ASRL TR 160-1

ANALYSIS OF FOLDING OF BLADDER STRUCTURES

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1. Introduction

Using a bladder or diaphragm as a means of obtaining positive expulsion of a propellant from a tank has attractive features. The volumetric efficiency, the fraction of total volume alloted to the propellant storage and expulsion system taken up by the propellant itself, is high, perhaps a maximum in comparison with other expulsion devices.* If the thin, flexible, shell is designed correctly, the expulsion efficiency, the fraction of total volume of propellant stored which is expelled, can be as high as 1.0. Correct design can also mean that a relatively low pressure differential is required to force the shell through the expulsion process. Furthermore, if either the shell material can withstand repeated folding or suitable control schemes are employed to achieve a desired deformation pattern restricting the extent of folding, then recycling of the bladder or diaphragm would be possible. Whether folding can be tolerated or not depends primarily on the material of the shell. The propellant's chemical properties determine, for the most part, the choice of material. Since the propellant may be stored for lengthy periods of time before expulsion, consideration must be given to the possibility of permeation of propellant through the shell walls or degradation of the shell material properties.

The design of a propellant expulsion bladder or diaphragm satisfying the aforementioned criteria (high volumetric and expulsion efficiency, low pressure differential driving force, recycle capability, propellant-material compatibility) remains a formidable problem. Most design to date has been via the seat of the pants. This is not surprising since the complexity of the deformation process experienced by an expulsion device of this type makes any rational analysis exceedingly difficult.

A glance at the collapsed configuration of a Teflon film, Teflon felt, aluminum foil, layered, hemispherical bladder, shown in Fig. 4 of Ref. 2, indicates the arbitrary, nonlinear nature of the deformation experience by this type of expulsion device. Other tests indicate that a bladder or diaphragm,

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*For a comprehensive survey of the characteristics of a more general range of expulsion devices, see Reference 1.

initially in the form of a shell of revolution, the convoluted hemispherical diaphragm for example (Ref. 3), does not deform in an axisymmetric way. At some point in the expulsion process nonaxisymmetric deformation is initiated, leading to the generation of creases, wrinkles, single or double folds. Even if the material can withstand a limited amount of folding or creasing, this so-called, "random deformation" may result in pockets of trapped propellant or premature closing of the exit port(s). In either case, the anticipated expulsion efficiency of the device may not be achieved. If the shell must be made of metal in order to satisfy the propellant-material compatibility requirement, then one preliminary analysis (Ref. 4) indicates that the occurrence of a double fold will lead to failure on the first expulsion cycle.

In order to establish a rational procedure for the design of expulsion devices, a coordinated program of appropriate analysis and testing, one must attempt to achieve some understanding of the fundamental mechanisms involved in the deformation process. In this investigation, attention is directed toward the behavior of the reversing diaphragm initially in the form of a shell of revolution. The hemispherical diaphragm shown in Fig. 1 is one possible configuration.

Observation of tests performed on a diaphragm of this kind, leads to the following characterization of the deformation process:

(1) Initially, the shell deforms in an axisymmetric fashion. A shallow portion near the apex becomes concave upwards.

(2) At some point, nonaxisymmetric deformation initiates.

(3) These wrinkles grow into folds in what appears to be a nondeterministic process as the diaphragm collapses.

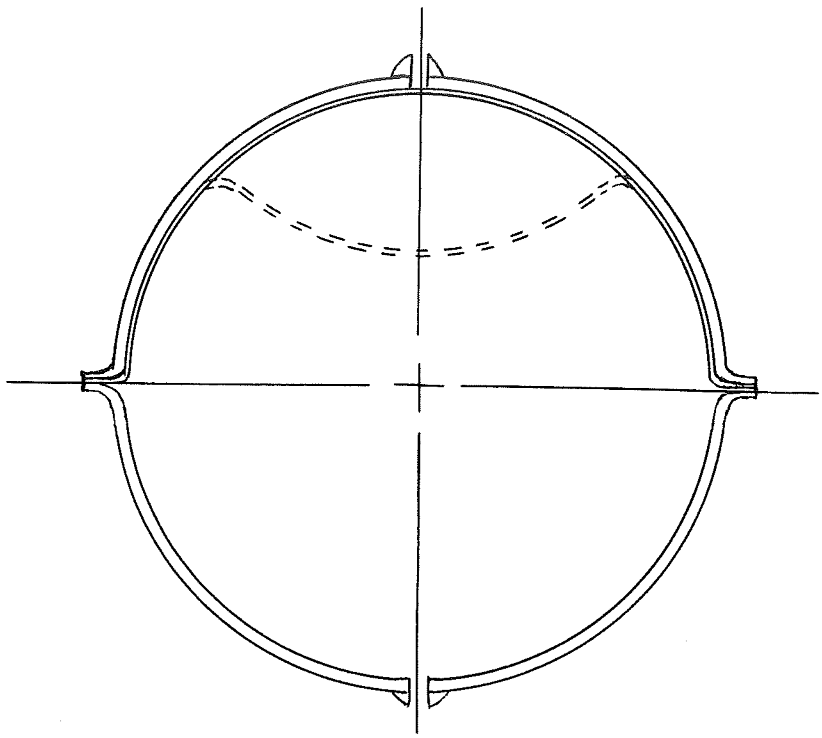

FIG. 1 REVERSING HEMISPHERICAL DIAPHRAGM

With this picture in mind, we may pose the following series of questions. What pressure differential is required to drive the shell through the axisymmetric stages of deformation? What mechanism is the cause of the initiation of nonaxisymmetric deformation? What is the extent of folding, i.e., single folds, double folds? Is recycling a possibility? What is the extent of plastic flow? When and where does it occur? What is the primary failure mechanism?

In order to evaluate a particular design, one would like to have available a set of appropriate analytical and experimental techniques which would provide answers to these questions in a definitive and quantitative way. These techniques should describe the effect of: variations in shell geometry and material properties, the introduction of stiffners, layering of different materials, etc., on the performance of the diaphragm.

The main thrust of this investigation has been to attempt to answer the first two questions listed above and to establish the necessary analytical techniques required to predict the effect of variations in design parameters on these answers. As such, this work represents a first step in the understanding of the deformation process experienced by the reversing diaphragm; a first element of a systematic plan for the determination of the efficiency and operational life of such metallic and nonmetallic propellant expulsion devices as bladders and diaphragms.

2. Analysis of the Axisymmetric Deformation of a Radially-Constrained, Spherical Shell under External Pressure

In this section, attention is directed to the problem involving the determination of the axisymmetric equilibrium configuration of a reversing diaphragm in the state shown in Fig. 2.1. Appropriate equations of equilibrium and compatibility of deformation have been deduced by E. Reissner (Refs. 5, 6). We show that this system of equations admits the possibility of the existence of an edge zone in the vicinity of the point where the detached portion of the shell comes in contact with the restraining tank surface. On either side of this edge zone, a solution is easily obtained. In the edge zone, however, the nonlinear system of equations describing the behavior of the shell, although simpler in form than the complete, general system, demand considerable effort in order to obtain a solution.

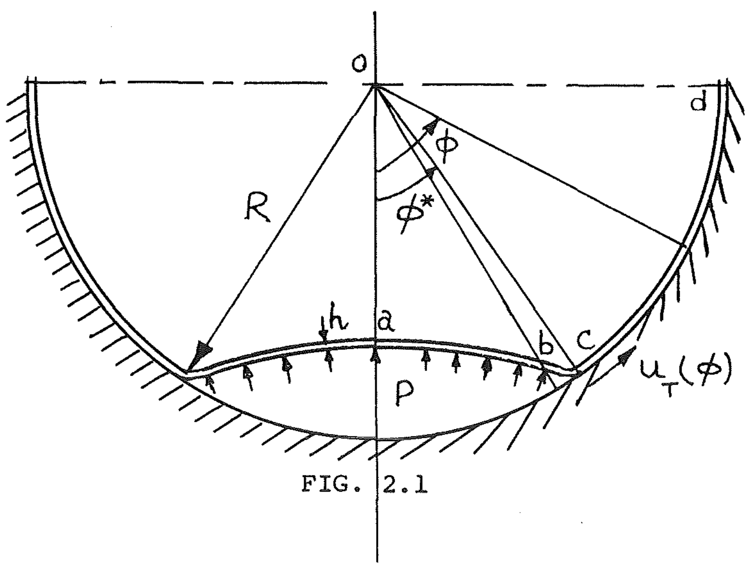

FIG. 2.1

2.1 Membrane Behavior - "Attached" Portion

Since the displacement normal to the midsurface of the shell must vanish along the attached portion, cd, the bending moment and transverse shear likewise are zero in this region. The shell behaves then as a membrane. Equilibrium of an element of the shell is satisfied if:

$$\frac{d}{d\phi}(N_{11} \sin\phi) - N_{22} \cos\phi = 0 \tag{2.1.1a}$$

$$\frac{N_{11} + N_{22}}{R} + P_3 = 0 \tag{2.1.1b}$$

where N_{11} is the meridional stress resultant, N_{22}, the circumferential stress resultant and P_3 the reactive normal force per unit area exerted on the shell by the constraining tank.

The stress-strain relations are

$$N_{11} = \frac{Eh}{(1-\nu^2)} (\varepsilon_{11} + \nu \varepsilon_{22}) \tag{2.1.2a}$$

$$N_{22} = \frac{Eh}{(1-\nu^2)} (\varepsilon_{22} + \nu \varepsilon_{11}) \tag{2.1.2b}$$

and the strain displacement relations are

$$\varepsilon_{11} = \frac{1}{R} \frac{du_T}{d\phi} \tag{2.1.3a}$$

$$\varepsilon_{22} = \frac{1}{R} \cot\phi \; u_T \tag{2.1.3b}$$

The first equilibrium equation, in terms of the tangential displacement, u_T, using Eqs. (2.1.2) and (2.1.3) becomes

$$\frac{d^2 u_T}{d\phi^2} + \frac{\cos\phi}{\sin\phi}\frac{d u_T}{d\phi} - \left(\frac{\cos^2\phi}{\sin^2\phi} + \nu\right) u_T = 0 \qquad (2.1.4)$$

The boundary conditions on u_T may be taken as:

at $\phi = \frac{\pi}{2}$ $\qquad u_T = 0$ (2.1.5a)

at $\phi = \phi^*$ $\qquad u_T = \Delta$ (assumed prescribed) (2.1.5b)

The solution to this problem was obtained by a regular series expansion about $\phi = \pi/2$. (See Ref. 7 for the details of this solution procedure.)

