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AN ANALYTICAL STUDY OF SOLID PROPELLANT COMBUSTION DURING RAPID DEPRESSURIZATION

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ABSTRACT

This paper discusses a modification of the solid propellant combustion model previously developed at Stanford Research Institute. The model was modified to contain two heat release zones in the gas phase, one adjacent to the wall governed by the wall temperature and a second primary flame governed by the final flame temperature. Comparisons are shown between predictions of the model and data obtained at both Stanford Research Institute and United Technology Center. It is demonstrated that the calculated behavior of six ammonium perchlorate propellants utilizing different binder systems agreed with the measured trends when only the burning rate, flame temperature, and total heat release were varied in the model from one propellant to another. Similar results were obtained when aluminum was added to one of the propellants.

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PREFACE

This study of solid propellant transient combustion was performed principally by the author. Thanks are due to Dr. G. A. Marxman for many stimulating discussions during the program and to Dr. D. C. Wooten who made valuable comments concerning the mathematical analysis. Finally, the author is deeply indebted to Miss Margery Brothers who was responsible for the long and tedious computer calculations.

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NOMENCLATURE

A	kinetics parameter
A_b	burning surface area
A_t	nozzle throat area
c_p	specific heat capacity of gas at constant pressure
c_s	specific heat capacity of solid
E_w	activation energy for pyrolysis at interface
E_D	activation energy for pressure-insensitive surface-coupled reactions in Eq. 2.
E_f	activation energy in the outer reaction zone
E_h	activation energy in the inner reaction zone
E_H	activation energy for pressure-sensitive surface-coupled reactions in Eq. 1
H_D	preexponential factor in Eq. 2
H_H	preexponential factor in Eq. 1
k_g	thermal conductivity of gas phase
k_s	thermal conductivity of solid
L	heat of vaporization per unit mass of propellant
m	exponent in Eq. 1
m_1	exponent in Eq. 22
$\dot{m}$	rate of external mass addition
m^*	nondimensional rate of external mass addition by pulser = $\dot{m}/\rho_s r_i A_b$
n_1	order of gas-phase reaction in the inner zone
n_2	order of gas-phase reaction in the outer zone

p	chamber pressure
q_g	nondimensional heat of reaction in the outer flame zone
q_s	nondimensional heat of reaction in the inner flame zone
Q_D	pressure-insensitive surface-coupled heat release in Eq. 2
Q_g	heat of combustion in the outer reaction zone
Q_H	pressure-sensitive surface-coupled heat release in Eq. 1
Q_s	heat of combustion in the inner reaction zone
Q_t	total heat of combustion
R	gas constant
r	burning rate
T	temperature
t	time
V	chamber volume
w_1	nondimensional rate of species production in the inner flame zone
w_2	nondimensional rate of species production in the outer flame zone
W_1	rate of species production in the inner flame zone
W_2	rate of species production in the outer flame zone
x	distance away from propellant surface
x_f	flame standoff distance (microns)
y^*	transformed distance

Greek Letters

α	kinetics parameter
β	W_1/W_2
γ	specific heat ratio, c_p/c_v
δ	Dirac delta function

ζ nondimensional location of the outer flame zone

$\varkappa$ thermal diffusivity of solid = $k/\rho_s c_s$

ρ_s density of propellant

ν propellant burning rate index

Subscripts

c chamber conditions

cr critical value

f gas-phase flame

g gas phase

i initial value at t = 0

ℓ limiting value

o conditions at $x \to \infty$

w conditions at wall (gas-solid interface)

* denotes nondimensional quantity normalized to steady-state value; e.g., $r^* = r/r_i$, $T_f^* = T_f/T_{f_i}$. Note one exception: $T_o^* = T_o/T_{w_i}$. Also, $t^* = r_i^2\, t/\varkappa$ and $x^* = r_i x/\varkappa$.

I INTRODUCTION

The understanding of composite solid propellant combustion has gradually advanced over the year with major contributions coming from several other groups[1-6] in addition to the work at Stanford Research Institute (SRI).[7] The theoretical models that have been advanced through the years have differed only in the treatment of the heat release in the gas phase and in the vicinity of the solid surface. The heat release process has usually been assumed to be quasi-steady in nature; i.e., time lags associated with the chemical reaction processes are assumed to be small in comparison with the thermal response time of the solid phase. The heat release acts as a boundary condition on the solid phase that in turn, has been treated in a one-dimensional way in spite of the heterogeneous nature of the propellant.

Thus, the main question is how to treat the heat release process in a quasi-steady way and still retain the important aspects of the transient combustion process. In general, the combustion process of a solid propellant is extremely complex. Chemical reactions may occur both in the solid phase near the regressing surface and throughout the gas-phase boundary layer up to the point where the final flame temperature T_f is reached. The approach at SRI, both in earlier work[7] and in the current model to be discussed in detail below, has been to consider the heat release as occurring in two thin zones, one associated with the surface temperature and one with the final flame temperature. This representation avoids the necessity of specifying the heat release distribution across the entire gas-phase thermal layer and replaces it by the simpler choice of the ratio of heat release in the two zones.

Early theoretical studies[7,11] showed that the presence of surface-coupled heat release would tend to make propellants more unstable and more

difficult to extinguish by rapid depressurization. Experimental studies confirmed these initial qualitative predictions. It was shown that axial-mode traveling wave instability could be induced with AP (ammonium perchlorate) propellants, but not KP (potassium perchlorate) propellants.[9] It was also demonstrated that AP propellants were more difficult to extinguish than KP propellants.[5,8,11]

The qualitative predictional success of the initial analytical concept of surface-coupled heat release led to an attempt to develop a quantitative transient combustion model that would predict the actual behavior of any given propellant formulation with a minimum of input information. Such a model was developed, based upon the stability limit line in the burning rate/pressure coordinate system which was derived in Ref. 12.

The initial analysis was patterned after the original burning rate behavior in terms of two parameters, the parameter A that described the pyrolysis behavior of the propellant surface and the parameter α that described the kinetics of the gas-phase heat release process. The SRI formulation extended the definition of α to include the kinetics of the surface-coupled heat release process as well. It was assumed that α has a constant value along the stability limit line, which will be discussed in more detail later. Knowing the transient response of any propellant at the pressure where its burning rate curve crosses the stability limit line allows the calculation of the response at any other pressure.

Reasonable agreement was obtained between the model and available experimental stability data.[9,10,12] However, Culick[2] later pointed out that the linearized versions of all theories could be shown to be inherently of the same mathematical form; i.e., only comparison with nonlinear experiments in which the pressure and burning rate changes were not perturbations could be expected to delineate important aspects of the theoretical model.

One nonlinear experiment of practical importance is that of combustion extinction by rapid depressurization. The development of a combustion model that could describe this phenomenon would be of great value in the correlation of data from different motor sizes and, therefore, in the

prediction of the critical depressurization rate for one motor using data from another or for one propellant using data from another. Up to the present time, depressurization data have generally been correlated by plotting the critical initial depressurization rate required for extinction $(dp/dt)_{cr}$ versus the initial pressure p_i. Such a correlation is very useful for a given configuration in an engineering sense because of its simplicity, but it cannot be expected to be successful when either the propellant type or the motor size is changed because it actually represents an attempted linearization of a nonlinear problem. Evidence to support this conclusion is shown by the data of United Technology Center (UTC),[5,6] in which it is shown that the exhaust pressure, the motor size, and the propellant type all have a strong influence on the $(dp/dt)_{cr}$ vs p_i results.

Similar depressurization data have been obtained at SRI, as well as pressurization data from an experiment that was devised to emphasize the role of the burning rate variation in the measured pressure variation.[8,11] Reference 9 compared these SRI experimental results to predictions of the initial SRI model that was based upon a modification of the Denison-Baum model[13] to account for heat release that is tied to the surface conditions. It was found that an unexpected nonlinear damping behavior was present in the SRI model, leading to an unsatisfactory nonlinear predictive ability including the observation that the required $(dp/dt)_{cr}$ calculated at a given value of p_i was always considerably less than the experimental value.[11] Because of this behavior the current program has been devoted to a reformulation of the SRI combustion model in a more self-consistent way and to a more comprehensive comparison of the predictions of the revised model with extinction data obtained at both SRI and UTC.

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II MATHEMATICAL FORMULATION OF THE COMBUSTION MODEL

To emphasize the importance of the surface-coupled portion of the heat release and, at the same time, to make the mathematical analysis tractable, it was assumed in the original SRI model[7] that the heat release zone of a solid propellant can be divided into two regions: one thin zone adjacent to the propellant surface in which the rate of chemical reaction is governed by the wall temperature T_w, and a second thin zone further out in the gas phase in which the rate of chemical reaction is governed by the final flame temperature T_f. It was assumed that the surface heat release acts as a boundary condition on the solid phase.

The new model developed here is based on the same assumptions. The difference from the original model comes in the mathematical representation of the surface-coupled heat release. The original formulation supposed the surface-coupled heat release to be composed of two parts, a pressure-sensitive part

$$Q_H = H_H \left(\frac{p}{T_w}\right)^m e^{-E_H/RT_w} \tag{1}$$

and a pressure-insensitive part

$$Q_D = H_D e^{-E_D/RT_w} \tag{2}$$

Qualitative comparison with traveling-wave instability data indicated that Q_H was, in general, an order of magnitude larger than Q_D.[11] This is an indication that gas-phase reactions near the surface that are coupled to the wall temperature T_w are more important than solid-phase reactions just below the surface.

In the original model it was also necessary to assume a function for the transient behavior of the gas-phase heat release during transition from one steady state to another in order to satisfy the steady-state

conditions at both ends of the calculation. In the new model this is not necessary because the steady-state conditions are met by variations in the reaction rates, but not the heat release, in the two flame zones.

The original formulation went on to use the flame speed equation, which related the instantaneous flow of reactant into the gaseous reaction zone to the instantaneous gas-phase reaction rate, originally proposed by Denison and Baum.[13] Two recent review papers by Culick[1,2], have emphasized the fact that the original gas-phase flame speed treatment by Denison and Baum,[13] which was utilized in the SRI model, is a valid analysis for the flame speed of a deflagration wave proceeding through a gas-filled container such as a tube, but does not account for heat transfer to a solid wall near the flame. In the solid propellant formulation, of course, the calculated flame speed in the gas phase is equated to the burning rate of the surface so that the surface and the flame travel along with a fixed distance between them (in the steady state).

Culick's analyses apply to steady-state conditions, but because the gas-phase portion of the transient propellant combustion problem can be considered quasi-steady, his results can be applied to that problem when the boundary conditions are modified appropriately to account for the transient behavior of the solid.

In view of the observation that most of the surface-coupled heat release appears to be pressure-sensitive ($Q_H >> Q_D$), the starting point for the new model is the assumption that the total heat release takes place in two infinitesimally thick gas-phase zones. One zone is located at the surface and governed by the wall temperature T_w and one is located further out in the gas phase and governed by the final flame temperature T_f.

The flame speed equation relates the instantaneous mass leaving the wall to the instantaneous mass being consumed in the two flame zones, based upon the assumption that the thermal relazation time in the gas phase is very small compared to the thermal relazation time of the solid. In this case

$$\rho_s r = \int \left[w_1 \delta (x-0) + w_2 \delta (x-x_f) \right] dx = W_1 + W_2 \qquad (3)$$

where W_1 is the rate of species production at the surface (x=0), W_2 is the rate of production in the outer flame zone ($x=x_f$), and δ represents the Dirac delta function. Equation (3) states that all of the material leaving the surface has been converted into combustion products at the end of the outer flame zone.

The thermal equation for the gas phase is

$$\rho_s r c_p \frac{dT}{dx} - \frac{d}{dx}\left(k_g \frac{dT}{dx}\right) = \begin{cases} Q_s W_1 & \text{in inner zone} \\ 0 & \text{between zones} \\ Q_g W_2 & \text{in outer zone} \end{cases} \qquad (4)$$

Integration of this equation across the gas phase from the outer edge of the inner flame zone to the outer edge of the outer flame zone gives

$$\rho_s r c_p (T_f - T_w) + \left(k_g \frac{\partial T}{\partial x}\right)_{w+} = Q_g W_2 \qquad (5)$$

where the x coordinate is measured outward from the surface. Across the inner flame zone, taking account of the changes in specific heat and thermal conductivity,

$$\rho_s r (c_p - c_s) T_w - \left(k_g \frac{\partial T}{\partial x}\right)_{w+} + \left(k_s \frac{\partial T}{\partial x}\right)_{w-} = Q_s W_1 \qquad (6)$$

Combining Eqs. (5) and (6) gives

$$\left(k_s \frac{\partial T}{\partial x}\right)_w = Q_s W_1 + Q_g W_2 - \rho_s r (c_p - c_s) T_w - \rho_s r c_p (T_f - T_w) \qquad (7)$$

where the superscript has now been dropped from w and it is understood that this is the boundary condition on the solid-phase temperature profile. It is convenient to add and substract $c_s T_0$, rearrange terms, and use Eq. (3) to get

$$\left(k_s \frac{\partial T}{\partial x}\right)_w = \rho_s r \left[-c_p (T_f - T_0) - (c_p - c_s) T_0 + c_s (T_w - T_0) + Q_s \frac{W_1}{W_1 + W_2} + Q_g \frac{W_2}{W_1 + W_2}\right] \qquad (8)$$

Using the steady-state relation for the derivative of Eq. (8),

$$\left(k_s \frac{\partial T}{\partial x}\right)_w = \rho_s r c_s (T_w - T_0) \qquad (9)$$

it can be seen that in the steady state it must also be true that

$$c_p (T_f - T_0) + (c_p - c_s) T_0 = Q_s \frac{W_1}{W_1+W_2} + \frac{W_2}{W_1+W_2} \tag{10}$$

The steady-state variation of T_f is predicted from thermochemical calculations. Thus, either Q_s and Q_g or W_1 or W_2 must be made to vary through a transient calculation in such a way that the proper steady-state conditions are approached after transition through the transient. In the present model, Q_s and Q_g are considered constant, and W_1 and W_2 are forced to vary in the proper way to satisfy the known steady-state variation of T_f.

The next step is to choose specific relationships for W_1 and W_2. Following the work of Denison and Baum,[13] in view of the fact that all reactions are now associated with the gas phase, let

$$W_1 = K_1 p^{n_1} T_w^{n_1+2} \exp(-E_h/RT_w) \tag{11}$$

and

$$W_2 = K_2 p^{n_2} T_f^{n_2+2} \exp(-E_f/RT_f) \tag{12}$$

In Eqs. (11) and (12), n_1 and n_2 are considered to be constants while E_h and E_f are assumed to be functions of pressure alone since the temperature occurs explicitly in the argument of the exponential. Since E_h and E_f are assumed to be functions of pressure alone, their instantaneous behavior can be calculated from the steady-state equations corresponding to the instantaneous pressure; this technique of solution automatically assures that the thermochemical prediction will be satisfied in the steady state.

Culick[1] discusses the fact it is an approximation to consider a delta function flame zone with a finite reaction rate such as that given by Eq. (11) or Eq. (12). Delta function representation is an approximation that will be valid if the standoff distance x_f is much larger than the thickness of the heat release zone, which is true under most circumstances of interest.

An expression for the outer flame zone location can be conveniently

obtained by use of the transformation of Culick[2] for the gas-phase thermal equation:

$$\zeta = \exp\ (\rho_s r c_p x/k_g) \tag{13}$$

If it is assumed that k_g is constant, the thermal equation between the two flame zones becomes

$$\frac{d^2T}{d\zeta^2} = 0 \tag{14}$$

i.e., the temperature profile is linear between the two heat release zones in the ζ-coordinate system. Using the heat transfer rate at the outer edge of the inner heat release zone and the known flame temperature as the two boundary conditions, one can solve Eq. (14) to obtain

$$\zeta_f = \frac{Q_g}{Q_g - c_p(T_f - T_w)\ (1+\beta)} \tag{15}$$

where $\beta = W_1/W_2$. Equation (15) reduces to Culick's result[2] when there is no heat release associated with the inner boundary; i.e., when Q_s and W_1 (and therefore β) are taken as zero. The physical location of the outer heat release zone x_f is then given by

$$x_f = \frac{k_g}{\rho_s r c_p}\ \ell n\ \zeta_f \tag{16}$$

The remaining point to be considered is the calculation of reference values for the activation energies E_h and E_f. According to the present analysis, reference values are calculated by choosing a value for the magnitude of the linear response at the point on the burning rate curve where an intersection with the "stability limit line" occurs. This is illustrated in Fig. 1 where the burning rate curves of the one SRI propellant (PU-269) and the five UTC propellants studied under this program are shown with the stability limit line from Ref. 10. (See also Refs. 9 and 12 for more details.) The limit line shown in Fig. 1 can be approximated for computational purposes by the equation

$$r_\ell = 0.00239\ p_\ell^{0.726} \tag{17}$$

where r_ℓ is given in in./sec and p_ℓ in psia.

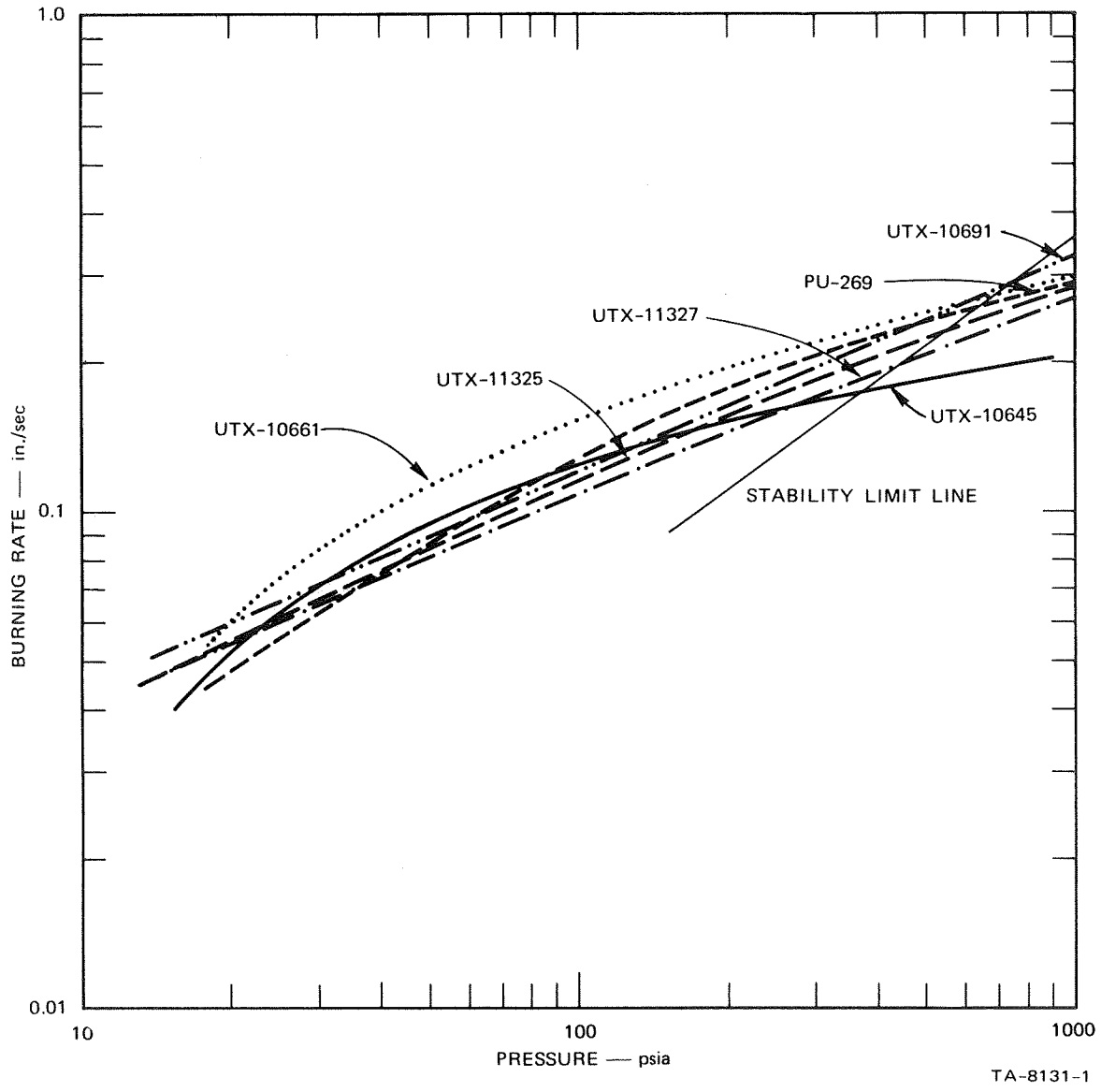

FIGURE 1 STRAND BURNING RATE VERSUS PRESSURE

Equation (17) is a least-squares fit of all the traveling wave instability data obtained at SRI.[9]

Two parameters govern the linear response, A and α. The parameter A is a pyrolysis kinetics parameter given by

$$A = \frac{E_w}{R\bar{T}_w}\left(1 - \frac{T_o}{\bar{T}_w}\right). \tag{18}$$

The parameter α, first introduced by Denison and Baum,[13] is a kinetics parameter related to the chemistry of the two reaction zones. It is obtained from a perturbation of the boundary condition at the wall, Eq. (8), where the perturbation is written in the form[7,13]

$$\frac{k_s}{\rho_s c_s \bar{r}}\left(\frac{\partial \tilde{T}_w}{\partial x}\right)_w = \left[1 + A\,(1-\alpha)\right]\tilde{T}_w + \nu\alpha\left(1 - \frac{T_o}{\bar{T}_w}\right)\tilde{\rho} \tag{19}$$

For a chosen constant level of linear response, the relationship between the two governing linear parameters A and α can be approximated by an expression of the form

$$\alpha = a_1 \ln A + a_2 \tag{20}$$

An expansion of Eq. (8) cast into the form of Eq. (19) using Eq. (18) for A, results in two expressions for α which can be solved simultaneously to obtain expressions for E_{f_ℓ} and E_{h_ℓ}, the variable activation energies, at the stability limit. Once values of E_{f_ℓ} and E_{h_ℓ} are established, values of E_f and E_h through any transient process can be established by steady-state equations since they are functions of pressure only. The value of α can also be calculated at any other pressure; the equation used for this in the numerical analysis is

$$\alpha = \frac{\bar{T}_w}{\bar{T}_w - T_o}\left\{ q_s w_1 + q_g w_2 - \frac{R\bar{T}_w}{E_w} q_s w_1 N_1 + \left[\frac{c_p \bar{T}_f}{c_s \bar{T}_w} - q_g w_2 N_2\right]\left[\frac{1 - \dfrac{R\bar{T}_w}{E_w} w_1 N_1}{w_2 N_2}\right]\right\} \tag{21}$$

where

$$q_s = \frac{Q_s}{c_s \bar{T}_w} \qquad q_g = \frac{Q_g}{c_s \bar{T}_w}$$

$$w_1 = \frac{\bar{W}_1}{\bar{W}_1+\bar{W}_2} \qquad w_2 = \frac{\bar{W}_2}{\bar{W}_1+\bar{W}_2} = 1-w_1$$

$$N_1 = n_1+2+\frac{\bar{E}_h}{R\bar{T}_w} \qquad N_2 = n_2+2+\frac{\bar{E}_f}{R\bar{T}_f}$$

A complete derivation of the technique for obtaining E_{h_ℓ} and E_{f_ℓ} and the expression shown in Eq. (21) is given in Appendix A.

The calculation of the transient behavior of any propellant requires starting calculations at both the limiting pressure and the initial pressure to establish initial values of the governing propellant parameters. The following steps are involved:

1. Choose reference values of T_w at a burning rate of 0.1 in./sec and of T_f at 100 psia. Any reference points could be chosen as long as they are used in a consistent manner.

2. Choose values for the propellant parameters ν, ρ_s, c_s, c_p, k_g, and γ.

3. Choose values for the kinetic parameters E_w, n_1, n_2, and m_1. The parameter m_1 relates the measured propellant burning rate at a given pressure to the calculated thermochemical flame temperature at the same pressure through the simple equation

$$T_f^* = r^{*m_1} \tag{22}$$

for the steady state.

