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LOCKHEED MISSILES & SPACE COMPANY
HUNTSVILLE RESEARCH & ENGINEERING CENTER
HUNTSVILLE RESEARCH PARK
4800 BRADFORD DRIVE, HUNTSVILLE, ALABAMA

EXTREME DISTRIBUTIONS OF GROUND WINDS (3 TO 150 METERS) AT CAPE KENNEDY, FLORIDA

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J.L. Wood
B.L. Palmer
B.C. Johnson
J.E. Tyson

APPROVED: E. A. Carter
E.A. Carter, Supervisor
Meteorology Section

Donald McDonald
D. McDonald, Manager
Structures & Mechanics Dept.

George D. Perry for J.S. Farrior
Resident Director

FOREWORD

This report presents the results of work performed by Lockheed's Huntsville Research & Engineering Center for Marshall Space Flight Center under Contract NAS8-24927, "Functions of Atmospheric Extremes."

The NASA technical coordinator for this study is Mr. O. E. Smith of the Aerospace Environmental Division of the Aero-Astrodynamics Laboratory, MSFC.

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Section 1
INTRODUCTION AND SUMMARY

The behavior of atmospheric extremes is a subject of continuous interest in aerospace applications. Much of this interest has been centered on the study of ground winds and their effect on space vehicle design and operations. Apollo and related launch operations impose a requirement for increasingly reliable wind profile information between the surface and 150 meters. Extremal statistics and the related distributions have been used to great advantage in establishing usable wind criteria. The purpose of this report is to extend work completed earlier (Ref. 1) by introducing new approaches and modifications to previous efforts. These additional considerations are warranted primarily by: (1) availability of new multi-level tower data and (2) adoption of a new wind reference level for use in studying design and launch wind criteria. Multi-level wind measurements make possible the use of a more direct approach in such studies, thereby increasing reliability of results and their applicability to design and operational problems.

Previous distributions of peak winds in the 10 to 150-m region over Cape Kennedy, Florida, were derived indirectly from the 10-m level winds and an empirical expression relating winds between levels (see Eq. 4). The present report develops probability distributions by using three years of Cape Kennedy tower winds available for seven levels from 3.0 to 152.4 m. The three-year sample of tower data is subjected to a statistical test which demonstrates its suitability in establishing such probabilities. The multi-level set of winds is fit to a Fisher-Tippett Type I (FTI) density function. The parameters of the distributions are compared with corresponding results from Ref. 1.

Because several levels of tower data are available, bivariate extremal functions can be considered. The bivariate techniques of Gumbel and Mustafi (Ref. 2) are applied to the wind samples. Resulting parameters describe the correlation between peak winds at two different levels for a particular exposure period.

There is continuing emphasis on launch area wind environments modeled in the form of synthetic wind profiles. These profiles provide critical information needed to evaluate such effects as dynamic loads and resistance to vehicle bending moments. The present study establishes profiles using the new reference level of 152.4 m (in contrast to earlier work (Ref. 1) which used the 18.3-m level). Also, earlier studies used one annual set of parameters to describe the wind profiles. This report extends these efforts by presenting profile parameters for several seasons and hourly class intervals.

The various applications of extremal distributions to the tower data and the development of synthetic profiles provide the following specific results:

- The probability of not exceeding the wind speed at each of seven levels between the surface and 152.4 m.
- The joint probability of not exceeding the wind speed at 152.4 m and any other level below.
- The joint probability of not exceeding a given wind speed at 152.4 m and a given profile for a particular exposure period.

Supporting tables, graphs, and examples of applying these results are included.

A preliminary analysis on the behavior of peak wind directions is presented. This compares the directions of the wind measured on the hour with the peak wind direction during the associated hour.

Finally, an appendix documents (1) the feasibility of integrating extreme distributions with analytical techniques; (2) the development of a sample distribution assuming Fisher-Tippett Type I parameters; and (3) a proposed method for determining the sample variance.

Section 2
STATISTICAL ANALYSIS OF CAPE KENNEDY TOWER WIND DATA

2.1 THE DATA SAMPLE

The three-year data sample of Cape Kennedy tower winds which are analyzed in this report includes measurements of peak winds at the 3.0-m and 18.3-m levels from a small tower, and peak winds from a higher tower at the 18.3, 30.5, 61.0, 91.4, 121.9 and 152.4-m levels. From a meteorological standpoint, however, three years is considered a very short time, and even an abundance of data over this period does not change the fact that one season characterized by unusually strong or light winds would greatly bias the data. In order to test the validity of using the relatively small sample available at this time, a comparative study was made of a twelve-year peak wind sample from Cape Kennedy against a three year sample, both of which consisted only of 10-m peak winds. The three-year sample was actually a subset of the twelve-year record, but the three years chosen were for the same time period as the multi-level sample of tower data. The hypothesis tested was that the twelve-year sample and its three-year subset represent the same population. An acceptance of this hypothesis lends support to the use of only three years of tower data in constructing bivariate and univariate extremal distributions. The 10-m tower data were not used in the comparison primarily because the tower winds were measured at a location different from the site of the twelve-year sample. Because of the different roughness characteristics of the underlying surface, the value of such a comparison would be suspect. Futhermore, a portion of the 10-m tower data was not available.

Specifically, the two samples compared consisted of daily peak wind speeds at the 10-m level and for the following time periods and sample sizes n_A and n_B.

Sample A: Dec 1956 – Dec 1968 (n_A = 4405)

Sample B: Jan 1966 – Dec 1968 (n_B = 1096 Same time period as multi-level tower data).

The two-sample Kolmogorov-Smirnov test was chosen to test the null hypothesis that the samples are from identical populations (Ref. 3). The basic statistic, D, employed in this test is defined as

$$D = \sup \left| F_A(x) - G_B(x) \right|$$

where the right-hand side expresses the maximum absolute difference in the distribution functions of samples A and B. In the above expression, the respective distribution function values are the cumulative relative frequencies of the peak wind speeds which can be readily computed from the data.

A test with level α is obtained by rejecting the hypothesis when the test statistic exceeds D. In the present case this is written

$$D > \left[-\frac{1}{2}\left(\frac{1}{n_A} + \frac{1}{n_B}\right) \ln \frac{\alpha}{2} \right]^{1/2} .$$

The samples compared gave D = 0.0309. For n_A = 4405, n_B = 1096, and α = 0.05, the right hand member of the above inequality is 0.04583. Thus, at the 5% level, the hypothesis of identical populations cannot be rejected. This is the conclusion desired from the comparison test. Therefore, it was permissible to assume that a three-year sample of peak winds at Cape Kennedy could be used in constructing probability distributions.

2.2 APPLICATIONS OF EXTREMAL FUNCTIONS TO THE TOWER DATA

The tower data introduced in the previous section can now be used to derive information about the extreme winds in the 3 to 150-m region.

Extreme winds, defined here as the peak wind registered during a predetermined period of exposure, have been found in previous studies to fit the Fisher-Tippett Type I distribution (Ref. 1). The Fisher-Tippett Type I density function for a peak wind speed u with parameters α and μ (determined from the mean and variance of the wind sample) is written*

$$f(u) = \alpha \exp\left[-e^{-\alpha(u-\mu)} - \alpha(u-\mu)\right] . \tag{1}$$

Figures 1 and 2 show examples of the plots of the three years of Cape Kennedy multi-level tower data. The lines on these plots, representing the Fisher-Tippett Type I theoretical distribution and 1σ confidence bands for the tower data, give some indication of the fit.

The data were divided into reference periods composed of four seasonal groupings (see appendix) and a composite annual reference period. For all reference periods, extremal parameters (α and μ) were found for: 1, 5, 10 and 15 day exposure periods. For the annual case, 30- and 60-day exposure periods were also studied. Gumbel's estimators of α and μ from Ref. 4 were used; and since the data sample was small, a correction, also available from Ref. 4, was added.

The results presented in Table 1 indicate that the exposure period and seasonal groups of data have an effect on the parameters calculated. Figures 3 through 42 show distributions at a particular level and season with varying exposure periods. These give the probability of not exceeding a given wind speed at a specified level.

A comparison of the present results for the annual reference period with corresponding results in Ref. 1 shows that the μ values in Ref. 1 are somewhat conservative. One explanation for this difference at the higher

*Gumbel, in (Ref. 4), presents a well-developed study of the FTI distribution.

levels and for longer exposure periods is the limitations inherent in the power-law relation (Eq. 4) for a 10-minute interval. The wind speeds measured over a 10-minute interval are likely to be relatively low, and inferences on wind speed behavior greater than about 32 knots were somewhat limited. Also, in Ref. 1 peak winds at each of the seven tower levels were assumed to occur within the same 10-minute interval, regardless of the exposure period. The validity of such an assumption decreases as the exposure period increases.

Another apparent discrepancy is an absence of a decreasing trend of α values over longer exposure periods in the present results. Such a trend was clearly indicated in the results of Ref. 1. Unfortunately, the α's in the present report could not be investigated for exposure periods beyond 60 days because of a lack of data. Conversely the α's for the base level of 10-m in Ref. 1 for short exposure periods were simply interpolations between one day and 30 day-values. Therefore, further investigation seems practical to establish actual trends of extremal parameters employing a more complete set of data (e.g., the twelve-year peak wind record for Cape Kennedy available at the 10-m level).

It can be seen, however, by examining the slopes shown on applicable graphs in Ref. 1 that α's (if not the μ's) fall within the confidence intervals established in the present report for the appropriate level and exposure period.

2.3 APPLICATION OF BIVARIATE DISTRIBUTIONS TO THE TOWER DATA

In Ref. 2, Gumbel and Mustafi present the following equations associating two extremes

$$F_{(1)}(x, y, a) = F(x)\,F(y)\,\exp\left[a\left(\frac{1}{-\ln_x F(x)} + \frac{1}{-\ln_y F(y)}\right)^{-1}\right] \tag{2}$$

$$F_{(2)}(x, y, m) = \exp\left[-\left\{\left(-\ln F(x)\right)^m + \left(-\ln F(y)\right)^m\right\}^{1/m}\right] \tag{3}$$

where F(x) and F(y) denote the Fisher-Tippett Type I probability function of extreme values x and y respectively, and the parameters of association, "a" and "m" behave such that $0 \leq a,\ 1/m \leq 1$.

The bivariate distribution, described by two random variables considered simultaneously, can be handled in a variety of ways in the present context of peak winds. Consider the two random variables to be peak wind speeds at two levels for the same exposure period. The peak winds are actually specified by a "parent" random variable which, in this case, is the epoch (exposure period) over which the peak winds occurred.

If, for example, the epoch is taken to be five days, then the random variable for one level is the five-day maximum of the hourly peak winds which occurred at that level. Similarly, the second random variable is the corresponding five-day maximum of the hourly peak winds at some other level. The epoch which specifies these two peak winds is the five-day interval of exposure. There is, of course, no guarantee that the five-day maxima for the two levels will occur in the same hour, and this becomes even more unlikely for longer epochs.

A more reasonable procedure is to define two random variables respectively as:

1. $u_{152.4}$: Maximum of the hourly peak winds at the reference level for a given epoch.
2. u_h: Hourly peak wind at another level h which occurs during the same hour as the peak wind in (1.) but which may not be the maximum wind at this level for the entire epoch.

This approach actually serves a dual purpose. First, it gives a more meaningful form of the bivariate; secondly, the results α and μ derived from this choice of variables give an indirect test of the validity of assuming that peak winds at two levels occur over an exposure period at the same time (i.e., same hour). Specifically, the winds described in (2.) above must be less than or equal to the peak wind at this second level for the given epoch. If the peak winds at both levels (reference level and second level) occur simultaneously for a particular epoch, then the extremal parameters (α's and μ's) calculated from the array described by (2.) will be the same as those given in Table 1 for the appropriate level, exposure period, and season. As discussed in subsection 2.2, Table 1 gives extremal parameters determined from a univariate distribution, but for several levels of tower data. Thus, as a comparison, the bivariate α and μ values determined from the definition (2.) above are presented in Table 2. Comparing Tables 1 and 2, it can be seen that as the exposure period and the distance between the two levels increase, the assumption of simultaneous occurrence of peak winds at two levels is weakened. Figures 43 and 44 are plots of the data as described in (2.) above. The figures indicate how well the data fit the Fisher-Tippett Type I distribution.

The quantities x and y as used in Eqs. (2) and (3) are called "reduced variates" and are calculated by

$$x = \alpha_1 (u_{152.4} - \mu_1)$$

$$y = \alpha_2 (u_h - \mu_2)$$

Here, $u_{152.4}$ represents the wind speed defined in (1.) above and α_1 and μ_1 are values for the 152.4-m level from Table 1. Similarly, u_h denotes wind values above in (2.), and α_2 and μ_2 are the values for a particular level h from Table 2. As can be seen in Figs. 1 through 44, there is a non-varying relationship between the reduced variate and probability.

In Ref. 2, several methods are presented for estimating the bivariate parameters a and m introduced in Eqs. (2) and (3) respectively. The first method depends on criteria which may be quantitatively determined by considering the number of pairs of reduced variates which satisfy certain inequalities. The criteria for both distributions are based on

$$F_{13} = (F_1 + F_3)/2N$$

Here N is the total number of pairs of reduced variates and F_1 is the number of pairs of variates whose members (x, y) satisfy the condition

$$x > 0.36651$$
$$y > 0.36651$$

and F_3 is the number of pairs whose members (x, y) satisfy

$$x < 0.36651$$
$$y < 0.36651$$

If $0.25 \leq F_{13} \leq 0.35355$, then the parameter "a" can be estimated using quadrant frequencies and,

If $0.25 \leq F_{13} \leq 0.50$, then the parameter "b" = 1/m can be estimated using quadrant frequencies.

In all but two of the 42 cases presented, the criterion for estimating "b" was satisfied. In only 4 of the 42 cases was the criterion for estimating "a" satisfied. Thus, only estimates for m(=1/b) are presented (Table 3), and it is assumed that only Eq. (3) fits the data satisfactorily. Table 3 also contains a "difference" estimate of m which is based on the standard deviation of the difference of the reduced variates x and y. Table 3 shows that, as the level approaches 152.4 m in each exposure period, the $\hat{m}$ (m estimate) increases. The parameter m is related to the correlation ρ between the wind speeds by

$$m = (1 - \rho)^{-1/2}.$$

The correlation between wind speeds at 152.4 m and any lower level would be expected to increase as this level approaches 152.4 m. The above relation shows that m increases in the same sense.

2.4 PEAK WIND PROFILES BASED ON THE 152.4-m REFERENCE LEVEL

Reference 1 presented a method for obtaining probabilities of winds at 18.3 m and wind profiles from 18.3 to 152.4 m. In this report, the levels are reversed; i.e., probabilities are given for 152.4-m winds and profiles constructed downward from 152.4 to 18.3m. This reflects the change which establishes 152.4 m as the reference level, in contrast to the old reference level of 18.3m.

The peak wind profiles require a distribution of the winds at 152.4 m and a relationship between winds at two levels. The distributions at 152.4 m are discussed in subsection 2.2 and distribution parameters are included in Table 1. The relationship between levels is such that given an operationally critical wind at some reference level, the peak wind speed at other levels can be prescribed by the power-law relationship

$$u_h = u_r\,(h/r)^k \tag{4}$$

where u_h is the peak wind speed at level h and u_r the peak wind speed at the reference level r.

Using a large sample of peak wind profiles for the Eastern Test Range, Fichtl (Ref. 5) has established that the exponent k can be expressed in the form

$$k = c\,(u_r)^p \,. \tag{5}$$

Statistical techniques have been applied to Eqs. (4) and (5) in constructing a wind profile extending upward from some lower reference level r, usually 18.3 m (Refs. 1 and 5). Using such an approach, Fichtl has found that k is very nearly normally distributed for any particular value of the peak wind speed at the 18.3-m level (Ref. 5).

In Eq. (5), p is an empirically determined exponent and the quantity c is a random variable distributed normally with mean $\bar{c}$ and standard deviation σ. Taking logarithms and expressing c in terms of this distribution, Eq. (5) becomes

$$\log k = \log(\bar{c} + n\sigma) + p \log u_r . \qquad (6)$$

The factor n in Eq. (6) determines the number of standard deviations from the mean and, for a normal distribution, has the value 0, 1.6, and 3.0 for the 0.50, 0.95, and 0.999 cumulative probabilities, respectively.

Consider now a procedure which will allow the construction of a peak wind profile downward from an upper reference level r = 152.4 m. For such a procedure, Eqs. (4) and (5) are rewritten with r = 152.4 m.

$$u_h = u_{152.4} (h/152.4)^k, \; h(m), \; u(m/sec) \qquad (7)$$

and

$$k = c u_{152.4}^p \qquad (8)$$

or

$$\log k = \log(\bar{c} + n\sigma) + p \log u_{152.4} . \qquad (9)$$

In constructing such profiles "downward" from a specified upper reference level, the variate c can now be interpreted as a risk. The term "risk" in this sense denotes the probability that the peak wind at level h is greater than u_h.

In order to derive a useful form of Eq. (8), wind profiles were investigated for each of 32 time divisions. These divisions consisted of eight 3-hour observational intervals for four seasons* of the year. These particular seasonal groupings were chosen in an attempt to ensure homogeneity of the samples

*See Appendix.

and to give the best description of winter, summer, and transitional periods of ground wind behavior. All sample profiles in each of these 32 time divisions were grouped into class intervals according to the peak wind speeds at 152.4 m. Equation (4) or (7) shows that each profile within such an interval corresponds to a particular value of k. Cumulative frequency values of k for the 50, 95, and 99.9 percentiles were plotted as a function of the 152.4 m wind speed. A curve was fit to each of the three percentile plots using the method of least squares. The equation of the resulting three curves can be written

$$\log k = \log \bar{c} + p_1 \log u_{152.4} \tag{10a}$$

$$\log k = \log(\bar{c} + 1.6\,\sigma) + p_2 \log u_{152.4} \tag{10b}$$

$$\log k = \log(\bar{c} + 3\,\sigma) + p_3 \log u_{152.4} \tag{10c}$$

If the k values are distributed in this manner, then the slope values p_1, p_2, and p_3 must be equal. In order to determine if the slope values for a given hour and seasonal division were (or were not) significantly different, the statistical F test was applied to the values p_1, p_2, p_3. All slopes for the 32 time divisions passed the F test with the exception of those associated with class intervals 1, 2, 3, 7 and 8 of Season 2, and class intervals 1, 2, 3 and 8 of Season 4 (see the Appendix for key to seasons and class intervals). Therefore, the results of the test indicated that the empirical Eq. (5) is acceptable.

Insofar as the slope values in Eq. (10) are not significantly different, the three equations contain only the two unknowns $\bar{c}$ and σ for the given sample values of k and $u_{152.4}$ available from ETR data. A maximum likelihood method was applied to Eq. (10) to determine $\bar{c}$ and σ. Thus, p, $\bar{c}$, and σ were found through statistical analyses and curve fitting methods of the sample profiles and 152.4-m peak wind data. They then become known quantities for a given time of day and season. A particular multiple of σ corresponds to a certain percentile value of the distribution of the wind profile parameter k. Equation (9) now provides a useful formulation of the distribution of k as a function of peak wind speed at 152.4-m. Once the parameter k is found, the complete wind profile is specified for a given peak wind speed at the upper reference level r = 152.4 m.

Figures 45a through 45c are graphs of the quantities $\bar{c}$, σ and p for four seasons plotted against eight three-hour class intervals. The results are also given in tabular form in Table 4. It can be seen that the hourly variations are much greater than the seasonal changes. Although there are some shifts in the times of maxima and minima, the general shapes of the curves for each parameter are preserved from season to season. However, there are large hourly amplitudes in most cases. This justifies the use of class interval divisions in determining the parameters $\bar{c}$, σ, and p; and thus represents an improvement over the use of average values as employed in Ref. 1.

Although wind profiles are constructed with reference to the 152.4-m level in this report, the basic theory described in Ref. 1 is still applicable. For a given value of $u_{152.4}$, the wind profile through lower levels is uniquely determined by the variable c; see Eqs. (7), (8), and (9). The probability that a profile will not be exceeded is equal to the probability that c will not be exceeded. Figures 46 and 47 represent the joint probability (ordinate value) that $u_{152.4}$ (abscissa value) and the profile indicated in the legend will not be exceeded. No special significance should be attached to the use of Fisher-Tippett Type I graphs in these figures; these graphs simply provide a convenient mode of display. In addition to the graphs, curve fit parameters β and γ were determined from the linear expression $y(c,t) = \beta(c,t)\left[u_{152.4} - \gamma(c,t)\right]$. These parameters appear in Table 5. Approximate joint distributions $F(u_{152.4}, c)$, for a specific 152.4-m wind and profile may be computed from

$$F(u_{152.4}, c) = \exp\left[-e^{-y(c,t)}\right].$$

Tables 6, 7 and 8 are α's and μ's corresponding to the three-hourly class intervals, but are given for monthly instead of seasonal divisions.

2.5 USE OF THE TABLES AND GRAPHS

Probabilities for the Fisher-Tippett Type I distribution can be determined through use of the reduced variate as defined in subsection 2.3. The relationship between the wind value and the reduced variate is linear. The relationship between a reduced variate y and the cumulative probability of y is one-to-one and does not change, as can be seen from the equation

$$P(y) = e^{-e^{-y}} .$$

This relationship is given on all of the ordinate scales in Figs. 1 through 44. When probabilities are being determined, it is convenient to find the reduced variate and then read the probability from the ordinate that corresponds to the particular variate.

Figures 3 through 42 show the calculated distributions of the peak winds at the seven levels of Cape Kennedy tower data. The graphs, or the corresponding α's and μ's in Table 1, can be used to calculate: (1) the probability that a wind A' will not be exceeded in a given exposure period; i.e., $P(A \leq A')$ or (2) the risk or probability that a wind A' will be exceeded in a given exposure period; i.e., $P(A > A')$.

- Example

Calculate the probabilities for a peak-wind in an exposure period of 10 days during season 3, and for a level of 61 m. Table 1 has $\alpha = 0.1428$ and $\mu = 33.0591$. Set A' equal to 34 knots, then the reduced variate is

$$\begin{aligned} y &= \alpha(u - \mu) \\ &= 0.1428\,(34.0 - 33.0591) \\ &= 0.1345 \end{aligned}$$

The probability corresponding to the reduced variate 0.1345 is 0.40. (See any ordinate scale Figs. 1 through 44.) Therefore, $P(A \leq A') = 0.40$. For the risk, $P(A > A') = 1 - P(A \leq A') = 0.60$.

Using the appropriate tables and graphs for the bivariate, peak wind profile, and univariate distributions, probabilities of the following forms can be found:

Case I: The probability that the wind A is less than a given value A′, and the wind (or profile) B is less than a given B′; in other words, neither A nor B is exceeded, is given symbolically by $P(A \leq A' \cap B \leq B')$.

Case II: The probability that at least one of the conditions in Case I is violated; i.e., either the wind A is exceeded or the wind (or profile) B is exceeded or both are exceeded is given by $P(A > A' \cup B > B') = 1 - P(A \leq A' \cap B \leq B')$.

Case III: The probability that both the wind A and the wind (or profile) B are exceeded is given by $P(A > A' \cap B > B') = 1 - P(A \leq A') - P(B \leq B') + P(A \leq A' \cap B \leq B')$.

2.5.1 The Bivariate Distribution

Parameters that represent the first variable of the bivariate (i.e., wind A in Cases I, II, III above) are those labeled 152.4 m under the annual case in Table 1. Figures 43 and 44 are examples of the second variable of the bivariate distribution. (This variable corresponds to B in Cases I, II, III above). Parameters for this second variable are given in Table 2.

In the bivariate procedure, first select the desired exposure period. From Table 1, the α and μ for that exposure period are selected for the annual case and the 152.4-m level. Next, a second level is chosen. Using the same exposure period, the α and μ are selected for the second level in Table 2. (All parameters in Table 2 are for an annual reference period.) The estimate for the parameter "m" is found in Table 3 for the appropriate exposure period and level. These parameters are then applied to Eq. (3), (subsection 2.3), which can be simplified for the Fisher-Tippett Type I distribution to

$$F(x, y, m) = \exp\left[-\left(e^{-mx} + e^{-my}\right)^{1/m}\right] \tag{11}$$

- Examples

Case I: As an example of the calculations for the bivariate distribution, consider an exposure period of one day, the second level equal to 18.3 m, the wind at 152.4-m equal to 37.5 knots, and the wind at 18.3-m equal to 32.2 knots. From Table 1, the parameters are $\alpha = 0.1548$, $\mu = 22.8319$. The reduced variate is $x = 0.1548$ $(37.5 - 22.8319) = 2.25$. From Table 2, the parameters are $\alpha = 0.1640$, $\mu = 17.1384$, and the reduced variate $y = 0.1640\ (32.2 - 17.1384) = 2.50$. From Table 3, $\hat{m} = 2.53885$. Applying these quantities to Eq. (11), $F(2.25, 2.50, 2.54) = 0.878$. Thus, 0.878 is the probability that the 152.4-m will not exceed 37.5 knots and that the 18.3-m wind will not exceed 32.2 knots. That is, $P(A \leq A' \cap B \leq B') = 0.878$.

Case II: The probability that one or both of the winds in Case I will be exceeded at their respective levels is 0.122. That is, $P(A > A' \cup B > B') = 1 - 0.878 = 0.122$.

Case III: Here it is necessary to obtain $P(A \leq A')$, which is the probability that the 152.4-m wind will not exceed 37.5 knots during a one day exposure period. Since the reduced variate for the 37.5 knot wind is 2.25, the corresponding probability value is 0.90. It is also necessary to have $P(B \leq B')$. In accordance with definitions of random variables in subsection 2.3, this is the probability that $B' = 32.2$ knots at the 18.3-m level will not be exceeded during the hour in which the 152.4-m daily peak wind occurs. Corresponding to the reduced variate 2.50, the probability is 0.92. Thus, $P(A > A' \cap B > B') = 1 - 0.900 - 0.920 + 0.878 = 0.058$. As expected, this is less than the probability 0.122 of one or both winds being exceeded.

2.5.2 Peak Wind Profiles

For these examples, consider the variable B in Cases I, II, III to be a peak wind profile instead of a single wind speed. From Table 5, find the β and γ corresponding to the desired profile and time; substitute these parameters along with the 152.4-m wind into the reduced variate equation $y = \beta(u_{152.4} - \gamma)$.

- Examples

Case I: Consider a 1σ wind profile and a 152.4-m wind of 31.2 knots during the month of January at 1200 EST. January is in Season 1, and 1200 falls in hour group 4. From Table 5, $\beta = 0.1408$, $\gamma = 16.6299$. Then $y = 0.1408\ (31.2 - 16.6299) = 2.05$. The probability corresponding to this reduced variate is 0.875; i.e., the probability that neither 31.2 knots at 152.4 m or the 1σ envelope of 31.2 knots is exceeded is equal to 0.875.

Case II: The probability that either the 152.4 m-wind or the 1σ envelope is exceeded, or both are exceeded, is 1 - 0.875 = 0.125.

Case III: First determine $P(A \leq A')$ and $P(B \leq B')$. A' is 31.2 knots, and the parameters for January (Season 1) and 1200 EST (hour group 4) found from Table 6 are $\alpha = 0.1399$, $\mu = 16.5100$. Thus, $y = 0.1399\ (31.2 - 16.5100) = 2.06$. The corresponding probability, $P(A \leq A') = 0.875$. Determine the quantity $u_{B'}$ from the power-law equation,

$$u_{B'} = u_{152.4} \left(\frac{18.3}{152.4}\right)^{(\bar{c} - n\sigma)\ (u_{152.4}/1.94)^{p}}$$

Table 4 has values for $\bar{c}$, σ, and p tabulated by seasons and hour groups. For this example, $\bar{c} = 0.09463$, $\sigma = 0.20820$, $p = -0.15599$. Substituting these values into the above power-law relation along with $u_{152.4} = 31.2$ knots yields $U_{B'} = 36.5$ knots. The α and μ for associating B' with a reduced variate are found in Table 7. Table 7 gives $\alpha = 0.1744$, $\mu = 12.8536$. Thus, $y = 0.1744\ (36.5 - 12.8536) = 4.1$. The corresponding probability, $P(B \leq B') = 0.983$. Thus, $P(A > A' \cap B > B') = 1 - 0.983 - 0.875 + 0.875 \doteq 0.017$. This is the probability that both the 152.4-m wind of 31.2 knots and the 1σ profile are exceeded.

2.6 STATISTICAL COMPARISON OF WIND DIRECTIONS

In addition to considering the behavior of peak wind speeds, the associated wind directions assume importance with regard to possible launch azimuths and vehicle shapes. A distribution which can usually be obtained is that of hourly peak winds plotted by speed and direction. An even more desirable distribution would consist of hourly peak winds for 10-degree directional classes, also plotted by speed and direction. Since the latter distribution is not readily available, it is practical to test the hypothesis that wind directions are fairly constant over an hour. To the extent that this is true, the direction associated with the hourly peak wind represents all the wind directions occurring during the hour.

For this test, an eight-year sample of 10-m wind directions from Cape Kennedy was subjected to statistical analysis. The data employed consisted of the direction of the wind (d_i), measured on the hour, and the direction of the peak wind (d_j), during the succeeding hour interval. The random variate to be analyzed was $\Delta d = d_i - d_j$, given in degrees. The distribution of the variate was characterized by the following statistics:

mean = 0.60
standard deviation = 27.40
skewness = 0.35
kurtosis = 15.28

These values show that the directional distribution by hourly peaks closely approximates the distribution of the hourly peaks by direction over the same time period. The mean is near zero. The dispersion about the mean indicated by the standard deviation of 27.4 degrees is also small. Thus, a large portion of the values Δd are grouped around the mean. The small skewness can be assumed to indicate a rather symmetrical distribution. The large kurtosis value generally shows that more values of the sample are close to the mean value than is true with a normal distribution. Thus, each statistic emphasizes that most sample values are very close to zero, indicating the small difference in d_i and d_j.

There are, of course, certain weather events which may cause d_i to be quite difference from d_j. These include frontal passages during the hour and pronounced wind directional changes associated with thunderstorms. It is not uncommon to experience brief directional changes of 180 degrees at locations affected by strong thunderstorms. No attempt was made to remove such cases from the sample under investigation. A further and useful refinement to the above results would be the directional analysis of stratified samples in which separate consideration could be given to: (1) thunderstorm and other strong wind regimes, (2) extremely light winds, and (3) time-of-day stratifications.

Section 3
CONCLUSIONS AND RECOMMENDATIONS

The results of this study indicate the practical value of using multi-level wind data to establish guiding probabilities for design and operational purposes. On the basis of appropriate tests, the validity of using a relatively small sample of tower data was justified. Even more definite conclusions could be derived from a larger sample; and it is hoped that a continuing program of multi-level wind measurements will provide the increased data sample required for further investigations in this area.

In applying Fisher-Tippett Type I distributions to several levels, parameters were obtained which are in general agreement with those derived for a single level in Ref. 1; however, definite trends in the parameter values were not as obvious in the present report. Further study is needed to determine if such trends are characteristic of the parameters developed from atmospheric extreme values.

Although the significance level was not determined for the bivariate distribution, 40 out of 42 cases passed the criteria for estimating the "b" (=1/m) parameter. Such results do not justify the rejection of the hypothesis that the data fit the bivariate distribution described by Eq. (3). In contrast, since only 4 out of 42 cases passed the criteria for estimating the "a" parameter, a data fit to Eq. (2) was rejected.

Variations in those parameters which define a synthetic wind profile indicate that an average value for these parameters may not be sufficient. More realistic profiles should result from considering seasonal and hourly variations. However, because of the divisions into 32 cases for the 152.4-m reference level, it was difficult to make a valid comparison of the resulting probabilities with Ref. 1, which used 18.3 m as the reference level. There is also some question as to the accuracy of subjecting the wind profile parameters to a linear fit at wind speeds greater than about 30 knots. A second degree fit may be more suitable. Further study of the power-law and its associated parameters is recommended to resolve such questions. In view of these uncertainties concerning the peak wind profile, the bivariate distribution is believed to give a more straightforward approach to the problem of bi-level winds.

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Appendix A

SAMPLE DISTRIBUTIONS FOR EXTREMAL FUNCTIONS

by

B. C. Johnson

Appendix

This Appendix presents theoretical topics related to extremal functions. Although emphasis is given to theoretical development, a consideration of these topics may contribute to future practical applications in the statistics of atmospheric extremes. Various integration techniques of extremal density functions will become important as these functions are used to solve aerospace design and operational problems. The feasibility of analytically integrating the Fisher-Tippett Type I density function is explored in Section A.1. In Section A.2, a procedure is developed which can be applied to test the goodness-of-fit of extreme value samples to a univariate Fisher-Tippett Type I distribution. A computer program is outlined for application of the technique. Section A.3 proposes a method for determining the sample variance. Further consideration of the methods discussed in these latter two sections may lead to useful testing procedures applicable to extremal statistics.

A.1 ANALYTICAL INVESTIGATION OF EXTREMAL FUNCTIONS

Consider the empirical power-law relationship (Ref. A.1)

$$u_h = u_{18.3} \left(\frac{h}{18.3}\right)^{c u_{18.3}^{-3/4}} \tag{A.1}$$

between u_h, the peak wind speed (m/sec) at level h(m) and $u_{18.3}$, the peak wind speed at the reference level, 18.3 m. Equation (A.1) represents a power-law profile with parameters c and 3/4 derived by statistical analysis of Cape Kennedy wind records for a specific exposure period.

