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DYNAMIC INTERACTION BETWEEN STRUCTURE AND LIQUID PROPELLANTS IN A SPACE SHUTTLE VEHICLE MODEL

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FINAL REPORT, PART I

Contract No. NAS1-9890

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SwRI Project No. 02-2820

Prepared for

National Aeronautics and Space Administration

Langley Research Center

Hampton, Virginia

October 15, 1970

SOUTHWEST RESEARCH INSTITUTE

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Approved:

H. Norman Abramson, Director
Department of Mechanical Sciences

PREFACE

This report constitutes the first of two volumes which summarize the work accomplished under Contract NAS1-9890. It contains the design of an experimental space shuttle vehicle model, supporting experimental data, the design of a corresponding analytical model, comparisons of results, and a listing of the digital computer program designed for predicting natural frequencies of a typical shuttle vehicle parallel-stage configuration which includes liquid propellants. The second part of the work, which deals with other liquid dynamics problems associated with space shuttle operation, is summarized in Final Report, Part II entitled "Propellant Dynamic Problems in Space Shuttle Vehicles."

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NOMENCLATURE

Symbol	Definition
a	distance between subsystems 2 and 11
$\underset{\sim}{a}$	coefficient matrix of constraint coordinates appearing in equations of motion
b	distance between subsystem 11 and the centerline of the Orbiter
c	distance between subsystems 12 and 13
$\underset{\sim}{c}$	constraint matrix
D_k	distance between the neutral axis of a reinforcing beam section and a cylinder shell ($k = 1, 2, 3, 4$)
d	distance between subsystems 15 and 16
$\underset{\sim}{d}$	coefficient matrix of mass coordinates appearing in equations of constraint
E_s	modulus of elasticity of cylinder shells ($s = 1, 2, 3, 5, 6, 7$)
$E^{(m)}$	modulus of elasticity of a reinforcing beam section ($m = 1, 2, 3, 4$)
e	distance between subsystem 5 and the rod $K^{(5)}$

$$F_i = \frac{4}{\pi} \frac{I_1\left(\frac{\pi R_i}{2h_i}\right)}{\frac{\pi R_i}{2h_i} I_o\left(\frac{\pi R_i}{2h_i}\right)}$$

Symbol	Definition
G_s	shear modulus of cylinder shell s
g	gravitational constant
H	total number of dynamic and constraint equations of the mechanical system

Symbol	Definition
h_i	height of liquid in cylinder i
$\tilde{h}_i$	height of horizontal sloshing mass in cylinder i
h_i^*	height of rigid mass in cylinder i
$\mathcal{I}_s$	moment of inertia of a cylinder cross section
I_o	modified Bessel function of order zero
I_1	modified Bessel function of order one
i	index
J_n	moment of inertia of mass element M_n (n = 1, 2, . . ., 8)
$J^{(m)}$	moment of inertia of a reinforcing beam cross section
J_i^*	moment of inertia of rigid mass m_i^*
$J^{(i)}_{rigid}$	moment of inertia of frozen liquid in cylinder i
j	index
K_i	equivalent longitudinal spring constant of cylinder i accounting for liquid effect
$K_j = \dfrac{2\pi R_j \delta_j E_j}{\ell_j}$	longitudinal spring constant of cylinder j
$\overline{K}_i = \dfrac{2\pi R_i \delta_i E_i}{\ell_i - \nu_i^2 h_i}$	longitudinal spring constant of cylinder i with liquid effect
$K^{(m)}, \overline{K}^{(3)}, \overline{K}^{(4)}$	longitudinal spring constants of the reinforcing beam sections
$K^{(6)}, K^{(7)}$	vertical and horizontal spring constants of the coupling spring system
k_i	vertical sloshing spring constant

Symbol	Definition
$\overline{k}_i$	vertical sloshing spring constant due to Poissons effect of the cylinder
$k_i' = \dfrac{4\pi R_i \delta_i E_i}{h_i}\left[\dfrac{I_1\left(\frac{\pi R_i}{2h_i}\right)}{\frac{\pi R_i}{2h_i} I_o\left(\frac{\pi R_i}{2h_i}\right)}\right]^2$	
$\tilde{k}_i$	lateral sloshing spring constant
k	index
L	number of dynamic equations
L'	number of constraint equations
ℓ_s	length of cylinder section
M_n	structure mass element
$M^{(\ )}, \overline{M}^{(\ )}$	end moments of a beam section
$M^{(\ )}_{[\ ]}, \overline{M}^{(\ )}_{[\ ]}$	net end moments of a shell-beam section
m_i	vertical sloshing mass in cylinder i
$\tilde{m}_i$	horizontal sloshing mass in cylinder i
m_i^*	rigid mass in cylinder i
$m_T^{(i)} = \pi R_i^2 h_i \rho_i$	total mass of the liquid in cylinder i
m	index
n	index
$P_{(\ )(\ )}$	mass matrix of the total system of dynamic and constraint equations

NOMENCLATURE (Cont'd)

Symbol	Definition
$\underset{\sim}{p}$	mass matrix of the dynamic equations only
$Q_{(\ )(\ )}$	stiffness matrix of the total system of dynamic and constraint equations
$\underset{\sim}{q}$	stiffness matrix of the dynamic equations only
r	index
R_s	R_B (Booster radius) for s - 1,2,3; R_O(Orbiter radius) for s = 5,6,7
S_r	torsional spring constant (r = 1,2,..., 7)
s	index
t	time
$V^{(\ )}$	lateral shearing force on an end cross section of a beam section
$V^{(\ )}_{[\ ]}$	net lateral shearing force on an end cross section of a shell-beam section
$W_{(\ )}$	vibration mode
$\underset{\sim}{X}$	coordinates associated with mass elements (see Table III)
$\underset{\sim}{x}$	coordinates associated with constraint conditions (see Table III)
$x_{(\ )}$	lateral displacement of a subsystem
$Y_{(\ )}$	amplitude of a vibration mode
$y_{(\ )}$	axis of rotation of a subsystem
$z_{(\ )}$	vertical displacement of a subsystem
$\alpha_{(\ )}$	rotation of a cross section of a shell beam where a horizontal sloshing spring or a rigid mass is attached

NOMENCLATURE (Cont'd)

Symbol	Definition
γ	phase angle
δ_s	cylinder shell thickness
$\zeta_{(\ )}$	vertical displacement of a vertical sloshing mass
$\eta_{(\ )}$	horizontal displacement of a lateral sloshing mass
ν_s	Poissons ratio of cylinder shell
$\xi_{(\ )}$	horizontal displacement of a shell beam cross section where the lateral sloshing spring or a rigid mass is attached
ρ_i	mass density of liquid in cylinder i
$\phi_{(\ )}$	rotation of a structure subsystem
Ω	eigenvalue of a vibration mode
ω	frequency

DYNAMIC INTERACTION BETWEEN STRUCTURE AND LIQUID PROPELLANTS IN A SPACE SHUTTLE VEHICLE MODEL

By Daniel D. Kana, William L. Ko,
Philip H. Francis, and Andrew Nagy
Southwest Research Institute

INTRODUCTION

Currently specified design requirements of a space shuttle vehicle are anticipated to present many new problems heretofore not encountered in aerospace systems. The dynamic interaction between elastic structure and liquid propellants has always been an important design criteria for launch vehicles and aircraft; however, it is surmised that potential problems posed by this interaction will become even more critical in presently envisioned space shuttle systems (ref. 1). Therefore, the purpose of this study is to examine the applicability of existing analytical techniques for studying the coupled liquid-structural dynamics of a typical space shuttle configuration--a parallel-stage design.

The program objective has been accomplished by developing a suitable experimental model that is capable of experiencing at least the most fundamental structural dynamics of a prototype system, measuring its natural frequencies of vibration for a range of various parameters, and comparing the results with those predicted from a corresponding analytically derived model. In selecting the model details from the outset, considerable effort was exercised to utilize components which were already available from previous research programs, in order to minimize fabrication costs. Further, existing concepts of spring-mass fluid models, which have been derived to simulate liquid reactions for decoupled lateral and longitudinal motions, are employed in a straightforward manner in a system which experiences strong coupling along these axes. The results of this study of a rather fundamental model will point toward the path to follow for more complex representations of a shuttle system. We begin with a description of the physical model, then outline the analysis, and finally present results and conclusions from the study.

DESCRIPTION OF PHYSICAL MODEL

A model consisting of a parallel-stage Booster and Orbiter, each consisting of two propellant tanks and appropriate intermediate skirts and rigid masses, was considered feasible to carry out the program objective. The major portion of the Booster was already on hand from a previous study (ref. 2) of longitudinal dynamics in axisymmetrical launch vehicles. As a result, as will be seen, it also included some components, such as stiffeners and baffles, which were not strictly essential to the present study. Nevertheless, the presence of these extra components did not alter the conclusions of the study.

A photograph of the completely assembled and suspended system is shown in Figure 1a, as it was used during most of the experiments. In order to provide a quick overall indication for model size and typical rigid masses, a schematic is shown in Figure 1b. Further details will be given now, as well as in later sections of the report.

The Booster comprises the major part of the model, and it consists of the following main components:

(1) Upper tank

(2) Lower tank

(3) Skirt

(4) Bulkheads

Both tanks in the Booster model were fabricated from 0.005-inch thick, type 302 Stainless Steel sheets, which were rolled and butt-welded along longitudinal seams. A flat steel disk was spot-welded to one of the tanks which serves as a top mass for the upper tank. The lower end of this upper tank and both ends of the lower tank were spot-welded to identical steel flanges, which can be bolted to the bulkheads and the skirt as required. Two rows of spot welds, each spot weld having a test strength of 25 pounds, were used at each end of the tanks. The spots were spaced 1/8-inch apart with 1/8-inch spacing between the rows. To provide for ullage pressure integrity, the ends of both tanks were sealed with epoxy cement.

The third main component in the Booster is the skirt. It was fabricated by rolling 0.025-inch thick, 6061-T6 aluminum sheet to the desired diameter and butt-welding it along a longitudinal seam. Two aluminum flanges were welded to the end of this cylinder with the same hole pattern as on the flanges of the tanks to provide for bolted assembly of these parts. Two small ports on the side of the skirt served as a pressure port for introducing ullage pressure to the lower tank and for filling the tank with liquid.

Flat, rigid bulkheads were machined from mild steel and 6061-T6 aluminum plates for the lower and upper tanks, respectively, with a shoulder

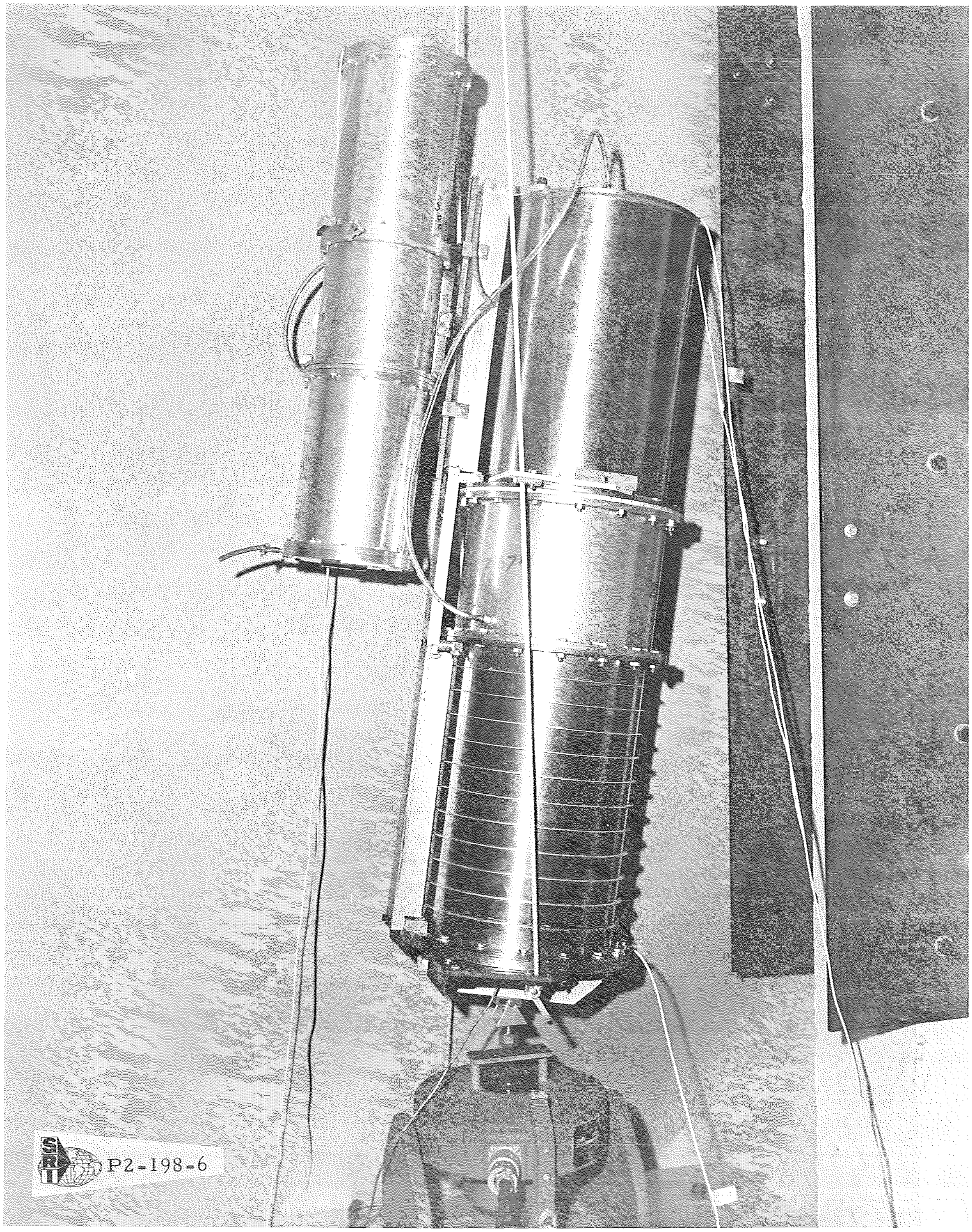


FIGURE 1a. --SwRI SPACE SHUTTLE VEHICLE DYNAMIC MODEL

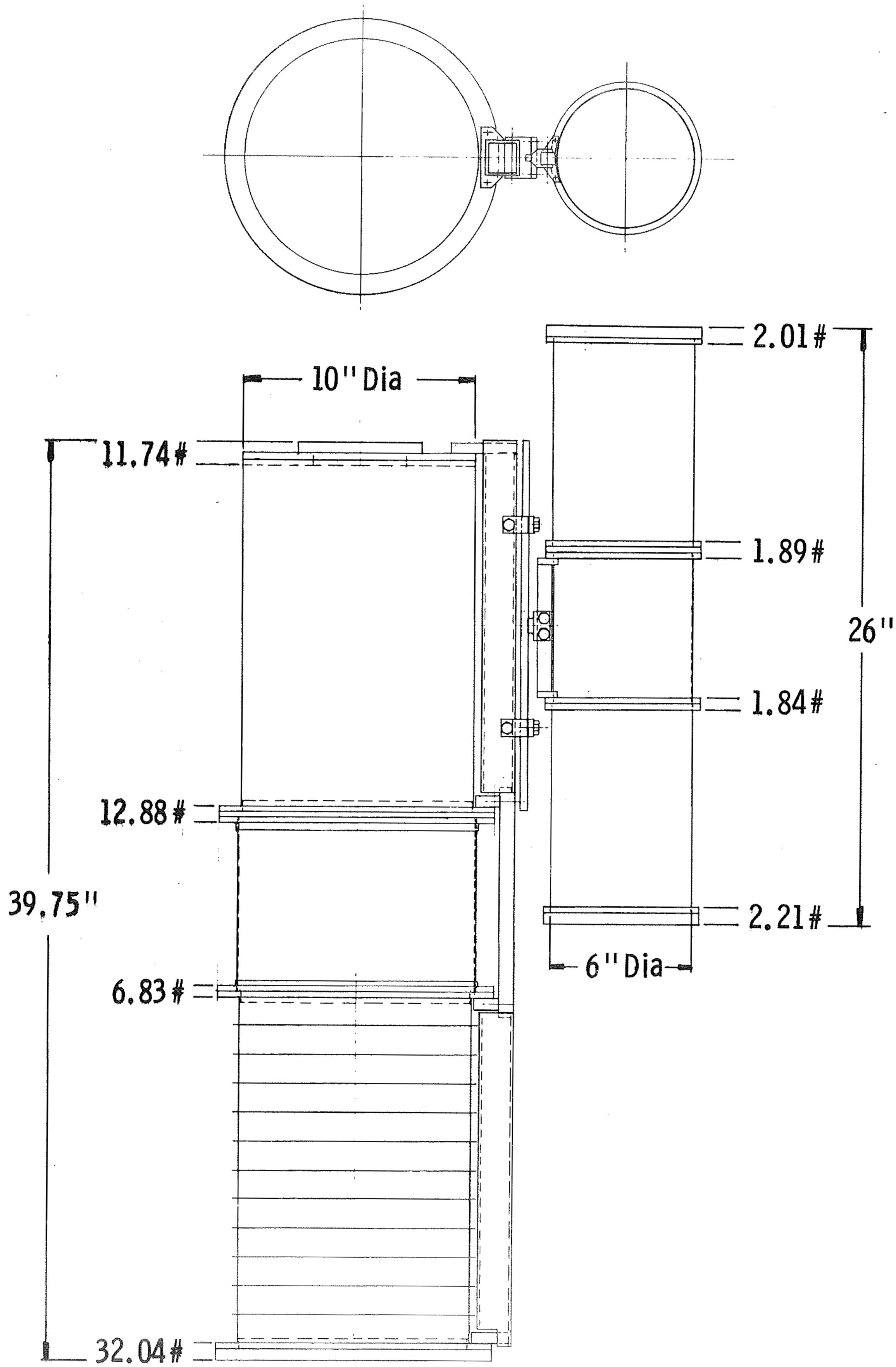


FIGURE 1b. --SCHEMATIC OF SPACE SHUTTLE VEHICLE MODEL

on them so they could partially fit inside the tanks. The shoulders and their hole patterns match the flanges on the tanks. The lower ends of the tanks were sealed by these flat, rigid bulkheads by bolting them to the corresponding flanges. The tanks in turn were joined by bolting them together using the skirt as a coupling.

The lower tank had stiffener rings, stringers, and baffles installed on it. Eleven 6061-T6 aluminum rings, 0.032-inch thick were cemented on the outside of the tank 1.25 inches apart, symmetrically about midspan. Twelve 6061-T6 aluminum stringers, 1/8 × 1/8 × 14 inches in dimensions, were fastened to the inside of the tank by use of epoxy. The stringers were equally spaced around the circumference of the tank and symmetrical about midspan. Tapped holes along the length of these stringers provided the means for installing eleven yellow brass ring baffles into the tank. Data pertaining to the Booster model are given in Table I.

The Orbiter model, similar to the Booster, consists of two tanks with flat, rigid bulkheads and a skirt. The tanks and the skirt were fabricated from 0.012-inch-thick and 0.020-inch-thick 1100-H14 aluminum sheets, respectively, which were rolled and butt-welded along a longitudinal seam in the same manner as the Booster. Identical flanges with 16-hole bolt patterns were welded to each end of both tanks and skirt by a continuous weld.

Flat, rigid bulkheads were machined from 6061-T6 aluminum plates with hole patterns matching the pattern on the flanges. These bulkheads, however, did not have shoulders on them as was the case for the Booster. Plates identical to the bulkheads were used for capping both of the tanks on the Orbiter with provisions for introduction of ullage pressure and modeling liquid.

The skirt, as in the case of the Booster, was used to join the two tanks. Data pertaining to the Orbiter model are given in Table II.

Coupling between the Booster and Orbiter was achieved by the strongback assembly. As can be seen in Figure 1, the strongback spans the full length of the Booster and is attached to it at four locations, namely, at each flange and the top. This part of the strongback was fabricated from a 1-1/2 × 1-1/2 × 1/8-inch, 6063-T5 aluminum square tube with appropriate altering to be attachable to the Booster.

A short backstrap was attached to the Orbiter spanning between the two flanges of the skirt. To this backstrap, a 1/4-inch square steel rod was fastened which fits into two guides on the strongback allowing adjustment of the relative position between the Booster and Orbiter.

TABLE I. --MATERIAL PROPERTIES AND GEOMETRY OF STRUCTURAL COMPONENTS OF BOOSTER MODEL

Structural Element	Effective Length(ℓ) (in.)	Inside Dia. (in.)	Wall Thickness (in.)	Material Density (#/in^3)	E $\times 10^6$ psi
Upper Tank	14.5	10.0	0.005	0.29	29
Lower Tank	14.5	10.0	0.005	0.29	29
Skirt	7.5	10.3	0.025	0.098	10

	Material Density (#/in^3)	E $\times 10^6$ psi	Dimensions (in.)
Flat Rigid Bulkhead Lower Tank	0.29	30	9.875 Dia., 1/2 Height, 12 Dia. shoulder, 1/2 Height
Flat Rigid Bulkhead Upper Tank	0.098	10	9.875 Dia., 1/2 Height, 12 Dia. shoulder, 1/2 Height

	Number Used on Tank	Material Density (#/in^3)	E $\times 10^6$ psi	Dimensions (in.)	Location	Spacing (in.)
Stiffener Ring	11	0.098	10	10.0 I.D. 10.5 O.D. 0.032 thick	Symmetrical about midspan	1.24
Stringer	12	0.098	10	0.125×0.125 × 14.0	Symmetrical about midspan	Equally spaced on inner circumference
Baffle	11	0.306	16	0.25 I.D. 9.68 O.D. 0.0125 thick	Symmetrical about midspan	1.25

TABLE II. --MATERIAL PROPERTIES AND GEOMETRY OF STRUCTURAL COMPONENTS OF ORBITER MODEL

Structural Element	Effective Length(ℓ) (in.)	Inside Dia. (in.)	Wall Thickness (in.)	Material Density (#/in^3)	E $\times 10^6$ psi
Upper Tank	8.5	6.0	0.012	0.098	10
Lower Tank	8.5	6.0	0.012	0.098	10
Skirt	6.0	6.0	0.020	0.098	10

	Diameter (in.)	Thickness (in.)	Material Density (#/in^3)	E $\times 10^6$ psi
Bulkheads	6.7	0.25	0.098	10
Caps	6.7	0.25	0.098	10

ANALYTICAL MODEL

Mechanical Model

In the modal analysis of free vibration of the model Shuttle Vehicle, the system is represented by the equivalent mechanical model shown in Figure 2a. The motion of the system will be limited to translations in x and z directions and pitching about an axis perpendicular to xz-plane. The cylindrical shells between any two neighboring mass elements will be represented by thin-walled beamlike tubes, or shell-beams, as shown in Figure 2b. Additional details are given in Figures 2c and 2d. Masses of the cylindrical shell sections, Booster and Orbiter strongback beam sections, and between any two neighboring mass elements are divided equally into two parts, each of which is lumped into each of the two mass elements. Thus, the inertia effect of the shells and the strongback beams will be otherwise neglected. The Booster and the Orbiter strongback reinforcing beams are pin-jointed, respectively, to the Booster and the Orbiter mass elements with one torsional spring attached to each joint. The two beams are then connected together through a coupling compound spring system which permits relative displacements in x and z directions, and one relative rotation about an axis perpendicular to the xz plane. The lower end of the Orbiter is connected to the Booster strongback beam through a rod $K^{(5)}$ both of whose ends are pin-jointed.*

The vertical and lateral sloshing motions of the liquid in a cylinder are represented independently by two sloshing models. The vertical sloshing model (ref. 3) consists of one vertical sloshing mass m_i ($i = 1, 3, 5, 7$) connected to the neighboring mass elements through two springs† k_i and $\bar{k}_i$. It will be assumed that the bending of the shell-beam does not interfere with the motion of m_i. The lateral sloshing model‡ (refs. 4, 5, 6) consists of one rigid mass m_i^* rigidly attached to the shell-beam, and one lateral sloshing mass $\tilde{m}_i$ connected to the shell-beam through two springs of spring constant $\tilde{k}_i/2$. For the vertical sloshing, quantities associated with the horizontal sloshing (i.e., m_i^* , $\tilde{m}_i$, and $\tilde{k}_i$) will be set to zero, and vice versa.

*This rod was removed from the experimental apparatus and, consequently, its effect was nullified in the numerical program by setting to zero its cross-sectional area.

†All springs in the model are massless and linear.

‡This model was originally developed for a rigid container.

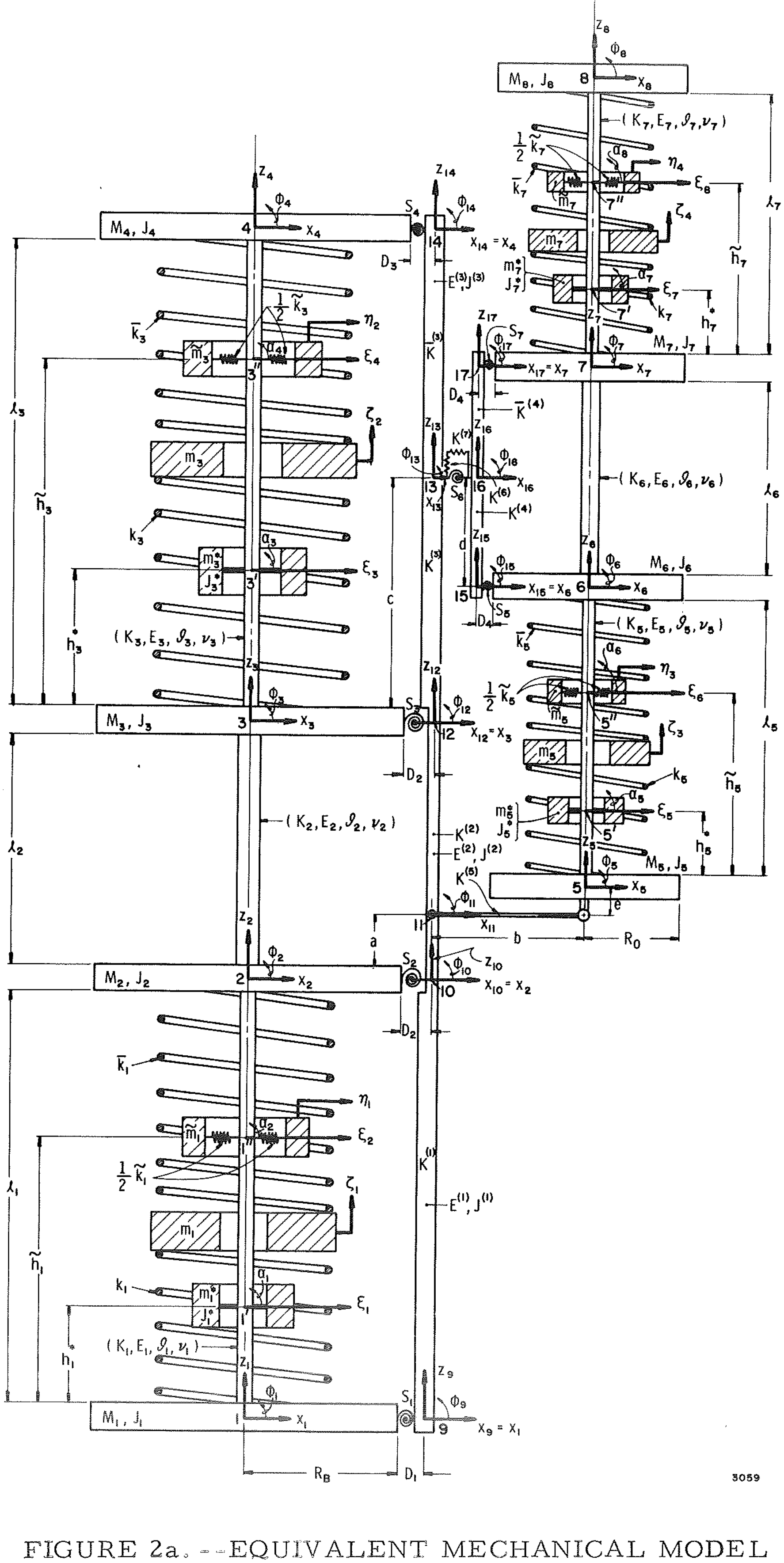
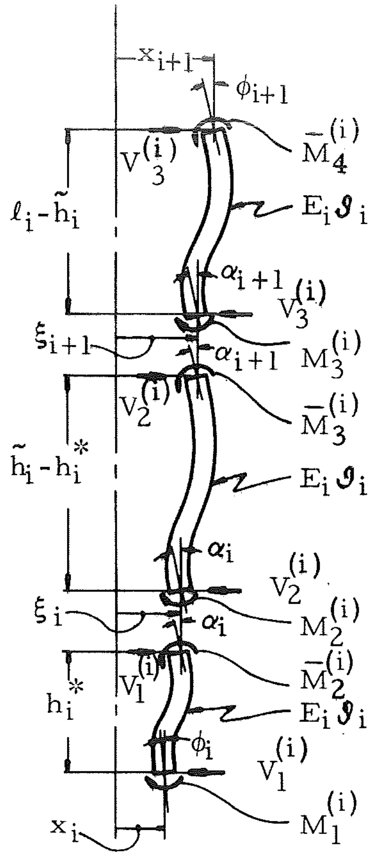


FIGURE 2a. --EQUIVALENT MECHANICAL MODEL



$$\bar{M}_4^{(i)} = \frac{2E_i\vartheta_i}{N_{i3}(\ell_i-\tilde{h}_i)^2}\left[3(x_{i+1}-\xi_{i+1})+(\ell_i-\tilde{h}_i)(2\phi_{i+1}A_{i3}+\alpha_{i+1}B_{i3})\right]$$

$$V_3^{(i)} = \frac{6E_i\vartheta_i}{N_{i3}(\ell_i-\tilde{h}_i)^3}\left[2(x_{i+1}-\xi_{i+1})+(\ell_i-\tilde{h}_i)(\alpha_{i+1}+\phi_{i+1})\right]$$

$$M_3^{(i)} = \frac{-2E_i\vartheta_i}{N_{i3}(\ell_i-\tilde{h}_i)^2}\left[3(x_{i+1}-\xi_{i+1})+(\ell_i-\tilde{h}_i)(2\alpha_{i+1}A_{i3}+\phi_{i+1}B_{i3})\right]$$

$$\bar{M}_3^{(i)} = \frac{2E_i\vartheta_i}{N_{i2}(\tilde{h}_i-h_i^*)^2}\left[3(\xi_{i+1}-\xi_i)+(\tilde{h}_i-h_i^*)(2\alpha_{i+1}A_{i2}+\alpha_iB_{i2})\right]$$

$$V_2^{(i)} = \frac{6E_i\vartheta_i}{N_{i2}(\tilde{h}_i-h_i^*)^3}\left[2(\xi_{i+1}-\xi_i)+(\tilde{h}_i-h_i^*)(\alpha_i+\alpha_{i+1})\right]$$

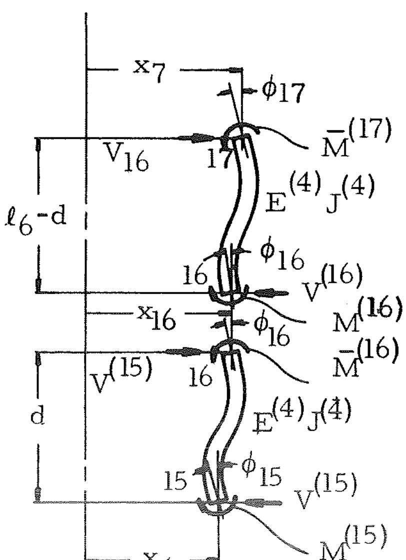
$$M_2^{(i)} = \frac{-2E_i\vartheta_i}{N_{i2}(\tilde{h}_i-h_i^*)^2}\left[3(\xi_{i+1}-\xi_i)+(\tilde{h}_i-h_i^*)(2\alpha_iA_{i2}+\alpha_{i+1}B_{i2})\right]$$

$$\bar{M}_2^{(i)} = \frac{2E_i\vartheta_i}{N_{i1}(h_i^*)^2}\left[3(\xi_i-x_i)+h_i^*(2\alpha_iA_{i1}+\phi_iB_{i1})\right]$$

$$V_1^{(i)} = \frac{6E_i\vartheta_i}{N_{i1}(h_i^*)^3}\left[2(\xi_i-x_i)+h_i^*(\alpha_i+\phi_i)\right]$$

$$M_1^{(i)} = \frac{-2E_i\vartheta_i}{N_{i1}(h_i^*)^2}\left[3(\xi_i-x_i)+h_i^*(2\phi_iA_{i1}+\alpha_iB_{i1})\right]$$

FIGURE 2b. --DETAIL OF i-th SHELL BEAM



$$\bar{M}^{(17)} = \frac{2E^{(4)}J^{(4)}}{(\ell_6-d)^2}\left[3(x_7-x_{16})+(\ell_6-d)(2\phi_{17}+\phi_{16})\right]$$

$$V^{(16)} = \frac{6E^{(4)}J^{(4)}}{(\ell_6-d)^3}\left[2(x_7-x_{16})+(\ell_6-d)(\phi_{16}+\phi_{17})\right]$$