4. Assign values to the heat releases Q_s and Q_g that are considered to be constant.

5. Assign values to the constants a_1 and a_2, corresponding to the chosen response; i.e., the chosen value of α at the threshold

Pressure. For the current calculations, $a_1 = 0.226$ and $a_2 = 0.170$ have been used, corresponding to a linear response level of

$$\left| \frac{1}{\nu} \frac{\tilde{r}}{\tilde{p}} \right| \simeq 7,$$

a reasonable value for most composite AP propellants.[7]

6. At the threshold pressure point:
   a. Calculate the threshold pressure p_ℓ by satisfying both the known burning rate/pressure dependence and Eq. (17).
   b. Using the procedure outlined in Appendix A, calculate the activation energies E_{h_ℓ} and E_{f_ℓ}.
7. At the chosen initial pressure:
   a. Calculate W_{1_i} and W_{2_i} from the two simultaneous equations

$$\left[c_p (T_{f_i} - T_0) + (c_p - c_s)T_0 - Q_s\right] W_{1_i} = \left[Q_g - c_p(T_{f_i} - T_0) - (c_p - c_s)T_0\right] W_{2_i} \quad (23)$$

$$\frac{r_i}{r_\ell} = \frac{W_{1_i} + W_{2_i}}{W_{1_\ell} + W_{2_\ell}} \quad (24)$$

   b. Use the calculated values of W_{1_i} and W_{2_i} with Eqs. (11) and (12) to calculate E_{h_i} and E_{f_i}.

A listing of the numerical program and a sample printout for a typical calculation are given in Appendix B.

A short comment should be made concerning the use of the transformation

$$y^* = 1 - e^{-x^*} \quad (25)$$

where $x^* = r_i x/\kappa$. (The distance x is measured positive into the propellant away from its surface for the solid phase calculation.) This transformation changes the infinite spatial domain in the solid $0 \leq x^* \leq \infty$ into a

finite domain $0 \leq y^* \leq 1$ in the y^* coordinate system. It was first used by the author in Ref. 11. Recently Merkle et al.[4] have pointed out that great care must be exercised in applying this transformation because the temperature derivative at the cold boundary ($y^* = 1$) is nonanalytic. This factor can easily be seen if we consider the temperature profile corresponding to the transformation of Eq. (25):

$$\frac{T^*-T_0^*}{T_w^*-T_0^*} = (1-y^*)^{r^*} \tag{26}$$

Another advantage of the transformation is shown by Eq. (26); i.e., initially the temperature profile is linear in the transformed coordinate. The temperature derivative is now given by

$$\frac{\partial T^*}{\partial y^*} \sim r^* \, (1-y^*)^{r^*-1} \tag{27}$$

For $r^* < 1$ and $y^* = 1$, the derivative then becomes infinite and one must take care in the numerical analysis to be sure that this singularity must not be allowed to affect the computation of the temperature profile. This computation is accomplished as described in detail in Appendix B of Ref. 11 by making use of the fact that $T^* = T_0^*$ at $y^* = 1$; i.e., only the derivatives at the other mesh points are needed to obtain the total solution and no difficulty has ever been encountered in obtaining the correct steady-state temperature profile after a decrease in burning rate. An example of such a calculation is shown in detail in Appendix C. In addition to the advantages cited previously, use of the transformation given by Eq. (25) allows an important saving of numerical computation time by reducing substantially the number of mesh points required inside the solid.

III PARAMETRIC STUDY OF THE MODEL

Before proceeding to a comparison of predicted and observed extinction behavior it is instructive to carry out a parametric study of the model in both the linear and nonlinear regimes to determine which of the chosen input parameters have the largest effects on the observed output. In this case the effects of varying the parameters Q_s/Q_t, Q_t, n_1, n_2, E_w, T_w, and κ have been studied under conditions of a step pressure change of 1% and 20% (the 20% change was actually applied as a steep ramp over a time period of 0.2 msec) at 1000 psia chamber pressure using typical values for the SRI PU-269 AP propellant. The computed thermochemical flame temperature of this propellant is shown in Fig. 2 with the flame temperatures of the five UTC propellants that will be considered in the next section.

In addition to the parameters listed above, a few other inputs are needed to start the program. Values used for these inputs are:

$$c_s = c_p = 0.4$$

$$T_0 = 300^0K$$

$$\gamma = 1.2$$

$$k_g = 0.0004$$

$$\rho_s = 0.06$$

It can be shown that variations in ρ_s, which may at first appear to be an important parameter, do not affect the response. Figure 3 shows the effect of varying Q_s/Q_t, the ratio of heat release associated with the surface reaction zone to the total heat release. At the chosen chamber pressure, which is higher than the pressure at which the burning rate curve crosses the stability limit line (see Fig. 1), the response increases as Q_s/Q_t increases. Thus, more surface heat release is destabilizing; the effect

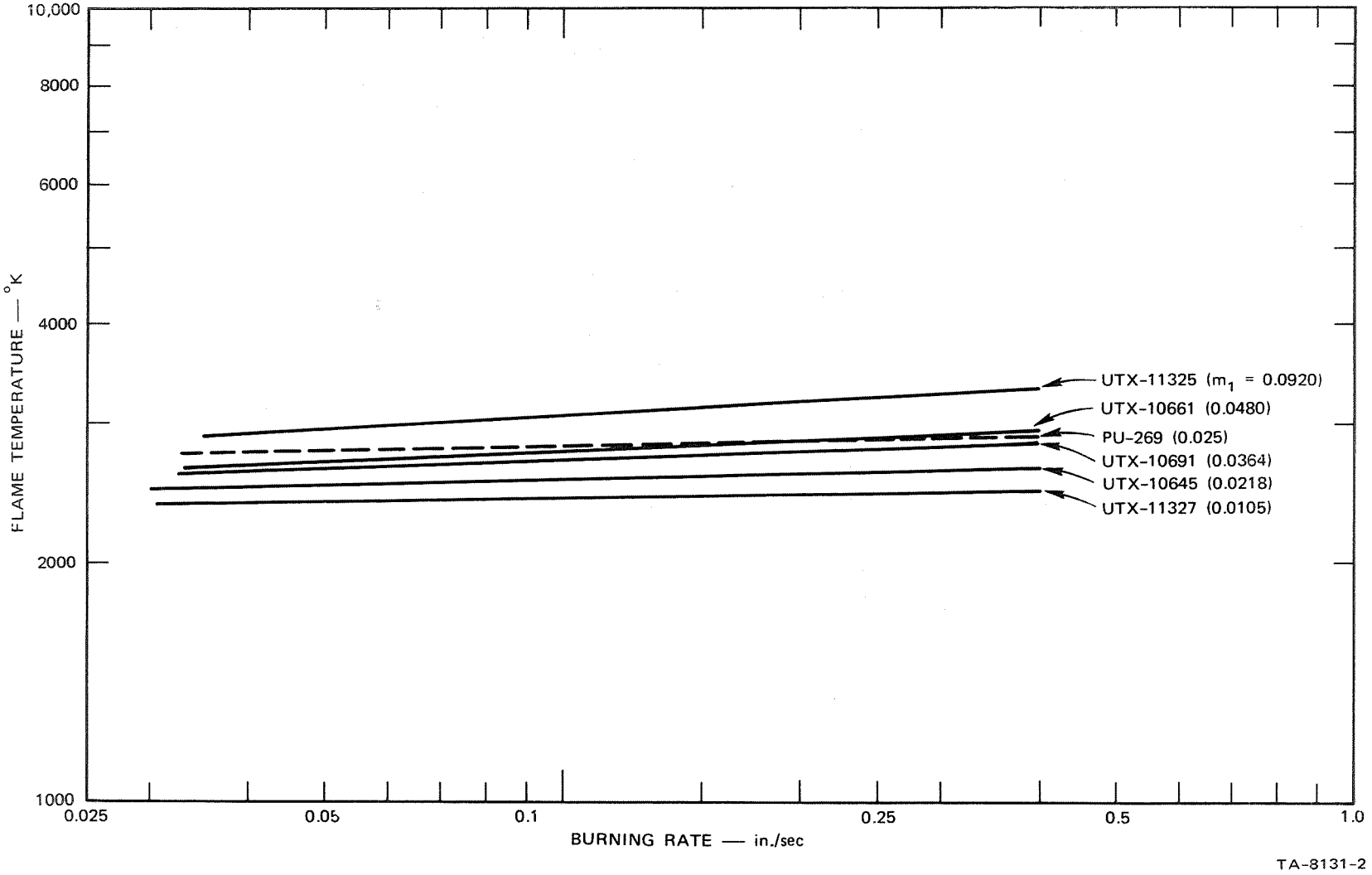

FIGURE 2 FLAME TEMPERATURE VERSUS BURNING RATE

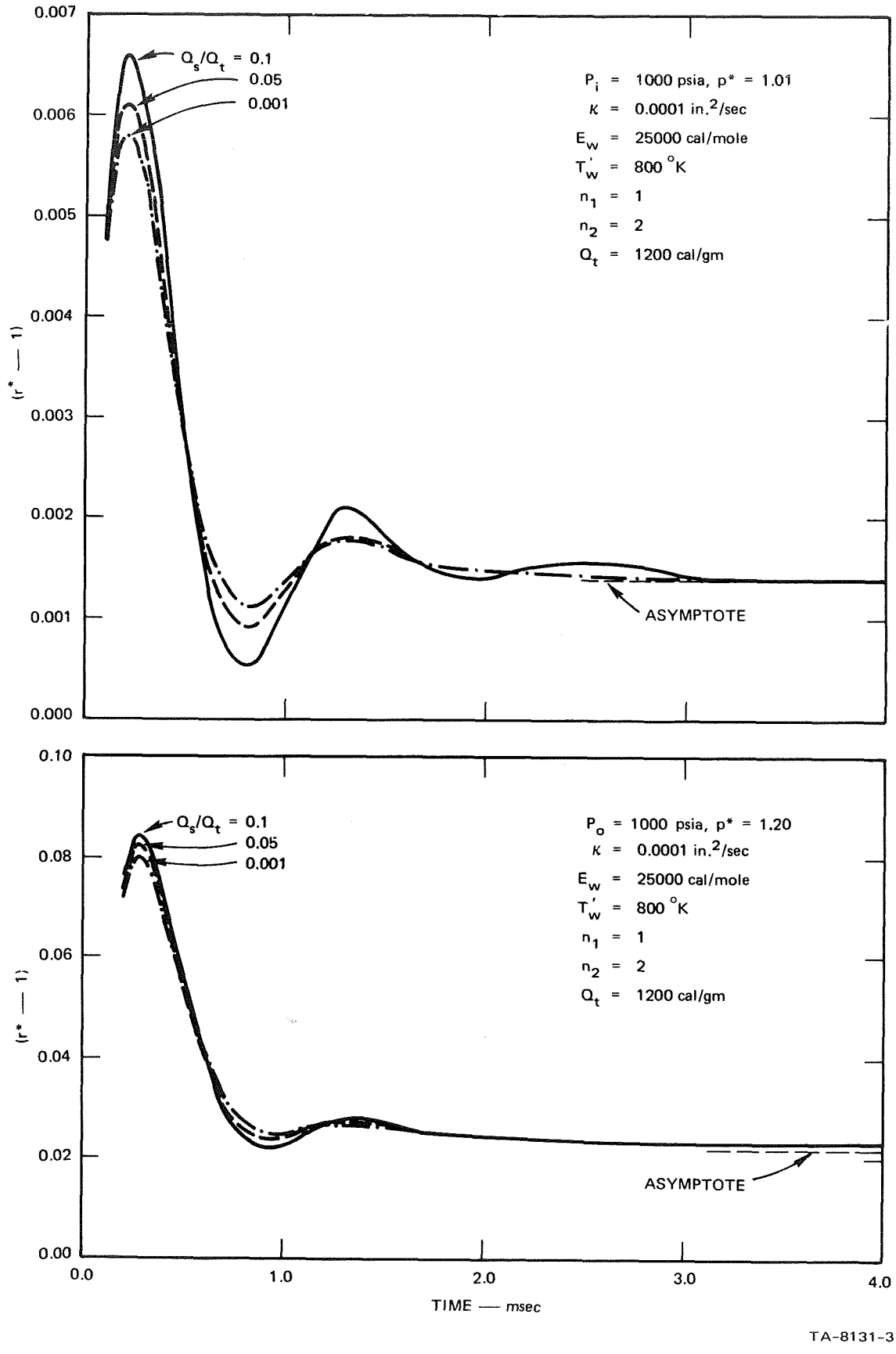

FIGURE 3 EFFECT OF VARIABLE Q_s/Q_t AT HIGH PRESSURE (PU-269)

is proportionately (but not absolutely) smaller for the larger pressure increase. Note that the asymptotic steady state is also shown.

An extremely interesting result is obtained when the effect of variable Q_s/Q_t at a pressure below the stability limit crossing is considered. In Fig. 4, where results are plotted for a chamber pressure of 400 psia, it can be seen that an increase in Q_s/Q_t reduces the response; in addition, the overall response level appears to be larger than that shown in Fig. 3a.

This behavior can be understood if we plot the burning rate response in its proper nondimensional form, shown in Fig. 5 for Q_s/Q_t = 0.05 and 0.1. Note that the curves cross at the pressure (645 psia) where the burning rate curve crosses the stability limit line in Fig. 1; this is built into the numerical analysis through the specification of the parameter α at that point. The salient fact is the increase in slope as Q_s/Q_t increases; the absolute value of the slope is governed by the chosen value of Q_t. This increase in slope may explain why propellants with surface-coupled heat release tend toward instability: not only does their response reach a certain level at the stability limit, but it also tends to increase more rapidly as the pressure is increased beyond that limit point. This result may help to explain the small scatter that is observed experimentally about the stability limit line.[9,10,12]

Figure 6 shows the effect of varying the total heat release on the observed burning rate response. It can be seen that increasing the heat release as the ratio Q_s/Q_t remains constant decreases the response somewhat. Numerical computations have shown that the allowable range of choice for Q_t is rather limited once values have been assigned to the other parameters. In the present case when Q_t is reduced below about 1100 cal/gm, the ratio W_1/W_2 becomes negative, implying that the main flame requires more species for consumption than are leaving the wall, a physically meaningless case. On the other hand, when Q_t is increased above about 1400 cal/gm, the slope of the curves shown in Fig. 5 becomes negative, which violates a basic assumption of the theory; namely, that the observed burning rate response should increase with increasing pressure. These results are in agreement with Eq. (10), which shows that for the values of c_p and T_f

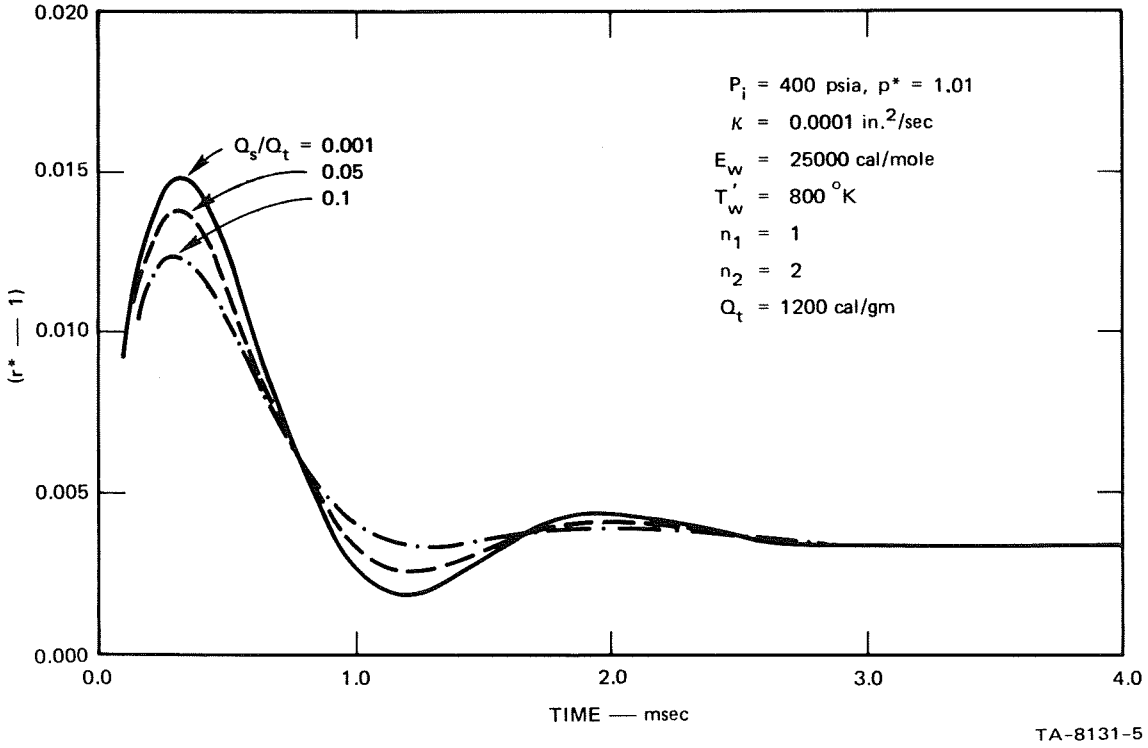

FIGURE 4 EFFECT OF VARIABLE Q_s/Q_t AT LOW PRESSURE (PU-269)

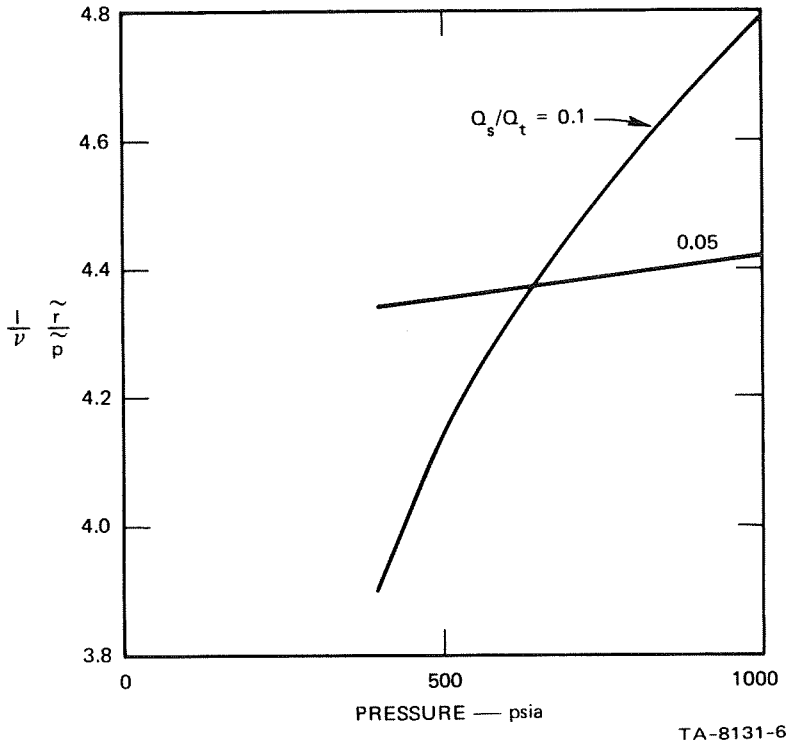

FIGURE 5 LINEAR BURNING RATE RESPONSE AS A FUNCTION OF PRESSURE (PU-269)

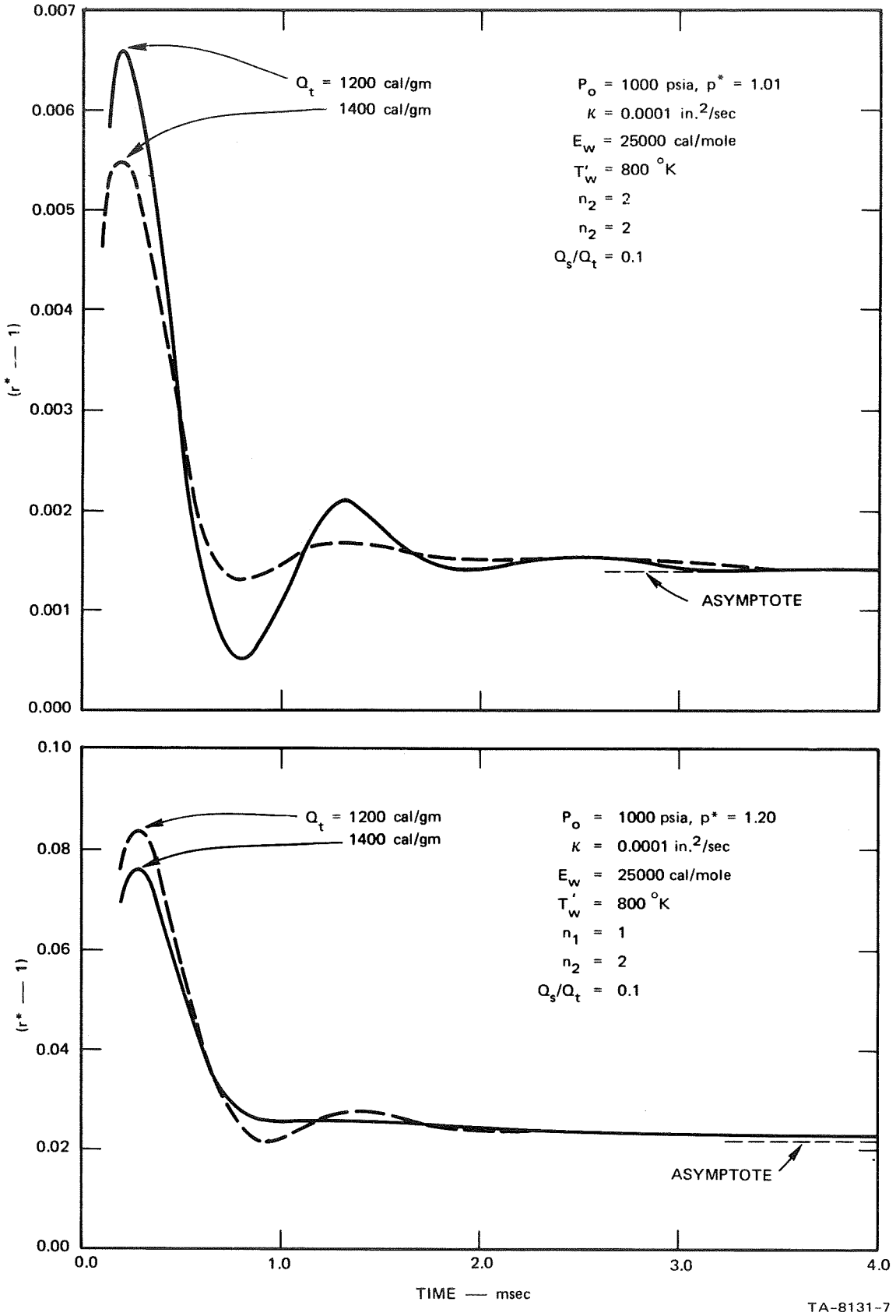

FIGURE 6 EFFECT OF VARIABLE Q_t AT HIGH PRESSURE (PU-269)

being used the total heat release should be in the neighborhood of 1200 cal/gm, lending further consistency to the calculations.

Figures 7 and 8 show the effects of varying the reaction rate pressure exponents n_1 and n_2. It can be seen that a variation in n_1 affects the burning rate response slightly whereas a variation in n_2 has practically no effect. This apparently occurs because changes in n_1 or n_2 are offset by changes in the calculated behavior of E_h and E_f with pressure, leading to nearly the same burning rate response.

The effect of varying the pyrolysis activation energy at the surface is shown by Fig. 9. Increasing the activation energy in general acts to decrease the burning rate response time and therefore to damp the observed oscillation. Decreasing the chosen surface temperature, on the other hand, also acts to decrease the response time but does not destroy the oscillation so readily, as shown by Fig. 10.

Figure 11 shows the effect of varying the thermal diffusivity $\varkappa$. As one would expect, the characteristic response time of the burning rate is proportional to $\varkappa$; i.e., the first peak for $\varkappa = 0.0005$ in.^{2}/sec occurs at a time that is approximately a multiple of five times the value of the time at which the curve for $\varkappa = 0.0001$ in.^{2}/sec exhibits its peak.

With the calculations just discussed as a background, it is now appropriate to turn to a discussion of experimental data.

