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FORTRAN PROGRAMS FOR CALCULATING LOWER IONOSPHERE ELECTRON DENSITIES AND COLLISION FREQUENCIES FROM ROCKET DATA

by

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E L E C T R O N D E N S I T I E S A N D C O L L I S I O N F R E Q U E N C I E S
F R O M R O C K E T D A T A

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ABSTRACT

This report is a record and discussion of computer programs used by personnel of the Aeronomy Laboratory for calculating D- and E-region electron densities and collision frequencies from Faraday rotation, differential absorption, and standing-wave data obtained from Nike Apache rocket-radio propagation experiments. All the programs are written in FORTRAN IV and are compatible with the IBM 360 system operated by the Department of Computer Science at the University of Illinois in Urbana.

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1. INTRODUCTION

The rocket-radio propagation experiment of the University of Illinois was first described by Bowhill (1965). Instrumentation of the experiment is discussed by Knoebel and Skaperdas (1966). Fundamentals of the method of analysis of Faraday rotation and differential absorption data are discussed by Mechtly et al. (1967), and of standing waves, by Monro et al. (1968).

The present report records and describes computer programs which execute the detailed numerical analysis of Faraday rotation, differential absorption, and standing-wave data from the rocket experiments. All of these programs are expressed in the language of FORTRAN IV.

2. SIMULTANEOUS ANALYSIS OF FARADAY AND ABSORPTION DATA

Ideally, both Faraday rotation (FR) and differential absorption (DA) data are measured at all altitudes of interest in the lower ionosphere. Unfortunately, in the region between about 55 and 85 km, only absorption data are generally measurable. Here, the electron concentrations are too small to cause a measurable Faraday rotation. However, both types of data are usually obtained between about 85 and 100 km.

We first consider, in this section, a computer program to analyze both types of data simultaneously. In Section 3, we consider the analysis of absorption data in the absence of Faraday data.

A program for the simultaneous analysis of FR and DA data is listed in Tables 1, 2, and 3. This program has no branches except to subroutines. Therefore, the program will be described sequentially without reference to a flow chart.

Table 1 lists alphabetically the names, definitions and units of the principal variables used in the program. Tables 2 and 3 are the FORTRAN IV statements of the main part of the program. The subroutines are discussed in Sections 4 and 5.

The values of physical constants are those listed by Taylor et al. (1969), and Mechtly (1969).

References to table numbers and statement numbers are made by enclosing the table number and the statement number in parentheses; the two numbers separated by a dash. For example, the notation (2-2) is a reference to Statement No. 2 of Table No. 2.

Statements (2-2) to (2-9) enter the equatorial radius and flattening factor of an oblate earth and generate constants from them for later use.

TABLE 1

Names of Variables

A = equatorial radius of the earth (meters)
 AIO = absorption index, ordinary mode
 AIX = absorption index, extraordinary mode
 AZD = azimuth angle of the rocket from the transmitter (deg)
 B = the total flux density of the geomagnetic field (tesla)
 BD = the downward component of the geomagnetic flux density (tesla)
 BN = the north component of the geomagnetic flux density (tesla)
 BP = the phi component of the geomagnetic flux density (tesla)
 BR = the radial component of geomagnetic flux density (tesla)
 BT = the theta component of the geomagnetic flux density (tesla)
 BW = the westward component of the geomagnetic flux density (tesla)
 CF = electron collision frequency, most probable (s^{-1})
 CPH = cosine of the geocentric longitude angle of the rocket
 CT = cosine of the geocentric polar angle of the rocket
 CTH = cosine of the angle between the wave normal and the geomagnetic field
 DA = calculated differential absorption rate (dB/s)
 DAE = an observed value of differential absorption rate (dB/s)
 DTR = the number of radians per angular degree
 ED = electron concentration (m^{-3})
 ELD = elevation angle of the rocket from the transmitter (deg)
 F = radio wave frequency of the propagation experiment (Hz)
 FLAT = flattening factor of an oblate earth
 FR = calculated Faraday rotation rate (deg/s)
 FRE = an observed value of Faraday rotation rate (deg/s)
 HL = altitude of the rocket above the earth (meters)
 R = the distance from the center of the earth to the rocket (meters)
 RIO = refractive index, ordinary mode
 RIX = refractive index, extraordinary mode
 RKTNO = rocket number
 RLATD = geodetic latitude of the rocket (deg, north (pos.), south (neg.))
 RLNGD = longitude of the rocket (deg, east (pos.), west (neg.))
 ROI = wave polarization, ordinary mode, imaginary part
 ROR = wave polarization, ordinary mode, real part
 RXI = wave polarization, extraordinary mode, imaginary part
 RXR = wave polarization, extraordinary mode, real part
 S = the angular cyclotron (gyro) frequency (rad/s)
 SPH = sine of the geocentric longitude angle of the rocket
 ST = sine of the geocentric polar angle of the rocket
 TM = time of the rocket shot (year and decimal part of a year)
 V = total velocity of the rocket (m/s)
 W = angular wave frequency of the propagation experiment (rad/s)
 X = the ratio of plasma frequency to wave frequency
 Y = ratio of gyro and wave frequencies, a negative number for the electron
 Z = the ratio of collision frequency to angular wave frequency

TABLE 2

```

1 C ANALYSIS OF ROCKET FR AND DA DATA
2 C ENTER PARAMETERS OF OBLATE EARTH
3   A=6.378165E6
4   A2 =A*A
5   A4 = A2*A2
6   FLAT=1.-1./298.3
7   B2=(A*FLAT)**2
8   A2B2=A2*(1.-FLAT**2)
9   A4B4=A4*(1.-FLAT**4)
10 C ENTER PARAMETERS
11   READ (5,50) RKTNO, TM, F
12 50  FORMAT (F10.3, F10.1, F10.0)
13   CALL COEFF(TM)
14   W = 6.2831853*F
15   DTR = 1.745329E-2
16 C INITIALIZE ELECTRON DENSITY AND COLLISION FREQUENCY
17   ED=1.E8
18   CF=1.E5
19 C ENTER VARIABLES
20 1   READ (5,51) AZD, ELD, HT, V, RLATD, RLNGD, FRE, DAE
21 51  FORMAT (8F10.4)
22 C CONVERT DEGREES TO RADIANS
23   AZ = AZD*DTR
24   EL = ELD*DTR
25   RLNGR = RLNGD*DTR
26   RLATR = RLATD*DTR
27   SINLA = SIN(RLATR)
28 20  SINLA2=SINLA*SINLA
29   COSLA2=1.-SINLA2
30   SPH = SIN(RLNGR)
31   CPH = COS(RLNGR)
32 C FIND GEOCENTRIC COORDINATES OF ROCKET
33   DEN2=A2-A2B2*SINLA2
34   DEN=SQRT(DEN2)
35   FAC=((HT*DEN)+A2)/((HT*DEN)+B2)**2
36   CT=SINLA/SQRT(FAC*COSLA2+SINLA2)
37   R=SQRT(HT*(HT+2.*DEN)+(A4-A4B4*SINLA2)/DEN2)
38 21  ST=SQRT(1.-CT**2)
39 C CALCULATE GEOMAGNETIC FIELD AT ROCKET
40   CALL FIELD (R, ST, CT, SPH, CPH, BR, BT, BP, B)
41 C TRANSFORM FIELD COMPONENTS, GEOCENTRIC TO GEODETIC.
42   SIND=SINLA*ST-SQRT(COSLA2)*CT
43   COSD=SQRT(1.-SIND**2)
44   BN=-BT*COSD-BR*SIND
45   BD=BT*SIND-BR*COSD
46   BW=-BP
47   S = -1.758803E11*B
48   Y=S/W
49 C CALCULATE COSINE OF PROPAGATION ANGLE
50   CEL = COS(EL)
51   CTH = (CEL*COS(AZ)*BN-SIN(EL)*BD-CEL*SIN(AZ)*BW)/B
52   TH=ARCOS(CTH)/DTR
53 C CALCULATE FR AND DA COEFFICIENTS
54   FV = F*V
55   FC = 6.004153E-7*FV
56   AC = 1.820428E-7*FV
57 C ITERATE TO MATCH FR AND DA
58   WRITE (6,67)
59 67  FORMAT (1H1)

```

TABLE 3

```

1      WRITE (6,68) RKTNO,TM
2 68   FORMAT (3X,F10.3,F10.1/)
3      I = 6
4      DO 2 N=1,I
5 C    CALCULATE REFRACTION AND ABSORPTION INDICES
6      CALL SENWYL (ED,CF,S,CTH,W,RIO,RIX,AIO,AIX,ROR,RXR,ROI,RXI)
7      FR = FC*(RIO-RIX)
8      DA=AC*ABS(AIX-AIO)
9      X = 3182.6018*ED/W/W
10     Z=CF/W
11     WRITE(6,61)AZD,BN,ROR,RIO,F,HT
12     WRITE(6,62)ELD,BD,RXR,RIX,FRE,ED
13     WRITE(6,63)RLATD,BW,ROI,AIO,FR,X
14     WRITE(6,64)RLNGD,B,RXI,AIX,DAE,CF
15     WRITE(6,65)V,Y,CTH,TH,DA,Z
16 61   FORMAT('  AZ ',1PE13.6,'  BN ',1PE13.6,'  ROR ',1PE13.6,
17     &      '  RIO ',1PE13.6,'  F ',1PE13.6,'  HT ',1PE13.6)
18 62   FORMAT('  EL ',1PE13.6,'  BD ',1PE13.6,'  RXR ',1PE13.6,
19     &      '  RIX ',1PE13.6,'  FRE ',1PE13.6,'  ED ',1PE13.6)
20 63   FORMAT('  LAT ',1PE13.6,'  BW ',1PE13.6,'  ROI ',1PE13.6,
21     &      '  AIO ',1PE13.6,'  FR ',1PE13.6,'  X ',1PE13.6)
22 64   FORMAT('  LNG ',1PE13.6,'  B ',1PE13.6,'  RXI ',1PE13.6,
23     &      '  AIX ',1PE13.6,'  DAE ',1PE13.6,'  CF ',1PE13.6)
24 65   FORMAT('  V ',1PE13.6,'  Y ',1PE13.6,'  COS ',1PE13.6,
25     &      '  TH ',1PE13.6,'  DA ',1PE13.6,'  Z ',1PE13.6)
26     WRITE (6,60)
27 60   FORMAT (1H )
28     ED=FRE/FR*ED
29     CF=FR/FRE*DAE/DA*CF
30 2    CONTINUE
31     GO TO 1
32     END

```

Statement (2-11) enters the rocket number, the time of the shot, and the frequency of the radio experiment from a data card.

The subroutine COEFF is called by statement (2-13) to obtain harmonic coefficients representing the geomagnetic field at the time TM. The geomagnetic field is represented by the numerical model of Cain et al. (1967), or Cain (1968). The harmonic coefficients of this model are for epoch 1960.0 and are designated by Cain as "GSFC (12/66)". The coefficients include time derivatives from which coefficients for some other time, TM, are generated by the subroutine COEFF. Subroutine COEFF, as listed in Table 6, is a modified form of a program originally provided by J. C. Cain (Cain, 1968).

Statements (2-17) and (2-18) specify initial values of electron density and collision frequency to begin the iterative process of calculating values of FR and DA which will match the experimentally measured values FRE and DAE, respectively.

Statement (2-20) enters coordinates of the rocket and experimentally measured values of Faraday rate FRE, and differential absorption rate DAE.

Statements (2-32) through (2-38) convert the geodetic coordinates of the rocket to geocentric coordinates for use by the subroutine FIELD which calculates the geomagnetic field components. Subroutine FIELD, as listed in Table 8, is also a modified form of a program originally provided by J. C. Cain (Cain, 1968). The field components are expressed in terms of north, west, and down components; and functions of the total field are computed by statements (2-41) to (2-48).

The numerical coefficient of (1-47) is the charge to mass ratio of the electron (e/m).

The angle between the wave normal and \vec{B} of the earth, required by the Sen-Wyller equations is found by statements (2-49) to (2-52).

To avoid repetitious evaluation, during the iterative process, of the coefficient of the FR and DA equations (Mechtly et al., 1967), the coefficient are evaluated, (2-53) to (2-56), before the D0 loop which performs the iterations. The numerical coefficient of (2-55) is $180/c$. The numerical coefficient of (2-56) is $2 \times (4.3429 \dots) \times 2\pi/c$.

The D0 loop is set for six iterations by (3-3).

Refraction and absorption indices are calculated, (3-6), by the subroutine SENWYL, and finally FR, and DA; and the conventional magnetic variables X and Z are obtained by statements (3-7) to (3-10). The numerical coefficient of (3-9) is $e^2/\epsilon_0 m$.

Statements (3-11) to (3-27) print all the parameters and variables of interest for a given iteration.

Statements (3-28) and (3-29) provide more correct values of electron density and collision frequency for the next iteration. These equations follow from first order approximations (Mechtly et al., 1967).

After the specified number of iterations, the program returns by (3-31) to the read statement (2-20) for the next set of data, which usually corresponds to one second later in flight time or about 1.5 km higher in altitude.

3. ANALYSIS OF DIFFERENTIAL-ABSORPTION DATA IN THE ABSENCE OF FARADAY DATA

The program for the analysis of DA data in the absence of FR data is listed in Tables 4 and 5. This program is, for the most part, identical to the program for the simultaneous analysis of FR and DA data. The two programs differ in the following particulars.

Statement (4-17) initializes only ED, and not CF. We assume that the collision frequency model of statement (4-22), $CF = CFM * P$, is valid at altitudes (below about 85 km) where only DA data are available. The proportionality constant CFM is the value of the ratio CF/P which is judged to best represent the values of CF from the FR and DA program (Tables 1, 2, and 3) and the values of atmospheric pressure P (N/m^2) from a selected CIRA or U.S. Standard Atmosphere model.

Values of pressure are entered from data cards, statement (4-20).

Iteration is done only on the variable DA, (5-26), and not on the variable CF as before, since CF is assumed to be known from the model (4-22).