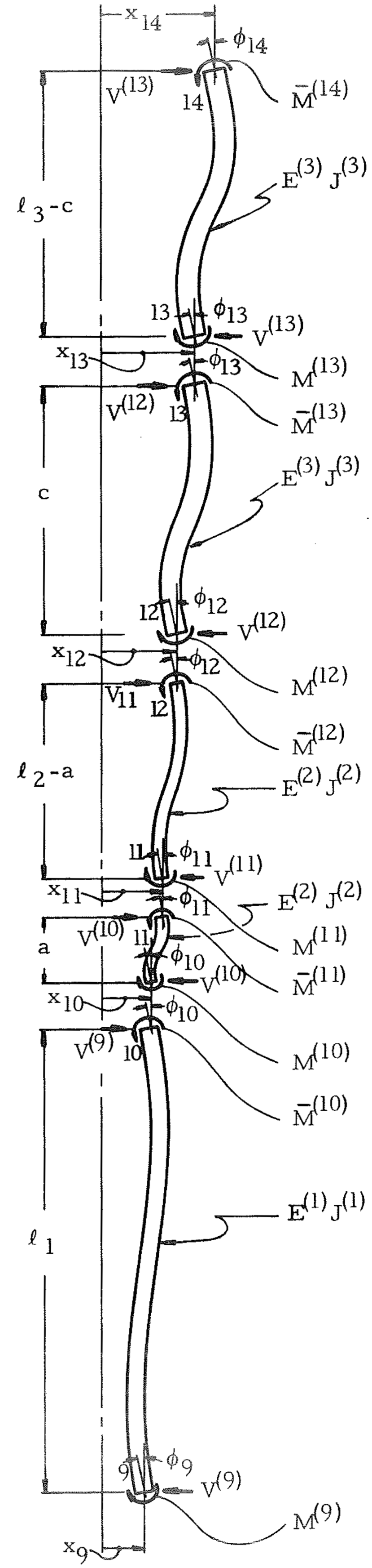
$$M^{(16)} = \frac{-2E^{(4)}J^{(4)}}{(\ell_6-d)^2}\left[3(x_7-x_{16})+(\ell_6-d)(2\phi_{16}+\phi_{17})\right]$$

$$\bar{M}^{(16)} = \frac{2E^{(4)}J^{(4)}}{d^2}\left[3(x_{16}-x_6)+d(2\phi_{16}+\phi_{15})\right]$$

$$V^{(15)} = \frac{6E^{(4)}J^{(4)}}{d^3}\left[2(x_{16}-x_6)+d(\phi_{15}+\phi_{16})\right]$$

$$M^{(15)} = \frac{-2E^{(4)}J^{(4)}}{d^2}\left[3(x_{16}-x_6)+d(2\phi_{15}+\phi_{16})\right]$$

FIGURE 2c. --DETAIL OF ORBITER REINFORCING BEAM



$$\bar{M}^{(14)} = \frac{2E^{(3)}J^{(3)}}{(\ell_3-c)^2}\left[3(x_4-x_{13})+(\ell_3-c)(2\phi_{14}+\phi_{13})\right]$$

$$V^{(13)} = \frac{6E^{(3)}J^{(3)}}{(\ell_3-c)^3}\left[2(x_4-x_{13})+(\ell_3-c)(\phi_{13}+\phi_{14})\right]$$

$$M^{(13)} = \frac{-2E^{(3)}J^{(3)}}{(\ell_3-c)^2}\left[3(x_4-x_{13})+(\ell_3-c)(2\phi_{13}+\phi_{14})\right]$$

$$\bar{M}^{(13)} = \frac{2E^{(3)}J^{(3)}}{c^2}\left[3(x_{13}-x_3)+c\,(2\phi_{13}+\phi_{12})\right]$$

$$V^{(12)} = \frac{6E^{(3)}J^{(3)}}{c^3}\left[2(x_{13}-x_3)+c\,(\phi_{12}+\phi_{13})\right]$$

$$M^{(12)} = \frac{-2E^{(3)}J^{(3)}}{c^2}\left[3(x_{13}-x_3)+c\,(2\phi_{12}+\phi_{13})\right]$$

$$\bar{M}^{(12)} = \frac{2E^{(2)}J^{(2)}}{(\ell_2-a)^2}\left[3(x_3-x_{11})+(\ell_2-a)(2\phi_{12}+\phi_{11})\right]$$

$$V^{(11)} = \frac{6E^{(2)}J^{(2)}}{(\ell_2-a)^3}\left[2(x_3-x_{11})+(\ell_2-a)(\phi_{11}+\phi_{12})\right]$$

$$M^{(11)} = \frac{-2E^{(2)}J^{(2)}}{(\ell_2-a)^2}\left[3(x_3-x_{11})+(\ell_2-a)(2\phi_{11}+\phi_{12})\right]$$

$$\bar{M}^{(11)} = \frac{2E^{(2)}J^{(2)}}{a^2}\left[3(x_{11}-x_2)+a\,(2\phi_{11}+\phi_{10})\right]$$

$$V^{(10)} = \frac{6E^{(2)}J^{(2)}}{a^3}\left[2(x_{11}-x_2)+a\,(\phi_{10}+\phi_{11})\right]$$

$$M^{(10)} = \frac{-2E^{(2)}J^{(2)}}{a^2}\left[3(x_{11}-x_2)+a\,(\phi_{11}+2\phi_{10})\right]$$

$$\bar{M}^{(10)} = \frac{2E^{(1)}J^{(1)}}{\ell_1^2}\left[3(x_2-x_1)+\ell_1(2\phi_{10}+\phi_9)\right]$$

$$V^{(9)} = \frac{6E^{(1)}J^{(1)}}{\ell_1^3}\left[2(x_2-x_1)+\ell_1(\phi_9+\phi_{10})\right]$$

$$M^{(9)} = \frac{-2E^{(1)}J^{(1)}}{\ell_1^2}\left[3(x_2-x_1)+\ell_1(\phi_{10}+2\phi_9)\right]$$

FIGURE 2d. --DETAIL OF BOOSTER REINFORCING BEAM

The vertical sloshing mass m_i and the sloshing spring constants k_i and $\overline{k}_i$ are defined as follows (ref. 3)

$$m_i = \frac{16}{\pi} \frac{I_1\left(\frac{\pi R_i}{2h_i}\right)}{\frac{\pi R_i}{2h_i} I_o\left(\frac{\pi R_i}{2h_i}\right)} m_T^{(i)} \quad ; \quad (i = 1,3,5,7) \tag{1}$$

$$k_i = k_i' - \nu_i \overline{K}_i F_i + \nu_i^2 \overline{K}_i F_i^2 \quad ; \quad \text{(no summation)} \tag{2}$$

$$\overline{k}_i = \nu_i \overline{K}_i F_i \tag{3}$$

where

I_1, I_o = modified Bessel functions of the first kind

R_i = R_B (Booster radius) for i = 1,3, = R_o (Orbiter radius) for i = 5,7

h_i = height of the liquid in cylinder i

$m_T^{(i)}$ = $\pi R_i^2 h_i \rho_i$ = total mass of the liquid in the cylinder

ρ_i = mass density of the liquid

$$k_i' = \frac{4\pi R_i \delta_i E_i}{h_i} \left[\frac{I_1\left(\frac{\pi R_i}{2h_i}\right)}{\frac{\pi R_i}{2h_i} I_o\left(\frac{\pi R_i}{2h_i}\right)} \right]^2 \tag{4}$$

ν_i = Poissons ratio of the cylindrical shell

$$\overline{K}_i = \frac{2\pi R_i \delta_i E_i}{\ell_i - \nu_i^2 h_i} \tag{5}$$

$$F_i = \frac{4}{\pi} \frac{I_1\left(\frac{\pi R_i}{2h_i}\right)}{\frac{\pi R_i}{2h_i} I_o\left(\frac{\pi R_i}{2h_i}\right)} \tag{6}$$

ℓ_i = length of the cylinder

δ_i = cylinder wall thickness

E_i = Young's modulus of the cylinder shell

The lateral sloshing mass for the first mode or fundamental slosh mass $\tilde{m}_i$, and its height $\tilde{h}_i$, the rigid mass m_i^* and its height h_i^* and the sloshing spring constant are defined in the following (refs. 4, 5, 6)

$$\tilde{m}_i = m_T^{(i)} \left(\frac{R_i}{2.2h_i} \right) \tanh 1.84 \frac{h_i}{R_i} \tag{7}$$

$$m_i^* = m_T^{(i)} - \tilde{m}_i \tag{8}$$

$$\tilde{k}_i = m_T^{(i)} \left(\frac{g}{1.19h_i} \right) \left(\tanh 1.84 \frac{h_i}{R_i} \right)^2 \tag{9}$$

$$\tilde{h}_i = h_i - \frac{R_i}{0.92} \tanh 0.92 \frac{h_i}{R_i} \tag{10}$$

= height of $\tilde{m}_i$

$$h_i^* = h_i - \frac{m_T^{(i)} h_i}{2m_i^*} + (h_i - \tilde{h}_i) \frac{\tilde{m}_i}{m_i^*} \tag{11}$$

= the height of m_i^*

g = gravitational constant, and the polar moment of inertia of m_i^* is defined as

$$J_i^* = J_{rigid}^{(i)} + m_T^{(i)} \frac{h_i^2}{4} - \frac{m_T^{(i)} R_i^2}{2} \left[1.995 - \frac{2.14R_i}{h_i} \tanh 0.92 \frac{h_i}{R_i} \right] - m_i^*(h_i - h_i^*)^2 - \tilde{m}_i(h_i - \tilde{h}_i)^2 \tag{12}$$

where

$$J_{rigid} = \frac{m_T^{(i)} R_i^2}{4} \left[\frac{1}{3} \frac{h_i^2}{R_i^2} + 1 \right] \tag{13}$$

Figures 2a and 2b show that $\tilde{m}_i$ is situated above m_i^*; however, as the depth of the liquid h_i decreases, $\tilde{m}_i$ will shift to a position below m_i^*.

Mathematical Formulation

The free dynamic behavior of the mechanical model can be described completely in mathematical terms by writing the equations of motion and equations of constraint for the system. In order to do this, the mechanical model is divided into subsystems for which equations of dynamic and static equilibrium for the free oscillation can easily be set up. The coordinate system at each subsystem is shown in Figure 2a.

In the model, the structure mass elements $M_n(n = 1, 2, \ldots, 8)$ will have three degrees of freedom in motion (two translations and one rotation) the rigid mass $m_i^*(i = 1, 3, 5, 7)$ two degrees of freedom (one horizontal translation and one rotation), the lateral sloshing mass $\tilde{m}_i$ one degree of freedom (horizontal translation), and the vertical sloshing mass m_i one degree of freedom (vertical translation). Thus, the entire system will have forty degrees of freedom and therefore forty equations of motion. However, only equations associated with subsystem 1 will be shown for the purpose of illustration.

Equations of motion. --At subsystem 1 there are two equations of motion for translation and one for rotation. They are

$$M_1 \ddot{x}_1 - V_1^{(1)} - V^{(9)} = 0 \tag{14}$$

$$M_1 \ddot{z}_1 - K^{(1)}[(z_2 - z_1) + (R_B + D_1)(\phi_2 - \phi_1)] - K_1(z_2 - z_1) - k_1(\zeta_1 - z_1) = 0 \tag{15}$$

$$J_1 \ddot{\phi}_1 - K^{(1)}(R_B + D_1)[(z_2 - z_1) + (R_B + D_1)(\phi_2 - \phi_1)] + S_1(\phi_1 - \phi_9) - M^{(1)} = 0 \tag{16}$$

where

D_1 = distance between the neutral axis of the Booster strongback beam section 9-10 and the Booster shell

K_1 = longitudinal spring constant of cylinder 1

$K^{(1)}$ = longitudinal spring constant of beam section 9-10

J_1 = polar moment of inertia of mass M_1 about y_1 axis

S_1 = torsional spring constant at joint 9

$$M_1^{(1)} = -\frac{2E_1\mathcal{J}_1}{(h_1^*)^2 N_{11}} [3(\xi_1 - x_1) + h_1^*(2A_{11}\phi_1 + B_{11}\alpha_1)] \quad (17)$$

= net moment at lower end of shell-beam 1

$$V^{(9)} = \frac{6E^{(1)}J^{(1)}}{\ell_1^3} [2(x_2 - x_1) + \ell_1(\phi_9 + \phi_{10})] \quad (18)$$

= end shearing force of the Booster reinforcing beam section at end 9 (ref. 7)

$$V_1^{(1)} = \frac{6E_1\mathcal{J}_1}{(h_1^*)^3 N_{11}} [2(\xi_1 - x_1) + h_1^*(\phi_1 + \alpha_1)] \quad (19)$$

= net shearing force at lower end of shell-beam 1 (see Appendix A)

$E^{(1)}$ = Young's modulus of the reinforcing beam section 9-10

$J^{(1)}$ = moment of inertia of the cross section of the reinforcing beam section 9-10

$\mathcal{J}_1$ = moment of inertia of the cylinder cross section and

$$N_{11} = 1 + \frac{12E_1 \mathscr{I}_1}{(h_1^*)^2 \pi R_B \delta_1 G_1}, \qquad A_{11} = 1 + \frac{3E_1 \mathscr{I}_1}{(h_1^*)^2 \pi R_B \delta_1 G_1},$$

$$B = 1 - \frac{6E_1 \mathscr{I}_1}{(h_1^*)^2 \pi R_B \delta_1 G_1} \qquad (20)$$

in which

G_1 = shear modulus

Equations (20) provide correction factors which arise from the analysis based on linear membrane theory of the shell. These factors are unity for an ordinary beam (see Appendix A). Similar equations of motion may be written for the rest of subsystems.

Equations of constraint.--As shown in Figure 2a, at each massless subsystem of 9, 10, 12, 14, 15, and 17 there will be one constraint condition in the ϕ direction; at subsystem 11 there are two constraint conditions (x and ϕ directions); at subsystems 13 and 16 there are three constraint conditions (x, z, ϕ directions) for each subsystem; at subsystem i (i = 1, 3, 5, 7) there are two constraint conditions (α, ξ directions). Thus there will be twenty-two constraint conditions associated with the model. For the purpose of illustration only the constraint condition at the massless subsystem 9 will be shown below.

$$0 \times \ddot{\phi}_9 - S_1(\phi_1 - \phi_9) - M^{(9)} = 0$$

where

$$M^{(9)} = -\frac{2E^{(1)}J^{(1)}}{\ell_1^2}\left[3(x_2 - x_1) + \ell_1(2\phi_9 + \phi_{10})\right] \qquad (21)$$

= end moment of the Booster reinforcing beam section at end 9

Similar constraint equations may be written for the rest of massless nodes.

Eigenvalue problem.--Since only free oscillations are being considered, the aforementioned system of H equations of motion and constraint is homogeneous. Taken together, the total system of dynamic and constraint equations can be written in compact notation (employing the summation convention) as

$$P_{rs}\ddot{W}_s + Q_{rs}W_s = 0 \qquad r, s = 1, 2, \ldots, H \tag{22}$$

It is assumed that the vibration modes are harmonic, i.e.,

$$W_s = Y_s \cos(\omega t + \gamma) \tag{23}$$

where each frequency ω is real. On substituting Eq. (23) into Eq. (22) one finds

$$(Q_{rs} - \omega^2 P_{rs})(Y_s) = 0 \tag{24}$$

There is a well-known principle (ref. 7) in vibration theory which states that a system of H equations consisting of L dynamic and L' constraint equations (L + L' = H) can always be reduced to a problem involving L equations. This can be accomplished by incorporating the constraint equations into the equations of motion. In the present problem, this can be done by segregating the set of generalized coordinates $\underset{\sim}{W}$ into two nonintersecting sets:

$$\underset{\sim}{W} = \underset{\sim}{X} \cup \underset{\sim}{x}$$

where

$\underset{\sim}{X}$ = set of coordinates associated with mass elements

$\underset{\sim}{x}$ = set of coordinates associated with constraint conditions

For the present problem, $\underset{\sim}{X}$ and $\underset{\sim}{x}$ are defined in Table III. Equations (24) can now be replaced by two sets of equations, the first of which gives the L equations of motion expressed in terms of the constraint coordinates $\underset{\sim}{x}$:

$$(q_{ij} - \Omega^2 p_{ij})(X_j) = -a_{in}x_n \qquad i, j = 1, 2, \ldots, L \qquad n = 1, 2, \ldots, L' \tag{25}$$

The second set of conditions relates the $\underset{\sim}{x}$ coordinates to the $\underset{\sim}{X}$ coordinates:

$$c_{lm}x_m = d_{lj}X_j \qquad l, m = 1, 2, \ldots, L' \tag{26}$$

TABLE III. --DEFINITION OF COORDINATES

Coordinates Associated with Mass Elements		Coordinates Associated with Constraint Conditions	
$X_1 = z_1$	$X_{21} = x_5$	$x_1 = z_{13}$	$x_{12} = x_{11}$
$X_2 = z_2$	$X_{22} = x_6$	$x_2 = z_{16}$	$x_{13} = x_{13}$
$X_3 = z_3$	$X_{23} = x_7$	$x_3 = \phi_9$	$x_{14} = x_{16}$
$X_4 = z_4$	$X_{24} = x_8$	$x_4 = \phi_{10}$	$x_{15} = \alpha_6$
$X_5 = z_5$	$X_{25} = \zeta_3$	$x_5 = \phi_{11}$	$x_{16} = \alpha_8$
$X_6 = z_6$	$X_{26} = \zeta_4$	$x_6 = \phi_{12}$	$x_{17} = \xi_6$
$X_7 = z_7$	$X_{27} = \alpha_5$	$x_7 = \phi_{13}$	$x_{18} = \xi_8$
$X_8 = z_8$	$X_{28} = \alpha_7$	$x_8 = \phi_{14}$	$x_{19} = \alpha_2$
$X_9 = \phi_1$	$X_{29} = \xi_5$	$x_9 = \phi_{15}$	$x_{20} = \alpha_4$
$X_{10} = \phi_2$	$X_{30} = \xi_7$	$x_{10} = \phi_{16}$	$x_{21} = \xi_2$
$X_{11} = \phi_3$	$X_{31} = \eta_3$	$x_{11} = \phi_{17}$	$x_{22} = \xi_4$
$X_{12} = \phi_4$	$X_{32} = \eta_4$		
$X_{13} = \phi_5$	$X_{33} = \zeta_1$		
$X_{14} = \phi_6$	$X_{34} = \zeta_2$		
$X_{15} = \phi_7$	$X_{35} = \alpha_1$		
$X_{16} = \phi_8$	$X_{36} = \alpha_3$		
$X_{17} = x_1$	$X_{37} = \xi_1$		
$X_{18} = x_2$	$X_{38} = \xi_3$		
$X_{19} = x_3$	$X_{39} = \eta_1$		
$X_{20} = x_4$	$X_{40} = \eta_2$		

This system of equations can be written such that the square matrix $\underset{\sim}{c}$ is symmetric and that $\underset{\sim}{d} = -\underset{\sim}{a}^T$. Solving for $\underset{\sim}{x}$:

$$x_n = c^{-1}_{nl} d_{lj} X_j \tag{27}$$

and substituting into Eq. (25):

$$\begin{aligned}(q_{ij} - \Omega^2 p_{ij})(X_j) &= - a_{in} c^{-1}_{nl} d_{lj} X_j \\ &= a_{in} c^{-1}_{nl} a^T_{lj} X_j\end{aligned} \tag{28}$$

Thus:

$$[(q_{ij} - a_{in} c^{-1}_{nl} a^T_{lj}) - \Omega^2 p_{ij}](X_j) = 0 \tag{29}$$

By premultiplying by $\underset{\sim}{p}^{-1}$ this equation takes the form of a standard eigenvalue problem

$$[p^{-1}_{ki}(q_{ij} - a_{in} c^{-1}_{nl} a^T_{lj}) - \Omega^2 \delta_{kj}]\ (X_j) = 0, \quad k = 1, 2, \ldots, L \tag{30}$$

Nontrivial solutions for the eigenvector $\underset{\sim}{X}$ exist if, and only if, the determinant of the coefficient matrix vanishes:

$$\left| p^{-1}_{ki}(q_{ij} - a_{in} c^{-1}_{nl} a^T_{lj}) - \Omega^2 \delta_{kj} \right| = 0 \tag{31}$$

Thus, the problem reduces to finding the eigenvalues of the $L \times L$ matrix $\underset{\sim}{p}^{-1}(\underset{\sim}{q} - \underset{\sim}{a}\underset{\sim}{c}^{-1}\underset{\sim}{a}^T)$. The eigenvalues found by this process are the natural frequencies of the Booster/Orbiter system, expressed in radians/second. Included in this set of frequencies are the zero frequencies identified with translation and rotation of the system as a rigid body.

In the present problem, the matrix $\underset{\sim}{q}$ and the matrix $\underset{\sim}{a}\underset{\sim}{c}^{-1}\underset{\sim}{a}^T$ both are symmetric, and the matrix $\underset{\sim}{p}$ is diagonal (and, hence, $\underset{\sim}{p}^{-1}$ is diagonal). However, the product matrix $\underset{\sim}{p}^{-1}(\underset{\sim}{q} - \underset{\sim}{a}\underset{\sim}{c}^{-1}\underset{\sim}{a}^T)$ is not symmetric. Mathematically, the eigenvalues of a real, nonsymmetric matrix may be complex, all or in part. On physical grounds, however, one knows that the mathematical model being solved represents a linear conservative system, and therefore a correct solution must result in real eigenvalues. Complex eigenvalues introduce growth and decay characteristics in the modal response which are inadmissible for the free vibrations of the problem under consideration. In the numerical solution for the present problem an eigenvalue routine was used which computes the complex eigenvalues of a nonsymmetric matrix. All nontrivial eigenvalues, however, were found to be real.

The nonzero elements of the matrices $\tilde{q}$, $\tilde{\tilde{a}}$, and $\tilde{c}$ are tabulated in Appendix B for three different cases: $h_i > h_i^*$, $h_i < h_i^*$ and the empty case.

Numerical Aspects

The physical quantities which are involved in the equations of motion and constraint for the analytical model consist of parameters which can be computed directly (masses, moments of inertia, etc.) and of terms which do not lend themselves to direct computation (effective* spring constants). These latter terms can be estimated from force-displacement calculations based on idealized models, but these estimates by no means serve as valid input data for computational purposes. Effective spring constants, in some cases, also can be determined experimentally by impedance techniques. The general approach used in this program for determining the input parameters for the analytical models was first to calculate by some means (quite approximately in some cases) all of the input data required of the model. Following this, certain of the parameters, least amenable to accurate calculation, were adjusted within certain narrow bounds in an attempt to match the frequencies computed theoretically with the experimental values for the empty tank case. This procedure is not the same as "curve fitting" where one takes much greater liberty with the number of parameters varied and disregards theoretical estimates on their magnitudes. The approach adopted here limits the amount of empiricism to a practical minimum in fixing the input data.

In the case of the Booster K_1 and K_2, the effective axial spring constants of the thin-walled shells were found experimentally by an axial vibration test, as described in the next section. The axial spring constants characterising the strongback were adjusted from their calculated values to match the empty, decoupled booster data. For the Orbiter, the axial spring constant of cylinder 5 was calculated, and a correction factor (a corrected shell thickness δ_5) introduced to force agreement with the experimental result for cylinder 5 alone in both axial and bending motion. This procedure was repeated using cylinders 5 and 6 together, to determine K_6. For cylinder 7, δ_7 was taken equal to δ_5. The parameters finally arrived at are listed. Once so determined, they were held constant throughout all computational work.

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*Note that effective spring constants include effects difficult to predict, such as bolted joint compliances and longitudinal stiffeners.

$K_1 = 3.48 \times 10^5$ lb/in.

$K_2 = 7.36 \times 10^5$ lb/in.

$K^{(1)} = 1.8333 \times 10^5$ lb/in.

$K^{(2)} = 5.3571 \times 10^5$ lb/in.

$K^{(3)} = 3.4921 \times 10^5$ lb/in.

$K^{(4)} = 1.2083 \times 10^6$ lb/in.

$\overline{K}^{(3)} = 3.8596 \times 10^5$ lb/in.

$\overline{K}^{(4)} = 1.2083 \times 10^6$ lb/in.

$\delta_5 = \delta_7 = 9.84 \times 10^{-3}$ in.

$K_6 = 1.885 \times 10^5$ lb/in.

$S_5 = S_7 = 2.60 \times 10^4$ in.-lb/rad

In the case of the coupled Booster/Orbiter system three additional spring constants were available for adjustment, within limits, to match the theoretical and experimental coupled empty tank condition. These three constants represented the torsional coupling spring S_6, the vertical coupling spring $K^{(6)}$, and the horizontal coupling spring $K^{(7)}$. All three of these spring constants were first calculated on the basis of idealized models, to serve as nominal values in the adjustment process. The values finally chosen were:

$S_6 = 3.9027 \times 10^4$ in.-lb/rad

$K^{(6)} = 2.677 \times 10^5$ lb/in.

$K^{(7)} = 9.9672 \times 10^3$ lb/in.

The calculations for the empty-tank condition omit the slosh models from the system of equations, and do not represent simply a degenerate case of vanishingly small liquid levels. The general system of equations, which does include the sloshing models, must predict frequencies compatible with the empty-tank results for small, but nonzero liquid levels. This criterion serves as a checkpoint on the accuracy of the numerical program. Also, a transition point occurs at a liquid level of 1.043 times the tank diameter, below which h_i^*, the location of the rigid mass, reverses its relative position with $\tilde{h}_i$, the location of the sloshing mass. On either side of this transition certain equations must be rewritten in a different form, with the result that the program differs according to whether one is considering a "near full" or a "near empty" tank. Continuity in the computed frequencies, of course, must be maintained across this transition point, and this criterion serves as an additional check on the programming accuracy.

The matrix eigenvalue problem was solved on a CDC-6400 computer using a standard eigenvalue routine (modified Jacobi method) for finding the eigenvalues of a real, nonsymmetric matrix. A listing of the computer program and instructions for use are given in Appendix C.

EXPERIMENTAL PROCEDURE

The test program performed on the previously described physical model can be divided into distinctive phases and may be listed as the determination of:

(1) Effective spring constants for subassemblies

(2) Natural bending frequencies of decoupled Booster and Orbiter models

(3) Natural frequencies of decoupled Booster and Orbiter models with longitudinal excitation

(4) Natural frequencies of Shuttle Vehicle Model.

The first part of the experimental program was to determine the effective spring constants of the Booster model components. This was accomplished by mounting the intermediate empty configurations illustrated in Figure 3 on an electrodynamic shaker and determining the natural frequencies of the components. Frequencies obtained by this test were used to calculate the effective spring constants. Similar procedure was used with the Orbiter model components, both for axial and lateral excitation. The resulting spring constants were tabulated in the previous section for both models.

The second phase of the test program was devoted to determining the natural bending frequencies of the decoupled models. The Booster model was vertically suspended by a nylon-rope, pulley, and spring combination which was designed to simulate a free-free condition. The rope was attached to the model at its bottom flange on the lower tank and was guided at the top of the skirt. A small electrodynamic shaker connected to the model at the

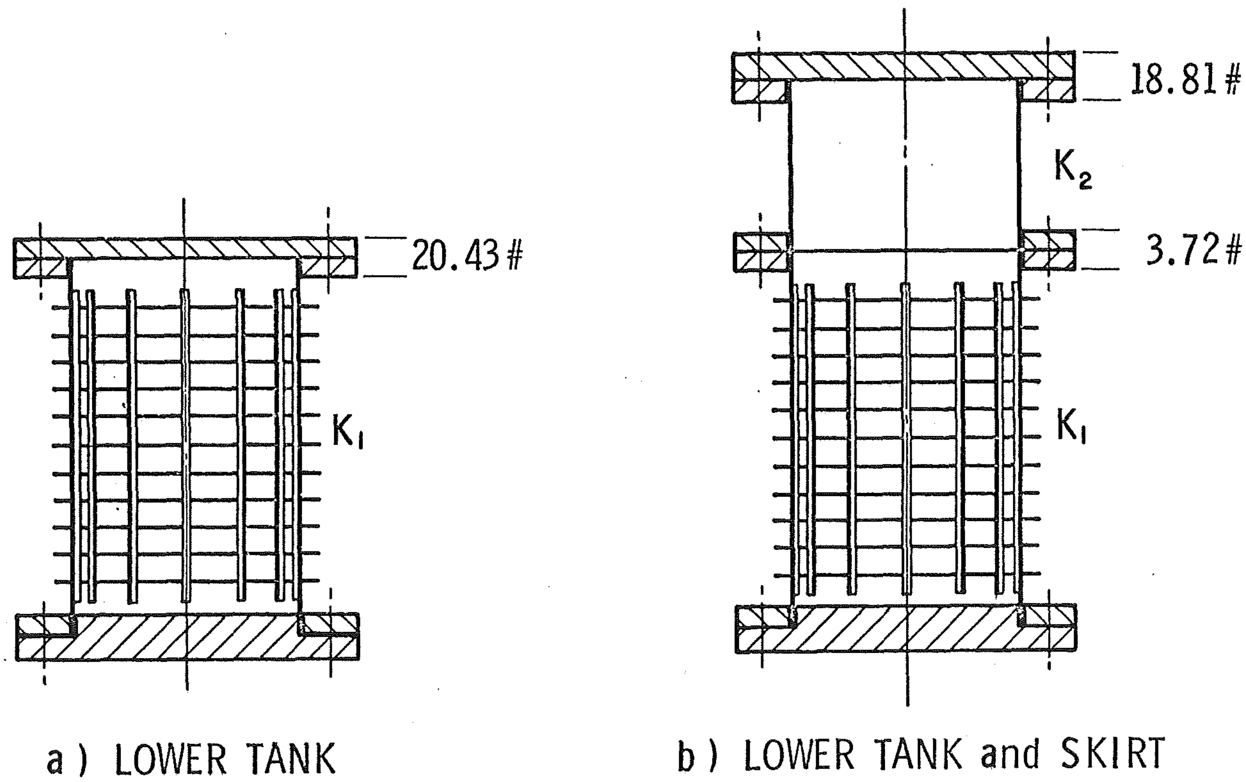

FIGURE 3.--INTERMEDIATE EMPTY CONFIGURATIONS OF BOOSTER MODEL

upper flange of the skirt was used to excite the model in a lateral direction. Four piezoelectric accelerometers were mounted on the model, one on each flange and one on top with their axes in line with the direction of excitation, and monitored simultaneously. The model was tested with empty, full, and intermediate liquid conditions and its natural frequencies were recorded. The information so obtained served a dual purpose; it provided data for comparison with the frequencies obtained by an analytical model discussed in the preceding section, and also allowed for the identification of bending modes when the model was later tested in its coupled configuration. This test procedure was also repeated using the Orbiter model.

To determine the natural frequencies of the Booster and Orbiter models with longitudinal excitation, the models were suspended as previously described and excited along their vertical axes by a small electrodynamic shaker. Four piezoelectric accelerometers, mounted on the flanges and the top of the models with their axes along the direction of excitation, were monitored together with pressure transducers installed in the center of each bulkhead. As in the preceding phase, each model was tested with empty, full, and intermediate liquid conditions.

The final step in the test program was the determination of natural frequencies of the complete shuttle vehicle model shown in Figure 1. The system was suspended in such a manner so that the driving force introduced to the model by the electrodynamic shaker always acted through the gravitational center of the model. Four piezoelectric accelerometers, located on the bottom flanges and the top of the models, were monitored measuring acceleration along the axes of the models, while two others were located at the tops recording acceleration in the lateral direction. In addition, four pressure transducers, one located in each bulkhead, were monitored. All tests were performed with the Orbiter tanks full. The liquid level in the Booster tanks was varied from empty to full with intermediate conditions. Thus, a normal operational sequence was simulated.

Throughout the entire test program, distilled water was used as a modeling liquid propellant. Ullage pressure was provided in all tanks to raise the natural frequencies of nonsymmetric shell modes above the frequency range used during the tests.

As a conclusion to the experimental program, the modeling liquid was replaced in the tanks by a granular substance with bulk density very closely equal to the modeling liquid and the model was tested at full and half-full levels in the Booster tanks, and similar levels in the Orbiter tanks. This substitution was implemented to facilitate identification of liquid and structural modes in the data obtained from tests completed on the coupled system, as well as to show more vividly the effects of liquid propellants.

COMPARISON OF RESULTS

The output from the theoretical model was in the form of natural frequencies representing:

(1) Rigid body motion of the system (zero frequencies)

(2) Sloshing frequencies of the four liquid-containing tanks

(3) Frequencies of the system.

Regarding the rigid body modes, the analytical model predicts three zero frequencies for the coupled Booster/Orbiter system, and six when the Booster/Orbiter coupling is set to zero. This prediction is consistent with rigid body motion in a plane. The four calculated sloshing frequencies were in the range of 1 to 3 Hz and are quite small when compared with system frequencies. Thus, the sloshing modes are essentially decoupled from the natural frequencies of the total system.

Figures 4, 5, and 6 present a comparison of the theoretically-predicted with the experimentally-determined results. Figure 4 shows the first five frequencies of the Booster alone, and Figure 5 shows the first three frequencies of the Orbiter alone. The analytical and experimental frequency values are quantitatively compared in Table IV. This agreement index was based upon the maximum (absolute value) percentage error between the theoretical and the experimental values, with the experimental values taken as the basis. The agreement is considered good if this error is within 10%, fair if between 10% and 20%, and poor if greater than 20%.

It is seen from this comparison, and from Figures 4 and 5, that the agreement between theory and experiment is generally better in the case of bending modes than in the longitudinal modes. There is a particularly significant lack of agreement in the second and third longitudinal Booster modes. Also, the theoretical frequencies tend to be somewhat higher than the experimental values (except in the case of the second Orbiter bending mode, where the theoretical and experimental curves cross each other).

Table V.A compares the theoretical and experimental frequencies for the coupled empty-tank condition. In terms of the comparison index mentioned above, the agreement is good for the first six modes, and poor for the next three higher modes. It is obvious that above the sixth mode, the relatively simple analytical model is no longer adequate to describe the motion of the system.

