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# BOUNDARY LAYER SEPARATION ON A LIQUID SPHERE

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16. Abstract

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## FOREWORD

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## SUMMARY

An experimental investigation was made to determine what happens to the inside boundary layer of a liquid sphere in a steady, uniform stream at high Reynolds number when the outside boundary layer separates. The results indicate that the inside boundary layer remains attached despite the outside separation. A physical model for the liquid sphere flow is suggested and the difficulties of a mathematical solution are discussed.

## INTRODUCTION

In Ref. 1 the authors extrapolated to high Reynolds numbers a small Reynolds number solution for the flow around a liquid sphere. This extrapolation suggests that the outside boundary layer separates except when the inside viscosity is much smaller than the outside viscosity. In Ref. 1 various aspects of the small Reynolds number extrapolation were supported by an early time solution to a problem in which the liquid sphere is impulsively accelerated from rest.

In Ref. 2 the early time approach was carried further in an effort to obtain insight into the nature of the inside boundary layer. In this latter reference, three terms of a power series in time are given for the following problem: an inviscid spherical vortex with an arbitrary vorticity is moving steadily in another inviscid fluid. Impulsively, at time $t = 0$, viscosity is switched on inside and outside of the sphere. When the viscosity is switched on, the flow begins adjusting at the sphere surface. The steady state flow in the core of the sphere will be a spherical vortex of unknown strength. Presumably, the flow will continue to adjust until, at large t, the steady state core strength has been attained. If at $t < 0$ the core already has its steady state strength, adjustment at $t > 0$ will only need to take place over the distance of the steady state boundary layer thickness, and, in the short times for which we can use the small time expansion, the flow might begin to resemble the steady state flow.

Unfortunately, the small time solution does not predict the steady state core strength. However, it turns out that the qualitative features of the velocity profiles are independent of the assumed core strength, and speculation about the inside steady state flow can be made based on the assumption that the profiles for one of the strengths represented is, in a broad sense, like the steady state profiles. The velocity profiles given by this small time solution are discussed below in conjunction with the results of an experiment in which velocities were obtained inside and outside of oblate, liquid drops falling in silicone oil.

## THEORY

### Separation

Separation of the boundary layer on a liquid sphere has been variously defined by writers on the subject. D. W. Moore (Ref. 3) and Harper and Moore (Ref. 4) speak of separation as what happens to the boundary layer when, because of continuity, it must turn from the sphere surface near the rear stagnation point. This does not imply any reverse flow in the boundary layer, nor even any large deviation of the potential flow from the inviscid flow around a sphere. Some authors have taken the criterion for separation on a stationary, solid wall and applied it to the velocity profile above the liquid surface to determine separation there. Winnikow and Chao (Ref. 5), finding that Chao's theoretical solution (Ref. 6) did not predict vanishing tangential shear anywhere on the surface, used the condition of vanishing velocity on the surface instead. F. K. Moore (Ref. 7) discussed the criterion for separation of the boundary layer on a surface moving downstream and concluded it is the simultaneous vanishing of the tangential shear and the tangential velocity at a point above the surface. This defines the location at which reverse flow first begins in the boundary layer. Brady and Ludwig (Ref. 8) made velocity measurements above a cylinder surface moving downstream and demonstrated that this is the correct, or nearly correct, criterion for separation in that case. The same criterion is used in this report to define the location of separation on the liquid sphere, although an experiment described below makes it clear that separation takes place on oblate drops some distance upstream of this condition. In the case of the sphere, though, the condition of simultaneous vanishing of shear and velocity probably exists fairly near, although downstream of, the separation point. We can see in the pictures by Taneda (Ref. 9) (see also Ref. 10, plate 3) that this is true for a solid sphere. The above criterion does correctly locate the separation ring on the liquid sphere at early times after the sphere begins to move.

Separation of the outside boundary layer has been observed in several liquid-liquid systems, although, in many cases, the cause of separation might indirectly have been the presence of surface active contaminants which slowed the internal circulation of the drop. Winnikow and Chao (Ref. 5) photographed the wakes behind several pure drops. They state that the boundary layers on all of the drops photographed separated and that the angles of separation fell between 120° and 160° from the front stagnation point. Although it is apparent that the flow has separated behind some of the drops shown, it is not clear that all the drops pictured in their paper meet their (or our) criterion for separation. Some of the drops pictured may not have a region of reverse flow behind them. Those systems for which the outside boundary layer does not separate can possibly be analyzed using the linearization technique of Harper and Moore in Ref. 4. Our report is concerned with those systems for which the outside flow separates at high Reynolds number.

## Impulsive viscosity solution

We define the separation profile as being a plot of tangential velocity against radial distance from the sphere surface at that time at which reverse flow is just beginning in the outside boundary layer at the angle being looked at. The separation profile given by the small time solution in Ref. 2 to the impulsive viscosity problem is used as a basis for speculations about the steady state flow inside liquid spheres.

At a time which depends on fluid properties and on the assumed core strength, the separation profile forms at the rear stagnation point and moves up the back of the sphere. After the appearance of this profile, the small time solution is no longer valid downstream of it because the outside boundary layer has separated from the sphere. The separation profiles for several angles and one core strength are shown in figure 1. The outside potential tangential velocity is $U = \frac{3}{2} U_\infty \sin\theta$, and the inside inviscid tangential velocity is $kU_\infty \sin\theta$. $\theta$ is the angle from the front stagnation point, and k is called the core strength. The shape of the profile is essentially invariant in time as it moves up the back of the sphere. Also, the general characteristics of the profile are not changed by varying the fluid properties (see figure 2), or by changing the initial core strength. Finally, the profile has the general shape of the one given by the small Reynolds number theory in Ref. 1 (see figure 3) and the outside portion of it looks like the experimentally obtained steady state separation profile above the wall moving downstream in an experiment by Brady and Ludwig (Ref. 8) (see figure 4) in which they measured the velocities in the boundary layer on a rotating cylinder in a uniform stream. For these reasons it seems reasonable to guess that here, as in the case of the solid sphere, the steady state separation profile is similar to the small time separation profile. The following discussion is based on the assumption that the profile in figure 2 "looks like" the steady state separation profile for high Reynolds number flow past a liquid sphere. This assumption is, simply,

a. that there is a downstream surface velocity at separation, and

b. that the inside profile has the general shape shown in figure 2 (there are no additional points of inflexion).

A solid sphere in a uniform stream at such large Reynolds number that the wake is turbulent has no pressure gradient on the surface downstream of the point of separation (see Schlicting (Ref. 11), p. 20). The pressure is constant past separation on the cylinder of Brady and Ludwig (Reynolds number = $6.7 \times 10^4$). It can be supposed that if a liquid sphere could be held in such a high velocity flow without moving unsteadily, it too would have no pressure gradient on its downstream surface.

If we assume that the steady state separation profile "looks like" the small time one and that the pressure gradient on the outside surface downstream of the point of separation is negligible, then we can say that the inside boundary layer separates, if at all, later than the outside boundary layer. This means that vorticity generation continues on the inside surface downstream of the point of separation. It is important to see what sign this generated vorticity has: does it tend to promote or to hinder separation of the inside boundary layer? Toward this end we can say that, because the point of separation is above the sphere surface, a portion of the outside boundary layer remains unseparated downstream to the point of separation. All the fluid which is between the point of separation and the surface continues flowing next to the surface. This can be inferred from the lack of any mechanism to push this fluid away from the surface. It has also been demonstrated experimentally in the case of the rotating cylinder mentioned previously (see Ref. 8, page 37). The dynamics of this partial boundary layer on the outside are controlled by generation of vorticity at the sphere surface and diffusion of vorticity through the layer. Thickening of the layer is caused by diffusion and by meeting the constraint of continuity. Because of the nearly stagnant region above it, there is not an effective shear force on the outside of this layer, and we have already assumed that there is not a significant pressure gradient acting on it. The only force on this layer is the shear force at the sphere surface, and, because the inside flow is being slowed by an adverse pressure gradient, the direction of the shear force is such that it will accelerate fluid in the inside boundary layer and slow down fluid in the outside layer. Thus, the momentum transfer across the surface downstream of the point of separation will hinder separation of the inside boundary layer, and it is not possible to predict, as has been done in the past, that separation of the outside boundary layer will cause the inside boundary layer to separate.

