

N 71-14071

NASA CR 111733

CASE FILE  
COPY



UNIVERSITY  
OF  
ARKANSAS

Graduate Institute of Technology

Department of Electronics and Instrumentation

Technical Report

SEPARATE REFERENCE BEAM HOLOGRAPHIC INTERFEROMETRY AND ITS APPLICATION  
TO THE MEASUREMENT OF SMALL PHASE VARIATIONS IN SUB-FRINGE SYSTEMS

November 1, 1970

A Technical Report

for

work performed under

NASA Research Grant NGL 04-001-007  
Ames Research Center  
Moffett Field, California

for that portion of the work entitled

SEPARATE REFERENCE BEAM HOLOGRAPHIC INTERFEROMETRY AND ITS APPLICATION  
TO THE MEASUREMENT OF SMALL PHASE VARIATIONS IN SUB-FRINGE SYSTEMS

by

Department of Electronics and Instrumentation  
University of Arkansas Graduate Institute of Technology  
Little Rock 72203



M. K. Testerman  
Principal Investigator

November 1, 1970

## TABLE OF CONTENTS

	Page
Abstract	
Introduction	1
Work Performed	4
Conclusions	35
References	36

LIST OF TABLES

	Page
1. Analysis of Phase Angles as Measured from Dual Trace Recording, First Setting, First Run	20
2. Analysis of Phase Angles as Measured from Dual Trace Recording, First Setting, Second Run	21
3. Analysis of Phase Angles as Measured from Dual Trace Recording, First Setting, Third Run	22
4. Phase Angles as Measured from Dual Trace Recording, Second Setting, First Run	23
5. Phase Angles as Measured from Dual Trace Recording, Second Setting, Second Run	24
6. Phase Angles as Measured from Dual Trace Recording, Second Setting, Third Run	25
7. Phase Angles as Measured from Recorded Average Phase Meter Output, First Run	28
8. Phase Angles as Measured from Recorded Average Phase Meter Output, Second Run	29
9. Phase Angles as Measured from Recorded Average Phase Meter Output, Third Run	30
10. Phase Angles as Measured from Recorded Average Phase Meter Output, Fourth Run	31
11. Phase Angles as Measured from Recorded Average Phase Meter Output, Fifth Run	32
12. Phase Angles as Measured from Recorded Average Phase Meter Output, Sixth Run	33
13. Mean Phase Angle and Standard Deviation Obtained from Tables 9 through 12	34

## LIST OF FIGURES

1. Recording a Double Exposure Hologram With Separate Reference Beams
2. Interferogram Reconstructed from Dual-Reference Hologram
3. Interferogram Reconstructed With Phase of Comparison Beam Shifted  $180^\circ$
4. Finite Fringe Interferogram Observed With Horizontal Change in Reconstructing Angle
5. Finite Fringe Interferogram Observed With Vertical Change in Reconstructing Angle
6. Reconstruction of Uniform Field
7. Real Time Interferogram
8. "Double Pass" Interferogram
9. Interferogram Showing Difference in Optical Components
10. Method of Measuring Small Phase Variations Using a Separate Reference Beam Hologram
11. Signals Produced When Measuring Phase Shift
12. Hologram Recording and Reconstructing Arrangement
13. Mechanical Device Used to Change the Path Length of One Reconstructing Beam
14. Amplifier Circuit Diagram
15. Dual Recording of Detector Output Signals
16. Recording of Averaged Phase Meter Output Signal

#### ACKNOWLEDGEMENTS

The study described in this report was supported by Research Grant NGL 04-001-007 from the National Aeronautics and Space Administration, Ames Research Center, Moffett Field, California. The personnel of the Department of Electronics and Instrumentation of the University of Arkansas Graduate Institute of Technology who assisted the Principal Investigator in this research work are:

G. S. Ballard, Research Associate

S. Sengupta, Graduate Research Assistant.

SEPARATE REFERENCE BEAM HOLOGRAPHIC INTERFEROMETRY AND ITS APPLICATION  
TO THE MEASUREMENT OF SMALL PHASE VARIATIONS IN SUB-FRINGE SYSTEMS

ABSTRACT

A method of dual exposure holographic interferometry which utilizes a separate reference beam for each of the two exposures is described. It is demonstrated that the holographically recorded interference fringes can be shifted for improved visibility and that finite and infinite fringe patterns can be obtained from a single hologram. The method can be used in recording interferograms of transient phenomena, for real time interferometry, and for comparing optical components. In addition, the method is the basis of a technique for the quantitative measurement of small phase differences in an optical field, whether the field exists in real time or is holographically stored. It is demonstrated that relative phase measurements have been accomplished with a standard deviation of as low as 1 degree for measurements made within a two minute period and of 1.4 degrees for measurements recorded over a thirty minute period.

SEPARATE REFERENCE BEAM HOLOGRAPHIC INTERFEROMETRY AND ITS APPLICATION  
TO THE MEASUREMENT OF SMALL PHASE VARIATIONS IN SUB-FRINGE SYSTEMS

G. S. Ballard and S. Sengupta

Introduction

Some of the most promising applications of holography have been in the field of interferometry. Using holograms, interferometric studies are now practical in areas which were not possible using conventional methods. One popular technique is known as real-time holographic interferometry<sup>1</sup>. With this method, a holographically reconstructed comparison beam is caused to interfere with some "real-time" test scene. Diffuse objects as well as transparent objects may be studied. The minute deformation of a structural member under load can be observed by superimposing the holographically reconstructed virtual image of the member upon the actual member itself and applying the load. This method is quite suitable for studying objects that are essentially static, allowing sufficient time for the interference fringes to be observed. Dynamic objects can also be used, provided the fringes can be recorded by some suitable means, such as a high speed movie camera. The hologram recording and reconstructing methods are straightforward, as the hologram is in reality a simple off-axis hologram. Precise optical components are not required, but all components must be stable for relatively long periods of time. One problem encountered with this method is that great care must be taken to insure that the photographic emulsion upon which the hologram has been recorded is in no way distorted during processing. Emulsion shrinkage or sagging will result in an imperfect match between the virtual image and the actual object and unwanted interference fringes will appear.



Since the interference pattern occurs in real-time, the system can be adjusted for either finite or infinite fringe patterns and is quite flexible.

Another widely used technique is that of double exposure holographic interferometry.<sup>2</sup> In this method both the test scene and the comparison beam are hologram reconstructions. First, the test scene is recorded using the normal off-axis hologram procedure. Then the test scene is removed and the comparison beam is recorded on the same photographic emulsion as a double exposure. After processing, the hologram is reconstructed by illuminating the emulsion with the off-axis reference beam. Both the test scene and the comparison beam are reconstructed and the interference between them is observed.

Some of the advantages of using this method are: (1) both transparent and diffuse subjects can be studied; (2) high quality optics are not required; (3) the components must be stable only during the time required to complete the two exposures; (4) emulsion instability affects both reconstructions equally, its effect being cancelled out during reconstruction; and (5) transient events can be studied by using a pulsed laser to record the dual exposure hologram. The main disadvantage of this method is that the reconstructions of the test scene and the comparison beam are inseparable, being linked together by the common reference beam. The particular interference presentation which is obtained is determined by the relative conditions present during the two exposures and cannot be changed by the observer. Either finite or infinite fringe interferograms are possible, but the decision must be made before the hologram recording is made. The interferogram which is obtained

may not show the desired features of the test scene to the best advantage, possibly a serious problem when recording transient phenomena.

With the addition of several components, a typical off-axis hologram arrangement can be modified into a three-beam system which can be used to study interference fields. The method can be described as dual exposure holographic interferometry with separate reference beams. The resulting system combines many of the advantages of both the real-time and double-exposure methods, while eliminating many of their respective disadvantages. Transparent and diffuse subjects can be studied. Precise optical components are not required. Emulsion stability is not a problem. Transient phenomena can be studied. Finite and infinite fringe interferograms are easily obtained from a single hologram, and the interference fringes can be adjusted after the hologram has been recorded. For transparent subjects, schlieren photographs and shadowgraphs can be obtained if desired, and the interference field can be observed in real time.

One disadvantage of all interferometric methods is that very small phase deviations are difficult to interpret. A number of methods have been proposed to increase the sensitivity of interferometric measurements. In addition to the capabilities pointed out above, the separate reference beam system to be described is the basis for an instrument which is capable of measuring small phase variations in an optical field. These phase measurements have been made with a standard deviation of as little as one degree, and this can be accomplished whether the field exists in real time or is holographically recorded.

In this report, the separate reference beam system will be described in detail and several examples will be presented to point out various

capabilities of the method. Finally, the application of the method to the measurement of very small phase variations within an optical field will be described.

### Work Performed

A typical arrangement for recording a double exposure hologram with separate reference beams is illustrated in Figure 1. Essentially this is a normal off-axis hologram recording arrangement to which an extra beam splitter has been added to provide three beams instead of two. With this arrangement, a hologram of  $R_{obj}$  could be recorded by using either  $R_1$  or  $R_2$  as the reference beam. In practice, this is exactly what is done.

To record a double exposure hologram, the test object of interest would be placed in the center beam,  $R_{obj}$ . A hologram of this object is then recorded using beam  $R_1$  as the reference beam.  $R_2$  would be stopped and would not illuminate the hologram recording plate during this first exposure. The dual-exposure would be completed by removing the test object from  $R_{obj}$  and recording a hologram of this beam using  $R_2$  as the reference. During this second exposure  $R_1$  would be stopped and thus would not illuminate the hologram recording plate. The only difference in this technique and the well known double exposure method is in the use of a different reference beam for each of the two exposures. This allows two distinct holograms of  $R_{obj}$  to be formed on a single emulsion, one hologram with the test object present and one without.

After processing, the hologram can be replaced in its original position. If it is illuminated with  $R_1$  alone, the original test scene will be reconstructed. This reconstruction can be used for obtaining shadowgrams or

schlieren photographs, or any other purpose for which a hologram may be suitable. (It is pointed out that if the test object were still located in the beam  $R_{obj}$ , illumination of the hologram with both  $R_1$  and  $R_{obj}$  would give the same results as the real-time type of interferometer described in the introduction of this report.) If the hologram is reconstructed using only beam  $R_2$ , then  $R_{obj}$  without the test subject will be reconstructed. Illumination by both  $R_1$  and  $R_2$  simultaneously will result in the reconstruction of  $R_{obj}$  both with and without the test scene present. These reconstructions are coaxial, and the interference pattern of the test object will be observed.

A hologram was recorded using the above procedure, utilizing a microscope slide as the test object. Figure 2 shows the interference pattern of the object which is obtained when the hologram is reconstructed with both  $R_1$  and  $R_2$  simultaneously. This is the pattern that would have resulted from a conventional double exposure hologram as described in the introduction to this report. One major difference, however, is that the two reconstructions are now independent of each other. If the resulting pattern does not show the desired information to the best advantage, the fringes can be shifted by any desired amount by merely altering the phase of either  $R_1$  or  $R_2$ . Figure 3 is the same reconstruction as shown in Figure 2, except that the phase of  $R_2$  has been shifted approximately 180 degrees during the reconstruction. This resulted in a corresponding phase shift in the beam reconstructed by  $R_2$ , but did not alter the phase of the beam reconstructed by  $R_1$ . It can be seen that certain details of the interference pattern near the upper edge of the slide are more easily distinguished in this figure. Had the phase variation over the entire test object been less than one-half wavelength, a sub-fringe interferogram would have resulted which would be very difficult

to interpret. By proper adjustment of the relative phases of  $R_1$  and  $R_2$  during reconstruction, the small phase variations could be displayed at maximum visibility. Several convenient methods can be used to adjust the phase of the reconstructing beam. These would include piezoelectric transducers, variable pressure chambers, and glass of varying thicknesses. For the preceding figures a very simple method was used. A glass microscope slide (actually, the identical slide which was used as the object of the hologram) was placed in one of the reconstructing beams prior to collimation. Adjustment of the slide perpendicular to the beam caused various thicknesses of the slide to be inserted into the beam, varying the path length of that beam.

