinic frontal zone are marked by heavy dashed lines, the tropopause by heavy solid lines. The shaded area within the "isentrope trough" indicates the region in which the occurrence of moderate to severe CAT (Λ) is most likely. The dotted line represents the northern boundary of extensive cirrus-cloud sheets (after Reiter, 1962c).

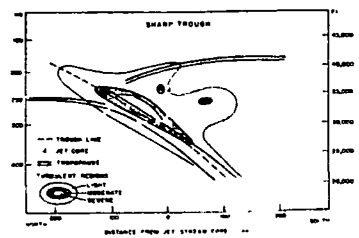


Fig. 11. Distribution of turbulence in the vicinity of a sharp trough and jet stream (after Endlich, 1963).

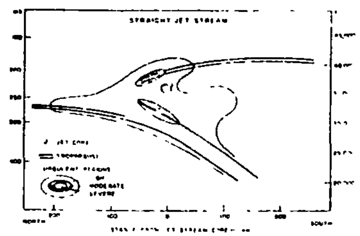


Fig. 12. Turbulent regions in straight jet streams (in the absence of mountain waves) (after Endlich, 1963).

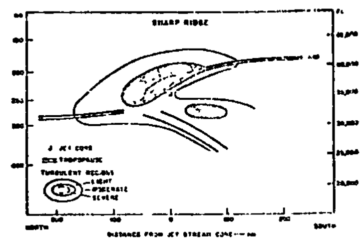


Fig. 13. Turbulent regions in sharply-curved anticyclonic jet streams (after Endlich, 1963).

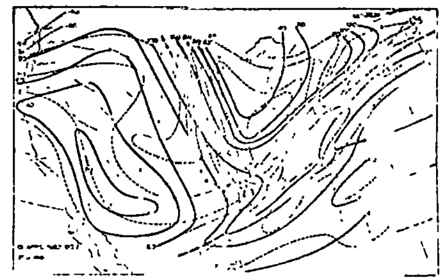


Fig. 14. 25-mb isotachs (areas > 100 knots shaded) and isotherms (°C), 13 April 1962, 0000 GCT, and moderate and severe cases of CAT observed within ± 6 hours of map time (after Reiter, 1962c).

of moderate and severe CAT relative to synoptic features on an upper-level chart for 13 April 1962. Figure 15 depicts the distribution about the jet stream of reported high-level turbulence over the North Atlantic Ocean.

The longitudinal distribution of CAT along the jet stream also has been investigated. Studies by Clem (1954) and Chambers (1955) indicate that turbulence may be somewhat more prevalent in the exit region of an isotach maximum. A study by Lennie (1969) shows that turbulence occurs near the exit region of the jet stream under conditions of cyclonic development. On the other hand, more turbulence was found in the entrance region of an isotach maximum by Bannon (1951) and by Balzer and Harrison (1959). Reiter (1962b) and Reiter and Nania (1964) have found evidence which suggests that CAT is concentrated at the confluence region of two jet streams. Over the oceans, the delta region seems to be more turbulent than any other section of the jet stream (Reiter, 1963).

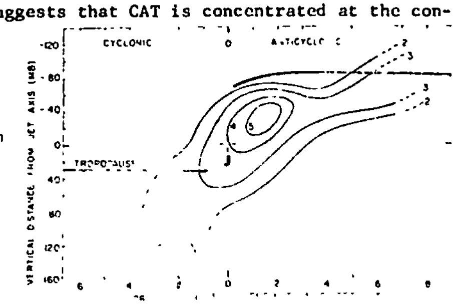


Fig. 15. Per cent occurrence of high-level turbulence about the jet stream over the North Atlantic Ocean in vertical "boxes" of 40 mb × 120 n mi (after Clodman <u>et al.</u>, 1961).

4.42 Troughs, Ridges, and Other Features

In order to develop practical forecasting techniques, numerous studies have been made in which CAT was correlated with synoptic features other than jet streams. Bannon (1952) found that, of those cases not related to the jet stream, most were associated with upper lows, upper troughs, or discontinuous tropopause surfaces. He reported also that less than 7 per cent of the cases appeared to have no obvious association with any configuration of the upper-air pattern. CAT associated with sharp trough lines aloft and deep upper-level cold lows has been reported by Clem (1957), Smigielski (1960), Briggs (1961a), Srinivasan (1961), and Endlich (1963).

Kronebach (1964) has studied the distribution of CAT with respect to troughs, ridges, and closed lows at 300 mb. His findings are shown in Table 4.

Table 4. Distribution of CAT at 300 mb (after Kronebach, 1964).

	Feature	% Total	% Within 5°Lat At or Behind	% Within 5°Lat At or Ahead	% Within 2.5°Lat
Cyclonic	Troughs	44	25	18	-
	Closed low	-	-	-	1
Anticyclonic	Ridges	13	7	6	-
	43% Undefined				

Surface synoptic features also have been related to occurrence of CAT. Turner (1959) found that over the land and coasts of the British Isles, only one-third of the cases were associated with fronts. On the other hand, three-fourths of the cases over the sea apparently were connected with fronts and half of these were warm fronts. In a study of satellite photographs, Weigman (1965) found that most turbulence below 20,000 ft was located in the warm air above the warm frontal boundary in advance of its surface position.

Sorenson (1964) studied all available CAT reports for December 1963 in the hope of finding means of delimiting CAT forecast areas. His study indicates that certain synoptic conditions and areas are favorable for CAT formation. Some of these are as follows:

a.) regions of increasing thermal gradients,

b.) areas of sharp horizontal shifting of wind aloft, and

c.) areas of sharp vertical direction shear (see also, Reiter and Nania, 1964).

Sorenson also points out that CAT seldom develops in areas of

a.) uniformly curved flow or straight flow,

b.) decreasing winds or decreasing thermal gradients, or

c.) above the tropopause east of 100° W. longitude unless there is fairly strong evidence of thermal convergence occurring above it.

Spectral Properties of CAT

In order to evaluate the effect of turbulence on aircraft, we must know turbulence energy is distributed over various eddy sizes (see Eq. 2.11(5)). spectrum of turbulent energy also is important for an understanding of the sical processes leading to the formation of CAT (Reiter and Burns, 1966; us et al., 1967).

Many of the early studies of the spectrum of turbulence in clear air were ducted at altitudes below 5,000 ft (Press, 1957; Houbolt et al., 1964). bulence spectra have been computed also for turbulence in cumulus clouds and thunderstorms (Steiner, 1965, 1966). Examples of typical power spectra of turbulence measured in these three weather conditions are given in Fig. 16.

More recently, the micro-structure of free-atmospheric turbulence has been estigated. Specially instrumented aircraft over the U.S.S.R. (Vinnichenko al., 1965), Australia (Burns and Rider, 1965), and the United States (Crooks al., 1968a) have produced sets of power spectra of CAT at the level of the stream. As pointed out by Pinus et al. (1967), data of this nature give indiions, for the first time, of processes that lead to the formation of turbulent ght conditions in clear air. Kao and Woods (1964) have obtained energ" ctra of mesoscale turbulence along across the jet stream. Thus, turence spectra are available for the stream level over wave lengths ging from approximately 50 m to 900 km (Pinus et al., 1967).

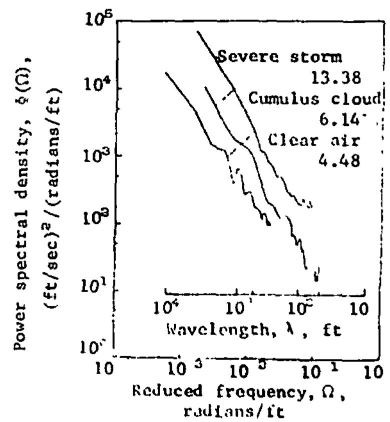


Fig. 16. Typical power spectra of vertical component of turbulence measured in clear air, cumulus cloud, and thunderstorm (after Houbolt et al., 1964).

The shape of the energy spec has been disputed for many years. as been shown by Kolmogorov (1941) Batchelor, 1959) that in the inal subrange, the energy is proional to the inverse five-thirds r of frequency. This can be exsed by

$$(\omega) = \alpha\epsilon^{2/3}\ \omega^{-5/3}, \qquad 4.5\ (1)$$

where E (ω) is the frequency-dependent energy of turbulence motion, α is a universal constant, ε is the rate of dissipation of energy, and ω is frequency (Lumley and Panofsky, 1964). The minus-five-thirds relationship has been well substantiated at low levels. Evidence that this shape holds for higher levels in the atmosphere has been presented by MacCready (1962), Houbolt et al. (1964), Reiter and Burns (1966), and others. Under certain conditions, the spectral curves seem to follow a minus two relationship (Lappe and Davidson, 1962; Kao and Woods, 1964), minus four-thirds, minus eleven-fifths (Reiter and Burns, 1966), as well as others (Ogura, 1958).

The power spectra obtained by some of the investigations mentioned above are shown in Fig. 17. Table 5 lists the sources and characteristics of the spectra in Fig. 17.

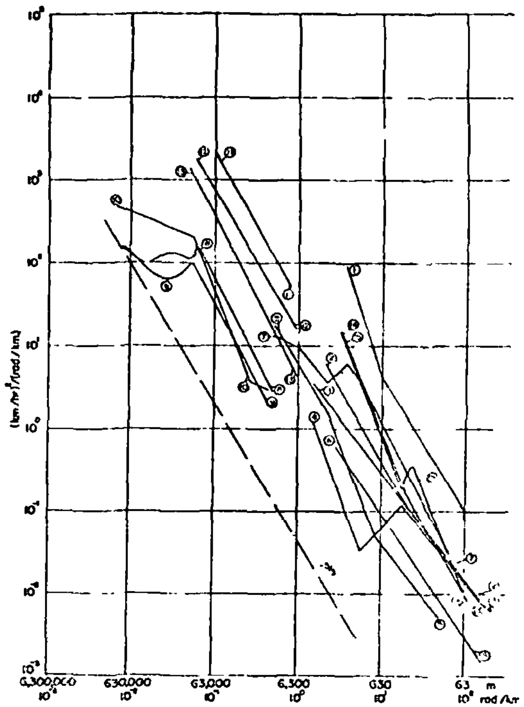


Fig. 17. Schematic presentation of spectral density, $(km/hr)^2/(rad/km)$, as a function of wave number (cycles/km), or wavelength in meters, for turbulence in the free atmosphere. Data sources of spectra are identified in Table V. (after Pinus et al., 1967).

Table 5. Power Spectra of Turbulence in the Free Atmosphere (Pinus, et al., 1966).

Spectrum No.	Source	Characteristics (turbulence components given with respect to course of aircraft)
1	Shur, 1962	w-component, severe CAT, near jet stream level, stable stratification.
2	Reiter and Burns 1965	u, v-components, moderate CAT, jet stream level, stable stratification.
3	Reiter and Burns 1965	w-component, flight parallel to wind, moderate CAT, jet stream level, stable stratification.
4	Reiter and Burns 1965	w-component, flight nearly normal to wind, moderate CAT, jet stream level, stable stratification.
5	Vinnichenko, Pinus and Shur, 1965	u-component, no CAT, near jet stream level, stable stratification.
6	Reiter and Burns 1965	u,v,w-components, light turbulence at 100 m altitude, unstable stratification.
7	Vinnichenko, Pinus and Shur, 1965	u-component, light turbulence at 1000 m altitude, unstable stratification.
8	Kao and Woods 1964	u-component, at jet stream level, flight parallel to jet stream.
9	Kao and Woods 1964	v-component, at jet stream level, flight parallel to jet stream.
10	Kao and Woods 1964	u,v-components, at jet stream level, flight normal to jet stream.
11	Pinus, 1963	u-component, severe CAT, under core of jet stream, flight normal to jet stream.
12	Pinus, 1963	u-component, moderate CAT, under core of jet stream, flight normal to jet stream.
13	Pinus, 1963	u-component, light CAT, over core of jet stream, flight normal to jet stream.
14	Shur, 1962	w-component, moderate CAT, at jet stream level, stable stratification.
15	Shur, 1962	w-component, moderate CAT, at jet stream level, stable stratification.