Equation (A.1) can also be written

$$u_{18.3}^{3/4}\left[\ell n\,\frac{u_h}{u_{18.3}}\bigg/\ell n\,\frac{h}{18.3}\right] = c \qquad \text{(A.2)}$$

In the statistical analysis from which (A.1) is derived, the quantity c is considered a normal random variable with standard deviation σ_c. This allows the expression of Eq. (A.2) in probability terms, or:

$$P\left\{u_h \leq u_{18.3}\left(\frac{h}{18.3}\right)^{u_{18.3}^{-3/4}\,c}\right\} = \int_{-\infty}^{c}\frac{1}{\sqrt{2\pi}\,\sigma_c}\,e^{-(c'-\bar{c})^2/2\sigma_c^2}\,dc' \qquad \text{(A.3)}$$

Now let

$$K = u_{18.3}\left(\frac{h}{18.3}\right)^{u_{18.3}^{-3/4}\,c}$$

The upper limit of integration for c then becomes

$$c = u_{18.3}^{3/4}\left[\ell n\,\frac{K}{u_{18.3}}\bigg/\ell n\,\frac{h}{18.3}\right]$$

A change of variables from c' to v is expressed by

$$c' = u_{18.3}^{3/4}\left[\ell n\,\frac{v}{u_{18.3}}\bigg/\ell n\,\frac{h}{18.3}\right]$$

Equation (A.3) becomes

$$P\left\{u_h \leq K\right\} = \int_o^K e^{-\left\{u_{18.3}^{3/4}\left[\ln \frac{v}{u_{18.3}} \Big/ \ln \frac{h}{18.3}\right] - \bar{c}\right\}^2 \Big/ 2\sigma_c^2}$$

$$\bullet \left[u_{18.3}^{3/4} \Big/ \sqrt{2\pi}\, \sigma_c\, v \ln\left(\frac{h}{15.3}\right)\right] dv \ . \qquad \text{(A.4)}$$

Equation (A.4) is the integral of the normal probability density function with the normal variate involving logarithms. Its value can be expressed in terms of the normal distribution function. However, the normal distribution cannot be expressed in terms of elementary functions. Consequently, both sides of Eq. (A.4) cannot be integrated analytically with respect to $u_{18.3}$. The integration, of course, may be carried out by numerical methods.

Another approach is to employ conditional probabilities. However, this approach also does not lend itself to an analytic solution, as shown in the following development.

For a given wind speed $u_{18.3}$, the derivative of Eq. (A.4) is the conditional probability density function $f(u_h \mid u_{18.3})$. The probability density function for any level is given by (Ref. A.1)

$$f(u_h) = \int_{-\infty}^{\infty} f(u_h, u_{18.3})\, du_{18.3}$$

$$= \int_{-\infty}^{\infty} f(u_h \mid u_{18.3})\, f(u_{18.3})\, du_{18.3} \qquad \text{(A.5)}$$

The function $f(u_{18.3})$ is assumed to be of the form described by the FTI density function

$$f(u_{18.3}) = \alpha \exp\left[-e^{-\alpha(u_{18.3}-\mu)} - \alpha(u_{18.3}-\mu)\right] \qquad (A.6)$$

where α and μ are parameters determined from the mean and variance of the wind sample.

The conditional probability can be written in terms of the normal variate c by the transformation

$$f(u_h \mid u_{18.3}) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(c-\bar{c})^2/2\sigma^2} \left|\frac{\partial c}{\partial u_h}\right|$$

where c and $\frac{\partial c}{\partial u_h}$ can be found from Eq. (A.2).

These substitutions when applied to Eq. (A.5) yield

$$f(u_h) = \int_{-\infty}^{\infty} \frac{u_{18.3}^{3/4}\,\alpha}{\sqrt{2\pi}\; u_h \,\ell n \frac{h}{18.3}\; \sigma_c}$$

$$\cdot \exp\left\{-e^{-\alpha(u_{18.3}-\mu)} - \alpha(u_{18.3}-\mu) \right. \qquad (A.7)$$

$$\left. - \left[u_{18.3}^{3/4}\left(\ell n \frac{u_h}{u_{18.3}} \Big/ \ell n \frac{h}{18.3}\right) - \bar{c}\right]^2 \Big/ 2\sigma_c^2 \right\} du_{18.3}$$

The immediate problem with Eq. (A.7) is that $\ell n\, u_{18.3}$ is not defined for negative values of $u_{18.3}$. Since $u_{18.3}$ is never negative, $f(u_{18.3})$ can be defined equal to zero for negative values of $u_{18.3}$. The lower integration limit then becomes zero.

The substitutions $\sigma_K = \ell n \frac{h}{18.3} \sigma_c$, $c_K = \bar{c}\, \ell n \frac{h}{18.3}$, and $u_{18.3} = e^{-y}$ transform Eq. (A.7) into

$$f(u_h) = \int_{-\infty}^{\infty} \frac{-\alpha e^{-3/4 y}}{\sqrt{2\pi}\, \sigma_K u_h} \exp \left[- e^{-\alpha(e^{-y} - \mu)} - \alpha(e^{-y} - \mu) - y \right]$$

$$\cdot \exp \left\{ - \left[e^{-3/4 y} (\ell n\, u_h + y) - \bar{c}_K \right]^2 \Big/ 2\sigma_K^2 \right\} dy . \qquad \text{(A.8)}$$

The last exponential in Eq. (A.8) is the normal probability density function with the variable equal to $e^{-3/4 y}(\ell n\, u_h + y)$. The integrand in Eq. (A.8) includes $e^{-3/4 y}$ in the first term of the product, but this is not related to the derivative of either of the other exponential terms in the product. The fact that the integrand contains the normal probability density function and other unrelated functions would in general preclude the possibility of expressing the integral in terms of elementary functions required for analytic integration.

A.2 SAMPLE DISTRIBUTION OF A VARIATE WITH A FISHER-TIPPETT TYPE I DISTRIBUTION

A.2.1 Development of Theory

It may be practical to test the goodness-of-fit of n sample values to an extreme value distribution, such as the Fisher-Tippet Type I (FTI), whose parameters α and μ are readily obtained from the given sample.

The density function for the FTI is Eq. (A.6). The corresponding distribution function is

$$F(x) = \exp\left\{- e^{-\alpha(x-\mu)}\right\} \tag{A.9}$$

where the variable x is used for generality instead of the wind speed u.

Hypothesize that a sample statistic, Z, is the sum of n independent extreme samples,

$$Z = x_1 + x_2 + \ldots + x_n = \sum_1^n x_i \tag{A.10}$$

The approach is to compute the probability that the sum of n independent samples from the FTI distribution is less than Z. The characteristic function of the FTI distribution is (Ref. A.2)

$$\phi(t) = \int_{-\infty}^{\infty} e^{itx} f(x)\, dx = \Gamma\left(1 - \frac{it}{\alpha}\right) e^{i\mu t} \tag{A.11}$$

The characteristic function of the sum of n sample values is the n^{th} power of the population characteristic function (Ref. A.3). If Z can be defined as in Eq. (A.10) then

$$F'(Z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-itz} \phi^n(t)\, dt \tag{A.12}$$

Except for n = 2 or 3, the integral in Eq. (A.12) cannot be evaluated in terms of classical functions. However, the integral can be evaluated numerically. For the sum of the n samples the density function can be expressed as

$$f(Z) = h(w)$$

where

$$w = e^{-\alpha(Z - n\mu)} \tag{A.13}$$

An appropriate series of substitutions* converts the integral in Eq. (A.12) into a form such that

$$\text{Probability}\left(\sum_{1}^{n} x_i < Z\right) = 1 - \int_{-\infty}^{\infty} h(w)\, dw \tag{A.14}$$

where:

$$h(w) = \frac{w}{2\pi\alpha} \frac{\left(\frac{\cos\phi}{\alpha} - v\sin\phi\right)}{\left(\frac{1}{\alpha^2} + v^2\right)} \exp\left\{-nP(1,v)\right\} \tag{A.14a}$$

and,

$$\phi = -\alpha\mu n v - \alpha v \ln w + n\theta(1,v) - nv\gamma \tag{A.14b}$$

$$v = \frac{t-1}{i} \text{ (here "i" refers to the imaginary unit)} \tag{A.14c}$$

$$\theta(1,v) = \sum_{s=o}^{\infty} \left(\frac{v}{1+s} - \arctan\frac{v}{1+s}\right) \tag{A.14d}$$

$$P(1,v) = \tfrac{1}{2}\sum_{s=o}^{\infty} \ln\left[1 + \frac{v^2}{(1+s)^2}\right] \tag{A.14e}$$

γ = Euler's constant

*See Refs. A.4, A.5 and A.6 for a more detailed treatment of the substitutions and transforms required in the above development.

It can be seen that the variable of integration in Eq. (A.14) is v, where $t = 1 + iv$. The quanity n is the sample size; α and μ are extremal parameters estimated from the sample values; and s is a parameter as specified in Eqs. (A.14d) and (A.14e). The quantities w and Z are defined in Eq. (A.13) and Eq. (A.10), respectively. Although complex variables were introduced in the substitution, it can be shown that the imaginary components of the integral in Eq. (A.14) for any positive increment of dv will be cancelled by the corresponding negative increment. Thus, the integral represents a real-valued function.

The limits of integration in Eq. (A.14) (say $-v_m$, v_m) must be chosen such that the contribution to the integral for $v < -v_m$ and $v > v_m$ are neglible. These limits can be estimated from the asymptotic expansion of the gamma function

$$\lim_{|v| \to \infty} \left| \Gamma(1 + iv) \right| e^{\frac{\pi}{2}|v|} |v|^{-\frac{1}{2}} = (2\pi)^{\frac{1}{2}}$$

(see Ref. A.4).

Also,

$$\left| \Gamma(1 + iv)^n \right| \sim (2\pi)^{\frac{1}{2}n} e^{-\frac{\pi}{2}n|v|} |v|^{n/2}$$

and the limit $v_m > 0$ is chosen such that

$$w (2\pi)^{\frac{1}{2}(n-1)} e^{(-\frac{\pi}{2} n v_m)} v_m^{(n/2 - 1)} < e^{-\epsilon} \qquad \text{(A.15)}$$

where ϵ is the desired accuracy in the distribution function.

A.2.2 Computer Program Outline

The following is a proposed program outline to compute the probability distribution discussed above:

1. The programmer will select the numerical integration scheme. Since the integrand contains the sine and cosine functions, the integrand will change sign frequently. The step size will in part be governed by the values of v which make ϕ a multiple of π.

2. The program will read n, μ, α and Z. The μ and α are parameters of the FTI distribution. The quantity n is the number of samples. The output of the program will be the probability that the sum of n independent samples, $x_1 + x_2, + \ldots + x_n$, from the Fisher-Tippett Type 1 distribution is less than Z.

3. Compute $w = e^{-\alpha(Z - n\mu)}$

4. Select limits of integration based upon the inequality (A.15).

5. Select integration step size.

6. Determine end points of integration interval, (v_{i-1}, v_i) and select v_i' in the interval.

7. Compute $P(1, v_i')$ and $\theta(1, v_i')$ from Eqs. (A.14d) and (A.14e)

8. Compute ϕ from Eq. (A.14b)

9. Compute sine and cosine of ϕ.

10. Compute

$$h(w) = \frac{w}{2\pi\alpha} \frac{\left(\frac{\cos\phi}{\alpha} - v_i' \sin\phi\right)}{\left(\frac{1}{\alpha^2} + v_i'^2\right)} \cdot \exp\left\{-nP(1, v_i')\right\}$$

11. Compute value of integral on the interval (v_{i-1}, v_i)

$$I = I + h(w) \cdot (v_i - v_{i-1})$$

or by the integration scheme adopted in step 1.

12. Repeat steps 5 through 11 until upper limit of integration is reached.

13. Probability = 1 - I

14. Write results.

A.2.3 Comments on Program Development:

1. The value of I for $n = 2$, $\mu = 0$, $\alpha = 1$ can be reached by summing the series

$$I = \sum_{m=0}^{\infty} \frac{2(-1)^{2m}}{m!} \frac{\psi(1+m)}{m!} \frac{(e^{-z})^{m+2}}{2+m} \qquad \text{(A.16)}$$

where, $$\psi(1+m) = -\gamma + \sum_{s=0}^{\infty} \left(\frac{1}{s+1} - \frac{1}{1+m+s} \right)$$

γ = Euler's constant.

The series in Eq. (A.16) is an application of the Cauchy residue theorem from complex variable theory. It is the integral of the sum of the residues in the integral representation of the density function (Ref. A.7). The numerical integration can also be checked for $n = 3$ with a similar but more complicated series expansion.

2. Since the computation of P and θ is time consuming, it may be expedient to compute I for several values of Z at the same time.

A.3 DEVELOPMENT OF THE SAMPLE VARIANCE

If a parent distribution is denoted dF(x), the simultaneous distribution of n values $x_1, x_2, \ldots, x_n$ is $dF(x_1)\, dF(x_2) \ldots dF(x_n)$; and if Z is a statistic

$$Z = Z(x_1, \ldots, x_n) = x_1^2 + x_2^2 + \ldots + x_n^2 \qquad \text{(A.17)}$$

the distribution function of Z is given by (see Ref. A.3)

$$F(Z_o) = \int \ldots \int dF(x_1) \ldots dF(x_n), \qquad \text{(A.18)}$$

the integration being taken over the domain of the x's such that $Z(x_1, \ldots, x_n) \leq Z_o$. A usual method of determining $F(Z_o)$ is to make a transformation to the variables $Z_1, \theta_1, \theta_2, \ldots, \theta_{n-1}$; so that the limits of integration for the θ_i's are fixed constants. Equation (A.18) becomes

$$F(Z_o) = \int \ldots \int f(x_1) \ldots f(x_2) \frac{\partial(x_1, \ldots, x_n)}{\partial(Z, \theta_1, \ldots, \theta_{n-1})}$$

$$\cdot \; dZ\, d\theta_1 \ldots d\theta_{n-1} .$$

A proposed transform is

$$-x_1 = \ln e^{-Z^{\frac{1}{2}}} \cos^2\theta_1 \cos^2\theta_2 \ldots \cos^2\theta_{n-1}$$

$$-x_2 = \ln e^{-Z^{\frac{1}{2}}} \cos^2\theta_1 \cos^2\theta_2 \ldots \cos^2\theta_{n-2} \sin^2\theta_{n-1}$$

$$\ldots$$

$$-x_j = \ln e^{-Z^{\frac{1}{2}}} \cos^2\theta_1 \cos^2\theta_2 \ldots \cos^2\theta_{n-j} \sin^2\theta_{n-j+1}$$

...

$$x_n = \ln e^{-Z^{\frac{1}{2}}} \sin^2 \theta_1$$

Substitution of the above variables into the equation

$$F(Z_o) = \int \ldots \int \exp \left\{ - \left(e^{-x_1} + e^{-x_2} + \ldots + e^{-x_n} \right) \right\}$$

$$\bullet \exp \left\{ - (x_1 + x_2 + \ldots + x_n) \right\} d_{x_1} d_{x_2} \ldots d_{x_n} \qquad \text{(A.20)}$$

yields

$$F(Z_o) = \int \ldots \int \left\{ \exp - \left(e^{-Z^{\frac{1}{2}}} \cos^2 \theta_1 \cos^2 \theta_2 \ldots \cos^2 \theta_{n-1} \right. \right.$$

$$+ e^{-Z^{\frac{1}{2}}} \cos^2 \theta_1 \cos^2 \theta_2 \ldots \cos^2 \theta_{n-2} \sin^2 \theta_{n-1} + \ldots$$

$$\left. \left. + e^{-Z^{\frac{1}{2}}} \sin^2 \theta_1 \right) \right\}$$

$$\bullet e^{-Z^{\frac{1}{2}}} (\cos^2 \theta_1 \cos^2 \theta_2 \ldots \cos^2 \theta_{n-1})$$

$$\bullet \quad (\cos^2 \theta_1 \cos^2 \theta_2 \ldots \cos^2 \theta_{n-2} \sin^2 \theta_{n-1}) \ldots .$$

$$\bullet \quad (\sin^2 \theta_1) \frac{\partial (x_1, \ldots x_n)}{\partial (Z, \theta_1, \ldots, \theta_n)} dZ\, d\theta_1 \ldots d\theta_{n-1} \qquad \text{(A.21)}$$

The last equation simplifies to

$$F(Z_o) = \int \ldots \int \exp \left\{ - e^{-Z^{\frac{1}{2}}} - nZ^{\frac{1}{2}} \right\}$$

$$\cdot\ (\cos^2\theta_1\ \cos^2\theta_2 \ldots \cos^2\theta_{n-1})$$

$$\cdot\ (\cos^2\theta_1\ \cos^2\theta_2 \ldots \cos^2\theta_{n-2}\ \sin^2\theta_{n-1}) \ldots$$

$$\cdot\ \sin^2\theta_1 \quad \frac{\partial(x_1, \ldots, x_n)}{\partial(Z, \theta_1, \ldots, \theta_{n-1})}\, dZ\, d\theta_1, \ldots d\theta_n\ . \tag{A.22}$$

The Jacobian of transform (A.19) is given by

$$\frac{\partial(x_1, \ldots, x_n)}{\partial(Z, \theta_1, \ldots, \theta_{n-1})}$$

which is equal to $\frac{Z^{\frac{1}{2}}}{2}$ times the determinant

$$\begin{vmatrix} 1 & 1 & 1 & & 1 \\ 2\tan\theta_1 & 2\tan\theta_1 & 2\tan\theta_1 & \cdots & -2\tan\theta_1 \\ 2\tan\theta_2 & 2\tan\theta_2 & 2\tan\theta_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 2\tan\theta_{n-2} & 2\tan\theta_{n-2} & -2\tan\theta_{n-1} & \cdots & 0 \\ 2\tan\theta_{n-1} & -2\tan\theta_{n-2} & 0 & \cdots & 0 \end{vmatrix}$$

It is conjectured that the integral in Eq. (A.22) can be reduced to the form

$$F(Z_o) = K \int_o^{Z_o} \exp\left\{ -e^{-Z^{\frac{1}{2}}} - n\, Z^{\frac{1}{2}} \right\} Z^{-\frac{1}{2}}\, dZ \ . \tag{A.23}$$

The preceding integration can be performed by n applications of the standard formula for integration by parts. The sample variance about the mean can be computed by the standard formulas.

A formal procedure for developing another sample statistic, the coefficient of variation, is given in Ref. A.3. The procedure depends on the sample variance whose distribution function was developed in the preceding discussion. It will also involve an integral of the type used for computing the distribution of the sample mean.

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Appendix B
TABLES AND FIGURES

Appendix B

Listed below are definitions of various legends which appear on some of the tables and figures presented in this appendix.

Seasons

Season 1 - December, January, February, March (Winter)
Season 2 - April, May (Spring)
Season 3 - June, July, August, September (Summer)
Season 4 - October, November (Fall)

Hour Groups (Class Intervals)

Hour Group 1: 0000-0200 EST
Hour Group 2: 0300-0500 EST
Hour Group 3: 0600-0800 EST
Hour Group 4: 0900-1100 EST
Hour Group 5: 1200-1400 EST
Hour Group 6: 1500-1700 EST
Hour Group 7: 1800-2000 EST
Hour Group 8: 2100-2300 EST

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Seasons

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Hour Group 6: 1500-1700 EST
Hour Group 7: 1800-2000 EST
Hour Group 8: 2100-2300 EST

Table 1
UNIVARIATE FISHER-TIPPETT PARAMETERS

	SEASON 1		SEASON 2		SEASON 3		SEASON 4		ANNUAL	
3.0 meter (m) Level	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu
1 DAY(S)	.1980	17.6862	.2281	19.1252	.2334	17.3724	.2292	16.2453	.2219	17.6601
5 DAY(S)	.2276	25.5303	.2083	25.6664	.1847	22.9542	.2056	21.7843	.2107	24.1730
10 DAY(S)	.2676	28.9075	.2043	28.8255	.1909	27.1348	.1757	24.5094	.2156	27.2305
15 DAY(S)	.2520	29.9834	.1938	30.9572	.2012	28.8273	.1539	25.3155	.2312	29.4385
30 DAY(S)									.2491	32.5136
60 DAY(S)									.2588	35.1433
18.3-m Level (Small Tower)										
1 DAY(S)	.1639	19.6085	.1879	20.8161	.1793	19.1755	.1905	18.3189	.1796	19.5721
5 DAY(S)	.1722	23.0930	.1713	29.3026	.1506	26.1547	.1932	25.2917	.1750	27.7174
10 DAY(S)	.1918	33.5404	.1646	31.9750	.1532	31.1315	.1724	28.3257	.1828	31.7676
15 DAY(S)	.1776	34.9346	.1623	34.0357	.1500	33.3059	.1676	30.3746	.1794	33.6532
30 DAY(S)									.2133	38.4312
60 DAY(S)									.2012	41.0234
18.3-m Level (Large Tower)										
1 DAY(S)	.1555	20.0870	.1816	21.5853	.1893	19.3263	.1856	19.3434	.1822	20.0155
5 DAY(S)	.1827	29.4499	.1612	29.3424	.1592	26.3965	.1846	26.8031	.1788	28.2168
10 DAY(S)	.1994	32.3912	.1529	33.1131	.1745	30.7303	.1454	28.3043	.1915	32.0476
15 DAY(S)	.1859	34.4975	.1591	36.2563	.1832	33.7431	.1554	31.9942	.1989	34.3866
30 DAY(S)									.2063	37.9565
60 DAY(S)									.2214	41.3155

Table 1 (Continued)

	SEASON 1		SEASON 2		SEASON 3		SEASON 4		NNUAL	
30.5-m Level	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu
1 DAY(S)	.1550	20.7166	.1707	21.6861	.1712	19.2487	.1848	19.7965	.1597	20.2933
5 DAY(S)	.1516	30.4067	.1454	29.9979	.1441	27.1968	.1799	26.7723	.1519	28.2736
10 DAY(S)	.1386	33.5950	.1240	33.5265	.1492	32.2300	.1441	28.3954	.1555	32.7044
15 DAY(S)	.1447	36.1877	.1157	36.2534	.1496	34.1653	.1427	31.5904	.1647	35.6758
30 DAY(S)									.1660	39.9897
60 DAY(S)									.1712	43.8426
61.0-m Level										
1 DAY(S)	.1566	22.5848	.1763	22.6912	.1682	20.3979	.1808	20.9437	.1693	21.6529
5 DAY(S)	.1453	31.5627	.1514	30.5726	.1365	27.2076	.1645	28.0252	.1567	30.0647
10 DAY(S)	.1526	36.8614	.1466	34.6543	.1428	33.0591	.1394	30.1464	.1572	34.3613
15 DAY(S)	.1379	37.6921	.1360	36.7166	.1482	35.1451	.1461	34.8405	.1625	36.6849
30 DAY(S)									.1912	41.3469
60 DAY(S)									.1995	45.4489
91.4-m Level										
1 DAY(S)	.1435	23.3464	.1705	23.0332	.1576	20.4785	.1649	22.5400	.1562	22.2811
5 DAY(S)	.1166	32.2934	.1440	30.1158	.1304	28.7280	.1543	29.9619	.1382	30.8765
10 DAY(S)	.1062	36.7226	.1330	34.7820	.1424	34.7290	.1392	33.2107	.1352	35.5785
15 DAY(S)	.0997	39.8340	.1057	35.2870	.1415	37.0430	.1240	35.5570	.1306	37.0531
30 DAY(S)									.1245	42.2598
60 DAY(S)									.1107	46.7773

Table 1 (Concluded)

	SEASON 1		SEASON 2		SEASON 3		SEASON 4		ANNUAL	
121.9-m Level	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu
1 DAY(S)	.1485	23.8716	.1729	23.4462	.1571	21.1340	.1910	22.4182	.1615	22.5953
5 DAY(S)	.1458	34.0139	.1406	31.0998	.1288	28.7976	.1743	28.7781	.1395	30.6353
10 DAY(S)	.1471	38.4955	.1348	35.3804	.1251	33.4353	.1468	30.4203	.1405	35.0139
15 DAY(S)	.1382	40.7446	.1313	38.9293	.1407	38.0943	.1242	34.0097	.1563	38.5405
30 DAY(S)									.1614	42.6849
60 DAY(S)									.1667	46.2697
152.4-m Level										
1 DAY(S)	.1474	24.6936	.1708	23.7952	.1555	20.6758	.1594	22.4727	.1548	22.8319
5 DAY(S)	.1560	34.8325	.1449	31.8098	.1253	29.0055	.1438	29.3745	.1459	31.3873
10 DAY(S)	.1609	39.0290	.1290	35.7644	.1358	35.0125	.1260	32.6426	.1522	36.3330
15 DAY(S)	.1524	41.1537	.1297	39.1298	.1543	38.5324	.1045	33.7381	.1485	38.6627
30 DAY(S)									.1644	44.0742
60 DAY(S)									.1870	48.9235

Table 2

FISHER-TIPPETT PARAMETERS FOR BIVARIATE VARIABLES
(ANNUAL REFERENCE PERIOD)

Level (m)	Days											
	1		5		10		15		30		60	
	α	μ	α	μ	α	μ	α	μ	α	μ	α	μ
3.0	.1810	14.2878	.1662	20.6778	.1769	24.6429	.1736	26.0837	.2052	30.2577	.1910	32.6667
18.3*	.1640	17.1384	.1515	24.8372	.1560	28.7317	.1534	30.7555	.1894	36.2384	.1883	39.7986
18.3**	.1652	17.4830	.1516	25.0676	.1527	29.0300	.1534	31.0774	.1872	36.7650	.1748	39.2638
30.5	.1623	18.3746	.1478	26.3781	.1491	30.6032	.1522	32.7241	.1677	37.7720	.1454	40.4169
61.0	.1591	20.4027	.1462	28.9721	.1453	33.0903	.1424	35.3118	.1633	41.0930	.2349	46.6340
91.4	.1585	21.6060	.1455	30.1034	.1517	34.7413	.1467	36.6726	.1688	42.8504	.2164	47.5267
152.4	.1596	22.2269	.1480	30.5050	.1510	35.0481	.1455	37.2572	.1666	42.9857	.1977	47.8719

*Small Tower

**Large Tower

Table 3

BIVARIATE PARAMETERS

Level (m)	Parameters	Days					
		1	5	10	15	30	60
3.0	$\hat{b}$*	.56969	.36294	.43289	.41158	.43683	+
	S. D.** of $\hat{b}$	.03319	.06428	.10020	.11962	.16898	+
	$1/\hat{b}$ = m	1.75534	2.75528	2.31006	2.42966	2.28922	+
	$\hat{m}$ (difference)	2.10511	2.35289	2.38506	2.20417	1.68561	1.55911
18.3 (Small Tower)	$\hat{b}$	.44632	.36450	.34117	.35626	.48122	.82993
	S. D. of $\hat{b}$	.02970	.06097	.08439	.10539	.16758	.28567
	$1/\hat{b}$ = m	2.24054	2.74348	2.93109	2.80694	2.07805	1.20492
	$\hat{m}$ (difference)	2.53885	2.68170	2.71930	2.54904	2.05584	1.69033
18.3 (Large Tower)	$\hat{b}$	.42578	.34117	.35469	.52044	.74680	.99999
	S. D. of $\hat{b}$	.02904	.05967	.08735	.12164	.19611	.30042
	$1/\hat{b}$ = m	2.34863	2.93109	2.81936	1.92145	1.33905	1.00001
	$\hat{m}$ (difference)	2.53189	2.74505	2.66802	2.42463	1.86083	1.79496
30.5	$\hat{b}$	.39285	.34117	.36681	.50083	.53854	.61401
	S. D. of $\hat{b}$	.02823	.05967	.08671	.11777	.17057	.25145
	$1/\hat{b}$ = m	2.54550	2.93109	2.72621	1.99669	1.85687	1.62864
	$\hat{m}$ (difference)	2.86603	2.99636	2.83738	2.59808	2.05660	1.89005
61.0	$\hat{b}$	.25586	.26440	.30120	.24418	.08596	.33689
	S. D. of $\hat{b}$	.02446	.05547	.08205	.09177	.08346	.21274

(Continued)

* ^ = Estimate

** S. D. = Standard Deviation

+ No Data Available

Table 3 - (Continued)

Level (m)	Parameters	Days					
		1	5	10	15	30	60
61.0	$1/\hat{b} = m$	3.90839	3.78215	3.32005	4.09534	11.63332	2.96833
	$\hat{m}$ (difference)	3.66767	3.84697	3.78167	3.52995	3.26068	3.25222
91.4	$\hat{b}$	.17722	.18587	.23795	.27857	.08596	.17060
	S. D. of b	.02053	.04663	.07317	.09584	.08346	.16096
	$1/\hat{b} = m$	5.64270	5.38010	4.20256	3.58976	11.63332	5.86166
	$\hat{m}$ (difference)	4.63840	4.84765	4.70826	4.68941	3.91131	4.45607
121.9	$\hat{b}$	.13190	.15551	.14252	.25946	+	.18587
	S. D. of b	.01839	.04446	.06071	.09702	+	.17446
	$1/\hat{b} = m$	7.58150	6.43045	7.01656	3.85416	+	5.38010
	$\hat{m}$ (difference)	6.75227	6.55409	6.63867	7.67962	6.78042	5.56431

*$\wedge$ = Estimate

**S. D. = Standard Deviation

+ No Data Available

Table 4
POWER-LAW PARAMETERS

HR Group		Seasons 1	2	3	4
1	p	-.21462	-.21406	-.16695	.05858
	$\overline{c}$	.43701	.43701	.35409	.21158
	σ	.26566	.07989	.14986	.06116
2	p	.11185	-.36407	.03017	.22149
	$\overline{c}$	.21659	.64234	.24597	.15327
	σ	.13806	.15223	.10604	.03804
3	p	.15735	.00236	.05304	-.04912
	$\overline{c}$	.15663	.19020	.15447	.25423
	σ	.11924	.09164	.10686	.08622
4	p	-.15599	.64863	.44611	.37386
	$\overline{c}$	.09463	.01172	.02425	.02434
	σ	.20820	.01472	.02802	.02128
5	p	.66795	-.22159	-.11499	.12271
	$\overline{c}$	.00920	.07826	.06279	.02840
	σ	.01312	.07669	.10824	.02268
6	p	.30195	-.37912	-.15348	.72094
	$\overline{c}$	.03857	.09798	.07583	.01471
	σ	.04290	.13380	.11513	.01240
7	p	-.09675	-.32209	-.26251	-.09029
	$\overline{c}$	.24682	.27933	.22946	.23852
	σ	.14055	.14976	.21505	.08621
8	p	.04346	-.19451	-.11263	.13888
	$\overline{c}$	.23350	.36757	.26270	.16746
	σ	.12614	.12106	.15159	.04144

Table 5
PEAK WIND PROFILE PARAMETERS

	Seasons							
	1		2		3		4	
HR Group 1	β	γ	β	γ	β	γ	β	γ
0 σ	.1156	17.4977	.1982	16.6153	.1970	12.0225	.1271	14.7881
1 σ	.1413	16.5633	.2057	16.1509	.2199	11.3656	.1383	14.3608
2 σ	.1495	16.3734	.2075	16.0332	.2276	11.2726	.1410	14.2542
3 σ	.1502	16.3338	.2077	16.0203	.2283	11.2548	.1413	14.2413
HR Group 2								
0 σ	.0909*	15.6637*	.1842	14.8119	.1770	10.8550	.1297	14.8598
1 σ	.1361*	16.3170*	.1914	14.2182	.2097	10.5237	.1441	14.6007
2 σ	.1496	16.3745	.1938	14.1035	.2184	10.4447	.1475	14.5276
3 σ	.1507	16.3519	.1940	14.0872	.2193	10.4321	.1478	14.5175
HR Group 3								
0 σ	.0898*	15.3327*	.1575	14.2544	.1797	8.2004	.1383	15.6891
1 σ	.1325*	16.0953*	.1770	13.6590	.2140	8.1154	.1501	15.1069
2 σ	.1445	16.1435	.1827	13.5638	.2230	8.0829	.1531	14.9676
3 σ	.1454	16.1178	.1832	13.5482	.2238	8.0751	.1534	14.9503
HR Group 4								
0 σ	.1151	17.4992	.1401	14.3857	.1847	13.4352	.1602	16.3330
1 σ	.1408	16.6299	.1626	14.4909	.2147	13.2431	.1740	16.0187
2 σ	.1487	16.4549	.1674	14.4740	.2229	13.2461	.1773	15.9545
3 σ	.1493	16.4148	.1678	14.4670	.2235	13.2377	.1776	15.9447

(Continued)

*Not valid for winds that give probability less than 0.80

Table 5 (Continued)

	Seasons							
	1		2		3		4	
HR Group 5	β	γ	β	γ	β	γ	β	γ
0 σ	.1105	16.3016	.2064	18.1298	.1927	15.8762	.1661	16.8306
1 σ	.1272	16.3683	.2141	17.5751	.2139	15.1458	.1713	16.5326
2 σ	.1309	16.3678	.2160	17.4420	.2204	15.0154	.1726	16.4634
3 σ	.1312	16.3605	.2162	17.4277	.2209	14.9949	.1727	16.4548
HR Group 6								
0 σ	.1222	15.8042	.2000	16.3750	.1768	14.6395	.1319	14.5622
1 σ	.1432	15.6559	.2076	15.7243	.1932	13.9872	.1593	15.0120
2 σ	.1483	15.6035	.2095	15.5626	.1974	13.8311	.1651	15.0454
3 σ	.1487	15.5899	.2097	15.5423	.1978	13.8118	.1656	15.0411
HR Group 7								
0 σ	.1308	17.1791	.1879	15.4179	.1771	12.4247	.1494	15.2955
1 σ	.1506	16.3800	.1984	14.7104	.2031	11.6861	.1596	14.7055
2 σ	.1562	16.2137	.2011	14.5375	.2110	11.5222	.1625	14.5910
3 σ	.1567	16.1878	.2019	14.5443	.2117	11.4968	.1627	14.5741
HR Group 8								
0 σ	.1194	17.2464	.1510	15.6843	.1804	12.1118	.1402	15.5036
1 σ	.1532	16.6485	.1609	15.0024	.2119	11.5841	.1517	15.1376
2 σ	.1638	16.5891	.1635	14.8436	.2210	11.4734	.1544	15.0426
3 σ	.1646	16.5645	.1640	14.8413	.2217	11.4524	.1546	15.0315

