COMPLEX OPTIMIZATION AND ITS APPLICATION TO DISTRIBUTED-LUMPED-ACTIVE NETWORKS

Prepared under Grant NGL-03-002-136 for the
Instrumentation Division of the Ames Research Center
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by

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__Abstract__: This report describes the use of a new optimization technique, complex optimization, which may be applied to the synthesis of distributed-lumped-active networks, i.e., networks comprised of elements which are distributed, lumped, or active in nature. A constraint procedure which may be used to insure the realizability of the network parameters and which avoids local optima in the optimization process is described. Several examples of the application of the process to specific network configurations are given.

March 1970

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Automated Synthesis of Distributed-Lumped-Active Networks Using Constrained Complex Optimization

I. Introduction

This is one of a series of reports concerning the use of digital computational techniques in the analysis and synthesis of DLA (Distributed-Lumped-Active) networks. This class of networks consists of three distinct types of elements, namely distributed elements (modeled by partial differential equations), lumped elements (modeled by algebraic equations and ordinary differential equations), and active elements (modeled by algebraic equations). Such a characterization is especially applicable to the broad class of circuits referred to as linear integrated circuits, since the required fabrication techniques for these circuits readily produce elements which may be referred to as "distributed", as well as producing elements which may be characterized as "lumped" and/or "active". The DLA class of networks is capable of realizing network functions with a wide range of properties. In addition, such realizations usually have fewer components and superior characteristics than realizations using only lumped elements, or realizations using lumped elements and active elements. DLA networks also have their disadvantages. One of the most significant of these is the difficulty of performing synthesis procedures, i.e., determining network topologies and element values such that the network meets some specified characteristics. Although synthesis procedures have been developed for networks containing only lumped elements, and for networks containing only distributed elements, these procedures are not, in general, applicable to the DLA class of networks. One of the most fruitful approaches to the synthesis of such networks has been the use of optimization techniques implemented on the digital computer. In this report a new approach to the application of such optimization methods is presented. The approach is called complex optimization. It is discussed in more detail in the following sections.

II. Complex Optimization

In a conventional optimization problem, if it is desired to have the response characteristic of a network match some specified sinusoidal steady-state magnitude curve, a methematical model of the network's response function is evaluated at a series of discrete sinusoidal frequencies, as indicated in