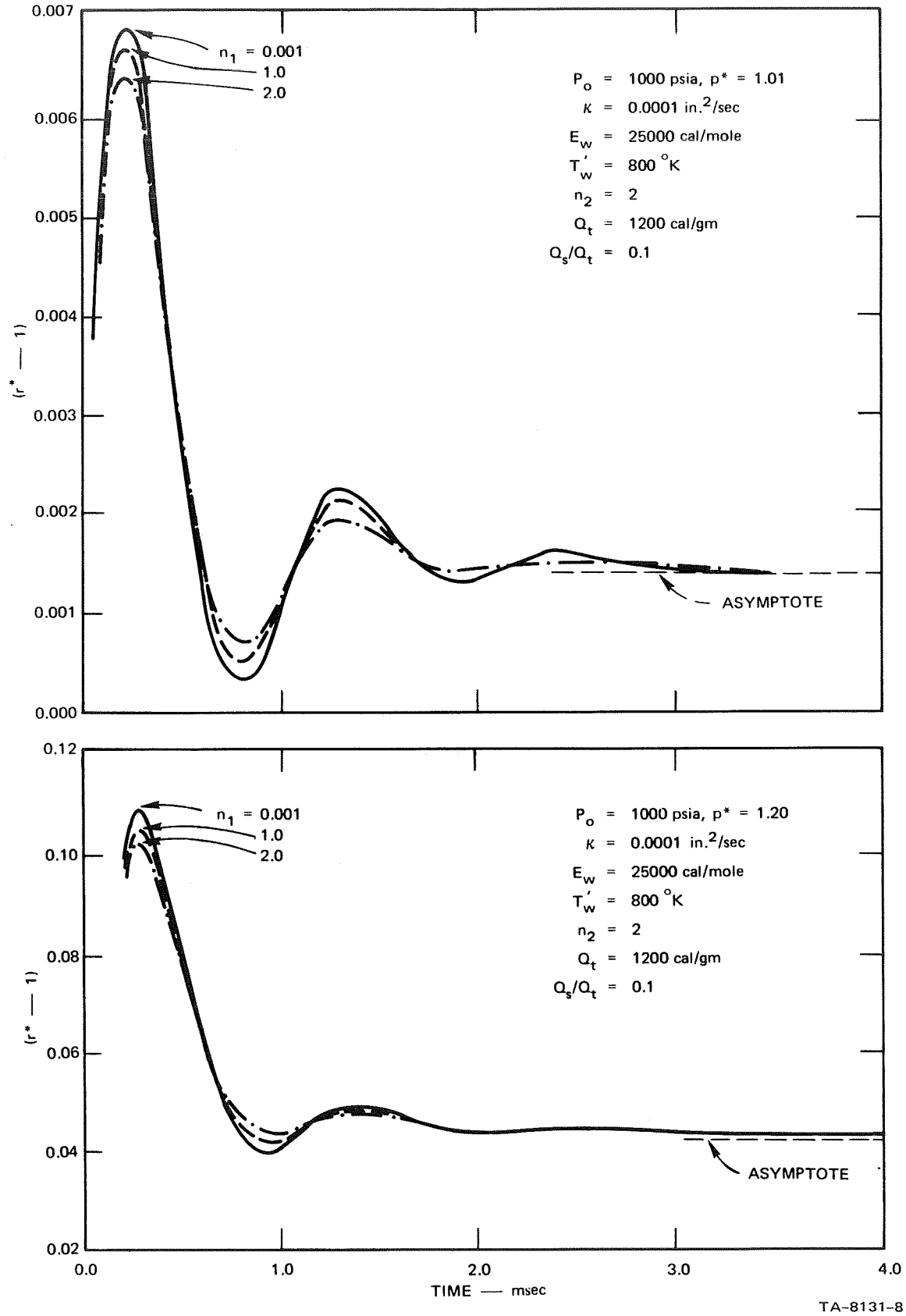

FIGURE 7 EFFECT OF VARIABLE n_1 AT HIGH PRESSURE (PU-269)

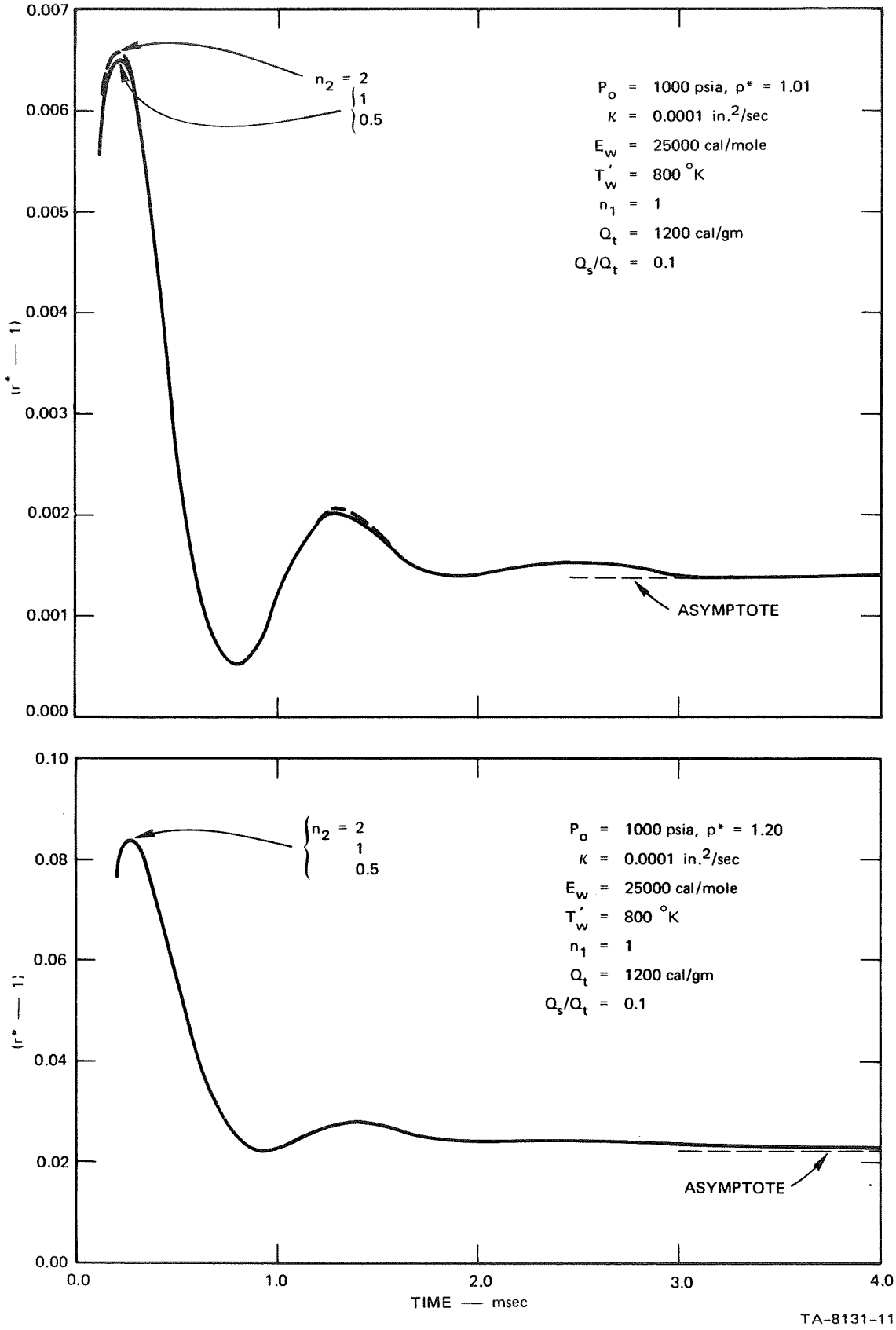

FIGURE 8 EFFECT OF VARIABLE n_2 AT HIGH PRESSURE (PU-269)

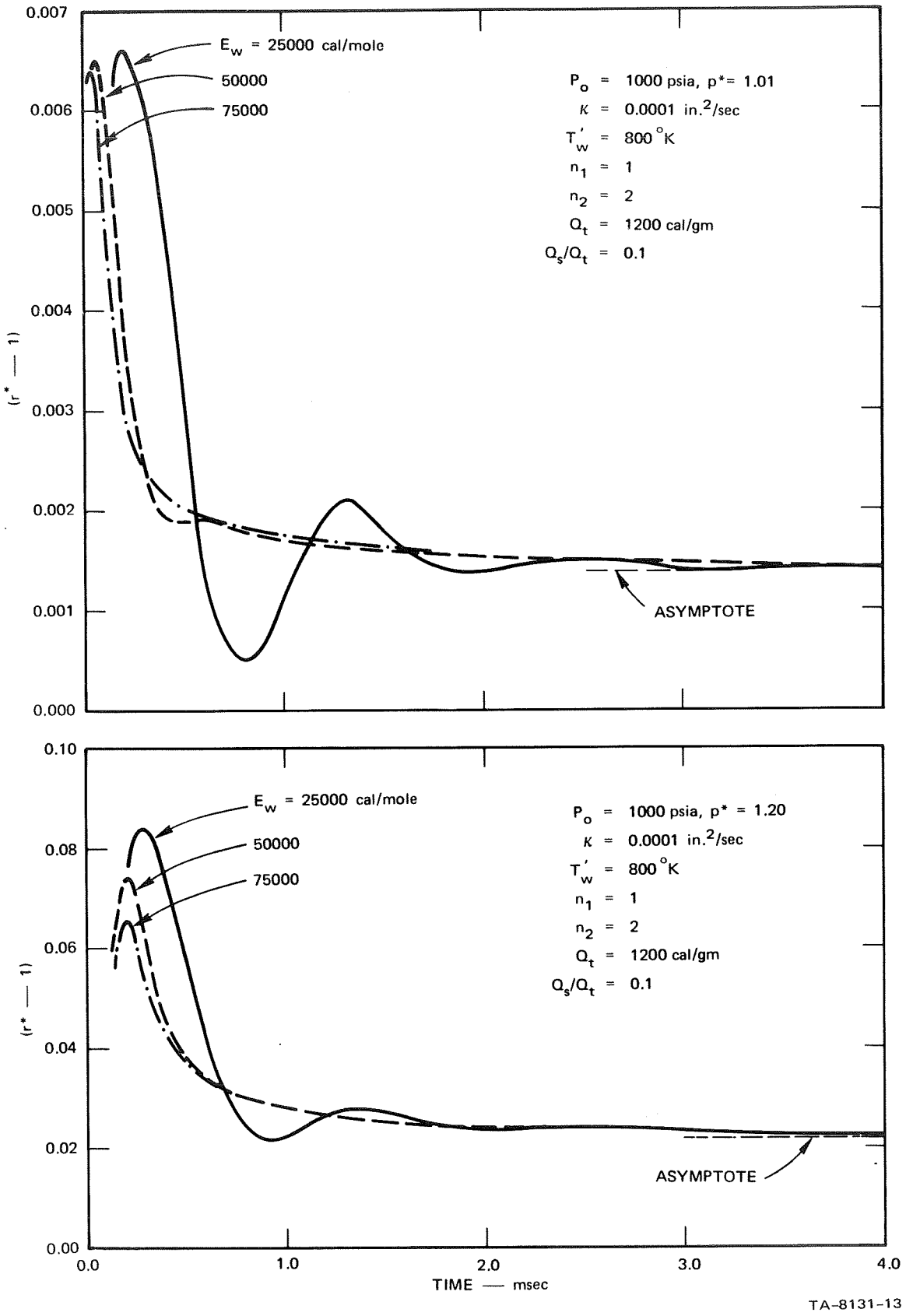

FIGURE 9 EFFECT OF VARIABLE E_w AT HIGH PRESSURE (PU-269)

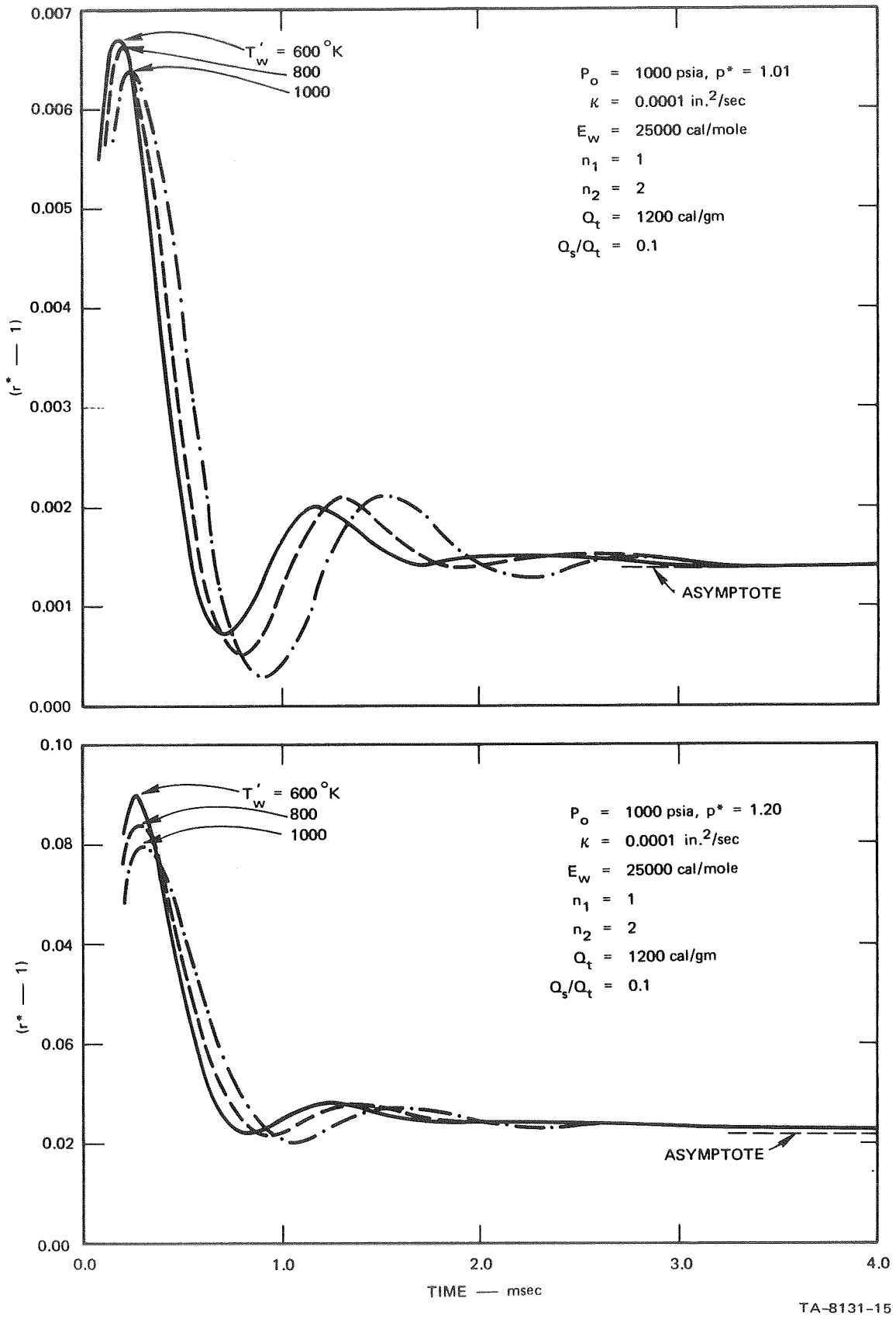

FIGURE 10 EFFECT OF VARIABLE T_w' AT HIGH PRESSURE (PU-269)

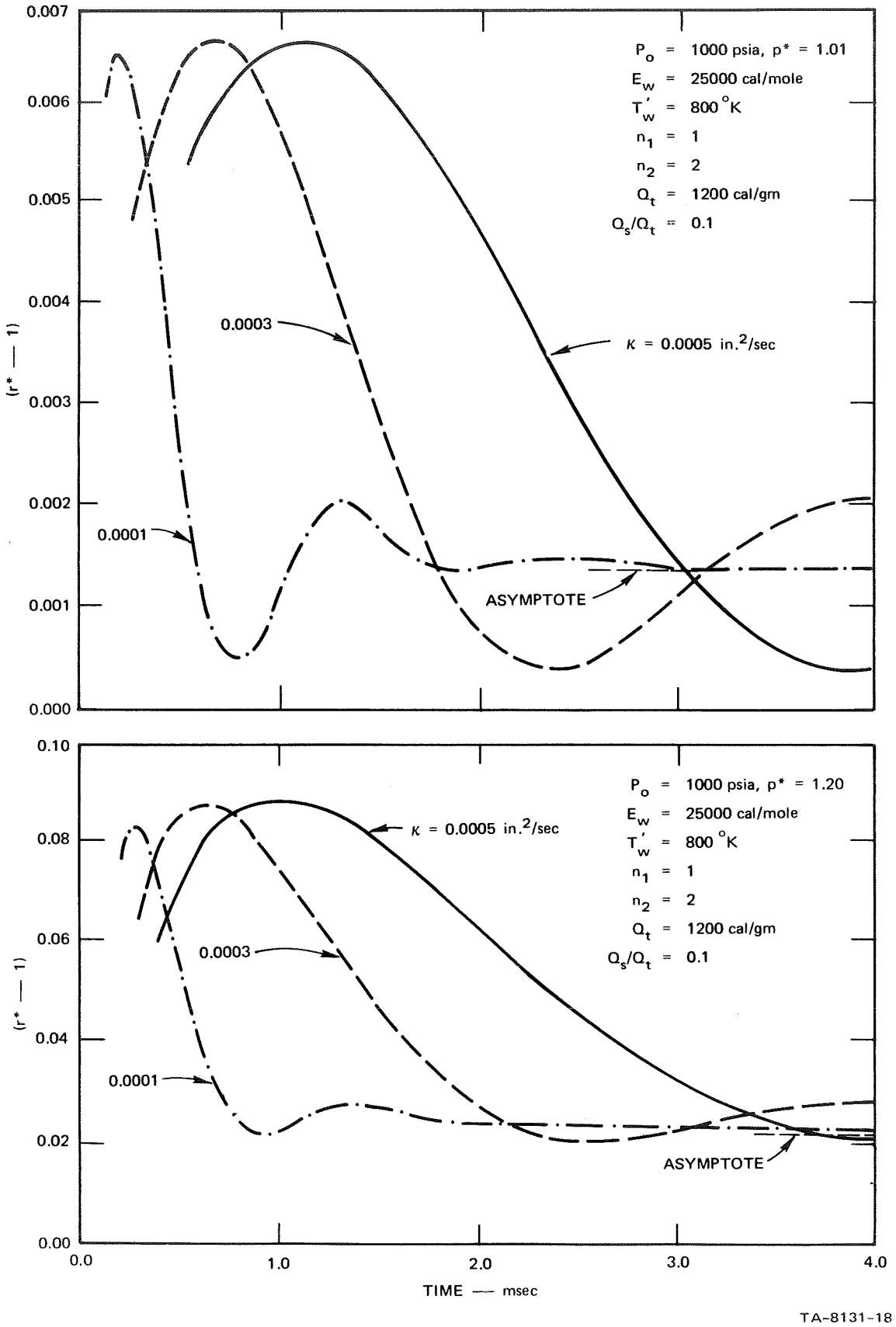

FIGURE 11 EFFECT OF VARIABLE κ AT HIGH PRESSURE (PU-269)

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IV COMPARISONS WITH EXPERIMENTAL DATA

Pressurization Data

Some pressure response data were previously obtained at SRI by rapidly pressurizing a chamber having a very small value of V/A_b ($\sim$0.05-0.1) with external mass addition.[11] The objective was to have a small enough free volume compared to the burning surface area so that the burning rate response would measurably affect the pressure response, and, in fact, an experimentally oscillating pressure response was obtained.[11] The previous version of the SRI model appeared to respond as though the burning rate followed the pressure too closely, and no oscillation was obtained.[11]

Figure 12 shows the previous data and the pressure response of the current model. In this case the calculated pressure is closer to the observed pressure early in the experiment, but clearly no undershoot is obtained. More experiments of this type with other propellants are obviously needed to determine whether the experimental procedure or the mathematical model is responsible for the poor agreement that is observed. The main thrust of this program was to compare the model predictions with the extensive depressurization data collected at UTC,[5,6] as well as those of SRI.[8,11] These comparisons follow.

Depressurization Data

Far more data are available in the case of depressurization because of its practical importance in extinguishment. To compare the theoretical model and these results with the least amount of numerical computing, it was decided to proceed by holding as many of the variables as possible constant. The results of the preceding section show that the two most influential variables are κ, which determines the time constant of the propellant response, and Q_t, which must lie between certain bounds in order both to assure the proper variation of α with pressure and to maintain a positive value for the ratio W_1/W_2.

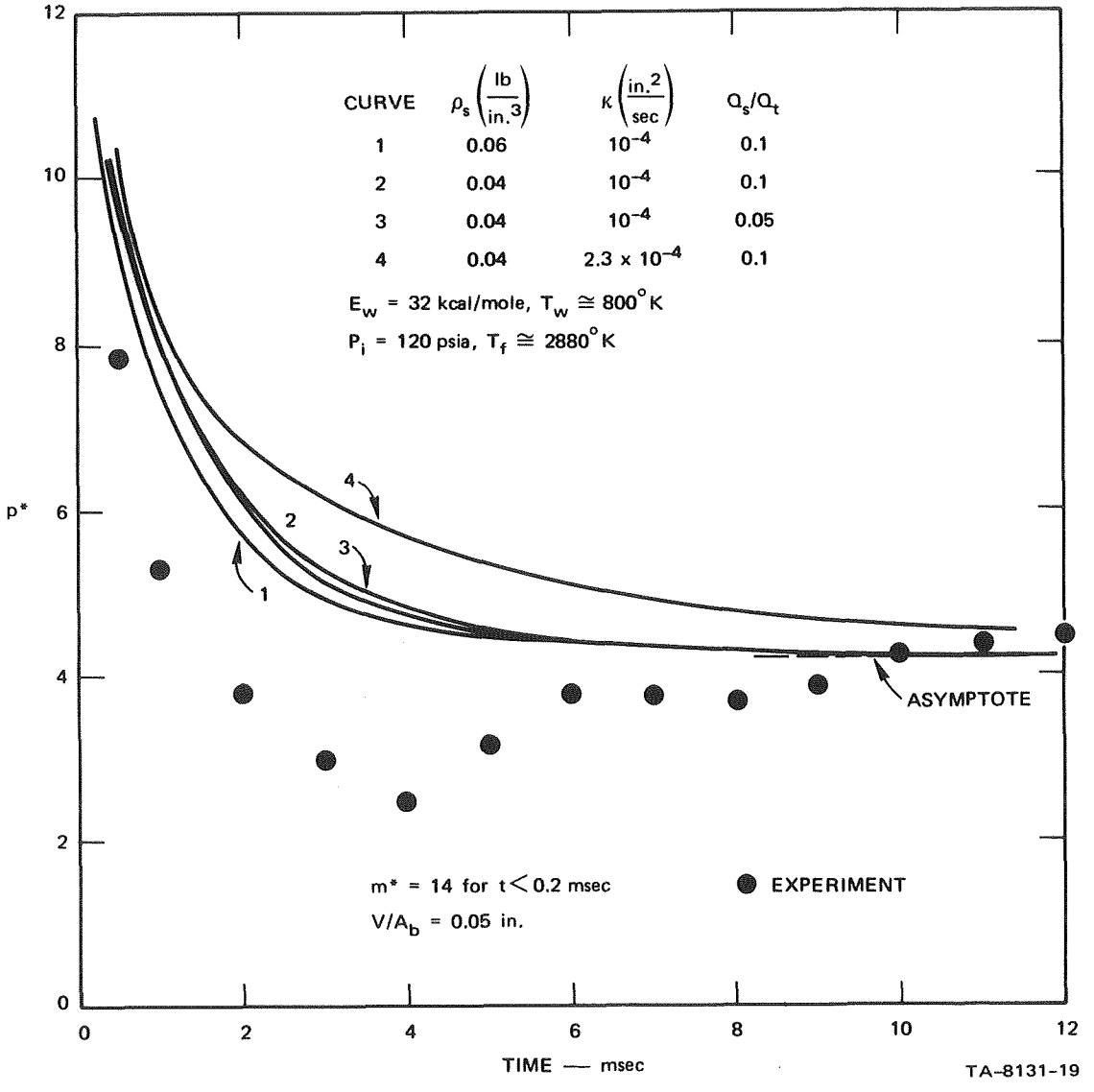

FIGURE 12 COMPARISON OF THEORY WITH OBSERVED PRESSURIZATION RESPONSE (PU-269)

Previous results obtained during depressurization tests on PU-269 can be used to establish values of the constants (excluding those already listed on Page 15); the values are

$$\kappa = 0.0001 \text{ in}^2/\text{sec}$$

$$T'_w = 800^\circ\text{K}$$

$$n_1 = 1$$

$$N_2 = 2$$

$$E_w = 50{,}000 \text{ cal/mole}$$

$$Q_s/Q_t = 0.1$$

Use of the above values together with $Q_t = 1200$ cal/gm for PU-269 gives good agreement between calculated and observed depressurization rates required for extinction.

