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Calculation of Flow of a Radiating Gas in a Shock Layer
O. M. Belotserkovskii and V. N. Fomin
(Translated for Jet Propulsion by A. D. McRonald of Section 373)

CALCULATION OF FLOW OF A RADIATING GAS IN A SHOCK LAYER

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Zhurnal Vychislitel'noi Matematiki e Matematicheskoi Fiziki
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CALCULATION OF FLOW OF A RADIATING GAS IN A SHOCK LAYER

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1. INTRODUCTION

In recent years, arising from the growing increase in the problem of flight of space vehicles in the atmosphere of the Earth and other planets, there has been intensive research in the field of physical gasdynamics. This is due to the fact that flight in dense layers of the atmosphere is accompanied by a number of physical and chemical processes occurring in the shock layer formed at the vehicle surface. Such processes include excitation of internal degrees of freedom, excitation of electronic levels, dissociation, ionization, radiation, etc.; in addition, the state of the gas can depart from thermodynamic equilibrium. The mechanisms listed have a strong influence on the characteristics and properties of a high-temperature gas stream, and it is, therefore, natural for engineers to be interested in a detailed study of these processes.

One of the important directions in which work has been done in contemporary physical gasdynamics is the investigation of hypersonic flow of radiating and absorbing gaseous media. Radiative transport in gaseous media has previously been of interest mainly to astrophysicists and meteorologists. In that context, methods were developed for calculating radiative transport mainly in plane gaseous layers, approximating the atmospheres of stars and planets. Applying this to the problem of flight in dense layers of the atmosphere, examination of radiative transport problems has raised a number of new questions of a different nature.

One such problem which has received attention from physicists for a number of years is the calculation of the absorption coefficients of air over a wide range of temperatures and pressures. This question has been discussed in a large number of experimental and theoretical papers in the USSR and abroad. Tables of absorption coefficients /1/ are presently available which allow gasdynamic calculations including radiation to be performed.

Another important problem of interest mainly to gasdynamicists is that of the influence of radiation on the flow over blunt bodies, as well as a simultaneous consideration of convective and radiative heating of bodies, allowing for coupling. The first aspect of this problem can be examined within the framework of solution of the inviscid problem of flow over a body, while the second part of the problem requires consideration of a system of equations of Navier-Stokes type, including radiation. In addition, in the study of hypersonic flight at high altitude (at Reynolds number of $Re \sim 10 - 10^2$), a physically valid solution of the problem of calculating the flow, including radiation, can be obtained only by including viscosity.

At the present time, there is a large number of Russian and foreign papers devoted to an investigation of the flow of a gas, allowing for radiation, in laminar boundary layers, in the region of the stagnation point of a blunt body. We mention only a few, i.e., /2,4/. In these papers, the radiation was computed by approximating the radiating layer of gas by a plane layer.

In the study of flows having axial symmetry (when considering the gas flow not only in the stagnation point region, but also considering the effect of blunting in the whole field), as well as multidimensional flows, an approximation by plane layers cannot be made, and the problem becomes extremely complex owing to the need to integrate the radiative transport equation in the gas stream with complex distribution of temperatures and pressures. Until recently, solution of problems of this kind was very laborious, but in recent years, due to the widespread development of numerical methods for the calculation of flow over bodies by means of computers, a number of approaches to these investigations has emerged.

This paper describes some of the results of applying numerical methods (the method of integral relations and the method of characteristics) to computation of supersonic flow over blunt bodies, allowing for radiation. Many calculations have been performed for various assumptions regarding the nature of the radiation (volume luminosity, selective luminosity, selective emission and absorption); the results of the computations are given. In conducting the computations, the selective nature of the radiation in the shock layer was taken into account, the transport of radiation being considered in a continuous spectrum, and in a set of weak spectral lines, taken into account by integration. The contribution of strong spectral lines to the energy balance in this paper was neglected, due to the absence of sufficiently reliable data in this region at the time the computations were made. All the calculations shown here were made under the assumption that local thermodynamic equilibrium of the gas prevailed.

2. SYSTEM OF EQUATIONS, BOUNDARY CONDITIONS AND VARIOUS NUMERICAL SCHEMES USED IN THE CALCULATIONS

1. The system of equations of gasdynamics, taking radiation into account, is well known. For inviscid non-conducting, equilibrium dissociated and ionized gas streams, this system of equations takes the form:

$$\frac{d\bar{w}}{dt} = -\frac{1}{\rho}\,\mathrm{grad}\,p, \qquad (1)$$

$$\mathrm{div}\,(\rho\bar{w}) = 0, \qquad (2)$$

$$\rho\frac{dh(p,T)}{dt} + \rho\bar{w}\frac{d\bar{w}}{dt} + \int_0^\infty k'_\nu\left(4\pi B_\nu - \int_\Omega I_\nu\, d\Omega\right)d\nu = 0, \qquad (3)$$

$$\frac{dI_\nu}{dr} = k_\nu'(B_\nu - I_\nu), \quad B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/kT)-1}, \qquad (4)$$

$$p = \frac{\rho RT}{\mu(p,T)}. \qquad (5)$$

Here $\bar{w}$ is the vector velocity; p is the pressure; ρ is the density; T is the temperature; h(p,T) is the specific enthalpy; μ(p,T) is the molecular weight; R is the universal gas constant: I_ν is the radiative intensity; $k_\nu' = k_\nu(1-\exp\{-h\nu/kT\})$ is the absorption coefficient, including induced emission; c is the speed of light; k is the Boltzmann constant: h is Planck's constant; r is the direction of radiative transfer; Ω is the element of solid angle; ν is the frequency.

The main difficulty in considering gasdynamic problems, taking radiation into account, is that it is necessary to deal with a very complicated integro-differential system of equations, Eq. (1) -- (5).