Having $u_T(\phi)$, the stress resultants N_{11} and N_{22} are obtained from Eqs. (2.1.2) and (2.1.3). In terms of u_T, we have

$$N_{11} = \frac{Eh}{(1-\nu^2)}\frac{1}{R}\left[\frac{du_T}{d\phi} + \nu u_T \cot\phi\right]$$

$$N_{22} = \frac{Eh}{(1-\nu^2)}\frac{1}{R}\left[u_T \cot\phi + \nu\frac{du_T}{d\phi}\right]$$

Of particular interest is the effective stiffness of this constrained portion as ϕ^* varies. Plotted in Fig. 2.2 is the way in which this stiffness, as measured by $(N_{11}/Eh)/(\Delta/R)$, changes as ϕ^* ranges from 15° to 90°. Also plotted is the ratio $(N_{22}/Eh)/(\Delta/R)$. Note that the nondimensionalized value of the meridional stress resultant N_{11}^*/Eh is of order Δ/R. (Recall that Δ is the tangential displacement at ϕ^*.)

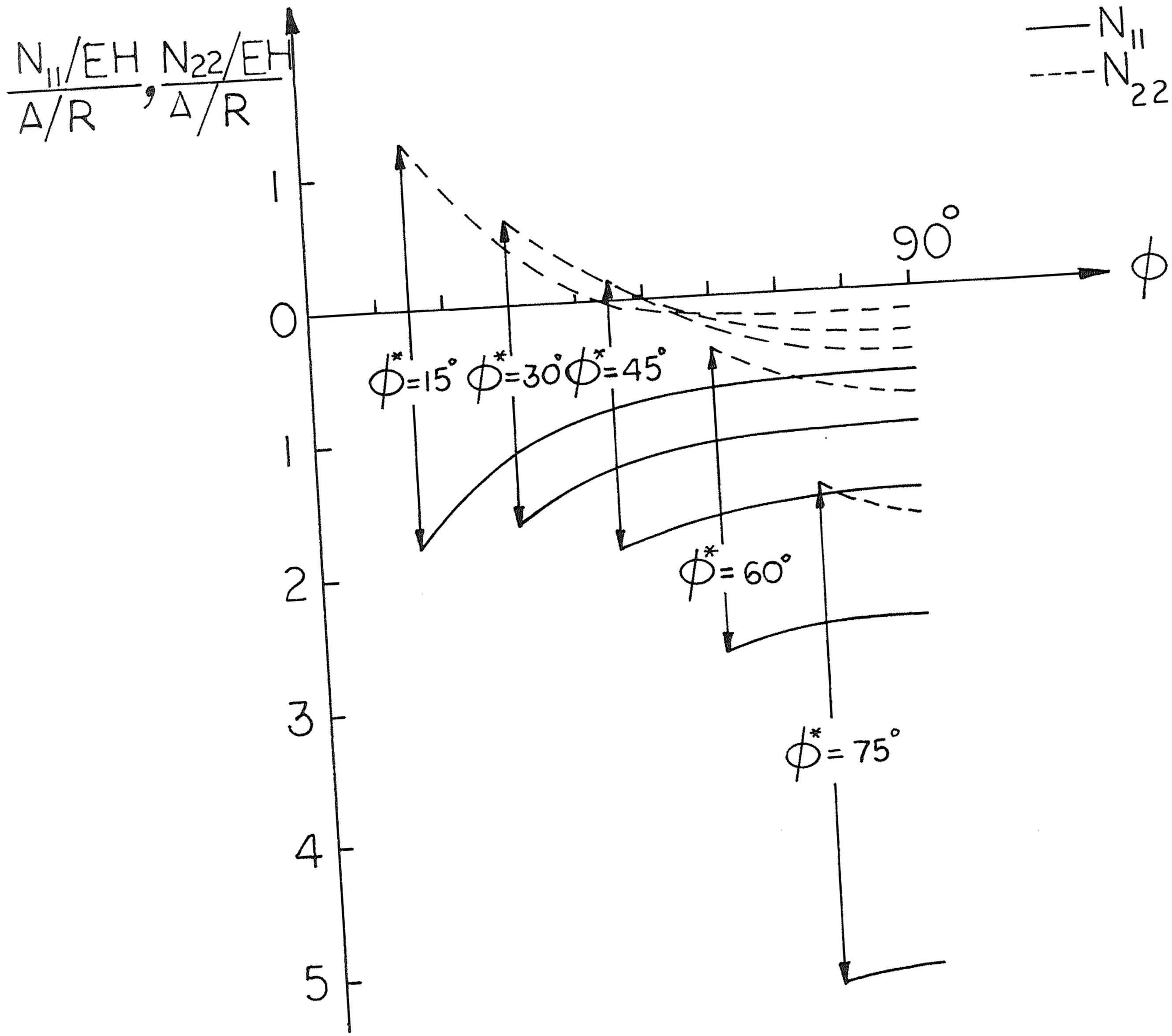

FIG. 2.2

2.2 Behavior of "Detached" Portion

For the detached portion of the shell, (ac), Reissner's equations apply for this case of finite displacements and rotations. The solution of this fourth order system of two coupled, nonlinear, differential equations presents a formidable problem. It is shown in the following that considerable simplification is achieved by pursuing an asymptotic solution.

For a spherical shell, under external pressure P, Reissner's equations may be put into the form:

Equilibrium

$$[\sin\phi \, V]' = -PR \sin\phi \cos\Phi \tag{2.2.1a}$$

$$[\sin\phi \, H]' - N_\theta - PR \sin\phi \sin\Phi = 0 \tag{2.2.1b}$$

$$[\sin\phi \, M_\xi]' - M_\theta \cos\Phi - R \sin\phi (V \cos\Phi - H \sin\Phi) = 0 \tag{2.2.1c}$$

Stress-Strain

(2.2.2a) $$\varepsilon_\xi = \frac{1}{Eh}(N_\xi - \nu N_\theta), \quad \varepsilon_\theta = \frac{1}{Eh}(N_\theta - \nu N_\xi)$$ (2.2.2b)

(2.2.2c) $$M_\xi = D(\kappa_\xi + \nu \kappa_\theta), \quad M_\theta = D(\kappa_\theta + \nu \kappa_\xi)$$ (2.2.2d)

Strain-Displacement

(2.2.3a) $$\varepsilon_\xi = \frac{u'}{R\cos\Phi} + \frac{\cos\phi - \cos\Phi}{\cos\Phi}, \quad \varepsilon_\theta = \frac{u}{R\sin\phi}$$ (2.2.3b)

(2.2.3c) $$\kappa_\xi = -\frac{1}{R}(\Phi' - 1), \quad \kappa_\theta = -\frac{(\sin\Phi - \sin\phi)}{R\sin\phi}$$ (2.2.3d)

Compatibility of Deformation

$$\left[\sin\phi\,\varepsilon_{\theta}\right]' = \varepsilon_{\xi}\cos\Phi + \left(\cos\Phi - \cos\phi\right) \qquad (2.2.4)$$

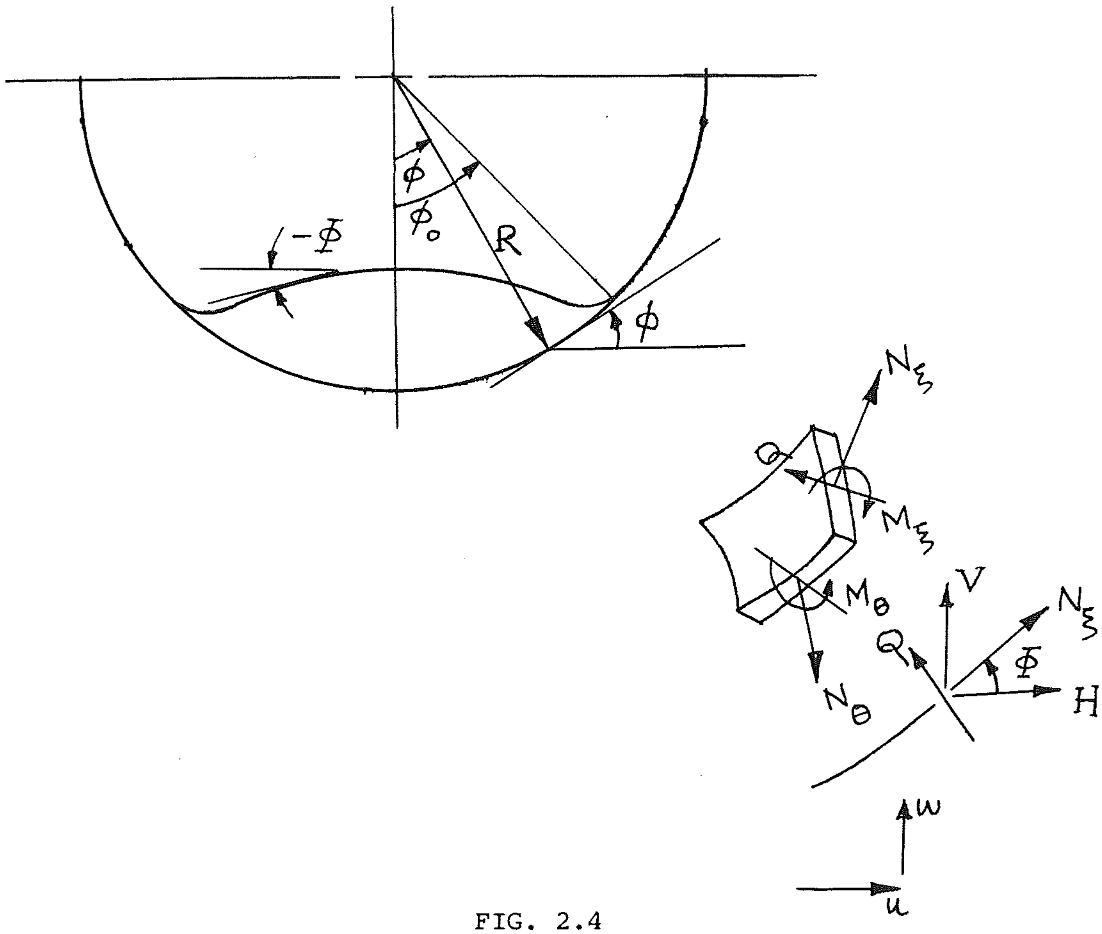

FIG. 2.4

In the above system, (refer to Fig. 2.4), V and H are vertical and horizontal stress resultants, respectively. They are related to the membrane stress resultant N_ξ and transverse shear Q via

