STUDY OF A NONSTATIONARY ADJOINT STATISTICAL WIND MODEL FOR FLIGHT CONTROL SYSTEMS ANALYSIS

ENGINEERING REPORT NO. 1621

28 FEBRUARY 1969

PREPARED FOR

GEORGE C. MARSHALL SPACE FLIGHT CENTER

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

HUNTSVILLE, ALABAMA

FINAL REPORT

CONTRACT NO. NAS8-21444

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INTERNATIONAL CORPORATION

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SUMMARY

Design and analysis of large booster flight control systems require techniques that account for the time varying character of the dynamic system. In addition, the techniques must account for system forcing functions that are random processes. The tools available for these studies have been limited. In general, they are subject to analytical formulation, but tend toward numerical schemes. Such a numerical scheme was presented in "Launch Vehicle Winds and Turbulence Response by Nonstationary Statistical Methods," NASA CR846.

This report is an extension of the work undertaken in NASA CR846 and presents the application of techniques there formulated, to large amounts of Jimsphere data with the random process frequency content included.

The work was performed at Hayes International Corporation from June 1968 to February 1969 under Contract No. NAS8-21444. "Study of a Non-stationary Adjoint Statistical Wind Model for Flight Control Systems Analysis," for Aero Astrodynamics Laboratory of George C. Marshall Space Flight Center.

Mr. Jerome R. Redus and Mr. William W. Heuser of the Optimal Control Theory Branch of the Astrodynamics and Guidance Theory Division were the NASA technical supervisors during the periods of performance of this contract.

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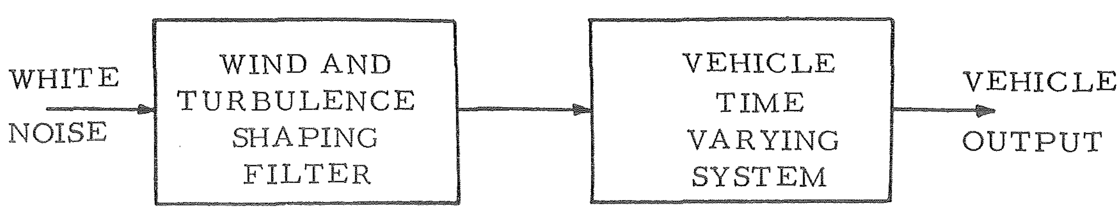
I. INTRODUCTION

For time varying systems that ascend the atmosphere, winds are a major consideration in design of flight control systems. Therefore, it becomes advantageous to have available a tool for design and analysis that can rapidly produce accurate vehicle response statistics. This tool is available in the winds and turbulence "shaping filter" and the "adjoint" techniques of analysis.

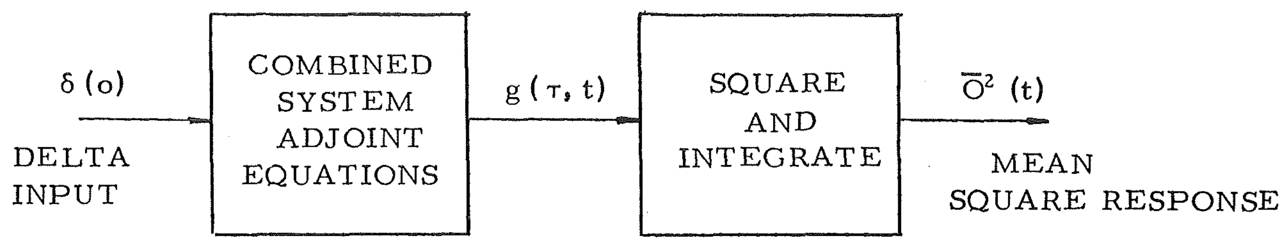
The shaping filter is designed for use with the adjoint technique of analysis, or programs formulated with optimal control techniques requiring mean square system response. By determining the appropriate coefficients, the shaping filter can be used to generate the winds and turbulence statistics that are necessary as an input to a dynamic system to obtain the system mean square response due to winds and turbulence input.

The shaping filter developed in this report is representative of the winds and turbulence over Cape Kennedy, Florida as measured by the Jimsphere Wind Sounding Program conducted by Marshall Space Flight Center, Huntsville, Alabama.

To use the shaping filter with an adjoint technique of analysis, the shaping filter is joined with the time varying system to form a combined system, that has a white noise input and a response which is the vehicle output that normally results from winds and turbulence.



The combined system equations are programmed as adjoint equations.



The impulse response of the adjoint system is the adjoint weighting function of the system, $g(\tau, t)$, as a function of the second argument τ. Application of linear system theory has resulted in the equation

$$\bar{\sigma}^2(t) = \int_0^t g(\tau, t)^2 \, d\tau,$$

and allows direct calculation of the same time varying system mean square response that would result from an ensemble of wind and turbulence profiles input to the time varying system.

The shaping filter, so named because it shapes a white noise input into a prescribed random process whose correlation function is known, was suggested for use with the adjoint technique of analysis by Lanning and Battin (1), for stationary processes, and applied to nonstationary processes by Bailey, Palmer, and Wheeler (2), using statistical dynamics based on the works of Batkov (3), and Solodovnikov (4).

The general synthesis of the shaping filter in the nonstationary case requires far more preparation than synthesizing the shaping filter for a stationary process developed in Ref. (1). This is primarily due to the fact that the power spectra and correlation of stationary processes have a direct transfer relationship and admit to an analytical solution. There are relationships in the nonstationary case, but at present there are no analytical solutions which allow the attainment of the filter equation through direct transformation as in the stationary case.

The shaping filter is defined as an n^{th} order differential equation with time or altitude varying coefficients that are developed from the winds and turbulence data. The input to the shaping filter is white noise and the output winds and turbulence statistics equivalent to ensemble statistics of the winds and turbulence used to develop the filter. These ensemble winds and turbulence nonstationary statistics, correlation or covariance, are related to the filter characteristics through the filter weighting function.

The winds and turbulence data are reduced and arranged in a form such that the statistics may be related to the filter. The order of the filter must be established and the necessary derivatives of the covariance determined. These derivatives are used to determine the coefficients of the filter equation.

Determination of the time varying coefficients of the shaping filter is equivalent to development of the shaping filter.

Section I of this report presents the basic equations that have been developed as foundation stones for the nonstationary statistical development. A complete development of these equations is given in References 3 and 4, with an excellent summary of the adjoint approach in Reference 5. Sections II, II and IV present the statistical development and underlying problems that are to be considered prior to undertaking the numerical task of development of the mean and covariance matrix.

Section V presents the approach taken in developing the derivatives of the covariance at each point along the altitude span of the covariance matrix.

Section VI presents development of the coefficients for both the left and right hand side of the equation (1) of Section II.

Section VII presents the filter coefficient development computer programs.

II. BASIC RELATIONSHIPS

To arrive at the basic equations that are to be used in the numerical and analytical presentation and development of the shaping filter, let the filter be defined as an n^{th} order differential equation with a white noise input

$$a_n(t) \frac{d^n x(t)}{dt^n} + \ldots + a_o(t) x(t) = b_m \frac{d^m \eta(t)}{d(t)^m} + \ldots + b_o(t)\eta(t)$$

or symbolically

$$D(p,t)x(t) = M(p,t)\eta(t) \qquad D(p,t) = \Sigma a_i(t) \frac{d}{dt} \qquad M(p,t) = \Sigma b_i(t) \frac{d}{dt} \qquad p = \frac{d}{dt} \tag{1}$$

$m<n$, $\eta(t)$ = White Noise

The solution to the shaping filter problem is determination of the $a_i(t)$, $b_i(t)$ of equation (1) such that x(t) has prescribed nonstationary statistics in the form of an arbitrary continuous covariance. A solution to the filter equation may be written as

$$X(t) = \int_{-\infty}^{t} g(t,\tau)\eta(t)\, d\tau. \tag{2}$$

X(t) is the random process desired and $g(t,\tau)$ is the filter weighting function, or impulse response. The impulse response $g(t,\tau)$ of the filter can be defined as the solution to

$$D(p,t)g(t,\tau) = M(p,t)\delta(t-\tau). \tag{3}$$

Equation (3) plays a major role in the development of relationships between the filter equation and the winds and turbulence, that the filter is to generate, for it delineates the form of the equations to be developed.

The form that is desired is a form that allows the solution to the filter equation to be in terms of winds and turbulence statistics, namely, correlation or covariance. The correlation of the solution to equation (1) is the average product of equation (2) at times t and τ.

$$R_x(t, \tau) = \overline{x(t)x(\tau)} = \int_{-\infty}^{t} g(t,\lambda)\eta(\lambda)d\lambda \int_{-\infty}^{\tau} g(\tau,\theta)\eta(\theta)\,d(\theta)$$
$$= \int_{-\infty}^{t}\int_{-\infty}^{\tau} g(t,\lambda)g(\tau,\theta)R_\eta(\theta,\lambda)d\lambda d\theta \tag{4}$$

where $R_\eta(\theta,\lambda)$ is the correlation of the input which is known to be $\delta(t)$, a result following from the input being white noise.

Substituting $\delta(t)$ into equation (4) and recalling that

$$\int_{0}^{t} g(t,\tau)\delta(\tau - \lambda)\,dx = g(\tau,\lambda) \tag{5}$$

equations for $R_x(t,\tau)$ are derived

$$R_x(t,\tau) = \int_{-\infty}^{t} g(t,\lambda)g(\tau,\lambda)d\lambda \quad \text{for } t<\lambda$$
$$= \int_{-\infty}^{\tau} g(t,\lambda)g(\tau,\lambda)d\lambda \quad \text{for } t<\tau \tag{6}$$

Operating on both sides of equation (6) with $D(p,t)$, equation (6), part two reduces to zero for $\tau < t$. The correlation, or covariance, of the output of the filter satisfies part two of equation (7), hence,

$$D(p,\tau)R_x(t,\tau) = 0 \qquad t<\tau$$
$$D(p,\tau)R_x(t,\tau) = M(p,\tau)\,g(t,\tau); \quad t>\tau; \quad p = \frac{d}{d\tau} \tag{7}$$

Equations (7) state the relationships between the correlation of the winds and turbulence, and the filter function. They are the basic equations from which the $a_i(t)$, the time varying coefficients of the left-hand side of equation (1) are developed. The values of equation (1) on the right-hand side, $b_i(t)$, are developed from the initial conditions placed on $g(t, \tau)$. The initial conditions used to develop the $b_i(t)$ have been thoroughly developed in Reference 2.

The numerical techniques that follow are developed in terms of altitudes h_i and h_j as opposed to t and τ in the development presented in Section II. Transformation from the altitude domain to the time domain can be accomplished by techniques described in Reference (6).

III. WINDS AND TURBULENCE DATA DEVELOPMENT

The shaping filter must have a random output which has the same correlation or covariance as the winds and turbulence measured by FPS/16-Jimsphere techniques. Figure 1 represents a Jimsphere zonal wind speed recording plotted over the altitude interval of interest.

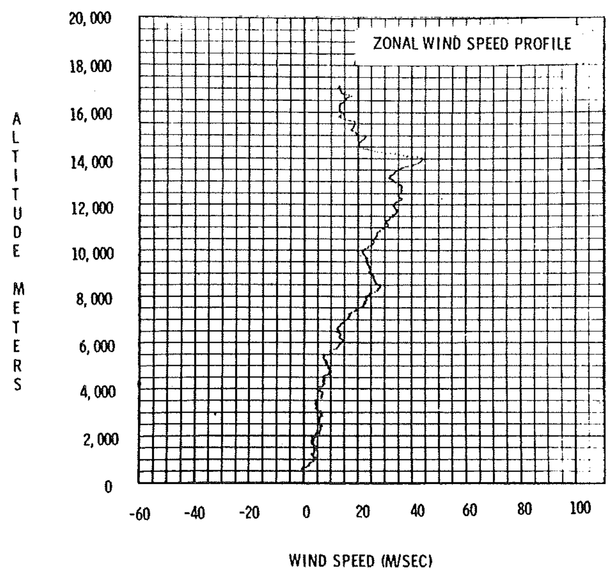


Figure 1

From an ensemble of such profiles, the mean and covariance matrices are to be formed. Utilization of the Jimsphere data to develop a shaping filter requires a careful evaluation of both Jimsphere data and characteristics of the vehicle to be analyzed. The Jimsphere data, at present, is represented by over 1200 records of Cape Kennedy winds, recorded at 25 meter intervals, beginning at approximately 200 meters and extending to approximately 18000 meters. Present records are zonal, meridional and scalar winds.

