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AN INVESTIGATION OF THE HIGH SPEED TURBULENT BOUNDARY LAYER WITH HEAT TRANSFER AND ARBITRARY PRESSURE GRADIENT

Part II - The Compressibility Transformation - General Considerations

by Constantino Economos

Prepared by
GENERAL APPLIED SCIENCE LABORATORIES, INC.
Westbury, L. I., N. Y. 11590
for Langley Research Center

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16. Abstract
Various properties of the Coles' compressibility transformation when modified and extended to include turbulent boundary layer flows with mass transfer and pressure gradient are examined. First the formulation for the mass transfer case in the absence of pressure gradient is developed and its applicability demonstrated by comparison with experimental data involving both homogeneous and heterogeneous injection as well as chemical reaction. This modified form of the transformation is then applied to the impermeable case and is shown to differ significantly from the earlier Baronti-Libby formulation. In particular, improved agreement with experimental skin friction data obtained under cold wall conditions is demonstrated. The implications of this result with regard to the more general case involving simultaneous mass transfer and pressure gradient are then discussed. In addition to these considerations the inability of the transformation to preserve the "wake component" of the velocity distribution is examined. An empirical method is proposed which provides good correlation of the transformed velocity profiles throughout the viscous layer and more realistic prediction for the streamwise variation of skin friction. Finally, the applicability of this technique to three-dimensional boundary layers is examined. In particular, the appropriate form of the transformation for flow over a swept cylinder is developed.

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FOREWORD

The present report is one of a series of three reports which describe analyses and computational procedures developed for describing the behavior of high speed-turbulent boundary layers under conditions involving both heat transfer and arbitrary pressure gradient. Part I,serves as a summary report and describes the analysis which is utilized in the numerical calculation scheme. In Part II, the fundamental properties of the compressibility transformation used in the analysis are examined in detail. Part III, describes the numerical and computational procedures involved and serves as a computer program manual.

The titles in the series are:

Part I Summary Report - "An Investigation of the High Speed Turbulent Boundary Layer with Heat Transfer and Arbitrary Pressure Gradient," by C. Economos and J. Boccio.

Part II- "The Compressibility Transformation - General Considerations," by C. Economos.

Part III- "Computer Program Manual," by J. Schneider and J. Boccio.

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TABLE OF CONTENTS

SYMBOLS

c_f, $\bar{c}_f$	local skin friction coefficients
F, $\bar{F}$	non-dimensionalized transpiration rate - $\frac{(\rho v)_w}{(\rho_e u_e)_o}$
g_1, g_2, g_1', g_2'	thermodynamic functions - see Appendix B
M	Mach number
$\dot{m}$	ablation rate
p, $\bar{p}$	pressure
$\dot{q}$	heat transfer rate
R_y, $R_{\bar{y}}$	Reynolds number based on normal coordinate
R_δ, $R_{\bar{\delta}}$	Reynolds number based on boundary layer thickness
R_δ^*, $R_{\bar{\delta}}^*$	Reynolds number based on displacement thickness
R_θ, $R_{\bar{\theta}}$	Reynolds number based on momentum thickness
T	temperature
T_t	total temperature
u, $\bar{u}$	streamwise (or chordwise) velocity component
$\bar{u}^+$	non dimensional velocity - $\bar{u}/\bar{u}_\tau$
$\bar{u}_\tau$	shearing velocity $(\bar{\tau}_w/\bar{\rho})^{1/2}$
U_e, $\bar{U}_e$	external velocity ratio - u_e/u_{e_o}, $\bar{u}_e/\bar{u}_{e_o}$
v, $\bar{v}$	normal velocity component
w, $\bar{w}$	spanwise velocity component
x, $\bar{x}$	streamwise (or chordwise) coordinates
y, $\bar{y}$	normal coordinates
$\bar{y}^+$	transformed normal coordinate $\overline{\rho u_\tau} \bar{y}/\bar{\mu}$
ζ, $\bar{\zeta}$	blowing parameter - $2F/c_f$, $2\bar{F}/\bar{c}_f$
η, ξ, σ	scaling parameters of the transformation
$\tilde{\eta}$	$\eta\rho_e/\bar{\rho}$
μ, $\bar{\mu}$	coefficients of viscosity
π	Coles wake parameter
ρ, $\bar{\rho}$	densities
$\tilde{\sigma}$	$\sigma\mu_e/\bar{\mu}$

$\tau, \bar{\tau}$ shear stresses (including Reynolds stresses)

$\chi, \bar{\chi}$ Reynolds number based on distance from arbitrary initial station

$\psi, \bar{\psi}$ stream function

ν kinematic viscosity

ω exponent of viscosity temperature variation

Subscripts:

BL value inferred using form II-B

e local condition external to boundary layer

M value inferred using form II-D

o initial value

R Reference value (direct measurement or Spalding-Chi)

s conditions at edge of laminar sub-layer

w conditions at wall

$(\,)$ variables in the variable property flow
$\bar{(\,)}$ variables in the constant property flow
$\tilde{(\,)}$ normalization with respect to corresponding external value; e.g., $\tilde{u}=u/u_e$

$(\,)'$ differentiation with respect to $\bar{\chi}$

THE COMPRESSIBILITY TRANSFORMATION - GENERAL CONSIDERATIONS

By Constantino Economos
General Applied Science Laboratories, Inc.

SUMMARY

Various properties of the Coles' compressibility transformation when modified and extended to include turbulent boundary layer flows with mass transfer and pressure gradient are examined. First the formulation for the mass transfer case in the absence of pressure gradient is developed and its applicability demonstrated by comparison with experimental data involving both homogeneous and heterogeneous injection as well as chemical reaction. This modified form of the transformation is then applied to the impermeable case and is shown to differ significantly from the earlier Baronti-Libby formulation. In particular, improved agreement with experimental skin friction data obtained under cold wall conditions is demonstrated. The implications of this result with regard to the more general case involving simultaneous mass transfer and pressure gradient are then discussed. In addition to these considerations the inability of the transformation to preserve the "wake component" of the velocity distribution is examined. An empirical method is proposed which provides good correlation of the transformed velocity profiles throughout the viscous layer and more realistic prediction for the streamwise variation of skin friction. Finally, the applicability of this technique to three-dimensional boundary layers is examined. In particular, the appropriate form of the transformation for flow over a swept cylinder is developed.

This report was originally prepared as a talk presented at the Symposium on Compressible Turbulent Boundary Layers at Langley Research Center on December 10-11, 1968, entitled "On the Applicability of Transformation Theory to Variable Property Turbulent Boundary Layers with Pressure Gradient and Heat and Mass Transfer."

I. INTRODUCTION

For a number of years the Coles compressibility transformation (Ref. 1) has been utilized by many investigators (Refs. 2,3,4,5) in studies related to variable property (VP) turbulent boundary layer flows on impermeable surfaces and at constant pressure. Exploitation of this technique for more complex flow configurations involving pressure gradient and/or mass transfer, on the other hand, has been quite limited. Furthermore, as will be indicated in the subsequent discussion, among those studies which have treated this more general problem (Refs. 6,7,8,9) fundamental differences regarding the properties of the transformation arise.

In the present paper a unified description is formulated by considering, at the outset, the most general problem, i.e., one in which mass transfer and pressure gradient occur simultaneously. The special cases wherein one or both of these disturbing influences vanish are then derived. The properties of the resulting formulations are compared with those derived by a less general approach, and the significant differences which arise are delineated and discussed qualitatively.

In order to provide a more quantitative indication of these effects, various forms of the transformation are applied to a series of zero pressure gradient flow configurations. These include;

(a) zero heat transfer with homogeneous and heterogeneous mass transfer,
(b) mass transfer with heat transfer and combustion,
(c) zero mass transfer with and without heat transfer.

In each case a comparison of the various predictions with experimental results is presented. It is found that one form of the transformation is clearly superior to the others. In particular, improved agreement with skin friction data under cold wall conditions is obtained.

Despite this result, detailed analysis of velocity profiles with this modified approach shows no corresponding improvement regarding the distortion of the "wake component" of the transformed velocity distribution. Accordingly, an empirical method is developed which serves to improve correlation of the transformed velocity profiles in the outer region. It should be emphasized here however, that this procedure does not represent a fundamen change in the transformation but is simply a "curve fit" of the available data. Accordingly, its applicability to flows involv pressure gradient is questionable at the present time.

Insofar as the gradient case is concerned, results of the type described above are not currently available. However, several velocity profiles obtained on curved ramps are analyzed utilizing the earlier formulation of Baronti-Libby (Ref. 2). The results which are obtained, although of a preliminary natur are encouraging in that good correlation of profiles in the inner region in terms of the law of the wall is obtained. Furthe work in this area is continuing under the support of NASA Langley Research Center, Contract No. NAS1-8424.

Finally, the applicability of the transformation technique to three-dimensional boundary layer flows is examined. In particular, the appropriate form of the transformation for flow over a swept infinite cylinder is developed and presented.

