Technical Report RSC-11

AN EXTENSION OF THE SLOPE-FACET MODEL OF RADAR BACKSCATTER FROM THE SEA

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KATZIN'S FACET-SLOPE MODEL

The scattering mechanism which Katzin develops consists of modeling the sea surface using perfectly conducting facets, that is, the surface of the sea is considered to be the superposition of facets of various sizes with orientations distributed about the mean sea contour. The facets are assumed to move randomly.

The theory is developed for small depression angles because of the comparative ease of dealing with the scattering equations. However, Katzin advances that the general approach is applicable to all depression angles.

At the offset of this development, four basic assumptions are made.

1. Backscatter due to diffraction is neglected.
2. The phases of the waves backscattered by the facets vary randomly, so that the backscattered powers are additive.
3. The facet size and slope are statistically independent of each other.
4. The distribution of both size and slope embrace a continuous range.

The development consists of obtaining a radar backscatter cross section per unit area, σ°, for the slope-facet model based on the above assumptions.

The backscatter cross section per unit is defined as:

$$\sigma^{\circ} = N_f \, E(\bar{\sigma}) \tag{1}$$

where: N_f = number of scatters per unit area

$E(\bar{\sigma})$ = mean value of the average cross section of each scatterer.

The average scattering cross section is a function of both the area of each facet and of its orientation (slope), hence:

$$E(\bar{\sigma}) = \iint p(s,A) \, \bar{\sigma}(s,A) \, dA \, ds \tag{2}$$

where: p(s,A) = joint probability density function

Since it has been assumed that slope and area are independent, substituting (2) into (1) yields:

$$\sigma^{\circ} = \iint N(A) \, p(s) \, \bar{\sigma}(s,A) \, dA \, ds \tag{3}$$

where: $N(A) = N_f p(A)$ = number density of facet areas.

The solution of (3) is obtained by (a) determining the average scattering cross section, $\bar{\sigma}(s,A)$, of specific types of flat plates, (b) determining a number density of facet areas consistant with the empirical measurements, and (c) employing a probability density function, p(s), for the sea determined by some independent means. The latter step in Katzin's work involves incorporating the probability density functions obtained by Cox and Munk (1954) and as such is incidental to the purpose of this report.

Faced with the problem of determining the scattering cross-section of various size facets, Katzin introduced an ingenious idea. The techniques for calculating the scattering cross-section for flat areas which are either large or small relative to the incident signed wavelength are well known. However, for areas on the order of a wavelength, no easily manageable methods are available. Therefore Katzin defined two distinct regions in which facets were either all "large" or all "small". In the region where facets are "small" the scattering cross section and scattering coefficient, σ^0, increases as the facet size increases. In the region where facets are large, the number of possible facets decreases as their area increases and hence the scattering coefficient, σ^0 decreases. The transition between these two regions is assumed to occur smoothly at some facet area A_1. The procedure is to integrate (3) with respect to A independently over the two regions and then sum the results.

The average scattering cross-section from a circular disc which is large relative to a wavelength is

$$\bar{\sigma} = f_1(\theta)\,\lambda\,A^{1/2} \tag{4}$$

where

$$f_1(\theta) = (4\pi^{3/2})^{-1} \tan^2\theta \sec\theta$$

for a disc which is small relative to a wavelength

$$\bar{\sigma} = f_2(\theta)\,A^3/\lambda^4 \tag{5}$$

where

$$f_2(\theta) = (4^5/9)(1 + \cos^2\theta/2)^2$$

The transition point is obtained when (4) and (5) are equal that is

$$A_1 = (f_1/f_2)^{2/5}\,\lambda^2 \tag{6}$$

EXTENSION OF SLOPE-FACET MODEL

The following introduces a modification to Katzin's method of handling the integration of (3) relative to slope variations. First Katzin's area integration operation is reviewed and then the point of departure from Katzin is explained.