Table 5 (Continued)

	Seasons							
	1		2		3		4	
HR Group 5	β	γ	β	γ	β	γ	β	γ
0 σ	.1105	16.3016	.2064	18.1298	.1927	15.8762	.1661	16.8306
1 σ	.1272	16.3683	.2141	17.5751	.2139	15.1458	.1713	16.5326
2 σ	.1309	16.3678	.2160	17.4420	.2204	15.0154	.1726	16.4634
3 σ	.1312	16.3605	.2162	17.4277	.2209	14.9949	.1727	16.4548
HR Group 6								
0 σ	.1222	15.8042	.2000	16.3750	.1768	14.6395	.1319	14.5622
1 σ	.1432	15.6559	.2076	15.7243	.1932	13.9872	.1593	15.0120
2 σ	.1483	15.6035	.2095	15.5626	.1974	13.8311	.1651	15.0454
3 σ	.1487	15.5899	.2097	15.5423	.1978	13.8118	.1656	15.0411
HR Group 7								
0 σ	.1308	17.1791	.1879	15.4179	.1771	12.4247	.1494	15.2955
1 σ	.1506	16.3800	.1984	14.7104	.2031	11.6861	.1596	14.7055
2 σ	.1562	16.2137	.2011	14.5375	.2110	11.5222	.1625	14.5910
3 σ	.1567	16.1878	.2019	14.5443	.2117	11.4968	.1627	14.5741
HR Group 8								
0 σ	.1194	17.2464	.1510	15.6843	.1804	12.1118	.1402	15.5036
1 σ	.1532	16.6485	.1609	15.0024	.2119	11.5841	.1517	15.1376
2 σ	.1638	16.5891	.1635	14.8436	.2210	11.4734	.1544	15.0426
3 σ	.1646	16.5645	.1640	14.8413	.2217	11.4524	.1546	15.0315

Table 6

EXTREMAL PARAMETERS FOR HOURLY PEAK WINDS (152.4-meter level)

Hour Group:	1		2		3		4		5		6		7		8	
	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu
Jan.	.1504	17.2883	.1446	16.9803	.1422	16.9076	.1399	16.5100	.1516	17.6054	.1561	17.1365	.1592	17.4445	.1459	17.2641
Feb.	.1349	17.3619	.1499	18.4648	.1255	17.5730	.1306	17.2964	.1532	17.9243	.1321	16.9735	.1376	16.2362	.1391	16.4409
Mar.	.1678	16.1605	.1600	15.7741	.1467	15.3573	.1625	17.3944	.1869	18.5823	.1574	17.0577	.1534	15.6951	.1676	16.7852
Apr.	.1837	16.5022	.1608	14.8626	.1556	14.0735	.1739	15.2018	.2215	17.8864	.2054	17.1157	.1802	15.6053	.1740	15.9530
May	.1937	14.3536	.2007	12.9213	.2122	13.0552	.2202	14.7617	.2126	17.1870	.2031	17.6040	.1977	16.1316	.1998	15.7463
June	.2067	11.9533	.1891	10.6635	.1740	10.4711	.1931	12.9271	.1834	15.6120	.1642	15.6800	.1843	14.0703	.1971	12.5027
July	.2351	11.4282	.2312	10.0472	.2498	9.5264	.2639	11.3136	.2357	14.4436	.2240	14.8258	.2053	12.8744	.2268	11.7561
Aug.	.2554	9.6358	.2292	8.8201	.2378	8.5297	.2056	11.0211	.2461	14.1016	.2157	14.6126	.3041	12.3959	.2977	11.0328
Sept.	.2207	12.0706	.2128	11.5759	.2018	11.3107	.2277	13.6453	.2494	15.9924	.1768	15.5763	.1764	14.1051	.1854	13.1614
Oct.	.1889	12.5850	.1819	12.3571	.1775	13.3411	.2069	15.0263	.2189	16.2234	.2108	14.7373	.1823	14.0603	.1920	13.4982
Nov.	.1328	15.6317	.1396	15.9492	.1479	15.5901	.1590	16.4205	.1512	16.9022	.1396	15.3490	.1433	14.3279	.1449	15.5120
Dec.	.1508	15.1049	.1461	14.9157	.1444	14.4330	.1524	14.6094	.1610	15.0023	.1645	13.2360	.1640	13.9316	.1566	14.9626

Table 7

EXTREMAL PARAMETERS FOR HOURLY PEAK WINDS (18.3-meter level)

Hour Group:	1		2		3		4		5		6		7		8	
	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu
Jan.	.1907	9.7065	.1686	9.1283	.1711	9.4440	.1744	12.8536	.2014	15.7052	.1876	14.2215	.1841	10.4965	.1816	9.8964
Feb.	.1730	10.3860	.1743	10.6488	.1920	11.3218	.1443	14.5626	.1795	17.4608	.1625	15.4340	.1599	11.4610	.1784	10.4311
Mar.	.2167	9.5591	.1983	9.0196	.1771	10.0577	.1918	15.5327	.2086	17.3449	.1829	15.6909	.1982	11.6076	.2232	10.3908
Apr.	.2257	10.3700	.2137	9.3083	.1945	9.5783	.2123	15.1484	.2365	17.5041	.2384	16.6880	.2296	12.6275	.2177	10.8933
May	.2333	9.0653	.2306	7.5398	.2203	9.1771	.2224	14.0513	.2194	16.4019	.2393	16.3511	.2471	12.7594	.2456	10.5647
June	.2754	7.9608	.2747	7.2627	.2277	6.6224	.2150	12.4689	.2300	15.0767	.1899	14.4377	.2022	10.9137	.2272	8.6063
July	.2849	7.3319	.3112	5.9362	.3054	7.2172	.2950	11.3946	.2488	14.3681	.2293	14.2732	.2278	10.8160	.2815	8.4962
Aug.	.3541	6.4022	.3509	5.5797	.3183	6.1840	.2496	10.8654	.2906	14.1574	.2302	14.1023	.2912	10.1470	.3946	8.1226
Sept.	.2456	6.9966	.2796	6.6273	.2321	7.0195	.2351	11.8118	.2563	14.3734	.2159	13.6382	.2013	10.2057	.2067	8.2101
Oct.	.2198	8.1143	.2193	8.4654	.2052	9.3826	.2286	13.6472	.2626	15.2052	.2513	13.5081	.2046	10.4687	.2124	9.5535
Nov.	.1800	9.2290	.1666	6.9548	.1832	9.0779	.1669	13.4917	.1829	15.3748	.1716	12.7881	.1802	6.6476	.1910	9.4900
Dec.	.2025	8.5708	.1837	6.2725	.1886	6.4352	.1906	12.3291	.1956	14.0766	.1999	11.4453	.2185	8.7770	.2061	8.8610

Table 8

EXTREMAL PARAMETERS FOR TEN-MINUTE PEAK WINDS (152.4-meter level)

Hour Group:	1		2		3		4		5		6		7		8	
	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu	Alpha	Mu
Jan.	.1559	15.7269	.1561	15.5320	.1435	15.1251	.1459	14.7140	.1544	15.6679	.1616	15.3545	.1613	15.6871	.1514	15.9517
Feb.	.1396	15.7811	.1557	16.7004	.1275	15.8406	.1286	15.0311	.1545	15.7959	.1386	15.1555	.1382	14.6051	.1441	14.7715
Mar.	.1727	14.9535	.1629	14.4999	.1478	14.0520	.1693	15.2326	.1939	16.8249	.1644	15.3311	.1602	14.3692	.1742	15.4147
Apr.	.1837	14.6055	.1620	13.5016	.1549	12.6386	.1793	12.8249	.2227	15.6932	.2038	15.1888	.1912	14.1026	.1737	14.2581
May	.1966	12.6650	.2031	11.6804	.2126	11.8586	.2144	13.1441	.2185	14.9070	.2156	15.4061	.1966	14.2694	.1962	14.0392
June	.2141	10.4313	.1975	9.5695	.1993	9.2230	.1869	11.0825	.2074	13.3311	.1873	13.3914	.1971	12.3005	.2072	10.8434
July	.2387	9.9786	.2335	8.7192	.2495	7.9244	.2715	9.4610	.2537	12.1632	.2693	12.5116	.2220	10.8864	.2352	10.1798
Aug.	.2649	8.5996	.2406	7.7043	.2483	7.2347	.2934	9.6647	.3240	12.3354	.2705	12.9287	.3034	[illegible]	.2963	9.6595
Sept.	.2260	10.7997	.2130	10.2748	.2157	10.0133	.2298	11.9175	.2561	14.0459	.1938	13.7609	.1661	12.6115	1945	11.8033
Oct.	.1863	11.2879	.1866	11.1524	.1602	12.1277	.2114	13.3238	.2207	14.2236	.2198	13.5234	.1613	12.5751	.1933	12.1326
Nov.	.1374	14.2360	.1450	14.6274	.1532	14.1412	.1633	14.7449	.1573	15.3964	.1466	13.9685	.1490	13.0333	.1486	13.9477
Dec.	.1555	13.9267	.1477	13.6105	.1515	12.9911	.1661	13.0602	.1797	13.5584	.1740	11.8353	.1702	12.7477	.1626	13.7365