II. THEORETICAL CONSIDERATIONS*

Fundamental Transformation Relations and Correspondence of Boundary Layer Parameters

The describing differential equations for the velocity field of interest here are assumed to be of the form

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0 \qquad (1)$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial \tau}{\partial y} - \frac{dp}{dx} \qquad (2)$$

It can be shown that this system of equations can be transformed identically to the constant property (CP) form:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \qquad (3)$$

$$\overline{\rho u} \frac{\partial \bar{u}}{\partial \bar{x}} + \overline{\rho v} \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{\partial \bar{\tau}}{\partial \bar{y}} - \frac{d\bar{p}}{d\bar{x}} \qquad (4)$$

by introduction of the following transformation relations**

$$\frac{d\bar{x}}{dx} = \xi(x) \qquad (5)$$

$$\frac{\bar{\rho} d\bar{y}}{\rho dy} = \eta(x) \qquad (6)$$

$$\frac{\bar{\psi}-\bar{\psi}_w}{\psi-\psi_w} = \sigma(x) \qquad (7)$$

providing that the following correspondences between gross boundary layer parameters are imposed

* Only a brief outline of the formal development is presented here. For more detailed derivations see Refs. 6 and 8.

**It is noted here that relation (7) represents a modification from the original Coles' stretching. The significance and motivation for this modification is discussed in Appendix A.

$$(\ln\tilde{\sigma})' - (\ln\tilde{\eta})' - (\ln\tilde{U}_e)' + \left(\frac{d\ln\tilde{U}_e}{d\chi} - \frac{d\ln\nu_e/\nu_{e_o}}{d\chi}\right)\chi' = 0 \qquad (8)$$

$$\frac{dR_\theta}{d\chi} + R_\theta\left(\frac{R_{\delta^*}}{R_\theta} + 2\right)\frac{d\ln\bar{U}_e}{d\chi} + R_\theta\frac{d\ln\rho_e/\rho_{e_o}}{d\chi} = \frac{c_f}{2} + \frac{F}{U_e}\frac{\rho_{e_o}}{\rho_e} \qquad (9)$$

$$(R_{\bar{\theta}})' + R_{\bar{\theta}}\left(\frac{R_{\bar{\delta}^*}}{R_{\bar{\theta}}} + 2\right)(\ln\bar{U}_e)' = \frac{\bar{c}_f}{2} + \frac{\bar{F}}{\bar{U}_e} \qquad (10)$$

$$\frac{u}{u_e} = \frac{\bar{u}}{\bar{u}_e} \equiv \hat{u} \qquad (11)$$

$$\frac{c_f}{\bar{c}_f} = \tilde{\sigma}\tilde{\rho}_w\tilde{\mu}_w \qquad (12)$$

$$\frac{R_\theta}{R_{\bar{\theta}}} = \frac{1}{\tilde{\eta}}\frac{\tilde{\eta}_o}{\tilde{\sigma}_o} \qquad (13)$$

$$\frac{R_{\delta^*} - R_\delta}{R_{\bar{\delta}^*} - R_{\bar{\delta}}} = \frac{1}{\tilde{\eta}}\frac{\tilde{\eta}_o}{\tilde{\sigma}_o} \qquad (14)$$

$$R_\delta = \frac{1}{\tilde{\eta}}\frac{\tilde{\eta}_o}{\tilde{\sigma}_o}\int_0^{R_{\bar{\delta}}}\frac{dR_{\bar{y}}}{\tilde{\rho}} \qquad (15)$$

Here the existence of mass transfer in both flow regimes is manifested by the presence of the terms $F \equiv \rho_w v_w/\rho_{e_o} u_{e_o}$ and $\bar{F} \equiv \bar{\rho}\bar{v}_w/\rho u_{e_o}$.

A. The CP Solutions

Evidently, for exploitation of this transformation approach to be feasible appropriate CP solutions must be presumed to be available. For purposes of the discussion in this section it is assumed that such a formulation exists and is of the form*

$$\tilde{u} = \tilde{u}(R_{\bar{y}};\ R_{\bar{\delta}},\ \bar{c}_f,\ \bar{F},\ \pi) \qquad (16)$$

$$\bar{c}_f = \bar{c}_f\ (R_{\bar{\delta}},\ \bar{F},\ \pi) \qquad (17)$$

$$R_{\bar{\theta}} = R_{\bar{\theta}}\ (R_{\bar{\delta}},\ \bar{F},\ \pi) \qquad (18)$$

$$R_{\bar{\delta}*} = R_{\bar{\delta}*}\ (R_{\bar{\delta}},\ \bar{F},\ \pi) \qquad (19)$$

$$R_{\bar{y}_s} = R_{\bar{y}_s}\ (R_{\bar{\delta}},\ \bar{F},\ \pi) \qquad (20)$$

$$\frac{d\pi}{d\bar{\chi}} = f(\bar{F},\ R_{\bar{\delta}},\ \bar{c}_f,\ \bar{U}_e,\ \frac{d\bar{U}_e}{d\bar{\chi}}\ ) \qquad (21)$$

Note that an additional parameter π has been introduced in the velocity representation (16) to reflect the dependence of the profiles on the varying external pressure distribution given by $\bar{U}_e=\bar{U}_e(\bar{\chi})$. Note also that Equation (21) anticipates the possible extension of the arbitrary pressure gradient formulation such as that developed in Refs. 10 or 11, to include the effect of mass transfer.

B. Completion of the System of Working Equations - The Sublayer Hypothesis and the Compatibility Conditions

With regard to the system of Equation (8)-(21) we consider that:

1) $U_e, F, \rho_e/\rho_{e_o}, \nu_e/\nu_{e_o}$ are prescribed functions of the VP Reynolds number χ.

2) the initial values of the stretching parameters $\tilde{\sigma}_o, \tilde{\eta}_o$ can be specified.

3) the density (and viscosity) distribution can be prescribed as a function of the local velocity ratio $\tilde{u}$; i.e., $\tilde{\rho}=\tilde{\rho}(\tilde{u})$ $\tilde{\mu}=\tilde{\mu}(\tilde{u})$

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* Actually very little has been accomplished in the way of developing such a general formulation for the CP case. This situation is compounded by the absence of reliable experimental data at low speeds. It is interesting to note that if the general validity of the transformation technique is ultimately demonstrated the desire to develop analytical tools which describe the VP behavior may give impetus to extensive work in the low speed area.

Then, examination of this system reveals that a total of 11 equations* are available for the determination of 14 dependent variables with the CP Reynolds number $\tilde{\chi}$ considered to be the sole independent variable. Accordingly, three additional equations are required. One is obtained by invoking the sublayer hypothesis which can be written (c.f., Ref. 2 or 6).

$$\tilde{\sigma} = \frac{\tilde{\rho}_s}{\tilde{\mu}_s} \frac{1}{R_{\overline{y}_s}} \int_0^{R_{\overline{y}_s}} \frac{dR_{\overline{y}}}{\tilde{\rho}}$$

In view of the assumptions made above this is an implicit relation of the form

$$\tilde{\sigma} = \tilde{\sigma} \left(R_{\overline{y}_s}, R_{\overline{\delta}}, \overline{c}_f, \overline{F}, \pi\right)$$

or in view of Equation (20)

$$\tilde{\sigma} = \tilde{\sigma} \left(R_{\overline{\delta}}, \overline{c}_f, \overline{F}, \pi\right)$$

The final two equations are obtained by satisfying the wall compatibility conditions in both flow regimes, i.e., we satisfy the momentum Equations (2) and (4) and their respective y derivatives at y, $\overline{y} \to 0$ taking into account the differentiation rules implied by the transformation and that $\tau \to \mu \frac{\partial u}{\partial y}$, $\overline{\tau} \to \overline{\mu} \frac{\partial u}{\partial \overline{y}}$ at $y, \overline{y} \to 0$. As a result of this procedure** there are obtained two relations of the form***

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* Here we have not included Equation (20) nor $R_{\overline{y}_s}$ as a dependent variable since the latter does not appear within the system at this point.

** This procedure is equivalent to the expansion method utilized by Lewis (Ref. 9) and the relations which are obtained here correspond to the second and third terms of that expansion. The relations which are obtained are not identical however, since in Ref. 9 the expansion is carried out with $F=\overline{F}=0$. As a result the final form of the transformation derived therein differs substantially from the one developed here.