The experiment described below was performed to resolve the questions about the fate of the inside boundary layer in a liquid sphere downstream of the outside separation point. Because of the problem of surface contaminants, the experiment was a study of oblate drops rather than spherical ones. In addition to the shape being an unknown and undesirable influence on the flow, surface tension forces due to surface contaminants were also present. However, it was deemed that the experiment would be useful if the inside flow in the drops used could be shown not to separate. This would imply that the inside flow in the ideal spherical drop discussed did not separate since distortion of the drop and contaminant caused surface forces make the inside flow less vigorous in the actual drops.

## EXPERIMENT

The experiment was designed to show the details of the flow in an axisymmetric plane of the drop. Flow visualization particles were added to both the drop and field fluids. The particles were illuminated by a vertical sheet of laser light, and high speed motion pictures were taken with a stationary camera at right angles to the light sheet. The movies were projected a frame at a time onto the back of a ground glass screen, and a tracing paper overlay was made of the drop and the particles. The locations relative to the drop of the particles were plotted for several frames so that streamline segments both inside and outside the drop were obtained as well as the magnitude of the velocity of points along the streamlines.

### Comparison of this experiment with previous ones

Several experimenters have used a sheet of light to illuminate an axisymmetric plane of a drop falling or rising in another fluid. Savic (Ref. 12) used a beam of light about 1/16" x 4" produced by a spotlight (presumably shining through a slit) to illuminate aluminum particles suspended in water drops falling in castor oil. A still camera at right angles to the beam and falling at the speed of the drop took a time exposure as the drop passed through the beam. This technique provided streamline patterns inside drops moving at Reynolds numbers around one. Other instances of the use of this darkfield particle trace technique are mentioned in the paper by Kintner, et al (Ref. 13). Horton, et al (Ref. 14) used a sheet of light produced by a mercury arc and a slit (slit opening not given) to record successive positions on film in a still camera of aluminum particles inside a drop suspended in an upward flowing fluid in a tapered glass tube. These drops were at Reynolds numbers below those at which a wake eddy is present.

Johnson and Braida (Ref. 15) used a high speed motion picture camera to record the motion of organic drops falling in glycerine-water solutions. They observed the circulation in the drops by adding aluminum particles to them. The camera moved with the drop. The entire drop was lit, and no attempt was made to record the motion in one axisymmetric plane, or to get quantitative information about the inside flow. In fact, only a brief verbal description of the inside flow is presented in their paper. No particles were added to the field fluid. Garner and Haycock (Ref. 16) also used a movie camera falling with the drop and observed the flow inside fully lit drops by suspending aluminum particles in the drop fluid. They obtained information about the inside flow by superimposing the projected outlines of the drop and following the motion of the aluminum particles relative to the drop. They did not observe the outside flow. The maximum Reynolds number used in their experiment was 3.7.

Our experiment has the following unique features:

a. it shows the flow both outside and inside the drop,

b. it allows the tracking of individual particles in an axisymmetric plane,

c. it shows the inside flow in the Reynolds number regime in which the drop has a wake eddy (all prior experiments which gave good information about the inside flow were run at lower Reynolds numbers where the drop moves slowly and steadily in a highly viscous fluid),

d. an extremely thin sheet of light was produced by using a laser as the light source.

## Apparatus

A Universal Laboratories helium-neon laser with 2 mw power was used as the light source. The laser beam was passed through the optical system shown in figure 5. The centerline of the drop tank was placed at the point where the beam focused horizontally. The dimensions of the beam at the tank centerline were .05 cm by 5 cm. The thickness of the beam was measured by passing it through an optical slit and then closing the slit until both sides of the slit were illuminated. The axis of the cylindrical lens was adjustable so that the light sheet could be made vertical within close tolerances. It was necessary to backlight the tank in order to record the drop outline on the film. This was done with a 15 watt fluorescent light which was reflected off a wall then through a large diffusing screen placed behind the tank (see figure 5).

A 16 mm, Fastax WF-3, high speed, movie camera was mounted in a stationary position on the drop tank stand. The 50 mm, f2 lens provided with the camera was used with a 0.65 inch extension. The lens was approximately 7.2 inches from the light sheet so that magnification was approximately 1/3, giving a vertical field of view of about 3/4 inches. A direct current power supply powered the two drive motors of the camera. A built-in timing light marked the edge of the film 120 times per second.

Kodak 4X, 16 mm, negative film in 100 foot rolls was used. It is a panchromatic film and so is red sensitive, a desirable feature when using the red light of the helium neon laser. The film speed is nominally ASA 500, but this was roughly doubled by developing for ten minutes in Kodak D11 (high contrast) developer at 72°F. The film was exposed at the rate of about 300 pictures per second, and this gave each picture an exposure time of about 1/900th of a second. For each run the lens was wide open at f2.

The inside dimensions of the drop tank were 4" x 4½" x 35½". It was constructed of ½" plexiglass with one side removable for cleaning. The tank was mounted on a metal stand with the lens of the movie camera mounted at about the half way height of the tank. Previous experimenters (for example Winnikow and Chao (Ref. 5)) have shown that the drop attains its terminal velocity close to the nozzle so that drops in this experiment had adequate distance to attain their terminal velocity before being photographed. Also the bottom and sides were far enough from the drop to reduce their influence to negligible proportions. The bottom of the tank contained a faucet. An adjustable nozzle-funnel combination and two plumb lines were attached to the top of the tank. The stainless steel nozzles screwed into a funnel block which could be traversed on two steel rods across the tank enabling the positioning of the nozzle so that the drop would intersect the sheet of laser light. The plumb lines were used to help locate the nozzle and to determine that the sheet of light was vertical. The weights at the end of the plumb lines were suspended in glycerine so that the lines were critically damped. A burette containing the drop fluid was suspended above the funnel block. The flow of drop fluid was controlled by a stopcock on the burette and could be adjusted without jostling the funnel block. When particles were added to the drop fluid for flow visualization, the burette could not be used because the particles clogged the stopcock. Then hypodermic syringes were used to supply fluid to the nozzle. A beaker was suspended at the bottom of the tank to receive the drops.

A Brookfield viscometer was used for viscosity determinations. A Polaroid Land Camera was used to determine exposure data for the movie film and to find the correct level of backlighting. A glass still was used for distilling carbon tetrachloride.

## Techniques

Several materials were tried as flow visualization particles, but the only two which combined long settling time with adequate light reflection at $90^{\circ}$ were fine aluminum powder and finely ground pliolite particles from the Goodyear Chemical Division. The obvious disadvantage of aluminum is that its relatively large density will cause the trajectory of the particle to deviate from a streamline. The results of its use in other experiments such as Taneda's observation of wake eddies behind solid spheres (see Ref. 10, plate 3) show that the deviation will not be so large as to obscure important details of the flow. Pliolite particles give less light reflection than the aluminum but their specific gravity is close to one. Particles which passed a #200 sieve were shaken vigorously with the field solution and allowed to settle out for one half hour. The residue was discarded. Enough particles were added to give the desired particle density. This density was limited by the requirement that an individual particle be identifiable from frame to frame of the movie. When system purity was a goal, particles passing a #200 sieve were shaken with distilled water in 500 ml cylinders. After one half hour, the fluid was removed from the cylinders with a transfer pipette, and the residue was thrown away. After three or four more hours of settling, the water was thrown away, and the residue was mixed with the field fluid. Particles were added to the drop fluid and the residue which settled out in one half hour was discarded; the residue which settled out in the next hour or so was used in the drop fluid.

Since the laser beam was thin, it had to be precisely vertical so that the particles which were in the beam when the drop hit it could be followed for a large portion of the drop's fall through the beam. The width of the string in the plumb line was about 0.5 mm, and this was just about the width of the beam there. When the string was put in front of the beam, a glow could be seen on both sides of the string when it was viewed from the rear (facing the laser, but looking down from above so as not to look into the laser beam). When the beam was vertical, this glow covered both sides of the string for the full height of the beam. When it was not vertical, each side of the string glowed for only a part of the height of the beam. Using this effect as a guide, it was possible to adequately position the cylindrical lens so that a particle could be observed through the camera lens to fall in the beam from top to bottom of the field of view.