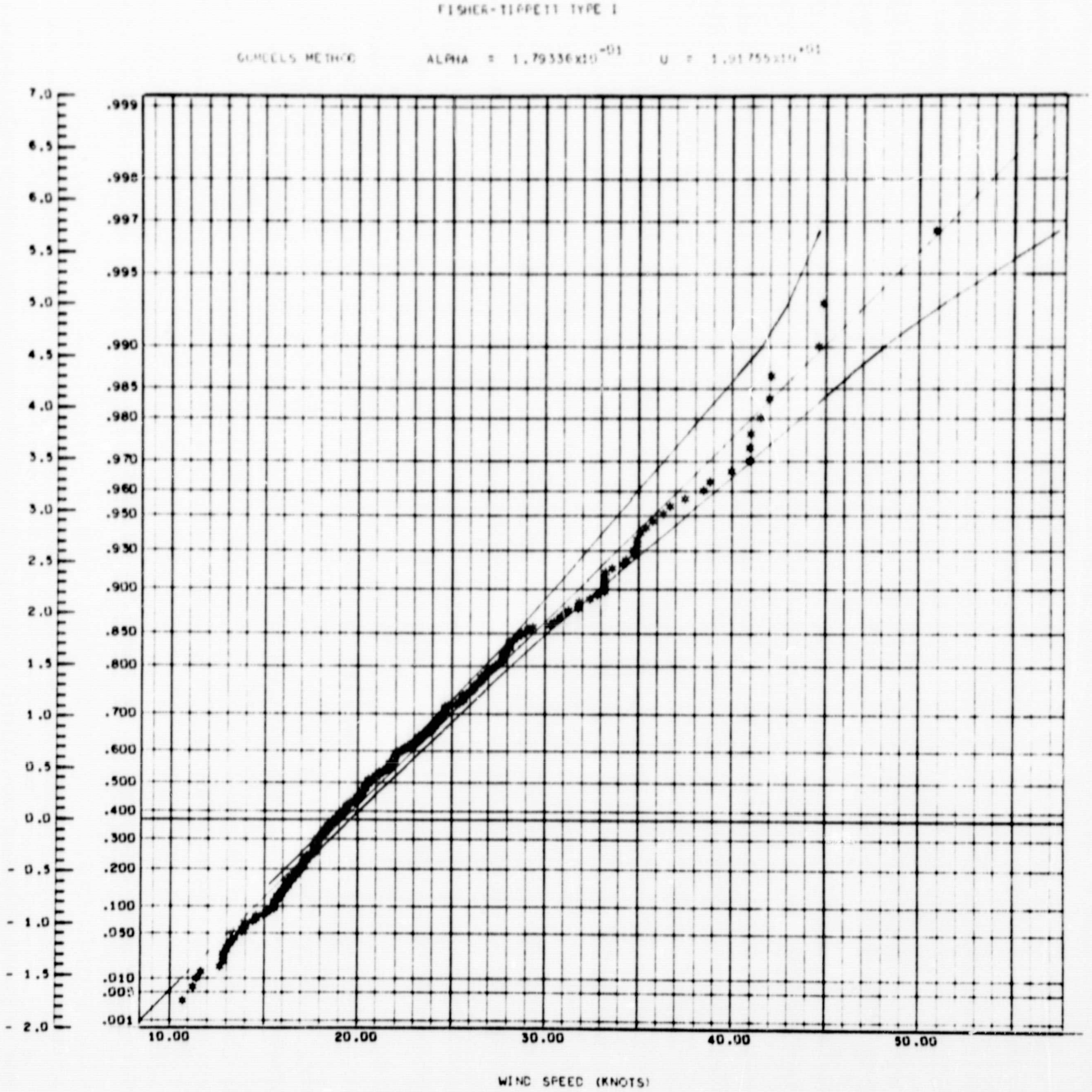

Fig. 1 - Data Fit of 1 Day Peak Winds in Summer at 18.3 Meters

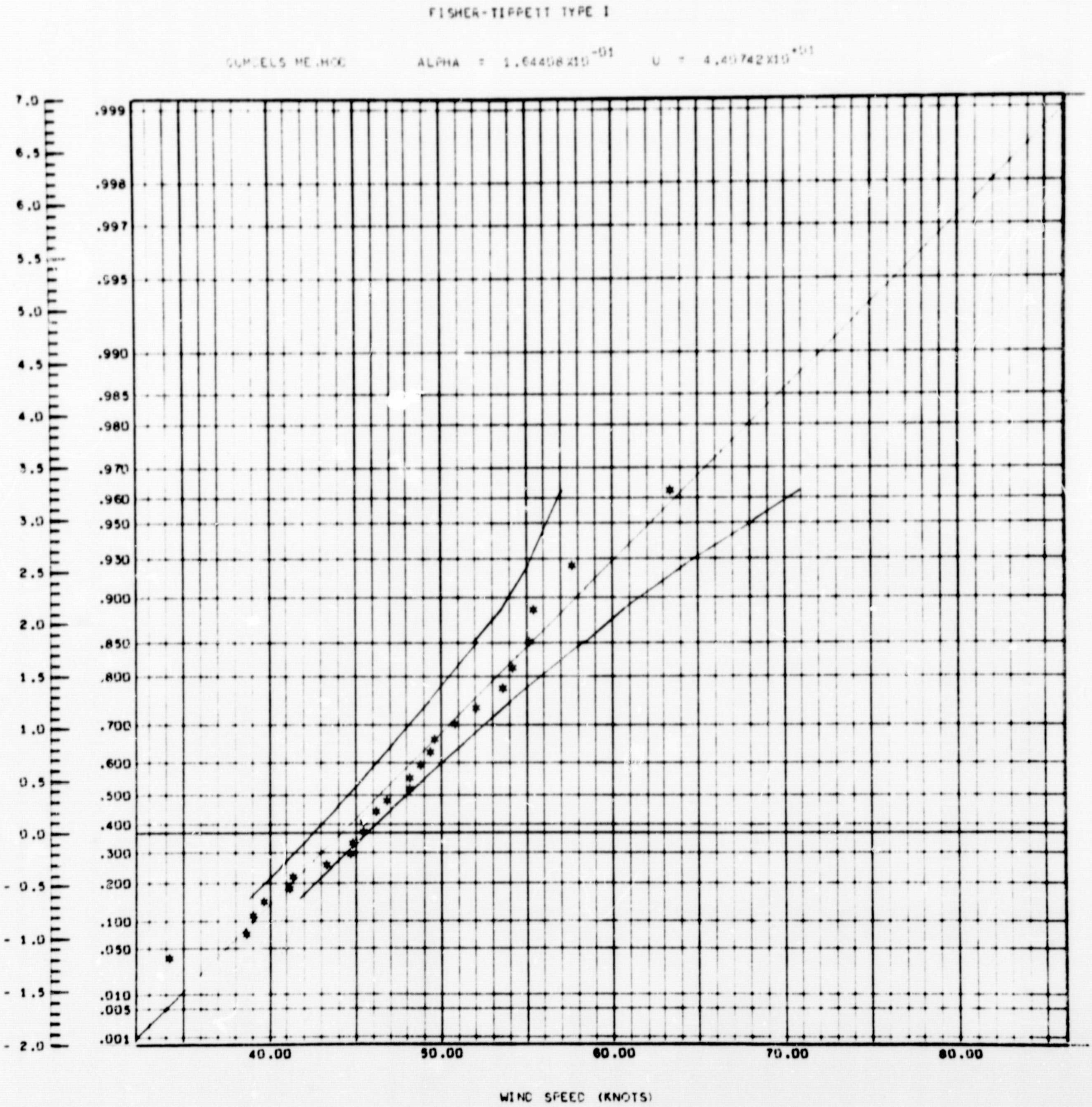

Fig. 2 - Data Fit of 30 Day Peak Winds All Year at 152.4 Meters

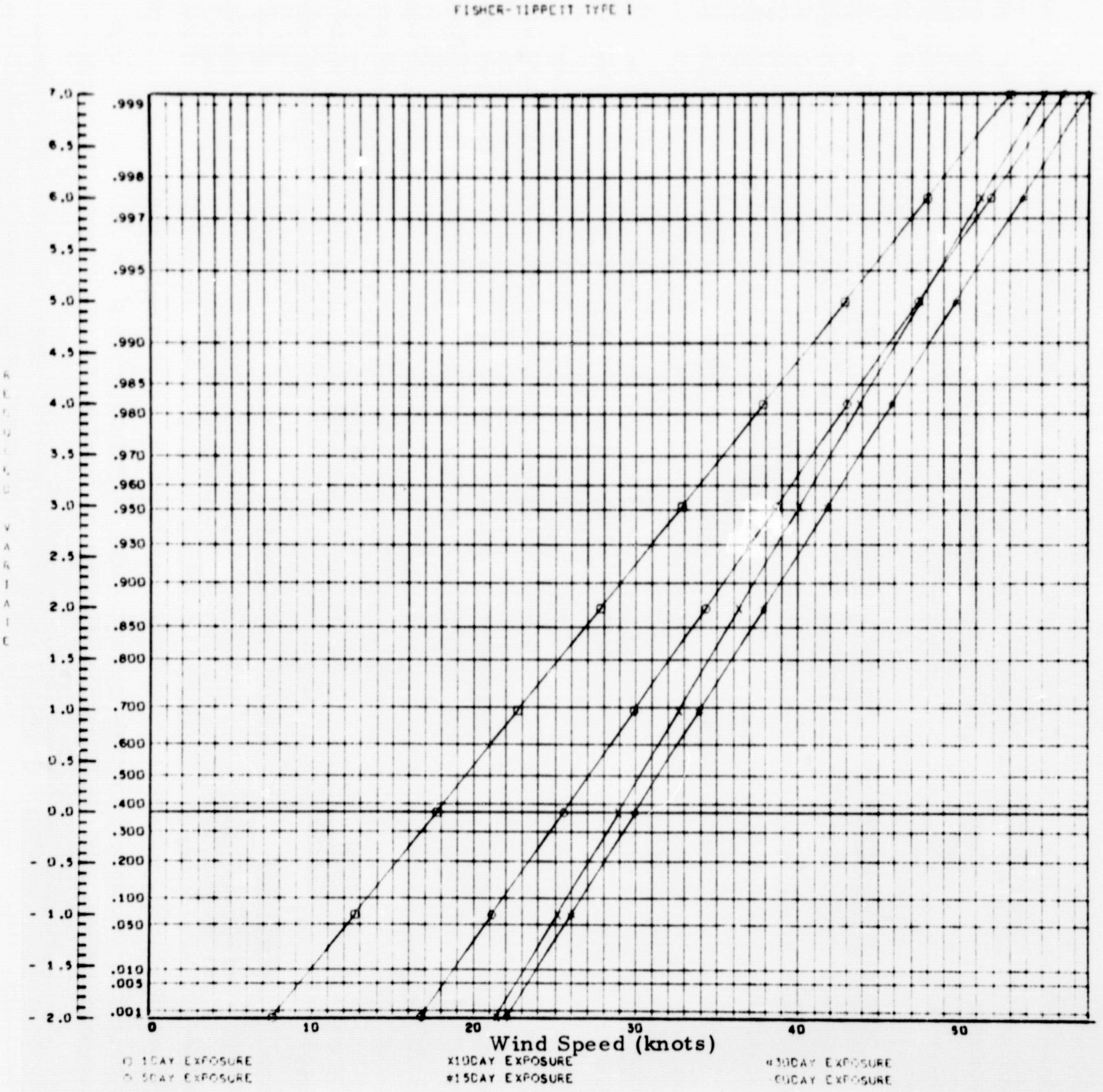

Fig. 3 - Cumulative Distributions in Winter at 3.0 Meters

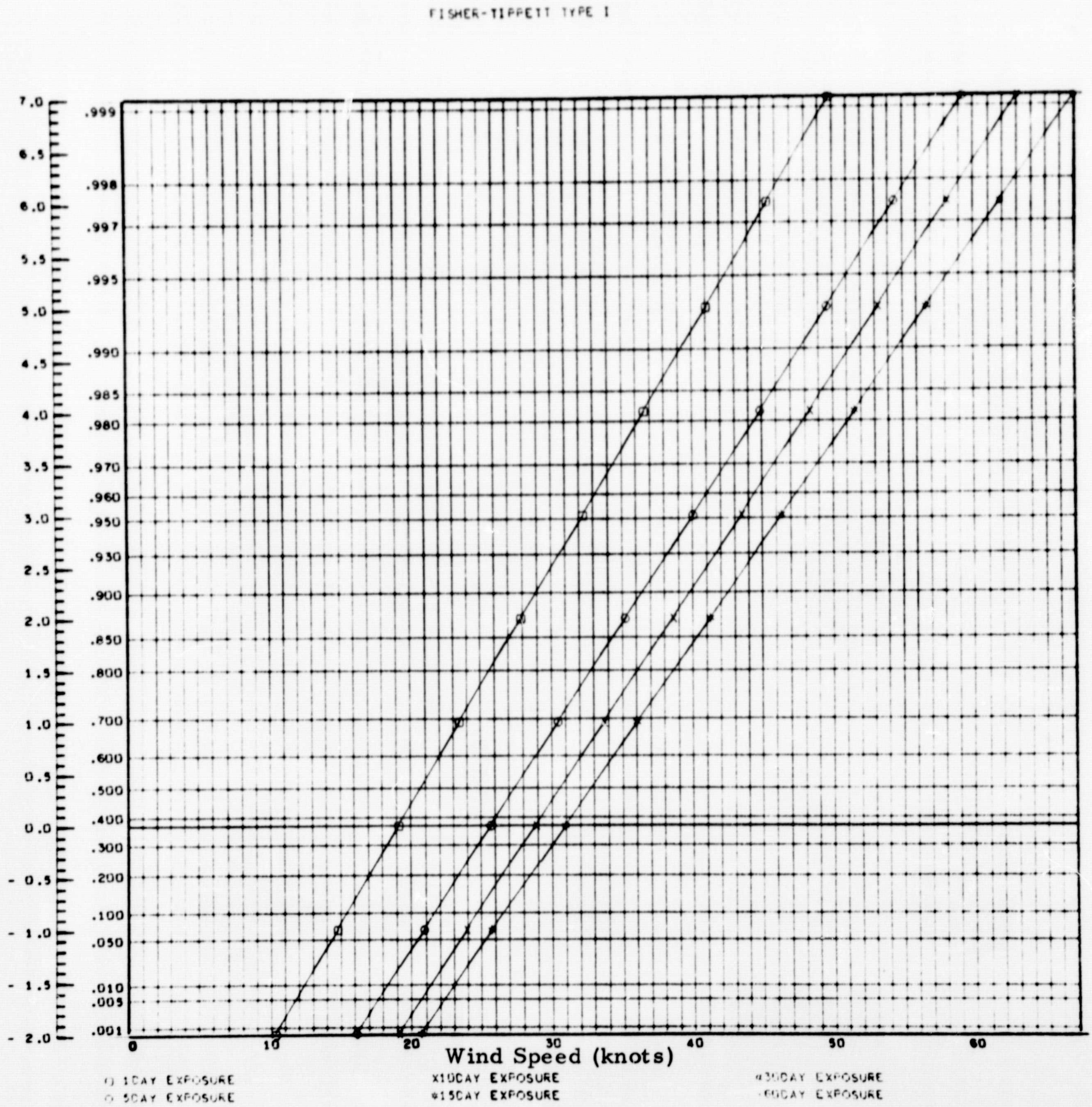

Fig. 4 - Cumulative Distributions in Spring at 3.0 Meters

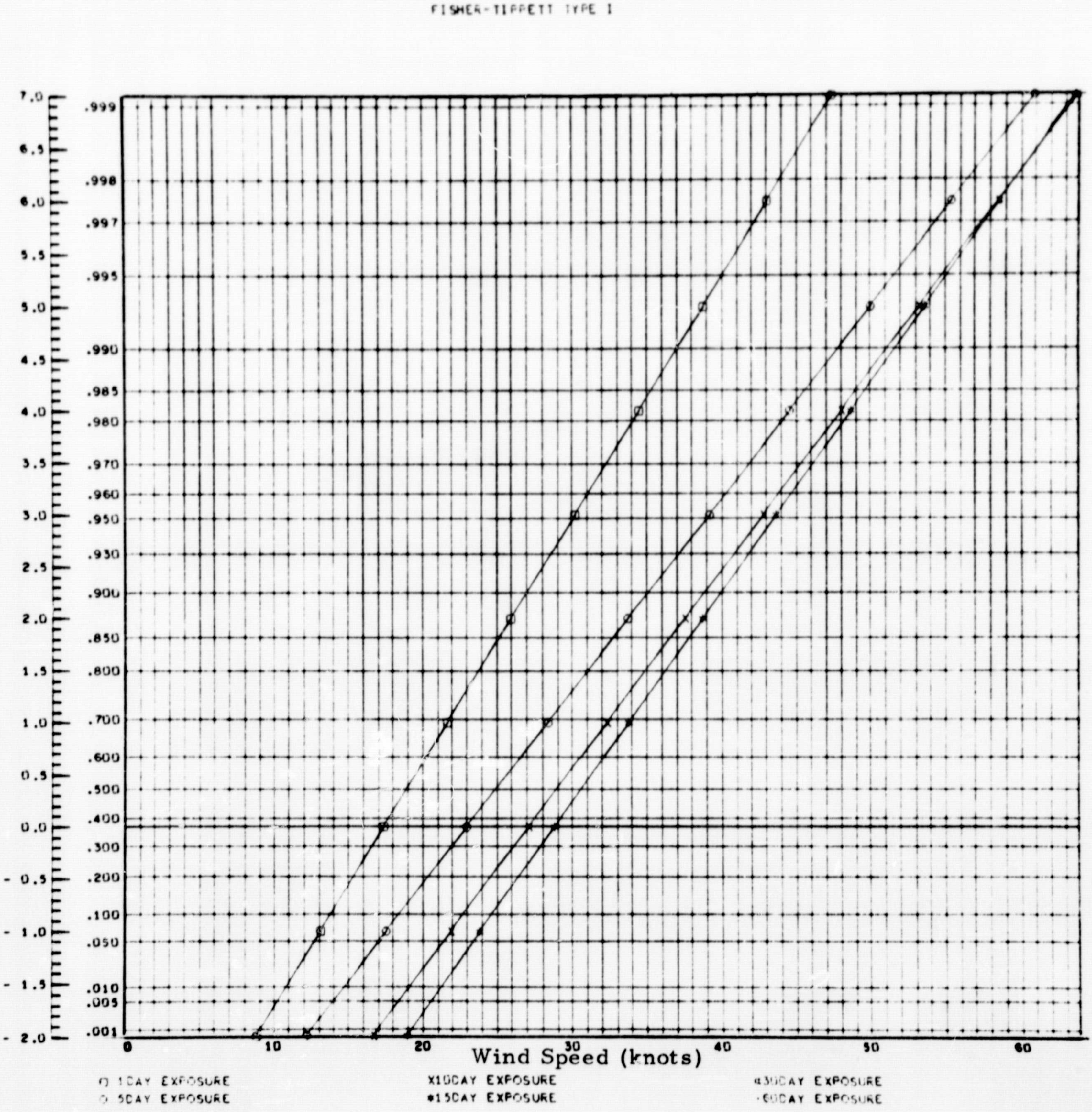

Fig. 5 - Cumulative Distributions in Summer at 3.0 Meters

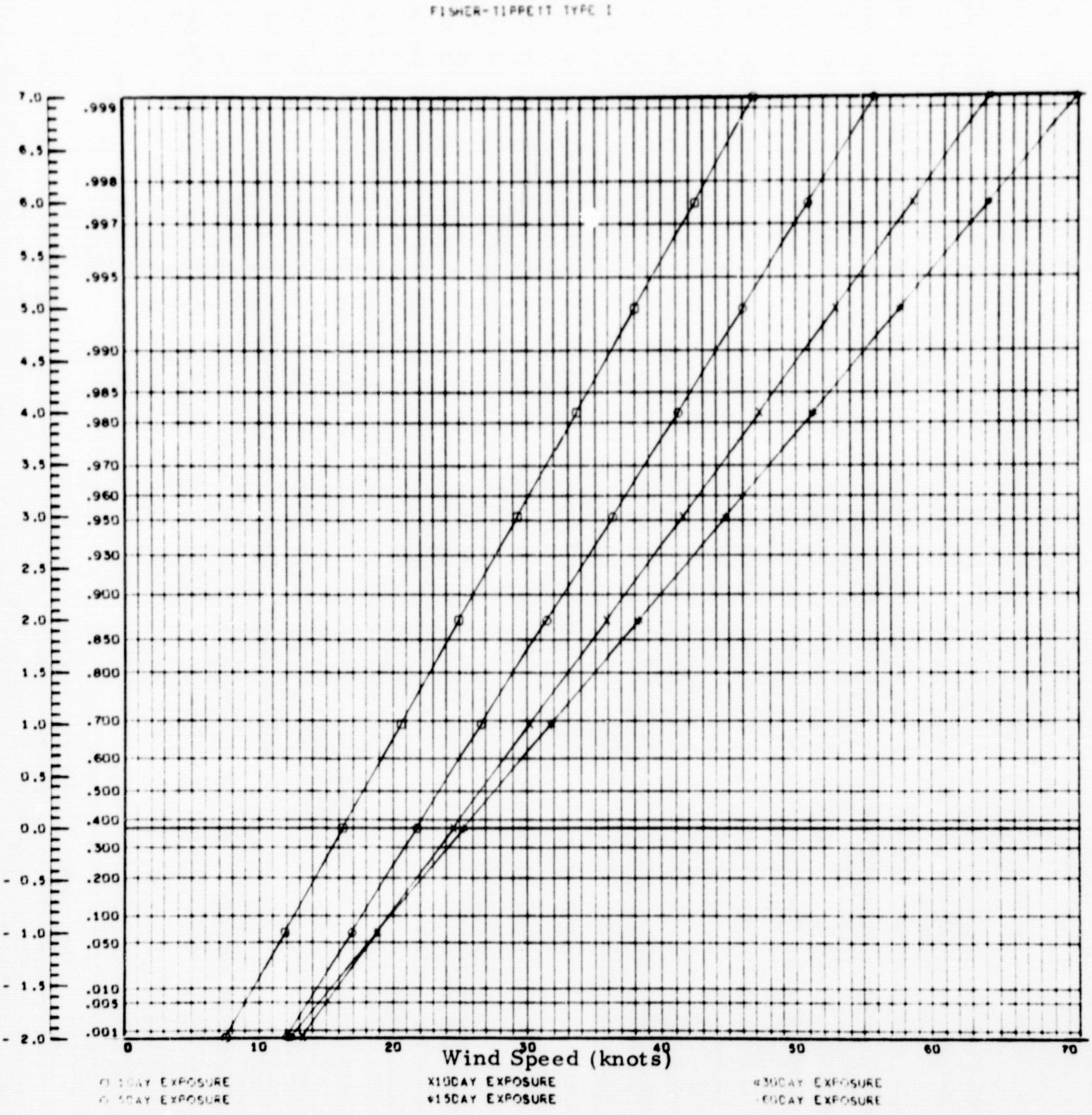

Fig. 6 - Cumulative Distributions in Fall at 3.0 Meters

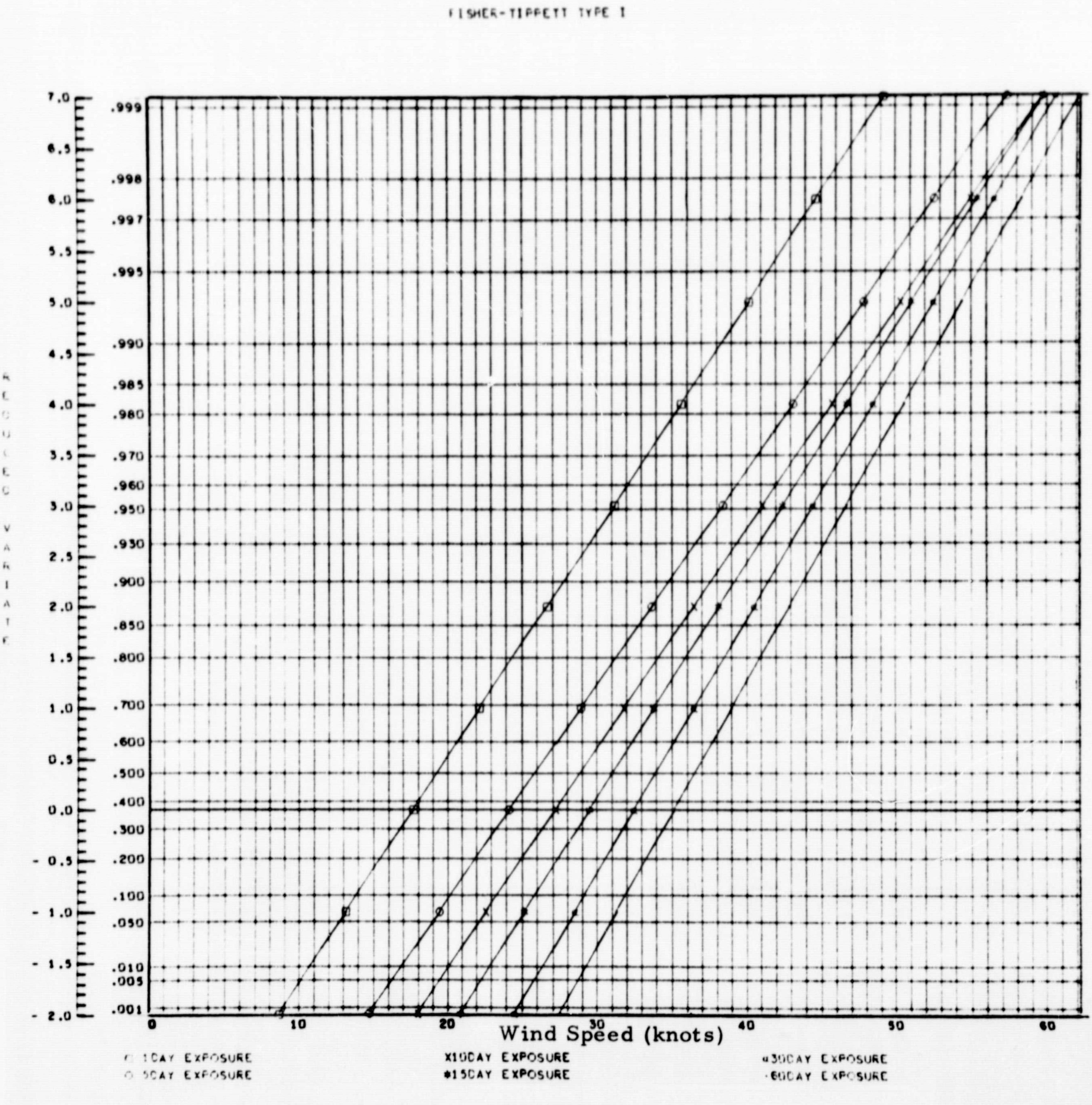

Fig. 7 - Cumulative Distributions for the Year at 3.0 Meters

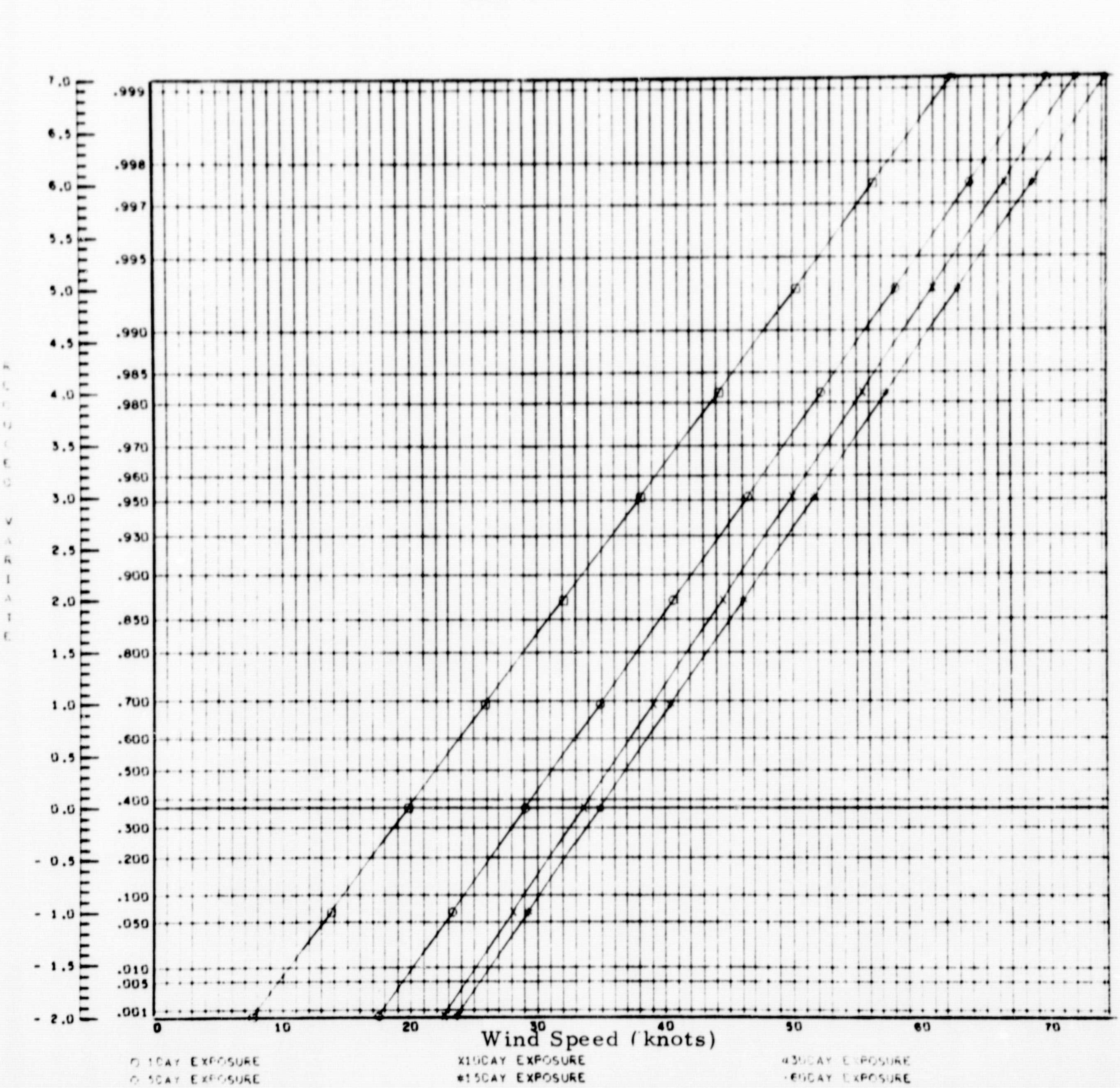

Fig. 8 - Cumulative Distributions in Winter at 18.3 Meters (Small Tower)

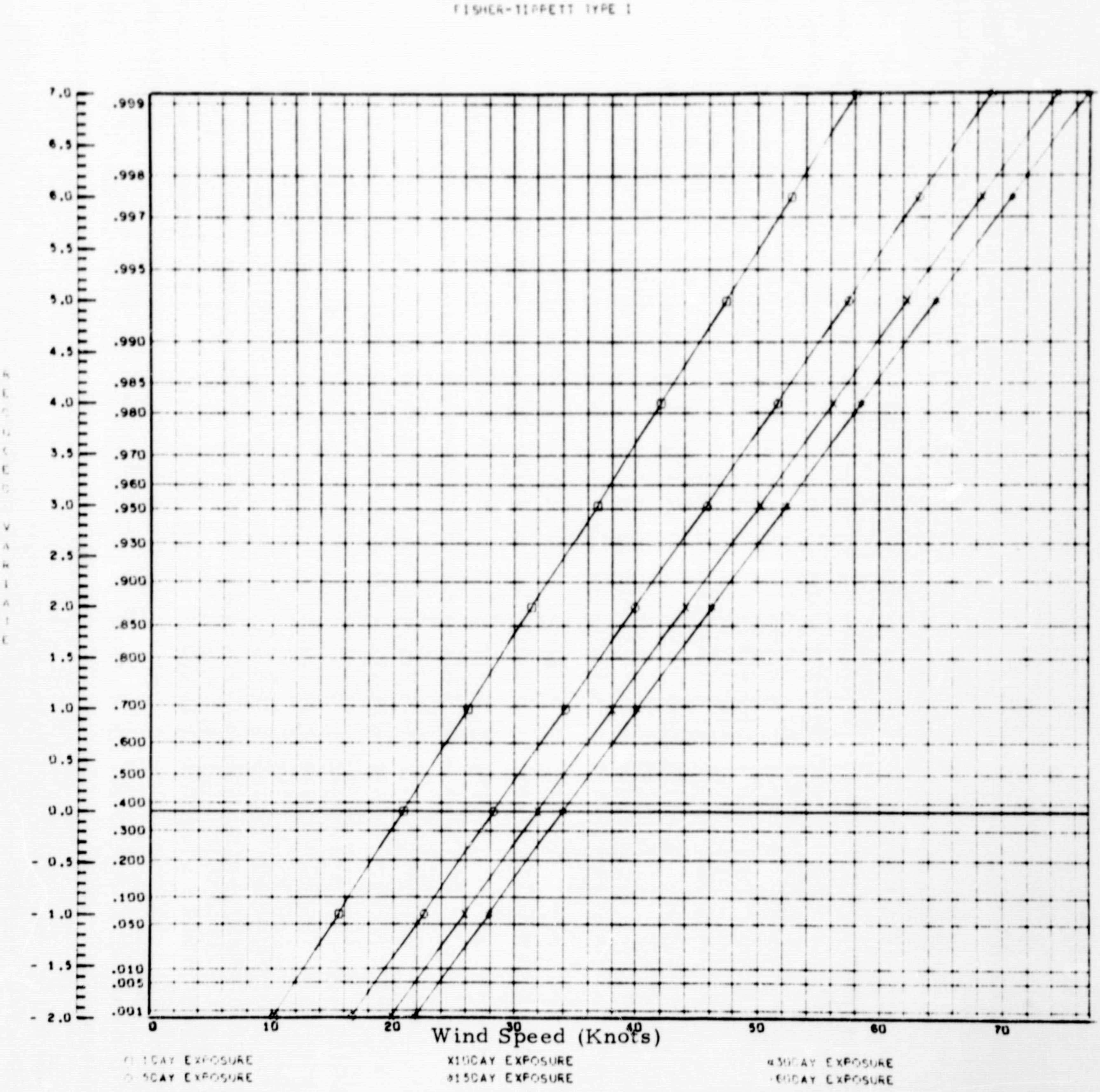

Fig. 9 - Cumulative Distributions in Spring at 18.3 Meters (Small Tower)

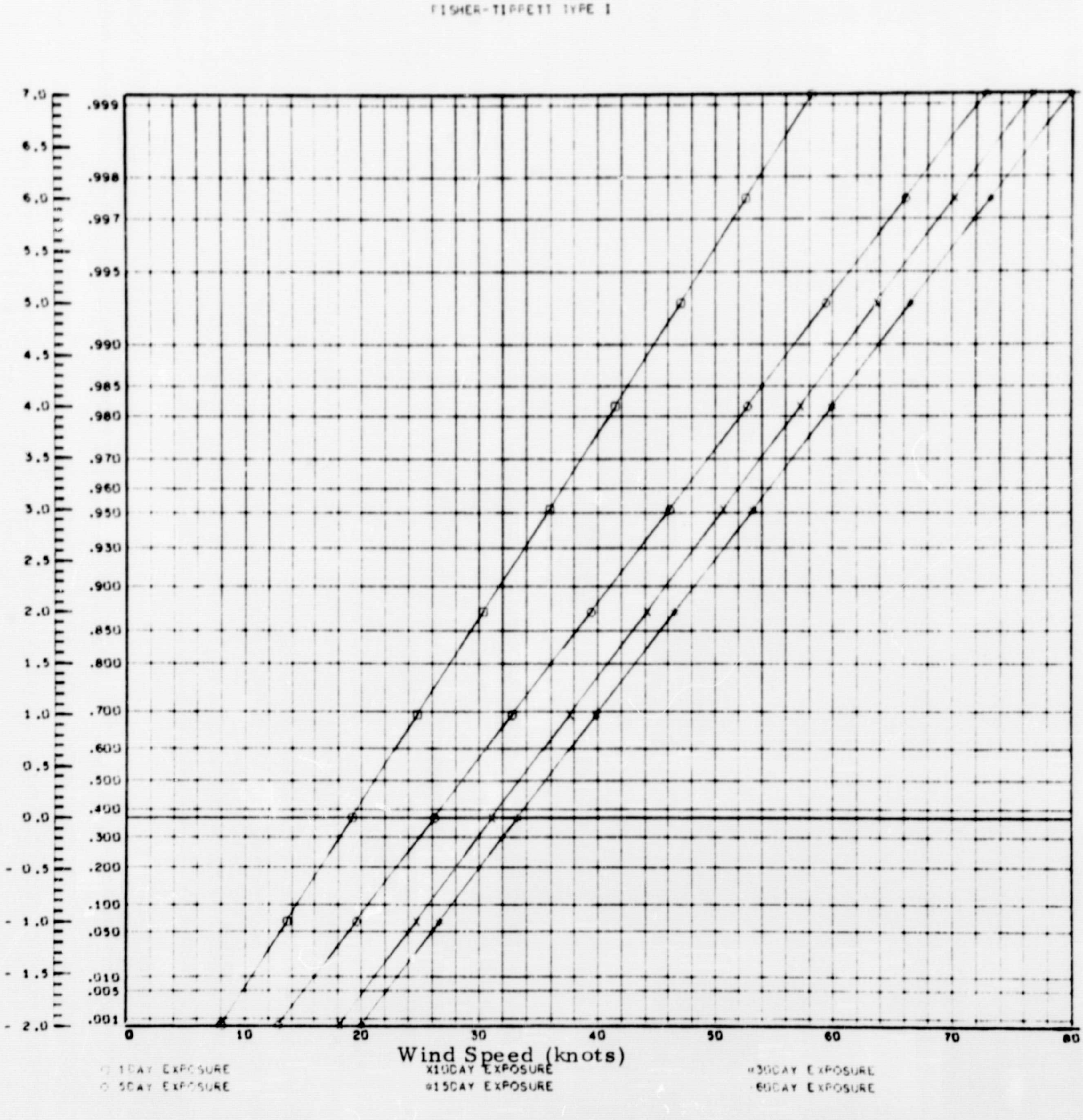

Fig. 10 - Cumulative Distributions in Summer at 18.3 Meters (Small Tower)

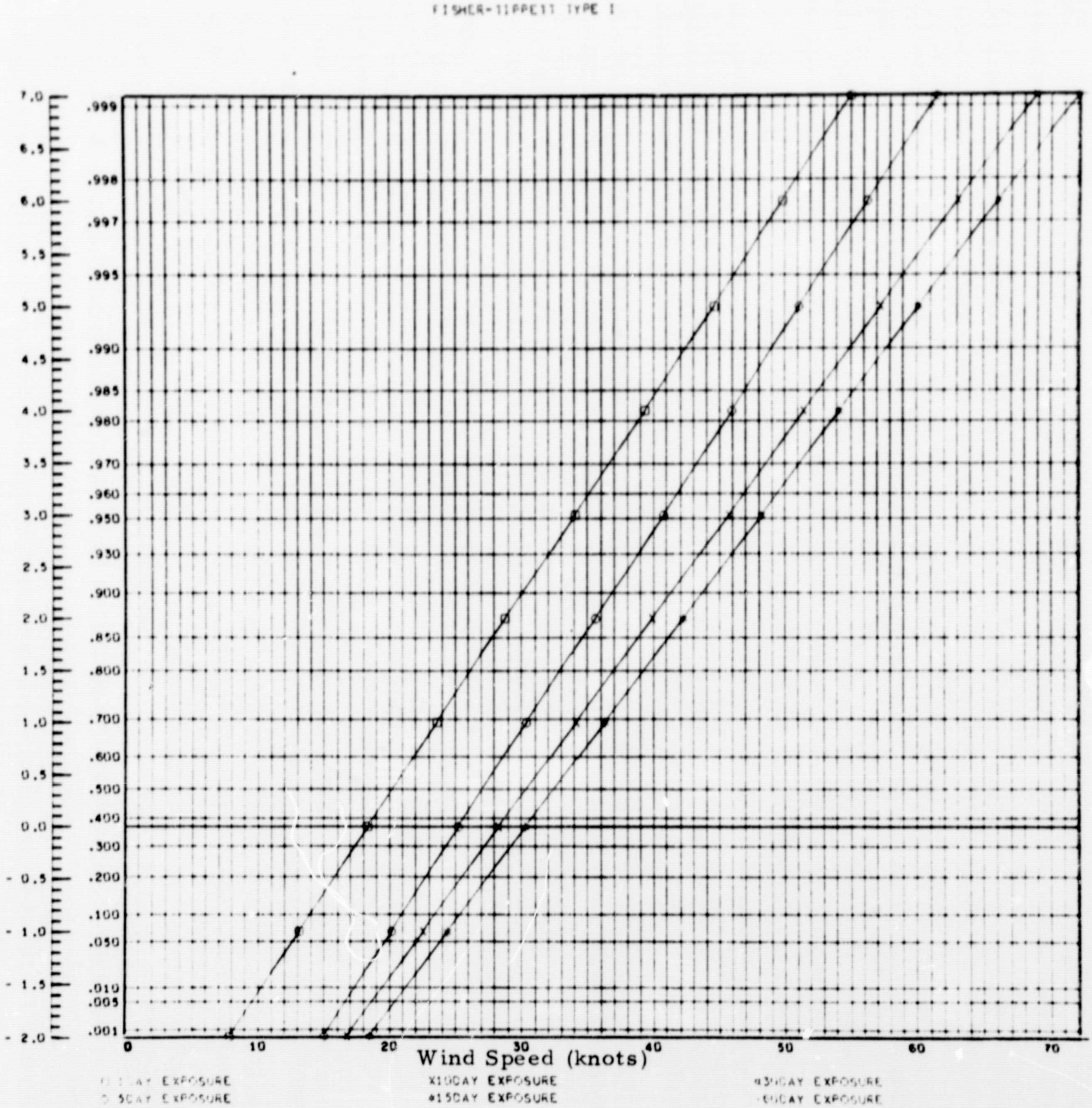

Fig. 11 - Cumulative Distributions in Fall at 18.3 Meters (Small Tower)

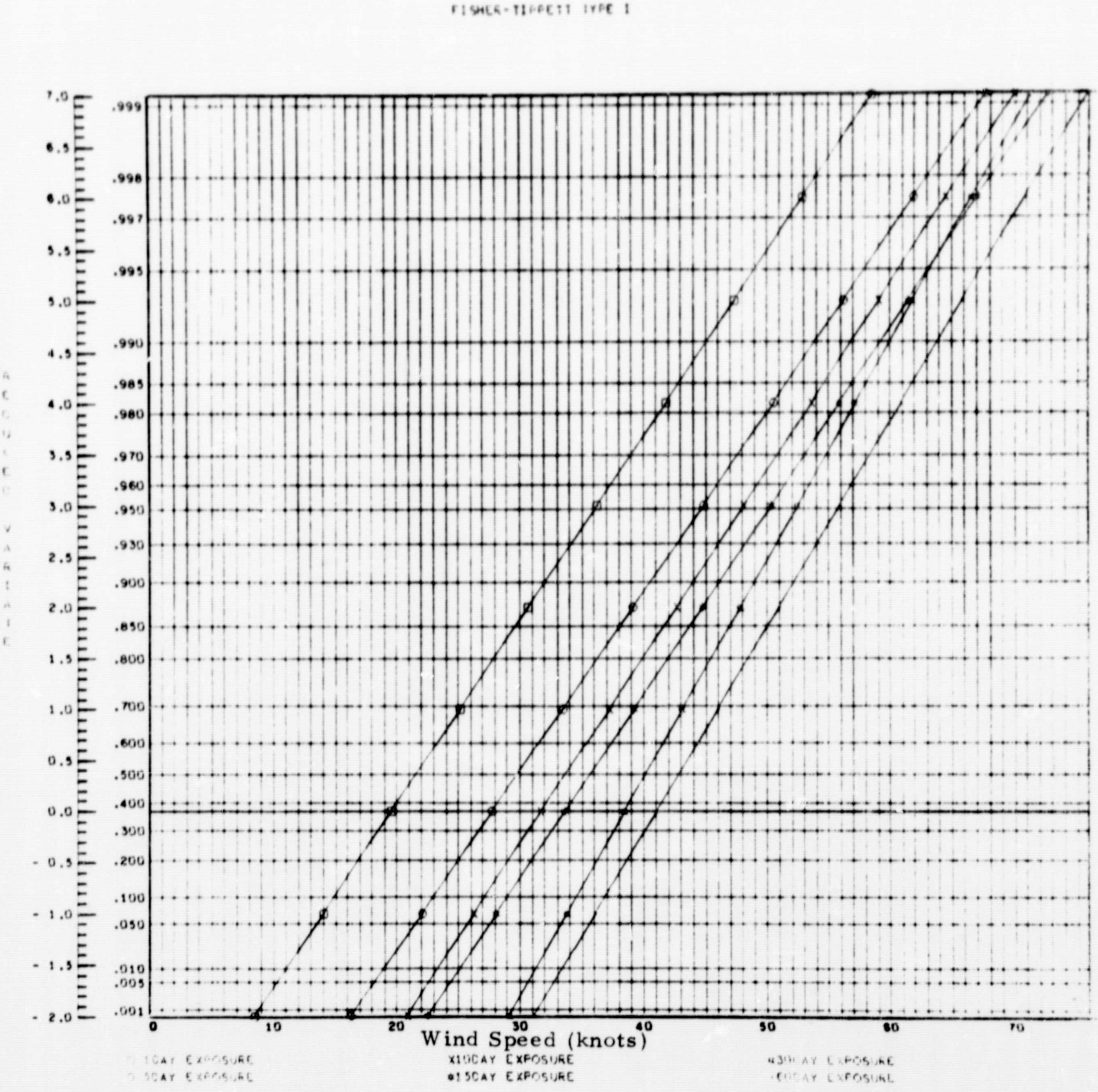

Fig. 12 - Cumulative Distributions for the Year at 18.3 Meters (Small Tower)

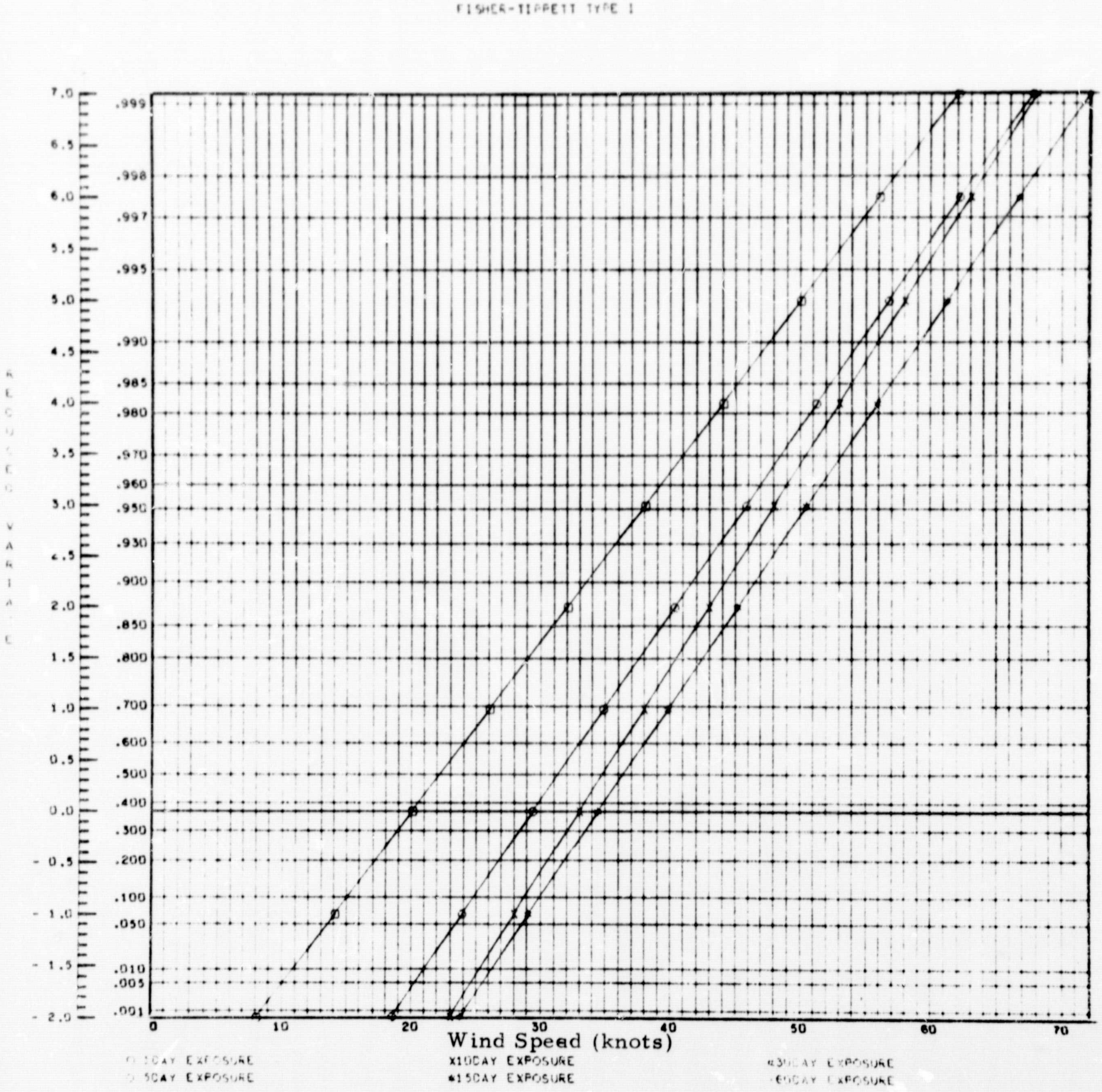

Fig. 13 - Cumulative Distributions in Winter at 18.3 Meters (Large Tower)

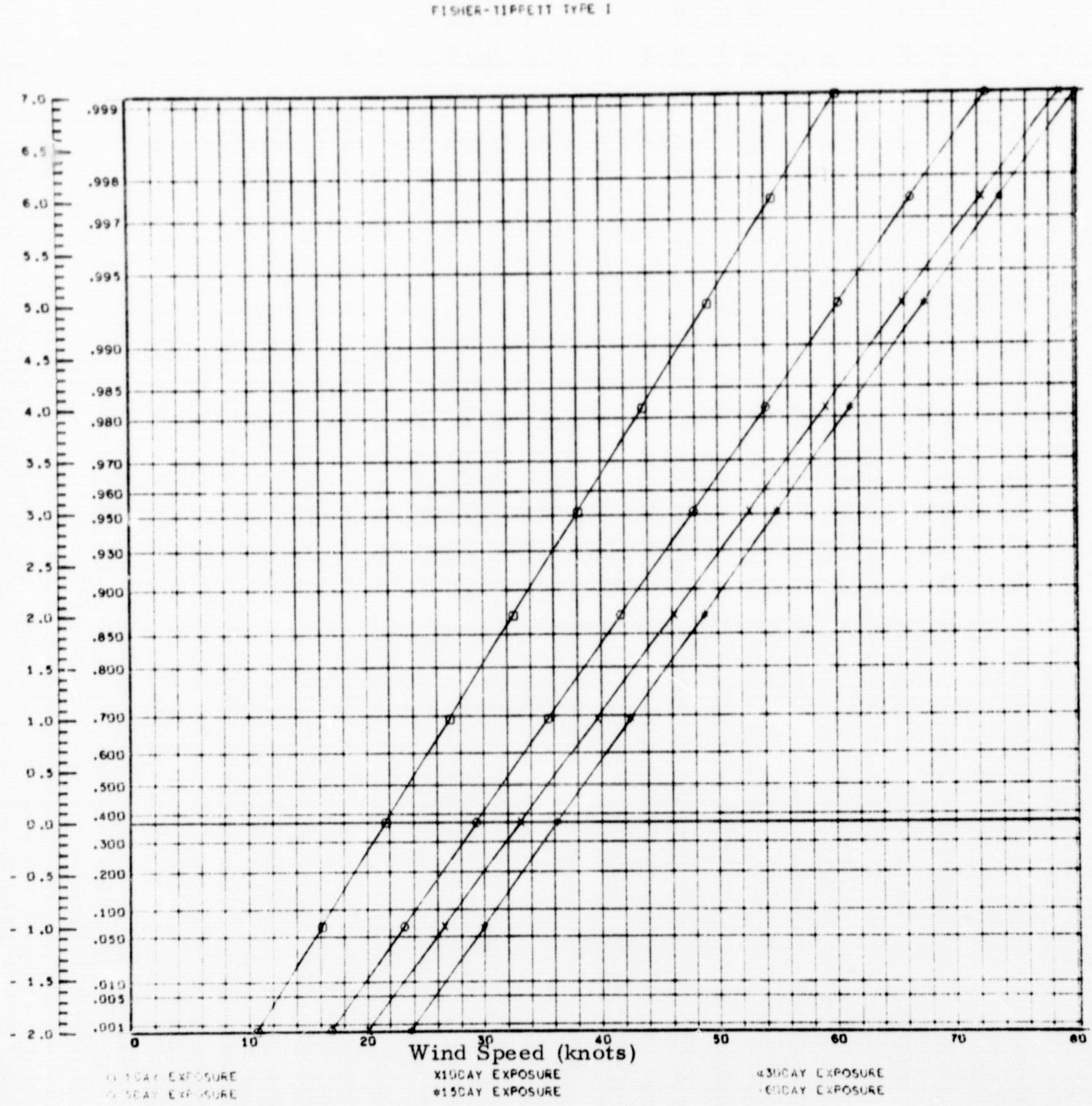

Fig. 14 - Cumulative Distributions in Spring at 18.3 Meters (Large Tower)

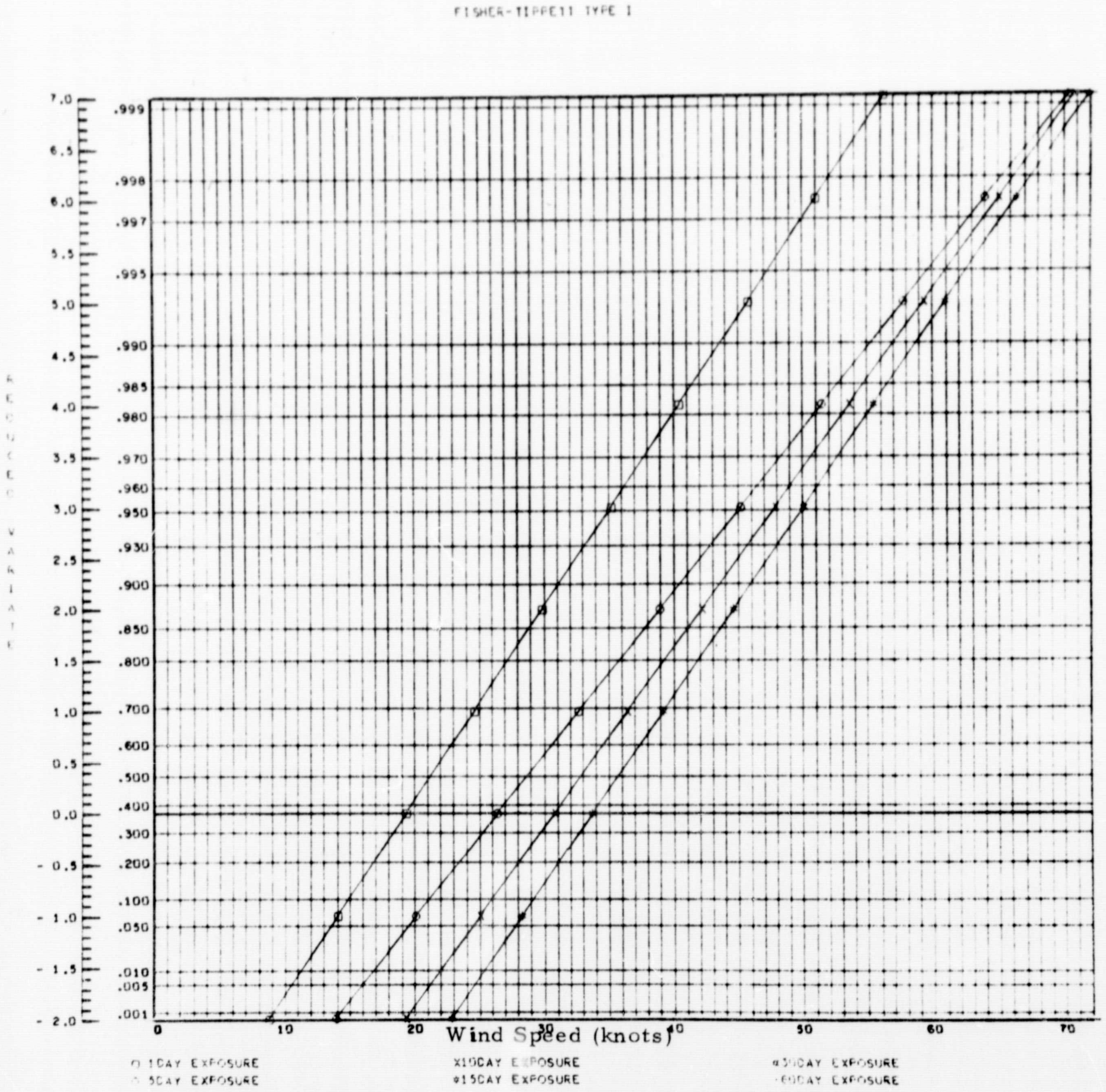

Fig. 15 - Cumulative Distributions in Summer at 18.3 Meters (Large Tower)