*** See Appendix B for the explicit form of these relations.

$$\bar{F} - \frac{2}{\bar{c}_f}(\ln \bar{U}_e)' = f_1\left[F, \frac{d\bar{U}_e}{dx}; \tilde{\rho}_w, \left(\frac{\partial\tilde{\rho}}{\partial\tilde{u}}\right)_w, \left(\frac{\partial^2\tilde{\rho}}{\partial\tilde{u}^2}\right)_w, \left(\frac{\partial^2\tilde{\rho}}{\partial\tilde{u}^2}\right)_w \ldots\right] \quad (22)$$

$$\bar{F}\left[\bar{F} - \frac{2}{\bar{c}_f}(\ln \bar{U}_e)'\right] = f_2\left[F, \frac{d\bar{U}_e}{dx}; \tilde{\rho}_w, \left(\frac{\partial\tilde{\rho}}{\partial\tilde{u}}\right)_w, \left(\frac{\partial^2\tilde{\rho}}{\partial\tilde{u}^2}\right)_w \ldots\right] \quad (23)$$

where the dependence of the right hand side of these relations on the thermodynamic behavior of the VP gas in the vicinity of the wall has been indicated. The properties of this general transformation and in particular, of Equations (22) and (23) are discussed in the next section.

III. COMPARISON OF VARIOUS FORMS OF THE COMPRESSIBILITY TRANSFORMATION

We consider now the possibility of treating the VP case corresponding to an impermeable flat plate with a uniform external pressure field by use of the unified transformation developed in the previous section. We find in this case that Equations (22) and (23) reduce to

$$\overline{F} - \frac{2}{\overline{c}_f}(\ln \overline{U}_e)' = \frac{\overline{c}_f}{2} \; g_1' \tag{24}$$

$$\overline{F}\,[\overline{F} - \frac{2}{\overline{c}_f}(\ln \overline{U}_e)'] = (\frac{\overline{c}_f}{2})^2 \; g_2' \tag{25}$$

where g_1' and g_2' which depend only on the thermodynamic behavior of the gas in the vicinity of the wall, do not vanish identically. It is easy to show that* for the adiabatic wall case Equations (24) and (25) are singular. On the other hand, for a viscosity temperature dependence of the form $\tilde{\mu} \sim \tilde{T}$ we find $g_1'=g_2' = 0$ which implies that $\overline{F} = d\overline{p}/dx=0$.

It would appear that the special case corresponding to $\mathring{q}_w$=F=dp/dx=0 should be treated with $\overline{F}=d\overline{p}/d\overline{x}=0$ together with the assumption that $\tilde{\mu}\sim\tilde{T}$ which is not an unreasonable model. On the other hand, for the heat transfer case a more general formulation is possible and would be of interest in view of some of the

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* See Appendix B for explicit expressions for g_1' and g_2'.

quantitative results which will be presented in the next section. However, we will first review briefly a few of the more specialized forms of the transformation which have previously been utilized and indicate how they differ from the present model.

In Reference 2 the case of $F=dp/dx=0$ was considered and it was tacitly assumed that $\overline{F}=d\overline{p}/d\overline{x}=0$. This form has been applied extensively to many cases involving heat transfer (cf., Refs. 2, 3,4). Accordingly, neither compatibility condition is explicitly satisfied by this formulation.

In Reference 8 the problem corresponding to $dp/dx \neq 0$ has been formulated. In this case it again is assumed that $\overline{F} = 0$ and the first compatibility condition (i.e., Eq. (22)) is satisfied to complete the system of describing equations. Thus the second compatibility condition is not satisfied here.

In Reference 6 the problem corresponding to $F \neq 0$, $dp/dx=0$ is formulated in an analogous way by assuming $d\overline{p}/d\overline{x} = 0$ and satisfying Equation (22) to complete the system. Again the second compatibility condition is not explicitly satisfied.

The matters discussed in this section have been summarized in Table I.

IV. COMPARISON OF THEORY WITH EXPERIMENT

A. Mass Transfer at Constant Pressure

In Reference 6 the mass transfer case at constant pressure was treated using the form of the transformation listed as IV-D in Table I. The CP solutions utilized were essentially those due to Stevenson (Ref. 12,13) and Crocco Integrals were used to determine energy and or species distributions. This method was extended to a case involving chemical reactions in Reference 14, by utilizing a flame sheet model to describe the chemistry (Ref. 15). Some comparisons of the theoretical predictions with experiment and with earlier prediction techniques are shown in Figure 1. In all cases improved agreement with the data is evident.

B. Impermeable Flow at Constant Pressure With and Without Heat Transfer

As indicated in the previous discussion and in Table I, this has been treated extensively utilizing form II-B of the transformation. Here we will compare these results with those obtained utilizing form II-D which follows from IV-D when F = 0.

To effect this comparison a total of 48 velocity profiles have been analyzed by both methods from the point of view of establishing whether correlation of the inner region by the law of the wall is obtained. These profiles are identified in Table II wherein the skin friction coefficient inferred from the transformed profiles by the two methods have also been listed. Note also that for each case a value of the incompressible blowing parameter $\overline{\zeta}=2\overline{F}/\overline{c}_f$ has also been listed. This value follows from Equation (24) with $\overline{U}_e{}' \equiv 0$*. The result of this analysis is shown in Figure 2. A typical comparison for a particular profile is shown in Figure 3. It is evident that the two methods give substantially different results, particularly with increasing heat transfer. This effect is demonstrated also by the behavior of the parameter $\overline{\zeta}$ as shown in Figure 4.

* For both methods we have utilized a Sutherland viscosity relation for evaluation of viscosity and Crocco integral for the total enthalpy variation.

It might be conjectured at this point that a similar improvement might be obtained by utilizing form II-C of the transformation which follows from form III-C for dp/dx=0. That this is not the case is evident since, insofar as the inner region is concerned, it is completely unaffected by non-zero values of dp/dx. Thus the values of skin friction that would be inferred using either II-B or II-C would be identical.

C. Correlation of Wake Parameter

Despite the improved agreement manifested by form II-D in the inner region, no corresponding improvement in the outer region is observed. In particular, following Reference 2 a value of Coles wake parameter π was inferred* for each of the profiles examined. These values are also listed in Table II. The considerable distortion which arises is apparent. Further analysis of these data however has revealed the existence of a correlating parameter $\rho_{min} T_w / T_{t_e}$ as indicated in Figure 5. Here ρ_{min} represents the minimum density ratio within the boundary layer**. Note that by maintaining the density in this parameter, correlation of the isothermal, low speed helium injection results obtained in Reference 6 is also possible, as shown in Figure 5.

D. Streamwise Prediction of Skin Friction

The possible improvement in prediction of skin friction variation by use of the modified transformation or the correlation proposed above, or both, has been examined. Calculations were made for three cases and are compared with the appropriate experimental data in Figures 6 and 7.

In all cases a considerable improvement in the prediction is obtained by use of the correlation. Furthermore, the agreement with experimental data is good, with the exception of one of the cases of Reference 16. The reason for this large disagreement is not known at this time. One possibility may be that the effective origins for the two data points do not coincide.

*See Reference 6 for method of calculating π when $\bar{F} \neq 0$.

** ρ_{min} was computed from the Crocco integral by evaluating the temperature ratio at $d\tilde{T}/d\tilde{u} = 0$.

E. Impermeable Flow With Variable Pressure and Heat Transfer

Here there are available only a few preliminary results obtained using form III-C of the transformation. Several velocity profiles obtained in adverse pressure gradient (Ref. 17) have been transformed and they appear to be correlated well by the law of the wall, as shown in Figure 8. However, this result can not be considered conclusive since no direct measurement (or other reliable estimate) is available for comparison with the inferred values of skin friction.

V. COMPRESSIBILITY TRANSFORMATION FOR TURBULENT BOUNDARY LAYER FLOW OVER A SWEPT INFINITE CYLINDER

Following Reference 18, the describing equations for the velocity field prevailing in the boundary layer on a swept infinite cylinder are assumed to be

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0 \tag{26}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = - \frac{dp}{dx} + \frac{\partial \tau^{(x)}}{\partial y} \tag{27}$$

$$\rho u \frac{\partial w}{\partial x} + \rho v \frac{\partial w}{\partial y} = \frac{\partial \tau^{(z)}}{\partial y} \tag{28}$$

where x is a (curvilinear) chordwise coordinate, y a coordinate normal to the surface of the cylinder and z a spanwise coordinate parallel to the generators of the cylinder. The quantities, u, v, w, of course, represent the corresponding velocity components and $\tau^{(x)}$ and $\tau^{(z)}$ represent the chordwise and spanwise components of shear respectively. We now seek a transformation which takes Equations (26)-(28) to a corresponding CP form given by

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{29}$$

$$\overline{\rho u} \frac{\partial \bar{u}}{\partial \bar{x}} + \overline{\rho v} \frac{\partial \bar{u}}{\partial \bar{y}} = - \frac{d\bar{p}}{d\bar{x}} + \frac{\partial \bar{\tau}^{(x)}}{\partial \bar{y}} \tag{30}$$

$$\overline{\rho u} \frac{\partial \bar{w}}{\partial \bar{x}} + \overline{\rho v} \frac{\partial \bar{w}}{\partial \bar{y}} = \frac{\partial \bar{\tau}^{(z)}}{\partial \bar{y}} \tag{31}$$