5. RELATIONSHIPS BETWEEN CAT AND SYNOPTIC PARAMETERS

We know from studies of the type covered in the preceding section that CAT is a microscale phenomenon. However, numerous correlations have been attempted to find a relation between CAT and macroscale parameters (Endlich, 1964; Endlich and Mancuso, 1964; Scoggins et al., 1969; etc.). As a result, correlation studies have shown large discrepancies. Furthermore, as pointed out by Moore and Krishnamurti (1966), the usual parameters such as the Richardson number, etc., are probably important in the region where turbulence is created, and are important only in that region at or just before the time the flow breaks down into turbulence. This means that once turbulence is initiated, it may be too late to look at these parameters since the flow field will be changed because of the onset of turbulence. In a similar fashion, others have pointed out that it is difficult to determine whether certain features (e.g., lapse rate of temperature, Richardson number, etc.) are responsible for turbulence or whether they themselves are caused by the turbulence (Gruzinova and Sofiev, 1964; Reed, 1969). In spite of the foregoing limitations, some success has been achieved using certain gross dynamic parameters.

5.1 CAT and Horizontal Wind Speed

Studies which have attempted to show a relation between CAT and horizontal wind speed have been largely unsuccessful. For example, Lake (1956) concluded that wind speed alone was a poor forecasting tool. Colson (1962), and Briggs and Roach (1963) reached this same conclusion. Others finding a poor relationship between CAT and horizontal wind speed include Hislop (1951), Davies (1951), Colquhoun and Bourke (1967a,b), and Colquhoun (1967a,b,c,). An exception to these general findings is reported by Ehernberger (1968a). He found that turbulence of significant intensity at SST supersonic climb and cruise altitudes is related in some degree to horizontal wind speeds in excess of 70 kt.

5.2 CAT Versus Horizontal and Vertical Wind Shear

Hislop (1951), Lake (1956), Clem (1957), Briggs (1961b), and George (1961) have all found that vertical shear is an important parameter in high-level turbulence. The results concerning horizontal shear are not so clear. Harrison (1959, 1961) states that horizontal shear is important in the forecasting of CAT. However, Briggs and Roach (1963) conclude in their study that horizontal wind shear is not highly correlated with the occurrence of CAT. Kronebach (1964) also obtained a poor relationship between the occurrence of CAT and horizontal shear of wind. According to Colson and Panofsky (1965), any correlation between Cat and horizontal shear is probably due to a statistical relation between horizontal and vertical wind shear.

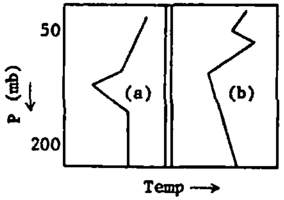


Fig. 18. Characteristic temperature profile produced by ageostrophic meridional wind component above and (a) on the poleward and (be) equatorward side of a sharp subtropical wind maximum (after Spillane, 1967).

5.3 CAT Vers s Hydrostatic Stability and/or Lapse Rate of Temperature

Lake (1956) found that moderate turbulence was most likely to be associated with lapse rates which were less than dry adiabatic but greater than isothermal. He also found that, with either the adiabatic or inversional class of lapse rates, no turbulence was likely. The data from Project Rough Rider (Clem, 1957) showed that a useful relationship existed between CAT and steep lapse rates of temperature. Endlich (1964) found that a discontinuity of lapse rate at an upper front or at the tropopause was important for the existence of CAT. Flights with Jindivik target aircraft over the Woomera Rocket Range in S. Australia have shown that CAT is most likely to occur when there is a sharp "kink" in the temperature profile between 50 and 200 mb (Spillane, 1967). The characteristic temperature profiles discussed by Spillane are shown schematically in Fig. 18.

Keitz (1959) found that a significant correlation existed between the observation of CAT and a forecasted change in the hydrostatic stability. The change in stability was determined to be caused by differential advection. Based on Keitz's observation, Schwerdtfeger and Radok (1959) have suggested a simple method using hodographic techniques to detect changes in static stability.

As indicated earlier, individual turbulence maxima coincide with the location of stable baroclinic zones. According to Danielsen (1959), the stable and baroclinic zones associated with CAT formation are rather thin. Hence, since accurate measurements of stability over thin layers are difficult to obtain, any relationship between hydrostatic stability and high-level turbulence is difficult to examine.

5.4 CAT Versus the Richardson Number (Ri)

A criterion widely used to describe the stability of fluid flow is the Richardson number (Ri) (Richardson, 1920; Calder, 1949; Dugstad, 1958a; etc.). Ri expresses the ratio of buoyancy forces to shearing stresses and can be written in the form

$$Ri = \frac{K_H}{K_M}\frac{g}{\bar{T}}\frac{\beta}{\gamma} \qquad 5.4(1)$$

In this equation, K_H is the eddy conductivity, K_M is the eddy viscosity, g the acceleration of gravity, and $\bar{T}$ the mean temperature. The factor β expresses the sum of the observed lapse rate of temperature and the dry adiabatic lapse rate, i.e., $\partial T/\partial z + \Gamma$, while γ is the vertical wind shear squared, viz., $\gamma = (\partial\bar{u}/\partial z)^2$. It should be noted that $\bar{u}$ is the mean vector wind.

Theoretically, if Ri is smaller than some critical value, Ri_c, laminar flow will break down into turbulence. Richardson suggested that turbulence would develop and grow if Ri< 1. Other values given for Ri_c are 0.5 by Prandtl (Scorer, 1956), 0.25 by Taylor (1931), 0.25 to 1.0 by Wanta (1953), plus many others (see, e.g., Lyons et al., 1964; Lumley and Panofsky, 1964; Webster, 1964; Panofsky et al., 1968).

It is generally assumed that the ratio K_H to K_M is equal to 1. However, this ratio is dependent on stability and other parameters. Values ranging from 0.83 to 3 have been reported (Lumley and Panofsky, 1964, p. 105-106), and a value of 0.65 has been reported for the free atmosphere by Petterssen and Swinbank (1947). Most studies which have attempted to correlate CAT with Ri have assumed that $K_H/K_M = 1$ so that Eq. 5.4(1) becomes simply

$$Ri = g\beta/\bar{T}\gamma. \qquad 5.4(2)$$

Many shortcomings are inherent in the evaluation or computation of Ri. These have been discussed by Scorer (1956), Colson and Panofsky (1965), Reiter and Lester (1967), and Scoggins et al. (1969). In summary, these shortcomings include:

a.) Poorly resolved data, i.e., significant vertical wind shears and lapse rates are often smoothed out (Reiter, 1963; Colson and Panofsky, 1965; Scoggins et al., 1969);
b.) improper choice of scale length (Reiter and Lester, 1967, Scorer, 1956);
c.) non-synchronous data collection (Zavarina and Yudin, 1960), and
d.) failure to take into account the sign of the vertical wind shear (Lake, 1956).

With these shortcomings, it is no wonder then that relationships between Ri and CAT have shown little consistency.

Table 6 is a compilation of the results obtained by several investigators as regards relationships between CAT and Ri. As expected, the results are highly divergent.

5.5 CAT Indices and Other Relationships

Since the Richardson number fails as a suitable indicator of turbulence, other dynamic parameters have been combined in the search for a consistent indicator of turbulence. One of these indices is due to Colson and Panofsky (1965), and the other to Endlich and Mancuso (1964).

The CAT index of Colson and Panofsky is really an index of CAT energy. The index is derived by using energy considerations, dimensional analysis, and similarity theory. By making several simplifying assumptions, one may write the index as

$$I = (\Delta \vec{V})^2 \ (1 - Ri/Ri_c), \qquad 5.5(1)$$

where $\Delta \vec{V}$ is the vector change of the horizontal wind in a layer. The parameters used in computing I may be taken from ordinary meteorological data. Analyses by Colquhoun (1967c) and Blackburn (1969) have shown that the index is, in general, a poor indicator of turbulence.

Endlich and Mancuso give their index as

$$V(\partial\alpha/\partial z)(\partial^2 T/\partial z^2), \qquad 5.5(2)$$

in which V is wind speed, α is wind direction, and T is temperature. Hence, this index incorporates the vertical directional shear and the curvature of the vertical profile of temperature. Both of these quantities have been shown to be of some importance in CAT processes (Sasaki, 1958; Reiter and Nania, 1964, Sorenson, 1964). Correlation studies have shown that this index also is unreliable (Endlich and Mancuso, 1965b; Colquhoun, 1967c).

Klemin and Pinus (1954), Pinus (1957), and Balzer and Harrison (1959) have found that cold air advection is more conducive to the occurrence of CAT than either warm or neutral advection. Sorenson (1964) has shown that either cold or warm advection is associated with CAT occurrence depending on the particular synoptic situation. Differential advection of temperature, which is indicated

Table 6. Results of the relationships between CAT and Ri

Investigator	Findings
Anderson (1956)	50% probability of CAT with Ri< 1.06, 82% with Ri< 6
Bannon (1951)	No relation between CAT and Ri in stratosphere. 30% probability of CAT with Ri< 3
Berenger and Heissat (1959)	70% probability of CAT with Ri≤ 1
Briggs (1961b)	Ri< 5 or horizontal shear > 0.3 hr^{-1} gives 80% success of CAT forecasts
Briggs and Roach (1963)	Significant increase in turbulence for decrease in Ri
Colquhoun (1967c)	Generally poor results
Colson (1962)	Probability of CAT occurrence increases with decreasing value of Ri
Colson (1963)	Fair correlation for flights below 29,000 feet
Durst (1943)	Weak relationship between CAT and Ri
Endlich (1964)	CAT regions with areas delimited by Ri_c = 1
Endlich and McLean (1965)	With Ri≤ 0.7, 50% increase in frequency of occurrence of all classes of CAT
Endlich and Mancuso (1964)	Ri = 0.6 correctly identified 28% of turbulent cases and 92% of non-turbulent cases. Ri_c = 1 "over forecasts" CAT
Gruzinova and Sofiev (1964)	No unequivocal relation between turbulence on a scale of 2-10 m and Ri. Small Ri are not prerequisites for turbulence.
Jaffe (1963)	Ri_c = 1.5 verified as CAT indicator
Kao and Woods (1964)	Found agreement between CAT and Ri~1.0
Kronebach (1964)	Ri_c = 1.0 useful for 12-hour forecast.
Lake (1956)	No clear relationship between CAT and Ri
Panofsky and McLean (1964)	ALLCAT reports occurred in regions of low Ri
Panofsky et al. (1968)	Ri_c in neighborhood of 0.5
Penn and Pisinski (1967)	For Ri≤ 0.5: 105 cases with CAT and 44 with no CAT. For Ri> 0.5: 44 cases with CAT and 246 with no CAT
Petterssen and Swinbank (1947)	Ri_c = 1.54 for free atmosphere

Table 6. (cont'd.)