TABLE 4

```

1 C ANALYSIS OF DA WITHOUT FR, GIVEN CF MODEL
2 C ENTER PARAMETERS OF OBLATE EARTH
3   A=6.378165E6
4   A2 =A*A
5   A4 = A2*A2
6   FLAT=1.-1./298.3
7   B2=(A*FLAT)**2
8   A2B2=A2*(1.-FLAT**2)
9   A4B4=A4*(1.-FLAT**4)
10 C ENTER PARAMETERS
11   READ (5,50) RKTNO, TM, F, CFM
12 50   FORMAT (F10.3, F10.1, F10.0, F10.0)
13   CALL COEFF (TM)
14   W = 6.2831853*F
15   DTR = 1.745329E-2
16 C INITIALIZE ELECTRON DENSITY
17   ED=1.E7
18 C ENTER VARIABLES
19 1   READ (5,51) AZD, ELD, HT, V, RLATD, RLNGD, FRE, DAE
20   READ (5,51) P
21 51   FORMAT (8F10.4)
22   CF = CFM*P
23 C CONVERT DEGREES TO RADIANS
24   AZ = AZD*DTR
25   EL = ELD*DTR
26   RLATR = RLATD*DTR
27   RLNGR = RLNGD*DTR
28   SINLA = SIN(RLATR)
29 20   SINLA2=SINLA*SINLA
30   COSLA2=1.-SINLA2
31   SPH = SIN(RLNGR)
32   CPH = COS(RLNGR)
33 C FIND GEOCENTRIC COORDINATES OF ROCKET
34   DEN2=A2-A2B2*SINLA2
35   DEN=SQRT(DEN2)
36   FAC=((HT*DEN)+A2)/((HT*DEN)+B2)**2
37   CT=SINLA/SQRT(FAC*COSLA2+SINLA2)
38   R=SQRT(HT*(HT+2.*DEN)+(A4-A4B4*SINLA2)/DEN2)
39 21   ST=SQRT(1.-CT**2)
40 C CALCULATE GEOMAGNETIC FIELD AT ROCKET
41   CALL FIELD (R, ST, CT, SPH, CPH, BR, BT, BP, B)
42 C TRANSFORM FIELD COMPONENTS, GEOCENTRIC TO GEODETIC.
43   SIND=SINLA*ST-SQRT(COSLA2)*CT
44   COSD=SQRT(1.-SIND**2)
45   BN=-BT*COSD-BR*SIND
46   BD=BT*SIND-BR*COSD
47   BW=-BP
48   S = -1.758803E11*B
49   Y=S/W
50 C CALCULATE COSINE OF PROPAGATION ANGLE
51   CEL = COS(EL)
52   CTH = (CEL*COS(AZ)*BN-SIN(EL)*BD-CEL*SIN(AZ)*BW)/B
53   TH = ARCCOS(CTH)/DTR
54 C CALCULATE FR AND DA COEFFICIENTS
55   FV = F*V
56   FC = 6.004153E-7*FV
57   AC = 1.820428E-7*FV
58 C ITERATE TO MATCH DA
59   WRITE (6,67)
60 67   FORMAT (1H1)

```

TABLE 5

```

1      WRITE (6,68) RKTNO, TM, CFM
2 68   FORMAT (3X, F10.3, F10.1, 1PE10.2/)
3      I = 6
4      DO 2 N=1, I
5      CALL SENWYL (ED, CF, S, CTH, W, RIO, RIX, AIO, AIX, ROR, RXR, ROI, RXI)
6      FR = FC*(RIO-RIX)
7      DA=AC*ABS(AIX-AIO)
8      X = 3182.6018*ED/W/W
9      WRITE(6,61) AZD, BN, ROR, RIO, F, HT
10     WRITE(6,62) ELD, BD, RXR, RIX, P, ED
11     WRITE(6,63) RLATD, BW, ROI, AIO, FR, X
12     WRITE(6,64) RLNGD, B, RXI, AIX, DAE, CF
13     WRITE(6,65) V, Y, CTH, TH, DA, Z
14 61   FORMAT('      AZ ', 1PE13.6, '      BN ', 1PE13.6, '      ROR ', 1PE13.6,
15     &      '      RIO ', 1PE13.6, '      F ', 1PE13.6, '      HT ', 1PE13.6)
16 62   FORMAT('      EL ', 1PE13.6, '      BD ', 1PE13.6, '      RXR ', 1PE13.6,
17     &      '      RIX ', 1PE13.6, '      P ', 1PE13.6, '      ED ', 1PE13.6)
18 63   FORMAT('      LAT ', 1PE13.6, '      BW ', 1PE13.6, '      ROI ', 1PE13.6,
19     &      '      AIO ', 1PE13.6, '      FR ', 1PE13.6, '      X ', 1PE13.6)
20 64   FORMAT('      LNG ', 1PE13.6, '      B ', 1PE13.6, '      RXI ', 1PE13.6,
21     &      '      AIX ', 1PE13.6, '      DAE ', 1PE13.6, '      CF ', 1PE13.6)
22 65   FORMAT('      V ', 1PE13.6, '      Y ', 1PE13.6, '      COS ', 1PE13.6,
23     &      '      TH ', 1PE13.6, '      DA ', 1PE13.6, '      Z ', 1PE13.6)
24     WRITE (6,60)
25 60   FORMAT (1H )
26     ED = ED*DAE/DA
27 2    CONTINUE
28     GO TO 1
29     END

```

4. SUBROUTINES FOR CALCULATING THE GEOMAGNETIC FIELD

The subroutine COEFF, listed in Table 6, and FIELD, listed in Table 7, follow directly from FORTRAN programs published by J. C. Cain and his coworkers of NASA's Goddard Space Flight Center. The Data Users' Note NSSDC 68-11 (Cain, 1968) and the paper by Cain et al. (1967) give a complete and detailed account of the foundations, development, and listing of programs for computing the geomagnetic field. We do not attempt to recapitulate the vast amount of work underlying these programs in the present report.

TABLE 6

```

1      SUBROUTINE COEFF(TM)
2      DIMENSION G(11,11),GT(11,11),SHMIT(11,11)
3      COMMON /COEFFS/TG(11,11)
4 C    READ SPHERICAL HARMONIC COEFFICIENTS
5 C    THE COEFFICIENTS IN THE DATA STATEMENT ARE GSFC(12/66)
6 C    THIS G ARRAY CONTAINS BOTH G AND H VALUES
7      DATA G/
8      Z0.,-30401.2,-1540.1,1307.1,949.3,-233.5,49.2,72.2,8.5,10.4,-2.9,
9      15778.2,-2163.8,2997.9,-1988.9,803.5,355.7,57.5,-53.7,6.5,5.8,-.9,
10     2-1932.,202.9,1590.3,1276.8,502.9,228.4,-.8,7.9,-9.3,7.5,-2.2,
11     3-425.4,227.8,-133.8,881.2,-397.7,-28.8,-238.3,15.6,-9.6,-15.1,.8,
12     4160.3,-274.3,2.3,-246.6,266.5,-157.9,-1.5,-24.3,-6.1,12.1,-2.8,
13     55.1,117.8,-114.8,-108.9,82.4,-62.2,-2.,-3.6,5.5,4.7,6.4,
14     6-12.1,104.4,56.6,-23.4,-14.8,-13.3,-108.9,15.5,-8.1,.2,4.7,
15     7-53.7,-27.4,-8.1,7.,24.3,-22.5,-21.4,3.6,13.,1.6,-.2,
16     85.4,-11.7,4.2,-15.3,4.6,21.9,-.7,-17.1,7.4,.9,1.8,
17     9-22.4,13.8,6.3,-3.,-1.9,9.,11.5,.1,-1.5,.2,2.,
18     D-.1,4.5,-1.,2.6,-4.4,-1.3,-3.6,4.,1.,-2.,1.1/
19     DATA GT/
20     Z0.,14.03,-23.29,-.93,1.45,1.61,-.42,-.57,.35,-.10,-.01,
21     1-3.71,8.76,-.09,-10.62,.9,.6,.82,-.34,.5,-.13,-.13,
22     2-14.31,-16.62,-4.56,2.31,-1.75,3.34,.82,-1.44,1.7,-1.2,.88,
23     35.2,2.53,-6.98,-5.89,.66,-.04,2.35,-.9,-.11,.08,-.18,
24     4-2.19,-.14,1.88,-6.52,-3.01,-.6,.83,.03,.34,-.08,.17,
25     52.24,1.59,-2.61,.5,-.12,1.76,.01,-.6,-.07,-.39,-.02,
26     6.05,.09,2.55,-1.19,.33,.84,.23,-.17,.43,-.36,.05,
27     7-.96,.01,.43,.75,-.33,.49,.9,-.64,-.15,.47,.17,
28     8-.5,-.21,.03,-.79,.05,.1,-.36,-.43,-.42,.37,.16,
29     9.66,.54,.03,.35,-.03,-.01,.45,-.05,.75,-.46,.31,
30     D-.61,-.64,.02,.05,-.63,-.07,.07,-.03,-.02,-.45,-.23/
31 C   CALCULATE NORMALIZATION CONVERSION FACTORS
32     SHMIT(1,1)=-1.
33     MAXN =11
34     DO 15 N=2,MAXN
35     SHMIT(N,1)=SHMIT(N-1,1)*FLOAT(2*N-3)/FLOAT(N-1)
36     SHMIT(1,N)=0.
37     JJ=2
38     DO 15 M=2,N
39     SHMIT(N,M)=SHMIT(N,M-1)*SQRT(FLOAT((N-M+1)*JJ)/FLOAT(N+M-2))
40     SHMIT(M-1,N)=SHMIT(N,M)
41 15   JJ=1
42 C   CONVERT COEFFICIENTS, SCHMIDT TO GAUSS
43     DO 16 N=2,MAXN
44     DO 16 M=1,N
45     G(N,M)=G(N,M)*SHMIT(N,M)
46     GT(N,M)=GT(N,M)*SHMIT(N,M)
47     IF (M.EQ.1) GO TO 16
48     G(M-1,N)=G(M-1,N)*SHMIT(M-1,N)
49     GT(M-1,N)=GT(M-1,N)*SHMIT(M-1,N)
50 16   CONTINUE
51 C   CONVERT COEFFICIENTS TO NEW TIME
52 17   T=TM-1960.0
53     DO 18 N=1,MAXN
54     DO 18 M=1,N
55     TG(N,M)=G(N,M)+T*GT(N,M)
56     IF (M.EQ.1) GO TO 18
57     TG(M-1,N)=G(M-1,N)+T*GT(M-1,N)
58 18   CONTINUE
59     RETURN
60     END

```

TABLE 7

```

1  SUBROUTINE FIELD (R,ST,CT,SPH,CPH,BR,BT,BP,B)
2  DIMENSION P(11,11), DP(11,11), CONST(11,11)
3  DIMENSION SP(11), CP(11), FN(11), FM(11)
4  COMMON/COEFFS/G(11,11)
5  NMAX = 11
6  1  P(1,1) = 1.
7  DP(1,1) = 0.
8  SP(1) = 0.
9  CP(1) = 1.
10 DO 2 N = 2,11
11 FN(N) = N
12 DO 2 M = 1,N
13 FM(M) = M-1
14 2  CONST(N,M) = FLOAT((N-2)**2-(M-1)**2)/FLOAT((2*N-3)*(2*N-5))
15 3  SP(2) = SPH
16 CP(2) = CPH
17 DO 4 M = 3,NMAX
18 SP(M) = SP(2)*CP(M-1)+CP(2)*SP(M-1)
19 4  CP(M) = CP(2)*CP(M-1)-SP(2)*SP(M-1)
20 ADR=6.3712E6/R
21 AR = ADR**2
22 BT = 0.
23 BP = 0.
24 BR = 0.
25 DO 8 N = 2,NMAX
26 AR = ADR*AR
27 DO 8 M = 1,N
28 IF(M.EQ.N) GO TO 5
29 P(N,M)=CT*P(N-1,M)
30 DP(N,M)=CT*DP(N-1,M)-ST*P(N-1,M)
31 IF(M.EQ.N-1) GO TO 7
32 P(N,M)=P(N,M)-CONST(N,M)*P(N-2,M)
33 DP(N,M)=DP(N,M)-CONST(N,M)*DP(N-2,M)
34 GO TO 7
35 5  P(N,N) = ST*P(N-1,N-1)
36 DP(N,N) = ST*DP(N-1,N-1)+CT*P(N-1,N-1)
37 7  PAR = P(N,M)*AR
38 IF (M.EQ.1) GO TO 9
39 TEMP = G(N,M)*CP(M)+G(M-1,N)*SP(M)
40 BP = BP-(G(N,M)*SP(M)-G(M-1,N)*CP(M))*FM(M)*PAR
41 GO TO 10
42 9  TEMP = G(N,M)*CP(M)
43 BP = BP-(G(N,M)*SP(M))*FM(M)*PAR
44 10 BT = BT+TEMP*DP(N,M)*AR
45 8  BR = BR-TEMP*FN(N)*PAR
46 BP = BP/ST
47 B = SQRT(BT*BT+BP*BP+BR*BR)
48 GTT=1.E-9
49 BR=BR*GTT
50 BT=BT*GTT
51 BP=BP*GTT
52 B=B*GTT
53 RETURN
54 END

```

5. A SUBROUTINE FOR GENERALIZED MAGNETOIONIC CALCULATIONS

In the region of the lower ionosphere between about 50 and 90 km, the collisions of electrons with neutral molecules appreciably influence the refraction and absorption indices of radio waves which are suitable for investigating this region. The proportionality of electron collision frequency and electron kinetic energy (Phelps and Pack, 1959) must be an intrinsic part of the equations for the numerical analysis of experimental data from this region. The Sen-Wyller equations (Sen and Wyller, 1960) are a convenient formulation of the generalized magnetoionic theory which includes the energy dependence of collision frequency.

Subroutine SENWYL, listed in Table 8, is a statement of the Sen-Wyller equations.

Statements (8-5) to (8-10) are polynomial approximations (Hara, 1963) of the semiconductor integrals appropriate for the ionosphere. Hara asserts that these approximations are in error by less than 1% at most.

Statement (8-11) calculates the square of the angular plasma frequency, P_2 . The numerical coefficient is $e^2/\epsilon_0 m$.

Convenient intermediate variables are calculated by (8-12) to (8-19).

Statements (8-20) to (8-25) are equations (55) from the paper by Sen and Wyller (1960).

Statements (8-26) to (8-28) are equation (56) of Sen and Wyller; (8-29) to (8-33) are equation (28); (8-34) to (8-44) are equation (27); and (8-45) to (8-49) separate the real and imaginary parts of the complex refractive indices for the two magnetoionic modes.

Wave polarizations are calculated by (8-50) to (8-57). These statements correspond to equation (35) of Sen and Wyller.

TABLE 8

```

1      SUBROUTINE SENWYL(ED,CF,S,CPH,W,RIO,RIX,AIO,AIX,ROR,RXR,ROI,RXI)
2  C    GENERALIZED MAGNETO-IONIC THEORY, SEN-WYLLER EQUATIONS.
3      COMPLEX EI,EII,EIII,AA,BB,CC,DD,EE,VC,VD,VE,VF,VG,VH,VO,VX
4      COMPLEX CMLX,CSQRT,BBVA,CCCPH,RO,RX
5  C    DEFINE BURKE-HARA FUNCTIONS FOR C3/2 AND C5/2.
6      C32(X) = (X*(X*(X*(X+24.653115)+113.9416)+11.287513)+.023983474)/(
7      1X*(X*(X*(X*(X+24.656819)+120.49512)+289.58085)+149.21254)+9.387
8      27372)+.018064128)
9      C52(X) = (X*(X*(X+6.6945939)+16.901002)+1.1630641)/(X*(X*(X*(X+
10     16.6314497)+35.355257)+68.920505)+64.093464)+4.3605732)
11     P2 = 3182.6018*ED
12     WPS = W+S
13     WMS = W-S
14     Q = P2/W/CF
15     R = Q/CF
16     T = 2.5*Q
17     A1 = W/CF
18     A2 = WMS/CF
19     A3 = WPS/CF
20     A = R*W*C32(A1)
21     B = T*C52(A1)
22     C = R*WMS*C32(A2)
23     D = T*C52(A2)
24     E = R*WPS*C32(A3)
25     F = T*C52(A3)
26     EI = CMLX (1.-A,-B)
27     EII = CMLX (.5*(F-D),.5*(C-E))
28     EIII = CMLX (A-.5*(C+E),B-.5*(F+D))
29     AA = 2.*EI*(EI+EIII)
30     BB = EIII*(EI+EIII)+EII**2
31     CC = 2.*EI*EII
32     DD = 2.*EI
33     EE = 2.*EIII
34     VB = CPH*CPH
35     VA = 1.-VB
36     VC = BB*BB*VA*VA-CC*CC*VB
37     VD=CSQRT(VC)
38     BBVA=BB*VA
39     VE=AA+BBVA
40     VF = DD+EE*VA
41     VG = (VE+VD)/VF
42     VH = (VE-VD)/VF
43     VO=CSQRT(VG)
44     VX=CSQRT(VH)
45  C    SEPARATE INDICES
46     RIO = REAL (VO)
47     AIO = -AIMAG(VO)
48     RIX = REAL (VX)
49     AIX = -AIMAG(VX)
50  C    CALCULATE WAVE POLARIZATIONS
51     CCCPH=CC*CPH
52     RO=-((BBVA-VD)/CCCPH)
53     ROR=REAL(RO)
54     ROI=AIMAG(RO)
55     RX=-((BBVA+VD)/CCCPH)
56     RXR=REAL(RX)
57     RXI=AIMAG(RX)
58     RETURN
59     END

```

6. RAY TRACING FOR THE ANALYSIS OF STANDING WAVES

6.1 Haselgrove Method

The Haselgrove (1954) method of ray tracing which provides wave-normal directions and ray coordinates at the end of each integration step is used, with some modification.