For this purpose, we proceed as in Appendix A and relate the chordwise and normal space coordinates and the chordwise and spanwise velocity components through a series of arbitrary stretching parameters according to

$$\frac{d\bar{x}}{dx} = \xi(x) \tag{32}$$

$$\frac{\bar{\rho} d\bar{y}}{\rho dy} = \eta(x) \tag{33}$$

$$\frac{u}{\bar{u}} = f_1(x) \qquad (34)$$

$$\frac{w}{\bar{w}} = f_2(x) \qquad (35)$$

It is now noted that Equations (26) and (29) are identically satisfied by stream functions defined by

$$\rho u = \frac{\partial \psi}{\partial y} \qquad \rho v = \frac{\partial \psi}{\partial x}$$

$$\overline{\rho u} = \frac{\partial \bar{\psi}}{\partial \bar{y}} \qquad \overline{\rho v} = \frac{\partial \bar{\psi}}{\partial \bar{x}}$$

In view of these definitions and Equations (33) and (34) it follows readily that*

$$\frac{\bar{\psi}}{\psi} = \frac{\int_0^{\bar{y}} \overline{\rho u} dy}{\int_0^{y} \rho u dy} = \eta \frac{\int_0^{y} \rho \bar{u} dy}{\int_0^{y} \rho u dy} = \frac{\eta}{f_1} \equiv \sigma(x)$$

so that

$$\frac{u}{\bar{u}} = \frac{\sigma}{\eta}$$

The normal velocity component can now be transformed according to

$$\rho v = -\frac{\partial \psi}{\partial x} = -\xi \frac{\partial (\bar{\psi}/\sigma)}{\partial \bar{x}} - \frac{\partial \bar{y}}{\partial x} \frac{1}{\sigma} \frac{\partial \bar{\psi}}{\partial \bar{y}} =$$

$$\rho v = \frac{\xi \bar{\psi}}{\sigma^2} \frac{d\sigma}{dx} + \frac{\xi}{\sigma} \overline{\rho v} - \frac{\overline{\rho u}}{\sigma} \frac{\partial \bar{y}}{\partial x}$$

In a similar manner the remaining terms appearing in the VP equations can be transformed and it can be shown that the desired CP form is obtained provided we take

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* For the VP flow we consider the swept cylinder to be impermeable. Also for simplicity, it has been arbitrarily assumed that $\bar{v}_w = 0$

$$\frac{d\bar{p}}{d\bar{x}} = \frac{\rho\sigma^2}{\xi\eta^2} \left(\frac{1}{\rho_e} \frac{dp}{dx} + u\tau^2 \quad \frac{d\ell n\eta/\sigma}{dx} \right)$$

$$\frac{\partial\bar{\tau}^{(\bar{x})}}{\partial\bar{y}} = \frac{\bar{\rho}\sigma^2}{\xi\eta^2} \left[\frac{1}{\rho_e} \frac{\partial\tau^{(x)}}{\partial y} - \frac{\psi}{\rho\sigma} \frac{\partial u}{\partial y} \frac{d\sigma}{dx} + \left(\frac{1}{\rho_e} - \frac{1}{\rho} \right) \frac{dp}{dx} + (u_e{}^2 - u^2) \frac{d\ell n\eta/\sigma}{dx} \right]$$

$$f_2 = \text{constant} = \frac{w_e}{\bar{w}_e}$$

$$\frac{\partial\bar{\tau}^{(\bar{z})}}{\partial\bar{y}} = \frac{\bar{\rho}}{\rho} \frac{\bar{w}_e}{w_e} \frac{\sigma}{\xi\eta} \left[\frac{\partial\tau^{(z)}}{\partial y} - \frac{\psi}{\sigma} \frac{\partial\bar{w}}{\partial\bar{y}} \frac{d\sigma}{dx} \right]$$

This is the desired result and would provide the means for generating predictions for the VP flow behavior by formulation of an appropriate CP solution (see e.g., Ref. 19).

VI. CONCLUDING REMARKS

It is concluded on the basis of the foregoing discussion and results that treatment of a rather wide variety of variable property turbulent boundary layer flows by means of a compressibility transformation is feasible. It appears to be essential however that for any choice of configuration the appropriate form of the transformation be utilized.

APPENDIX A

In the original Coles formulation (Ref. 1) the stream function stretching is taken to be

$$\frac{\bar{\psi}}{\psi} = \sigma(x)$$

Taking into account the differentiation rules implied by Equations (5) and (6) and the usual definitions of the respective stream functions, it then follows that the normal velocity components in the two flow regimes are related by

$$\rho v = \frac{\xi}{\sigma}\,\overline{\rho v} + \psi\,\frac{d\ln\sigma}{dx} - \frac{\overline{\rho u}}{\sigma}\,\frac{\partial \bar{y}}{\partial x}$$

at y, $\bar{y} = 0$ this relation yields

$$\rho_w v_w = \frac{\xi}{\sigma}\,\overline{\rho v}_w + \psi_w\,\frac{d\ln\sigma}{dx}$$

which appears to be an independent relation for the determination of F. In the original effort to develop an analysis for the transpired case, this was indeed utilized as a working equation but was found to introduce a singularity into the system. Accordingly a modified stretching for the stream function was deduced by requiring that

$$\frac{\bar{\rho}d\bar{y}}{\rho dy} = \eta(x)$$

and

$$\frac{\bar{u}}{u} = f(x)$$

Then it follows immediately that the stream functions are related according to:

$$\frac{\bar{\psi}-\bar{\psi}_w}{\psi-\psi_w} = \frac{\int_0^{\bar{y}} \bar{\rho}\bar{u}dy}{\int_0^{y} \rho udy} = \frac{\eta\int_0^{y} \rho\bar{u}dy}{\int_0^{y} \rho udy} = \eta f\,\frac{\int_0^{y} \rho udy}{\int_0^{y} \rho udy}$$

i.e.,

$$\frac{\overline{\psi}-\overline{\psi}_w}{\psi-\psi_w} = \eta f \equiv \sigma(x)$$

and that

$$\frac{\overline{u}}{u} = \frac{\sigma}{\eta}$$

In this case we find that the normal velocity components are related by

$$\rho v-\rho_w v_w = \frac{\xi}{\sigma}(\overline{\rho v}-\overline{\rho v}_w)+(\psi-\psi_w)\ \frac{d\ln\sigma}{dx} - \frac{\overline{\rho u}}{\sigma}\ \frac{\partial\overline{y}}{\partial x}$$

which reduces to the zero identity at y,y=0. It would appear therefore that the original stretch of the stream function is too restrictive for the mass transfer case. In this connection, it is interesting to note that Jeromin (Ref.7) retained the original stretching and used as a working equation the relation

$$\frac{\overline{\psi}_w}{\psi_w} = \sigma$$

which was satisfied empirically. He found however, that the resulting formulation did not reduce uniformly to the impermeable case.

APPENDIX B

Equations (22) and (23) have the explicit form

$$\bar{F} - \frac{2}{\bar{c}_f}(\ell n\bar{U}_e)' = (\frac{\tilde{\eta}_o}{\tilde{\eta}}\ \frac{\tilde{\sigma}}{\tilde{\sigma}_o}\ \frac{\bar{c}_f}{2})[\frac{U_e\nu_{e_o}}{\nu_e}g_1 - \tilde{\mu}_w(\frac{2}{c_f})^2\frac{d\ell nU_e}{d\chi}]$$

$$\bar{F}[\bar{F} - \frac{2}{\bar{c}_f}(\ell nU_e)'] = (\frac{\tilde{\eta}_o}{\tilde{\eta}}\ \frac{\tilde{\sigma}}{\tilde{\sigma}_o}\ \frac{\bar{c}_f}{2})^2\{\frac{U_e\nu_{e_o}}{\nu_e}g_2 - \tilde{\mu}_w(\frac{2}{c_f})^2\frac{d\ell nU_e}{d\chi}g_3\}$$

where

$$g_1 = \frac{2F}{c_f}\ \frac{\rho_{e_o}u_{e_o}}{\rho_e\nu_e} - (\frac{\partial\ell n\tilde{\rho}\tilde{\mu}}{\partial\tilde{u}})_w$$

$$g_2 = 3(\frac{\partial\ell n\tilde{\rho}}{\partial\tilde{u}})^2 + [\frac{2F}{c_f}\ \frac{\rho_{e_o}u_{e_o}}{\rho_e\nu_e} - (\frac{\partial\ell n\mu}{\partial\tilde{u}})_w][\frac{2F}{c_f}\ \frac{\rho_{e_o}u_{e_o}}{\rho_e\nu_e} - 3(\frac{\partial\ell n\mu}{\partial\tilde{u}})_w$$

$$-4(\frac{\partial\ell n\tilde{\rho}}{\partial\tilde{u}})_w] - [\frac{1}{\tilde{\rho}_w}(\frac{\partial^2\tilde{\rho}}{\partial\tilde{u}^2})_w + \frac{1}{\tilde{\mu}_w}(\frac{\partial^2\tilde{\mu}}{\partial\tilde{u}^2})_w]$$

$$g_3 = \frac{2F}{c_f}\ \frac{\rho_{e_o}u_{e_o}}{\rho_e\nu_e} - 3(\frac{\partial\ell n\tilde{\mu}}{\partial\tilde{u}})_w - 4(\frac{\partial\ell n\tilde{\rho}}{\partial\tilde{u}})_w$$

also $g_1' = -(\frac{\partial\ell n\tilde{\rho}\tilde{\mu}}{\partial\tilde{u}})_w$

$$g_2' = 3(\frac{\partial\ell n\tilde{\rho}}{\partial\tilde{u}})_w^2 + (\frac{\partial\ell n\tilde{\mu}}{\partial\tilde{u}})_w[3(\frac{\partial\ell n\tilde{\mu}}{\partial\tilde{u}})_w + 4(\frac{\partial\ell n\tilde{\rho}}{\partial\tilde{u}})_w] - [\frac{1}{\tilde{\rho}_w}(\frac{\partial^2\tilde{\rho}}{\partial\tilde{u}^2})_w + \frac{1}{\tilde{\mu}_w}(\frac{\partial^2\tilde{\mu}}{\partial\tilde{u}^2})_w]$$