Investigator	Findings
Pinus (1957)	Ri≤ 4 gave weak aircraft bumpiness while Ri≤ 0.5 gave moderate to strong bumpiness
Pinus and Shmeter (1962)	85% probability of CAT with Ri≤ 4
Pinus and Shmeter (1965)	Concluded that the smaller the value of Ri, the greater the probability of turbulence. Ri does not give necessary and sufficient conditions for CAT
Reiter and Lester (1967)	Concluded that there is usually a strong dependence of Ri on layer thickness L. It does not appear fruitful to specify Ri_c for CAT occurrence unless the dependence Ri = f(L) is known for the time and vicinity of the atmospheric level in question
Rustenbeck (1963)	In general, CAT frequency 64% for Ri≤ 5. In the region 4,000 feet above to 10,000 feet below the level of maximum winds 79% of CAT with Ri≤ 5. Found poor correlation in the stratosphere
Scoggins (1963a)	No correlation
Scoggins et al. (1969)	Poor relationship with turbulence occurring with about equal frequency for all Ri's< 50
Scorer (1957)	CAT probability approaches 100% with Ri≤ 0.01
Stinson et al. (1964)	Showed that layers with Ri< 1 were common and persisted for many hours
Zavarina and Yudin (1960)	Found good correlation with Ri_c = 1

by a turning of the wind with height, has been shown to be important by Keitz (1959), Reiter (1964c), Reiter and Nania (1964), and others.

Several investigations have shown that more turbulence is found with cyclonic curvature and shear than with anticyclonic conditions (Pinus, 1957; Clodman, 1958a; Balzer and Harrison, 1959). This is in agreement with the findings cited in Section 4.42. Sorenson (1964) reported that he found a significant number of CAT cases during December 1963, which occurred in connection with anticyclonic curvature. According to Clodman et al. (1961), turbulence over the ocean is favored by anticyclonic shear and curvature. Some of the evidence supporting these conclusions is given in Tables 7 and 8.

Table 7. Recurrence (%) of aircraft bumping in relation to the character of the barometric field (after Pinus, 1957)

Character of Current	Cyclonic Curvature of the Height Contours in the Layer			Anticyclonic Curvature of the Height Contours in the Layer		
	3-8 km	8-14 km	3-14 km	3-8 km	8-14 km	3-14 km
Convergence	57.4	50.0	[illegible]	[illegible]	[illegible]	[illegible]
Divergence	17.3	8.0	15.6	7.3	5.3	6.8
Parallel Structure	17.2	9.6	14.9	8.7	4.4	7.0

Table 8. Turbulence cases (all intensities) for amount of flying (%) (after Clodman et al., 1961)

300-mb Contour Curvature	Season	Horizontal Wind Shear Anticyclonic	Cyclonic	Total
Anticyclonic	APR. - SEP.	2.84	7.80	10.64
	OCT. - MAR.	21.80	16.31	38.11
	YEAR	24.64	24.11	48.75
Cyclonic	APR. - SEP.	2.41	1.91	4.32
	OCT. - MAR.	8.86	3.87	12.73
	YEAR	11.27	5.78	17.05
Total	APR. - SEP.	5.25	9.71	15.00
	OCT. - MAR.	30.66	20.18	51.00
	YEAR	35.91	29.89	66.00

Other parameters that have been related to CAT include strong horizontal gradients of temperature (Reiter, 1962c; Sorenson, 1964; George, 1965; Moore and Krishnamurti 1966; Kadlec, 1969), curvature of the vertical profile of wind (Lake, 1956; Endlich and Mancuso, 1965a), vorticity patterns and advection of vorticity (Colquhoun and Bourke, 1967a,b; Colquhoun, 1967c), deformation of the wind field (Mancuso and Endlich, 1966), horizontal and vertical gradients of kinetic energy (Briggs and Roach, 1963; Kronebach, 1964), and the Scorer parameter (Lake, 1956). None of these parameters have shown a consistent and reliable relationship with the occurrence of CAT.

McKay (1968) suggested that CAT might be associated with atmospheric electric phenomena. He correlated reports of CAT with radar-detected sporadic E(Es) activity. The correlation can only be described as inferential because of the extremely small sample.

6. FORECASTING CAT

6.1 A Brief History of CAT Forecasting

As was pointed out in the introduction, the phenomenon of CAT was first discovered in the latter stages of World War II. Formal forecast procedures were not in use, however, until 1959 when commercial flights with turbojet aircraft were initiated. At this time, United Air Lines began to issue routine CAT forecasts (Sorenson, 1964). Early procedures were based on a meandering jet-stream model related to horizontal wind shear as outlined by Harrison (1959). In 1960, Eastern Air Lines adopted the procedures formulated by George (1960b, 1961). Procedures for forecasting CAT were introduced by the U.S. Weather Bureau in 1962 (Hanson et al., 1962). The USAF Air Weather Service established its CAT Forecast Section at Kansas City, Mo., in 1961. The procedures used by AWS were basically those of Harrison and George (Kronebach, 1964). Modifications to some of the above procedures have been introduced as new knowledge has been gained (Harrison, 1961; Kronebach, 1964; Reiter, 1964c; Sorenson, 1964).

6.2 Routine Forecast Procedures

The forecast procedures and rules which are presented in this section lend themselves to routine usage by the operational forecaster. The only requirement for application of the techniques is access to ordinary meteorological data, viz., standard synoptic reports, rawinsonde or pibal data, upper-level charts, etc. Success of the forecasts is dependent on several factors, viz.,

a.) accuracy of the data,
b.) accurate data analysis,
c.) degree of correlation between CAT and forecast parameters, and
d.) skill of the forecaster.

A technique developed by Pinus (1957) is based on the thermal wind equation along with an expression for Ri. It can be shown, for a stable current of air of sufficiently large scale, that

$$Ri = \frac{f^2 T}{\rho} \frac{\Gamma - \gamma}{\theta_1^2}, \qquad 6.2(1)$$

where f is the coriolis parameter, ρ the density of the air (average value for the atmospheric level under consideration), T the observed temperature of the air, Γ the dry adiabatic lapse rate, γ the observed lapse rate, and β_1 the horizontal temperature gradient. Values of γ and β_1 are determined from analyzed charts and these values are then used to enter the nomogram shown in Fig. 19. The success of this technique is highly dependent on the relationship between CAT and Ri, which, as was seen in Section 5.4, is unreliable.

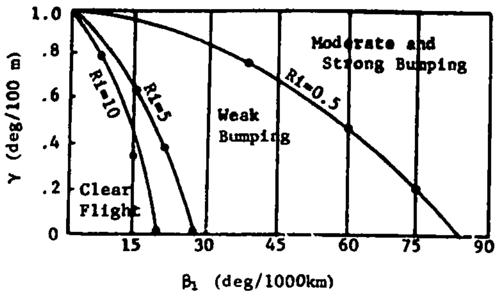


Fig. 19. Nomogram for forecasting bumpiness in flight (after Pinus, 1957).

Following a study of 76 cases of severe CAT occurrence, Harrison (1959) proposed a set of forecasting rules. His rules are as follows:

Rule 1. Severe CAT occurs in a layer of strong horizontal wind shear.

Rule 2. Three out of four severe CAT occurrences are found to the left of the jet core facing downwind.

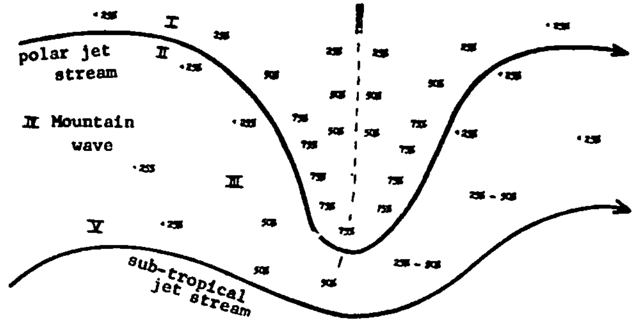
Rule 3. CAT has a strong tendency to be associated with a meandering polar jet stream.

Rule 4. About one severe CAT case in ten occurs between the polar and subtropical jet streams when they are in close proximity.

Rule 5. At least one severe CAT case in 20 is mountain wave in origin.

The forecast procedures of United Air Lines are based upon these rules in addition to some modifications proposed by Harrison (1961) and Sorenson (1964). The UAL forecasting model is shown in Fig. 20. Figure 21 is the UAL mountain wave nomogram which is used for forecasting Type IV CAT occurrences. According to Sorenson, CAT zones also should be indicated in the following regions:

1. In troughs where horizontal shear is ≥ 40 kt per 150 n mi and the thermal gradient is steepening.
   a.) Between the thermal trough and jet axis.
   b.) In areas where the wind direction changes by 30° or more in 4° of latitude.



1. Horizontal wind shear should be 40 k/150 n mi
2. Vertical wind shear may be substituted for Types II and III (6 k/1,000 ft)
3. Use Fig. 21 for Type IV - Mountain Wave cases

Fig. 20. UAL CAT forecasting model - 1961 Revision

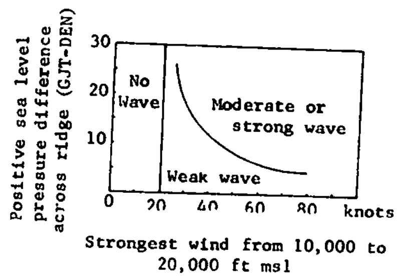


Fig. 21. UAL Mountain wave nomogram.

   c.) Ahead of the trough when southerly winds at 700 or 500 mb turn sharply eastward.
   d.) In areas of pronounced "undercutting".
   e.) In areas of cold advection at 200 mb.
2. In sharp ridges, when winds at LMW (level of maximum wind) are at least

40 kt and the radius of curvature at 700 or 500 mb is smaller than 4° of latitude (and tending to sharpen further). Include the area 3° of latitude on either side of the ridge line with the required curvature.

3. In tightening thermal gradients along mountain ranges when winds are nearly normal to the range.
4. Limit CAT to 3,000 ft above the tropopause (5,000 ft over mountain areas) unless significant thermal convergence is occurring above these limits.

Vertical wind shear has been incorporated into a technique proposed by George (1961). His investigations show that in severe CAT cases, it is usually possible to draw smooth isolines connecting equal altitudes of the base of significant shear layers. Wind shears greater than 6 kt per 1000 ft are deemed as being significant. George has found that severe CAT occurs where large gradients of horizontal wind shear (50 kt per 150 n mi) cross the centers of the vertical shear isotachs (defined as the inner two closed isotachs when drawn for intervals of 3 kt per 1000 ft). CAT usually occurs within a few thousand feet above the base of the vertical shear layer. George indicates that it is best to forecast CAT occurrence in a layer about 10,000 ft deep from 3,000 ft below to 7,000 ft above the base of the shear layer. The details of chart preparation to use this method can be found in <u>Shell Aviation News</u>, No. 273, March 1961.

The U.S. Weather Bureau (ESSA) issues CAT forecasts on the basis that there is more than a 50 per cent chance for moderate-severe CAT in areas of strong vertical vector shear where the atmosphere is unstable or rapidly becoming so (ESSA, 1966). Areas where these conditions are most likely to occur are:

1. To left of strong jet (one isotach ≥ 125 kt at 300 mb) looking downstream between the trough and upstream ridge where the averaged vertical scalar shear is greater than 6 kt per 1000 ft. This area is most likely to have moderate or greater CAT when on the latest 300 mb chart, an area of cold air advection also is indicated as impinging on the left side of the jet stream.
2. In the confluent region between two jet cores where the cores are less than 5° of latitude apart.
3. In sharp, V-shaped troughs which slope rapidly with elevation below 300 mb.
4. In the "neck" of cut-off lows.
5. To the right (looking downstream) of strong anticyclonically curved jet streams.

6. Over mountain ranges, when the jet core crosses at right angles to the major axis of the range, and winds at the top of the mountain are at least 25 kt.