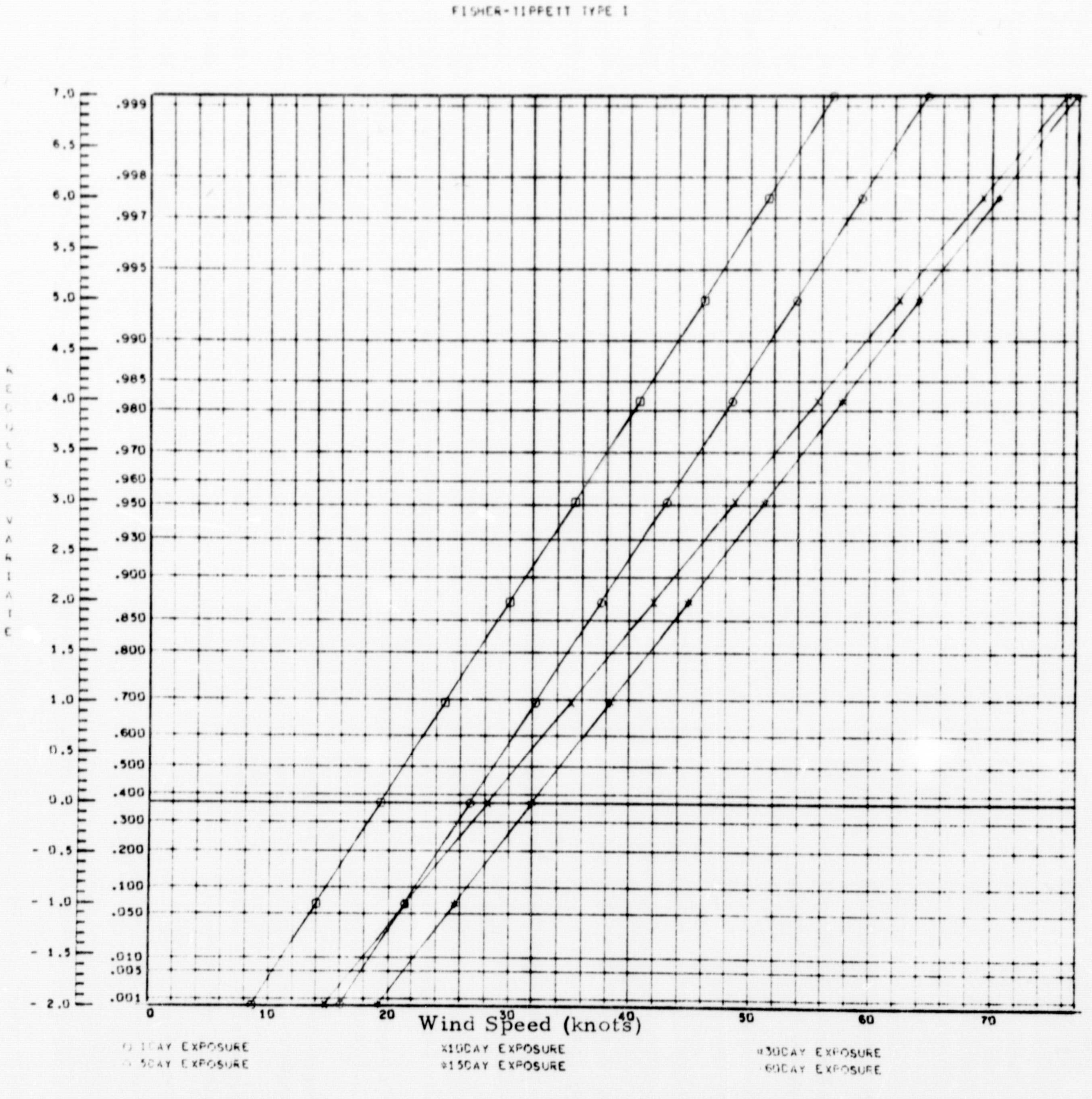

Fig. 16 - Cumulative Distributions in Fall at 18.3 Meters (Large Tower)

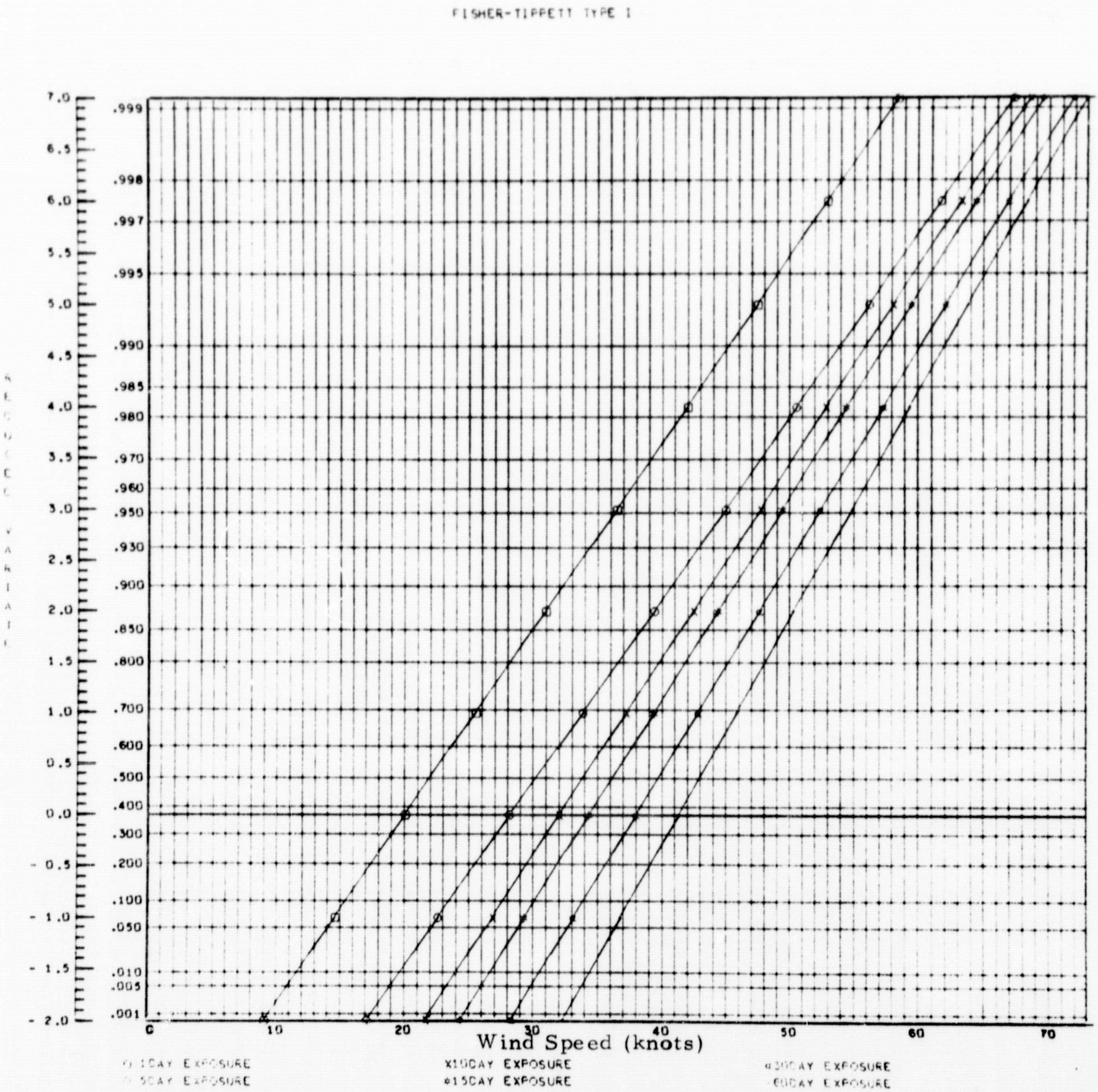

Fig. 17 - Cumulative Distributions for the Year at 18.3 Meters (Large Tower)