In calculating flow of a radiating gas, the optically thin layer approximation has been widely used. This approximation corresponds to the condition $k_\nu' l \ll 1$ (here l is the characteristic dimension of the flow). This approach is not always physically valid, especially for real gas media, but it leads to considerable simplification of the original system of equations, since in this case, the gas can be considered to be radiating (volume luminosity), but not absorbing energy, which leads to the disappearance of the integral term in Eq. (3), and the system (1) -- (5) becomes purely differential. Using such simple models, the possibilities of different numerical schemes in the method of integral relations can be investigated.

In general, the assumption of an optically thin gas layer is not satisfied, and then we must consider the emission and absorption of energy in the gas, allowing for selectivity. The solution of this problem in a rigorous formulation is very difficult. However, we can use an approximate method for considering selective radiation and absorption of energy, without imposing limitations on the optical thickness of the layer.

In neutron transport theory /5/, the method of spherical harmonics is widely used to obtain an approximate system of differential equations, equivalent to the transport equation. Following this same approach, we represent the radiative intensity in the form of a series in spherical harmonics:

$$I_\nu(\Omega, r) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} A_{\nu l}^{m}(r) Y_{\nu l}^{m}(\Omega). \qquad \text{--------6}$$

Now we substitute Eq. (6) into Eq. (4), and multiply the resulting equation by $Y_{\nu l}^{m}(\Omega)$, and thereafter integrate over the solid angle. The equations resulting from this procedure are an infinite system of differential equations with an infinite number of unknown functions $A_{\nu l}^{m}$, completely equivalent to the exact transport equation. Since we know, from the theory of neutron transport, that the first approximation in the method of spherical harmonics (which is called the P_1-M approximation) is sufficiently accurate in the majority of problems, we restrict ourselves here to only this approximation (in what follows, we call this the differential approximation).

Thus, the radiative transfer equation, Eq. (4) is replaced by the approximate system of equations:

$$\begin{aligned} \operatorname{div} \Pi_\nu &= -k_\nu'(I_{\nu 0} - 4\pi B_\nu), \\ \operatorname{grad} I_{\nu 0} &= -3k_\nu' \Pi_\nu, \end{aligned} \qquad (7)$$

where $I_{\nu_0} = \int_\Omega I_\nu d\Omega,\ \overline{\Pi}_\nu = \int_\Omega I_\nu \cos\vartheta\, d\Omega$ is the radiative energy flux.

We now turn to the question that Eq. (7) assumes the validity of the differential approximation for any frequency ν, i.e., the first equation in Eq. (7) is exact, and the second equation is approximate. We note that Eq. (7) reduces to the equations for an optically thin gas layer (or optically thick), if $1/k_\nu'$ is larger (or considerably smaller, respectively) than the characteristic flow dimension 2.

Since the P_1 approximation restricts consideration to the first terms of the series expansion of the intensity I_ν in spherical harmonics, it is clear that:

$$I_\nu(r, \Omega) = \frac{1}{4\pi}[I_{\nu_0}(r) + 3\vec{I}_\nu\Omega]. \qquad (8)$$

The boundary conditions for the original system of equations includes the conventional gasdynamic relations, given on the shock wave and on the body, and boundary conditions for the radiation, which are determined by assigning the radiative intensity at the front of the shock wave and on the body surface.

We note that the differential approximation has been used successfully to calculate the flow of radiating hydrogen in axisymmetric nozzles /6/.

2. We use the method of integral relations to calculate the flow of a radiating gas in the region influenced by the bluntness.

We shall consider Scheme I of the method of integral relations /7/. The system of equations describing the flow of the gas behind a shock wave, allowing for radiation, in dimensionless form, can be written in s, n (fig. 1) coordinates as follows:

$$\frac{\partial}{\partial s}\bar{Z} + \frac{\partial}{\partial n}A\bar{H} = \bar{Y}, \qquad \frac{\partial}{\partial s}\bar{G} + \frac{\partial}{\partial n}A\bar{Z} = \bar{X},$$
$$\frac{\partial}{\partial s}\bar{g} + \frac{\partial}{\partial n}A\bar{l} = 0, \qquad \frac{d}{dt}\left(h + \frac{w^2}{2}\right) + \frac{2E}{\rho} = 0, \qquad (9)$$
$$\rho = \rho(p, T), \qquad h = h(p, T).$$

The notation and dimensions are conventional for Scheme I/7/. In the case of volume emission, the quantity E is the amount of radiant energy passing through unit volume of the gas in unit time, divided by $\rho_\infty w_{max}^3 / R_0$ (where R_0 is the radius of the body).

Using the well-developed apparatus of Scheme I, we construct an app:oxi-
ate system of equations for numerical integration.

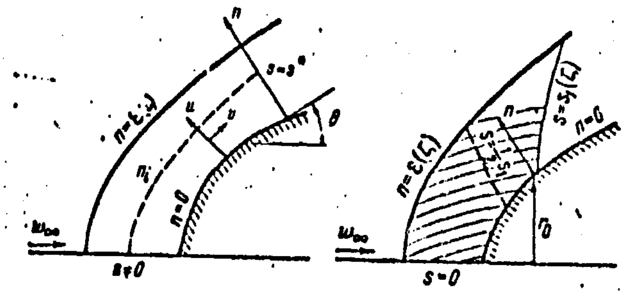

Fig. 6

The differences from Scheme I, which does not take account of the effect
radiation, reduce to the following: Since the flow of the gas with radia-
ion behind a shock wave is non-isentropic and non-adiabatic, then, as was
rue in calculating non-equilibrium flows in Scheme I, the condition that
ntropy is conserved along the stream lines in not used. Since the line of
ymmetry s = 0 (Fig. 1) and the body surface are stream lines, the energy
quation is used along these in the form of Eq. (9). In the second (and
igher) approximations, where the intermediate lines n_i are not stream lines,
he energy equation is written approximately along these lines n_i in terms
the known values of the corresponding functions on the stream line /7/.