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TABLE I COMPARISON OF VARIOUS FORMS OF THE COMPRESSIBILITY TRANSFORMATION

Form	Specification of VP Flow Configuration			Assumed CP Flow Configuration			Number of Additional Relations Required	Form of Relations	Comments
		$\frac{dp}{dx}$	F		$\frac{d\bar{p}}{d\bar{x}}$	$\bar{F}$			
Unified	I	SNZ	SNZ	A	?	?	2	1st & 2nd Compatibility.	-
II-B	II	0	0	B	0	0	0	-	Formulated in Refs. 1,2. Exploited, e.g., in Refs. 1-4.
III-C	III	SNZ	0	C	?	0	1	1st compatibility for $d\bar{p}/d\bar{x}$.	Formulated in Ref. 8.
IV-D	IV	0	SNZ	D	0	?	1	1st compatibility for $\bar{F}$.	Formulated and Exploited in Ref. 6.
II-D	-	0	0	-	0	?	1	1st compatibility for $\bar{F}$.	Follows from IV-D.
II-C	-	0	0	-	?	0	1	1st compatibility for $d\bar{p}/d\bar{x}$.	Follows from IV-B
II-A	-	0	0	-	?	?	2	1st & 2nd Compatibility.	Follows from Unified Form

SNZ = Specified Non-Zero

TABLE II SUMMARY OF RESULTS-ANALYSIS OF ZERO PRESSURE GRADIENT VELOCITY PROFILES BY TRANSFORMATION METHOD

Ref.	M_e	$R_\theta(-3)$	$\tilde{T}_w$	T_w/T_{t_e}	$C_{f_R}(+3)$	$C_{f_{BL}}(+3)$	ζ	$C_{f_M}(+3)$	Π
20	1.97	2.98	1.69	.95	2.72	2.48	.016	2.50	.337
	1.98	6.47	1.70	.95	2.18	2.02	.016	2.05	.414
	1.98	8.57	1.70	.95	2.02	1.88	.015	1.90	.431
	2.54	2.19	2.16	.94	2.42	2.42	.019	2.45	.019
	2.57	6.60	2.18	.94	1.81	1.64	.020	1.66	.381
	2.58	10.2	2.18	.94	1.66	1.48	.021	1.50	.392
	3.70	4.10	3.45	.92	1.62	1.50	.026	1.48	-.010
	3.70	7.56	3.44	.92	1.38	1.23	.026	1.22	+.165
	4.51	3.47	4.69	.92	1.48	1.28	.027	1.32	-.123
	4.55	6.59	4.74	.92	1.22	1.06	.026	1.02	-.167
21	2.95	8.05	2.46	.90	1.54	1.60	.036	1.60	+.134
	2.95	21.57	2.50	.91	1.29	1.29	.030	1.30	+.062
	4.20	6.15	3.81	.84	1.27	1.32	.055	1.32	-.339
	4.20	22.75	4.06	.90	.952	.992	.034	.978	-.206
	4.20	37.58	4.03	.89	.868	.880	.040	.820	-.185
	5.19	5.37	8.10	.81	.673	.896	.082	.810	-.57
22	1.74	8.43	1.54	.96	2.11	2.06	.017	2.05	+.37
	1.74	12.49	1.53	.95	1.99	1.90	.014	1.80	+.380
	2.96	2.74	2.55	.93	2.16	2.20	.022	2.15	-.160
	2.74	11.90	2.33	.93	1.53	1.50	.024	1.53	+.073
	3.67	7.99	3.38	.91	1.29	1.22	.031	1.24	+.123
	3.68	10.18	3.40	.92	1.24	1.16	.027	1.15	+.099
	1.74	3.44	1.53	.95	2.56	2.64	.013	2.65	+.134
23	2.50	6.08	2.14	.95	1.80	2.03	.017	2.00	0
	2.53	9.64	2.16	.95	1.58	1.82	.017	1.80	-.121
	2.45	18.81	2.09	.95	1.56	1.57	.015	1.55	+.051

TABLE II (Contd)

Ref.	M_e	$R_\theta(-3)$	$\tilde{T}_w$	T_w/T_{t_e}	$C_{f_R}(+3)$	$C_{f_{BL}}(+3)$	ζ	$C_{f_M}(+3)$	Π
24	5.01	6.48	4.29	.713	1.10	1.14	.120	1.06	-.175
	5.06	7.37	3.27	.533	1.11	1.19	.263	1.10	-.185
	6.78	7.96	4.64	.455	.814	.860	.362	.775	-.175
	6.83	8.55	6.34	.613	.750	.760	.193	.700	-.206
25	5.29	4.30	5.56	.840	1.08	1.09	.055	1.12	-.309
	5.24	3.79	4.97	.765	1.25	1.22	.091	1.16	-.288
	5.17	1.05	3.89	.615	1.62	1.90	.027	1.50	-.411
26	9.07	2.276	8.3	.475	.725	.874	.333	.775	-.191
3	6.00	49.0	5.3	.63	.646	.638	.168	.62	-.432
	6.00	24.0	5.3	.63	.717	.720	.168	.70	-.432
	6.00	19.0	5.1	.63	.760	.750	.184	.71	-.227
	6.80	30.0	5.1	.50	.620	.610	.305	.57	-.467
	6.80	13.0	5.1	.50	.716	.690	.305	.64	+.021
3	20.2	5.24	137.	.99	.115	.149	3.213	.149	-1.84
27	7.53	2.05	19.9	1.0	.587	.620	.005	.66	-1.245
28	6.00	12.9	3.7	.45	.865	.930	.373	.82	-.339
29	6.00	7.1	7.3	.89	.830	.800	.034	.78	-.380
30	18.4	1.25	11.3	.16	.313	.564	1.539	.388	-.760
16	7.87	58.0	3.88	.29	.540	.630	.766	.50	-.800
	7.61	14.0	3.69	.29	.620	.820	.751	.65	-.610
	7.16	2.69	.760	.094	1.09	1.65	4.146	.79	-.473
	7.21	3.40	.900	.105	1.09	1.88	3.50	1.02	-.950