The coordinates of the ray are denoted by $y(1)$, $y(2)$, $y(3)$ and the direction cosines of the wave normal by $AP(1)$, $AP(2)$, $AP(3)$ in the Cartesian coordinate system. The coordinate system is oriented such that $y(3)$ is the vertical axis and the wave normal is in the $y(1)$, $y(3)$ plane, this will be called the propagation plane. See Figure 6.1.

The propagation plane [$y(1)$, $y(3)$] makes an angle AZI with magnetic north and DIP is the magnetic dip. AZI is determined by the plane of the rocket trajectory. The direction cosines of B are then:

$$B(1) = \cos DIP \cdot \cos AZI$$

$$B(2) = \cos DIP \cdot \sin AZI$$

$$B(3) = - \sin AZI$$

Both DIP and AZI are input parameters for the ray-tracing program.

6.2 Differential Equations of the Ray

The ray paths are calculated by integrating the following simultaneous differential equations

$$\frac{du_i}{dt} = \frac{\partial G}{\partial y_i} \quad (1)$$

$$\frac{dy_i}{dt} = \frac{\partial G}{\partial U_i} \quad (2)$$

(In the FORTRAN program u_i is denoted by $Y(I + 3)$)

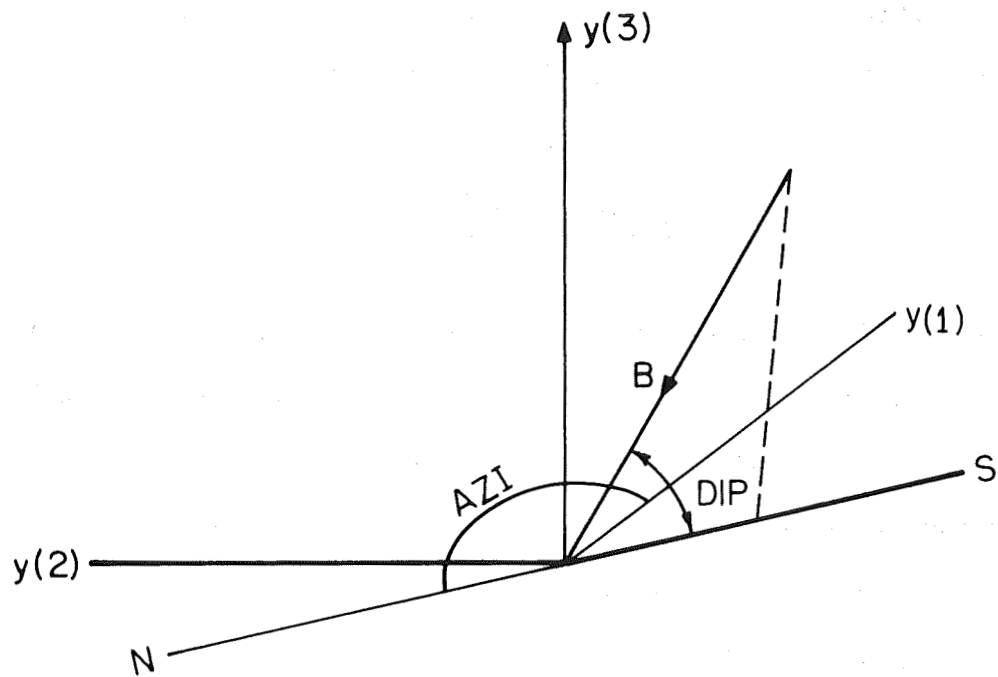


Figure 6.1 Orientation of the coordinate system for the ray-tracing problem.

The parameter, G , in these equations is used in defining the normal index surface. This surface is constructed about a point, Q (say), in the medium by first drawing a vector from Q in the direction of a wave normal. Then the locus of the point on the vector which is displaced from Q by a distance equal to the phase refractive index, μ , generates the normal index surface. In deriving the equations Haselgrove (1954) defined the normal index surface by

$$G(y_i, u_i) = 1$$

where

$$G \equiv \frac{u}{\mu} \equiv \frac{(u_1^2 + u_2^2 + u_3^2)^{1/2}}{\mu(y_i, u_i)} \equiv 1 \quad (3)$$

Thus the end of the vector, \vec{u} (which is in the direction of wave normal), generates the normal index surface since the condition, $\frac{u}{\mu} = 1$, implies that the magnitude of \vec{u} is equal to μ .

In equations (1) and (2), t is a variable of integration and, in the sense used here, it has the dimension of length. The significance of this variable is such that if, in a small interval of time, the phase of the radio wave advances a distance dt in free space, then in the medium it advances a distance $ds = (dy_1^2 + dy_2^2 + dy_3^2)^{1/2}$ in the direction of the ray, where dy_i are given by equation (2).

Neglecting collisions and using equation (3) and

$$D = 2(1-X) - y_T^2 \pm \{y_T^4 + 4y_L^2(1-X)^2\}^{1/2}$$

$$\mu^2 = 1 - \frac{2X(1-X)}{D}$$

$$\cos\theta = \frac{1}{u} \sum_{i=1}^3 u_i b_i,$$

and

$$\sin\theta = (1 - \cos^2\theta)^{1/2} \text{ (always positive),}$$

$$\frac{dy_i}{dt} = \frac{1}{\mu} \left[\frac{u_i}{\mu} - Fl(b_i - \cos\theta \frac{u_i}{u}) \right] \quad (4)$$

$$\frac{du_i}{dt} = - F_2 \frac{\partial X}{\partial y_i} \quad (5)$$

where

$$F_1 = \frac{2X(1-X)}{\mu^2 D^2} \left\{ 1 + \frac{2(1-X)^2 - y_T^2}{D - 2(1-X) + y_T^2} \right\} y y_L$$

and

$$F_2 = \frac{u}{D\mu^3} \left[1 - 2X + \frac{2X(1-X)}{D} \left\{ 1 + \frac{2y_L^2(1-X)}{D - 2(1-X) + y_T^2} \right\} \right]$$

Thus (4) and (5) are a set of six differential equations. They are integrated simultaneously to obtain the parameters of the ray path.

6.3 Method of Integrating

The method of integrating equations (4) and (5) is the Runge-Kutta fourth order process as modified by Gill (1951).

The set of six simultaneous differential equations are written in generalized form:

$$\frac{dy_i}{dt} = f_i(y_1, y_2, y_3, y_4, y_5, y_6) \quad (i = 1, 2, \dots, 6).$$

If the initial values of y_i are y_{i0} then four approximations to the new values corresponding to an interval h in t are

$$k_{i0} = hf_i(y_{10}, y_{20}, \dots, y_{60})$$

$$k_{i1} = hf_i(y_{11}, y_{21}, \dots, y_{61})$$

$$k_{i2} = hf_i(y_{12}, y_{22}, \dots, y_{62})$$

$$k_{i3} = hf_i(y_{13}, y_{23}, \dots, y_{63})$$

in which

$$y_{i1} = y_{i0} + k_{i0}/2$$

$$y_{i2} = y_{i1} + (1 - \sqrt{1/2}) (k_{i1} - q_{i1})$$

where

$$q_{i1} = k_{i0}$$

$$y_{i3} = y_{i2} + (1 + \sqrt{1/2}) (k_{i2} - q_{i2})$$

where

$$q_{i2} = (2 - \sqrt{2}) k_{i1} - (2 - 3\sqrt{1/2}) q_{i1}.$$

The adopted value of y_i at the end of the step is then

$$y_{i4} = y_{i3} + 1/6 k_{i3} - 1/3 q_{i3}$$

where

$$q_{i3} = (2 + \sqrt{2}) k_{i2} - (2 + 3\sqrt{1/2}) q_{i2}.$$

To economize in the computer storage requirements, a different notation is used in the FORTRAN program. Suppose that we are at the n th stage of approximations, that is y_{ij} , k_{ij} , q_{ij} have been calculated for $j = n - 1$ and they are to be calculated for $j = n$. In the FORTRAN program (y_{ij} , k_{ij} , q_{ij} , $i = 1, 2, \dots, 6$, $j = n-1$) are denoted by $(AY(I), AK(I), AQ(I), I = 1, 2, \dots, 6)$ and (y_{ij} , k_{ij} , q_{ij} , $i = 1, 2, \dots, 6$, $j = n$) are denoted by $(Y(I), K(I), Q(I), I = 1, 2, \dots, 6)$.

6.4 Initial Conditions for the Ray

Let A be the take-off angle of the ray then the initial direction cosines of the wave normal are:

$$AP(1) = \cos A$$

$$AP(2) = 0$$

$$AP(3) = \sin A$$

The coordinates (y_1, y_2, y_3) at the base height of the ionosphere are called (H_1, H_2, H_3) and are given by $(\frac{HB}{\tan A}, 0, HB)$.

Because of the method of integration used, ray tracing starts at a distance Q (taken as 1 km; an input parameter to the program) beyond HB in the direction of the wave normal. Thus the initial coordinates of the ray are:

$$y(i) = H(i) + Q \cdot AP(i) \quad i = 1, 2, 3$$

6.5 Ray Tracing near the Reflection Height

The refractive index μ and hence the parameters $y(i + 3) = \mu AP(i)$ vanish rapidly near the reflection height, therefore the accumulated errors in the calculated values of $y(4)$, $y(5)$, $y(6)$ would become serious. Therefore, the tracing of the upgoing ray is truncated when it reaches within 0.2 km of the reflection height. Then the program starts tracing the downgoing ray. The

initial position $y(1)$, $y(2)$, $y(3)$ of the downgoing ray is taken to be the position of the upgoing ray at truncation. The initial values for $y(4)$, $y(5)$, $y(6)$ of the downgoing ray are determined from the roots of the Booker quartic equation, Budden (1961).

6.6 The Computer Program

The computer program is written in FORTRAN IV and consists of a main program and four subroutines. The main program includes the integration instructions. The first subroutine returns the initial coordinates of the ray and the direction-cosines of the wave normal at the base of the ionosphere. The second subroutine returns the values of X and $\frac{\partial X}{\partial y(i)}$ at each point of the integration, the third solves the Booker quartic equation and returns its roots, and the fourth increments the take-off angle at the end of each integration and senses when all rays have been computed.

A simplified form of the flow diagram of the program is shown in Figure 6.2. Constant parameters are defined first (9-3) to (9-18), next, frequency and the magnetic field data are read (9-19), and then the program branches to the subroutine DETXP. On entering this subroutine the first time, the parameters describing the ionosphere are read and these instructions, (13-6) to (13-41), are by-passed on succeeding occasions. DETXP also determines the initial direction cosines of the wave normal and the starting coordinates of the ray at the level where $y(3)=H_B$, (13-47) to (13-53). These values are returned to the master program which then determines the starting point for the integration. The values Y and Y_L and Y_T are then determined (9-47) to (9-53), before branching to the subroutine DETX which returns the values of X and its spatial derivatives. This information then allows the calculation of the refractive index, AMU from the Appleton-Hartree equations, (9-56) to (10-20). The program then proceeds through a loop involving the

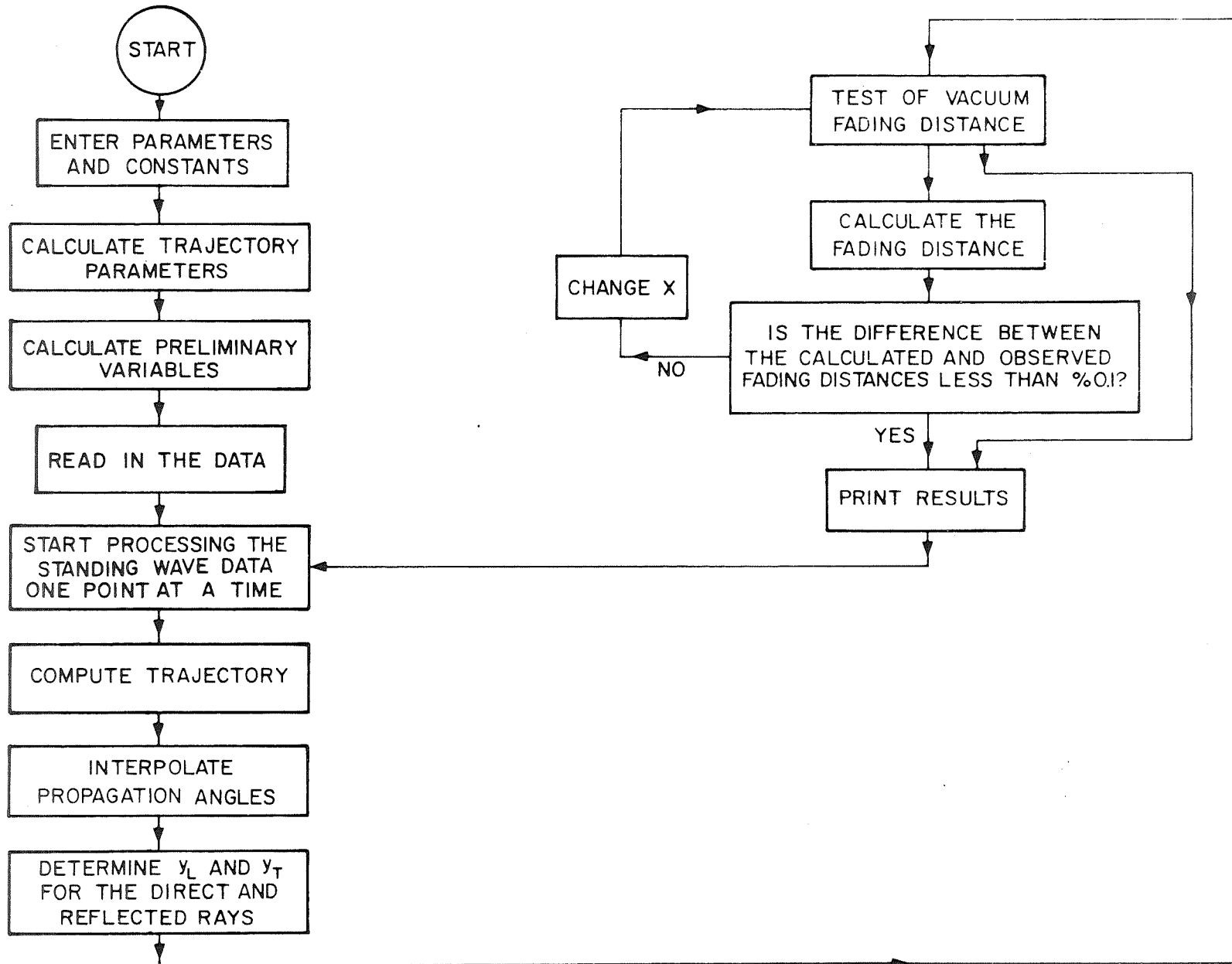


Figure 6.2 Simplified flow diagram for the ray tracing computer program.