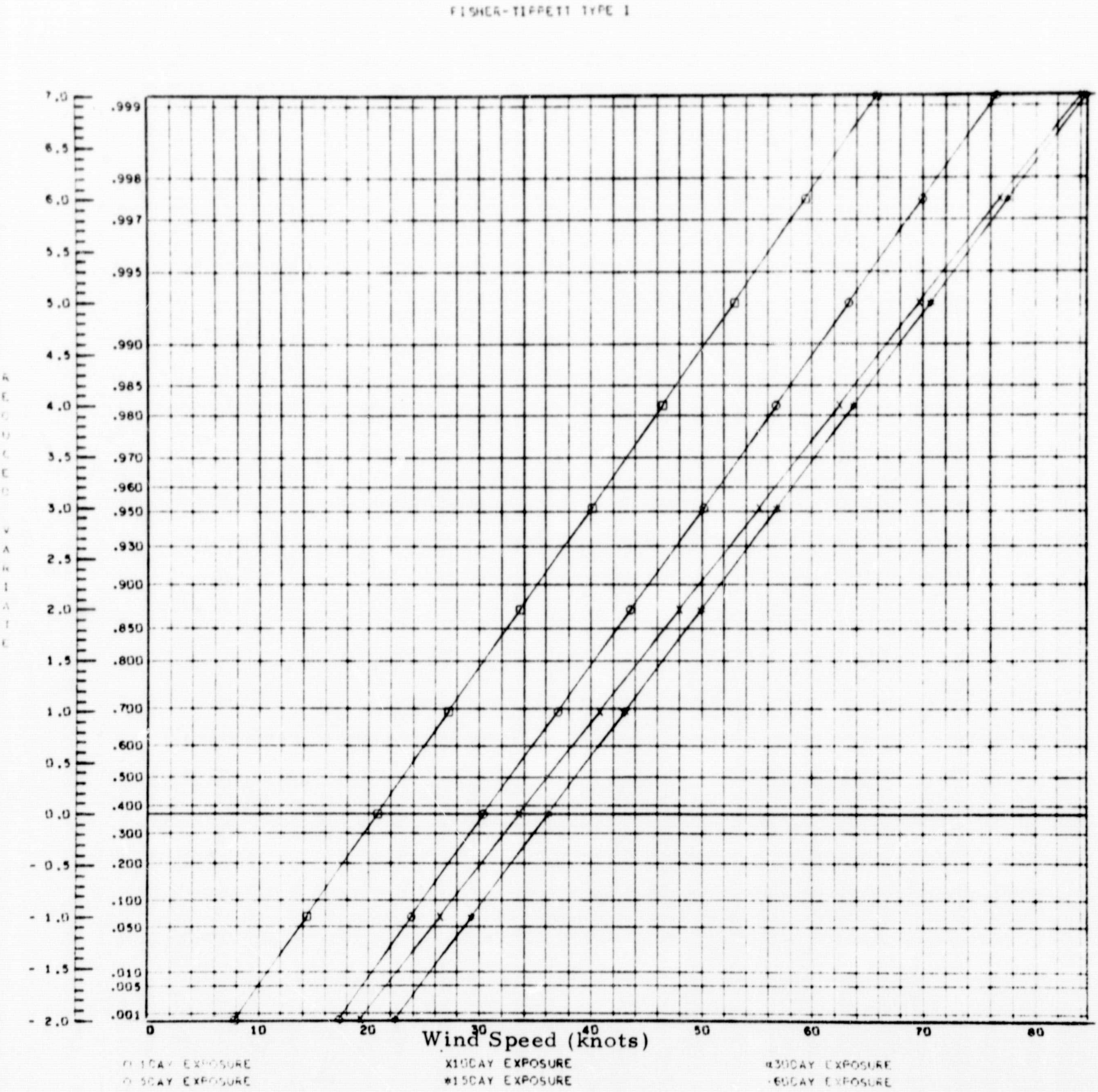

Fig. 18 - Cumulative Distributions in Winter at 30.5 Meters

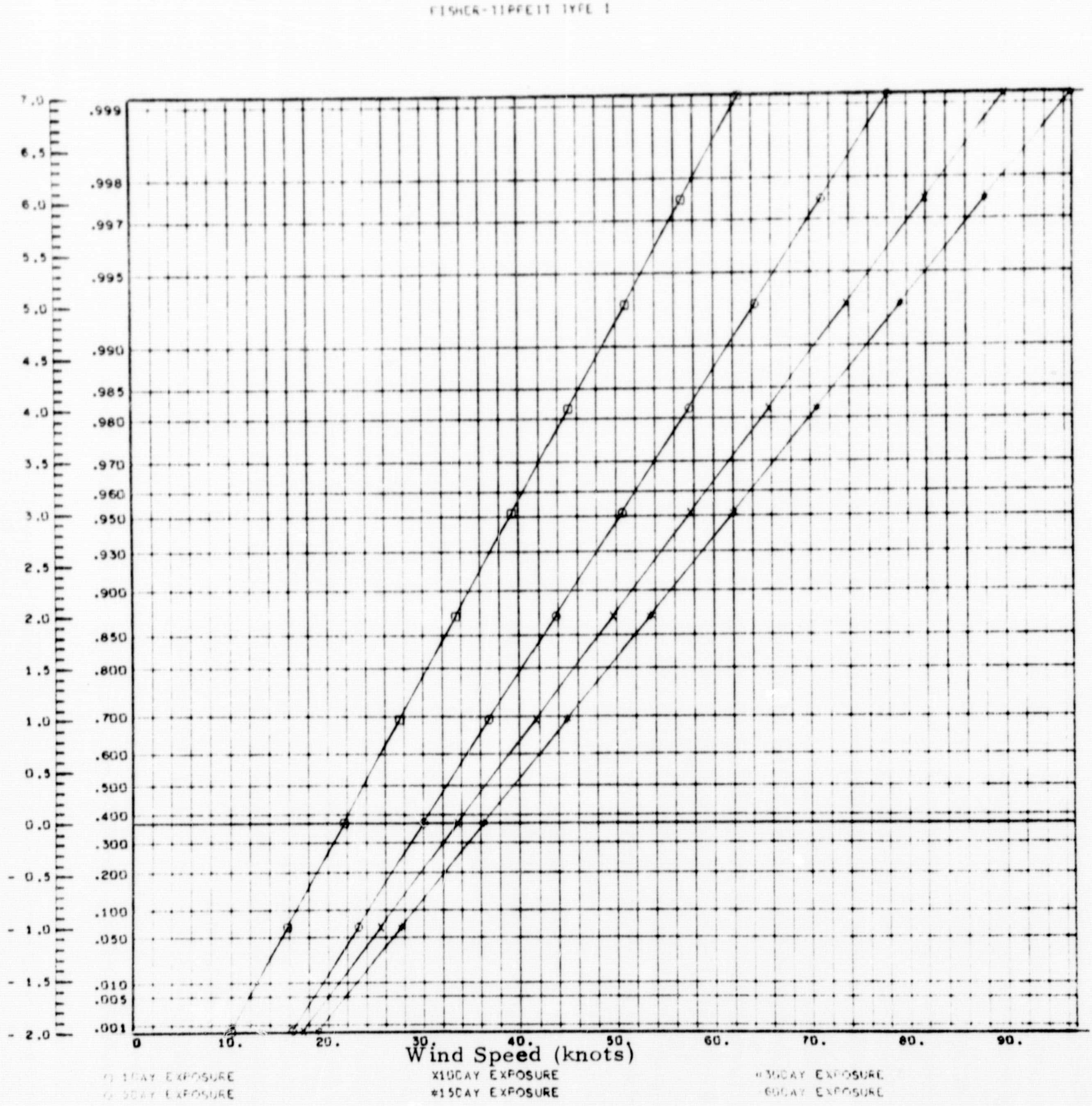

Fig. 19 - Cumulative Distributions in Spring at 30.5 Meters

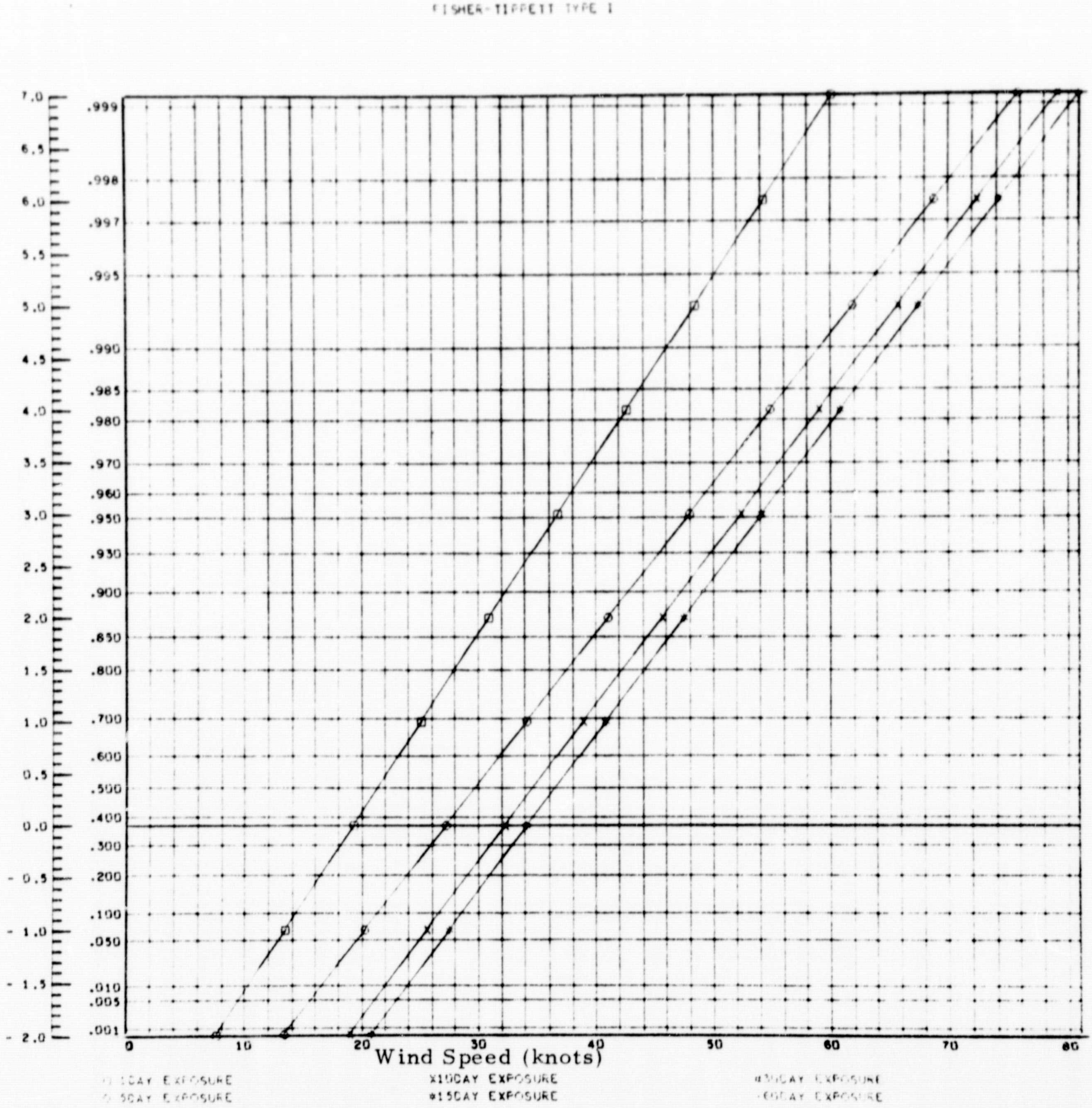

Fig. 20 - Cumulative Distributions in Summer at 30.5 Meters

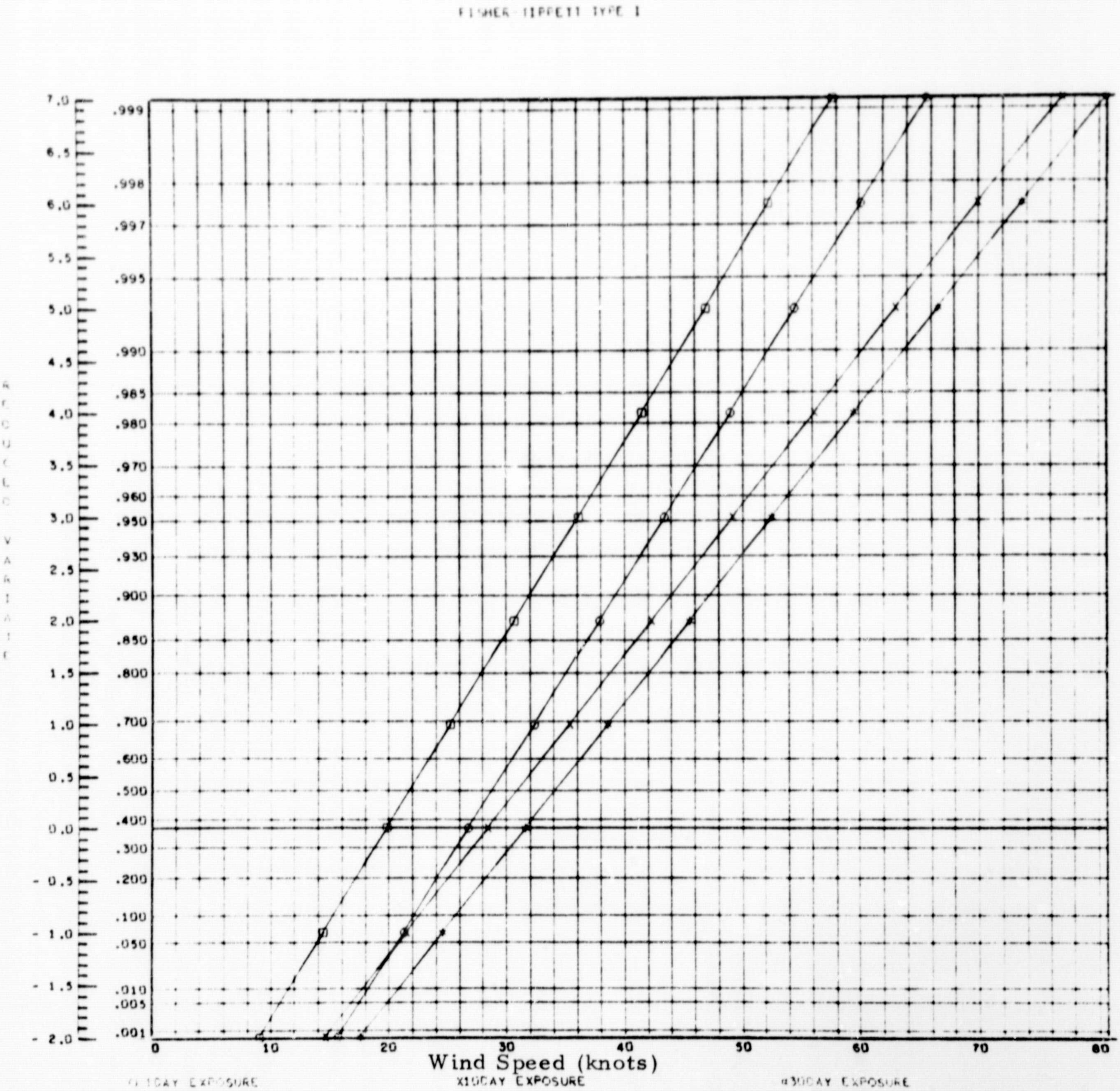

Fig. 21 - Cumulative Distributions in Fall at 30.5 Meters

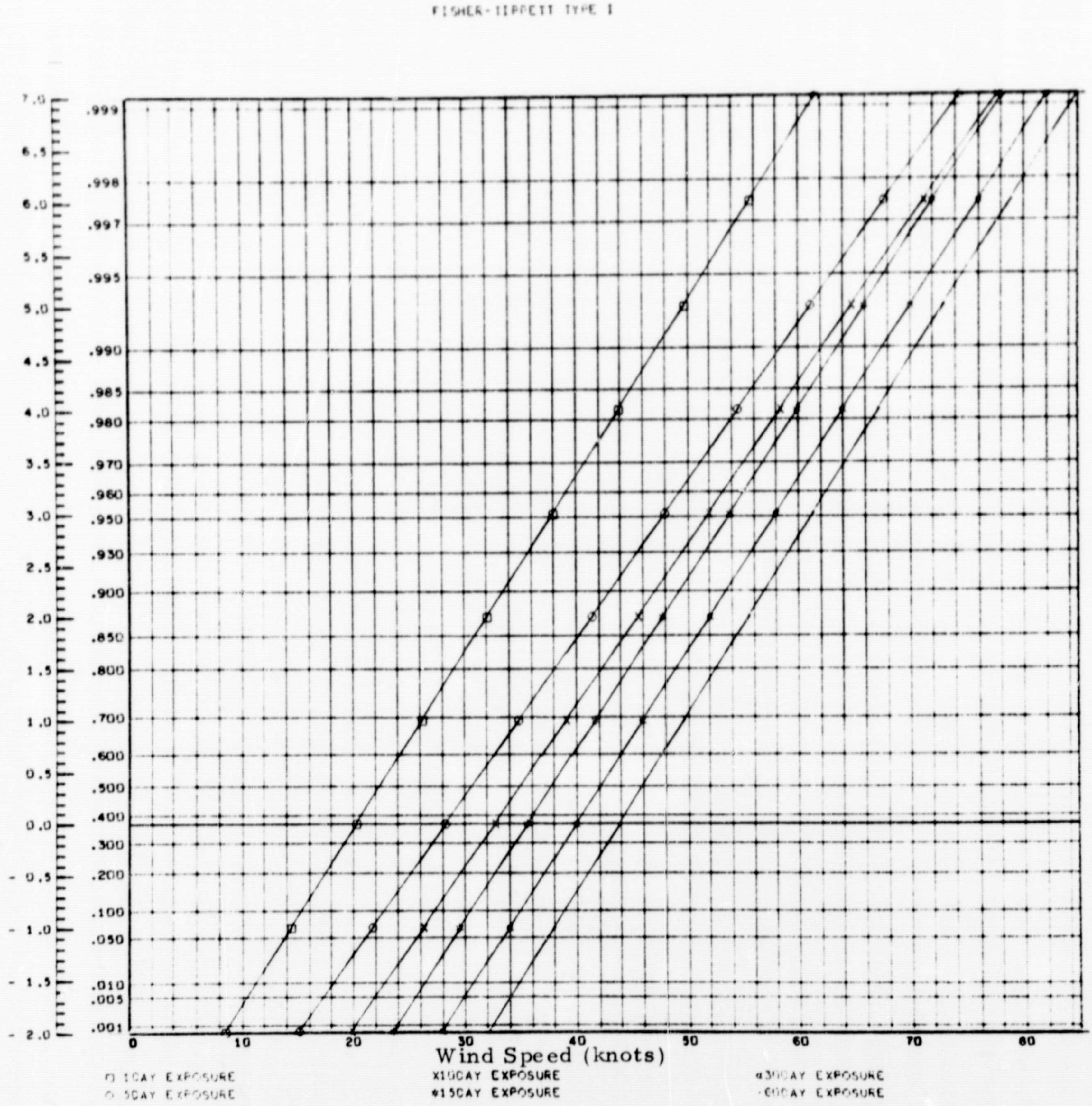

Fig. 22 - Cumulative Distributions for the Year at 30.5 Meters

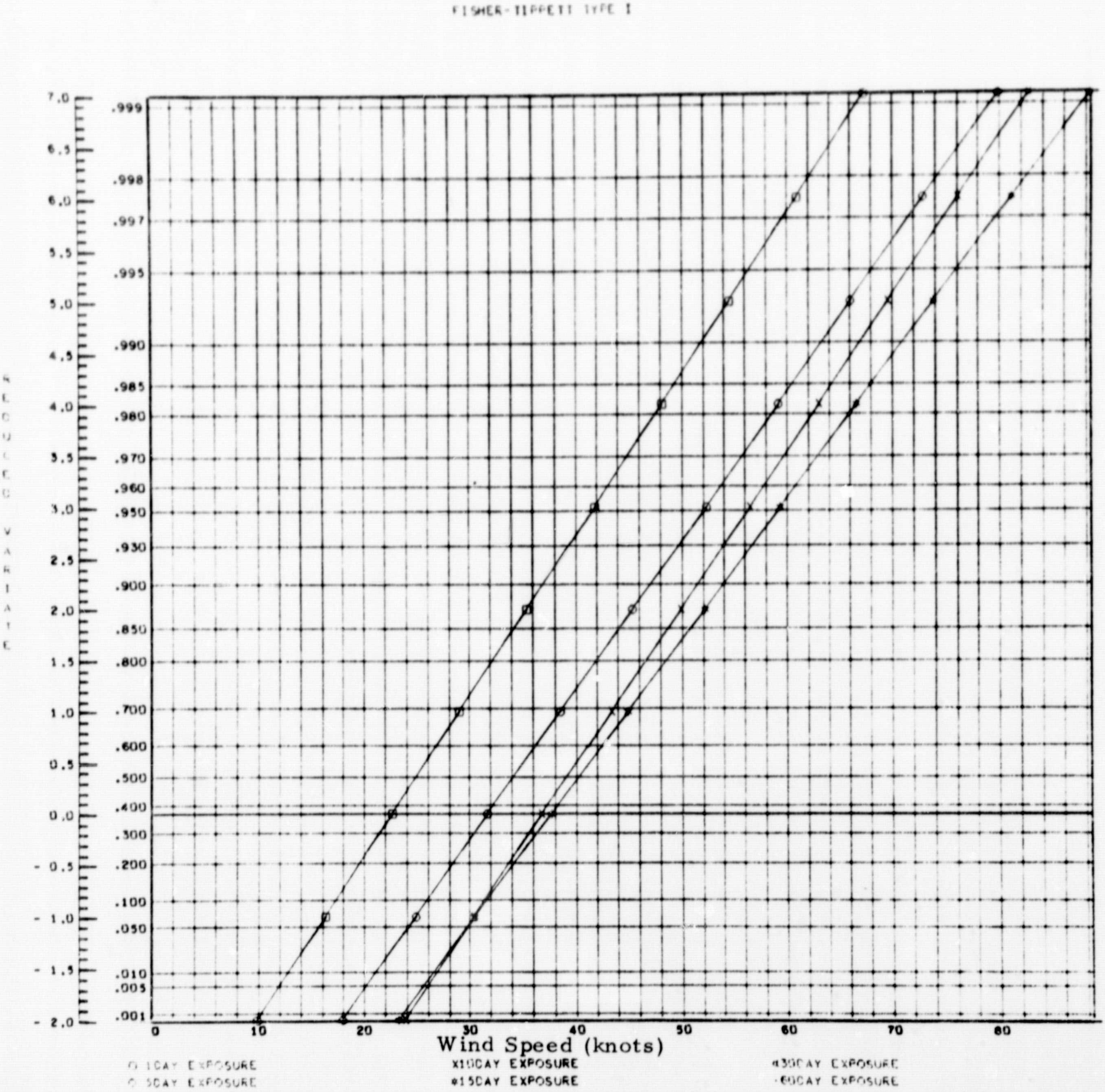

Fig. 23 - Cumulative Distributions in Winter at 61.0 Meters

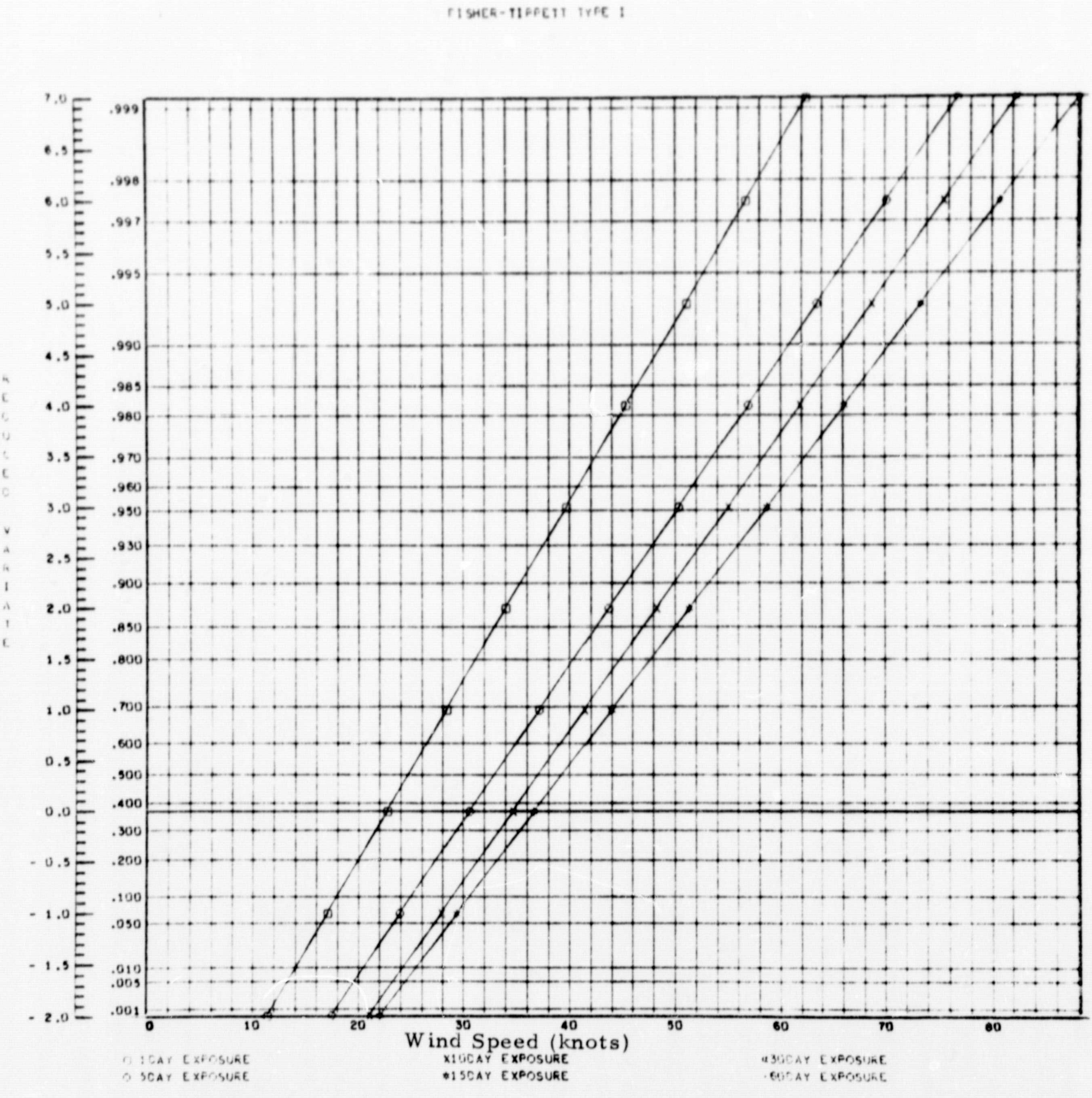

Fig. 24 - Cumulative Distributions in Spring at 61.0 Meters

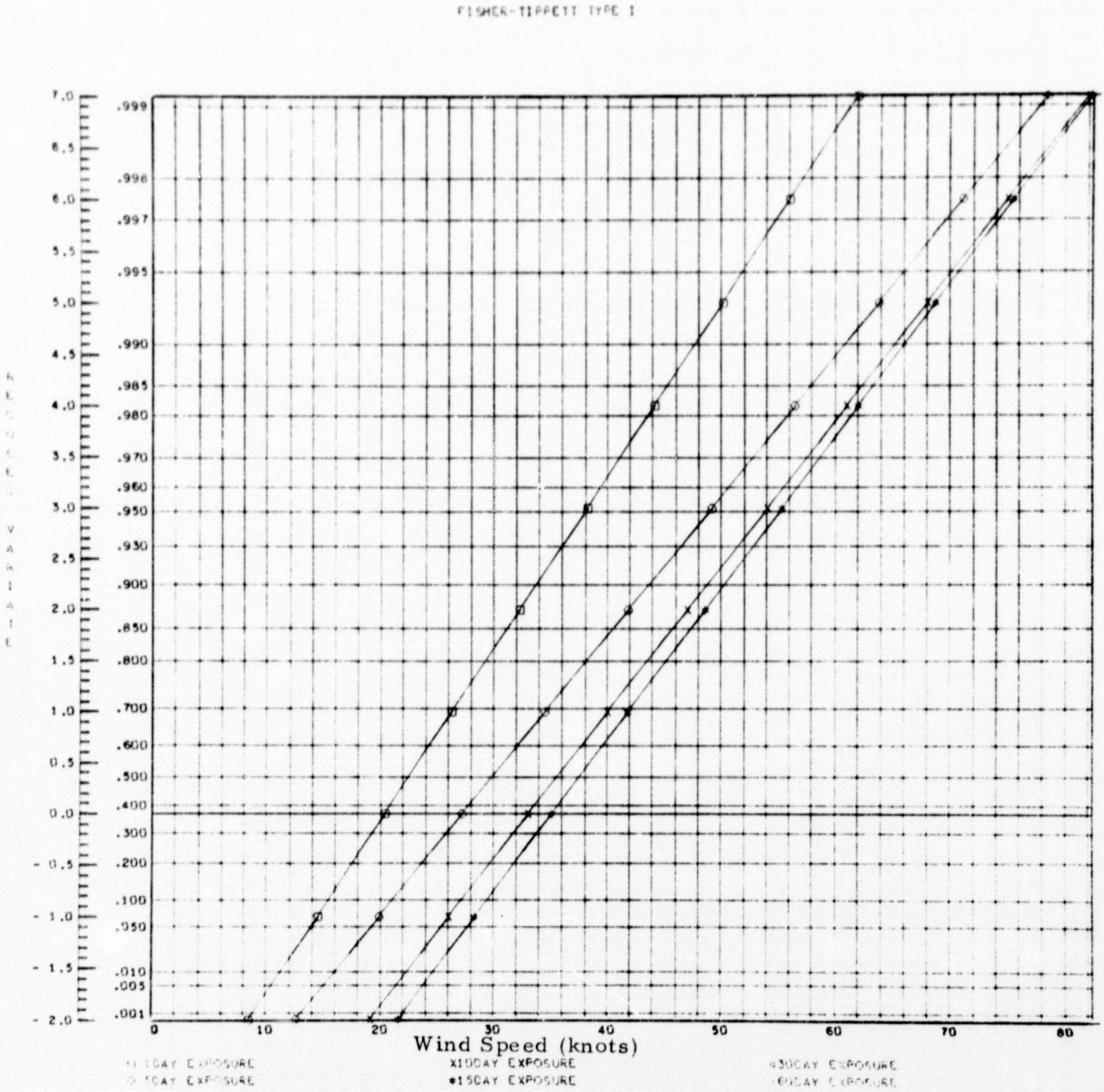

Fig. 25 - Cumulative Distributions in Summer at 61.0 Meters

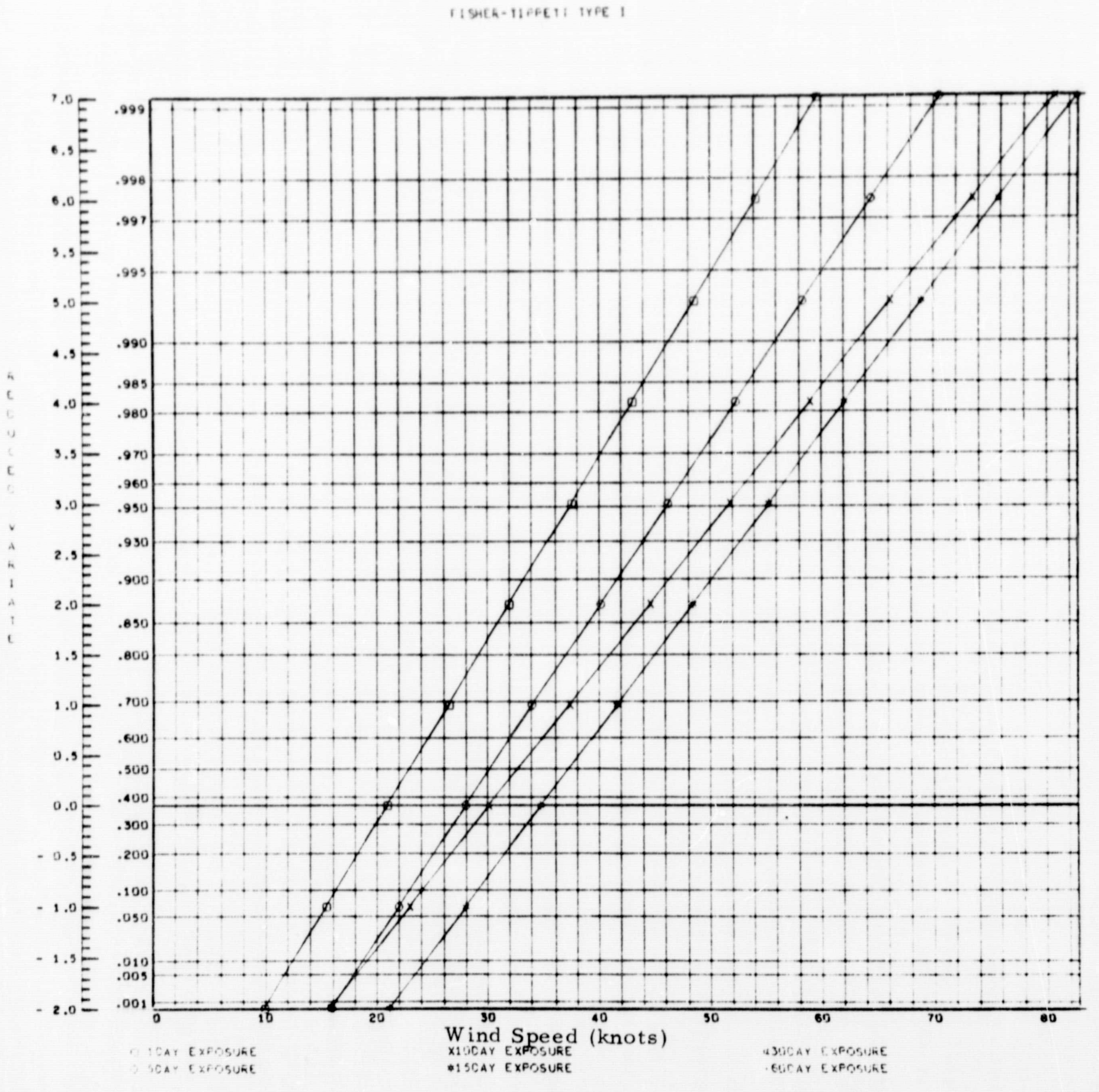

Fig. 26 - Cumulative Distributions in Fall at 61.0 Meters

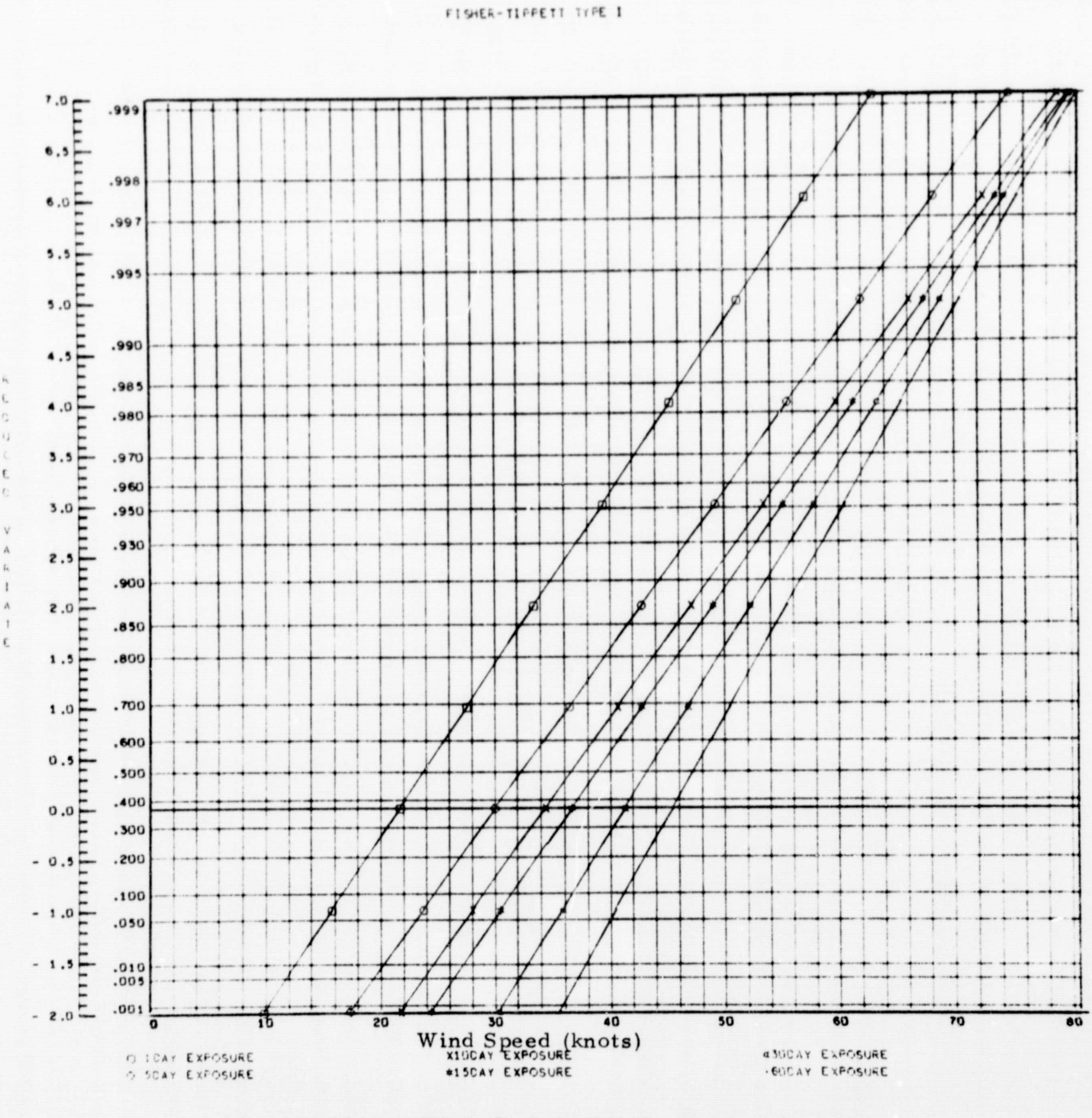

Fig. 27 - Cumulative Distributions for the Year at 61.0 Meters

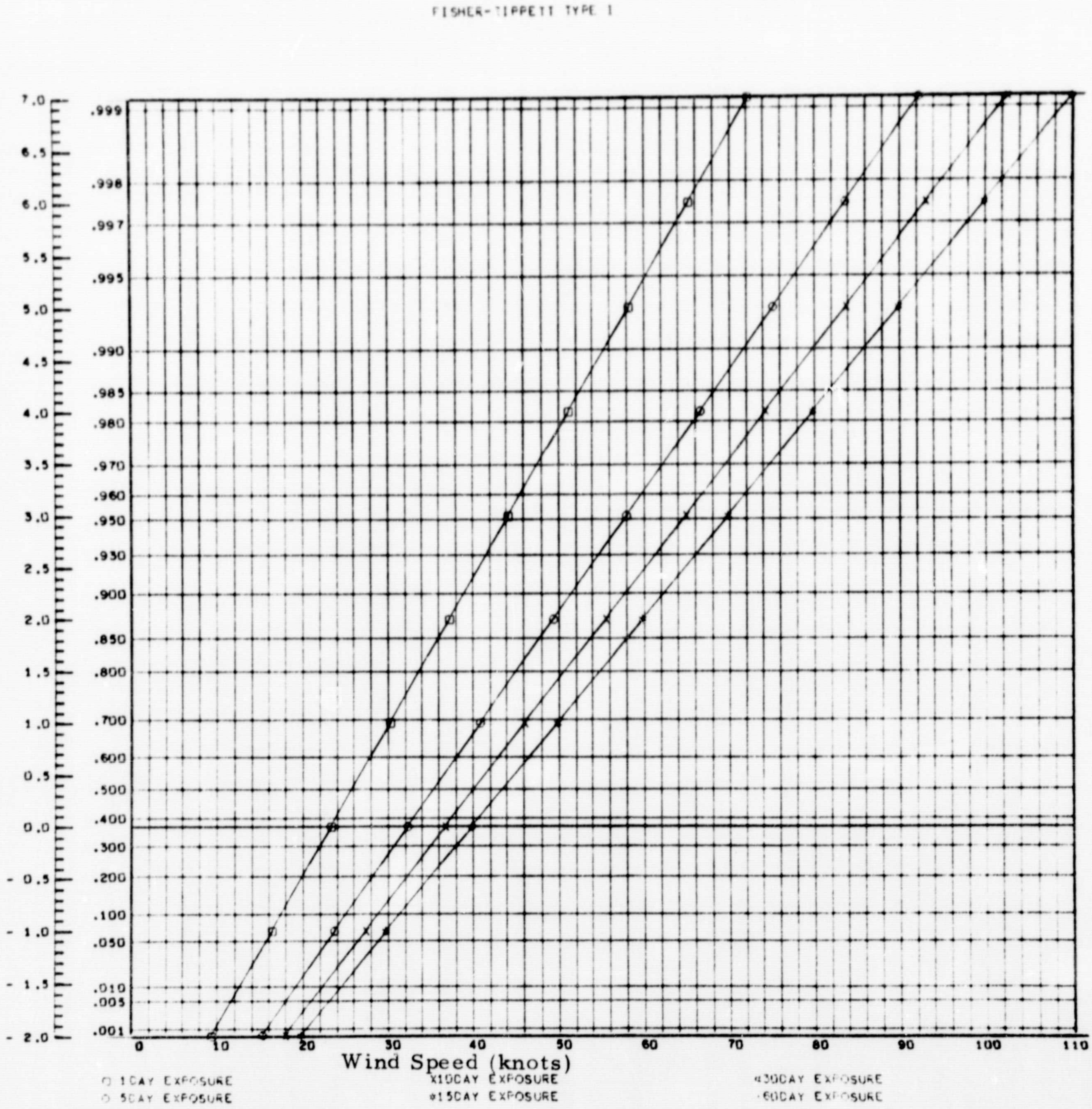

Fig. 28 - Cumulative Distributions in Winter at 91.4 Meters

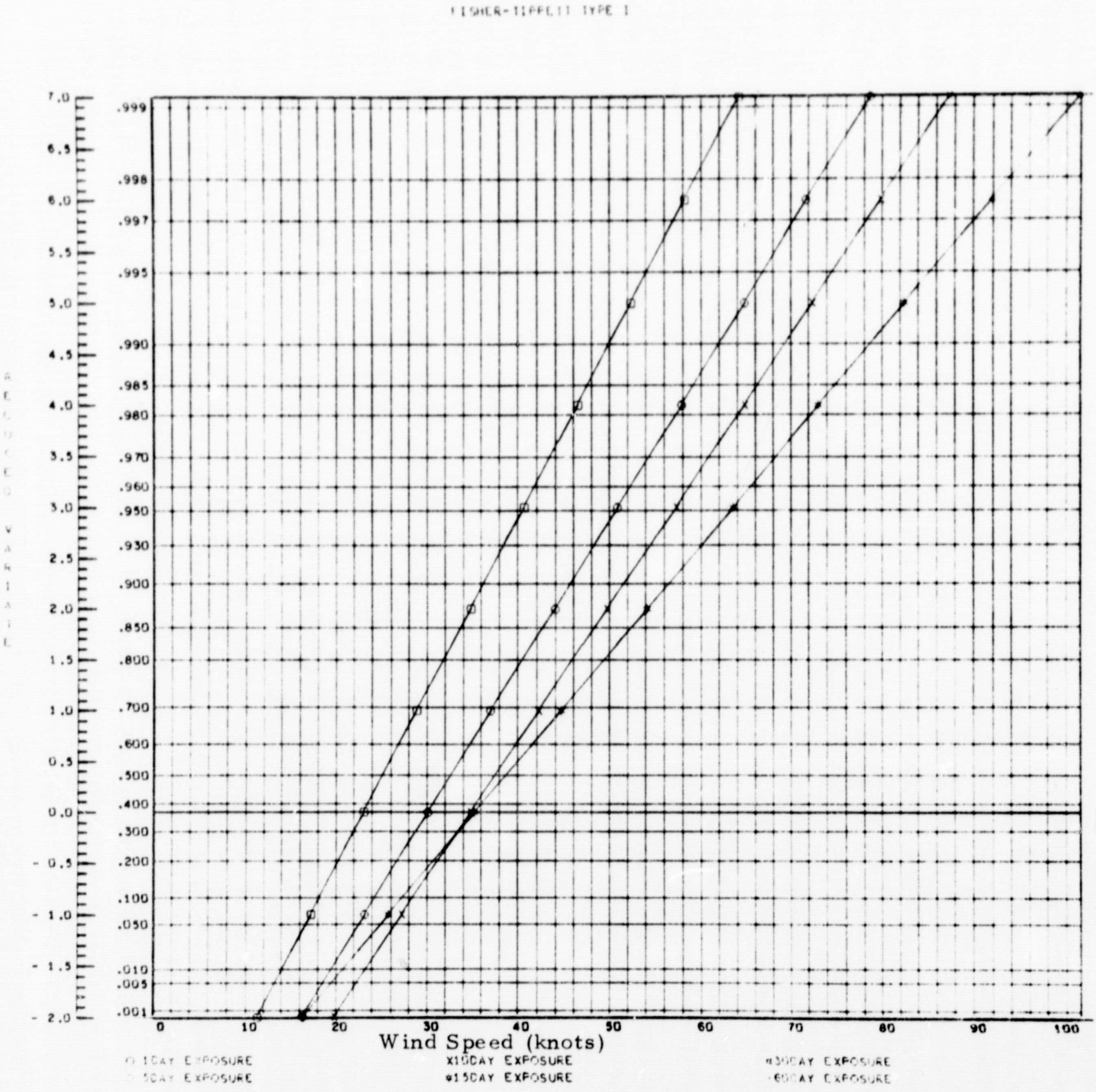