Figure 13 shows the calculated behavior of the burning rate and pressure of this propellant at two initial depressurization rates in a small chamber containing an end-burning grain. Extinction data for this propellant in the SRI test apparatus have previously been reported; the critical initial depressurization rate required to reach extinction is somewhat in excess of 100,000 psi/sec.[11] Figure 13 is interesting because of the oscillatory behavior exhibited by the burning rate at a depressurization rate of 100,000 psi/sec. Imposition of a higher rate merely leads to extinction sooner and, furthermore, the burning rate oscillations are much less intense.

The procedure followed during the remaining calculations, then, was to vary only the total heat release Q_t in order to satisfy the restrictions mentioned before. Such an approach assumes that the only propellant characteristics in addition to Q_t that affect the transient response are the measured burning rate behavior and the calculated thermochemical flame temperature behavior, since these are the only additional inputs (except for the motor geometry) to the numerical program.

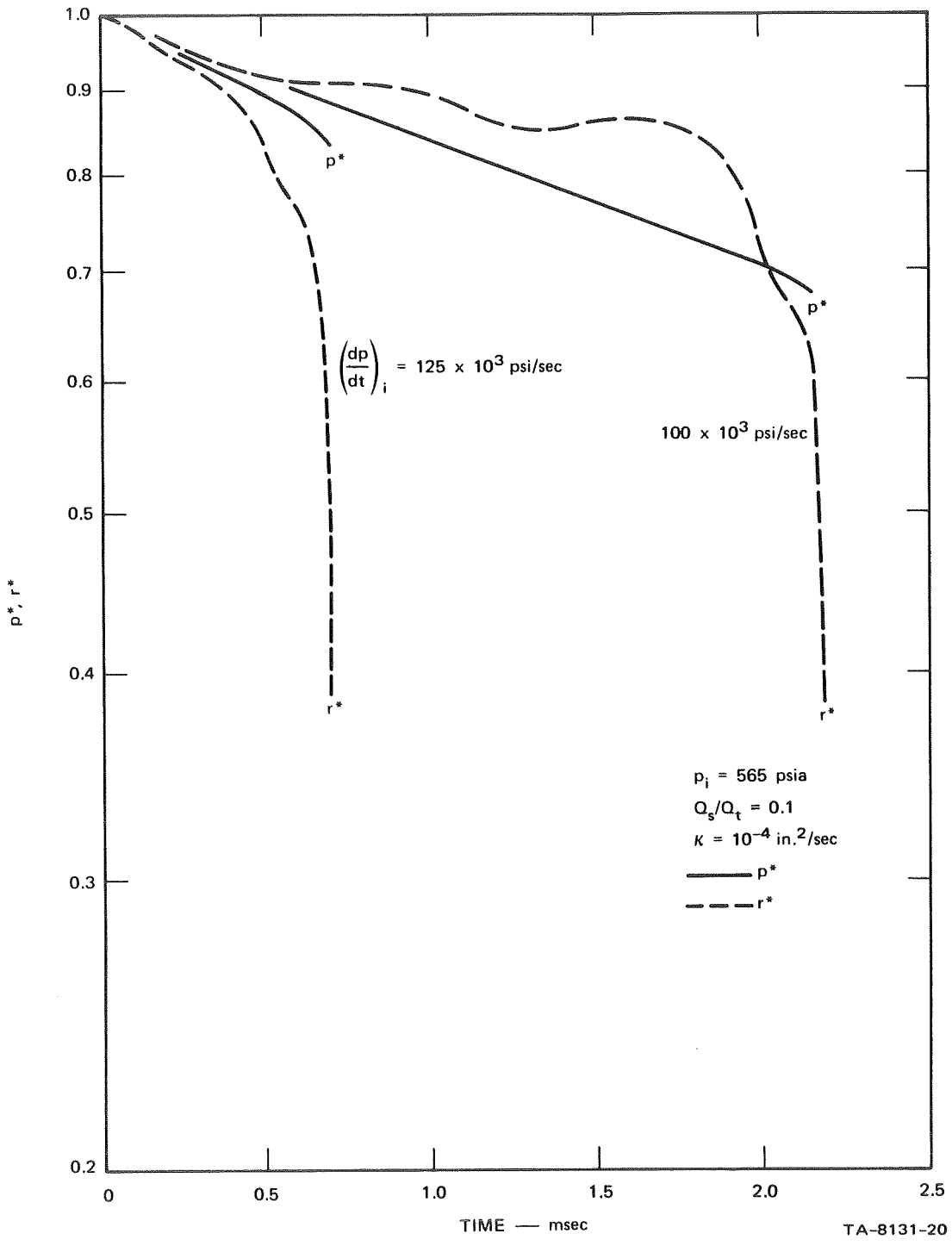

FIGURE 13 CALCULATED EXTINCTION BEHAVIOR (PU-269)

A great deal of extinction data has been obtained for NASA at UTC; a complete description of their studies is given in Refs. 5 and 6. The UTC study was concerned with the effects of systematic variations in propellant binder, oxidizer loading level, burning rate catalyst, metal loading, and exhaust pressure level. From the myriad results presented in Ref. 6, five propellants and three motor geometries were chosen for comparison with and three motor geometries were chosen for comparison with the SRI theory. A description of these propellants is given in Table I (the SRI propellant is included for completeness); they will hereafter be cited by their numerical designation.

Table I

PROPELLANT FORMULATIONS CONSIDERED DURING THIS PROGRAM

Designation	Binder (%)	Oxidizer (%)	Metal Loading (%)
UTX-10645	16.2 CTPIB	83.6 AP	— — —
UTX-10661	16.7 PU	83.1 AP	— — —
UTX-10691	16.2 CTPB	83.1 AP	— — —
UTX-11325	16.2 CTPB	67.6 AP	16% Al
UTX-11327	20.2 CTPB	79.6 AP	— — —
PU -269	20 PU	80 AP	— — —

The CTPIB (carboxy-terminated polyisobutylene), PU (polyurethane), and CTPB (carboxy-terminated polybutadiene) propellants were selected for comparison with the model because of their diverse thermal and oxidative degradation characteristics. In addition, one propellant containing aluminum was chosen for comparison. According to Jensen,[6] the CTPIB and PU formulations are less subject to exothermic oxidative degradation of the binder than are the CTPB formulations, but are more subject to endothermic thermal decomposition. The AP used in each propellant consisted of a 65:35 coarse-to-fine ratio with the mean particle diameter of the coarse AP being 190 μ and that of the fine approximately 6 μ. In the case of the aluminized propellant (UTX-11325), the aluminum replaced part of the coarse AP. Each of the UTC propellants contained 0.2% carbon.

Three motor geometries were used in the UTC work: a slab motor, a window motor, and a swing-nozzle motor. The slab motor was a small motor containing 1-lb slabs and having $V/A_b = 9.8$. The window motor also contained slabs, but had a volume to burning surface area ratio $V/A_b = 4.4$. Finally, the swing-nozzle motor utilized internal-burning cylindrical grains and contained a much higher propellant loading with $V/A_b = 0.30$ for the 5-in. long grain configuration and 0.24 for the 15-in. long configuration. A more detailed description of these motors is given in Ref. 6.

Figure 14 compares calculated and observed results for the standard CTPIB propellant (UTX-10645) burned in the slab motor and the window motor. For this propellant, a value of Q_t=1100 cal/gm was used. Note that, in general, the model suggests that a larger depressurization rate should be required in the window motor because of its smaller value of V/A_b; however, such a rate is not discernable experimentally.

The calculated transient behavior of the burning rate and pressure in the slab motor at a pressure of 200 psia is shown in Fig. 15; note that in this case no oscillation is visible in the burning rate. As will presently be shown, it occurs only under certain depressurization conditions.

Similar results are shown for the UTC polyurethane formulation (UTX-10661) in the slab and window motors in Figs. 16 and 17. For this propellant a value of Q_t=1200 cal/gm was used, equal to the value used for PU-269. Note once again that better agreement is obtained with the slab motor data and that no oscillation is observed in the calculated burning rate.

Extinguishment characteristics of the UTX-10645 CTPIB propellant considered previously and of the UTX-10691 CTPB propellant (for which 1200 cal/gm was used for Q_t in the calculations) are shown in Fig. 18 for the swingnozzle motor in which V/A_b is an order of magnitude lower. Note that the required depressurization rates are correspondingly an order of magnitude higher than those shown in Fig. 14. In this case the calculated values fall somewhat below the experimental values, perhaps because the numerical calculation is strictly one-dimensional, whereas the apparatus uses cylindrical internal-burning grains.

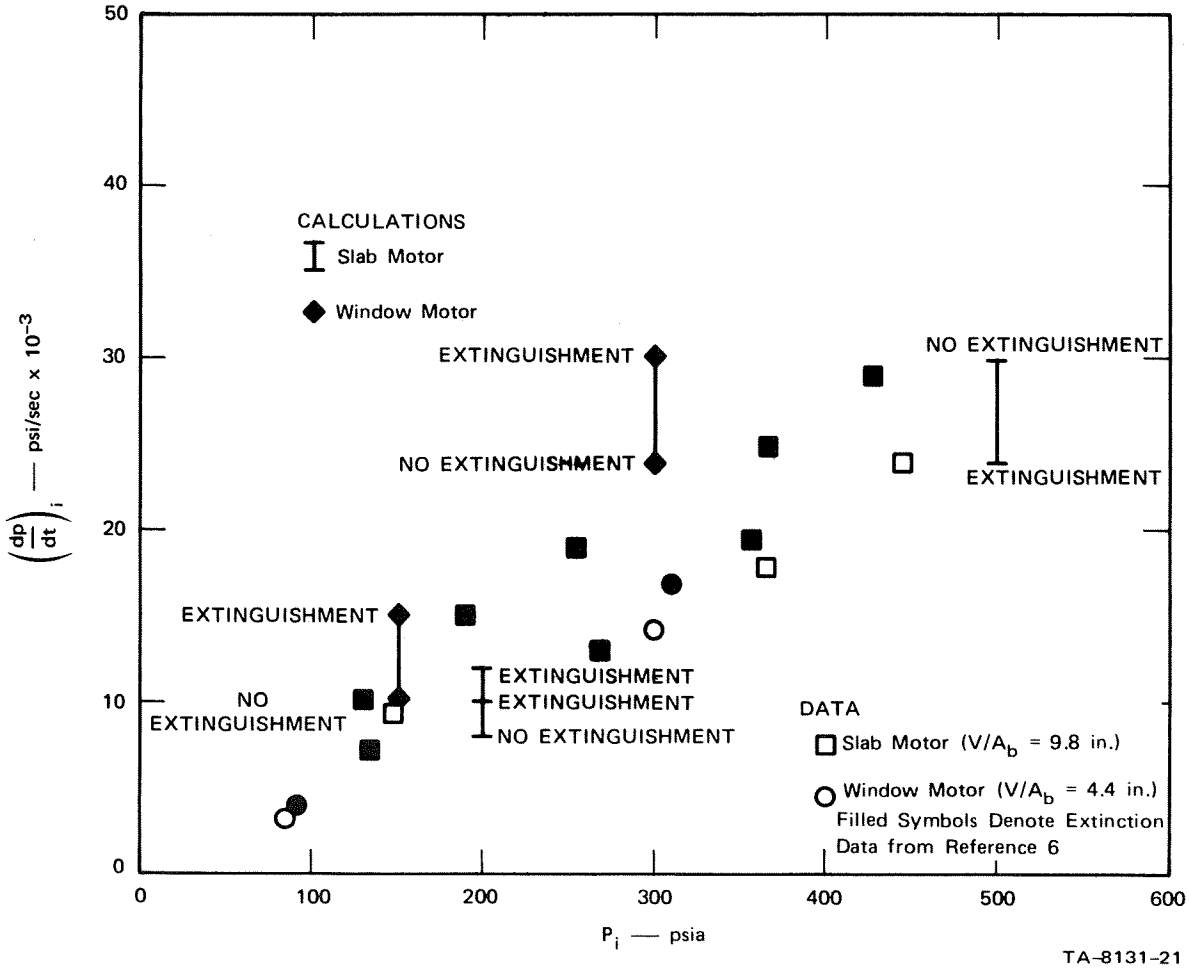

FIGURE 14 COMPARISON OF CALCULATED AND OBSERVED EXTINCTION BEHAVIOR (UTX-10645)

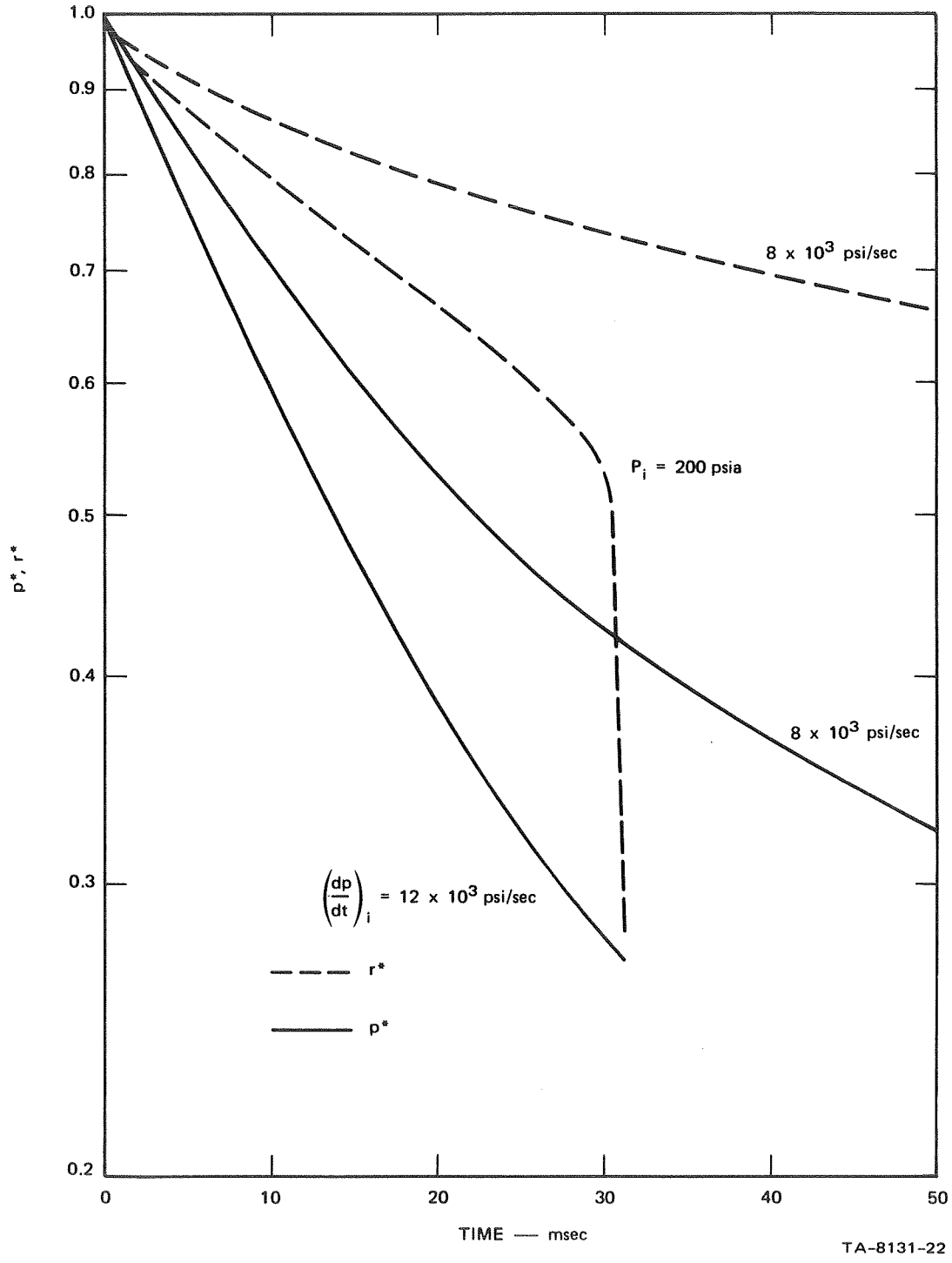

FIGURE 15 CALCULATED EXTINCTION BEHAVIOR IN THE SLAB MOTOR (UTX-10645)

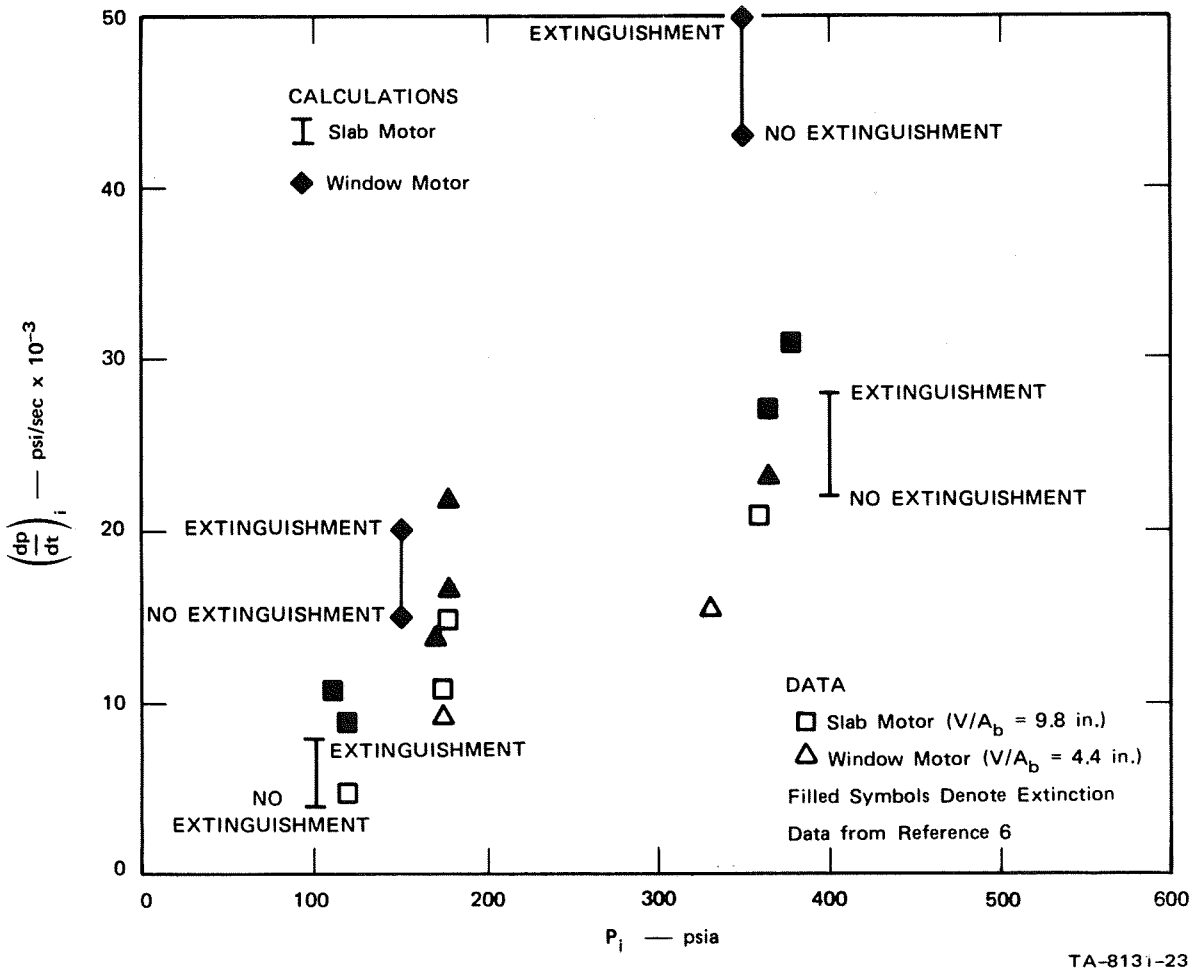

FIGURE 16 COMPARISON OF CALCULATED AND OBSERVED EXTINCTION BEHAVIOR (UTX-10661)

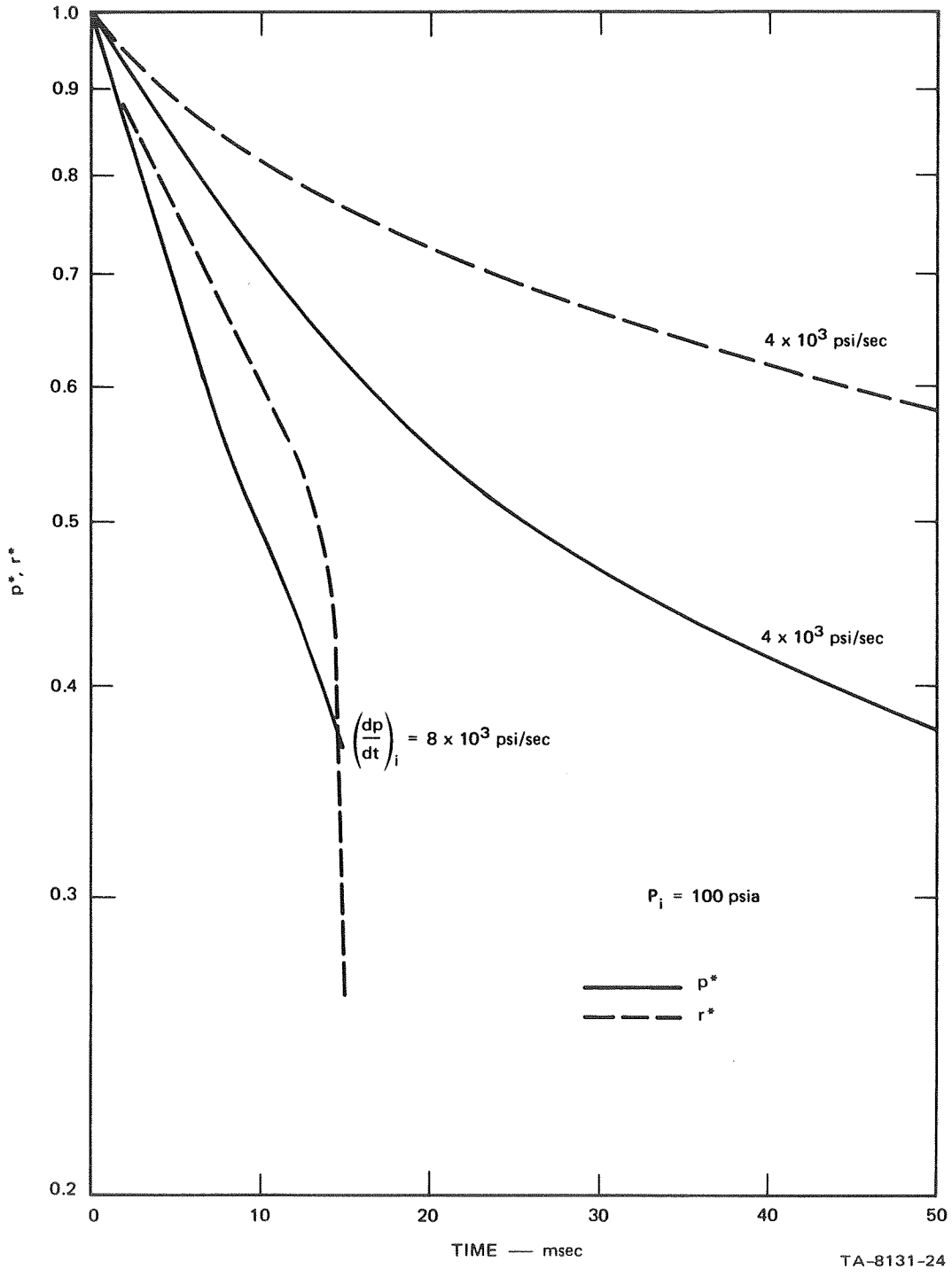

FIGURE 17 CALCULATED EXTINCTION BEHAVIOR IN THE SLAB MOTOR (UTX-10661)

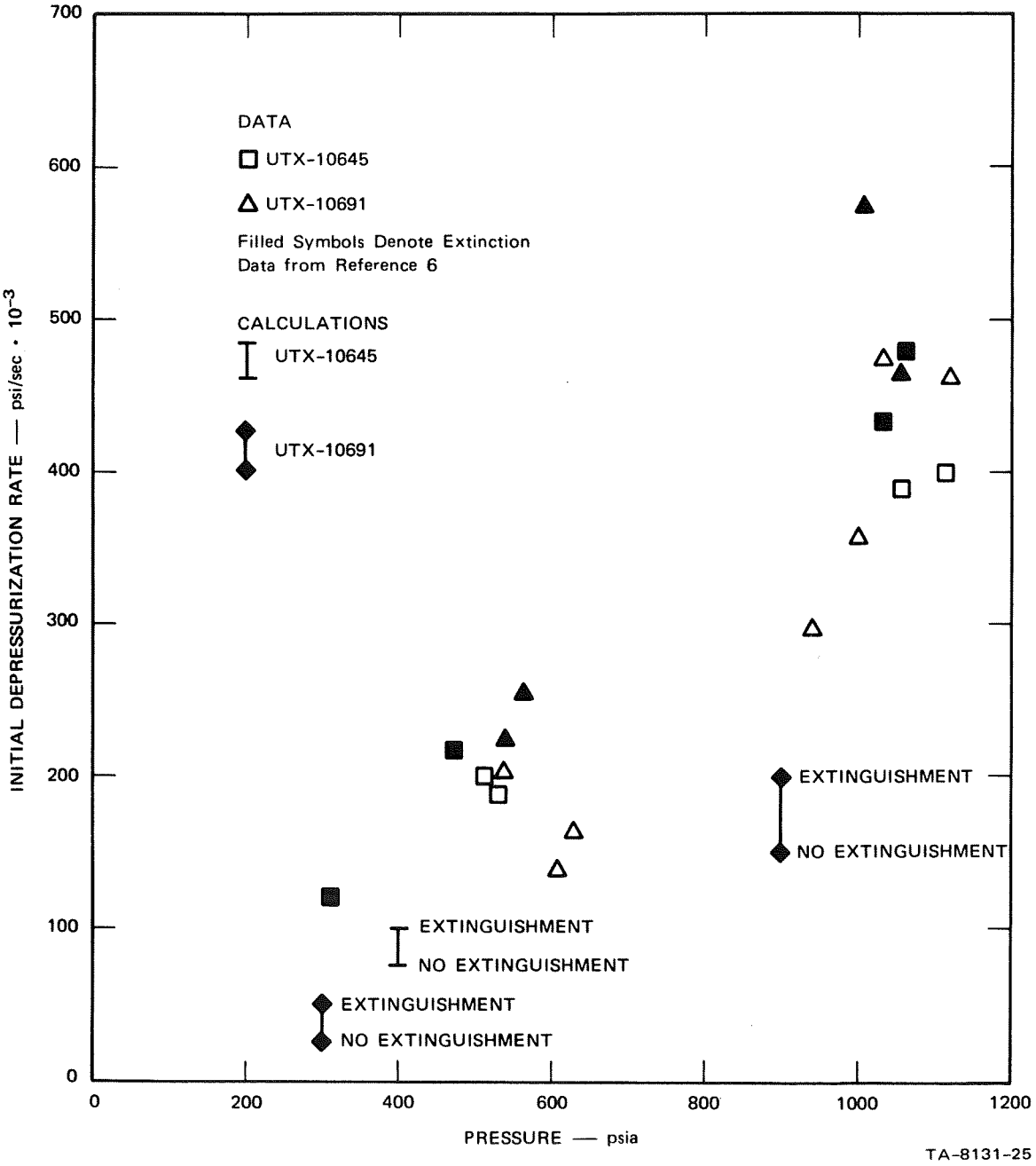

FIGURE 18 EXTINGUISHMENT CHARACTERISTICS OF UTX-10645 AND UTX-10691 IN THE SWING-NOZZLE MOTOR

Also of interest is the calculated pressure and burning rate behavior of the UTX-10645 propellant shown in Fig. 19. In this case an oscillation of the burning rate is evident, whereas it was previously absent at the lower depressurization rate (Fig. 15).