The beam was aligned with the camera lens by a micrometer screw on the laser mount. Focusing of the camera was critical due to the f2 lens opening and the proximity of the lens to the drop. The lens was adjusted until sharp images of the particles in the laser beam were observed on an opaque film strip in the position at which the actual film was exposed.

If the light beam did not intersect the axis of symmetry of the drop, the movie record of its motion was not usable since the particles illuminated by the beam had a component of velocity in the direction of the camera and would not remain in the beam for a sufficient number of frames. The nozzle was aligned before the movies were taken by observing how the light was refracted on passing through the drop. Those drops which did not refract the light were centered on the beam. Even after the nozzle was located as precisely as possible, not every drop recorded on the film was centered; thus, during the filming, it was necessary to keep a record of which drops were centered. To minimize the waste of film the Reynolds number was kept low enough for the path of the drop to be fairly predictable. For some of the systems used, however, the drops either did not circulate or did so only partially at these Reynolds numbers.

A typical roll of film ran through the camera in 15 to 20 seconds and recorded about 3/4 of an inch of the fall of five or six drops. The film was projected onto the 12" x 13" ground glass screen of a Recordak film reader. A tracing was made of the drop, and frame by frame the location relative to the drop of some of the particles was plotted on the tracing. With this information plotted some partial streamlines relative to the drop could be drawn. A grid was photographed at the location of the beam with the Fastax camera so that the drop size could be determined and, using the marks left by the timing light, the velocities of the drop and the particles could be calculated. Thus, it was possible to obtain the Reynolds number of the drop, the relative strength of the circulation inside the drop, an estimate of the shape of the separation profile and sketches of partial streamlines (including velocity magnitudes) both outside and inside the drop, although the latter, unless they are near the center of the drop, are distorted by the non-uniform refraction of light through the drop.

## Systems

The first liquid-liquid system used consisted of organic solvent drops in glycerine plus water solution. Using a 3/8" bore nozzle the drops could be made quite large, but because of surface contaminants the drops were only weakly and partially circulating so that there was an extensive stagnant region in the back of the drop.

The next system tried was pure carbon tetrachloride drops in distilled water. Care was taken to remove surface contaminants from the liquids and from the apparatus the liquids came in contact with. The glass burette which held the drop fluid was cleaned with chromic acid. The brass funnel block was soaked in a 1:1 solution of concentrated hydrochloric acid and water, and the nozzles were soaked in a 1:1 solution of concentrated nitric acid and water. After the soaking, the funnel block and the nozzles were rinsed in distilled water. The AR grade carbon tetrachloride was distilled twice in a glass still, and only the middle batch of distillate was retained each time. The water for the field fluid was distilled in a stainless steel still.

Thorsen, et al (Ref. 17) have published data on the terminal velocity of carbon tetrachloride drops in water for both pure and impure systems. These data show a difference in the velocities of the two systems beginning at a drop size of about 1mm. This difference is caused by the internal circulation of the pure drops. Unfortunately, carbon tetrachloride drops of this size have such a high Reynolds number in water that their motion is unsteady making their path unpredictable. The only way these drops could be used in this experiment was by sending a large number of drops toward the light beam in a short time. A nozzle was fashioned out of hypodermic tubing, and it formed drops of about 1.4 mm equivalent diameter at a spacing of about one per inch. The velocities of these drops ranged from 12 cm/sec to a little over 15 cm/sec. The faster drops were on the "pure" curve, and the slow drops were on the "impure" curve of figure 1 in Thorsen, et al (Ref. 17). When movies showed aluminum particles sticking to the drops, this system was abandoned.

The system finally used was chosen because it did not contain water as a component (and might not, therefore, be so plagued by troubles arising from surface active contaminants) and because the viscosity of the field solution was great enough for the path of a large drop to be fairly predictable. Silicone oil was used for the field fluid, and ethylene glycol was used for the drop. The viscosity of the silicone oil was varied by blending different grades of the oil. A preliminary visual test of the system using aluminum particles in the drop showed the inside circulation to be vigorous and apparently complete.

## Results

The silicone oil-glycol system used was less troubled by tangential surface tension forces than was the first system tried, but the problem was still present because small glycol drops did not circulate. The extent of its influence is not known.

Figure 6 in McDonald's paper (Ref. 18) shows a suggested flow pattern around an oblate rain drop in air. This flow pattern is sketched in figure 6(a). It shows separation occurring at the drop shoulder with the separated streamlines going straight downstream. The movies show that this is what happens in the flow around glycol drops in silicone oil. Figure 6(a) shows a reverse flow on the surface of the drop in the entire separated region. A secondary eddy would be required inside the drop to make this condition possible. Winnikow and Chao (Ref. 5) state that separation implies that all the fluid inside the drop downstream of the separation ring is stagnant. Their model is sketched in figure 6(b). These two representations of the inside and outside flow downstream of the separation ring are typical of the explanation which are current concerning the effect of separation on the flow.

The small Reynolds number theory in Ref. 1 and the small time theory in Ref. 2 and the speculations based on them suggest that the surface of the drop will continue moving downstream past the point of separation, and that the inside flow downstream of the separation ring will not be a secondary eddy or a stagnation region. The small Reynolds number theory says that the opposing flows in the drop and in the wake eddy will be buffered by an almost stagnant region between the two. This is borne out by the experiment with glycol drops in silicone oil. The movies of this system show separation occurring at the drop shoulder. Some distance downstream of this a stagnation ring is made visible by the accumulation there of flow visualization particles convected from upstream. The fact that these particles have been convected past the separation ring proves that the surface is still moving downstream at separation. Figure 7 shows the slow approach of a particle along the surface to the stagnation ring. Figure 8 is a frame by frame plot of the location of several particles relative to two consecutive drops which were photographed about three seconds apart. The spacing of the dots along the streamline gives the velocity of the fluid there. The leftmost streamline in drop (a) clearly shows the separation at the shoulder. The stagnation ring mentioned is about half way between the separation ring and the rear stagnation point. The leftmost streamline in drop (b) shows that the surface velocity at separation is the same order as the free stream velocity. The movies show the separation ring is at $\theta = 90°$. Possibly the development on the experimental drops of profiles of outside tangential velocity between $\theta = 90°$ and $125°$ is similar to the speculative one in figure 9. The development from the separation profile (a) to profile (b) in figure 9 is like the one found experimentally by Brady and Ludwig above the surface of the rotating cylinder.

The nearly stagnant region between the surface and the separated streamlines has been confirmed by the experiment. The tangential velocity of the particle labeled "p" in figure 8(b) is small indeed. Figure 10 is a composite of several drops in which information on either side of the drop has been reflected to the other side. It shows some streamline segments in the near wake region and inside, and it defines the approximate limits of the wake eddy as well as the strength of the circulation within it and within the drop. Stagnant particles are shown on the back of each of these drops. The entire surface area between the stagnation ring at "S" in figure 10 and the rear stagnation point "R" is motionless. There is not a reverse flow there as would be the case if the wake eddy were attached to the back of the drop.

Figure 11 is a composite of information obtained in the vicinity of the rear stagnation point of several drops of about the same size and Reynolds number. It shows that the fluid in the wake adjacent to this surface is moving slowly in the direction of the main inside circulation. The inside flow does not separate, and the circulation in the wake eddy is detached from it by a stagnant region wholly outside the drop. This is essentially the flow given by the small Reynolds number and small time theories. In the theories the flow is nearly stagnant in this detachment region. That it is completely so in the case of the glycol drops could be explained to be a result of the drop shape or the surface tension forces or a combination of the two.

The effect of distortion of the drop on the internal circulation has been commented on by F. K. Moore (Ref. 20). He concludes that distortion considerably lessens the strength of the inside circulation. This effect along with the early triggering of separation by the drop shoulder and the upstream directed surface tension forces resulting from contamination of the drop surface would make the flow less vigorous in the back of the actual drop than would exist in the back of the ideal, spherical drop of the theoretical model. These facts make the agreement of theory and experiment seem adequate in so far as both indicate that the inside flow will not separate despite the separation of the outside flow. The secondary eddy inside implied by McDonald's model and the inside stagnation region assumed by Winnikow and Chao are certainly incorrect in the limit of small, contaminant-caused, tangential surface tension forces.

