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# SATELLITE AND MESOMETEOROLOGY RESEARCH PROJECT <br> Department of the Geophysical Sciences The University of Chicago 

# DYNAMICAL ANALYSIS OF OUTFLOW FROM: TORNADO-PRODUCING THUNDERSTORMS AS REVEALED BY ATS III PICTURES 

by
K. Ninomiya

The University of Chicago

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# DYNAMICAL ANALYSIS OF `UTFLOW FROM TORNADO-PRODUCING THUNDERSTORMS AS REVEALED BY ATS III PICTURES ${ }^{1}$ 

by<br>K. Ninomiya ${ }^{2}$<br>The University of Chicago


#### Abstract

Detailed synoptic and dynamic analyses of outflow from tornadoproducing tiunderstorms of April 23, 1968 were made by using conventional rawinsonde data combined with ATS III pictures. It was found that the pre-existing flow at he cirrus level over storm areas changed dramatically into outflow as the storms developed. When che storms reached their mature stage, the horizontal dimensions of the outflow increa.ied io about 500 km . Detailed analyses of rawinsonde data inside the outflow area revealed the existence of a mid-tropospheric warm core accompanied by a significant field of convergence below the $700-\mathrm{mb}$ surface.


Quantitative analysis of the thermodynamical and dynamical aspects of the outflow field showed that the outflow was induced and maintained by convective warming.

1. Introduction

In order to und stand the dynamical aspects of severe loc-1 storms many researchers have trit' to describe the three-dimensional wind field making use of both aerological and aircraft data (see Fujita (1963) and Newton (1963 and 1967)).

It has been pointed out that mesoscale outflow is found over developed thunderstorms. Using a Project Jetstream flight into a squall line of April 23, 1957, McLean (1961) obtained an outflow pattern in the layer between 36,000 and $40,000 \mathrm{ft}$ over the thunderstorm. In the analysis of a huge cumulonimbus, Fujita and Arnold (1963) showed that the anvil

[^0]cloud grew rapidly from the echo area. By using wind data obtained by high-level observation flights, staff members of the National Severe Storms Project (1963) showed that the high level wind speed increased over the lee side of severe storms. The use of $b \cdot h$ aircraft and rawinsonde data, however, is limited for the determination of the outflow field as a function of time, because the outflow from the thunderstorms is much smaller in area and shorter in life than that of hurricanes.

Series of photographs obtained by the geosynchronous ATS satellite have been found useful in determining the cloud velocity field. In the spring of 1968, NASA and ESSA conducted the Tornado Watch Experiment. On days when severe storms were expected, ATS III took pictures at 14 minute intervals. Fujita and Bradbury (1969) determined the mass outflow from a thunderstorm complex on April 19, 1968 by computing the high clouds' displacement observed by the ATS III pictures. It is the special advantage of ATS observations that the cloud velocity field can be determined during the entire period of thunderstorm development.

In this paper, the upper level outflow from the tornado-producing thunderstorms on April 23, 1968 will be analysed both synoptically and dynamically in detall by using rawinsonde data combined with a series of ATS III pictures of the Tornado Watch Experiment. It is the purpose of the study to clarify the role of the convective warming in the dynamical process of the formation of the outflow.
2. Description of the Severe Storms Synoptic Situation of 23 April 1968

On the morning of 23 April 1968 there was a moderately developed extratropiccl cyclone over southwest Wisconsin with a weak trough line extending through central Illinois, western Kentucky and Tennessee. During the day, the cyclone remained nearly stationary and as the trough line moved eastward a secondary low pressure center developed. By 1800 CST this secondary low senter had become intense and was located over eastern Michigan.

In the mid-troposphere a cut-off cold core ( $500-400: \mathrm{ib}$ ) was centered south of the surface cyclone center during the morning of 23 April. As this cold core moved northeastward over the Great Lakes region, it passed over the very moist and warm lower
tropospheric air mass which was being advected ahead of the surface trough by strong southerly flow. This fits the model f $\cap$ a typical synoptic altuation for severe storm development described by Fawbush, Miller and Starret (1951). Figure 1 shows the condition, that existed at 1200 CST. Surface isotherms and mixing ratios are shown in the flgure with the $1,000 \mathrm{ft}$. winds. Also included is the movement of the cut-off cold core during the 12 -hour period between 0600 CST and 1800 CST.

The remarkably unstable stratification was brought over southeastern Indiana in the early afternoon. The initial development of the thunderstorms occurred between 1100 CST and 1200 CST . The thunderstorms grew rapldly as they moved northeastward with the speed of $50-60$ knots. The maximum storm uctivity occurred between 1400 CST and 1700 CST , and the activity of the storms continued late into the night of the 23 rc .

## 3. High-Level Flow Obtained from Cloud Movement

Detailed analysis of the cioud displacement in the upper layers was made by tracing cloud movements from a film loop of consesutive ATS III pictures taken at 14 minute intervals. The film loop was put on a "loop movie projector" designed by Fujita and the cloud displacement between first and last picture represented the cluan motion during that interval of time. The velocity in the cloud layer was assumed to be equal to the velocity of the cloud movement. The moist layers were found from rawinsonde observations in the convective layer and also in the higher troposphere. The wind field in the higher troposphere was obtained by tracing the movements of high clouds.

The ATS III pictures and the displacement of clouds for the three stages of the thunderstorm complexes, i.e., the initial stage, the growing stage and the mature stage, are presented in Fig. 2, 3, and 4, respectively. Shown in Fig. 2a is the ATS III picture at 1100 CST. The bright cloud mass which was located in southern Indiana at this time grew explosively into a huge triangle-shaped cloud in two hours as observed in Fig. 3a. Fig. 2 b shows the cloud displacement in the initial stage of the storms. No significan* mesoscale difluence flow pattern or mesoscale divergence field is oiserved prior to and in the initial stage of the storms' development.

The velocity field of high clouds in Fig. 3b shows a remarkable change from the field
two hours earlier. The flow around the huge triangle-shaped cloud became a remarkable mesoscale difluence flow.

As several mesosystems developed successively, the high cloud sheet grew from each storm and they formed a large nephsystem as shown in Fig. 4a. This large nephsystem is considered to be a large-scale nephsystem rather than a mesoscale one because its horizontal scale is about 500 km or more. The cloud veloctty field at the time corresponding to the picture in Fig. 4 a is shown in Fig. 4b. A mesoscale difluence flow pattern is observed not only over each thunderstorm, but also a large-scale difluence flow pattern is observed over the whole area of mature thunderstorms.

It might be questioned whether or not the high clouds' movement does, indeed, represent the wind velocity field at a certain isobaric surface. However, a comparison of the clouds' movement with the wind velocity observed by the rawinsonde shows a strong correlation. The comparisons made at HTS and BUF for 1800 CST are shown in Fig. 5 as examples. We can see that the high clouds' movement coincides almost with the wind in the layer between 300 and 200 mb . Of course, the heights of all individual clouds are not necessarily uniform. The height difference among these clouds would be two thousand meters at most. The height difference would not make, however, serious erzor in the wind velocity estimation in the outflow layer because the vertical wind shear in the layer is fortunately small (see Fig. 5 and Fig. 10). Because some irregular variation in the velocity field given in Figs. 2b, 3b, and 4 b is due to the height difference, in all quantitative analyses in this section, the irregularity of the wind field was eliminated by areal smoothing.

In order to describe characteristics of the nutflow quantitatively, it is necessary to evaluate divergence, vorticity and deformation in the outflow layer. The evaluation of the mean divergence is made over the areas indicated in Figs. 6a and 6b, where boundary $\widehat{A B}$ and $\widehat{C D}$ are normal to the stream lines everywhere while $\widehat{B C}$ and $\widehat{D A}$ are parallel to the stream lines. The mean divergence in the area is therefore written as

$$
\begin{equation*}
\overline{\operatorname{div} V}=\frac{1}{s} \oint V_{n} d s=\frac{1}{s}\left[\int_{B}^{A} V d s+\int_{D}^{C} V d s\right] \tag{1}
\end{equation*}
$$

where $V_{n}$, is the component of wind normal to the boundary, $d s$, the line element along the boundary and $S$, the dimension of the area.

