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ADVANCED STUDY OF VIDEO SIGNAL PROCESSING IN LOW SIGNAL TO NOISE ENVIRONMENTS

By

Frank Carden
William Osborne
Alton Gilbert

A Quarterly Progress Report
Submitted to
NATIONAL AERONAUTICAL SPACE ADMINISTRATION
WASHINGTON, D. C.

NASA RESEARCH GRANT NGR-32-003-037

Electrical Engineering Department
Communication Research Group

October 1969 - January 1970

ENGINEERING EXPERIMENT STATION

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ABSTRACT

This work is concerned with establishing the proper parameter space for a PLL to lock onto a modulated carrier. The results of the work are adequately described in the two papers of the report. The first paper is to be published in the Proceedings of the IEEE Region V Record, Dallas, Texas, 1970. The second paper will be published as a special supplement of the Aerospace and Electronic Systems Transaction by the National Aerospace Electronics Conference (NAECON-70).

PHASE LOCK LOOP ACQUISITION OF A CARRIER MODULATED BY A SINGLE SINUSOID

by

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ABSTRACT

The required parameter values for acquisition and tracking of a carrier frequency modulated by a single sinusoid are established for a second order loop. A three variable parameter space is examined consisting of the modulation index, the ratio of the modulation frequency to the loop natural frequency, and the normalized initial frequency offset. Conclusions are drawn from data accumulated through the use of a mathematical model. Simulation of the model on a digital computer was used to obtain numerical results. Verification of the numerical results was accomplished using a 455 KHz second order PLL in the laboratory. Allowable parameter limits are examined to insure acquisition of the modulated carrier instead of its sidebands. Acquisition of the sidebands is discussed along with general comments on distortion.

INTRODUCTION

The phase lock loop has not uncommonly been rejected by telemetry engineers as a demodulation circuit for two principal reasons. The first is that the PLL is known to lock onto sidebands [1,2] in some regions of the modulation parameter space, and the second is that it has been argued that when the modulation index, β, is such that $J_o(\beta)$ is zero the carrier amplitude goes to zero and loss of lock on the carrier must, of necessity, occur. The principal interest of the authors' research into this problem was to establish the regions of the modulation parameter space for acquisition and proper demodulation of the sinusoidally modulated carrier.

A mathematical model of the PLL was derived for this problem and solved by numerical methods. A laboratory model was constructed and experimental data accumulated to establish the regions of interest. An investigation was undertaken into the effects of the damping ratio, ζ, on the region of interest and these results are presented with observations and numerical solutions to clarify the results. The derivation of the mathematical model is presented in a paper by the same authors for presentation at NAECON [3], and the laboratory set-up used for the experimental work is presented in Appendix I along with a brief discussion of the methods used in the measurement of the parameters.

TEXT

It can be seen from the Fourier series expansion of the FM modulated carrier that the loop has a choice of frequencies to acquire, namely ω_c and $\omega_c \pm k\omega_m$ where generally k is any small integer less than or on the order of β. A necessary condition for proper demodulation is that the modulation parameters be such that in every instance the loop will acquire and track the carrier. If the parameters are such that the acquisition of the carrier is not assured, but rather acquisition and tracking of a sideband results, then these parameter values represent an area of the modulation parameter space to be avoided in most applications of the PLL as a demodulator.

The differential equation that describes the acquisition behavior of the second order PLL is

$$\ddot{\phi}_o(t) = 2\zeta\omega_n[\omega_\varepsilon + \dot{\phi}_m(t) - \dot{\phi}_o(t)]\cos[\omega_\varepsilon t + \phi_m(t) - \phi_o(t)] + \omega_n^2\sin[\omega_\varepsilon t + \phi_m(t) - \phi_o(t)]$$

where $\phi_o(t) = \int_0^t \dot{\phi}_o(t)dt$

and ω_ε is the frequency difference of the VCO and the carrier. This equation is derived in [3]. This differential equation was solved by numerical methods for various values of the modulation parameters and loop initial conditions. A boundary between the region of the parameter space where the loop will acquire the carrier and the region where it will acquire the sidebands was determined. Acquisition of a sideband was defined as having occurred when the input to the VCO

$$\frac{\dot{\phi}_o(t)}{K_o}$$

developed a D.C. shift approximately equal to

$$\frac{k\omega_m}{K_o}$$

where K_o is the constant of the VCO. In Figure I the results of the investigation by Carden, Kelly and Hintz [4] into the loss of lock of an initially locked loop with an increase of the modulation index are presented as a comparison to the separation of the two regions for $\zeta = .707$ and $\omega_\varepsilon = 0.0$. In this paper ω_m and ω_ε are normalized by ω_n.

It may be noted that acquisition and tracking of the carrier for modulating frequencies up to the loop natural frequency compare closely, and that up to this limiting value the loop will acquire and demodulate any signal it can track. Beyond $\omega_m = 1.0$, however, a divergence in the results occur. It can be shown that the initial

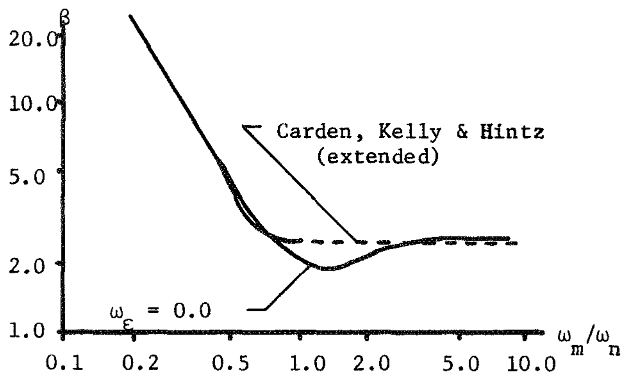

FIGURE I

phase of the modulation has a strong influence on this behavior. In Figure II the minimum value of β for acquisition of a sideband (i.e. the maximum β for acquisition and tracking the carrier has been exceeded) is plotted as a function of the initial modulation angle, θ_i, for $\zeta = .707$ and $\omega_\varepsilon = 0.0$.