Extinguishment characteristics of the other CTPB propellant (UTX-11327) in the window motor, using Q_t=1100 cal/gm, are shown in Fig. 20. In this case the calculated results fall somewhat below the data, perhaps showing the sensitivity to the choice of Q_t. (See the discussion in connection with Fig. 6).

Finally, results are shown in Fig. 21 for the one aluminized propellant (UTX-11325) burned in the window motor. In this case Q_t was chosen as 1400 cal/gm for the numerical calculation and the program was modified to account for the mass of gaseous products that is lost to the formation of aluminum oxide. Considering the simplifications involved in the theory, the agreement is remarkably good.

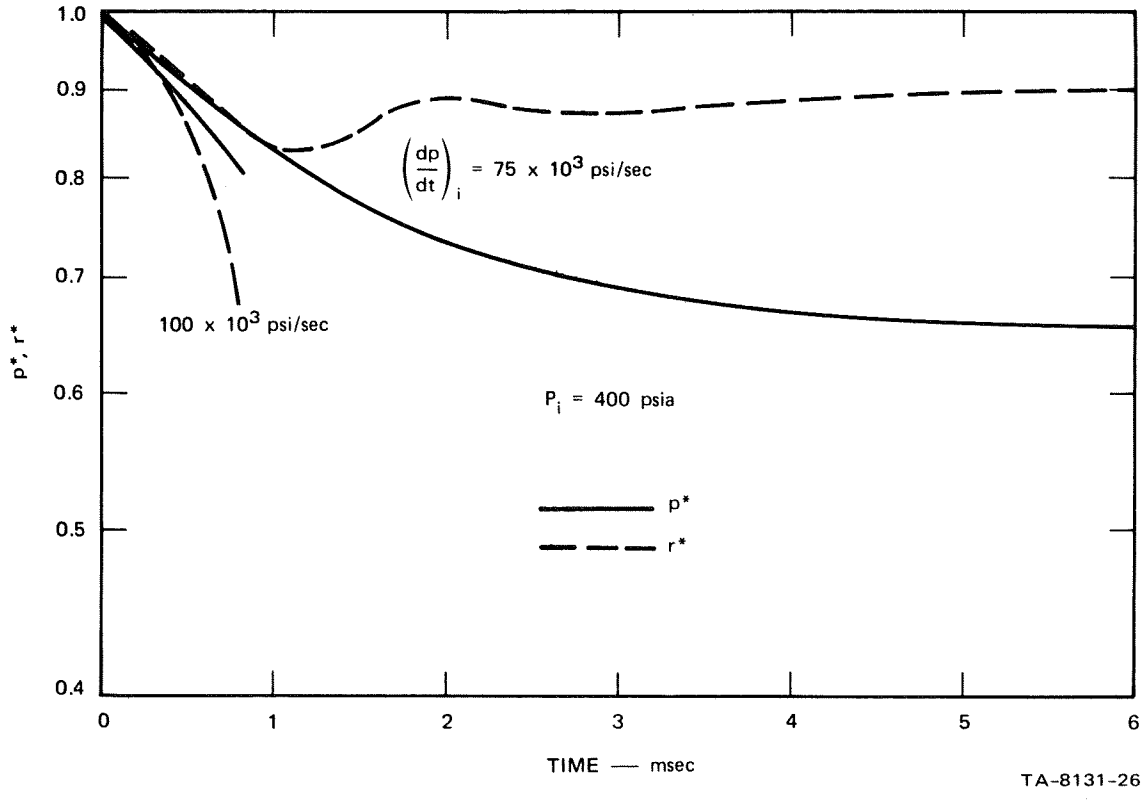

FIGURE 19 CALCULATED EXTINCTION BEHAVIOR IN THE SWING-NOZZLE MOTOR (UTX-10645)

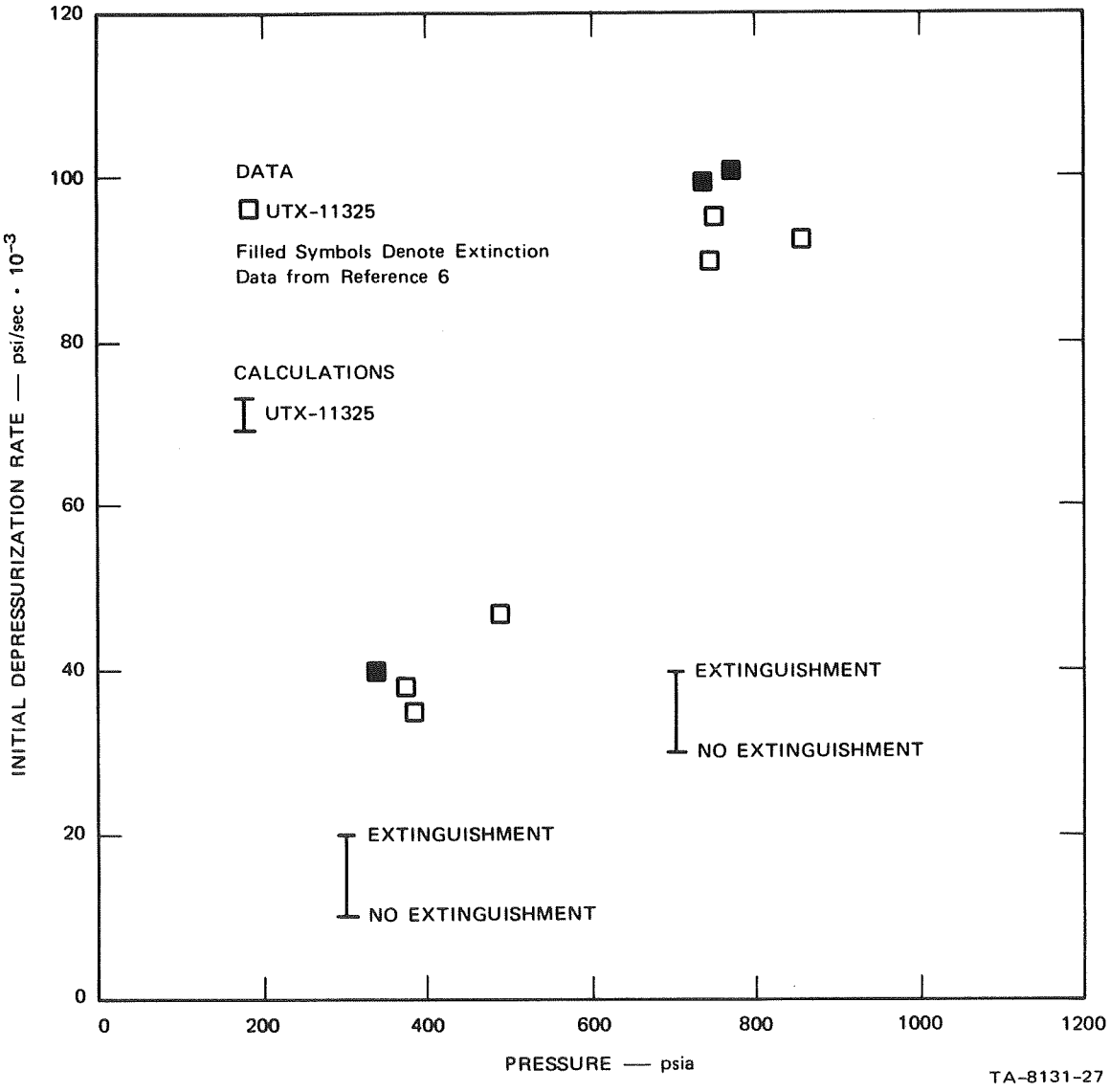

FIGURE 20 EXTINGUISHMENT CHARACTERISTICS OF UTX-11327 IN THE WINDOW MOTOR

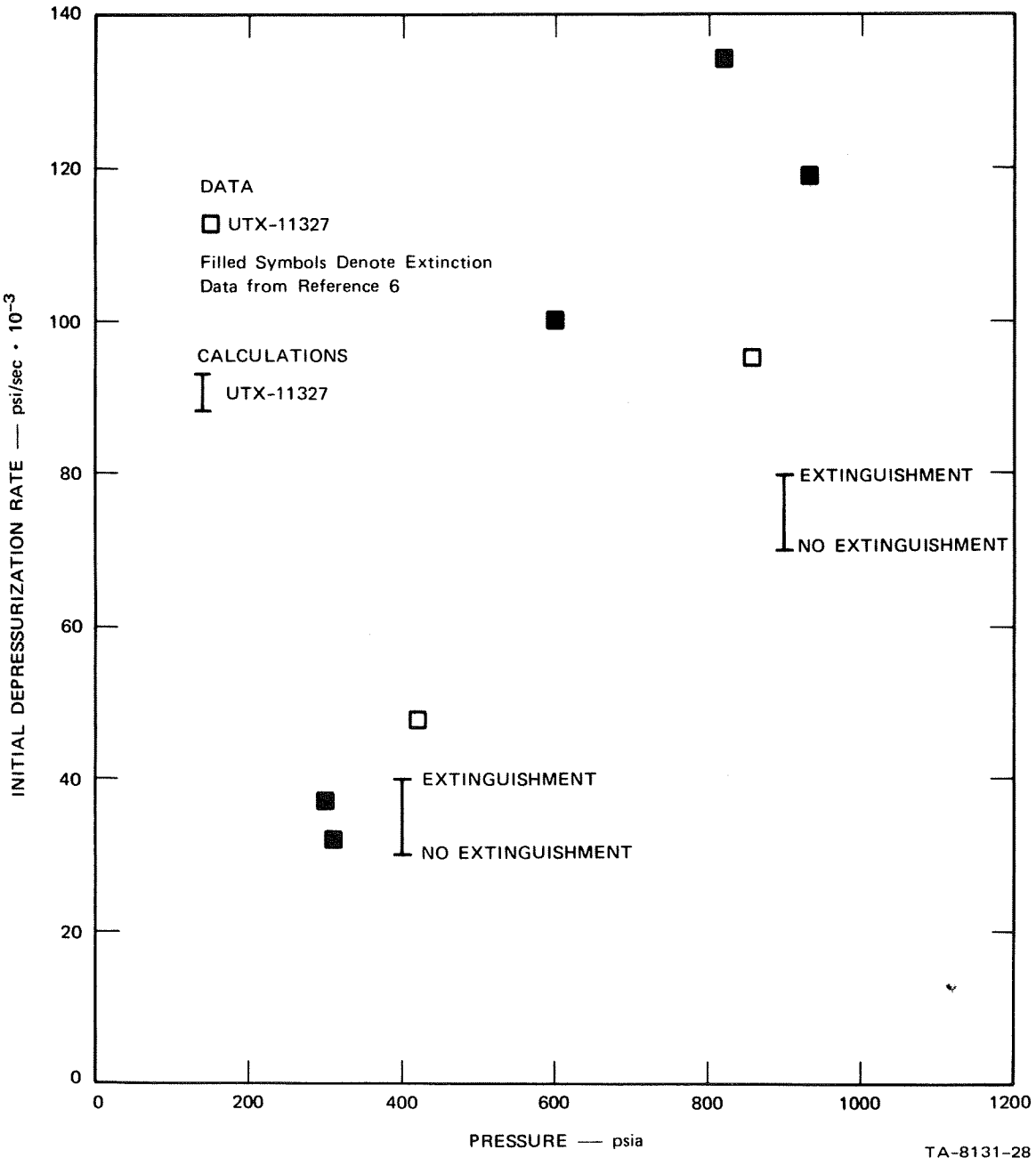

FIGURE 21 EXTINGUISHMENT CHARACTERISTICS OF UTX-11325 IN THE WINDOW MOTOR

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V CONCLUDING REMARKS

During this program an attempt was made to simplify the original SRI combustion model in a way that would still retain the essential features of past observations, the most salient of which is that ammonium perchlorate composite propellants exhibit a "stability limit" in the burning rate/pressure coordinate system. The calculations presented here have shown that the use of this limit line concept alone, together with the burning rate and flame temperature behavior of any given propellant, can predict the extinction behavior of a wide variety of propellants remarkably well without regard for changes in the other parameters of the model, once these parameters have been set by the behavior of one propellant.

Of course, much more could be learned through more extensive computations. One can only say that the limit line concept appears to be as useful as any other that is available today, and the resulting numerical model is a good deal simpler than many others in existence. The ultimate utility of the model cannot be demonstrated until comparisons with large-scale tests are made.

Appendix A

PERTURBATION ANALYSIS OF THE MODEL

The perturbation analysis described here is similar mathematically to that carried out for the previous SRI model.[7,8] The task is to perturb the solid phase boundary condition and relate the result to the perturbation parameter α, according to the procedure of Denison and Baum.[13]

The solid phase boundary condition is Eq. (8).

$$\left(k_s \frac{\partial T}{\partial x}\right)_w = \rho_s r \left[-c_p(T_f-T_0)-(c_p-c_s)T_0+c_s(T_w-T_0)+Q_s \frac{W_1}{W_1+W_2} + Q_g \frac{W_2}{W_1+W_2}\right] \quad (A1)$$

Expand each of the variables in Eq. (A1) by letting $r = \bar{r}\ (1+\tilde{r})$, $T_w = \bar{T}_w (1+\tilde{T}_w)$, etc. to obtain the perturbation boundary condition:

$$\frac{k_s}{\rho_s \bar{r} c_s}\left(\frac{\partial \tilde{T}}{\partial x}\right)_w = \tilde{r}\left(1-\frac{T_0}{\bar{T}_w}\right) - \frac{c_p \bar{T}_f}{c_s \bar{T}_w}\tilde{T}_f+\tilde{T}_w+q_s w_1\ (\tilde{W}_1-\tilde{r}) + q_g w_2\ (\tilde{W}_2-\tilde{r}) \quad (A2)$$

Insertion of the perturbation pyrolysis equation

$$\tilde{r} = \frac{E_w}{R\bar{T}_w}\ \tilde{T}_w \quad (A3)$$

gives

$$\frac{k_s}{\rho_s \bar{r} c_s}\left(\frac{\partial \tilde{T}}{\partial x}\right)_w = \left[\frac{E_w}{R\bar{T}_w}\left(1 - \frac{T_0}{\bar{T}_w} - q_s w_1 - q_g w_2\right) + 1\right]\tilde{T}_w$$

$$- \frac{c_p \bar{T}_f}{c_s \bar{T}_w}\ \tilde{T}_f + q_s w_1 \tilde{W}_1 + q_g w_2 \tilde{W}_2 \quad (A4)$$

Perturbations of Eqs. (3), (11), and (12) give

$$\tilde{r} = w_1 \tilde{W}_1 + w_2 \tilde{W}_2 \tag{A5}$$

$$\tilde{W}_1 = n_1 \tilde{p} + N_1 \tilde{T}_w - \frac{\bar{E}_h}{R\bar{T}_w} \tilde{E}_h \tag{A6}$$

$$\tilde{W}_2 = n_2 \tilde{p} + N_2 \tilde{T}_f - \frac{\bar{E}_f}{R\bar{T}_f} \tilde{E}_f \tag{A7}$$

Combining Eqs. (A3) and (A5-7) gives an expression for $\tilde{T}_f$ for use in Eq. (A4):

$$\tilde{T}_f = \frac{1}{w_2 N_2}\left\{\left(\frac{E_w}{R\bar{T}_w} - w_1 N_1\right)\tilde{T}_w - \left[w_1\left(n_1 - \frac{\tilde{E}_h}{R\bar{T}_w}\frac{\tilde{E}_h}{\tilde{p}}\right) + w_2\left(n_2 - \frac{\tilde{E}_f}{R\bar{T}_f}\frac{\tilde{E}_f}{\tilde{p}}\right)\right]\tilde{p}\right\} \tag{A8}$$

It has been explicitly assumed in Eq. (A8) that the activation energies E_h and E_f are functions of pressure only. Expressions for the quantities

$$X = n_1 - \frac{\bar{E}_h}{R\bar{T}_w}\frac{\tilde{E}_h}{\tilde{p}} \tag{A9}$$

and

$$Y = n_2 - \frac{\bar{E}_f}{R\bar{T}_f}\frac{\tilde{E}_f}{\tilde{p}} \tag{A10}$$

can be obtained from the simultaneous solution of the steady-state equations that are the equivalents of Eqs. (A4) and (A5):

$$\frac{c_p \bar{T}_f}{c_s \bar{T}_w} m_1 \upsilon = q_s w_1 \left(X + N_1 \frac{R\bar{T}_w}{E_w}\upsilon\right) + q_g w_2 \quad (Y + N_2 m_1 \upsilon)$$

$$- (q_s w_1 + q_g w_2)\upsilon \tag{A11}$$

$$\upsilon = w_1 \left(X + N_1 \frac{R\bar{T}_w}{E_w} \upsilon \right) + w_2 \left(Y + N_2 m_1 \upsilon\right) \tag{A12}$$

If Eq. (A4) is now considered to be of the form

$$D = E\tilde{T}_w + F\tilde{p} \tag{A13}$$

it must be true[13] that

$$E = 1 + A\,(1-\alpha) \tag{A14}$$

$$F = \alpha\upsilon \left(1 - \frac{T_0}{\bar{T}_w} \right) \tag{A15}$$

The essentials of the task have now been completed; by collecting terms in $\tilde{T}_w$ and $\tilde{p}$ in Eq. (A4) and using the representation of Eqs. (A14) and (A15), two expressions are obtained for α. From the $\tilde{T}_w$ coefficient, the algebraic manipulation gives Eq. (21) of the main body of the report:

$$\alpha \left(1 - \frac{T_0}{\bar{T}_w} \right) = q_s w_1 + q_g w_2 - \frac{R\bar{T}_w}{E_w} \; q_s w_1 N_1$$

$$+ \frac{1}{w_2 N_2} \left(\frac{c_p \bar{T}_f}{c_s \bar{T}_w} - q_g w_2 N_2 \right) \left(1 - \frac{R\bar{T}_w}{E_w} \; w_1 N_1 \right) \tag{A16}$$

From the $\tilde{p}$ coefficient

$$\alpha \left(1 - \frac{T_0}{\bar{T}_w} \right) = q_s w_1 \left(N_1 \frac{R\bar{T}_w}{E_w} - 1 \right) + q_g w_2 \quad (N_2 m_1 - 1)$$

$$+ \frac{c_p \bar{T}_f}{c_s \bar{T}_w} \left(\frac{1}{w_2 N_2} - \frac{w_1 N_1}{w_2 N_2} \frac{R\bar{T}_w}{E_w} - 2m_1 \right) \tag{A17}$$

Since the parameter α is specified at the limiting pressure, Eqs. (A16) and (A17) actually represent two equations for the two unknowns N_1 and N_2, i.e., E_{h_ℓ} and E_{f_ℓ}. When they have been solved as indicated in the numerical analysis (Appendix B), the solution is complete in the sense that the desired linear response at the stability limit line has been introduced (through specifying α_ℓ) and that the proper steady-state behavior will always be achieved after passage through a transient.

Appendix B

COMPUTER CODE AND REPRESENTATIVE OUTPUT FORMAT

This appendix includes a printout of the numerical program and of a representative output format. Comments are included throughout the program to make its use self-explanatory. Four input formats are provided for the calculation of the burning rate response during:

1. depressurization of a chamber
2. pressurization of a chamber
3. depressurization with an exponential pressure decay
4. a sinusoidal pressure variation.

Any other desired pressure-time history could easily be introduced into the program.