The boundary conditions are the conventional gasdynamic conditions for
cheme I, which, in the case of volume emission, do not contain additional
onditions for the radiation, since the gas transmits energy without absorbing
t. Because of their difficulty, the approximation systems were not written
ere, but some results of the calculations are given in Part 3 for the case
volume emission, as obtained according to this scheme.

Calculations of the flow in the bluntness influence region were performed
lso, using Scheme II of the method of integral relations, and different variants
this scheme were investigated. References /7--8/ have made a detailed

examination of the question of applying Scheme II to calculation of the flow of a radiating gas. By constructing an approximating system, a representation has been formulated for the equations of motion and continuity along the shock layer by means of ordinary differential equations, and an approximate form of the energy equation along the boundaries of the strips has been written in terms of the known form of this equation along the stream lines.

Afterwards, another variant was developed for constructing an approximate system within the framework of Scheme II. In contrast with that examined above, the form of the energy equation here is written in the divergent form:

$$\frac{\partial}{\partial s}(\bar{u}\varepsilon\bar{\mathcal{H}}) + \frac{\partial}{\partial \zeta}(\bar{N}\bar{\mathcal{H}}) + 2ErA\varepsilon = 0, \qquad (10)$$

where $\bar{\mathcal{H}} = h + w^2$, and s and ζ are the coordinate system in which a solution is usually developed according the Scheme II (see Fig. 1); the remaining symbols are conventional for Scheme II /7/.

In constructing the approximating system, the energy equation, Eq. (10) was integrated, in addition to the equations of continuity and motion. The integrated functions here are approximated by polynomials in the coordinate s along the shock layer, taking account of the symmetry about s = 0. By integrating Eq. (10) we obtain the two missing relations necessary to close the approximation system of equations. Because of its cumbersome nature we shall not show the approximation system here.

The results of calculations according to those variants of Scheme II and a comparison of them are presented in Section 3; results of calculations according to Scheme I also presented there. By comparing the results of calculations employing the different methods of approximate representation of the original equations (and particularly of the energy equation), we can assess the possibilities of the various numerical schemes and the reliabiltiy of the results obtained. When the results of calculations using the various approaches converge, we can be assured as to the adequate accuracy of the numerical schemes and the reliability of the results. This is particularly important when we are considering problems of a complex physical nature (in particular, with radiation), where a comparison with experimental data cannot be made in order to assess the accuracy of the calculations, because such data are totally absent at present.

The above variant of Scheme II was also used to calculate flow over blunt bodies, allowing for radiation, in carbon dioxide and in gas mixtures of the type $CO_2 + N_2$. Some of the results of these calculations are presented in Section 3.

3. For a more rigorous calculation of radiative transfer in a shock layer, a numerical scheme has been developed in which no restrictions are imposed on the optical thickness of the gas, and which permits calculation not only of radiative emission, but also of absorption of energy by the gas. This is accomplished by means of Scheme II of the method of integral relations and the above differential approximation.

A short account was given in /7/ of a scheme for calculating supersonic flow over a blunt body, allowing for selective emission and absorption of energy. A more detailed description of this scheme will be given below.

The equations of motion, continuity and state are employed in the ordinary form for Scheme II. In dimensionless form, the energy equation is:

$$dh(p, T) - \frac{2}{\rho} dp + \frac{2}{\rho} E\, dt = 0, \qquad (11)$$

where

$$E = \sum_{k=1}^{n} [k_\nu' (4\pi B_\nu - I_{\nu 0})]_k \Delta\nu_k$$

is the difference between the amounts of energy emitted and absorbed by unit volume of gas in unit time.

We note that Eq. (11) differs from Eq. (3) in that the frequency range examined is subdivided into n intervals, the coefficient within each interval being assumed constant (but varying from interval to interval). This permits the operation of integration to be replaced by summation in the energy equation.

Equation (7) in the s and ζ coordinate system can be written in dimensionless form as follows:

$$\begin{gathered}\frac{\partial}{\partial s}(r\varepsilon H_{\nu s})+\frac{\partial}{\partial \zeta}(rAH_{\nu\zeta}-r\zeta\varepsilon_s' H_{\nu s})+Ar\varepsilon k_\nu'(I_{\nu 0}-4\pi B_\nu)=0,\\ \frac{\partial I_{\nu 0}}{\partial \zeta}=-3k_\nu'\varepsilon H_{\nu\zeta},\\ \frac{\partial I_{\nu 0}}{\partial s}=-3k_\nu'(AH_{\nu s}+\zeta\varepsilon_s' H_{\nu\zeta}),\end{gathered} \qquad (12)$$

where $H_{\nu s}$ and $H_{\nu\zeta}$ are components of the radiative flux vector along the s and ζ axes, respectively. Equation (12) is solved n times, i.e., the number of intervals into which the frequency range is subdivided.

In order to construct a closed solution for the system of equations comprising the equations of continuity, motion, state, energy (11) and the relations of Eq. (12), for example, in the second approximation, a variant of Scheme II was used, based on an approximate representation of the energy equation along the strip boundaries.