6.3 <u>Forecasting CAT by Numerical Methods</u>

The techniques of predicting CAT by numerical methods generally fall into two groups. One group utilizes automated procedures to compute various quantities such as Ri, static stability, etc., from current upper-air data. Forecasts are then based upon the currently existing situation. The CAT Forecast Section of AWS utilizes Ri as an indicator of CAT, and areas within isopleths of Ri ≤ 1.0 are assumed to locate the probable area of CAT for the following 12 hours (Kronebach, <u>op</u>. <u>cit</u>.).

The second group of forecasts is based upon the prognosticated values of atmospheric parameters. Various parameters such as vertical shear, Ri, and vertical wind speed are computed over a grid network. Forecasts for turbulence are then made on the basis of statistical correlations between CAT and the various parameters (Endlich and Mancuso, 1965b).

Numerical techniques generally are favorable from the standpoint of both speed and accuracy. The techniques are highly dependent, however, upon the atmospheric model that is employed (see e.g., Shuman and Hovermale, 1968), the input data, and of course, upon the various statistical correlations used in making the forecasts.

6.4 <u>Other Methods</u>

A method of making short-period warnings over mountainous terrain recently has been proposed by Foltz (1967). The method is predicated on the assumption that CAT with "wavelengths" of approximately 100 m is caused by unstable gravity waves. Foltz further assumes that the turbulence is isotropic and that the energy spectral distribution follows the "-5/3 law" over wavelengths of 0.05 to 20 km. Utilizing various aspects of the theory of air flow over mountains (see Krishnamurti, 1964b), Foltz derives an expression for the maximum vertical velocity to be expected in a lee wave with wavelength λ (Reiter and Foltz, 1967), viz.,

$$w_o \approx \frac{2\pi hb}{e^{kb}} u_1 \frac{4\pi^2 A}{\lambda} . \qquad 6.4(1)$$

In this equation, w_o is the vertical velocity (maximum), h the mountain height, b the half-width of the mountain, u_1 the wind speed at gradient-wind level, and k the wave number ($k = 2\pi\lambda^{-1}$). The parameter A is equal to $\tan^2\alpha(\eta\pi-\alpha-\tan\alpha)^{-1}$, where α is an angle between 0 and 90 deg (Corby and Wallington, 1956), and η is a positive integer greater than zero. Reiter and Foltz show that A is approximately equal to 1 (maximum value) when $\alpha \approx 65$ deg, and $\eta = 1$. Hence, all the parameters in Eq. 6.4(1) can be estimated using topographic maps, synoptic charts, and satellite photographs (to determine λ, which might also be determined visually). Figure 22 is a nomogram used in conjunction with Eq. 6.4(1) to determine the intensity of CAT associated with mountain wave activity. The spectra for various intensity levels of CAT, extrapolated with "-5/3 slope" to lee-wave lengths, are shown in Fig. 23.

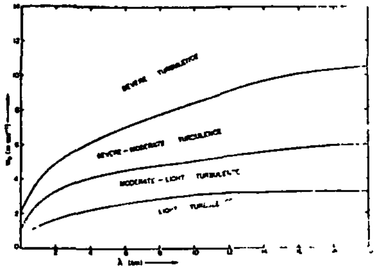


Fig. 22. Forecast degree of CAT. When the maximum vertical velocity in the lee wave is determined from Eq. 25 and the lee wavelength is determined from $\lambda=2\pi/k$, the degree of CAT can be determined from this graph (after Foltz, 1967).

6.5 Evaluation of Forecast Methods

Sorenson (1964) has summarized the problems affecting most techniques of forecasting CAT. The problems are that the techniques

1. over-forecast to a large extent (so as to be certain of "hitting" the areas of CAT),
2. miss some important areas of turbulence and
3. are based on symptoms rather than causes.

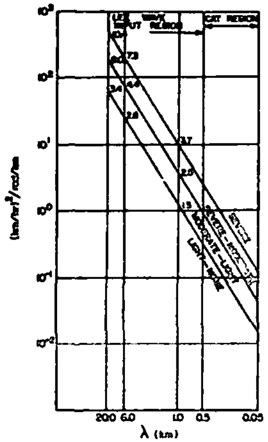


Fig. 23. Spectra of lee wave energy. The values entered on the spectral curves of -5/3 slope are the maximum vertical velocities contained in the lee waves in m sec $^{-1}$ at the lee wavelength necessary to result in the level of energy (w component) in the CAT wavelength region to cause various degrees of CAT (after Foltz, 1967).

Reiter (1964a) has suggested that forecast procedures might be improved significantly if we: a) first look for physical causes of CAT rather than for multi-discriminate correlations with atmospheric parameters, and b) as a second step, search for a combination of parameters that might bring about the necessary conditions to make the physical causes effective. But, as pointed out by Reiter and Foltz (1967), we will never be able to pin-point isolated patches of turbulence in space and time because of the vast difference of scales involved in the forecasting problem.

7. THEORIES OF CAT

To the present day, no single theory is fully capable of explaining the phenomenon of CAT. There are several reasons why this is so. First, if by CAT one means "bumpiness in flight through clear air" (Reiter, 1962e), the physical nature of the phenomenon is multi-faceted. Atmospheric motions which affect aircraft are known to occur under both stable and unstable flow regimes. They also occur under conditions of both laminar and turbulent flow. Second, truly turbulent flow in clear air may at times be of a scale so as to be completely unimportant to aerospace vehicles. But, CAT of this scale can be, and is, important to the astronomer, radio engineer, and others (see Section 1.).

Before going into the various theories which have been advanced to explain CAT, a distinction must be made between turbulence and undulance. The motion of a fluid particle in an undulating medium can be described rather specifically, while in a turbulent medium, the motion can be described only in statistical terms. Furthermore, the process of diffusion is not caused by undulance (Hildreth et al., 1963). Another important difference between the two is that, under conditions of static stability, undulant flow can persist whereas turbulence is greatly inhibited (Lumley and Panofsky, 1964). According to Hildreth et al. (1963), undulance can be induced in the atmosphere by one or more of the following:

a.) flow over mountains,
b.) convective activity in adjacent layers, and
c.) shear flow across a stable layer.

The mechanisms which produce turbulence are as follows:

a.) mechanical stirring caused by the flow of air over terrain or other "rough" surfaces,
b.) convection caused by differential heating or cooling, and
c.) *instability in shear flow.*

From the foregoing, then, it appears that the applicability of any one CAT theory is dependent upon several factors. These factors seem to be one's definition of CAT and the specific area of application.

The theories which have been advanced to explain the occurrence of upper-level turbulence can be put in one of three broad classes:

a.) theories of turbulent fluid flow,
b.) theories of dynamic or inertial instability, and

c.) theories of wave instability.

Airflow of an undulant nature is treated by means of wave theory.

7.1 Turbulent Flow of Fluids

The study of turbulence in fluids is due in large part to Osborne Reynolds (Brunt, 1952). Assuming an incompressible fluid, Reynolds concluded that the kinetic energy of the turbulent motion would increase, be stationary, or diminish depending on whether the rate at which work is done by the eddy stresses is greater than, equal to, or less than the rate at which work has to be done to maintain the turbulence against the stabilizing forces (Lamb, 1945). Reynold's theory of the criterion of turbulence was extended to the atmosphere by Richardson (1920). The Richardson number and many of its limitations were discussed earlier in Section 5.4.

The Richardson number has been modified by several workers to account for factors which were omitted in its original derivation. Dugstad (1958a) investigated the effects of horizontal wind shear and the dynamic stability of the mean flow on Richardson's criterion. With the inclusion of these two aspects of fluid flow, Dugstad shows that the general criterion for increasing turbulence is given by

$$(G-A)\tan^2\alpha - 2(B+C-F)\tan\alpha + H-D > 0, \qquad 7.1(1)$$

where:

$A = C_e g T^{-1}(\partial T/\partial z + \Gamma_d)$
$B = C_e g (2T)^{-1}(\partial T/\partial x)$
$C = \frac{1}{2}\mu_e(f + 2Vr^{-1})\,\partial V/\partial z$
$D = \mu_e(f + 2Vr^{-1})(f + \partial V/\partial x + Vr^{-1})$
$F = \frac{1}{2}\mu_e(f + 2\,\partial V_\theta/\partial r)\,\partial V_\theta/\partial z$
$G = \mu_e(\partial V_\theta/\partial z)^2$
$H = \mu_e(f + \partial V_\theta/\partial r + V_\theta r^{-1})(\partial V_\theta/\partial r - V_\theta r^{-1})$
f = Coriolis parameter = $2\Omega\sin\phi$, where Ω is Earth's angular velocity
l = characteristic mixing length
r = radius of curvature of mean flow
α = angle between the x-axis and l (xz-plane)
θ = azimuth angle of cyclindrical coordinate system
V_θ, V_r = tangential and radial velocities, respectively
C_e = eddy conductivity in direction of l

μ_e = eddy viscosity along 1

g = acceleration of gravity

Γ_d = dry-adiabatic lapse rate of temperature.

Special cases also are given by Dugstad. Specifically, he shows that the criterion for isotropic turbulence is given by

$$\left(\frac{\partial V_\theta}{\partial z}\right)^2 + \left(f + \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\right)\left(-f + \frac{\partial V_\theta}{\partial r} - 3\frac{V_\theta}{r}\right) \begin{matrix} \geq C_e \\ < \mu_e \end{matrix} \; \frac{g}{T}\left(\frac{\partial T}{\partial z} + \Gamma_d\right) \qquad 7.1(2)$$

This equation is quite similar to the Richardson number with the exception of the second term on the left-hand side. For vertical turbulence, Dugstad's criterion is identical to that of Richardson's (see Eq. 5.4(1)).

As mentioned in Section 4.41, observations have shown that there may be an increase in the frequency of CAT within stable and baroclinic layers. This is clearly contradictory to the implications of Richardson's criterion. Pinus (1957), Reed (1960), Reiter (1963), and others have shown that, by combining the thermal wind equation with Ri, one arrives at

$$Ri^* = f^2\theta(g\,\partial\theta/\partial z)^{-1}\left[(\partial z/\partial n)_\theta - (\partial z/\partial n)_p - (\theta/g)(\partial\dot{V}/\partial\theta)\right]^{-2}, \qquad 7.1(3)$$

where θ is potential temperature, n is the coordinate measured normal to the wind current, the subscripts θ and P indicate differentiation on isentropic or isobaric surfaces, respectively, and $\dot{V}$ is the acceleration of the vector wind, and Ri* is a modified Richardson number. Other symbols used in the equation follow standard notation. The term $\partial\dot{V}/\partial\theta$ is extremely difficult to measure and $(\partial z/\partial n)_p$ is usually small in comparison to $(\partial z/\partial n)_\theta$. Hence, if we neglect these terms, we get a simplified form for Ri* which states that perturbations are likely to amplify in stable and baroclinic layers, viz.,

$$Ri^* \approx \frac{f^2\,\theta}{g(\partial\theta/\partial z)(\partial z/\partial n)^2_\theta} \qquad 7.1(4)$$

It should be noted that this equation goes far beyond the original assumptions made by Richardson, in that broad-scale flow has been introduced, i.e., inertia effects are now important.

Scorer (1969) and Badgley (1969) have discussed the changes in Ri associated with broad-scale flow. The conclusion reached by Scorer is that the greatest decreases in Ri occur where air with positive shear ($\partial V/\partial z > o$) is decelerated and flows upslope, or where air with negative shear is decelerated while flowing downslope. Badgley reaches the same conclusion whereby he suggests that breeding places for CAT might well be regions where the air is descending and spreading out normal to the positive shear vector and simultaneously slowing up in the direction of the negative shear vector.