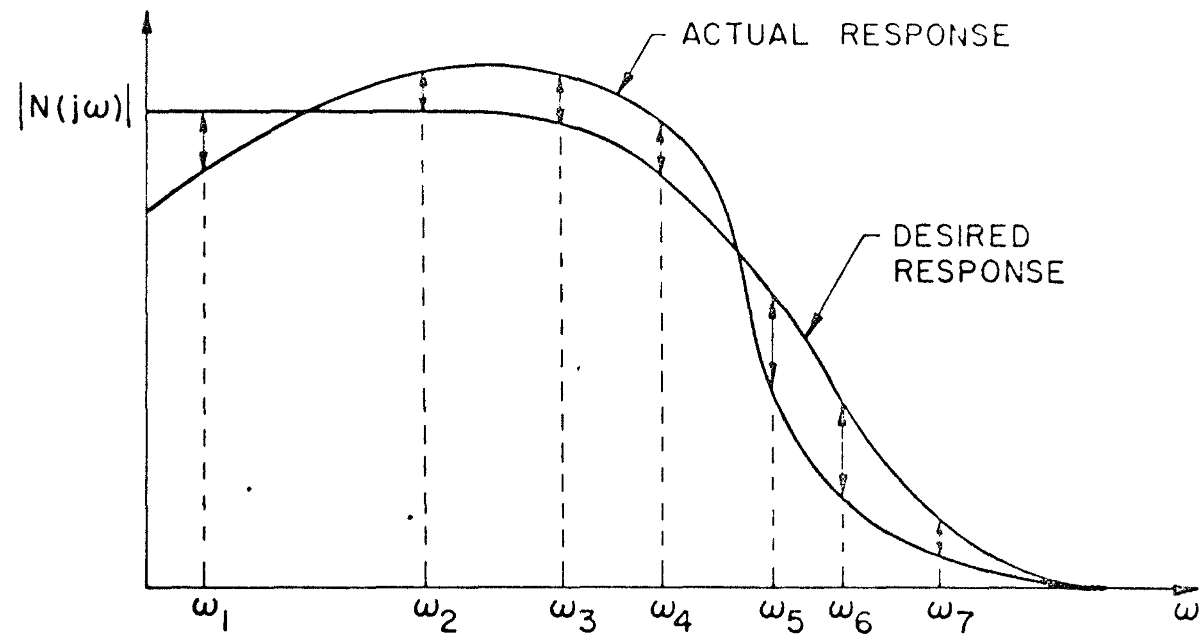

ACTUAL RESPONSE
|N(jω)|
DESIRED RESPONSE
ω1
ω2
ω3
ω4
ω5
ω6
ω7
ω

FIG. 1

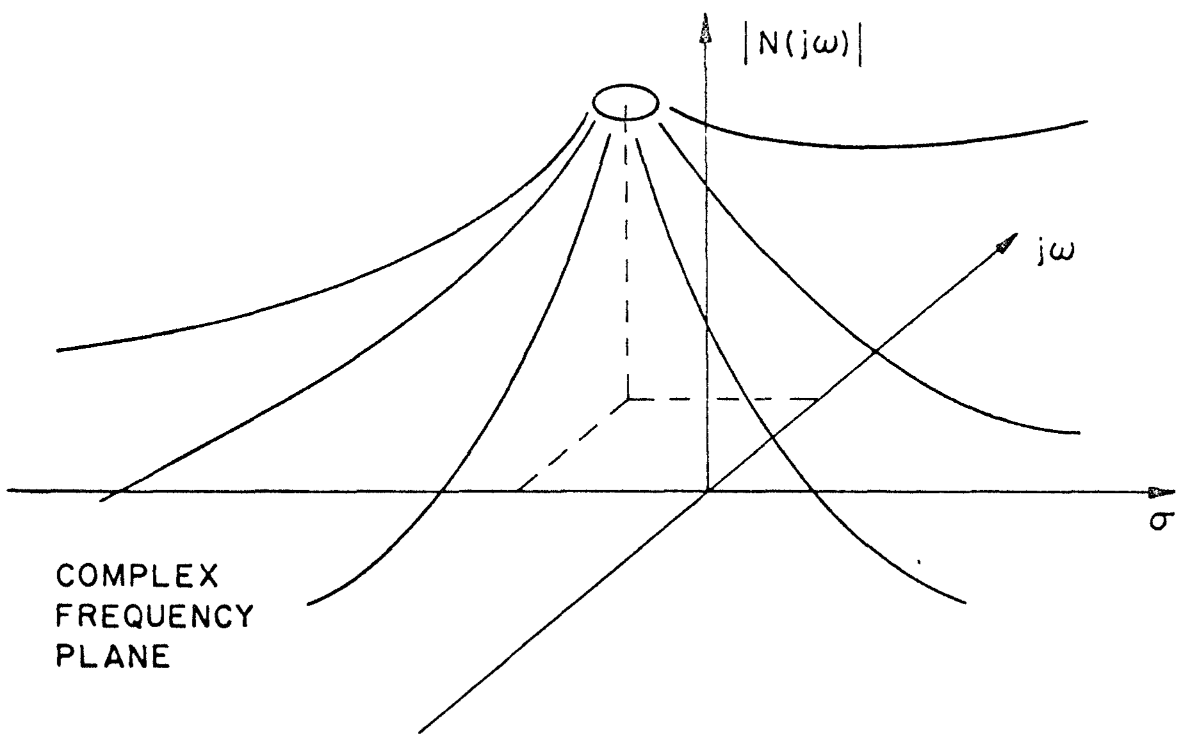

|N(jω)|
jω
σ
COMPLEX FREQUENCY PLANE

FIG. 2

Fig. 1. Next, a scalar error function is computed, usually as a weighted sum of the differences between the desired and actual curves, as evaluated at the specified frequencies. Finally, an optimization strategy is utilized in such a manner as to change the network parameters to decrease the error function. One of the disadvantages of this type of an approach is that it requires several evaluations of the network response function for each application of the optimization strategy. In the complex optimization procedure presented in this report the network response function is defined for complex frequencies. Thus, if a network characteristic corresponding with a specified pole or zero location is desired, the optimization strategy need merely maximize or minimize the magnitude of the network response function at the desired complex values of its frequency argument. A sketch of the process is shown in Fig. 2.