The first and third equations of Eq. (12) are integrated from s = 0 to $s = s_1$, and from s = 0 to $s = 0.5s_1$, the integrand functions being polynomials of fourth degree in s for even functions, and of third degree for odd functions. This gave four ordinary differential equations, approximating the original system, and four other differential equations were obtained by representing the total derivatives of I_{ν_0} with respect to ζ at the strip boundaries, and the total derivative of H_{ν_s} at s = 0 in terms of the known form of the partial derivatives of these functions with respect to s and ζ. These equations are enough to determine the 8n unknowns, connected with radiative transfer, $H^{(k)}_{\nu s_i}$, $H^{(k)}_{\nu\zeta_i}$,

$$I^{(k)}_{\nu 0_i} \quad (H^{(k)}_{\nu s_0}=0); \quad i=0,1,2$$

is the index of the strip boundary, $k = 1, 2, \ldots, n$ is the index of the frequency interval. In addition to the system of 12 equations for determining the gasdynamic parameters at the strip boundaries, /8/, we obtain a closed approximation system of 12 + 8n equations.

We shall write down the 8n equations, dropping the subscript k to simplify the notation:

$$
\begin{aligned}
I'_{\nu 0_0} &= -3k_{\nu_0}'\varepsilon_0 H_{\nu\zeta_0},\\
I'_{\nu 0_1} &= -3k_{\nu_1}'H_{\nu\zeta_1} - 3k_{\nu_1}'(A_1 H_{\nu s_1} - \zeta_1\varepsilon_{s_1}'H_{\nu\zeta_1})s_1',\\
I'_{\nu 0_2} &= -3k_{\nu_2}'H_{\nu\zeta_2} - 3k_{\nu_2}'(A_2 H_{\nu s_2} - \zeta_2\varepsilon_{s_2}'H_{\nu\zeta_2})s_2',\\
H'_{\nu\zeta_0} &= -\varepsilon_0 k_{\nu_0}'(I_{\nu 0_0} - 4\pi B_{\nu_0}),
\end{aligned}
$$

$$
\begin{aligned}
&-N_0'D_1 + N_1'D_2 + N_2'D_3 + 2\frac{s_1'}{s_1}[-N_0D_4 + N_1(D_5 - 3D_{21}) + N_2D_6] +\\
&\qquad + 6\tilde{t}\frac{\varepsilon_1}{s_1^2} - \bar{A}_0D_1 + \bar{A}_2D_2 + \bar{A}_3D_3 = 0,\\
&N_0'D_7 - N_1'D_8 + N_2'D_9 + \frac{s_1'}{s_1}[N_0D_{10} - N_1D_{11} + N_2(D_{12} - 12D_{22})] +\\
&\qquad + 24\tilde{t}_2\frac{\varepsilon_2}{s_1^2} + \bar{A}_0D_7 - \bar{A}_1D_8 + \bar{A}_2D_9 = 0,\\
&{}^2/_3 I'_{\nu 0_1} + {}^{16}/_3 I'_{\nu 0_2} - 6I'_{\nu 0_0} + 6s_1\mathscr{E}_2' + 6s_1'\mathscr{E}_2 = 0,\\
&{}^7/_3 I'_{\nu 0_1} - {}^{10}/_3 I'_{\nu 0_2} + 3I'_{\nu 0_0} + {}^3/_2 s_1\mathscr{E}_1' + {}^3/_2 s_1'\mathscr{E}_1 = 0.
\end{aligned}
\tag{13}
$$

Here,

$$
\begin{aligned}
&N_i = (A_i H_{\nu\zeta_i} - \zeta_i\varepsilon_{s_i}'H_{\nu s_i})P_i, \quad \mathscr{E}_i = k_{\nu_i}'(A_i H_{\nu s_i} + \zeta_i\varepsilon_{s_i}'H_{\nu\zeta_i}),\\
&\bar{A}_0 = m_{20}\varepsilon_0 k_{\nu_0}'(I_{\nu 0_0} - 4\pi B_{\nu_0}), \quad \bar{A}_1 = m_{21}\varepsilon_1 k_{\nu_1}'(I_{\nu 0_1} - 4\pi B_{\nu_1}),\\
&A_2 = m_{22}\varepsilon_2 k'_{\nu_2}(I_{\nu 0_2} - 4\pi B_{\nu_2}), \quad \tilde{t}_i = r_i H_{\nu s_i}.
\end{aligned}
$$

The remaining symbols are the same as in /8/. By solving Eq. (13) with respect to derivatives of the functions $H_{\nu s_i}$, $H_{\nu\zeta_i}$, $I_{\nu 0_i}$, we obtain a system of equations used for numerical integration, along with the 12 equations from /8/, and connected with them via the energy equation.

We now discuss the boundary conditions of this problem. The gasdynamic boundary conditions were formulated in /8/, and remain unchanged. The boundary conditions for the radiation are assigned approximately under the assumption that no radiation enters the shock layer from the flow upstream; the body surface absorbs all the energy supplied to it from the shock layer, and radiates an amount of energy that is negligibly small in comparison with the emission from the shock layer.

These assumptions allow us to obtain the approximate conditions:

At the shock wave $$I_{\nu 0_i} = 2H_{\nu\zeta_i}. \qquad (14)$$

On the body $$I_{\nu 0_i} + 2H_{\nu\zeta_i} = 0. \qquad (15)$$

In addition, the following approximate boundary conditions can be used at the shock wave:

$$\begin{aligned} H_{\nu s_1} &= -\frac{1}{k_{\nu_i}' A_1 s_1}\left(\frac{14}{9} I_{\nu 0_1} - \frac{32}{9} I_{\nu 0_2} + 2I_{\nu 0_3} + k_{\nu_i}' e_{s_1}' s_1 H_{\nu\zeta_1}\right), \\ H_{\nu s_2} &= -\frac{1}{k_{\nu_i}' A_2 s_2}\left(\frac{1}{18} I_{\nu 0_1} + \frac{8}{9} I_{\nu 0_2} - I_{\nu 0_3} + k_{\nu_i}' e'_{s_2} s_2 H_{\nu\zeta_2}\right). \end{aligned} \qquad (16)$$

These conditions were obtained from the third equation of Eq. (12) with $\zeta = 1$.