TABLE 9

```

1 C OBLIQUE INCIDENCE RAY-TRACING
2   DIMENSION B(3),AP(3),Y(8),AY(8),AK(8),AQ(8),H(3),DX(3),W(8)
3 C ENTER CONSTANTS FOR R-K-G SOLUTIONS
4   ST=SQRT(2.)
5   S1=1.-1./ST
6   S2=1.+1./ST
7   S3=2.-ST
8   S4=2.-3./ST
9   S5=2.+ST
10  S6=2.+3./ST
11  PI2=6.2831854
12 C ENTER PARAMETERS
13  CO=PI2/360.
14  Q= 1.0
15  SG = 0.5
16  JP = 8.0
17  M = 1.0
18  JF = 4.0
19 1  READ(5,2) F,FH,DIP,AZI
20 2  FORMAT(F7.0,F8.0,F5.0,F4.0)
21  WRITE(6,3) F,FH,DIP,AZI
22 3  FORMAT(1H1,6HFREQ =,-6PF6.3,10X8HG.FREQ =,-6PF6.3,10X5HDIP =,OPF5.
23 11,10X10HAZIMUTH = ,F4.0)
24  READ (5,85) SE,SF
25 85  FORMAT(2F6.0)
26 C FIND FIELD DIRECTION COSINES
27  DIP=DIP*CO
28  AZI=AZI*CO
29  BY=-FH/F
30  C=COS(DIP)
31  B(1)=C*COS(AZI)
32  B(2)=C*SIN(AZI)
33  B(3)=-SIN(DIP)
34  J=0
35 C START POINT FOR EACH RAY ELEMENT
36 4  CALL DETXP(AP,H,J,IOX,HB,HR,F,PI2,CO,BY)
37  IM=JP
38 C FIND INITIAL VALUES OF Y1 TO Y3
39  DO 20 I=1,3,1
40 20  Y(I)=H(I)+Q*AP(I)
41  S=AP(1)
42  CF=AP(3)
43  K=0
44  L=1
45  SD=SG
46 C FIND Y,YL,YT
47 5  C=0.
48  DO 6 I=1,3,1
49 6  C=C+AP(I)*B(I)
50  YL=BY*C
51  SC=SQRT(1.-C*C)
52  YT=BY*SC
53  YT2=YT*YT
54 C FIND X AND DX/DH
55  CALL DETX(X,DX,Y,SD,K,L)
56 C FIND INDICES FROM A-H EQ
57  X1=1.-X
58  X2=2.*X1
59  X3=YL*YL*X2*X2

```

TABLE 10

```

1      X4=X2-YT2
2      X5=2.*X*X1
3      X6=SQRT(YT2*YT2+X3)
4      DEX=X4-X6
5      IF(IOX)7,7,8
6      7 D=X4+X6
7      IF(K-1) 70,70,9
8 70   IF(SF-Y(3))89,89,90
9 89   IF(IM-JF)9,10,10
10 90  IF(IM-JP)9,10,10
11 10  AMUE=1.-X5/DEX
12    IF(AMUE)11,11,12
13 11  AMUE=-1.
14    GO TO 9
15 12  AMUE=SQRT(AMUE)
16    GO TO 9
17    8 D=DEX
18    9 AMU=1.-X5/D
19    IF(AMU)13,13,14
20 14  AMU=SQRT(AMU)
21    IF(K-1)15,16,17
22    15 IF(L)18,18,19
23 C PRINT OUTPUT COLUMN HEADINGS
24 19  WRITE(6,21)
25    21 FORMAT(5X1HX,10X1HY,11X1HH,9X2HMU,10X1HU,9X2HEL,7X5HTHETA,6X5HRATI
26      10,5X6HINCLIN,7X3HMUX)
27      L=0
28 96  U=AMU
29 C FIND INITIAL VALUES OF Y4 TO Y6
30    DO 22 I=1,3,1
31    22 Y(I+3)=AMU*AP(I)
32 C PRESERVE ALL INITIAL VALUES OF Y
33    DO 23 I=1,6,1
34    23 AY(I)=Y(I)
35      K=1
36      GO TO 24
37 18  IM=JF
38      GO TO 96
39 16  IF(L-1)24,24,25
40 25  L=0
41      GO TO 17
42 C CALCULATE STEP SIZE
43 24  DN=AMU+2.*((1.-AMU*AMU)*(1.+(1.-X*X)*YL*YL/(D-X4))-X*X)/D/AMU
44      DN=SG/DN
45      IF(SF-Y(3))87,87,88
46 87  IF(IM-JF)13,76,76
47 88  IF(IM-JP)13,76,76
48 C PREPARE RESULTS FOR OUTPUT
49 76  EI=ATAN(Y(3)/Y(1))/CO
50      THE=ATAN(Y(6)/Y(4))/CO
51      P1=YT2/X2
52      P2=SQRT(YT2*YT2/X2/X2+YL*YL)
53      IF(IOX)55,55,56
54 55  P1=(P2-P1)/YL
55      GO TO 57
56 56  P1=-P1/YL
57 57  R=2.*P1/(1.+P1*P1)
58      R=(1.-SQRT(1.-R*R))/R
59      AIN = 1.
60      IF (IOX) 26,26,27

```


determination of the appropriate values of AK_{in} (11-52) to (11-57), AQ_{in} and Y_{in} (11-59) to (12-3) and (12-30) to (12-42), until the integration step is complete. Each integration step requires the program to run through this loop four times; during this cycle each new value of ray elevation, $y(3)$, is compared with the ordinary-wave reflection height (12-11) if the former is higher the integration increment (DN) is reduced by 1/2, and the integration step is redone (12-5) to (12-11). When the integration step is complete the various parameters of the ray are printed out and the program then determines if the ray-tracing procedure has gone far enough (11-9) and (11-12), in which case the integration is terminated. If the ray is not complete, the ray tracing is continued in this manner until the ray is traced up to within 0.2 km of the ordinary-wave reflection height, whereupon the upgoing ray is truncated (11-10) to (11-11); then a downgoing ray is traced starting from where the upgoing ray was truncated. The initial values of the refractive index and wave normal direction cosines of the downgoing ray are determined from the roots of the Booker quartic equation. The parameters of the Booker quartic equation are determined first (11-14) to (11-30), then the program branches to the subroutine QUAR (11-31) which returns the four roots of the quartic equation, and finally the refractive index, $y(4)$ and $y(6)$ are calculated from the proper root of the B.Q. equation (11-32) to (11-47). Then the program goes back to the integration cycle.

When a ray is complete the program branches to the subroutine REP. This increments the take-off angle so that another ray can be computed after branching back to DETXP. REP also senses when all rays are computed and the program is stopped at this point.

For the purposes of ray tracing the electron density is assumed to be a function of height only, so that in equation (5) $\frac{\partial X}{\partial y(1)}$ and $\frac{\partial X}{\partial y(2)}$ are zero. The

TABLE 11

```

1 26 WRITE(6,28) (Y(I),I=1,3),AMU,U,EL,THE,R,AIN,AMUE
2 C PRINT ORD-MODE RESULTS
3 28 FORMAT(F8.1,F12.1,F11.1,2F11.3,F10.1,F11.1,F11.2,F11.1,F12.3)
4 C PRINT X-MODE RESULTS
5 GO TO 75
6 27 WRITE(6,29) (Y(I),I=1,3),AMU,U,EL,THE,R,AIN
7 29 FORMAT(F8.1,F12.1,F11.1,2F11.3,F10.1,F11.1,F11.2,F11.1)
8 75 IM=1.
9 13 IF(SE-Y(1))33,86,86
10 86 IF(HR-Y(3)-.2)30,30,31
11 31 IF(Y(6))32,17,17
12 32 IF(HB-Y(3)+.5)17,17,33
13 30 IF(Y(6))17,17,34
14 34 Y2=BY*BY
15 T1=1.-Y2
16 T2=B(3)*B(3)*Y2-1.
17 T3A=2.*B(1)*B(3)*Y2
18 S9=S*S
19 C2=CF*CF
20 T3=T3A*S
21 T4=Y2*(1.-B(3)*B(3)*C2+S9*B(1)*B(1))
22 T5=-T3A*C2*S
23 T6=B(1)*B(1)*S9*C2*Y2
24 V=C2-X
25 C FIND COEFFS OF QUARTIC
26 AL=T1+X*T2
27 BE=T3*X
28 GA=-X2*V+2.*Y2*V+X*T4
29 DE=T5*X
30 EP=X1*V*V-C2*Y2*V-T6*X
31 CALL QUAR(AL,BE,GA,DE,EP,W)
32 I=2
33 102 IF(W(I)) 100,101,100
34 101 I=I+2
35 IF(I-8)102,102,103
36 103 QU=W(3)
37 GO TO 107
38 100 I=2
39 106 IF(W(I))104,105,104
40 104 I=I+2
41 GO TO 106
42 105 QU=W(I-1)
43 107 TT=-S/QU
44 TT1=SQRT(TT*TT+1.)
45 AMU=SQRT(QU*QU+S*S)
46 Y(4)=AMU*TT/TT1
47 Y(6)=-AMU/TT1
48 L=0
49 K=0
50 GO TO 35
51 C PART OF RT MEMBER OF EQ 4
52 17 F1=X*X2*(1.+(X1*X2-YT2)/(D-X4))*BY*YL/AMU/AMU/D/D
53 C PART OF RT MEMBER OF EQ 5
54 F2=(1.-2.*X+X2*X*(1.+X3/X2/(D-X4))/D)*U/D/AMU/AMU/AMU
55 DO 36 I=1,3,1
56 AK(I)=(AP(I)-F1*(B(I)-C*AP(I)))/AMU*DN
57 36 AK(I+3)=-F2*DX(I)*DN
58 GO TO(37,38,39,40),K
59 C FIND Y1-Y6 IN FIRST APPROX
60 37 DO 41 I=1,6,1

```

TABLE 12

```

1      Y(I)=Y(I)+AK(I)/2.
2      41 AQ(I)=AK(I)
3      50 K=K+1
4      53 IF(Y(3)-HB)42,43,43
5      42 DO 44 I=1,8,1
6      44 Y(I)=AY(I)
7      DN=DN/2.
8      K=1
9      L=2
10     GO TO 35
11     43 IF(Y(3)-HR)35,42,42
12     35 U=0.
13     DO 45 I=4,6,1
14     45 U=U+Y(I)*Y(I)
15     U=SQRT(U)
16     DO 46 I=1,3,1
17     46 AP(I)=Y(I+3)/U
18     IF(K-1)5,47,5
19     47 BQ=0.
20     DO 84 I=1,3,1
21     84 BQ=BQ+(Y(I)-AY(I))*(Y(I)-AY(I))
22     IF(BQ)95,64,95
23     95 SD=(Y(3)-AY(3))*DN/SQRT(BQ)
24     IF(SD)63,64,64
25     63 SD=-SD
26     64 DO 48 I=1,6,1
27     48 AY(I)=Y(I)
28     IM=IM+1
29     GO TO 5
30     C FIND Y1-Y6 IN SECOND APPROX
31     38 DO 49 I=1,6,1
32     Y(I)=Y(I)+S1*(AK(I)-AQ(I))
33     49 AQ(I)=S3*AK(I)-S4*AQ(I)
34     GO TO 50
35     C FIND Y1-Y6 IN THIRD APPROX
36     39 DO 51 I=1,6,1
37     Y(I)=Y(I)+S2*(AK(I)-AQ(I))
38     51 AQ(I)=S5*AK(I)-S6*AQ(I)
39     GO TO 50
40     C FIND Y1-Y6 IN FOURTH APPROX
41     40 DO 52 I=1,6,1
42     52 Y(I)=Y(I)+AK(I)/6.-AQ(I)/3.
43     K=1
44     GO TO 53
45     33 CALL REP(J,M)
46     GO TO(1,4,54),J
47     54 CONTINUE
48     END

```

TABLE 13

```

1      SUBROUTINE DETXP(AP,H,J,IOX,HB,HR,F,PI2,CO,Y)
2      DIMENSION AP(3),H(3),AH(200),X(200),DX(200),AX(200)
3      COMMON A,AMAX,DA,AH,X,DX
4      IF(J)1,1,2
5      C DO 1 TO 16 FIRST TIME ONLY
6          1 J=1
7      C FACTOR TO CONVERT N TO X
8      CON=1.6021E-19*1.7588E11/8.854E-12/PI2/PI2/F/F*1.E6
9      IOX = 0
10     READ (5,3) AMIN,AMAX,DA,N
11     3   FORMAT(F4.0,F5.0,F5.0,I2)
12     AMIN=CO*AMIN
13     AMAX=CO*AMAX
14     DA=CO*DA
15     READ(5,4) (AH(I),X(I), I=1,N)
16     4   FORMAT(4(F6.0,E9.0))
17     C CONVERT N TO X
18     DO 5 I=1,N,1
19     5   X(I)=X(I)*CON
20     N=N-1
21     DO 6 I=1,N,1
22     C FIND CHANGE IN X BETWEEN DATA POINTS
23     6   DX(I)=(X(I+1)-X(I))/(AH(I+1)-AH(I))
24     HB=AH(1)
25     IF(IOX)7,7,8
26     7   B=1.
27     WRITE (6,9)
28     9   FORMAT(3X5HO-RAY)
29     GO TO 10
30     8   B=1.+Y
31     WRITE(6,11)
32     11  FORMAT(3X5HX-RAY)
33     10  I=1
34     15  IF(X(I)-B)12,13,14
35     12  I=I+1
36     GO TO 15
37     13  HR=AH(I)
38     GO TO 16
39     14  I=I-1
40     HR=(B-X(I))/DX(I)+AH(I)
41     16  A=AMIN
42     C ABOVE 1 TO 16 FIRST TIME ONLY
43     2  A1=A/CO
44     WRITE(6,17) A1
45     17  FORMAT(3X16HTAKE-OFF ANGLE =,F5.1//)
46     C DIRECTION COSINES OF RAY NORMAL
47     AP(1)=COS(A)
48     AP(2)=0.
49     AP(3)=SIN(A)
50     TA=AP(3)/AP(1)
51     H(1)=HB/TA
52     H(2)=0.
53     H(3)=HB
54     RETURN
55     END