As a general example of the application of this procedure consider the use of complex optimization to realize a single dominant complex conjugate pole pair for a given network. Let p be the complex location of either of the conjugate poles, let $\underline{x}$ be a vector defining the value of the network parameters, and let $f(p,\underline{x})$ be the square of the reciprocal of the network function magnitude. The basic problem is to find some vector $\underline{x}$ such that the value of $f(p,\underline{x})$ is arbitrarily small, i.e., the magnitude of the network function is very large. In Fig. 3, the basic complex optimization algorithm for accomplishing this is presented. As shown in this figure, first a starting vector $\underline{x}_0$ is chosen. Next, an optimization strategy is employed to modify $\underline{x}_0$ in such a way as to minimize the value of $f(p,\underline{x})$. When the value of $f(p,\underline{x}_0)$ is less than some specified small magnitude, the resulting $\underline{x}_0$ is the desired solution vector $\underline{x}$.

There are several major advantages that result from the use of complex optimization rather than the more usual optimization methods. First of all, the number of evaluations that need be made for each cycle of the optimization strategy is considerably reduced. For example, if only a single pole is to be synthesized, then only a single (complex) evaluation is required for each cycle, an obvious computational advantage over the 5-10 evaluations at different sinusoidal frequencies that might be required in conventional optimization. Second, it is possible to provide control over the phase characteristics as well as the magnitude characteristics of the network, since the synthesis is in terms of a set of pole and zero locations rather than a magnitude curve defined over some

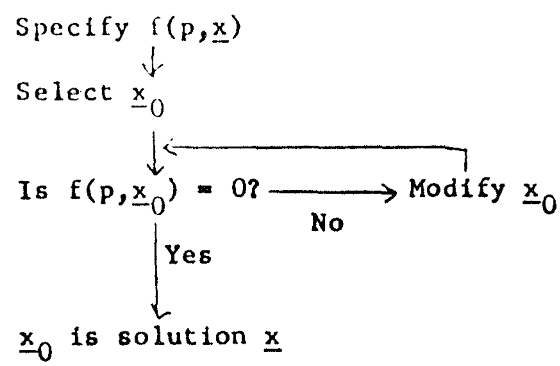
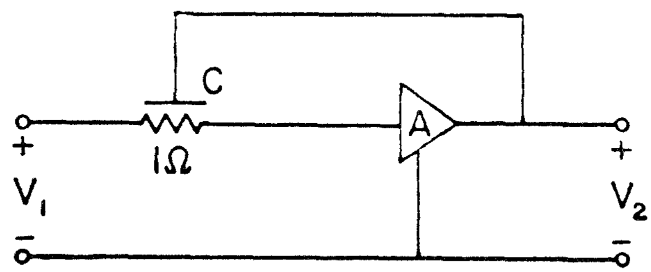


Fig. 3 Basic Complex Optimization Procedure



	Pole Location	R(ohms)	C(farads)
Starting Point	0 + j.1	0.056	11.19
Problem	-1 + j0.1	?	?

Fig. 4 Complex Optimization Example Problem

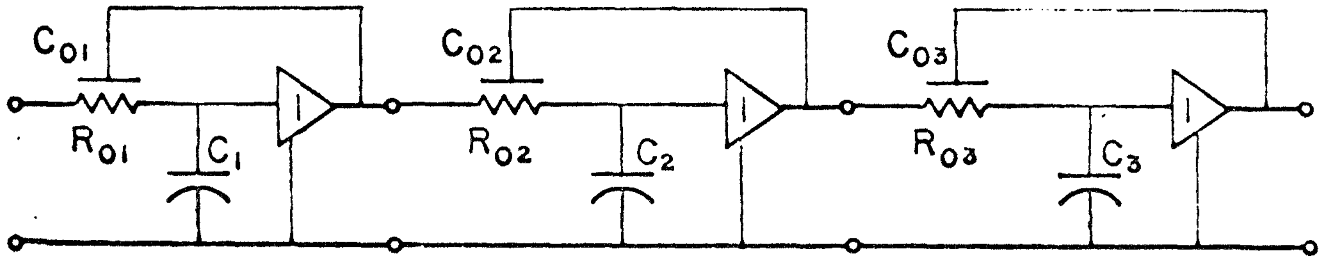


Fig. 5 Complex Optimization Example Problem

limited frequency range. Finally, all the well known techniques of approximation theory may be used to determine the desired pole and/or zero locations for a desired network trnasfer characteristic. Such approximation is considerably more difficult to apply in terms of a sinusoidal magnitude characteristic.