$$H = N_\xi \cos\Phi - Q\sin\Phi \qquad (2.2.5a)$$

$$V = N_\xi \sin\Phi + Q\cos\Phi \qquad (2.2.5b)$$

Prime denotes differentiation with respect to the independent variable ϕ, the slope of the meridional in the undeformed, spherical state. Φ is this slope in the deformed state. R is the radius of the sphere, h its thickness; E, Young's modulus; ν, Poisson's ratio and D is the shell bending stiffness $D = Eh^3/12(1 - \nu^2)$.

As shown in Refs. 5 and 6, this system of equations may be reduced to two coupled, second-order nonlinear differential equations for the horizontal stress resultant, H, and the slope of the meridional tangent in the deformed state, Φ. For our spherical shell, these two equations become:

$$\left[\sin\phi\left\{(\sin\phi\, H)' - PR\sin\phi\sin\Phi - \nu(H\cos\Phi + V\sin\Phi)\right\}\right]'$$
$$- \left[H\cos\Phi + V\sin\Phi - \nu\left\{(\sin\phi\, H)' - PR\sin\phi\sin\Phi\right\}\right]\cos\Phi$$
$$= Eh(\cos\Phi - \cos\phi) \tag{2.2.6a}$$

$$-\left[\sin\phi\left(\Phi' - 1 + \nu\,\frac{\sin\Phi - \sin\phi}{\sin\phi}\right)\right]'$$
$$+ \left[\frac{\sin\Phi - \sin\phi}{\sin\phi} + \nu(\Phi' - 1)\right]\cos\Phi$$
$$= \frac{R^2}{D}\sin\phi\,(V\cos\Phi - H\sin\Phi) \tag{2.2.6b}$$

<u>2.2.1 Detached Portion - Far-Field Solution</u>

For the detached portion of the shell, region ab in Fig. 2.1, the far field solution (the state of affairs away from the edge zone) is obtained by considering the relative orders of magnitudes of terms in our system of equations. An appropriate balance of terms, leading to the existence of an edge zone, is obtained via the following set of assumptions.

(i) Assume, in the far field, that derivatives with respect to ϕ are of order 1.0.

(ii) From Eq. (2.2.1a), V is of order P.R. Since, for a spherical shell under external pressure, the buckling load, p, is of order $E(h^2/R^2)$, we assume $P(R^2/Eh^2)$ is of order 1.0 at most.

(iii) Assume H is of order Eh(h/R). It will be seen that this yields an appropriate balance of terms and the existence of an edge zone.

Dividing Eq. (2.2.6a) through by the order of magnitude of H, we see that all terms on the left-hand side of this relationship are of order 1.0 at most, whereas the right-hand side is of order R/h. Similarly, all terms on the left-hand side of Eq. (2.2.6b) are of order 1.0, whereas the right-hand side is, again, of order R/h.

This implies that, in the far field away from the edge zone, the stress and deformation state of the shell is described approximately by

$$\cos \Phi - \cos \phi = 0 \tag{2.2.7a}$$

$$H \sin \Phi - V \cos \Phi = 0 \tag{2.2.7b}$$

The solution to the first of these two equations we take as

$$\Phi = -\phi \tag{2.2.8}$$

That is, in the detached portion away from the edge zone, the configuration of the shell is a reflection of its undeformed state about a plane perpendicular to the radius 0 a and passing through b.

With this, the state of stress, strain and deformation within this portion of the shell is found to be:

$$V = -\frac{PR}{2}\sin\phi \qquad N_\theta = N_\xi = +\frac{PR}{2}$$
$$H = +\frac{PR}{2}\cos\phi$$

$$\varepsilon_\theta = \frac{PR}{2Eh}(1-\nu) \quad , \quad \varepsilon_\xi = \frac{PR}{2Eh}(1-\nu)$$

$$K_\theta = \frac{2}{R} \quad , \quad K_\xi = \frac{2}{R}$$

$$M_\theta = \frac{2D}{R}(1+\nu) \quad , \quad M_\xi = \frac{2D}{R}(1+\nu)$$

$$u = -\frac{PR^2}{2Eh}(\nu-1)\sin\phi$$

As far as the contribution of the membrane stress resultants is concerned, we see that we have a state of stress equivalent to a spherical shell under <u>internal</u> pressure.

2.2.2 Detached Portion - Edge-Zone Formulation

It is obvious that the far field solution cannot fit the required conditions at that point where the shell comes in contact with the restraining tank. In order to make things right, we assume that there exists an edge zone within which derivatives with respect to ϕ are of order $\sqrt{R/h}$. This restores the presence of the highest derivatives in Eqs. (2.2.6a) and (2.2.6b), which, in turn, enables us to satisfy the appropriate boundary conditions at the contact point.

Thus, in the edge zone we have, noting that $\sin\phi \simeq \sin\phi_o$ and $\cos\phi \simeq \cos\phi_o$, in this region.

$$\sin^2\phi_o \, H'' = Eh(\cos\Phi - \cos\phi_o) \tag{2.2.9a}$$

$$\Phi'' = -\frac{R^2}{D}(V\cos\Phi - H\sin\Phi) \tag{2.2.9b}$$

These are the same equations developed by E. Reissner (Ref. 8) in obtaining nonlinear influence coefficients for edge loading of deep shells of revolution.

Proceeding now to a nondimensionalization of the edge zone problem, we set

$$\zeta = \lambda(\phi_0 - \phi) \tag{2.2.10a}$$

where

$$\lambda^2 = \frac{R \sqrt{12(1-\nu^2)}}{2h} . = O\left(\frac{R}{h}\right) \tag{2.2.10b}$$

$$H = g_0\, g(\zeta) \quad , \quad g_0 = Eh \frac{h}{R\sqrt{12(1-\nu^2)}\ \sin\phi_0} \tag{2.2.11a}$$

and

$$p^* = \frac{pR^2\sqrt{12(1-\nu^2)}\ \sin^2\phi_0}{2\,Eh^2} \tag{2.2.11b}$$

With these, our system of equations becomes

$$\frac{d^2 g}{d\zeta^2} - \frac{2}{\sin\phi_0}\left(\cos\Phi - \cos\phi\right) = 0 \tag{2.2.12a}$$

$$\frac{d^2 \Phi}{d\zeta^2} - \frac{2}{\sin\phi_0}\left(p^*\cos\Phi + g\sin\Phi\right) = 0 \tag{2.2.12b}$$

The complete statement of the edge zone problem, requires that boundary conditions be prescribed.

Since these conditions will involve the horizontal displacement, u, bending moment M_ξ and membrane stress resultants N_ξ and N_θ, we must consider their role in this edge zone system.

From Eq. (2.2.1b) we must have

$$N_\theta = \sin\phi_0 \, \lambda \, g_0 \frac{dg}{d\xi} = O\left(Eh\sqrt{\frac{h}{R}}\right)$$

From Eqs. (2.2.5a) and (2.2.5b) we have

$$N_\xi = g_0\left(g \cos\Phi - p^* \sin\Phi\right) = O\left(Eh \cdot \frac{h}{R}\right)$$

From Eq. (2.2.2b), then

$$\varepsilon_\theta = \frac{\sin\phi_0 \, \lambda \, g_0}{Eh} \, \frac{dg}{d\xi} = O\left(\sqrt{\frac{h}{R}}\right)$$

From Eq. (2.2.3b), it follows

$$u = \frac{R}{Eh} \sin^2\phi_0 \, \lambda \, g_0 \cdot \frac{dg}{d\xi} = O\left(\sqrt{hR}\right)$$

With regard to the bending moments, we have from (2.2.3c) and (2.2.3d)

$$K_\xi = + \frac{\lambda}{R} \frac{d\Phi}{d\xi} = O\left(\frac{1}{\sqrt{Rh}}\right)$$

$$K_\theta = \frac{-\left(\sin\Phi - \sin\phi_0\right)}{R \sin\phi_0} = O\left(\frac{1}{R}\right)$$

From (2.2.3a) then

$$M_\xi = D \frac{\lambda}{R} \frac{d\Phi}{d\xi} = O\left(\frac{Eh^3}{\sqrt{Rh}}\right) = O\left(Eh^2\sqrt{\frac{h}{R}}\right)$$

Recall that away from the edge zone, the far field solution yields

$$u = O(h)$$

$$M_\xi = O\left(Eh^2 \cdot \frac{h}{R}\right)$$

Requiring that the edge zone solution match the far field solution as ζ approaches infinity, we obtain

$$\Phi \rightarrow -\phi_0 \tag{2.2.13a}$$

$$\frac{dg}{d\zeta} \rightarrow 0 \quad \text{or} \quad \frac{d\Phi}{d\zeta} = 0 \tag{2.2.13b}$$

At the other boundary, $\zeta=0$, at that point where the shell comes into contact with the restraining tank wall, we require that the membrane stress resultant be continuous across the contact point ($\zeta=+\varepsilon$ to $\zeta=-\varepsilon$)

$$N_{\xi^+} = N_{\xi^-}$$

We also require continuity of displacement

$$u_+ = u_-$$

and finally, that Φ and Φ' be continuous.