Two output formats are provided, one including the temperature profile behavior in the solid and one excluding it. The output Z* is the non-dimensional heat transfer (i.e., temperature derivative) at the surface with the spatial coordinate considered positive into the grain. The printout excluding the temperature profile is shown here. For an example of the temperature profile printout, see Appendix C.

```
                PROGRAM COMB(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)                    0010
          C     THIS PROGRAM COMPUTES THE TRANSIENT BURNING RATE RESPONSE OF           0020
          C     COMPOSITE SOLID PROPELLANTS                                            0030
000003          COMMON W,W0,W2,W4, Z,Z1, V,I1,J5,J9,K,K5,L1,L2,N,N2,P5,R5,U2,Y1,      0040
               1EPS3,EPS4,EPS1,EPS2,DI,C9,JERROR,IPLUS, JPRINT,C0,C,P0,R0,QRL,QGP,     0050
               2C1,C2,C3,C5,U5,POW,N22,W44,E1,C4,U1,W1,M,EHS,EFS,WRR,ZFI,             0060
               3CP,CS,QQG,QQS,EFI,EHI,NN2,NN1,WF1,QS,QG,U0,W1I,W2I                    0070
000003          REAL K,K1,K2,K3,K4,K5,K6,KAPPA,N,M,M1,M2,L1,L2,KIN                     0080
000003          REAL L2L1,KL,J3,J7,KK,NN2,NN1,NU,NUM                                   0090
000003          DIMENSION AA(100),BB(100),CC(100),DD(100),AVEC(100),BVEC(100),         0100
               1          CVEC(100),DVEC(100)                                         0110
000003          DIMENSION W(100), V(100), Z(100),DATE(12),PROP(12),Q(100),GG(100)      0120
000003          DIMENSION AXZ(4,4),BXZ(4),PXZ(4),BBB(5,5),CXZ(4  ),BXY(4),PXY(4)       0130
000003          JRR=0                                                                  0140
000004          JERROR=0                                                               0150
000005          EPS1=0.0001                                                            0160
000006          EPS2=0.001
000010          EPS3=1.0E-07                                                           0180
000011          EPS4=1.0E-08                                                           0190
000013          JPRINT=0                                                               0200
          C     JPRINT=0 EXCLUDES PRINTOUT OF TEMPERATURE PROFILE IN SOLID             0210
          C     JPRINT=1 INCLUDES PRINTOUT OF TEMPERATURE PROFILE IN SOLID             0220
000014          N2= 0                                                                  0230
000015          THETA=0.50                                                             0240
000016          Y1= 0.04                                                               0250
          C     Y1 IS NONDIMENSIONAL INCREMENT OF DISTANCE IN SOLID PHASE              0260
000020          N22=5                                                                  0270
          C     VALUE OF N22 ESTABLISHES NUMBER OF POINTS                              0280
          C     IN TEMPERATURE PROFILE PRINTOUT                                        0290
000021          J3=1.0                                                                 0300
000022          J5=1                                                                   0310
000023          J6=1                                                                   0320
000024          J9=1                                                                   0330
000025          READ(5,500) (DATE(I),I=1,12)                                           0340
000037    500   FORMAT(12A6)                                                           0350
000037          READ(5,550) (PROP(I),I=1,12)                                           0360
000051    550   FORMAT(12A6)                                                           0370
000051    202   READ(5,10)C4,KAPPA,TSTOP,G,U0,CP,P0,E1,NN1,NN2,QQT,QRL,RHS,KK,        0380
               1           FR,POW,  TSTEP,TCHANGE,TIMFAC,J7,RDL,WRR                  0390
000131          IF(EOF,5)1002,8880                                                     0400
000134    8880  WRITE(6,8888)                                                          0410
          C     C4 IS RATIO CP/CS                                                      0420
          C     KAPPA IS THERMAL DIFFUSIVITY OF SOLID (IN**2/SEC)                      0430
          C     TSTOP IS QUIT TIME (SEC)                                               0440
          C     G IS GAMMA                                                             0450
          C     U0 IS AMBIENT TEMPERATURE OF SOLID (DEG K)                             0460
          C     CP IS SPECIFIC HEAT AT CONSTANT PRESSURE (CAL/GM-DEG K)                0470
          C     P0 IS INITIAL PRESSURE (PSIA)                                          0480
          C     E1 IS ACTIVATION ENERGY OF WALL DECOMPOSITION (CAL/MOLE)               0490
          C     NN1 IS GAS PHASE ORDER AT WALL                                         0500
          C     NN2 IS GAS PHASE ORDER IN MAIN FLAME                                   0510
          C     QQT IS TOTAL HEAT RELEASE PER UNIT MASS (CAL/GM)                       0520
          C     QRL IS RATIO OF SURFACE TO GAS PHASE HEAT RELEASE                      0530
          C     RHS IS DENSITY OF PROPELLANT (LB/IN**3)                                0540
          C     KK IS THERMAL CONDUCTIVITY OF THE GAS PHASE (CAL/CM-SEC-DEG K)         0550
          C     FR IS FLAME TEMPERATURE AT 100 PSIA (DEG K)                            0560
          C     POW IS POWER IN FLAME TEMPERATURE/BURNING RATE EQUATION                0570
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      C     ICHANGE ALLOWS CHANGE IN TIME STEP (SEC)                                0580
      C     TIMFAC IS FACTOR BY WHICH TIME STEP IS CHANGED                          0590
      C     J7 GIVES PRINTOUT AT EACH (J7+1) TIME INCREMENT                         0600
      C     RDL IS LOWER DEFLAGRATION LIMIT                                         0610
      C     WRR IS WALL TEMPERATURE WHEN THE BURNING RATE IS .1 IN/SEC (DEG K)      0620
000140  8888 FORMAT(1H1)                                                           0630
000140       WRITE(6,501) (DATE(I),I=1,12)                                         0640
000152   501 FORMAT(1H ,50X,12A6)                                                  0650
000152       WRITE(6,551) (PROP(I),I=1,12)                                         0660
000164   551 FORMAT(47H SOLID PROPELLANT TRANSIENT COMBUSTION BEHAVIOR/            0670
                11H ,12A6)                                                       0680
000164       READ(5,100)(BXY(I),I=1,4),(PXY(I),I=1,4)                              0690
      C     BXY AND PXY ARE FOUR BURNING RATES (IN/SEC) AND FOUR PRESSURES          0700
      C     (PSIA) CHOSEN TO FIT MEASURED CURVE                                     0710
000204   100 FORMAT(8F10.4)                                                        0720
000204       DO 5 I=1,4                                                            0730
000206       BXZ(I)=ALOG(BXY(I))                                                   0740
000212       PXZ(I)=ALOG(PXY(I))                                                   0750
000216       AXZ(I,1)=1.0                                                          0760
000220       AXZ(I,2)=PXZ(I)                                                       0770
000221       AXZ(I,3)=(PXZ(I))**2                                                  0780
000223       AXZ(I,4)=(PXZ(I))**3                                                  0790
000225     5 CONTINUE                                                              0800
000226       CALL SOLVE(AXZ,BXZ,4,4,CXZ,5,BBB,ISOL)                                0810
000236       C1=CXZ(1)                                                             0820
000240       C2=CXZ(2)                                                             0830
000241       C3=CXZ(3)                                                             0840
000243       C5=CXZ(4)                                                             0850
000244       PA=500.0                                                              0860
000246   560 F6=C1+C2*ALOG(PA)+C3*(ALOG(PA))**2+C5*(ALOG(PA))**3                   0870
000266       F6=EXP(F6)                                                            0880
000270       PB=PA                                                                 0890
000271       PA=(F6/0.00239)**(1.0/0.726)                                          0900
000300       IF(PA.LT.(0.999*PB))GO TO 560                                         0910
000304       IF(PA.GT.(1.001*PB))GO TO 560                                         0920
000307       PL=PA                                                                 0930
      C     PL IS THRESHOLD PRESSURE FOR TRAVELING WAVE INSTABILITY                 0940
000307       JEXT=1                                                                0950
000310       RDL=C1+C2*ALOG(3.0)+C3*(ALOG(3.0))**2+C5*(ALOG(3.0))**3
000330       RDL=EXP(RDL)
000332       R0=C1+C2*ALOG(P0)+C3*(ALOG(P0))**2+C5*(ALOG(P0))**3                   0960
000351       R0=EXP(R0)                                                            0970
      C     R0   IS THE INITIAL BURNING RATE (IN/SEC)                               0980
000353       UN=C2+2.0*C3*ALOG(PL)+3.0*C5*(ALOG(PL))**2                            0990
      C     UN IS THE SLOPE OF THE BURNING RATE/PRESSURE CURVE AT LIMITING          1000
      C     PRESSURE                                                                1010
000366       NU=C2+2.0*C3*ALOG(P0)+3.0*C5*(ALOG(P0))**2                            1020
      C     NU IS THE SLOPE OF THE BURNING RATE/PRESSURE CURVE AT INITIAL           1030
      C     PRESSURE                                                                1040
000401       RR=C1+C2*ALOG(100.0)+C3*(ALOG(100.0))**2.0+C5*(ALOG(100.0))**3.0      1050
000423       RR=EXP(RR)                                                            1060
      C     RR   IS THE REFERENCE BURNING RATE AT 100 PSIA (IN/SEC)                 1070
000426       U2=FR*(R0/RR)**POW                                                    1080
      C     U2   IS THE INITIAL FLAME TEMPERATURE (DEG K)                           1090
000433       U1=WRR  /(1.0-WRR  *ALOG(10.0*R0)/E1)                                 1100
      C     U1   IS THE INITIAL WALL TEMPERATURE (DEG K)                            1110
000443       W0=U0/U1                                                              1120
000444       A =E1*(1.0-W0)/U1                                                     1130
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      C     A IS A PYROLYSIS FACTOR ASSOCIATED WITH THE LINEAR BEHAVIOR OF THE      1140
      C     PROPELLANT                                                              1150
000447      K5=R0/KAPPA                                                           1160
000451      K6=R0*K5                                                              1170
000452      K=K6*TSTEP/(Y1*Y1)                                                    1180
000455      KIN=K                                                                 1190
000456      TST=TSTOP                                                             1200
000457      TSTOP=K6*TSTOP                                                        1210
000460       RL=C1+C2*ALOG(PL)+C3*(ALOG(PL))**2+C5*(ALOG(PL))**3                  1220
000477      RL=EXP(RL)                                                            1230
      C     RL   IS THE VALUE OF BURNING RATE (IN/SEC) AT THE THRESHOLD PRESS     1240
000501      FL=FR*(RL/RR)**POW                                                    1250
      C     FL   IS THE VALUE OF FLAME TEMP. (DEG K) AT THE THRESHOLD PRESSURE    1260
000506      WL=WRR  /(1.0-WRR  *ALOG(10.0*RL)/E1)                                 1270
      C     WL   IS THE VALUE OF WALL TEMP. (DEG K) AT THE THRESHOLD PRESSURE     1280
000516      AL=E1*(1.0-U0/WL)/WL                                                  1290
000522      ALPHA=0.226*ALOG(AL)+0.170                                            1300
      C     THIS IS ALPHA AT THE THRESHOLD PRESSURE                               1310
000526      WR=WRR  /(1.0-WRR  *ALOG(10.0*RR)/E1)                                 1320
      C     WR   IS THE REFERENCE WALL TEMP. AT 100 PSIA                          1330
000537      CS=CP/C4                                                              1340
      C     CS IS SPECIFIC HEAT OF THE SOLID PHASE (CAL/GM-DEG K)                 1350
000541      RHO= RHS*454.0/(2.54)**2                                              1360
000544      QQG=QQT/(1.0+QRL)                                                     1370
000547      QGP=QQG/CP                                                            1380
000550      NUM=QGP-FL+U0-(1.0-1.0/C4)*U0                                         1390
000556      DEN=FL-U0+(1.0-1.0/C4)*U0-QRL*QGP                                     1400
000563      WRL=NUM/DEN                                                           1410
000564      ZFL=1.0+1.0/(QGP/((FL-WL)*(1.0+WRL))-1.0)                             1420
000573      XFL=(KK/(RHO*RL*CP))*ALOG(ZFL)*10000.0                                1430
      C     XFL IS THE DISTANCE FROM THE WALL TO THE GAS PHASE FLAME AT THE       1440
      C     LIMITING PRESSURE (MICRONS)                                           1450
000602      QQS=QRL*QQG                                                           1460
000604      QG=QQG/(CS*WL)                                                        1470
000606      QS=QQS/(CS*WL)                                                        1480
000610      W2L=RHO*RL/(1.0+WRL)                                                  1490
000613      W1L=WRL*W2L                                                           1500
000614      B11=W1L/(W1L+W2L)                                                     1510
000616      B22=W2L/(W1L+W2L)                                                     1520
000620      BNA=B22*POW*QG*(QG-QS)                                                1530
000625      BNB=-C4*FL/WL*POW*(3.0*QG-2.0*QS)-QG*B22*(QG-QS)-ALPHA*(1.0-U0/WL)    1540
     1        *(QG-2.0*QS)                                                       1550
000650      BNC=2.0*C4*FL/WL*(QG-QS+POW/B22*C4*FL/WL)                             1560
000662      BN2=(-BNB+(BNB**2-4.0*BNA*BNC)**0.5)/(2.0*BNA)                        1570
000674      BN1=(-2.0*QS*B11-2.0*QG*B22+QG*B22*BN2*POW+QG-2.0*POW*C4*FL/WL)/      1580
     1        (QG*B11*WL/E1-2.0*QS*B11*WL/E1)                                    1590
000716      EFL=FL*(BN2-NN2-2.0)                                                  1600
      C     EFL IS GAS PHASE ACTIVATION ENERGY AT THE THRESHOLD PRESSURE          1610
000722      EHL=WL*(BN1-NN1-2.0)                                                  1620
      C     EHL IS WALL REACTION ACTIVATION ENERGY AT THE THRESHOLD PRESSURE       1630
000725      SS1=C4*U2/WL-QS-U0/WL                                                 1640
000732      SS2=C4*U2/WL-QG-U0/WL                                                 1650
000735      SS4=R0/RL*(W1L+W2L)                                                   1660
000740      W2I=-SS1*SS4/(SS2-SS1)                                                1670
000744      W1I=SS2*SS4/(SS2-SS1)                                                 1680
000746      WRI=W1I/W2I                                                           1690
000750      EHS=(1.0-WL/EHL*(ALOG(W1I/W1L)-NN1*ALOG(P0/PL)-(NN1+2.0)*             1700
     1        ALOG(U1/WL)))*U1/WL                                                1710
```

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000777          EFS=(1.0-FL/EFL*(ALOG(W2I/W2L)-NN2*ALOG(P0/PL)-(NN2+2.0)*      1720
                1     ALOG(U2/FL)))*U2/FL                                       1730
001026          QS=QQS/(CS*U1)                                                  1740
001031          QG=QGG/(CS*U1)                                                  1750
001033          EFI=EFS*EFL                                                     1760
001035          EHI=EHS*EHL                                                     1770
001037          ZFI=1.0+1.0/(QGP/((U2-U1)*(1.0+WRI))-1.0)                       1780
001046          XFI=(KK/(RHO*RL*CP))*ALOG(ZFI)*10000.0                          1790
          C     XFI IS THE DISTANCE FROM THE WALL TO THE GAS PHASE FLAME AT THE  1800
          C     INITIAL PRESSURE (MICRONS)                                      1810
001055          E11=2.0*E1                                                      1820
001057          E55=2.0*EFI                                                     1830
001060          E33=2.0*EHI                                                     1840
001062          E44=2.0*E4                                                      1850
001063          READ(5,11) J                                                    1860
          C     J DETERMINES CASE TO BE CALCULATED                              1870
001071       10 FORMAT(7F10.4)                                                  1880
001071       11 FORMAT(2I1)                                                     1890
001071          GO TO (12,13,14,15),J                                           1900
001101       12 READ(5,16) DECAY,V1,M1,PF
001115          K4=1.0+(V1*M1*DECAY)/(1545.0*12.0*1.8*RHS*R0*G*U2*0.7)
          C     DECAY IS THE INITIAL DECAY RATE IN PSI/SEC                      1920
          C     K4 IS THE RATIO AT/ATI=AT*                                      1930
          C     PF IS THE EXTERNAL PRESSURE                                     1940
001127          GO TO 17                                                        1950
001127       13 READ(5,16) TAU,PF                                               1960
          C     TAU IS TIME CONSTANT OF DEPRESSURIZATION CURVE                  1970
          C     PF IS THE FINAL PRESSURE                                        1980
001137          GO TO 17                                                        1990
001140       14 READ(5,16) P8,F3                                                2000
          C     P8 IS THE AMPLITUDE CHOSEN FOR P*                               2010
          C     F3 IS THE FREQUENCY IN CPS                                      2020
001150          GO TO 17                                                        2030
001151       15 READ(5,16) TPULSE,V1,M1,M2,K4,PF                                2040
          C     TPULSE IS THE DURATION OF EXTERNAL INJECTION                    2050
          C     V1 IS RATIO OF CHAMBER VOLUME TO BURNING SURFACE AREA           2060
          C     M1 IS MOLECULAR WEIGHT OF PRODUCT GASES                         2070
          C     M2 IS RATIO OF MASS INJECTED TO MASS EVOLVED BY PROPELLANT       2080
          C     K4 IS THE RATIO AT/ATI=AT*                                      2090
          C     PF IS THE EXTERNAL PRESSURE                                     2100
001171       16 FORMAT(7F10.4)                                                  2110
001171       17 WRITE(6,18)C1,C2,C3,C5,C4,CP,CS,KK,NN1,NN2,U0,FR,WRR,POW,RHS,WRL,  2120
                1           QRL,KAPPA,G,TST,TIMFAC,TCHANGE,TSTEP              2130
001253       18 FORMAT(/16H INPUT CONSTANTS/4H C1=,E15.8/4H C2=,E15.8/4H C3=,E15.8  2140
                1      /4H C5=,E15.8/4H C4=,E15.8/4H CP=,E15.8/4H CS=,E15.8/4H KG=,  2150
                2      E15.8/4H N1=,E15.8/4H N2=,E15.8/4H T0=,E15.8/5H TFR=,E15.8/  2160
                3      5H TWR=,E15.8/5H POW=,E15.8/5H RHO=,E15.8/5H WRL=,E15.8/  2170
                4      5H QRL=,E15.8/7H KAPPA=,E15.8/7H GAMMA=,E15.8/7H TSTOP=,  2180
                5      E15.8/8H TIMFAC=,E15.8/9H TCHANGE=,E15.8/11H TIME STEP=,  2190
                6      E15.8)                                                    2200
001253          EF1=2.0*EFL                                                     2210
001255          EH1=2.0*EHL                                                     2220
001257          WRITE(6,40)PL,RL,    AL,ALPHA,    EF1,EH1,FL,WL,XFL,UN          2230
001306       40 FORMAT(/19H LIMIT POINT VALUES/4H PL=,E15.8/4H RL=,E15.8/       2240
                1           4H AL=,E15.8/8H ALPHAL=,E15.8/                    2250
                2    5H EFL=,E15.8/5H EHL=,E15.8/5H TFL=,E15.8/5H TWL=,E15.8/  2260
                3    5H XFL=,E15.8/5H NU =,E15.8)                              2270
001306          I1=INT(1.0/Y1+0.5)                                              2280
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001312          B11=W1I/(W1I+W2I)                                                2290
001314          B22=W2I/(W1I+W2I)                                                2300
001316          BN2=NN2+2.0+EFI/U2                                               2310
001322          BN1=NN1+2.0+EHI/U1                                               2320
001325          ALPHA=(QS*B11+QG*B22-U1/E1*QS*B11*BN1+(C4*U2/U1-QG*B22*BN2)*     2330
                1       ((1.0-U1/E1*B11*BN1)/(B22*BN2)))/(1.0-U0/U1)             2340
001356          WRITE(6,41)P0,R0,E11,E55,E33,U2,U1,QQG,QQS,A,ALPHA,      NU,XFI  2350
001413       41 FORMAT(/21H INITIAL POINT VALUES/4H P0=,E15.8/4H R0=,E15.8/4H EW=,2360
                1       E15.8/4H EF=,E15.8/4H EH=,E15.8/4H TF=,E15.8/4H TW=,E15.8/2370
                2       4H QG=,E15.8/4H QS=,E15.8/4H A =,E15.8/7H ALPHA=,E15.8/  2380
                3                          8H NU    =,E15.8/8H XFI   =,E15.8)    2390
001413          ALPHAL=1.0+(1.0-SQRT(1.0+8.0*A))/(A+A)                           2400
001425          WRITE(6,8888)                                                    2410
001430          WRITE(6,501) (DATE(I),I=1,12)                                    2420
001442          IF(ALPHA-ALPHAL) 720,720,136                                     2430
001445      136 P5=1.0                                                           2440
001447          R5=1.0                                                           2450
001450          R3=1.0                                                           2460
001451          K44=1.0                                                          2470
001452          W1=1.0                                                           2480
001453          W2=1.0                                                           2490
001454          W3=1.0                                                           2500
001455          W44=1.0                                                          2510
001456          Z1=-(W1-W0)                                                      2520
001460          T00=0.0                                                          2530
001461          G0=0.0                                                           2540
001462          G00=1.0                                                          2550
          C     G00 IS THE INTEGRAL OF THE INITIAL TEMPERATURE PROFILE.          2560
001463          EFS=1.0                                                          2570
001464          EHS=1.0                                                          2580
001465          T=K*Y1*Y1                                                        2590
001467          IMINUS=I1-1                                                      2600
001471          IPLUS=I1+1                                                       2610
001472          GG1=(G-1.0)/2.0*(2.0/(G+1.0))**((G+1.0)/(G-1.0))                 2620
001505          PRF=((G+1.0)/2.0)**(-G/(G-1.0))                                  2630
001514          IF(J-1) 52,51,52                                                 2640
001516       52 IF(J-2) 171,238,171                                              2650
001520      171 IF(J-3) 172,268,172                                              2660
001522      172 WRITE(6,173) V1, K4, M2, M1, TPULSE,PF                           2670
001542      173 FORMAT(24H EXTERNAL PULSE RESPONSE//8H V/AB  =,E15.8/             2680
                1       8H AT/ATI=,E15.8/ 8H M.R.  =,E15.8/8H M.W.  =,E15.8/     2690
                2       8H TPULSE=,E15.8/8H PF    =,E15.8/)                      2700
001542          K1=(P0*V1*R0*M1)/(1545.0*12.0*1.8*RHS *KAPPA*U2)                 2710
001552          K2=K1/G                                                          2720
001554          IF(JPRINT) 27,26,27                                              2730
001555       26 WRITE(6,28)                                                      2740
001561       28 FORMAT(9H     TIME,11X,2HP*,11X,2HE*,11X,2HZ*,11X,2HR*,11X,3HTW*,2750
                1      10X,3HTF*,10X,3HTC*,10X,3HEF*,10X,3HEH*/)                2760
001561          WRITE(6,29)T00,P5,G00,Z1,R5,W1,W2,W3,EFS,EHS                     2770
001611       29 FORMAT(10E13.4)                                                  2780
001611       27 CONTINUE                                                         2790
001611          CALL SUB600                                                      2800
001612          GO TO 210                                                        2810
001613      200 IF(T-K6*TPULSE) 210,210,206                                      2820
001617      206 M2=0.0                                                           2830
001620      210 PFR=PF/(P5*P0)                                                   2840
001623          IF (PFR-PRF) 2111,2111,2101                                      2850
001625     2101 K44=(GG1/((1.0-PFR**((G-1.0)/G))*PFR**(2.0/G)))**0.5             2860
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      C     K44 IS PSEUDO THROAT AREA RATIO TO ACCOUNT FOR NOZZLE UNCHOKING          2870
001647     2111 IF(J9-1)207,207,208                                                  2880
001652      207 X5=(R5*W2+M2-(K4/K44)*P5*W3**0.5)/K2                                 2890
001666          X6=(K1*X5/W3-R5-M2+(K4/K44)*P5/(W3**0.5))*W3*W3/(K1*P5)              2900
001706          P50=P5                                                               2910
001707          W30=W3                                                               2920
001710          P5=P50+X5*K*Y1*Y1                                                    2930
001714          W3=W30+X6*K*Y1*Y1                                                    2940
001717          GO TO 365                                                            2950
001717      208 X50=(R5*W2+M2-(K4/K44)*P5*W3**0.5)/K2                                2960
001733          X60=(K1*X50/W3-R5-M2+(K4/K44)*P5/(W3**0.5))*W3*W3/(K1*P5)            2970
001753          P5=P50+(X5+X50)*K*Y1*Y1/2.0                                          2980
001762          W3=W30+(X6+X60)*K*Y1*Y1/2.0                                          2990
001770          GO TO 365                                                            3000
001770      238 RATE=(P0-PF)/TAU                                                     3010
001773          WRITE(6,244)   TAU,P0,PF,RATE                                        3020
002007      244 FORMAT(1H /27H EXPONENTIAL DECAY RESPONSE/                           3030
                1 15H TIME CONSTANT=,E15.8/                                        3040
                24H P0=,E15.8/4H PF=,E15.8/20H INITIAL DECAY RATE=,E15.8/)          3050
002007          IF(JPRINT) 900,901,900                                               3060
002010      901 WRITE(6,28)                                                          3070
002014          WRITE(6,29)T00,P5,G00,Z1,R5,W1,W2,W3,EFS,EHS                         3080
002044      900 CONTINUE                                                             3090
002044          CALL SUB600                                                          3100
002045      906 P5=PF/P0+(1-PF/P0)*EXP(-T/(K6*TAU))                                  3110
002060          GO TO 365                                                            3120
002061       51 WRITE(6,50) DECAY,K4                                                 3130
002071       50 FORMAT(34H CHAMBER DEPRESSURIZATION RESPONSE//                       3140
              120H INITIAL DECAY RATE=,E15.8/8H AT/ATI=,E15.8/)                    3150
002071          IF (JPRINT) 902,903,902                                              3160
002072      903 WRITE(6,28)                                                          3170
002076          WRITE(6,29)T00,P5,G00,Z1,R5,W1,W2,W3,EFS,EHS                         3180
002126      902 CONTINUE                                                             3190
002126          CALL SUB600                                                          3200
002127          K2=-P0*R0*R0*(1-K4)/(KAPPA*DECAY)                                    3210
002137          K1=G*K2                                                              3220
002140          GO TO 210                                                            3230
002141      268 P81=1.0+P8*TSTEP                                                     3240
002144          WRITE(6,275) P81,F3                                                  3250
002154      275 FORMAT(20H SINUSOIDAL RESPONSE/4H P*=,E15.8/4H F*=,E15.8/)           3260
002154          IF(JPRINT) 904,905,904                                               3270
002155      905 WRITE(6,28)                                                          3280
002161          WRITE(6,29)T00,P5,G00,Z1,R5,W1,W2,W3,EFS,EHS                         3290
002211      904 CONTINUE                                                             3300
002211          CALL SUB600                                                          3310
002212          F3=6.2832*F3/K6                                                      3320
002215      280 IF((T/K6).GT.(TCHANGE-20.0*TSTEP))GO TO 365                          3330
002224            P5=1.0+P8*(T/K6)                                                   3340
002226      365 CALL SUB500                                                          3350
002227          IF(JERROR-1) 366,745,366                                             3360
002231      366 W2=W4                                                                3370
      C         W2 IS FLAME TEMPERATURE RATIO TF*                                  3380
002233          WF2=R5*DI-WF1                                                        3390
002236          Z1=R5*(C4*(U2/U1*W2-W0)+(C4-1.0)*W0-(W1-W0)-QS*WF1/(WF1+WF2)-QG*     3400
              1   WF2/(WF1+WF2))                                                   3410
002261          W44=W4                                                             3420
002262          FNUM=K*(1.0-THETA)                                                 3430
002265          CONS=FNUM*FNUM                                                     3440
```