For some applications it is more desirable to study a finite fringe interferogram. This type presentation can be obtained during reconstruction by altering the angle at which either  $R_1$  or  $R_2$  illuminates the hologram. The effect is the same as would be obtained by slightly rotating one of the mirrors in a Michelson or other standard type laboratory interferometer. Figure 4 illustrates the result of a slight horizontal change in the angle at which  $R_2$  illuminates the hologram during reconstruction. The greater the change, the more closely spaced the fringe pattern will become. The effect of a small vertical change in  $R_2$  is shown in Figure 5. Combinations of adjustments both horizontally and vertically can produce any desired orientation and spacing of the finite fringes.

Figures 2 through 5, all reconstructed from the same hologram, are representative of the infinite variation of fringe presentation available to an observer during reconstruction, with no prior knowledge of the test subject or special procedures required during the hologram exposure. This

capability would be a great asset in studying various transient phenomena, in which flexibility of the reconstruction could be useful in assuring that the maximum amount of information would be derived from a single recording of the event.

Although the sample chosen for these figures is transparent, the method works equally well for diffuse objects. This fact has been ascertained in this laboratory and has also been independently described at length in the literature<sup>3</sup>. For this reason it was not felt necessary to duplicate the preceding figures using a diffuse object.

The separate reference beam technique has proved quite useful in the laboratory for the study of various transparent or reflective objects in real time, with the assurance that emulsion distortion will not produce unwanted interference. For real-time interferometry, a double exposure hologram is recorded with separate reference beams in the same manner as described previously, except that no object is placed in  $R_{obj}$  for either of the two exposures. After processing and upon reconstruction of this hologram a uniform optical field will be observed. Figure 6 shows the reconstruction of such a hologram. Since there was no difference in  $R_{obj}$  in the two exposures, no fringes would be expected. This will be true as long as the wavefronts used to reconstruct the hologram are identical to those wavefronts which originally formed  $R_1$  and  $R_2$ . If either of the two reconstructing beams is perturbed in any manner, the same perturbation will appear in the reconstruction. If, then, some object is placed in  $R_2$  during reconstruction, any phase variations in this object will be present in the beam reconstructed by  $R_2$ . Reconstruction of the hologram with the original

$R_1$  and the perturbed  $R_2$  will result in the interference pattern of the perturbing object appearing in the reconstruction.

Figure 7 is identical with Figure 6 in every respect, except that a transparent test object has been placed in beam  $R_2$  during reconstruction. The test object was the same slide used in previous figures, and its interference pattern can be observed in the reconstruction in real time. These real-time fringes can be shifted by the same means as described for the holographically stored image which was discussed previously, and finite fringe interferograms can also be obtained.

A variation of the real-time technique would require that the test object be placed in one of the two reference beams when the hologram is recorded. If the object is left in place during the reconstruction process, a uniform field similar to Figure 6 will be reconstructed. If, however, the object is removed during reconstruction, the reconstructing beam is perturbed. This perturbation will be present in the reconstruction, and the interference pattern of the test object will be observed. For example, the microscope slide used previously could be placed in reference beam  $R_2$  when the hologram is recorded. If the slide remains in  $R_2$  during the reconstruction, a uniform field will be observed. If the slide is removed from  $R_2$  during reconstruction, an interference pattern similar to Figure 7 will be observed. This procedure is not, of course, real-time and there would be no apparent advantage in obtaining an interferogram in this manner. However, it does help to describe a method for doubling the sensitivity of the interferometer for certain suitable subjects.

Figure 8 is an interference pattern of the now familiar slide used in previous examples, except that this pattern exhibits twice the number of

fringes as previously observed. To obtain this increase in sensitivity the slide was placed in reference beam  $R_1$  when the hologram was recorded. By removing the slide from  $R_1$  during reconstruction the reconstructed beam was altered and the interference pattern of the slide could be observed. The slide was then placed in reconstruction beam  $R_2$  and aligned so that its shadow on the hologram fell exactly upon the shadow that was present when the hologram was exposed. The placement of the slide in  $R_2$  resulted in that beam being altered by exactly the same degree as  $R_1$ , but in the opposite direction. The result of these two equal but opposite alterations is the increased sensitivity seen in Figure 8.

The separate reference beam system which has been described would appear to lend itself well to applications which would require the comparison of various optical components. If, for example, it was desired to compare or match the output waveforms of several collimators, the following procedure could be used. A hologram producing a uniform reconstruction such as that in Figure 6 could be recorded using the "standard" collimator to form reference beam  $R_1$ . During reconstruction the "standard" would be replaced by one of the collimators to be tested. Any difference between the test unit and the "standard" would be observed in real-time as an interference pattern in the reconstruction. The test collimator could then be adjusted until the fringes were eliminated or minimized, after which other units could be similarly tested.

In order to illustrate this application and also to determine the actual difficulty involved in switching various components, the wavefronts of two commercial collimators have been compared. The same hologram used for Figures 6 and 7 was placed in the system and aligned so that a uniform field similar



to Figure 6 was obtained. The collimators which were used to form beams  $R_1$  and  $R_2$  were then exchanged and the system re-aligned. The resulting interference pattern is shown in Figure 9. If the two wavefronts had been identical, no fringe pattern would have been observed. After Figure 9 was recorded on film, the collimators were returned to their original locations and adjusted until the uniform field similar to Figure 6 was again obtained. This procedure was more difficult than that required to replace and adjust only one collimator, yet the total time consumed in alignment, changing both collimators, photographing the fringes, and restoring the system to its original condition did not exceed ten minutes. In other cases components of the system have been replaced by other units in even less time.

The double exposure holographic interferometer with separate reference beams has proven to be a useful, versatile and simple method of holographic interferometry for many applications, offering several unique capabilities. The hologram system is slightly more complicated than a conventional two beam system due to the addition of a second reference beam, but even this small inconvenience arises only once, when the various components are originally arranged, and the resulting advantages make this effort negligible. The resulting technique, in common with other forms of holographic interferometry, does not require precise optical components. Diffuse as well as transparent subjects can be studied. As with the standard double-exposure technique, high speed transient events can be recorded for study, and emulsion stability during processing is of no consequence. However, the method also possesses the flexibility of the real-time holographic laboratory type interferometers when they are used with static test objects.

This enables even transient sub-fringe systems to be displayed to their maximum visibility and best advantage.

As attractive as the dual reference beam technique appears to be considering the various characteristics which have been described, its widest application may well be in the area of sub-fringe interferometry. This method, with its unique properties and capabilities, is the basis for an instrument with which it is possible to measure very small phase variations within an optical field, whether the field exists in real time or is holographically stored. The resulting instrument combines simplicity with a reasonable degree of precision, relative phase measurements having been made with a standard deviation of only 1 degree.

One method for utilizing dual reference beam holograms for the measurement of phase variations was discussed in the Informal Status Progress Report for NASA Research Grant Nsg 713 (NGL 04-001-007) for the period ending September 30, 1968. In this method, the light intensity at a point of interest within the interference field was compared with a standard intensity. The standard used was the reconstructed comparison beam. The beam reconstructing the test scene beam was interrupted by means of a mechanical chopper, causing a light detector placed within the reconstruction to detect alternately the comparison beam alone and then the interference field. The resulting alternating signal was nulled out by changing the phase of one of the two reconstructing beams. The device used to change the phase of the reconstructing beam was a small pressure chamber, which permitted the path length of the beam passing through the chamber to be increased or decreased as the pressure inside the chamber was adjusted. The pressure within the chamber necessary to null out the alternating detector signal

was a measure of the optical phase at that point, relative to any other point within the field. With this method, a precision of measurement of  $\pm 1/60$  of a wavelength was achieved. The precision was limited mainly by the fact that thermal gradients within the system caused the path lengths of the reconstructing beams to vary by as much as  $1/60$  of a wavelength during the time required to null the detector signal. Nevertheless, the feasibility of the instrument was proven.

A new method was required for detecting the phase within the interference field which would not be affected by the small gradients which limited the precision of the original method. Of course, measures could have been taken to control the thermal variations by operating at a carefully controlled constant temperature, but this approach was not consistent with the overall goal of the project: a simple, inexpensive system of reasonable precision.

The method chosen is shown in Figure 10. This figure illustrates the reconstruction of a double-exposed hologram which has been recorded as shown in Figure 1. Two right-angle prisms, one of which can be moved in the direction of the arrows, have been placed in reconstructing beam  $R_2$  just prior to collimation. As the prism is caused to move, the path length of  $R_2$  is changed. In this manner the relative phase of  $R_2$  is continuously varied with respect to  $R_1$ , and as a result the relative phase of the two beams reconstructed by  $R_1$  and  $R_2$  are also continuously varied. The phase variation is periodic in nature, one cycle being completed as the optical path length of  $R_2$  is altered by a distance equal to one wavelength. The frequency of the phase variation is determined by the rate at which the prism is moved.

If a light detector is placed at some point in the reconstructed optical field (Detector 1, Figure 10), it will produce an electrical signal which is proportional to the instantaneous light intensity at that point. As the prism is moved, the intensity measured by the detector will vary as alternate conditions of constructive and destructive interference occur. The output signal of the detector will be a sine wave, the frequency of which depends again upon the rate at which the prism moves. A second detector placed at any other point in the reconstructed field will produce a similar signal. For purposes of illustration, let us assume that Detector 1 of Figure 10 happened to be placed upon a dark fringe in the reconstruction and Detector 2 was located upon a bright fringe. The optical phase of these two points would be 180 degrees out of phase. As the prism is caused to move, both of these detectors would experience alternate conditions of constructive and destructive interference and will produce alternating electrical signals. These two electrical signals will not be in phase, however. Due to the location of the two detectors within the optical field, one signal will be at a maximum when the other is at a minimum, and vice-versa. The two electrical signals will be "out of phase" by 180 degrees, or by the exact amount corresponding to the difference in optical phase between the two points at which the detectors are located.

A dual reference beam hologram was reconstructed as shown in Figure 10, and an effort was made to locate the two detectors 180 degrees apart in optical phase. As the prism was moved, the output of each of these detectors was displayed on a dual beam oscilloscope. A photograph of these two signals is shown in Figure 11. It will be observed that the

two signals are not exactly 180 degrees out of phase. This is the result of an error in the positioning of the two detectors in the interference field. The relative phase of these two signals is a true measure of the actual optical phase difference between the two points at which the detectors were located. In addition to showing the actual signals obtained from the detectors, this figure also adds more weight to the already well known fact that the human eye is not a reliable quantitative detector.

If the two detector signals are fed into an electronic phase meter, the relative phase of the two points could be read directly and recorded. A phase map of the entire optical field can be obtained by permitting one of the two detectors to remain stationary at some arbitrary reference point while the second detector is moved to all points of interest within the field. Any low frequency drift between the two paths which make up the two reconstructing beams will not have an adverse effect upon the phase measurement. This drift will affect the signal from both detectors equally, and will appear as a slight frequency modulation on both detector signals.