Shown in Fig. 6a is the evaluation of divergence over the developing thunderstorm complex. The amount of the outflow across the downstream side boundary is $9 \times 10^{6} \mathrm{~m}^{2} \mathrm{sec}^{-1}$ while that oif inflow across the upstream side boundary is only $2 \times 10^{6} \mathrm{~m}^{2} \mathrm{sec}^{-1}$. The mean divergence over this mesoscale area ( $S=6 \times 10^{10} \mathrm{~m}^{2}$ ) is, thus, evaluated as $12 \times 10^{-5} \mathrm{sec}^{-1}$.

The evaluation for the mature stage is made over the relatively larger area (Fig. 6b). As the dimension of the area is $29 \times 10^{10} \mathrm{~m}^{2}$, the result of this evaluation should be considered as the value of the large-scale divergence fleld rather than that of the mesoscale field. We have $28 \times 10^{6} \mathrm{~m}^{2} \mathrm{sec}^{-1}$ of the outflow across the downstream boundary against $7 \times 10^{6} \mathrm{~m}^{2} \mathrm{sec}^{-1}$ of the inflow across the upstream boundary. In spite of the large area for the calculation, the magnitude of the mean divergence is as large as $7 \times 10^{-5} \mathrm{sec}^{-1}$. The above results indicate that there was strong upward motion just below the outflow layer.

The values of divergence mentioned above are the value of mean divergence defined in the area whose dimension is on the order of $1 \times 10^{4} \mathrm{~km}^{2}$ and should not be confused with the value of the upper divergence over each cumulonimbus. As reported by McLean (1961), Fujita and Arnold (1963), and Matsumoto, Ninomiya and Nakagaki (1967), the magnitude of the divergence and/or the vertical velocity in a cumulonimbus is larger by one order of magnitude or more than the mean value evaluated in a whole storm area.

Another interesting feature of the upper level flow pattern is the appearance of strong winds along the northwest boundary of the storm area (see Figs. 3b and 4b). These strong winds are not only detected by using the clouds' movement but also observed by rawinsonde. As found in Fig. 9 of the next section, the wind speed at DAY exceeds 120 knots even at 500 mb . (The wind observation above 440 mb was missing at this station.) The vorticity field in the outflow layer was characterized by strong cyclonic vorticity to the north of the storm areu. The magnitude of the relative vorticity is given by

$$
\begin{equation*}
\zeta=K_{s} V-\frac{\partial V}{\partial n} \tag{2}
\end{equation*}
$$

where $V, K_{g}$, and $n$ are the wind speed, the curvature of the stream line and the distance normal to the stream line, respectively. The evaluation is made along the line $\widehat{A G}$ and $\widehat{a h}$ in Fig. 7a for the mature thunderstorm complex. To obtain the smoothed wind speed along these lines, the cloud movements used were within two 200 km wide zones centered on $\widehat{A G}$ and $\widehat{a h}$. The smoothed wind speed and calculated relative vorticity are presented in the lower part of Fig. 7a. The maximum of the cyclonic and anticyclonic vorticity is observed to the north and to the south of the northern boundary of the storm area, respectively, Decause the wind speed is largest along the northern boundary as mentiored.

The deformation fleld was analyzed in the outflow layer. It can be expressed as

$$
\begin{equation*}
a=\frac{\partial V}{\partial z}-V K_{n} \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
b=V K_{s}+\frac{\partial V}{\partial n} \tag{4}
\end{equation*}
$$

where $V, K_{s}, K_{n}, s$ and $n$ are the wind speed, the curvature of the stream line, the curvature of the orthogonal line to the stream line, the distance along the stream line and the distance along the normal to the stream line, respectively. The magnitude of deformation is given by

$$
\begin{equation*}
d=a \cos 2 \theta+b \sin 2 \theta \tag{5}
\end{equation*}
$$

or

$$
\begin{equation*}
|d|=\sqrt{a^{2}+b^{2}} \tag{6}
\end{equation*}
$$

where $\theta$, the angle between the axis of ditatation and the orthogonal line, is calculated as

$$
\begin{equation*}
\theta=\frac{1}{2} \tan ^{-}\left(\frac{b}{a}\right) \tag{7}
\end{equation*}
$$

The evaiuation of deformation for the mature thunderstorm complex is made along the line $\widehat{K Q}$ in Fig. 7b while smoothed wind profiles along lines $\widehat{A G}$ and $\widehat{a i}$ in Fig. Te are used. Results of the analysis show that the magnitude of deformacion in the outilow layer is about $6 \times 10^{-5} \mathrm{sec}^{-1}$, and the axis of dilatation is almost normal to the streamline.

It is noceworthy that the axis of dilatation is not parallel to the stream line but normal to it. It is also rather haid to assume an elongation owards the lateral direction for the isolated cumulonimbus from tile growth of the anvil cloud. The remarkable lateral elongation or difluence in the upper layer would be the characteristic observed only over the group of thunderstorms.

In order to verify the results of this section, a comparison should be made with the results in the following section which were obtained by using rawinsonde data only. After 1700 CST as the darkness of the earth's shadow began to cover the eastern part of the area under consideration, it became too hard to evaluate the cloud movement. Therefore, only the comparison between the results of the analysis for 1600 CST by using the ATS cloud data and the results for 1800 CST by using rawinsonde data, is possible. The comparison seems, however, to he adequate because the life time of the me'n outflow, whose horizontal scale is more than 500 km , would be sufficiently longer than the 2 hour time difference mentioned above.

## 4. Vertical Structure of the Outflow

Although the detail of the outflow and its grc wth could be studied by using high cloud movements as done in the previous section, the wind field in the layer under the outflow layer could not be studied by using ATS pictures. This was because the velocity of middle and low clouds can not be evaluated beneath dense hijgil clouds. As a result, the vertical structure of the wind field in and under the cuitifiow layer was analyzed by using the data of rawinsonde observations at 1800 CST.

A cloud distribution map at 1700 CST and the surface weather map at 1800 CST are shown in Fig. 8. Charts for the $250,300,500$, and 700 mb surfaces at 1800 CST are shown in Fig. 9. As its spacing is more than 300 km , the mesoscale field can not be aralyzed by using only the regular rawinsonde network. The large scale difluence
flow in the upper troposphere, however, was sbserved clearly around the storm arca. The strongest difluence is observed on the $250-\mathrm{mb}$ and $300-\mathrm{mb}$ surfaces The difluence in the upper level is mere clearly recognized in Fig. 10 by comparing the wind hodograph at the stations BUF and DIA which are situated on the aorth and south siues of the center of the convective storms. At 300 mb , the difference between the wind direction at BUF and DIA is as large as $60^{\circ}$. Fig. 10 also demonstrates the confluence in the lower troposphere between 650 and 750 mb . The confluence is strongest at 700 mb . Since the convective warm core exists in the layer between 600 and 400 mb , the wind field around the thunderstorm complex was characterized by the difluence above the warm core and the confluence below the warm core (for a example of a similar wind field, see Matsumoto and Ninomiya (1967)).

By using the wind observation at PIT, HTS, GSO, and DIA (see Fig. 8) the mean horizontal divergence over the quadrangle area enclosed by these stations is calculated as follows:

$$
\begin{equation*}
\overline{c . V V}=\frac{1}{s} \oint V_{n} d s=\frac{1}{s} \sum_{i=1}^{4} V_{n_{i}} \cdot L_{i} \tag{8}
\end{equation*}
$$

where $L_{i}$ and $V_{n}$ are the length of the side and the wind component normal to $L_{i}$ respectively. The dimension of the area, $S$, is about $10.1 \times 10^{4} \mathrm{~km}^{2}$. The vertica! distribution of the mean horizontal divergence is presented in Fig. 11. The vertical velocity $\omega\left(=\frac{d P}{d t}\right) \quad$ is, then, also calculated by using the continuity equation. The result is also shown in Fig. 11.

Upper divergence exists in a shallow layer between the 300 and $200-\mathrm{mb}$ surface. The magnitude of the maximum divergence observed at 250 mb is as large as $7 \times 10^{-5} \mathrm{sec}^{-1}$. The layer of the horizontal convergi nce is observed below 700 mb and the mid-troposphere is characterized as the non-divergence layer. The larg : upward motion is, therefore, in the layer between 700 mb and 40 nb . Above 300 mb , the upward motion decreased rapid!y ard almost vanished at the $200-\mathrm{mb}$ surface.