```
002266          THK=K*THETA                                                     3450
002267          DO 20 I=1,I1                                                    3460
002271          FI=I                                                            3470
002272          CTERM=1.0-(FI-1.0)*Y1                                           3480
002276          CTERM=CTERM*CTERM*CONS                                          3490
002300          AA(I)=1.0+CTERM                                                 3500
002303       20 BB(I)=1.0-CTERM                                                 3510
002306          CONS=0.5*(R5-1.0)*Y1                                            3520
002311          DO 21 I=1,I1                                                    3530
002313          FI=I                                                            3540
002314          CTERM=1.0-(FI -1.0)*Y1                                          3550
002320          CC(I)=CTERM*(CTERM+CONS)                                        3560
002324          DD(I)=CTERM*(CTERM-CONS)                                        3570
002325       21 BVEC(I)=1.0                                                     3580
002331          CVEC(1)=-(FNUM+FNUM)/AA(1)                                      3590
002334          DO 22 I=2,IMINUS                                                3600
002335       22 CVEC(I)=-FNUM*CC(I)/AA(I)                                       3610
002342          DO 23 I=1,IMINUS                                                3620
002344          IP=I+1                                                          3630
002346       23 AVEC(I)=-FNUM*DD(IP)/AA(IP)                                     3640
002354          DVEC(1)=(2.0*K*(THETA*V(2)-Y1*Z1*DD(1))+BB(1)*V(1))/AA(1)       3650
002365          DO 24 I=2,IMINUS                                                3660
002366       24 DVEC(I)=(THK*(DD(I)*V(I-1)+CC(I)*V(I+1))+BB(I)*V(I))/AA(I)      3670
002377          DVEC(I1)=(FNUM*CC(I1)*W0+BB(I1)*V(I1)+THK*(DD(I1)*V(I1-1)+CC(I1)*  3680
              1          V(IPLUS)))/AA(I1)                                     3690
002414          CALL TRIDAG(I1,AVEC,BVEC,CVEC,DVEC,W)                           3700
002420          W(IPLUS)=W0                                                     3710
002422          W1=W(1)                                                         3720
        C       W1 IS WALL TEMPERATURE RATIO TW*                                3730
002424          R8=R5                                                           3740
002425          R5=EXP(-E1*(1.0-W1)/(U1*W1))                                    3750
        C       R5 IS THE BURNING RATE RATIO R*                                 3760
002436          RTEST=R8/R5                                                     3770
002437          IF(RTEST-(1.0-EPS2)) 465,403,403                                3780
002443       403 IF(RTEST-(1.0+EPS2)) 441,441,465                               3790
002447       441 J9=1                                                           3800
002450          GO TO 680                                                       3810
002451       446 J3=J3+1.0                                                      3820
002453       447 TY=T+K*Y1*Y1                                                   3830
002457          IF(TY-K6*TCHANGE)449,448,448                                    3840
002462       448 K=KIN*TIMFAC                                                   3850
002464       449 T=T+K*Y1*Y1                                                    3860
002467          IF(T-TSTOP) 455,455,8222                                        3870
002472      8222 CALL SECOND(TME)                                               3880
002474          WRITE(6,8224) TME                                               3890
002502      8224 FORMAT(1H ///14H RUNNING TIME=,E15.8)                          3900
002502          GO TO 202                                                       3910
002503       455 DO 460 I=1, IPLUS                                              3920
002505       460 V(I)=W(I)                                                      3930
002511          R50=R5*R0                                                       3940
        C       R50 IS THE INSTANTANEOUS BURNING RATE                           3950
002513          IF(R50-RDL) 756,756,461                                         3960
002515       461 GO TO (200,906,280,200),J                                      3970
002525       465 J9=J9+1                                                        3980
002527       496 IF(J9-20) 461,461,716                                          3990
002532       680 IF(J3-J7) 446,446,685                                          4000
002535       685 TK6=T/K6                                                       4010
002537          N2=0                                                            4020
```

```
002540          DO 6000 I=1, IPLUS                                              4030
002541          Q(I)=(W(I)-W0)/(1.0-W0)                                         4040
002546     6000 CONTINUE                                                        4050
002550          DO 6010 I=1, I1                                                 4060
002551          GG(I)=(Q(I+1)+Q(I))/2.0*Y1                                      4070
002555     6010 CONTINUE                                                        4080
002557          DO 6020 I=1, I1                                                 4090
002561          G0=G0+GG(I)                                                     4100
002563     6020 CONTINUE                                                        4110
002565          G0=2.0*G0                                                       4120
           C    G0 IS THE INTEGRAL OF THE TEMPERATURE PROFILE                   4130
002567          IF(JPRINT) 175,174,175                                          4140
002570      175 WRITE(6,686) TK6                                                4150
002576      686 FORMAT(6H TIME=,E15.8)                                          4160
002576          DO 696 I=1, IPLUS                                               4170
002600          L=I-1                                                           4180
002602          IF(L) 613,613,622                                               4190
002603      613 WRITE(6,614)                                                    4200
002607      614 FORMAT(8H         /9H        Y*,13X,7HX (IN.), 6X,14H(T-T0)/(TW-T0))  4210
002607      622 IF(L-N2) 696,694,694                                            4220
002612      694 FL=L                                                            4230
002614          PRNT1=FL*Y1                                                     4240
002616          IF(PRNT1 -1.0) 616,615,616                                      4250
002620      615 ARG=0.001                                                       4260
002622          PRNT3=0.001                                                     4270
002623          GO TO 617                                                       4280
002623      616 ARG=1.0-PRNT1                                                   4290
002625          PRNT3=(W(I)-W0)/(W(1)-W0)                                       4300
002633      617 PRNT2=-ALOG(ARG)/(K5*R5)                                        4310
002641          WRITE(6,620) PRNT1,PRNT2,PRNT3                                  4320
002653      620 FORMAT(3E16.8)                                                  4330
002653          N2=N2+N22                                                       4340
002655      696 CONTINUE                                                        4350
002660          WRITE(6,629)                                                    4360
002663      629 FORMAT(1H )                                                     4370
002663      475 WRITE(6,476) P5,G0,  Z1,R5,W1,W2,W3,EFS,EHS                     4380
002711      476 FORMAT(8H       P*,13X,2HE*,11X,2HZ*,                           4390
              112X,2HR*,12X,3HTW*,11X,3HTF*,11X,3HTC*,11X,3HEF*,11X,3HEH*/      4400
              29E14.5)                                                        4410
002711          WRITE(6,610)                                                    4420
002715      610 FORMAT(1H0)                                                     4430
002715          GO TO 30                                                        4440
002716      174 WRITE(6,29)TK6,P5,G0,  Z1,R5,W1,W2,W3,EFS,EHS                   4450
002746       30 CONTINUE                                                        4460
002746          J3=1.0                                                          4470
002750          G0=0.0                                                          4480
002751          GO TO 447                                                       4490
002751      716 WRITE(6,717)                                                    4500
002755      717 FORMAT(34H FINAL ITERATION DOES NOT CONVERGE)                   4510
002755          WRITE(6,7000)R5,R8,TK6                                          4520
002767     7000 FORMAT(4H R5=,E15.8/4H R8=,E15.8/4H T =,E15.8)                  4530
002767          GO TO 8222                                                      4540
002770      720 WRITE(6,721)                                                    4550
002774      721 FORMAT(27H LINEAR LIMIT LINE EXCEEDED)                          4560
002774          GO TO 8222                                                      4570
002775      745 L2L1=L2-L1                                                      4580
002777          WRITE(6,746) W4,L2L1                                            4590
003007      746 FORMAT(31H TF ITERATION DOES NOT CONVERGE/4H W4=,E15.8,8H  F(TF)=,  4600
```

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                1          E15.8)                                                  4610
003007      747 GO TO 8222                                                         4620
003010      756 WRITE(6,758)                                                       4630
003014      758 FORMAT(44H BURNING RATE BELOW LOWER DEFLAGRATION LIMIT)             4640
003014     1000 GO TO 8222                                                         4650
003015     1001 CONTINUE                                                           4660
003015     1002 STOP                                                               4670
003017          END                                                                4680
```

```
                SUBROUTINE SOLVE(AA, Y, N, NMX, X, NP1, A, ISOL)                        4690
C               SUBROUTINE TO SOLVE AN N BY N SYSTEM OF EQUATIONS OF THE FORM           4700
C               AA * X = Y  ---  WRITTEN IN FORTRAN II                                  4710
C                                                                                       4720
C               AA     CONTAINS THE GIVEN N BY N COEFICIENT MATRIX                      4730
C               Y      CONTAINS THE GIVEN N BY 1 RIGHT HAND SIDE                        4740
C               N      IS A POSITIVE INTEGER                                            4750
C               NMX    IS THE GIVEN DIMENSION OF AA, X, AND Y IN THE CALLING PROGRAM    4760
C               X      IS THE N BY 1 SOLUTION VECTOR                                    4770
C               NP1    IS A POSITIVE INTEGER GREATER THAN OR EQUAL TO  N+1              4780
C               A      IS AN  NP1  BY  NP1  SCRATCH ARRAY                               4790
C               ISOL   IS RETURNED AS 1 IF A SOLUTION EXISTS, 0 IF NO SOLUTION EXIST    4800
000013          DIMENSION AA(NMX, 1), Y(NMX), X(NMX), A(NP1, 1)                         4810
000013          NM = N + 1                                                              4820
000015          DO 1 I = 1,N                                                            4830
000016          A(I,NM) = Y(I)                                                          4840
000024          DO 1 J = 1,N                                                            4850
000025        1 A(I,J) = AA(I,J)                                                        4860
000044          DO 6 I=1,N                                                              4870
000046          IE=0                                                                    4880
000047        2 IF(A(I,I))3,400,3                                                       4890
000054        3 DIAGST=A(I,I)                                                           4900
000061          DO 4 J=I,NM                                                             4910
000062        4 A(I,J)=A(I,J)/DIAGST                                                    4920
000072          IX=I+1                                                                  4930
000074          DO 6 K=IX,N                                                             4940
000075          ROWMUL=A(K,I)                                                           4950
000102          DO 6 L=I,NM                                                             4960
000103        6 A(K,L)=A(I,L)*ROWMUL*(-1.)+A(K,L)                                       4970
000126          M=N                                                                     4980
000127          X(M)=A(M,NM)                                                            4990
000135        7 M=M-1                                                                   5000
000137          DO 8 IA=1,M                                                             5010
000140        8 A(IA,NM)=A(IA,NM)-A(IA,M+1)*A(M+1,NM)                                   5020
000162          X(M)=A(M,NM)                                                            5030
000166          IF (M-1) 502,502,7                                                      5040
000171      400 IE = IE+1                                                               5050
000173          IZ=I+IE                                                                 5060
000174          IF (I+IE-N-1) 401,481,481                                               5070
000177      401 DO 404 ID=I,NM                                                          5080
000201          SWITCH = A(I,ID)                                                        5090
000206          A(I,ID)=A(IZ,ID)                                                        5100
000215      404 A(IZ,ID)=SWITCH                                                         5110
000223          GO TO 2                                                                 5120
000223      481 ISOL = 0                                                                5130
000224          RETURN                                                                  5140
000225      502 ISOL = 1                                                                5150
000227          RETURN                                                                  5160
000227          END                                                                     5170
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                      SUBROUTINE SUB500                                                  5180
                C     SUBROUTINE TO SOLVE FLAME SPEED EQUATION FOR TF*                   5190
000002                COMMON W,W0,W2,W4, Z,Z1, V,I1,J5,J9,K,K5,L1,L2,N,N2,P5,R5,U2,Y1,   5200
                     1EPS3,EPS4,EPS1,EPS2,DI,C9,JERROR,IPLUS, JPRINT,C0,C,P0,R0,QRL,QGP, 5210
                     2C1,C2,C3,C5,US,POW,N22,W44,E1,C4,U1,W1,M,EHS,EFS,WRR,ZFI,          5220
                     3CP,CS,QQG,QQS,EFI,EHI,NN2,NN1,WF1,QS,QG,U0,W1I,W2I                5230
000002                DIMENSION W(100), V(100), Z(100)                                   5240
000002                REAL K,K5,L1,L2,N,M,NN2,NN1                                        5250
000002                REAL LTEST                                                         5260
000002                W4=W2                                                              5270
000003                P6=P5*P0                                                           5280
000005                RS=C1+C2*ALOG(P6)+C3*(ALOG(P6))**2+C5*(ALOG(P6))**3                5290
000024                RS=EXP(RS)                                                         5300
000026                US=U2*(RS/R0)**POW                                                 5310
000033                WS=WRR  /(1.0-WRR  *ALOG(10.0*RS)/E1)                              5320
000043                SX1=C4*U2/U1*US/U2-QS-U0/U1                                        5330
000052                SX2=C4*U2/U1*US/U2-QG-U0/U1                                        5340
000060                SX4=RS/R0*(W1I+W2I)                                                5350
000064                WW2=-SX1*SX4/(SX2-SX1)                                             5360
000070                WW1=SX2*SX4/(SX2-SX1)                                              5370
000071                WRW=WW1/WW2                                                        5380
000073                EHS=(1.0-U1/EHI*(ALOG(WW1/W1I)-NN1*ALOG(P6/P0)-(NN1+2.0)*          5390
                     1    ALOG(WS/U1)))*WS/U1                                          5400
000122                EFS=(1.0-U2/EFI*(ALOG(WW2/W2I)-NN2*ALOG(P6/P0)-(NN2+2.0)*          5410
                     1    ALOG(US/U2)))*US/U2                                          5420
                C     EF* IS ASSUMED TO BE A FUNCTION OF PRESSURE ONLY                   5430
                C     AND IS CALCULATED FROM STEADY STATE CONDITIONS                     5440
000151                DI=W1I+W2I                                                         5450
000153                WF1=W1I*P5**NN1*W1**(NN1+2.0)*EXP(EHI/U1*(W1-EHS)/W1)              5460
000174                L1=ALOG(R5*DI-WF1)-NN2*ALOG(P5)-ALOG(W2I)                          5470
000212    510         L2=(NN2+2.0)*ALOG(W4)+(EFI/U2)*(1.0-EFS/W4)                        5480
000226                LTEST=L2/L1                                                        5490
000227                IF(LTEST-(1.0-EPS3)) 530,404,404                                   5500
000233    404         IF(LTEST-(1.0+EPS3)) 522,522,530                                   5510
000237    522         J5=1                                                               5520
000240                GO TO 525                                                          5530
000241    530         WSQ=W4*W4                                                          5540
000242                F1=(NN2+2.0)/W4+(EFI/U2)*(EFS/WSQ)                                 5550
                C     F1 IS FIRST DERIVATIVE OF L2                                       5560
000252                F2=-(NN2+2.0)/WSQ-2.0*(EFI/U2)*(EFS/(WSQ*W4))                      5570
                C     F2 IS SECOND DERIVATIVE OF L2                                      5580
000264                W4=W4-(L2-L1)/(F1-F2*(L2-L1)/(F1+F1))                              5590
                C     W4 IS FLAME TEMPERATURE RATIO TF*                                  5600
000275    513         J5=J5+1                                                            5610
000277                IF(J5-50) 510,510,524                                              5620
000301    524         JERROR=1                                                           5630
000302    525         CONTINUE                                                           5640
000302                RETURN                                                             5650
000303                END                                                                5660
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                SUBROUTINE SUB600                                                       5670
          C     SUBROUTINE TO SOLVE FOR INITIAL TEMPERATURE PROFILE IN SOLID            5680
000002          COMMON W,W0,W2,W4, Z,ZI, V,I1,J5,J9,K,K5,L1,L2,N,N2,P5,R5,U2,Y1,      5690
               1EPS3,EPS4,EPS1,EPS2,DI,C9,JERROR,IPLUS, JPRINT,C0,C,P0,R0,QRL,QGP,     5700
               2C1,C2,C3,C5,US,POW,N22,W44,E1,C4,U1,W1,M,EMS,EFS,WRR,ZFI,              5710
               3CP,CS,QQG,QQS,EFI,EHI,NN2,NN1,WF1,QS,QG,U0,W1I,W2I                     5720
000002          DIMENSION W(100), V(100), Z(100)                                        5730
000002          REAL K,K5,L1,L2,N,M                                                     5740
000002          DO 625 I=1,IPLUS                                                        5750
000004          L=I-1                                                                   5760
000005          FL=L                                                                    5770
000007          V(I)=(1.0-FL*Y1)*(1.0-W0)+W0                                            5780
000017          IF(JPRINT) 618,625,618                                                  5790
000020      618 IF(J9-1) 612,612,625                                                    5800
000023      612 IF(L) 613,613,622                                                       5810
000025      613 WRITE(6,614)                                                            5820
000031      614 FORMAT(8H  TIME=0/9H        Y*,13X,7HX (IN.), 6X,14H(T-T0)/(TW-T0))     5830
000031      622 IF(L-N2) 625,623,623                                                    5840
000034      623 FL=L                                                                    5850
000035          PRNT1=FL*Y1                                                             5860
000037          IF(PRNT1-1.0) 616,615,616                                               5870
000042      615 ARG=0.001                                                               5880
000043          PRNT3=0.001                                                             5890
000045          GO TO 617                                                               5900
000045      616 ARG=1.0-PRNT1                                                           5910
000047          PRNT3=(V(I)-W0)/(1.0-W0)                                                5920
000055      617 PRNT2=-ALOG(ARG)/(K5*R5)                                                5930
000063          WRITE(6,620) PRNT1,PRNT2,PRNT3                                          5940
000075      620 FORMAT(3E16.8)                                                          5950
000075          N2=N2+N22                                                               5960
000077      625 CONTINUE                                                                5970
000102      630 DO 634 I=1, IPLUS                                                       5980
000104      634 W(I)=V(I)                                                               5990
000111          RETURN                                                                  6000
000112          END                                                                     6010
```

```
                SUBROUTINE TRIDAG(N,AVEC,BVEC,CVEC,DVEC,SVEC)                    6020
      C         SUBROUTINE TO SOLVE FOR INSTANTANEOUS TEMPERATURE PROFILE IN SOLID 6030
      C         SOLVES TRIDIAGIONAL MATRIX                                       6040
000011          DIMENSION AVEC(1),BVEC(1),CVEC(1),DVEC(1),SVEC(1)                6050
000011          DVEC(1)=DVEC(1)/BVEC(1)                                          6060
000012          W=BVEC(1)                                                        6070
000013          DO 2 J=2,N                                                       6080
000015          JJ=J-1                                                           6090
000016          BVEC(JJ)=CVEC(JJ)/W                                              6100
000022          W=BVEC(J)-AVEC(JJ)*BVEC(JJ)                                      6110
000026        2 DVEC(J)=(DVEC(J)-AVEC(JJ)*DVEC(JJ))/W                            6120
000036          IM=N-1                                                           6130
000040          DO 3 J=1,IM                                                      6140
000041          JJ=N-J                                                           6150
000042        3 DVEC(JJ)=DVEC(JJ)-BVEC(JJ)*DVEC(JJ+1)                           6160
000051          DO 4 J=1,N                                                       6170
000052        4 SVEC(J)=DVEC(J)                                                  6180
000056          RETURN                                                           6190
000056          END                                                              6200
```

JULY 20 1970

SOLID PROPELLANT TRANSIENT COMBUSTION BEHAVIOR
UTX 11327

INPUT CONSTANTS
C1=-3.08174274E+00
C2=-2.70948178E-01
C3= 1.44544251E-01
C5=-9.98599071E-03
C4= 1.00000000E+00
CP= 4.00000000E-01
CS= 4.00000000E-01
KG= 4.00000000E-04
N1= 1.00000000E+00
N2= 2.00000000E+00
T0= 3.00000000E+02
TFR= 2.41900000E+03
TWR= 8.00000000E+02
POW= 1.05000000E-02
RHO= 8.00000000E-02
WRL= 1.94071918E-01
QRL= 1.00000000E-01
KAPPA= 1.00000000E-04
GAMMA= 1.20000000E+00
TSTOP= 7.00000000E-02
TIMFAC= 1.00000000E+00
TCHANGE= 1.00000000E-03
TIME STEP= 1.00000000E-05

LIMIT POINT VALUES
PL= 4.27591898E+02
RL= 1.94264798E-01
AL= 1.93599454E+01
ALPHAL= 8.39684615E-01
EFL= 5.37787134E+05
EHL= 1.02230756E+05
TFL= 2.43430861E+03
TWL= 8.17368812E+02
XFL= 1.35302233E+01
NU = 3.80898941E-01

INITIAL POINT VALUES
P0= 7.00000000E+02
R0= 2.32014316E-01
EW= 5.00000000E+04
EF= 5.42757819E+05
EH= 1.03395534E+05
TF= 2.43885176E+03
TW= 8.22142032E+02
QG= 1.00000000E+03
QS= 1.00000000E+02
A = 1.93123427E+01
ALPHA= 8.38358051E-01
NU = 3.37197914E-01
XFI = 1.34515667E+01