The basic relationship of concern is

$$\sigma^{o} = \int N(A)\, p(s)\, \bar{\sigma}(s,A)\, ds\, dA \qquad (3)$$

The area integration is handled independently

$$\sigma^{o} = \int p(s) \left[\int N(A)\, \bar{\sigma}(s,A)\, dA \right] ds$$

or

$$\sigma^{o} = \int p(s)\, I(A)\, ds \qquad (A\text{-}1)$$

In order to evaluate I(A), some functional relationships must be employed to represent N(A) and $\bar{\sigma}$(s,A). Following Katzin

$$N(A) = N_0\, A^{-(n+4)/2}$$

and

$$\bar{\sigma}(s,A) = f_2(\theta)\, A^{3} \lambda^{-4} \qquad A < A_1$$

$$\bar{\sigma}(s,A) = f_1(\theta)\, A^{1/2} \lambda^{1} \qquad A > A_1$$

hence

$$I(A) = \int_{A_0}^{A_1} N_0 A^{-(n+4)/2} f_2(\theta) A^3 \lambda^{-4} \, dA$$

$$+ \int_{A_1}^{A_2} N_0 A^{-(n+4)/2} f_1(\theta) A^{1/2} \lambda \, dA \qquad \text{(A-2)}$$

$$I(A) = I_1(A) + I_2(A) \qquad \text{(A-3)}$$

Consider $I_1(A)$

$$I_1(A) = N_0 \int_{A_0}^{A_1} f_2(\theta) \lambda^{-4} A^{-\frac{n}{2}+1} \, dA$$

$$= \frac{N_0 f_2(\theta) \lambda^{-4}}{[2 - (n/2)]} \left[A_1^{(2-n/2)} - A_0^{(2-n/2)} \right] \qquad \text{(A-4)}$$

If $(2 - n/2) > 0$, i.e. $n < 4$, then $I_1(A)$ is finite, and A_0 can be assumed to be zero under the assumption that the contribution to $I_1(A)$ from very small facets is negligible. Note however that n cannot exceed 4 otherwise $I_1(A)$ does not converge.

Using Katzin's value for A_1, i.e. $A_1 = (f_1/f_2)^{2/5} \lambda^2$

$$I_1(A) = \frac{N_0 f_2}{(2 - n/2)} \left(\frac{f_1}{f_2}\right)^{\left(\frac{4}{5} - \frac{n}{5}\right)} \lambda^{-n} \tag{A-5}$$

In a similar manner it is easily shown that

$$I_2(A) = \frac{N_0 \lambda f_1(\theta)}{(-n/2 - 1/2)} \left[A_2^{(-n/2 - 1/2)} - A_1^{(-n/2 - 1/2)} \right] \tag{A-6}$$

If $(-n/2 - 1/2) < 0$, i.e. $n > -1$, then $I_2(A)$ is finite, and A_2 can be assumed to be infinity under the assumption that the number density of facet slopes that are large approaches zero, hence the contributions to $I_2(A)$ for large areas is negligible. Note however that n must not be less than -1 or $I_2(A)$ will not converge. Consequently in Katzin method, n is restricted to the range $-1 < n < 4$.

Substituting for A_1 as before

$$I_2(A) = \frac{2 N_0 f_1}{(n+1)} \left(\frac{f_1}{f_2}\right)^{-\left(\frac{n+1}{5}\right)} \lambda^{-n} \tag{A-7}$$

$$I(A) = I_1(A) + I_2(A)$$

$$= \frac{10 N_o}{(n+1)(4-n)} f_1^{\left(\frac{4-n}{5}\right)} f_2^{\left(\frac{n+1}{5}\right)} \lambda^{-n}$$

$$= N_o F(\theta) \lambda^{-n} \tag{A-8}$$

From Katzin,

$$f_1 = (4\pi^{3/2})^{-1} \tan^2\theta \sec\theta \tag{A-9}$$

$$f_2 = (4^5/9)(1 + \cos^2\theta/2)^2 \tag{A-10}$$

The problem now is to evaluate the slope integral

$$\sigma^\circ = \int p(s)\, I(A)\, ds \tag{A-11}$$

Katzin handles this problem by employing the following assumptions

$$\sec\theta \doteq 1 \tag{A-12}$$

$$(1 + \cos^2\theta/2) \doteq 2 \tag{A-13}$$

That is, he evaluated σ^0 for small θ, (A-12), and assumed that the slope dependence for small facets is not a major factor (A-13). This would seem reasonable except for the fact that subsequently Katzin attempts to evaluate the integral (A-11) for a particular p(s) by employing slope limits from $-\infty$ to $+\infty$. This is apparently justified because p(s) is a filtering function which suppresses the contributions to σ^0 for slopes outside a select range.