This last requirement yields

$$\frac{d\Phi}{d\zeta} = 0 \quad , \quad \zeta = 0$$

since, everywhere within the attached portion this quantity is of order $1/\lambda = \sqrt{h/R}$.

Now, as indicated by Fig. 2.2

$$u_- = O\left(\frac{R}{Eh} N_{\xi^-}\right)$$

With the above, this may be written as

$$u_- = O\left(\frac{R}{Eh} N_{\xi^+}\right) = O(h)$$

Thus we conclude that, at $\zeta = 0$

$$\frac{dg}{d\zeta} = 0$$

Summarizing, we have as our problem the solution of the fourth order system

$$\frac{d^2 g}{d\zeta^2} - \frac{2}{\sin\phi_o}\left(\cos\Phi - \cos\phi_o\right) = 0 \qquad (2.2.12a)$$

$$\frac{d^2 \Phi}{d\zeta^2} - \frac{2}{\sin\phi_o}\left(P^*\cos\Phi + g\sin\Phi\right) = 0 \qquad (2.2.12b)$$

with the following boundary conditions

As $\zeta \to +\infty$

$$\Phi \to -\phi_o \qquad (2.2.13a)$$

$$\frac{dg}{d\zeta} \to 0 \quad \text{or} \quad \frac{d\Phi}{d\zeta} \to 0 \qquad (2.2.13b)$$

At $\zeta = 0$

$$\Phi = +\phi_o \qquad (2.2.14a)$$

$$\frac{dg}{d\zeta} = 0 \qquad (2.2.14b)$$

$$\frac{d\Phi}{d\zeta} = 0 \qquad (2.2.14c)$$

Note that five boundary conditions are required since ϕ_o is not known a priori. (We assume P* is prescribed.)

2.2.3 Solution of the Edge Zone Problem

In solving the system of equations (2.2.12a) through (2.2.14c), one would expect to obtain a relationship between the pressure differential P* and the contact angle ϕ_o. In the following we show that consistent with the order of approximation of our asymptotic procedure, P* is zero. Proof of this follows from the generation of a first integral of our system.

This first integral is obtained by multiplying Eq. (2.2.12a) by $dg/d\zeta$; Eq. (2.2.12b) by $d\Phi/d\zeta$ and forming the difference. In this way we obtain

$$\left(\frac{d\Phi}{d\zeta}\right)^2 - \left(\frac{dg}{d\zeta}\right)^2 + \frac{4}{\sin\phi_o}\left[g(\cos\Phi - \cos\phi_o) + P^*\sin\Phi\right] = C \qquad (2.2.15)$$

where C is a constant of integration.

Applying the boundary conditions at $\zeta = 0$, Eqs. we obtain

$$+4P^* = C$$

On the other hand, as $\zeta \rightarrow \infty$, we expect $d\Phi/d\zeta$ and $dg/d\zeta$ to approach zero with Φ approaching $-\phi_o$. This yields

$$-4P^* = C$$

These last two relationships imply

$$C = P^* = 0$$

Apparently, then, in order to have equilibrium configurations of the form displayed in Fig. 1, P* is not of order 1.0. Alternatively, PR^2/Eh^2 is of order $\sqrt{h/R}$ at most.

The solution procedure employed has been presented in Ref. 9. Of interest here are the results obtained. Having obtained Φ and g as functions of ζ, stress and moment resultants and the displacement $u(\zeta)$ are given by the following:

$$N_\xi = \frac{Eh^2}{\sqrt{12(1-\nu^2)}} \frac{\cos\Phi(\zeta)}{R\sin\phi_o} g(\zeta) \qquad (2.2.16a)$$

$$N_\theta = -\frac{\lambda}{R}\frac{Eh^2}{\sqrt{12(1-\nu^2)}} \cdot \frac{dg}{d\zeta} \qquad (2.2.16b)$$

$$M_\xi = \frac{D\lambda}{R} \cdot \frac{d\Phi}{d\zeta} \qquad (2.2.17)$$

$$u(\zeta) = -\sin\phi_o\left(\frac{DR^2}{4Eh}\right)^{1/4} \frac{dg}{d\zeta} \qquad (2.2.18)$$

Where, as before

$$\lambda^2 = \frac{R}{2}\left(\frac{Eh}{D}\right)^{1/2}$$

The results of this analysis are displayed in Figs. 2.4 through 2.11.

In the first three figures are plotted nondimensional values of N_ξ, N_θ, and M_ξ as functions of ζ for various values of separation angle ϕ_o. In

$$n_\xi = \frac{N_\xi}{Eh^2/\sqrt{12(1-\nu^2)}\,R} = g(\varsigma)\left(\cot\phi_o \cos\beta - \sin\beta\right)$$

$$n_\theta = \frac{N_\theta}{\lambda\; Eh^2/\sqrt{12(1-\nu^2)}\,R} = \frac{dg}{d\varsigma}$$

$$m_\xi = \frac{M_\xi}{\frac{D\lambda}{R}} = +\frac{d\Phi}{d\varsigma}$$

Observe that critical stress conditions are due to the action of n_θ and m_β, i.e.,

$$\sigma\Big|_{N_\theta} = O\left(\frac{N_\theta}{h}\right) = O\left(E\sqrt{\frac{h}{R}}\right)$$

$$\sigma\Big|_{M_\xi} = O\left(\frac{M_\xi}{h^2}\right) = O\left(E\sqrt{\frac{h}{R}}\right)$$

$$\sigma\Big|_{N_\xi} = O\left(\frac{N_\xi}{h}\right) = O\left(E\cdot\frac{h}{R}\right) << \sigma\Big|_{M_\xi}$$

The deformed configuration of the shell in the vicinity of the contact point for various values of ϕ_o is shown in the remaining five figures.

A cautionary note must be introduced here. In the formulation of the problem, it was assumed that ϕ_o was of order 1.0. If ϕ_o is a smaller order of magnitude, the problem must be reformulated within the framework of shallow shell theory. Thus, the validity of the results of the analysis for small ϕ_o; e.g., $\phi_o = 10°$, is subject to question.