Fig. 29 - Cumulative Distributions in Spring at 91.4 Meters

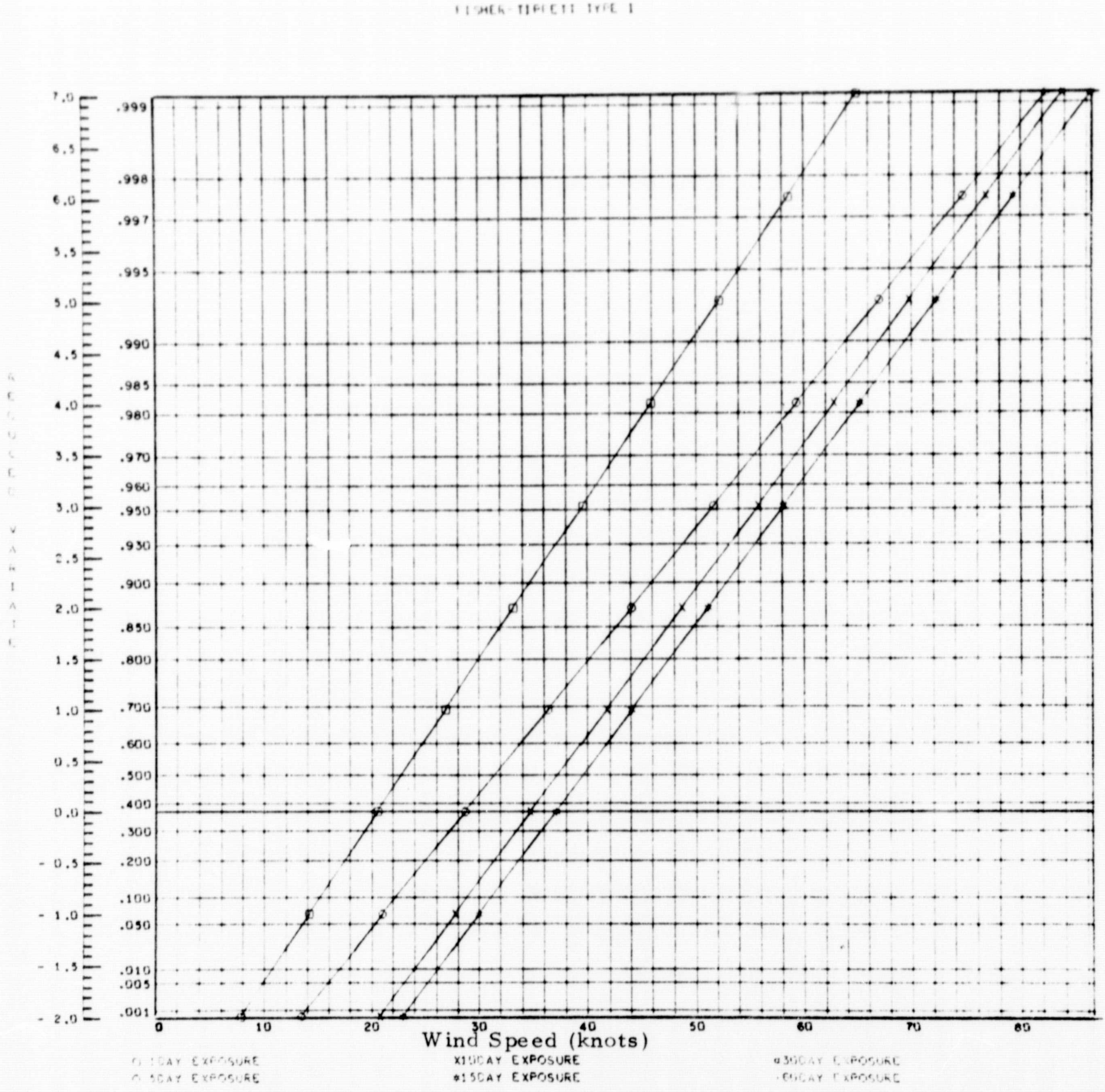

Fig. 30 - Cumulative Distributions in Summer at 91.4 Meters

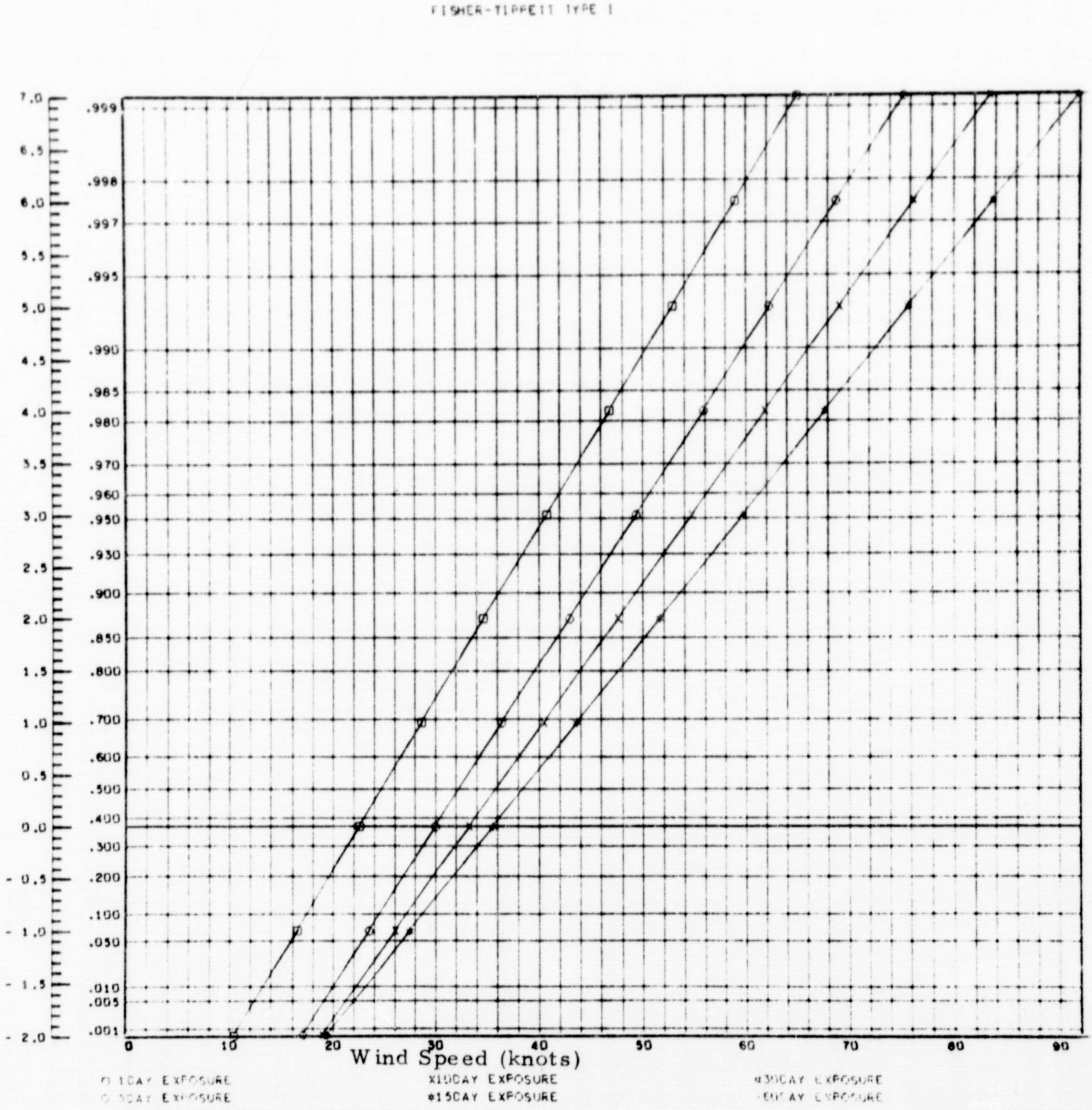

Fig. 31 - Cumulative Distributions in Fall at 91.4 Meters

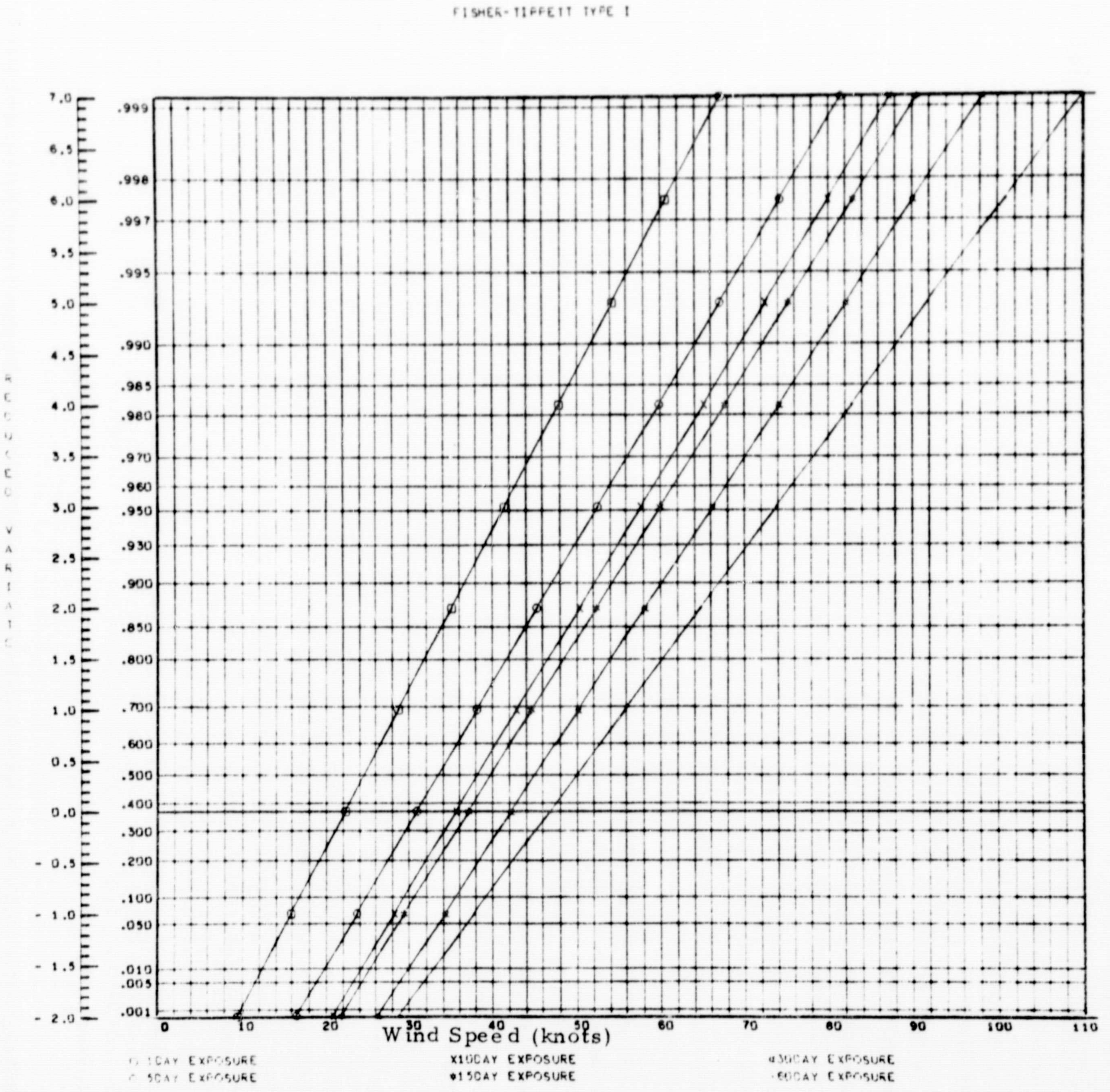

Fig. 32 - Cumulative Distributions for the Year at 91.4 Meters

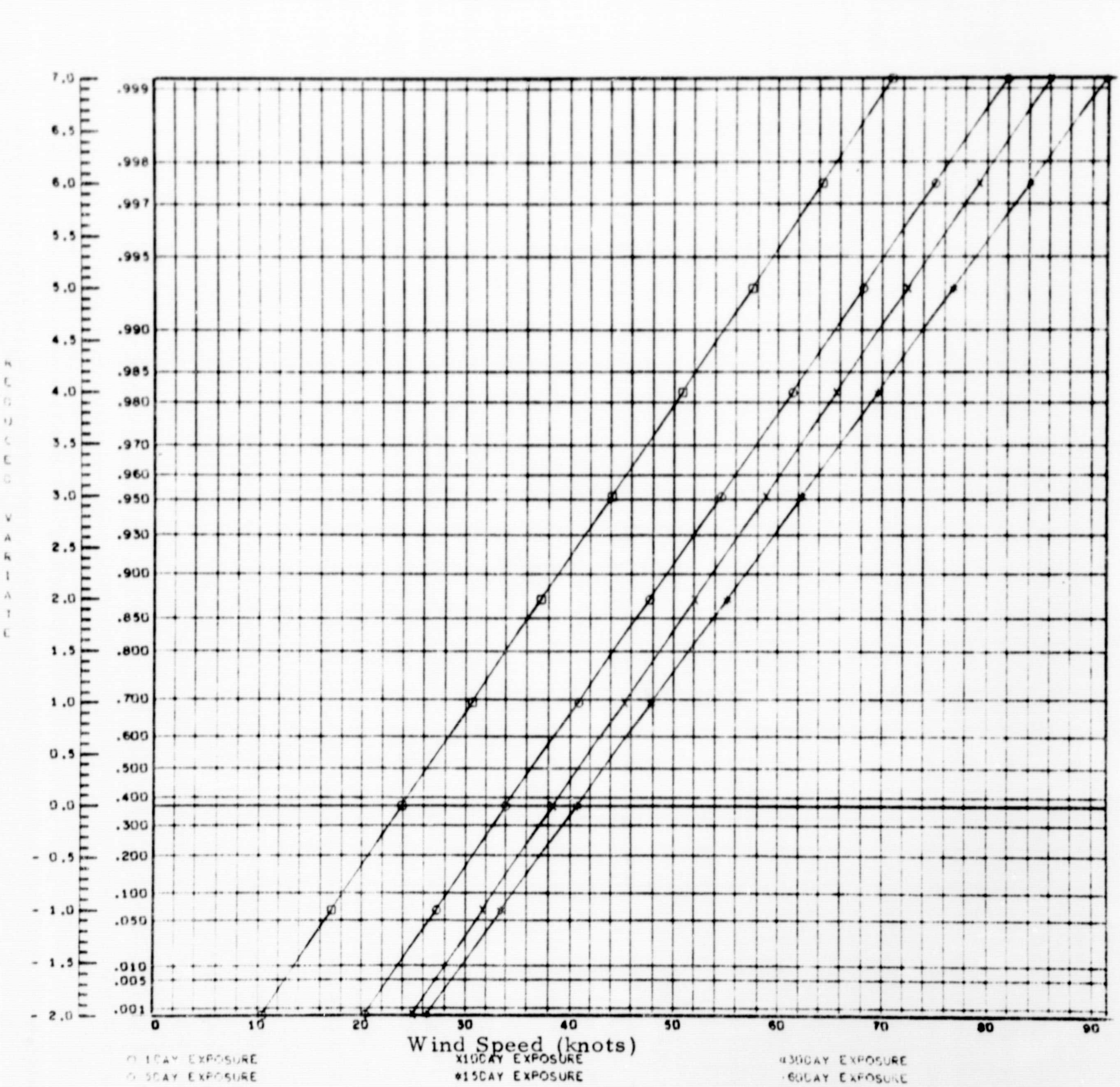

Fig. 33 - Cumulative Distributions in Winter at 121.9 Meters

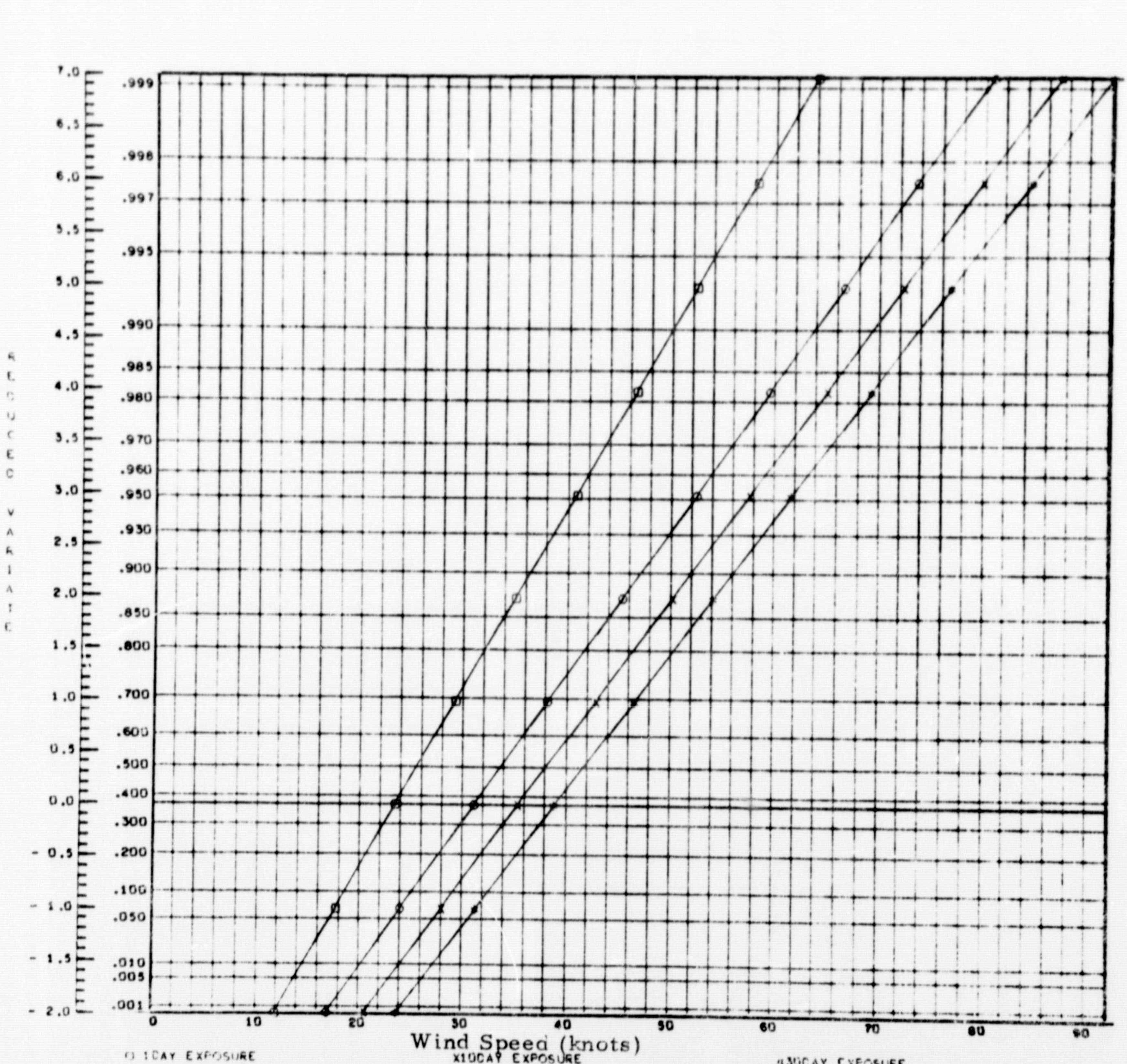

Fig. 34 - Cumulative Distributions in Spring at 121.9 Meters

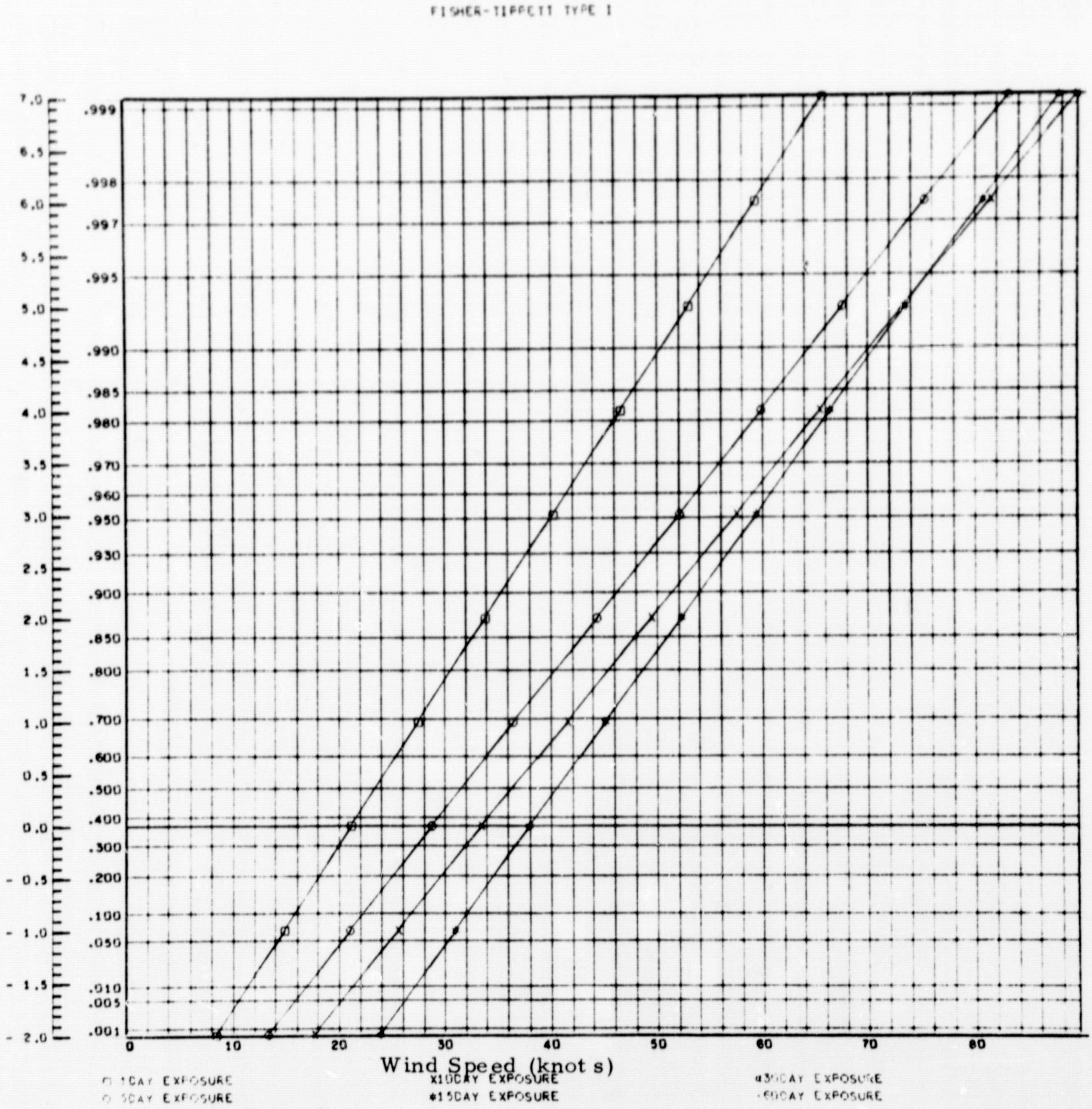

Fig. 35 - Cumulative Distributions in Summer at 121.9 Meters

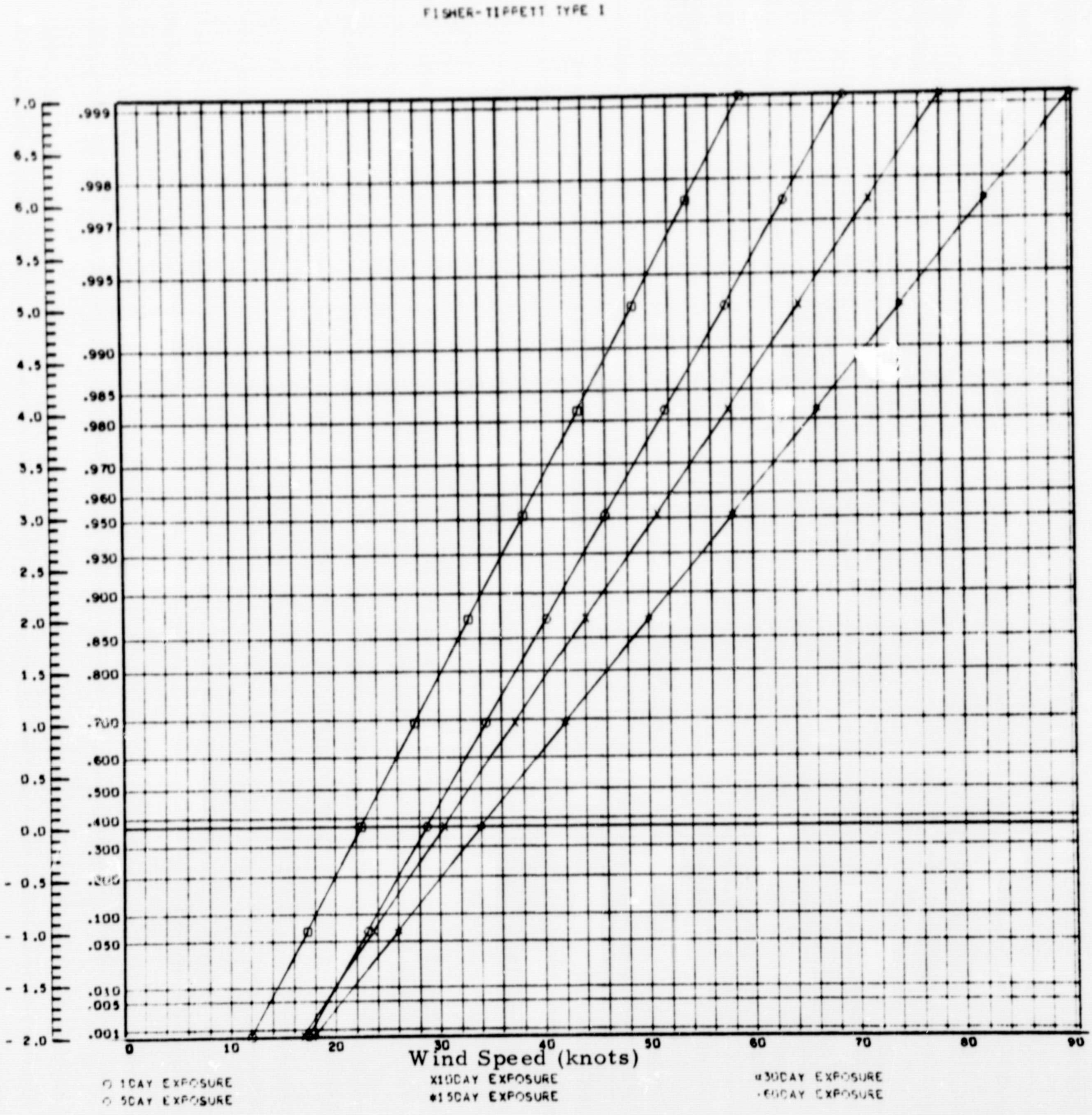

Fig. 36 - Cumulative Distributions in Fall at 121.9 Meters

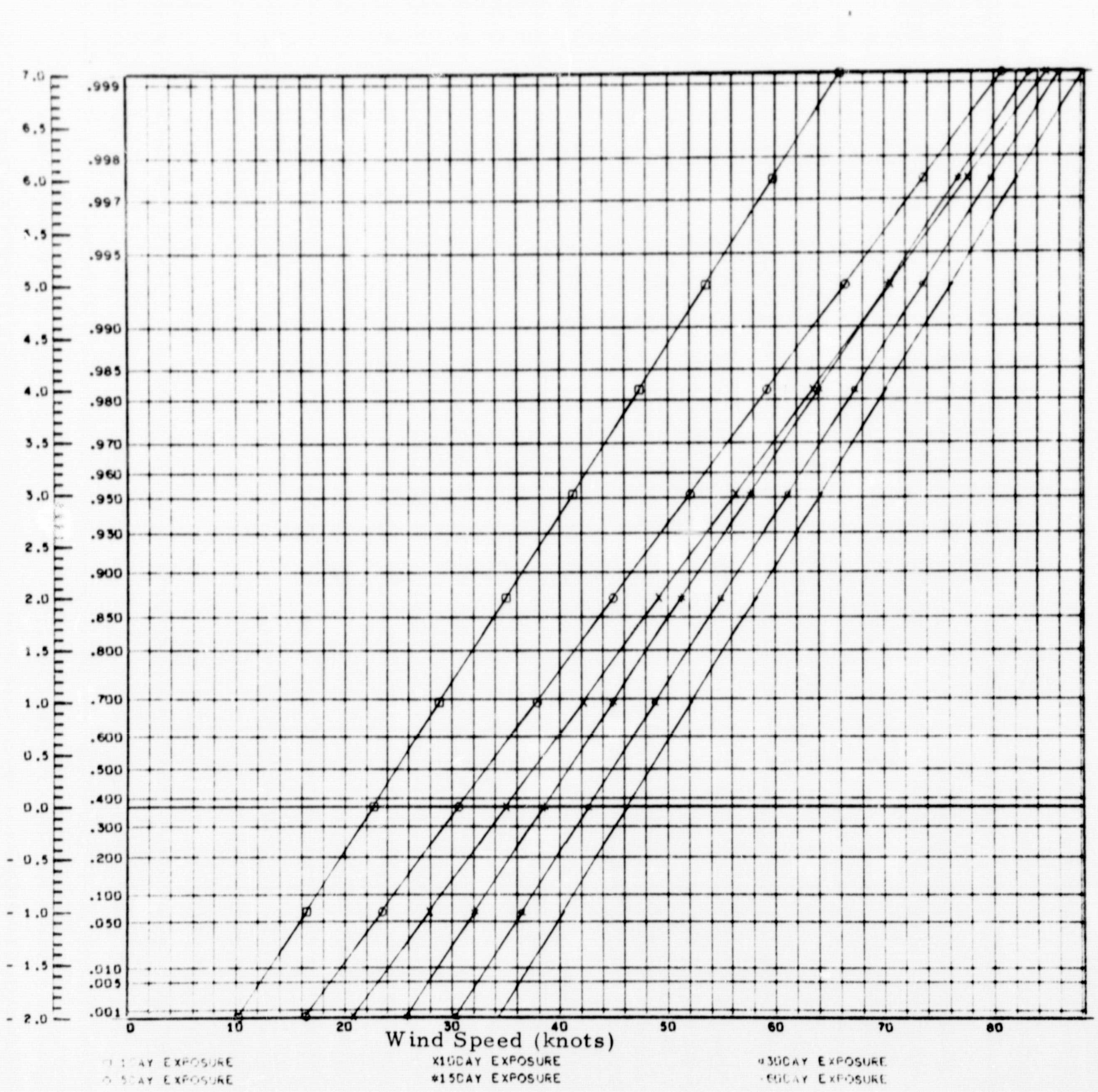

Fig. 37 - Cumulative Distributions for the Year at 121.9 Meters

FISHER-TIPPETT TYPE I

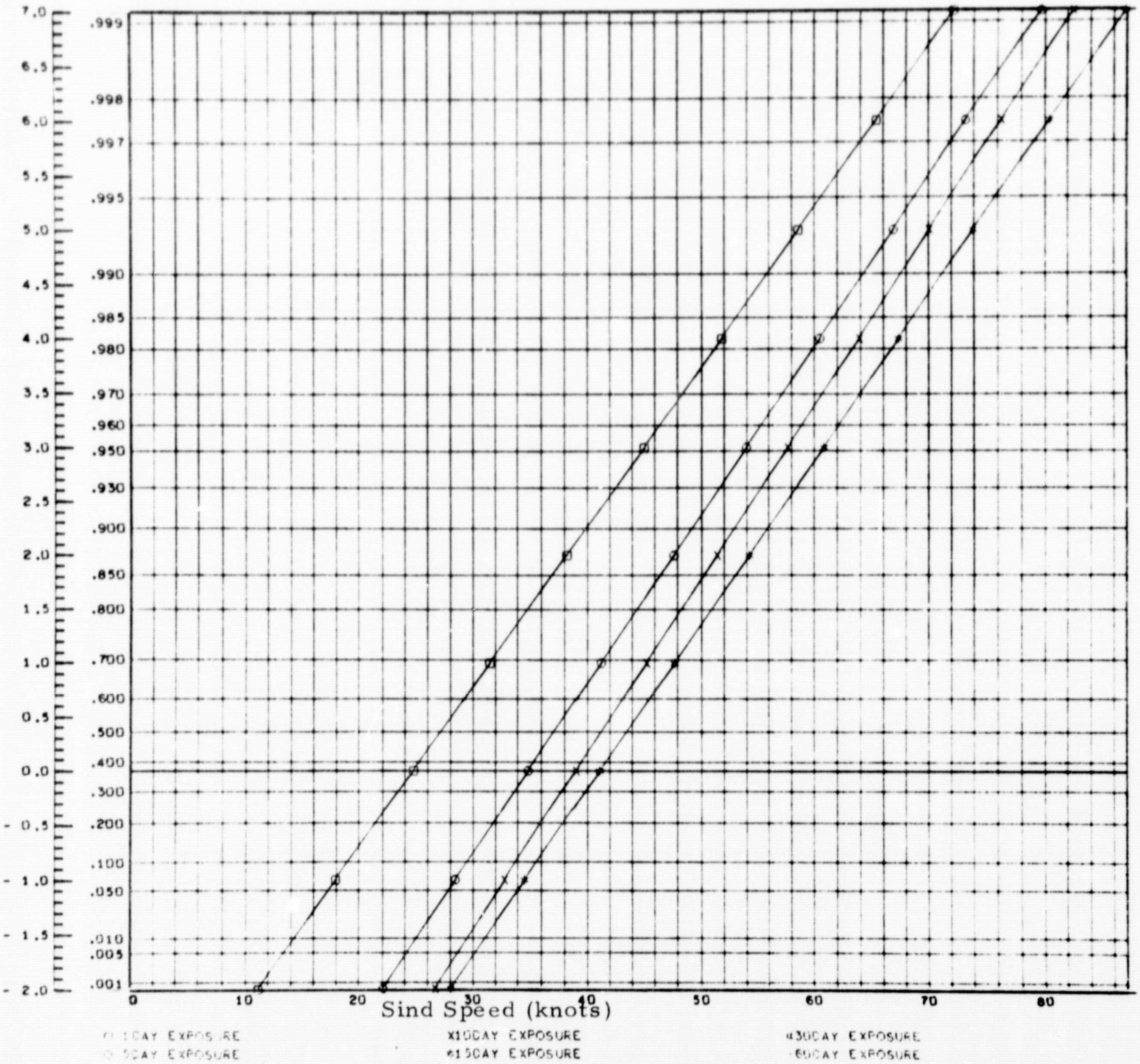

Fig. 38 - Cumulative Distributions in Winter at 152.4 Meters

FISHER-TIPPETT TYPE I

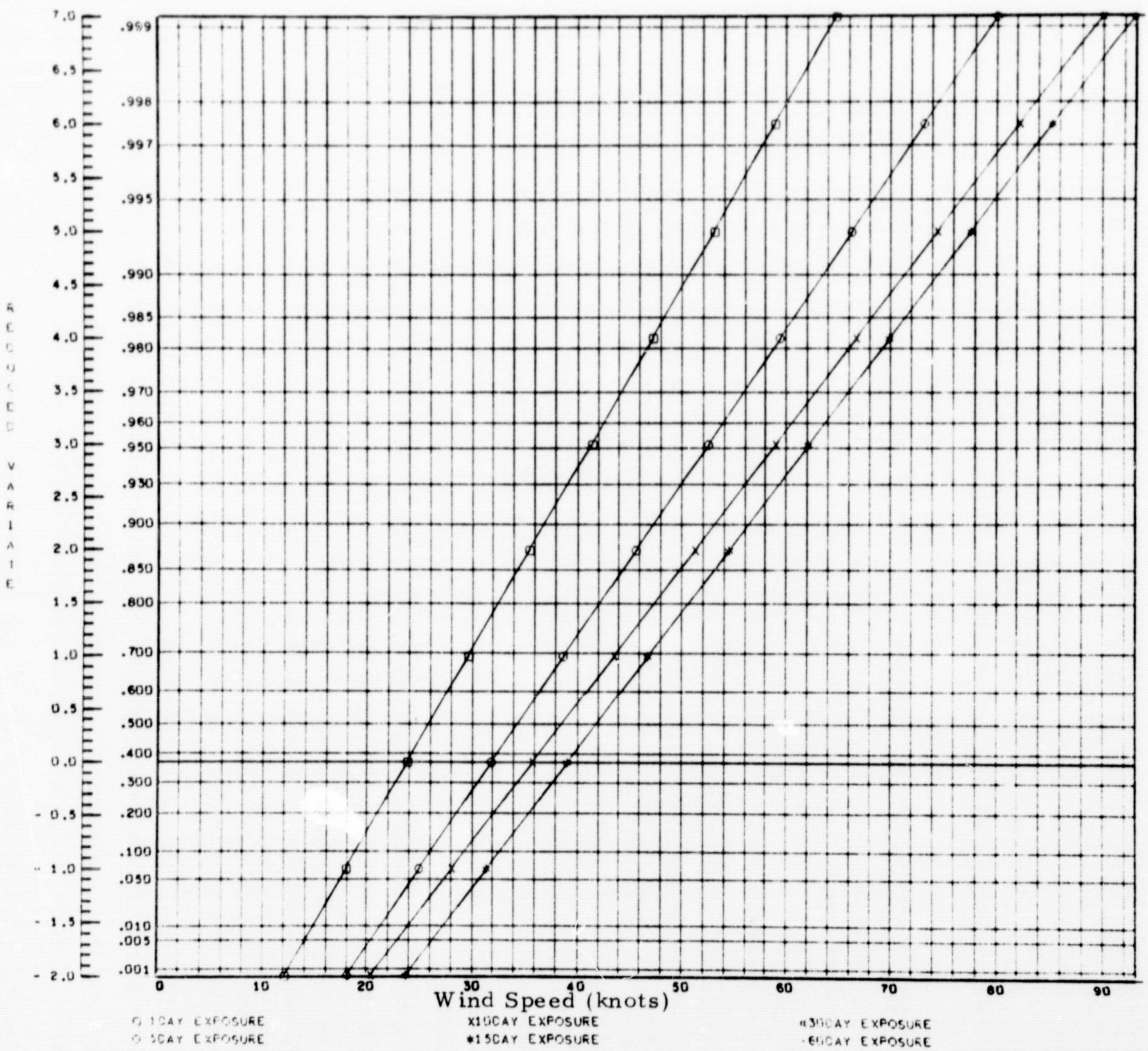

Fig. 39 - Cumulative Distributions in Spring at 152.4 Meters

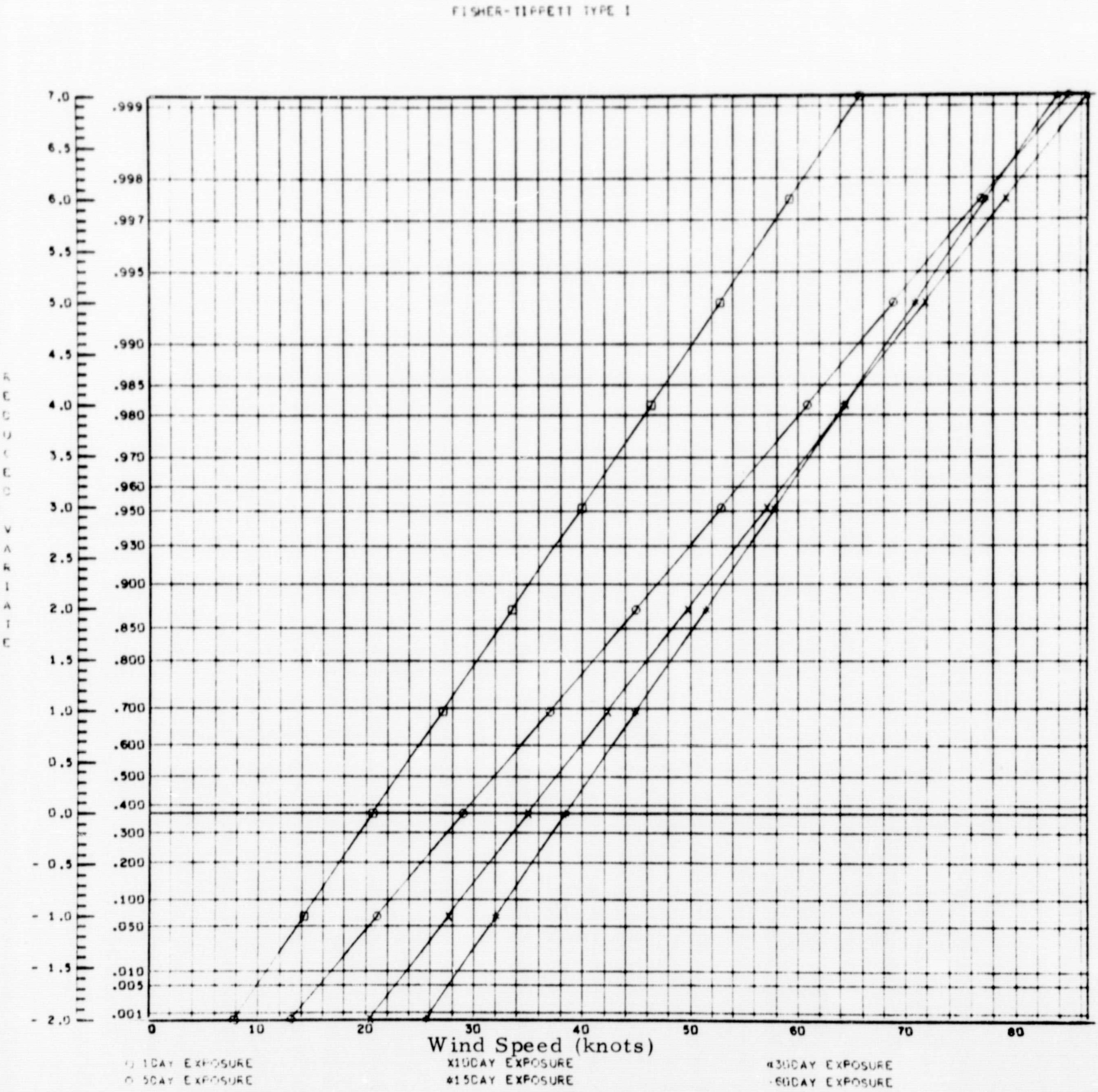