Figure 12 which shows superimposed drop images nine frames apart gives details about the wake eddy. For this run the external flow visualization particles were largely confined to the top few inches of the field fluid where some of them were entrapped by the wake eddy. All the external particles downstream of the shoulder in the two frames were plotted in figure 12. The absence of particles in the triangular region between the back of the drop and the wake eddy indicate that there are no secondary vortices there and that the region is slowly washed out by the external flow as predicted by the theories. The limits of the wake eddy, as well as the magnitudes of the velocities in it are clearly shown in this figure.

## CONCLUSIONS

### Limits of the linearized boundary layer

Harper and Moore (Ref. 4) have developed a boundary layer solution for the flow in and around a liquid sphere which is based on the assumption that the velocities in the boundary layers are small perturbations of the inviscid flow around and in a spherical vortex. This scheme presumes that the slip at the surface between the outside and the inside tangential velocities is much smaller than the free stream velocity. The

The range of validity of this assumption is not known since no direct method of relating this slip to the parameters of the problem has been found. It seems plausible that when the viscosity inside the sphere gets large enough, the slip will be $O(U_\infty)$. It is certainly of that order in the limit of the solid sphere.

Clearly the small perturbation scheme cannot support the phenomenon of separation of the outside boundary layer. Whenever the liquid sphere flow is in a regime where separation occurs, the velocity perturbations in the boundary layer must be $O(U_\infty)$. The absence of separation does not imply that the boundary layer perturbations are a smaller order than $U_\infty$, so that the use of the criterion of separation to delimit the range of validity of the small perturbation model will probably make that range appear larger than it really is. In Refs. 1 and 2 information about separation was obtained from the small Reynolds number, Stokes-Oseen type theory and the small time theory. The small Reynolds number theory predicts that the outside boundary layer will separate from the liquid sphere at large Reynolds numbers except when the ratio of outside to inside viscosity is very large. This makes the range of validity of the small perturbation model quite small.

The small time expansion in Ref. 2 and the experiment in this report support the unexpected prediction of the small Reynolds number theory that the wake eddy is detached from the sphere and, hence, that the inside flow does not separate when the outside flow does.

## Model for the flow with separation

Previous authors have assumed that separation of the outside boundary layer from a drop must imply either a reverse eddy inside the drop or a stagnation region inside which encompasses all of that inside fluid which is downstream of the separation ring. This latter hypothesis is undoubtedly based on the observation of systems containing significant tangential surface tension forces. These forces result from a gradient in surface tension caused by a gradient in concentration of surface contaminants. These contaminants are always present in large enough concentration to cause the tangential surface tension forces to dominate the dynamics of the drop surface when the drop is small. The larger the drop, the less significance these forces have. The fluid inside small drops is stagnant. As the drops are made larger, inside cirulcation starts at the front of the drop. The vigor of this circulation and the region of the inside which is circulating increases with drop size. However, with many systems the drop cannot be made large enough (without it becoming unsteady) for this region of circulation to include the whole drop. Then it is possible to have the outside flow separate with the inside fluid remaining stagnant downstream of the separation ring. This stagnant region which is really caused by the upstream - acting surface tension forces had been considered to be the result of the separation of the outside boundary layer.

The small Reynolds number theory says that separation of the outside flow does not cause separation of the inside flow. This result is confirmed by both the small time expansion of Ref. 2 and by the experiment described in this report. The small time expansions suggest that part of the outside boundary layer never does separate, but rather that that part of the outside boundary layer which is between the point of separation and the surface continues moving downstream between the wake eddy and the drop while the outer portion of the outside layer separates from the sphere and moves as a free shear layer around a wake eddy. This concept of an attached outside layer past the point of separation agrees with the experimental results of Brady and Ludwig (Ref. 8). The presence of such an attached outside layer enhances the ability of the inside flow to remain attached all the way to the rear stagnation point. If the separation profile given by the small time expansion is similar to the steady state profile, then the separation of the inside flow is unlikely, and the inside boundary layer continues on past the outside separation ring and exchanges momentum across the surface with the remaining part of the outside boundary layer. This suggestion that the inside flow does not separate received strong support from the experiment described in this report. Since the flow inside a liquid sphere can be assumed to be more vigorous than the flow inside the oblate and contaminated drops used in the experiment, the absence of inside separation in the experimental drops is a clear indication that outside separation does not cause inside separation in the ideal, liquid sphere.

The model being suggested for the flow when there is separation of the outside boundary layer is:

(1) The point of initial separation of the outside boundary layer is above the sphere surface so that only a portion of the outside boundary layer moves away from the surface and around the wake eddy; the inner portion remains attached to the surface and moves downstream between the wake eddy and the sphere.

(2) The inside flow does not separate. It has sufficient momentum when the outside flow separates that, with the momentum obtained from the unseparated part of the outside layer, it can continue all the way to the rear stagnation point.

(3) The vorticity distribution across the inside boundary layer in the vicinity of the rear stagnation point will not be a fully diffused one so that there will be a wake connecting the rear of the inside boundary layer with the front. Because continuity requires that the breadth of this wake be an order of magnitude greater than the boundary layer thickness, the wake will be inviscid to first order, and the vorticity distribution in the vicinity of the rear stagnation point will be convected almost unchanged to the vicinity of the front stagnation point (as in Harper and Moore, Ref. 4).

(4) The core flow of the sphere contains all those streamlines not in the boundary layer or the wake and is a Hill's spherical vortex with a strength that depends on the viscosities and densities of the two fluids.

(5) If the inside boundary layer is defined as being the region in which the distribution of vorticity/cyl. radius is $O(U_\infty/\text{radius}^2)$ different from the core flow, then the edge of the boundary layer is a streamline.

## The steady state solution

We can discuss what must be involved in obtaining a steady state boundary layer solution. The problem is apparent: It is not possible, as it was in the case of the solid sphere, to get determinate and fairly accurate results by carrying the solution only so far as the separation ring. When there is a wake eddy, even the flow in the wake eddy must be known before the problem can be made determinate since the solution in the attached portion of boundary layer on the back of the sphere must be known so that the net vorticity generation on the entire inside surface can be calculated. Hence, obtaining a steady state boundary layer solution for the liquid sphere involves all of these problems:

(1) Boundary layer solutions inside and outside the sphere must be matched.

(2) Both boundary layer solutions must continue past the outside separation ring to the rear stagnation point.

(3) The flow adjoining the outside of that part of the outside boundary layer which remains attached must be determined and must be matched to that outside layer.

(4) An interior wake solution must be found joining the flow at the rear stagnation point to the flow at the front stagnation point.

(5) The strength of the inside core flow and the initial conditions for the inside boundary layer must be adjusted until the terminal conditions match the initial conditions, so that there is no net vorticity generation on the inside sphere surface, and the solution is a steady one.

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## APPENDIX - SYMBOLS

| | |
|---|---|
| $a$ | sphere radius |
| $k$ | $\dfrac{\text{inside inviscid tangential velocity at surface}}{U_\infty \sin\theta}$ |
| $R$ | Reynolds number based on sphere radius and outside fluid properties |
| $t$ | time |
| $U$ | outside potential tangential velocity at sphere surface |
| $U_\infty$ | free stream velocity |
| $\theta$ | angle from front stagnation point |
| $\sigma$ | $\dfrac{\text{outside viscosity}}{\text{inside viscosity}}$ |