Such large quantities of data require selectivity in use such that the model contains statistics that excite the important vehicle modes that would be excited during an actual flight. Selecting data for building the filter can best be explained by an example. Consider the Saturn class vehicle during actual flight. The rigid body mode is in the neighborhood of 1 radian per second and the first the three elastic modes are approximately 6, 13, and 18 radians per second. The frequency range of importance then will be between approximately 0.5 and 20 radians per second.

The model can be developed around statistics that contain these frequencies if the proper lag is utilized in development of the covariance matrix. In order to determine a lag size in the covariance matrix that will insure incorporating frequencies up to 20 rad/sec into the problem, consider a vehicle velocity of 525 m/sec, and wind data recorded at 25 meter intervals. Then,

$$\lambda(\text{wind profile wavelength}) = \frac{V\ (\text{vehicle velocity at max } Q)}{f\ (\text{vehicle frequency of interest})}.$$

$$= \frac{525\ \text{m/sec}}{3.18\ \text{c/s}} = 165\ \text{meters} \qquad (8)$$

Therefore, a wavelength of 165 meters is required to introduce frequencies of up to 20 rad/sec into the vehicle problem in the maximum dynamic pressure region. When building a covariance matrix, three points are required to define one wavelength. Hence, the lag size must be one-half (1/2) the wavelength requirement to avoid averaging out the desired frequencies. Therefore, to use the Jimsphere data without interpolation, a lag size of 75 meters would be appropriate in the example considered.

By considering the frequency range of interest, the analyst can select from the bulk of Jimsphere data only the wind direction desired, at 75 meter intervals, thereby reducing the bulk of data to be handled by from 3 to 5 times. Proper selection of data can be crucial where the filter is to be developed on a small digital computer, i.e., 5K storage.

The data selected for this model development were the EAST-WEST (zonal) component of all Jimsphere recordings available. The data were taken in 50 meter steps, beginning at 200 meters and extending through 16,050 meters. To facilitate the analysis, all E-W (zonal) data at 50 meter intervals on 5 Jimsphere record tapes were transferred to a single E-W tape at 50 meter intervals, reducing the data handling from 5 tapes to 1 tape.

The single tape contains record lengths of 1251 words: altitude, and 1250 recordings at a given altitude, extending over several years, for a grand total of 320 records each of 1251 words. From the ensemble of such profiles, on the single tape, separate statistics may be readily calculated for any altitude range. The required statistics of interest are:

mean wind at altitude h_r

$$V_w(h_r) = \sum_{j=1}^{N} V_w(h_r)_j / N, \tag{9}$$

covariance of the wind profile

$$C(V_w(h_r), V_w(h_s)) = \sum_{j=1}^{N} (V_w h_r - \overline{V}_w h_r)_j (V_w(h_s) - \overline{V}_w(h_s))_j / N, \tag{10}$$

and correlation defined as

$$R(V_w(h_r) V_w(h_s)) = C(V_w(h_s) V_w(h_s)) \tag{11}$$

when the average of the process is equal to zero.

Either covariance or correlation may be used in the filter development. However, previous developments have indicated that covariance is a more tractable number and will be used in the filter development.

Utilizing equations (1) and (3), and the reduced data tape, a covariance and mean matrix can be formed.

IV DEVELOPING THE COVARIANCE MATRIX

Referring to Figure 2, a typical set of N wind profiles, and applying standard ensemble techniques, a matrix of covariance values may be formed.

A sample average or expected value of the wind velocity at any altitude h_r may be calculated by use of equation (9).

$$AV(Vh_r) = E(V(h_r)) = \frac{1}{N}\sum_{j=1}^{N} V_j(h_r) \qquad (12)$$

$\frac{1}{N}\sum_{j=1}^{N} V_j(h_r)$ may also be expressed $< V_j(h_r) >$, the ensemble average of wind velocity at altitude h_r.

Let $V'(h) = V(h) - < V_j(h) >$. Then the covariance is defined as

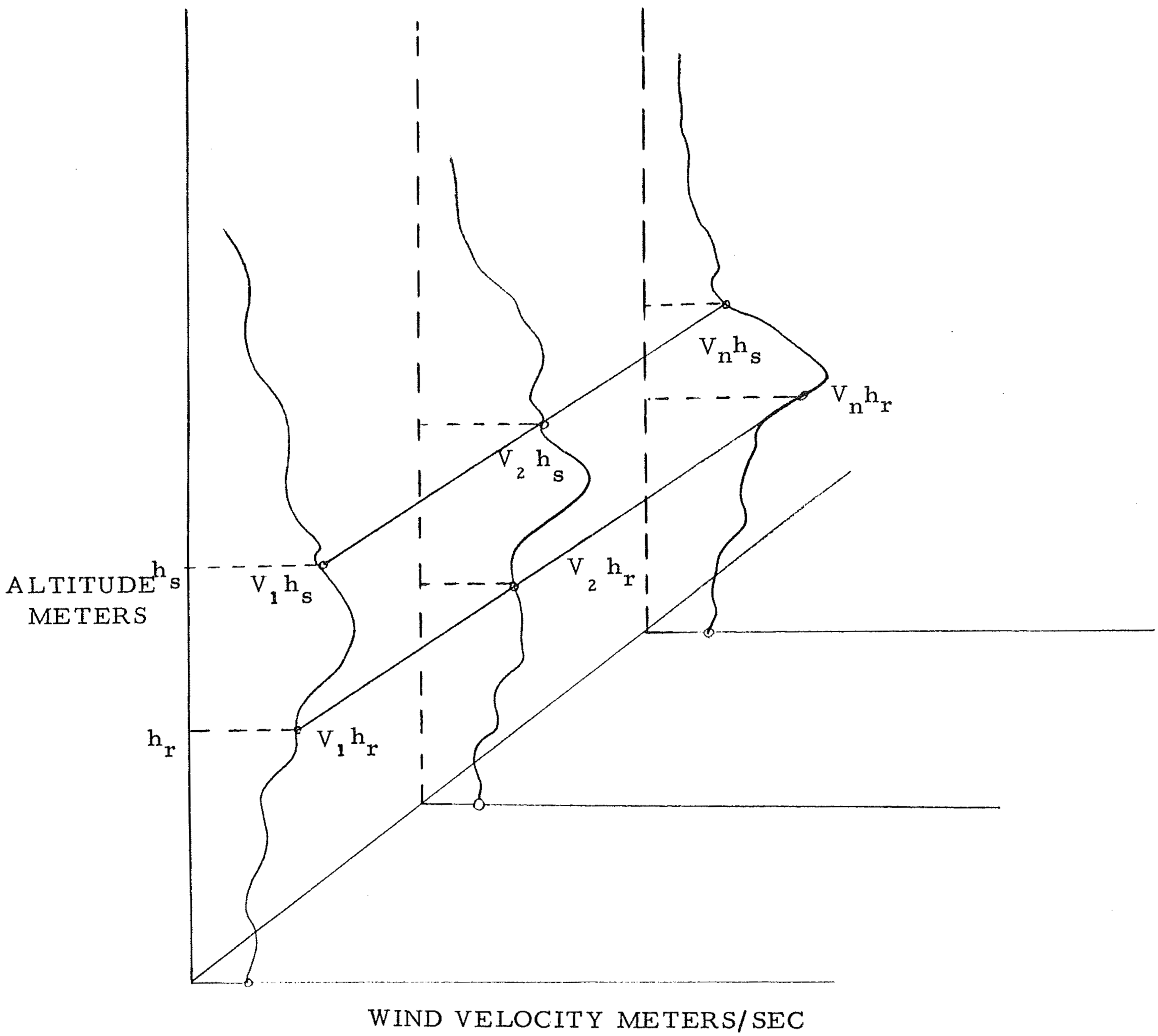
$$\begin{aligned} Cov(h_r h_s) &= < V'(h_r)\, V'(h_s) > = \{V(h_r) - < V_j(h_r) >\}\{V(h_s) - < V_j(h_s) >\} \\ &= < V(h_r)\, V(h_s) > - < V_j(h_r) >< V_j(h_s) > \end{aligned} \qquad (13)$$

Equation (13) indicates the covariance value between wind velocity at altitudes h_r and h_s is given by the correlation of wind velocities at altitudes h_r and h_s averaged across the wind profile sample, minus the product of the average values of wind velocity at altitudes h_r and h_s. By taking altitudes h_r and h_s throughout the wind profile, from the lowest to the highest altitude, a matrix of covariance values will be formed. The matrix is shown in Figure 3, and is an n x n symmetric matrix with n depending on the lag size selected.

The numerical scheme that is used with equations (12) and (13) depends on the computational facility available, and as more Jimsphere data becomes available, the computational scheme will play an even more important part. The scheme used in computing the matrix of this program was to "split out" the 320 Jimsphere records into 4 tapes of 80 records each, thus saving computer time in search through the tape as the record number increases.

There were 1251 words read into computer storage for 2 altitudes, beginning with 200 meters altitude. These records were correlated with themselves and all records above 200 meters. This procedure is repeated throughout the 320 profiles. Any number of records may be read in initially, depending on available storage space of the computational facility. The technique described required 92 minutes on an IBM 7094 computer. An IBM 360-30 of 5K storage was tried but proved prohibitive timewise. This machine appeared to be execute bound.

The covariance matrix,generated and stored rowwise, is now ready for calculating the n derivatives for use with regression analysis to develop the coefficients.



h_r LOWER ALTITUDE

h_s UPPER ALTITUDE

N NUMBER OF WIND PROFILES

Figure 2

COVARIANCE MATRIX

$$h_j$$

$$h_i \quad \begin{bmatrix} C(h_1,h_1) & C(h_1,h_2) & C(h_1,h_3) & & C(h_1,h_n) \\ & C(h_2,h_2) & C(h_2,h_3) & & C(h_2,h_n) \\ & & C(h_3,h_3) & & C(h_3,h_n) \\ & & & \ddots & \vdots \\ & & & & C(h_n,h_n) \end{bmatrix}$$

N x N Symmetric Matrix

Figure 3

V. DETERMINATION OF THE DERIVATIVES OF THE COVARIANCE MATRIX

A method for developing $\frac{d}{dh_j} C(h_i h_j)$ and $\frac{d^2 (h_i, h_j)}{dh_j^2}, \ldots, \frac{d^n(h_i, h_j)}{dh_j^n}$ when $C(h_i, h_j)$ is given in tabulated form as shown in Figure 3 is to represent $C(h_i, h_j)$, as shown in Figure 4, as a polynomial and evaluate $\frac{dC}{dh}$ and $\frac{d^2C}{dh^2}$, $\ldots$, $\frac{d^n C}{dn^n}$ along the span of the polynomial. The polnomial that represents $C(h_i, h_j)$ is called an interpolating polynomial and may be found from the tabulated data through the use of a finite difference table to develop the elements of the polynomial.

When tabulated data is given as in the case of $C(h_i, h_j)$ of Figure 3, h_i is held constant for any tabulation of $C(h_i, h_j)$ resulting in a $C(h_i, h_j)$ that has h_i as a dependent variable and h_j as an independent variable.

The elements of the finite difference table are developed from the tabulated $C(h_j)$, called the zero order difference. Representing the first difference in the finite difference table as $\Delta C_o = C(h_{j+1}) - C(h_j)$, the difference in $C(h_j)$ over a span of altitude h (lag size of C), $\Delta^2 C_o$ as the second difference etc., the table takes the form shown in equation (14). The whole curve made up of all covariance values for a fixed $h_i = 200$ meters is shown in Figure 4. Determination of the values of $\frac{dC}{dh_j}$ by fitting this curve with a single polynomial would require a polynomial of prohibitive degree if an accuracy of interpolation of 1% is desired. This problem was circumvented by using a floating point polynomial fit of small segments of the total curve, in this case 2000 meters and and a fourth degree polynomial.