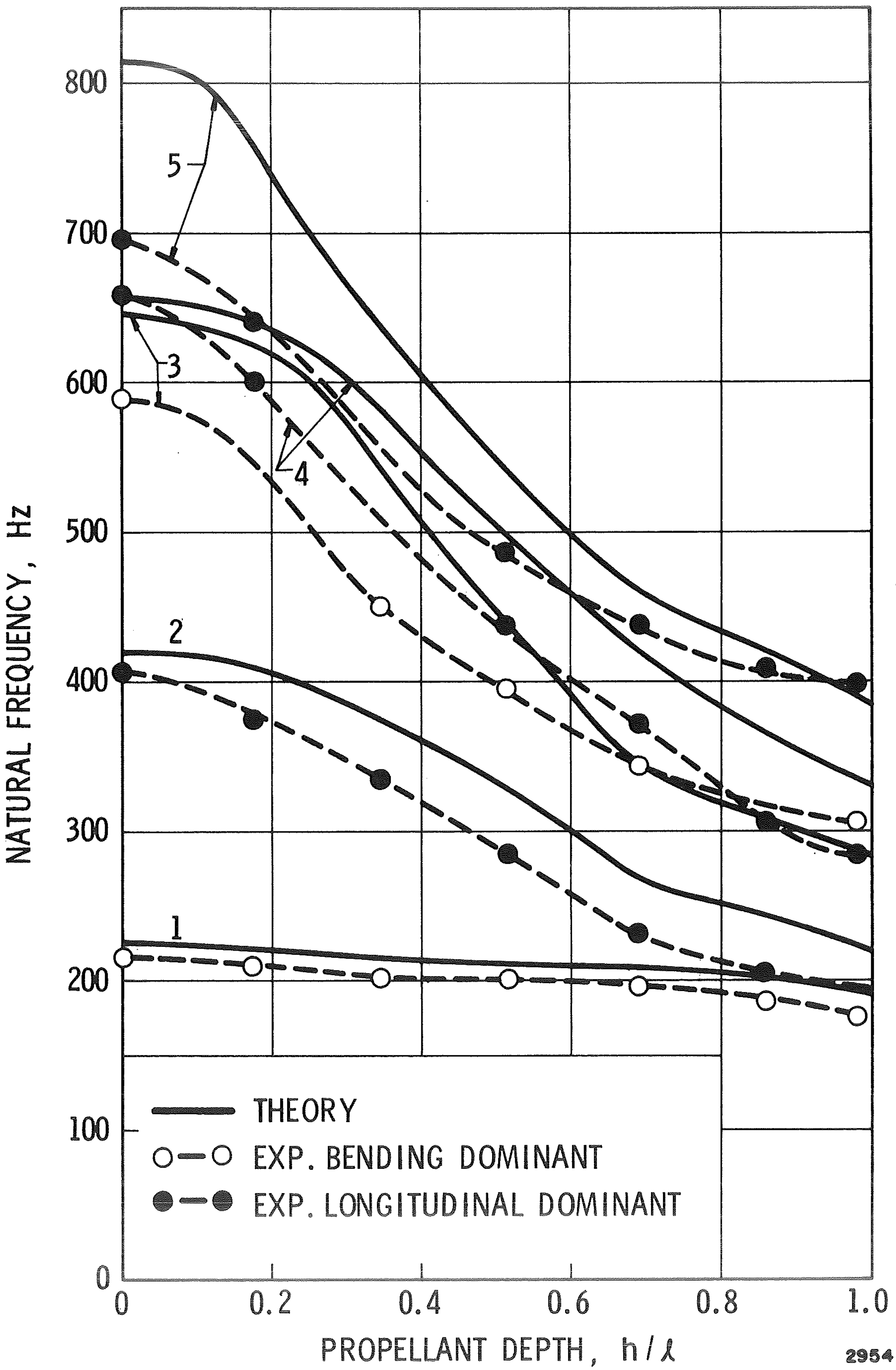

FIGURE 4. --UNCOUPLED BOOSTER NATURAL FREQUENCIES

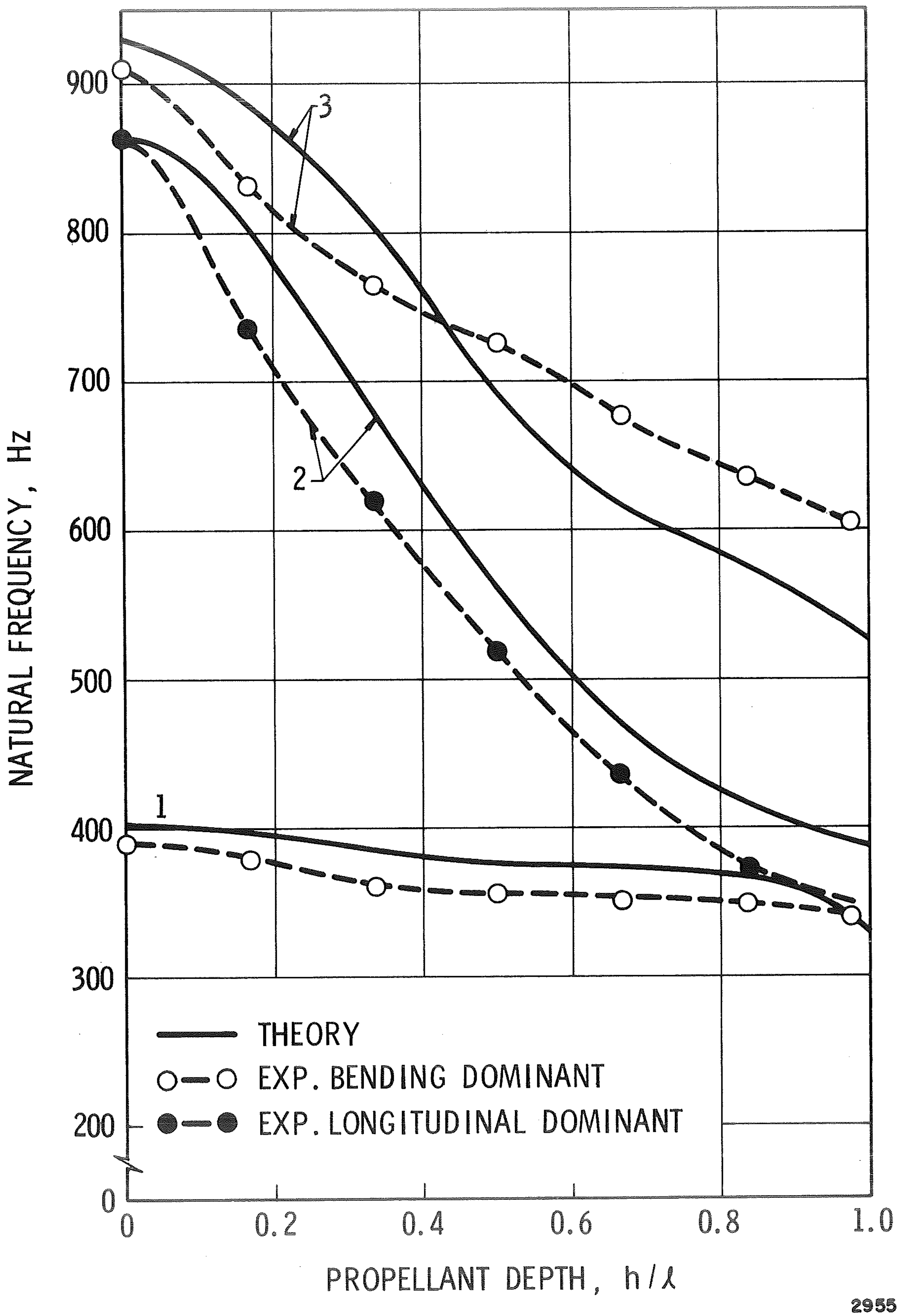

FIGURE 5.--UNCOUPLED ORBITER NATURAL FREQUENCIES

TABLE IV. --AGREEMENT INDEX FOR DECOUPLED MODELS

BOOSTER

Mode Number	Mode Form	Agreement
1	1st bending	Good
2	1st longitudinal	Fair
3	2nd bending	Good
4	2nd longitudinal	Poor
5	3rd longitudinal	Poor

ORBITER

Mode Number	Mode Form	Agreement
1	1st bending	Good
2	1st longitudinal	Good
3	2nd bending	Good

Figure 6 compares the theoretical and experimental frequencies for the coupled Booster/Orbiter system for full Orbiter and various Booster propellant levels. The two lowest modes, approximately constant at 15 Hz and 70 Hz, represent modes in which the Booster and Orbiter act essentially as rigid bodies, but vibrate relative to each other through the torsional coupling spring and the lateral coupling spring, respectively. The third mode is dominantly Booster bending, while the fourth mode is the remaining rigid body mode, with relative Booster/Orbiter motion resisted through the longitudinal coupling spring. The fifth and sixth modes exchange motions of dominantly Orbiter bending and Booster longitudinal motion. Above this, the discrepanies become quite large.

Additional results are shown in Tables V.B and V.C where frequencies are given for the case of a solid-like fluid. A mixture of soil and flour was used to produce a substance having a bulk specific gravity of 1.0. Only the first two modes remained relatively unaltered, while most higher modes disappeared. Apparently, considerably more damping was displayed by this mixture than experienced with water. Thus, in this case a better study of the effects of liquid rather than solid propellant simulation could have been obtained from the analytical model.

In Tables V.D and V.E, results are given for alternate positions of the Orbiter on the Booster, when all tanks were full.* In these cases the difference was surprisingly small. Other such results at various liquid levels would be highly desirable.

It appears reasonable to conclude that the liquid propellant models, as derived, provide only a fair overall prediction of frequencies for the decoupled models and a somewhat better prediction for the coupled case. However, possibilities for refinement of the models can immediately be considered. For example, the use of additional modes for the longitudinal liquid model could very likely improve the results at the higher frequencies. A better estimation of joint compliances in the structural model would also help.

Finally, Figure 7 shows an instability that occurs in the system for both Orbiter and Booster full and excitation through the system center of gravity at 453 Hz. The oscilloscope traces show only the pulsating envelope of the high frequency responses. The origin or cause of this type of instability remains to be investigated. However, inspection of the liquid surfaces showed no apparent slosh coupling with the low frequency pulsation, even though it was near the frequency for those modes.

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*Because of the coupling design, it was subsequently determined that these results include the effects of the indicated amount of Orbiter position change, as well as effects of an undetermined amount of variation in the coupling springs $K^{(6)}$, $K^{(7)}$, and S_6.

TABLE V. --NATURAL FREQUENCIES FOR SPACE SHUTTLE VEHICLE MODEL

A. All Tanks Empty
(Orbiter Position c = 7.87 in.)

Experimental (Hz)	Theoretical (Hz)		Mode
23.0	23.2	Good	Torsional Coupling
109	105	Good	Lateral Coupling
212	221	Good	Booster Bending
343	355	Good	Longitudinal Coupling
402	417	Good	Orbiter Bending
431	463	Good	Booster-Orbiter Bending
471	649	Poor	
546	659	Poor	
597	815	Poor	
708	863	Poor	

B. Booster and Orbiter $h/\ell = 0.983$ - Granular Propellant
(Orbiter Position c = 7.87 in.)

Experimental (Hz)	Mode
14.7	Torsional Coupling
60.0	Lateral Coupling
436	
845	

C. Booster and Orbiter $h/\ell = 0.517$ - Granular Propellant
(Orbiter Position c = 7.87 in.)

Experimental (Hz)	Mode
14.7	Torsional Coupling
64.0	Lateral Coupling
420	
869	

TABLE V. --NATURAL FREQUENCIES FOR SPACE SHUTTLE VEHICLE MODEL (Cont'd)

D. All Tanks Full
(Orbiter Position c = 4.06 in.)

Experimental (Hz)	Mode
15.5	Torsional Coupling
121	Lateral Coupling
171	Booster Bending
191	Booster Longitudinal
216	
300	
310	
416	
458	
476	

E. All Tanks Full
(Orbiter Position c = 11.69 in.)

Experimental (Hz)	Mode
15.2	Torsional Coupling
124	Lateral Coupling
173	Booster Bending
190	Booster Longitudinal
221	
299	
312	

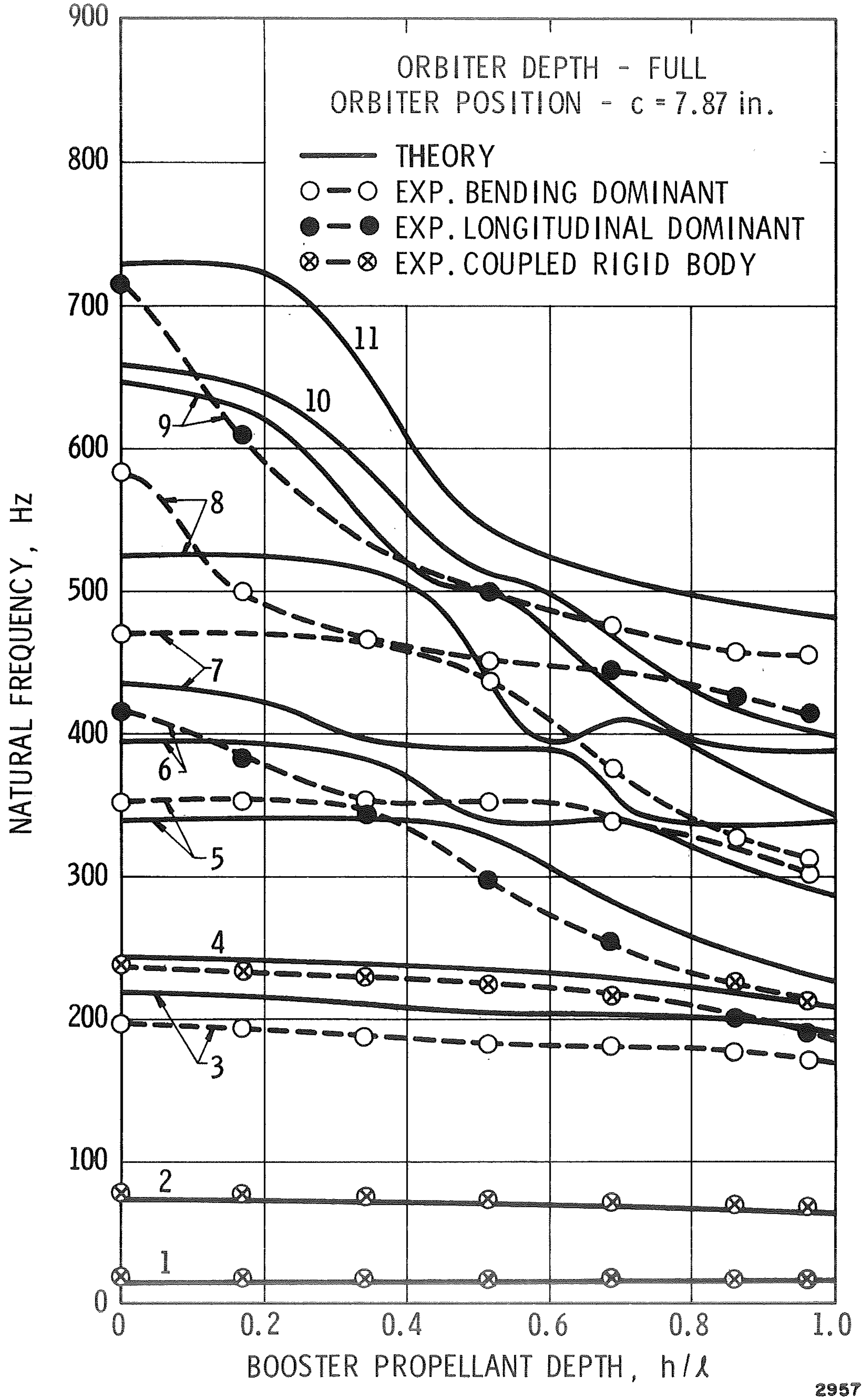

FIGURE 6. --NATURAL FREQUENCIES FOR SPACE SHUTTLE VEHICLE MODEL

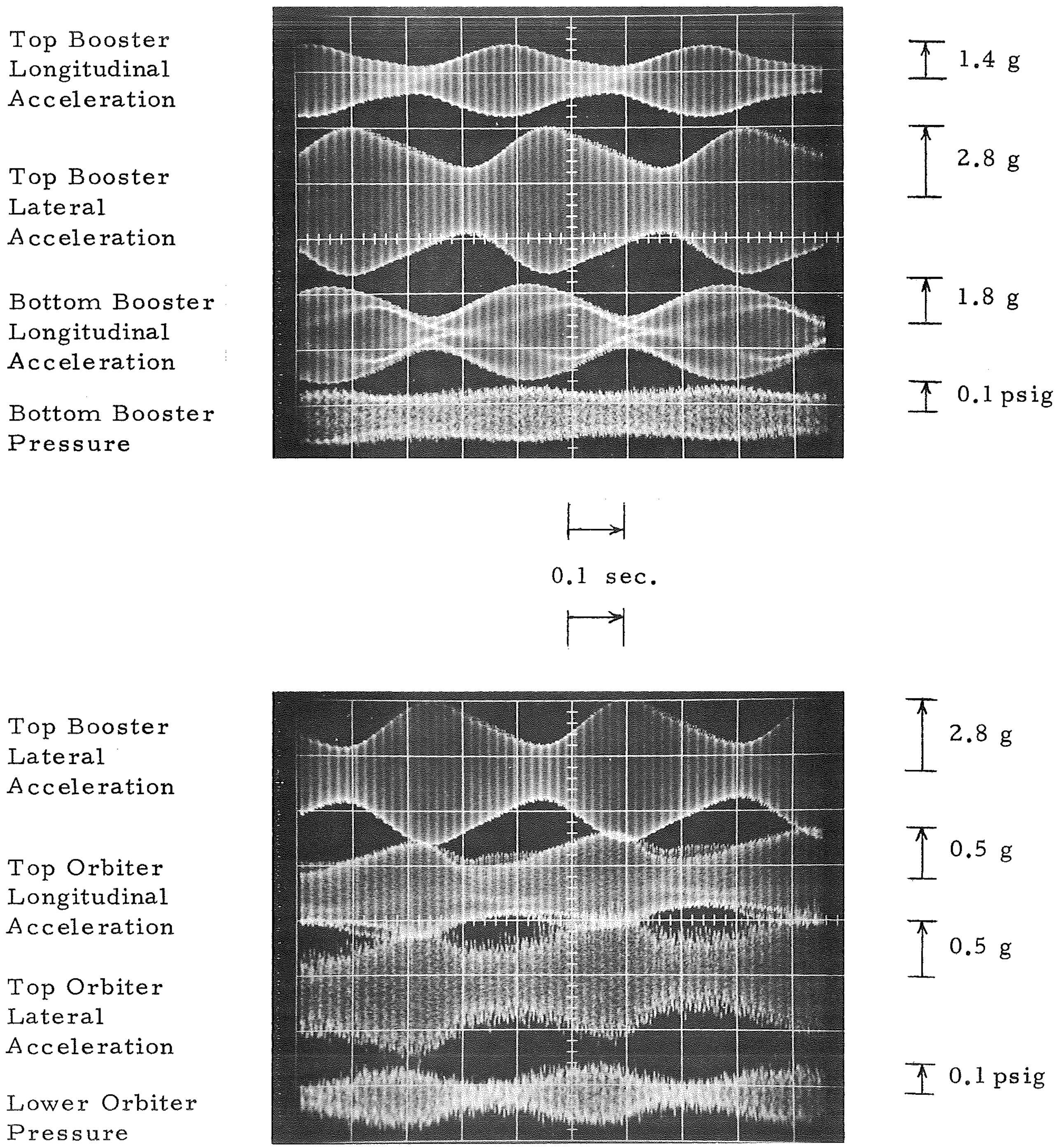

FIGURE 7. --SPACE SHUTTLE VEHICLE MODEL INSTABILITY

SUGGESTED FURTHER STUDIES

The results of this study indicate that a very effective, yet rather simple model of a typical space shuttle system has been developed, whereby many potential problems can be studied. Desirable steps to follow in this process are as follows:

(1) Minor refinement of the analytical model is in order. This can be done by incorporating higher longitudinal liquid modes and getting better estimates of effective spring constants.

(2) The analytical model should be used to compute results for a wide variety of parameters including other Booster-Orbiter liquid depth combinations, Orbiter positions, coupling springs, etc. The assumption of solid propellants can easily be made and results determined from the analytical model. Some experiments should be performed to verify select cases of the results.

(3) Experimental transfer functions should be run between various response points and the excitation. A better description of coupling between longitudinal and lateral motions would result. Various gimbal angles on the excitation should be used along with this.

(4) The present analytical model incorporates motion only in the plane of symmetry or system pitch plane. Experiments should be conducted to determine system response for both yaw and roll excitation. The derivation of an analytical model for these types of motion is also appropriate.

(5) The origin of the pulsating instability should be identified and its significance explored.

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APPENDIX A

REVISED BEAM EQUATIONS

END MOMENT AND SHEAR EQUATIONS FOR BEAM-LIKE THIN-WALLED CIRCULAR TUBES

Based on the linear membrane theory, the governing equations for beam-like, thin-walled circular cylinder following the sign convention shown in Figure A-I(a) are given by (ref. 8)

$$\frac{dV}{dz} = 0 \tag{A.1}$$

$$\frac{dM}{dz} = V \tag{A.2}$$

$$\frac{d\phi}{dz} = \frac{M}{E_i \mathcal{J}_i} \tag{A.3}$$

$$\frac{dx}{dz} = -\phi + \frac{V}{\pi R_i \delta_i G_i} \tag{A.4}$$

where

V = net shearing force on a cross section

M = net moment on a cross section

ϕ = measure of net rotation

E_i = modulus of elasticity

G_i = shear modulus

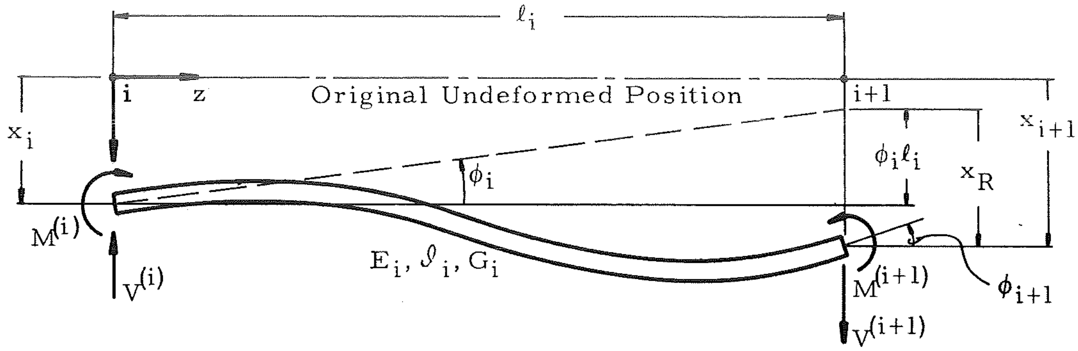
$\mathcal{J}_i$ = moment of inertia of the cylinder cross section

R_i = radius of the cylinder

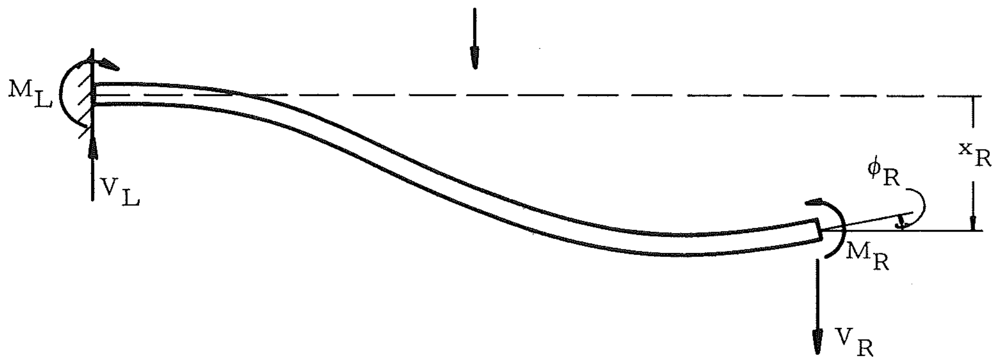
δ_i = thickness of cylinder wall

x = displacement in x-direction

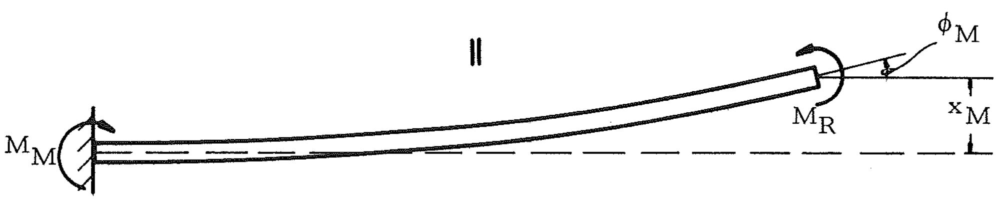
The general deformation of a beam-like thin-walled cylinder subjected to end moments $M^{(i)}$, $M^{(i+1)}$ and end shear $V^{(i)}$, $V^{(i+1)}$ [see Figure A-I(a)] may be represented by an equivalent cantilever beam subjected to equivalent end moment M_R and end shear V_R [see Figure A-I(b)], with the conditions:



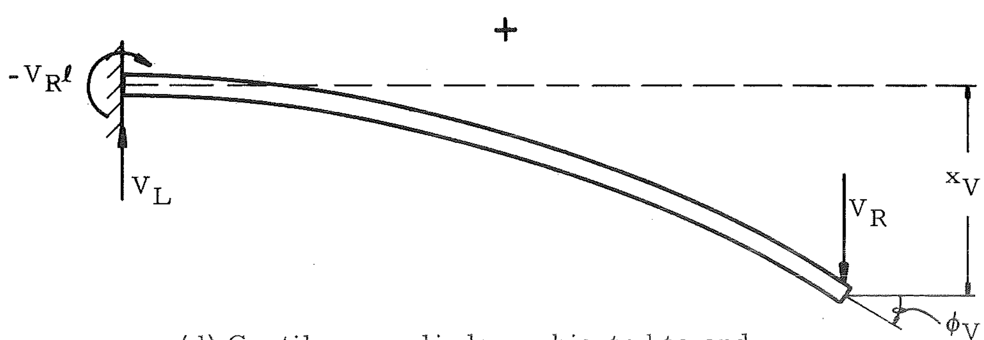
(a) Free body diagram of deformed (net) cylindrical shell section between stations (i-1) and i.



(b) Equivalent cantilever cylinder subjected to end moment and shear



(c) Cantilever cylinder subjected to end moment



(d) Cantilever cylinder subjected to end shearing force

3060

FIGURE A. 1. --SIGN CONVENTION FOR BENDING LOADS

$$x_R = x_{i+1} - x_i + \ell_i \phi_i \tag{A.5}$$

$$\phi_R = \phi_{i+1} - \phi_i \tag{A.6}$$

If we decompose the deformation of the equivalent cantilever beam into that due to end moment M_R only and that due to end shear V_R only [see Figure A-I(d)], the governing equations (A.1 to A.4) written for the two cases are

(1) Due to M_R alone

$$V' = 0 \tag{A.7}$$

$$\frac{dM'}{dz} = 0 \tag{A.8}$$

$$\frac{d\phi'}{dz} = \frac{M'}{E_i \mathcal{I}_i} \tag{A.9}$$

$$\frac{dx'}{dz} = -\phi' \tag{A.10}$$

(2) Due to V_R alone

$$\frac{dV''}{dz} = 0 \tag{A.11}$$

$$\frac{dM''}{dz} = V_R \tag{A.12}$$

$$\frac{d\phi''}{dz} = \frac{M''}{E_i \mathcal{I}_i} \tag{A.13}$$

$$\frac{dx''}{dz} = -\phi'' + \frac{V_R}{\pi R_i \delta_i G_i} \tag{A.14}$$

Solving the above two sets of equations with proper boundary conditions one obtains

$$x_M = -\frac{M_R \ell_i^2}{2E_i \mathcal{I}_i} \tag{A.15}$$

$$\phi_M = \frac{M_R \ell_i}{E_i \mathcal{I}_i} \tag{A.16}$$

$$x_V = \frac{V_R \ell_i^3}{3E_i \mathcal{I}_i} + \frac{V_R \ell_i}{\pi R_i \delta_i G_i} \tag{A.17}$$

$$\phi_V = -\frac{V_R \ell_i^2}{2E_i \mathcal{I}_i} \tag{A.18}$$

Substitution of Equations (A. 15) to (A. 18) into the following equations

$$x_R = x_M + x_V \tag{A.19}$$

$$\phi_R = \phi_M + \phi_V \tag{A.20}$$

and solving for M_R and V_R, there results

$$M_R = \frac{2E_i \mathcal{I}_i}{\ell_i^2 N_i} [3x_R + 2\ell_i A_i \phi_R] \tag{A.21}$$

$$V_R = \frac{6E_i \mathcal{I}_i}{\ell_i^3 N_i} [2x_R + \ell_i \phi_R] \tag{A.22}$$

where

$$N_i \equiv 1 + \frac{12E_i \mathcal{I}_i}{\ell_i^2 \pi R_i \delta_i G_i} \tag{A.23}$$

$$A_i \equiv 1 + \frac{3E_i \mathcal{I}_i}{\ell_i^2 \pi R_i \delta_i G_i} \tag{A.24}$$

Using Equations (A. 21) and (A. 22), one obtains

$$M_L = -\frac{2E_i \mathcal{I}_i}{\ell_i^2 N_i} [3x_R + \ell_i B_i \phi_R] \tag{A.25}$$

$$V_L = V_R \tag{A.26}$$

where

$$B_i \equiv 1 - \frac{6E_i \mathscr{I}_i}{\ell_i^2 \pi R_i \delta_i G_i} \tag{A.27}$$

Now writing $M_L = M^{(i)}$, $M_R = M^{(i+1)}$, $V_R = V^{(i)} = V^{(i+1)}$, and using Equations (A.5) and (A.6), the end moments and end shear of the original beam section are obtained

$$M^{(i)} = -\frac{2E_i \mathscr{I}_i}{\ell_i^2 N_i} [3(x_{i+1} - x_i) + \ell_i(2A_i\phi_i + B_i\phi_{i+1})] \tag{A.28}$$

$$M^{(i+1)} = \frac{2E_i \mathscr{I}_i}{\ell_i^2 N_i} [3(x_{i+1} - x_i) + \ell_i(2A_i\phi_{i+1} + B_i\phi_i)] \tag{A.29}$$

$$V^{(i)} = V^{(i+1)} = \frac{6E_i \mathscr{I}_i}{\ell_i^3 N_i} [2(x_{i+1} - x_i) + \ell_i(\phi_i + \phi_{i+1})] \tag{A.30}$$

For ordinary beams one simply sets A_i, B_i, and N_i to unity.