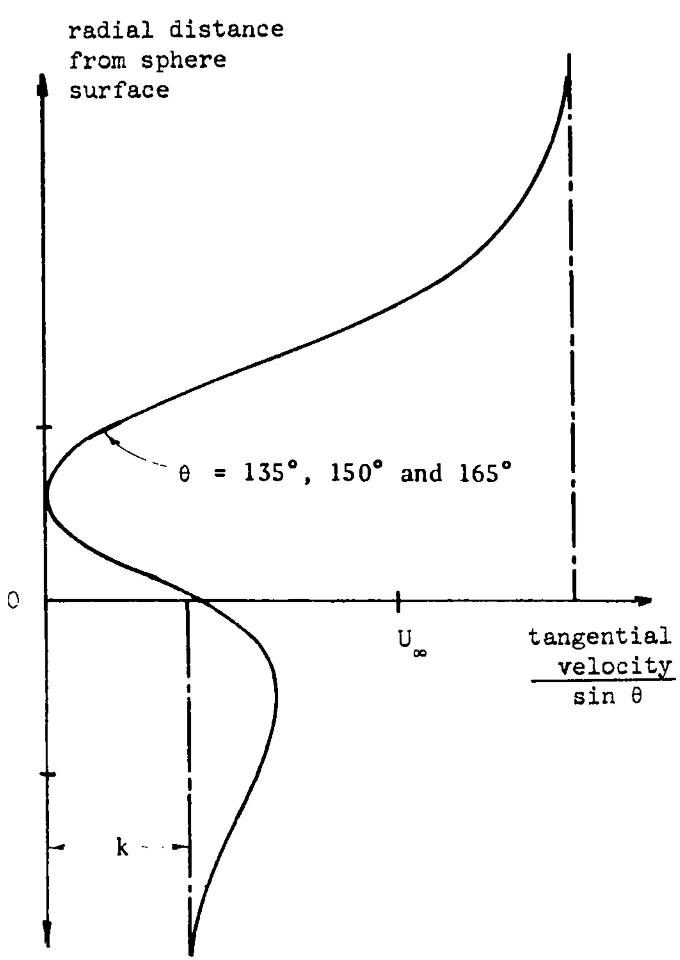



Figure 1. Separation profile at various angles for $\sigma = 0.3$ and $k = 0.4$. Each angle represents a different time. $\sigma = \frac{\text{outside viscosity}}{\text{inside viscosity}}$

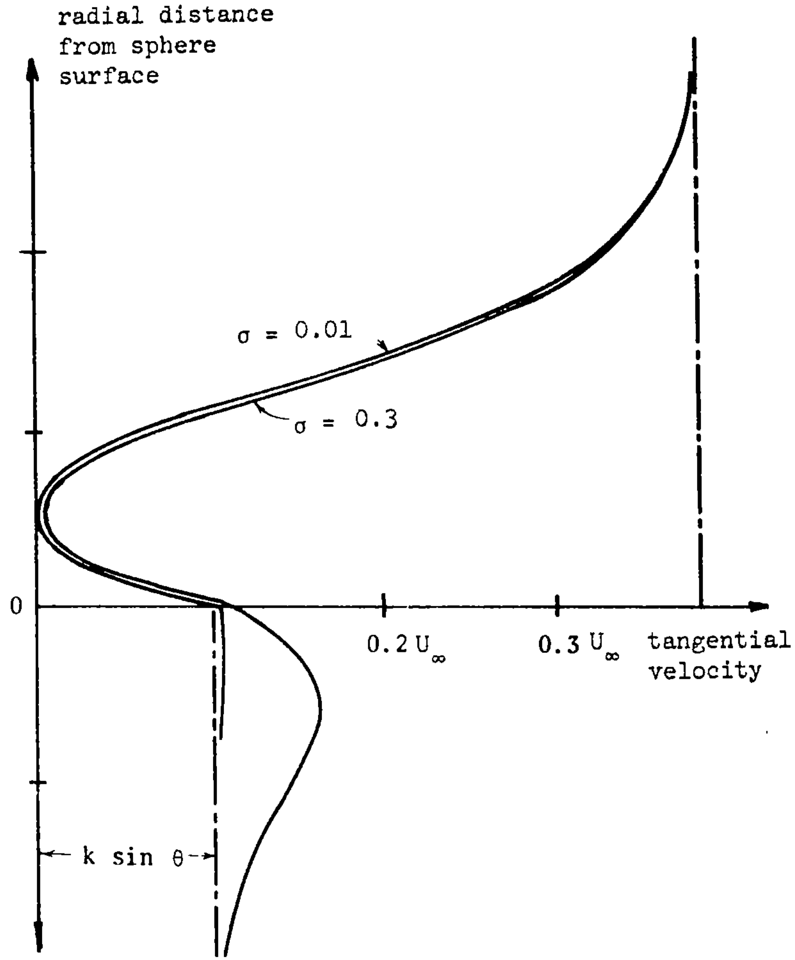



Figure 2. Separation profile at $165^\circ$ for $\sigma = 0.01$ and $\sigma = 0.3$ and $k = 0.4$.

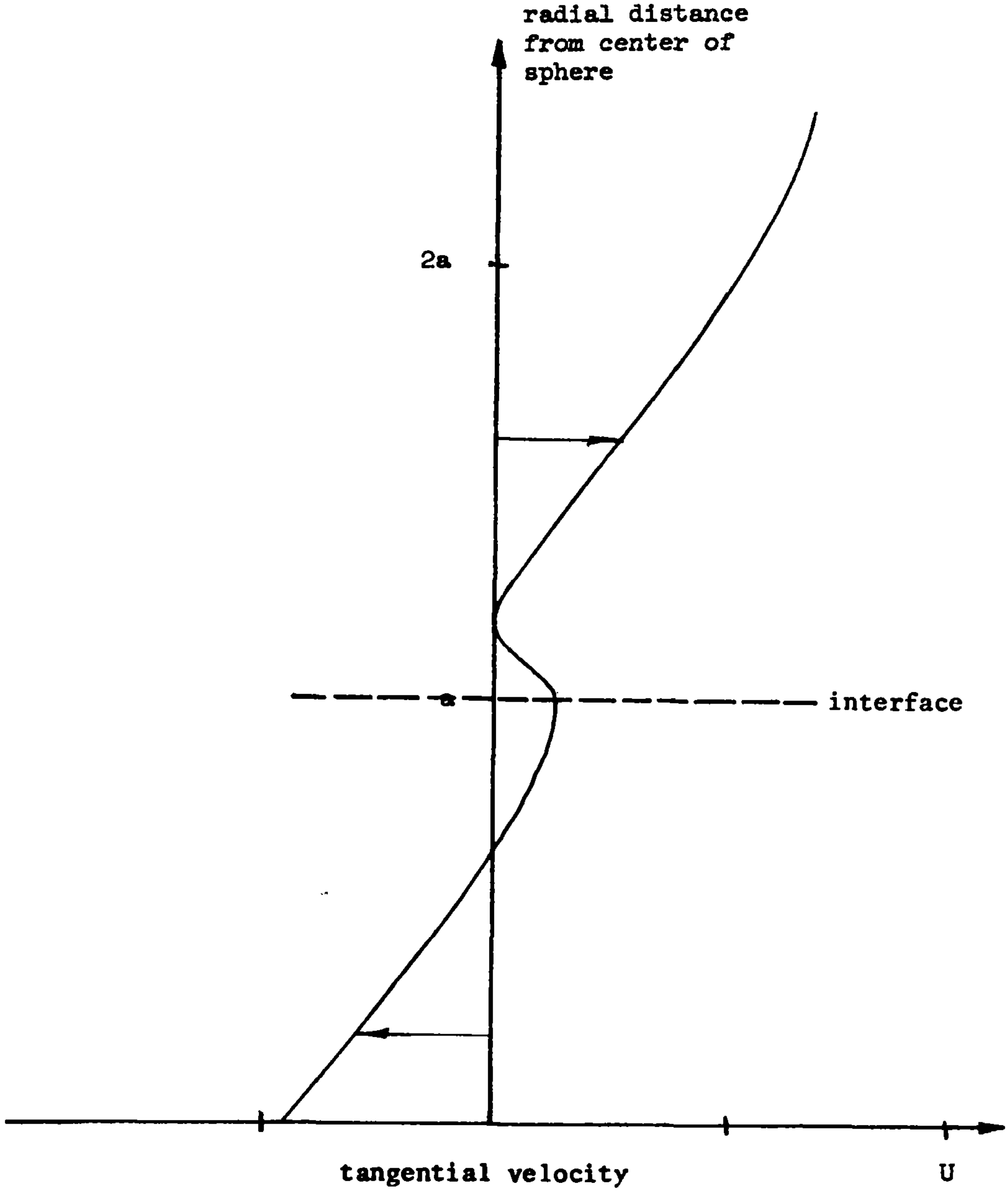



Figure 3. Separation profile from small Reynolds number theory. $R = 60$, $\sigma = 0.1$, $\theta = 25^{\circ}$. R is the Reynolds number based on the sphere radius, a.