With the basic system that has been described, the relative phase of any two points within a reconstructed field has been determined with a one degree standard deviation. This figure is the overall precision for the entire system, including all components. It is important to point out that this precision is possible with an interference field using the separate reference beam technique, whether the field exists in real-time or is stored in the hologram. This would make it possible to record high-speed projectiles in rarified atmospheres using pulsed laser-separate reference beam holography and study the resultant interference field in detail and with precision at leisure after the event has taken place. The

phase measurement technique can be used in conjunction with the method for comparing optical components as suggested earlier in this report with the same degree of precision and in real time. A discussion of the methods and equipment used to achieve these measurements, as well as some of the problems which have been encountered, follows.

The experimental arrangement which has been used for both recording and reconstructing double exposure holograms with separate reference beams is shown in Figure 12. This arrangement differs from the schematic shown in Figures 1 and 10 in that the two reference beams are not arranged symmetrically on either side of the object beam. This arrangement has been used only for a more efficient utilization of the available work area. The large diameter collimator seen to the right in Figure 12 is the beam which has been used for  $R_{obj}$ . After the hologram has been recorded, this collimator is no longer needed and may be removed from the table, resulting in a more compact system than if  $R_{obj}$  were placed in the middle. The two smaller diameter collimators which can be seen to the left of  $R_{obj}$  are used to form  $R_1$  and  $R_2$ . These collimators illuminate the hologram, which is in the lower right of the figure. Two detectors are in place near the hologram and within the reconstruction of  $R_{obj}$ . In the background of the figure can be seen the Spectra Physics Model 112 HeNe laser which is used as the coherent light source. This laser has a rated output of 10 milliwatts CW at 6328 Angstroms. Also visible are the beam splitters and mirrors necessary to the system. These components are all mounted upon a 3 foot x 5 foot granite table. No provision has been made to isolate the table from room vibrations. The movable prism used to continuously change the optical path length of

reconstructing beam  $R_2$  is mounted upon a mechanical drive unit which can be seen to the left of the small collimators. This is only one of several methods which are available for changing the relative phase of the two reconstructing beams.

In the Informal Status Progress Report for NASA Research Grant NGL 04-001-007 for the period ending September 30, 1969, one method is described wherein the desired phase shift is obtained by using a form of diffraction grating instead of a beam splitter to obtain the various beams necessary for the system, and then moving this grating laterally during the reconstruction. The gratings used were photographic recordings of linear interference patterns and, after normal processing, did not exhibit the required diffraction efficiency. Bleaching of these gratings increased the diffraction efficiency, but caused the emulsion to be altered in such a fashion as to produce extremely non-uniform beams of light which were useless for forming the collimated reconstructing beams.

An attempt was made to change the optical pathlength of reconstructing beam  $R_2$  by slowly moving an optical wedge into the beam just prior to collimation. Results showed that the movement of the mechanical drive unit was not uniform, causing the resulting output signals and phase measurements to be erratic. The vibration of the drive motor also contributed to poor results.

In the Informal Status Progress Report for NASA Research Grant NGL 04-001-007 for the period ending April 1, 1970, another method for changing the optical path length of reconstructing beam  $R_2$  was discussed which utilized the change of index of refraction within a chamber as the pressure inside the chamber is varied. This method would seem to overcome the difficulties

encountered with a mechanical drive unit as it contains no moving parts. The use of this method pointed out several disadvantages, however. First, data could be obtained only during the relatively brief periods when the chamber was being evacuated or allowed to fill to atmospheric pressure. Also, multiple passes through the chamber were required in order to alter the path length by reasonable amounts. Mirrors which provided the multiple passes were mounted on the end plates of the chamber for ease of alignment, and, despite the use of heavy aluminum, the end plates were distorted slightly under varying vacuum conditions. The result was a slight change in the angle at which the reconstructing beam illuminated the hologram, causing a variation in the phase measurements during the evacuation of the chamber. This latter fault could have been easily designed out of the system, but a more serious problem was the fact that the rapid pump-down of the chamber caused eddies of turbulence to exist, resulting in a non-uniform change of index of refraction within the chamber.

The use of piezoelectric transducers in one of the reconstructing beams to alter the path length has been considered, but the requirements for a continuous change over a relatively large number of wavelengths does not make their application attractive at this time.

The final method used and the one with which the best precision has thus far been obtained is shown in Figure 12 and, in more detail, in Figure 13. The drive motor is mounted upon rubber pads so that its vibration will not be transmitted to the granite optical table. This motor drives a lead screw by means of a flexible belt. As the lead screw is turned, it causes an aluminum block to be displaced. The motion of this block is guided by two cylindrical rods. Any eccentricity of the lead screw can be transmitted



to the block in the form of a vertical displacement. To minimize this possibility, the aluminum block pushes a platform through an arrangement that allows vertical play between the two. The platform, whose motion is also guided by a pair of cylindrical rods, supports the right-angle prism. By this means it is intended that vibration from the drive motor and vertical movement caused by the lead screw will be avoided. The use of a right-angle prism is intended to eliminate any horizontal rotation of the laser beam. The effect of this rotation would be an alteration of the angle at which the hologram is reconstructed and accompanying errors in the phase measurements.

The light detectors in use are two identical LS-400-NPN silicon planar photo devices. The relative spectral response of these devices at 6328A is approximately 55%. The sensitive area of these devices has a diameter of 1/32 inch, or essentially a point.

The output signals from the detectors were of such amplitude that they could not be directly introduced into an available electronic phase meter without amplification. This was accomplished by utilizing two identical amplifiers, the circuit for which is shown in Figure 14. These were operated with a gain of approximately  $10^4$ .

The phase difference between the two points at which the detectors are located is measured with an AN/URM-67 phase monitor. This instrument measures the phase angle between two periodic signals over the range of 20 to 20,000 Hz to an accuracy of  $\pm 1$  degree. The frequency of the input signals is determined by the rate at which the path length of the reconstructing beam is varied, and can be changed by altering the speed of the prism drive motor. This speed was selected so as to give a frequency of approximately 280 Hz at the detectors. The phase monitor also requires input signals with amplitudes

of 4.0 to 60 volts peak-to-peak. It was this requirement that necessitated the use of the matched amplifiers mentioned previously. The specific amplitude and frequency of the two signals applied to the phase monitor were approximately 10 volts peak-to-peak at 280 Hz. These values are not critical, as long as they are within the specified operating range of the phase monitor.

With the equipment described, it was found that the phase measurements made with the system were erratic, noisy, and difficult to interpret visually. Observation of these data did indicate that the recorded phase values were fluctuating about some mean value. With all parts of the system apparently functioning normally, it was necessary to find the cause of the erratic readings, and, if possible, eliminate them.

As a first step, the two detector signals were simultaneously recorded on a double stylus instrument. Typical signals are shown in Figure 15. It was immediately obvious that the signals were not true sine waves, and that irregularities in the shape of the curves would make it impossible for the phase monitor to give a smooth, steady reading. In order to determine the actual range of information the phase meter was attempting to analyze, the phase of the recorded signals was measured directly from the recording one cycle at a time. The results of this analysis are shown in Table 1. A similar recording, made several minutes after the first, was also analyzed. The results are listed in Table 2. Comparison of these two analyses indicated that, although the variation of individual phase angles varied widely, the average values were in close agreement. A third recording, obtained several hours after the first, still showed good agreement as far as the mean phase angle was concerned. This analysis is shown in Table 3.

Table 1

Analysis of Phase Angles as Measured from Dual Trace Recording, First Setting,  
First Run

---

---

<u>Cycle Number</u>	<u>Phase Angle (degrees)</u>	<u>Deviation from Mean <math>X - \bar{X}</math> (degrees)</u>	<u><math>(X - \bar{X})^2</math></u>
1	57.3	3.0	9.0
2	57.8	3.5	12.2
3	48.2	-6.1	37.2
4	47.2	-7.1	50.4
5	52.5	-1.8	3.2
6	41.6	-12.7	161.3
7	37.9	-16.4	269.0
8	64.0	9.7	94.1
9	62.5	8.2	67.2
10	74.1	19.8	392.0

$$\bar{X} = 54.3 \text{ degrees}$$

$$\Sigma(X-\bar{X})^2 = 1095.6$$

$$\sigma = \sqrt{\frac{\Sigma(X-\bar{X})^2}{N}} = 10.5 \text{ degrees}$$

Table 2

Analysis of Phase Angles as Measured from Dual Trace Recording, First Setting,  
Second Run

---

---

<u>Cycle Number</u>	<u>Phase Angle (degrees)</u>	<u>Deviation from Mean <math>X - \bar{X}</math> (degrees)</u>	<u><math>(X - \bar{X})^2</math></u>
1	58.4	1.5	2.2
2	52.4	-4.5	20.2
3	51.5	-5.4	29.2
4	59.7	2.8	7.8
5	59.4	2.5	6.2
6	60.0	3.1	9.6

$$\bar{X} = 56.9 \text{ degrees}$$

$$\Sigma(X-\bar{X})^2 = 75.2$$

$$\sigma = \sqrt{\frac{\Sigma(X-\bar{X})^2}{N}} = 3.5 \text{ degrees}$$

Table 3

Analysis of Phase Angles as Measured from Dual Trace Recording, First Setting,  
Third Run

---

---

<u>Cycle Number</u>	<u>Phase Angle (degrees)</u>	<u>Deviation from Mean <math>X - \bar{X}</math> (degrees)</u>	<u><math>(X - \bar{X})^2</math></u>
1	55.4	-5.5	30.2
2	60.3	-0.6	0.4
3	72.4	11.5	132.2
4	68.4	7.5	56.2
5	60.0	-0.9	0.8
6	56.6	-4.3	18.5
7	55.8	-5.1	26.0
8	54.4	-6.5	42.2
9	62.1	1.2	1.4
10	63.3	2.4	5.8

---

$\bar{X} = 60.9$  degrees       $\Sigma(X - \bar{X})^2 = 313.7$        $\sigma = \sqrt{\frac{\Sigma(X - \bar{X})^2}{N}} = 5.6$  degrees

---

One of the detectors was then moved to a different location within the reconstructed field and another series of signal recordings was made. These recordings were all obtained within an hour of each other. The analyses of these curves are listed in Tables 4, 5, and 6. Again, the agreement between the mean values was quite good, considering the small number of cycles recorded and the relatively long time interval between the recordings.