In conducting the calculations with selective emission and absorption of energy, we used data from /1/. Here the range of variation of $k_\nu'(p, T)$ with respect to frequency was divided into 20 intervals and a table of values of k'_ι as a function of p, T, and ι was constructed. The necessary values of k'_ι for the calculation which did not coincide with the entry points of the table were determined by interpolation of k'_ι with respect to p and T on a logarithmic scale.

A numerical integration of the system Eq. (13), together with Eq. (25) from /8/, was carried out in a similar way as was done in /8/. There is a difference in that, in starting the calculation from the shock wave, values of $I_{\iota 0i}$ (and $H_{\nu\zeta_i}$ and $H_{\nu s_i}$) were determined from these, using Eqs. (14) and (16)) must be assigned in such a way as to satisfy condition (15) at the body surface. The necessary values of $I\iota_{0i}$ were chosen during the calculation. Some of the results of the calculations are shown in Section 3.

4. On the basis of Schemes I and II of the method of integral relations, Scheme III /7/ has been developed, which has definite advantages over Schemes I and II.

It is very expedient to use Scheme III to calculate flows of radiating gas (clearly, this is particularily favorable in calculating viscous flow of a radiating gas). This work is currently being conducted by V. I. Lebedev and V. N. Fomin, on the basis of Ref. /9/.

5. The schemes considered above for calculating flow of the radiating gas in the region influenced by bluntness were constructed using the method of integral relations. The solution of a problem in the region affected by bluntness can be continued into the supersonic region. For this, the method of characteristics is used, adapted for calculating flow of a radiating gas. This makes it possible to calculate the flow over blunt cones, allowing for radiation.

In order to determine the flow parameters on the inclined surface of a cone, the initial data are taken to be the conditions at the limiting characteristic, obtained in the calculation of the flow over the blunt region. A scheme for calculating the flow on the conical surface is described in /7/. The calculations made assumed volume luminescence. We note that the finite difference relations of the numerical method of characteristics for computing flow with radiation differ from the numerical scheme for calculating supersonic flow of equilibrium dissociated and ionized gas streams, in that additional terms appear /7/. Some results of the calculations are shown in Section 3.

6. In calculating subsonic and supersonic flow of the radiating gas, to

determine the thermodynamic functions of air, we used the universal analytical representation due to V. V. Mikhailov /7/, which allowed us to make calculations, both in air and in gaseous mixtures of the type $CO_2 + N_2$. In performing the calculations for air, assuming volume luminescence, we used the data of Reference /10/, which can be represented in the form:

$$E = 10^5 \left(\frac{\rho}{\rho_0}\right)^{1.3} [10^{0.55\,T/1000} - 10^{1.1}] \quad \text{kg/m.sec}^3 \qquad (17)$$

for $2000° K \leqslant T \leqslant 12\,000° K,\ 10^{-4} \leqslant \rho/\rho_0 \leqslant 1,$

or more accurately in the form:

$$E = 10^{6-4600/T} \left[10^{0.5\,T/1000} - 2.6\left(\frac{\rho}{\rho_0}\right)\right]\left(\frac{\rho}{\rho_0}\right)^{1.3} \quad \text{kg/m.sec}^3 \qquad (18)$$

for $3000° K \leqslant T \leqslant 10\,000° K$ и and $E = 10^{6+0.6\,T/1000-(T/14750)^2}\left(\frac{\rho}{\rho^0}\right)^{1.2+1000/T}$ kg/m.sec^3

for $10\,000° K \leqslant T \leqslant 16\,000° K,\ 10^{-4} \leqslant \rho/\rho_0 \leqslant 1$, where ρ_0 is the air density under standard conditions.

In performing the calculations for gas mixtures of the type $CO_2 + N_2$, under the assumption of volume luminescence, we used data of Reference /11/, which can be approximated in the form:

$$E = 10^{0.201\,T/1000+7.12}\left(\frac{\rho}{\rho_0}\right)^{2.24-0.133\,T/1000} \quad \text{kg/m.sec}^3 \qquad (19)$$

for $6000° K \leqslant T \leqslant 12\,000° K,\ 10^{-3} \leqslant \rho/\rho_0 \leqslant 10^{-2},\ \rho_0 = 1.29$ kg/m^3

For the calculations with selective emission and absorption, we used data of Reference /1/. If we put $I\nu_0 = 0$ in Eq. (11), the result is a calculation corresponding to the case of volume luminescence with allowance for selective emission (selective luminescence).

3. RESULTS OF COMPUTATIONS AND CONCLUSIONS

Numerous calculations were performed according to the numercial schemes examined in Section 2. We now present some of the results and an analysis of them.

The majority of the results refer to an sample calculation of supersonic flow of a radiating and absorbing gas over a sphere of radius $R_o = 1$ m, the incident stream conditions being $M_\infty = 33$, $p_\infty = 0.0020$ aTm $T_\infty = 257°$ K.