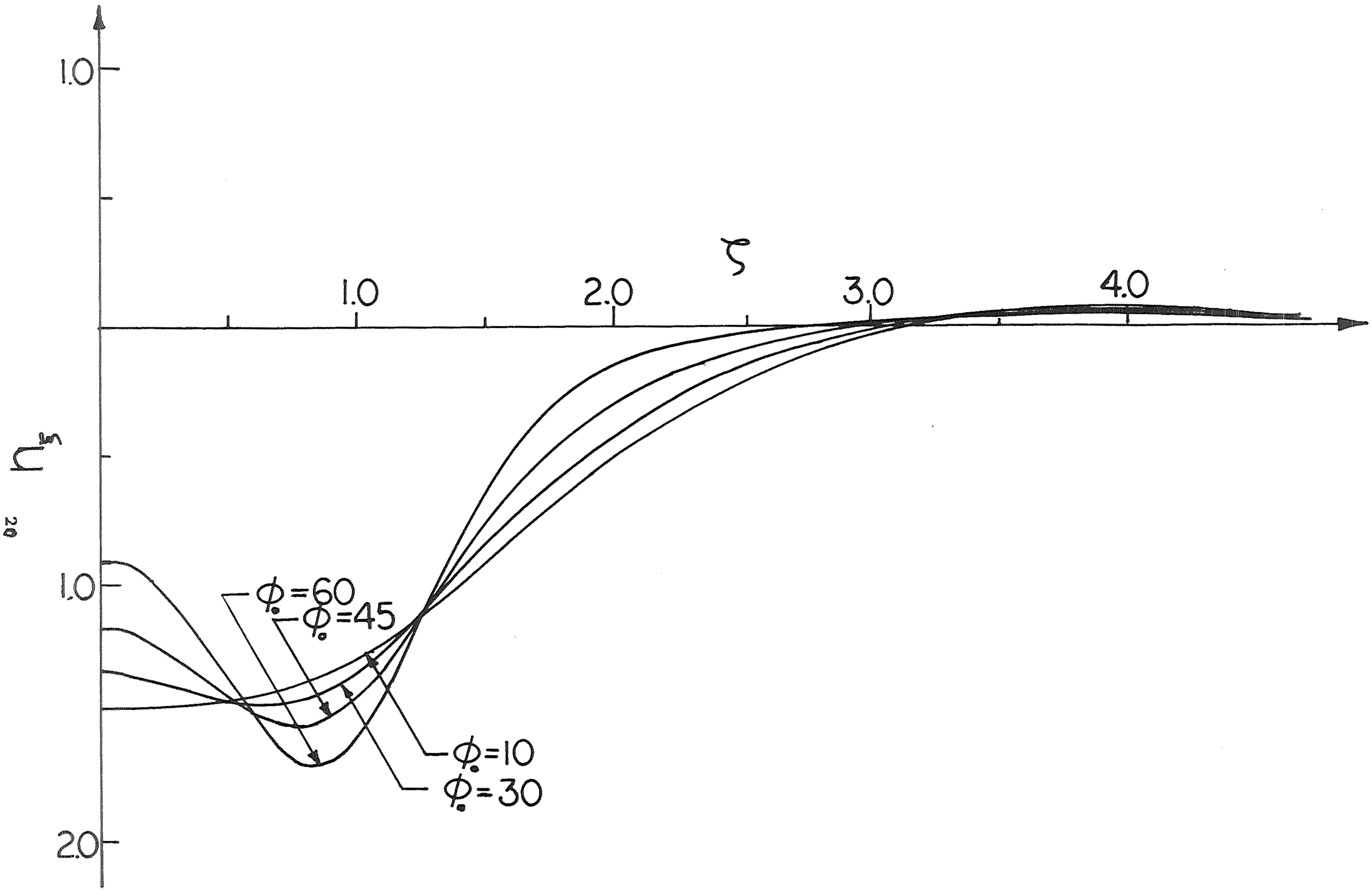

FIG..2.4 NONDIMENSIONALIZED MERIDIONAL STRESS-RESULTANT

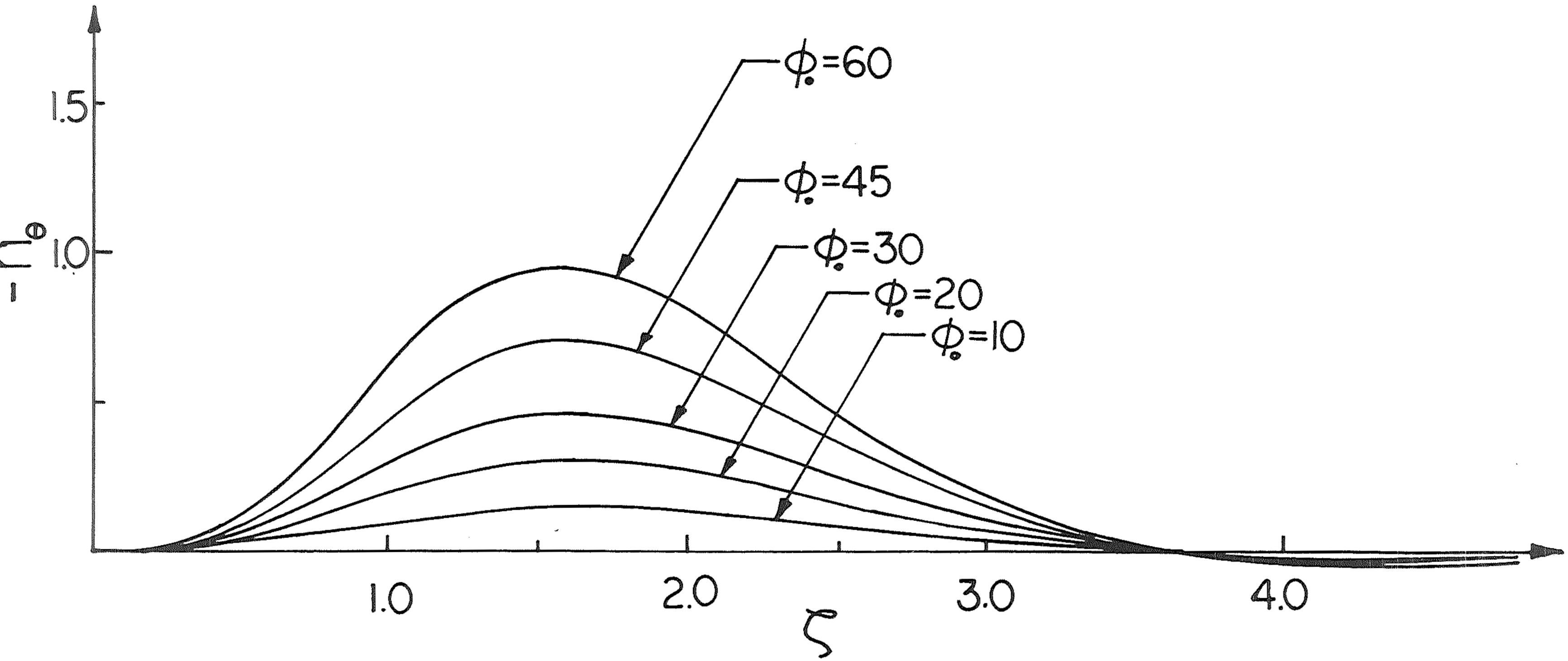

FIG. 2.5 NONDIMENSIONALIZED CIRCUMFERENTIAL STRESS-RESULTANT

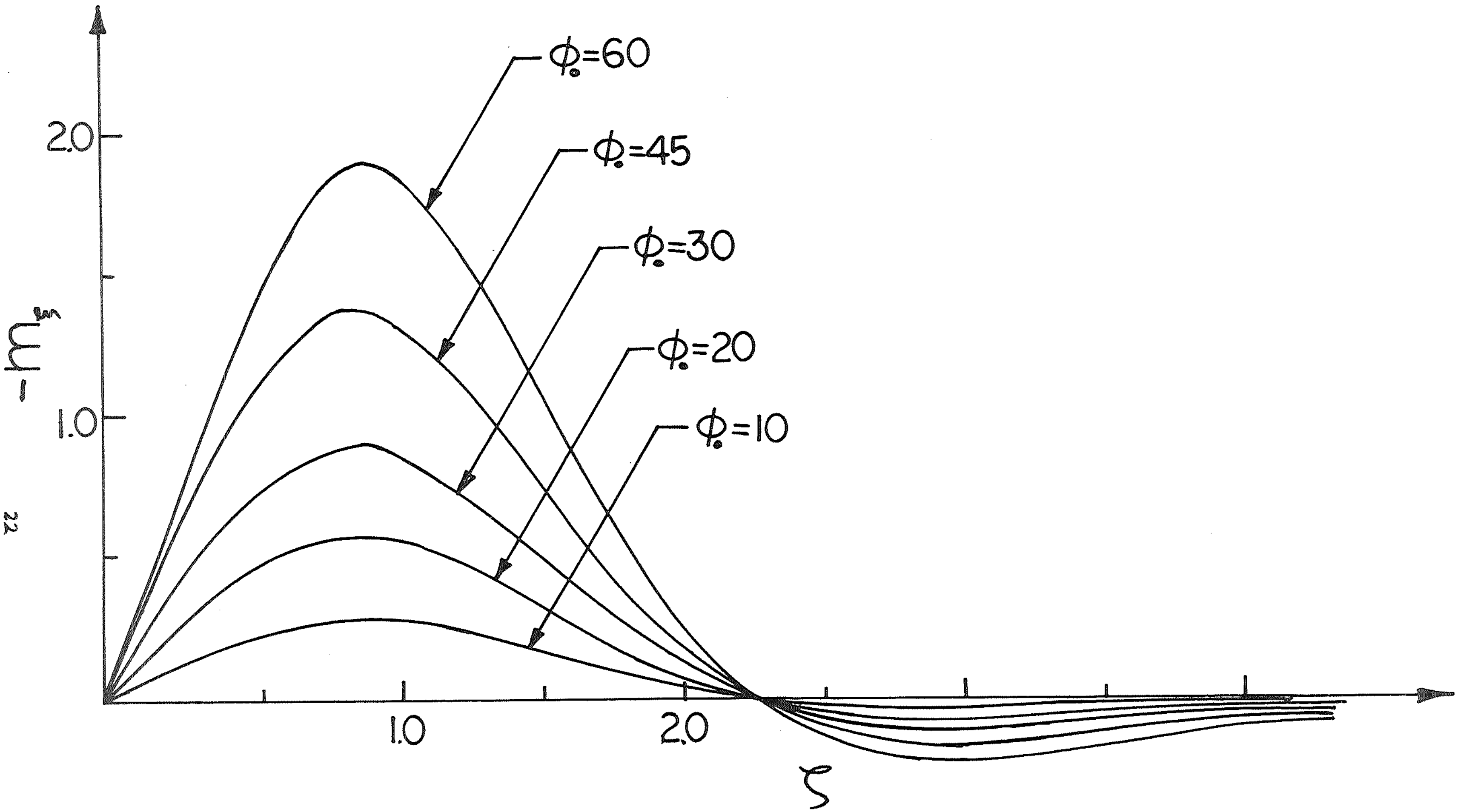

FIG. 2.6 NONDIMENSIONALIZED MERIDIONAL MOMENT

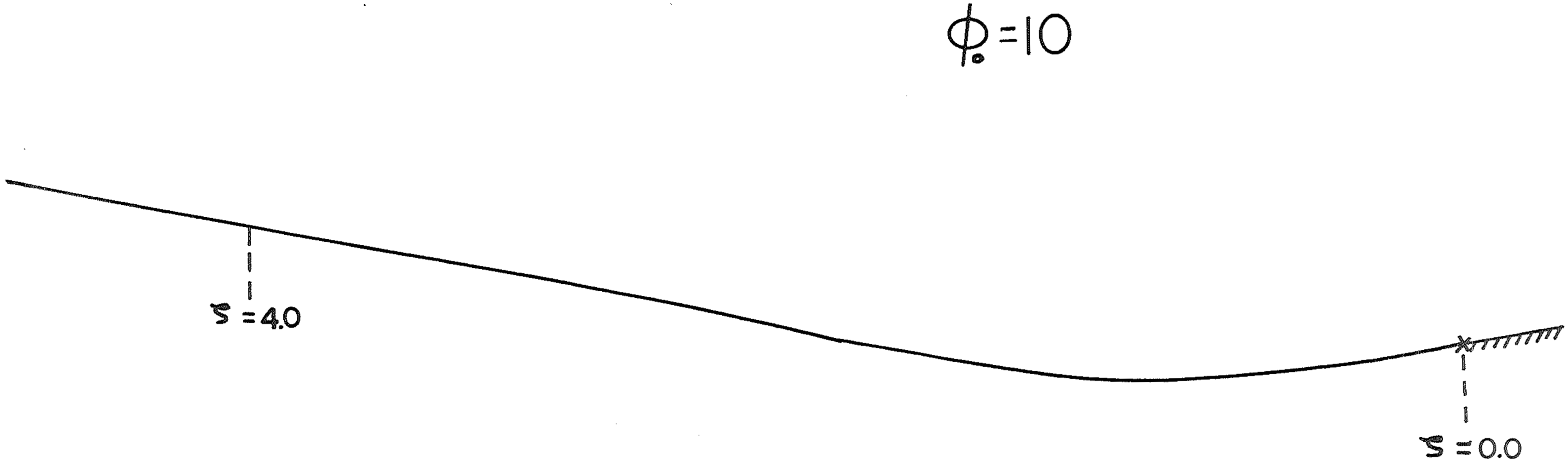

FIG. 2.7 DEFORMED CONFIGURATION OF THE SHELL AT $\phi_o = 10°$

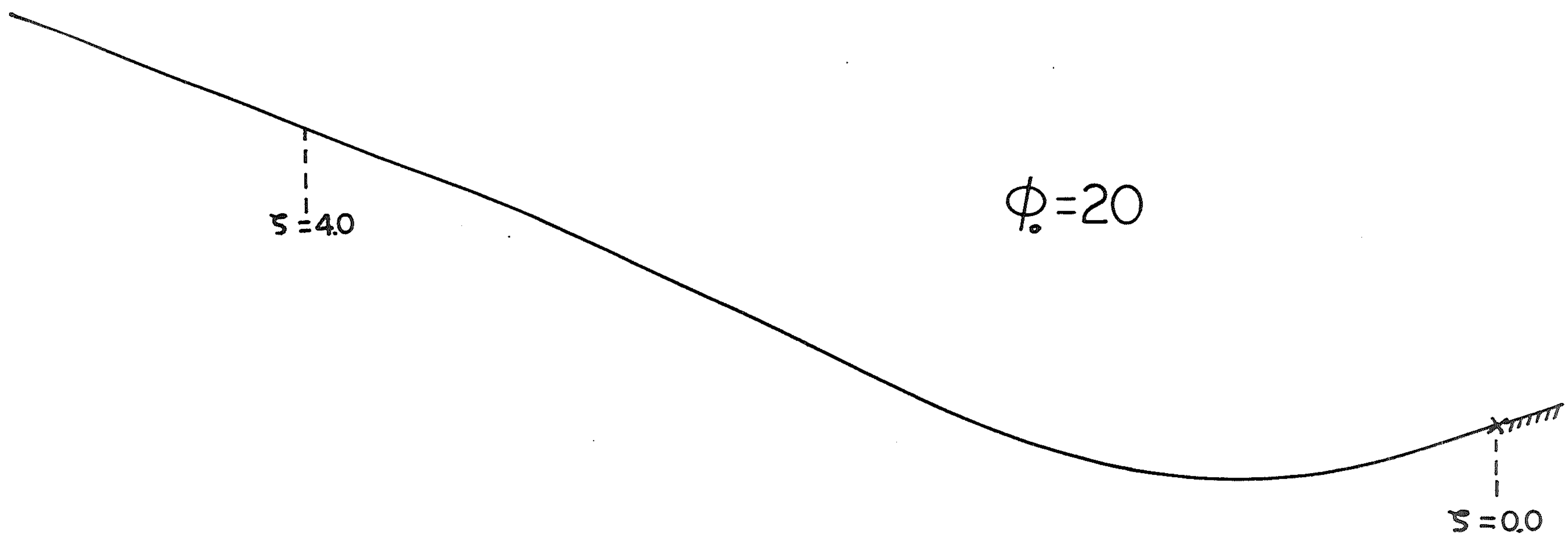

FIG. 2.8 DEFORMED CONFIGURATION OF THE SHELL AT ϕ_o = 20°

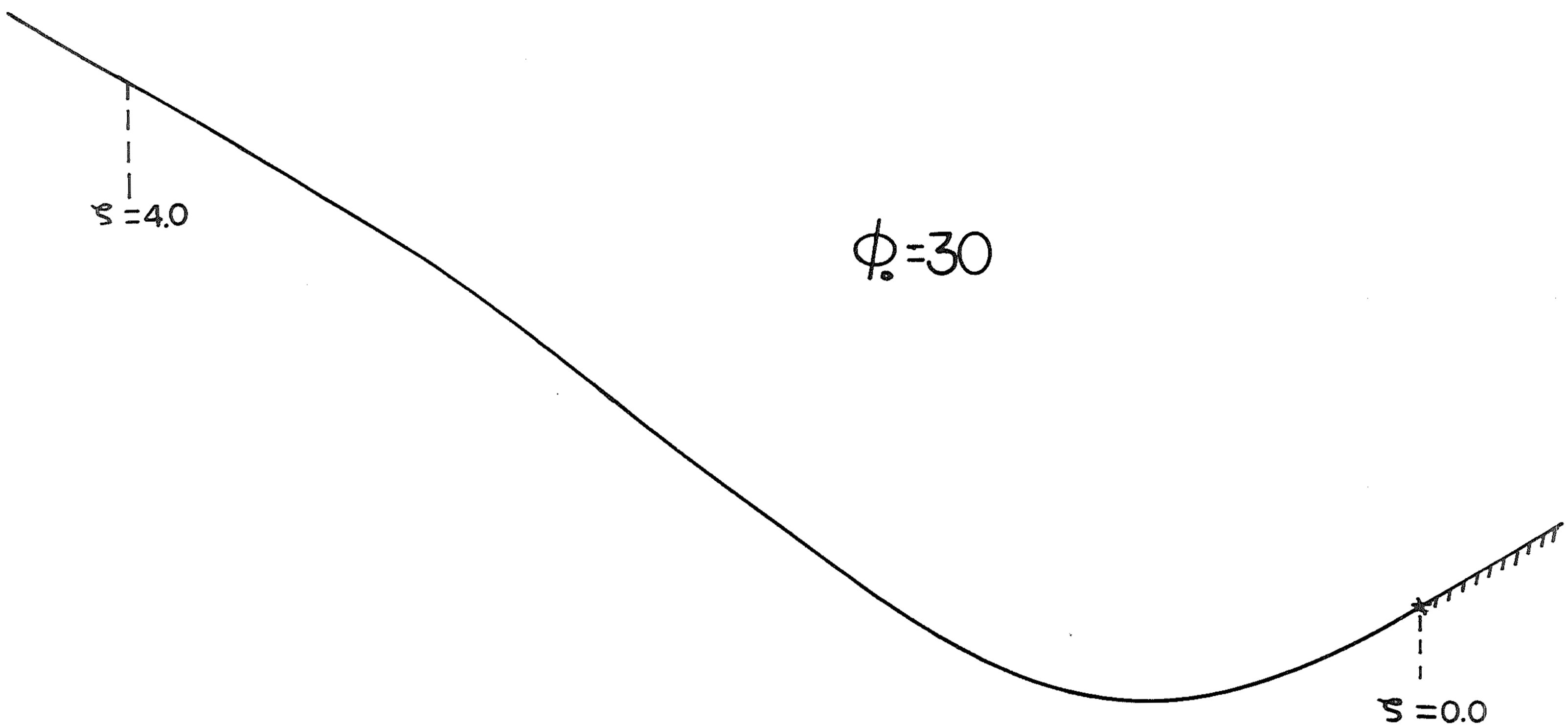

FIG. 2.9 DEFORMED CONFIGURATION OF THE SHELL AT $\phi_o = 30°$

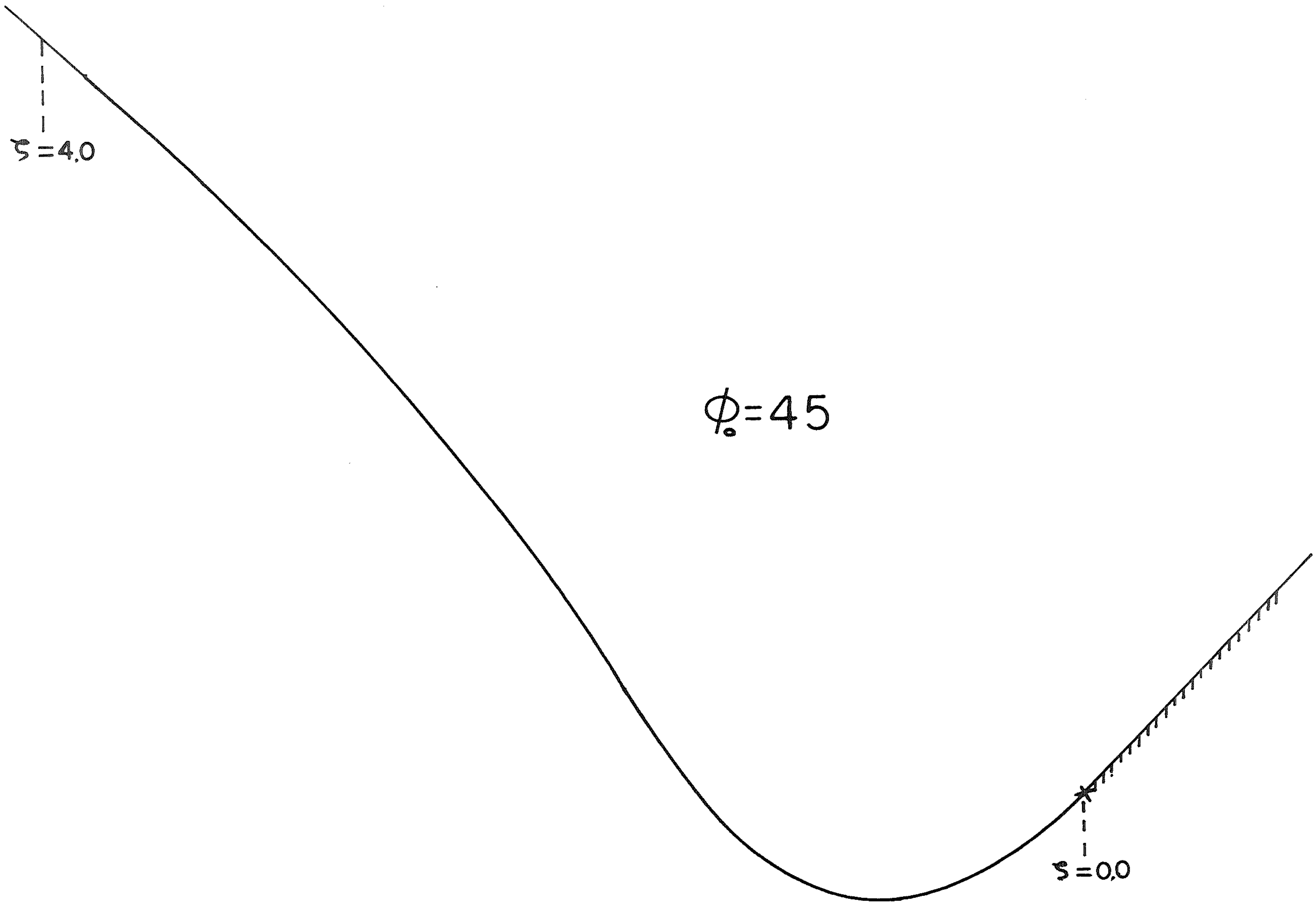

FIG. 2.10 DEFORMED CONFIGURATION OF THE SHELL AT $\phi_o = 45°$

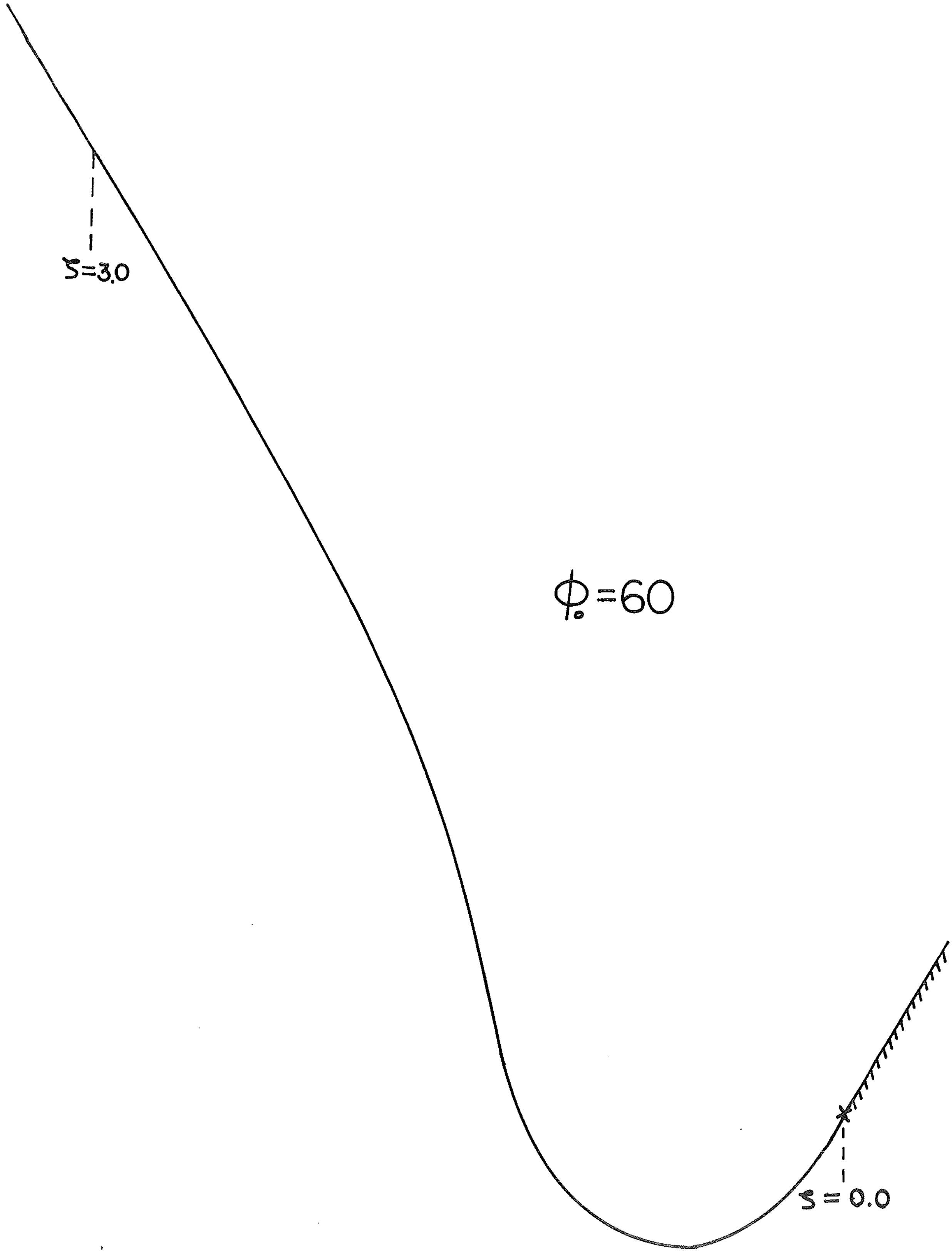

FIG. 2.11 DEFORMED CONFIGURATION OF THE SHELL AT $\phi_o = 60°$

3. The Initiation of Nonaxisymmetric Deformation-Problem Formulation