Another stability criterion which is applicable in the vicinity of the jet-stream has been derived by Sasaki (1958); this incorporates the curvature of the vertical profiles of wind and temperature. The criterion can be written as

$$S_i \equiv \Delta T + \Gamma - m_i\,\Delta V^2 \begin{matrix} < \\ > \end{matrix} 0 \quad \begin{matrix} \text{unstable} \\ \text{stable} \end{matrix}, \qquad 7.1(5)$$

where m_i is a statistical factor $(\bar{T}/g\Delta T)(K_m/K_H)$ equal to 0.0198 °C/kt² (empirically determined). After the manner of Reiter (1963), S_i can be used to investigate the conditions of critical flow at the tropopause and at the level of the jet stream. Use is made of the following definitions:

$$\tau_i \equiv \overline{\Delta\Delta T} + 2m_i\,\overline{\Delta V}\,(\overline{\Delta\Delta V})$$

and

$$j \equiv \overline{\Delta\Delta V} = \frac{\Delta V_{+1} - \Delta V_{-1}}{2},$$

where the subscripts +1, 0, and -1 refer to the upper, middle, and lower layers of a three-layer model of the atmosphere. Furthermore, $\Delta V_{+1} \equiv V_{+1} - V_o$ and $\Delta V_{-1} \equiv V_o - V_{-1}$. With this notation, the results of Sasaki may be summarized as follows:

1.) At the tropopause, $|\tau_i| > m_i j^2$, for which $0 < S_i < m_i j^2$ implies stability and $S_i < 0$ implies instability.

2.) At jet-stream level $|\tau_i| < m_i j^2$ so that stable conditions exist for $m_i j^2 - |\tau_i| < R < m_i j^2$ and unstable conditions for $R < m_i j^2 - |\tau_i|$

7.2 Dynamic (Inertial) Instability

Dynamic instability has been used to explain the occurrence of CAT in regions of strong horizontal and vertical wind shears (Arakawa 1951, 1953, 1958; Kao, 1964). If a fluid particle, embedded in a broad geostrophic current, is originally in a stable state, it will describe oscillations about its equilibrium state if removed from its equilibrium position. The period of the oscillations described by the particle in this stable state will be $T = 2\pi/f = \frac{1}{2}$ pendulum day (Hess, 1959). Conditions may exist, however, in which a fluid particle moving into certain flow configurations will experience unstable accelerations. Under these conditions, any small perturbation may amplify rapidly and finally break down into turbulence.

Solberg (1939) has discussed the vanishing of absolute vorticity in the case of transition to instability (also see Kleinschmidt, 1941, Van Mieghem, 1944a,b, 1945, 1946, 1948, 1950). The vertical component of the absolute vorticity η in a zonal current is given by

$$\eta = -\frac{\partial u}{\partial y} + f + \frac{u}{R}\tan\phi \qquad 7.2(1)$$

where y is the meridional coordinate pointing northward, f the Coriolis parameter, u the speed of the west wind, R the radius of Earth, and ϕ the latitude. With neglect of the last term in 7.2(1), the stability criterion becomes

$$f - \left(\frac{\partial u}{\partial y}\right) \begin{matrix} > \\ = \\ < \end{matrix}\ 0 \quad \begin{matrix}\text{stable} \\ \text{neutral} \\ \text{unstable,}\end{matrix} \qquad 7.2(2)$$

for the anticyclonic side of strong jet streams. Dynamic instability also is possible on the cyclonic side of the jet-stream according to the criterion developed by Arakawa (1951). According to his theory, the critical cyclonic shear is given by

$$-\frac{\partial u}{\partial y} = \omega \sin\phi + \frac{2u}{R}\tan\phi, \qquad 7.2(3)$$

in which ω is the angular velocity of Earth. Arakawa (1958) has also shown that dynamic turbulence is related to a critical value of the vertical gradient

of wind speed. He gives as the critical negative wind shear (wind speed decreasing with height) the following approximate relationship

$$\frac{\partial u}{\partial z} = -f\cot\phi - \left(\frac{\partial T}{\partial z} + \Gamma\right)\frac{g}{fT}\tan\phi, \qquad 7.2(4)$$

where T, Γ, and g are the temperature, dry-adiabatic lapse rate, and acceleration of gravity, respectively.

The instability criterion of Arakawa expressed by Eq. 7.2(3) has been disputed by Kao (1964). Using energy considerations and the assumption that the motion of a fluid will be stable or unstable if the change of the eddy kinetic energy is increasing or decreasing with time, Kao found that the stability criterion can be given by

$$-\langle\overline{u'v'}\rangle\left(\frac{1}{r}\frac{\partial\bar{u}}{\partial\phi} + \frac{\bar{u}}{r}\tan\phi\right) - \langle\overline{u'w'}\rangle\left(\frac{\partial\bar{u}}{\partial r} - \frac{\bar{u}}{r}\right) + \frac{g}{T}\langle\overline{w'T'}\rangle \begin{matrix} > \\ < \end{matrix}\ 0 \quad \begin{matrix}\text{unstable} \\ \text{stable}\end{matrix} \ . \qquad 7.2(5)$$

The various quantities in 7.2(5) are defined as follows:

- u', v', w': perturbation wind speeds positive to the west, north, and vertically upward, respectively; and
- r, ϕ: radial and latitudinal coordinates of a spherical coordinate system.

Quantities within < > are time integral values and barred quantities are average values. If the perturbation value of the vertical velocity is equal to zero, i.e., $w' = 0$, Kao shows that the stability criterion is given by

$$\left(\frac{1}{r}\frac{\partial\bar{u}}{\partial\phi} + \frac{\bar{u}}{r}\tan\phi\right)\left(\frac{1}{r}\frac{\partial\bar{u}}{\partial\phi} - f - \frac{\bar{u}}{r}\tan\phi\right) \begin{matrix} > o \\ < o \end{matrix} \quad \begin{matrix}\text{unstable} \\ \text{stable}\end{matrix} \ . \qquad 7.2(6)$$

For a positive vortex (westward-flowing zonal current) with positive lateral velocity shear toward the pole, Eq. 7.2(6) becomes

$$\frac{1}{r}\frac{\partial\bar{u}}{\partial\phi} \begin{matrix} > \\ < \end{matrix}\ f + \frac{\bar{u}}{r}\tan\phi \quad \begin{matrix}\text{unstable} \\ \text{stable}\end{matrix} \ , \qquad 7.2(7)$$

and with negative lateral velocity shear toward the pole, the criterion is

$$\frac{1}{r}\frac{\partial \bar{u}}{\partial \phi} \begin{matrix} < \\ > \end{matrix} - \frac{\bar{u}}{r} \tan \phi \begin{matrix} \text{unstable} \\ \text{stable} \end{matrix} \quad . \qquad 7.2(8)$$

Equation 7.2(7) is equivalent to the criterion of Solberg and Klienschmidt (7.2(2)), but Eq. 7.2(8) differs considerably from that of Arakawa, 7.2(3). Dugstad (1958a) has shown also that Arakawa's criterion, expressed by 7.2(3), is in error. For geostrophic flow conditions, the critical wind shears of Arakawa would be $\partial v_\theta/\partial r = -f$ and $\partial v_\theta/\partial r = \frac{1}{2} f$, which quite clearly are physically impossible.

7.3 Atmospheric Wave Motions

The hydrodynamic equations of motion hold a wealth of information from which an explanation for a wide range of physical phenomena may be derived. Mathematical solutions for most problems of dynamic meteorology are exceedingly difficult to obtain for a variety of reasons, viz., non-linearity of the differential equations, correctly specified boundary conditions, incomplete specification of the problem, etc. The difficulty arising from nonlinear equations can be overcome for a wide range of problems by applying the method of perturbations (see, e.g., Bjerknes et al., 1933; Haurwitz, 1941, 1951). The method of perturbations "linearizes" the nonlinear equations through the assumption that atmospheric motions consist of small perturbations superimposed on a steady, mean field of flow. Several simplifying assumptions (see Appendix II) about the mean flow field permit the resulting perturbation equations to have solutions that have the form of waves. Problems involving sound waves, gravity waves, waves in the upper westerlies, shearing waves, and flow over hills and mountains have been solved successfully using perturbation techniques.

A wave traveling in the x-direction with phase velocity c and not undergoing a change in shape can be represented mathematically by A cos k(x-ct) or A sin k(x-ct), where A is the amplitude of the wave, k the wave number (equal to $2\pi\lambda^{-1}$), λ the wave length, and t the time. Simple harmonic waveforms like the above can be generalized to allow A, c, and k to be functions of time as well as of other coordinate directions (Kasahara, 1967). Another, and in many ways more useful, wave function has the form

$$A \exp ik(x-ct), \qquad 7.3(1)$$

in which exp = e, the base of the natural logarithms, $i = \sqrt{-1}$, and A, x, c, and t are the same as defined above. The phase velocity, c, in 7.3(1) may be complex, i.e.,

$$c = c_r + ic_i \qquad 7.3(2)$$

in which case, the wave function takes the form

$$A \exp\{kc_i t\} \exp\{ik(x-c_r t)\}. \qquad 7.3(3)$$

The factor, $\exp\{kc_i t\}$, determines whether the wave form is stable, neutral, or unstable according to the following:

$c_i > 0$ amplified waves, unstable,

$c_i = 0$ neutral waves, and

$c_i < 0$ damped waves, stable.

The rate of amplification for those cases where $c_i > 0$ is important since very small amplification would represent little development (Haltiner and Martin, 1957). A convenient measure of wave instability is the doubling time, which is the time required for the amplitude of a wave disturbance to double its magnitude.

7.31 Gravity Waves

Gravity waves are an example of wave motion in a single layer. The assumptions used to obtain a mathematical description of these waves include: (1) fluid is incompressible, (2) coriolis force and friction are neglected, (3) wave motion is two-dimensional, and (4) only the undistrubed fluid is in hydrostatic equilibrium. Under these conditions, gravity is the prime restoring force. It should be noted that in this system, no vorticity can be created and, since the undisturbed motion has no vorticity, the wave motion is irrotational.

Pure gravity waves are stable as long as the wave height is small, but they may be unstable if the undisturbed fluid velocity is a nonlinear function of height. The speed of gravity waves depends on the wavelength, λ, and on the thickness, h, of the fluid in which they occur. Wave speed may be calculated from the solution of the perturbation equations, which is given by

$$c = U \pm \left(\frac{g\lambda}{2\pi}\tanh\frac{2\pi h}{\lambda}\right)^{\frac{1}{2}}. \qquad 7.31(1)$$

In this equation, c is the wave speed and U is the velocity of the undisturbed fluid. If the fluid depth exceeds the wave length by about 60 per cent, then, the wave speed is very closely approximated by

$$c \approx U \pm \left(\frac{g\lambda}{2\pi}\right)^{\frac{1}{2}}. \qquad 7.31(2)$$

Another special case exists for the case in which λ is much larger than h, i.e., $h < \lambda/25$. These are the conditions for long waves in a shallow ocean or atmosphere and the wave speed can be calculated from

$$c \approx U \pm (gh)^{\frac{1}{2}}. \qquad 7.31(3)$$

Pure gravity waves are of little interest in the formation of CAT because of their relatively long wave length and the fact that they are stable. However, it must be realized that a complete spectrum of waves is possible through superposition and interference patterns by a number of gravity waves and by gravity waves interacting with other small-scale waves.

7.32 *Waves on a Surface of Discontinuity*

A great deal of evidence has been collected which tends to show that CAT is associated with waves forming on stably stratified interfaces. Observations by Reiter (1962a,b), Endlich (1964), Reiter and Nania (1964), Reiter and Burns (1966), and many others support this contention.