APPENDIX B

MATRIX ELEMENTS

ADDITIONAL NOMENCLATURE

Symbol	Definition
$A_{i1} = 1 + \dfrac{3E_i \mathscr{I}_i}{(h_i^*)^2 \pi R_i \delta_i G_i}$	= Correction factor in modified beam theory for thin-walled beam-like cylinders (i = 1, 3, 5, 7, no summation)
$A_{i2} = 1 + \dfrac{3E_i \mathscr{I}_i}{(\tilde{h}_i - h_i^*)^2 \pi R_i \delta_i G_i}$	"
$A_{i3} = 1 + \dfrac{3E_i \mathscr{I}_i}{(\ell_i - \tilde{h}_i)^2 \pi R_i \delta_i G_i}$	"
$A_j = 1 + \dfrac{3E_j \mathscr{I}_j}{\ell_j^2 \pi R_j \delta_j G_j}$	" (j = 2, 6)
$\overline{A}_{i1} = A_{i1}\Big\|_{h_j^* \to \tilde{h}_i}$	"
$\overline{A}_{i2} = A_{i2}$	"
$\overline{A}_{i3} = A_{i3}\Big\|_{\tilde{h}_i \to h_i^*}$	"
$A_{i1}' = A_{i1}\Big\|_{h_i^* \to \ell_i}$	"
$B_{i1} = 1 - \dfrac{6E_i \mathscr{I}_i}{(h_i^*)^2 \pi R_i \delta_i G_i}$	= Correction factor in modified beam theory for thin-walled beam-like cylinders
$B_{i2} = 1 - \dfrac{6E_i \mathscr{I}_i}{(\tilde{h}_i - h_i^*)^2 \pi R_i \delta_i G_i}$	"

ADDITIONAL NOMENCLATURE (Cont'd)

Symbol	Definition
$B_{i3} = 1 - \dfrac{6E_i \mathscr{I}_i}{(\ell_i - \tilde{h}_i)^2 \pi R_i \delta_i G_i}$	= Correction factor in modified beam theory for thin-walled beam-like cylinders
$B_j = 1 - \dfrac{6E_j \mathscr{I}_j}{\ell_j^2 \pi R_j \delta_j G_j}$	"
$\overline{B}_{i1} = B_{i1}\Big\vert_{h_i^* \to \tilde{h}_i}$	"
$\overline{B}_{i2} = B_{i2}$	"
$\overline{B}_{i3} = B_{i3}\Big\vert_{\tilde{h}_i \to h_i^*}$	"
$B'_{i1} = B_{i1}\Big\vert_{h_i^* \to \ell_i}$	"
$N_{i1} = 1 + \dfrac{3E_i \mathscr{I}_i}{(h_i^*)^2 \pi R_i \delta_i G_i}$	= Correction factor arising from modified beam theory
$N_{i2} = 1 + \dfrac{3E_i \mathscr{I}_i}{(\tilde{h}_i - h_i^*)^2 \pi R_i \delta_i G_i}$	"
$N_{i3} = 1 + \dfrac{3E_i \mathscr{I}_i}{(\ell_i - \tilde{h}_i)^2 \pi R_i \delta_i G_i}$	"
$N_j = 1 + \dfrac{3E_j \mathscr{I}_j}{\ell_j^2 \pi R_j \delta_j G_j}$	"
$\overline{N}_{i1} = N_{i1}\Big\vert_{h_i^* \to \tilde{h}_i}$	"

ADDITIONAL NOMENCLATURE (Cont'd)

Symbol	Definition
$\overline{N}_{i2} = N_{i2}$	= Correction factor arising from modified beam theory
$\overline{N}_{i3} = N_{i3}\Big\vert_{\tilde{h}_i \to h_i^*}$	"
$N_{i1}' = N_{i1}\Big\vert_{h_i^* \to \ell_i}$	"

NONZERO ELEMENTS OF MATRICES $\underset{\sim}{P}$ (DIAGONAL) AND $\underset{\sim}{q}$ (SYMMETRIC)

P_{ij} (i,j=1, 40)

$P_{11} = M_1$
$P_{22} = M_2$
$P_{33} = M_3$
$P_{44} = M_4$
$P_{55} = M_5$
$P_{66} = M_6$
$P_{77} = M_7$
$P_{88} = M_8$
$P_{99} = J_1$
$P_{1010} = J_2$
$P_{1111} = J_3$
$P_{1212} = J_4$
$P_{1313} = J_5$
$P_{1414} = J_6$
$P_{1515} = J_7$
$P_{1616} = J_8$
$P_{1717} = M_1$
$P_{1818} = M_2$
$P_{1919} = M_3$
$P_{2020} = M_4$
$P_{2121} = M_5$
$P_{2222} = M_6$
$P_{2323} = M_7$
$P_{2424} = M_8$
$P_{2525} = m_5$
$P_{2626} = m_7$
$P_{2727} = J_5^*$
$P_{2828} = J_7^*$
$P_{2929} = m_5^*$
$P_{3030} = m_7^*$
$P_{3131} = \tilde{m}_5$
$P_{3232} = \tilde{m}_7$
$P_{3333} = m_1$
$P_{3434} = m_3$
$P_{3535} = J_1^*$
$P_{3636} = J_3^*$

P'_{ij}

$P_{3737} = m_1^*$
$P_{3838} = m_3^*$
$P_{3939} = \tilde{m}_1$
$P_{4040} = \tilde{m}_3$

q_{ij} (i,j=1,---40) ($k > k^*$)

$q_{1-1} = K_1 + K^{(1)} + k_1$
$q_{1-2} = -(K_1 + K^{(1)})$
$q_{1-9} = K^{(1)}(R_B + D_1)$
$q_{1-10} = -K^{(1)}(R_B + D_1)$
$q_{1-33} = -k_1$
$q_{2-1} = -(K_1 + K^{(1)})$
$q_{2-2} = K_1 + K_2 + K^{(1)} + K^{(2)} + \bar{k}_1$
$q_{2-3} = -(K_2 + K^{(2)})$
$q_{2-9} = -K^{(1)}(R_B + D_1)$
$q_{2-10} = K^{(1)}(R_B + D_1) + K^{(2)}(R_B + D_2)$
$q_{2-11} = -K^{(2)}(R_B + D_2)$
$q_{2-33} = -\bar{k}_1$
$q_{3-2} = -(K_2 + K^{(2)})$
$q_{3-3} = K_2 + K_3 + K^{(2)} + K^{(3)} + k_3$
$q_{3-4} = -K_3$
$q_{3-10} = -K^{(2)}(R_B + D_2)$
$q_{3-11} = K^{(2)}(R_B + D_2) + K^{(3)}(R_B + D_3)$
$q_{3-34} = -k_3$
$q_{4-3} = -K_3$
$q_{4-4} = K_3 + \bar{K}^{(3)} + \bar{k}_3$
$q_{4-12} = \bar{K}^{(3)}(R_B + D_3)$
$q_{4-34} = -\bar{k}_3$
$q_{5-5} = K_5 + k_5$

q'_{ij} (empty) (i',j'=1, ---)

SAME AS q_{ij}

$\bar{q}_{ij}$ (i,j=1,---40) ($k < k^*$)

SAME AS q_{ij}

q_{ij}

$q_{5-6} = -K_5$
$q_{5-25} = -k_5$
$q_{6-5} = -K_5$
$q_{6-6} = K_5 + K_6 + K^{(4)} + \bar{k}_5$
$q_{6-7} = -K_6$
$q_{6-14} = -K^{(4)}(R_0 + D_4)$
$q_{6-25} = -\bar{k}_5$
$q_{7-6} = -K_6$
$q_{7-7} = K_6 + K_7 + \bar{K}^{(4)} + k_7$
$q_{7-8} = -K_7$
$q_{7-15} = -\bar{K}^{(4)}(R_0 + D_4)$
$q_{7-26} = -k_7$
$q_{8-7} = -K_7$
$q_{8-8} = K_7 + \bar{k}_7$
$q_{8-26} = -\bar{k}_7$
$q_{9-1} = K^{(1)}(R_B + D_1)$
$q_{9-2} = -K^{(1)}(R_B + D_1)$
$q_{9-9} = K^{(1)}(R_B + D_1)^2 + S_1 + 4E_1 I_1 A_{11}/k_1^* N_{11}$
$q_{9-10} = -K^{(1)}(R_B + D_1)^2$
$q_{9-17} = -\dfrac{6E_1 I_1}{(k_1^*)^2 N_{11}}$
$q_{9-35} = \dfrac{2E_1 I_1 B_{11}}{k_1^* N_{11}}$
$q_{9-37} = \dfrac{6E_1 I_1}{(k_1^*)^2 N_{11}}$ →
$q_{10-1} = -K^{(1)}(R_B + D_1)$

q'_{ij}

SAME AS q_{ij}

$q'_{9-9} = q_{9-9}\big|_{k_1^* = l_1}$
$q'_{9-10} = q_{9-10} + \dfrac{2E_1 I_1 B'_{11}}{l_1 N'_{11}}$
$q'_{9-17} = q_{9-17}\big|_{k_1^* = l_1}$
$q'_{9-35} = 0$
$q'_{9-18} = \dfrac{6E_1 I_1}{l_1^2 N'_{11}}$
$q'_{10-1} = q_{10-1}$

$\bar{q}_{ij}$

SAME AS q_{ij}

$\bar{q}_{9-9} = K^{(1)}(R_B + D_1)^2 + S_1 + \dfrac{4E_1 I_1 \bar{A}_{11}}{\bar{k}_1 \bar{N}_{11}}$
$\bar{q}_{9-10} = q_{9-10}$
$\bar{q}_{9-17} = -\dfrac{6E_1 I_1}{(\bar{k}_1)^2 \bar{N}_{11}}$
$\bar{q}_{9-35} = 0$
$\bar{q}_{9-37} = 0$
$\bar{q}_{10-1} = q_{10-1}$

NONZERO ELEMENTS OF $\mathfrak{q}$ MATRIX (SYMMETRIC)

q_{ij}	q'_{ij}	$\bar{q}_{ij}$
$q_{10-2} = K^{(1)}(R_B+D_1) + K^{(2)}(R_B+D_2)$	SAME AS q_{ij}	SAME AS q_{ij}
$q_{10-3} = -K^{(2)}(R_B+D_2)$	SAME AS q_{ij}	SAME AS q_{ij}
$q_{10-9} = -K^{(1)}(R_B+D_1)^2$	$q'_{10-9} = q_{10-9} + \frac{2E_1 J_1 B'_{11}}{l_1 N'_{11}}$	SAME AS q_{ij}
$q_{1010} = K^{(1)}(R_B+D_1)^2 + K^{(2)}(R_B+D_2)^2 + S_2 + \frac{4E_1 J_1 A_{13}}{(l_1-\hat{r}_1)N_{13}} + \frac{4E_2 J_2 A_2}{l_2 N_2}$	$q'_{1010} = q_{1010}\big\vert_{\hat{r}_1=0}$	$\bar{q}_{1010} = K^{(1)}(R_B+D_1)^2 + K^{(2)}(R_B+D_2)^2 + S_2 + \frac{4E_1 J_1 \bar{A}_{13}}{(l_1-\hat{r}_1^*)\bar{N}_{13}} + \frac{4E_2 J_2 A_2}{l_2 N_2}$
$q_{1011} = -K^{(2)}(R_B+D_2)^2 + \frac{2E_2 J_2 B_2}{l_2 N_2}$	$q'_{1011} = q_{1011}$	$\bar{q}_{1011} = q_{1011}$
$q_{1017} = 0$	$q'_{1017} = -\frac{6E_1 J_1}{l_1^2 N'_{11}}$	$\bar{q}_{1017} = 0$
$q_{1018} = 6\left[\frac{E_1 J_1}{(l_1-\hat{r}_1)^2 N_{13}} - \frac{E_2 J_2}{l_2^2 N_2}\right]$	$q'_{1018} = q_{1018}\big\vert_{\hat{r}_1=0}$	$\bar{q}_{1018} = 6\left[\frac{E_1 J_1}{(l_1-\hat{r}_1^*)^2\bar{N}_{13}} - \frac{E_2 J_2}{l_2^2 N_2}\right]$
$q_{1019} = \frac{6E_2 J_2}{l_2^2 N_2}$	SAME AS q_{ij}	$\bar{q}_{1019} = q_{1019}$
		$\bar{q}_{1035} = 2E_1 J_1 \bar{B}_{13}/[(l_1-\hat{r}_1^*)\bar{N}_{13}]$
		$\bar{q}_{1037} = -6E_1 J_1/[(l_1-\hat{r}_1^*)^2\bar{N}_{13}]$
$q_{11-2} = -K^{(2)}(R_B+D_2)$	SAME AS q_{ij}	$\bar{q}_{11-2} = q_{11-2}$
$q_{11-3} = K^{(2)}(R_B+D_2) + K^{(3)}(R_B+D_3)$	SAME AS q_{ij}	$\bar{q}_{11-3} = q_{11-3}$
$q_{1110} = -K^{(2)}(R_B+D_2)^2 + \frac{2E_2 J_2 B_2}{l_2 N_2}$	SAME AS q_{ij}	$\bar{q}_{1110} = q_{1110}$
$q_{1111} = K^{(2)}(R_B+D_2)^2 + K^{(3)}(R_B+D_3)^2 + S_3 + \frac{4E_2 J_2 A_2}{l_2 N_2} + \frac{4E_3 J_3 A_{31}}{\hat{r}_3^* N_{31}}$	$q'_{1111} = q_{1111}\big\vert_{\hat{r}_3^*=l_3}$	$\bar{q}_{1111} = K^{(2)}(R_B+D_3)^2 + K^{(3)}(R_B+D_3)^2 + S_3 + \frac{4E_2 J_2 A_2}{l_2 N_2} + \frac{4E_3 J_3 \bar{A}_{31}}{\hat{r}_3 \bar{N}_{31}}$
$q_{1118} = -\frac{6E_2 J_2}{l_2^2 N_2}$	$q'_{1118} = q_{1118}$	$\bar{q}_{1118} = q_{1118}$
$q_{1119} = 6\left[\frac{E_2 J_2}{l_2^2 N_2} - \frac{E_3 J_3}{(\hat{r}_3^*)^2 N_{31}}\right]$	$q'_{1119} = q_{1119}\big\vert_{\hat{r}_3^*=l_3}$	$\bar{q}_{1119} = 6\left[\frac{E_2 J_2}{l_2^2 N_2} - \frac{E_3 J_3}{(\hat{r}_3)^2 \bar{N}_{31}}\right]$
$q_{1136} = \frac{2E_3 J_3 B_{31}}{\hat{r}_3^* N_{31}}$	$q'_{1112} = \frac{2E_3 J_3 B'_{31}}{l_3 N'_{31}}$	$\bar{q}_{1136} = 0$
$q_{1138} = \frac{6E_3 J_3}{(\hat{r}_3^*)^2 N_{31}}$	$q'_{1120} = \frac{6E_3 J_3}{l_3^2 N'_{31}}$	$\bar{q}_{1138} = 0$
$q_{12-4} = \bar{K}^{(3)}(R_B+D_3)$	$q'_{12-4} = q_{12-4}$	$\bar{q}_{12-4} = q_{12-4}$
$q_{1211} = 0$	$q'_{1211} = \frac{2E_3 J_3 B'_{31}}{l_3 N'_{31}}$	$\bar{q}_{1211} = q_{1211}$

q_{ij}	q'_{ij}	$\bar{q}_{ij}$
$q_{1212} = \bar{K}^{(3)}(R_B+D_3)^2 + S_4 + \frac{4E_3 J_3 A_{33}}{(l_3-\hat{r}_3)N_{33}}$	$q'_{1212} = q_{1212}\big\vert_{\hat{r}_3=0}$	$\bar{q}_{1212} = \bar{K}^{(3)}(R_B+D_3)^2 + S_4 + \frac{4E_3 J_3 \bar{A}_{33}}{(l_3-\hat{r}_3^*)\bar{N}_{33}}$
$q_{1219} = 0$	$q'_{1219} = -\frac{6E_3 J_3}{l_3^2 N'_{31}}$	$\bar{q}_{1219} = 0$
$q_{1220} = \frac{6E_3 J_3}{(l_3-\hat{r}_3)^2 N_{33}}$	$q'_{1220} = q_{1220}\big\vert_{\hat{r}_3=0}$	$\bar{q}_{1220} = 6E_3 J_3/[(l_3-\hat{r}_3^*)^2\bar{N}_{33}]$
		$\bar{q}_{1236} = 2E_3 J_3 \bar{B}_{33}/[(l_3-\hat{r}_3^*)\bar{N}_{33}]$
		$\bar{q}_{1238} = -6E_3 J_3/[(l_3-\hat{r}_3^*)^2\bar{N}_{33}]$
$q_{1313} = \frac{4E_5 J_5 A_{51}}{\hat{r}_5^* N_{51}}$	$q'_{1313} = q_{1313}\big\vert_{\hat{r}_5^*=l_5}$	$\bar{q}_{1313} = \frac{4E_5 J_5 \bar{A}_{51}}{\hat{r}_5 \bar{N}_{51}}$
$q_{1321} = K^{(5)}e - \frac{6E_5 J_5}{(\hat{r}_5^*)^2 N_{51}}$	$q'_{1321} = q_{1321}\big\vert_{\hat{r}_5^*=l_5}$	$\bar{q}_{1321} = K^{(5)}e - \frac{6E_5 J_5}{(\hat{r}_5)^2\bar{N}_{51}}$
$q_{1327} = \frac{2E_5 J_5 B_{51}}{\hat{r}_5^* N_{51}}$	$q'_{1314} = \frac{2E_5 J_5 B'_{51}}{l_5 N'_{51}}$	$\bar{q}_{1327} = 0$
$q_{1329} = \frac{6E_5 J_5}{(\hat{r}_5^*)^2 N_{51}}$	$q'_{1322} = \frac{6E_5 J_5}{l_5^2 N'_{51}}$	$\bar{q}_{1329} = 0$
$q_{14-6} = -K^{(4)}(R_0+D_4)$	$q'_{14-6} = q_{14-6}$	$\bar{q}_{14-6} = q_{14-6}$
$q_{1412} = 0$	$q'_{1413} = \frac{2E_5 J_5 B'_{51}}{l_5 N'_{51}}$	$\bar{q}_{1413} = 0$
$q_{1414} = K^{(4)}(R_0+D_4)^2 + S_5 + \frac{4E_5 J_5 A_{53}}{(l_5-\hat{r}_5)N_{53}} + \frac{4E_6 J_6 A_6}{l_6 N_6}$	$q'_{1414} = q_{1414}\big\vert_{\hat{r}_5=0}$	$\bar{q}_{1414} = K^{(4)}(R_0+D_4)^2 + S_5 + \frac{4E_5 J_5 \bar{A}_{53}}{(l_5-\hat{r}_5^*)\bar{N}_{53}} + \frac{4E_6 J_6 A_6}{l_6 N_6}$
$q_{1415} = \frac{2E_6 J_6 B_6}{l_6 N_6}$	$q'_{1415} = q_{1415}$	$\bar{q}_{1415} = q_{1415}$
$q_{1421} = 0$	$q'_{1421} = -\frac{6E_5 J_5}{l_5^2 N'_{51}}$	$\bar{q}_{1421} = 0$
$q_{1422} = 6\left[\frac{E_5 J_5}{(l_5-\hat{r}_5)^2 N_{53}} - \frac{E_6 J_6}{l_6^2 N_6}\right]$	$q'_{1422} = q_{1422}\big\vert_{\hat{r}_5=0}$	$\bar{q}_{1422} = 6\left[\frac{E_5 J_5}{(l_5-\hat{r}_5^*)^2\bar{N}_{53}} - \frac{E_6 J_6}{l_6^2 N_6}\right]$
$q_{1423} = \frac{6E_6 J_6}{l_6^2 N_6}$	SAME AS q_{ij}	$\bar{q}_{1423} = q_{1423}$
		$\bar{q}_{1437} = 2E_5 J_5 \bar{B}_{53}/[(l_5-\hat{r}_5^*)\bar{N}_{53}]$
		$\bar{q}_{1439} = -6E_5 J_5/[(l_5-\hat{r}_5^*)^2\bar{N}_{53}]$
$q_{15-7} = -\bar{K}^{(4)}(R_0+D_4)$	SAME AS q_{ij}	$\bar{q}_{15-7} = q_{15-7}$
$q_{1514} = \frac{2E_6 J_6 B_6}{l_6 N_6}$	SAME AS q_{ij}	$\bar{q}_{1514} = q_{1514}$
$q_{1515} = \bar{K}^{(4)}(R_0+D_4)^2 + S_7 + \frac{4E_6 J_6 A_6}{l_6 N_6} + \frac{4E_7 J_7 A_{71}}{\hat{r}_7^* N_{71}}$	$q'_{1515} = q_{1515}\big\vert_{\hat{r}_7^*=l_7}$	$\bar{q}_{1515} = \bar{K}^{(4)}(R_0+D_4)^2 + S_7 + \frac{4E_6 J_6 A_6}{l_6 N_6} + \frac{4E_7 J_7 \bar{A}_{71}}{\hat{r}_7 \bar{N}_{71}}$
$q_{1522} = -\frac{6E_6 J_6}{l_6^2 N_6}$	$q'_{1522} = q_{1522}$	$\bar{q}_{1522} = q_{1522}$

NONZERO ELEMENTS OF q MATRIX (SYMMETRIC)

q_{ij}

$$q_{1523} = 6\left[\frac{E_6 I_6}{\ell_6^2 N_6} - \frac{E_7 I_7}{(h_7^*)^2 N_{71}}\right]$$
$$q_{1528} = \frac{2E_7 I_7 B_{71}}{-h_7^* \; N_{71}}$$
$$q_{1530} = \frac{6E_7 I_7}{(h_7^*)^2 N_{71}}$$
$$q_{1615} = 0$$
$$q_{1616} = \frac{4E_7 I_7 A_{73}}{(\ell_7 - \tilde{h}_7) N_{73}}$$
$$q_{1623} = 0$$
$$q_{1624} = \frac{6E_7 I_7}{(\ell_7 - \tilde{h}_7)^2 N_{73}}$$
$$q_{17\text{-}9} = -\frac{6E_1 I_1}{(h_1^*)^2 N_{11}}$$
$$q_{1717} = 12\left[\frac{E_1 I_1}{(h_1^*)^3 N_{11}} + \frac{E^{(1)}J^{(1)}}{\ell_1^3}\right]$$
$$q_{1718} = -\frac{12E^{(1)}J^{(1)}}{\ell_1^3}$$
$$q_{1735} = -\frac{6E_1 I_1}{(h_1^*)^2 N_{11}}$$
$$q_{1737} = -\frac{12E_1 I_1}{(h_1^*)^3 N_{11}}$$
$$q_{18\text{-}9} = 0$$
$$q_{1810} = 6\left[\frac{E_1 I_1}{(\ell_1 - \tilde{h}_1)^2 N_{13}} - \frac{E_2 I_2}{\ell_2^2 N_2}\right]$$
$$q_{1811} = -\frac{6E_2 I_2}{\ell_2^2 N_2}$$
$$q_{1817} = -\frac{12E^{(1)}J^{(1)}}{\ell_1^3}$$
$$q_{1818} = 12\left[\frac{E_1 I_1}{(\ell_1 - \tilde{h}_1)^3 N_{13}} + \frac{E^{(1)}J^{(1)}}{\ell_1^3} + \frac{E_2 I_2}{\ell_2^3 N_2} + \frac{E^{(2)}J^{(2)}}{a^3}\right]$$
$$q_{1819} = -\frac{12E_2 I_2}{\ell_2^3 N_2}$$
$$q_{1910} = \frac{6E_2 I_2}{\ell_2^2 N_2}$$

$q'_{i'j'}$

$$q'_{1523} = q_{1523}\big|_{h_7^* = \ell_7}$$
$$q'_{1516} = \frac{2E_7 I_7 B'_{71}}{\ell_7 \; N'_{71}}$$
$$q'_{1524} = \frac{6E_7 I_7}{\ell_7^2 N'_{71}}$$
$$q'_{1615} = \frac{2E_7 I_7 B'_{71}}{\ell_7 \; N'_{71}}$$
$$q'_{1616} = q_{1616}\big|_{\tilde{h}_7 = 0}$$
$$q'_{1623} = -\frac{6E_7 I_7}{\ell_7^2 N'_{71}}$$
$$q'_{1624} = q_{1624}\big|_{\tilde{h}_7 = 0}$$
$$q'_{17\text{-}9} = q_{17\text{-}9}\big|_{h_1^* = \ell_1}$$
$$q'_{1717} = q_{1717}\big|_{h_1^* = \ell_1}$$
$$q'_{1718} = q_{1718} - \frac{12E_1 I_1}{\ell_1^3 N'_{11}}$$
$$\rightarrow \; q'_{1710} = -\frac{6E_1 I_1}{\ell_1^2 N'_{11}}$$
$$q'_{1737} = 0$$
$$q'_{18\text{-}9} = \frac{6E_1 I_1}{\ell_1^2 N'_{11}}$$
$$q'_{1810} = q_{1810}\big|_{\tilde{h}_1 = 0}$$
$$q'_{1811} = q_{1811}$$
$$q'_{1817} = q_{1817} - \frac{12E_1 I_1}{\ell_1^3 N'_{11}}$$
$$q'_{1818} = q_{1818}\big|_{\tilde{h}_1 = 0}$$

↑ SAME AS q_{ij} ↓

$\bar{q}_{ij}$

$$\bar{q}_{1523} = 6\left[\frac{E_6 I_6}{\ell_6^2 N_6} - \frac{E_7 I_7}{(\tilde{h}_7)^2 \bar{N}_{71}}\right]$$
$$\bar{q}_{1528} = 0$$
$$\bar{q}_{1530} = 0$$
$$\bar{q}_{1615} = 0$$
$$\bar{q}_{1616} = \frac{4E_7 I_7 \bar{A}_{73}}{(\ell_7 - h_7^*) \bar{N}_{73}}$$
$$\bar{q}_{1623} = 0$$
$$\bar{q}_{1624} = \frac{6E_7 I_7}{(\ell_7 - h_7^*)^2 \bar{N}_{73}}$$
$$\bar{q}_{1628} = 2E_7 I_7 \bar{B}_{73} / [(\ell_7 - h_7^*)\bar{N}_{73}]$$
$$\bar{q}_{1630} = -6E_7 I_7 / [(\ell_7 - h_7^*)^2 \bar{N}_{73}]$$
$$\bar{q}_{17\text{-}9} = -\frac{6E_1 I_1}{(\tilde{h}_1)^2 \bar{N}_{11}}$$
$$\bar{q}_{1717} = 12\left[\frac{E_1 I_1}{(\tilde{h}_1)^3 \bar{N}_{11}} + \frac{E^{(1)}J^{(1)}}{\ell_1^3}\right]$$
$$\bar{q}_{1718} = q_{1718}$$
$$\bar{q}_{1735} = 0$$
$$\bar{q}_{1737} = 0$$
$$\bar{q}_{18\text{-}9} = 0$$
$$\bar{q}_{1810} = 6\left[\frac{E_1 I_1}{(\ell_1 - h_1^*)^2 \bar{N}_{13}} - \frac{E_2 I_2}{\ell_2^2 N_2}\right]$$
$$\bar{q}_{1811} = q_{1811}$$
$$\bar{q}_{1817} = q_{1817}$$
$$\bar{q}_{1818} = 12\left[\frac{E_1 I_1}{(\ell_1 - h_1^*)^3 \bar{N}_{13}} + \frac{E^{(1)}J^{(1)}}{\ell_1^3} + \frac{E_2 I_2}{\ell_2^3 N_2} + \frac{E^{(2)}J^{(2)}}{a^3}\right]$$
$$\bar{q}_{1819} = q_{1819}$$
$$\bar{q}_{1835} = 6E_1 I_1 / [(\ell_1 - h_1^*)^2 \bar{N}_{13}]$$
$$\bar{q}_{1837} = -12E_1 I_1 / [(\ell_1 - h_1^*)^3 \bar{N}_{13}]$$
$$\bar{q}_{1910} = q_{1910}$$

q_{ij}

$$q_{1911} = 6\left[\frac{E_2 I_2}{\ell_2^2 N_2} - \frac{E_3 I_3}{(h_3^*)^2 N_{31}}\right]$$
$$q_{1918} = -\frac{12E_2 I_2}{\ell_2^3 N_2}$$
$$q_{1919} = 12\left[\frac{E_2 I_2}{\ell_2^3 N_2} + \frac{E^{(2)}J^{(2)}}{(\ell_2 - a)^3} + \frac{E_3 I_3}{(h_3^*)^3 N_{31}} + \frac{E^{(3)}J^{(3)}}{c^3}\right]$$
$$q_{1936} = -\frac{6E_3 I_3}{(h_3^*)^2 N_{31}}$$
$$q_{1938} = -\frac{12E_3 I_3}{(h_3^*)^3 N_{31}}$$
$$q_{2011} = 0$$
$$q_{2012} = \frac{6E_3 I_3}{(\ell_3 - \tilde{h}_3)^2 N_{33}}$$
$$q_{2019} = 0$$
$$q_{2020} = 12\left[\frac{E_3 I_3}{(\ell_3 - \tilde{h}_3)^3 N_{33}} + \frac{E^{(3)}J^{(3)}}{(\ell_3 - c)^3}\right]$$
$$q_{2113} = K^{(5)}e - \frac{6E_5 I_5}{(h_5^*)^2 N_{51}}$$
$$q_{2121} = K^{(5)} + \frac{12E_5 I_5}{(h_5^*)^3 N_{51}}$$
$$q_{2127} = -\frac{6E_5 I_5}{(h_5^*)^2 N_{51}}$$
$$q_{2129} = -\frac{12E_5 I_5}{(h_5^*)^3 N_{51}}$$
$$q_{2213} = 0$$
$$q_{2214} = 6\left[\frac{E_5 I_5}{(\ell_5 - \tilde{h}_5)^2 N_{53}} - \frac{E_6 I_6}{\ell_6^2 N_6}\right]$$
$$q_{2215} = -\frac{6E_6 I_6}{\ell_6^2 N_6}$$
$$q_{2221} = 0$$
$$q_{2222} = 12\left[\frac{E_5 I_5}{(\ell_5 - \tilde{h}_5)^3 N_{53}} + \frac{E_6 I_6}{\ell_6^3 N_6} + \frac{E^{(4)}J^{(4)}}{d^3}\right]$$
$$q_{2223} = -\frac{12E_6 I_6}{\ell_6^3 N_6}$$

$q'_{i'j'}$

$$q'_{1911} = q_{1911}\big|_{h_3^* = \ell_3}$$
$$q'_{1918} = q_{1918}$$
$$q'_{1919} = q_{1919}\big|_{h_3^* = \ell_3}$$
$$q'_{1912} = -\frac{6E_3 I_3}{\ell_3^2 N'_{31}}$$
$$q'_{1920} = -\frac{12E_3 I_3}{\ell_3^3 N'_{31}}$$
$$q'_{2011} = \frac{6E_3 I_3}{\ell_3^2 N'_{31}}$$
$$q'_{2012} = q_{2012}\big|_{\tilde{h}_3 = 0}$$
$$q'_{2019} = -\frac{12E_3 I_3}{\ell_3^3 N'_{31}}$$
$$q'_{2020} = q_{2020}\big|_{\tilde{h}_3 = 0}$$
$$q'_{2113} = q_{2113}\big|_{h_5^* = \ell_5}$$
$$q'_{2121} = q_{2121}\big|_{h_5^* = \ell_5}$$
$$q'_{2114} = -\frac{6E_5 I_5}{\ell_5^2 N'_{51}}$$
$$q'_{2122} = -\frac{12E_5 I_5}{\ell_5^3 N'_{51}}$$
$$q'_{2213} = \frac{6E_5 I_5}{\ell_5^2 N'_{51}}$$
$$q'_{2214} = q_{2214}\big|_{\tilde{h}_5 = 0}$$
$$q'_{2215} = q_{2215}$$
$$q'_{2221} = -\frac{12E_5 I_5}{\ell_5^3 N'_{51}}$$
$$q'_{2222} = q_{2222}\big|_{\tilde{h}_5 = 0}$$
$$q'_{2223} = q_{2223}$$

$\bar{q}_{ij}$

$$\bar{q}_{1911} = 6\left[\frac{E_2 I_2}{\ell_2^2 N_2} - \frac{E_3 I_3}{(\tilde{h}_3)^2 \bar{N}_{31}}\right]$$
$$\bar{q}_{1918} = q_{1918}$$
$$\bar{q}_{1919} = 12\left[\frac{E_2 I_2}{\ell_2^3 N_2} + \frac{E^{(2)}J^{(2)}}{(\ell_2 - a)^3} + \frac{E_3 I_3}{(\tilde{h}_3)^3 \bar{N}_{31}} + \frac{E^{(3)}J^{(3)}}{c^3}\right]$$
$$\bar{q}_{1936} = 0$$
$$\bar{q}_{1938} = 0$$
$$\bar{q}_{2011} = 0$$
$$\bar{q}_{2012} = \frac{6E_3 I_3}{(\ell_3 - h_3^*)^2 \bar{N}_{33}}$$
$$\bar{q}_{2019} = 0$$
$$\bar{q}_{2020} = 12\left[\frac{E_3 I_3}{(\ell_3 - h_3^*)^3 \bar{N}_{33}} + \frac{E^{(3)}J^{(3)}}{(\ell_3 - c)^3}\right]$$
$$\bar{q}_{2036} = 6E_3 I_3 / [(\ell_3 - h_3^*)^2 \bar{N}_{33}]$$
$$\bar{q}_{2038} = -12E_3 I_3 / [(\ell_3 - h_3^*)^3 \bar{N}_{33}]$$
$$\bar{q}_{2113} = K^{(5)}e - \frac{6E_5 I_5}{(\tilde{h}_5)^2 \bar{N}_{51}}$$
$$\bar{q}_{2121} = K^{(5)} + \frac{12E_5 I_5}{(\tilde{h}_5)^3 \bar{N}_{51}}$$
$$\bar{q}_{2127} = 0$$
$$\bar{q}_{2129} = 0$$
$$\bar{q}_{2213} = 0$$
$$\bar{q}_{2214} = 6\left[\frac{E_5 I_5}{(\ell_5 - h_5^*)^2 \bar{N}_{53}} - \frac{E_6 I_6}{\ell_6^2 N_6}\right]$$
$$\bar{q}_{2215} = q_{2215}$$
$$\bar{q}_{2221} = 0$$
$$\bar{q}_{2222} = 12\left[\frac{E_5 I_5}{(\ell_5 - h_5^*)^3 \bar{N}_{53}} + \frac{E_6 I_6}{\ell_6^3 N_6} + \frac{E^{(4)}J^{(4)}}{d^3}\right]$$
$$\bar{q}_{2223} = q_{2223}$$
$$\bar{q}_{2227} = 6E_5 I_5 / [(\ell_5 - h_5^*)^2 \bar{N}_{53}]$$

NONZERO ELEMENTS OF q MATRIX (SYMMETRIC)

q_{ij}

$$q_{2314}=\frac{6E_6J_6}{l_6^2N_6}$$
$$q_{2315}=6\left[\frac{E_6J_6}{l_6^2N_6}-\frac{E_7J_7}{(h_7^*)^2N_{71}}\right]$$
$$q_{2322}=-\frac{12E_6J_6}{l_6^3N_6}$$
$$q_{2323}=12\left[\frac{E_6J_6}{l_6^3N_6}+\frac{E_7J_7}{(h_7^*)^3N_{71}}+\frac{E^{(4)}J^{(4)}}{(l_6-d)^3}\right]$$
$$q_{2328}=-\frac{6E_7J_7}{(h_7^*)^2N_{71}}$$
$$q_{2330}=-\frac{12E_7J_7}{(h_7^*)^3N_{71}}$$
$$q_{2415}=0$$
$$q_{2416}=\frac{6E_7J_7}{(l_7-\bar{h}_7)^2N_{73}}$$
$$q_{2423}=0$$
$$q_{2424}=\frac{12E_7J_7}{(l_7-\bar{h}_7)^3N_{73}}$$

$$q_{25-5}=-k_5$$
$$q_{25-6}=-\bar{k}_5$$
$$q_{2525}=k_5+\bar{k}_5$$
$$q_{26-7}=-k_7$$
$$q_{26-8}=-\bar{k}_7$$
$$q_{2626}=k_7+\bar{k}_7$$
$$q_{2713}=\frac{2E_5J_5B_{51}}{h_5^*N_{51}}$$
$$q_{2721}=-\frac{6E_5J_5}{(h_5^*)^2N_{51}}$$
$$q_{2727}=4E_5J_5\left[\frac{A_{51}}{h_5^*N_{51}}+\frac{A_{52}}{(\bar{h}_5-h_5^*)N_{52}}\right]$$
$$q_{2729}=6E_5J_5\left[\frac{1}{(h_5^*)^2N_{51}}-\frac{1}{(\bar{h}_5-h_5^*)^2N_{52}}\right]$$
$$q_{2815}=\frac{2E_7J_7B_{71}}{h_7^*N_{71}}$$

$q'_{i'j'}$

$$q'_{2314}=q_{2314}$$
$$q'_{2315}=q_{2315}\big|_{h_7^*=l_7}$$
$$q'_{2322}=q_{2322}$$
$$q'_{2323}=q_{2323}\big|_{h_7^*=l_7}$$
$$q'_{2316}=-\frac{6E_7J_7}{l_7^2N'_{71}}$$
$$q'_{2324}=-\frac{12E_7J_7}{l_7^3N'_{71}}$$
$$q'_{2415}=\frac{6E_7J_7}{l_7^2N'_{71}}$$
$$q'_{2416}=q_{2416}\big|_{\bar{h}_7=0}$$
$$q'_{2423}=-\frac{12E_7J_7}{l_7^3N'_{71}}$$
$$q'_{2424}=q_{2424}\big|_{\bar{h}_7=0}$$