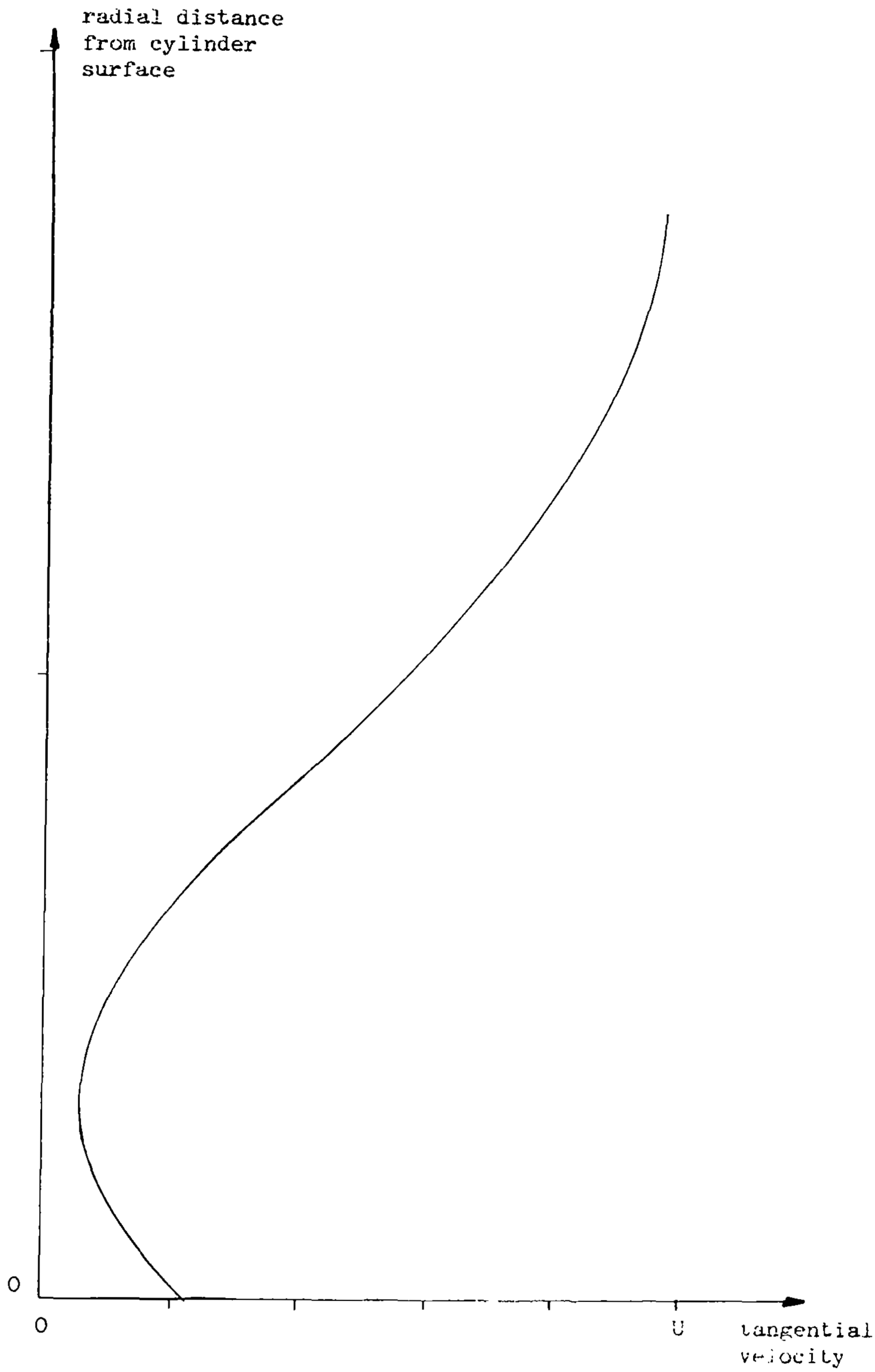
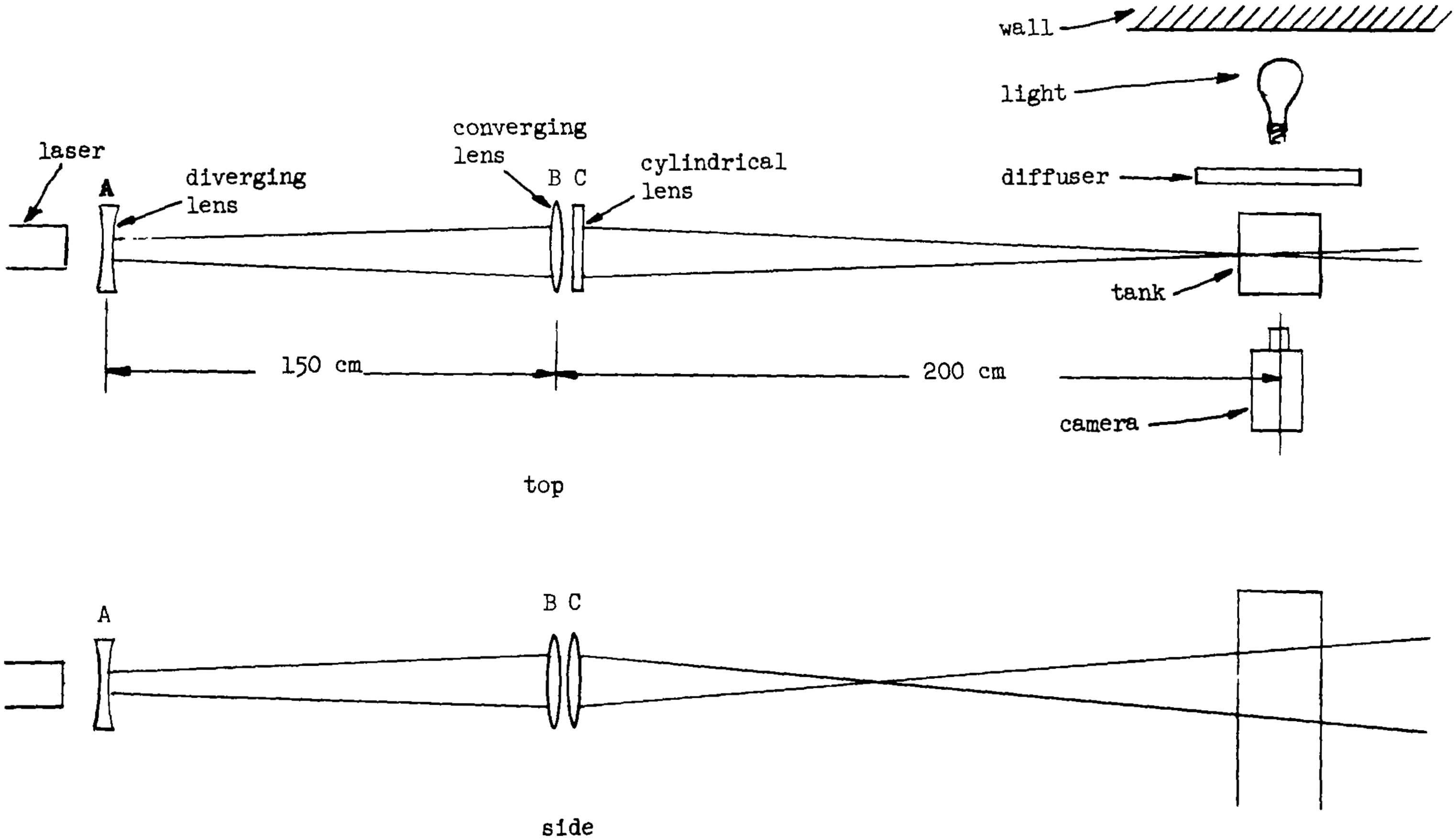



Figure 4. Profile near separation point above rotating cylinder from Brady and Ludwig.




Focal lengths:

$f_A$ = - 50 cm

$f_B$ = 100 cm

Figure 5. Optical arrangement in the experiment.