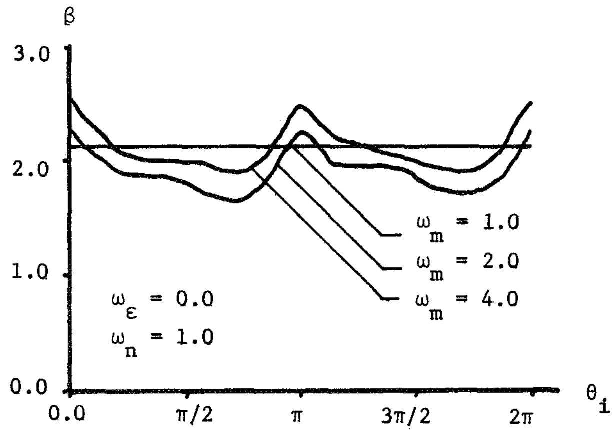

FIGURE II

Since the plot in Figure I is made for an initial modulation phase of zero it is apparent that the plot does not, in fact, represent the worst possible condition. It is also interesting to note that for ω_m up to 1.0 the initial phase of the modulation has little or no effect on the problem. The initial phase was investigated for $\omega_\varepsilon = 2.0$ with even more extreme variations but the same general patterns. The results of the experimental investigation could be expected to compare closely with the numerical solution for modulating frequencies up to the loop natural frequency. No control was exercized over the initial angle of the modulating signal in the experimental work thus a divergence in the results should occur beyond this point. This divergence should not exceed the variance due to the initial phase. The experimental set-up shown in Appendix I was used along with the evaluation criteria set forth therein. In Figure III the experimental and numerical solution curves for $\zeta = .707$ and $\omega_\varepsilon = 2.0$ are given. The three points in Figure III which represent worse

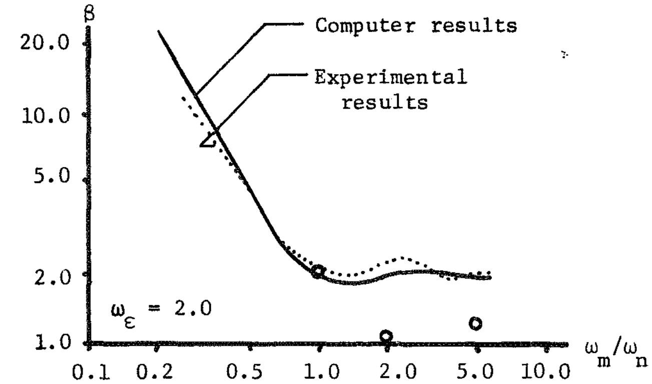

FIGURE III

case β over all θ_i should lie on or below the experimental curve. It can be seen that the experimental curve does conform to these theoretical bounds.

It was determined during this investigation that a small variation in the damping ratio, ζ, does not appreciably affect the acquisition behavior. The results of an investigation with $\zeta = .400$ compared to $\zeta = .707$ both for $\omega_\varepsilon = 0.0$ are shown in Figure IV. the curves are both from data accumulated on the computer.

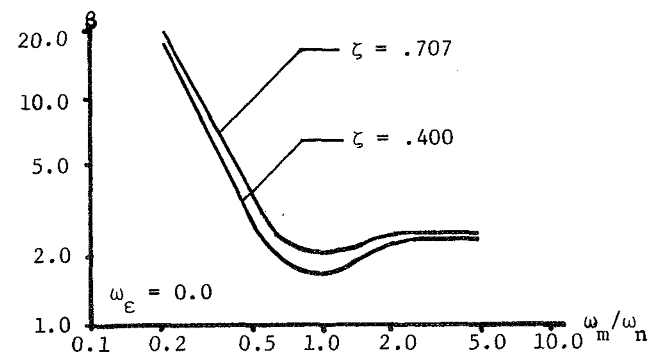

FIGURE IV

In Figure V the bulk of the results are presented. It will be noted that the experimental results are presented for values of $\omega_\varepsilon = 0.0$ and 2.0 along with the results of the numerical solutions for these values. Since $\omega_\varepsilon = 2.0$ represents an initial offset of twice the natural frequency of the loop, and since in most applications involving the demodulation of a carrier and/or subcarriers this value should be quite within the tolerance of the VCO and carrier frequency stabilities, it can be seen that for most applications the initial offset should not be a significant factor.

It must be noted that approaching the boundary of the two regions from below most generally produces an increasing amount of distortion in the demodulated output [5]. In general it was observed that when the first sideband was acquired a beat note [6] equal in frequency to the modulation frequency appeared at the positive or negative maximum of the demodulated output. Higher sidebands