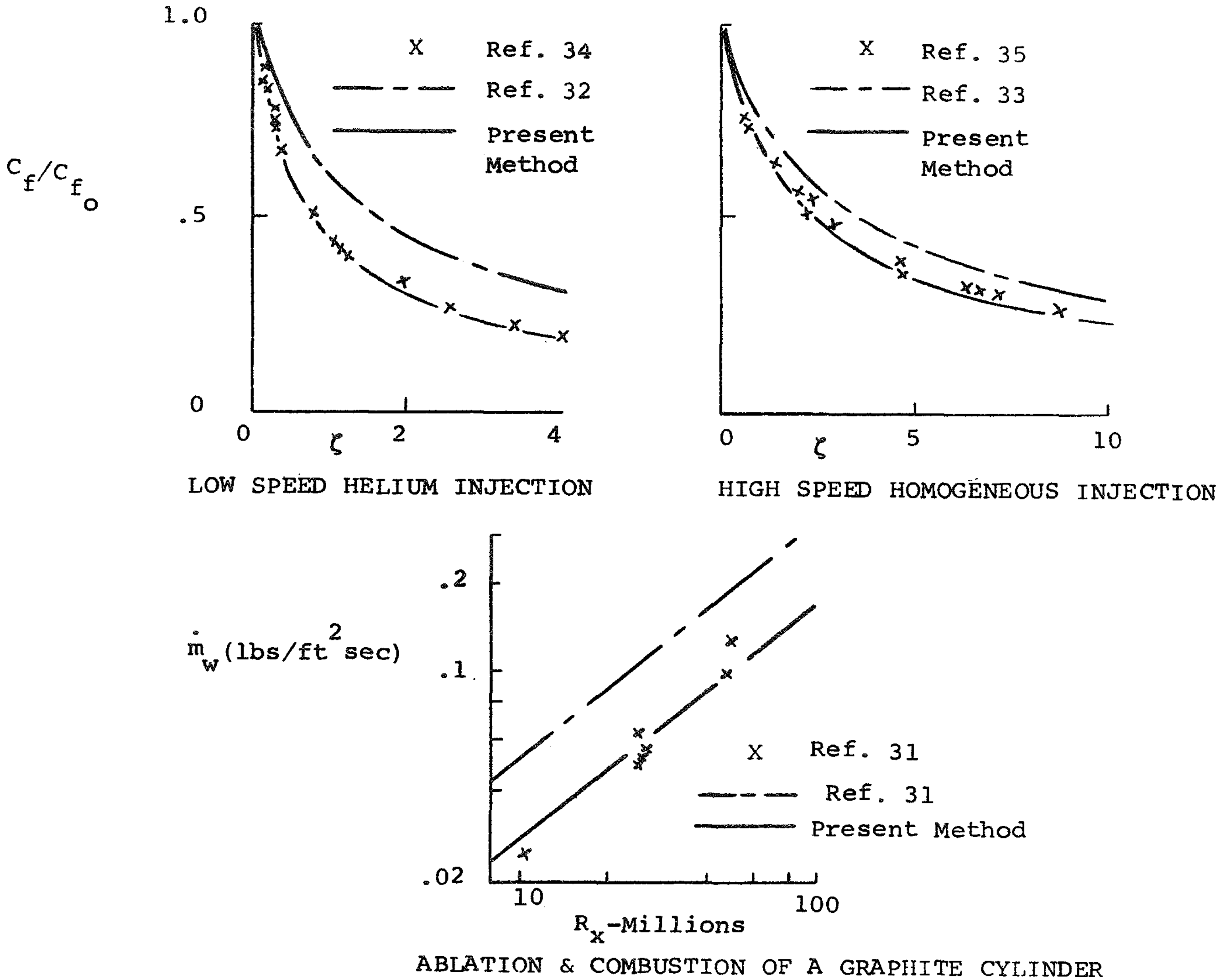

FIGURE 1 - COMPARISON OF THEORY AND EXPERIMENT FOR FLOW WITH MASS TRANSFER AT CONSTANT PRESSURE

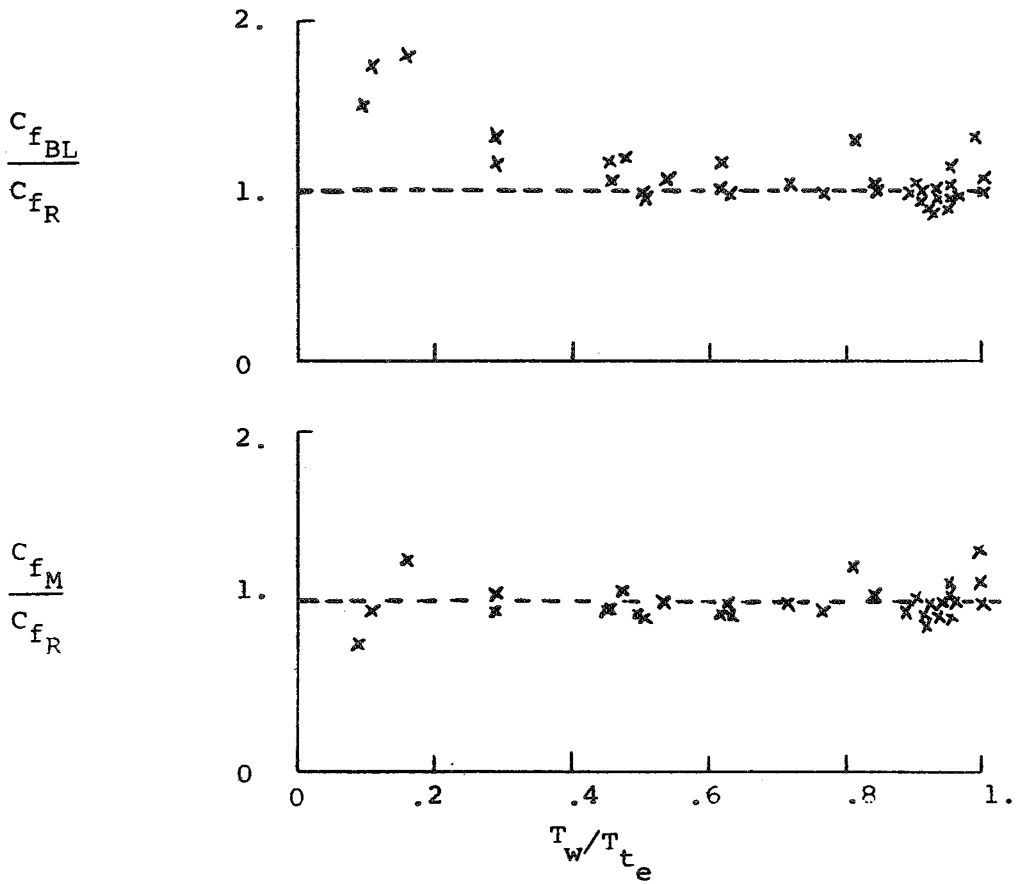

FIGURE 2 - COMPARISON OF SKIN FRICTION INFERRED FROM TWO FORMS OF THE TRANSFORMATION

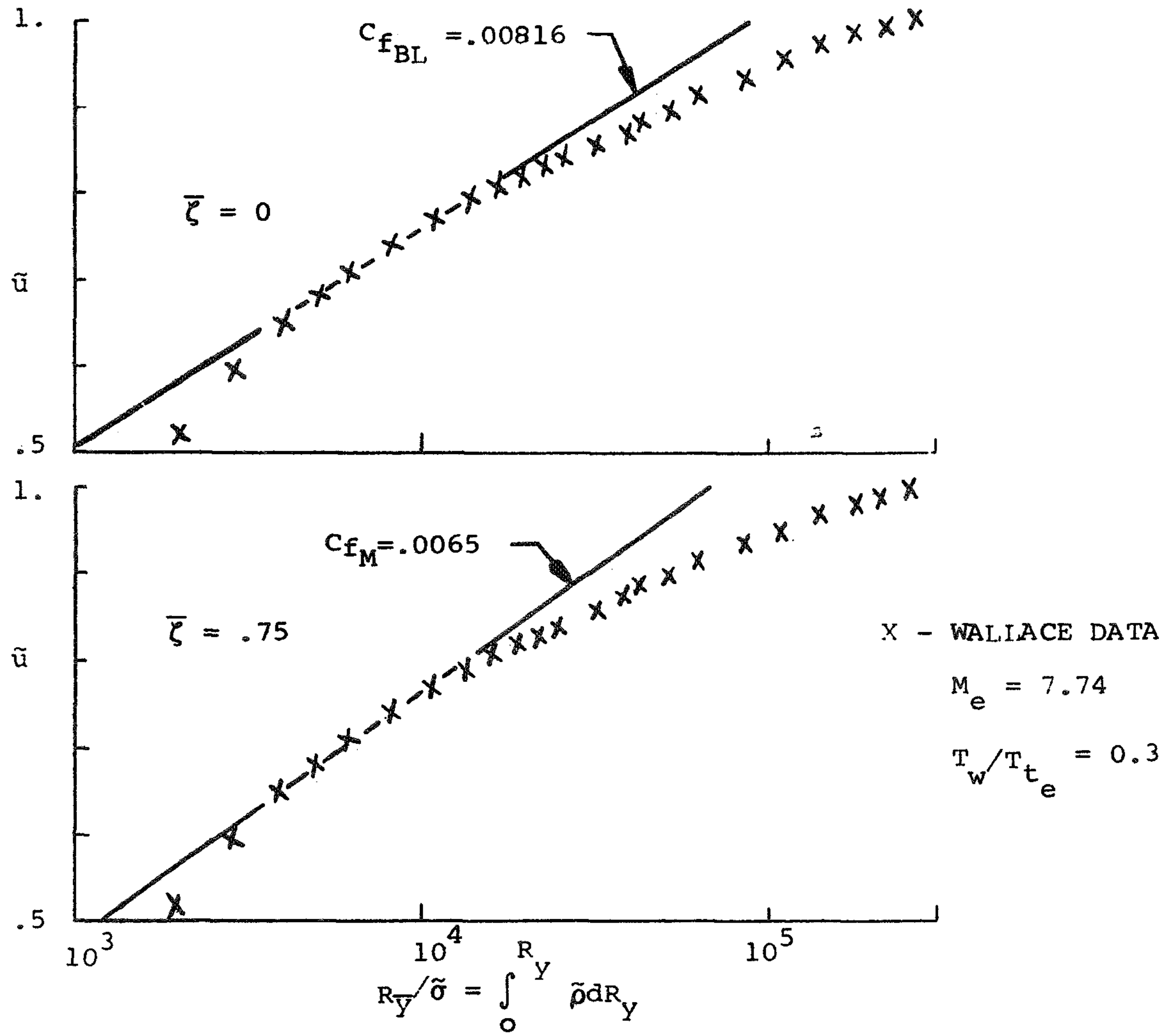