Table 4  
Phase Angles as Measured from Dual Trace Recording,  
Second Setting, First Run

<u>Cycle Number</u>	<u>Phase Angle (degrees)</u>	<u>Deviation from Mean <math>X - \bar{X}</math> (degrees)</u>	<u><math>(X - \bar{X})^2</math></u>
1	79.3	-4.6	21.2
2	80.9	-3.0	9.0
3	74.6	-9.3	86.5
4	104.3	20.4	416.2
5	80.4	-3.5	12.2

$$\bar{X} = 83.9 \text{ degrees} \quad \Sigma(X - \bar{X})^2 = 545.1 \quad \sigma = \sqrt{\frac{\Sigma(X - \bar{X})^2}{N}} = 10.4 \text{ degrees}$$

Table 5

Phase Angles as Measured from Dual Trace Recording,  
Second Setting, Second Run

---

---

<u>Cycle Number</u>	<u>Phase Angle (degrees)</u>	<u>Deviation from Mean <math>X - \bar{X}</math> (degrees)</u>	<u><math>(X - \bar{X})^2</math></u>
1	88.7	6.1	37.2
2	88.8	6.2	38.4
3	82.8	0.2	0
4	75.2	-7.4	54.8
5	79.6	-3.0	9.0
6	79.6	-3.0	9.0
7	79.7	-2.9	8.4
8	82.1	-0.5	0.2
9	83.6	1.0	1.0
10	85.9	3.3	10.9

$$\bar{X} = 82.6 \text{ degrees}$$

$$\Sigma(X - \bar{X})^2 = 168.9$$

$$\sigma = \sqrt{\frac{\Sigma(X - \bar{X})^2}{N}} = 4.1^\circ$$

Table 6

Phase Angles as Measured from Dual Trace Recording,  
 Second Setting, Third Run

<u>Cycle Number</u>	<u>Phase Angle (degrees)</u>	<u>Deviation from Mean <math>X - \bar{X}</math> (degrees)</u>	<u><math>(X - \bar{X})^2</math></u>
1	71.5	-7.7	59.3
2	71.5	-7.7	59.3
3	75.2	-4.0	16.0
4	84.1	4.9	24.0
5	83.8	4.6	21.2
6	81.4	2.2	4.8
7	87.1	7.9	62.4

$$\bar{X} = 79.2 \text{ degrees} \quad \Sigma(X-\bar{X})^2 = 247.0 \quad \sigma = \sqrt{\frac{\Sigma(X-\bar{X})^2}{N}} = 5.0 \text{ degrees}$$

These data indicated that the erratic phase measurements were due to variation of the signals from the ideal. These variations were caused by incomplete damping of the prism's drive-motor vibrations, vibrations and irregular motion of the moving prism mount itself, building vibrations reaching the interferometer, and various transient conditions within the interferometer. The data also indicated that the average phase angle,



measured over a significant number of cycles, could be expected to give reasonably precise results.

With this in mind, the DC output of the phase meter has been averaged by placing a very simple filter between the phase meter and the recorder. This filter consists of a 10k resistor in a series and a 200 microfarad capacitor in parallel across the recorder input terminals. The time constant of this combination is 2 seconds, giving DC signal proportional to the phase angle as averaged over more than 500 cycles.

With the averaging circuit in use, two previously unnoticed sources of error became apparent. These were both associated with the mechanical drive unit used to move the prism and change the optical path length of one reconstructing beam.

One source of error was due to the "sag" of the cylindrical rods which guide the prism platform. This sag results in the prism mount traveling up or down hill as it moves, and the result of this is a slow drift of the indicated phase angle. This has been overcome by experimentally locating a portion of the drive unit which did not exhibit this drift and assuring that all subsequent measurements were made with the prism moving over this identical portion of the drive unit each time.

The second error showed up as a spike in the recorded phase information, and appeared to be caused by a sudden irregularity in the movement of the drive unit. It was found that a number of these spikes could be repeated by retracing the movement of the drive unit over a particular region. They were undoubtedly the result of some slight mechanical flaw in a portion of the drive unit itself. Other similar spikes were recorded which did not repeat, and they were attributed to a non-recurring bind in the mechanism.

Fortunately, the data recorded during these incidents is in obvious error and can immediately be discounted as having no significance with regard to the measured phase angle.

It was also found that the mechanical drive unit operated more smoothly in one direction than when it was reversed. In view of this, all measurements were recorded with the prism moving in such a manner so as to decrease the optical path length of the reconstructing beam. It would be desirable to record all phase angles with the prism moving in one specified direction in any case. If the carriage were to be reversed, the indicated phase angle would be the conjugate of that obtained when the drive unit was moving in the forward direction. This is due to the fact that one detector signal is taken by the phase meter as a reference and the phase of the second signal is measured with respect to that reference. By changing the direction of travel of the movable prism, what was originally a phase lead will become a phase lag and vice-versa, resulting in the conjugate angle being recorded.

Using the total system which has been described, a series of measurements was made to establish the precision and stability of the instrument. For these measurements the phase between two points in the reconstructed field was monitored for two minutes. The measurements were repeated several times for a total elapsed time of over 30 minutes. It was desired to establish the precision of the instrument over each of the 2 minute periods and then compare the results to determine the long term precision and stability.

A recording of the measured phase angle is shown in Figure 16. This curve represents the entire two minute period during which a phase measurement was being taken. For purposes of analysis, the phase angle indicated by the recording was read at  $7\frac{1}{2}$  second intervals. These angles and the accompanying

analysis are listed in Tables 7 through 13. Some of the  $7\frac{1}{2}$  second time periods have been omitted from the tables because the measurements at those particular points were in obvious error. These were points at which "spikes" mentioned earlier had occurred.

Table 7  
Phase Angles as Measured from Recorded Average Phase Meter  
Output, First Run

<u>Time (seconds)</u>	<u>Phase Angle (degrees)</u>	<u>Deviation from Mean <math>X - \bar{X}</math></u>	<u><math>(X - \bar{X})^2</math></u>
30.0	132.8	6.6	44.1
37.5	130.5	4.4	19.3
45.0	130.5	4.4	19.3
52.5	134.1	8.0	63.8
60.0	126.9	0.8	0.6
67.5	128.2	2.1	4.6
75.0	123.8	-2.4	5.6
82.5	121.5	-4.6	21.3
90.0	121.5	-4.6	21.3
97.5	120.6	-5.5	30.4
105.0	121.5	-4.6	21.3
120.0	121.5	-4.6	21.3

$$\bar{X} = 126.1 \text{ degrees} \quad \Sigma(X - \bar{X})^2 = 272.6 \quad \sigma = \sqrt{\frac{\Sigma(X - \bar{X})^2}{N}} = 4.8 \text{ degrees}$$

Note: System was reaching equilibrium while measurements were being recorded

Table 8

Phase Angles as Measured from Recorded Average Phase Meter

Output, Second Run

---

---

<u>Time</u> <u>(seconds)</u>	<u>Phase Angle</u> <u>(degrees)</u>	<u>Deviation from Mean</u> <u><math>X - \bar{X}</math> (degrees)</u>	<u><math>(X - \bar{X})^2</math></u>
30.0	121.5	2.4	5.8
37.5	121.5	2.4	5.8
45.0	119.7	0.6	0.4
52.5	119.7	0.6	0.4
60.0	119.7	0.6	0.4
67.5	119.7	0.6	0.4
75.0	119.7	0.6	0.4
82.5	117.0	-2.1	4.4
90.0	117.0	-2.1	4.4
97.5	117.9	-1.2	1.4
105.0	117.9	-1.2	1.4
112.5	117.9	-1.2	1.4

$$\bar{X} = 119.1 \text{ degrees}$$

$$\Sigma(X - \bar{X})^2 = 26.5$$

$$\sigma = \sqrt{\frac{\Sigma(X - \bar{X})^2}{N}} = 1.5 \text{ degrees}$$

Table 9

Phase Angles as Measured from Recorded Average Phase Meter

Output, Third Run

<u>Time (seconds)</u>	<u>Phase Angle (degrees)</u>	<u>Deviation from Mean <math>X - \bar{X}</math> (degrees)</u>	<u><math>(X - \bar{X})^2</math></u>
7.5	121.5	2.2	5.1
15.0	119.7	0.4	0.2
22.5	116.6	-2.7	7.3
30.0	116.6	-2.7	7.3
37.5	117.0	-2.2	5.1
45.0	117.0	-2.2	5.1
75.0	121.5	2.2	5.1
82.5	121.5	2.2	5.1
90.0	121.5	2.2	5.1
97.5	119.7	0.4	0.2
105.0	119.2	0	0
112.5	119.2	0	0
120.0	119.2	0	0

$$\bar{X} = 119.2 \text{ degrees}$$

$$\Sigma(X - \bar{X})^2 = 45.4$$

$$\sigma = \sqrt{\frac{\Sigma(X - \bar{X})^2}{N}} = 1.9 \text{ degrees}$$

Table 10

Phase Angles as Measured from Recorded Average Phase Meter

Output, Fourth Run

<u>Time (seconds)</u>	<u>Phase Angle (degrees)</u>	<u>Deviation from Mean <math>X - \bar{X}</math> (degrees)</u>	<u><math>(X - \bar{X})^2</math></u>
15.0	121.0	3.2	10.1
22.5	119.3	1.4	1.9
30.0	119.3	1.4	1.9
37.5	117.0	-0.9	0.8
45.0	117.0	-0.9	0.8
52.5	117.0	-0.9	0.8
60.0	117.0	-0.9	0.8
67.5	117.9	0	0
75.0	117.0	-0.9	0.8
82.5	117.9	0	0
90.0	117.9	0	0
97.5	117.0	-0.9	0.8
105.0	117.9	0	0
112.5	117.0	-0.9	0.8

$$\bar{X} = 117.9 \text{ degrees}$$

$$\Sigma(X - \bar{X})^2 = 19.2$$

$$\sigma = \sqrt{\frac{\Sigma(X - \bar{X})^2}{N}} = 1.2 \text{ degrees}$$

Table 11

Phase Angles as Measured from Recorded Average Phase Meter

Output, Fifth Run

<u>Time (seconds)</u>	<u>Phase Angle (degrees)</u>	<u>Deviation from Mean <math>X - \bar{X}</math> (degrees)</u>	<u><math>(X - \bar{X})^2</math></u>
22.5	119.2	2.5	6.1
30.0	117.4	0.7	0.4
37.5	117.9	1.1	1.3
45.0	117.0	0.1	0
52.5	117.0	0.1	0
60.0	116.1	-0.7	0.5
67.5	116.1	-0.7	0.5
75.0	116.1	-0.7	0.5
82.5	116.1	-0.7	0.5
90.0	116.1	-0.7	0.5
97.5	116.1	-0.7	0.5
112.5	116.1	-0.7	0.5

$$\bar{X} = 116.8 \text{ degrees} \quad \Sigma(X - \bar{X})^2 = 11.1 \quad \sigma = \sqrt{\frac{\Sigma(X - \bar{X})^2}{N}} = 1.0 \text{ degree}$$

Table 12

Phase Angles as Measured from Recorded Average Phase Meter

Output, Sixth Run

---

---

<u>Time (seconds)</u>	<u>Phase Angle (degrees)</u>	<u>Deviation from Mean <math>X - \bar{X}</math> (degrees)</u>	<u><math>(X - \bar{X})^2</math></u>
30.0	117.9	-0.5	0.2
37.5	117.9	-0.5	0.2
45.0	117.9	-0.5	0.2
52.5	117.9	-0.5	0.2
90.0	121.0	2.6	7.0
97.5	119.2	0.8	0.7
105.0	118.4	0	0
112.5	117.9	-0.5	0.2
120.0	117.4	-1.0	0.9

---

$$\bar{X} = 118.4 \text{ degrees} \quad \Sigma(X - \bar{X})^2 = 9.9 \quad \sigma = \sqrt{\frac{\Sigma(X - \bar{X})^2}{N}} = 1.0^\circ$$

---



Table 13

Mean Phase Angle and Standard Deviation Obtained from All Points in  
Tables 8 Through 12, 30 Minutes Elapsed Time

---

---

$$\bar{X} = 7096.5/60 = 118.3 \text{ degrees}$$

$$\Sigma(X - \bar{X})^2 = 112.0$$

$$\sigma = \sqrt{\frac{\Sigma(X - \bar{X})^2}{N}} = \sqrt{\frac{112.0}{60}} = 1.4 \text{ degrees}$$

---

---

Note: Data from Table 7 not included due to non-equilibrium condition.