Analyses of rawinsonde data confirmed the conclusions about the vorticity field that were derived from the ATS pictures. It was shown in the previous section that the largest cloud velocity was found along the north wistern boundary of the storm area.

As seen in Fig. 9, the wind speed at DAY excerded 120 knots even the thormb surface. Since there were no wind observations it DAY obve 4.0-mb, only the valuee of the geostrophic wind velocity above this height can be ued. The maximum peostrophic wind speed of 130 knots was at $300-\mathrm{mb}$ ovef CAY. The magitude of ine reitive vortictity in the area to the northwest and to the southeat of the axif of the nirompet wind are evaluated by using the geostrophic wind velocty a DAY and the nweifed wind relocities at FNT, PIA, PIT and HTS. The resulte of the evaluation are prewentid in Table 1. Since the jet stream is located along the northwentem boundary of the trorn area, the upper tropospheric vorticity field it the vicinity of the torm area wes charecterized by the strong cyclonic vorticity to the nortiment of the storm erea and the itrong anticyeloaic vorticity over the storm aree.

Analyses of the deformation field using the rawturode date verified the conclusion of the previous section obrained from the cloud velocify feld. The antyale is made over the same area inwhich divergence and vertical velocity were eviluated. The maphide of deformation $d$ and the angle ( between the axim of dilatation and the $y$ axis (here the calculation is made in the rectainwlar coordinates) are fuen ae

$$
d=A \cos 2 \theta+8 \sin 2 \theta
$$

and

$$
\begin{equation*}
\theta=\frac{1}{2} \tan ^{-1} \frac{B}{A} \tag{10}
\end{equation*}
$$

where

$$
\begin{equation*}
A=\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y} \tag{11}
\end{equation*}
$$

and

$$
\begin{equation*}
B=\frac{\partial y}{\partial x}+\frac{\partial u}{\partial y} \tag{i}
\end{equation*}
$$

The angle between the axis of dilatation and the nomal to the atream linc. $\theta$ is obtained from the wind direction and 8 . As presented in table 2. the megnitude of deformation in the outflow layer it an large an that of deffence ane the axim of

Table 1. The magnitude of relative vorticity in the area to the northwest and to the southeast of the jetstream.

| mb | Northwest | Southeast |
| :--- | :--- | :--- |
| 200 | $10 \times 10^{-5} \mathrm{sec}^{-1}$ | $-7 \times 10^{-5} \mathrm{sec}^{-1}$ |
| 250 | 14 | -8 |
| 300 | 18 | -8 |
| 400 | 15 | -8 |

Table 2. The magnitude of deformation, $d$, and the angle between the ebngation axis and the normal to the stream line, $\theta$

| mb | $\|\mathrm{d}\|, 10^{-5} \mathrm{sec}^{-1}$ | $\theta \mathrm{deg}$ |
| :---: | :---: | :---: |
| 206 | 6 | 10 |
| 250 | 8 | 10 |
| 300 | 11 | 5 |
| 400 | 6 | 15 |

of dilatation is almost normal to the wind direction.
The conclusions obtained in the quantitative analyses of the upper wind field in this section coincide well with the results obtained using the high cloud movements it: the previous section and support our confidence in the validity of those analyses.

## 5. The Warm Core in the Mid-Troposphere

The most remarkable feature in the thermal field over the storm area was a warm core in the mid-troposphere. As seen in Fig. 9, the difference between the temperature inside and outside the warm core is as great as 3 C or more at the $\mathbf{5 0 0} \mathbf{- m b}$ surface. In addition. Fig. 9 shows high relative humidity within the core, itself. Fig. 12 represents the mean temperature sounding in the warm core which was obtained by averaging the rawinsonde observations at PIT, GSO, and HTS and clearly shows that the lapse rate in the layer between the $700-\mathrm{mb}$ and the $400-\mathrm{mb}$ surface is almost equal to the wet adiabatic lapse rate. Since the layer inside the warm core is characterized by high relative humidity, a wet-adiabatic lapse rate and strong mean upward motion (see Fig. 11), it is infered that the warm core was caused by the release of latent heat in the condensation process.

Because the thermal gradient of the large-scale field is so strong around the cut-off cold vortex (see Section 2) the thermal field of the warm core should be separated from the large-scale field. The temperature anomaly in the storm area is expressed by

$$
\begin{equation*}
\Delta T=\frac{1}{2}\left(T_{P I T}+T_{H T S}\right)-\frac{1}{2}\left(T_{D I A}+T_{D A Y}\right) \tag{13}
\end{equation*}
$$

Since stations DAY and DIA are located just north and south, respectively, of the storm area, $\frac{1}{2}\left(T_{D I A}+T_{D A Y}\right) \quad$ would give the temperature of the large-scale field. The vertical distribution of the temperature anomaly obtained by using equation (13) is presented in Fig. 13. The figure shows a temperature anomaly of 2 C in the warm core between the $600-\mathrm{mb}$ and the $400-\mathrm{mb}$ surface. Above this warm core there is cold air
${ }^{3}$ An example of a similar temperature field was suggested by Fujita and Byers (1960) for a single huge cumulonimbus and another was analysed by Matsumoto and Ninomiya (1967) for mesoscale convective storms in wintertime. Yet another analogous situation is the thermal structure of the warm core of a tropical cyclone and the associated upper outflow (see Yanai 1968) even though there are differences between the wind field of a tropical cyclone and that of a severe storm.
as seen in the 300 and the $250-\mathrm{mb}$ maps in Fig. 8. This cold air is a result of the overshooting of convective motions.

Because convective warming in the deep layer of the mid-troposphere increases the layer's thickness between the $700-\mathrm{mb}$ and the $300-\mathrm{mb}$ surfaces, there is local anomaly of the geopotential height of the isobaric surface in a warm core. The Laplacian of the geopotential height of the isolaric surface, $\nabla^{2} \boldsymbol{\phi}$ would be a good indication of the geopotential height anomaly because it expresses the curvature of the isobaric surface. The vertical distribution of $\nabla^{\mathbf{2}} \boldsymbol{\phi}$ calculated over the same area on which the temperature anomaly was evaluated, is $p_{\perp}$ esented in Fig. 13. The figure shows that the maximum of the negative value, i.e., the maximum anomaly, was just above the warm core (c.f. Matsumoto, Ninomiya and Akiyama 1967) (for the relationship of the negative value of $\nabla^{2} \boldsymbol{\phi}$ o the upper divergence field, see Section 7).

As mentioned previously, the large-scale thermal gradient is strong to the south of the cut-off cold core. As the convective warm core is situated to the southeast of the cut-off cold air, the strongest thermal gradient appears to the northwest of the storm area and has a maximum value of $2 \mathrm{C} / 100 \mathrm{~km}$. Although the reverse temperature gradient in the southern part of the warm core is partly canceled out by the large-scale gradient, still a weak reverse thermal gradient can be seen in the $500-\mathrm{mb}$ surface (Fig. 9).

Hodographs of the geostrophic wind are shown in Fig. 14 for the northside and the southside of the storm area. The northside hodograph was obtained by using geopotential height 'at DAY, HTS, and PIT and the southside hodograph was calculated by using observations at HTS, PIT and DIA. The figure shows that to the north of the warm core the geostrophic wind speed increases rapidly with height up to the 300 mb surface. Then it decreases with height due to the reverse thermal gradient which is caused by warm air over the cut-off cold air and the cold air over the storm area. To the south of the warm core, the speed of the geostrophic wind is almost unchanged, but its direction varies with height from southerly in the lower levels to westerly in the upper levels.

The results of the analysis show that the strong wind in the outflow layer along the northwest boundary of the cirrus sheet over the thunderstorms, the relatively weak wind to the south of the storms, and the difluence flow over the storm (see Fig. 14
and also 5 b and 9 ) are caused by the characteristic thermal field around the storm area.
6. Thermodynamical Aspects in the Upper Outflow Layer and the Warm Core

Several analyses have been made to clarify the role of convective transfer of heat and moisture in the thermodynamical process of the atmosphere. In their budget analysis of a tropical storm Riehl and Malkus (1961) stressed the role of convective transfer (i. e., the hot tower hy pothesis), while Matsumoto (1967) and Ninomiya (1968a, b) analyzed quantitatively the convective transfer in convective storms in wintertime.