END OF $q'_{i'j'}$

$\bar{q}_{ij}$

$$\bar{q}_{2229}=-12E_5J_5/[(l_5-h_5^*)^3\bar{N}_{53}]$$
$$\bar{q}_{2314}=q_{2314}$$
$$\bar{q}_{2315}=6\left[\frac{E_6J_6}{l_6^2N_6}-\frac{E_7J_7}{(\bar{h}_7)^2\bar{N}_{71}}\right]$$
$$\bar{q}_{2322}=q_{2322}$$
$$\bar{q}_{2323}=12\left[\frac{E_6J_6}{l_6^3N_6}+\frac{E_7J_7}{(\bar{h}_7)^3\bar{N}_{71}}+\frac{E^{(4)}J^{(4)}}{(l_6-d)^3}\right]$$
$$\bar{q}_{2328}=0$$
$$\bar{q}_{2330}=0$$
$$\bar{q}_{2415}=0$$
$$\bar{q}_{2416}=\frac{6E_7J_7}{(l_7-h_7^*)^2\bar{N}_{73}}$$
$$\bar{q}_{2423}=0$$
$$\bar{q}_{2424}=\frac{12E_7J_7}{(l_7-h_7^*)^3\bar{N}_{73}}$$
$$\bar{q}_{2428}=6E_7J_7/[(l_7-h_7^*)^2\bar{N}_{73}]$$
$$\bar{q}_{2430}=-12E_7J_7/[(l_7-h_7^*)^3\bar{N}_{73}]$$

SAME AS q_{ij}

$$\bar{q}_{2714}=\frac{2E_5J_5\bar{B}_{53}}{(l_5-h_5^*)\bar{N}_{53}}$$
$$\bar{q}_{2722}=\frac{6E_5J_5}{(l_5-h_5^*)^2\bar{N}_{53}}$$
$$\bar{q}_{2727}=4E_5J_5\left[\frac{\bar{A}_{53}}{(l_5-h_5^*)\bar{N}_{53}}+\frac{\bar{A}_{52}}{(h_5^*-\bar{h}_5)\bar{N}_{52}}\right]$$
$$\bar{q}_{2729}=6E_5J_5\left[\frac{1}{(h_5^*-\bar{h}_5)^2\bar{N}_{52}}-\frac{1}{(l_5-h_5^*)^2\bar{N}_{53}}\right]$$
$$\bar{q}_{2816}=\frac{2E_7J_7\bar{B}_{73}}{(l_7-h_7^*)\bar{N}_{73}}$$

q_{ij}

$$q_{2823}=-\frac{6E_7J_7}{(h_7^*)^2N_{71}}$$
$$q_{2828}=4E_7J_7\left[\frac{A_{71}}{h_7^*N_{71}}+\frac{A_{72}}{(\bar{h}_7-h_7^*)N_{72}}\right]$$
$$q_{2830}=6E_7J_7\left[\frac{1}{(h_7^*)^2N_{71}}-\frac{1}{(\bar{h}_7-h_7^*)^2N_{72}}\right]$$
$$q_{2913}=\frac{6E_5J_5}{(h_5^*)^2N_{51}}$$
$$q_{2921}=-\frac{12E_5J_5}{(h_5^*)^3N_{51}}$$
$$q_{2927}=6E_5J_5\left[\frac{1}{(h_5^*)^2N_{51}}-\frac{1}{(\bar{h}_5-h_5^*)^2N_{52}}\right]$$
$$q_{2929}=12E_5J_5\left[\frac{1}{(\bar{h}_5-h_5^*)^3N_{52}}+\frac{1}{(h_5^*)^3N_{51}}\right]$$
$$q_{3015}=\frac{6E_7J_7}{(h_7^*)^2N_{71}}$$
$$q_{3023}=-\frac{12E_7J_7}{(h_7^*)^3N_{71}}$$
$$q_{3028}=6E_7J_7\left[\frac{1}{(h_7^*)^2N_{71}}-\frac{1}{(\bar{h}_7-h_7^*)^2N_{72}}\right]$$
$$q_{3030}=12E_7J_7\left[\frac{1}{(\bar{h}_7-h_7^*)^3N_{72}}+\frac{1}{(h_7^*)^3N_{71}}\right]$$
$$q_{3131}=\tilde{k}_5$$
$$q_{3232}=\tilde{k}_7$$
$$q_{33-1}=-k_1$$
$$q_{33-2}=-\bar{k}_1$$
$$q_{3333}=k_1+\bar{k}_1$$
$$q_{34-3}=-k_3$$
$$q_{34-4}=-\bar{k}_3$$
$$q_{3434}=k_3+\bar{k}_3$$

$\bar{q}_{ij}$

$$\bar{q}_{2824}=\frac{6E_7J_7}{(l_7-h_7^*)^2\bar{N}_{73}}$$
$$\bar{q}_{2828}=4E_7J_7\left[\frac{\bar{A}_{73}}{(l_7-h_7^*)\bar{N}_{73}}+\frac{\bar{A}_{72}}{(h_7^*-\bar{h}_7)\bar{N}_{72}}\right]$$
$$\bar{q}_{2830}=6E_7J_7\left[\frac{1}{(h_7^*-\bar{h}_7)^2\bar{N}_{72}}-\frac{1}{(l_7-h_7^*)^2\bar{N}_{73}}\right]$$
$$\bar{q}_{2914}=-\frac{6E_5J_5}{(l_5-h_5^*)^2\bar{N}_{53}}$$
$$\bar{q}_{2922}=-\frac{12E_5J_5}{(l_5-h_5^*)^3\bar{N}_{53}}$$
$$\bar{q}_{2927}=6E_5J_5\left[\frac{1}{(h_5^*-\bar{h}_5)^2\bar{N}_{52}}-\frac{1}{(l_5-h_5^*)^2\bar{N}_{53}}\right]$$
$$\bar{q}_{2929}=12E_5J_5\left[\frac{1}{(h_5^*-\bar{h}_5)^3\bar{N}_{52}}+\frac{1}{(l_5-h_5^*)^3\bar{N}_{53}}\right]$$
$$\bar{q}_{3016}=-\frac{6E_7J_7}{(l_7-h_7^*)^2\bar{N}_{73}}$$
$$\bar{q}_{3024}=-\frac{12E_7J_7}{(l_7-h_7^*)^3\bar{N}_{73}}$$
$$\bar{q}_{3028}=6E_7J_7\left[\frac{1}{(h_7^*-\bar{h}_7)^2\bar{N}_{72}}-\frac{1}{(l_7-h_7^*)^2\bar{N}_{73}}\right]$$
$$\bar{q}_{3030}=12E_7J_7\left[\frac{1}{(h_7^*-\bar{h}_7)^3\bar{N}_{72}}+\frac{1}{(l_7-h_7^*)^3\bar{N}_{73}}\right]$$

SAME AS q_{ij}

NONZERO ELEMENTS OF $\underset{\sim}{q}$ MATRIX (SYMMETRIC)

q_{ij}	$\bar{q}_{ij}$
$q_{35\text{-}9} = \frac{2E_1 g_1 B_{11}}{k_1^* N_{11}}$	$\bar{q}_{3510} = \frac{2E_1 g_1 \bar{B}_{13}}{(\ell_1 - k_1^*)\bar{N}_{13}}$
$q_{3517} = -\frac{6E_1 g_1}{(k_1^*)^2 N_{11}}$	$\bar{q}_{3518} = \frac{6E_1 g_1}{(\ell_1 - k_1^*)^2 \bar{N}_{13}}$
$q_{3535} = 4E_1 g_1 \left[\frac{A_{11}}{k_1^* N_{11}} + \frac{A_{12}}{(\tilde{k}_1 - k_1^*) N_{12}}\right]$	$\bar{q}_{3535} = 4E_1 g_1 \left[\frac{\bar{A}_{13}}{(\ell_1 - k_1^*)\bar{N}_{13}} + \frac{\bar{A}_{12}}{(k_1^* - \tilde{k}_1)\bar{N}_{12}}\right]$
$q_{3537} = 6E_1 g_1 \left[\frac{1}{(k_1^*)^2 N_{11}} - \frac{1}{(\tilde{k}_1 - k_1^*)^2 N_{12}}\right]$	$\bar{q}_{3537} = 6E_1 g_1 \left[\frac{1}{(k_1^* - \tilde{k}_1)^2 \bar{N}_{12}} - \frac{1}{(\ell_1 - k_1^*)^2 \bar{N}_{13}}\right]$
$q_{3611} = \frac{2E_3 g_3 B_{31}}{k_3^* N_{31}}$	$\bar{q}_{3612} = \frac{2E_3 g_3 \bar{B}_{33}}{(\ell_3 - k_3^*)\bar{N}_{33}}$
$q_{3619} = -\frac{6E_3 g_3}{(k_3^*)^2 N_{31}}$	$\bar{q}_{3620} = \frac{6E_3 g_3}{(\ell_3 - k_3^*)^2 \bar{N}_{33}}$
$q_{3636} = 4E_3 g_3 \left[\frac{A_{31}}{k_3^* N_{31}} + \frac{A_{32}}{(\tilde{k}_3 - k_3^*) N_{32}}\right]$	$\bar{q}_{3636} = 4E_3 g_3 \left[\frac{\bar{A}_{33}}{(\ell_3 - k_3^*)\bar{N}_{33}} + \frac{\bar{A}_{32}}{(k_3^* - \tilde{k}_3)\bar{N}_{32}}\right]$
$q_{3638} = 6E_3 g_3 \left[\frac{1}{(k_3^*)^2 N_{31}} - \frac{1}{(\tilde{k}_3 - k_3^*)^2 N_{32}}\right]$	$\bar{q}_{3638} = 6E_3 g_3 \left[\frac{1}{(k_3^* - \tilde{k}_3)^2 \bar{N}_{32}} - \frac{1}{(\ell_3 - k_3^*)^2 \bar{N}_{33}}\right]$
$q_{37\text{-}9} = \frac{6E_1 g_1}{(k_1^*)^2 N_{11}}$	$\bar{q}_{3710} = -\frac{6E_1 g_1}{(\ell_1 - k_1^*)^2 \bar{N}_{13}}$
$q_{3717} = -\frac{12E_1 g_1}{(k_1^*)^3 N_{11}}$	$\bar{q}_{3718} = -\frac{12E_1 g_1}{(\ell_1 - k_1^*)^3 \bar{N}_{13}}$
$q_{3735} = 6E_1 g_1 \left[\frac{1}{(k_1^*)^2 N_{11}} - \frac{1}{(\tilde{k}_1 - k_1^*)^2 N_{12}}\right]$	$\bar{q}_{3735} = 6E_1 g_1 \left[\frac{1}{(k_1^* - \tilde{k}_1)^2 \bar{N}_{12}} - \frac{1}{(\ell_1 - k_1^*)^2 \bar{N}_{13}}\right]$
$q_{3737} = 12E_1 g_1 \left[\frac{1}{(\tilde{k}_1 - k_1^*)^3 N_{12}} + \frac{1}{(k_1^*)^3 N_{11}}\right]$	$\bar{q}_{3737} = 12E_1 g_1 \left[\frac{1}{(k_1^* - \tilde{k}_1)^3 \bar{N}_{12}} + \frac{1}{(\ell_1 - k_1^*)^3 \bar{N}_{13}}\right]$
$q_{3811} = \frac{6E_3 g_3}{(k_3^*)^2 N_{31}}$	$\bar{q}_{3812} = -\frac{6E_3 g_3}{(\ell_3 - k_3^*)^2 \bar{N}_{33}}$
$q_{3819} = -\frac{12E_3 g_3}{(k_3^*)^3 N_{31}}$	$\bar{q}_{3820} = -\frac{12E_3 g_3}{(\ell_3 - k_3^*)^3 \bar{N}_{33}}$
$q_{3836} = 6E_3 g_3 \left[\frac{1}{(k_3^*)^2 N_{31}} - \frac{1}{(\tilde{k}_3 - k_3^*)^2 N_{32}}\right]$	$\bar{q}_{3836} = 6E_3 g_3 \left[\frac{1}{(k_3^* - \tilde{k}_3)^2 \bar{N}_{32}} - \frac{1}{(\ell_3 - k_3^*)^2 \bar{N}_{33}}\right]$
$q_{3838} = 12E_3 g_3 \left[\frac{1}{(\tilde{k}_3 - k_3^*)^3 N_{32}} + \frac{1}{(k_3^*)^3 N_{31}}\right]$	$\bar{q}_{3838} = 12E_3 g_3 \left[\frac{1}{(k_3^* - \tilde{k}_3)^3 \bar{N}_{32}} + \frac{1}{(\ell_3 - k_3^*)^3 \bar{N}_{33}}\right]$
$q_{3939} = \tilde{k}_1$	$\bar{q}_{3939} = q_{3939}$
$q_{4040} = \tilde{k}_3$	$\bar{q}_{4040} = q_{4040}$

NONZERO ELEMENTS OF $\underset{\sim}{a}$ MATRIX (NONSYMMETRIC)

a_{in} ($\tilde{h} > h^\circ$) ($i=1,\dots 40;\ n=1,\dots 22$)	$a'_{i'n'}$ (empty) ($i'=1,\dots 24,\ n'=1,\dots 14$)	$\bar{a}_{in}$ ($\tilde{h} < h^\circ$)
$a_{3-1} = -K^{(3)}$	SAME AS a_{in}	SAME AS a_{in}
$a_{4-1} = -\bar{K}^{(3)}$	SAME AS a_{in}	SAME AS a_{in}
$a_{6-2} = -K^{(4)}$	SAME AS a_{in}	SAME AS a_{in}
$a_{7-2} = -\bar{K}^{(4)}$	SAME AS a_{in}	SAME AS a_{in}
$a_{9-3} = -\mathcal{S}_1$	SAME AS a_{in}	SAME AS a_{in}
$a_{9-19} = 0$	SAME AS a_{in}	$\sqrt{\ }\ \bar{a}_{9-19} = \frac{2E_1 \vartheta_1 \bar{B}_{11}}{\tilde{h}_1 \bar{N}_{11}}$
$a_{9-21} = 0$	SAME AS a_{in}	$\sqrt{\ }\ \bar{a}_{9-21} = \frac{6E_1 \vartheta_1}{(\tilde{h}_1)^2 \bar{N}_{11}}$
$a_{10-4} = -\mathcal{S}_2$	SAME AS a_{in}	$\bar{a}_{10-4} = a_{10-4}$
$a_{1019} = \frac{2E_1 \vartheta_1 B_{13}}{(l_1 - \tilde{h}_1) N_{13}}$	$a'_{1019} = 0$	$\bar{a}_{1019} = 0$
$a_{1021} = -\frac{6E_1 \vartheta_1}{(l_1 - \tilde{h}_1)^2 N_{13}}$	$a'_{1021} = 0$	$\bar{a}_{1021} = 0$
$a_{11-1} = -K^{(3)}(R_B + D_3)$	SAME AS a_{in}	$\bar{a}_{11-1} = a_{11-1}$
$a_{11-6} = -\mathcal{S}_3$	SAME AS a_{in}	$\bar{a}_{11-6} = a_{11-6}$
$a_{1120} = 0$	SAME AS a_{in}	$\bar{a}_{1120} = \frac{2E_3 \vartheta_3 \bar{B}_{31}}{\tilde{h}_3 \bar{N}_{31}}$
$a_{1122} = 0$	SAME AS a_{in}	$\bar{a}_{1122} = \frac{6E_3 \vartheta_3}{(\tilde{h}_2)^2 \bar{N}_{31}}$
$a_{12-1} = -\bar{K}^{(3)}(R_B + D_3)$	SAME AS a_{in}	$\bar{a}_{12-1} = a_{12-1}$
$a_{12-8} = -\mathcal{S}_4$	SAME AS a_{in}	$\bar{a}_{12-8} = a_{12-8}$
$a_{1220} = \frac{2E_3 \vartheta_3 B_{33}}{(l_3 - \tilde{h}_3) N_{33}}$	$a'_{1220} = 0$	$\bar{a}_{1220} = 0$
$a_{1222} = -\frac{6E_3 \vartheta_3}{(l_3 - \tilde{h}_3)^2 N_{33}}$	$a'_{1222} = 0$	$\bar{a}_{1222} = 0$
$a_{1312} = -K^{(5)} e$	SAME AS a_{in}	$\bar{a}_{1312} = a_{1312}$
$a_{1315} = 0$	SAME AS a_{in}	$\sqrt{\ }\ \bar{a}_{1315} = \frac{2E_5 \vartheta_5 \bar{B}_{51}}{\tilde{h}_5 \bar{N}_{51}}$
$a_{1317} = 0$	SAME AS a_{in}	$\sqrt{\ }\ \bar{a}_{1317} = \frac{6E_5 \vartheta_5}{(\tilde{h}_5)^2 \bar{N}_{51}}$

a_{in}	$a'_{i'n'}$	$\bar{a}_{in}$
$a_{14-2} = K^{(4)}(R_0 + D_4)$	SAME AS a_{in}	$\bar{a}_{14-2} = a_{14-2}$
$a_{14-9} = -\mathcal{S}_5$	SAME AS a_{in}	$\bar{a}_{14-9} = a_{14-9}$
$a_{1415} = \frac{2E_5 \vartheta_5 B_{53}}{(l_5 - \tilde{h}_5) N_{53}}$	$a'_{1415} = 0$	$\bar{a}_{1415} = 0$
$a_{1417} = -\frac{6E_5 \vartheta_5}{(l_5 - \tilde{h}_5)^2 N_{53}}$	$a'_{1417} = 0$	$\bar{a}_{1417} = 0$
$a_{15-2} = \bar{K}^{(4)}(R_0 + D_4)$	SAME AS a_{in}	$\bar{a}_{15-2} = a_{15-2}$
$a_{1511} = -\mathcal{S}_7$	SAME AS a_{in}	$\bar{a}_{1511} = a_{1511}$
$a_{1516} = 0$	SAME AS a_{in}	$\bar{a}_{1516} = \frac{2E_7 \vartheta_7 \bar{B}_{71}}{\tilde{h}_7 \bar{N}_{71}}$
$a_{1518} = 0$	SAME AS a_{in}	$\bar{a}_{1518} = \frac{6E_7 \vartheta_7}{(\tilde{h}_7)^2 \bar{N}_{71}}$
$a_{1616} = \frac{2E_7 \vartheta_7 B_{73}}{(l_7 - \tilde{h}_7) N_{73}}$	$a'_{1616} = 0$	$\bar{a}_{1616} = 0$
$a_{1618} = -\frac{6E_7 \vartheta_7}{(l_7 - \tilde{h}_7)^2 N_{73}}$	$a'_{1618} = 0$	$\bar{a}_{1618} = 0$
$a_{17-3} = -\frac{6E^{(1)}J^{(1)}}{l_1^2}$	SAME AS a_{in}	$\bar{a}_{17-3} = a_{17-3}$
$a_{17-4} = -\frac{6E^{(1)}J^{(1)}}{l_1^2}$	SAME AS a_{in}	$\bar{a}_{17-4} = a_{17-4}$
$a_{1719} = 0$	SAME AS a_{in}	$\bar{a}_{1719} = -\frac{6E_1 \vartheta_1}{(\tilde{h}_1)^2 \bar{N}_{11}}$
$a_{1721} = 0$	SAME AS a_{in}	$\bar{a}_{1721} = -\frac{12E_1 \vartheta_1}{(\tilde{h}_1)^3 \bar{N}_{11}}$
$a_{18-3} = \frac{6E^{(1)}J^{(1)}}{l_1^2}$	SAME AS a_{in}	SAME AS a_{in}
$a_{18-4} = 6\left[\frac{E^{(1)}J^{(1)}}{l_1^2} - \frac{E^{(2)}J^{(2)}}{a^2}\right]$	SAME AS a_{in}	SAME AS a_{in}
$a_{18-5} = -\frac{6E^{(2)}J^{(2)}}{a^2}$	SAME AS a_{in}	SAME AS a_{in}
$a_{1812} = -\frac{12E^{(2)}J^{(2)}}{a^3}$	SAME AS a_{in}	SAME AS a_{in}
$a_{1819} = \frac{6E_1 \vartheta_1}{(l_1 - \tilde{h}_1)^2 N_{13}}$	$a'_{1819} = 0$	$\bar{a}_{1819} = 0$
$a_{1821} = -\frac{12E_1 \vartheta_1}{(l_1 - \tilde{h}_1)^3 N_{13}}$	$a'_{1821} = 0$	$\bar{a}_{1821} = 0$

a_{in}	$a'_{i'n'}$	$\bar{a}_{in}$
$a_{19-5} = \frac{6E^{(2)}J^{(2)}}{(l_2 - a)^2}$	SAME AS a_{in}	SAME AS a_{in}
$a_{19-6} = 6\left[\frac{E^{(2)}J^{(2)}}{(l_2 - a)^2} - \frac{E^{(3)}J^{(3)}}{c^2}\right]$	SAME AS a_{in}	SAME AS a_{in}
$a_{19-7} = -\frac{6E^{(3)}J^{(3)}}{c^2}$	SAME AS a_{in}	SAME AS a_{in}
$a_{1912} = -\frac{12E^{(2)}J^{(2)}}{(l_2 - a)^3}$	SAME AS a_{in}	SAME AS a_{in}
$a_{1913} = -\frac{12E^{(3)}J^{(3)}}{c^3}$	SAME AS a_{in}	SAME AS a_{in}
$a_{1920} = 0$	SAME AS a_{in}	$\bar{a}_{1920} = -\frac{6E_3 \vartheta_3}{(\tilde{h}_3)^2 \bar{N}_{31}}$
$a_{1922} = 0$	SAME AS a_{in}	$\bar{a}_{1922} = -\frac{12E_3 \vartheta_3}{(\tilde{h}_3)^3 \bar{N}_{31}}$
$a_{20-7} = \frac{6E^{(3)}J^{(3)}}{(l_3 - c)^2}$	SAME AS a_{in}	SAME AS a_{in}
$a_{20-8} = \frac{6E^{(3)}J^{(3)}}{(l_3 - c)^2}$	SAME AS a_{in}	SAME AS a_{in}
$a_{2018} = -\frac{12E^{(3)}J^{(3)}}{(l_3 - c)^3}$	SAME AS a_{in}	SAME AS a_{in}
$a_{2020} = \frac{6E_3 \vartheta_3}{(l_3 - \tilde{h}_3)^2 N_{33}}$	$a'_{2020} = 0$	$\bar{a}_{2020} = 0$
$a_{2022} = -\frac{12E_3 \vartheta_3}{(l_3 - \tilde{h}_3)^3 N_{33}}$	$a'_{2022} = 0$	$\bar{a}_{2022} = 0$
$a_{2112} = -K^{(5)}$	SAME AS a_{in}	$\bar{a}_{2112} = a_{2112}$
$a_{22-9} = -\frac{6E^{(4)}J^{(4)}}{d^2}$	SAME AS a_{in}	$\bar{a}_{2115} = -6E_5 \vartheta_5 / [(\tilde{h}_3)^2 \bar{N}_{51}]$
$a_{2210} = -\frac{6E^{(4)}J^{(4)}}{d^2}$	SAME AS a_{in}	$\bar{a}_{2117} = -12E_5 \vartheta_5 / [(\tilde{h}_5)^3 \bar{N}_{51}]$
$a_{2214} = -\frac{12E^{(4)}J^{(4)}}{d^3}$	SAME AS a_{in}	SAME AS a_{in}
$a_{2215} = \frac{6E_5 \vartheta_5}{(l_5 - \tilde{h}_5)^2 N_{53}}$	$a'_{2215} = 0$	$\bar{a}_{2215} = 0$
$a_{2217} = -\frac{12E_5 \vartheta_5}{(l_5 - \tilde{h}_5)^3 N_{53}}$	$a'_{2217} = 0$	$\bar{a}_{2217} = 0$
$a_{2310} = \frac{6E^{(4)}J^{(4)}}{(l_6 - d)^2}$	SAME AS a_{in}	SAME AS a_{in}
$a_{2311} = \frac{6E^{(4)}J^{(4)}}{(l_6 - d)^2}$	SAME AS a_{in}	SAME AS a_{in}

NONZERO ELEMENTS OF $\underset{\sim}{a}$ MATRIX (NONSYMMETRIC)

a_{ik}	$a'_{i'n'}$	$\bar{a}_{ik}$	a_{ik}	$\bar{a}_{ik}$
$a_{2314} = -\frac{12E^{(4)}J^{(4)}}{(l_6-d)^3}$	↓	$\bar{a}_{2314} = a_{7314}$	$a_{3622} = \frac{6E_3J_3}{(\tilde{h}_3-h_3^*)^2N_{32}}$	$\bar{a}_{3622} = -\frac{6E_3J_3}{(h_3^*-\tilde{h}_3)^2\bar{N}_{32}}$
$a_{2316} = 0$	End of $a'_{i'n'}$	$\bar{a}_{2316} = -\frac{6E_7J_7}{(\tilde{h}_7)^2\bar{N}_{71}}$	$a_{3719} = -\frac{6E_1J_1}{(\tilde{h}_1-h_1^*)^2N_{12}}$	$\bar{a}_{3719} = \frac{6E_1J_1}{(h_1^*-\tilde{h}_1)^2\bar{N}_{12}}$
$a_{2318} = 0$		$\bar{a}_{2318} = -\frac{12E_7J_7}{(\tilde{h}_7)^3\bar{N}_{71}}$	$a_{3721} = -\frac{12E_1J_1}{(\tilde{h}_1-h_1^*)^3N_{12}}$	$\bar{a}_{3721} = -\frac{12E_1J_1}{(h_1^*-\tilde{h}_1)^3\bar{N}_{12}}$
$a_{2416} = \frac{6E_7J_7}{(l_7-\tilde{h}_7)^2N_{73}}$	$a'_{2416} = 0$	$\bar{a}_{2416} = 0$	$a_{3820} = -\frac{6E_3J_3}{(\tilde{h}_3-h_3^*)^2N_{32}}$	$\bar{a}_{3820} = \frac{6E_3J_3}{(h_3^*-\tilde{h}_3)^2\bar{N}_{32}}$
$a_{2418} = -\frac{12E_7J_7}{(l_7-\tilde{h}_7)^3N_{73}}$	$a'_{2418} = 0$	$\bar{a}_{2418} = 0$	$a_{3822} = -\frac{12E_3J_3}{(\tilde{h}_3-h_3^*)^3N_{32}}$	$\bar{a}_{3822} = -\frac{12E_3J_3}{(h_3^*-\tilde{h}_3)^3\bar{N}_{32}}$
$a_{2715} = \frac{2E_5J_5B_{52}}{(\tilde{h}_6-h_5^*)N_{52}}$		$\bar{a}_{2715} = \frac{2E_5J_5\bar{B}_{52}}{(h_6^*-\tilde{h}_5)\bar{N}_{52}}$	$a_{3921} = -\tilde{k}_1$	$\bar{a}_{3921} = a_{3921}$
$a_{2717} = \frac{6E_5J_5}{(\tilde{h}_5-h_5^*)^2N_{52}}$		$\bar{a}_{2717} = -\frac{6E_5J_5}{(h_5^*-\tilde{h}_5)^2\bar{N}_{52}}$	$a_{4022} = -\tilde{k}_3$	$\bar{a}_{4022} = a_{4022}$
$a_{2816} = \frac{2E_7J_7B_{72}}{(\tilde{h}_7-h_7^*)N_{72}}$		$\bar{a}_{2816} = \frac{2E_7J_7\bar{B}_{72}}{(h_7^*-\tilde{h}_7)\bar{N}_{72}}$		
$a_{2818} = \frac{6E_7J_7}{(\tilde{h}_7-h_7^*)^2N_{72}}$		$\bar{a}_{2818} = -\frac{6E_7J_7}{(h_7^*-\tilde{h}_7)^2\bar{N}_{72}}$		
$a_{2915} = -\frac{6E_5J_5}{(\tilde{h}_5-h_5^*)^2N_{52}}$		$\bar{a}_{2915} = \frac{6E_5J_5}{(h_5^*-\tilde{h}_5)^2\bar{N}_{52}}$		
$a_{2917} = -\frac{12E_5J_5}{(\tilde{h}_6-h_5^*)^3N_{52}}$		$\bar{a}_{2917} = -\frac{12E_5J_5}{(h_5^*-\tilde{h}_5)^3\bar{N}_{52}}$		
$a_{3016} = -\frac{6E_7J_7}{(\tilde{h}_7-h_7^*)^2N_{72}}$		$\bar{a}_{3016} = \frac{6E_7J_7}{(h_7^*-\tilde{h}_7)^2\bar{N}_{72}}$		
$a_{3018} = -\frac{12E_7J_7}{(\tilde{h}_7-h_7^*)^3N_{72}}$		$\bar{a}_{3018} = -\frac{12E_7J_7}{(h_7^*-\tilde{h}_7)^3\bar{N}_{72}}$		
$a_{3117} = -\tilde{k}_5$		$\bar{a}_{3117} = a_{3117}$		
$a_{3218} = -\tilde{k}_7$		$\bar{a}_{3218} = a_{3218}$		
$a_{3519} = \frac{2E_1J_1B_{12}}{(\tilde{h}_1-h_1^*)N_{12}}$		$\bar{a}_{3519} = \frac{2E_1J_1\bar{B}_{12}}{(h_1^*-\tilde{h}_1)\bar{N}_{12}}$		
$a_{3521} = \frac{6E_1J_1}{(\tilde{h}_1-h_1^*)^2N_{12}}$		$\bar{a}_{3521} = -\frac{6E_1J_1}{(h_1^*-\tilde{h}_1)^2\bar{N}_{12}}$		
$a_{3620} = \frac{2E_3J_3B_{32}}{(\tilde{h}_3-h_3^*)N_{32}}$		$\bar{a}_{3620} = \frac{2E_3J_3\bar{B}_{32}}{(h_3^*-\tilde{h}_3)\bar{N}_{32}}$		

NONZERO ELEMENTS OF $\underset{\sim}{C}$ MATRIX (SYMMETRIC)

$C_{n\ell}$ $(\hat{h}_i > h_i^*)$ $(n, \ell = 1, \cdots 22)$	$\overline{C}_{n\ell}$ $(\hat{h}_i < h_i^*)$	$C_{n\ell}$	$\overline{C}_{n\ell}$
$C_{11} = K^{(3)} + \bar{K}^{(3)} + K^{(6)}$	SAME AS $C_{n\ell}$	$C_{77} = S_6 + \frac{4E^{(3)}J^{(3)}\ell_3}{c(\ell_3 - c)}$	SAME AS $C_{n\ell}$
$C_{12} = -K^{(6)}$		$C_{78} = \frac{2E^{(3)}J^{(3)}}{\ell_3 - c}$	
$C_{21} = -K^{(6)}$		$C_{7-10} = -S_6$	
$C_{22} = K^{(4)} + \bar{K}^{(4)} + K^{(6)}$		$C_{7-13} = 6E^{(3)}J^{(3)}\left[\frac{1}{c^2} - \frac{1}{(\ell_3 - c)^2}\right]$	
$C_{33} = S_1 + \frac{4E^{(1)}J^{(1)}}{\ell_1}$		$C_{87} = \frac{2E^{(3)}J^{(3)}}{\ell_3 - c}$	
$C_{34} = \frac{2E^{(1)}J^{(1)}}{\ell_1}$		$C_{88} = S_4 + \frac{4E^{(3)}J^{(3)}}{\ell_3 - c}$	
$C_{43} = \frac{2E^{(1)}J^{(1)}}{\ell_1}$		$C_{8-13} = -\frac{6E^{(3)}J^{(3)}}{(\ell_3 - c)^2}$	
$C_{44} = S_2 + 4\left[\frac{E^{(1)}J^{(1)}}{\ell_1} + \frac{E^{(2)}J^{(2)}}{a}\right]$		$C_{99} = S_5 + \frac{4E^{(4)}J^{(4)}}{d}$	
$C_{45} = \frac{2E^{(2)}J^{(2)}}{a}$		$C_{9-10} = \frac{2E^{(4)}J^{(4)}}{d}$	
$C_{4-12} = \frac{6E^{(2)}J^{(2)}}{a^2}$		$C_{9-14} = \frac{6E^{(4)}J^{(4)}}{d^2}$	
$C_{54} = \frac{2E^{(2)}J^{(2)}}{a}$		$C_{10-7} = -S_6$	
$C_{55} = \frac{4E^{(2)}J^{(2)}\ell_2}{a(\ell_2 - a)}$		$C_{10-9} = \frac{2E^{(4)}J^{(4)}}{d}$	
$C_{56} = \frac{2E^{(2)}J^{(2)}}{\ell_2 - a}$		$C_{1010} = S_6 + \frac{4E^{(4)}J^{(4)}\ell_6}{d(\ell_6 - d)}$	
$C_{5-12} = 6E^{(2)}J^{(2)}\left[\frac{1}{a^2} - \frac{1}{(\ell_2 - a)^2}\right]$		$C_{1011} = \frac{2E^{(4)}J^{(4)}}{\ell_6 - d}$	
$C_{65} = \frac{2E^{(2)}J^{(2)}}{\ell_2 - a}$		$C_{1014} = 6E^{(4)}J^{(4)}\left[\frac{1}{d^2} - \frac{1}{(\ell_6 - d)^2}\right]$	
$C_{66} = S_3 + 4\left[\frac{E^{(2)}J^{(2)}}{\ell_2 - a} + \frac{E^{(3)}J^{(3)}}{c}\right]$		$C_{1110} = \frac{2E^{(4)}J^{(4)}}{\ell_6 - d}$	
$C_{67} = \frac{2E^{(3)}J^{(3)}}{c}$		$C_{1111} = S_7 + \frac{4E^{(4)}J^{(4)}}{\ell_6 - d}$	
$C_{6-12} = -\frac{6E^{(2)}J^{(2)}}{(\ell_2 - a)^2}$		$C_{1114} = -\frac{6E^{(4)}J^{(4)}}{(\ell_6 - d)^2}$	
$C_{6-13} = \frac{6E^{(3)}J^{(3)}}{c^2}$			
$C_{76} = \frac{2E^{(3)}J^{(3)}}{c}$			

NONZERO ELEMENTS OF $\underset{\sim}{C}$ MATRIX (SYMMETRIC)