Fig. 40 - Cumulative Distributions in Summer at 152.4 Meters

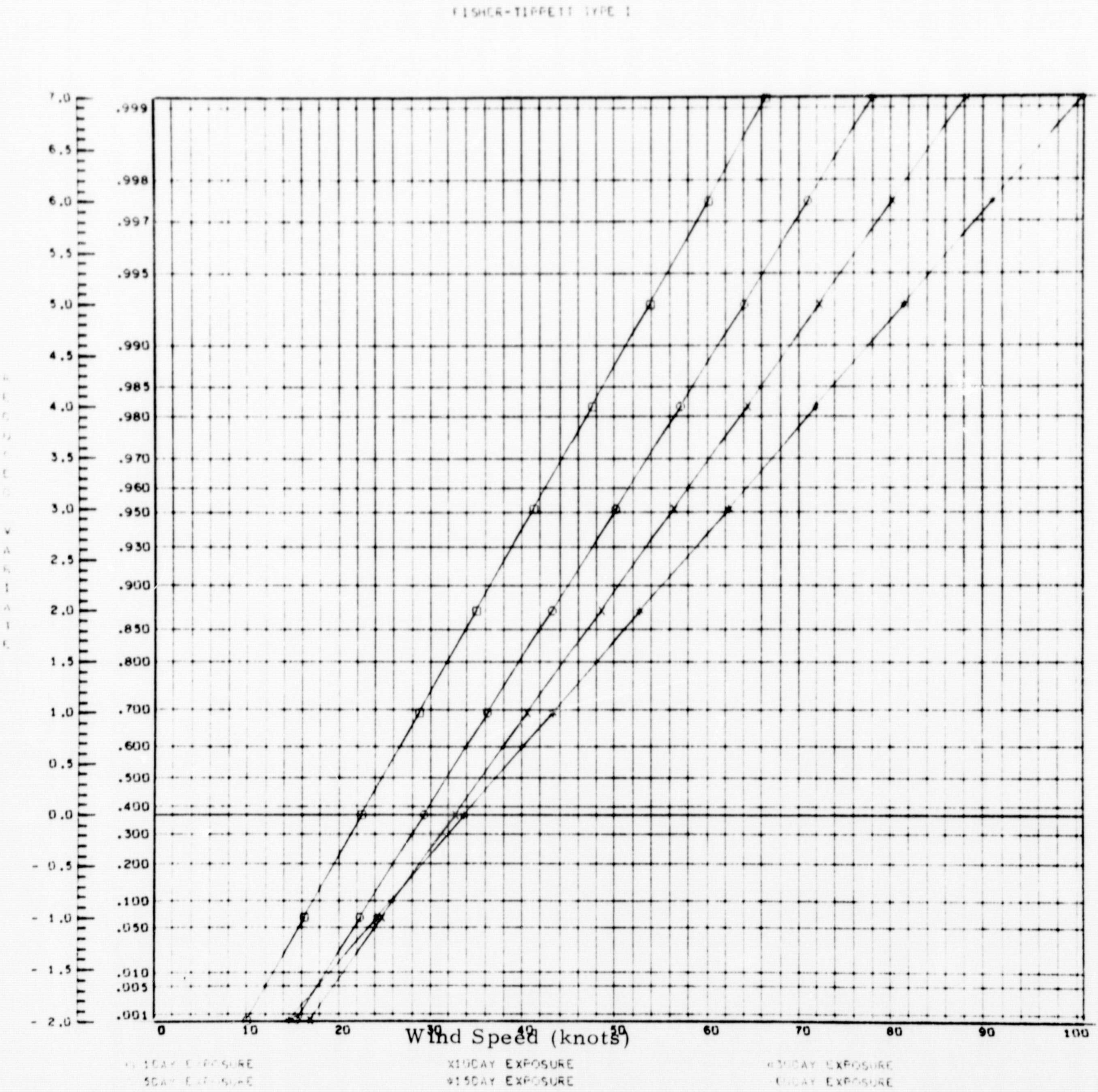

Fig. 41 - Cumulative Distributions in Fall at 152.4 Meters

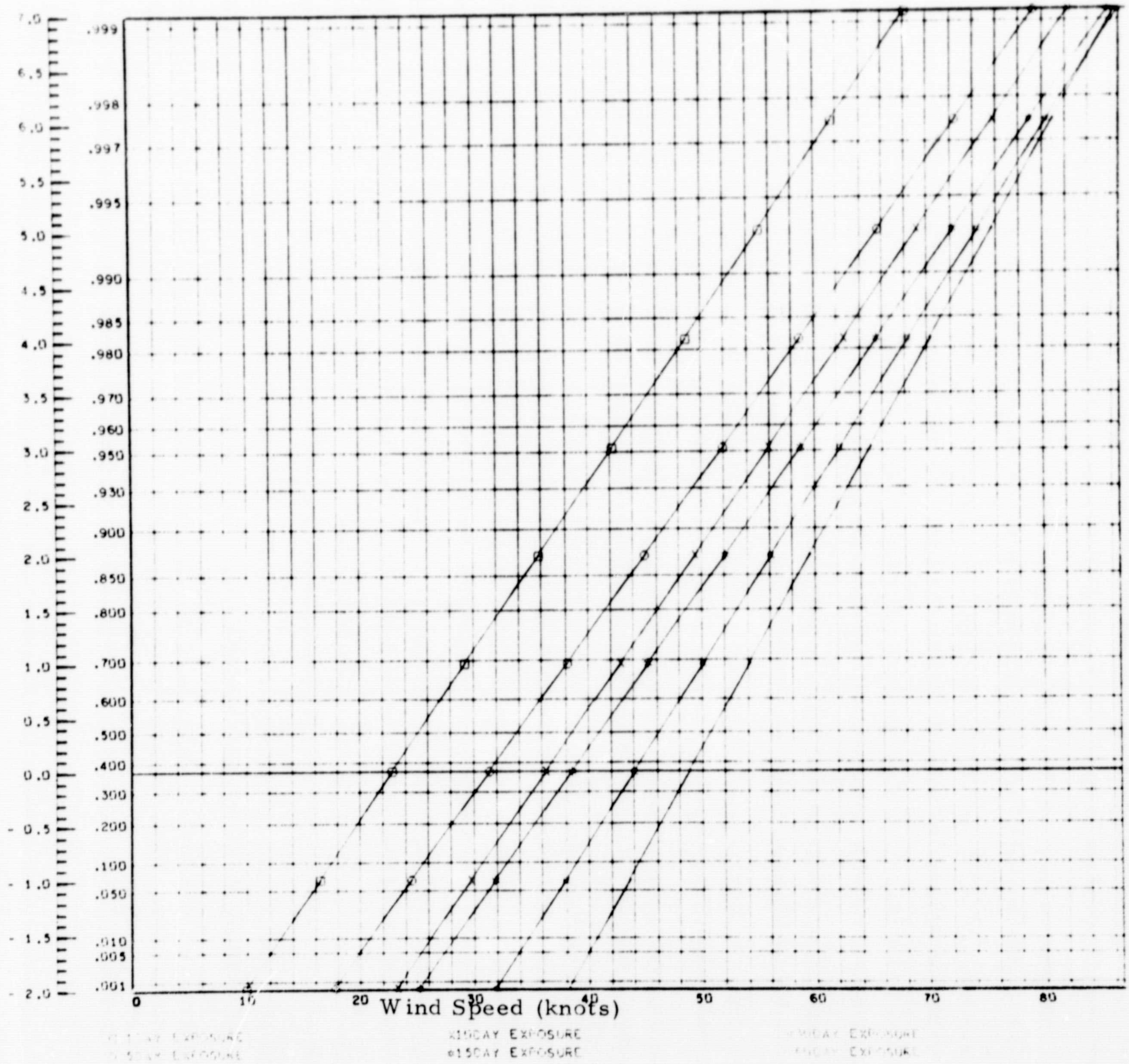

Fig. 42 - Cumulative Distributions for the Year at 152.4 Meters

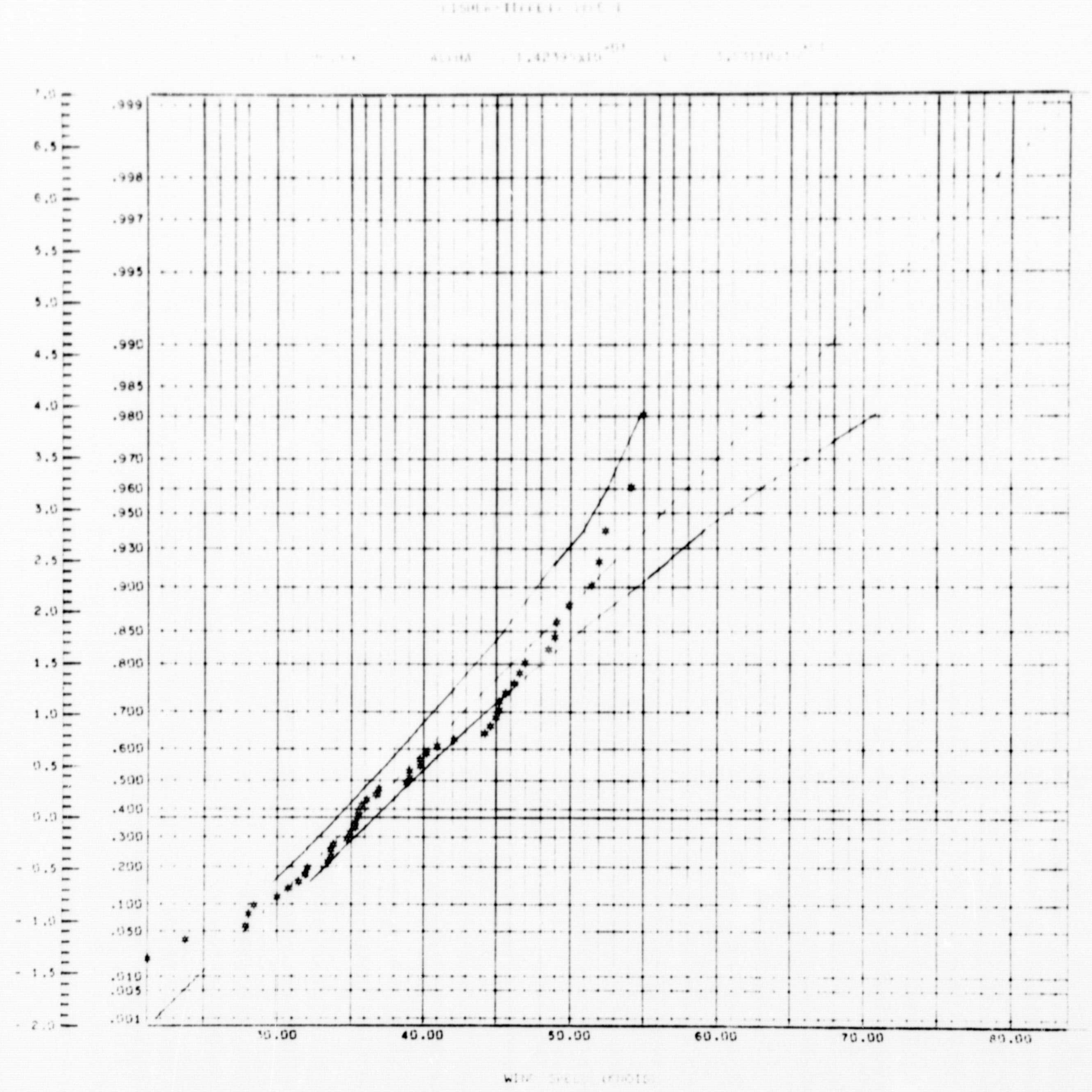

Fig. 43 - Data Fit for the Second Bivariate Variable over 15 Days Exposure Period at 61.0 Meters

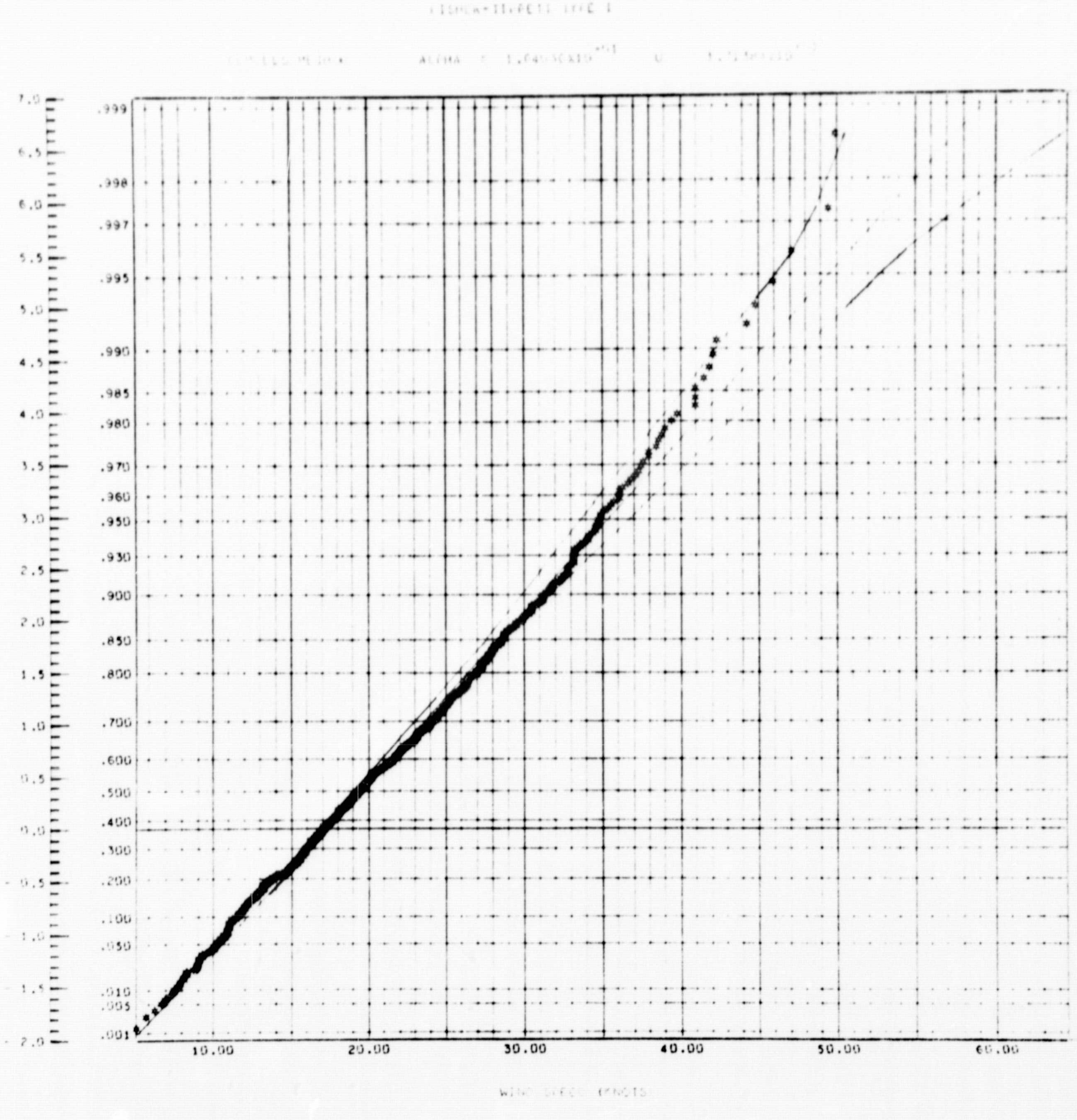

Fig. 44 - Data Fit for the Second Bivariate Variable over 1 Day Exposure Period at 18.3 Meters (Small Tower)

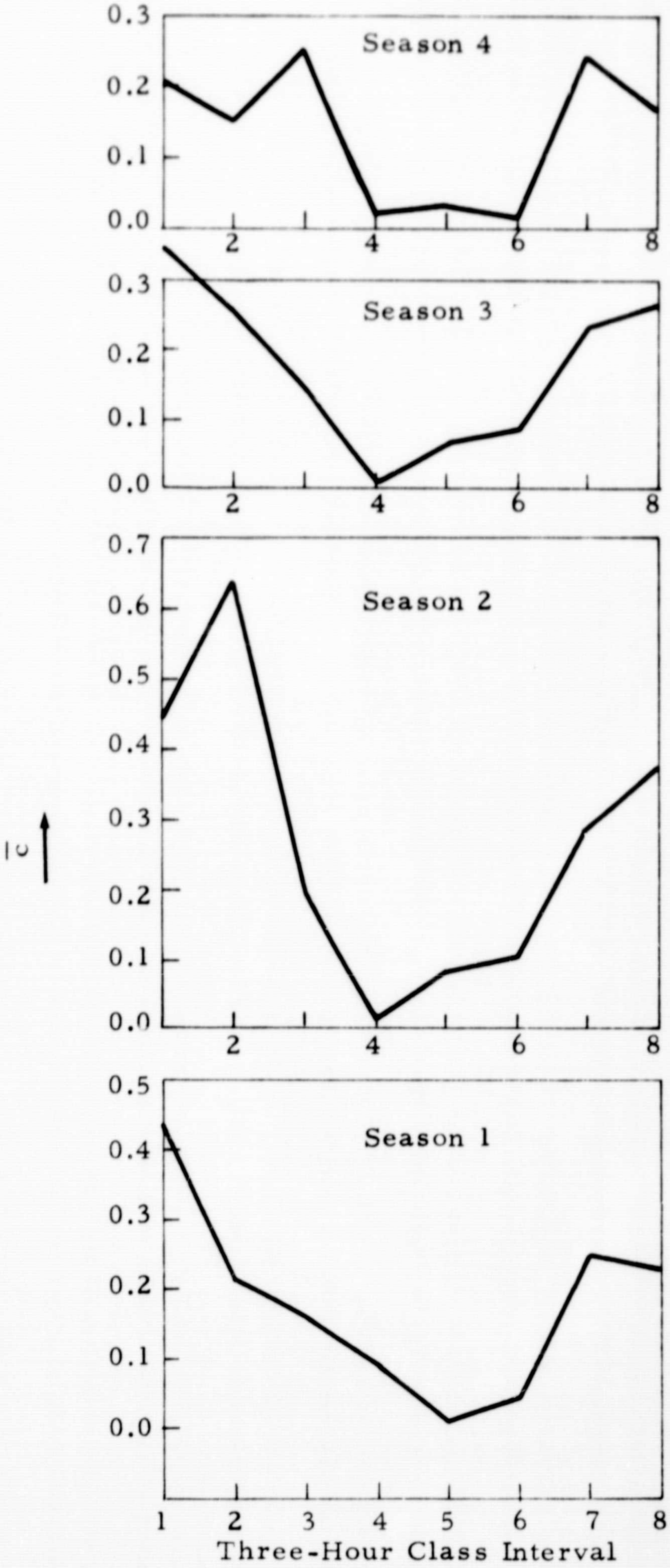

Fig. 45a - Wind Profile Parameter, $\overline{c}$

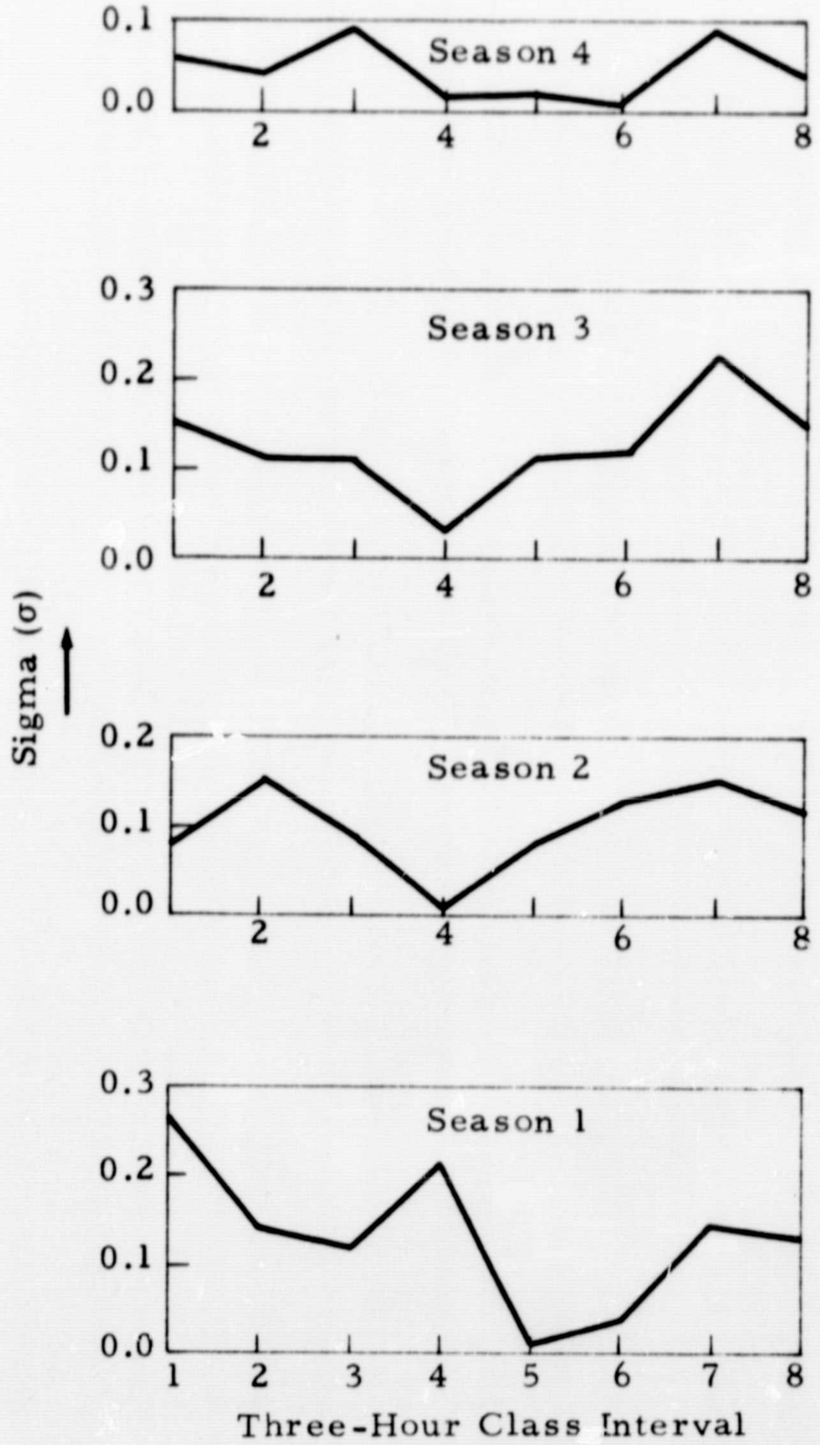

Fig. 45b - Wind Profile Parameter, σ

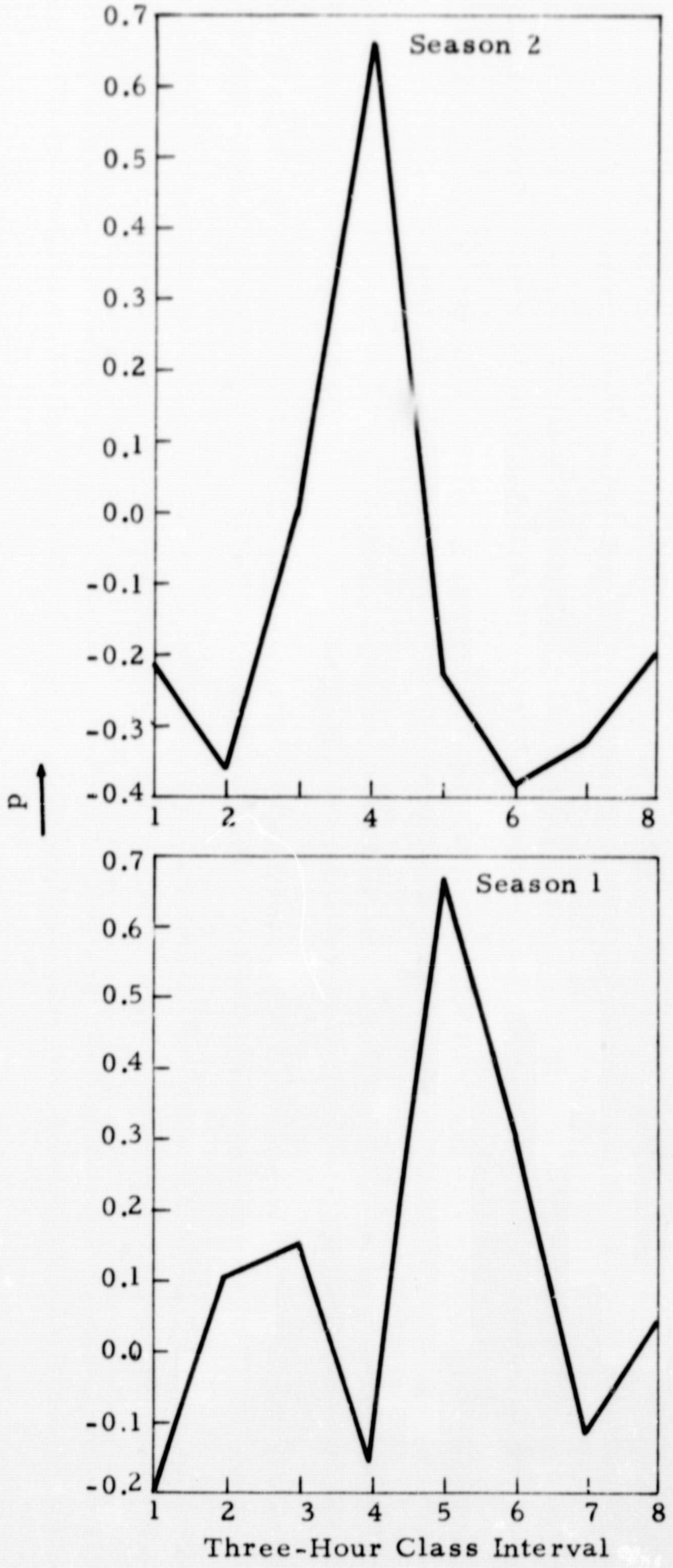

Fig. 45c - Wind Profile Parameter, p

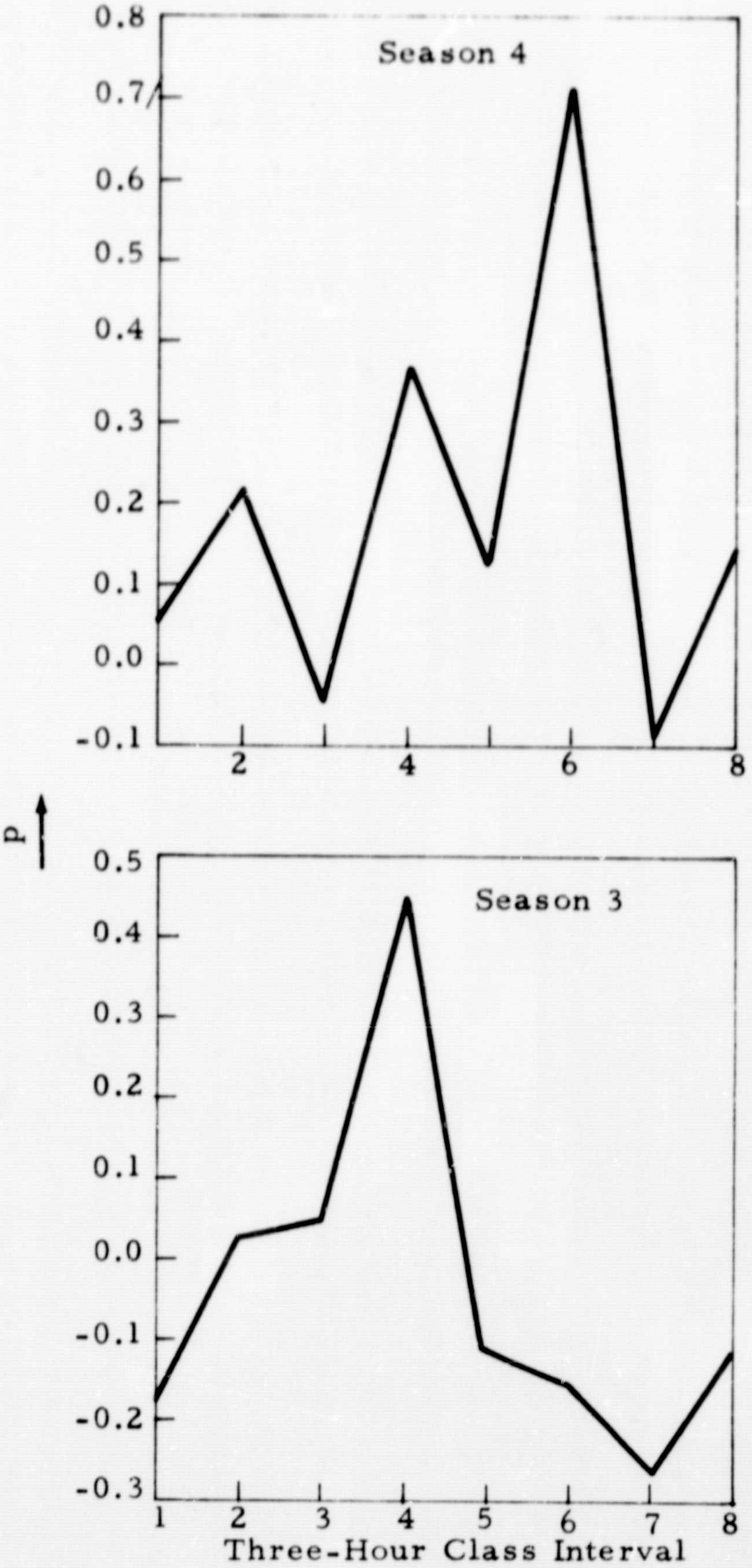

Fig. 45c - Wind Profile Parameter, p (Continued)

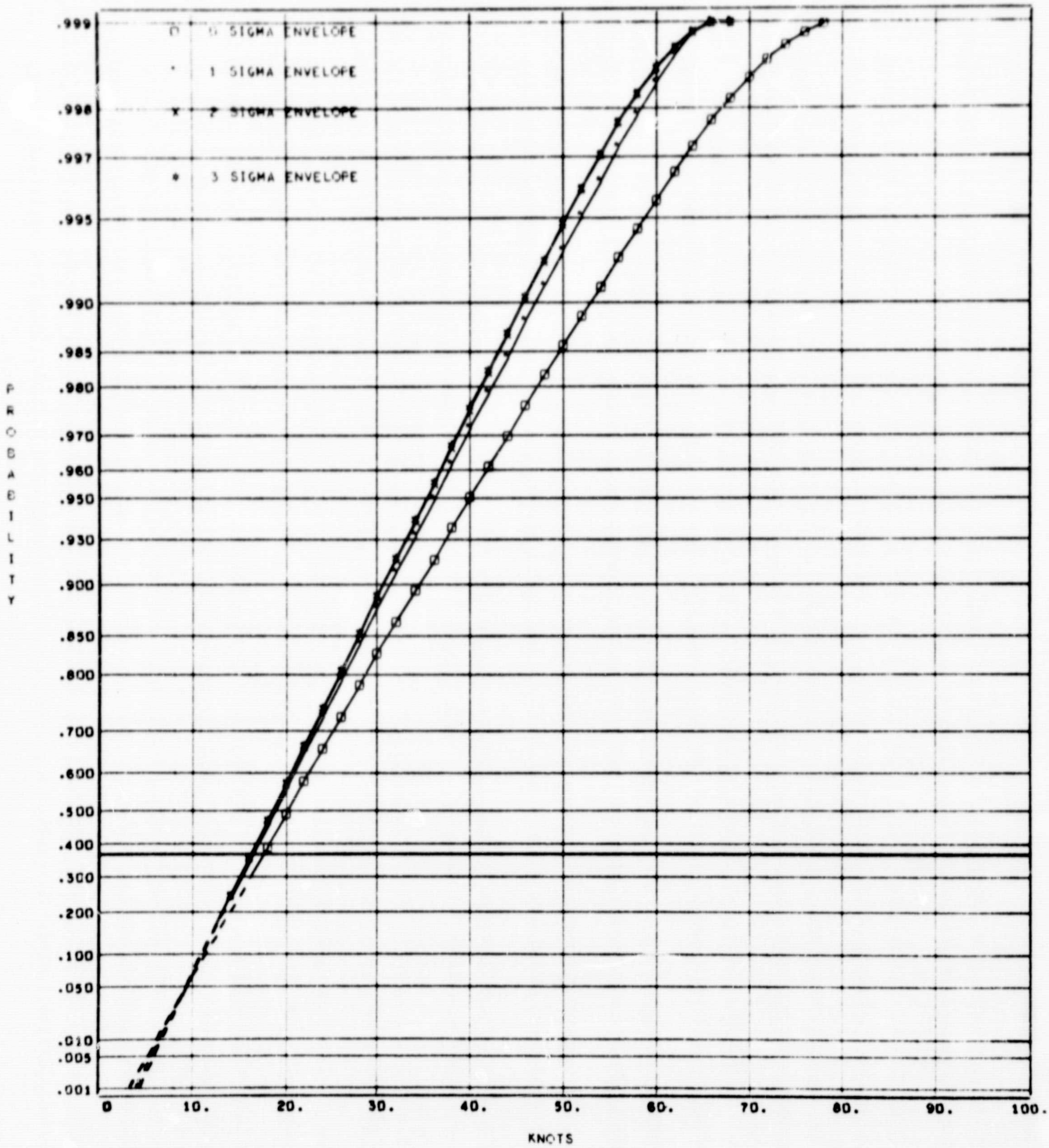

Fig. 46 - Wind Profile Cumulative Distributions for Winter, Hours 1800 - 0200 EST

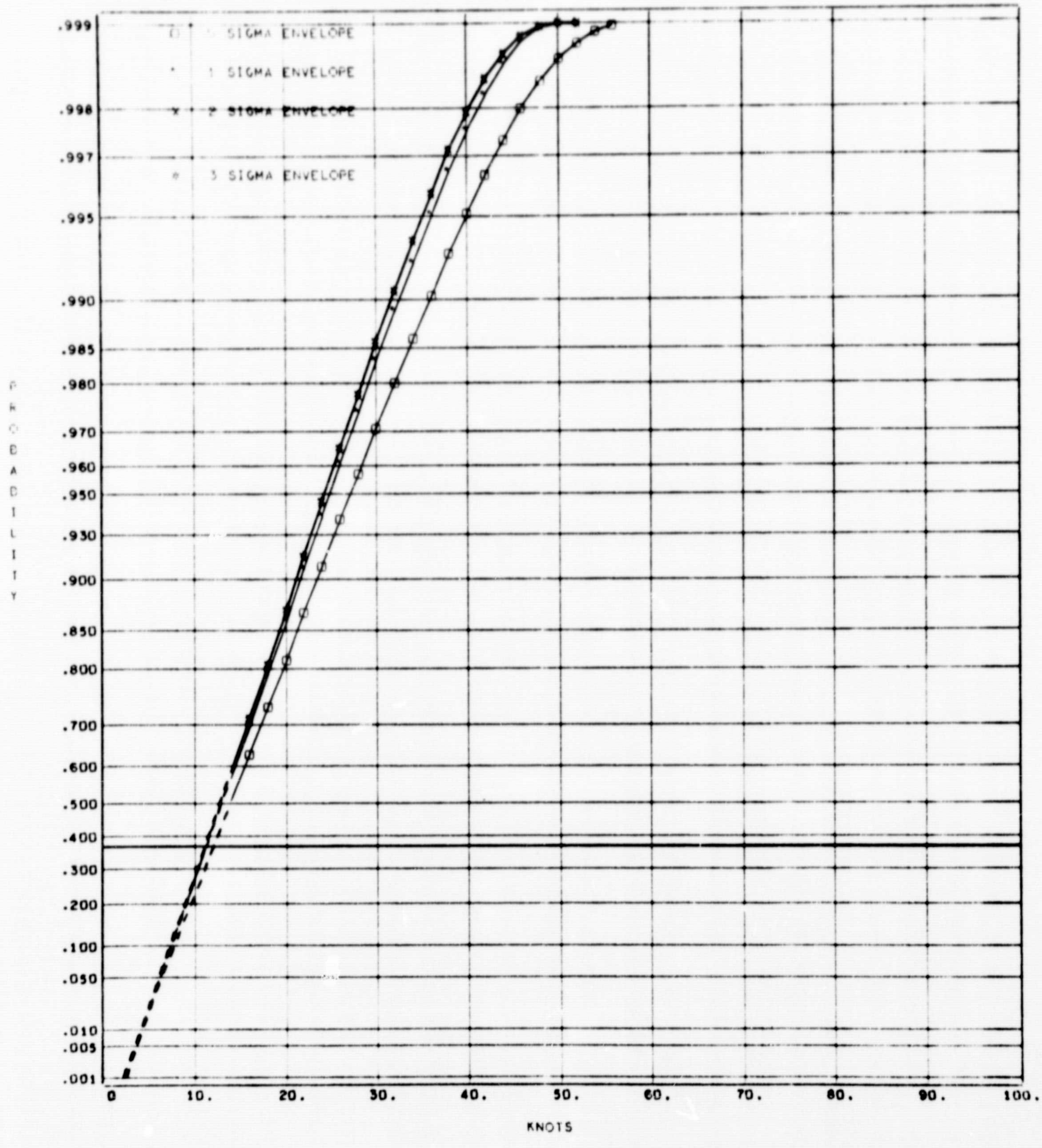

Fig. 47 - Wind Profile Cumulative Distributions for Summer, Hours 0000 - 0200 EST