Readings for the first two-minute period are listed in Table 7. These measurements were made immediately after a small adjustment was made upon the optical system of the interferometer. The slight change in measured phase angle as the interferometer slowly attained equilibrium after this adjustment has been measured. This recording was a fortunate accident, brought about by an over-eagerness to obtain data, but it does dramatically point out the sensitivity of the method and the system.

Identical measurements were repeated five more times, and the results of these are tabulated. The data for Table 10 were obtained from the curve illustrated in Figure 16. The minimum time interval from the termination of one run to the beginning of the next was two minutes. This interval was necessary to return the prism drive unit to its original position. The

total elapsed time for the five measurements tabulated in Tables 8 through 12 was 30 minutes.

As can be seen from the tables, the minimum short term standard deviation for a two minute period was 1.0 degree, with a maximum deviation from the mean of +2.5 degrees and -0.7 degrees.

The stability of the system can be ascertained by comparing the calculations from Tables 8 through 12. Table 13 lists a compilation of all the data from the five observation periods. The long term standard deviation for the entire 30 minute period was 1.4 degrees, with a maximum deviation from the mean of +3.2 degrees and -2.2 degrees.

The extreme stability of the system has been attained with no measures taken other than those normally associated with any holographic recording system. As has been pointed out, the granite table upon which the instrument is located is not isolated from building vibrations. This, coupled with the fact that the laser laboratory of the Graduate Institute of Technology is located on the fourth floor of the GIT building, and that this building is immediately adjacent to an Interstate Highway with its associated heavy truck traffic makes the performance of the system quite remarkable. It was found necessary to cover the system during phase measurements to prevent air currents from affecting the results.

### Conclusions

The dual exposure hologram with separate reference beams has been demonstrated to be a flexible, versatile, and simple method of holographic interferometry. The unique combination of advantages possessed by this method make it ideally suited for a number of applications, such as the real time

comparison of optical components and the study of holographically recorded transient systems resulting in sub-fringe interferograms.

It has also been demonstrated that the double exposure technique with separate reference beams can be utilized as the basis for an instrument that is capable of measuring small phase variations within an optical field, whether that field exists in real time or is holographically stored. The instrument, as described, has the demonstrated capability of measuring the relative phase between points with a standard deviation of as small as 1.0 degree. This precision would allow entire sub-fringe interference fields to be mapped and studied in detail, even though no fringes were visible to the naked eye. The application of this technique to transient, sub-fringe systems is but one of many possible uses.

#### References

1. K. A. Haines and B. P. Hildebrand, "Surface Deformation Measurements Using The Wavefront Reconstruction Technique", Appl. Opt., 5; 595 (1966).
2. L. O. Heflinger, R. F. Wuerker, and R. E. Brooks, "Holographic Interferometry", J. Appl. Phys., 37; 642 (1966).
3. T. Tsuruta, N. Shiotake and Y. Itoh, "Hologram Interferometry Using Two Reference Beams", Japanese J. Appl. Phys., 7; No. 9 (1968).