III. Examples of Complex Optimization

As an example of the application of the technique described above, a solution for a synthesis problem using the network shown in Fig. 4 was found. The vector $\underline{x}$ was chosen as $(C, A)^t$, where C is the value of the the total capacitance of the distributed network, and A is the value of the gain of the VCVS represented by the triangle in the figure. The problem was the realization of the low-pass network function

$$V_2(p)/V_1(p) = N(p) = .9048/(p^2 + .4p + 1.04) \qquad (1)$$

thus, the reciprocal of the magnitude of the voltage transfer function at $p_0 = -.2 + j1$ was to be minimized. The resulting values of the parameters found by using the complex optimization algorithm with a Fletcher-Powell optimization strategy are C = 1[illegible].48 F and A = 0.87. The error, defined as $|1/N(p_0)|^2$ is 5.237×10^{-8}. A corresponding synthesis using a design chart developed by Kerwin[1] which matches the network magnitude characteristic at various sinusoidal frequencies gave C = 16 F, A = .87, and an error of 9.413×10^{-3}. A comparison of $N(j\omega)$, the actual values of the second degree function, with $N_1(j\omega)$, the optimized results, and $N_2(j\omega)$, the results from Kerwin, at 50 logarithmically spaced values of sinusoidal frequency ω_k from 0.1 to 10.0 rad/sec using an error criteria defined as

$$E_i = \sum_{k=1}^{50} \left| \frac{|N(j\omega_k)| - |N_i(j\omega_k)|}{|N(j\omega_k)|} \right|^2 \qquad (2)$$

gave $E_1 = 23.74$ and $E_2 = 2.263$, indicating the improvement obtained from the complex optimization approach to synthesis. A small improvement was also noted in a similarly defined phase error criteria.

As a second example of the power and utility of the complex optimization approach to network synthesis the circuit shown in Fig. 5 consisting of three cascaded DLA networks, each capable of realizing a dominant complex-conjugate pole pair, was used to approximate a 6th order maximally-flat-magnitude Butterworth low-pass function. A second purpose of this example was to investigate

Parameter	Values found by Bunker	Values found by Complex Optimization
C_1	1.06422	1.07095
P_1	2.04376	2.05419
C_2	0.75723	0.76383
P_2	2.27479	2.27714
C_3	0.34973	0.35186
P_3	2.84680	2.84341

Fig. 6 Parameter Values for the Network shown in Fig. 5

$$-.2588 \pm j.9659$$
$$-.7071 \pm j.7071$$
$$-.9659 \pm j.2588$$

Fig. 7 Pole Locations for 6th-order Maximally-Flat-Magnitude Butterworth Low-Pass Filter

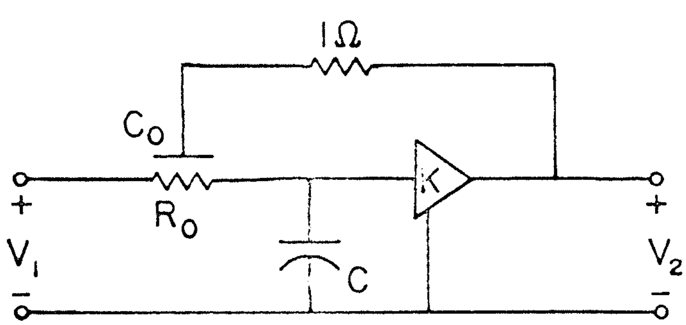


Fig. 8 Complex Optimization Example Problem for Simultaneous Realization of a Pole and a Zero

the effect of inaccuracies which may result when DLA network configurations are used to realize high-order network functions. Regarding this latter, Bunker has claimed that the exact realization of such a maximally-flat-magnitude characteristic requires the specification of dominant poles which are displaced from the traditionally accepted pole locations for a 6th order Butterworth low-pass filter.[2] The values of a set of parameters for the network shown in Fig. 5 which provide a realization based on the displaced Butterworth poles are given in Fig. 6. These values were used as a starting point for a complex optimization procedure designed to minimize the reciprocal magnitude of the voltage transfer function for the entire network at the pole positions normally used for a 6th order maximally-flat-magnitude Butterworth low-pass filter. These complex locations are tabulated in Fig. 7. The reciprocal voltage transfer function for the ith stage of the circuit shown in Fig. 5 is