$C_{n\ell}$	$\bar{C}_{n\ell}$
$C_{12-4} = \frac{6E^{(2)}J^{(2)}}{a^2}$	SAME AS $C_{n\ell}$
$C_{12-5} = 6E^{(2)}J^{(2)}\left[\frac{1}{a^2} - \frac{1}{(\ell_2-a)^2}\right]$	
$C_{12-6} = -\frac{6E^{(2)}J^{(2)}}{(\ell_2-a)^2}$	
$C_{1212} = K^{(6)} + 12E^{(2)}J^{(2)}\left[\frac{1}{a^3} + \frac{1}{(\ell_2-a)^3}\right]$	
$C_{13-6} = \frac{6E^{(3)}J^{(3)}}{c^2}$	
$C_{13-7} = 6E^{(3)}J^{(3)}\left[\frac{1}{c^2} - \frac{1}{(\ell_3-c)^2}\right]$	
$C_{13-8} = -\frac{6E^{(3)}J^{(3)}}{(\ell_3-c)^2}$	
$C_{1313} = K^{(7)} + 12E^{(3)}J^{(3)}\left[\frac{1}{c^3} + \frac{1}{(\ell_3-c)^3}\right]$	
$C_{1314} = -K^{(7)}$	
$C_{14-9} = \frac{6E^{(4)}J^{(4)}}{d^2}$	
$C_{1410} = 6E^{(4)}J^{(4)}\left[\frac{1}{d^2} - \frac{1}{(\ell_6-d)^2}\right]$	
$C_{1411} = -\frac{6E^{(4)}J^{(4)}}{(\ell_6-d)^2}$	
$C_{1413} = -K^{(7)}$	
$C_{1414} = K^{(7)} + 12E^{(4)}J^{(4)}\left[\frac{1}{d^3} + \frac{1}{(\ell_6-d)^3}\right]$	
$C_{1515} = 4E_5\vartheta_5\left[\frac{A_{52}}{(\tilde{h}_5-h_5^*)N_{52}} + \frac{A_{53}}{(\ell_5-\tilde{h}_5)N_{53}}\right]$	$\bar{C}_{1515} = 4E_5\vartheta_5\left[\frac{\bar{A}_{52}}{(h_5^*-\tilde{h}_5)\bar{N}_{52}} + \frac{\bar{A}_{51}}{\tilde{h}_5\bar{N}_{51}}\right]$
$C_{1517} = 6E_5\vartheta_5\left[\frac{1}{(\tilde{h}_5-h_5^*)^2_{N_{52}}} - \frac{1}{(\ell_5-\tilde{h}_5)^2_{N_{53}}}\right]$	$\bar{C}_{1517} = 6E_5\vartheta_5\left[\frac{1}{(\tilde{h}_5)^2_{\bar{N}_{51}}} - \frac{1}{(h_5^*-\tilde{h}_5)^2_{\bar{N}_{52}}}\right]$
$C_{1616} = 4E_7\vartheta_7\left[\frac{A_{72}}{(\tilde{h}_7-h_7^*)N_{72}} + \frac{A_{73}}{(\ell_7-\tilde{h}_7)N_{73}}\right]$	$\bar{C}_{1616} = 4E_7\vartheta_7\left[\frac{\bar{A}_{72}}{(h_7^*-\tilde{h}_7)\bar{N}_{72}} + \frac{\bar{A}_{71}}{\tilde{h}_7\bar{N}_{71}}\right]$
$C_{1618} = 6E_7\vartheta_7\left[\frac{1}{(\tilde{h}_7-h_7^*)^2_{N_{72}}} - \frac{1}{(\ell_7-\tilde{h}_7)^2_{N_{73}}}\right]$	$\bar{C}_{1618} = 6E_7\vartheta_7\left[\frac{1}{(\tilde{h}_7)^2_{\bar{N}_{71}}} - \frac{1}{(h_7^*-\tilde{h}_7)^2_{\bar{N}_{72}}}\right]$

$C_{n\ell}$	$\bar{C}_{n\ell}$
$C_{1715} = 6E_5\vartheta_5\left[\frac{1}{(\tilde{h}_5-h_5^*)^2_{N_{52}}} - \frac{1}{(\ell_5-\tilde{h}_5)^2_{N_{53}}}\right]$	$\bar{C}_{1715} = 6E_5\vartheta_5\left[\frac{1}{(\tilde{h}_5)^2_{\bar{N}_{51}}} - \frac{1}{(h_5^*-\tilde{h}_5)^2_{\bar{N}_{52}}}\right]$
$C_{1717} = \tilde{k}_5 + 12E_5\vartheta_5\left[\frac{1}{(\ell_5-\tilde{h}_5)^3_{N_{53}}} + \frac{1}{(\tilde{h}_5-h_5^*)^3_{N_{52}}}\right]$	$\bar{C}_{1717} = \tilde{k}_5 + 12E_5\vartheta_5\left[\frac{1}{(\tilde{h}_5)^3_{\bar{N}_{51}}} + \frac{1}{(h_5^*-\tilde{h}_5)^3_{\bar{N}_{52}}}\right]$
$C_{1816} = 6E_7\vartheta_7\left[\frac{1}{(\tilde{h}_7-h_7^*)^2_{N_{72}}} - \frac{1}{(\ell_7-\tilde{h}_7)^2_{N_{73}}}\right]$	$\bar{C}_{1816} = 6E_7\vartheta_7\left[\frac{1}{(\tilde{h}_7)^2_{\bar{N}_{71}}} - \frac{1}{(h_7^*-\tilde{h}_7)^2_{\bar{N}_{72}}}\right]$
$C_{1818} = \tilde{k}_7 + 12E_7\vartheta_7\left[\frac{1}{(\ell_7-\tilde{h}_7)^3_{N_{73}}} + \frac{1}{(\tilde{h}_7-h_7^*)^3_{N_{72}}}\right]$	$\bar{C}_{1818} = \tilde{k}_7 + 12E_7\vartheta_7\left[\frac{1}{(\tilde{h}_7)^3_{\bar{N}_{71}}} + \frac{1}{(h_7^*-\tilde{h}_7)^3_{\bar{N}_{72}}}\right]$
$C_{1919} = 4E_1\vartheta_1\left[\frac{A_{12}}{(\tilde{h}_1-h_1^*)N_{12}} + \frac{A_{13}}{(\ell_1-\tilde{h}_1)N_{13}}\right]$	$\bar{C}_{1919} = 4E_1\vartheta_1\left[\frac{\bar{A}_{12}}{(h_1^*-\tilde{h}_1)\bar{N}_{12}} + \frac{\bar{A}_{11}}{\tilde{h}_1\bar{N}_{11}}\right]$
$C_{1921} = 6E_1\vartheta_1\left[\frac{1}{(\tilde{h}_1-h_1^*)^2_{N_{12}}} - \frac{1}{(\ell_1-\tilde{h}_1)^2_{N_{13}}}\right]$	$\bar{C}_{1921} = 6E_1\vartheta_1\left[\frac{1}{(\tilde{h}_1)^2_{\bar{N}_{11}}} - \frac{1}{(h_1^*-\tilde{h}_1)^2_{\bar{N}_{12}}}\right]$
$C_{2020} = 4E_3\vartheta_3\left[\frac{A_{32}}{(\tilde{h}_3-h_3^*)N_{32}} + \frac{A_{33}}{(\ell_3-\tilde{h}_3)N_{33}}\right]$	$\bar{C}_{2020} = 4E_3\vartheta_3\left[\frac{\bar{A}_{32}}{(h_3^*-\tilde{h}_3)\bar{N}_{32}} + \frac{\bar{A}_{31}}{\tilde{h}_3\bar{N}_{31}}\right]$
$C_{2022} = 6E_3\vartheta_3\left[\frac{1}{(\tilde{h}_3-h_3^*)^2_{N_{32}}} - \frac{1}{(\ell_3-\tilde{h}_3)^2_{N_{33}}}\right]$	$\bar{C}_{2022} = 6E_3\vartheta_3\left[\frac{1}{(\tilde{h}_3)^2_{\bar{N}_{31}}} - \frac{1}{(h_3^*-\tilde{h}_3)^2_{\bar{N}_{32}}}\right]$
$C_{2119} = 6E_1\vartheta_1\left[\frac{1}{(\tilde{h}_1-h_1^*)^2_{N_{12}}} - \frac{1}{(\ell_1-\tilde{h}_1)^2_{N_{13}}}\right]$	$\bar{C}_{2119} = 6E_1\vartheta_1\left[\frac{1}{(\tilde{h}_1)^2_{\bar{N}_{11}}} - \frac{1}{(h_1^*-\tilde{h}_1)^2_{\bar{N}_{12}}}\right]$
$C_{2121} = \tilde{k}_1 + 12E_1\vartheta_1\left[\frac{1}{(\ell_1-\tilde{h}_1)^3_{N_{13}}} + \frac{1}{(\tilde{h}_1-h_1^*)^3_{N_{12}}}\right]$	$\bar{C}_{2121} = \tilde{k}_1 + 12E_1\vartheta_1\left[\frac{1}{(\tilde{h}_1)^3_{\bar{N}_{11}}} + \frac{1}{(h_1^*-\tilde{h}_1)^3_{\bar{N}_{12}}}\right]$
$C_{2220} = 6E_3\vartheta_3\left[\frac{1}{(\tilde{h}_3-h_3^*)^2_{N_{32}}} - \frac{1}{(\ell_3-\tilde{h}_3)^2_{N_{33}}}\right]$	$\bar{C}_{2220} = 6E_3\vartheta_3\left[\frac{1}{(\tilde{h}_3)^2_{\bar{N}_{31}}} - \frac{1}{(h_3^*-\tilde{h}_3)^2_{\bar{N}_{32}}}\right]$
$C_{2222} = \tilde{k}_3 + 12E_3\vartheta_3\left[\frac{1}{(\ell_3-\tilde{h}_3)^3_{N_{33}}} + \frac{1}{(\tilde{h}_3-h_3^*)^3_{N_{32}}}\right]$	$\bar{C}_{2222} = \tilde{k}_3 + 12E_3\vartheta_3\left[\frac{1}{(\tilde{h}_3)^3_{\bar{N}_{31}}} + \frac{1}{(h_3^*-\tilde{h}_3)^3_{\bar{N}_{32}}}\right]$

APPENDIX C

COMPUTER PROGRAM

The computer program consists of a main program SHUTTLE together with eight function and subroutine subprograms. The purpose of SHUTTLE is to accept input data, compute certain constant data, generate and manipulate the requisite matrices and set up the final matrix for the computation of eigenvalues and ordered printout of frequencies. SHUTTLE has one basic option, viz., whether the Booster and Orbiter tanks are all empty or whether all tanks contain liquid. The input data required of SHUTTLE are listed below:

Mathematical Symbol	FORTRAN Symbol	Meaning
ρ_i , i = 1, 3, 5, 7	RHO(I) , I = 1, 3, 5, 7	Density of liquids in each of the four tanks
a_s	AS	Constant, = 4.375 (see footnote)
h_i , i = 1, 3, 5, 7	HI(I) , I = 1, 3, 5, 7	Height of liquids in each of the four tanks
K_1	KCAP1	Longitudinal spring constant of lower Booster system, = 3.48E+05
--	TANK	Tank code, = 0.0 if all tanks empty; = 1.0 if all tanks contain liquid
E_s	ES	Coupling parameter, = 0.0 for uncoupled Booster/Orbiter; = 3.0E+07 for coupled Booster/Orbiter

footnote: a_s is the distance from the lower support of the strongback to the location of the coupling spring.

The input order is RHO(I), AS, HI(I), KCAP1, TANK, ES. The format is (8F10. 0), so that two cards per case are required.

The names and purposes of the eight subprograms are given below:

FORTRAN Name	Purpose
SINCH	Computes sinh(x)
COSCH	Computes cosh(x)
MMATRX	Multiplies two matrices AA and BB and stores result in CC
MPRINT	Prints matrices (call to MPRINT optional)
BESNIS	Computes $I_0(x)$ and $I_1(x)$
SRTCYC	Sorts eigenvalues in ascending order and converts to Hz
EIGEN	Computes eigenvalues (and, optionally, eigenvectors) of real nonsymmetric matrix
MATINV	Computes inverse of square matrix

```
                PROGRAM SHUTTLE (INPUT,OUTPUT,TAPE6=INPUT)
00003           REAL  NNN,NNN2,NNN6
00003           REAL I1,I2,I3,I5,I6,I7,L1,L2,L3,L5,L6,L7,LS,JUP1,JUP2,JUP3,JUP4,
               *IS,LL,MM,MT,MTDA,JSTR,JRIG,MSTR,KCAP1,KCAP2,KCAP3,
               3        KCAP5,KCAP6,KCAP7,KUP1,KUP2,KUP3,KUP4,KUP5,KUP6,KUP7,KBUP3,
               4        KBUP4,K1,K3,K5,K7,KB1,KB3,KB5,KB7,KT1,KT3,KT5,KT7,NU,KP,KCB,
               5        NUST,NUAL,MASS,
               *        LW,KR,KW
          C****
00003           DIMENSION AAA(7,3),BETA(7,3),NNN(7,3),BBB(7,3)
00003           DIMENSION CYC(40,2)
00003           DIMENSION LL(7),DEL(7),NU(7),EE(7),RHO(7),HI(7),QARR(40,40),
               1             MASS(40,40),F(7),ANS(50),T1(40),T(40),PDUCT(40,40),R(7),
               *KCB(7),MSTR(7),MTDA(7),JSTR(7),JRIG(7),MT(7),MM(7),KP(7),
               *             DARR(40,40),ARR(40,40),CARR(40,40)
00003       900 FORMAT(8F10.0)
00003       500 FORMAT(1H039X,10HINPUT DATA//5X,
               *             * RHO1 =*E13.4,4X,* RHO3 =*E13.4,4X,* RHO5 =*E13.4,4X,
               *             * RHO7 =*E13.4,/5X,
               *             * H1   =*E13.4,4X,* H3   =*E13.4,4X,* H5   =*E13.4,4X,
               *             * H7   =*E13.4,/5X,
               *             * AS   =*E13.4,4X,* KCAP1=*E13.4,4X,* ES   =*E13.4)
00003       550 FORMAT(1H1)
00003       600 FORMAT(1H038X,14HROW  FREQUENCY//
               *      40(39X,F2.0,1X,E10.2,/))
00003        50 CONTINUE
          C****       ZERO ALL ELEMENTS
00003           JC=40
00004           JR=40
00005           DO 5 I=1,JR
00007           DO 5 J=1,JC
00010           ARR(I,J)=0.0
00013           CARR(I,J)=0.0
00016           DARR(I,J)=0.0
00021           MASS(I,J)= 0.0
00024           PDUCT(I,J) =0.0
00027           QARR(I,J)= 0.0
00032         5 CONTINUE
          C
          C****       OPTIONS  DESIRED  FROM  EIGENVALUE  SUBROUTINE
          C
00037           MXX=40
00040           IJ=JI=22
00042           TOL=1.E-5
00043           PI = 3.1415927
00045           N1OPT= 0
00046           N2OPT= 0
00047           MAXIT=50
00050           NDEC = 2
          C
          C     DATA  STATEMENTS  FOR  PROGRAM  CONSTANTS
          C****
00051           DATA  GST,GAL/1.2E7,4.E6/
00051           DATA  AW,TO,EW,LW/0.0,1.0,2.9E7,7./
00051           DATA  A,B,C,D,E/2.25,5.0,7.875,3.0,1.0/
00051           DATA  R0,RB,D1,D2,D3,D4/3.0,5.0,1.125,1.5,1.125,0.375/
00051           DATA  E1,E2,E3,E5,E6,E7/2.9E7,1.E7,2.9E7,3*1.E7/
```

```
      DATA  EUP1,EUP2,EUP3,EUP4,EUP5/3*1.E7,2*2.9E7/
      DATA  I1,I2,I3,I5,I6,I7/3.487,10.59,1.963,.84577,1.40934,0.84577/
      DATA  L1,L2,L3,L5,L6,L7,LS/15.,7.,15.,8.5,6.,8.5,8.75/
      DATA  JUP1,JUP2,JUP3,JUP4/2.175E-1,3.060E-2,2.175E-1,1.020E-2/
      DATA  AUP1,AUP2,AUP3/.6875,.9375,.6875/
      DATA  AUP4,AUP5,ACAPS/.3125,0.000,.0976/
      DATA  NUAL,NUST    /.36,.26/
      DATA IS,G,S1,S2,S3/7.94E-4,386.4,3*8.67E4/
      DATA S4,S5,S7/8.67E4,2*2.60E4/
      DATA (R(I),I=1,7,2)/2*5.0,2*3.0/
      DATA (LL(I),I=1,7)/15.,7.,15.,0.,8.5,6.,8.5/
      DATA (NU(I),I=1,7)/0.,0.,.26,0.,.36,0.,.36/
      DATA (DEL(I),I=1,7)/5.E-3,2.5E-2,5.E-3,0.0,9.84E-3,2.E-2,9.84E-3/
      DATA (EE(I),I=1,7)/0.,0.,2.9E7,0.,1.E7,0.,1.E7/
C
C****       READ IN THE CASE CONSTANTS
      READ 900,(RHO(I),I=1,7,2),AS,(HI(I),I=1,7,2),KCAP1,TANK,ES
      IF(EOF,6)702,55
   55 CONTINUE
      CALL SECOND(X)
      IF(TANK.NE.0.0)GO TO 3001
      DO 905 I=3,7,2
  905 KCB(I)=2.*PI*R(I)*EE(I)*DEL(I)/LL(I)
      KCB(1)=KCAP1
      KCAP2=7.36E5
      KCAP3=KCB(3)
      KCAP5=KCB(5)
      KCAP6 = 2.*PI*R0*E6*DEL(6)/L6*.30
      KCAP7=KCB(7)
      K1=K3=K5=K7=0.0
      KT1=KT3=KT5=KT7=0.0
      KB1=KB3=KB5=KB7=0.0
      HT1=HT3=HT5=HT7=0.0
      HST1=L1
      HST3=L3
      HST5=L5
      HST7=L7
      JR=JC=24
      JI=14
 3001 CONTINUE
C****                  COMPUTED  DATA
      KR    = AUP5*EUP5/B
      KW    = (TO*AW*EW)/LW
      KUP1  = (AUP1*EUP1)/L1        *.4
      KUP2  = (AUP2*EUP2)/L2        *.4
      KUP3  = (AUP3*EUP3)/C         *.4
      KBUP3= (AUP3*EUP3)/(L3-C)    *.4
      KUP4  = (AUP4*EUP4)/D         *.4
      KBUP4= (AUP4*EUP4)/(L6-D)    *.4
      KUP5  = KR*KW/(KR+KW)
      KUP6  =  ACAPS*ES*LS/(AS*(LS-AS)) *.2
      KUP7  = 3.*ES*IS*LS**3/(AS**3*(LS-AS)**3) *1.46
      S6    = 4.*ES*IS*LS/(AS*(LS-AS)) *.896
      IF(TANK.EQ.0.0)GO TO 4001
      DO 100 I=3,7,2
      ARG = PI*R(I)/(2.*HI(I))
      CALL  BESNIS(ARG,2,ANS)
      F(I)=((4./PI)*ANS(2))/((PI*R(I)*ANS(1))/(2.*HI(I)))
```

```
      KCB(I)=2.*PI*R(I)*EE(I)*DEL(I)/(LL(I)-NU(I)**2*HI(I))
      KP(I)=(4.*PI*R(I)*EE(I)*DEL(I)/HI(I))*(ANS(2)/((PI*R(I)*ANS(1))/
     1(2.*HI(I))))**2
  100 CONTINUE
C****      CALCULATIONS FOR F(1), KCB(1), KB1, KP(1), K1
      ARG = PI * RB/(2.*HI(1))
      CALL BESNIS(ARG,2,ANS)
      F(1)= ((4./PI)*ANS(2))/((PI*RB*ANS(1))/(2.*HI(1)))
      KCB(1)=KCAP1/(1.-NUST*F(1))
      KB1=NUST*KCB(1)*F(1)
      KP(1)=(2.*KCB(1)*(L1-NUST**2*HI(1))/HI(1))
     1 *(ANS(2)/((PI*RB*ANS(1))/(2.*HI(1))))**2
      K1=KP(1)-NUST*KCB(1)*F(1)+NUST**2*KCB(1)*F(1)**2
      DO 200 J=1,7,2
      ARG = PI*R(J)/(2.*HI(J))
      CALL  BESNIS(ARG,2,ANS)
      MT(J)= PI*R(J)**2*HI(J)*RHO(J)
      MM(J)= 16./(PI**2)*((ANS(2)*MT(J))/(PI*R(J)*ANS(1)/(2.*HI(J))))
  200 CONTINUE
C****      CALCULATE THE KCAP
      KCAP2=7.36E5
      KCAP3 = (1.-NU(3)*F(3))*KCB(3)
      KCAP5 = (1.-NU(5)*F(5))*KCB(5)
      KCAP6 = 2.*PI*R0*E6*DEL(6)/L6*.30
      KCAP7 = (1.-NU(7)*F(7))*KCB(7)
C****      CALCULATE THE SMALL KS  AND  KBS
      K3 = KP(3)-NU(3)*KCB(3)*F(3)+NU(3)**2*KCB(3)*F(3)**2
      K5 = KP(5)-NU(5)*KCB(5)*F(5)+NU(5)**2*KCB(5)*F(5)**2
      K7 = KP(7)-NU(7)*KCB(7)*F(7)+NU(7)**2*KCB(7)*F(7)**2
      KB3= NU(3)*KCB(3)*F(3)
      KB5= NU(5)*KCB(5)*F(5)
      KB7= NU(7)*KCB(7)*F(7)
C****      CALCULATE  DATA FOR MASS MATRIX
      DO 300 I=1,7,2
      ARG =1.84*HI(I)/R(I)
      MTDA(I)=MT(I)*R(I)/(2.2*HI(I))*TANH(ARG)
      MSTR(I)=MT(I)-MTDA(I)
      JRIG(I)=MT(I)*R(I)**2*(1./3.*(HI(I)/R(I))**2+1.)/4.
  300 CONTINUE
      ARG=1.84*HI(1)/R(1)
      HT1=HI(1)-R(1)/.92*TANH(.92*HI(1)/R(1))
      KT1=MT(1)*G/(1.19*HI(1))*(TANH(ARG))**2
      HST1=HI(1)-MT(1)* HI(1)/(2.*MSTR(1))
     1 +(HI(1)-HT1)*MTDA(1)/MSTR(1)
      JSTR(1)=JRIG(1)+MT(1)*(HI(1)/2.)**2 -MT(1)*R(1)**2/2.*(1.995-
     1 2.*R(1)/HI(1)*((1.07*COSCH(ARG)-1.07)/SINCH(ARG)))-MSTR(1)*(HI(1)
     2-HST1)**2 - MTDA(1)*(HI(1)-HT1)**2
      ARG=1.84*HI(3)/R(3)
      HT3=HI(3)-R(3)/.92*TANH(.92*HI(3)/R(3))
      KT3=MT(3)*G/(1.19*HI(3))*(TANH(ARG))**2
      HST3=HI(3)-MT(3)* HI(3) /(2.*MSTR(3))
     1 +(HI(3)-HT3)*MTDA(3)/MSTR(3)
      JSTR(3)=JRIG(3)+MT(3)*(HI(3)/2.)**2-MT(3)*R(3)**2/2.*(1.995-
     1 2.*R(3)/HI(3)*((1.07*COSCH(ARG)-1.07)/SINCH(ARG)))-MSTR(3)*(HI(3)
     2-HST3)**2 - MTDA(3)*(HI(3)-HT3)**2
      ARG=1.84*HI(5)/R(5)
      HT5=HI(5)-R(5)/.92*TANH(.92*HI(5)/R(5))
      KT5=MT(5)*G/(1.19*HI(5))*(TANH(ARG))**2
```

```
00735          HST5=HI(5)-MT(5)* HI(5)                  /(2.*MSTR(5))
              1 +(HI(5)-HT5)*MTDA(5)/MSTR(5)
00745          JSTR(5)=JRIG(5)+MT(5)*(HI(5)/2,)**2-MT(5)*R(5)**2/2,*(1,995-
              1 2,*R(5)/HI(5)*((1,07*COSCH(ARG)-1,07)/SINCH(ARG)))-MSTR(5)*(HI(5)
              2-HST5)**2 - MTDA(5)*(HI(5)-HT5)**2
01003          ARG=1,84*HI(7)/R(7)
01005          HT7=HI(7)-R(7)/,92*TANH(,92*HI(7)/R(7))
01015          KT7=MT(7)*G/(1,19*HI(7))*(TANH(ARG))**2
01024          HST7=HI(7)-MT(7)* HI(7)                  /(2,*MSTR(7))
              1 +(HI(7)-HT7)*MTDA(7)/MSTR(7)
01034          JSTR(7)=JRIG(7)+MT(7)*(HI(7)/2,)**2-MT(7)*R(7)**2/2,*(1,995-
              1 2,*R(7)/HI(7)*((1,07*COSCH(ARG)-1,07)/SINCH(ARG)))-MSTR(7)*(HI(7)
              2-HST7)**2 - MTDA(7)*(HI(7)-HT7)**2
01072     4001 CONTINUE
         C
         C     COMPLTED DATA  FOR FACTORS OF MULTIPLICATION
01072          IF(TANK,EQ,0,0)GO TO 2999
01073          IF(HT1,GE,HST1)GO TO 2999
         C
         C                       INVERTED CASE HT1 LT HST1
         C              INVERTED  BOOSTER
         C
01076          BETA(1,1)= 3,*EE(3)*I1/( HT1**2*PI*R(1)*DEL(1)*GST)
01105          BETA(3,1)= 3,*EE(3)*I3/( HT3**2*PI*R(3)*DEL(3)*GST)
01114          BETA(1,2)= 3,*EE(3)*I1/((HST1-HT1)**2*PI*R(1)*DEL(1)*GST)
01124          BETA(3,2)= 3,*EE(3)*I3/((HST3-HT3)**2*PI*R(3)*DEL(3)*GST)
01135          BETA(1,3)= 3,*EE(3)*I1/((L1-HST1)**2*PI*R(1)*DEL(1)*GST)
01145          BETA(3,3)= 3,*EE(3)*I3/((L3-HST3)**2*PI*R(3)*DEL(3)*GST)
         C
         C              INVERTED  ORBITER
         C
01156          IF(HT5,GE,HST5)GO TO 4002
01160          BETA(5,1)= 3,*EE(5)*I5/( HT5**2*PI*R(5)*DEL(5)*GAL)
01167          BETA(7,1)= 3,*EE(7)*I7/( HT7**2*PI*R(7)*DEL(7)*GAL)
01176          BETA(5,2)= 3,*EE(5)*I5/((HST5-HT5)**2*PI*R(5)*DEL(5)*GAL)
01207          BETA(7,2)= 3,*EE(7)*I7/((HST7-HT7)**2*PI*R(7)*DEL(7)*GAL)
01217          BETA(5,3)= 3,*EE(5)*I5/((L5-HST5)**2*PI*R(5)*DEL(5)*GAL)
01230          BETA(7,3)= 3,*EE(7)*I7/((L7-HST7)**2*PI*R(7)*DEL(7)*GAL)
01240          GO TO  4003
         C
01241     2999 CONTINUE
         C                       REGULAR CASE HT1 GT HST1
         C
01241          BETA(1,1)= 3,*EE(3)*I1/(HST1**2*PI*R(1)*DEL(1)*GST)
01251          BETA(3,1)= 3,*EE(3)*I3/(HST3**2*PI*R(3)*DEL(3)*GST)
01260          BETA(1,2)= 3,*EE(3)*I1/((HT1-HST1)**2*PI*R(1)*DEL(1)*GST)
01270          BETA(3,2)= 3,*EE(3)*I3/((HT3-HST3)**2*PI*R(3)*DEL(3)*GST)
01301          BETA(1,3)= 3,*EE(3)*I1/((L1-HT1)**2*PI*R(1)*DEL(1)*GST)
01311          BETA(3,3)= 3,*EE(3)*I3/((L3-HT3)**2*PI*R(3)*DEL(3)*GST)
01322     4002 CONTINUE
01322          BETA(5,1)= 3,*EE(5)*I5/(HST5**2*PI*R(5)*DEL(5)*GAL)
01332          BETA(7,1)= 3,*EE(7)*I7/(HST7**2*PI*R(7)*DEL(7)*GAL)
01341          BETA(5,2)= 3,*EE(5)*I5/((HT5-HST5)**2*PI*R(5)*DEL(5)*GAL)
01351          BETA(7,2)= 3,*EE(7)*I7/((HT7-HST7)**2*PI*R(7)*DEL(7)*GAL)
01362          BETA(5,3)= 3,*EE(5)*I5/((L5-HT5)**2*PI*R(5)*DEL(5)*GAL)
01372          BETA(7,3)= 3,*EE(7)*I7/((L7-HT7)**2*PI*R(7)*DEL(7)*GAL)
01403     4003 CONTINUE
01403          BETA*2    = 3,*EE(7)*I2/(L2**2*PI*RB*DEL(2)*GAL)
```

```
01413          BETAN6    = 3.*EE(7)*I6/(L6**2*PI*RO*DEL(6)*GAL)
01422          DO 90 I=1,7,2
01423          DO 90 J=1,3
01424          NNN(I,J) = 1.  +  4.*BETA(I,J)
01433          AAA(I,J) = 1.  +     BETA(I,J)
01436          BBB(I,J) = 1.  -  2.*BETA(I,J)
01442       90 CONTINUE
01446          NNN2 = 1. + 4. * BETAN2
01451          NNN6 = 1. + 4. * BETAN6
01453          AAA2 = 1. +      BETAN2
01454          AAA6 = 1. +      BETAN6
01455          BBB2 = 1. - 2. * BETAN2
01457          BBB6 = 1. - 2. * BETAN6
        C
        C****      FILL THE P-MATRIX (MASS) WITH ITS  NON-ZERO ELEMENTS
        C
01461          MASS(1,1)=MASS(17,17)=8.291E-2
01464          MASS(2,2)=MASS(18,18)=1.768E-2
01467          MASS(3,3)=MASS(19,19)=3.334E-2
01472          MASS(4,4)=MASS(20,20)=3.037E-2
01475          MASS(5,5)=MASS(21,21)=5.715E-3
01500          MASS(6,6)=MASS(22,22)=4.755E-3
01503          MASS(7,7)=MASS(23,23)=4.885E-3
01506          MASS(8,8)=MASS(24,24)=5.208E-3
01511          MASS(33,33)= MM(1)
01513          MASS(34,34)= MM(3)
01514          MASS(25,25)= MM(5)
01516          MASS(26,26)= MM(7)
01517          MASS( 9, 9)=7.65E-1
01521          MASS(10,10)=2.09E-1
01522          MASS(11,11)=3.255E-1
01524          MASS(12,12)=1.721E-1
01525          MASS(13,13)=1.782E-2
01527          MASS(14,14)=2.546E-2
01530          MASS(15,15)=2.582E-2
01532          MASS(16,16)=1.643E-2
01533          MASS(35,35)= JSTR(1)
01535          MASS(36,36)= JSTR(3)
01536          MASS(27,27)= JSTR(5)
01540          MASS(28,28)= JSTR(7)
01541          MASS(37,37)= MSTR(1)
01543          MASS(38,38)= MSTR(3)
01544          MASS(29,29)= MSTR(5)
01546          MASS(30,30)= MSTR(7)
01547          MASS(39,39)= MTDA(1)
01551          MASS(40,40)= MTDA(3)
01552          MASS(31,31)= MTDA(5)
01554          MASS(32,32)= MTDA(7)
        C**** INVERT  THE  DIAGONAL  P-MATRIX  (MASS)
        C****
01555          DO  75  I=1,JC
01557       72 MASS(I,I)=1./MASS(I,I)
01563       75 CONTINUE
        C      THE  BEGINNING  OF  PROGRAM  CALCULATIONS  FOR  THE  NON-ZERO
        C      ELEMENTS  OF THE SYMMETRIC Q-MATRIX ( QARR )
        C**** ASSIGNING  VARIABLE  NAMES  TO  OFTEN  USED  CONSTANTS
        C****
01565          RBD1 = RB*D1
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```
01567          RBD2 = RB*D2
01571          RBD3 = RB*D3
01572          ROD4 = RO*D4
01574          EIL2   = E2*I2/L2
01577          E1I1   = E1*I1
01601          E2I2   = E2*I2
01602          E3I3   = E3*I3
01604          E5I5   = E5*I5
01606          E6I6   = E6*I6
01610          E7I7   = E7*I7
01612          EIHST1 = E1*I1/HST1
01614          EIHST3 = E3*I3/HST3
01617          EIHST5 = E5*I5/HST5
01621          EIHST7 = E7*I7/HST7
01624          EILHT3 = E3*I3/(L3-HT3)
01627          EILHT5 = E5*I5/(L5-HT5)
01633          EILHT7 = E7*I7/(L7-HT7)
01636          EUJUP1 = EUP1*JUP1
01640          EUJUP2 = EUP2*JUP2
01642          EUJUP3 = EUP3*JUP3
01644          EUJUP4 = EUP4*JUP4
01646          QARR( 1, 1) = KCAP1+KUP1+K1
01651          QARR( 1, 2) =-KCAP1-KUP1
01653          QARR( 1, 9) = KUP1*RBD1
01654          QARR( 1,10) =-KUP1*RBD1
01655          QARR( 1,33) =-K1
01656          QARR( 2, 2) = KCAP1+KCAP2+KUP1+KUP2+KB1
01662          QARR( 2, 3) =-KCAP2-KUP2
01663          QARR( 2, 9) = QARR(1,10)
01665          QARR( 2,10) = QARR(1,9)+KUP2*RBD2
01667          QARR( 2,11) =-KUP2*RBD2
01670          QARR( 2,33) =-KB1
01671          QARR( 3, 3) = KCAP2+KCAP3+KUP2+KUP3+K3
01676          QARR( 3, 4) =-KCAP3
01677          QARR( 3,10) = QARR(2,11)
01700          QARR( 3,11) =-QARR(2,11)+KUP3*RBD3
01702          QARR( 3,34) =-K3
01703          QARR( 4, 4) = KCAP3+KBUP3+KB3
01706          QARR( 4,12) = KBUP3*RBD3
01707          QARR( 4,34) =-KB3
01710          QARR( 5, 5) = KCAP5+K5
01712          QARR( 5, 6) =-KCAP5
01713          QARR( 5,25) =-K5
01714          QARR( 6, 6) = KCAP5+KCAP6+KUP4+KB5
01717          QARR( 6, 7) =-KCAP6
01720          QARR( 6,14) =-KUP4*ROD4
01722          QARR( 6,25) =-KB5
01723          QARR( 7, 7) = KCAP6+KCAP7+KBUP4+K7
01726          QARR( 7, 8) =-KCAP7
01727          QARR( 7,15) =-KBUP4*ROD4
01730          QARR( 7,26) =-K7
01731          QARR( 8, 8) = KCAP7+KB7
01732          QARR( 8,26) =-KB7
01733          QARR( 9, 9) = QARR(1,9)*RBD1+S1+4,*EIHST1*AAA(1,1)/NNN(1,1)
01742          QARR( 9,10) =-QARR(1,9)*RBD1
01744          QARR( 9,17) =-6,*E1I1/HST1**2*1,/NNN(1,1)
01751          QARR( 9,35) = 2,*EIHST1*BBB(1,1)/NNN(1,1)
01754          QARR( 9,37) =-QARR(9,17)
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01755        QARR(10,10) =-QARR(9,10)+KUP2 *RBD2**2+S2+4.*(E1I1/(L1-HT1)*AAA
            *(1,3)/NNN(1,3)+EIL2*AAA2/NNN2)
01773        QARR(10,11) =-KUP2 *RBD2**2+2.*EIL2*BBB2/NNN2
02000        QARR(10,18) = 6.*(E1I1/(L1-HT1)**2*1./NNN(1,3)-EIL2/(L2*NNN2))
02011        QARR(10,19) = 6.*EIL2/L2*1./NNN2
02014        QARR(11,11) = KUP2*RBD2**2+KUP3*RBD3**2+S3+4.*(EIL2*AAA2/NNN2+
            *EIHST3*AAA(3,1)/NNN(3,1))
02030        QARR(11,18) =-QARR(10,19)
02031        QARR(11,19) = QARR(10,19)-6.*E3I3/HST3**2*1./NNN(3,1)
02040        QARR(11,36) = 2.*EIHST3*BBB(3,1)/NNN(3,1)
02043        QARR(11,38) = 6.*EIHST3/HST3*1./NNN(3,1)
02047        QARR(12,12) = QARR(4,12)*RBD3+S4+4.*EILHT3*AAA(3,3)/NNN(3,3)
02055        QARR(12,20) = 6.*EILHT3/(L3-HT3)*1./NNN(3,3)
02062        QARR(13,13) = 4.*EIHST5*AAA(5,1)/NNN(5,1)
02066        QARR(13,21) = KUP5*E-6.*EIHST5/HST5*1./NNN(5,1)
02074        QARR(13,27) = 2.*EIHST5*BBB(5,1)/NNN(5,1)
02077        QARR(13,29) = 6.*EIHST5/HST5*1./NNN(5,1)
02102        QARR(14,14) = KUP4*ROD4**2+S5+4.*EILHT5*AAA(5,3)/NNN(5,3)+4.*E6I6*
            *AAA6/(L6*NNN6)
02117        QARR(14,15) = 2.*E6I6/L6*BBB6/NNN6
02123        QARR(14,22) = 6.*(EILHT5/(L5-HT5)*1./NNN(5,3)-E6I6/(L6**2*NNN6))
02134        QARR(14,23) = 6.*E6I6/(L6**2*NNN6)
02137        QARR(15,15) = KBUP4*ROD4**2+S7+4.*(E6I6/L6*AAA6/NNN6+E7I7/HST7*
            *AAA(7,1)/NNN(7,1))
02153        QARR(15,22) =-QARR(14,23)
02154        QARR(15,23) = QARR(14,23)-EIHST7/HST7*6.*1./NNN(7,1)
02160        QARR(15,28) = 2.*EIHST7*BBB(7,1)/NNN(7,1)
02163        QARR(15,30) = 6.*EIHST7/HST7*1./NNN(7,1)
02166        QARR(16,16) = 4.*EILHT7*AAA(7,3)/NNN(7,3)
02172        QARR(16,24) = 6.*EILHT7/(L7-HT7)*1./NNN(7,3)
02177        QARR(17,17) = 12.*(E1I1/HST1**3*1./NNN(1,1)+EUJUP1/L1**3)
02207        QARR(17,18) =-12.*EUJUP1/L1**3
02211        QARR(17,35) =-QARR(9,37)
02213        QARR(17,37) =-12.*EIHST1/HST1**2*1./NNN(1,1)
02217        QARR(18,18) = 12.*(E1I1/((L1-HT1)**3*NNN(1,3))+EUJUP1/L1**3 +
            *                 E2I2/(L2**3*NNN2) + EUJUP2/A**3)
02236        QARR(18,19) =-12.*EIL2/L2**2*1./NNN2
02243        QARR(19,19) =-QARR(18,19)+12.*(EUJUP2/(L2-A)**3+EIHST3/HST3**2
            **1./NNN(3,1)+ EUJUP3/C**3)
02257        QARR(19,36) =-6.*EIHST3/HST3*1./NNN(3,1)
02263        QARR(19,38) =-12.*EIHST3/HST3**2*1./NNN(3,1)
02266        QARR(20,20) = 12.*(EILHT3/(L3-HT3)**2*1./NNN(3,3)+EUJUP3/(L3-C)
            ***3)
02301        QARR(21,21) = KUP5+ 12.*EIHST5/HST5**2*1./NNN(5,1)
02306        QARR(21,27) =-QARR(13,29)
02307        QARR(21,29) =-12.*EIHST5/HST5**2*1./NNN(5,1)
02314        QARR(22,22) = 12.*(E5I5/((L5-HT5)**3*NNN(5,3))+E6I6/(L6**3*NNN6)
            *               + EUJUP4/D**3)
02331        QARR(22,23) =-12.*E6I6/L6**3*1./NNN6
02335        QARR(23,23) = 12.*(E6I6/(L6**3*NNN6)+E7I7/(HST7**3*NNN(7,1))+
            *                 EUJUP4/(L6-D)**3)
02353        QARR(23,28) =-QARR(15,30)
02354        QARR(23,30) = 2.*QARR(23,28)/HST7
02357        QARR(24,24) = 2.*QARR(16,24)/(L7-HT7)
02362        QARR(25,25) = K5+KB5
02364        QARR(26,26) = K7+KB7
02366        QARR(27,27) = 4.*E5I5*(AAA(5,1)/(HST5*NNN(5,1))+AAA(5,2)/((HT5-HST
            *                 5)*NNN(5,2)))
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02377          QARR(27,29) = QARR(13,29)-6.*E5I5/(HT5-HST5)**2*1./NNN(5,2)
02406          QARR(28,28) = 4.*E7I7*(AAA(7,1)/(HST7*NNN(7,1)) + AAA(7,2)/((HT7-
              *              HST7)*NNN(7,2)))
02417          QARR(28,30) = QARR(15,30)-6.*E7I7/(HT7-HST7)**2*1./NNN(7,2)
02426          QARR(29,29) = 12.*E5I5*(1./((HT5-HST5)**3*NNN(5,2))+1./(HST5**3*
              *              NNN(5,1)) )
02440          QARR(30,30) = 12.*E7I7*(1./((HT7-HST7)**3*NNN(7,2))+1./(HST7**3*
              *              NNN(7,1)) )
02452          QARR(31,31) = KT5
02453          QARR(32,32) = KT7
02455          QARR(33,33) = K1+KB1
02457          QARR(34,34) = K3+KB3
02461          QARR(35,35) = 4.*E1I1*(AAA(1,1)/(HST1*NNN(1,1))+AAA(1,2)/
              *              ((HT1-HST1)*NNN(1,2)))
02472          QARR(35,37) = 6.*E1I1*(1./(HST1**2*NNN(1,1))-1./((HT1-HST1)**2*
              *              NNN(1,2)))
02503          QARR(36,36) = 4.*E3I3*(AAA(3,1)/(HST3*NNN(3,1)) + AAA(3,2)/
              *              ((HT3-HST3)*NNN(3,2)))
02514          QARR(36,38) = 6.*E3I3*(1./(HST3**2*NNN(3,1))-1./((HT3-HST3)**2*
              *              NNN(3,2)))
02525          QARR(37,37) = 12.*E1I1*(1./((HT1-HST1)**3*NNN(1,2))+1./(HST1**3*
              *              NNN(1,1)))
02537          QARR(38,38) =12.*E3I3*(1./((HT3-HST3)**3*NNN(3,2))+1./(HST3**3*NNN
              *              (3,1)))
02551          QARR(39,39) = KT1
02552          QARR(40,40) = KT3
        C
        C       GENERATE REMAINDER OF QARR MATRIX FROM SYMMETRY
        C
02554          K=1
02555          DO 352 I=2,40
02556          DO 351 J=1,K
02557      351 QARR(I,J)=QARR(J,I)
02571          K=K+1
02572      352 CONTINUE
        C
02574          IF(TANK.NE.0.0)GO TO 2001
        C                                   ZERO CASE
        C
02575          QARR( 9,10) = QARR(9,10)+2.*E1I1/L1*BBB(1,1)/NNN(1,1)
02602          QARR( 9,18) = QARR(9,37)
02604          QARR( 9,35) = 0.0
02605          QARR(10,17) =-6.*E1I1/L1**2*1./NNN(1,1)
02610          QARR(11,12) = QARR(11,36)
02612          QARR(11,20) = QARR(11,38)
02613          QARR(12,19) =-6.*E3I3/L3**2*1./NNN(3,1)
02620          QARR(13,14) = QARR(13,27)
02622          QARR(13,22) = QARR(13,29)
02623          QARR(14,21) =-6.*E5I5/L5**2*1./NNN(5,1)
02630          QARR(15,16) = QARR(15,28)
02632          QARR(15,24) = QARR(15,30)
02633          QARR(16,23) =-6.*E7I7/L7**2*1./NNN(7,1)
02640          QARR(17,18) = QARR(17,18)-12.*E1I1/L1**3*1./NNN(1,1)
02646          QARR(17,37) =0.0
02647          QARR(19,20) = QARR (19,38)
02650          QARR(21,22) = QARR(21,29)
02652          QARR(23,24) = QARR(23,30)
02653     2001 CONTINUE
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        C
02653         IF(TANK,EQ,0,0)GO TO 2009
02654         IF(HT1,GE,HST1)GO TO 2011
        C                        INVERTED CASE
        C                       INVERTED  BOOSTER
        C
02657         QARR( 9, 9) = KUP1*RBD1**2+S1*4,*E1I1*AAA(1,1)/(HT1*NNN(1,1))
02667         QARR( 9,17) =-6,*E1I1/(HT1**2*NNN(1,1))
02673         QARR( 9,35) = 0,0
02674         QARR( 9,37) = 0,0
02675         QARR(10,10) = KUP1*RBD1**2+KUP2*RBD2**2+S2*4,*E1I1*AAA(1,3)/
             *                ((L1-HST1)*NNN(1,3))+4,*E2I2*AAA2/(L2*NNN2)
02716         QARR(10,18) = 6,*(E1I1/((L1-HST1)**2*NNN(1,3))-E2I2/(L2**2
             *                *NNN2))
02726         QARR(10,35) = 2,*E1I1*BBB(1,3)/((L1-HST1)*NNN(1,3))
02734         QARR(10,37) =-6,*E1I1/((L1-HST1)**2*NNN(1,3))
02741         QARR(11,11) = KUP2*RBD2**2+KUP3*RBD3**2+S3*4,*EIL2*AAA2/NNN2
             *                + 4,*E3I3*AAA(3,1)/(HT3*NNN(3,1))
02757         QARR(11,19) = 6,*(EIL2/(L2*NNN2)-E3I3/(HT3**2*NNN(3,1)))
02766         QARR(11,36) = 0,0
02767         QARR(11,38) = 0,0
02770         QARR(12,12) = KBUP3*RBD3**2+S4*4,*E3I3*AAA(3,3)/((L3-HST3)
             *                *NNN(3,3))
03002         QARR(12,20) = 6,*E3I3/((L3-HST3)**2*NNN(3,3))
03007         QARR(12,36) = 2,*E3I3*BBB(3,3)/((L3-HST3)*NNN(3,3))
03014         QARR(12,38) =-6,*E3I3/((L3-HST3)**2*NNN(3,3))
03022         QARR(17,17) = 12,*(E1I1/(HT1**3*NNN(1,1))+EUJUP1/L1**3)
03030         QARR(17,35) = 0,0
03031         QARR(17,37) = 0,0
03032         QARR(18,18) = 12,*(E1I1/((L1-HST1)**3*NNN(1,3))+EUJUP1/L1**3+
             *                E2I2/(L2**3*NNN2)+EUJUP2/A**3)
03051         QARR(18,35) = 6,*E1I1/((L1-HST1)**2*NNN(1,3))
03055         QARR(18,37) =-12,*E1I1/((L1-HST1)**3*NNN(1,3))
03062         QARR(19,19) = 12,*(E2I2/(L2**3*NNN2)+EUJUP2/(L2-A)**3 +
             *                E3I3/(HT3**3*NNN(3,1))+EUJUP3/C**3)
03103         QARR(19,36) = 0,0
03104         QARR(19,38) = 0,0
03105         QARR(20,20) = 12,*(E3I3/((L3-HST3)**3*NNN(3,3))+EUJUP3/(L3-C)**3)
03117         QARR(20,36) = 6,*E3I3/((L3-HST3)**2*NNN(3,3))
03124         QARR(20,38) =-12,*E3I3/((L3-HST3)**3*NNN(3,3))
03130         QARR(35,35) = 4,*E1I1*(AAA(1,3)/((L1-HST1)*NNN(1,3))+
             *                AAA(1,2)/((HST1-HT1)*NNN(1,2)))
03142         QARR(35,37) = 6,*E1I1*(1,/((HST1-HT1)**2*NNN(1,2))-1,/
             *                ((L1-HST1)**2*NNN(1,3)))
03154         QARR(36,36) = 4,*E3I3*(AAA(3,3)/((L3-HST3)*NNN(3,3))+
             *                AAA(3,2)/((HST3-HT3)*NNN(3,2)))
03166         QARR(36,38) = 6,*E3I3*(1,/((HST3-HT3)**2*NNN(3,2))-1,/
             *                ((L3-HST3)**2*NNN(3,3)))
03200         QARR(37,37) = 12,*E1I1*(1,/((HST1-HT1)**3*NNN(1,2))+1,/
             *                ((L1-HST1)**3*NNN(1,3)))
03212         QARR(38,38) = 12,*E3I3*(1,/((HST3-HT3)**3*NNN(3,2))+1,/
             *                ((L3-HST3)**3*NNN(3,3)))
03224         IF(HT5,GE,HST5)GO TO 2009
        C
        C                       INVERTED  ORBITER
        C
03226         QARR(13,13) = 4,*E5I5*AAA(5,1)/(HT5*NNN(5,1))
03232         QARR(13,21) = KUP5*E - 6,*E5I5/( HT5**2*NNN(5,1))
```

```
          QARR(13,27) = 0.0
          QARR(13,29) = 0.0
          QARR(14,14) = KUP4*ROD4**2+S5+4.*E5I5*AAA(5,3)/
     *                  ((L5-HST5)*NNN(5,3)) + 4.*E6I6*AAA6/(L6*NNN6)
          QARR(14,22) = 6.*(E5I5/((L5-HST5)**2*NNN(5,3))-
     *                  E6I6/(L6**2*NNN6))
          QARR(14,27) = 2.*E5I5*BBB(5,3)/((L5-HST5)*NNN(5,3))
          QARR(14,29) =-6.*E5I5/((L5-HST5)**2*NNN(5,3))
          QARR(15,15) = KBUP4*ROD4**2+S7+4.*E6I6*AAA6/(L6*NNN6)+
     *                  4.*E7I7*AAA(7,1)/(HT7*NNN(7,1))
          QARR(15,23) = 6.*(E6I6/(L6**2*NNN6)-E7I7/(HT7**2*NNN(7,1)))
          QARR(15,28) = 0.0
          QARR(15,30) = 0.0
          QARR(16,16) = 4.*E7I7*AAA(7,3)/((L7-HST7)*NNN(7,3))
          QARR(16,24) = 6.*E7I7/((L7-HST7)**2*NNN(7,3))
          QARR(16,28) = 2.*E7I7*BBB(7,3)/((L7-HST7)*NNN(7,3))
          QARR(16,30) =-6.*E7I7/((L7-HST7)**2*NNN(7,3))
          QARR(21,21) = KUP5 + 12.*E5I5/( HT5**3*NNN(5,1))
          QARR(21,27) = 0.0
          QARR(21,29) = 0.0
          QARR(22,22) = 12.*(E5I5/((L5-HST5)**3*NNN(5,3))+E6I6/(L6**3*NNN6)
     *                  + EUJUP4/D**3)
          QARR(22,27) = 6.*E5I5/((L5-HST5)**2*NNN(5,3))
          QARR(22,29) =-12.*E5I5/((L5-HST5)**3*NNN(5,3))
          QARR(23,23) = 12.*(E6I6/(L6**3*NNN6)+E7I7/( HT7**3*NNN(7,1))
     *                  + EUJUP4/(L6-D)**3)
          QARR(23,28) = 0.0
          QARR(23,30) = 0.0
          QARR(24,24) = 12.*E7I7/((L7-HST7)**3*NNN(7,3))
          QARR(24,28) = 6.*E7I7/((L7-HST7)**2*NNN(7,3))
          QARR(24,30) =-12.*E7I7/((L7-HST7)**3*NNN(7,3))
          QARR(27,27) = 4.*E5I5*(AAA(5,3)/((L5-HST5)*NNN(5,3))
     *                  + AAA(5,2)/((HST5-HT5)*NNN(5,2)))
          QARR(27,29) = 6.*E5I5*(1./((HST5-HT5)**2*NNN(5,2))-
     *                  1./((L5-HST5)**2*NNN(5,3)))
          QARR(28,28) = 4.*E7I7*(AAA(7,3)/((L7-HST7)*NNN(7,3))
     *                  + AAA(7,2)/((HST7-HT7)*NNN(7,2)))
          QARR(28,30) = 6.*E7I7*(1./((HST7-HT7)**2*NNN(7,2))-
     *                  1./((L7-HST7)**2*NNN(7,3)))
          QARR(29,29) = 12.*E5I5*(1./((HST5-HT5)**3*NNN(5,2))+
     *                  1./((L5-HST5)**3*NNN(5,3)))
          QARR(30,30) = 12.*E7I7*(1./((HST7-HT7)**3*NNN(7,2))+
     *                  1./((L7-HST7)**3*NNN(7,3)))
     2009 CONTINUE
          K=1
          DO 354 I=2,JC
          DO 353 J=1,K
      353 QARR(I,J)=QARR(J,I)
          K=K+1
      354 CONTINUE
     2011 CONTINUE
          ARR( 3, 1) =-KUP3
          ARR( 4, 1) =-KBUP3
          ARR( 6, 2) =-KUP4
          ARR( 7, 2) =-KBUP4
          ARR( 9, 3) =-S1
          ARR(10, 4) =-S2
          ARR(10,19) = 2.*E1I1/(L1-HT1)*BBB(1,3)/NNN(1,3)
```

```
03610          ARR(10,21) =-6.*E1I1/((L1-HT1)**2*NNN(1,3))
03614          ARR(11, 1) =-KUP3*RBD3
03616          ARR(11, 6) =-S3
03620          ARR(12, 1) =-KBUP3*RBD3
03621          ARR(12, 8) =-S4
03623          ARR(12,20) = 2.*EILHT3*BBB(3,3)/NNN(3,3)
03626          ARR(12,22) =-6.*EILHT3/(L3-HT3)*1./NNN(3,3)
03633          ARR(13,12) =-KUP5*E
03635          ARR(14, 2) = KUP4*ROD4
03637          ARR(14, 9) =-S5
03641          ARR(14,15) = 2.*EILHT5*BBB(5,3)/NNN(5,3)
03644          ARR(14,17) =-6.*EILHT5/(L5-HT5)*1./NNN(5,3)
03651          ARR(15, 2) = KBUP4*ROD4
03653          ARR(15,11) =-S7
03655          ARR(16,16) = 2.*EILHT7*BBB(7,3)/NNN(7,3)
03660          ARR(16,18) =-6.*EILHT7/(L7-HT7)*1./NNN(7,3)
03665          ARR(17, 3)  =-6.*EUJUP1/L1**2
03670          ARR(17, 4) = ARR(17,3)
03671          ARR(18, 3) = 6.*EUJUP1/L1**2
03673          ARR(18, 4) = 6.*(EUJUP1/L1**2-EUJUP2/A**2)
03677          ARR(18, 5) =-6.*EUJUP2/A**2
03701          ARR(18,12) =-12.*EUJUP2/A**3
03703          ARR(18,19) = 6.*E1I1/(L1-HT1)**2*1./NNN(1,3)
03711          ARR(18,21) =-12.*E1I1/(L1-HT1)**3*1./NNN(1,3)
03717          ARR(19, 5) = 6.*EUJUP2/(L2-A)**2
03723          ARR(19, 6) = ARR(19,5)-6.*EUJUP3/C**2
03727          ARR(19, 7) =-6.*EUJUP3/C**2
03731          ARR(19,12) =-2.*ARR(19,5)/(L2-A)
03734          ARR(19,13) =-12.*EUJUP3/C**3
03737          ARR(20, 7) = 6.*EUJUP3/(L3-C)**2
03742          ARR(20, 8) = ARR(20,7)
03743          ARR(20,13) =-2.*ARR(20,7)/(L3-C)
03746          ARR(20,20) = 6.*EILHT3/(L3-HT3)*1./NNN(3,3)
03753          ARR(20,22) =-2.*ARR(20,20)/(L3-HT3)
03756          ARR(21,12) =-KUP5
03760          ARR(22, 9) =-6.*EUJUP4/D**2
03763          ARR(22,10) = ARR(22,9)
03764          ARR(22,14) = -12.*EUJUP4/D**3
03766          ARR(22,15) = 6.*EILHT5/(L5-HT5)*1./NNN(5,3)
03773          ARR(22,17) =-12.*EILHT5/(L5-HT5)**2*1./NNN(5,3)
04001          ARR(23,10) = 6.* EUJUP4/(L6-D)**2
04005          ARR(23,11) = ARR(23,10)
04006          ARR(23,14) =-2.*ARR(23,10)/(L6-D)
04011          ARR(24,16) =-ARR(16,18)
04013          ARR(24,18) = 2.*ARR(16,18)/(L7-HT7)
04017          ARR(27,15) = 2.*E5I5/(HT5-HST5)*BBB(5,2)/NNN(5,2)
04024          ARR(27,17) = 6.*E5I5/(HT5-HST5)**2*1./NNN(5,2)
04032          ARR(28,16) = 2.*E7I7/(HT7-HST7)*BBB(7,2)/NNN(7,2)
04037          ARR(28,18) = 6.*E7I7/(HT7-HST7)**2*1./NNN(7,2)
04045          ARR(29,15) =-ARR(27,17)
04047          ARR(29,17) = 2.*ARR(29,15)/(HT5-HST5)
04052          ARR(30,16) =-6.*E7I7/(HT7-HST7)**2*1./NNN(7,2)
04060          ARR(30,18) = 2.*ARR(30,16)/(HT7-HST7)
04063          ARR(31,17) =-KT5
04065          ARR(32,18) =-KT7
04066          ARR(35,19) = 2.*E1I1/(HT1-HST1)*BBB(1,2)/NNN(1,2)
04074          ARR(35,21) = 6.*E1I1/(HT1-HST1)**2*1./NNN(1,2)
04102          ARR(36,20) = 2.*E3I3/(HT3-HST3)*BBB(3,2)/NNN(3,2)
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```
04107          ARR(36,22) = 6.*E3I3/(HT3-HST3)**2*1./NNN(3,2)
04115          ARR(37,19) =-ARR(35,21)
04117          ARR(37,21) = 2.*ARR(37,19)/(HT1-HST1)
04122          ARR(38,20) =-6.*E3I3/(HT3-HST3)**2*1./NNN(3,2)
04130          ARR(38,22) = 2.*ARR(38,20)/(HT3-HST3)
04133          ARR(39,21) =-KT1
04135          ARR(40,22) =-KT3
04136          IF(TANK.NE.0.0)GO TO 3002
04137          ARR(10,19)=0.0
04140          ARR(10,21)=ARR(12,20)=ARR(12,22)=ARR(14,15)= ARR(14,17)=0.0
04146          ARR(16,16)=ARR(16,18)=ARR(18,19)=ARR(18,21)=ARR(20,20)=0.0
04154          ARR(20,22)=ARR(22,15)=ARR(22,17)=ARR(24,16)=ARR(24,18)=0.0
04162          GO TO 3006
04163     3002 CONTINUE
04163          IF(HT1.GE.HST1)GO TO 3006
        C
        C                          INVERTED  BOOSTER
        C
04166          ARR( 9,19) = 2.*E1I1*BBB(1,1)/(HT1*NNN(1,1))
04172          ARR( 9,21) = 6.*E1I1/(HT1**2*NNN(1,1))
04175          ARR(10,19) = 0.0
04176          ARR(10,21) = 0.0
04177          ARR(11,20) = 2.*E3I3*BBB(3,1)/(HT3*NNN(3,1))
04203          ARR(11,22) = 6.*E3I3/(HT3**2*NNN(3,1))
04206          ARR(12,20) = 0.0
04207          ARR(12,22) = 0.0
04210          ARR(17,19) =-6.*E1I1/(HT1**2*NNN(1,1))
04213          ARR(17,21) =-12.*E1I1/(HT1**3*NNN(1,1))
04217          ARR(18,19) = 0.0
04220          ARR(18,21) = 0.0
04221          ARR(19,20) =-6.*E3I3/(HT3**2*NNN(3,1))
04224          ARR(19,22) =-12.*E3I3/(HT3**3*NNN(3,1))
04230          ARR(20,20) = 0.0
04231          ARR(20,22) = 0.0
04232          ARR(35,19) = 2.*E1I1*BBB(1,2)/((HST1-HT1)*NNN(1,2))
04237          ARR(35,21) =-6.*E1I1/((HST1-HT1)**2*NNN(1,2))
04245          ARR(36,20) = 2.*E3I3*BBB(3,2)/((HST3-HT3)*NNN(3,2))
04252          ARR(36,22) =-6.*E3I3/((HST3-HT3)**2*NNN(3,2))
04260          ARR(37,19) = 6.*E1I1/((HST1-HT1)**2*NNN(1,2))
04264          ARR(37,21) =-12.*E1I1/((HST1-HT1)**3*NNN(1,2))
04271          ARR(38,20) = 6.*E3I3/((HST3-HT3)**2*NNN(3,2))
04276          ARR(38,22) =-12.*E3I3/((HST3-HT3)**3*NNN(3,2))
04303          IF(HT5.GE.HST5)GO TO 3006
        C
        C                          INVERTED  ORBITER
        C
04306          ARR(13,15) = 2.*E5I5*BBB(5,1)/(HT5*NNN(5,1))
04312          ARR(13,17) = 6.*E5I5/(HT5**2*NNN(5,1))
04315          ARR(14,15) = 0.0
04316          ARR(14,17) = 0.0
04317          ARR(15,16) = 2.*E7I7*BBB(7,1)/(HT7*NNN(7,1))
04323          ARR(15,18) = 6.*E7I7/(HT7**2*NNN(7,1))
04326          ARR(16,16) = 0.0
04327          ARR(16,18) = 0.0
04330          ARR(21,15) =-6.*E5I5/(HT5**2*NNN(5,1))
04333          ARR(21,17) =-12.*E5I5/(HT5**3*NNN(5,1))
04337          ARR(22,15) = 0.0
04340          ARR(22,17) = 0.0
```

```
04717         CARR(19,19) = 4.*E1I1*(AAA(1,2)/((HT1-HST1)*NNN(1,2))+AAA(1,3)/
             *             ((L1-HT1)*NNN(1,3)))
04731         CARR(19,21) = 6.*E1I1*(1./((HT1-HST1)**2*NNN(1,2))-1./((L1-HT1)
             *         **2*NNN(1,3)))
04743         CARR(20,20) = 4.*E3I3*(AAA(3,2)/((HT3-HST3)*NNN(3,2))+AAA(3,3)/
             *            ((L3-HT3)*NNN(3,3)))
04755         CARR(20,22) = 6.*E3I3*(1./((HT3-HST3)**2*NNN(3,2))-1./((L3-HT3)**2
             *             *NNN(3,3)))
04767         CARR(21,21) = KT1+12.*E1I1*(1./((L1-HT1)**3*NNN(1,3))+1./((HT1-HST
             *             1)**3*NNN(1,2)))
05003         CARR(22,22) = KT3+12.*E3I3*(1./((L3-HT3)**3*NNN(3,3))+1./((HT3-HST
             *             3)**3*NNN(3,2)))
05017         IF(TANK.EQ.0.0)GO TO 3010
05020         IF(HT1.GE.HST1)GO TO 3010
        C
        C                            INVERTED  BOOSTER
        C
05023         CARR(19,19) = 4.*E1I1*(AAA(1,2)/((HST1-HT1)*NNN(1,2))+
             *             AAA(1,1)/(HT1*NNN(1,1)))
05033         CARR(19,21) = 6.*E1I1*(1./(HT1**2*NNN(1,1))-1./
             *             ((HST1-HT1)**2*NNN(1,2)))
05044         CARR(20,20) = 4.*E3I3*(AAA(3,2)/((HST3-HT3)*NNN(3,2))+
             *             AAA(3,1)/((HT3*NNN(3,1))))
05055         CARR(20,22) = 6.*E3I3*(1./(HT3**2*NNN(3,1))-1./
             *             ((HST3-HT3)**2*NNN(3,2)))
05066         CARR(21,21) = KT1+12.*E1I1*(1./(HT1**3*NNN(1,1))+1./
             *             ((HST1-HT1)**3*NNN(1,2)))
05100         CARR(22,22) = KT3+12.*E3I3*(1./(HT3**3*NNN(3,1))+1./
             *             ((HST3-HT3)**3*NNN(3,2)))
05113         IF(HT5.GE.HST5)GO TO 3010
        C
        C                            INVERTED  ORBITER
        C
05116         CARR(15,15) = 4.*E5I5*(AAA(5,2)/((HST5-HT5)*NNN(5,2))+
             *             AAA(5,1)/(HT5*NNN(5,1)))
05126         CARR(15,17) = 6.*E5I5*(1./(HT5**2*NNN(5,1))-1./
             *             ((HST5-HT5)**2*NNN(5,2)))
05137         CARR(16,16) = 4.*E7I7*(AAA(7,2)/((HST7-HT7)*NNN(7,2))+
             *             AAA(7,1)/(HT7*NNN(7,1)))
05150         CARR(16,18) = 6.*E7I7*(1./(HT7**2*NNN(7,1))-1./
             *             ((HST7-HT7)**2*NNN(7,2)))
05161         CARR(17,17) = KT5+12.*E5I5*(1./(HT5**3*NNN(5,1))+1./
             *             ((HST5-HT5)**3*NNN(5,2)))
05173         CARR(18,18) = KT7 + 12.*E7I7*(1./(HT7**3*NNN(7,1))+1./
             *             ((HST7-HT7)**3*NNN(7,2)))
05206    3010 CONTINUE
        C     GENERATE  SYMMETRY  FOR  THE C-MATRIX
05206         K=1
05207         DO 375 I=2,JI
05211         DO 374 J=1,K
05212     374 CARR(I,J)=CARR(J,I)
05224         K=K+1
05225     375 CONTINUE
        C     GENERATE  THE  D-MATRIX  AS THE  NEGATIVE  TRANSPOSE  OF A-MATRIX
        C
05227         DO  95 I=1,JI
05231         DO  95 J=1,JC
05232      95 DARR(I,J)=-ARR(J,I)
```

```
04341          ARR(23,16) =-6.*E7I7/(HT7**2*NNN(7,1))
04344          ARR(23,18) =-12.*E7I7/(HT7**3*NNN(7,1))
04350          ARR(24,16) = 0.0
04351          ARR(24,18) = 0.0
04352          ARR(27,15) = 2.*E5I5*BBB(5,2)/((HST5-HT5)*NNN(5,2))
04357          ARR(27,17) =-6.*E5I5/((HST5-HT5)**2*NNN(5,2))
04365          ARR(28,16) = 2.*E7I7*BBB(7,2)/((HST7-HT7)*NNN(7,2))
04372          ARR(28,18) =-6.*E7I7/((HST7-HT7)**2*NNN(7,2))
04400          ARR(29,15) = 6.*E5I5/((HST5-HT5)**2*NNN(5,2))
04404          ARR(29,17) =-12.*E5I5/((HST5-HT5)**3*NNN(5,2))
04411          ARR(30,16) = 6.*E7I7/((HST7-HT7)**2*NNN(7,2))
04416          ARR(30,18) =-12.*E7I7/((HST7-HT7)**3*NNN(7,2))
04423     3006 CONTINUE
04423          CARR( 1, 1) = KUP3+KBUP3+KUP6
04426          CARR( 1, 2) =-KUP6
04427          CARR( 2, 2) = (KUP4+KBUP4+KUP6)
04432          CARR( 3, 3) = S1+4.*EUJUP1/L1
04436          CARR( 3, 4) = 2.*EUJUP1/L1
04437          CARR( 4, 4) = S2+4.*(EUJUP1/L1+EUJUP2/A)
04444          CARR( 4, 5) = 2.*EUJUP2/A
04446          CARR( 4,12) = 6.*EUJUP2/ A**2
04450          CARR( 5, 5) = 4.*EUJUP2*L2/(A*(L2-A))
04454          CARR( 5, 6) = 2.*EUJUP2/(L2-A)
04456          CARR( 5,12) = 6.*EUJUP2*(1./A**2-1./(L2-A)**2)
04463          CARR( 6, 6) = S3+4.*(EUJUP2/(L2-A)+EUJUP3/C)
04472          CARR( 6, 7) = 2.*EUJUP3/C
04474          CARR( 6,12) =-6.*EUJUP2/(L2-A)**2
04500          CARR( 6,13) = 6.*EUJUP3/ C**2
04503          CARR( 7, 7) = S6+4.*EUJUP3*L3/(C*(L3-C))
04511          CARR( 7, 8) = 2.*EUJUP3/(L3-C)
04513          CARR( 7,10) =-S6
04515          CARR( 7,13) = 6.*EUJUP3*(1./C**2-1./(L3-C)**2)
04522          CARR( 8, 8) = S4+4.*EUJUP3/(L3-C)
04527          CARR( 8,13) =-6.*EUJUP3/(L3-C)**2
04532          CARR( 9, 9) = S5+4.*EUJUP4/D