In this section the role of the convective transfer of heat and moisture in the thermodynamical process in the upper outflow layer and the warm core will be clairfied by analyzing the continuity equation of water vapor and the thermodynamical equation for the mean motion using the rawinsonde observation data at 1800 CST. For the purpose of the analysis, convective terms should be introduced in these two equations. They are written as (Ninomiya 1968a)

$$
\begin{equation*}
\left[\frac{\partial q}{\partial f}+\overline{\nabla \cdot V q}+\frac{\partial}{\partial p} \bar{\omega} \bar{q}\right]+\frac{\partial}{\partial p} \overline{\omega^{\prime} q^{\prime}}=-m^{*} \tag{14}
\end{equation*}
$$

and

$$
\begin{equation*}
c_{p}\left[\frac{\overline{\partial T}}{\partial t}+\overline{\nabla \cdot V T}+\frac{\partial}{\partial p} \bar{\omega} \bar{T}-\frac{R}{c_{p} \bar{T} \bar{\omega}}\right]+c_{p} \frac{\partial}{\partial P} \overline{\omega^{\prime} T^{\prime}}=L m^{*} \tag{15}
\end{equation*}
$$

where $q$ is the mixing ratio of water vapor, $m *$ the condensation amount in the unit air mass, $R$ the gas constant, $C_{p}$ the specific heat of the air in a corstant-pressure process, and $L$ is the latent heat of vaporization. The bar - and the prime ' in eqs. (14) and (15) denote the mean value in the area under consideration and the deviation from it, respectively. Since the deviation field is due to the sub-grid size convective motions, the terms $-\frac{1}{9} \overline{\omega^{\prime} q^{2}}$ and $-\frac{C_{p}}{g} \overline{\omega T^{\prime}} \quad$ represent the vertical convective transfer of the water vapor and of the seusible heat, respectively. The mean upward velocity $\bar{\omega}$ is given as

$$
\begin{equation*}
\bar{\omega}=\nabla \cdot \int_{\mu}^{p_{s}} V d p+\frac{\partial P_{s}}{\partial t} \tag{16}
\end{equation*}
$$

where $P_{s}$ indicates the surface pressure.
It is more convenient for the following discussion to rewrite the eqs. (14) and (15)
as

$$
\begin{equation*}
\left(\bar{\delta} \frac{\delta}{\delta t}\right) \bar{q}+\frac{\partial}{\partial p} \overline{\omega^{\prime} q^{\prime}}=-m * \tag{17}
\end{equation*}
$$

and

$$
\begin{equation*}
\left.c_{p}\left(\overline{\delta_{1}}\right) \overline{\delta t}\right)+c_{p} \frac{\partial}{\partial P} \overline{\omega^{\prime} T^{\prime}}=L m^{*} \tag{18}
\end{equation*}
$$

where the net increment of the mixing ratio due to the mean motion ( $\left.\frac{\bar{\delta}}{\delta t}\right) \bar{q}$ and of the temperature $\left(\frac{\overline{\delta_{1}}}{\delta \dagger}\right) \bar{T} \quad$ are defined as

$$
\begin{equation*}
\left(\overline{\frac{\delta}{\delta t}}\right) \bar{q}=\frac{\overline{\partial q}}{\partial t}+\overline{\nabla \cdot V q}+\frac{\partial}{\partial p} \bar{\omega} \bar{q} \tag{19}
\end{equation*}
$$

and

$$
\begin{equation*}
\left(\overline{\delta_{1}}\right) \overline{\delta t}=\frac{\overline{\partial T}}{\partial t}+\overline{\nabla \cdot V T}+\frac{\partial}{\partial p} \bar{\omega} \bar{T}-\frac{R}{c_{p}} \frac{\bar{\omega} \bar{T}}{p} \tag{20}
\end{equation*}
$$

respectively. ${ }^{4}$
In the continuity equation of water vapor, eq. (14), only the terms in the brackets are evaluated by using the rawinsonde observation data, while both $\frac{\partial}{\partial p} \overline{\omega^{\prime} q^{\prime}}$ and $m^{*}$
${ }^{4}$ Instead of eqs. (17) and (18), or (15) and (16),

$$
\frac{\overline{\partial q}}{\partial!}+\overline{\nabla \cdot V q}+\frac{\partial}{\partial p} \bar{\omega} \bar{q}=-m^{*}
$$

and

$$
c_{p}\left[\frac{\overline{\partial T}}{\partial t}+\overline{\nabla \cdot V T}+\frac{\partial}{\partial p} \bar{\omega} \bar{T}-\frac{R}{C_{p}} \frac{\bar{\omega} \bar{T}}{p}\right]=L m *
$$

are sometimes used for the mean field. It is not correct, however, in the convective area because the most important process in the distribution of heat energy, i.e., the convective transfer of heat energy, is left out.
could not be evaluated with rawinsonde data. A similar situation occurs in the thermodynamic equation, eq. (15), where $c_{p} \frac{\partial}{\partial p} \overline{\omega^{\prime} T^{\prime}}$ and $m *$ can not be evaluated with rawinsonde data, either. In order to evaluate separately the vertical convective transport of water vapor and heat, it is necessary, therefore, to know the vertical distribution of the amount of condensation. As we do not have the necessary information on the vertical distribution of $m$ * ${ }_{\text {* }}^{\text {the }}$ equation of total heat energy is used instead, which is obtained by multiplying eq. (14) by $L$ and adding eq. (15) as

$$
\begin{align*}
& c_{p}\left[\frac{\partial T}{\partial t}+\overline{\nabla \cdot V T}+\frac{\partial}{\partial p} \bar{\omega} \bar{T}-\frac{R}{c_{p}} \frac{\overline{\omega T}}{\bar{p}}\right]  \tag{21}\\
& +L\left[\frac{\partial q}{\partial t}+\overline{\nabla \cdot V q}+\frac{\partial}{\partial p} \bar{\omega} \bar{q}\right]+\frac{\partial}{\partial p}\left[c_{p} \overline{\omega^{\prime} T^{\prime}}+L \overline{\omega^{\prime} q^{\prime}}\right]=0
\end{align*}
$$

or

$$
\begin{equation*}
c_{p}\left(\frac{\overline{\delta_{1}}}{\delta t}\right) \bar{T}+L\left(\frac{\bar{\delta}}{\delta t}\right) \bar{q}+\frac{\partial}{\partial p}\left[c_{p} \overline{\omega^{\prime} T^{\prime}}+L \overline{\omega^{\prime} q^{\prime}}\right]=0 \tag{22}
\end{equation*}
$$

The convective term $\frac{\partial}{\partial D}\left[C_{p} \overline{\omega^{\prime} T^{\prime}}+L \overline{\omega^{\prime} q^{\prime}}\right] \quad$ in eq. (21) or (22) can be evaluated as the residual term once the mean terms are evaluated by using the rawinsonde observation data. The evaluations are made in the quadrangle bounded by the four ravinsonde stations PIT, HTS, GSO and DIA. The mean horizontal divergence of water-vapor flux and sensible heat flux are calculated as

$$
\overline{\nabla \cdot V q}=\frac{1}{S} \sum_{i=1}^{4} V_{n_{i}} q_{i} L_{i}
$$

and

$$
c_{p} \overline{\nabla \cdot V T}=c_{p} \frac{1}{S} \sum_{i=1}^{4} V_{n_{i}} T_{i} L_{i}
$$

where $L_{i}$ is the length of the side, $V_{n_{1}}$ the wind component normal to $L_{i}$ and $Q_{1}$ and $T_{i}$ are the mean value of $q$ and $T$ along $L_{i}$ respectively. The dimension of the quadrangle, $S$, is about $10.1 \times 10^{4} \mathrm{~km}^{2}$. The value of the mean vertical velocity $\bar{\omega}$ given by eq. (16) and the areal averaged values $\bar{T}$ and $\bar{q}$ are used to calculate $\frac{\partial}{\partial P} \bar{\omega} \bar{T}, \frac{R}{C_{P}} \frac{\bar{\omega} T}{P}$ and $\frac{\partial}{\partial P} \bar{\omega} \bar{q}$. The terms involving local time
change in eq. (21) in the layer below 800 mb are evaluated by using the surface observation both in plain and mountainous areas and these terms in the layer above 800 mb are calculated by using the space-time transformation technique,

$$
\begin{equation*}
\frac{\partial}{\partial t}=-C \frac{\partial}{\partial s} \tag{23}
\end{equation*}
$$

where $\mathbf{C}$ and $\mathbf{S}$ are the propagation velocity and direction of the system.
Thus the evaluated magnitude of each mean term in eq. (21) is integrated in each layer between two isobaric surfaces. The results ${ }^{5}$ of the evaluation are presented in Figs. 15 and 16.