$$\frac{V_1(p)}{V_2(p)} = 1 + C_i \sqrt{p} \sinh P_i \sqrt{p} \qquad (3)$$

where $P_i = R_{oi}C_{oi}$, and R_{oi} and C_{oi} are the total resistance and capacitance of the distributed RC uniform line used for the ith stage. The error criterion E used for this problem is

$$E = \sum_{i=1}^{3} G_i^2 \qquad (4)$$

where

$$G_i = \frac{V_1}{V_2}(p)\Big|_{p = p_i} \qquad (5)$$

and the p_i are the upper-half-plane Butterworth pole locations. Using Bunker's values from Fig. 6 for the network parameters gave an error $E = 7.487 \times 10^{-5}$ for the network. Starting from these values, and using complex optimization, after eight iterations of a Fletcher-Powell optimization strategy this error was reduced to 4.830×10^{-12}. The corresponding optimized parameter values are given in Fig. 6. It is readily observed that these values differ only slightly from the ones originally determined by Bunker. To compare the performance of the network shown in Fig. 5 using the two different sets of parameter values, namely, Bunker's original values and ones found by complex optimization, an evaluation was made of the magnitude of the actual network function, the network with Bunker's parameter values and the network with the values determined by complex optimization for 40 sinusoidal frequencies logarithmically spaced from 0.1 to 10 rad/sec. An error criterion defined as

$$E_i = \sum_{k=1}^{40} \left| \frac{|B(j\omega_k)| - |N_i(j\omega_i)|}{|B(j\omega_i)|} \right|^2 \tag{6}$$

where the ω_k are the sinusoidal frequencies, $|B(j\omega_i)|$ is the actual magnitude of the Butterworth function evaluated at ω_k, $|N_1(j\omega_k)|$ is the Bunker network transfer function magnitude evaluated at the frequency ω_k, and $|N_2(j\omega_k)|$ is the corresponding magnitude of the network produced by complex optimization. The respective errors are $E_1 = 1289$ (for the Bunker network parameter values) and $E_2 = 1365$ (for the complex optimization network parameter values). The difference between the two errors is obviously relatively small. It is of even less significance when one examines the data and notes that the major contribution to the difference occurs in the high frequency region where attenuations of over 100 db occur.

As a third example of the use of complex optimization, and one which requires the generation of both poles and zero, consider the network shown in Fig. 8. Let us assume that it is desired to use this network configuration to produce a complex-conjugate pole-pair at $p_1 = 0.1 \pm j0.6$ and a complex-conjugate zero-pair on the $j\omega$ axis at $z_1 = \pm j1$. Choosing the value of the lumped resistor as unity, four parameters of the network remain to be found. In Fig. 9 the values of the arbitrarily chosen starting point for the vector $\underline{x}$ representing these parameters are tabulated. The error E corresponding with this choice of parameter values was 1.249, where

$$E = 2 \left[\left| \frac{V_1}{V_2}(p) \right|_{p = p_1} + \left| \frac{V_2}{V_1}(p) \right|_{p = z_1} \right] \tag{7}$$

After 14 iterations of a Fletcher-Powell optimization algorithm the error was reduced to 2.766×10^{-7}. The resulting values of the parameters for this error are also tabulated in Fig. 9. To verify the validity of these results a comparable set of values for the network parameters were directly determined from a

Items	Starting Value for Complex Optimization	Final Value after Complex Optimization	Values obtained by Kerwin
R_o	15.0	18.6443	17.78
C_o	0.5	0.61288	0.629
C	0.1	0.05401	0.05
K	1.0	1.05601	1.05
Error	1.249	2.766×10^{-7}	0.0258