```

TABLE 14

```

1      SUBROUTINE DETX(X,DX,Y,SD,I,L)
2  C FIND X AND DX/DH
3      DIMENSION DX(3),Y(6),AH(200),AX(200),ADX(200)
4      COMMON A,AMAX,DA,AH,AX,ADX
5      IF(I-1)1,2,3
6  1      IF(L)4,4,5
7  5      X1=0.
8          J=2
9          DX(3)=AX(J)/(AH(J)-AH(1))
10         H2=Y(3)
11         H1=AH(1)
12         J1=J
13  3      X=X1+DX(3)*(Y(3)-H1)
14  4      DX(1)=0.
15         DX(2)=0.
16         RETURN
17  2      J=1
18  8      IF(AH(J)-Y(3))6,6,7
19  6      J=J+1
20         GO TO 8
21  7      IF(H2-Y(3))9,9,10
22  9      H2=Y(3)
23         IF(J-J1)11,12,13
24  11     J=J1
25         GO TO 3
26  12     IF(AH(J)-Y(3)-SD)14,3,3
27  14     J=J+1
28  13     M=J
29  16     J1=J
30         X=X1+DX(3)*(Y(3)-H1)
31         X1=X
32         DX(3)=(AX(M)-X)/(AH(M)-Y(3))
33         H1=Y(3)
34         GO TO 4
35  10     H2=Y(3)
36         IF(J-J1)20,17,11
37  20     IF(J-1)3,3,15
38  15     M=J-1
39         GO TO 16
40  17     IF(Y(3)-AH(J-1)-SD)18,3,3
41  18     IF(J-1)3,3,19
42  19     J=J-1
43         GO TO 15
44     END

```

TABLE 15

```

1  SUBROUTINE QUAR(A,B,C,D,E,S)
2  DIMENSION S(8),QR(2),RR(2)
3  PI=3.1415927
4  PI2=PI*2.
5  A1=B/A
6  B1=C/A
7  C1=D/A
8  D1=E/A
9  J= 1
10 A2=A1*A1
11 PA=B1-3.*A2/8.
12 QA=C1-A1*B1/2.+A1*A2/8.
13 RA=D1-A1*C1/4.+B1*A2/16.-3.*A2*A2/256.
14 P=1.
15 Q=-PA/3.
16 R=-4.*RA/3.
17 W=4.*PA*RA-QA*QA
18 AQ=P*R-Q*Q
19 AR=(3.*P*Q*R-P*P*W)/2.-Q*Q*Q
20 Z=AR*AR+AQ*AQ*AQ
21 IF(AQ)71,72,72
22 71 TH=AR/SQRT(-AQ*AQ*AQ)
23 IF(TH)75,76,76
24 75 TH=-TH
25 76 IF(Z)73,74,74
26 73 TH=ATAN(SQRT(1.-TH*TH)/TH)
27 TH=TH/3.
28 81 Y1=2.*SQRT(-AQ)*COS(TH)
29 IF(AR)77,78,78
30 77 Y1=-Y1
31 78 Y1=(Y1-Q)/P
32 IF(Y1-PA)79,80,80
33 79 TH=TH+PI2/3.
34 J=J+1
35 IF(J-3)81,81,80
36 74 TG=TH+SQRT(TH*TH-1.)
37 TH = ALOG(TG)
38 TG=TH/3.
39 TH=-TG
40 Y1=SQRT(-AQ)*(EXP(TG)+EXP(TH))
41 IF(AR)82,83,83
42 82 Y1=-Y1
43 83 Y1=(Y1-Q)/P
44 GO TO 80
45 72 TH=AR/SQRT(AQ*AQ*AQ)
46 IF(TH)86,87,87
47 86 TH=-TH
48 87 TG=TH+SQRT(TH*TH+1.)
49 TH = ALOG(TG)
50 TG=TH/3.
51 TH=-TG
52 Y1=SQRT(AQ)*(EXP(TG)-EXP(TH))
53 IF(AR)84,85,85
54 84 Y1=-Y1
55 85 Y1=(Y1-Q)/P
56 80 TH=SQRT(Y1-PA)
57 QR(1)=TH
58 QR(2)=-TH
59 RR(1)=Y1/2.-QA/2./TH
60 RR(2)=Y1/2.+QA/2./TH

```

TABLE 16

```

1      DO 99 I=1,2,1
2      SR=QR(I)*QR(I)-4.*RR(I)
3      IF(SR)57,58,58
4      58  ST=0.
5          CT=1.
6          R=SQRT(SR)
7          GO TO 59
8      57  ST=1.
9          CT=0.
10         R=SQRT(-SR)
11      59  IF(I-1)96,96,97
12      96  J=1
13         GO TO 98
14      97  J=5
15      98  S(J)=-QR(I)/2.+R*CT/2.-A1/4.
16         S(J+1)=R*ST/2.
17         S(J+2)=-QR(I)/2.-R*CT/2.-A1/4.
18      99  S(J+3)=-R*ST/2.
19      32  J=1
20      33  IF(S(J)-S(J+2))30,30,31
21      31  P=S(J)
22         S(J)=S(J+2)
23         S(J+2)=P
24         P=S(J+1)
25         S(J+1)=S(J+3)
26         S(J+3)=P
27         GO TO 32
28      30  J=J+2
29         IF(J-5)33,33,34
30      34  RETURN
31      END

```

TABLE 17

```
1      SUBROUTINE REP(J,M)
2      COMMON A,AMAX,DA
3      A=A+DA
4      IF(A-AMAX)2,2,1
5      1 IF(M)3,3,4
6      3 J=1
7      GO TO 5
8      4 J=3
9      GO TO 5
10     2 J=2
11  5  WRITE(6,6)
12     6 FORMAT(1H1)
13     RETURN
14     END
```


electron densities are provided to the subroutine DETXP at a series of heights and between these heights the electron density is assumed to vary linearly.

Input data for the program includes, besides the electron-density parameters and operating frequency (F), parameters describing the magnetic field, of which any variation with height is neglected. These field parameters are the electron gyrofrequency (FH), dip angle (DIP) and the azimuth angle (AZI) which determines the propagation plane. A parameter (SG) which determines the size of each integration step is also supplied and this is selected so that, in general, an integration step in free space would be about 0.5 km. However, this parameter is multiplied by μ/n' at each point in the integration; multiplying by μ would give approximately equal integration steps and further dividing by n' gives shorter integration steps in regions where the refractive index is changing more rapidly.

Figure 6.3 represents the ray-tracing results for a typical rocket shot, the rocket path and only portions of the rays which are of interest for the standing-wave analysis are shown. The standing-wave analysis formulated in section 6 requires the propagation angles (of both the upgoing and downgoing rays) for a number of points on the rocket trajectory. An important feature of the ray-tracing results is that the propagation angles vary very slowly as a function of position. Hence, when the rays are shifted in position due to ray-tracing errors, the corresponding error in propagation angles and hence the error in electron-density profile is small.

The standing-wave analysis, formulated from the Booker quartic equation, Section 8, requires a family of take-off angles of the upgoing and downgoing rays (α and β in Figure 6.3). It should be noted from the figure that these take-off angles also vary slowly as functions of position.

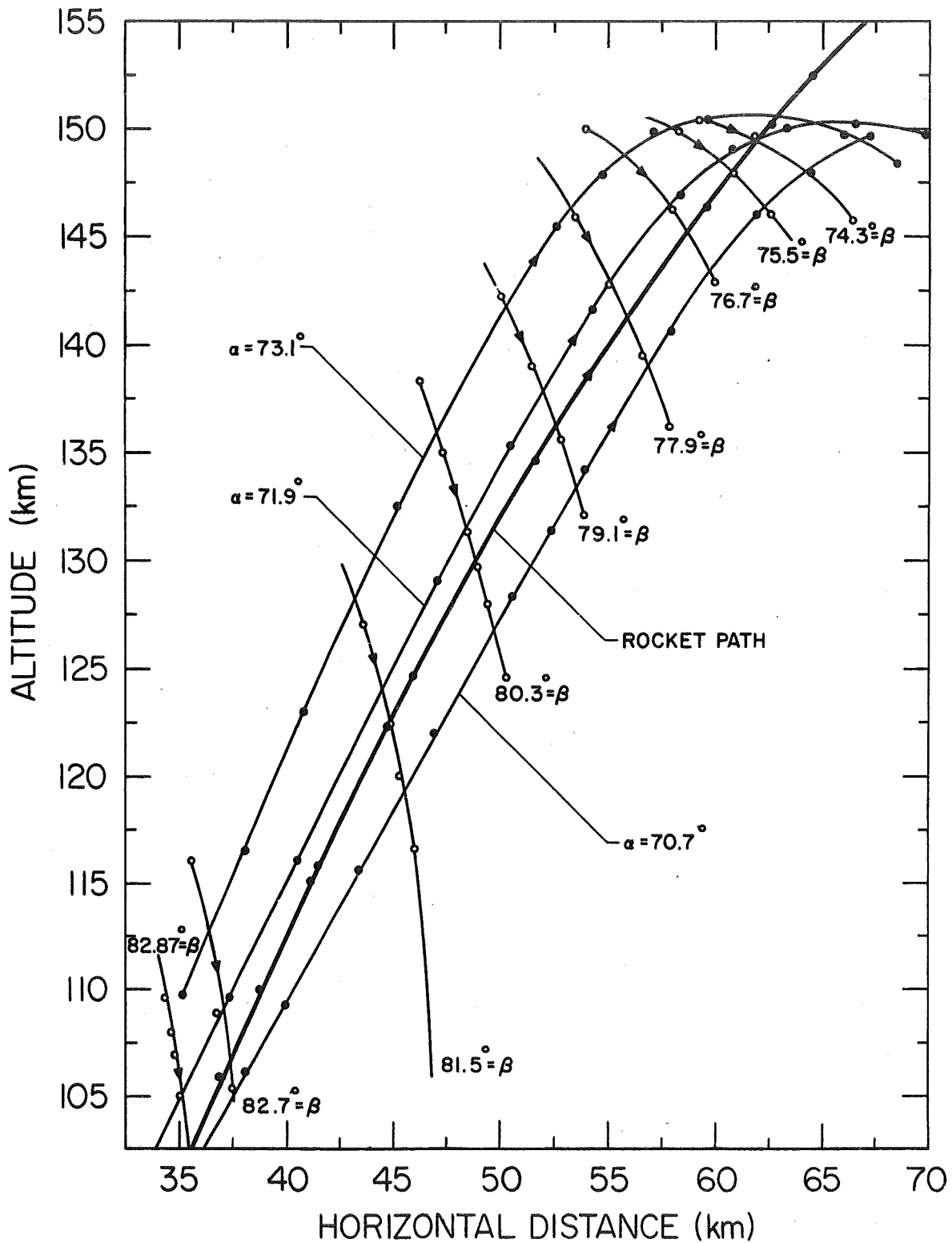


Figure 6.3 Rays traced for a series of take-off angles. Only those sections of the rays which intersect the rocket trajectory are shown. (Nike-Apache 14.143).

7. STANDING-WAVE ANALYSIS, FORMULATED FROM THE APPLETON-HARTREE EQUATION

7.1 Method of Analysis

Monro et al. (1968) have described a method of determining electron densities from standing-wave patterns between the X-ray and O-ray reflection levels. Two waves of the same frequency but differing in wavelength are considered in the r_1 and r_2 directions, Figure 7.1. When the X - Y axes are oriented such that

$$\phi_1 = \tan^{-1} \frac{\mu_1 - \mu_2 \cos \theta}{\mu_2 \sin \theta} \quad (\text{or equivalently } \cos \phi_1 = \frac{\mu_1 \sin \theta}{\mu'}),$$

then the planes of maximum (or minimum) amplitude are normal to Y, the separation between adjacent planes being

$$d = \frac{\lambda_0}{\mu'} \quad (1)$$

where

λ_0 = free space wavelength

μ_1 = refractive index for the direct ray

μ_2 = refractive index for the reflected ray

$$\mu' = (\mu_1^2 + \mu_2^2 - 2\mu_1 \mu_2 \cos \theta)^{1/2}.$$

The distance between standing wave minima observed along the rocket trajectory is given by

$$d' = \frac{d}{\cos \phi_3} \quad (2)$$

From the geometry of Figure 7.1, ϕ_3 is determined by:

$$\phi_1 = \tan^{-1} \frac{\mu_1 - \mu_2 \cos \theta}{\mu_2 \sin \theta} \quad (3)$$

$$\phi_2 = \left(\frac{\pi}{2} - \phi_1\right) + \text{TH1} \quad (4)$$

$$\phi_3 = \phi_2 - \text{PHR} \quad (5)$$

The refractive indices are given by the A-H equation:

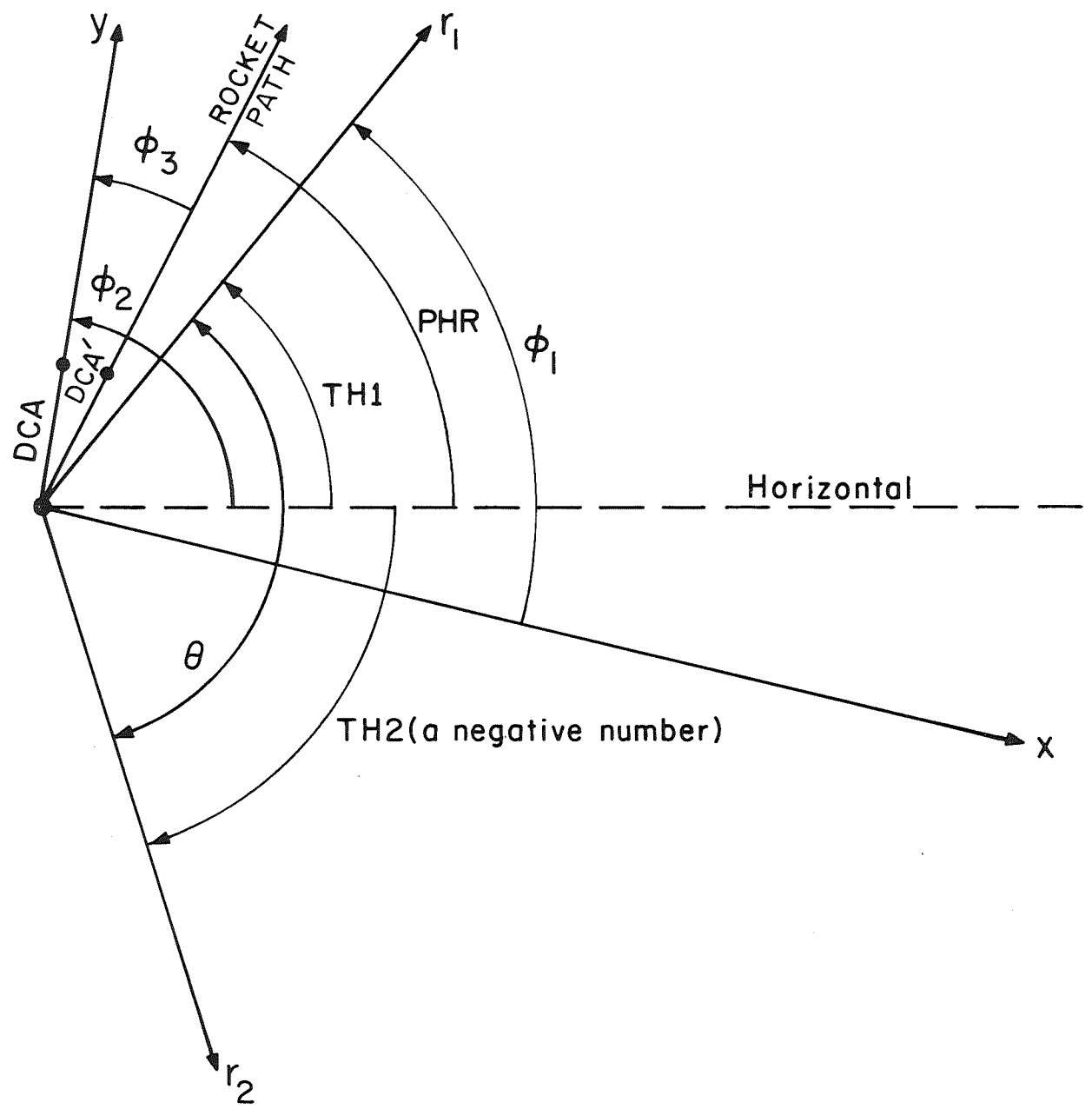


Figure 7.1 Geometry used in the standing analysis, formulated from the Appleton-Hartree equation.

$$\mu_1 = \frac{1 - 2X(1-X)}{2(1-X) - (y_{T_1})^2 + [y_{T_1}^2 + 4 y_{L_1}^2(1-X)^2]^{1/2}} \quad (6)$$

$$\mu_2 = \frac{1 - 2X(1-X)}{2(1-X) - (y_{T_2})^2 + [y_{T_2}^2 + 4 y_{L_2}^2(1-X)^2]^{1/2}} \quad (7)$$

where

$$y_{L_1} = y \text{ C01}$$

$$y_{L_2} = y \text{ C02}$$

$$y_{T_1} = y \text{ ST1}$$

$$y_{T_2} = y \text{ ST2}$$

C01, ST1 = cosine, and sine of the angle between \vec{B} and \vec{r}_1

C02, ST2 = cosine, and sine of the angle between \vec{B} and \vec{r}_2 .