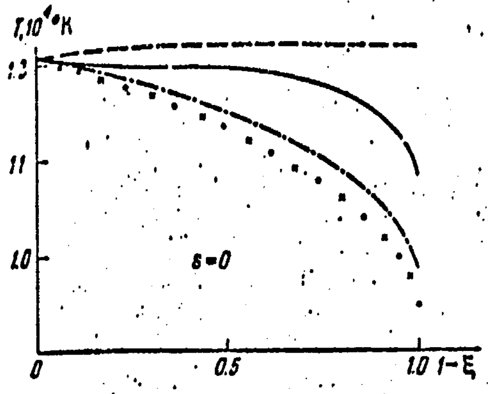

Fig. 2

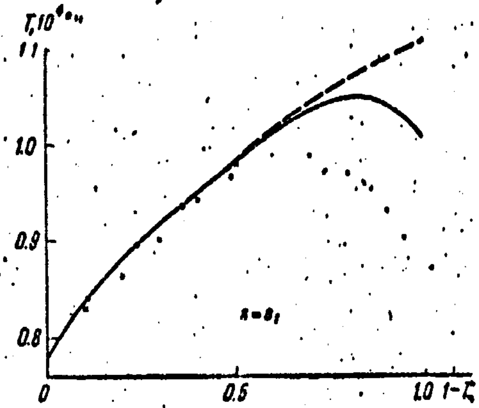

Fig. 3

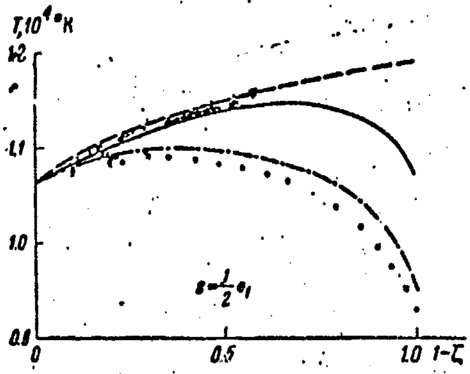

Fig. 4

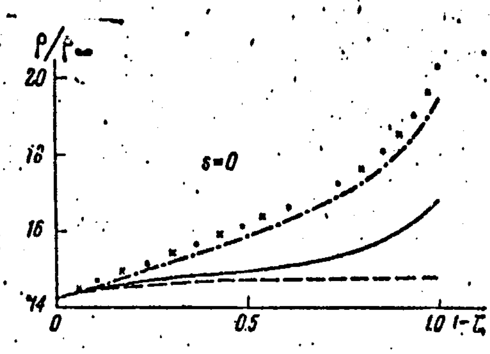

Fig. 5

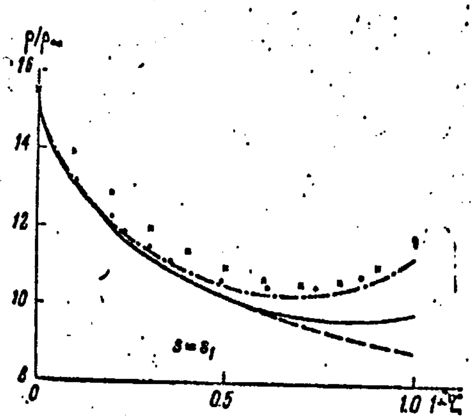

Fig. 6

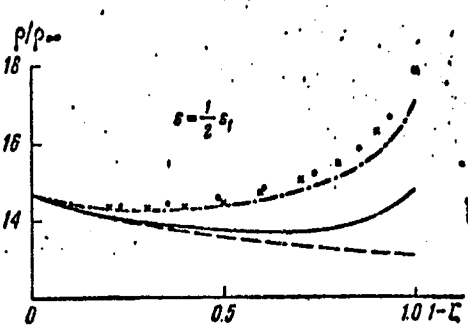

Fig. 7

Figure 2 shows the variation of temperature along the axis of symmetry $s = 0$ from the shock wave to the body, for calculations according to Scheme II in the second approximation. (It seemed inappropriate here to use high-order approximations in the numerical scheme, since the accuracy of the second approximation is considerably greater than the accuracy of the physical quantities entering into the original system.) The broken line shows the temperature profile obtained without energy loss in the shock layer due to radiation ($E = 0$); the continuous line is the temperature profile for the calculations with allowance for selective emission and absorption of energy; the results of calculations of selective luminescence using the data of Reference /1/ are shown by dot-dash lines; the points show the results of calculations of volume luminescence, calculating E according to Eq. (17) (the crosses indicate data obtained in calculating volume luminescence according to a variant of Scheme II, using integration of the energy equation.)

Figs. 3 and 4 show the data for the line s_1 (limiting characteristic) and $s = \frac{1}{2}s_1$. Figs. 5,6 and 7 show data on the variation of density in the shock layer on the same lines and for the same assumptions regarding the nature of the radiation.

We can see that, in all the cases examined, the temperature drops in the shock layer, and the density increases. The results of the calculations show that the pressure p remains practically unchanged, while the velocity $\overline{w}$ decreases somewhat near the body surface, due to radiation. Hence, we can deduce the important conclusion that the radiation can have an appreciable influence on the radiative heat flux reaching the body surface, but does not have an appreciable influence on the aerodynamic characteristics of the body.

A comparison of the results of the calculations made according to the different variants of Scheme II (points and crosses) permits us to conclude that the results of the calculations by the different methods give quite good accuracy, which is evidence of the reliability of the data obtained. The greatest differences are observed at the limiting characteristic, but it should be borne in mind here that the position of the limiting characteristic alters somewhat for the various forms of the approximations, so that the comparison should be made on different lines. The curves of variation of T and ρ on an intermediate line give a maximum difference between them of $\sim 0.3\%$ for T and $\sim 0.6\%$ for ρ. On the limiting characteristic, at a distance of $\sim 0.25E$ from the shock wave, the difference for T constitutes ~ 3--4%, and for ρ, $\sim 4\%$, and then this difference decreases, being no more than 1%, as we approach the body.

An examination of the calculated results allows us to draw an important conclusion about the goodness of convergence for the various methods of approximating to the energy equation. In /7/ the error in calculations according to Scheme II and the second approximation was determined to be $\sim 2\%$. It can be seen from the results here that the error of calculations in this paper does not exceed 1%, at least in the major part of the flow region examined. (This region is shaded in Fig. 1.) In the unshaded region, the error in determining some of the parameters can reach as high as $\sim 4\%$.