Fig. 9 Complex Optimization at Both a Pole and a Zero

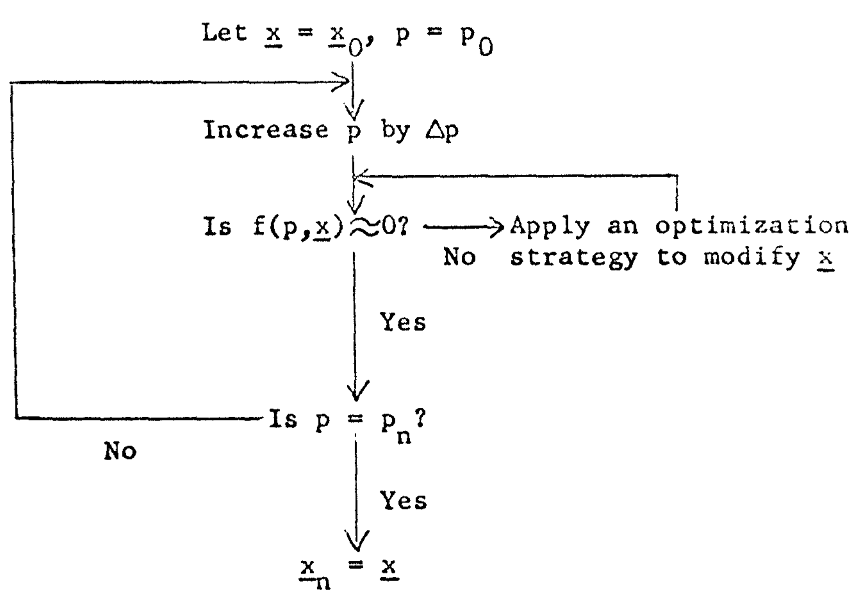


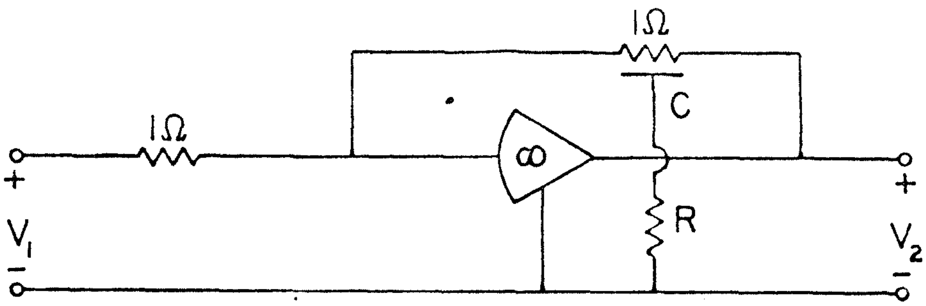
Fig. 10 Basic Constrained Complex Optimization Procedure

design chart for this network presented by Kerwin[1]. The network parameter values are shown in the right column of the table in Fig. 9. Comparing these values with those obtained by complex optimization it is obvious that good agreement is attained. Actually, the error associated with the network parameter values found by Kerwin (which were derived by matching the magnitude characteristic along the $j\omega$ axis) is 0.0258. This is considerably higher than the error obtained by the use of complex optimization.

IV. Constrained Complex Optimization

One problem that arrises in all applications of optimization methods, including complex optimizations, is the tendency of optimization strategies to converge toward a local rather than a global optimum. In addition, such a local optimum may represent physically unrealizable sets of network parameters, for example, negative-valued ones. In this section a technique is presented for imposing constraints on the general complex optimization procedure such that only global optimal solutions with physically realizable values for the network parameters are obtained. The method is called constrained complex optimization. It is covered in more detail in the following paragraphs.

The method of constrained complex optimization may be explained as follows: Let $f(p,\underline{x})$ be a function of some complex scalar variable p and some vector set of variables $\underline{x}$ which gives the value of the reciprocal of the magnitude squared of a network function. The basic complex optimization procedure may be stated as requiring that the function equal zero when evaluated at some value p_n of the functional variable p. Thus we wish to find a specific vector $\underline{x}_n$ which satisfies the relation $0 = f(p,\underline{x}_n)$. The usual optimization approach for doing this is to start with some vector $\underline{x}_0$ for which $\underline{f}(p_n,\underline{x}_0) = E \neq 0$ where E is an error criteria. A conventional optimization strategy is then applied in an effort to minimize E by varying $\underline{x}$. Such a procedure faces several possible difficulties, for example, non-convergence, convergence to a local rather than a global optimum, the possibility of a non-physically realizable solution vector $\underline{x}$, etc. To avoid these difficulties we may apply a constraint procedure. We begin by choosing a physically realizable vector set of variables $\underline{x}_0$ which is known to produce a pole at p_0. Thus we define $0 = f(p_0,\underline{x}_0)$. We next define a set of intermediate quantities $p_i = p_0 + i\,\Delta p$, where $\Delta p = (p_n - p_0)/n$. Using $\underline{x}_0$ as a starting point, we then apply complex optimization to find a vector $\underline{x}_1$



	Pole Location	R(ohms)	C(farads)
Starting point	0 + j.1	0.056	11.19
Problem	-0 + j0.1	?	?