Electron concentrations are deduced from standing wave pattern by an iterative process which calculates the distance between the fades for a given value of X, and keeps changing X until the calculated distance agrees with the observed value.

7.2 Rocket Trajectory

In order to eliminate the necessity of entering rocket coordinates for a large number of heights, the rocket trajectory is calculated internally as follows.

The acceleration, g, due to gravity, is assumed to decrease linearly over a limited height range so that

$$g = g_0 - g_1(h - h_0)$$

where $g = g_0$ at $h = h_0$. This is a good approximation over the standing-wave region.

The equation of vertical motion is then

$$\frac{d^2h}{dt^2} = -g.$$

Solving this and using the boundary conditions that

$$v_h = v_c \text{ and } h = h_1 \text{ at } t = t_0,$$

we get for the time variation of the height of the rocket:

$$h = (h_1 - h_0 - \frac{g_0}{g_1}) g_1^{1/2} \sinh \{g_1^{1/2}(t - t_0)\} + v_c \cosh \{g_1^{1/2}(t - t_0)\}.$$

Variation of horizontal distance with time is given by

$$(x - x_1) = (t - t_0)v_x$$

where $x = x_1$ at $t = t_0$ and v_x is the horizontal velocity (assumed constant).

The coordinates h_1 , x_1 , v_c , v_x at time t_0 (time after launch) are input parameters to the program.

7.3 The Computer Program

The computer program is written in FORTRAN IV and will be discussed with the aid of the flow diagram of Figure 7.2. The names of variables and their definitions are listed in Table 18. Parameters are entered by statements (19-1) to (19-22), next, the trajectory parameters are calculated (19-23) to (19-31) and then part of the magnetoionic parameters are determined (19-32) to (19-46). The input data consisting of propagation angles for the direct and reflected rays and the elapsed time between successive fades are entered at this point (19-47) to (19-53). For each item of the standing-wave data a corresponding electron density is determined in the following manner: First the position and velocity of the rocket is calculated (20-5) to (20-12), next, the observed distance between the fades is determined (20-13) to (20-15), the propagation angles at the rocket are interpolated from the input data (20-16) to (20-26), and then the magnetoionic parameters Y_L and Y_T for the A.H. equation are

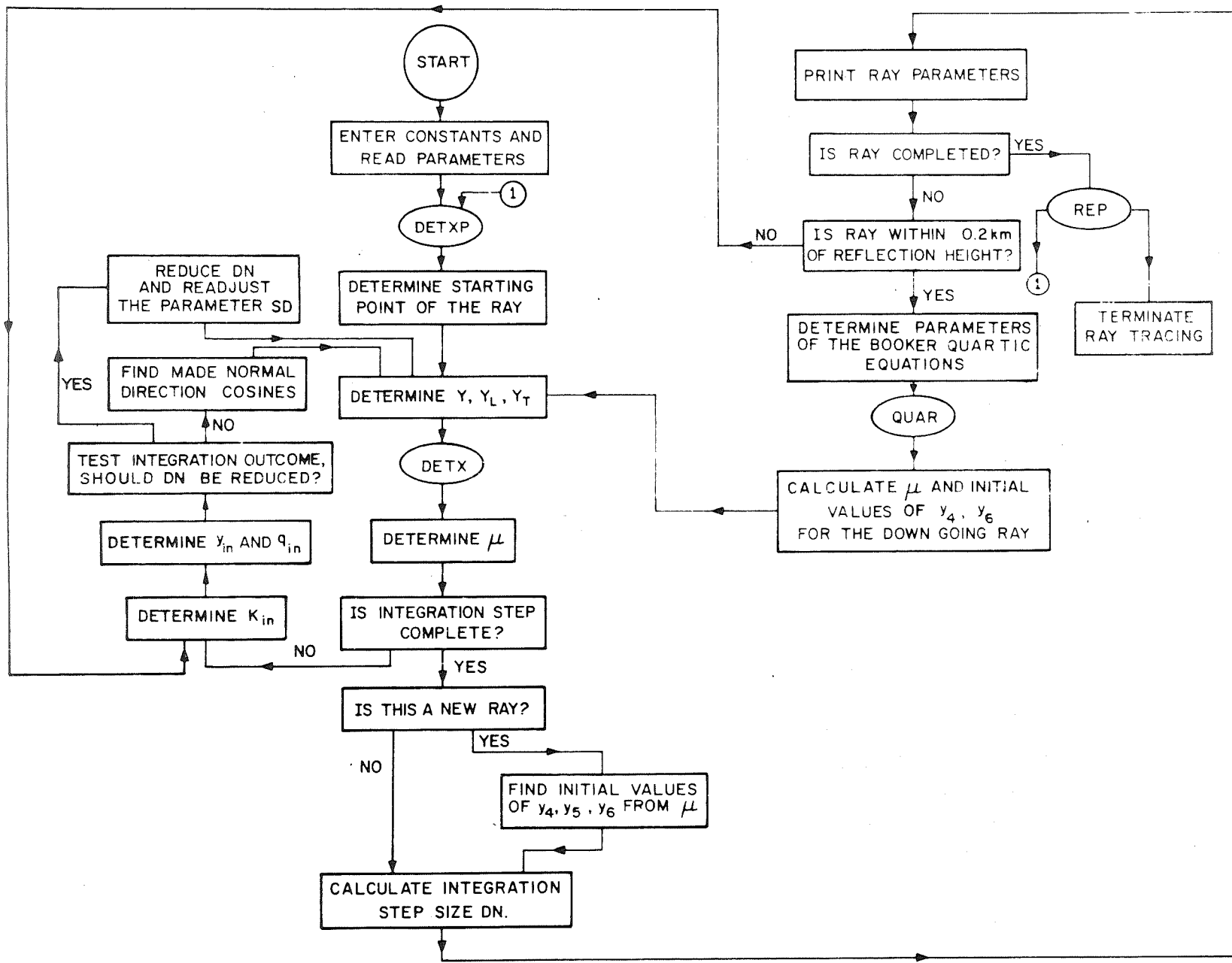


Figure 7.2 Simplified flow diagram for the standing-wave analysis computer program.

TABLE 18

Names of Variables

AZ	Azimuth angle of the propagation plane (an input parameter)
AZI	Geomagnetic azimuth angle of the propagation plane
B1,B2,B3	Direction cosines of \vec{B}
BD,BN,BW	Vertical, northward, and westward components of \vec{B} (input parameters)
BT	Magnitude of \vec{B} (an input parameter)
CLI	$\cos(\text{TH1})$
CL2	$\cos(\text{TH2})$
CN1	$\sin(\text{TH1})$
CN2	$\sin(\text{TH2})$
CO1	Cosine of the angle between the direct ray and \vec{B}
CO2	Cosine of the angle between the reflected ray and \vec{B}
CON	$\pi/180$
DCA	Calculated distance between fades, measured along y-axis
DCA	Calculated distance between fades, measured along the rocket trajectory
DIP	Magnetic dip angle
DM	Distance between fades if both rays were propagating in vacuum
DOB	Observed distance between fades
DX	Increment for changing X
EOM	Charge to mass ratio for electron
ED	Electron density (cm^{-3})
F	Frequency of the propagating wave
FH	
FR	Cosine of the angle between the direct and reflected rays
GO	Acceleration due to gravity, at height H_0
GI	Rate of decrease of gravitational acceleration per meter elevation
HO	See GO
HI	Elevation of the rocket at time T_0
HD(I)	See THD(I)
HU(I)	See THU(I)
N1	Number of points where propagation angles for the direct ray is given (an input parameter)
N2	Number of points where propagation angles for the reflected ray is given (an input parameter)
N3	Number of points where standing wave data is given (an input parameter)
PI	π
PHR	Rocket propagation angle measured from the horizontal axis
ST1	Sine of the angle between the direct ray and \vec{B}
ST2	Sine of the angle between the reflected ray and \vec{B}
SW(I)	Time lapse between two fades, occurring at mean time $T(I)$ (input data)
T(I)	See SW(I)
T0	Initial time (seconds after launch) for calculating the rocket trajectory
TH1	Interpolated value of the direct ray propagation angle at the rocket
TH2	Interpolated value of the reflected ray propagation angle at the rocket
THD(I)	Propagation angle of the reflected ray at a height HD(I) (input data)
THU(I)	Propagation angle of the direct ray at a height HU(I)
UM1	Square of the refractive index of direct ray
UM2	Square of the refractive index of reflected ray

TABLE 18 (Continued)

VC	Vertical velocity of the rocket at time T0 (an input parameter)
VX	Horizontal velocity of the rocket at time T0
WH	Gyrofrequency (rad/sec)
WL	Wave length in vacuum
X	$\omega_n^2/\omega^2 = Ne^2/\epsilon_0 m \omega^2$
X1	Horizontal coordinate of the rocket at time T0 (an input parameter)
XN	Ratio of X to electron density
Y	Ratio of gyrofrequency to propagation frequency
YL1,YL2	Longitudinal component of Y for the direct and reflected rays
YT1,YT2	Transverse component of Y for the direct and reflected rays

TABLE 19

```

1 C  STANDING WAVE ANALYSIS, A-H
2    DIMENSION HU(50),THU(50),HD(50),THD(50),T(200),SW(200)
3 C  ENTER CONSTANTS
4    PI = 3.1415927
5    CON = PI/180.
6    EOM = 1.758796E11
7 C  ENTER PARAMETERS
8    F = 3.385E06
9    BD = 4.894E-05
10   BN = 1.723E-05
11   BW = 3.549E-06
12   BT = 5.201E-05
13   AZ = 94.67
14   N1 = 12
15   N2 = 10
16   N3 = 105
17   T0 = 88.33
18   H1=106742.
19   X1=27726.
20   V0=1139.3
21   VX=367.94
22   H0=152400.
23 C  CALCULATE TRAJECTORY PARAMETERS
24   G0=9.351
25   G1=2.76E-6
26   C1=G0/G1+H0
27   GS=SQRT(G1)
28   C2=((H1-C1)*GS+V0)/2.
29   C3=((C1-H1)*GS+V0)/2.
30   C4=(H1-C1-V0/GS)/2.
31   C5=C2/GS
32 C  CALCULATE PRELIMINARY VARIABLES
33   WL=2.997925F8/F
34   W=2.*PI*F
35   XN = 3.182407E3/W/W
36   DIP = ATAN (BD/SQRT(BN*BN+BW*BW))
37   DEC = ATAN(BW/BN)
38   AZ = CON *AZ
39   AZI = AZ+DEC
40   WH = -BT*EOM
41   Y = WH/W
42   C=COS(DIP)
43 C  DIRECTION COSINES OF FIELD
44   B1 = C*COS(AZI)
45   B2 = C*SIN(AZI)
46   B3 = -SIN(DIP)
47 C  ENTER DATA
48   READ (5,3) (HU(I),THU(I), I=1,N1)
49 3   FORMAT(10F7.0)
50   READ (5,5) (HD(I), THD(I), I=1,N2)
51 5   FORMAT(10F7.0)
52   READ (5,7) (T(I), SW(I),I=1,N3)
53 7   FORMAT(10F7.0)
54 C  CONVERT KM TO M, DEG TO RAD.
55   DO 4 I=1,N1,1
56   HU(I)=HU(I)*1.E3
57 4   THU(I)=THU(I)*CON
58   DO 6 I=1,N2,1
59   HD(I)=HD(I)*1.E3

```

TABLE 20

```

1 6      THD(I)=THD(I)*CON
2 C      PROCESS ALL DATA, N3 CASES.
3        DO 8 M=1,N3,1
4          TD=T(M)
5 C      COMPUTE TRAJECTORY
6 47     EX=GS*(T(M)-T0)
7         E1=EXP(EX)
8         F2=EXP(-EX)
9         ALT=C4*E2+C5*E1+C1
10        VH=C2*E1+C3*E2
11        V=SQRT(VH*VH+VX*VX)
12        PHR=ATAN(VH/VX)
13 C     DISTANCE OBSERVED BETWEEN FADES
14        SPF=-SW(M)
15        DOB=V*SPF
16 C     INTERPOLATE ARRIVAL ANGLES
17        I=1
18 12     IF(HU(I)-ALT)9,10,10
19 9      I=I+1
20        GO TO 12
21 10     J=1
22 14     IF(HD(J)-ALT)11,13,13
23 11     J=J+1
24        GO TO 14
25 13     TH1=THU(I-1)+(ALT-HU(I-1))*(THU(I)-THU(I-1))/(HU(I)-HU(I-1))
26        TH2=THD(J-1)+(ALT-HD(J-1))*(THD(J)-THD(J-1))/(HD(J)-HD(J-1))
27 C     CALCULATE PROPAGATION ANGLES
28        CL1=COS(TH1)
29        CN1=SQRT(1.-CL1*CL1)
30        IF(TH1)15,16,16
31 15     CN1=-CN1
32 16     CL2=COS(TH2)
33        CN2=SQRT(1.-CL2*CL2)
34        IF(TH2)17,18,18
35 17     CN2=-CN2
36 18     C01 = B1*CL1+B3*CN1
37        C02 = B1*CL2+B3*CN2
38        YL1 = Y*C01
39        YL2 = Y*C02
40        S11=SQRT(1.-C01*C01)
41        S12=SQRT(1.-C02*C02)
42        YT1 = Y*S11
43        YT2 = Y*S12
44        YT12=YT1*YT1
45        YT22=YT2*YT2
46        FR=CN1*CN2+CL1*CL2
47        TL=SQRT(1.-FR*FR)
48 C     TEST OF VACUUM FADING DISTANCE
49        DM=WL/SQRT(2.-2.*FR)
50        PH=(TH1+TH2+PI)/2.-PHR
51        DM=DM/COS(PH)
52        IF(DM)52,53,53
53 52     DM=-DM
54 53     IF(DM-DOB)70,70,66
55 66     FD=-1.
56        GO TO 8
57 C     REFRACTIVE INDICES FROM A-H EQS.
58 70     X=.5
59        DX=.11
60        I=1

```

calculated (Y_{L1} , Y_{T1} for the direct ray and Y_{L2} , Y_{T2} for the reflected ray) (20-36) to (20-45). At this point the vacuum fading distance DM (the distance that would be observed between fades if the rays were propagating in vacuum) is calculated and compared with the observed distance DOB (20-48) to (20-56), if the latter is equal to or smaller than the former no meaningful electron density can be calculated and the program goes to the next data. If the observed fading distance is larger than the vacuum fading distance the corresponding electron density is determined by an iterative process which varies X until the observed and calculated distance between the fades agree. The iteration loop starts with calculating the refractive indices for the direct and reflected rays (21-3) to (21-12), next, the distance between the fades DCA is calculated (21-13) to (21-20), and then the observed and calculated distances are compared, if they differ by more than 0.1%, X is incremented (21-21) to (21, 42) and DCA is recalculated, the iteration continues until the observed and calculated distances agree within 0.1 %; then the electron density ED is determined from the final value of X (21-43) and the result printed (21-44). Now the program returns to (20-3) to process the next standing-wave data.