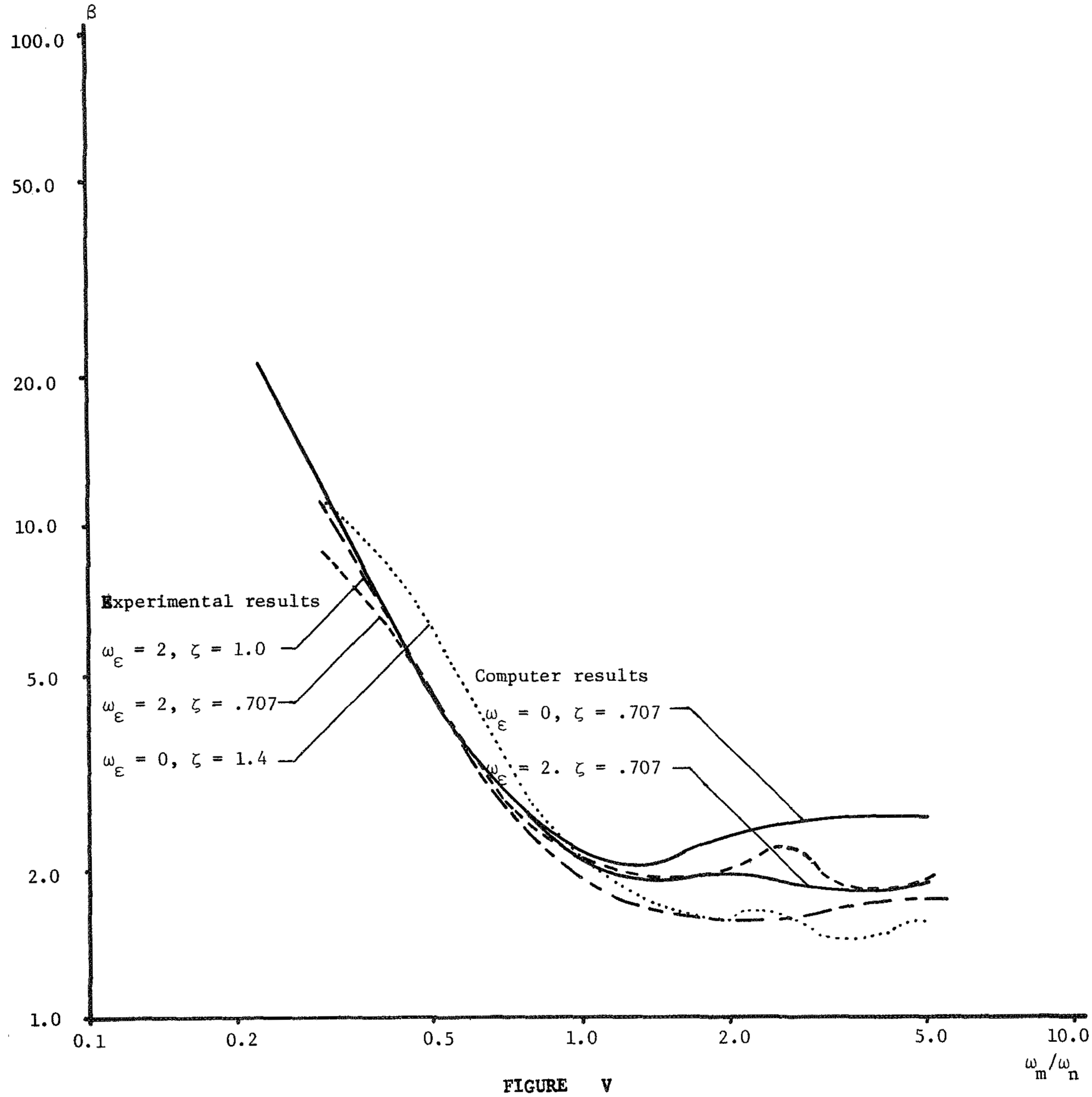

FIGURE V

produced beat notes twice, three times, etc., each cycle of the modulation.

In conclusion the investigation provides a separation of the modulation parameter space into two regions. In the first region the loop will acquire and track the carrier fulfilling a necessary condition for demodulation. In the second region the loop will acquire a sideband and the demodulated output will be highly distorted. The effects of small variations of the loop damping ratio are not of particular significance in the acquisition problem. Initial phase of the modulation, however, is. The fear that $\beta = 2.4$ is, in itself, sufficient to insure the loss of the carrier is dispelled. These results should provide a guideline to the engineer interested in acquiring and demodulating a FM modulated carrier with the PLL.

APPENDIX - I

The experimental work involved in the research presented in this paper was performed with the laboratory set-up shown in Figure A -I. Of particular importance to this research was the method of determining the modulation index of the modulated carrier, and the method of determining the acquisition and tracking of a sideband instead of the carrier. The modulation index was determined by observing the modulated carrier on a spectrum analyzer and comparing the normalized (to unity) amplitude components of the sidebands to the values computed in standard tables of the Bessel functions

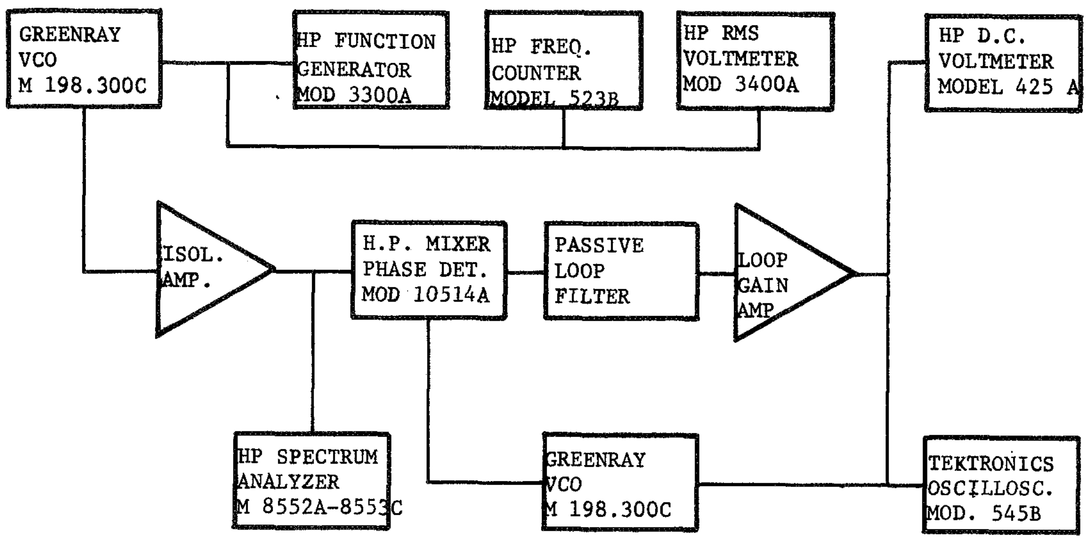

FIGURE A - I

of the proper order. This method yielded quite satisfactory results. Determination of the sideband tracking was made by observing the D.C. level at the input to the loop VCO, as well as the demodulated output on an oscilloscope. It was observed that a noticeable distortion in the demodulated output occured in the sideband acquisition region [5] and a D.C. shift equal to that predicted always occurred in this region. This joint observation defined the acquisition of a sideband.

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PHASE LOCK LOOP ACQUISITION OF AN ANGLE MODULATED CARRIER

by

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ABSTRACT

In this paper a baseband model which describes the behavior of the output signal from a second order PLL during acquisition and subsequent demodulation of an angle modulated carrier is developed. The non-linear differential equation resulting from the model is solved using a digital computer for frequency modulation by a single sine wave and by a square wave. The result is the definition of a region in the parameter space consisting of the modulation index, the ratio of the modulation frequency to the loop natural frequency, and the ratio of the initial difference in the VCO and carrier frequencies to the natural frequency of the loop, such that the PLL will acquire and properly demodulate the carrier. An experimental investigation was made and the results can be seen to compare closely with the theoretical results.