The waves forming on the boundary separating two layers having different wind speeds are called shear waves. These waves are always unstable, as can be seen from Eq. 7.32(1). If the densities in the two layers also are different, the resulting waves are called shear-gravity waves. The phase velocity of shear-gravity waves in an incompressible fluid is given by

$$c = \frac{\rho U + \rho' U'}{\rho + \rho'} \pm \left\{\frac{g\lambda}{2\pi}\,\frac{\rho-\rho'}{\rho+\rho'} - \frac{\rho\rho'(U-U')^2}{(\rho+\rho')^2}\right\}^{\frac{1}{2}}. \qquad 7.32(1)$$

The primed symbols in this equation refer to the upper fluid.

Shear-gravity waves are automatically unstable if the uppermost fluid is more dense than the lower. These waves also may be unstable if the wind shear, U-U', is sufficiently large so as to make the radical in Eq. 7.32(1) imaginary. The critical wave length for which the latter condition may arise is found by equating the last term in Eq. 7.32(1) to zero. Hence, λ_{crit} is given by

$$\lambda_{crit} = \frac{2\pi}{g}\,\frac{(U-U')^2\rho\rho'}{\rho^2-\rho'^2}. \qquad 7.32(2)$$

Reiter (1962e) uses temperature, T, instead of density to obtain values of λ_{crit}, viz.,

$$\lambda_{crit} = \frac{2\pi}{g}\,\frac{(U-U')^2 TT'}{(T'+T)(T'-T)}. \qquad 7.32(3)$$

If we let $\bar{T} = (T + T')/2$, $\Delta T = T' - T$, $\Delta U = U - U'$, and note that $\frac{1}{4}(\Delta T)^2 << \bar{T}^2$, then, Eq. 7.32(3) can be written as

$$\lambda_{crit} = \frac{\pi}{g}\left\{\frac{(\Delta U)^2\ \bar{T}}{\Delta T}\right\}. \qquad 7.32(4)$$

Table 9 shows the sensitivity of λ_{crit} on vertical wind shear for different discontinuities of temperature (after Reiter, 1962c). The dependence of Eq. 7.32(4) on the meso-structure of the atmosphere clearly shows that the patchy nature of CAT can be explained by shear-gravity waves.

Table 9. Vertical wind shear ΔU (m/sec) for different temperature discontinuities and critical wave lengths at an interface (after Reiter, 1962c).

ΔT	L_c = 200 m	L_c = 100 m	L_c = 50 m
2°	2.3 m/sec	1.6 m/sec	1.2 m/sec
4°	3.3	2.3	1.6
6°	4.0	2.9	2.0
8°	4.7	3.3	2.3
10°	5.2	3.7	2.6

Stable waves of the shear-gravity type also may be responsible for aircraft-observed CAT. Because of their short wave length, an aircraft "plowing" through these waves could well experience vertical accelerations of the order produced by true turbulence or gusts (Reiter, 1962e).

It should be noted that shear-gravity waves are not dependent on terrain features for their initiation. That this is true has been shown by the observation of billow clouds over relatively level terrain (Haurwitz, 1941; Ludlam, 1967).

7.33 Mountain Waves

As observed by Harrison (1959), at least one case in 20 of severe CAT is associated with wave formations in the lee of mountains. All mountain waves are not turbulent as evidenced by numerous pilot reports (Alaka, 1958); however, extremely strong vertical and horizontal gusts have been observed (Colson, 1954; Dutton, 1967). Typically, the mountain wave is evidenced by the occurrence of several types of clouds, particularly the cap cloud, the rotor cloud, and the lenticular (Moatzagotl) cloud. Occasionally, if the air is very dry, waves may exist without the presence of any clouds. Figure 24 is a schematic presentation showing typical flow and cloud patterns associated with mountain waves. An example of the atmospheric structure observed during the "Sierra Wave Project" is shown in Fig. 25. (Holmboe and Klieforth, 1957).

The effect of mountains on the air flow at great heights, as evidenced by wave patterns in nacreous clouds, has been reported by Dieterichs (1950), Palm and Foldvik (1960), and others. As previously indicated, the mountain effect has even been detected in the ozonosphere (Paetzold and Zschorner, 1955).

The framework for the theoretical treatment of airflow over mountains was begun in the 1880's by Rayleigh (1883) and Kelvin (1886). It was not applied to the atmosphere, however, until the 1940's when theories were offered by Lyra (1940) and Queney (1947). In 1949, Scorer (1949) investigated a two-layer model of the atmosphere in which he permitted a vertical variation of wind and temperature. Other theoretical work on the mountain-wave problem has been done by Hoiland (1951), Wurtele (1953), Long (1953a,b, 1958), Palm (1958), Krishnamurti (1964a), Das (1964), and many others. A detailed summary of this work, plus many other aspects of the problem, may be found in reviews by Corby (1954), Queney et al. (1960), Krishnamurti (1964b), and Alaka (1958).

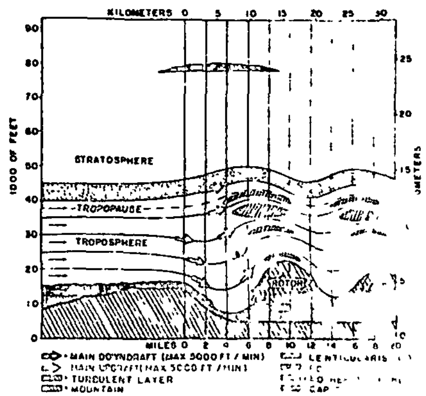


Fig. 24. Cross section of conditions associated with a typical mountain wave (after Jenkins, 1958).

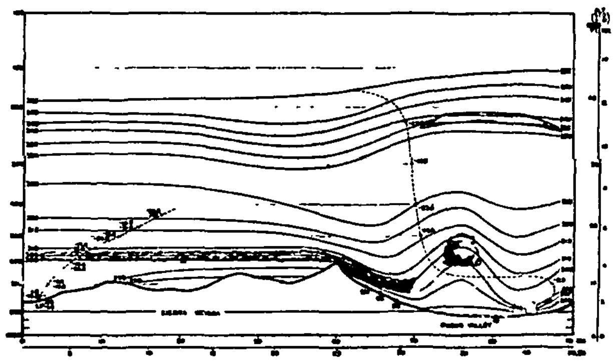


Fig. 25. Vertical cross-section at 1000P December 18, 1951 from Sequoia to the western slope of the Inyo Mountains. Isentropes are drawn for every 5°C. Both T and T_D are plotted for the Lodgepole (Sequoia) sounding. Locations of barograph stations are shown by small circles. Dashed lines indicate the paths of the Radiosonde balloon and of flight 2006. The ratio of the vertical scale to the horizontal is 2.5 to 1 (after Holmboe and Klieforth, 1957).

The works of Scorer (1949, 1951a,b,c, 1953a,b, 1954a,b, 1955, 1957, 1958a, b, 1959), Corby (1954), Corby and Wallington (1956), Corby and Sawyer (1958), and Scorer and Klieforth (1959), to name a few, reveal some of the properties of the mean flow over mountains that are necessary (but not sufficient) for the perturbation wave solutions to contain lee waves. These properties can be determined relatively easily from radiosonde data and, hence, are important in a practical as well as theoretical sense.

Disturbances of the size applicable to the mountain-wave problem are described by the wave equation,

$$\frac{\partial^2 Y}{\partial z^2} - A\frac{\partial Y}{\partial z} + (\ell^2 - k^2)Y = 0, \qquad 7.33(1)$$

where $A = gC^{-2} + \beta$, C is the speed of sound, β the vertical stability, and Ψ is a stream function. ℓ^2 is known as the "Scorer parameter" and is equal to

$$\frac{g\beta}{U^2} - \frac{1}{U}\frac{\partial^2 U}{\partial z^2} \quad . \qquad 7.33(2)$$

The "Scorer parameter" characterizes the dynamic properties of the atmosphere at each level.

Scorer found that standing lee waves are possible only if ℓ^2 is less in some fairly-deep upper layer than in a layer below. For wave formation, the two-layer model requires that the decrease in ℓ^2 from the lower layer to the upper layer should attain a certain minimum magnitude. This magnitude is dependent on the depth of the lower layer, h, with the limiting condition given by

$$\ell_L^2 - \ell_u^2 > \frac{\pi^2}{4h^2} \quad . \qquad 7.33(3)$$

The condition that ℓ^2 decrease with height may be met by a wide range of vertical profiles of wind and temperature. Figure 26 gives an example of the

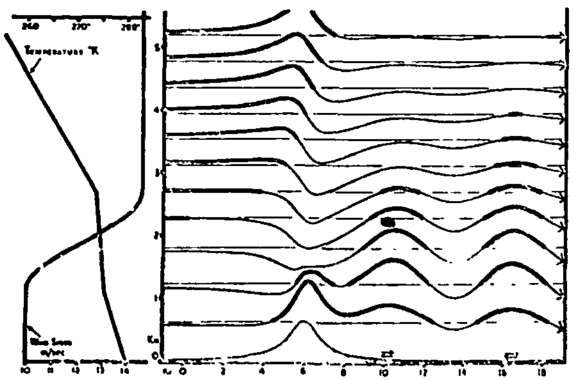


Fig. 26. An example of a train of lee waves. ℓ is greater in the lower layers than higher up. Each stream-line may contain many wave crests. (After Scorer, 1949).

streamline pattern computed by Scorer, using the wind and temperature profiles shown on the left of the figure. Corby and Wallington (1956) have shown that for identical ℓ^2 profiles, the largest amplitude waves are produced by strong winds at the surface (mountain top). Large amplitude waves are associated with narrow but steep obstacles and also with long waves which fulfill the requirement specified by Eq. 7.33(3).

Turbulence associated with lee waves has been discussed theoretically by Scorer (1954b, 1955, 1959, 1969), Scorer and Wilson (1963), Reiter (1963), and Pao (1969). General discussions of mountain wave turbulence with emphasis on aviation aspects can be found in Jenkins (1952), Corby (1954), Alaka (1958) and Queney et al. (1960).

8. THE REMOTE DETECTION OF CAT

Aviation interests have long sought means whereby advanced warning of the location and/or probable occurrence of CAT may be obtained. Since all present-day forecasting techniques essentially are probabilistic in nature, numerous airborne and land-based, passive and active systems have been devised to obtain the desired forewarning. Paulsen (1966) has listed the techniques which have been investigated or which offer practical utility in the near future:

a. sonic techniques,
b. stellar scintillation or microwave scintillation techniques,
c. infrared and microwave radiometry,
d. low-frequency radar,
e. microwave radar,
f. optical-laser radar, and
g. electric-field and corona-discharge techniques.

8.1 Passive Techniques

Kadlec (1964, 1966) has discussed the possibility of using inflight sensors to detect horizontal changes in the ambient temperature. This technique is based on thermal gradients supposedly associated with the atmospheric structure in the neighborhood of CAT. Reiter (1963), McLean (1965), and Atlas (1969) have shown that this method is highly unreliable. Techniques using infrared and microwave radiometers have been proposed by Astheimer (1965), Norman and Macoy (1966), and others. These methods obtain integrated path temperatures over paths whose lengths vary as a function of the proximity of the instrument's center frequency to the peak of an appropriate absorption band (Paulsen, 1966). From these measurements a discontinuity of temperature in the flight path of the aircraft can be computed. CAT warnings then are based on the assumption that CAT is associated with these discontinuities. This technique has been evaluated by Mather (1967), Flint (1968, 1969), and Weiss (1968). The results are similar to those obtained by direct temperature sensing, i.e., highly unreliable.