TABLE 21

```

1      J=0
2 32   X1=(1.-X)*2.
3      D1=X1-YT12+SQRT(YT12*YT12+YL1*YL1*X1*X1)
4      D2=X1-YT22+SQRT(YT22*YT22+YL2*YL2*X1*X1)
5      UM1=1.-X*X1/D1
6      UM2=1.-X*X1/D2
7      IF(UM1)33,63,63
8 63   IF(UM2)33,64,64
9 64   ER=2.*FR*SQRT(UM1*UM2)
10     DCA=WL/SQRT(UM1+UM2-ER)
11     UM1S=SQRT(UM1)
12     UM2S=SQRT(UM2)
13     PH=ATAN((UM1S-UM2S*FR)/UM2S/TL)
14     IF(PH)19,20,20
15 19   PH=PH+PI
16 20   PH=TH1-PH+PI/2.
17     PH=PH-PHR
18     DCA=DCA/COS(PH)
19     IF(DCA)60,61,61
20 60   DCA=-DCA
21 C   MATCH CALCULATED AND OBSERVED DISTANCES.
22 61   P=(DCA-DOB)/DOB
23     IF(I-1)21,21,22
24 22   DX=DX/2.
25 21   IF(P)23,24,24
26 23   IF(P+.001)25,26,26
27 25   X=X+DX
28     IF(J-1)27,28,29
29     I=2
30 27   J=2
31 29   IF(X)30,31,31
32 31   IF(X-1.)32,33,33
33 24   IF(P-.001)26,26,35
34 35   X=X-DX
35     IF(J-1)34,29,36
36 36   I=2
37 34   J=1
38     GO TO 29
39 30   DX=DX/2.
40     GO TO 25
41 33   DX=DX/2.
42     GO TO 35
43 26   ED=X/XN
44 8    WRITE(6,67)TD,ALT,ED,DOB,DCA,UM1S,UM2S,PHR
45 67   FORMAT(1P8E15.5)
46     END

```

8. STANDING-WAVE ANALYSIS, FORMULATED FROM THE BOOKER QUARTIC EQUATION

An alternate, but physically equivalent, method of analyzing the standing-wave data makes use of the Booker quartic equation. Roots of the Booker quartic equation (Booker, 1938; Budden, 1961; Kelso, 1964) conveniently relate refractive indices and arrival angles of waves propagating obliquely in a doubly refracting ionosphere.

Assuming horizontal stratification of the ionosphere, we may write Snell's law for the direct and reflected rays respectively as follows.

$$\mu_i \sin \theta_i = \mu_{i0} \sin \theta_{i0} \quad (8.1)$$

$$\mu_r \sin \theta_r = \mu_{r0} \sin \theta_{r0} \quad (8.2)$$

or in the computer notation, defined in Table 22,

$$UI \sin \text{THI} = \sin \text{THIC} \equiv \text{SI}, \quad (8.3)$$

$$\text{and } UR \sin \text{THR} = \sin \text{THRC} \equiv \text{SR}. \quad (8.4)$$

For a trial value of X (if sufficiently close to the correct value), two roots of the Booker equation are pure real numbers, corresponding to the upgoing and downgoing rays, from which refractive indices are obtained by the equations,

$$UI = \sqrt{(\text{QIR})^2 + (\text{SI})^2}, \quad (8.5)$$

$$\text{and } UR = \sqrt{(\text{QRR})^2 + (\text{SR})^2}. \quad (8.6)$$

Then, since THIC and THRC are known, in first approximation, from an initial set of ray tracings,

$$\text{THI} = \arcsin (\text{SI}/\text{UI}), \quad (8.7)$$

$$\text{and } \text{THR} = \arcsin (\text{SR}/\text{UR}). \quad (8.8)$$

Wavelengths in the directions of the direct and reflected rays are also obtained from the refractive indices and the equations,

$$\text{WLI} = C/(F * \text{UI}), \quad (8.9)$$

TABLE 22

Names of Variables

Column No.			
1	2	3	
AL	-	α_i	Coefficient of Booker quartic equation for direct wave
AL	-	α_r	Coefficient of Booker quartic equation for reflected wave
BEI	-	β_i	Coefficient of Booker quartic equation for direct wave
BER	-	β_r	Coefficient of Booker quartic equation for reflected wave
BT	B	B	The total flux density of the geomagnetic field (tesla)
BX	BX	B_x	Geomagnetic field component parallel to trajectory plane (tesla)
BY	BY	B_y	Geomagnetic field component perpendicular to trajectory plane (tesla)
BZ	BA	B_z	Upward component of geomagnetic field (tesla)
C	-	c	Speed of light in vacuum
CI	-	-	cos (THIC)
CR	-	-	cos (THRC)
D	N	N	Electron density (m^{-3})
DEI	-	δ_i	Coefficient of Booker quartic equation for direct wave
DER	-	δ_r	Coefficient of Booker quartic equation for reflected wave
E	-	e	Charge of an electron, a positive number (Coulomb)
EPI	-	ϵ_i	Coefficient of Booker quartic equation for direct wave
EPR	-	ϵ_r	Coefficient of Booker quartic equation for reflected wave
F	F	f	Frequency of propagating wave (Hz)
GAI	-	γ_i	Coefficient of Booker equation for direct wave
GAR	-	γ_r	Coefficient of Booker quartic equation for reflected wave
H	H	z	Altitude above the earth (km)
N	N	n	The number of iterations performed
P	-	ϵ_0	Permittivity of free space (farad/meter)
PH	-	ϕ	Elevation angle from the transmitter to the rocket (radians)
PHD	EL	ϕ	Elevation angle from the transmitter to the rocket (degrees)
QII	-	q_i	A root of the Booker equation, imaginary part, direct wave
QIR	QD	q_i	A root of the Booker equation, real part, direct wave
QRI	-	q_r	A root of the Booker equation, imaginary part, reflected wave
QRR	QR	q_r	A root of the Booker equation, real part, reflected wave
RL	RL	ℓ	Calculated distance along rocket path between minima (meters)
RLO	RLO	-	Observed distance along rocket path between minima (meters)
SI	-	-	sin (THIC)
SR	-	-	sin (THRC)
THI	-	θ_i	Arrival angle of the direct wave (radians)
THIC	-	θ_{i0}	Incident angle of the direct wave (radians)
THICD	THD	θ_{i0}	Incident angle of the direct wave (degrees)
THID	PSD	θ_i	Arrival angle of the direct wave (degrees)
THR	-	θ_r	Arrival angle of the reflected wave (radians)
THRC	-	θ_{r0}	Incident angle of the reflected wave (radians)
THRCD	THR	θ_{r0}	Incident angle of the reflected wave (degrees)
THRD	PSR	θ_r	Arrival angle of the reflected wave (degrees)
UI	MUD	μ_i	Refractive index of the direct wave

TABLE 22 (Continued)

UR	MUR	μ_r	Refractive index of the reflected wave
V	-	-	$e/2\pi fm$
VC	-	c	Speed of light in vacuum (meter/second)
W	-	m	Mass of the electron (kilogram)
WLI	WLD	λ_i	Wavelength of the direct wave (meters)
WLR	WLR	λ_r	Wavelength of the reflected wave (meters)
X	X	X	Square of the ratio of plasma frequency to wave frequency
Y	-	Y	Ratio of gyrofrequency to wave frequency

Column No. 1 lists the names of variables used in the source program (Tables 23, 24, and 25, Column No. 2 lists corresponding names used to identify output variables, and Column No. 3 lists conventional mathematical symbols corresponding to the variables of Columns 1 and 2. Definitions of each of the variables follow to the right.

$$\text{and WLR} = C/(F * UR). \quad (8.10)$$

Constant phase fronts of the direct and reflected waves are represented in Figure 8.1 as they intersect in the vicinity of the rocket and establish surfaces of minimum field strength. The geometry of Figure 8.1 dictates the following sequence of equations:

$$SH = \sin (THR + THI) \quad (8.11)$$

$$CH = \cos (THR + THI) \quad (8.12)$$

$$SOL = \frac{WLI}{SH} \quad (8.13)$$

$$SSH = \frac{WLR}{SH} \quad (8.14)$$

$$DIAG = \{SOL^2 + SSH^2 + 2 SOL * SSH * CH\}^{1/2} \quad (8.15)$$

$$\cos TA \equiv CTA = \frac{SOL^2 + DIAG^2 - SSH^2}{2 * DIAG * SOL} \quad (8.16)$$

$$\sin TA \equiv STA = \sqrt{1 - CTA ** 2} \quad (8.17)$$

$$TA = \arctan \frac{STA}{CTA} \quad (8.18)$$

$$S = SOL * STA \quad (8.19)$$

$$SRL = \sin(PH + TA - THR) \quad (8.20)$$

$$RL = \frac{S}{SRL} \quad (8.21)$$

The calculated value of RL, the distance along the rocket path between two adjacent minima of signal strength, is of principal interest. A correct value of X will bring RL into agreement with the observed distance between minima, RLO. The trial value of X is changed until

$$(RLO - RL)/RLO \leq 0.001, \quad (8.22)$$

An iteration equation, for changing the value of X until (8.22) is satisfied, is derived as follows. For the ordinary mode of propagation,

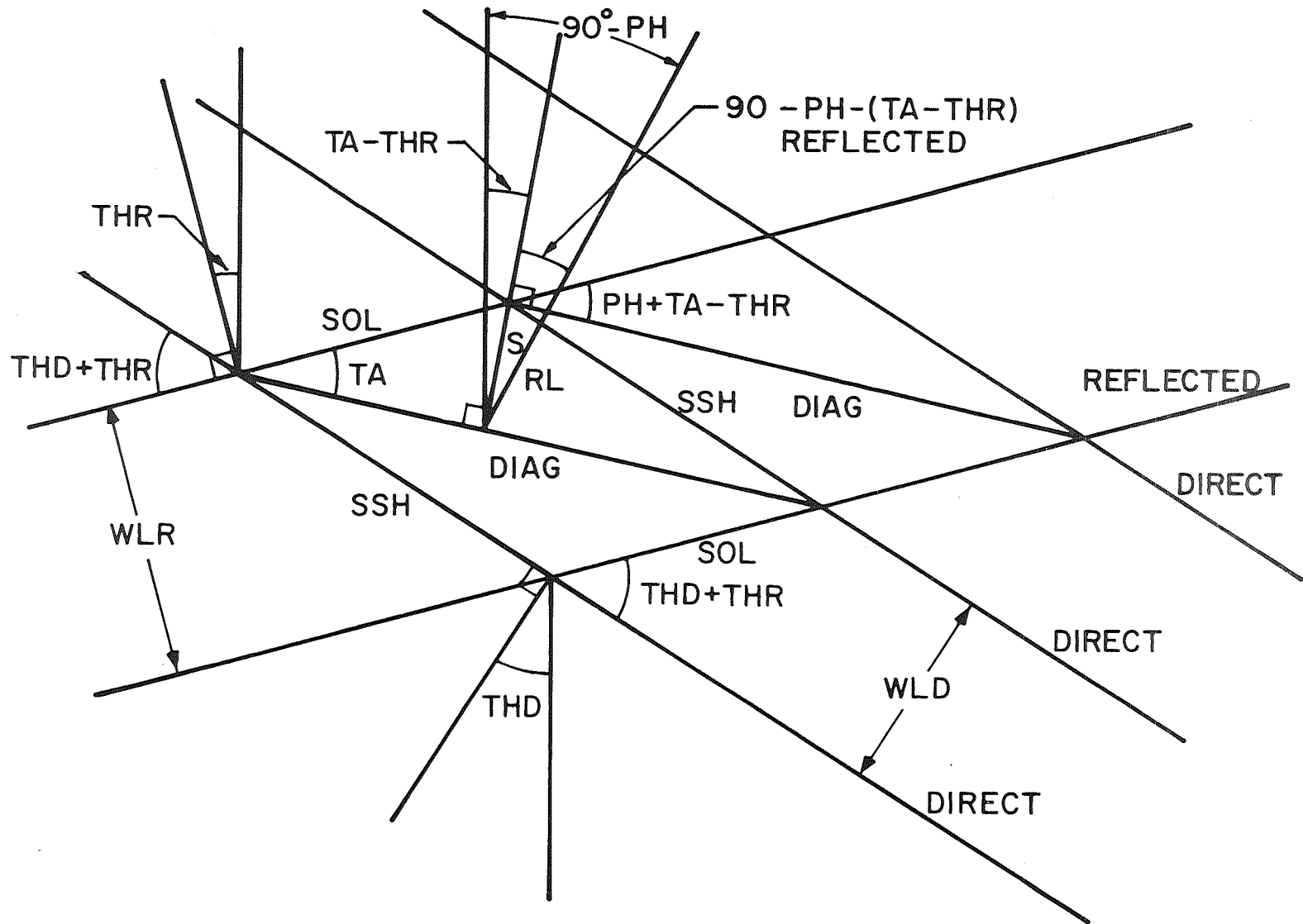


Figure 8.1 Intersection of direct and reflected waves.

$$\lambda = \frac{c}{f} \frac{1}{\mu} = \frac{c}{f} \left[1 - \frac{X}{1 + Y_L} \right]^{1/2}. \quad (8.23)$$

Thus, changes in λ and X are related by the equation

$$\Delta\lambda = \frac{c}{2f} \frac{\Delta X}{(1 + Y_L) \left(1 - \frac{X}{1 + Y_L} \right)^{3/2}} \quad (8.24)$$

From general studies of standing waves in the E region, we have observed that

$$RL \approx 1/2 \lambda \quad \text{or} \quad \Delta(RL) \approx 1/2 \Delta\lambda \quad (8.25)$$

where λ is the wavelength of the direct or upgoing wave. According to this approximation, the change in X required to produce a desired change in RL is given by the equation

$$\Delta X = \frac{4f(1+Y_L)}{c} \left(1 - \frac{X}{1+Y_L} \right)^{3/2} (RLO - RL) \quad (8.26)$$

The FORTRAN equivalent of (8.26), which iterates X until (8.22) is satisfied, is

$$X = 2. * F/VC * (1.+YL) * (1. - X/(1.+YL)) ** 1.5 * (RLO-RL) + X \quad (8.27)$$

The convergence of RL to RLO was found to improve by a change of the numerical coefficient 4 of (8.26) to 2 of (8.27). This reduction in the magnitude of X tends to limit "over correction" of RL . Convergence is usually attained in three to five iterations.