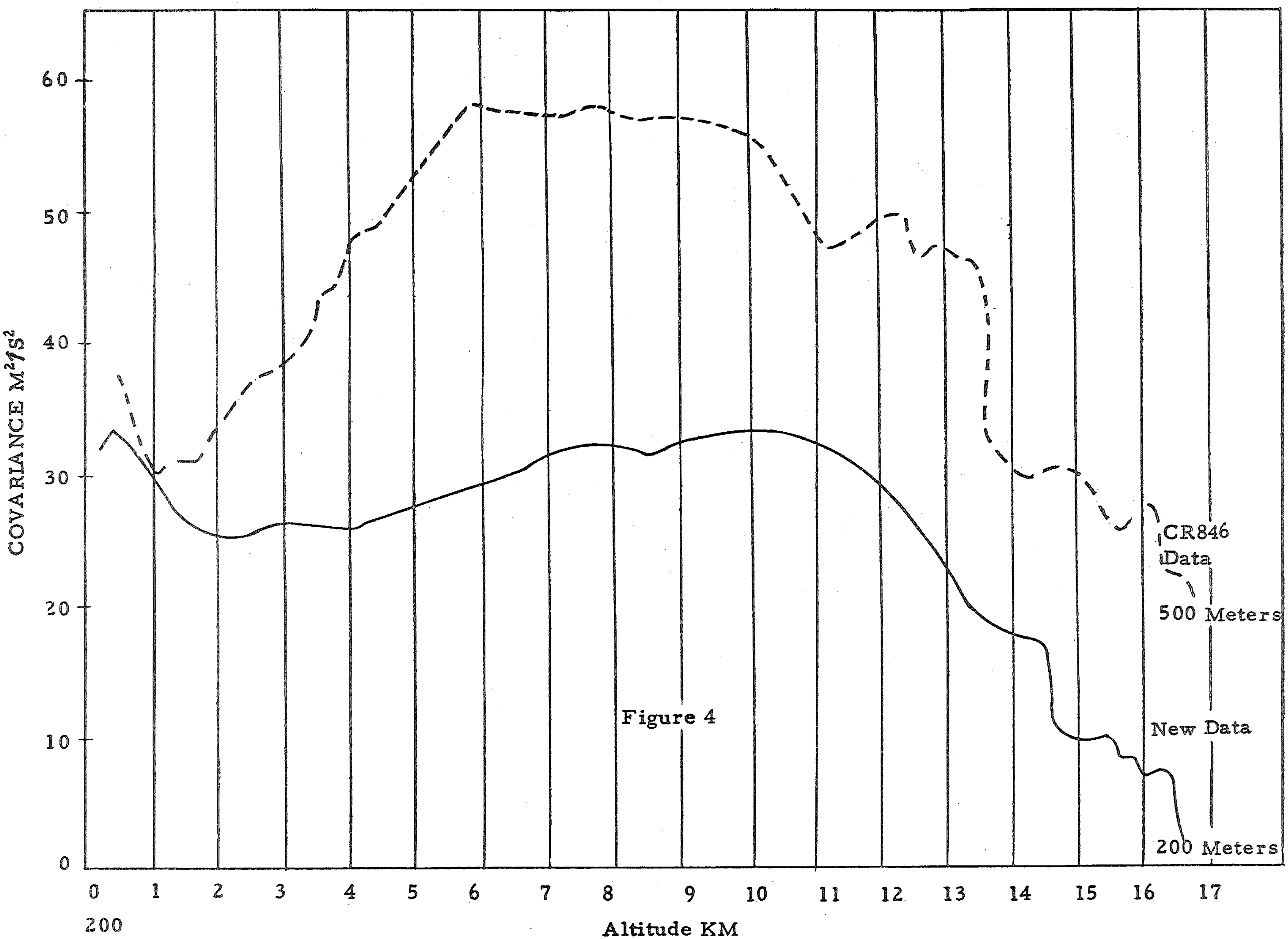

COVARIANCE M²/S²
0
10
20
30
40
50
60
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
200
Altitude KM
CR846
Data
500 Meters
New Data
200 Meters

Figure 4

Interval of Interpolation

$$
\begin{array}{lllll}
C_o(h_o) & & & & \\
 & \Delta C_o & & & \\
C_1(h_1) & & \Delta^2 C_o & & \\
 & \Delta C_1 & & \cdot & \\
C_2(h_2) & & \Delta^2 C_1 & & \\
 & \Delta C_2 & & \cdot & \Delta^n C_o \\
 & & \cdot & & \\
C_3(h_3) & & & & \\
\cdot & \cdot & \cdot & \cdot & \\
\cdot & \cdot & \cdot & & \\
\cdot & \cdot & & & \\
C(h_n) & & & &
\end{array}
\qquad (14)
$$

From this finite difference table, the interpolation of C may be accomplished. The Gregory-Newton formula for forward interpolation will be used. This equation is

$$P_n(C) = \sum_{r=o}^{n} \frac{u^{[r]}}{r\,!} \Delta^r C_o \qquad (15)$$

$$u^{[r]} = u(u-1)(u-2)\ldots(u-r+1)$$

The polynomial is some function of $C(h_j)$, $f(C)$. It is approximated by the polynomial $P_n(u)$ where $u = \frac{C_1 - C_o}{h}$, $h = .05$ kilometer.

$$\frac{df(C)}{dh_j} = \left(\frac{dP_n(u)}{du}\right) \quad \frac{du}{dh_j} \tag{16}$$

$$\frac{du}{dh_j} = \frac{1}{h}$$

Differentiation of $P_n(u)$ results in derivative matrices similar to the covariance matrix. The derivative matrices have as elements the first and second, or more, derivatives of covariance corresponding to the elements of the covariance matrix. The N derivative matrices are developed from the covariance matrix by using the row-wise differentiation of the covariance matrix. The elements of the covariance, first, second, third and etc., derivative matrices are used with the regression technique of Section VI to develop the $a_i(t)$ of equation (1).

As in the initial selection and organization of the winds and turbulence data, the matrices must be arranged in a convenient form if a small computer is to be used for the filter development.

The elements of all matrices are formed by row-wise operation whereas the regression analysis of Section VI, used for developing the $a_i(t)$, is performed column-wise. To facilitate the regression analysis, data sorting is performed on the three matrices to form a single matrix. The single matrix is unique in that each single element of this new matrix has several parts, C_{ij}, $\frac{dC_{ij}}{dh_j}, \ldots,$ $\frac{d^nC_{ij}}{dh^2_j}$. Therefore, when the regression analysis is performed at a given h_j, this matrix furnishes all necessary data values in a single record read by the computer. Regression analysis begins with the second record, since the first record contains only one point i. e. (Cov (200, 200), $\frac{dC}{dn}$ (Cov 200, 200), $\frac{d^n}{dh^n}$ (Cov 200, 200)), and the regression analysis does not hold (is equal to zero) for a single point.

VI. REGRESSION ANALYSIS FOR THE SHAPING FILTER EQUATION COEFFICIENTS

In general theory, multivariate regression plane is represented by

$$X_{1r} = k_1 + k_2 X_2 + k_3 X_3 \ldots k_n X_n \tag{17}$$

where X_1 is a regression on $X_2 \ldots X_n$. In other words, X_1 is the dependent variable and it remains to determine the k_i such that the equation best approximates the data represented by the X variables. By relating the variables $C(h_i, h_j)$, $C'(h_i, h_j)$, ... , $C^n(h_i, h_j)$ of the problem to the regression equation, the $a_i(t)$ of equation (1) can be found as the k_i are found in the general problem. The filter order is selected as 2 for the problem filter. The regression equation and the filter equation as related to the covariance is $C^n(h_i, h_j)$.

$$C'' + a_i C + a_o C = 0 \tag{18}$$

This equation is similar to the regression equation

$$X_{1r} = k_2 X_2 + k_3 X_3 \tag{19}$$

where the X_i are referred to their mean value and k_1 becomes zero. The $a_i(h)$ are to found such that the sum of the squares of the deviations between the observed and predicted values, as given by

$$D^2 = \sum_{j=1}^{N} \{C''_j + a_i C_j + a_o C_j\}^2 , \tag{20}$$

is a minimum. By taking the partial derivative of D^2, a set of normal equations are arrived at such that their solution results in the a_i given in terms of products and squares of known values of C, C', C'' that were stored in the single matrix generated in the final stages of the derivative program. The equations that result from differentiation of equation (20) may be solved as to equations with two unknowns, resulting in expressions for $a_o(h_i)$, $a_1(h_i)$, i = 200, 15950 meters, by 50 meter steps.

$$a_o(h_i) = \frac{-\sum_{j=1}^{i} C''(h_i,h_j) \sum_{j=1}^{i} C'(h_i,h_j)^2 + \sum_{j=1}^{i} C'(h_i,h_j)\, C''(h_i,h_j) \sum_{j=1}^{i} C(h_i,h_j)\, C'(h_i,h_j)}{\sum_{j=1}^{i} C(h_i,h_j)^2 \sum_{j=1}^{i} C'(h_i,h_j)^2 - \left(\sum_{j=1}^{i} C(h_i,h_j)\, C'(h_i,h_j)\right)^2}$$

(21)

$$a_1(h_i) = \frac{-\sum_{j=1}^{i} C(h_i,h_j)^2 \sum_{j=1}^{i} C'(h_i,h_j)\, C''(h_i,h_j) + \sum_{j=1}^{i} C(h_i,h_j)\, C'(h_i,h_j) \sum_{j=1}^{i} C(h_i,h_j) C'(h_i,h_j)}{\sum_{j=1}^{i} C(h_i,h_j)^2 \sum_{j=1}^{i} C'(h_i,h_j)^2 - \left(\sum_{j=1}^{i} C(h_i,h_j)\, C'(h_i,h_j)\right)^2}$$

The numerical approach taken to develop the $b_i(h)$ can be more easily understood by referring to Figure 5 during the following development. When the $C(h_i, h_j)$ of Figure 3 is plotted with h_i as the dependent variable, the curves for $h_i > h_j$ are obtained and when the $C(h_i, h_j)$ are plotted with h_j, the dependent variable, the curves of $h_i < h_j$ are obtained. They are identical due to the symmetry of the matrix. Either form may be used to generate the filter coefficients. For the case taken, h_i is the dependent variable and the curves for $h_j > h_i$, the curves to the right of $h_i = h_j$ are used.

The line $h_i = h_j$ is of importance in evaluating $\left.\frac{dc}{dh_j}\right|_{h_i = h_j^-}$ and $\left.\frac{dc}{dh_j}\right|_{h_i = h_j^+}$.

$\left.\frac{dc}{dh_j}\right|_{h_i = h_j^+}$ is evaluated along the curves B at a point $h_i = h_j^+$. The value of $\left.\frac{dc}{dh_j}\right|_{h_i = h_j^-}$ is evaluated along the curves A at a point $h_i = h_j^-$, or can be more readily found by differenting curves C since the values of $C(h_i, h_j)$ etc. are taken along the curves B at 50 meter intervals. The values of $C(h_i, h_j)$ at $h_i = h_j$ appear to be zero (0) in Figure 5. This, however, is not the case as is shown in the exploded view of Figure 5, where a discontinuity is evident.