Fig. 11 Complex Optimization
Example Problem

Iteration	R(ohms)	C(farads)	Error
0	0.056	11.19	25.380
1	18.58	11.33	0.125
2	18.60	9.61	0.0290
⋮	⋮	⋮	⋮
40	18.58	9.63	0.0289

$$\text{Error} = \left| \frac{V_1}{V_2}(p) \right|^2 \quad p = -1 + j0.1$$

Fig. 12 Results of Using Non-Constrained
Optimization on Complex Optimization
Example Problem

which satisfies $f(p_1, \underline{x}_1) = 0$. The vector $\underline{x}_1$ is then used as a starting point for the determination of $\underline{x}_2$, the solution of $f(p_2, \underline{x}_2) = 0$. The process is continued until $\underline{x}_n$ is found. By choosing a sufficiently large value of n, the value p_1 can be positioned so closely to p_0 (for which a global optimum solution $\underline{x}_0$ is known), that $\underline{x}_1$ is a global optimum solution with physically realizable parameters, similarly the other solution vectors $\underline{x}_i$ satisfying the relations $0 = f(p_i, \underline{x}_i)$ will also be global optimal solutions. Thus we substitute n small optimization problems for the large one originally posed. The basic constrained complex optimization strategy for the realization of a single dominant pole at p_n is shown in Fig. 10.

As an example of the application of constrained complex optimization to a typical DLA network synthesis problem consider the network shown in Fig. 11. The circuit consists of two lumped resistors, a distributed RC network and an operational (infinite gain) amplifier. The properties of this circuit will be discussed in more detail in a future report. Here the circuit will be used as a media for illustrating constrained complex optimization. The parameter vector for this circuit may be defined as $\underline{x} = (R,C)^t$. The portion of the circuit on the right in Fig. 11 consisting of the distributed RC network and the resistor R is well-known as a null network. Specifically, for the values R = 0.056 ohm and C = 11.19 F, there is no transmission through the distributed RC network at a frequency of 1 rad/sec. Thus, at this frequency there is no negative feedback around the operational amplifier and the gain of the network is directly equal to that of the amplifier. In effect this closely approximates the condition where the network has a pole at the corresponding complex frequency, namely, $0 \pm j1$. Thus, we have defined an initial vector $\underline{x}_0$ which has the value $\underline{x}_0 = (0.056, 11.19)^t$ which is known to produce such a pair of poles. Suppose that we now desire to modify the parameter vector $\underline{x}_0$ so as to produce poles at $-1 \pm j0.1$. First let us attempt to do this by using conventional (non-constrained) complex optimization. The results of such an attempt using a Fletcher-Powell strategy are shown in Fig. 12. From the data it is clear that the optimization strategy converges rapidly (on the second iteration) to a non-global optimum, and, even after many more iterations, it has been unable to reduce the error to anything approaching the value of 10^{-8} which was specified as the criterion for the convergence of this problem. Now let us see how the constrained complex optimization technique described above operates on this problem. Using the same Fletcher-Powell optimization strategy, and ten intermediate steps, the results obtained are shown in Fig. 13. For each intermediate change for the required pole position the optimization strategy rapidly converges to an error less than the specified minimum value of 10^{-8}. In addition, a final solution vector $\underline{x}_{10} = (0.5454, 4.11)^t$ is found satisfying the requirements of the problem. A comparison of the results obtained by applying non-constrained and constrained complex optimization is given in Fig. 14.

V. Automated Preparation of DLA Network Design Charts

In generating meaningful results describing dominant pole positions for DLA networks frequent recourse is made to two-dimensional charts which give the location of one of the dominant complex conjugate poles as a function of two of the network parameters. The concept of constrained complex optimization described above is readily adapted to the development of such design charts. In such an application, having found a set of variables $\underline{x}_0$ such that $0 = f(p_0, \underline{x}_0)$, we proceed to reapply our optimization strategy to find a vector $\underline{x}_1$ which satisfies $0 = f(p_1, \underline{x}_1)$, where p_1 is some value of the complex frequency variable in the vicinity of p_0. Repeating this process for a series of values p_i, such that a grid of points covering some desired area in the complex frequency plane is treated, we may readily interpolate between the values of the elements of the vectors $\underline{x}_i$ so as to construct equal-valued contour lines for the network variables. Such a procedure is readily implemented on the digital computer, and it provides an effective approach to the automated preparation of design charts for DLA networks. As an example of the application of such an automated design procedure let us consider the network shown in Fig. 11. The general problem is illustrated schematically in Fig. 15. Using complex optimization a grid on the complex frequency plane was defined by using 11 values of Im p from 1 to 0 and 11 values of le p from -1 to 0. Complex optimization was applied at each of these points on the complex plane, thus, 121 separate optimization problems were specified. The starting point p_0 was chosen as 0 + j1 using the known solution vector $\underline{x}_0 = (0.056, 11.19)$ defined in Sec. IV. A summary of the resulting data obtained is shown in Fig. 16. Using this data to locate equal-valued contours for the network parameters R and C leads to the design chart shown in Fig. 16. Thus, the network shown in Fig. 11 may be used to produce a dominant pair of complex conjugate poles anywhere in the left-half of the complex frequency plane by choosing the values of the lumped resistor R and the distributed capacitance