INTRODUCTION

A significant problem in the use of phase lock loops as demodulators for angle modulated carriers is the tendency of the PLL to lock onto sidebands of the modulated carrier [1]. The principal effort of this research was to determine parameter values for the second order PLL which would insure that sideband lockup would not occur.

The existence of circuits to prevent a PLL from acquiring a sideband instead of the carrier are well known (see for example [2], [3]). However, these circuits are in general quite complex and not well suited to applications requiring either economic or size constraints.

There has been a rather large amount of literature written on the problem of acquiring an unmodulated carrier with a PLL (see for example [4,5,6]). However, none of this literature sheds much light on the problem of acquiring a modulated carrier.

The approach to the problem of acquisition of a modulated carrier by a PLL used by the authors was to derive a differential equation which would describe the behavior of the output phase of the PLL during acquisition and demodulation. The equation was solved using numerical techniques on a digital computer.

The behavior of these solutions was examined as a function of the modulation parameters to determine under what conditions the loop acquired the carrier and under what conditions it acquired a sideband. This investigation of the loop behavior was carried out for both sinusoidal and square wave modulation. These results are presented later in the paper.

DIFFERENTIAL EQUATION

Baseband Model for Output Phase

In this section a baseband model of the PLL which describes the output during acquisition and subsequent demodulation of an angle modulated carrier will be derived. Figure 1 is an IF model of a second order PLL.

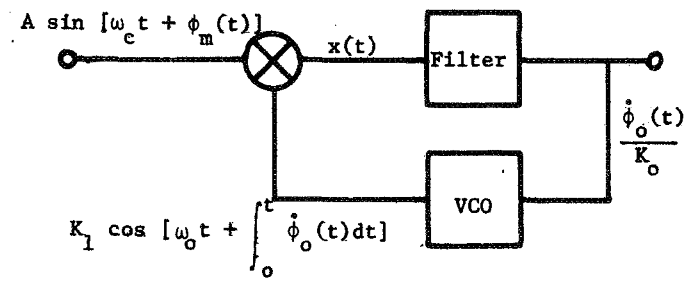

FIGURE 1

Performing the multiplication indicated in Figure 1, x(t) can be written as

$$x(t) = \frac{AK_1}{2} \sin[(\omega_c - \omega_o)t + \phi_m(t) - \phi_o(t)]$$
$$+ \text{ second harmonic term} \qquad (1)$$

where the $\int_o^t \dot{\phi}_o(t)dt$ has been denoted by $\phi_o(t)$.

Then denoting the impulse response of the filter by $h(\tau)$ the filter output can be written as,

$$\frac{\dot{\phi}_o(t)}{K_o} = \int_o^\infty x(u)\, h(t-u)du \qquad (2)$$

The impulse response of the ideal second order PLL is given by,

$$h(\tau) = u_o(\tau) + au_{-1}(\tau) \qquad (3)$$

where $u_o(\tau)$ denotes the Dirac delta function and $u_{-1}(\tau)$ denotes the step function.

Substituting equation 3 into 2 and simplifying gives

$$\frac{\dot{\phi}_o(t)}{K_o} = x(t) + a\int_0^t x(u)\,du \qquad (4)$$

Differentiating equation 4 with respect to t and applying Leibniz's theorem to the integral, a differential equation describing $\phi_o(t)$ is obtained as

$$\ddot{\phi}_o(t) = K_o\dot{x}(t) + ax(t) \qquad (5)$$

Substituting equation 1 into equation 5, neglecting the second harmonic terms gives

$$\ddot{\phi}_o(t) = \frac{AK_oK_1}{2}[(\omega_c - \omega_o) + \dot{\phi}_m(t) - \dot{\phi}_o(t)]\cos[(\omega_c - \omega_o)t + \phi_m(t) - \phi_o(t)] + \frac{AK_oK_1a}{2}\sin[(\omega_c - \omega_o)t + \phi_m(t) - \phi_o(t)] \qquad (6)$$

Finally, in order to comply with standard PLL notation define

$$\frac{AK_oK_1}{2} = 2\zeta\omega_n \qquad (7)$$

$$\frac{AK_oK_1a}{2} = \omega_n^2 \qquad (8)$$

Then substituting equations 7 and 8 into 6 the final form of the differential equation for $\phi_o(t)$ is

$$\ddot{\phi}_o(t) = 2\zeta\omega_n[\omega_\varepsilon + \dot{\phi}_m(t) - \dot{\phi}_o(t)]\cos[\omega_\varepsilon t + \phi_m(t) - \phi_o(t)] + \omega_n^2\sin[\omega_\varepsilon t + \phi_m(t) - \phi_o(t)] \qquad (9)$$

where ω_ε denotes $\omega_c - \omega_o$, the initial frequency offset.

The criterion for proper acquisition and demodulation used in this paper is a direct result of equation 9. Consider the case where $\phi_m(t) = 0$ (normal unmodulated carrier acquisition problem) and assume

$$\phi_o(t) = \omega_\varepsilon t \qquad (10)$$

Then substitution of equation 10 into 9 reveals that $\phi_o(t) = \omega_\varepsilon t$ is a solution of the differential equation.

The fact that $\phi_o(t) = \omega_\varepsilon t$ is a solution implies that the output of the PLL, $\dot{\phi}_o(t)/K_o$ is ω_ε/K_o. Now if $\phi_m(t)$ is a periodic signal with zero average value an obvious conjecture would be that ω_ε/K_o is the average value of the PLL output, E_{oDC}, when the loop is properly demodulating the carrier, i.e.,

$$E_{oDC} = \omega_\varepsilon/K_o \, . \qquad (11)$$

This conjecture is based on the assumption that the output due to modulation will be a periodic signal with average value equal to zero when the loop is properly demodulating.

Following the same line of reasoning the average output voltage would be

$$E_{oDC} = \frac{\omega_\varepsilon \pm k\omega_m}{K_o} \qquad (12)$$

when the loop is tracking a sideband where ω_m is a fundamental frequency of the modulation and k is the number of the sideband the loop is tracking.

This conjecture was born out completely by computer solutions of equation 9 and by laboratory experiments. For a complete description of the experimental work see [7]. An abbreviated discussion of the experimental set-up is given in Appendix I.