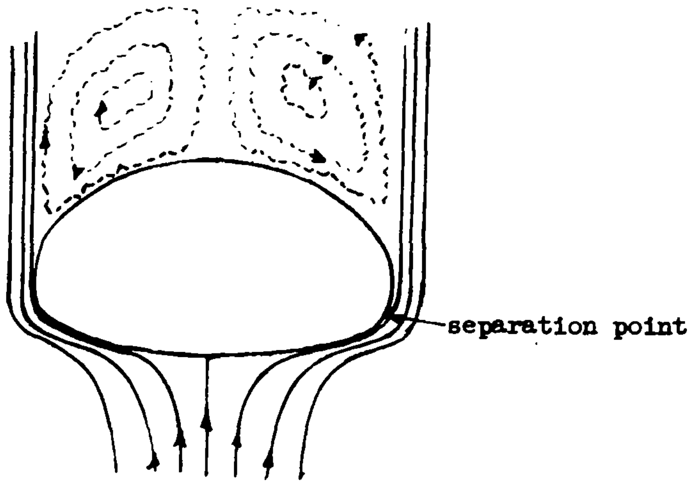



Figure 6(a). McDonald's suggested flow pattern around a large raindrop.

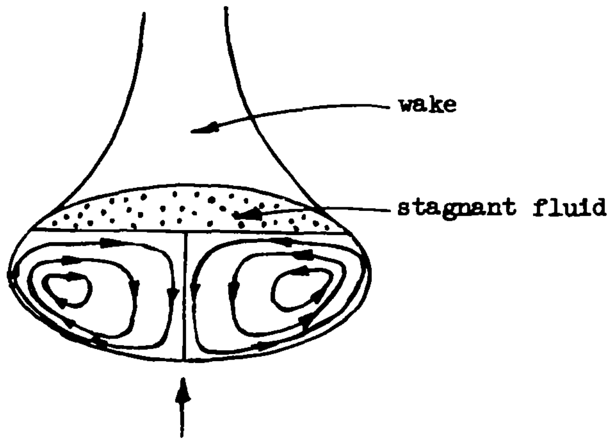



Figure 6(b). Winnikow and Chao's concept of the inside flow with separation.

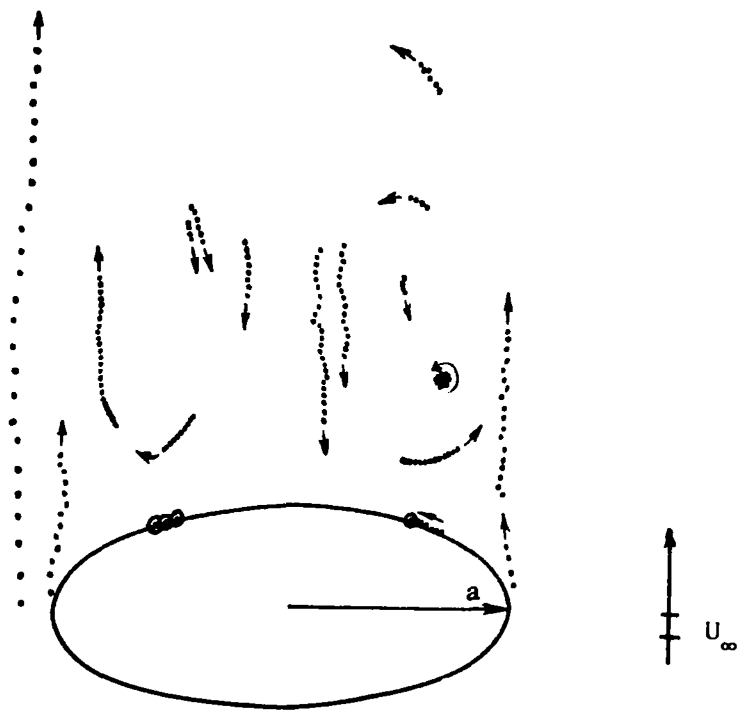



Figure 7. Flow behind 7.7 x 3.6 mm glycol drop in 5.5 cp silicone oil. R = 100. $U_\infty = 15.8 \frac{cm}{sec.}$ Circled dots indicate location of stagnant particles.

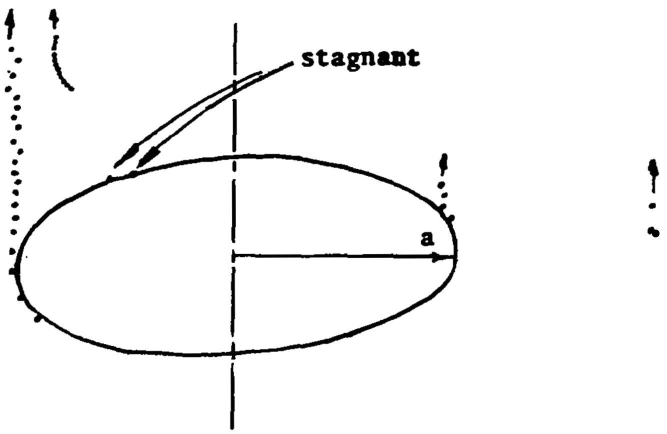



Figure 8(a). Path of pliolite particles relative to a 3.8 mm by 7.7 mm glycol drop in 5.5 cp silicone oil.
$U_\infty = 15.8 \frac{cm}{sec.}$, $R = 105$ $\sigma \approx 0.3$.

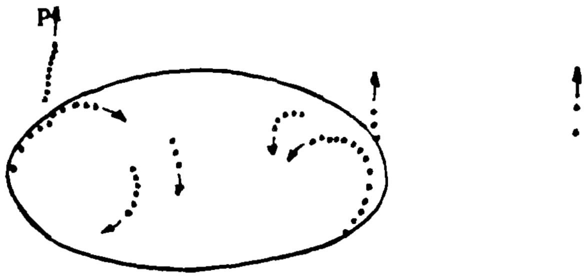



Figure 8(b). Path of pliolite particles relative to a 3.7 mm by 6.7 mm glycol drop in 5.5 cp silicone oil.
$U_\infty = 16.2 \frac{cm}{sec.}$ $R = 94.$ $\sigma \approx 0.3$.

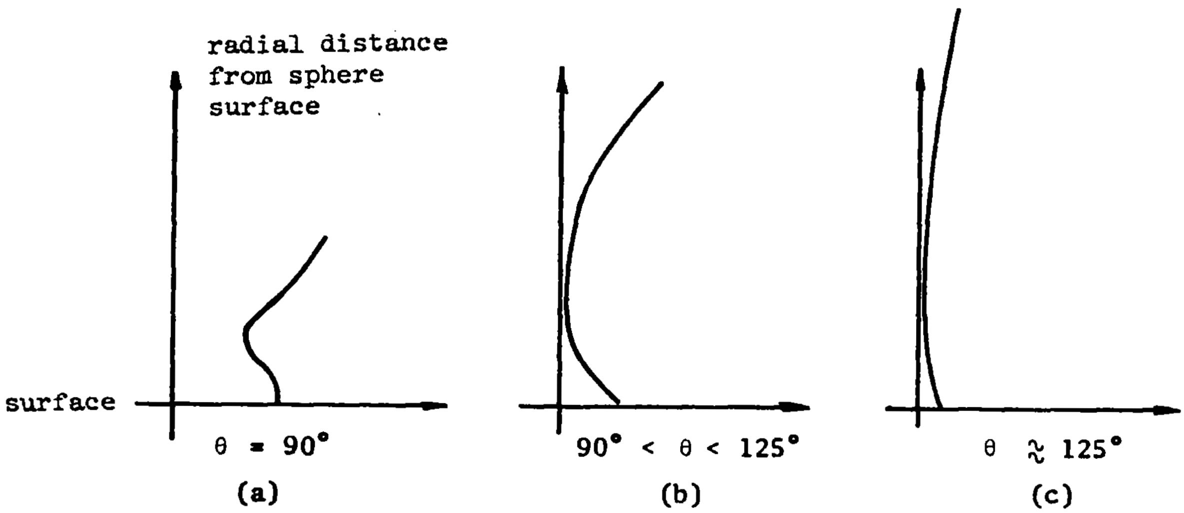



Figure 9. Possible development past separation of outside tangential velocity profile on oblate drop.