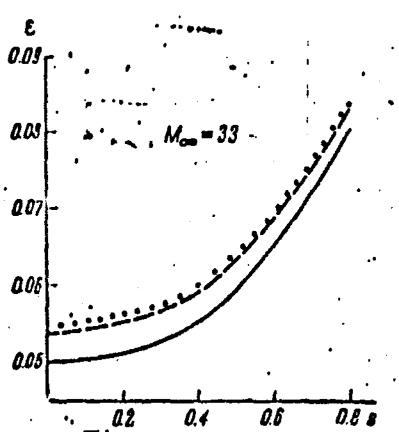

Fig. 8. — $E \neq 0$, $R_0 = 1$ m; – – – $E \neq 0$, $R_0 = 0.1$ m; ... $E = 0$

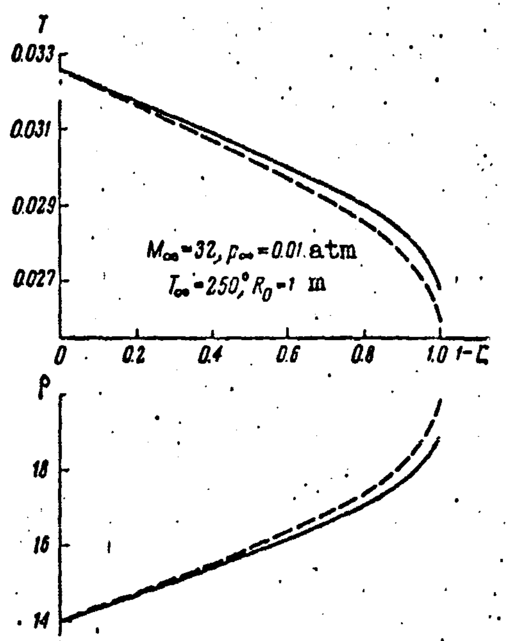

Fig. 9

Because of the decrease of temperature and the increase of density in the shock layer, the divergence of the shock wave is decreased (Fig. 8); this leads, in turn, to a reduced radiative heat flux. Fig. 9 shows a comparison of results of calculations of temperature and density according to Scheme II (second approximation, full line).

We now discuss the following circumstance. When examining Scheme I and Scheme II for calculating the flow of the radiating gas over a sphere, a preference for Scheme II was given. This arises from several causes. Calculations according to Scheme I involved some specific difficulties in passing through the singular points found at points in the flow where $v = a_s$. In addition, in Scheme I, the integration is carried out along the body, while, in the case of volume luminescence, the energy equation has a logarithmic singularity on the body at the stagnation point, and so we cannot obtain realistic values of temperature at the body surface within the framework of volume luminescence. For calculations with selective emission and absorption of energy, within the limitations of the differential approximation, we do not obtain singularities at the stagnation point, but the values of temperature at the body, even in this case, are evidently determined with a large error. For this reason, the schemes considered gi·e the true behavior of temperature in the major part of the shock layer, apart from a narrow region near the body surface. It is clear that an improvement in the temperature is this region is possible by solving the viscous problem. It is possible that it would be more favorable to use Scheme I when considering complex shapes of space vehicles.

Figures 10 and 11 (on following page) show the variation of temperature and density in the shock layer in a gas mixture of $9\%\ CO_2 + 91\%\ N_2$. We see that qualitatively, the picture of the variation of ρ and T is the same as for air $(M_\infty = 40,\ p_\infty = 0.005$ atm $T_\infty = 300° K,\ R_0 = 1$ m)