In view of the fact that the method of dealing with the slope integral is somewhat arbitrary, which is understandable since little is known about the slope variations of an actual surface, it seems reasonable to deal with (A-11) in much the same way as was done to obtain I(A), (A-2). That is, assume that there exists two distinct slope behavioral regions synonomous with the two area regions. This may be expressed as

$$\sigma^{o} = \int_{S_0}^{S_2} p(s) \int_{A_0}^{A_2} N(A)\, \bar{\sigma}(s,A)\, dA\, ds \qquad \text{(A-14)}$$

To handle (A-14) requires a slightly different approach when expressing $\bar{\sigma}$ (s,A). It is arbitrarily assumed that

$$\bar{\sigma}_1 = k_1 s^r \lambda A^{1/2} \qquad A > A_1 \qquad \text{(A-15)}$$

$$\bar{\sigma}_2 = k_2 s^t \lambda^{-4} A^3 \qquad A < A_1 \qquad \text{(A-16)}$$

where r and t are positive constants. The relationships are implied in Katzin's work, where t=0 and r=2. These expressions, (A-15) and (A-16), show that the scattering cross-section is somehow dependent upon the slope of the facet, and it is assumed that the dependence can be expressed as a coefficient of s just as the area dependence was expressed as a coefficient of A; no assumption is made as to the values of r or t except that they are positive; nor are k_1 or k_2 known. Using (A-15) and (A-16) the transition area, A_1, becomes

$$A_1 = \left(\frac{k_1}{k_2}\right)^{2/5} s^{2/5(r-t)} \lambda^2 \tag{A-17}$$

Equation (A-6) becomes

$$I_1(A) = \frac{2N_0 k_2 s^t}{(4-n)} \left(\frac{k_1}{k_2}\right)^{\frac{4-n}{5}} s^{\left(\frac{4-n}{5}\right)(r-t)} \lambda^{-n} \tag{A-18}$$

Equation (A-7) becomes

$$I_2(A) = \frac{2N_0 k s^r}{(n+1)} \left(\frac{k_1}{k_2}\right)^{-\frac{n+1}{5}} s^{-\left(\frac{n+1}{5}\right)(r-t)} \lambda^{-n} \tag{A-19}$$

and the sum reduces to

$$I(A) = \frac{10 N_0 \lambda^{-n} s^{\left[\frac{r(4-n)+t(n+1)}{5}\right]} k_1^{\frac{4-n}{5}} k_2^{\frac{n+1}{5}}}{(n+1)(4-n)} \tag{A-20}$$

Consequently

$$\sigma^\circ = \int p(s)\, I(A)\, ds = K \int p(s) s^a\, ds \tag{A-21}$$

where

$$a = \frac{r(4-n) + t(n+1)}{5} \qquad \text{(A-22)}$$

In the same manner that N(A) is continuous and independent of the two area regions on each side of A_1, p(s) is continuous regardless of which facet region is being considered. Based on the measurements of Cox and Munk (1954) it is reasonable to assume that p(s) is Gaussian. According to Cox and Munk the mean of the distribution is near zero for crosswind slopes and slightly removed from zero for upwind or downwind slopes. Regardless of the exact nature of p(s), it is safe to assume that the integral (A-21) converges regardless of the values of r and t, because p(s) must approach zero for finite values of s. Since p(s) does serve as a "filter" function in a bandpass sense, the limits on s can be extended so that $s_0 = -\infty$ and $s_2 = +\infty$ in (A-14).