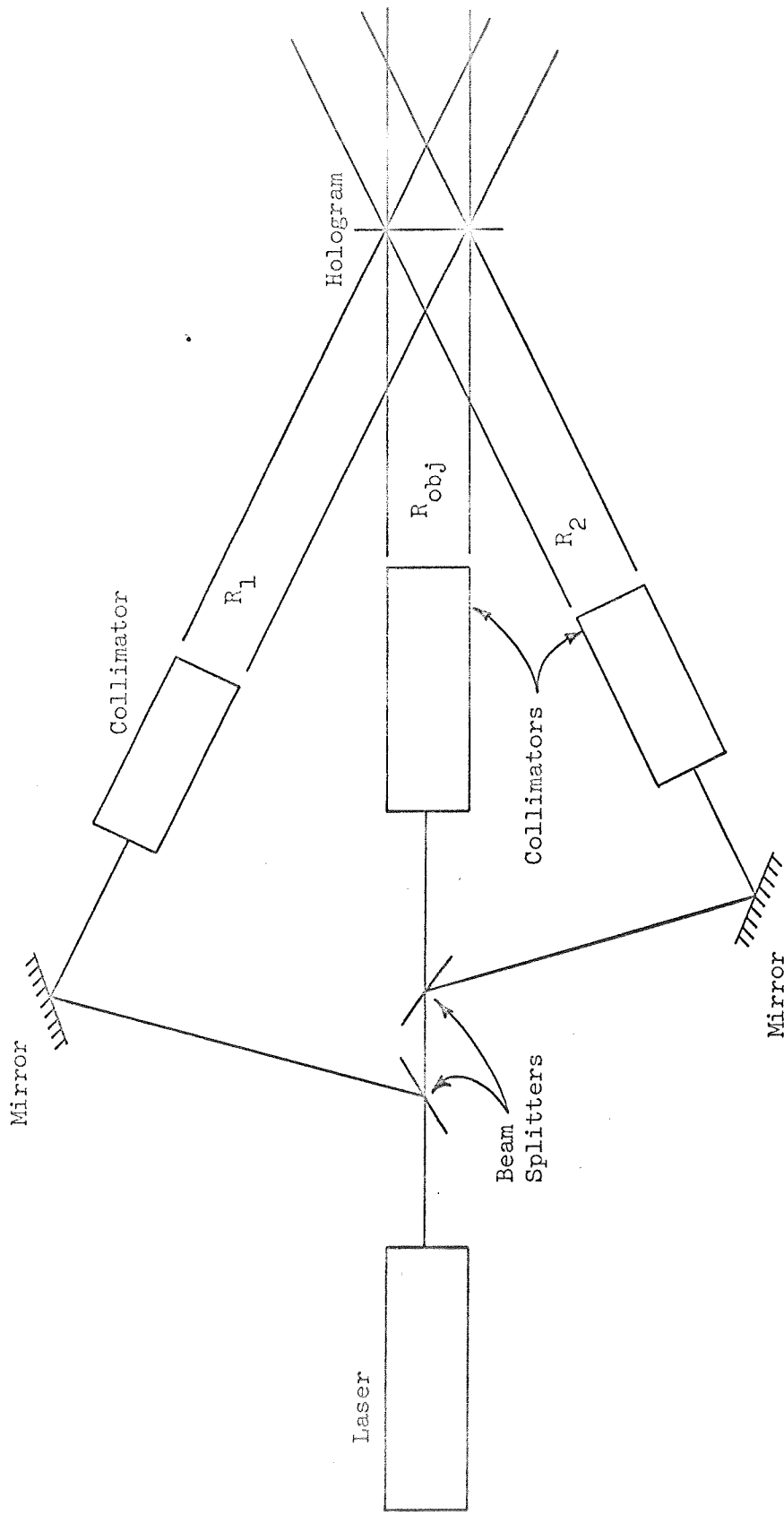


Figure 1

Recording a Double Exposure Hologram With Separate Reference Beams

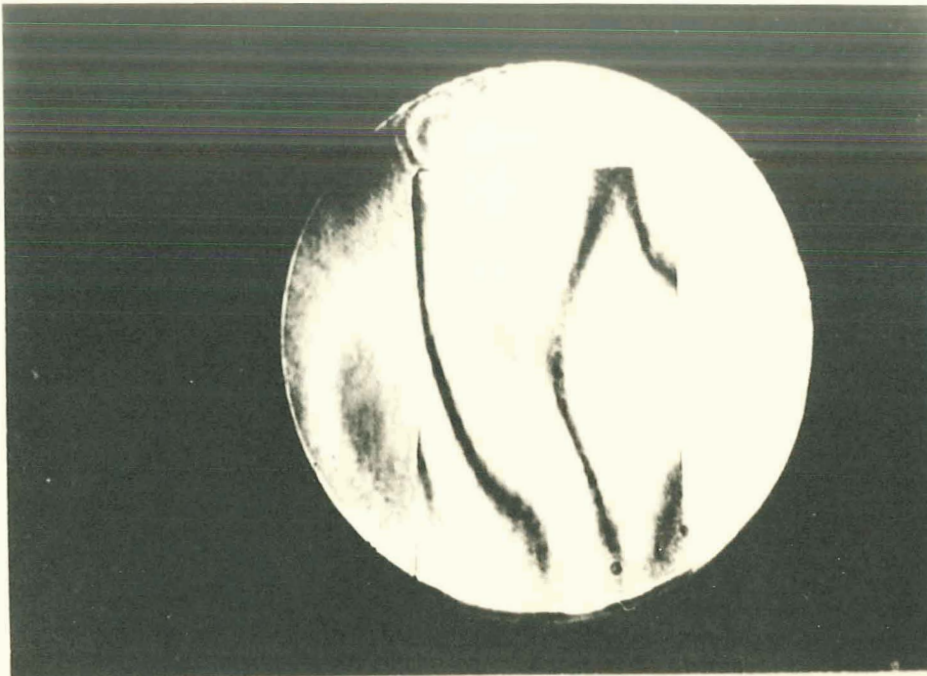


Figure 2

Interferogram Reconstructed from Dual-Reference Hologram

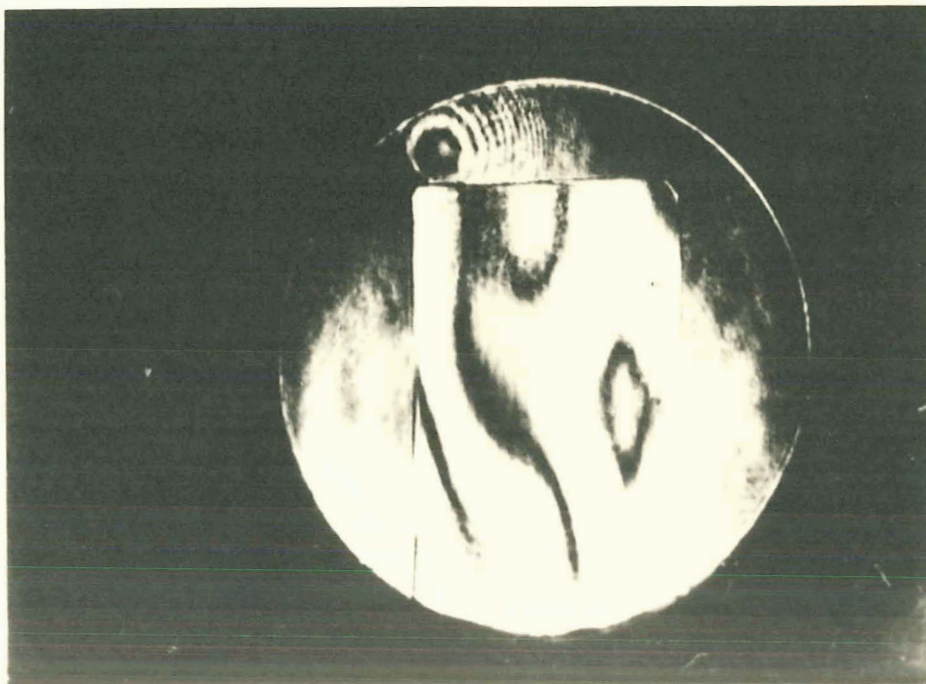


Figure 3

Interferogram Reconstructed With Phase of Comparison Beam Shifted  $180^\circ$

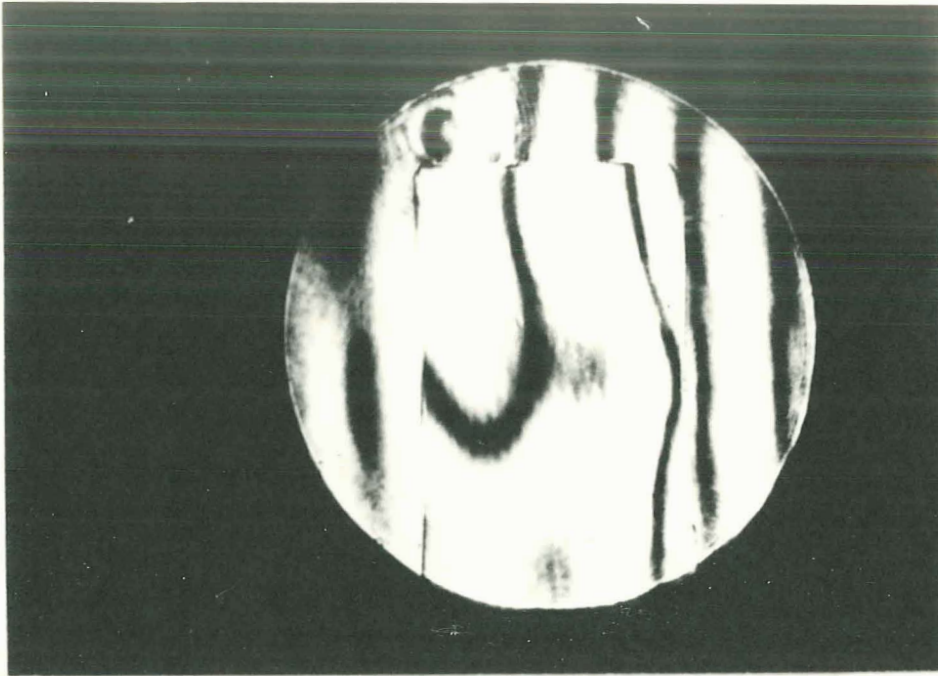


Figure 4

Finite Fringe Interferogram Observed With Horizontal  
Change in Reconstructing Angle

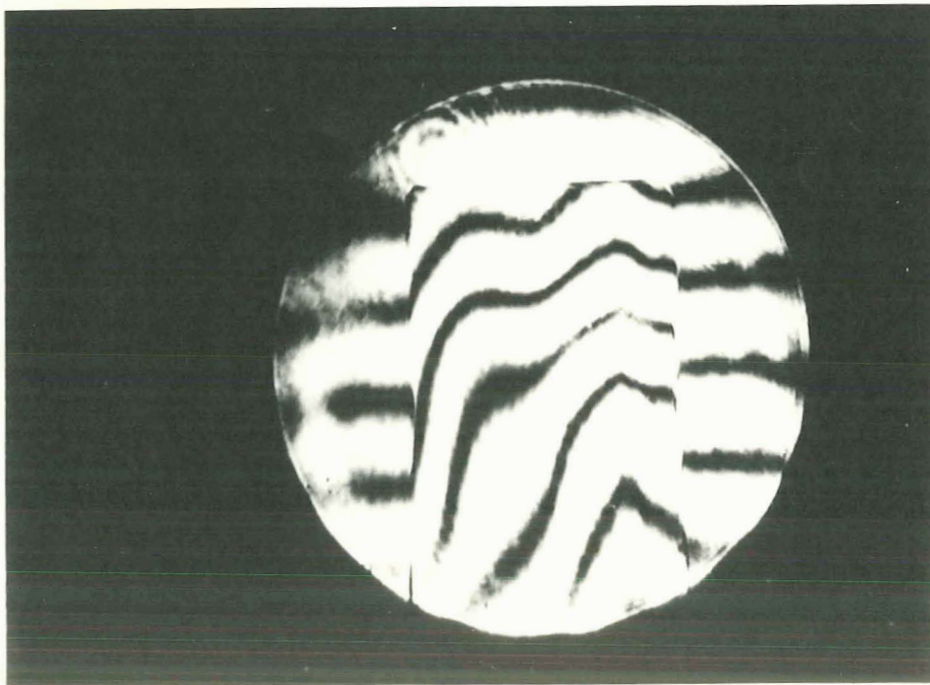


Figure 5

Finite Fringe Interferogram Observed With Vertical  
Change in Reconstructing Angle

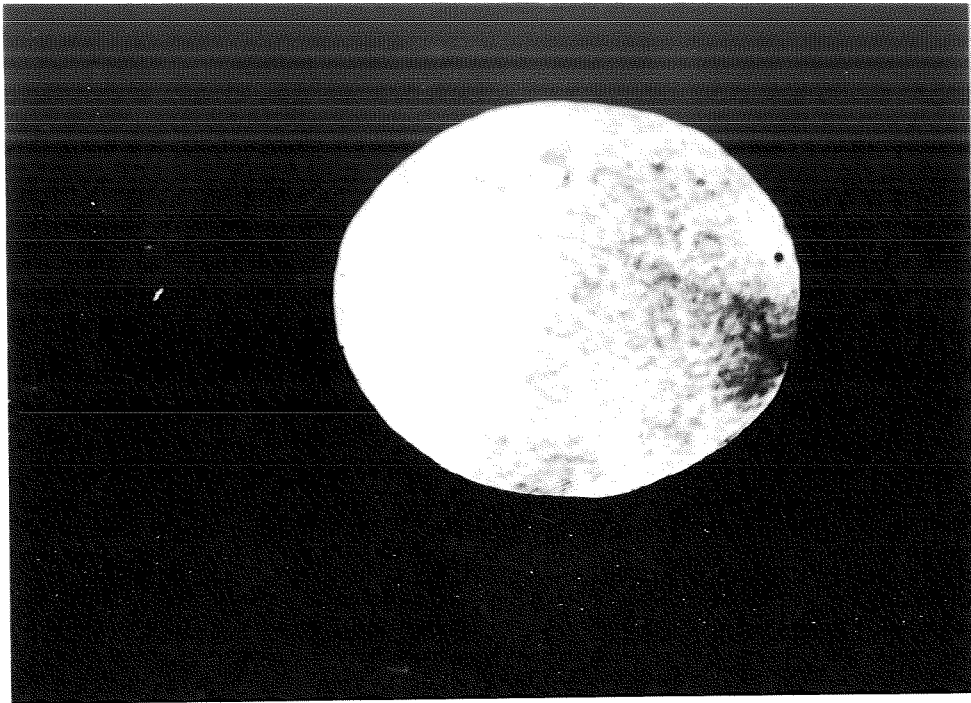


Figure 6

Reconstruction of Uniform Field



Figure 7

Real Time Interferogram



Figure 8

"Double Pass" Interferogram

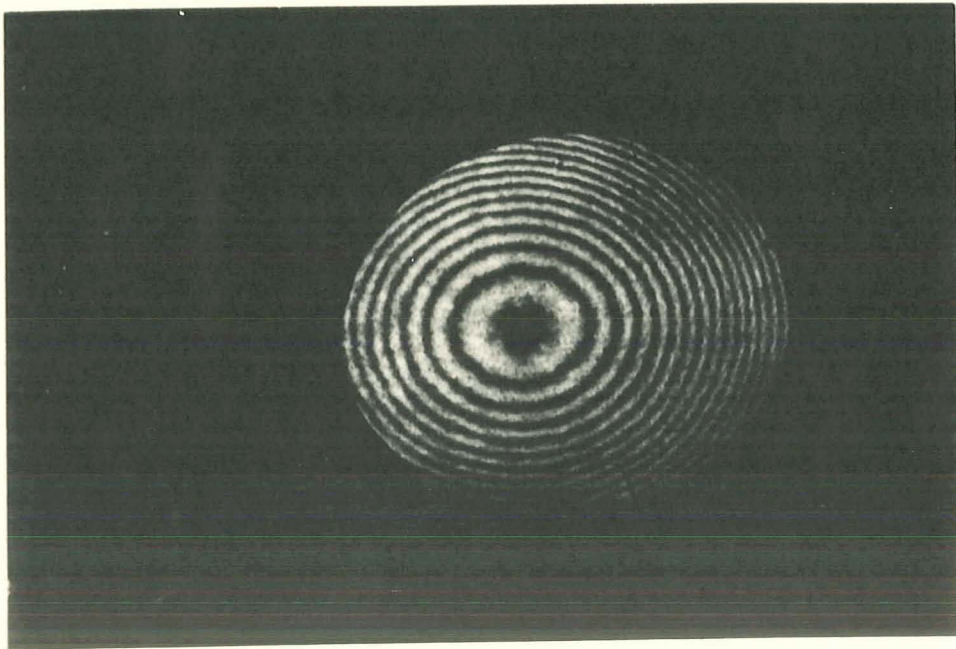


Figure 9

Interferogram Showing Difference in Optical Components



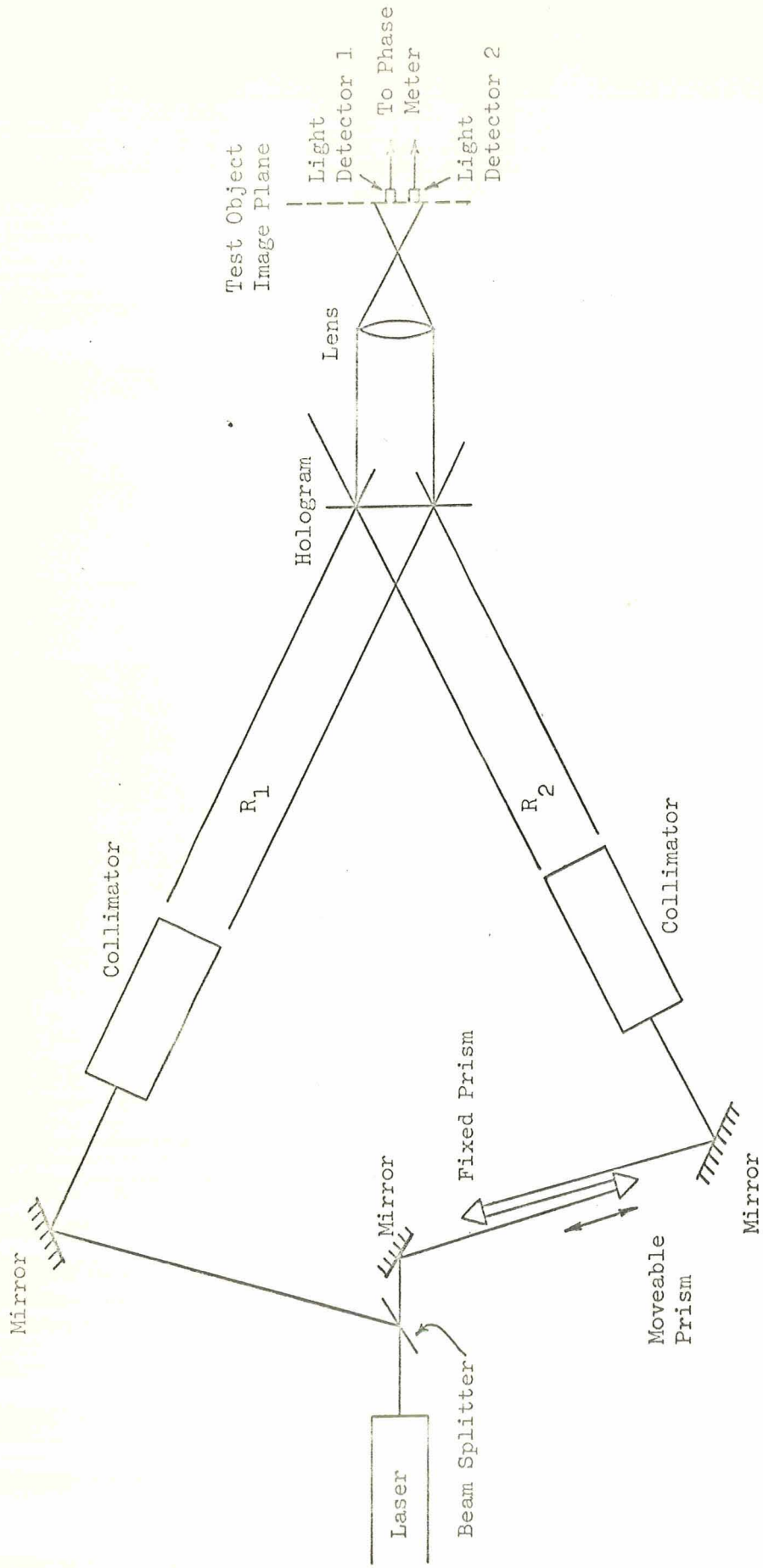


Figure 10

Method of Measuring Small Phase Variations Using a Separate Reference Beam Hologram