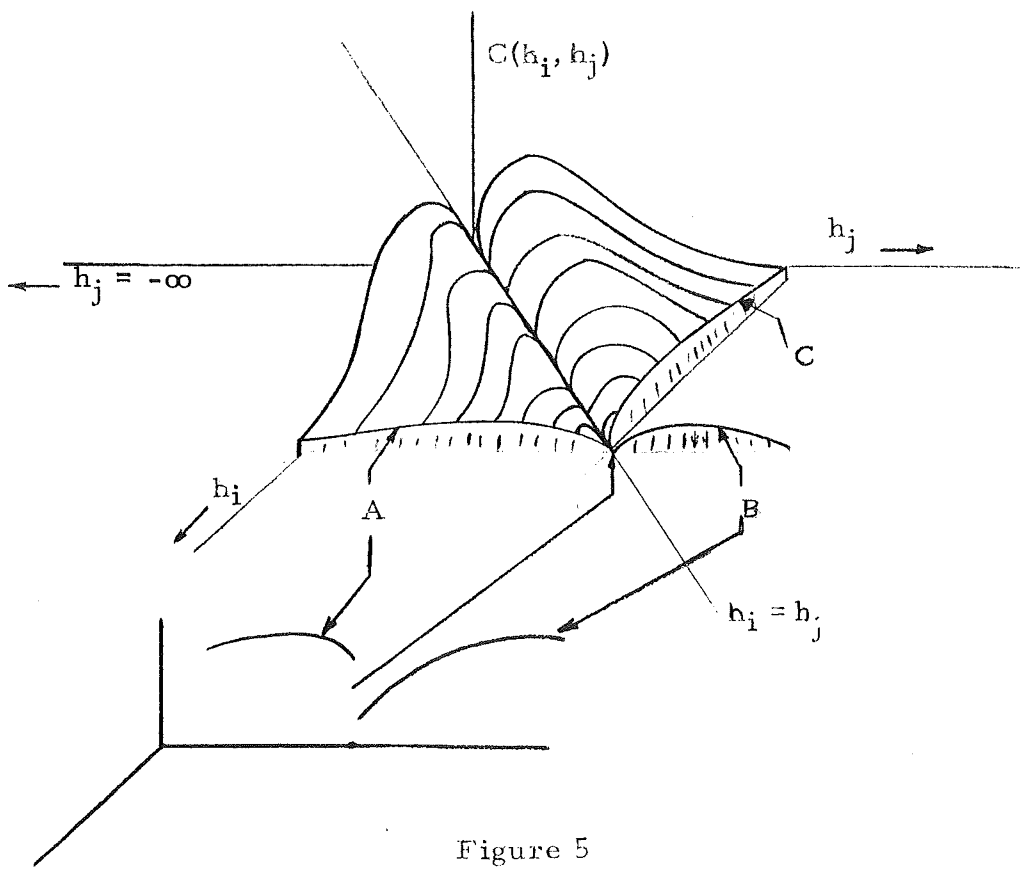


Figure 5

The curves A and C are obtained by taking a plane through the $C(h_i, h_j)$ curves at points h_i or h_j.

Since $g(h_i, h_j)$ is the impulse response function or weighing function for this filter

$$a_o(h_i)\, g(h_i, h_j) + a_1(h_i) \frac{dg(h_i, h_j)}{dh_j} + \frac{d^2\, g(h_i, h_j)}{dh_i^2} = 0 \text{ for } h_i > h_j .$$

By approximating $\frac{dg(h_i, h_j)}{dh_j}$ and $\frac{d^2\, g(h_i, h_j)}{dh_j}$ in terms of differences representing three points, a system of equations may be obtained to be utilized for numerically obtaining $g(h_i, h_j)$ for points near $h_i = h_j$. If h_{io}, h_{i1} and h_{i2} are three consecutive values of h_i near $h_i = h_j$ taken along the span of h_j, then

$$\left. \frac{d\, g(h_i, h_j)}{dh_j} \right|_{h_i = h_{io}} = \frac{1}{2} [\ -3\, g(h_{io}, h_j) + 4\, g(h_{i1}, h_j) - g(h_{i2}, h_j)]$$

$$\left. \frac{d\, g(h_i, h_j}{dh_j} \right|_{h_i = h_{i1}} = \frac{1}{2} [\ -g(h_{io}, h_j) + g(h_{i2}, h_j)]$$

$$\left. \frac{d\, g(h_i, h_j)}{dh_j} \right|_{h_i = h_{i2}} = \frac{1}{2} [\ g(h_{io}, h_j) - 4\, g(h_{i1}, h_j) + 3\, g(h_{i2}, h_j)]$$

and

$$\left. \frac{d_2\ g(h_i, h_j)}{dh_j} \right|_{h_i = h_{io}} = g(h_{io}, h_j) - 2\, g(h_{i1}, h_j) + g(h_{i2}, h_j)$$

In terms of three consecutive points the second derivatives are assumed constant. The points are taken close enough together to minimize error.

From the preceding numerical substitutions for the first and second derivatives of $g(h_i, h_j)$ in the differential equation of the shaping filter, the following system of difference equations may be obtained.

$$[\, a_o h_{i1}) - 2\, a_1(h_{i1})]\; g(h_{i1}, h_j) + a_1(h_{i1})\, g(h_{i2}, h_j) =$$

$$[\, a_o(h_{io}) - 2\, a_1(h_{io}) + a\,(h_{i1})]\; g(h_{io}, h_j)$$

$$(\, a_o(h_{i1}) + 1\, a_1(h_{i2})]\; g(h_{i1}, h_j) + (\, a_1(h_{i1}) - \underset{1}{a_o}(h_{i2}) - 2\, a_1(h_{i2}) - 2\, a_1(h_{i2})]$$

$$g(h_{i2}, h_j) = a_1(h_{i1})\, g(h_{io}, h_j).$$

as developed in reference (3).

$$g(h_{io}, h_j)^2 = \left(\left. \frac{d\, C(h_{io} h_j)}{dh_{io}} \right|_{h_{io} = h_j^-} - \left. \frac{d\, C(h_{io}, h_j)}{dh_{io}} \right|_{h_{io} = h_j^+} \right)$$

Thus, the system of two equations and two unknowns can be solved for $g(h_{i1}, h_j)$ and $g(h_{j2}, h_j)$. Substituting $g(h_{io}, h_j)$, $g(h_{i1}, hj)$, $g(h_{i2}, h_j)$ when $h_j = h_{io}$ in the formula for $\left. \frac{d\, g(hi, hj)}{dh_j} \right|_{\substack{h_i = h_{io} \\ h_j = h_{io}}}$

enables one to compute $b_1(h_{io})$ from the formulas,

$$b_1(h_{io}) = g(h_i, h_j) \Big|_{\substack{h_i = h_{io} \\ h_j = h_{io}}} \qquad \text{and}$$

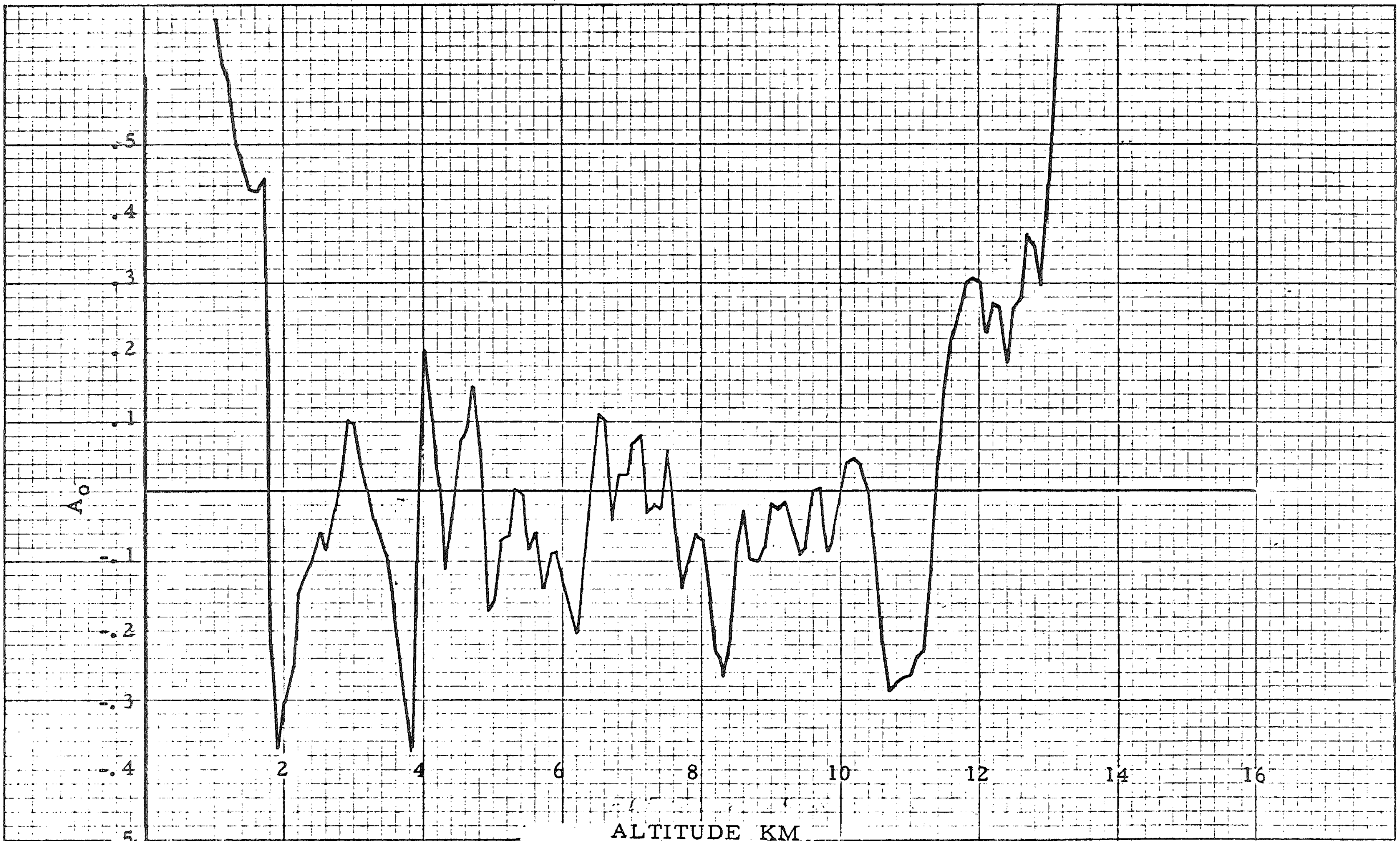
$$b_o(h_{io}) = \left. \frac{dg(hi, hj)}{dh_j} \right|_{\substack{h_i = h_{io} \\ h_j = h_{io}}} + a_1(h_{io})\, b_1(h_{io}).$$

VII. SHAPING FILTER COEFFICIENTS

The coefficients of the shaping filter differential equation (1) are given in this section. Figures 6 through 9 are $A\phi$, A1, $B\phi$, B1 respectively. Computer tabulations of each graph are given following each graphical display.

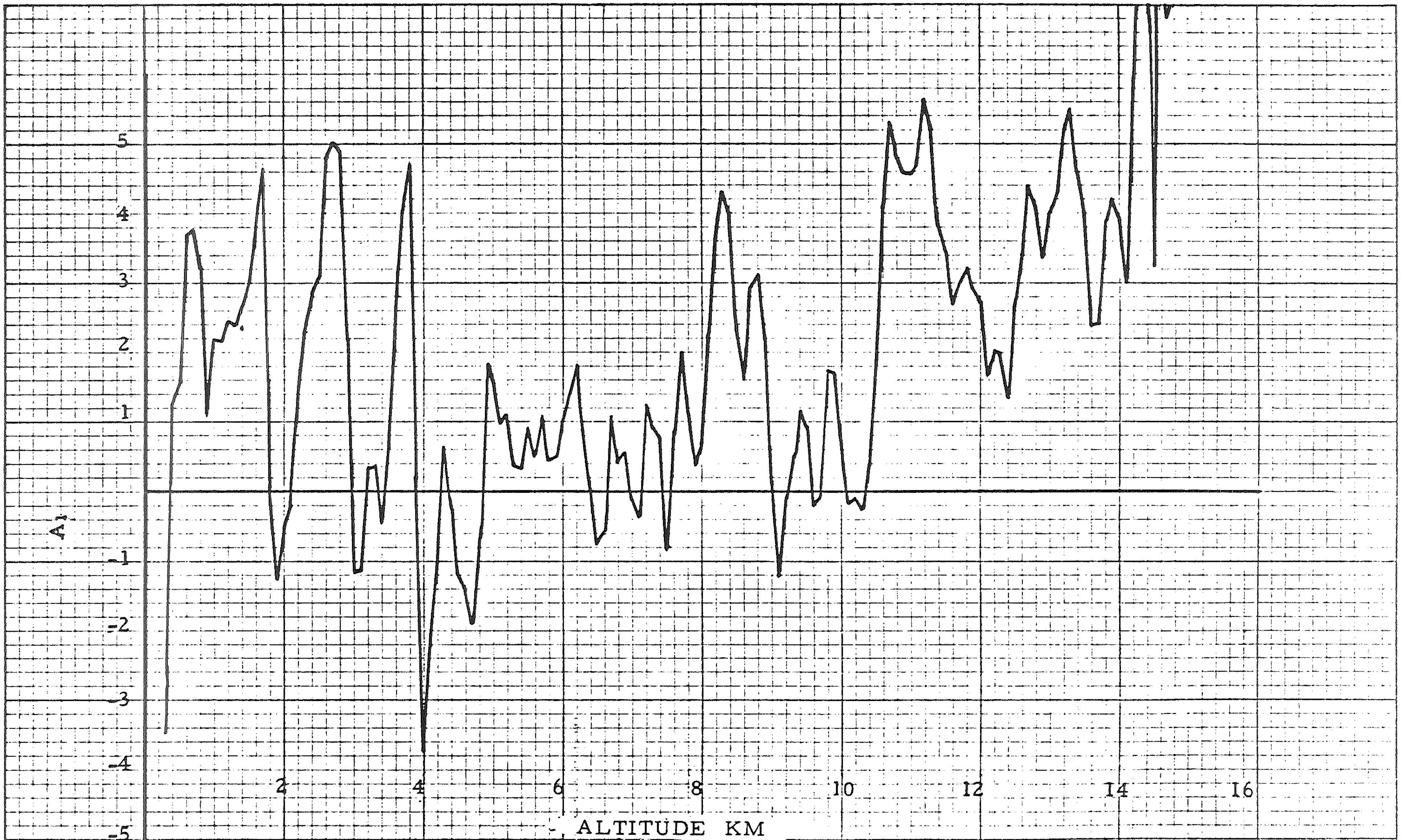
No effort has been made to utilize the coefficients in an analog computer program, as was done during the development program CR846. No data smoothing was performed on the covariance data to insure smooth curves of $A\phi$, A1, $B\phi$ and B1, rather, an effort was made to retain as much of the natural shape of the covariance as possible.