Once the vertical distribution of the value of $-\frac{1}{g}\left(c_{p} \overline{\omega^{\prime} T^{\prime}}+L \overline{\omega^{\prime} q^{\prime}}\right)$ surface by integrating $\left[C_{p}\left(\frac{\overline{\delta_{1}}}{\delta \dagger}\right) \bar{T}+L\left(\frac{\bar{\delta}}{\delta \dagger}\right) \bar{q}\right]$ $C_{p}\left(\overline{\delta_{1}} \frac{\delta t}{\delta t}+L\left(\overline{\frac{\delta}{\delta t}}\right) \bar{q}\right] \quad$ is obtained, can be obtained at an arbitrary pressure layer to the pressure surfacc with the boundary condition that the convective transfer vanishes at the top of the oufflow layer $\rho_{1}$ as

$$
\begin{equation*}
-\frac{1}{g}\left(c_{p} \overline{\omega^{\prime} T^{\prime}}+L \overline{\omega^{\prime} q^{\prime}}\right)=\frac{1}{g} \int_{p}^{p}\left[c_{p}\left(\overline{\frac{\delta_{1}}{\delta t}}\right) \bar{T}+L\left(\overline{\frac{\delta}{\delta t}}\right) \bar{q}\right] d p \tag{24}
\end{equation*}
$$

The results are also presented in Fig. 16.
The most interesting features of the vertical distribution of $\left[c_{p}\left(\frac{\overline{\delta_{1}}}{\delta t}\right) \bar{T}+L\left(\frac{\bar{\delta}}{\delta \dagger}\right) \bar{q}\right]$ are its large positive value in the upper troposphere and its negative value in the lower troposphere (c. f. , Matsumoto 1968). Eq. (22) and Fig. 16 show that the net increment of the total heat energy in the upper outflow layer is balanced by the vertical convergence of the convective transfer of the heat energy. The results of the analysis show that the large amount of convective transfer of heat energy across the base of the outflow layer thermodynamically maintains the mean motion in the outflow layer.
${ }^{5}$ Since $\nabla \cdot V T_{a}+\frac{\partial}{\partial D} \omega T_{0}$
is zero for an arbitrary constant $T_{a}$, the value of the three dimensional flux divergence $\quad \nabla \cdot V T+\frac{\partial}{\partial P} \omega T$ is equal


Previously, it was mentioned that the net increment of the total heat energy appears in the outflow layer and that the large amount of heat energy is transported into the outflow layer by the convective motion. The following discussion would explain the physical meaning of the results of the heat energy budget analysis in the outflow layer.

The equation of total heat energy eq. (2i) is expressed by using the equivalent potential temperature ${ }^{6} \quad \theta_{e}=\pi\left(T+\frac{L}{C_{p}} q\right)$
where $\pi \quad$ indicates
$\left(\frac{1000}{p}\right)^{k} \quad$ and then integrating from the bottom of the outflow layer $P_{1}$ to the top of the layer $P_{2}$,

$$
\begin{equation*}
\frac{C_{p}}{g} \int_{p_{1}}^{P_{2}} \frac{\overline{\partial \theta}}{\partial t} e d p+\frac{C_{p}}{g} \int_{p_{1}}^{P_{2}} \overline{\nabla \cdot V \theta_{e}} d p+\frac{C_{p}}{g} \bar{\theta}_{\theta_{2}} \bar{\omega}_{2}+\frac{C_{\rho}}{g} \overline{\left.\theta_{e}^{F} \omega^{\prime}\right)_{2}}=0 \tag{25}
\end{equation*}
$$

(Both the mean upward motion $\bar{\omega}$ and the convective transfer $\frac{C_{p}}{g} \overline{\theta_{e}^{\prime} \omega^{\prime}} \quad$ vanish at the top of the outflow layer.) In order to simplify the discussion, eq. (25) is rewritten with the aid of eq. (23) and the continuity relation

$$
\begin{equation*}
\widehat{\nabla \cdot V \Delta p+\bar{\omega}_{2}=0} \tag{26}
\end{equation*}
$$

as

$$
\begin{equation*}
\left[\frac{C_{p}}{g}(\hat{V}-C) \cdot \hat{\nabla} \hat{\theta}_{e} \Delta p\right]+\left[\frac{C_{p}}{g}\left(\hat{\theta_{\theta}}-\bar{\theta}_{e_{2}}\right) \hat{\nabla \cdot V \Delta p}\right]+\frac{C_{p}}{g}\left(\bar{\theta}_{e}^{\prime} \omega^{\prime}\right)_{2}=0 \tag{27}
\end{equation*}
$$

In eqs. (26) and (27), $\wedge$ indicates the vertical mean value in the outflow layer and $\Delta P$ is defined as $P_{2}-P_{1}$. Since the magnitude of the second term in eq. (27) is two or three times greater than that of the first term, the second term seems to be balanced mainly by the third term of eq. (27). The difference between $\widehat{\theta}_{e}$, the mean equivalent potential temperature in the outflow layer,

[^1]and $\overline{\theta_{0}}$, the equivalent pusential temperature at the base of the layer, is positive and its value is 5 K in the area under consideration. Thus we have approximately 25 ly hour ${ }^{-1}$ for the second term since the mean divergence in the outflow layer is $5 \times 10^{-5} \mathrm{sec}^{-1}$. In other words, the horizontal out-flux of equivalent potential temperature from the storm area associated with the horizontal mass divergence in the outflow layer is larger than the amount of the mean upward transfer of equivalent potential temperature across the base of the outflow layer because $\frac{\partial \theta_{0}}{\partial \rho}$ is negative in the layer. (The difference. between $\hat{\theta}_{\theta}$ and $\bar{\theta}_{\theta_{2}}$, i.e. $\hat{\theta}_{\theta_{0}}-\bar{\theta}_{\theta_{2}}$ in eq. (27), is expressed as
$\widehat{\theta}_{\mathrm{e}}-\bar{\theta}_{\mathrm{e}_{2}} \cong \frac{1}{2}\left(\bar{\theta}_{\theta_{1}}+\bar{\theta}_{\mathrm{e}_{2}}\right)-\bar{\theta}_{\theta_{2}} \cong-\frac{1}{2} \frac{\partial}{\partial \mathrm{p}} \theta_{\mathrm{e}} \Delta \mathrm{p}$. $\quad$ The difference between the horizontal outflow of total heat energy and the mean vertical transfer of the heat energy into the outflow laye. is supplied mainly by the convective transfer of the heat energy. Likewise it is easily shown that the convergence in the lower troposphere should increase the convective transfer of water vapor (Ninomiya 1968b).

At the end of this section, the value of the convective transfer in Figs. 15 and 16, which is evaluated as the residual term in the budget calculation, should be related to the characteristic thermodynamical quantities of the convective motion. As shown in the Appendix, ve can express the convective transfer of total heat energy as

$$
\begin{align*}
-\frac{1}{g}\left(c_{p} \overline{\omega^{\prime} T^{\prime}}+L \overline{\omega^{\prime} q^{\prime}}\right) & \cong-\frac{1}{g} \dot{\sigma}_{c} \omega_{c}\left(c_{p} \Delta_{c} T+L \Delta_{c} q\right)  \tag{28}\\
& \cong-\frac{1}{g} \bar{\omega}\left(c_{p} \Delta_{c} T+L \Delta_{c} q\right)
\end{align*}
$$

where $\sigma_{c}$ is the area ratio of the convective upward motion, $\omega_{c}$ the convective upward velocity, $\Delta_{c} T$ and $\Delta_{c} q$ are the convective excess temperature and excess mixing ratio, respectively. Although we do not have precise information al it $\Delta_{c} T$ and $\Delta_{C} q, 2 \mathrm{~K}$ of the excess temperature and $2 \mathrm{gm} \mathrm{kg}^{-1}$ of the excess mixing ratio have not been considered unusual (see for instance, Fujita and Byers 1960, Matsumoto, Ninomiya and Nakagaki 1967). As mentioned in section 4, the mean vertical velocity in the mid-tronosphere is about $25 \mathrm{mb}^{\text {hour }}{ }^{-1}$. By using these values in eq. (28), the magnitude of the convective transfer of the heat energy is evaluated to be approximately

40 ly hour ${ }^{-1}$. This value coincides rather well with the value of the convective tranefer obtained from the budget calculation and indicates the validity of the budget calculation.