Other passive systems are based on the method of measuring optical and microwave scintillations of stars (Greene et al., 1966; Fried, 1968). These methods suffer from the limitation of being useful only during the night or at least during the time that star fields can be observed (Atlas, 1969).

Observations of coronal discharge have been correlated with CAT by Nanevicz et al., (1966). A significant correlation was found to exist between CAT encounters and periods of electrical discharge. A somewhat similar technique

een investigated by Ten Broek and Seashore (1966). They, however, have iered the possible relationship between CAT and atmospheric quasi-electro-c fields. Initial results for both of these techniques seem promising.

Active Techniques

The use of radar to detect CAT shows the most promise of any method tested e past few years. Echoes from CAT arise because of scattering caused by :ions in the refractive index. The intensity of the echo depends on the :ude of the mean gradient of refractive index and, in a complex manner, the .ty of the turbulence (Hicks et al., 1967). The theoretical basis for CAT :ion by radar has been discussed by numerous workers (Stewart, 1961; Bates Zirkle, 1966; Atlas et al., 1966; Atlas and Naito, 1966; Hardy et al., Hicks et al., 1967; Stevens and Reiter, 1967; Buehler et al., 1968; 1968; Hardy and Ottersten, 1968). Simultaneous observations of CAT by and aircraft (Hicks et al., 1967) have confirmed the possibility of using idar as a forewarning tool.

ptical radars and laser measurements also have been proposed for use in tection (Collis, 1964; Gibson, 1966; Breece et al., 1966; Zirkle, 1966). and optical radar methods involve spectral analysis of the doppler shift ht back-scattered by moving particles in the atmosphere. From these ements, the average magnitude of gusts and the gust spectrum of the ty components can be obtained. Another approach involves the mapping of le formations and relating these patterns to rough flying conditions e, 1966).

easurements of atmospheric winds, and hence various gust components, also accomplished by tropospheric radio scatter (Atlas, 1969). As indicated as (1969), tropo-scatter systems provide enhanced signal levels over ponding radar systems. This advantage means that a CAT warning system on this technique would be highly reliable.

ost of the systems covered in this section can be used in either a ground- or airborne operation. The state-of-the-art ultra-sensitive radar can be nly from the ground, however. Atlas et al., (1966) have shown that a improvement in the best airborne radar is needed to detect most CAT. This ement in sensitivity would permit CAT detection at a range of approximately i, although not with 100 per cent confidence (Atlas, 1969).

n excellent collection of papers on the individual methods discussed in ection may be found in two bound works:

a.) ION/SAE Conference Proceedings, 1966; Society of Automotive Engineers, Inc. 485 Lexington Ave., New York, New York 10017.

b.) Clear Air Turbulence and Its Detection. Pao and Goldburg, Editors, Plenum Press: New York, 1969.

ABBREVIATIONS FOR REFERENCES

- Astronautics and Aeronautics
- Aerospace Engineering (Astronautical/Aerospace Engineering)
- Aeronautical Engineering Review, New York
- Air Force Cambridge Research Center, Cambridge Massachusetts
- Air Force Cambridge Research Laboratories, Cambridge, Massachusetts
- Air Force Flight Dynamics Laboratory, Wright-Patterson AFD, Ohio
- Air Force Surveys in Geophysics, Cambridge Research Center
- Annalen der Hydrographie, Berlin
- American Institute of Aeronautics and Astronautics
- Annalen der Meteorologie, Hamburg
- Archiv Fur Meteorologie, Geophysik und Bioklimatologie, Serie A, Wien
- Australian Meteorological Magazine, Melbourne
- American Meteorological Society, Boston
- Air Weather Service, U.S. Air Force, Scott AFB, Illinois
- Atmospheric Science (Technical) Paper
- Bulletin of the AMS, Boston
- Bulletin of the Academy of Sciences, U.S.S.R., Geophysics Series
- Berichte des Deutschen Wetterdienstes, Offenbach
- British European Airways
- Beitrage zur Physik der (frein) Atmosphare, Frankfort a.m.
- Boeing Scientific Research Laboratories, Seattle
- Compendium of Meteorology, AMS, Boston
- Colorado State University, Department of Atmospheric Science, Ft. Collins
- Meteorology Branch, Department of Transport, Technical Circular, Toronto
- Deutsche Versuchsanstalt fur Luft-und Raumfahrt, Munich
- Environmental Science Services Administration, Washington, D.C.
- Final Report
- Great Britain Meteorological Research Committee, London
- Glavnaia Geof. Observatory, U.S.S.R.
- Geofysiske Publikasjoner, Oslo
- Geophysics Research Directorate, AFCRC, Cambridge, Massachusetts
- Geophysical Research Papers, AFCRC, Cambridge, Massachusetts
- Hydrometeorological Publishing House, Moscow
- Izvestiia, Fizika Atmosfera i Okeana, Akademiia Nauk SSSR
- Indian Journal of Meteorology and Geophysics, Delhi

ION	-	Institute of Navigation
ISG	-	Izvestiia, Ser. Geof., Akademiia Nauk SSSR
ISGG	-	Izvestiia, Ser. Geograf. i Geof., Akademiia Nauk SSSR
JA	-	Journal of Aircraft
JAM	-	Journal of Applied Meteorology, AMS, Boston
JAS	-	Journal of the Atmospheric Sciences, AMS, Boston
JFM	-	Journal of Fluid Mechanics, Cambridge, England
JGR	-	Journal of Geophysical Research
JINL	-	Institute of Navigation, Journal, London
JM	-	Journal of Meteorology, AMS, Boston
JMR	-	Journal of Meteorological Research, Tokyo
JRAS	-	Journal of the Royal Aeronautical Society, London
LMS	-	London Mathematical Society, Proceedings, London
MA	-	Meteorologiske Annaler, Oslo
MM	-	Meteorological Magazine, London
MR	-	Meteorologische Rundschau, Offenbach
MRP	-	Meteorological Research Report
MWR	-	Monthly Weather Review, Washington, D.C.
N	-	Nature, London
NAA	-	North American Aviation, Space and Information Systems Division
NACA	-	National Advisory Committee on Aeronautics, Washington, D.C.
NASA	-	National Aeronautics and Space Administration, Washington, D.C.
NCAR	-	National Center for Atmospheric Research, Boulder, Colorado
NRLR	-	Naval Research Laboratory Report
NSSL	-	National Severe Storms Laboratory
NYU	-	New York University
PF	-	Physics of Fluids
PM	-	Philosophical Magazine, London
PMG	-	Papers in Meteorology and Geophysics, Tokyo
PSU	-	Pennsylvania State University, University Park
QJRMS	-	Royal Meteorological Society, Quarterly Journal, London
RAE	-	Royal Aircraft Establishment, Farnborough, England
RG	-	Reviews of Geophysics, Washington, D.C.
RM	-	Research Memorandum
RP	-	Research Paper
RR	-	Research Report
RSLA	-	Royal Society of London, Proceedings, Ser. A.

S	-	Science, American Association for the Advancement of Science, Washington, D.C.
SAE	-	Society of Automotive Engineers, New York
SP	-	Special Publication
SR	-	Scientific Report
SRI	-	Stanford Research Institute, Menlo Park, California
T	-	Tellus, Stockholm
TAM	-	Texas A&M (College) University, College Station
TIP	-	Tsentral′naia Institut Prognozov, U.S.S.R.
TM	-	Technical Memorandum
TN	-	Technical Note
TP	-	Technical Paper
TR	-	Technical Report
TsAO	-	Tsentral′naia Aerologicheskaia Observatory, U.S.S.R.
UALMC	-	United Air Lines, Meteorology Circular, Chicago
UCLA	-	University of California at Los Angeles, Department of Meteorology
USWB	-	U.S. Weather Bureau, Washington, D.C.
W	-	Weather, London
WMOTN	-	World Meteorological Organization, Technical Note, Geneva
ZM	-	Zeitschrift fur Meteorologie, Berlin

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APPENDIX I

SYMBOLS AND USEFUL EQUATIONS

A. List of Symbols

A, B, D, μ, η - amplitudes of wave forms

c - wave speed, $c = c_r + ic_i$ where $i = \sqrt{-1}$, c_r is real part of c and c_i the imaginary part

g - acceleration of gravity

k - wave number, $k = 2\pi\lambda^{-1}$

P - pressure

R - radius of Earth

t - time

u, v, w - components of motion in the x, y, and z directions, respectively

x, y, z - coordinates of a right-hand Cartesian coordinate system with x positive to the east, y to the north, and z vertically upward

ρ, α - density and specific volume, respectively, $\rho\alpha = 1$

λ - wave length

Ω_y, Ω_z - components of Earth's angular velocity in y and z directions, respectively

B. System of Equations

The three hydrodynamic equations of motion (scalar) are given by

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + 2\Omega_y w - 2\Omega_z v = -\frac{1}{\rho}\frac{\partial P}{\partial x} \qquad \text{(AI-1)}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + 2\Omega_z u = -\frac{1}{\rho}\frac{\partial P}{\partial y} \qquad \text{(AI-2)}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} - 2\Omega_y u = -\frac{1}{\rho}\frac{\partial P}{\partial y} - g \qquad \text{(AI-3)}$$

Terms containing factors R^{-1} and those representing friction have been omitted from Eqs. (AI-1) to (AI-3). The equation of continuity for an incompressible fluid may be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \text{(AI-4)}$$

APPENDIX II

PERTURBATION THEORY

A. The Perturbation Equations

Derivation of the perturbation equations is dependent on the following assumptions:

(1) the mean or steady-state motion must satisfy the basic equations, i.e., Eqs. (AI-1) to (AI-4);
(2) the total motion must satisfy the basic equations; and
(3) second-order perturbations appearing in the equations may be neglected in comparison with first order terms.

We define the total motion quantities as follows:

$$\begin{aligned} u &= U + u', & v &= V + v', & w &= W + w', \\ P &= \bar{P} + P', & \rho &= \bar{\rho} + \rho', & \alpha &= \bar{\alpha} + \alpha', \end{aligned} \tag{AII-1}$$

where U, V, W, $\bar{P}$, $\bar{\rho}$, and $\bar{\alpha}$ refer to the undisturbed current; and u', v', w', P', ρ', and α' refer to the perturbation quantities.

Substituting the quantities in (AII-1) into Eq. (AI-1) gives the equation for total motion in the x direction, viz.,

$$\begin{aligned} &\frac{\partial(U+u')}{\partial t} + (U+u')\frac{\partial(U+u')}{\partial x} + (V+v')\frac{\partial(U+u')}{\partial y} + (W+w')\frac{\partial(U+u')}{\partial z} + \\ &2\Omega_y(W+w') - 2\Omega_z(V+v') = -(\bar{\alpha}+\alpha')\frac{\partial(\bar{P}+P')}{\partial x} \end{aligned} \tag{AII-2}$$

The corresponding equation for the basic motion is

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} + W\frac{\partial U}{\partial z} + 2\Omega_y W - 2\Omega_z V = -\bar{\alpha}\frac{\partial \bar{P}}{\partial x}. \tag{AII-3}$$

Subtracting Eq. (AII-3) from (AII-2) and neglecting second-order perturbations yields the perturbation equations in the x direction. Results for the other two coordinate directions, as well as for the continuity equation, are obtained in the same manner. The set of perturbation equations is thus given by

$$\begin{aligned} &\frac{\partial u'}{\partial t} + U\frac{\partial u'}{\partial x} + V\frac{\partial u'}{\partial y} + W\frac{\partial u'}{\partial z} + u'\frac{\partial U}{\partial x} + v'\frac{\partial U}{\partial y} + w'\frac{\partial U}{\partial z} + \\ &2\Omega_y w' - 2\Omega_z v' = -\bar{\alpha}\frac{\partial P'}{\partial x} - \alpha'\frac{\partial \bar{P}}{\partial x} \end{aligned} \tag{AII-4}$$

$$\frac{\partial v'}{\partial t} + U\frac{\partial v'}{\partial x} + V\frac{\partial v'}{\partial y} + W\frac{\partial v'}{\partial z} + u'\frac{\partial V}{\partial x} + v'\frac{\partial V}{\partial y} + w'\frac{\partial V}{\partial z} + 2\Omega_y u' = -\bar{\alpha}\frac{\partial P'}{\partial y} - \alpha'\frac{\partial \bar{P}}{\partial y} \qquad \text{(AII-5)}$$

$$\frac{\partial w'}{\partial t} + U\frac{\partial w'}{\partial x} + V\frac{\partial w'}{\partial y} + W\frac{\partial w'}{\partial z} + u'\frac{\partial W}{\partial x} + v'\frac{\partial W}{\partial y} + w'\frac{\partial W}{\partial z} - 2\Omega_y u' = -\bar{\alpha}\frac{\partial P'}{\partial z} - \alpha'\frac{\partial \bar{P}}{\partial z} \qquad \text{(AII-6)}$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \qquad \text{(AII-7)}$$

For most problems, the terms $2\Omega_y w'$ and $2\Omega_y u'$ may be omitted.