Having verified the above conjecture the procedure for determining when the PLL acquired the modulated carrier and when it acquired a sideband is now clear; the differential equation (equation 9) is solved numerically for various modulations and the resulting $\dot{\phi}_o(t)_{DC}$ obtained by averaging the output. Then apply equation 12 to determine k and the loop has acquired the modulated carrier if and only if k = 0.

ACQUISITION OF A SINUSOIDALLY MODULATED CARRIER

For the case of sinusoidal modulation $\phi_m(t)$ and $\dot{\phi}_m(t)$ are given by

$$\phi_m(t) = \beta\sin(\omega_m t + \theta_i) \qquad (13)$$

and

$$\dot{\phi}_m(t) = \Delta\omega\cos(\omega_m t + \theta_i) \qquad (14)$$

Equations 13 and 14 were substituted into equation 9 and the resulting differential equation was solved numerically for various values of the modulation index, β, the modulating frequency, ω_m, and the initial offset ω_ε. For this series of solutions θ_i was set at zero, ζ at .707 and ω_n was 1.0. The results are plotted in Figure 2. Figure 2 shows the division of the parameter space into two regions. The first region (below the curves) is the region in which the loop will acquire and demodulate the carrier. The second region (above the curves) is a region in which the loop will acquire sidebands.

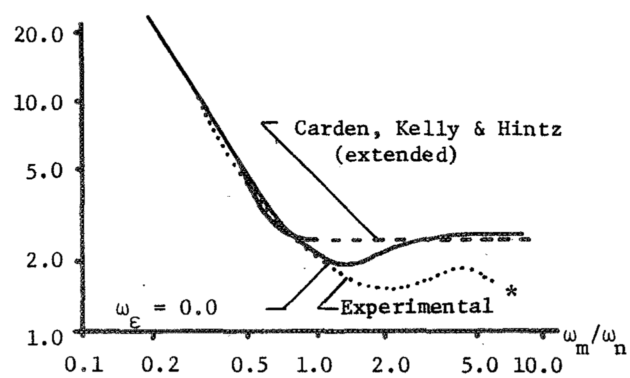

FIGURE 2

There are several points to be made about Figure 2. First, in the region $\omega_m/\omega_n \leq 1$ and $\omega_\varepsilon = 0$ the agreement with [8] is very good. This was a

* See Appendix I for comments on experimental parameters.

little surprising since Carden, Kelly and Hintz [8] obtained their results by assuming the loop was initially tracking the carrier and then the modulation was applied. The immediate conclusion would be that the β required to cause the loop to lose lock is almost the same as that required to prevent acquisition of the carrier. This implies that for sinusoidal modulation and very little offset the loop will acquire anything it can track.

The second point of interest is the apparent disagreement between the experimental and theoretical results in the region $\omega_m/\omega_n > 1.0$. This disagreement can be explained by the use of some initial phase, θ_i, not equal to zero. In the experimental portion of the investigation the initial phase of the modulation was not controlled and hence was not in general zero. Figure 3 is a plot of the critical value of β as a function of θ_i.

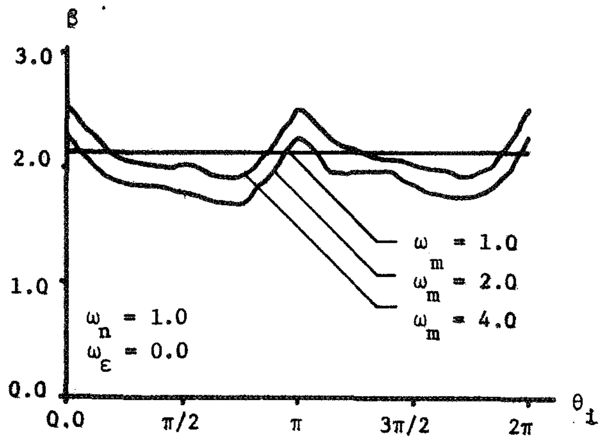

FIGURE 3

For the case of $\omega_m/\omega_n = 1.0$ which is within the region of good experimental and theoretical agreement the critical value of β is almost constant and equal to the measured value. However, for $\omega_m = \omega_\varepsilon = 2.0$ which is in the region of question the critical value of β varies from .9 to 2.4 as a function of θ_i. The experimental value of β for this ω_m/ω_n is 2.0. Thus the experimental results are between the extremes of the theoretical solution and hence the agreement is still quite good if various values of θ_i are considered. Due to the additional amount of computer time involved and the fact that this isn't an extremely interesting region the authors didn't investigate the effect of θ_i any further.

The final point of interest about Figure 2 is that for $\omega_\varepsilon/\omega_n = 2.0$ the critical values of β are very near those of $\omega_\varepsilon/\omega_n = 0$, as illustrated in Figure 5. Hence, for moderate values of frequency offset the loop can still acquire almost anything it can demodulate.

In order to study the effect of small changes in the zeta of the loop equation 9 was solved with $\zeta = .400$ under all of the above conditions. The results are shown in Figure 4. The principal point of interest concerning Figure 4 is that small variations in zeta have essentially no effect and hence figures 2 and 5 should be applicable for all zetas between .400 and 1.00.

In Figure 5 the results of numerical and experimental investigation for $\omega_\varepsilon = 0$ and 2 and various

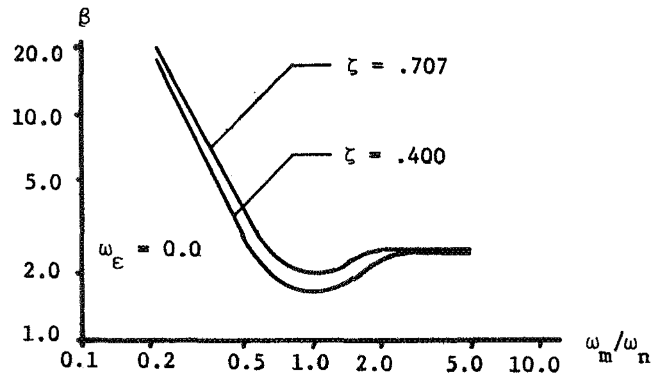

FIGURE 4

zetas is given. These curves provide a summary of the research into the acquisition behavior for the case of modulation by a single sinusoid. Due to the size of Figure 5 it is presented on the next page.