However, it does not seem reasonable to give equal weight to both positive and negative slopes. Clearly for large facets the areas having a positive slope will have a greater influence on σ^0 than those with negative slopes. This will also be true for small facets, but possibly to a lesser extent. However, it has been noted that t will likely be small so that for all practical purposes the negative slopes can be excluded. Consequently it will be assumed that p(s) is a Gaussian function, and that the limits $s_1 = 0$ to $s_2 = +\infty$ enclose the region of primary concern for determining the scattering coefficient. Assume

$$p(s) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-(s-\eta)^2}{2\sigma^2}\right] \tag{A-23}$$

where: σ^2 = variance of the slopes

η = mean

Using (A-23), the integral (A-21) reduces to

$$\sigma^0 = \frac{K\sigma^{+(a+1)}}{\sqrt{2\pi\sigma^2}} \Gamma(a+1) D_{-(a+1)}(-\eta/\sigma) \exp(-\eta^2/4\sigma^2) \tag{A-24}$$

for the limits zero to infinity. $D_{-(a+1)}(-\eta/\sigma)$ is the Parabolic Cylinder Function. The mean of the distribution, η, is approximately zero for the crosswind direction, and is generally small for the upwind, downwind directions. The mean may be positive or negative depending upon the wind direction. For a zero mean and r = 2, t = 0, and n = 1, (A-24) reduces to

$$\sigma^{o} \doteq \frac{K\sigma^{3.2}}{\sqrt{2\pi}} \tag{A-25}$$

From (A-20) and (A-24) the scattering coefficient is found to be

$$\sigma^{o} = \frac{10N_0\lambda^{-n}}{(n+1)(4-n)} k_1^{\left(\frac{4-n}{5}\right)} k_2^{\left(\frac{n+1}{5}\right)} \sigma^{a} \cdot \Gamma(a+1)\, D_{-(a+1)}(-\eta/\sigma) \exp(-\eta^2/4\sigma^2) \tag{A-26}$$

It will be assumed that k_1 and k_2 can be related to f_1, (A-9), and f_2, (A-10), by an constant. These constants combine with N_0 and will be denoted as N_1. It is further assumed for the following calculations that the angle θ in (A-9) and (A-10) is the depression angle. This assumption is not strictly true, but it is reasonably valid on the average and is consistant with the approach used by Katzin.

RESULTS

In figure 1 the function (A-26) is plotted for X-band using r = 2, t = 0, and n = 0, 1, 2. The normalizing factor has been arbitrarily adjusted for convenience such that the curves join at 85° depression angle. Actually the curves are displaced vertically approximately 10 db at 85° for each unity charge in n.

The values of r and t used for figure 1 were selected to agree with Katzin. In figure 2 the curves for n = 2 are shown for the four combinations of r = 1, 2 and t = 0, 1. In general the effect is a vertical shift, and it is to be noted that the changes in r and t do not appreciably effect the angular dependence of the normalized radar cross section.

In figure 3 are shown data points reported by Guinard and Daley (1970) obtained with an X-band, vertical polarization radar over relatively high sea-states near Iceland. It is evident that the curve for n = 0 provides a reasonably good fit to the data, especially for the mid-angles. The fact that (A-26) led to a value of n = 0 when fit to the angular dependence of actual sea clutter measurements is important since two recently reported radar experiments have confirmed that high windspeed sea clutter is wavelength independent (Wright, 1968; Guinard and Daley, 1970).

It is particularly interesting to note in (A-26) that changes in the wavelength of the incident signal, λ, do not effect the angular dependence of σ^0, but variation in n, the wavelength dependence, do alter the angular behaviour. That is, a change in the angular dependence of σ^0 strictly implies a change in the wavelength dependence.

Schooley (1962) has shown that variations in windspeed change the probability distribution of the surface slopes. The distribution width broadens as the wind velocity increases. The term σ^a in (A-26) accounts for this wind dependence and the predicted "saturation effect" at high wind velocities at which point σ, the standard deviation of slopes, no longer increases with windspeed, will lead to "saturation" of σ^0.

The change in the value of the mean sea slope, η, due to wind direction and velocity are contained in the parabolic cylinder function (A-26). The upwind-downwind ratio is approximately the ratio of two parabolic cylinder functions having arguments which differ in sign. From the Guinard and Daley (1970) X-band data, the average upwind-downwind ratio for relatively high windspeeds was found to be approximately 1.0 db over the range of depression angles from 30° to 45°, using measurements from six different flights. This upwind-downwind ratio is achieved from (A-26) for an η/σ ratio of 0.1, which is in general agreement with measurements reported by Schooley (1962).