FIGURE 3 - SKIN FRICTION ESTIMATE USING TWO FORMS OF THE TRANSFORMATION

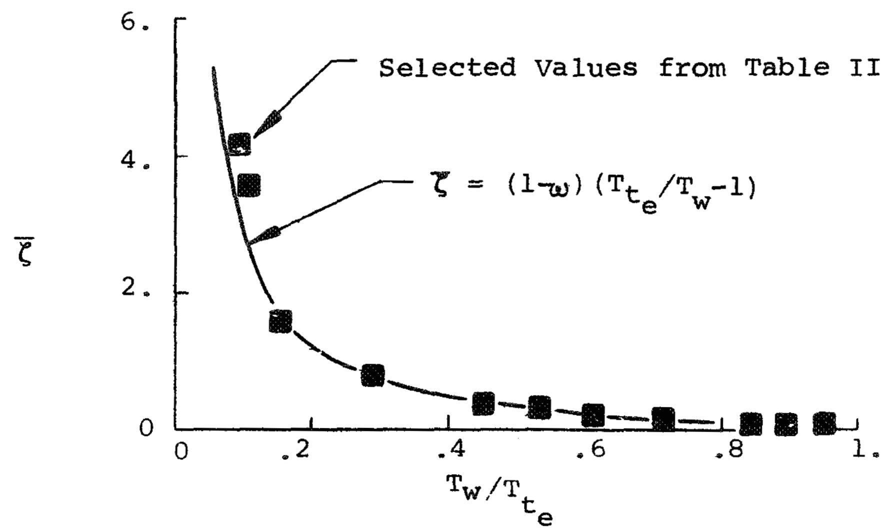

FIGURE 4 - VARIATION OF CP BLOWING PARAMETERS WITH WALL TEMPERATURE

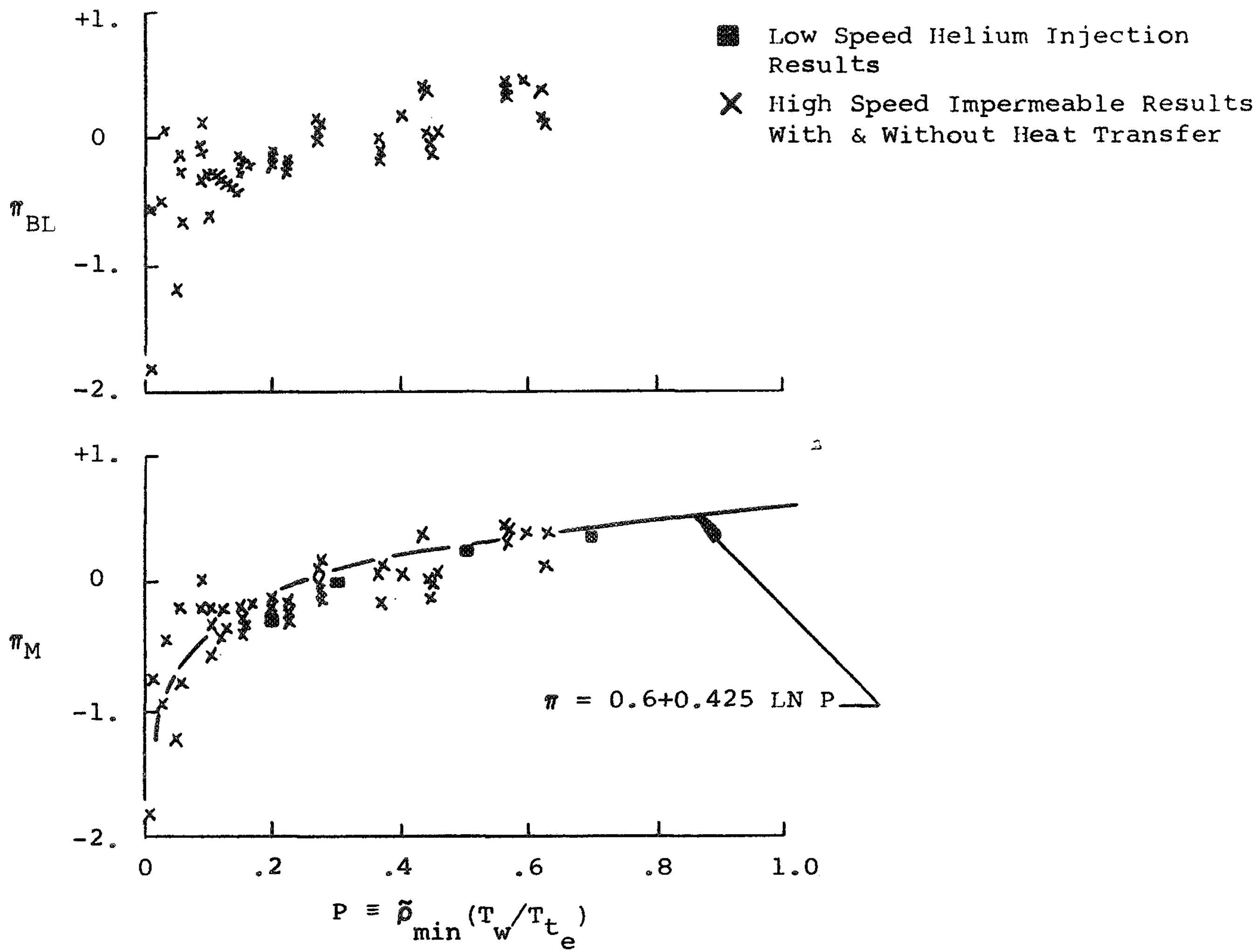

FIGURE 5 - CORRELATION OF THE WAKE PARAMETER

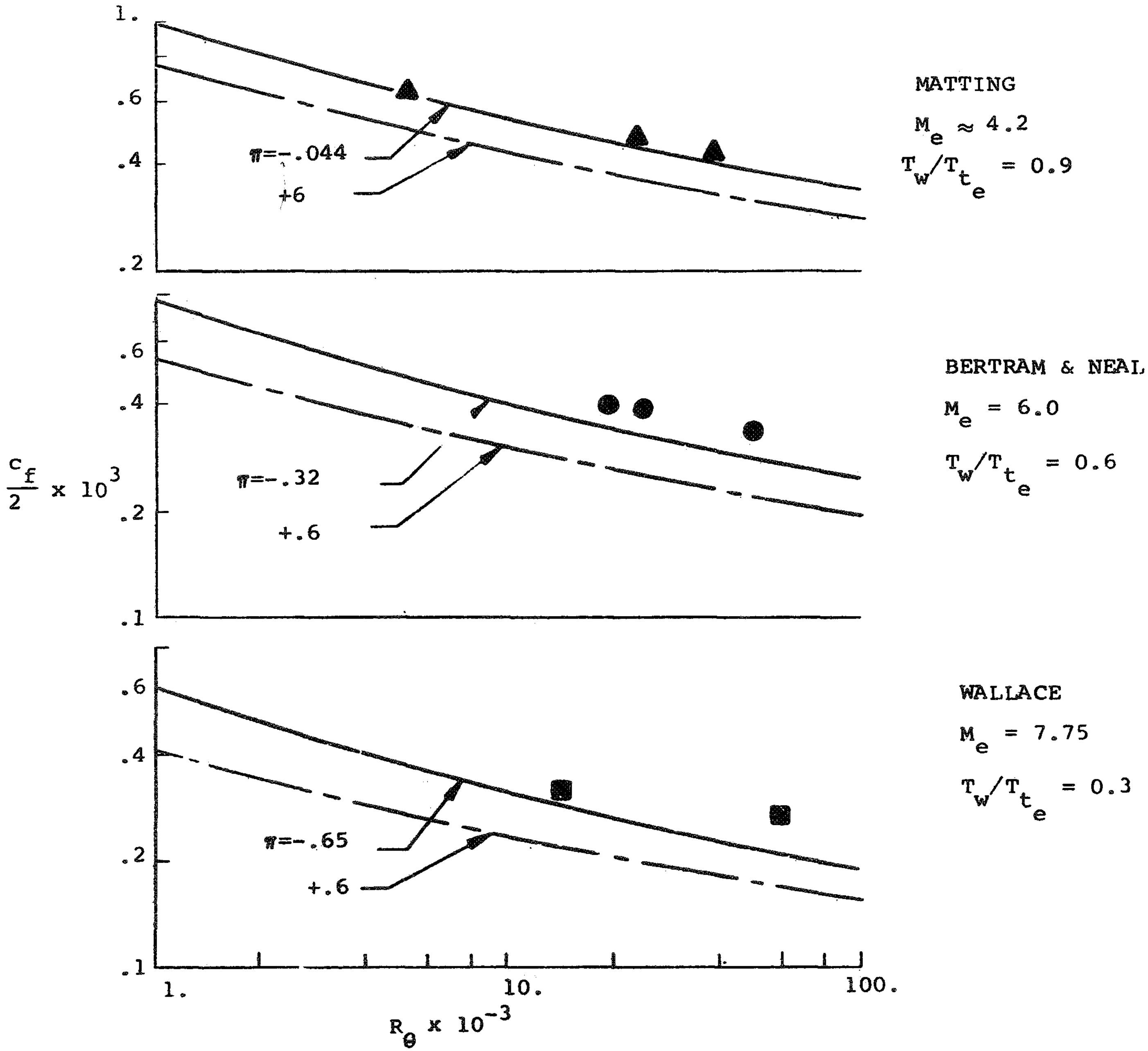

FIGURE 6 - COMPARISON OF SKIN FRICTION PREDICTION WITH EXPERIMENT