Given this solution for the axisymmetric deformation of the constrained sphere, one may proceed to investigate the possibility of asymmetric deformation; the onset of wrinkling. The extreme compressive stress conditions existing within the edge zone suggests that nonaxisymmetric deformation will be initiated within this same region. To determine when wrinkling initiates, we admit the possibility of the existence of small, nonaxisymmetric, displacements and rotations in the neighborhood of our previously determined, axisymmetric configuration. An appropriate system of equations describing this situation is the Donnell-Mushtari-Vlosov system written with respect to the deformed, axisymmetric state.

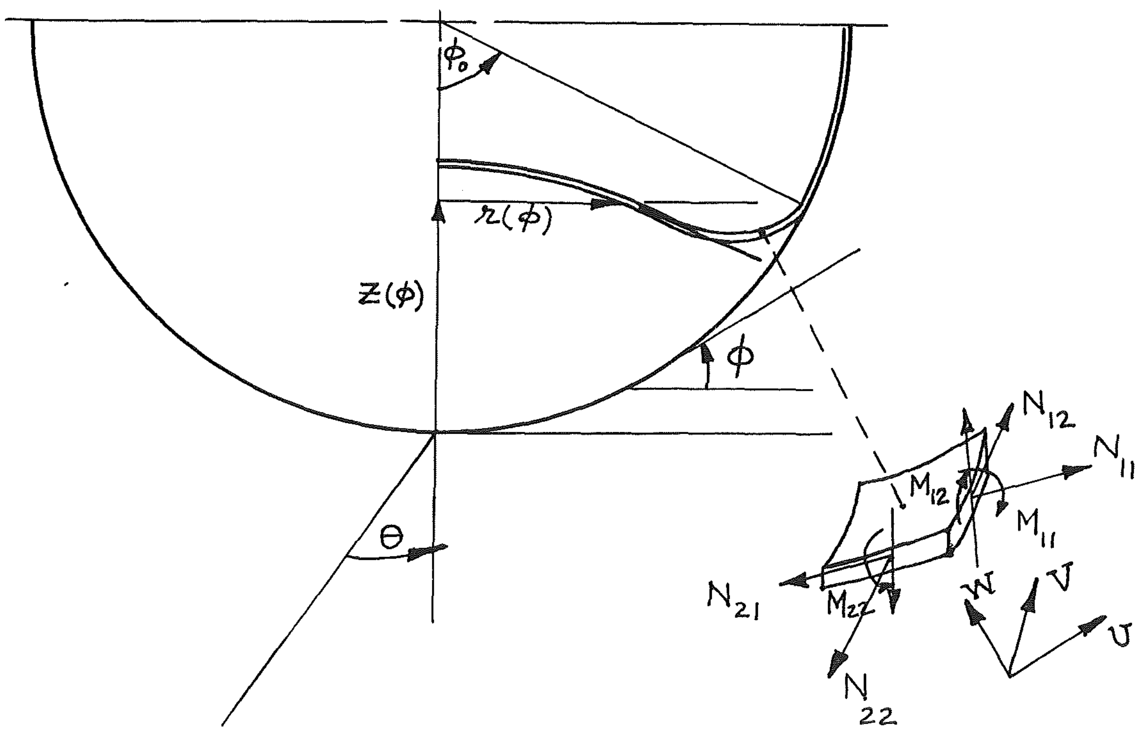

FIG. 3.1

In the following, we take as our independent variable in the meridional direction the slope of the meridional in the undeformed configuration, ϕ, and in the circumferential direction, the polar angle, θ.

Defining the geometric parameters α, R_1 and R_2 by

$$\frac{dz}{d\phi} = \alpha \sin \Phi \qquad \frac{dr}{d\phi} = \alpha \cos \Phi \tag{3.1a}$$