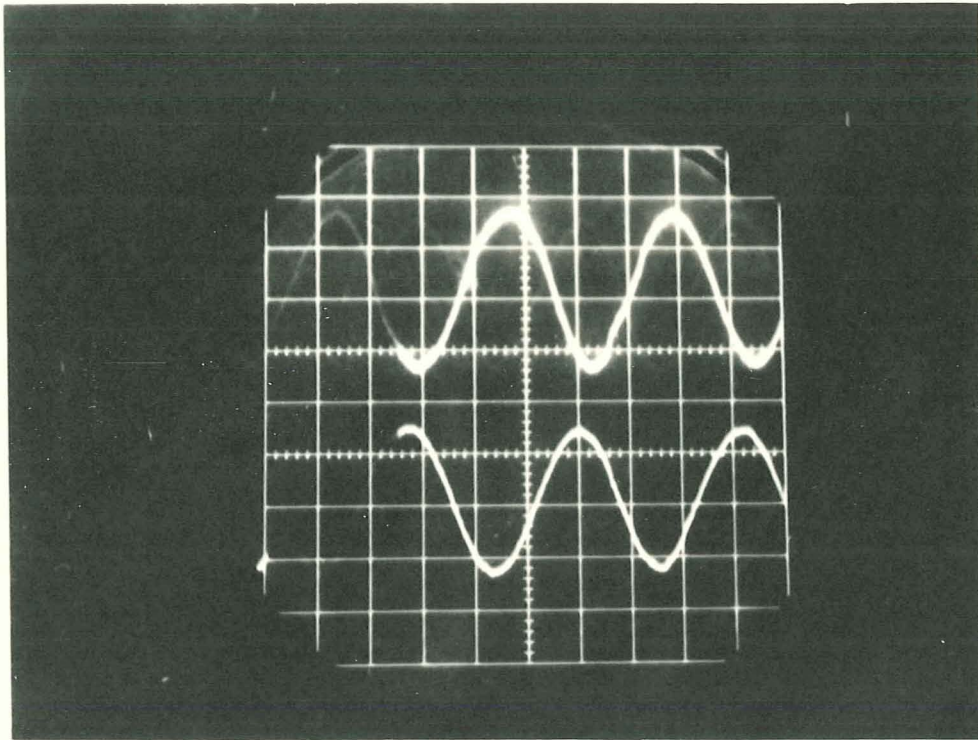


Figure 11

Signals Produced When Measuring Phase Shift

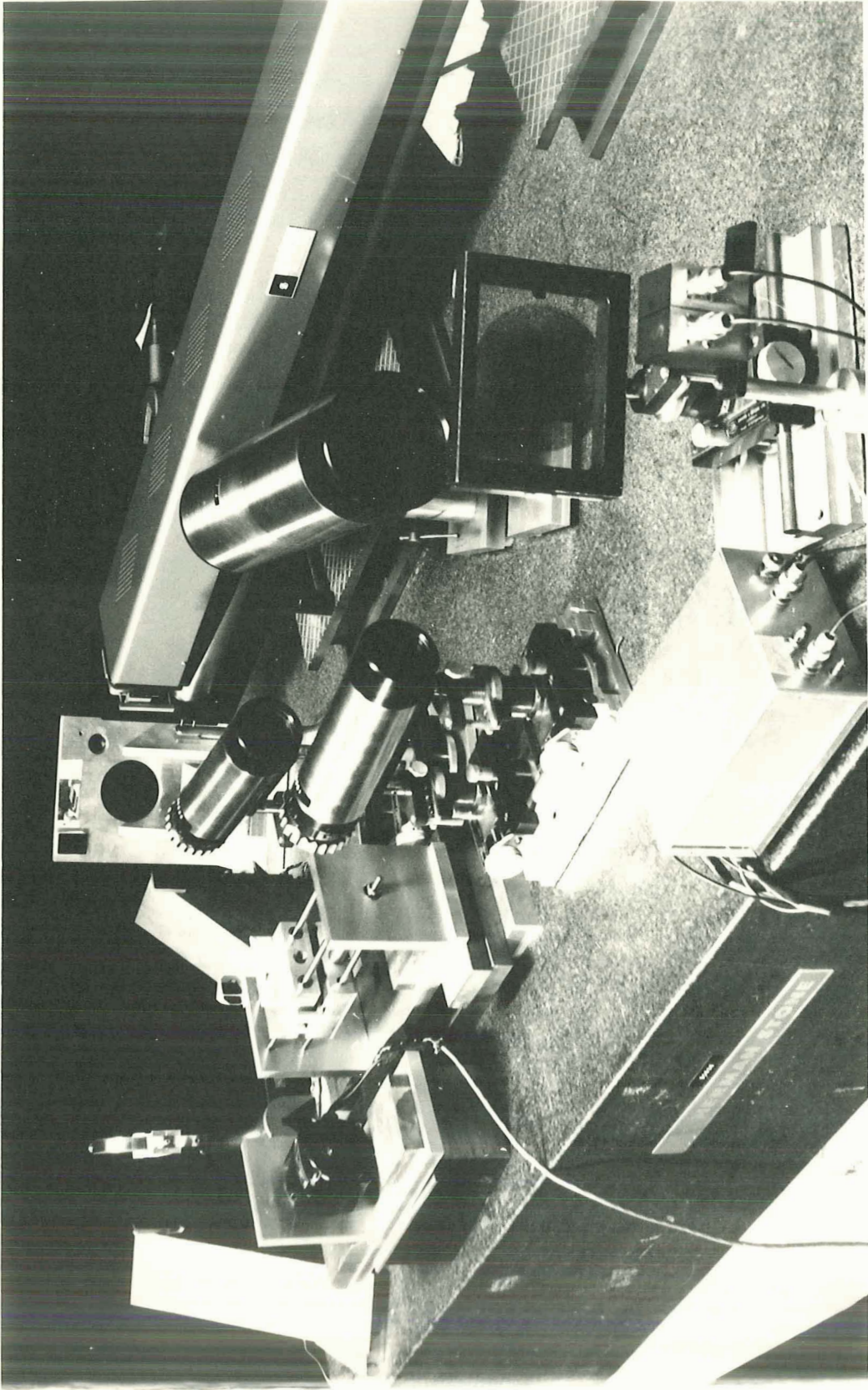


Figure 12  
Hologram Recording and Reconstructing Arrangement

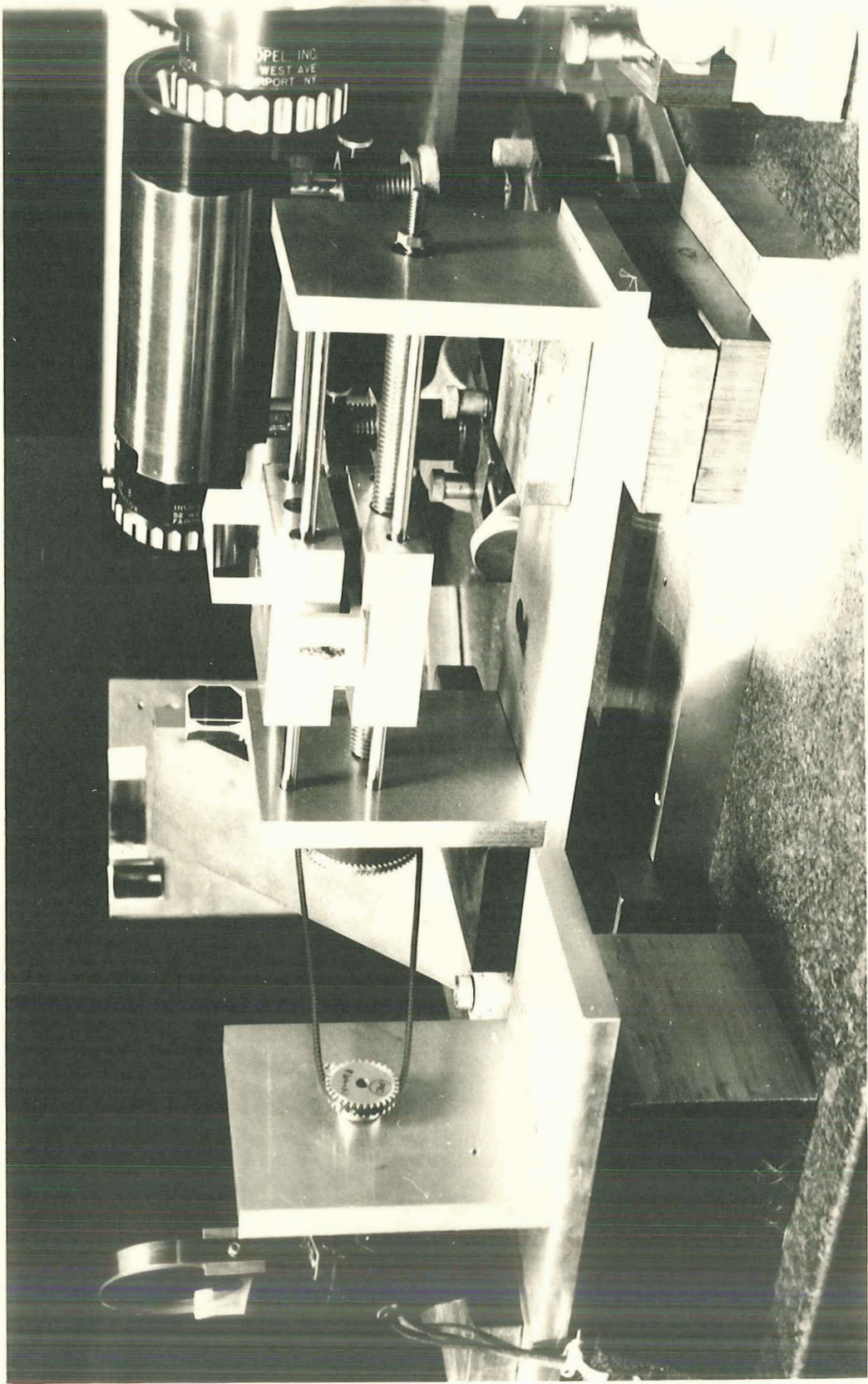
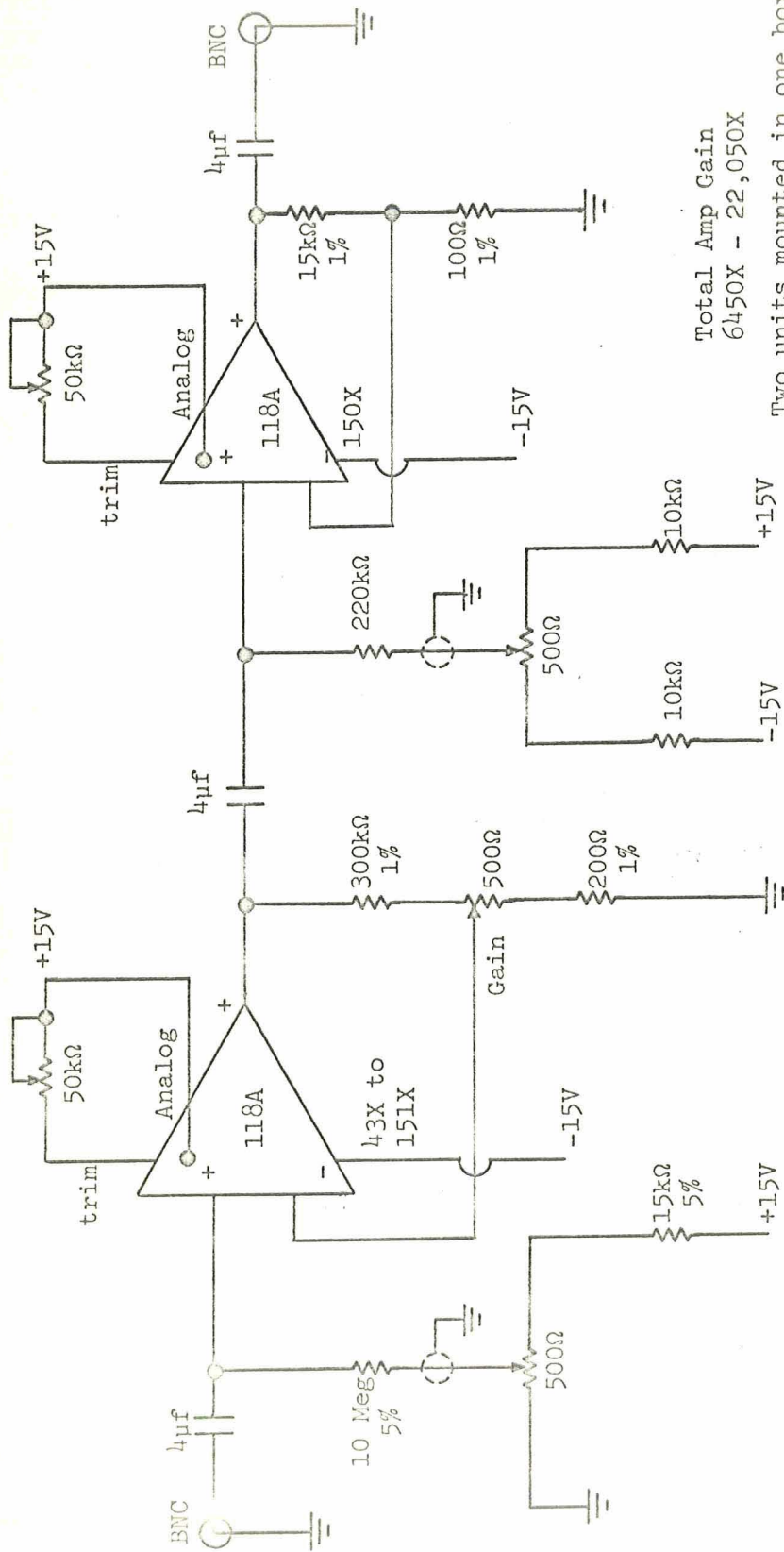