The covariance matrix will not be presented in this report due to the number of pages that would be required to fully represent all values utilized.



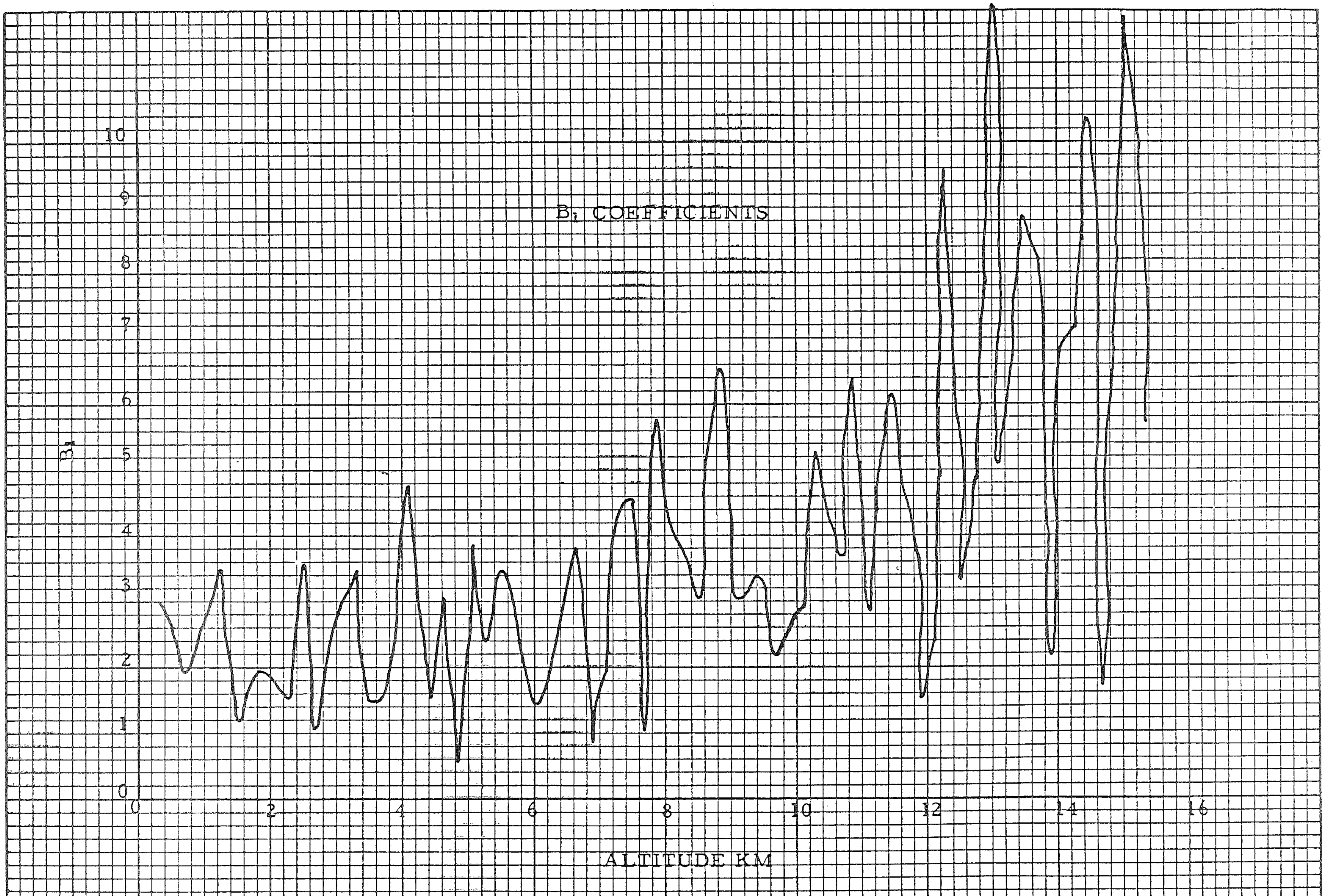
A_o COEFFICIENTS

Figure 6



A_1 COEFFICIENTS

Figure 7


B1 COEFFICIENTS
B1
0
1
2
3
4
5
6
7
8
9
10
0
2
4
6
8
10
12
14
16
ALTITUDE KM
Figure 8

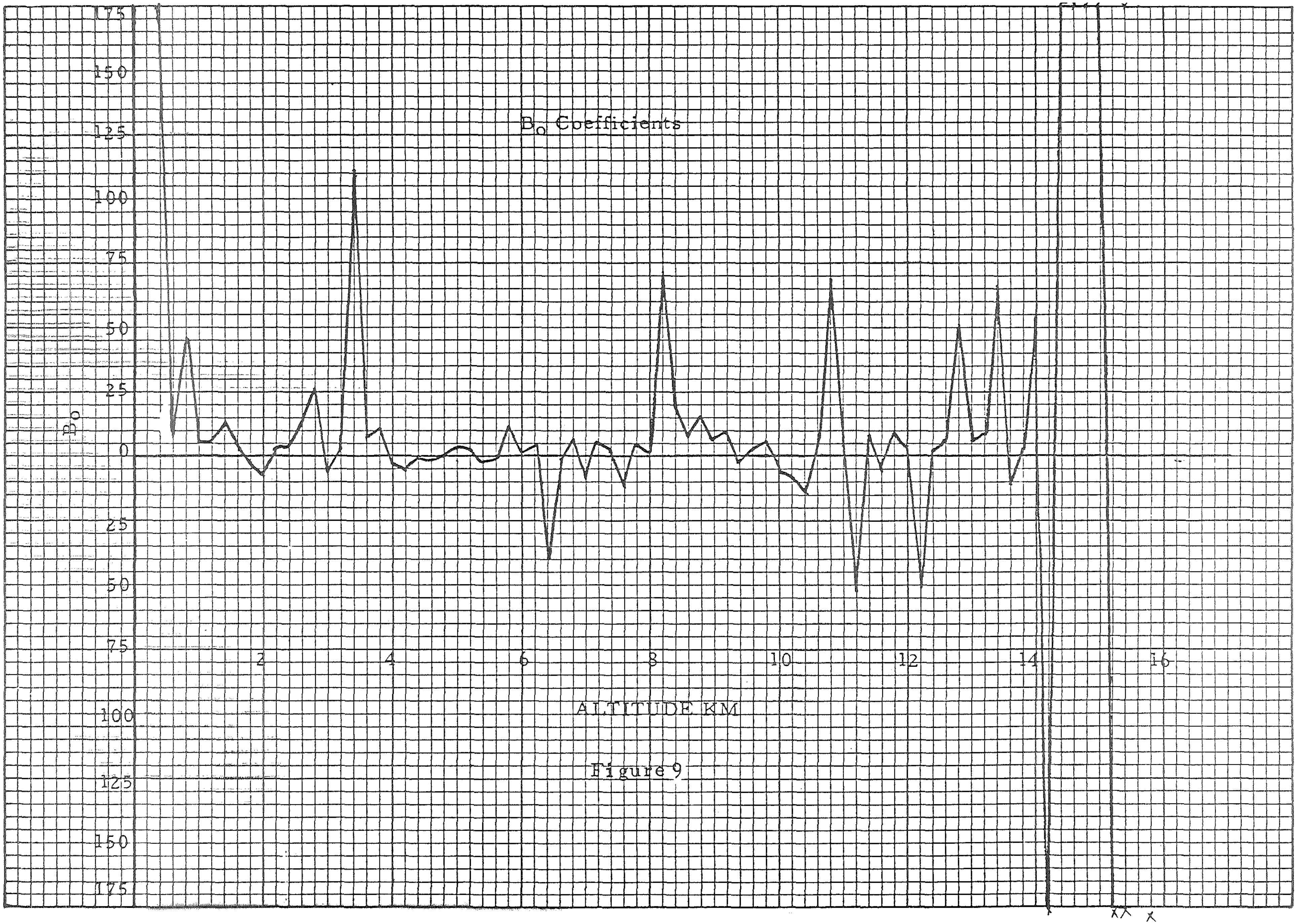

B$_o$ Coefficients
B_o
175
150
125
100
75
50
25
0
25
50
75
100
125
150
175
2
4
6
8
10
12
14
16
ALTITUDE KM

Figure 9

Wind Model coefficients beginning 400 meters and increasing by 50 meter steps.