$$\frac{1}{R_1} = \frac{1}{\alpha}\frac{d\Phi}{d\phi} \qquad \frac{1}{R_2} = \frac{\sin \Phi}{r} \tag{3.1b}$$

The equilibrium equations of the Donnell system become (Ref. 10)

$$(r N_{11})^{\prime} + (\alpha N_{12})^{\circ} - \alpha \cos \Phi \, N_{22} = 0 \tag{3.2a}$$

$$(\alpha N_{22})^{\circ} + (r N_{21})^{\circ} + \alpha \cos \Phi \, N_{21} = 0 \tag{3.2b}$$

$$(r Q_1)^{\prime} + (\alpha Q_2)^{\circ} - r\alpha\left(\frac{N_{11}}{R_1} + \frac{N_{22}}{R_2}\right) - (r \Psi_1 N_{\xi})^{\prime} - (\alpha \psi_2 N_{\theta})^{\circ} = 0 \tag{3.2c}$$

$$(r M_{11})^{\prime} + (\alpha M_{12})^{\circ} - \alpha \cos \Phi \, M_{22} - \alpha r Q_1 = 0 \tag{3.2d}$$

$$(\alpha M_{22})^{\circ} + (r M_{21})^{\prime} + \alpha \cos \Phi \, M_{21} - \alpha r Q_2 = 0 \tag{3.2e}$$

In the above equations prime denotes differentiation with respect to ϕ, and dot denotes differentiation with respect to θ. N_{ξ} and N_{θ} are the meridional and circumferential stress resultants existing in the axisymmetric, deformed state as defined by Eqs. (2.2.16a) and (2.2.16b).