B. Boundary Conditions

Wave solutions of Eqs. (AII-4) to (AII-6) are correct only if they satisfy certain boundary conditions. Boundaries may be free, rigid, or internal, or there may be no boundaries at all. Irrespective of the type of boundary, the perturbations must remain small throughout the fluid. A free boundary is exemplified by the top of the homogeneous atmosphere or surface of the ocean (in the case of water waves), a rigid boundary by the ground, and an internal boundary by a density discontinuity. A parcel of fluid at a boundary must remain there since, otherwise, the boundary would dissolve. At a boundary, two conditions apply, namely, the kinematic and dynamic boundary conditions.

The inematic condition requires that (1) the velocity normal to the rigid boundary be zero; and (2) the component velocities be the same on ooth sides of an internal boundary. The dynamic condition requires that the pressure be the same on both sides of a boundary. At the top of a free surface such as the homogeneous atmosphere, the pressure and its changes with time are equal to zero.

C. Solution of the Perturbation Equations

1. Gravity waves

Consider a fluid with a free surface in the absence of friction and rotation. In this case, gravity is the primary restoring force. Let the depth of the fluid be h, U the undisturbed horizontal velocity between $z = 0$ and $z = h$, let the waves have an infinite lateral extent in the y direction. Only the undisturbed fluid is in hydrostatic equilibrium, i.e., $0 = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial z} - g$, $0 \le z \le h$. For a homogeneous incompressible fluid, Eqs. (AII-4), (AII-6), and (AII-7) become

$$\frac{\partial u'}{\partial t} + U\frac{\partial u'}{\partial x} = -\frac{1}{\rho}\frac{\partial P'}{\partial x}$$
$$\frac{\partial w'}{\partial t} + U\frac{\partial w'}{\partial x} = -\frac{1}{\rho}\frac{\partial P'}{\partial z} \qquad \text{(AII-8)}$$
$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0.$$

Suitable conditions are $w = 0$ at $z = 0$, and $\bar{P} + P' = 0$ at the free surface. Assume a wave solution of the form

$$u' = A(z) \exp\{ik(x-ct)\}$$
$$w' = B(z) \exp\{ik(x-ct)\} \qquad \text{(AII-9)}$$
$$P' = D(z) \exp\{ik(x-ct)\}.$$

Substituting (AII-9) into (AII-8) gives the system

$$(U-c)\,A(z) = -D(z)\alpha \qquad \text{(AII-10)}$$
$$ik(U-c)\,B(z) = -A'(z)\alpha \qquad \text{(AII-11)}$$
$$ik\,A(z) + B'(z) = 0. \qquad \text{(AII-12)}$$

Now, differentiating (AII-10) with respect to z and equating like members of the result with (AII-11) gives

$$(U-c)\,A'(z) = ik(U-c)\,B(z). \qquad \text{(AII-13)}$$

From (AII-12), we obtain the result

$$ik\,A'(z) + B''(z) = 0,$$

so that (AII-13) becomes

$$(U-c)\,\{B''(z) - k^2 B(z)\} = 0. \qquad \text{(AII-14)}$$

Equation (AII-14) is satisfied by the case $c = U$, i.e., the phase velocity is the same as the undisturbed current, but the general solution is

$$B(z) = \mu_1 \exp(kz) + \mu_2 \exp(-kz),$$

where μ_1 and μ_2 are constants. Since $w' = 0$ at $z = 0$, it follows that $\mu_1 = -\mu_2 = \mu$ and

$$B(z) = \mu\{\exp(kz) - \exp(-kz)\}.$$

Hence, from Eqs. (AII-9) and (AII-10) to (AII-12), we get

$$\begin{aligned} u' &= i\mu\{\exp(kz) + \exp(-kz)\} \exp\{ik(x-ct)\} \\ w' &= \mu\{\exp(kz) - \exp(-kz)\} \exp\{ik(x-ct)\} \\ P' &= -i\mu\rho(U-c)\{\exp(kz) + \exp(-kz)\} \exp\{ik(x-ct)\}. \end{aligned} \quad \text{(AII}$$

Since the pressure at the top of the fluid, i.e., where $z = h$, must be zero, we get

$$\frac{d}{dt}(\bar{P}+P') = 0 \quad \text{or (neglecting second-order terms)}$$

$$\frac{\partial P'}{\partial t} + U\frac{\partial P'}{\partial x} + w'\frac{\partial \bar{P}}{\partial z} = 0. \quad \text{(AII-}$$

Recall that $\frac{\partial \bar{P}}{\partial z} = -g\rho$. Since the perturbations are assumed to be small, Eq. (AII-16) is a very close approximation to the conditions at $z = h$. Equat (AII-15) may be rewritten as follows:

$$\begin{aligned} u' &= i\mu(\cosh kz) \exp\{ik(x-ct)\} \\ w' &= \mu(\sinh kz) \exp\{ik(x-ct)\} \\ P' &= i\rho(c-U)\ \mu(\cosh kz) \exp\{ik(x-ct)\}. \end{aligned} \quad \text{(AII-}$$

Differentiating the last of Eqs. (AII-17) and substituting into Eq. (AII-16) yields

$$\rho(c-U)ck \cosh kz - Uk\rho(c-U) \cosh kz - g\rho \sinh kz = 0.$$

Evaluating this last equation at $z = h$ yields the result

$$(c-U)^2 = \frac{\lambda g}{2\pi} \tanh kh$$

or finally

$$c = U \pm \left(\frac{\lambda g}{2\pi} \tanh \frac{2\pi h}{\lambda}\right)^{\frac{1}{2}}. \quad \text{(AII-}$$

Equation (AII-18) is the "wave equation" satisfying the assumptions which were stipulated for this problem.

2. Shearing-gravitational waves on an internal surface of discontinuity

An internal surface of discontinuity refers to marked changes in T, ρ, and $\hat{V}$, or a combination of these. Consider a fluid, with an internal boundary at $z = 0$, that is homogeneous and infinitely deep on either side of the boundary. Let the undisturbed flow in the x direction be given by U_0 and U_1, where subscript 1 refers to the upper layer. If the trajectories of the fluid particles do not range over too great an area, then the effect of Earth's rotation can be neglected. For the upper layer, we assume $\frac{\partial \bar{P}_1}{\partial z} = -\rho_1 g$ and for the lower layer, $\frac{\partial \bar{P}_0}{\partial z} = -\rho_0 g$. Of course, at $z = 0$, the dynamic boundary condition requires that $P_1 = P_0$.

The perturbation Eqs. (AII-8) and their solutions (AII-9) apply to the fluids in each layer. The general solution to the perturbation equations is

$$\begin{aligned} u' &= i\{\mu \exp(kz) - \eta \exp(-kz)\} \exp\{ik(x-ct)\} \\ w' &= \{\mu \exp(kz) + \eta \exp(-kz)\} \exp\{ik(x-ct)\} \\ P' &= i\rho(c-U)\{\mu \exp(kz) - \eta \exp(-kz)\} \exp\{ik(x-ct)\}. \end{aligned} \quad \text{(AII-19)}$$

Boundary conditions require that $w_0' = 0$ at $z = -\infty$ and $w_1' = 0$ at $z = \infty$. Hence, for $w_0' = 0$, $\eta = 0$, and for $w_1' = 0$, $\mu = 0$. The following solutions hold for the indicated layers:

$$\begin{aligned} u_0' &= i\mu \exp(kz) \exp\{ik(x-ct)\} \\ w_0' &= \mu \exp(kz) \exp\{ik(x-ct)\} \\ P_0' &= i(c-U)\rho_0\mu \exp(kz) \exp\{ik(x-ct)\} \end{aligned} \quad \text{(AII-20)}$$

and

$$\begin{aligned} u_1' &= -i\eta \exp(-kz) \exp\{ik(x-ct)\} \\ w_1' &= \eta \exp(-kz) \exp\{ik(x-ct)\} \\ P_1' &= -i(c-U)\rho_1\eta \exp(-kz) \exp\{ik(x-ct)\}. \end{aligned} \quad \text{(AII-21)}$$

The dynamic boundary condition is given by

$$\frac{\partial}{\partial t}(P_0'-P_1') + U\frac{\partial}{\partial x}(P_0'-P_1') + w'\frac{\partial}{\partial z}(\bar{P}_0-\bar{P}_1) = 0.$$

Differentiating Eqs. (AII-20) and (AII-21), and then invoking the dynamic boundary condition, yields

$$\{\rho_0(c-U_0)^2k + g(\rho_1-\rho_0)\}\mu + \{(c-U_1)\rho_1 k(c-U_0)\eta\} = 0, \tag{AII-22}$$

and

$$\{\rho_0 k(c-U_0)(c-U_1)\mu\} + \{(c-U_1)^2\rho_1 k + g(\rho_1-\rho_0)\}\eta = 0. \tag{AII-23}$$

Equations (AII-22) and (AII-23) are satisfied if the determinant of the system is zero, i.e.,

$$\begin{vmatrix} \rho_0(c-U_0)^2k + g(\rho_1-\rho_0) & (c-U_1)(c-U_0)\rho_1 k \\ \rho_0 k(c-U_0)(c-U_1) & (c-U_1)^2\rho_1 k + g(\rho_1-\rho_0) \end{vmatrix} = 0.$$

Solving this determinant, we get

$$c^2 - \frac{2(\rho_0U_0 + \rho_1U_1)}{\rho_0 + \rho_1}c + \frac{\rho_1U_1^2 + \rho_0U_0^2}{\rho_0 + \rho_1} + \frac{g}{k}\left(\frac{\rho_0-\rho_1}{\rho_0+\rho_1}\right) = 0,$$

which yields for c:

$$c = \frac{\rho_0U_0 + \rho_1U_1}{\rho_0 + \rho_1} \pm \left\{\frac{g\lambda}{2\pi}\left(\frac{\rho_0-\rho_1}{\rho_0+\rho_1}\right) - \rho_0\rho_1\frac{(U_0-U_1)^2}{(\rho_0\ \rho_1)^2}\right\}^{\frac{1}{2}}. \tag{AII-24}$$

Equation (AII-24) gives the phase velocity of a shear-gravity wave. The dynamic term may be imaginary if the second term in the bracket exceeds the first. Under these conditions, the wave will be unstable.

References: Haltiner and Martin (1957), Haurwitz (1941), and Panofsky (1958)*.
* See Additional References.