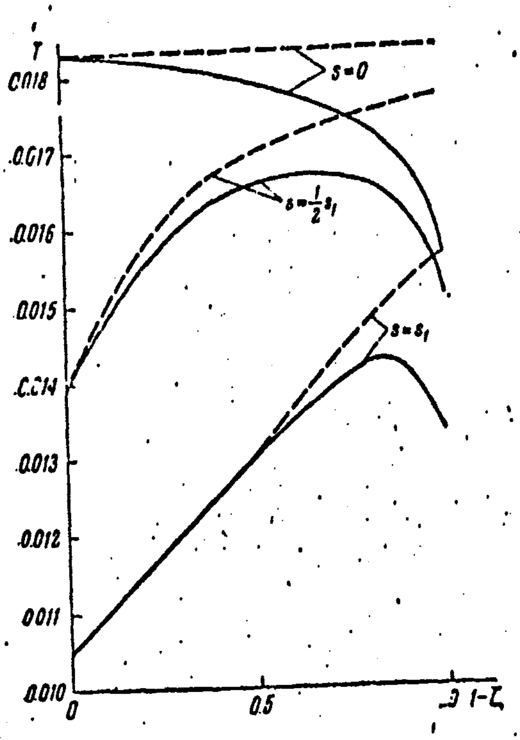

Fig. 10

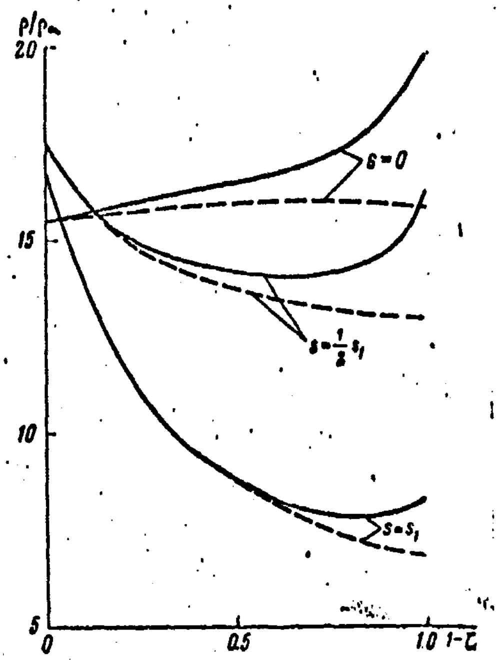

Fig. 11

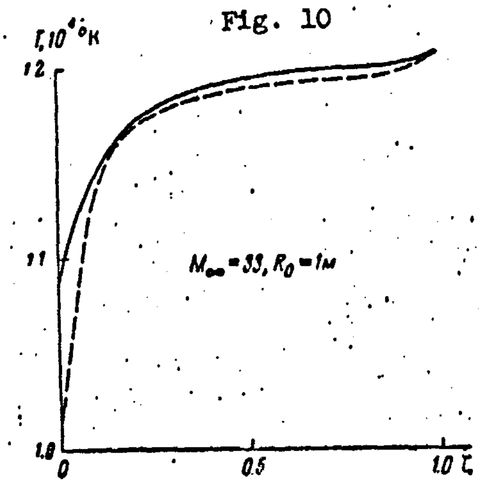

Fig. 12

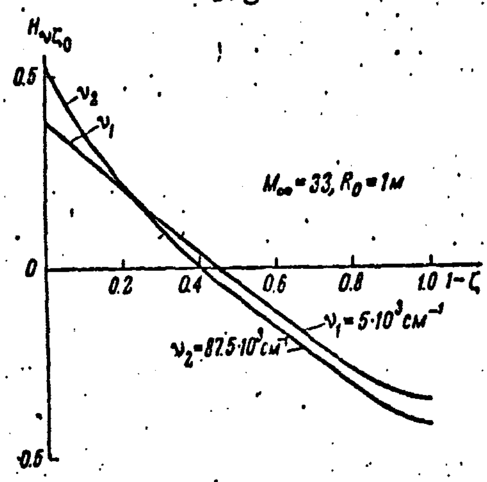

Fig. 13

Figure 12 shows a comparison of the results of calculations of the temperature profile along s = 0, according to the data of Reference /2/ (broken line), in which the shock layer is approximated by a plane layer, with the results of calculations using a differential approximation (full line). We see that the agreement between the results is quite good over the major part of the shock layer, apart from a narrow region near the body surface, where the accuracy of the differential approximation possibly decreases. Figure 13 shows, by way of example, the variation of $H_{\nu\zeta_0}$ along s = 0 for various values of ι, and Fig. 14 gives the radiant heat flux $H_{\nu\zeta_0}$ on the body surface at the stagnation point (ζ = 0, s = 0), and the nature of the variation of the flux with frequency (full line). The broken lines indicate the value of $H_{\nu\zeta_0}$ for no losses due to radiation in the shock layer.

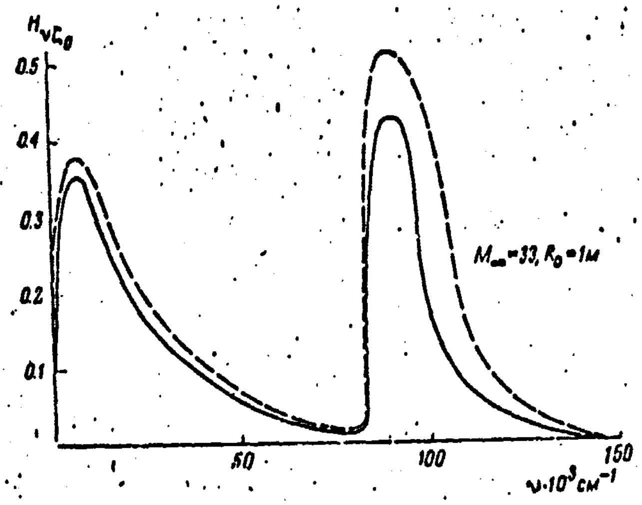

Fig. 14

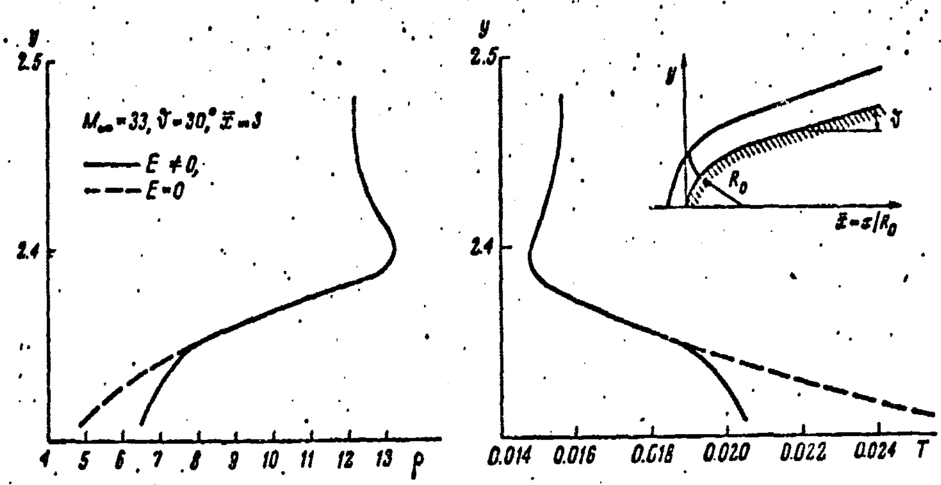

Fig. 15

Figure 15 shows the variation of the parameters ρ and T in the shock layer on the inclined surface of the cone. We see that calculation of the radiation leads to a fall in temperature and an increase in density in a narrow region located near the cone surface, while the gas parameters remain unchanged in the major part of the shock layer.

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