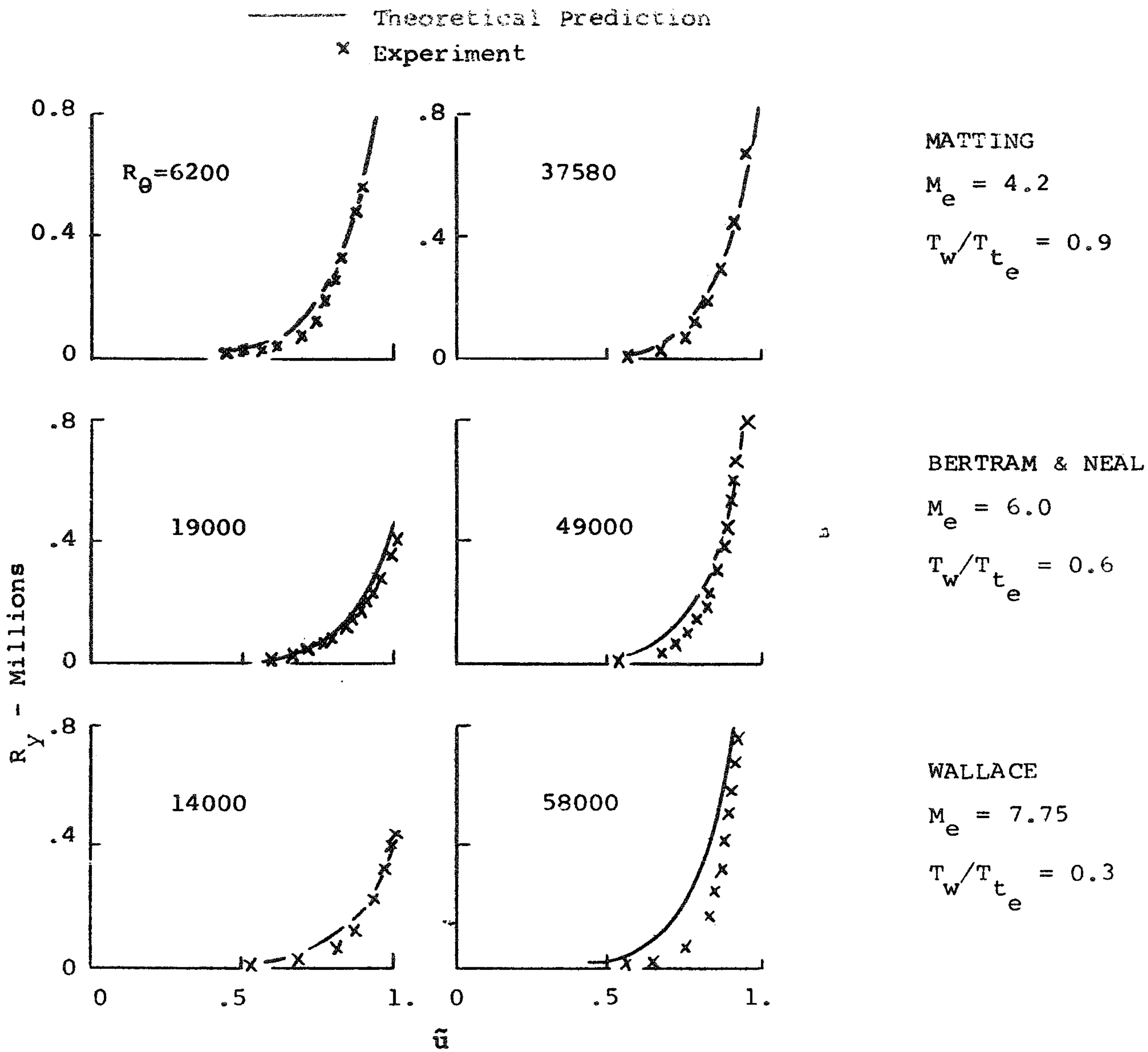

FIGURE 7 - COMPARISON OF VELOCITY PROFILES IN PHYSICAL COORDINATES

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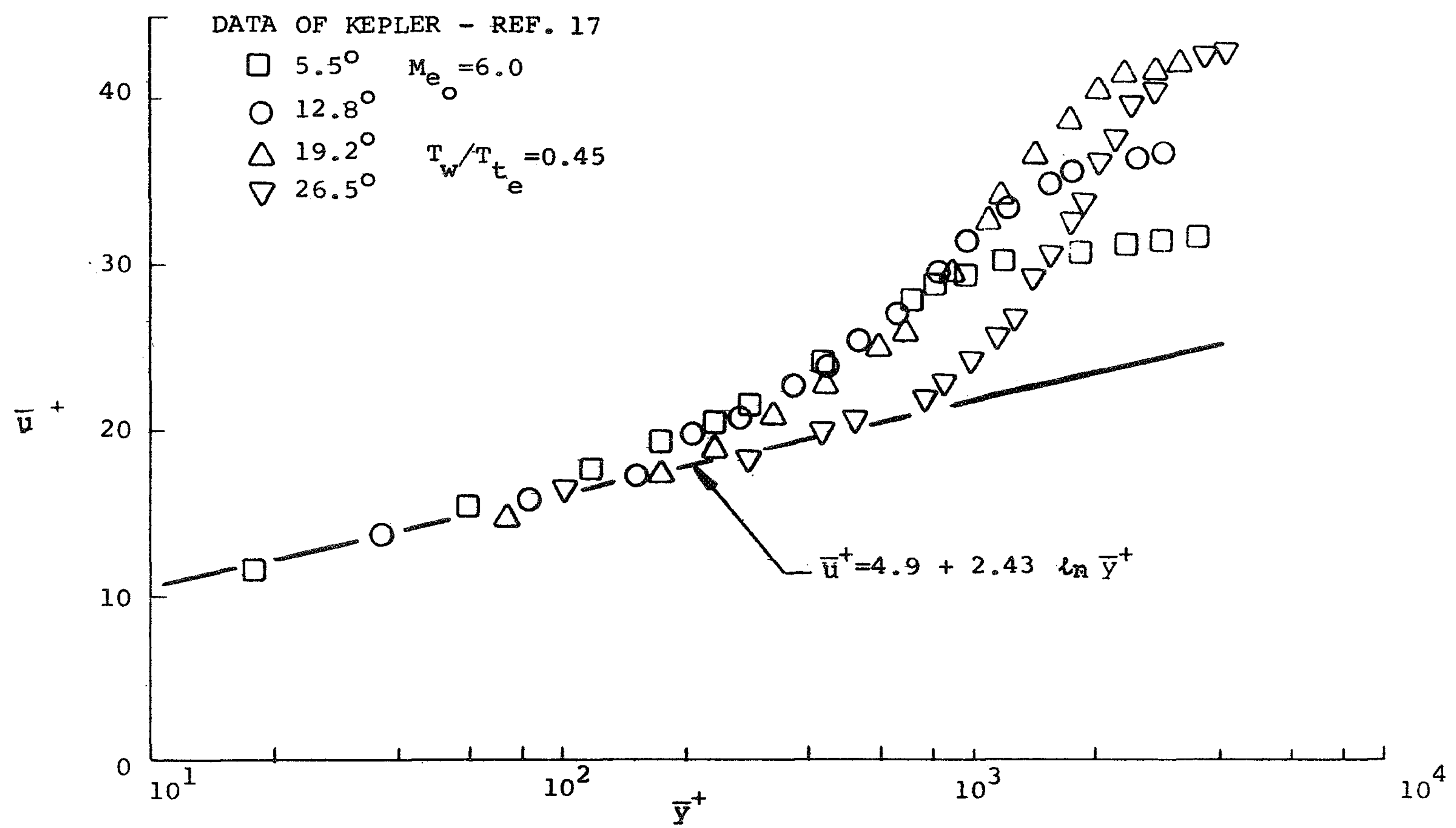

FIGURE 8 - CORRELATION OF VELOCITY PROFILES OBTAINED IN VARIABLE PRESSURE FLOWS

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