ACQUISITION OF A SQUARE WAVE MODULATED CARRIER

For the case of frequency modulation by a square wave, it is easier to sketch $\phi_m(t)$ and $\dot{\phi}_m(t)$ than it is to write analytical expressions for them. The modulating functions used in this portion of the study are shown in Figure 6.

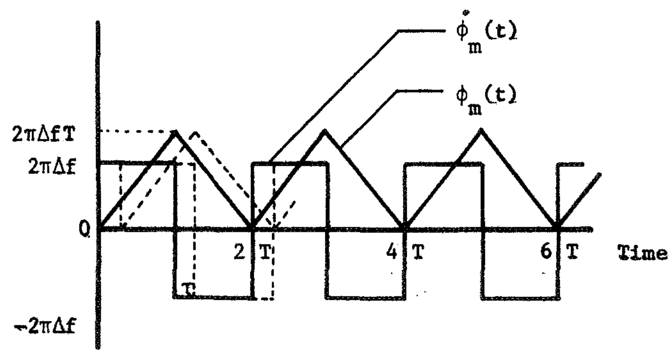

FIGURE 6

The parameter τ in Figure 6 serves the same purpose as θ_i in the expressions for the sinusoidal modulation.

Using the functions shown in Figure 6, equation 9 was solved numerically using equation 12 again to determine whether or not the loop acquired the carrier. A parameter space consisting of ΔfT and $\pi/T\omega_n$ (note $\pi/T\omega_n = \omega_m/\omega_n$ where ω_m is the fundamental frequency of the square wave) was investigated to determine the region in which the loop will acquire and demodulate the carrier. The results of this investigation are shown in Figure 7 which is also placed on a later page due to its size.

In examining Figure 7 in the region of $\pi/T\omega_m > 1.0$ the apparent disagreement between theoretical and experimental results again is due to the effect of the starting point of the modulation in the cycle, i.e., the parameter τ. Figure 8 is a plot of ΔfT critical as a function of τ.

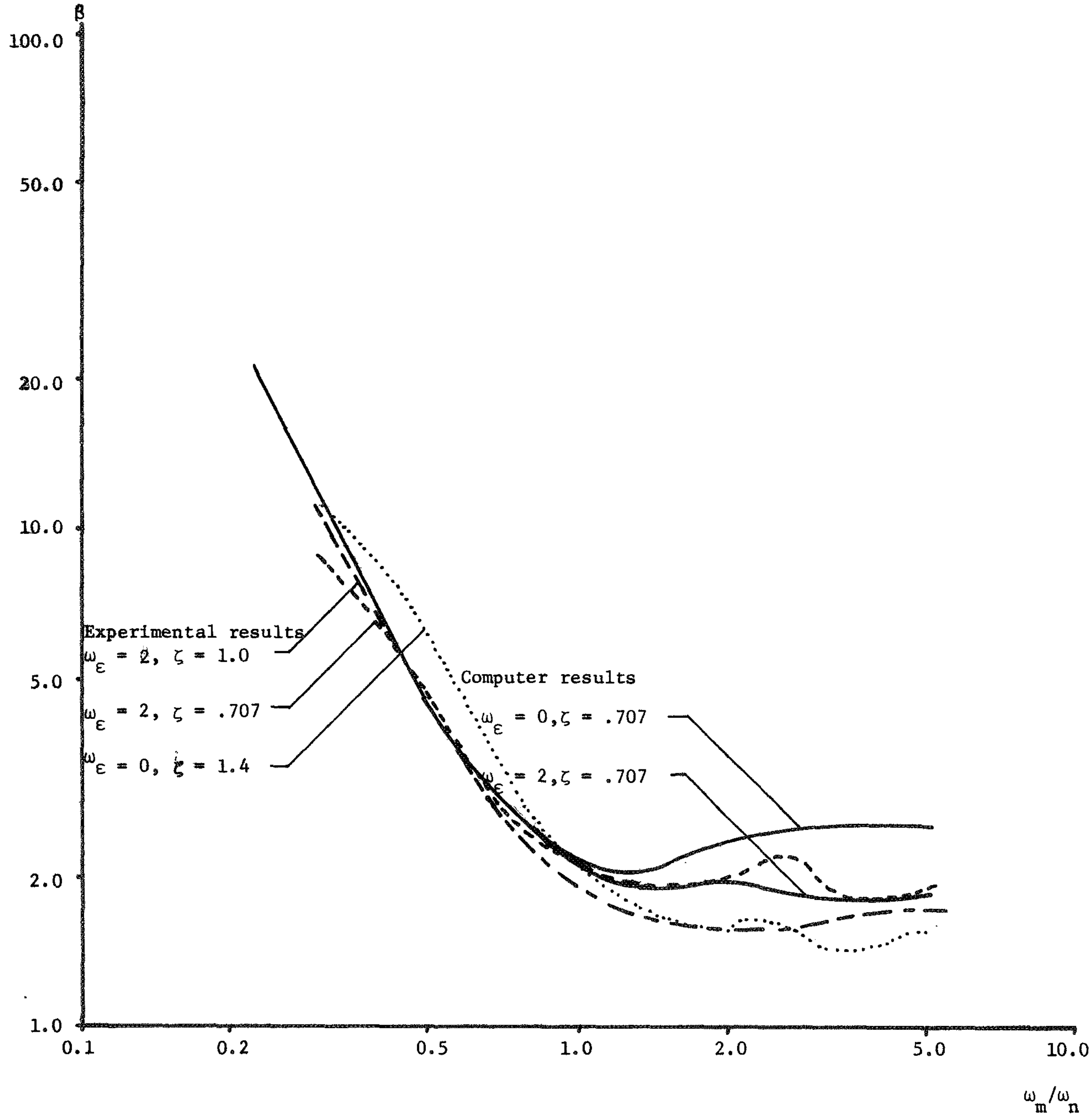

FIGURE 5

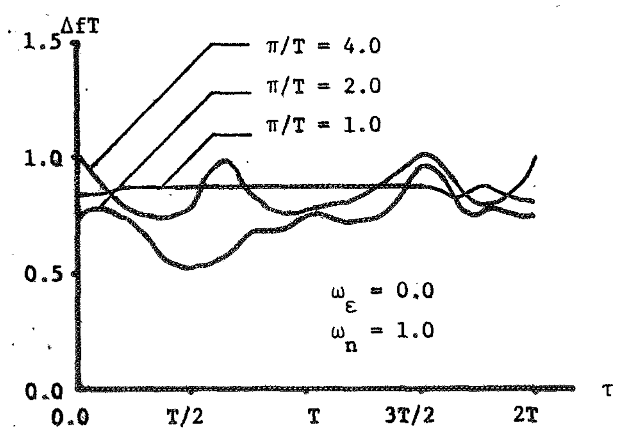

FIGURE 8

By exactly the same line of reasoning as was used for sinusoidal modulation, it can be seen that the experimental results and theoretical results are actually in close agreement if variations in τ are considered. However, due to the additional computer time the authors again chose to hold $\tau = 0$ *for the majority of the work.*