## 7. Dynamical Aspects of the Upper Divergence

In this section some dynamical aspects of the wind field of the large scale upper outflow will be analysed. Since the wind field in the upper cutflow layer is characterized by strong divergence, it is appropriate to use the divergence equation instead of the momentum equation for discussing the dynamical mechanism of the outflow. The divergence equation relevant to the mean motion in the outflow layer is written as

$$
\begin{align*}
& \frac{\overline{\partial D}}{\partial t}+\overline{V \cdot \nabla D}+\bar{\omega} \frac{\partial \bar{D}}{\partial P}+\left(\frac{\partial \bar{\omega}}{\partial x} \frac{\partial \bar{u}}{\partial P}+\frac{\partial \bar{\omega}}{\partial y} \frac{\partial \bar{v}}{\partial P}\right) \\
& +\frac{1}{2}\left(\bar{D}^{2}+\bar{A}^{2}+\bar{B}-\bar{\zeta}^{2}\right)-f \bar{\zeta}+\nabla^{2} \phi  \tag{29}\\
& +\left[\overline{\frac{\partial D^{\prime}}{\partial P}}+\frac{1}{2} \overline{D^{2}}\right]=0
\end{align*}
$$

where $D$ is divergence, $\zeta$ is relative vorticity, and $A=\frac{\partial u}{\partial X}-\frac{\partial v}{\partial y} \quad$ and $B=\frac{\partial V}{\partial X}+\frac{\partial U}{\partial y} \quad$ are deformations (see Mats $\quad$ moto, Ninomiya and Akiyama 1967). The bar - and prime ' are used in the same way as in eqs. (14) and (15) in section 6. The convective term in eq. (29), to be precise, should be

$$
\left[\overline{\omega^{\prime} \frac{\partial D^{\prime}}{\partial P}}+\left(\overline{\frac{\partial \omega^{\prime}}{\partial x} \frac{\partial u^{\prime}}{\partial P}}+\overline{\frac{\partial \omega^{\prime}}{\partial y} \frac{\partial v^{\prime}}{\partial P}}\right)+\frac{1}{2}\left(\overline{D^{2}}+\overline{A^{2}}+\overline{B^{2}}-\overline{\zeta^{\prime}}\right)\right]
$$

Although we do not have enough information on these correlation terms, it is inferied from the dynamical nature of convective motion that $\frac{\omega^{\prime} \frac{\partial D^{\prime}}{\partial P}}{}$ and $\frac{D^{,^{2}}}{}$ are the most important.

Thus, in the divergence equation, eq. (29), $\left[\overline{\omega^{\prime} \frac{\partial D^{\prime}}{\partial P}}+\frac{1}{2} \overline{D^{\prime 2}}\right] \quad$ is introduced as the most important of the convective terms. The mean terms in eq. (29) are evaluated over the storm area by using rawinsonde observation at 1800 CST. (The time-space transformation technique given in eq. (23) was applied to the first term of eq. (29). ) The estimated value of each mean term over the storm area is given in Table 3.

Table 3. The estimation of the divergence equation (unit $=10^{-8} \mathrm{sec}^{-2}$ )

|  | 300 mb | 250 mb |
| :--- | :--- | :--- |
| $\frac{\partial D}{\partial \dagger}+\bar{Y} \cdot \overline{\nabla D}$ | 0.1 | 0.3 |
| $\bar{\omega} \frac{\overline{\partial D}}{\partial P}$ | 0.3 | 0.0 |
| $\frac{\partial \bar{\omega}}{\partial x} \frac{\partial \bar{v}}{\partial P}-\frac{\partial \bar{\omega}}{\partial y} \frac{\partial \bar{u}}{\partial P}$ | 0.1 | 0.0 |
| $\frac{1}{2}\left(\bar{D}^{2}+\frac{2}{A}+\frac{2}{B}-\frac{2}{\zeta}\right)-f \bar{\zeta}$ | 0.6 | 0.8 |
| $\nabla^{2} \phi$ | -1.9 | -1.0 |

On the $250-\mathrm{mb}$ surface, the surface of maximum divergence, the main terms of eq. (29) are $\nabla^{2} \phi, \frac{\overline{\partial D}}{\partial t}-\frac{,}{\bar{i}} \overline{V \cdot \nabla D}$ and $\left[\frac{1}{2}\left(\bar{D}^{2}+\bar{A}^{2}+\bar{B}^{2}-\bar{\zeta}^{2}\right)-\bar{f}\right]$. Both the twisting terms and $\bar{\omega} \frac{\overline{\partial D}}{\partial P} \quad$ are negligibly small on this surface, it should be emphasized that the large scale divergence in the outflow layer is due to the negative value of $\nabla^{2} \phi \quad$ which is caused by the convective warming in the midtroposphere.

The eddy terms in eq. (29) could not be evaluated directly by using rawinsonde observation and so were evaluated as the residual terms. The value of $\left[\omega^{\prime} \frac{\partial \bar{D}^{9}}{\partial P}+\frac{1}{2} D^{\frac{, 2}{2}}\right]$ was $0.1 \times 10^{-8} \mathrm{sec}^{-2}$. This value is smaller than the ralue of $\nabla^{2} \phi \quad$ by one order of magnitude, and hence the role of the convective edd $\gamma$ terms seems to be less important at the $250-\mathrm{mb}$ surface than on the surface of maximum $\nabla^{2} \phi$, the $300-\mathrm{mb}$ surface, where the role of the convective eddy terms seems to be significant because the estimated value of the eddy terms is $1 \times 10^{-8} \mathrm{sec}^{-2}$, which is obtained by the residual calculations. This large value of the convective eddy terms on the $300-\mathrm{mb}$ surface is the reason why the maximum divergence appears in a higher level than the level of the maximum of the negative $\nabla^{2} \phi$.

The vertical distribution of the magnitude of the eonvective eddy terma. which is estimated in the residual calculation. is related to the dyamical properfy of the
 as

$$
\begin{equation*}
\frac{1}{2} \overline{D^{p^{2}}} \cong \frac{1}{2} \alpha_{c}\left(D_{c}-\bar{D}\right) \cdot\left(D_{z}=D_{i}\right) \approx \frac{1}{2} \bar{D}\left(D_{k}=D_{1}\right) \tag{70}
\end{equation*}
$$

and

$$
\begin{equation*}
\omega^{\prime} \frac{\partial D^{\prime}}{\partial D} \cong \bar{\omega}\left(\frac{\partial D_{s}}{\partial P}-\frac{\partial D_{i}}{\partial D}\right) \tag{14}
\end{equation*}
$$

 respectively. Although we do not have much information of the ariderive motion in
 can be inferred that $\omega_{c}$ is almost constan is the mid:tropospitfe bexuet the








$\omega_{c}$ and $D_{c}$ are larger than $\bar{\omega}$ and $\bar{D}$ feeptively, Ey one ondef of
 terms will be $10^{-8} \mathrm{sec}^{-\dot{غ}}$.






8. The Influence of the Convective Warming on the Mean Vertical Motion in the Storm Area

There were several ways to analyze the influence of the convective warming on the wind field. Manabe (1956), for example, analysed the change of the potential vorticity in the large scale field due to the released latent heat in the area of intense precipitation. Solving the $\omega$ - equation which included the terms of diabatic heating, Danard (1964) a lso showed the infiuence of the released latent heat on the large-scale wind field. He estimated the amount of diabatic heating by using the amount of condensation which was evaluated from the kinematically computed vertical velocity.