The stress-strain relations are

$$N_{11} = \frac{Eh}{(1-\nu^2)}\left(E_{11} + \nu E_{22}\right), N_{22} = \frac{Eh}{(1-\nu^2)}\left(E_{22} + \nu E_{11}\right) \quad (3.3a,b)$$

$$N_{12} = N_{21} = \frac{Eh}{(1+\nu)} E_{12} \quad (3.3c)$$

$$M_{11} = D(K_{11} + \nu K_{22}) \quad , \quad M_{22} = D(K_{22} + \nu K_{11}) \quad (3.4a,b)$$

$$M_{12} = M_{21} = D(1-\nu) K_{12} \quad (3.4c)$$

The strain-displacement relationships are:

$$E_{11} = \frac{1}{\alpha} U' + \frac{W}{R_1} \quad , E_{22} = \frac{V^{\circ}}{r} + \frac{r'}{r\alpha} U + \frac{W}{r} \sin \Phi \quad (3.5a,b)$$

$$E_{12} = E_{21} = \frac{1}{2}\left[\frac{V'}{\alpha} + \frac{U^{\circ}}{r} - \frac{r'}{\alpha r} V\right] \quad (3.5c)$$

$$K_{11} = \frac{1}{\alpha} \Psi_1' \quad , \quad K_{22} = \frac{1}{r} \Psi_2^{\circ} + \frac{r'}{r\alpha} \Psi_1 \quad (3.6a,b)$$

$$K_{12} = K_{21} = \frac{1}{2}\left[\frac{1}{\alpha} \Psi_2' + \frac{1}{r} \Psi_1^{\circ} - \frac{r'}{\alpha r} \Psi_2\right] \quad (3.6c)$$

Where the rotations ψ_1 and ψ_2 are related to the normal displacement W by

$$\Psi_1 = -\frac{1}{\alpha} W' \quad , \quad \Psi_2 = -\frac{1}{r} W^{\circ} \quad (3.7a,b)$$

The boundary conditions on an edge ϕ = constant take the form

$$N_{11} = 0 \quad \text{or} \quad U = 0 \quad (3.8a)$$

$$N_{12} = 0 \quad \text{or} \quad V = 0 \quad (3.8b)$$

$$Q_1 + \frac{1}{\alpha}(M_{12})^{\circ} - \Psi_1 N_\xi = 0 \quad \text{or} \quad W = 0 \tag{3.8c}$$

$$M_{11} = 0 \quad \text{or} \quad \Psi_1 = 0 \tag{3.8d}$$

This system of homogeneous equations (3.2) through (3.8) constitutes an eigenvalue problem. The eigenvalue, which we may take as ϕ_o, is imbedded in the "prebuckling" stress resultants N_ξ and N_θ. We expect that nonaxisymmetric deformation states, nontrivial solutions to this system, exist only at discrete values of the separation angle ϕ_o. A first step in the solution of this problem is accomplished by elimination of dependence on the polar angle θ.

Setting

$$\{N_{11}, N_{22}, Q_1, M_{11}, M_{22}, E_{11}, E_{22}, K_{11}, K_{22}, \Psi_1, U, W\}$$

$$= \{n_{11}, n_{22}, q_1, m_{11}, m_{22}, e_{11}, e_{22}, \kappa_{11}, \kappa_{22}, \psi_1, u, w\} \sin k\theta$$

$$\{N_{12}, Q_2, M_{12}, E_{12}, K_{12}, \Psi_2, V\}$$

$$= \{n_{12}, q_2, m_{12}, e_{12}, \kappa_{12}, \psi_2, v\} \cos k\theta$$

where k is an integer and n_{11}, n_{22} ... , n_{12}, q_2 ... are functions only of ϕ. Our system of equations becomes:

Equilibrium

$$(r n_{11})' - k\alpha\, n_{12} - \alpha \cos\Phi\, n_{22} = 0 \tag{3.9a}$$

$$k\alpha n_{22} + (r n_{12})' + \alpha \cos\Phi\, n_{12} = 0 \qquad (3.9b)$$

$$(r q_1)' - k\alpha q_2 - r\alpha\left(\frac{n_{11}}{R_1} + \frac{n_{22}}{R_2}\right) - (r\psi_1 N_\xi)' + k\alpha \psi_2 N_\theta = 0 \qquad (3.9c)$$

$$(r m_{11})' - k\alpha m_{12} - \alpha\cos\Phi\, m_{22} - r\alpha q_1 = 0 \qquad (3.9d)$$

$$k\alpha m_{22} + (r m_{12})' + \alpha\cos\Phi\, m_{12} - r\alpha q_2 = 0 \qquad (3.9e)$$

Stress-Strain

$$n_{11} = \frac{Eh}{(1-\nu^2)}(e_{11} + \nu e_{22}), \quad n_{22} = \frac{Eh}{(1-\nu^2)}(e_{22} + \nu e_{11}) \qquad (3.10a,b)$$

$$n_{12} = \frac{Eh}{1+\nu} e_{12} \qquad (3.10c)$$

$$m_{11} = D(K_{11} + \nu K_{22}), \quad m_{22} = D(K_{22} + \nu K_{11}) \qquad (3.11a,b)$$

$$m_{12} = D(1-\nu) K_{12} \qquad (3.11c)$$

Strain-Displacement

$$e_{11} = \frac{1}{\alpha} u' + \frac{w}{R_1}, \quad e_{22} = -\frac{kv}{r} + \frac{r' u}{r\alpha} + \frac{w}{r}\sin\Phi \qquad (3.12a,b)$$

$$e_{12} = \frac{1}{2}\left[\frac{v'}{r} + \frac{k}{r}u - \frac{r'}{\alpha r} v\right] \qquad (3.12c)$$

$$K_{11} = \frac{1}{\alpha}\psi_1', \quad K_{22} = -\frac{k}{r}\psi_2 + \frac{r'}{r\alpha}\psi_1 \qquad (3.13a,b)$$

$$K_{12} = \frac{1}{2}\left[\frac{1}{\alpha}\psi_2' + \frac{k}{r}\psi_1 - \frac{r'}{\alpha r}\psi_2\right] \qquad (3.13c)$$

In the above equations

$$\psi_1 = -\frac{1}{\alpha}w' \qquad (3.14a)$$

$$\psi_2 = -\frac{k}{r}w \qquad (3.14b)$$

Boundary Conditions

On ϕ = constant

$$n_{11} = 0 \quad \text{or} \quad u = 0 \qquad (3.15a)$$

$$n_{12} = 0 \quad \text{or} \quad v = 0 \qquad (3.15b)$$

$$q_1 - \frac{k}{\alpha}m_{12} - \psi_1 N_\xi = 0 \quad \text{or} \quad w = 0 \qquad (3.15c)$$

$$m_{11} = 0 \quad \text{or} \quad \psi_1 = 0 \qquad (3.15d)$$

This completes the formulation of the eigenvalue problem. The solution of this problem defines that value of separation angle, ϕ_o, when nonaxisymmetric deformation is possible; i.e., when folding is initiated. It is expected that some simplification in the solution to this rather formidable problem may be achieved by the same asymptotic techniques used to determine the axisymmetric deformation states.

4. Summary of Results

The preceding effort has focused on the understanding of one phase of the diaphragm deformation process; the axisymmetric, nonlinear behavior of the constrained hemisphere. The results of this analysis show that, even at moderate separation angles, extreme stress and strain states are generated in a narrow edge zone in the shell near the separation point. An important consequence of this asymptotic procedure is that it is only the geometry of the shell in the vicinity of the separation point which is important in the evaluation of the edge zone behavior. Hence, our results apply to any simply-connected shell of revolution.

Although it remains to be shown, we feel that it is the extreme stress state in this edge zone which leads to the initiation of nonaxisymmetric deformation. The solution of the eigenvalue problem posed in Section 3 will yield values of separation angle at which folding initiates. It will also provide the preferred circumferential wave number k. This information might be of use in the designing of meridional stiffners.

An important result of this analysis is the fact that the pressure differential required to deform the constrained shell, once it has assumed the configuration depicted in Fig. 2.1, is essentially zero. More precisely,

$$\frac{pR^2}{Eh^2} = O\left(\sqrt{\frac{h}{R}}\right) \qquad \text{at most.}$$

5. Recommendations for Future Work

The results obtained in this effort represent a first step in the understanding of the deformation of a reversing diaphragm. Attention has been limited to the axisymmetric, elastic behavior of an isotropic shell of revolution; the initial phase in the deformation process. It is recommended that the following analysis be attempted for this same phase.

(1) Consider the effect of employing a layered shell construction on the edge zone state. This might require consideration of transverse shear deformation.

(2) Consider the effect of orthotropic construction in the shell surface. The introduction of preferred direction of stiffness might alleviate the extreme conditions existing in the edge zone.

(3) Consider the possibility and consequences of plastic flow.

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13. ABSTRACT

The large, axisymmetrically deformed state of a constrained spherical shell under external pressure is determined. Results show that there exists a narrow zone along the meridional wherein extreme stress levels are attained. It is postulated that when wrinkling (asymmetric deformation) initiates, it does so within this zone. These results provide insight into the collapse mechanism of propellant expulsion bladders and diaphragms in the form of shells of revolution.

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