The points in Figure 7 labeled by small squares are the result of a calculation based on the tracking performance of the PLL. Carden, Lucky and Swinson [9] obtained results for the magnitude of a step in frequency such that a loop which was tracking a carrier would skip one cycle and then continue to track. It was conjectured that for $\omega_m/\omega_n << 1.0$ each transition of the square wave could be considered independent and hence the output of the loop would be a series of step responses. If this were the case, then the value of $\Delta\omega$

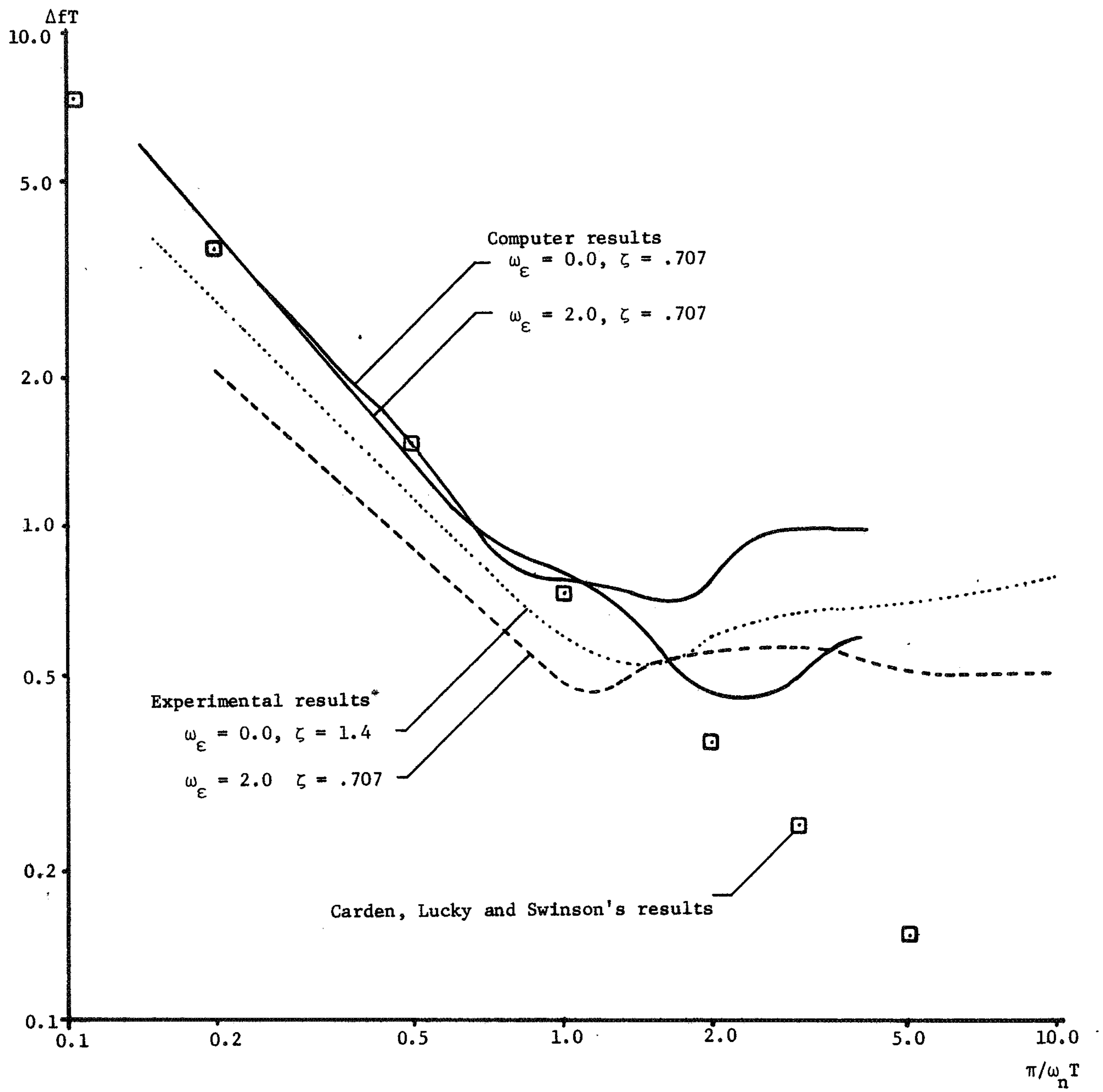

FIGURE 7

such that the loop would skip one cycle per transition might also be the $\Delta\omega$ **for which the** loop would acquire a sideband. From [9] the critical value of $\Delta\omega/\omega_n$ for $\zeta = .707$ is

$$\Delta\omega/\omega_n = 1.5. \tag{15}$$

Manipulation of equation 15 gives the critical ΔfT as

$$\Delta fT = .75\left(\frac{1}{\pi/\omega_n T}\right). \tag{16}$$

The points on Figure 7 labeled with squares are solutions of equation 16. It can be seen that in the region $\pi/\omega_n T > 1.0$ the analogy of the step response breaks down and the resulting curves diverge.

The final point of interest about Figure 7 is that for a frequency offset $\omega_\varepsilon/\omega_n = 2.0$ there are no essential differences in the curves. This indicates that for moderate offsets the region in which the loop will acquire and demodulate a square wave is essentially the same as with no offset, i.e., $\omega_\varepsilon = 0$.