In the case presented bere, the influence of the convective warming on the mean upward motion will be analysed by means of the $\boldsymbol{\omega}$ - equation. When the vorticity equation and the thermodynamic equation are combined to eliminate the terms of time change of the geopotential and temperature, the familiar $\omega$ - equation is obtained,

$$
\begin{align*}
& \sigma \nabla^{2} \bar{\omega}+\beta^{2} \frac{\partial^{2} \bar{\omega}}{\partial p^{2}}+\frac{R}{c_{p}} \frac{1}{p} \nabla^{2}\left(\frac{\overline{\delta Q}}{\delta t}\right) \\
& \quad+\left\{\nabla^{2}\left[V \cdot \nabla\left(\frac{\partial \phi}{\partial p}\right)\right]-f \frac{\partial}{\partial p}[V \cdot \nabla(f+\zeta)]\right\}=0 \tag{32}
\end{align*}
$$

where $\sigma$ is defined as $-\theta \frac{\partial}{\partial P} \ln \theta$ and $\left(\frac{\overline{8 Q}}{\delta t}\right)$ is defined as diabatic hearing for the mean motion. The vertical motion caused by the third term and the last term are the thermally and dynamically forced upward motion, respectively.

In order to clarify the discussion, the thermaily and the dynamically forced tupard motion should be analysed separately. We will evaluate the thermally forced upward motion by using the $\omega$ - equation which involves the term of the diabatic heating,

$$
\begin{equation*}
\sigma \nabla^{2} \bar{\omega}+f^{2} \frac{\partial^{2} \bar{\omega}}{\partial p^{2}}+\frac{R}{C_{\phi}} \frac{1}{p} \nabla^{2}\left(\frac{\overline{\delta Q}}{\delta i}\right)=0 \tag{33}
\end{equation*}
$$

As already discussed in section 6 , the diabatic heating for the mean motion $\left(\frac{\overline{8 Q}}{8 \dagger}\right)$ is defined as

$$
\begin{align*}
\left(\overline{\frac{\delta Q}{\delta t}}\right) & \equiv c_{p}\left[\frac{\partial T}{\partial t}+\overline{\partial \cdot V T}+\frac{\partial}{\partial p} \bar{\omega} \bar{T}-\frac{R}{c_{p}} \frac{1}{p} \overline{\omega T}\right]  \tag{34}\\
& =L \overline{m^{*}}-c_{p} \frac{\partial}{\partial p} \overline{\omega^{\prime} T^{\prime}}
\end{align*}
$$

where $\mathrm{m}^{*}$ means the amount of condensation per unit mass of the air. It is important to note that the diabatic heating for the mean motion is caused not only by the release of the latent heat $\mathrm{Lm}^{*}$ but also by the vertical convergence of the convective transfer of the sensible heat $C_{p} \frac{\partial}{\partial P} \omega^{\prime} T^{\prime}$

In order to evaluate the mean vertical velocity, the $\boldsymbol{\omega}$-equation, eq. (33) was rewritten in the form of simultaneous linear equations of $\bar{\omega}$ at each grid point, $\bar{\omega}_{i, j, k}$ by using the grid systern shown in Fig. 17, where the area of the diabatic heating is indicated by stippling. The amount of the diabatic heating in each layer evaluated over the storm area in section 6 is adopted as the amount of diabatic hearing in the heating area. Adiabatic motion is assumed outside this heating area. It was also assumed that the mean vertical velocity $\bar{\omega}$ vanishes at the lateral boundaries of the volume over which calculations are performed and on the $1000-\mathrm{mb}$ and $100-\mathrm{mb}$ surfaces. The values of $\bar{\omega}$ were obtained by solving the simultaneous equations. The horizontal divergence at eacn grid point was also calculated from the vertical velocity by using the continuity equation. The reaults for the center of the heating area are presented in Fig. 18. A comparison of these results (Fig. 18) with the mean vertical velocity and divergence obtained kinematically from the obse.ved wind (Fig. 11) verifies the fact that the Wertical velocity and the divergence obtained by solving ey. (33) show the characteristic features of the vertical distributions of the upward velocity and horizontal divergence in the storm area. The results indicate that the strong upward motion in the warm core and the remarkable high level divergence over the warm core are mainly induced and/or maintained by the convective warming.

Lastly we will study the dynamically forced vertical motion in the storm area by using the $\omega$ - equation for adiabatic motion

$$
\begin{align*}
& \sigma \nabla^{2} \bar{\omega}+f^{2} \frac{\partial^{2} \bar{\omega}}{\partial p^{2}}  \tag{35}\\
& \quad+\left\{\nabla^{2}\left[V \cdot \nabla\left(\frac{\partial \phi}{\partial \bar{p}}\right)\right]-f \frac{\partial}{\partial p}[V \cdot \nabla(f+\zeta)]\right\}=0
\end{align*}
$$

It should be noted that the dynamically forced vertical motion is also influenced by the convective warming because the wind velocity and the vorticity, which are involved in the last term of eq. (35), have been modified in the vicinity of the warm core as shown in sections 4 and 5. In order to simplify the discussion, eq. (35) is rewritten by using the geostropnic wind approximation

$$
\begin{equation*}
\sigma \nabla^{2} \bar{\omega}+f^{2} \frac{\partial^{2} \bar{\omega}}{\partial p^{2}}=f \frac{\partial V}{\partial p} \cdot \nabla(f+\zeta) \tag{36}
\end{equation*}
$$

In rewriting the equation, the terms which are insignificantly smaller than the term in the right side of eq. (36) are dropped. Since the left side of eq. (36) is negatively correlated with $\bar{\omega}$ itself, eq. (36) indicates that upward motion tends to occur in the region in which the thermal wind blows from the area of the maximum absolute vorticity to the area of the minimum vorticity and vice versa (Eliassen 1964).

It was shows in section 4 that the core of strong cyclonic vorticity is located to the northwest of the storm area and that the thermal wind is very strong along the northwestern boundary of the storm area. Therefore, the downward and upward motion are induced dynamically behind and ahead of the core of the cyclonic vorticity, respectively. This downward motion seems to intensify the dry area in the rear of the storm area. (see the $700-\mathrm{mb}$ map in Fig. 9).

Although the core of anticyclonic vorticity was located within the storm area as mentioned in section 4, the dynamically forced vertical motion in the storm area was weak because the thermal wind was very weak in the storm area.

## 9. Concluding Remarks

On the upper level outflow and the midtropospheric warm core over the tornadoproducing thunderstorms of April 23, 1968, a detailed synoptic and dynamic analysis
was made by using the conventional rawinsonde data combined with A.TS III pictures obtained by the Tornado Watch Experiment of 1968.

The main results are summarized as follows:

1) No unusual mesoscale divergence in the upper level was found in the period prior to the development of the thunderstorms.
2) The velocity field of high-level clouds obtained by using the series of ATS pictures revcaled the existence of mesoscale outflow from the developing thunderstorms. The magnitude of the mean divergence evaluated in the area whose horizontal dimension was on the order of $10 \times 10^{4} \mathrm{~km}^{2}$ was about $10^{-4} \mathrm{sec}^{-1}$.
3) As the number of the developing thunderstorms increased, the mesoscale outflow patterns modified the general flow and a large-scale outflow pattern was formed over the storm area.
4) A warm core was observed in the mid-troposphere beneath the large-scale outflow. The high humidity and also the strong upward motion in the warm core suggest that the warm core was due to the convective warming.
5) The wind field in the storm area was characterized by strong divergence and difluence above the warm core and also by strong convergence below the warm core.
6) The warm core intensified the thermal gradient to the northwest of the storm area. As explained by the thermal wind equation, the strong thermal gradier: causes in part the strong vertical wind shear a.ld strong wind observed to the northwest of the storm area.
7) Heat budget analysis indicates that the large amount of diabatic warming was necessary to maintain the mean motion in the ouflow layer. This diabatic warming was due to the upward convective transfer of heat energy across the base of the outflow layer. This convective transfer was caused mainly by the convergence of water-vapor flux in the lower troposphere.
8) The dynamical aspects of the large-scale upper outflow was studied by using the divergence equation in the upper layer and also by using the $\omega$ - equation with diabatic heating. The strong mean upward motion in the mid-troposphere and the strong mean divergence in the upper outflow layer were induced and maintained by the convective warming in the storm area.