The surface slopes are determined relative to the angle of incidence of the incoming signal, which accounts for the fact that the upwind-downwind ratio is angle dependent. In addition, the range of facet areas which contributes to the slope statistics is restricted by the wavelength "size-filtering" effect (Rouse, 1968) such that the upwind-downwind ratio is also wavelength dependent.

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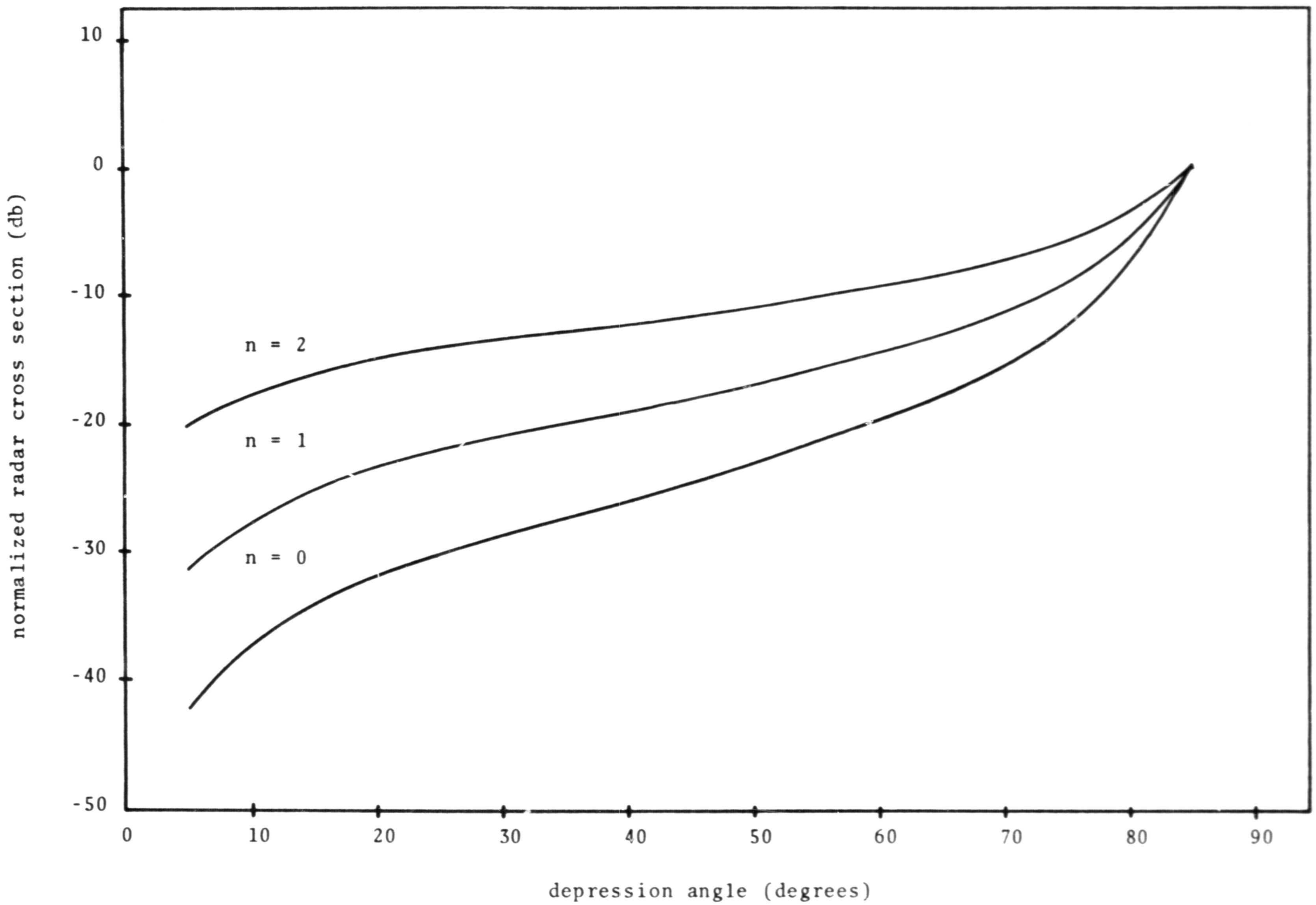