$A_o(h)$	$A_1(h)$	$B_o(h)$	$B_1(h)$
0.943709E 00	-0.211279E 01	0.317744E 03	0.325791E 01
0.802144E 00	0.157822E 01	0.102805E 03	0.283626E 01
0.828959E 00	0.346543E 01	0.279399E 01	0.622888E 00
0.843599E 00	0.369723E 01	0.517014E 01	0.280986E 01
0.869565E 00	0.380628E 01	-0.196222E 01	0.261020E 01
0.888715E 00	0.378451E 01	-0.133538E 02	0.320822E 01
0.893160E 00	0.352549E 01	-0.649815E 02	0.969438E 00
0.879876E 00	0.320951E 01	-0.209659E 03	0.318848E 01
0.108513E 00	-0.121552E 01	0.697845E 02	0.191760E 01
0.583673E 00	0.100762E 01	0.445114E 02	0.324926E 01
0.744088E 00	0.236039E 01	-0.228122E 01	0.147955E 01
0.682485E 00	0.218749E 01	-0.257396E 01	0.278551E 01
0.631291E 00	0.208067E 01	0.320753E 01	0.226791E 01
0.614490E 00	0.218301E 01	0.376675E 01	0.180602E 01
0.606613E 00	0.234383E 01	0.511350E 01	0.392960E 01
0.589931E 00	0.245439E 01	-0.579306E 00	0.116943E 01
0.538930E 00	0.239308E 01	0.334284E 00	0.296740E 01
0.494154E 00	0.239092E 01	0.754026E 00	0.214086E 01
0.452656E 00	0.240968E 01	0.714017E 01	0.217995E 01
0.454203E 00	0.265291E 01	0.696000E 01	0.229465E 01
0.447347E 00	0.286617E 01	0.954346E 01	0.358303E 01
0.430088E 00	0.303655E 01	0.383235E 01	0.961039E 00
0.419509E 00	0.325960E 01	0.290048E 02	0.308477E 01
0.429190E 00	0.367066E 01	0.467322E 02	0.254762E 01
0.505635E 00	0.458601E 01	0.552612E 00	0.117103E 01
0.448293E 00	0.464117E 01	0.166026E 03	0.255130E 01
0.147697E 00	0.283752E 01	0.707506E 02	0.175903E 01
-0.231535E 00	0.398151E-01	0.149830E 02	0.202026E 01
-0.429863E 00	-0.174198E 01	-0.117261E 01	0.174727E 01
-0.375097E 00	-0.126482E 01	-0.863995E 01	0.398003E 01
-0.315105E 00	-0.639294E 00	0.163908E 01	0.138501E 01
-0.300412E 00	-0.516356E 00	-0.518522E 01	0.245332E 01
-0.263514E 00	-0.571142E-01	-0.887778E 01	0.188162E 01
-0.251260E 00	-0.200119E 00	0.177309E 02	0.227703E 01
-0.174032E 00	0.105719E 01	0.336270E 01	0.147708E 01
-0.147314E 00	0.150594E 01	0.974294E 00	0.202946E 01
-0.136513E 00	0.164821E 01	0.124524E 02	0.180706E 01
-0.118806E 00	0.229773E 01	0.984760E 01	0.245956E 01
-0.107625E 00	0.270200E 01	0.224566E 01	0.186950E 01
-0.948648E-01	0.285227E 01	0.301852E 01	0.114634E 01
-0.815233E-01	0.313020E 01	-0.263678E 01	0.149140E 01
-0.572561E-01	0.308738E 01	0.769665E 02	0.262379E 01
-0.106617E 00	0.460042E 01	0.580705E 01	0.200069E 01
-0.890381E-01	0.478895E 01	0.746592E 01	0.236154E 01
-0.671857E-01	0.499752E 01	-0.226526E 01	0.356677E 01
-0.378602E-01	0.501886E 01	-0.121078E 02	0.227820E 01
-0.298590E-02	0.480692E 01	0.700622E 00	0.100238E 01
0.161719E-01	0.490334E 01	0.131163E 01	0.335748E 01
0.488736E-01	0.436243E 01	0.423120E 02	0.100214E 01
0.101570E 00	0.215354E 01	0.309789E 02	0.180770E 01
0.116154E 00	0.601752E-01	0.841065E 01	0.298941E 01
0.986002E-01	-0.117030E 01	-0.409474E 01	0.187342E 01
0.686666E-01	-0.923914E 00	0.182336E 01	0.215730E 01
0.389109E-01	-0.114997E 01	0.925683E 00	0.228207E 01
0.134006E-01	-0.471046E 00	0.793175E 01	0.279629E 01
-0.689258E-02	0.338095E 00	0.735691E 00	0.142143E 01
-0.314399E-01	0.771772E 00	-0.220435E 01	0.304047E 01
-0.545792E-01	0.368309E 00	0.213950E 01	0.246462E 01
-0.676400E-01	-0.309014E 00	0.112275E 03	0.438917E 00
-0.800424E-01	-0.428428E 00	0.126323E 01	0.209116E 01
-0.938338E-01	0.380518E-01	0.230223E 02	0.347595E 01
-0.112281E 00	0.655090E 00	0.384108E 02	0.214221E 01
-0.157654E 00	0.199363E 01	0.101884E 02	0.816395E 00

-0.199861E 00	0.281284E 01	0.393311E 02	0.345138E 01
-0.243638E 00	0.346806E 01	0.184097E 02	0.153076E 01
-0.293965E 00	0.405991E 01	0.144685E 02	0.123853E 01
-0.354257E 00	0.470163E 01	0.164699E 01	0.240846E 01
-0.381615E 00	0.473598E 01	-0.222885E 03	0.232406E 01
-0.292293E 00	0.301283E 01	0.937383E 02	0.153164E 01
-0.104789E 00	0.159907E 00	0.270512E 02	0.513614E 00
0.956720E-01	-0.249995E 01	0.125484E 02	0.173146E 01
0.203343E 00	-0.373976E 01	-0.244337E 02	0.298146E 01
0.130900E 00	-0.263270E 01	-0.369059E 01	0.193461E 01
0.109123E 00	-0.223724E 01	-0.576640E 00	0.118695E 01
0.707732E-01	-0.169739E 01	-0.102937E 01	0.193471E 01
0.165342E-01	-0.101622E 01	0.537357E 01	0.145935E 01
-0.914965E-01	0.267229E 00	0.323680E 02	0.472098E 01
-0.119734E 00	0.638621E 00	-0.961770E 00	0.274235E 01
-0.669824E-01	0.124399E 00	0.166821E 00	0.207993E 01
-0.268649E-01	-0.252290E 00	0.129107E 01	0.249531E 01
-0.928986E-02	-0.428739E 00	0.108764E 02	0.277480E 01
0.652044E-01	-0.116330E 01	0.130569E 01	0.210529E 01
0.851464E-01	-0.135310E 01	-0.551410E-01	0.805345E 00
0.955754E-01	-0.141925E 01	0.431789E 00	0.676609E 00
0.112430E 00	-0.155327E 01	0.124526E 01	0.154210E 01
0.153925E 00	-0.188800E 01	0.364275E 00	0.156582E 01
0.985109E-01	-0.120286E 01	0.109198E 01	0.104092E 01
0.534475E-01	-0.666931E 00	0.206858E 02	0.190134E 01
-0.554763E-01	0.436444E 00	0.258248E 02	0.314967E 01
-0.176818E 00	0.156574E 01	0.308060E 00	0.356925E 00
-0.195050E 00	0.180132E 01	-0.251288E 01	0.256123E 01
-0.158473E 00	0.154293E 01	-0.328275E 01	0.329153E 01
-0.111641E 00	0.121141E 01	-0.953826E 00	0.589831E 00
-0.727124E-01	0.944011E 00	-0.159492E 00	0.390000E 00
-0.658601E-01	0.983772E 00	0.101610E 01	0.158131E 01
-0.681559E-01	0.108831E 01	-0.547214E 00	0.198072E 01
-0.731279E-02	0.552186E 00	-0.391883E 10	0.394663E 10
0.926723E-02	0.351162E 00	-0.282795E 00	0.137870E 01
-0.645760E-02	0.401369E 00	-0.333934E 01	0.269969E 01
-0.701579E-02	0.327856E 00	0.269266E 01	0.267354E 01
-0.558947E-01	0.702544E 00	0.685685E 00	0.235336E 01
-0.883603E-01	0.918793E 00	0.228752E 00	0.565152E 00
-0.874392E-01	0.812315E 00	-0.319728E 01	0.190258E 01
-0.634038E-01	0.466120E 00	0.182124E 00	0.178538E 01
-0.893636E-01	0.623129E 00	0.289520E 00	0.339844E 00
-0.142388E 00	0.106653E 01	0.456813E 01	0.451491E 01
-0.129098E 00	0.848542E 00	0.955779E 01	0.300561E 01
-0.877656E-01	0.412275E 00	-0.649456E 01	0.312864E 01
-0.568658E-01	0.137209E 00	0.162416E 01	0.321392E 01
-0.877489E-01	0.485996E 00	0.146184E 01	0.266874E 01
-0.111217E 00	0.752009E 00	0.807066E 00	0.936857E 00
-0.133013E 00	0.100761E 01	0.115610E 01	0.990456E 00
-0.161121E 00	0.129956E 01	0.613590E 00	0.176255E 01
-0.171295E 00	0.142216E 01	0.471102E 01	0.270732E 01
-0.203459E 00	0.173446E 01	0.130343E 00	0.706038E 00
-0.207558E 00	0.180690E 01	-0.426128E 00	0.252709E 01
-0.152783E 00	0.138752E 01	0.193483E 01	0.137393E 01
-0.881863E-01	0.863871E 00	-0.411499E 02	0.214294E 01
-0.287331E-01	0.375039E 00	-0.991124E 00	0.398373E 01
0.162784E-01	-0.550087E-02	0.680920E 00	0.852164E 00
0.668823E-01	-0.403240E 00	0.326638E 01	0.278390E 01
0.110078E 00	-0.751574E 00	0.193382E 01	0.313986E 01
0.151361E 00	-0.109838E 01	-0.277548E 01	0.123052E 01
0.999856E-01	-0.576802E 00	0.152623E 02	0.314059E 01
0.357825E-01	0.119017E 00	0.624935E 01	0.324824E 01
-0.473252E-01	0.107978E 01	0.995210E 00	0.343629E 01
-0.164699E-01	0.804549E 00	0.730338E-01	0.248308E 01
0.230375E-01	0.433262E 00	-0.185026E 01	0.106137E 01
0.445789E-01	0.244383E 00	0.101309E 01	0.378020E 01

0.216552E-01	0.575047E 00	-0.978428E 01	0.203553E 01
0.364634E-01	0.376144E 00	0.692974E 00	0.324578E 01
0.693445E-01	-0.122011E 00	0.557342E 00	0.278598E 01
0.872145E-01	-0.461336E 00	-0.222190E 01	0.903273E 00
0.814008E-01	-0.369683E 00	0.848892E 01	0.191606E 01
0.417363E-01	0.185902E 00	0.463671E 01	0.853111E 00
-0.369772E-01	0.125684E 01	0.328524E 00	0.245756E 01
-0.454301E-01	0.132104E 01	-0.384703E 01	0.189262E 01
-0.200614E-01	0.895527E 00	-0.790643E 00	0.322772E 01
-0.301916E-01	0.905357E 00	0.455513E 01	0.700930E 00
-0.295018E-01	0.761288E 00	0.124755E 02	0.402727E 01
0.401461E-02	0.111611E 00	0.201698E 02	0.422955E 01
0.597702E-01	-0.828778E 00	-0.103616E 02	0.206980E 01
0.767367E-01	-0.116737E 01	0.709525E 02	0.354020E 01
-0.566310E-01	0.804821E 00	0.263114E 02	0.355419E 01
-0.121824E 00	0.174355E 01	0.300694E 01	0.447622E 01
-0.141348E 00	0.196478E 01	-0.659660E 01	0.342481E 01
-0.114037E 00	0.135897E 01	0.189831E 01	0.188050E 01
-0.107600E 00	0.109238E 01	-0.284977E 00	0.662118E 00
-0.719016E-01	0.511497E 00	-0.247157E 00	0.103004E 01
-0.609699E-01	0.377398E 00	-0.288800E 01	0.229401E 01
-0.524013E-01	0.308032E 00	0.175538E 01	0.369627E 01
-0.725371E-01	0.632188E 00	0.366867E 01	0.314600E 01
-0.775989E-01	0.778019E 00	0.115925E 03	0.567662E 01
-0.158677E 00	0.221241E 01	0.739513E 02	0.600991E 01
-0.203522E 00	0.299609E 01	0.201337E 02	0.188141E 01
-0.240030E 00	0.361584E 01	0.213955E 02	0.252301E 01
-0.270395E 00	0.414413E 01	0.631605E 01	0.414474E 01
-0.267146E 00	0.427736E 01	-0.402133E 01	0.244168E 01
-0.245547E 00	0.420930E 01	-0.396657E 01	0.168645E 01
-0.212260E 00	0.400075E 01	-0.892814E 01	0.218177E 01
-0.145870E 00	0.322869E 01	-0.266944E 00	0.372284E 00
-0.736722E-01	0.227735E 01	-0.932379E 01	0.110032E 01
-0.715723E-02	0.127533E 01	0.330320E 01	0.115291E 01
-0.315233E-01	0.160231E 01	0.197694E 02	0.278054E 01
-0.617113E-01	0.211575E 01	0.310139E 02	0.280087E 01
-0.101106E 00	0.293162E 01	0.540204E 01	0.156713E 01
-0.113019E 00	0.326312E 01	-0.291784E 01	0.250966E 01
-0.105511E 00	0.312396E 01	-0.450763E 02	0.867888E 01
-0.834538E-01	0.246546E 01	0.235457E 01	0.566376E 01
-0.822704E-01	0.218585E 01	0.206248E 02	0.909125E 00
-0.518410E-01	0.114781E 01	0.232841E 02	0.261740E 01
-0.200445E-01	0.780971E-01	0.165116E 02	0.274157E 01
0.228227E-01	-0.114292E 01	0.359894E 00	0.651823E 01
0.292208E-01	-0.124870E 01	0.384215E 02	0.419043E 01
0.246053E-01	-0.109441E 01	-0.633273E 01	0.651096E 01
-0.150532E-01	-0.117035E 00	-0.120604E 01	0.300918E 01
-0.319057E-01	0.137392E 00	0.134157E 01	0.305087E 01
-0.569459E-01	0.474103E 00	0.213976E 01	0.318600E 01
-0.727367E-01	0.724660E 00	0.278712E 01	0.273670E 01
-0.949331E-01	0.114677E 01	-0.438580E 01	0.510471E 01
-0.857830E-01	0.985494E 00	0.202020E 01	0.287352E 01
-0.827649E-01	0.913476E 00	-0.344831E 01	0.226104E 01
-0.516514E-01	0.457636E 00	0.273400E 00	0.435788E 00
-0.848176E-03	-0.203705E 00	0.620077E 01	0.256575E 01
0.196514E-01	-0.402445E 00	0.741951E 00	0.346681E 01
0.280312E-02	-0.108226E 00	0.281501E 02	0.364394E 01
-0.350578E-01	0.687286E 00	0.993534E 01	0.446738E 01
-0.887074E-01	0.171410E 01	-0.189468E 02	0.323725E 01
-0.440592E-01	0.113393E 01	0.273308E 01	0.224143E 01
-0.726798E-01	0.166357E 01	-0.130608E 01	0.203472E 01
-0.347394E-01	0.112260E 01	-0.140451E 01	0.257482E 01
-0.803917E-02	0.726241E 00	-0.109383E 02	0.287472E 01
0.390208E-02	0.508030E 00	0.874653E 01	0.259108E 01
0.416689E-01	-0.167922E 00	0.874102E 01	0.892755E 00
0.843589E-01	-0.965042E 00	0.301740E 01	0.265774E 01