## APPENDIX

## THE CONVECTIVE TRANSFER IN THE MEAN FIELD

The ratios of area of the core of upward convective motion and that of area outside of the core are denoted by $\sigma_{c}$ and $\sigma_{f}$ respectively. The values inside and outside of the core are indicated by the suffixes $c$ and $f$. The areal mean is denoted by a bar - and the deviation from the mean by a prime ' . According to the definition we have

$$
\begin{align*}
& \sigma_{c}+\sigma_{f}=1  \tag{A-1}\\
& \bar{\omega}=\sigma_{c} \omega_{c}+\sigma_{f} \omega_{f} \tag{A-2}
\end{align*}
$$

and

$$
\begin{equation*}
\bar{a}=\sigma_{c} a_{c}+\sigma_{f} a_{f} \tag{A-3}
\end{equation*}
$$

where $\omega$ is vertical velocity and $\alpha$ is an arbitrary scalar quantity. The total upward transfer of the quantity $d$ is written as

$$
\begin{align*}
\bar{\omega} \alpha & =\sigma_{c} \omega_{c} a_{c}+\sigma_{f} \omega_{f} a_{f}  \tag{A-4}\\
& =\bar{\omega} \bar{a}+\sigma_{c} \cdot \sigma_{f}\left(\omega_{c}-\omega_{f}\right)\left(a_{c}-a_{f}\right)
\end{align*}
$$

or

$$
\begin{equation*}
\overline{\omega \bar{a}}=\bar{\omega} \bar{a}+\sigma_{f}\left(\bar{\omega}-\omega_{f}\right)\left(a_{c}-G_{f}\right) \tag{A-5}
\end{equation*}
$$

or

$$
\begin{equation*}
\overline{\omega a}=\bar{\omega} \bar{a}+\sigma_{c}\left(\omega_{c}-\bar{\omega}\right)\left(a_{c}-a_{f}\right) \tag{A-6}
\end{equation*}
$$

'Therefore the eddy term is written as

$$
\begin{equation*}
\overline{\omega^{\prime} a^{\prime}}=\sigma_{c}\left(\omega_{c}-\bar{\omega}\right)\left(a_{c}-a_{f}\right) \tag{A-7}
\end{equation*}
$$

(see also Yanai (1964), Matsumoto (1967) and Ninomiya (1968a)). As the $\omega_{c}$ is one order of magnitude or more larger than $\bar{\omega}$ and as the mean upward motion is mainly due to the convective motion, eq. (A-7) is roughly

$$
\begin{equation*}
\overline{\omega^{\prime} a^{\prime}} \cong \sigma_{c} \cdot \omega_{c}\left(a_{c}-a_{f}\right) \cong \overline{\omega^{\prime}}\left(a_{c}-a_{f}\right) \tag{A-8}
\end{equation*}
$$

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Fig. 1. Surface temperature ( C , thin line), mixing ratio (gm. $\mathrm{kg}^{-1}$, thin dashed line) and $1,300-\mathrm{ft}$ winds at $1200 \mathrm{CST}, 23$ April 1968. The storm (tornado and hail) areas reported between 1200 CST and 1500 CST are indicated by the stipple. The movement of the cut-off cold core is also shown in this figure. The isotherms at 400 mb are drawn with thick solid lines.


Fig. 2(A). A digitized ATS III picture at 1100 CST, April 23, 1963.
Fig. \& (B). The velocity field of clouds for the initial stage of the thunderstorm complex. The clouds' movements are obtained for a $41-\mathrm{min}$ period from 1047 to 1128 CST. The figures indicate the wind speed (knot). The distribution of clouds (area of thin stipples) and radar echoes (area of dense stipples) is for 1100 CST .




 1304 CST .


Fig. 4(A). A iligtized ATS III picture at 1605 CST, April 23, 1968.
Fig. 4(B). The velocity helid of cioads for the mature atage of the chunderstorm comples. The clouls' movements are oblained for a 41 -min period fromit 158 s is 1619 CST. The distritration of clouds and radar echocs is for Leos CST.


Fig. 5. The comparison between wind speed and velocity of high clouds in the neighborhood of the radiosonde station, HIS and BUF.


Fig. $6(A)$. Divergence in the outflow layer of the thunderstorm complex in the growing stage. Calculation is made by using the high clouds' movements for a $41-\mathrm{min}$ period from 1236 to $\$ 317$ CST.
Fig. 6(B). Divergence in the outflow layer of the mature thunderstorm complex. Caiculation is made by using the high clouds* movements for a 41-min period from 1538 to 1619.


Fig. 7(A). Relative vorticity calculated along the line $\widehat{A G}$ and the line $\widehat{\text { gh }}$ by using the high clouds' movernents for a $41-\mathrm{min}$ period from 1538 to 1519 CST.
Fig. $7(B)$. The deformation field in the outflow layer evaiuated along the line $\widehat{K P}_{\text {by }}$ using the high clouds' movements for a $41-\mathrm{min}$ period irom 1538 to 1619 CST.


Fig. 8. Left: Distributions of clouds (indicated with thin stipples) and radar echoes (indicated with dense stipples) at 1700 CST, April 23, 1968. Right: The surfacw weather map at 1800 CST. April 23, 1968. Isobars, isotherms and isolines of mixing ratio are indicated by thin solid, solid and dashed lines, respectively. The isolines of the hourly precipitation amount (mm hour ${ }^{-1}$ ) ending at 1800 CST are shown by the heavy solid lines.


Fig. 9. $250,300,500$ and 700 mb charts at 1800 CST, April 23, 1968. Height contours, isotherms and isolines of mixing are indicated by thin solid, solid and dashed line, respectiveiy.


Fig. 10. Wind hodographs at BUF and DIA for 1800 CST, April 23, BI F and DIA were situated to the north and to the south of the storm area at the time. Note that the maximum difluence is at the $300-\mathrm{mb}$ surface and the maximum confluence is at the $700-\mathrm{mb}$ surface.


Fig. 11. Vertical distribution of the horizontal divergence and the vertical velocity calculated by using the upper wind data at PIT, HTS, GSO, and DIA for 1800 CST', April $_{23}$.


Fig. 12. The mean sounding in the Warm sore obtained by averaging ine radiosonde observations at PIT, GSO, and HTS for 1800 CST.


Fig. 13. Vertical distributions of the temperature anomaly in the warm core and $\nabla^{2} \phi$ calculated in the area of thunderstorm complex.


Fig. 14. Hodographs of the geostrophic wind in the north and the south side of the storm area,

HEAT ENERGY BUDGET
unit $=\mid y \cdot$ hour $^{-1}$


Fig. 15. Total heat energy budget over the mature thunderstorm complex.


Fig. 16. The net increment of the toal heat energy for the mean motion in each layer over the mature thundezstorm complex $\frac{1}{g} \int\left[c_{p}\left(\frac{\overline{\delta_{1}}}{\delta t}\right) \bar{T}+L\left(\frac{\frac{\delta}{\delta t}}{\frac{\delta}{q}} \bar{q}\right] d p\right.$

$$
=\frac{1}{g} \int\left\{c_{p}\left[\frac{\overline{\partial T}}{\partial t}+\overline{\nabla \cdot V T}+\frac{\partial}{\partial p} \bar{\omega} \bar{T}-\frac{R}{c} \frac{\bar{\omega} \bar{T}}{p}\right]+L\left[\frac{\overline{\partial q}}{\partial t}+\overline{\nabla \cdot V q}+\frac{\partial}{\partial p} \cdot \bar{\omega} \bar{q}\right]\right\} d p
$$

and the convective transfer of total he it energy $\frac{1}{g} \overline{\omega^{\prime}}\left(C_{p} T^{\prime}+L q^{\prime}\right)$


Fig. 17. The grid system used for solving eq. (34) and the vertical distri-
bution of diabatic heating $\left(\frac{8 Q}{\delta t}\right)$ in the storm area.

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    ${ }^{2}$ On leave from the Meteorological Research Institute, Tokyo, Japan.

[^1]:    ${ }^{6}$ For the small value of the mixing ratio, the equivalent potential temperature
    $\theta_{\mathrm{e}}=\pi T \exp \left(\frac{\frac{L}{}}{C_{p} T}\right)$
    is written as $\theta_{e}=\pi T\left(1+\frac{1}{C_{p} T}\right)$