JULY 20 1970

CHAMBER DEPRESSURIZATION RESPONSE

INITIAL DECAY RATE= 6.00000000E+04
AT/ATI= 4.49511536E+00

TIME	P*	E*	Z*	R*	TW*	TF*	TC*	EF*	EM*
0.	1.0000E+00	1.0000E+00	-6.3510E-01	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00
4.0000E-04	9.6628E-01	1.0019E+00	-6.2238E-01	9.5506E-01	9.9849E-01	9.9965E-01	9.9432E-01	9.9936E-01	9.9923E-01
8.0000E-04	9.3382E-01	1.0060E+00	-6.1334E-01	9.3621E-01	9.9784E-01	9.9947E-01	9.8874E-01	9.9873E-01	9.9845E-01
1.2000E-03	9.0270E-01	1.0102E+00	-6.0483E-01	9.1980E-01	9.9726E-01	9.9930E-01	9.8328E-01	9.9810E-01	9.9769E-01
1.6000E-03	8.7286E-01	1.0148E+00	-5.9656E-01	9.0459E-01	9.9671E-01	9.9915E-01	9.7794E-01	9.9747E-01	9.9692E-01
2.0000E-03	8.4426E-01	1.0195E+00	-5.8856E-01	8.9046E-01	9.9620E-01	9.9900E-01	9.7273E-01	9.9686E-01	9.9616E-01
2.4000E-03	8.1683E-01	1.0242E+00	-5.8071E-01	8.7686E-01	9.9570E-01	9.9885E-01	9.6763E-01	9.9624E-01	9.9541E-01
2.8000E-03	7.9053E-01	1.0290E+00	-5.7303E-01	8.6383E-01	9.9521E-01	9.9870E-01	9.6265E-01	9.9563E-01	9.9466E-01
3.2000E-03	7.6531E-01	1.0339E+00	-5.6549E-01	8.5116E-01	9.9473E-01	9.9856E-01	9.5780E-01	9.9503E-01	9.9392E-01
3.6000E-03	7.4111E-01	1.0388E+00	-5.5809E-01	8.3886E-01	9.9425E-01	9.9841E-01	9.5306E-01	9.9443E-01	9.9319E-01
4.0000E-03	7.1789E-01	1.0436E+00	-5.5081E-01	8.2686E-01	9.9379E-01	9.9827E-01	9.4844E-01	9.9384E-01	9.9246E-01
4.4000E-03	6.9561E-01	1.0485E+00	-5.4365E-01	8.1513E-01	9.9332E-01	9.9813E-01	9.4394E-01	9.9326E-01	9.9173E-01
4.8000E-03	6.7422E-01	1.0534E+00	-5.3662E-01	8.0364E-01	9.9286E-01	9.9799E-01	9.3955E-01	9.9268E-01	9.9101E-01
5.2000E-03	6.5368E-01	1.0583E+00	-5.2969E-01	7.9238E-01	9.9240E-01	9.9785E-01	9.3527E-01	9.9210E-01	9.9030E-01
5.6000E-03	6.3395E-01	1.0632E+00	-5.2288E-01	7.8132E-01	9.9195E-01	9.9771E-01	9.3111E-01	9.9153E-01	9.8959E-01
6.0000E-03	6.1499E-01	1.0681E+00	-5.1617E-01	7.7046E-01	9.9150E-01	9.9758E-01	9.2706E-01	9.9097E-01	9.8889E-01
6.4000E-03	5.9678E-01	1.0729E+00	-5.0957E-01	7.5978E-01	9.9105E-01	9.9744E-01	9.2311E-01	9.9041E-01	9.8820E-01
6.8000E-03	5.7926E-01	1.0777E+00	-5.0307E-01	7.4928E-01	9.9060E-01	9.9730E-01	9.1928E-01	9.8986E-01	9.8751E-01
7.2000E-03	5.6243E-01	1.0825E+00	-4.9667E-01	7.3894E-01	9.9015E-01	9.9716E-01	9.1555E-01	9.8931E-01	9.8682E-01
7.6000E-03	5.4624E-01	1.0873E+00	-4.9038E-01	7.2876E-01	9.8970E-01	9.9703E-01	9.1193E-01	9.8876E-01	9.8615E-01
8.0000E-03	5.3066E-01	1.0920E+00	-4.8417E-01	7.1873E-01	9.8926E-01	9.9689E-01	9.0841E-01	9.8823E-01	9.8547E-01
8.4000E-03	5.1566E-01	1.0967E+00	-4.7807E-01	7.0884E-01	9.8881E-01	9.9675E-01	9.0499E-01	9.8769E-01	9.8481E-01
8.8000E-03	5.0123E-01	1.1014E+00	-4.7205E-01	6.9909E-01	9.8836E-01	9.9662E-01	9.0167E-01	9.8717E-01	9.8415E-01
9.2000E-03	4.8734E-01	1.1060E+00	-4.6613E-01	6.8947E-01	9.8792E-01	9.9648E-01	8.9845E-01	9.8664E-01	9.8350E-01
9.6000E-03	4.7395E-01	1.1107E+00	-4.6029E-01	6.7998E-01	9.8747E-01	9.9635E-01	8.9533E-01	9.8613E-01	9.8285E-01
1.0000E-02	4.6106E-01	1.1152E+00	-4.5455E-01	6.7061E-01	9.8703E-01	9.9621E-01	8.9230E-01	9.8561E-01	9.8221E-01
1.0400E-02	4.4863E-01	1.1198E+00	-4.4888E-01	6.6135E-01	9.8659E-01	9.9607E-01	8.8937E-01	9.8511E-01	9.8157E-01
1.0800E-02	4.3665E-01	1.1243E+00	-4.4329E-01	6.5219E-01	9.8614E-01	9.9594E-01	8.8653E-01	9.8460E-01	9.8094E-01
1.1200E-02	4.2510E-01	1.1287E+00	-4.3778E-01	6.4314E-01	9.8569E-01	9.9580E-01	8.8378E-01	9.8411E-01	9.8031E-01
1.1600E-02	4.1396E-01	1.1332E+00	-4.3235E-01	6.3420E-01	9.8525E-01	9.9567E-01	8.8112E-01	9.8361E-01	9.7969E-01
1.2000E-02	4.0321E-01	1.1376E+00	-4.2700E-01	6.2535E-01	9.8480E-01	9.9553E-01	8.7854E-01	9.8312E-01	9.7908E-01
1.2400E-02	3.9283E-01	1.1419E+00	-4.2170E-01	6.1654E-01	9.8434E-01	9.9539E-01	8.7605E-01	9.8264E-01	9.7847E-01
1.2800E-02	3.8281E-01	1.1463E+00	-4.1645E-01	6.0772E-01	9.8389E-01	9.9526E-01	8.7364E-01	9.8216E-01	9.7787E-01
1.3200E-02	3.7312E-01	1.1506E+00	-4.1125E-01	5.9895E-01	9.8342E-01	9.9512E-01	8.7132E-01	9.8168E-01	9.7727E-01
1.3600E-02	3.6377E-01	1.1548E+00	-4.0615E-01	5.9043E-01	9.8297E-01	9.9498E-01	8.6907E-01	9.8121E-01	9.7668E-01
1.4000E-02	3.5473E-01	1.1591E+00	-4.0125E-01	5.8244E-01	9.8254E-01	9.9485E-01	8.6690E-01	9.8075E-01	9.7609E-01
1.4400E-02	3.4600E-01	1.1632E+00	-3.9653E-01	5.7498E-01	9.8213E-01	9.9472E-01	8.6481E-01	9.8028E-01	9.7551E-01
1.4800E-02	3.3756E-01	1.1673E+00	-3.9175E-01	5.6708E-01	9.8169E-01	9.9459E-01	8.6280E-01	9.7983E-01	9.7493E-01
1.5200E-02	3.2939E-01	1.1714E+00	-3.8612E-01	5.5592E-01	9.8106E-01	9.9441E-01	8.6086E-01	9.7937E-01	9.7436E-01
1.5600E-02	3.2141E-01	1.1756E+00	-3.7569E-01	5.2920E-01	9.7950E-01	9.9403E-01	8.5893E-01	9.7892E-01	9.7378E-01

BURNING RATE BELOW LOWER DEFLAGRATION LIMIT

RUNNING TIME= 3.35110000E+01

Appendix C

TRANSIENT COMBUSTION BEHAVIOR WITH A DECREASE IN BURNING RATE

In this appendix, a test calculation is shown that was run early in the program to ensure that correct behavior of the temperature profile was being attained as the combustion process moves from one steady state to another in which $r^* < 1$. This calculation must be carefully examined because of the nonanalyticity of the temperature profile at the cold end in the transformed coordinate system.

In the case considered here, the pressure was reduced from 200 to 150 psia over a period of 10 msec and then then the computer was allowed to run for an additional 20 msec. At that time the burning rate has very nearly achieved its asymptotic value and one can compare the numerical temperature profile with the equation obtained from the transformation (Eq. (26) of the main body):

$$\frac{T-T_0}{T_w-T_0} = (1-y^*)^{r^*} \qquad \text{(C1)}$$

Note that $y^* = 1$ corresponds to $x = \infty$ and $T = T_0$; what has been printed out instead opposite $y^* = 1$ is the value of x for which $\frac{T-T_0}{T_w-T_0} = 0.001$ in order to give some measure of the temperature profile thickness.

The calculated asymptotic profile from Eq. (C1) is shown opposite the last printout; it can be seen that there is no difficulty of the type encountered by Merkle, et al.[4]

JUNE 26 1970

SOLID PROPELLANT TRANSIENT COMBUSTION BEHAVIOR
PU 269

INPUT CONSTANTS
C1=-5.07776492E+00
C2= 6.36824580E-01
C3= 3.57458855E-02
C5=-6.93449490E-03
C4= 1.00000000E+00
CP= 4.00000000E-01
CS= 4.00000000E-01
KG= 4.00000000E-04
N1= 1.00000000E+00
N2= 2.00000000E+00
T0= 3.00000000E+02
TFR= 2.82800000E+03
TWR= 8.00000000E+02
POW= 2.50000000E-02
RHO= 6.00000000E-02
WRL= 6.39611342E-02
GRL= 1.00000000E-01
KAPPA= 1.00000000E-04
GAMMA= 1.20000000E+00
TSTOP= 3.00000000E-02
TIMFAC= 1.00000000E+00
TCHANGE= 1.02000000E-02
TIME STEP= 1.00000000E-05

LIMIT POINT VALUES
PL= 6.45228635E+02
RL= 2.62007004E-01
AL= 1.92793167E+01
ALPHAL= 8.38741423E-01
EFL= 2.51478200E+05
EHL= 1.77375374E+05
TFL= 2.87971517E+03
TWL= 8.25442135E+02
ZFL= 5.03552841E+00
 NU= 2.28601891E-01

INITIAL POINT VALUES
P0= 2.00000000E+02
R0= 1.77000000E-01
EW= 5.00000000E+04
EF= 2.37747253E+05
EH= 1.73490974E+05
TF= 2.85161600E+03
TW= 8.14889121E+02
QG= 1.09090909E+03
QS= 1.09090909E+02
A = 1.93845929E+01
ALPHA= 9.73183177E-01

NU   = 4.31611778E-01
ZFI   = 5.11151233E+00

SINUSOIDAL RESPONSE
P*= 9.99750000E-01
F*= 0.

TIME=0

Y*	X (IN.)	(T-T0)/(TW-T0)
0.	-0.	1.00000000E+00
2.00000000E-01	1.26069803E-04	8.00000000E-01
4.00000000E-01	2.88602047E-04	6.00000000E-01
6.00000000E-01	5.17678380E-04	4.00000000E-01
8.00000000E-01	9.09286956E-04	2.00000000E-01
1.00000000E+00	3.90268660E-03	1.00000000E-03

TIME= 2.00000000E-04

Y*	X (IN.)	(T-T0)/(TW-T0)
0.	-0.	1.00000000E+00
2.00000000E-01	1.26839058E-04	8.00264957E-01
4.00000000E-01	2.90363044E-04	6.00271804E-01
6.00000000E-01	5.20837158E-04	4.00189648E-01
8.00000000E-01	9.14835258E-04	2.00094979E-01
1.00000000E+00	3.92650008E-03	1.00000000E-03

P*	E*	Z*	R*	TW*	TF*	TC*	EF*	EH*
9.95000E-01	1.00003E+00	-6.30228E-01	9.93935E-01	9.99802E-01	9.99883E-01	1.00000E+00	9.99754E-01	9.99892E-01

TIME= 4.00000000E-04

Y*	X (IN.)	(T-T0)/(TW-T0)
0.	-0.	1.00000000E+00
2.00000000E-01	1.27683922E-04	8.00616398E-01
4.00000000E-01	2.92297127E-04	6.00725858E-01
6.00000000E-01	5.24306409E-04	4.00549867E-01
8.00000000E-01	9.20928895E-04	2.00279796E-01
1.00000000E+00	3.95265415E-03	1.00000000E-03

P*	E*	Z*	R*	TW*	TF*	TC*	EF*	EH*
9.90000E-01	1.00025E+00	-6.28519E-01	9.87358E-01	9.99585E-01	9.99749E-01	1.00000E+00	9.99506E-01	9.99784E-01

TIME= 6.00000000E-04

Y*	X (IN.)	(T-T0)/(TW-T0)
0.	-0.	1.00000000E+00
2.00000000E-01	1.28399548E-04	8.00989416E-01
4.00000000E-01	2.93935355E-04	6.01252728E-01
6.00000000E-01	5.27244972E-04	4.01011598E-01
8.00000000E-01	9.26090396E-04	2.00529304E-01
1.00000000E+00	3.97480746E-03	1.00000000E-03

P*	E*	Z*	R*	TW*	TF*	TC*	EF*	EH*
9.85000E-01	1.00063E+00	-6.26847E-01	9.81856E-01	9.99403E-01	9.99634E-01	1.00000E+00	9.99258E-01	9.99675E-01

TIME= 8.00000000E-04

Y*	X (IN.)	(T-T0)/(TW-T0)
0.	-0.	1.00000000E+00
2.00000000E-01	1.29012739E-04	8.01370931E-01
4.00000000E-01	2.95339088E-04	6.01817694E-01
6.00000000E-01	5.29762910E-04	4.01539777E-01

```
 8.00000000E-01  9.30513080E-04  2.00831452E-01
 1.00000000E+00  3.99378975E-03  1.00000000E-03

     P*              E*              Z*              R*              TW*             TF*             TC*             EF*             EH*
 9.80000E-01   1.00112E+00  -6.25214E-01   9.77189E-01   9.99248E-01   9.99533E-01   1.00000E+00   9.99008E-01   9.99566E-01


TIME= 1.00000000E-03

       Y*              X (IN.)         (T-T0)/(TW-T0)
 0.              -0.             1.00000000E+00
 2.00000000E-01  1.29560372E-04  8.01756964E-01
 4.00000000E-01  2.96592743E-04  6.02406231E-01
 6.00000000E-01  5.32011648E-04  4.02114737E-01
 8.00000000E-01  9.34462923E-04  2.01177760E-01
 1.00000000E+00  4.01074260E-03  1.00000000E-03

     P*              E*              Z*              R*              TW*             TF*             TC*             EF*             EH*
 9.75000E-01   1.00168E+00  -6.23608E-01   9.73058E-01   9.99111E-01   9.99441E-01   1.00000E+00   9.98756E-01   9.99456E-01


TIME= 1.20000000E-03

       Y*              X (IN.)         (T-T0)/(TW-T0)
 0.              -0.             1.00000000E+00
 2.00000000E-01  1.30067604E-04  8.02146325E-01
 4.00000000E-01  2.97753911E-04  6.03011580E-01
 6.00000000E-01  5.34094485E-04  4.02724988E-01
 8.00000000E-01  9.38121366E-04  2.01561804E-01
 1.00000000E+00  4.02644474E-03  1.00000000E-03

     P*              E*              Z*              R*              TW*             TF*             TC*             EF*             EH*
 9.70000E-01   1.00230E+00  -6.22019E-01   9.69264E-01   9.98983E-01   9.99355E-01   1.00000E+00   9.98504E-01   9.99345E-01


TIME= 1.40000000E-03

       Y*              X (IN.)         (T-T0)/(TW-T0)
 0.              -0.             1.00000000E+00
 2.00000000E-01  1.30549433E-04  8.02538634E-01
 4.00000000E-01  2.98856926E-04  6.03630198E-01
 6.00000000E-01  5.36073013E-04  4.03363395E-01
 8.00000000E-01  9.41596592E-04  2.01978580E-01
 1.00000000E+00  4.04136051E-03  1.00000000E-03

     P*              E*              Z*              R*              TW*             TF*             TC*             EF*             EH*
 9.65000E-01   1.00297E+00  -6.20438E-01   9.65686E-01   9.98863E-01   9.99273E-01   1.00000E+00   9.98250E-01   9.99234E-01


TIME= 1.60000000E-03

       Y*              X (IN.)         (T-T0)/(TW-T0)
 0.              -0.             1.00000000E+00
 2.00000000E-01  1.31014601E-04  8.02933722E-01
 4.00000000E-01  2.99921800E-04  6.04259963E-01
 6.00000000E-01  5.37983125E-04  4.04025222E-01
 8.00000000E-01  9.44951649E-04  2.02424122E-01
 1.00000000E+00  4.05576052E-03  1.00000000E-03

     P*              E*              Z*              R*              TW*             TF*             TC*             EF*             EH*
 9.60000E-01   1.00366E+00  -6.18863E-01   9.62258E-01   9.98748E-01   9.99194E-01   1.00000E+00   9.97995E-01   9.99122E-01


TIME= 1.80000000E-03

       Y*              X (IN.)         (T-T0)/(TW-T0)
```

0.	-0.	1.00000000E+00
2.00000000E-01	1.31468373E-04	8.03331476E-01
4.00000000E-01	3.00960584E-04	6.04899457E-01
6.00000000E-01	5.39846439E-04	4.04707137E-01
8.00000000E-01	9.48224505E-04	2.02895247E-01
1.00000000E+00	4.06980771E-03	1.00000000E-03

P*	E*	Z*	R*	TW*	TF*	TC*	EF*	EH*
9.55000E-01	1.00439E+00	-6.17289E-01	9.58936E-01	9.98635E-01	9.99116E-01	1.00000E+00	9.97739E-01	9.99009E-01

TIME= 2.00000000E-03

Y*	X (IN.)	(T-T0)/(TW-T0)
0.	-0.	1.00000000E+00
2.00000000E-01	1.31914123E-04	8.03731805E-01
4.00000000E-01	3.01981005E-04	6.05547665E-01
6.00000000E-01	5.41676814E-04	4.05406685E-01
8.00000000E-01	9.51439505E-04	2.03389369E-01
1.00000000E+00	4.08360659E-03	1.00000000E-03

P*	E*	Z*	R*	TW*	TF*	TC*	EF*	EH*
9.50000E-01	1.00515E+00	-6.15715E-01	9.55696E-01	9.98525E-01	9.99039E-01	1.00000E+00	9.97481E-01	9.98896E-01

TIME= 2.80000000E-02

Y*	X (IN.)	(T-T0)/(TW-T0)
0.	-0.	1.00000000E+00
2.00000000E-01	1.44196698E-04	8.22016328E-01
4.00000000E-01	3.30098574E-04	6.38287900E-01
6.00000000E-01	5.92112553E-04	4.46540115E-01
8.00000000E-01	1.04002841E-03	2.41753537E-01
1.00000000E+00	4.46383279E-03	1.00000000E-03

P*	E*	Z*	R*	TW*	TF*	TC*	EF*	EH*
7.50000E-01	1.05582E+00	-5.50932E-01	8.74290E-01	9.95640E-01	9.96678E-01	1.00000E+00	9.85892E-01	9.93702E-01

TIME= 2.82000000E-02

Y*	X (IN.)	(T-T0)/(TW-T0)

0.	-0.	1.00000000E+00
2.00000000E-01	1.44195474E-04	8.22016468E-01
4.00000000E-01	3.30095772E-04	6.38288500E-01
6.00000000E-01	5.92107527E-04	4.46541579E-01
8.00000000E-01	1.04001958E-03	2.41756251E-01
1.00000000E+00	4.46379490E-03	1.00000000E-03

P*	E*	Z*	R*	TW*	TF*	TC*	EF*	EH*
7.50000E-01	1.05582E+00	-5.50932E-01	8.74298E-01	9.95640E-01	9.96678E-01	1.00000E+00	9.85892E-01	9.93702E-01

TIME= 2.84000000E-02

Y*	X (IN.)	(T-T0)/(TW-T0)
0.	-0.	1.00000000E+00
2.00000000E-01	1.44194314E-04	8.22016601E-01
4.00000000E-01	3.30093117E-04	6.38289068E-01
6.00000000E-01	5.92102765E-04	4.46542966E-01
8.00000000E-01	1.04001122E-03	2.41758822E-01
1.00000000E+00	4.46375900E-03	1.00000000E-03

P*	E*	Z*	R*	TW*	TF*	TC*	EF*	EH*
7.50000E-01	1.05582E+00	-5.50932E-01	8.74305E-01	9.95641E-01	9.96678E-01	1.00000E+00	9.85892E-01	9.93702E-01

TIME= 2.86000000E-02

Y*	X (IN.)	(T-T0)/(TW-T0)
0.	-0.	1.00000000E+00
2.00000000E-01	1.44193215E-04	8.22016727E-01
4.00000000E-01	3.30090601E-04	6.38289607E-01
6.00000000E-01	5.92098252E-04	4.46544280E-01
8.00000000E-01	1.04000329E-03	2.41761259E-01
1.00000000E+00	4.46372497E-03	1.00000000E-03

P*	E*	Z*	R*	TW*	TF*	TC*	EF*	EH*
7.50000E-01	1.05583E+00	-5.50932E-01	8.74312E-01	9.95641E-01	9.96679E-01	1.00000E+00	9.85892E-01	9.93702E-01

TIME= 2.88000000E-02

Y*	X (IN.)	(T-T0)/(TW-T0)
0.	-0.	1.00000000E+00
2.00000000E-01	1.44192174E-04	8.22016846E-01
4.00000000E-01	3.30088217E-04	6.38290117E-01
6.00000000E-01	5.92093975E-04	4.46545526E-01
8.00000000E-01	1.03999578E-03	2.41763568E-01
1.00000000E+00	4.46369274E-03	1.00000000E-03

P*	E*	Z*	R*	TW*	TF*	TC*	EF*	EH*
7.50000E-01	1.05583E+00	-5.50933E-01	8.74318E-01	9.95641E-01	9.96679E-01	1.00000E+00	9.85892E-01	9.93702E-01

TIME= 2.90000000E-02

Y*	X (IN.)	(T-T0)/(TW-T0)
0.	-0.	1.00000000E+00
2.00000000E-01	1.44191187E-04	8.22016960E-01
4.00000000E-01	3.30085958E-04	6.38290601E-01
6.00000000E-01	5.92089923E-04	4.46546707E-01
8.00000000E-01	1.03998866E-03	2.41765756E-01
1.00000000E+00	4.46366219E-03	1.00000000E-03

P*	E*	Z*	R*	TW*	TF*	TC*	EF*	EH*
7.50000E-01	1.05583E+00	-5.50933E-01	8.74324E-01	9.95641E-01	9.96679E-01	1.00000E+00	9.85892E-01	9.93702E-01

TIME= 2.92000000E-02

Y*	X (IN.)	(T-T0)/(TW-T0)
0.	-0.	1.00000000E+00
2.00000000E-01	1.44190252E-04	8.22017067E-01
4.00000000E-01	3.30063818E-04	6.38291059E-01
6.00000000E-01	5.92086084E-04	4.46547825E-01
8.00000000E-01	1.03998192E-03	2.41767829E-01
1.00000000E+00	4.46363324E-03	1.00000000E-03

P*	E*	Z*	R*	TW*	TF*	TC*	EF*	EH*
7.50000E-01	1.05583E+00	-5.50933E-01	8.74330E-01	9.95642E-01	9.96679E-01	1.00000E+00	9.85892E-01	9.93702E-01

TIME= 2.94000000E-02

Y*	X (IN.)	(T-T0)/(TW-T0)
0.	-0.	1.00000000E+00
2.00000000E-01	1.44189366E-04	8.22017168E-01
4.00000000E-01	3.30081790E-04	6.38291494E-01
6.00000000E-01	5.92082446E-04	4.46548886E-01
8.00000000E-01	1.03997553E-03	2.41769793E-01
1.00000000E+00	4.46360582E-03	1.00000000E-03

P*	E*	Z*	R*	TW*	TF*	TC*	EF*	EH*
7.50000E-01	1.05583E+00	-5.50933E-01	8.74335E-01	9.95642E-01	9.96679E-01	1.00000E+00	9.85892E-01	9.93702E-01

TIME= 2.96000000E-02

Y*	X (IN.)	(T-T0)/(TW-T0)
0.	-0.	1.00000000E+00
2.00000000E-01	1.44168526E-04	8.22017265E-01
4.00000000E-01	3.30079868E-04	6.38291905E-01
6.00000000E-01	5.92078998E-04	4.46549890E-01
8.00000000E-01	1.03996947E-03	2.41771655E-01
1.00000000E+00	4.46357983E-03	1.00000000E-03

P*	E*	Z*	R*	TW*	TF*	TC*	EF*	EH*
7.50000E-01	1.05584E+00	-5.50933E-01	8.74340E-01	9.95642E-01	9.96679E-01	1.00000E+00	9.85892E-01	9.93702E-01

TIME= 2.98000000E-02

Y*	X (IN.)	(T-T0)/(TW-T0)
0.	-0.	1.00000000E+00
2.00000000E-01	1.44187731E-04	8.22017356E-01
4.00000000E-01	3.30078046E-04	6.38292296E-01
6.00000000E-01	5.92075731E-04	4.46550842E-01
8.00000000E-01	1.03996373E-03	2.41773419E-01
1.00000000E+00	4.46355520E-03	1.00000000E-03

P*	E*	Z*	R*	TW*	TF*	TC*	EF*	EH*
7.50000E-01	1.05584E+00	-5.50933E-01	8.74345E-01	9.95642E-01	9.96679E-01	1.00000E+00	9.85892E-01	9.93702E-01

TIME= 3.00000000E-02

Y*	X (IN.)	(T-T0)/(TW-T0)	$\frac{(1-y^*)^{r^*}}{}$
0.	-0.	1.00000000E+00	1.0
2.00000000E-01	1.44186977E-04	8.22017443E-01	0.823
4.00000000E-01	3.30076321E-04	6.38292665E-01	0.640
6.00000000E-01	5.92072636E-04	4.46551744E-01	0.449
8.00000000E-01	1.03995829E-03	2.41775091E-01	0.245
1.00000000E+00	4.46353186E-03	1.00000000E-03	0

P*	E*	Z*	R*	TW*	TF*	TC*	EF*	EH*
7.50000E-01	1.05584E+00	-5.50934E-01	8.74349E-01	9.95642E-01	9.96679E-01	1.00000E+00	9.85892E-01	9.93702E-01

RUNNING TIME= 5.13460000E+01

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