0.416532E-01	-0.103511E 00	-0.147324E 02	0.339566E 01
0.842536E-02	0.339265E 00	0.214469E 01	0.289133E 01
0.345767E-01	-0.254715E 00	-0.351384E 01	0.285443E 01
0.258097E-01	-0.127857E 00	0.563259E 01	0.343492E 01
-0.313023E-02	0.390412E 00	0.110062E 02	0.255603E 01
-0.292308E-01	0.820610E 00	0.284168E 02	0.522755E 01
-0.876366E-01	0.189888E 01	-0.110463E 02	0.164073E 01
-0.788873E-01	0.163347E 01	0.363614E 02	0.663775E 00
-0.219552E 00	0.410369E 01	0.924857E 02	0.530615E 01
-0.267449E 00	0.490261E 01	0.340276E 02	0.422478E 01
-0.287308E 00	0.531620E 01	-0.702180E 01	0.832201E 01
-0.292084E 00	0.527293E 01	-0.338406E 02	0.443831E 01
-0.277724E 00	0.480535E 01	-0.170577E 02	0.279550E 01
-0.270394E 00	0.448562E 01	0.334339E 01	0.365841E 01
-0.271627E 00	0.456470E 01	-0.137689E 02	0.463350E 01
-0.263149E 00	0.442933E 01	0.409989E 01	0.311957E 01
-0.265333E 00	0.457145E 01	-0.131566E 03	0.579451E 01
-0.205690E 00	0.362649E 01	0.147910E 03	0.641682E 01
-0.235240E 00	0.473098E 01	0.287904E 02	0.325106E 01
-0.227334E 00	0.514200E 01	0.366364E 02	0.376599E 01
-0.227942E 00	0.563281E 01	-0.281987E 01	0.167123E 01
-0.185201E 00	0.561348E 01	-0.212245E 02	0.261058E 01
-0.112666E 00	0.519499E 01	-0.247223E 02	0.286463E 01
-0.324127E-01	0.460507E 01	-0.806165E 01	0.825717E 00
0.491175E-01	0.383808E 01	-0.311395E 02	0.305179E 01
0.115468E 00	0.325733E 01	0.526248E 01	0.472477E 01
0.153479E 00	0.346851E 01	-0.556439E 02	0.646884E 01
0.202516E 00	0.277998E 01	-0.709221E 01	0.377810E 01
0.221717E 00	0.268193E 01	0.142144E 01	0.356651E 01
0.235652E 00	0.275793E 01	0.181530E 02	0.610192E 01
0.261008E 00	0.302703E 01	0.118298E 01	0.314013E 01
0.278677E 00	0.309844E 01	0.358224E 01	0.320618E 01
0.297059E 00	0.323103E 01	-0.289945E 01	0.182932E 01
0.307888E 00	0.314721E 01	-0.120120E 02	0.440774E 01
0.306443E 00	0.292469E 01	-0.127094E 02	0.278675E 01
0.288999E 00	0.259618E 01	0.112091E 01	0.309122E 01
0.303658E 00	0.273263E 01	-0.127026E 03	0.102252E 02
0.186435E 00	0.133410E 01	0.277612E 02	0.150097E 02
0.231985E 00	0.164275E 01	0.376237E 01	0.785726E 01
0.247574E 00	0.175129E 01	0.758515E 01	0.486343E 01
0.272628E 00	0.203112E 01	-0.109227E 02	0.445193E 01
0.251562E 00	0.182161E 01	0.256207E 01	0.362642E 01
0.264645E 00	0.198973E 01	-0.391174E 01	0.272127E 01
0.231318E 00	0.173206E 01	-0.107253E 02	0.327157E 01
0.187696E 00	0.134694E 01	0.504668E 01	0.264535E 01
0.191121E 00	0.151334E 01	-0.234074E 02	0.951927E 01
0.266658E 00	0.269114E 01	-0.234022E 03	0.417892E 01
0.785594E-01	0.555499E 00	0.489014E 03	0.903209E 01
0.280812E 00	0.337952E 01	0.635261E 02	0.515073E 01
0.334078E 00	0.403810E 01	0.185508E 02	0.331071E 01
0.372052E 00	0.438840E 01	-0.594028E 01	0.271704E 01
0.366775E 00	0.428111E 01	-0.888763E 01	0.255884E 01
0.353632E 00	0.410568E 01	-0.615656E 01	0.651342E 01
0.364029E 00	0.405692E 01	-0.532421E 02	0.588143E 01
0.295977E 00	0.336630E 01	-0.269034E 02	0.478814E 01
0.290954E 00	0.316892E 01	0.101105E 02	0.283401E 01
0.442194E 00	0.400850E 01	-0.400098E 03	0.958163E 01
0.288827E 00	0.248105E 01	0.513667E 03	0.126596E 02
0.580688E 00	0.429913E 01	0.106781E 03	0.816671E 01
0.695055E 00	0.488798E 01	0.563036E 02	0.745197E 01
0.774086E 00	0.522821E 01	0.340874E 02	0.685092E 01
0.847600E 00	0.547511E 01	-0.476597E 01	0.513980E 01
0.906111E 00	0.548923E 01	-0.111494E 02	0.166127E 01
0.941966E 00	0.520077E 01	-0.107618E 03	0.870726E 01
0.939967E 00	0.455639E 01	-0.591021E 02	0.671453E 01
0.947013E 00	0.410481E 01	-0.146933E 02	0.684433E 01

0.968542E 00	0.397681E 01	-0.170658E 03	0.118796E 02
0.831059E 00	0.266810E 01	-0.332233E 02	0.951173E 01
0.786933E 00	0.237258E 01	0.839044E 01	0.848803E 01
0.798301E 00	0.245338E 01	-0.282529E 01	0.883469E 01
0.796542E 00	0.241754E 01	0.207576E 03	0.101573E 02
0.101496E 01	0.342709E 01	0.627974E 02	0.644307E 01
0.111458E 01	0.387480E 01	0.524840E 02	0.982513E 01
0.112336E 01	0.406166E 01	0.404784E 02	0.820096E 01
0.110866E 01	0.422185E 01	0.121770E 02	0.847326E 01
0.988621E 00	0.416457E 01	-0.176658E 04	0.433247E 01
0.770002E 00	0.389730E 01	-0.406168E 04	0.172576E 02
0.657279E 01	0.179491E 02	-0.834116E 04	0.223200E 02
0.433429E-02	0.304213E 01	0.787306E 03	0.162744E 02
0.904204E 00	0.492957E 01	0.439958E 03	0.878639E 01
0.867487E 00	0.597022E 01	0.158979E 04	0.783944E 01
0.200934E 01	0.106794E 02	-0.635603E 03	0.696089E 01
0.162938E 01	0.103614E 02	0.287901E 05	0.104156E 02
0.366376E 01	0.209056E 02	0.381437E 05	0.613517E 01
0.757446E 01	0.461476E 02	0.407709E 05	0.857945E 01
0.819826E 01	0.518227E 02	-0.760323E 05	0.711271E 01
0.553436E 00	0.221404E 01	0.493831E 05	0.142592E 02
0.381869E 01	0.275444E 02	0.522609E 03	0.935415E 01
0.421557E 01	0.298740E 02	0.633534E 03	0.389944E 01
0.299537E 01	0.178853E 02	-0.166736E 04	0.103369E 02
0.920998E 00	0.685388E 01	-0.178671E 04	0.616817E 01
0.106475E 01	0.487155E 01	0.831537E 04	0.737039E 01
0.250372E 01	0.165377E 02	0.324611E 04	0.710284E 01
0.337774E 01	0.196197E 02	-0.102391E 05	0.169322E 01
0.482319E 01	0.292515E 02	-0.145028E 06	0.480233E 01
-0.144521E 01	-0.676340E 01	-0.413870E 05	0.428825E 01
0.166828E 02	0.762554E 02	-0.110797E 06	0.103385E 02
0.441480E 01	0.143055E 02	0.102068E 05	0.166851E 02
0.611192E 01	0.282888E 02	-0.247734E 05	0.957319E 01
0.569515E 00	0.109599E 01	0.170902E 05	0.886191E 01
0.381050E 01	0.112204E 02	-0.744473E 04	0.960273E 01
0.878811E 01	0.330381E 02	-0.263733E 05	0.101284E 02
0.131514E 01	0.540148E 01	0.111555E 05	0.915977E 01
0.438483E 01	0.135823E 02	0.111862E 05	0.835442E 01
0.667309E 01	0.258263E 02	-0.850298E 05	0.104735E 02
-0.233651E 00	0.200355E 00	-0.740939E 04	0.563203E 01
0.990260E 01	0.321878E 02	-0.148469E 05	0.100922E 02
0.162218E 01	0.543645E 01	0.237776E 04	0.114209E 02
0.205831E 01	0.989410E 01	0.265788E 04	0.902429E 01
-0.908996E 00	0.120616E 00	0.717033E 02	0.855989E 01
-0.217763E 01	-0.332186E 01	-0.113591E 02	0.757271E 01

VIII. COMPUTER PROGRAMS

The computer programs furnished are the programs to calculate the covariance matrix, $A\phi$, $A1$, $B\phi$ and $B1$. The computer program to calculate the derivatives by polynomial curve fit are not furnished since this routine is standard in computation libraries and depends on the researchers preferences. The first program consisting of three pages is the computer program for calculating the covariance matrix. The second program consisting of one page is the program to calculate the A coefficients. The third program consists of one page and is the program to develop the B coefficients. The covariance program is developed for the IBM 7094 computer whereas the A and B coefficient programs were written for the IBM 360 computer.