Figure 13

Mechanical Device Used to Change the Path Length of One Reconstructing Beam



Total Amp Gain  
6450X - 22,050X

Two units mounted in one box  
(separated and shielded)

Figure 14

Amplifier Circuit Diagram

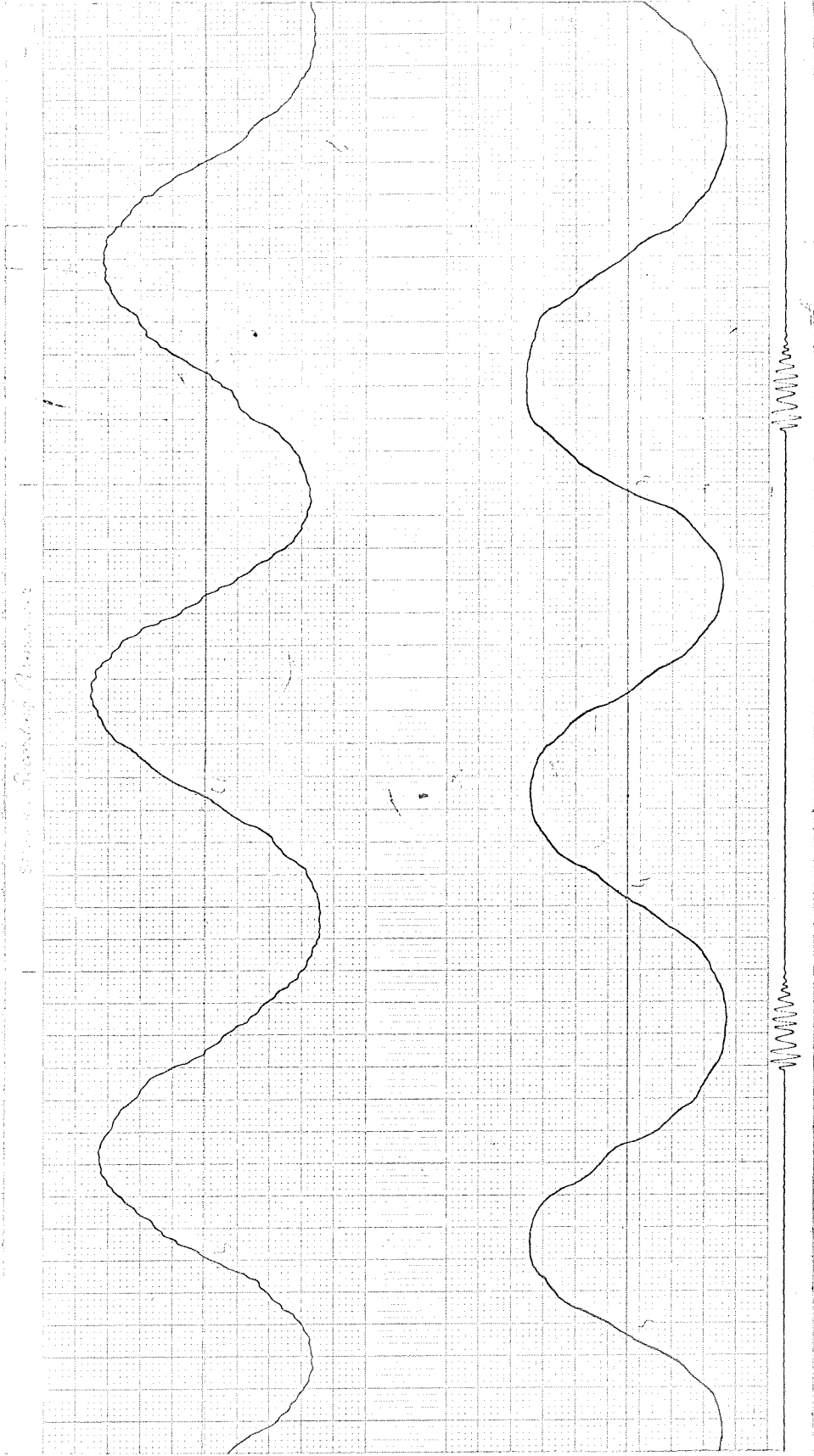


Figure 15  
Dual Recording of Detector Output Signals

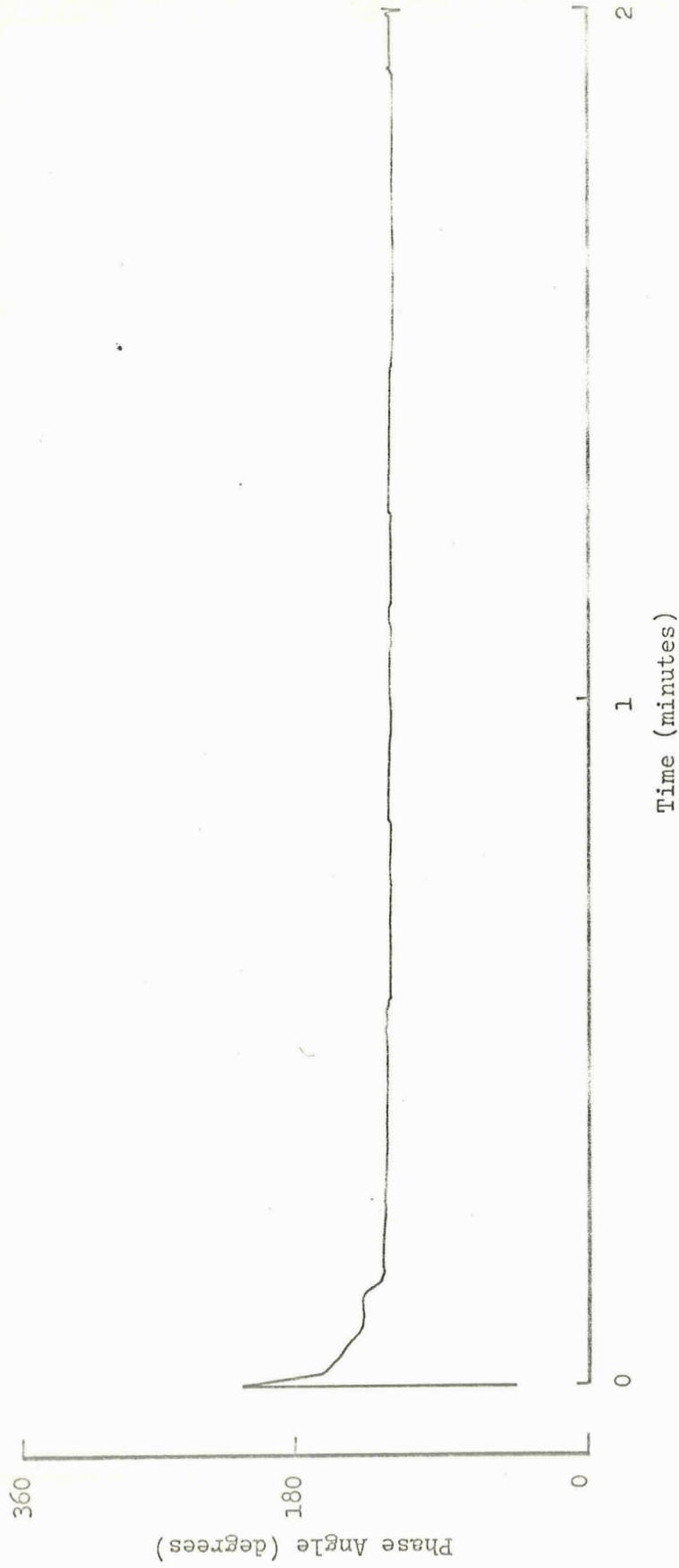


Figure 16  
Recording of Averaged Phase Meter Output Signal