ACQUISITION TIME

In order to investigate acquisition behavior the authors took a different definition of acquisition time than is normally used in the study of the unmodulated case. The definition chosen was the time of the first zero crossing of $\dot{\phi}_e(t)$ where $\dot{\phi}_e(t) = \omega_\varepsilon + \dot{\phi}_m(t) - \dot{\phi}_o(t)$. The reason for this definition can best be explained by a phase plane portriat. Figure 9 is a phase plane diagram of $\phi_e(t)$ versus $\dot{\phi}_e(t)$ for the case of sine wave modulation and $\omega_\varepsilon = 0$.

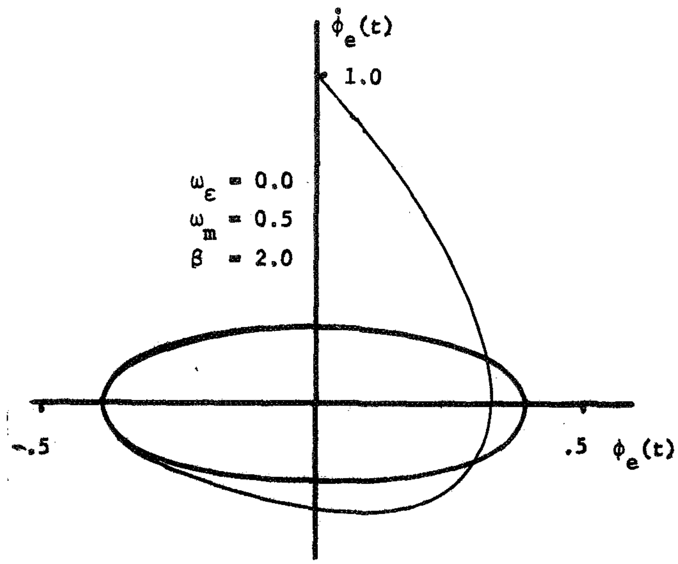

FIGURE 9

From Figure 9 it can be seen that the loop enters its stable trajectory at approximately the first zero crossing of $\phi_e(t)$ and this is the motivation for the definition of acquisition time chosen by the authors. Some typical acquisition times are presented in Table 1, all for values of $\Delta\omega \cdot \omega_m \leq 0.9$, since it was not determined that the definition of acquisition time was meaningful for larger values of $\Delta\omega \cdot \omega_m$.

TABLE 1

ω_m	β	ω_ε	Time (sec.)
.50	2.0	0.0	.099
.50	2.0	0.5	.106
.50	2.0	1.0	.115
.40	2.0	0.5	.109

CONCLUSION

To summarize, the authors have presented a study of the acquisition behavior of a second order phase lock loop when the input is an angle modulated carrier. The baseband differential equation can be used to solve similar acquisition problems for other applications since it is the result of a general development.

The primary results of this study are shown in the parametric regions of Figures 5 and 7. This result is to divide a parameter space consisting of appropriate modulation parameters into two regions. The first region is a region in which the loop will acquire and subsequently demodulate an angle modulated carrier. This information can be used to design phase lock loops which do not need acquisition aids or special complex circuits to prevent locking onto a sideband. In addition, such loops can operate unattended and with no external control. This feature should make them attractive for application in commercial systems.

APPENDIX I

The experimental work involved in the research presented in this paper was performed with the laboratory set-up shown in Figure A - 1.

Of particular importance to this research was the method of determining the modulation index, β, of the modulated carrier, and the method of deter-

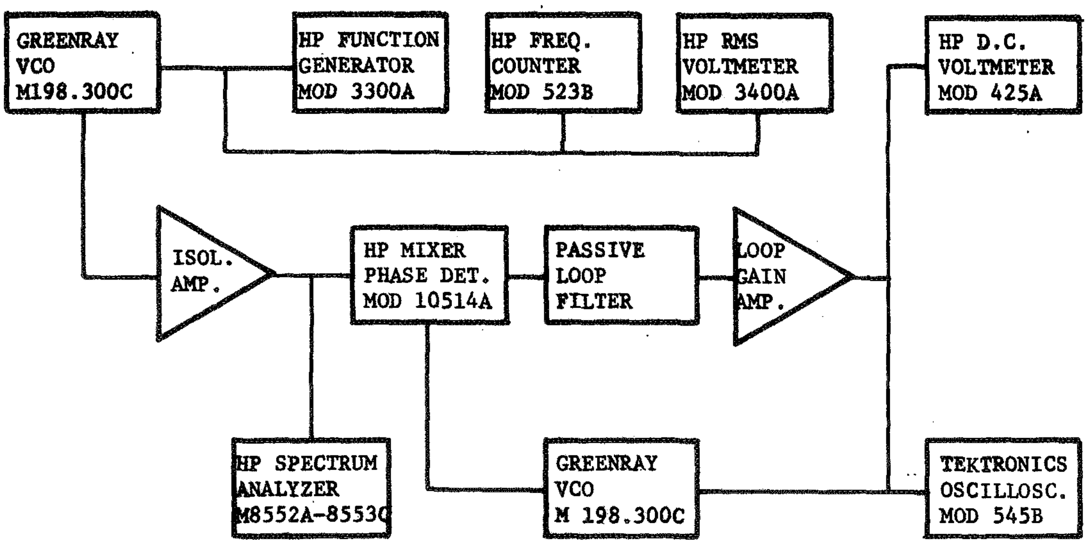

FIGURE A - 1

mining the acquisition and tracking of a sideband instead of the carrier. The modulation index for a sine wave was determined by observing the modulated carrier on a spectrum analyzer and comparing the normalized(to unity) amplitude components of the sidebands to the values computed in standard tables of the Bessel functions of the proper order. For square wave modulation Δf was determined from the modulating voltage and the carrier generation VCO constant. These methods yielded quite satisfactory results. Determination of sideband tracking was made by observing the D.C. level at the input to the VCO of the loop, as well as the demodulated output on an oscilloscope. It was noted that a noticeable distortion in the demodulated output occurred in the sideband acquisition region [7] and a D.C. shift equal to that predicted always occurred in this region. This joint observation served as the basis for defining the acquisition and demodulation of a sideband.

It should be noted that immediately prior to publication the authors discovered that the PLL used to obtain the experimental data for Figure 2 had a zeta of 1.0 instead of .707. However, due to the fact that actual values of zeta near .7 have little effect on the acquisition problem this does not represent a serious error.

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