COMPUTER SYMBOLS

AV1	Average wind speed
AV2	
AVW	
AVS	
AVPIJ	Product of the average wind speed at altitudes I, J
CORR	Correlation of wind speeds at altitudes I, J
COV	Covariance of wind speeds at altitudes I, J
C	Covariance
DC	Derivative of covariance
DDC	Second Derivative of covariance
IT	Tape Numbers
S	Sum
W(I, K)	Wind velocity at altitude I, record K

MAIN2 - EFN SOURCE STATEMENT - IFN(S) -

```
      DIMENSION W(3,1251),                  AVW(2),COV(2,350),CORR(3),
     1  CORRS(2)
    1 FORMAT(1H1)
    2 FORMAT(F10.1 /(10F13.4))
    4 FORMAT(8F10.4)
      DO 100 IB=36,159
      IB2=2*IB
      IF(IB-40) 202,202,200
  200 IF(IB-80) 203,203,201
  201 IF(IB-120) 204,204,205
  202 IT=8
      ISKIP=2*(IB-1)
      GO TO 206
  203 IT=9
      ISKIP=2*(IB-41)
      GO TO 206
  204 IT=10
      ISKIP=2*(IB-81)
      GO TO 206
  205 IT=11
      ISKIP=2*(IB-121)
  206 IF(ISKIP) 209,209,207
  207 DO 208 I=1,ISKIP
  208 READ(IT)
  209 READ(IT) (W(1,I),I=1,1251)
      READ(IT) (W(2,I),I=1,1251)
      WRITE(6,1)
      I1=0
      I2=0
      AV1=0.
      AV2=0.
      DO 14 K=2,1251
       IF(W(1,K)) 12,11,12
   11 I1=I1+1
      GO TO 13
   12 AV1=AV1+W(1,K)
   13 IF(W(2,K)) 132,131,132
  131 I2=I2+1
      GO TO 14
  132 AV2=AV2+W(2,K)
   14 CONTINUE
      I1=1250-I1
      I2=1250-I2
      AI=I1
      AVW(1)=AV1/AI
      AI=I2
      AVW(2)=AV2/AI
      AVP11=AVW(1)*AVW(1)
      AVP12=AVW(1)*AVW(2)
      AVP22=AVW(2)*AVW(2)
      DO 22 I=1,3
      CORR(I)=0.
      K=0
      IF(I-2) 15,16,17
   15 L=1
```

MAIN2 - EFN SOURCE STATEMENT - IFN(S) -

```
      M=1
       GO TO 18
   16 L=1
      M=2
       GO TO 18
   17 L=2
      M=2
   18 DO 21 J=2,1251
      COR=W(L,J)*W(M,J)
      IF(COR) 20,19,20
   19 K=K+1
       GO TO 21
   20 CORR(I)=COR+CORR(I)
   21 CONTINUE
      K=1250-K
      AI=K
      CORR(I)=CORR(I)/AI
   22 CONTINUE
      COV11=CORR(1)-AVP11
      COV12=CORR(2)-AVP12
      COV22=CORR(3)-AVP22
      ALT=W(1,1)
      ALT2=W(2,1)
      ISF=320-IB2
      DO 90 IS=1,ISF
      ITEST=IB2+IS
      IF(ITEST-80) 216,216,2100
 2100 CALL CLOSE (8,4)
      CALL REW08
  210 IF(ITEST-160) 213,213,2110
 2110 CALL CLOSE (9,4)
      CALL REW09
  211 IF(ITEST-240) 214,214,2150
 2150 CALL CLOSE (10,4)
      CALL REW10
      GO TO 215
  213 IT=9
      GO TO 216
  214 IT=10
      GO TO 216
  215 IT=11
  216 READ(IT) (W(3,I),I=1,1251)
      J=0
      AVS=0.
       DO 25 I=2,1251
      IF(W(3,I)) 24,23,24
   23 J=J+1
      GO TO 25
   24 AVS=AVS+W(3,I)
   25 CONTINUE
      J=1250-J
      AI=J
      AVS=AVS/AI
      DO 30 I=1,2
      CORRS(I)=0.
      K=0
```

```
      DO 29 J=2,1251
      CORS=W(1,J)*W(3,J)
      IF(CORS) 27,26,27
   26 K=K+1
      GO TO 29
   27 CORRS(I)=CORRS(I)+CORS
   29 CONTINUE
       K=1250-K
      AI=K
      CORRS(I)=CORRS(I)/AI
   30 CONTINUE
      AVP31=AVW(1)*AVS
      AVP32=AVW(2)*AVS
      COV(1,IS)=CORRS(1)-AVP31
      COV(2,IS)=CORRS(2)-AVP32
   90 CONTINUE
      WRITE(6,2) ALT,AVW(1),COV11,COV12,(COV(1,I),I=1,ISF)
      WRITE(7,4) ALT,AVW(1),COV11,COV12,(COV(1,I),I=1,ISF)
      WRITE (6,1)
      WRITE(6,2) ALT2,AVW(2),COV22,(COV(2,I),I=1,ISF)
      WRITE(7,4) ALT2,AVW(2),COV22,(COV(2,I),I=1,ISF)
      CALL CLOSE (11,4)
      CALL REW11
  100 CONTINUE
      STOP
      END
```

```
C      A COEFFICIENT PROGRAM
       DIMENSION A0%320<,A1%320<,C%320<,DC%320<,DDC%320<
C      S=SUM C=COV,DC=DERIVATIVES OF COV
     3 FORMAT(1H1)
     4 FORMAT(6E15.7)
     5 FORMAT(5E15.7)
     6 FORMAT(1X9H A0 COEFF   )
     7 FORMAT(1X9H A1 COEFF   )
     8 FORMAT%40X110 <
       READ(7)
       IT#10
       DO 40 I=2,IT
       WRITE%3,8< I
       READ( 7 )(C(L),DC(L),DDC(L),L=1,I)
       SCC=0.
       SCDC=0.
       SCDDC=0.
       SDCDC=0.
       SDCDDC=0.
       DO 15 J=1,I
       P=C(J)
       SCC=C(J)*P+SCC
       SCDC=DC(J)*P+SCDC
    15 SCDDC=DDC(J)*P+SCDDC
       DO 30 J=1,I
       P=DC(J)
       SDCDC=DC(J)*P+SDCDC
    30 SDCDDC=DDC(J)*P+SDCDDC
       TOP1=-SCC*SDCDDC+SCDC*SCDDC
       BOT1=SCC*SDCDC-SCDC*SCDC
       TOP2=-SCDDC*SDCDC+SCDC*SDCDDC
       BOT2=BOT1
       A1(I)=TOP1/BOT1
       WRITE%3,3<
       WRITE%3,9< SCC,SCDC,SCDDC,SDCDC,SDCDDC,TOP1,BOT1,TOP2,BOT2
     9 FORMAT%///10E13.5<
       WRITE%3,10< %C%L<,DC%L<,DDC%L<,L#1,I<
    10 FORMAT%//%3E20.7<<
    40 A0(I)=TOP2/BOT2
       WRITE(3,3)
       WRITE(3,6)
       WRITE(3,4) (A0(J),J=1,IT)
       WRITE(2,5) (A0(J),J=1,IT)
       WRITE(3,3)
       WRITE(3,7)
       WRITE(3,4) (A1(J),J=1,IT)
       WRITE(2,5) (A1(J),J=1,IT)
       STOP
       END
```

```
BPS FORTRAN IVD  COMPILER  VERSION  3  LEVEL  0  DEC  1966

/JOB      SINGLE,GO
BEGIN COMPILATION
          C      PROGRAM TO CALCULATE B0,B1
 S.0001          DIMENSION A0%320<,A1%320<,B0%320<,B1%320<,DSTARC%320<,
                1DC%320<,G%3<,C%320<
 S.0002          IALL#306
 S.0003          READ%1,1<%A0%J<,J#1,IALL<
 S.0004          READ%1,1<%A1%J<,J#1,IALL<
 S.0005          WRITE(3,2) (A0(J),A1(J),J=1,IALL)
 S.0006        1 FORMAT(60X E15.7/(5E15.7))
 S.0007          READ%1,7<%DSTARC%J<,J#1,IALL<
 S.0008        7 FORMAT%6E12.6<
 S.0009          READ%1,2<%DC%J<,J#1,IALL<
 S.0010          WRITE(3,2) (DSTARC(J),DC(J),J=1,IALL)
 S.0011        2 FORMAT%6E13.6<
 S.0012          DO 10 I#1,IALL
 S.0013          J#I&1
 S.0014          K#J&1
 S.0015          CON1#A0%J<-2.*A1%K<
 S.0016          CON2#A1%J<
 S.0017          CON3#A0%I<-2.*A1%I<&A1%J<
 S.0018          CON4#A0%J<&2.*A1%K<
 S.0019          CON5#A1%J<-A0%K<-2.*A1%K<
 S.0020          ALFA=ABS(DSTARC(I)-DC(I))
 S.0021          WRITE(3,2) CON1,CON2,CON3,CON4,CON5,ALFA
 S.0022        5 G%I<=SQRT%ALFA<
 S.0023          G%K<#%CON3*CON4-CON1*CON2/CON2*CON4-CON1*CON5<*G%I<
 S.0024          G%J<#%CON2*G%I<-CON5*G%K<</CON4
 S.0025          B1(I)=G(I)
 S.0026          B0%I<#.5*%-3.*G%I<&4.*G%J<-G%K<<&A1%I<*B1%I<
 S.0027       10 CONTINUE
 S.0028          WRITE%3,8<
 S.0029        8 FORMAT%1H1<
 S.0030          WRITE%3,9<
 S.0031        9 FORMAT%7X2HB012X2HB1<
 S.0032          WRITE%3,6<%B0%I<,B1%I<,I#1,IALL<
 S.0033        6 FORMAT(20X2E13.6)
 S.0034          STOP
 S.0035          END
```

CRITIQUE OF RESULTS

The results obtained, as shown in Figures 6, 7, 8, and 9, show an unusual amount of variability. This variability appears to be directly proportional to the variability in the covariance derivatives, $\frac{dC}{dh}$, $\frac{d^2C}{dh^2}$. An important conclusion results from this proportionality: that this technique will not produce accurate results on turbulence data. A cursory view of turbulence data obtained by separating turbulence frequencies from Jimsphere records reveals the impracticality in attempting to obtain these derivatives. To obtain accurate results on the filter development, accurate derivatives are required. This accuracy is difficult to obtain using polynomial techniques; however, any other technique results in smoothing of the data, thereby losing the effect of the selection of an appropriate lag size. Lag size was discussed in Section III.

The dependence on covariance derivative variability can be seen in Figures 6, 7, 8, and 9. The data up to 14000 meters were segmented for curve fitting every 2000 meters (500 meters between points), introducing some smoothing in the data. The data above 14000 meters were curve fit at 250 meter increments (50 meters between each point). The latter case detects each change of sign in the derivatives along the span of the covariance, resulting in the degree of variability as shown above 14000 meters; whereas, the former maintains the same derivative sign for large increments due to some smoothing introduced by the 2000 meter increment selection, resulting in less variability as shown below 14000 meters. A case for highly smoothed data which exhibits no variability can be seen in CR846.

The values of the covariance matrix of the Jimsphere data obtained during this analysis appears to be accurate. The data sample was adequate and sufficient care was taken such that the present matrix has the necessary elements (accuracy, sample size, etc.) such that validity can be placed on its use in further analyses.

Attempts were made during the present analysis to expand the order of the differential equation to an order greater than two (2). These were unsuccessful, primarily due to the order of the right hand side of Equation (1), Section II.

The coefficients of the right hand side were developed from results obtained by Solodovnikov[4], based on the works of Batkov[3], particularly Solodovnikov's Equations (10.27), page 383 and (10.60), page 390, Reference (4). The major difficulty that arises was pointed out by Stear[7], that the Equation (10.27) is not a recursive relationship as stated by Batkov. Further, as became evident during the analysis, Solodovnikov's Equation (10.60) puts a restraint on the data, forcing the value of $\left.\frac{dC(h_i, h_j)}{dh_j}\right|_{h_i = h_j^-} - \left.\frac{dC(h_i, h_j)}{dh_j}\right|_{h_i = h_j^+}$ to be positive. When analysing the data, it is seen that there is no rational basis for such a constraint.

Equation (10.60) is equal to the value of the difference between the derivatives at $h_i = h_j$, or $(-1)^{n-m-1} \frac{b^2{}_m(t)}{a^2{}_n(t)}$, the value of the discontinuity at $h_i = h_j$, which has meaning when viewed as the initial value of $g(h_i, h_j)$ at $h_i = h_j$. When considered as the initial value of $g(h_i, h_j)$, (all values of $g(h_i, h_j)$ for $h_i > h_j = 0$), only the magnitude of Equation (10.60) has significance. Therefore, this report utilized the magnitude of Equation (10.60).

CONCLUSIONS

1. The shaping filter can be developed using a large volume of winds and turbulence data.

2. The Jimsphere data handling requires a computer with large storage capacity.

3. Floating point polynomial fitting of the covariance curves produces derivatives of sufficient accuracy for filter development, except the end points of each segment fit.

4. The technique can be very effective for smaller more tractable data samples, but for large, highly nonstationary processes of high frequency content, accuracy of coefficients becomes critical with small lag sizes.

5. This technique is not readily useable on data of high variability, such as turbulence.

RECOMMENDATIONS

1. The accuracy of the coefficients should be validated prior to assigning credibility to analyses utilizing the filter coefficients.
2. Consideration should be given to use of this technique with partitions of the covariance matrix.
3. Some smoothing of the covariance matrix should be considered.
4. When using a polynomial curve fit to determine the covariance derivatives, the segments that are curve fit should overlap to eliminate the derivative error on the polynomial end points.

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