Step	P	Iterations	R(ohms)	C(farads)	Error
0	0+j1	0	0.0561	11.19	9.69×10^{-10}
1	-0.1+j0.91	5	0.0805	10.57	3.77×10^{-11}
2	-012+j0.82	5	0.1167	9.82	1.99×10^{-13}
⋮	⋮	⋮	⋮	⋮	⋮
9	-0.9+j0.19	6	0.5316	4.54	2.22×10^{-11}
10	-1+jo.1	7	0.5454	4.11	1.16×10^{-11}

$$\text{Error} = \frac{V_1}{V_2}(p)^{2}$$

Fig. 13 Results of Using Constrained Optimization on Complex Optimization Example Problem using Ten Intermediate Steps

Optimization Algorithm	R(ohms)	C(farads)	Error
Non-Constrained	18.55	9.63	2.89×10^{-2}
Constrained	0.545	4.11	1.16×10^{-11}

Fig. 14 Comparison of Results from Constrained and Non-Constrained Optimization on the Complex Optimization Example Problem

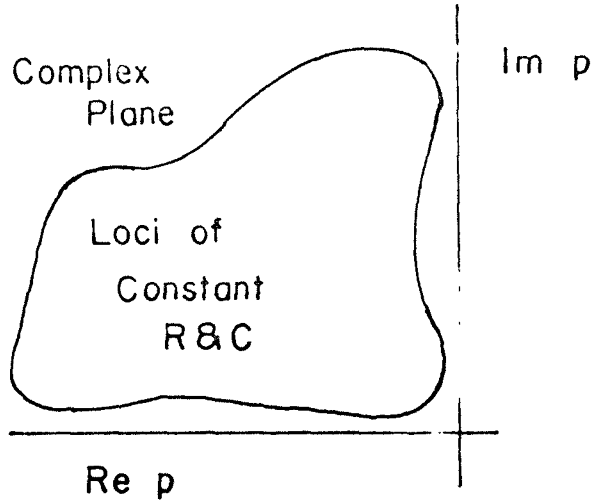


Fig. 15 The Design Chart Problem.

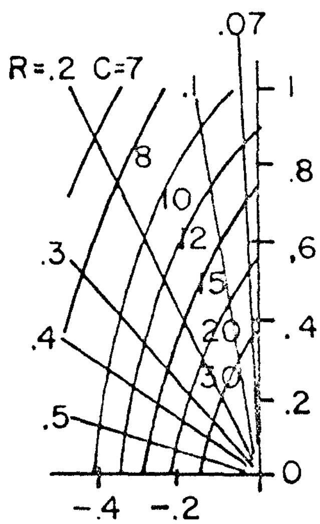


Fig. 16 Design Chart for the Network shown in Fig. 11.

C shown in the chart in Fig. 16. The method is readily applied to a wide range of other DLA synthesis problems.

VI. Conclusion

This report has described two new results which have been found to be of significant importance in the synthesis of DLA networks. The first of these, namely complex optimization, is a simple method for achieving an approximation to some desired network performance using a DLA network. The performance specifications are given in terms of the pole and zero configuration for the desired characteristics. The computational effort required at each application of the optimization strategy is minimized over that required by other methods. The second result, namely constrained complex optimization, is a general technique which may be applied to a wide varity of optimization problems. It provides a means of preventing convergence to a local minimum and helps to constrain the values of the network variables in such a way that the possibility of a solution requiring unrealizable parameter values is minimized.

Acknowledgment

The author wishes to acknowledge the support given to this research by the Instrumentation Division, Ames Research Center, National Aeronautics and Space Administration, Moffett Field, California, under Grant NGL-03-002-136.

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