Figure 1 Calculated Cross Section Angular Variations For Three Different Wavelength Dependence Coefficients

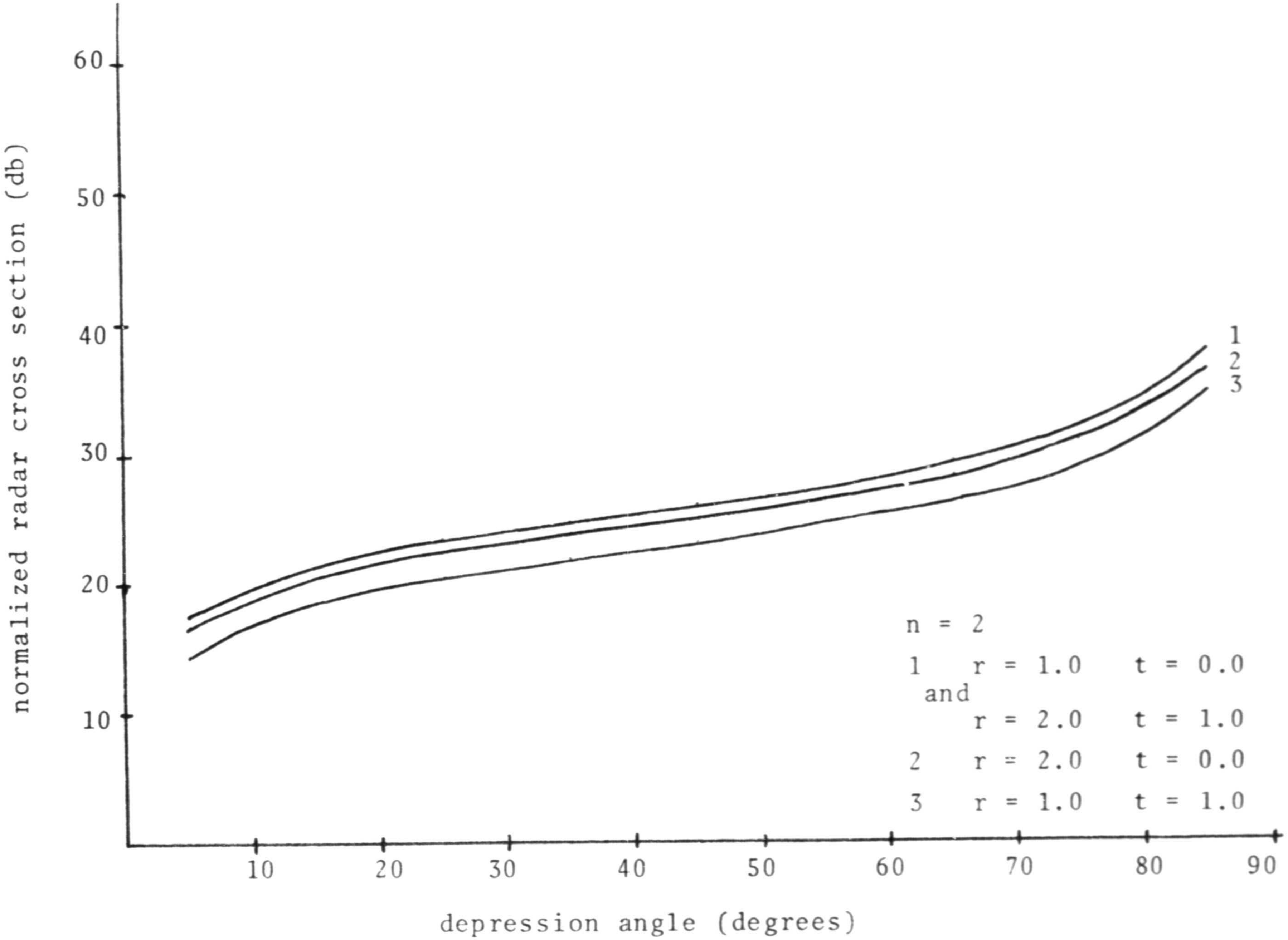

Figure 2 Variations Of The Calculated Cross Section As A Function Of Slope Dependence