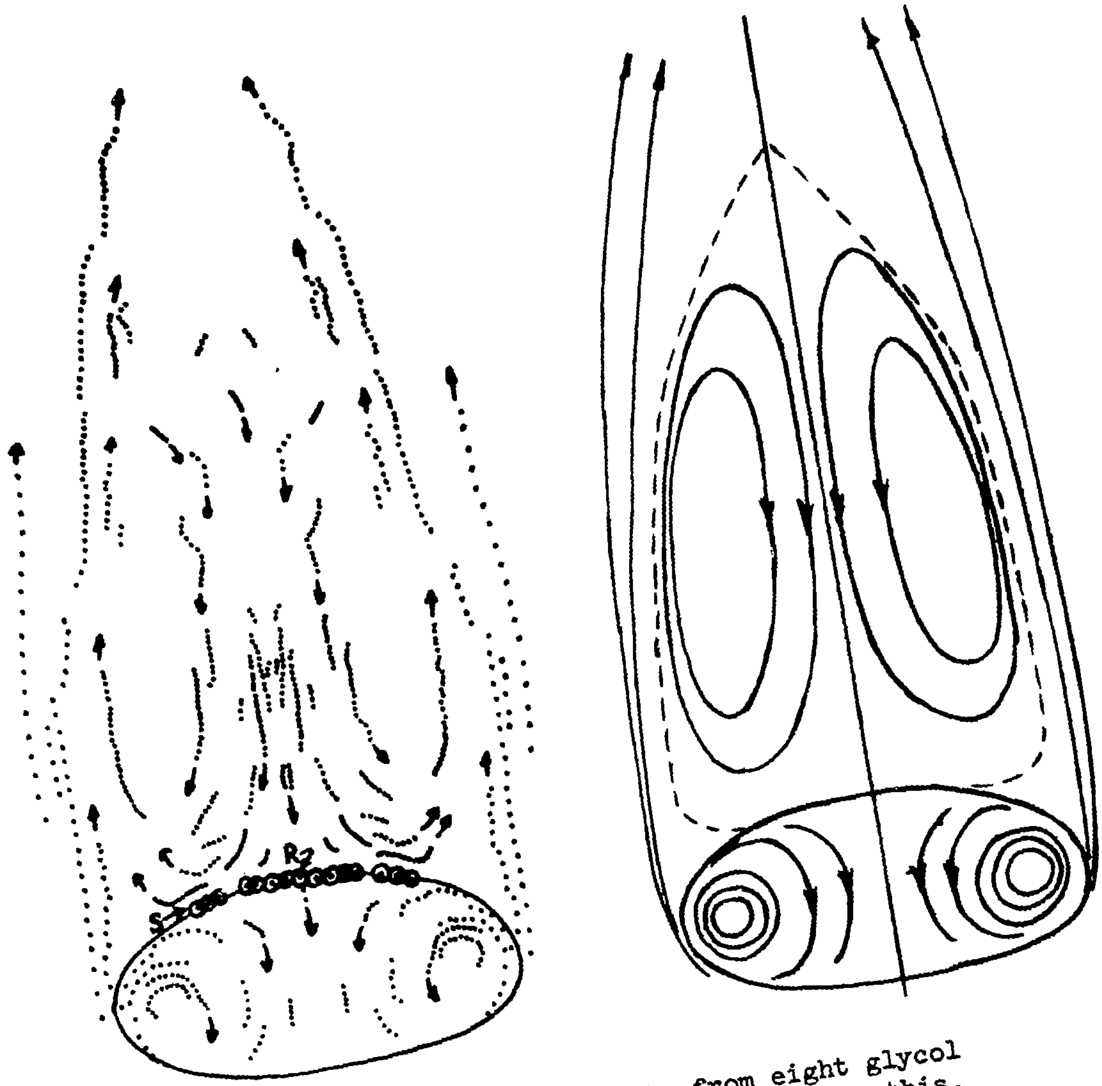
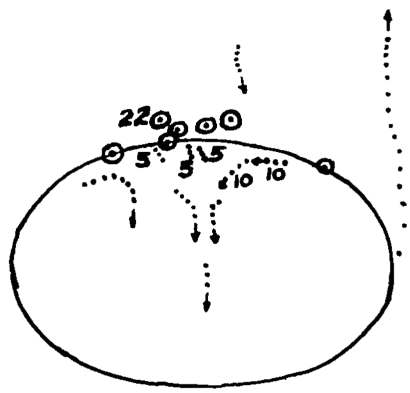



Figure 10. A composite of the plotted data from eight glycol drops in 5.5 cp silicone oil, and, based on this, a guess at the streamline pattern. Distortion due to the different indices of refraction has not been corrected. Reynolds number based on 1/2 major diameter ≈ 100. Circled dots indicate location of stagnant particles. Dashed line is guessed limit of wake eddy.



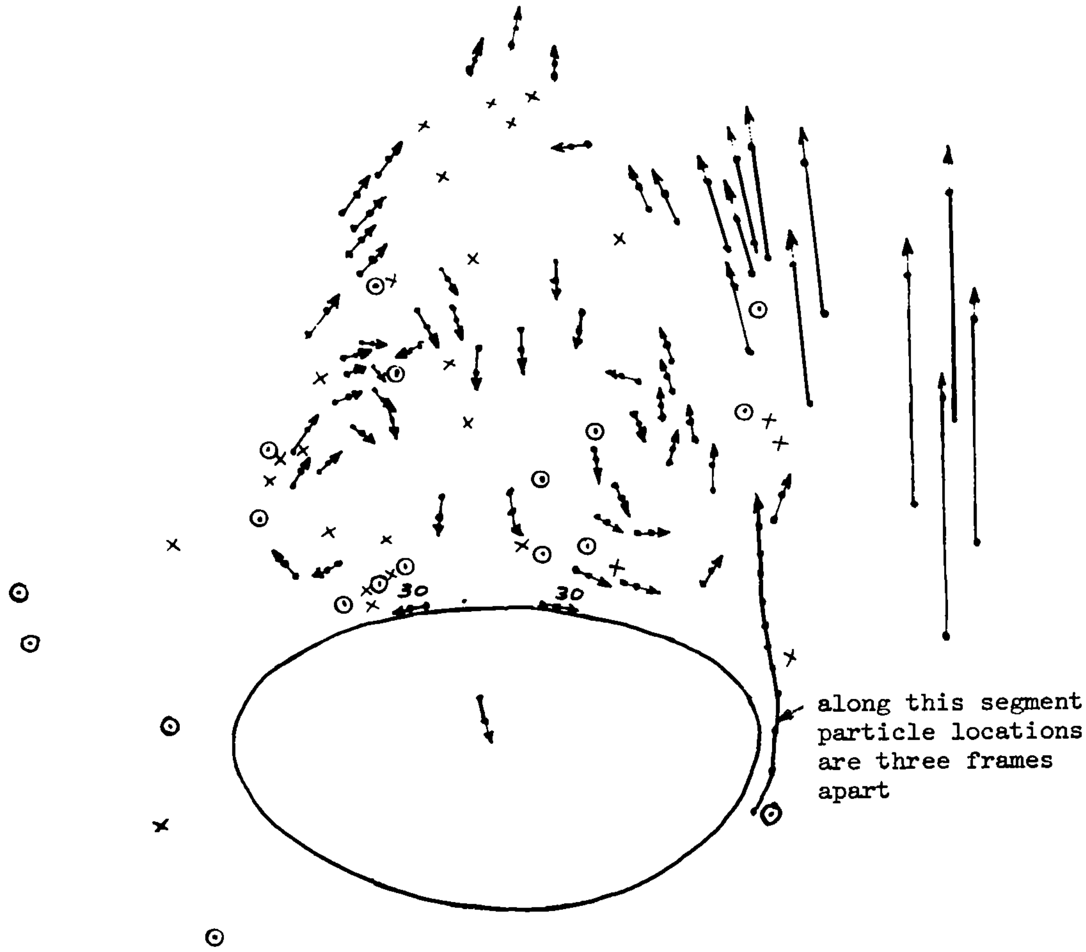

**legend:**

**⊙ stagnant particle.**

**n ⁝ n frames between dots.**

**22⊙ particle location for 22 frames .**

Figure 11. Composite of particle tracks in the vicinity of the rear stagnation point of seven glycol drops in 10 cp silicone oil. R ≈ 45.




Legend:

↑ distance traveled by one particle in nine frames.

30↑ distance traveled by one particle in 30 frames.

⊙ particle observed in first frame of nine frame sequence but not last frame.

× particle observed in last frame of nine frame sequence but not first frame.

Figure 12. Motion of particles in the wake of a 9.5 x 5.5 mm glycerine drop falling in 20 cp silicone oil. All outside particles downstream of the shoulder in the first and ninth frames of a sequence have been recorded.



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