04535          CARR( 9,10) = 2.*EUJUP4/D
04537          CARR( 9,14) = 6.*EUJUP4/D**2
04541          CARR(10,10) = S6+4.*L6*EUJUP4/(D*(L6-D))
04546          CARR(10,11) = 2.*EUJUP4/(L6-D)
04551          CARR(10,14) = 6.*EUJUP4*(1./D**2-1./(L6-D)**2)
04556          CARR(11,11) = S7+4.*EUJUP4/(L6-D)
04563          CARR(11,14) =-6.*EUJUP4/(L6-D)**2
04566          CARR(12,12) = KUP5+12.*EUJUP2*(1./A**3+1./(L2-A)**3)
04576          CARR(13,13) = KUP7+12.*EUJUP3*(1./C**3+1./(L3-C)**3)
04606          CARR(13,14) =-KUP7
04607          CARR(14,14) = KUP7+12.*EUJUP4*(1./D**3+1./(L6-D)**3)
04617          CARR(15,15) = 4.*E5I5*(AAA(5,2)/((HT5-HST5)*NNN(5,2)) +AAA(5,3)/
              *                 ((L5-HT5)*NNN(5,3)))
04631          CARR(15,17) = 6.*E5I5*(1./((HT5-HST5)**2*NNN(5,2))-1./((L5-HT5)**2
              *                *NNN(5,3)))
04643          CARR(16,16) = 4.*E7I7*(AAA(7,2)/((HT7-HST7)*NNN(7,2))+AAA(7,3)/
              *                 ((L7-HT7)*NNN(7,3)))
04655          CARR(16,18) = 6.*E7I7*(1./((HT7-HST7)**2*NNN(7,2))-1./((L7-HT7)**2
              *                *NNN(7,3)))
04667          CARR(17,17) = KT5+12.*E5I5*(1./((L5-HT5)**3*NNN(5,3))+1./((HT5-HST
              *                 5)**3*NNN(5,2)))
04703          CARR(18,18) = KT7+12.*E7I7*(1./((L7-HT7)**3*NNN(7,3))+1./((HT7-HST
              *                 7)**3*NNN(7,2)))
```

```
05246          CALL MATINV(CARR,T,T1,JI,40,TOL,IERR)
05255          CALL MMATRX(ARR,CARR,PDUCT,JR,JI,JI,MXX)
05264          CALL MMATRX(PDUCT,DARR,ARR,JR,JI,JR,MXX)
05273          K=1
05274          DO 8 I=2,JC
05276          DO   7 J=1,K
05277          IF(ABS(ARR(I,J)).LT.1.E-15)GO TO 6
05305          DIFF=ARR(I,J)-ARR(J,I)
05312          PERCENT=DIFF/ARR(I,J)
05316          IF(ABS(PERCENT).GT.1.E-6)GO TO 9
05322        6 CONTINUE
05322          GO TO 7
05323        9 CONTINUE
05323          PRINT 550
05327          CALL MPRINT(ARR,JR,JC,40)
05332          GO TO 700
05333        7 CONTINUE
05336          K=K+1
05337        8 CONTINUE
05341          DO 400 I=1,JR
05343          DO 400 J=1,JC
05344          QARR(I,J)= QARR(I,J)*ARR(I,J)
05354      400 CONTINUE
      C****      MULTIPLY  THE  INVERSE  OF  THE  MASS  MATRIX  AND  THE
      C                          QARR  MATRIX
05360          CALL MMATRX(MASS,QARR,PDUCT,JR,JR,JR,MXX)
05367          PRINT 2000,KR,KW,KUP1,KUP2,KUP3,KBUP3,KUP4,KBUP4,KUP5,KUP6,KUP7,S6
05423     2000 FORMAT(1X,*KR=*E11.4,*KW=*E11.4,*KUP1=*E11.4,*KUP2=*E11.4,*KUP3=*
              *E11.4,*KBUP3=*E11.4,*KUP4=*E11.4,*KBUP4=*E11.4,/1X,*KUP5=*E11.4,
              *      *KUP6=*E11.4,*KUP7=*E11.4,*S6=*E11.4)
05423     3000 FORMAT(1X,*F1=*E11.4,*KCB1=*E11.4,*KP1=*E11.4,/
              *          1X,*F3=*E11.4,*KCB3=*E11.4,*KP3=*E11.4,/
              *          1X,*F5=*E11.4,*KCB5=*E11.4,*KP5=*E11.4,/
              *          1X,*F7=*E11.4,*KCB7=*E11.4,*KP7=*E11.4)
05423          PRINT 3000,(F(I),KCB(I),KP(I),I=1,7,2)
05441     5000 FORMAT(1X,*MT1=*E11.4,*MM=*E11.4,*MTDA=*E11.4,*MSTR=*E11.4,/
              *          1X,*MT3=*E11.4,*MM=*E11.4,*MTDA=*E11.4,*MSTR=*E11.4,/
              *          1X,*MT5=*E11.4,*MM=*E11.4,*MTDA=*E11.4,*MSTR=*E11.4,/
              *          1X,*MT7=*E11.4,*MM=*E11.4,*MTDA=*E11.4,*MSTR=*E11.4)
05441          PRINT 5000,(MT(I),MM(I),MTDA(I),MSTR(I),I=1,7,2)
05461     4000 FORMAT(1X,*K1=*E11.4,*KB1=*E11.4,*KCAP2=*,E11.4,*KCAP6=*E11.4,/
              *          1X,*K3=*E11.4,*KB3=*E11.4,*KCAP3=*,E11.4,/
              *          1X,*K5=*E11.4,*KB5=*E11.4,*KCAP5=*,E11.4,/
              *          1X,*K7=*E11.4,*KB7=*E11.4,*KCAP7=*,E11.4)
05461          PRINT 4000,K1,KB1,KCAP2,KCAP6,K3,KB3,KCAP3,K5,KB5,KCAP5,K7,
              * KB7,KCAP7
05517     6000 FORMAT(1X,*HT1=*E11.4,*KT1=*E11.4,*HST1=*E11.4,*JSTR1=*E11.4,
              * *JRIG=*E11.4,/
              *          1X,*HT3=*E11.4,*KT3=*E11.4,*HST3=*E11.4,*JSTR3=*E11.4,
              * *JRIG=*E11.4,/
              *          1X,*HT5=*E11.4,*KT5=*E11.4,*HST5=*E11.4,*JSTR5=*E11.4,
              * *JRIG=*E11.4,/
              *          1X,*HT7=*E11.4,*KT7=*E11.4,*HST7=*E11.4,*JSTR7=*E11.4,
              * *JRIG=*E11.4)
05517          PRINT 6000,HT1,KT1,HST1,JSTR(1),JRIG(1),
              *            HT3,KT3,HST3,JSTR(3),JRIG(3),
              *            HT5,KT5,HST5,JSTR(5),JRIG(5),
              *            HT7,KT7,HST7,JSTR(7),JRIG(7)
```

```
05573          CALL EIGEN(JC,PDUCT,CARR,QARR,MAXIT,NDEC,N1OPT,N2OPT,40)
05604          PRINT 500,(RHO(I),I=1,7,2),(HI(I),I=1,7,2),AS,KCAP1 ,ES
05632          DO 450 I=1,JC
05634          CYC(I,1)=PDUCT(I,I)
05640          CYC(I,2)=FLOAT(I)
05641      450 CONTINUE
05644          CALL SRTCYC(CYC,JR,40)
05646          PRINT 600,(CYC(I,2),CYC(I,1),I=1,JC)
05663          CALL SECOND(Y)
05665          TIME=Y-X
05667          PRINT 551,TIME
05675      551 FORMAT(118X,5HTIME F7.2)
05675      700 CONTINUE
05675          GO TO 50
05676      702 CONTINUE
05676          STOP
05700          END
```

```
              FUNCTION SINCH(X)
000003        SINCH= .5*(EXP(X)-EXP(-X))
000015        RETURN
000015        END
```

```
              FUNCTION COSCH(X)
000003        COSCH= .5*(EXP(X)+EXP(-X))
000015        RETURN
000015        END
```

```
              SUBROUTINE MMATRX(AA,BB,CC,NR,JR,JC,MD)
         C    SUBROUTINE MULTIPLIES THE AA AND BB MATRICES AND PLACES THE ANSWER IN
000012        DIMENSION AA(MD,MD),BB(MD,MD),CC(MD,MD)
000012        DO  102  I=1,NR
000013        DO  102  J=1,JC
000014        ELEM = 0.D0
000015        DO  95   K=1,JR
000017        ELEM = ELEM + AA(I,K)*BB(K,J)
000032     95 CONTINUE
000034        CC(I,J)=ELEM
000040    102 CONTINUE
000044        RETURN
000044        END
```

```
              SUBROUTINE MPRINT ( A , M , N , MD )                                        MP
        C     MATRIX PRINT SUBROUTINE                                                     MP
        C     THE CALL FOR THIS SUBROUTINE IS AS FOLLOWS,                                 MP
        C     CALL MPRINT (A,M,N,MD)                                                      MP
        C       WHERE A IS THE MATRIX TO BE PRINTED                                       MP
        C             M IS THE NUMBER OF ROWS                                             MP
        C             N IS THE NUMBER OF COLUMNS                                          MP
        C            MD IS DIMENSIONED NO. OF ROWS OF MATRIX A                            MP
000007        DIMENSION A ( 1 ) , JT ( 6 ) , C ( 6 )                                      MP
000007        EQUIVALENCE ( JT , C )                                                      MP
000007        N1 = N
000010        N2 = 6
000011        N3 = 6
000012        N4 = 1
000013    100 IF ( N3 - N1 )  120, 120, 110
        C
000016    110 N2 = N1 - N3 + 6
000021        N3 = N1
000021    120 K = 0
000022        DO  130 I = N4, N3
000024        K = K + 1
000026    130 JT ( K ) = I
000032        PRINT  1, ( JT ( I ) , I = 1 , N2 )
000045        DO  150 I = 1, M
000051        K = 0
000052        L = MD * ( N4 - 1 ) + I
000056        DO  140 J = N4, N3
000057        K = K + 1
000061        C ( K ) = A ( L )
000064    140 L = L + MD
000070    150 PRINT  2, I , ( C ( K ) , K = 1 , N2 )
000111        IF ( N3 - N1 )  160, 170, 170
        C
000113    160 N3 = N3 + 6
000115        N4 = N4 + 6
000116        GO TO  100
        C
000116    170 RETURN
        C
000117      1 FORMAT    (1H , 4X, 6( 6X, 7HCOLUMN   1I4   )   /        )
000117      2 FORMAT    (1H  1I4, X,    (6E 17.8)    )
000117        END
        C3 BKY BESMIS
```

```
                 FORTRAN IV SUBROUTINE BESNIS (X,NMAX,FI)                         BSIS
          C      X- A FLOATING POINT SINGLE PRECISION VARIABLE
          C      J- IS THE NUMBER OF VALUES
          C      FI- IS A ONE-DIMENSIONAL ARRAY OF ANSWERS
          C      TO EVALUATE EXP(-X)I(X) MAKE THE FOLLOWING CHANGE
          C            A=1./(PI(1)+SUM)
000006           DIMENSION FI(50), PI(200)                                        BSIS
000006           SUM=0.                                                           BSIS
000006           I=X                                                              BSIS
000010           JMAX=I+21                                                        BSIS
000012           TZ=2./X                                                          BSIS
000013           PI(JMAX+2)=0.                                                    BSIS
000015           PI(JMAX+1)=1.E-20                                                BSIS
000017           DO 1 J=1,JMAX                                                    BSIS
000020           K=JMAX+2-J                                                       BSIS
000022           DK=K-1                                                           BSIS
000024           PI(K-1)=DK*TZ*PI(K)+PI(K+1)                                      BSIS
000032         1 SUM=SUM+PI(K)                                                    BSIS
000037           SUM=SUM+SUM                                                      BSIS
000040           A=EXP(X)/(PI(1)+SUM)                                             BSIS
000047           DO 2 N=1,NMAX                                                    BSIS
000050         2 FI(N)=A*PI(N)                                                    BSIS
000055           RETURN                                                           BSIS
000056           END                                                              BSIS
```

```
                 SUBROUTINE SRTCYC(A,NROWS,NDIM)
           C     SUBROUTINE SORTS THE FIRST COLUMN OF THE IN COMING ARRAY IN
           C     ASCENDING ORDER WHILE THE CORRESPONDING ROW IN THE SECOND
           C     COLUMN IS MOVED ALONG WITH IT.   THE SUBROUTINE ALSO DIVIDES
           C     THE SQRT OF THE SORTED ELEMENT BY  2*PI IF THE ELEMENT IS NOT NEGATIVE
           C     IF THE ELEMENT IS NEGATIVE IT IS PLACED BACK INTO THE ARRAY.
000006           DIMENSION A(1)
000006           DO 15 I=1,NROWS
000007           DO 14 J=I,NROWS
000010           IF(A(I)-A(J))14,14,11
000014        11 L=I-NDIM
000015           LL=J-NDIM
000017           DO 12 K=1,2
000021           L=L+NDIM
000022           LL=LL+NDIM
000023           F=A(L)
000025           A(L)=A(LL)
000027        12 A(LL)=F
000034        14 CONTINUE
000037        15 CONTINUE
000041           DO 20 I=1,NROWS
000042           IF(A(I).LT.0.0)GO TO 20
000044           A(I)=SQRT(A(I))/6.2831
000054        20 CONTINUE
000057           RETURN
000057           END
```

```
                SUBROUTINE EIGEN ( N , B , TI , T , MAXIT , NDEC , N1OPT , N2OPT ,EI
               * NDIM )                                                           EI
          C     SUBROUTINE FOR GENERATING THE EIGENVALUES AND EIGENVECTORS        EI
          C     OF A REAL SYMMETRIC OR NON-SYMMETRIC MATRIX.                      EI
          C     THIS PROGRAM GENERATES THE EIGENVALUE MATRIX(REAL OR COMPLEX),    EI
          C     AND AS OPTIONS, THE EIGENVECTOR MATRIX AND ITS INVERSE,           EI
          C       THE CALL FOR THIS SUBROUTINE IS AS FOLLOWS,                     EI
          C     CALL EIGEN (N,B,TI,T,MAXIT,NDEC,N1OPT,N2OPT,NDIM)                 EI
          C       WHERE N IS THE ORDER OF THE MATRIX                              EI
          C             B IS THE MATRIX WHOSE EIGENVALUES ARE DESIRED             EI
          C            TI IS THE INVERSE OF THE EIGENVECTOR MATRIX                EI
          C             T IS THE EIGENVECTOR MATRIX                               EI
          C         MAXIT IS MAX NO. OF ITERATIONS FOR GENERATING EIGENVALUES     EI
          C          NDEC IS THE NUMBER OF TIMES RESULT IS REFINED                EI
          C         N1OPT IS 1 IF OPTION =1 IS DESIRED, OTHERWISE 0.              EI
          C     OPTION =1 PROVIDES FOR GENERATING THE EIGENVECTOR MATRIX INVERSE  EI
          C         N2OPT IS 1 IF OPTION =2 IS DESIRED, OTHERWISE 0.              EI
          C     OPTION =2 PROVIDES FOR GENERATING THE EIGENVECTOR MATRIX          EI
          C          NDIM IS DIMENSIONED NO. OF ROWS OF MATRIX (B)                EI
          C       THE ORGINAL MATRIX B IS LOST DURING THE COMPUTATIONS AND IS     EI
          C     REPLACED BY THE EIGENVALUE MATRIX.                                EI
000014          DIMENSION B ( 1 ) , TI ( 1 ) , T ( 1 )                            EI
          C     INITIALIZE COUNTERS FOR NO. OF ITERATIONS AND YR,YS REDUCTIONS    EI
000014          PRINT  1, N , NDEC , MAXIT                                        EI
000025          IT = 0                                                            EI
000026          NTIMES = 0                                                        EI
000026          ANORM = 0 . 0                                                     EI
000030          DO  100 I = 1, N                                                  EI
000035          DO  100 J = 1, N                                                  EI
000036          IJ = ( J - 1 ) * NDIM + I                                         EI
000042      100 ANORM = ANORM + B ( IJ ) * * 2                                    EI
000052          ANORM = SQRTF ( ANORM )                                           EI
000054          DO  110 I = 1, N                                                  EI
000061          DO  110 J = 1, N                                                  EI
000062          IJ = ( J - 1 ) * NDIM + I                                         EI
000066      110 B ( IJ ) = B ( IJ ) / ANORM                                       EI
          C     FORM IDENTITY MATRIX IN TI LOCATION IF OPTION 1 IS DESIRED        EI
000076      120 IF ( N1OPT )  160, 160, 130                                       EI
          C                                                                       EI
000100      130 DO  150 I = 1, N                                                  EI
000102          II = ( I - 1 ) * NDIM + I                                         EI
000106          DO  140 J = 1, N                                                  EI
000107          IJ = ( J - 1 ) * NDIM + I                                         EI
000113      140 TI ( IJ ) = 0 .                                                   EI
000120      150 TI ( II ) = 1 . 0                                                 EI
          C     FORM IDENTITY MATRIX IN T LOCATION IF OPTION 2 DESIRED            EI
000124      160 IF ( N2OPT )  200, 200, 170                                       EI
          C                                                                       EI
000126      170 DO  190 I = 1, N                                                  EI
000130          II = ( I - 1 ) * NDIM + I                                         EI
000134          DO  180 J = 1, N                                                  EI
000135          IJ = ( J - 1 ) * NDIM + I                                         EI
000141      180 T ( IJ ) = 0 .                                                    EI
000146      190 T ( II ) = 1 . 0                                                  EI
000152      200 CONTINUE                                                          EI
000152          YR = 10 . 0 E - 7                                                 EI
000153          YS = 10 . 0 E - 7                                                 EI
```

```
000155      210 NO = N - 1                                          EI
000156          SUM = 10 . 0 E 20                                   EI
000160      220 TAU = 0 . 0                                         EI
000161          EN = 0 . 0                                          EI
000162          DO  230 I = 1, NO                                   EI
000163          JO = I + 1                                          EI
000165          DO  230 J = JO, N                                   EI
000167          IJ = ( J - 1 ) * NDIM + I                           EI
000173          JI = ( I - 1 ) * NDIM + J                           EI
000176      230 TAU = TAU + B ( IJ ) * * 2 + B ( JI ) * * 2         EI
000211          DO  240 I = 1, N                                    EI
000213          II = ( I - 1 ) * NDIM + I                           EI
000216          TE = B ( II )                                       EI
000221      240 EN = EN + TE * * 2                                  EI
000225      250 EN = EN + TAU                                       EI
000227      260 DELN = SUM - EN                                     EI
000231      270 IF ( DELN )  290, 290, 280                          EI
          C                                                         EI
000233      280 SUM = EN                                            EI
000234          IT = IT + 1                                         EI
000236          IF ( MAXIT - IT )  970, 970, 310                    EI
          C                                                         EI
000241      290 CONTINUE                                            EI
000241          IF ( NDEC - NTIMES )  970, 970, 300                 EI
          C                                                         EI
000243      300 YR = YR / 100 . 0                                   EI
000245          YS = YS / 100 . 0                                   EI
000247          PRINT  2, NTIMES , IT                               EI
000256          NTIMES = NTIMES + 1                                 EI
000260          IT = IT + 1                                         EI
000261      310 DO  960 K = 1, NO                                   EI
000266          KK = ( K - 1 ) * NDIM + K                           EI
000271          KO = K + 1                                          EI
000273      320 DO  960 M = KO, N                                   EI
000275          MM = ( M - 1 ) * NDIM + M                           EI
000300          KM = ( M - 1 ) * NDIM + K                           EI
000303          MK = ( K - 1 ) * NDIM + M                           EI
000307          H = 0 . 0                                           EI
000307          G = 0 . 0                                           EI
000310          HJ = 0 . 0                                          EI
000312      330 DO  450 I = 1, N                                    EI
000314      340 IF ( I - K )  350, 450, 350                         EI
          C                                                         EI
000316      350 IF ( I - M )  360, 450, 360                         EI
          C                                                         EI
000320      360 IK = ( K - 1 ) * NDIM + I                           EI
000324          KI = ( I - 1 ) * NDIM + K                           EI
000327          IM = ( M - 1 ) * NDIM + I                           EI
000333          MI = ( I - 1 ) * NDIM + M                           EI
000335          BO = B ( IK )                                       EI
000340      370 BR = B ( KI )                                       EI
000343      380 BQ = B ( IM )                                       EI
000346      390 BS = B ( MI )                                       EI
000351      400 H = H + BR * BS - BO * BQ                           EI
000356      410 TEP = BO * BO + BS * BS                             EI
000361      420 TEM = BR * BR + BQ * BQ                             EI
000364      430 G = G + TEP + TEM                                   EI
000367      440 HJ = HJ - TEP + TEM                                 EI
```

```
000156        ITEMP = ICOL ( MAXC )                                    MA
000160        ICOL ( MAXC ) = ICOL ( 1 )                               MA
000162        ICOL ( 1 ) = ITEMP                                       MA
000164    200 TEMP = A ( ITER )                                        MA
000166        ITEMP = ICOL ( 1 )                                       MA
000170        DO  210 J = 2, N                                         MA
000172        ITJM1 = ( J - 2 ) * NDIM + ITER                          MA
000175        ITJ = ( J - 1 ) * NDIM + ITER                            MA
000200        A ( ITJM1 ) = A ( ITJ ) / TEMP                           MA
000204    210 ICOL ( J - 1 ) = ICOL ( J )                              MA
000211        ITN = ( N - 1 ) * NDIM + ITER                            MA
000214        A ( ITN ) = 1 . 0 / TEMP                                 MA
000217        ICOL ( N ) = ITEMP                                       MA
000221        DO  240 I = 1, N                                         MA
000223        IF ( I - ITER )  220, 240, 220                           MA
       C                                                               MA
000225    220 TEMP = A ( I )                                           MA
000230        DO  230 J = 2, N                                         MA
000231        IJM1 = ( J - 2 ) * NDIM + I                              MA
000234        IJ = ( J - 1 ) * NDIM + I                                MA
000237        ITJM1 = ( J - 2 ) * NDIM + ITER                          MA
000242        A ( IJM1 ) = A ( IJ ) - A ( ITJM1 ) * TEMP               MA
000251    230 CONTINUE                                                 MA
000254        IN = ( N - 1 ) * NDIM + I                                MA
000257        ITN = ( N - 1 ) * NDIM + ITER                            MA
000261        A ( IN ) = - ( TEMP * A ( ITN ) )                        MA
000267    240 CONTINUE                                                 MA
000274        DO  290 I = 1, N                                         MA
000275        DO  250 J = I, N                                         MA
000276        IF ( IROW ( J ) - I )  250, 260, 250                     MA
       C                                                               MA
000301    250 CONTINUE                                                 MA
000304    260 IF ( I - J )  270, 290, 270                              MA
       C                                                               MA
000306    270 DO  280 L = 1, N                                         MA
000310        LI = ( I - 1 ) * NDIM + L                                MA
000313        LJ = ( J - 1 ) * NDIM + L                                MA
000316        TEMP = A ( LI )                                          MA
000321        A ( LI ) = A ( LJ )                                      MA
000324    280 A ( LJ ) = TEMP                                          MA
000331        IROW ( J ) = IROW ( I )                                  MA
000334    290 CONTINUE                                                 MA
000337        DO  340 I = 1, N                                         MA
000340        DO  300 J = I, N                                         MA
000341        IF ( ICOL ( J ) - I )  300, 310, 300                     MA 1
       C                                                               MA 1
000344    300 CONTINUE                                                 MA 1
000347    310 IF ( I - J )  320, 340, 320                              MA 1
       C                                                               MA 1
000351    320 DO  330 L = 1, N                                         MA 1
000353        IL = ( L - 1 ) * NDIM + I                                MA 1
000356        JL = ( L - 1 ) * NDIM + J                                MA 1
000361        TEMP = A ( IL )                                          MA 1
000364        A ( IL ) = A ( JL )                                      MA 1
000367    330 A ( JL ) = TEMP                                          MA 1
000374        ICOL ( J ) = ICOL ( I )                                  MA 1
000377    340 CONTINUE                                                 MA 1
000402        IROW ( NP1 ) = 0                                         MA 1
```

```
000613          JK = ( K - 1 ) * NDIM + J                                          EI 1
000617          JM = ( M - 1 ) * NDIM + J                                          EI 1
000622      860 BO = B ( JK )                                                      EI 1
000625      870 BR = B ( JM )                                                      EI 1
000630      880 B ( JK ) = BO * C2 - BR * S2                                       EI 1
000635      890 B ( JM ) = - BO * S1 + BR * C1                                     EI 1
000643      900 IF ( N1OPT )  930, 930, 910                                        EI 1
          C                                                                        EI 1
000645      910 DO  920 J = 1, N                                                   EI 1
000647          KJ = ( J - 1 ) * NDIM + K                                          EI 1
000653          MJ = ( J - 1 ) * NDIM + M                                          EI 1
000656          BQ = TI ( KJ )                                                     EI 1
000661          BS = TI ( MJ )                                                     EI 1
000663          TI ( KJ ) = C1 * BQ + S1 * BS                                      EI 1
000670      920 TI ( MJ ) = S2 * BQ + C2 * BS                                      EI 1
000676      930 IF ( N2OPT )  960, 960, 940                                        EI 1
          C                                                                        EI 1
000700      940 DO  950 J = 1, N                                                   EI 1
000702          JK = ( K - 1 ) * NDIM + J                                          EI 1
000706          JM = ( M - 1 ) * NDIM + J                                          EI 1
000711          BO = T ( JK )                                                      EI 1
000713          BR = T ( JM )                                                      EI 1
000716          T ( JK ) = BO * C2 - BR * S2                                       EI 1
000723      950 T ( JM ) = - BO * S1 + BR * C1                                     EI 1
000731      960 CONTINUE                                                           EI 1
000736          GO TO  220                                                         EI 1
          C                                                                        EI 2
000737      970 DO  980 I = 1, N                                                   EI 2
000741          DO  980 J = 1, N                                                   EI 2
000742          IJ = ( J - 1 ) * NDIM + I                                          EI 2
000746      980 B ( IJ ) = B ( IJ ) * ANORM                                        EI 2
000756          PRINT  3, NTIMES , IT                                              EI 2
000765          RETURN                                                             EI 2
          C                                                                        EI 2
000766        1 FORMAT(*1EIGEN SUBROUTINE STATISTICS*/ * SIZE OF MATRIX IS*,I4,    EI 2
             * * MAX. NUM. OF REFN. IS *,I3, * MAX. NUM. OF ITER. IS *,I4,/)      EI 2
000766        2 FORMAT(*0REFN.*,I3,* FINISHED ON ITER.*,I4)                        EI 2
000766        3 FORMAT(*0EXIT ON REFN.*,I3,* ITER.*,I4)                            EI 2
000766          END                                                                EI 2
```

```
              SUBROUTINE MATINV ( A , IROW , ICOL , N , NDIM , SMLST , IERR )    MA
000012        DIMENSION A ( 1 ) , IROW ( 1 ) , ICOL ( 1 )                        MA
          C   709-16065                                                          MA
          C   709-16065  SUBROUTINE MATINV - MATRIX INVERSION ROUTINE            MA
          C                                                                      MA
          C       A = ARRAY NAME OF MATRIX                                       MA
          C    IROW = DIMENSIONED AT N+1 OR GREATER                              MA
          C    ICOL = DIMENSIONED AT N OR GREATER                                MA
          C      N  = NUMBER OF EQUATIONS                                        MA
          C    NDIM = VALUE OF I IN DIMENSION A(I,J) , I AND J MAY DIFFER        MA
          C   SMLST = SMALLEST LEADING ELEMENT ALLOWED BEFORE CALLING THE        MA
          C                SYSTEM SINGULAR , USUALLY = 1.0 E-04 OR 1.0 E-05      MA
          C    IERR = ERROR INDICATOR                                            MA
000012        IERR = 0                                                           MA
000012        NP1 = N + 1                                                        MA
000015        DO  100 I = 1, N                                                   MA
000016        ICOL ( I ) = I                                                     MA
000017    100 IROW ( I ) = I                                                     MA
000022        DO  240 ITER = 1, N                                                MA
000024        MAXR = ITER                                                        MA
000025        MAXC = 1                                                           MA
000026        TEMP = ABSF ( A ( MAXR ) )                                         MA
000031        LIMITC = NP1 - ITER                                                MA
000033        DO  120 I = ITER, N                                                MA
000034        DO  120 J = 1, LIMITC                                              MA
000035        IJ = ( J - 1 ) * NDIM + I                                          MA
000040        IF ( TEMP - ( ABSF ( A ( IJ ) ) ) )  110, 120, 120                 MA
          C                                                                      MA
000045    110 MAXR = I                                                           MA
000046        MAXC = J                                                           MA
000050        TEMP = ABSF ( A ( IJ ) )                                           MA
000053    120 CONTINUE                                                           MA
000060        IF ( TEMP - SMLST )  130, 130, 140                                 MA
          C                                                                      MA
000062    130 IROW ( NP1 ) = ITER                                                MA
000065        PRINT 1, ITER , SMLST                                              MA
000074        IERR = 1                                                           MA
000076        RETURN                                                             MA
          C                                                                      MA
000076    140 IF ( MAXR - ITER )  150, 170, 150                                  MA
          C                                                                      MA
000104    150 DO  160 J = 1, N                                                   MA
000106        MAXRJ = ( J - 1 ) * NDIM + MAXR                                    MA
000111        ITJ = ( J - 1 ) * NDIM + ITER                                      MA
000114        TEMP = A ( MAXRJ )                                                 MA
000117        A ( MAXRJ ) = A ( ITJ )                                            MA
000122    160 A ( ITJ ) = TEMP                                                   MA
000127        ITEMP = IROW ( MAXR )                                              MA
000131        IROW ( MAXR ) = IROW ( ITER )                                      MA
000134        IROW ( ITER ) = ITEMP                                              MA
000136    170 IF ( MAXC - 1 )  180, 200, 180                                     MA
          C                                                                      MA
000140    180 DO  190 I = 1, N                                                   MA
000142        IMAXC = ( MAXC - 1 ) * NDIM + I                                    MA
000145        TEMP = A ( I )                                                     MA
000147        A ( I ) = A ( IMAXC )                                              MA
000153    190 A ( IMAXC ) = TEMP                                                 MA
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000372      450 CONTINUE                                              EI 1
000375      460 H = 2 . 0 * H                                         EI 1
000377          D = B ( KK ) - B ( MM )                               EI 1
000403          TEP = B ( KM )                                        EI 1
000405          TEM = B ( MK )                                        EI 1
000410          C = TEP + TEM                                         EI 1
000411          E = TEP - TEM                                         EI 1
000413      470 IF ( ABSF ( C ) - YR )  480, 480, 510                 EI 1
          C                                                           EI 1
000417      480 CC = 1 . 0                                            EI 1
000421      490 SS = 0 . 0                                            EI 1
000422      500 GO TO  620                                            EI 1
          C                                                           EI 1
000423      510 BY = D / C                                            EI 1
000425      520 IF ( BY )  530, 540, 540                              EI 1
          C                                                           EI 1
000427      530 SIG = - 1 . 0                                         EI 1
000431          GO TO  550                                            EI 1
          C                                                           EI 1
000431      540 SIG = 1 . 0                                           EI 1
000433      550 COT = BY + ( SIG * SQRTF ( BY * BY + 1 . 0 ) )        EI 1
000442      560 SS = SIG / SQRTF ( COT * COT + 1 . 0 )                EI 1
000450      570 CC = SS * COT                                         EI 1
000452      580 TEP = CC * CC - SS * SS                               EI 1
000455      590 TEM = 2 . 0 * SS * CC                                 EI 1
000460      600 D = D * TEP + C * TEM                                 EI 1
000464      610 H = H * TEP - HJ * TEM                                EI 1
000470      620 CONTINUE                                              EI 1
000470      630 ED = 2 . 0 * E * D                                    EI 1
000473      640 EDH = ED - H                                          EI 1
000475      650 DEN = G + 2 . 0 * ( E * E + D * D )                   EI 1
000502      660 TEE = EDH / ( DEN + DEN )                             EI 1
000505      670 CONTINUE                                              EI 1
000505      680 IF ( ABSF ( TEE ) - YS )  690, 690, 710               EI 1
          C                                                           EI 1
000514      690 CH = 1 . 0                                            EI 1
000516      700 SH = 0 . 0                                            EI 1
000517          GO TO  730                                            EI 1
          C                                                           EI 1
000520      710 CH = 1 . 0 / SQRTF ( 1 . 0 - TEE * TEE )              EI 1
000526      720 SH = TEE * CH                                         EI 1
000530      730 C1 = CH * CC - SH * SS                                EI 1
000534      740 C2 = CH * CC + SH * SS                                EI 1
000540      750 S1 = CH * SS + SH * CC                                EI 1
000544      760 S2 = - CH * SS + SH * CC                              EI 1
000550      770 CONTINUE                                              EI 1
000550      780 IF ( S1 )  800, 790, 800                              EI 1
          C                                                           EI 1
000555      790 IF ( S2 )  800, 960, 800                              EI 1
          C                                                           EI 1
000556      800 DO  840 J = 1, N                                      EI 1
000560          KJ = ( J - 1 ) * NDIM + K                             EI 1
000564          MJ = ( J - 1 ) * NDIM + M                             EI 1
000570      810 BO = B ( KJ )                                         EI 1
000573      820 BR = B ( MJ )                                         EI 1
000576      830 B ( KJ ) = C1 * BO + S1 * BR                          EI 1
000603      840 B ( MJ ) = S2 * BO + C2 * BR                          EI 1
000611      850 DO  890 J = 1, N                                      EI 1
```

```
000404          RETURN                                                                MA 1
           C                                                                          MA 1
000404        1 FORMAT (7HOON THEI3,63HTH ITERATION ALL THE REMAINING TERMS WERE LMA 1
               *ESS THAN OR EQUAL TO E11.4,18H IN ABSOLUTE VALUE)                     MA 1
000404          END                                                                   MA 1
```