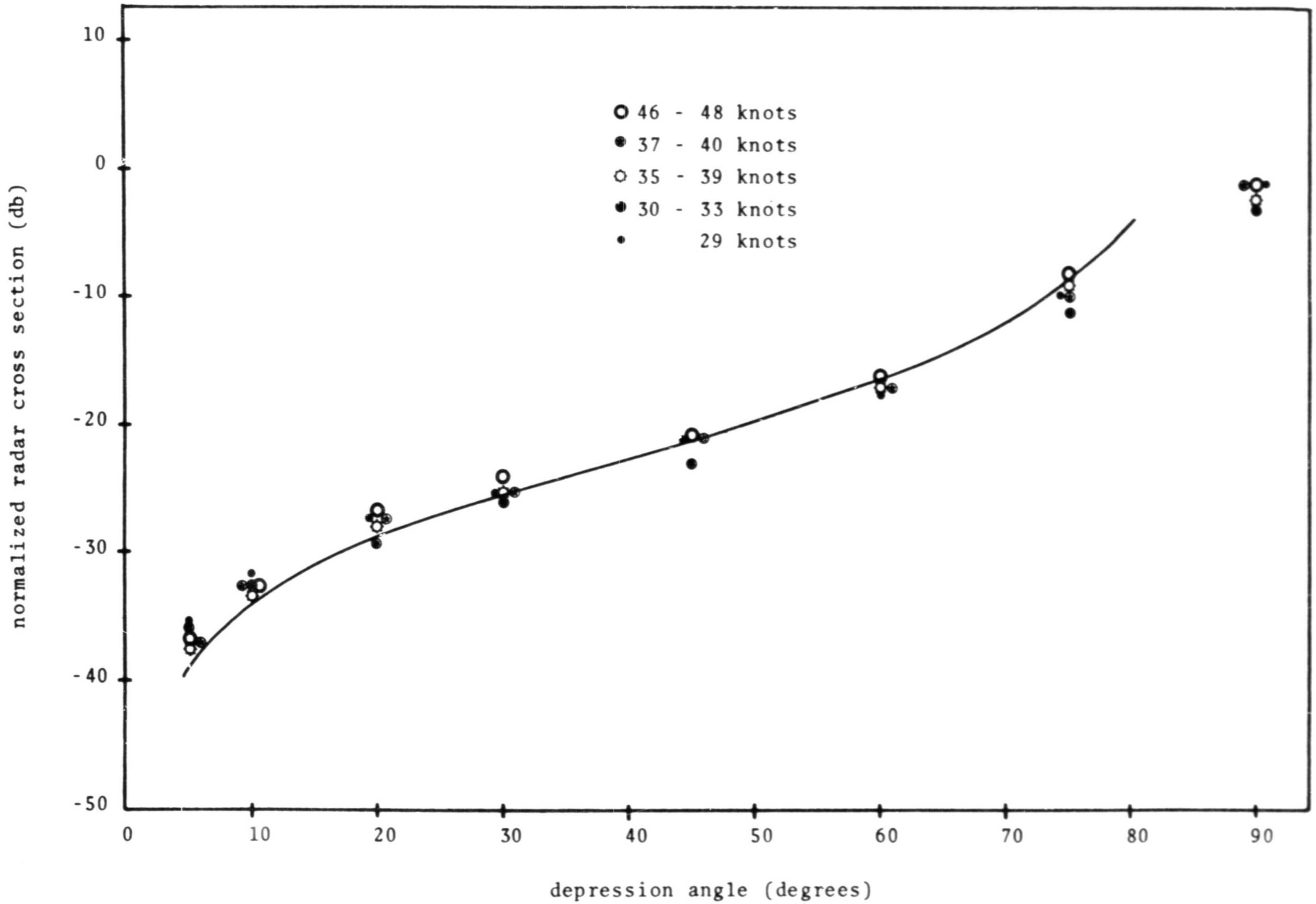

Figure 3 X-band Vertical Polarization Sea Clutter Measurements And Theoretical Results For n = 0