However, difficulties are encountered when X is made to advance into the forbidden region, $X > 1$, by the iteration equation, (8.27). Under these circumstances, X is forced to remain less than one by repeated subtraction of the number 0.01, but convergence is still not attained in some cases. If convergence is not achieved, RL is calculated and plotted as a function of X . The value of X corresponding to RLO is then read from the graph.

Values of electron density, obtained by the procedure described above, are used for a second ray tracing to calculate more accurate angles of incidence for

the direct and reflected rays. These, in turn, are used in (8.5) through (8.27) to obtain more correct values of X and of electron density.

The changes of incidence and arrival angles resulting from a third ray tracing are generally less than $\pm 1^\circ$, and have a relatively insignificant influence on the calculated values of electron density. Numerical calculations of variations are illustrated graphically in Figure 8.2. From these calculations we conclude that:

1. A 1° positive change in the value of THI results in an 0.5% decrease in electron density, at most.
2. A 1° positive change in the value of THR results in a 1.5% decrease in electron density, at most.
3. A 1% positive change in the value of the quantity, RLO, results in a 3% increase in electron density.

It is evident that the limiting factor is the accuracy of the observed distances between fades, RLO, and not the determination of arrival angles. A third ray tracing cannot improve the accuracy of electron densities.

The FORTRAN IV program which implements (8.5) through (8.27) is listed in Tables 23, 24, and 25. Parameters for a given rocket shot are entered by Statements (23-10) to (23-18). Altitude, incident angles of the direct and reflected rays, and the observed spacing of fades are entered for each data point by (23-19). Coefficients of the Booker equation are calculated by (23-44) through (24-22). Equations (8.5) through (8.21) are represented by Statements (24-24) through (24-46), and the iteration and output statements follow from (24-47) through (25-11).

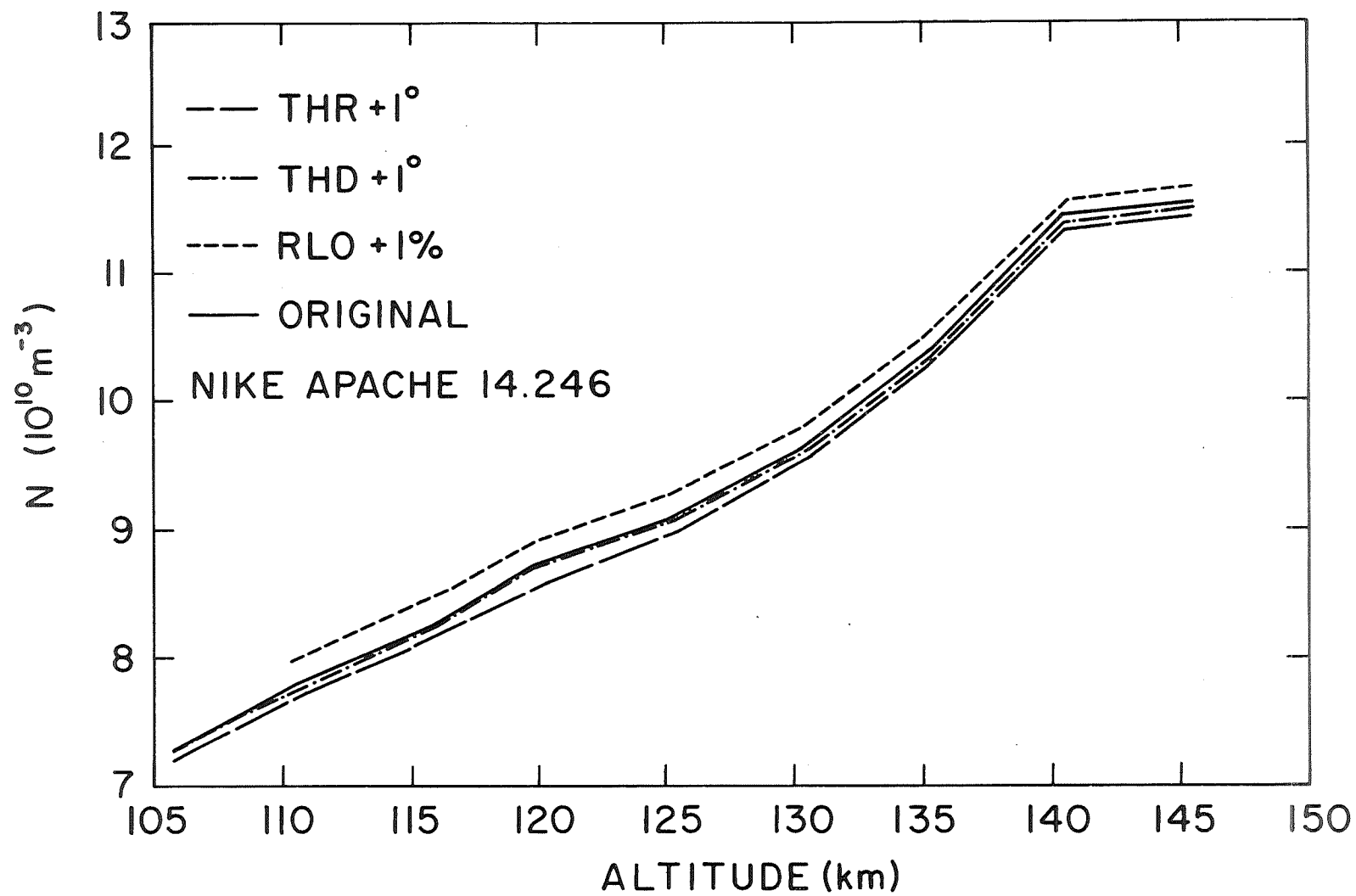


Figure 8.2 Variation of electron density with parameters of the analysis.

TABLE 23

```

1  C  STANDING WAVE ANALYSIS BY BOOKER QUARTIC
2      DIMENSION R(8)
3  C  READ CONSTANTS
4      E=1.602E-19
5      P=8.8542E-12
6      W=9.1091E-31
7      VC=2.9979E8
8      PI=3.1415927
9      RD = 1.74533E-02
10 C  READ PARAMETERS
11     PHD = 7.7599E01
12     AZD = 1.5895E02
13     BZ = 1.3369E-05
14     BS = -2.2563E-05
15     BE = -2.5310E-06
16     BT = 2.6348E-05
17     YL = -6.9662E-02
18     F = 3.3850E06
19 21  READ(5,42) H,THRC,THIC,RLO
20 42  FORMAT (4F10.2)
21 C  CONVERT DEGREES TO RADIANS
22     PH=PHD*RD
23     THIC=THICD*RD
24     THRC=THRC*RD
25     AZ = AZD*RD
26 C  COMPUTE INTERMEDIATE VARIABLES
27     BX = BS*SIN(AZ)+BE*COS(AZ)
28     BY = -BS*COS(AZ)+BE*SIN(AZ)
29     V = 1.758796E11/2./PI/F
30     YX = V*BX
31     YY = V*BY
32     YZ = V*BZ
33     Y = V*BT
34     AK=E*E/((W*F**2)*(P*4.*PI**2))
35     X=.7
36     N=0
37 35  N=N+1
38     IF(N-35) 36,38,38
39 36  CONTINUE
40     SI = SIN(THIC)
41     SR = SIN(THRC)
42     CI = COS(THIC)
43     CR = COS(THRC)
44 C  CALCULATE COEFFICIENTS OF QUARTIC EQUATION
45     AL=(1.-Y**2)-X*(1.-YZ**2)
46     BEI=SI*X*YY*YZ*2.
47     BER=SR*X*YY*YZ*2.
48     GAI=(CI**2*Y**2-(1.-X)*(CI**2-X))*2.-X*(YX**2+CI**2*YY**2+(1.+CI
49 1**2)*YZ**2)
50     GAR=(CR**2*Y**2-(1.-X)*(CR**2-X))*2.-X*(YX**2+CR**2*YY**2
51 1+(1.+CR**2)*YZ**2)
52     DEI=-CI**2*SI*YZ*YY*2.
53     DER=-CR**2*SR*YZ*YY*2.
54     EPI=(1.-X)*((CI**2-X)**2-YY**2*CI**4)-CI**2*(CI**2
55 1-X)*(YX**2+YZ**2)
56     EPR=(1.-X)*((CR**2-X)**2-YY**2*CR**4)-CR**2*(CR**2-X)*(YX**2
57 1+YZ**2)
58 C  FIND ROOTS OF QUARTIC EQUATION FOR DIRECT WAVE
59     CALL QUAR(AL,BEI,GAI,DEI,EPI,R)
60     IF(R(8)) 5,3,5

```

TABLE 24

```

1   3   QD=R(7)
2   GO TO 61
3   5   IF (R(6)) 6,14,6
4   14  QD=R(5)
5   GO TO 61
6   6   IF(R(4)) 7,2,7
7   2   QD=R(3)
8   GO TO 61
9   7   QD=R(1)
10  61  CONTINUE
11  C   FIND ROOTS OF QUARTIC EQUATION FOR REFLECTED WAVE
12  CALL QUAR(AL,BER,GAR,DER,EPR,R)
13  IF(R(2))17,16,17
14  16  QR=R(1)
15  GO TO 91
16  17  IF(R(4)) 19,18,19
17  18  QR=R(3)
18  GO TO 91
19  19  IF(R(6)) 22,20,22
20  20  QR=R(5)
21  GO TO 91
22  22  QR=R(7)
23  91  CONTINUE
24  C   CALCULATE DIRECT WAVE INDEX, ARRIVAL ANGLE, AND WAVELENGTH
25  8   UI=SQRT(QD**2+SI*SI)
26  THI=ARSIN(SI/UI)
27  THID=THI/RD
28  WLI=VC/F/UI
29  C   CALCULATE REFLECTED WAVE INDEX, ARRIVAL ANGLE, AND WAVELENGTH
30  UR=SQRT(QR**2+SR*SR)
31  THR=ARSIN(SR/UR)
32  THRD=THR/RD
33  WLR=VC/F/UR
34  C   COMPUTE SUPERPOSITION OF DIRECT AND REFLECTED WAVES
35  4   SH = SIN(THR+THI)
36  CH = COS(THR+THI)
37  SOL=WLI/SH
38  SSH=WLR/SH
39  DIAG2=SOL**2+SSH**2+CH*SOL*SSH*2.
40  DIAG=SQRT(DIAG2)
41  CTA = (SOL**2+DIAG2-SSH**2)/(DIAG*SOL*2.)
42  STA=SQRT(1.-CTA**2)
43  TA =ATAN(STA/CTA)
44  S=SOL*STA
45  SRL = SIN(PH+TA-THR)
46  RL=S/SRL
47  IF(ABS((RLO-RL)/RLO)-0.001) 38,38,39
48  38  CONTINUE
49  C   PRINT VALUES OF IMPORTANT QUANTITIES
50  WRITE(6,71)F,PHD,D,X,H
51  WRITE(6,72)BT,BX,BY,BZ
52  WRITE(6,73)THICD,THID,THRCD,THRD
53  WRITE(6,74)QD,QR,UI,UR
54  WRITE(6,75)WLI,WLR,S,RL
55  WRITE(6,76)SOL,SSH,N,RLO
56  GO TO 21
57  39  X = 2.*F/VC*(1.+YL)*(1.-X/(1.+YL))**1.5*(RLO-RL)+X
58  25  IF (X-0.98) 70,70,80
59  80  X = X-0.01
60  GO TO 25

```

TABLE 25

```
1 70 CONTINUE
2   D=X/AK
3 71 FORMAT(6H F =1PE10.3,5X,5H EL =OPF5.1,10X,5H N =1PE10.3,5X,
4   15H X =OPF6.3,6X,4HH = F6.2)
5 72 FORMAT(6H B =1PE14.7,1X,5H BX =1PE14.7,1X,5H BY =1PE14.7,1X,
6   15H BZ =1PE14.7)
7 73 FORMAT(6H THD =F6.3,9X,5HPSD =F6.3,9X,5HTHR =F5.1,10X,
8   15HPSR =F5.1)
9 74 FORMAT(6H QD =F6.4,9X,5H QR =F7.4,8X,5HMUD =F6.4,9X,5HMUR =F6.4)
10 75 FORMAT(6H WLD =F6.1,9X,5HWLR =F6.1,9X,5H S =F6.1,9X,5H RL =F6.2)
11 76 FORMAT(6H SOL =F6.1,9X,5SSH =F6.1,10X,4H N =I2,13X,5HRLO =F6.2/)
12 GO TO 35
13 END
```


REFERENCES

- Booker, H. G. (1938), Propagation of wave-packets incident obliquely upon a stratified doubly refracting ionosphere, *Phil. Trans. A* 237, 411-450.
- Bowhill, S. A. (1965), IQSY rocket studies of the C, D, and E regions of the ionosphere, NASA University Program Review Conference, NASA Special Publication SP-85 (U. S. Govt. Printing Office, Washington, D. C.), 269-275.
- Budden, K. G. (1961), *Radio Waves in the Ionosphere*, Cambridge University Press, Great Britain.
- Cain, J. C., S. J. Hendricks, R. A. Langel and W. V. Hudson (1967), A proposed model for the international geomagnetic reference field-1965, *J. Geomagn. Geoelect.* 19, 335-355.
- Cain, S. H., J. C. Cain, W. E. Daniels and D. C. Jensen (1968), Computation of the main geomagnetic field from spherical harmonic expansions, NSSDC 68-11, National Space Science Data Center, Greenbelt, Maryland.
- Gill, S. (1951), A process for the step-by-step integration of differential equations in an automatic digital computing machine, *Proc. Camb. Phil. Soc.* 47, 96-108.
- Hara, E. H. (1963), Approximations to the semiconductor integrals $C_p(x)$ and $D_p(x)$ for use with the generalized Appleton-Hartree magnetoionic formulas, *J. Geophys. Res.* 68, 4388-4389.
- Haselgrove, J. (1954), Ray theory and a new method of ray tracing, *The Physics of the Ionosphere*, The Physics Society, London, 355-364.
- Kelso, J. M. (1964), *Radio Ray Propagation in the Ionosphere*, McGraw-Hill Book Co., New York.
- Knoebel, H. W. and D. O. Skaperdas (1966), Rocket measurements of Faraday rotation and differential absorption, *Rev. Sci. Instr.* 37, 1395-1400.
- Mechtly, E. A., S. A. Bowhill, L. G. Smith and H. W. Knoebel (1967), Lower ionosphere electron concentration and collision frequency from rocket measurements of Faraday rotation, Differential absorption, and probe current, *J. Geophys. Res.* 72, 5239-5245.
- Mechtly, E. A. (1969), The international system of units, physical constants, and conversion factors, NASA SP 7012, revised, U. S. Govt. Printing Office.
- Monro, P. E., E. A. Mechtly and R. S. Sastry (1968), Analysis of ordinary mode standing waves observed by rockets in the E-region, *J. Geophys. Res.* 73, 4291-4297.

- Phelps, A. V. and J. L. Pack (1959), Electron collision frequencies in nitrogen and in the lower ionosphere, *Phys. Rev. Letters* 3, 340-342.
- Sen, H. K. and A. A. Wyller (1960), On the generalization of the Appleton-Hartree magnetoionic formulas, *J. Geophys. Res.* 65, 3931-3950.
- Taylor, B. N., W. H. Parker and D. N. Langenberg (1969), Determination of e/h , using macroscopic quantum phase coherence in superconductors: implications for quantum electrodynamics and the fundamental physical constants, *Rev. of Modern Physics* 41, 375-496.