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CONSIDERATIONS OF ELECTRON BEAM PROPAGATION FROM SPACE VEHICLES

W. McNeal, R. Cheever and W. C. Beggs

Prepared for:

National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland

Contract No. NAS5-3935

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ION PHYSICS CORPORATION

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A Subsidiary of High Voltage Engineering Corporation

BURLINGTON, MASSACHUSETTS

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SECTION 1
INTRODUCTION

The following sections comprise the theoretical considerations given to the problem of propagating an electron beam array from a space vehicle. The first few sections consider the velocity associated with the beam front, the maximum beam current which is compatible with vehicle neutralization via plasma return currents, and the beam spreading caused by space charge blow-up (with the earth's magnetic field being treated in first approximation). Finally, considerations are made of the possibility of further beam blow-up caused by a two-stream instability. Conclusions and recommendations for further studies are made at the end.

SECTION 2

BEAM FRONT VELOCITY CALCULATIONS

During the intial portion of the pulse, the vehicle potential builds up positively as negative charge depletion occurs. Without any return current the vehicle potential would build up to the acceleration voltage, and propagation of the beam into space would stop.

If the beam were injected into a medium where the ambient plasma density were greater than the beam electron density, for the beam parameters used, the return current electrons would be supplied by the plasma. If the beam electron density were much greater than the ambient plasma density, the return current electrons would be provided by electrons from a beam-generated plasma. In this section a one dimensional model for the latter case will be discussed, and based on the model a propagation velocity for the beam front will be derived.

In the one dimensional model the vehicle is the cathode and the beam is taken to be infinite in extent in the transverse direction:

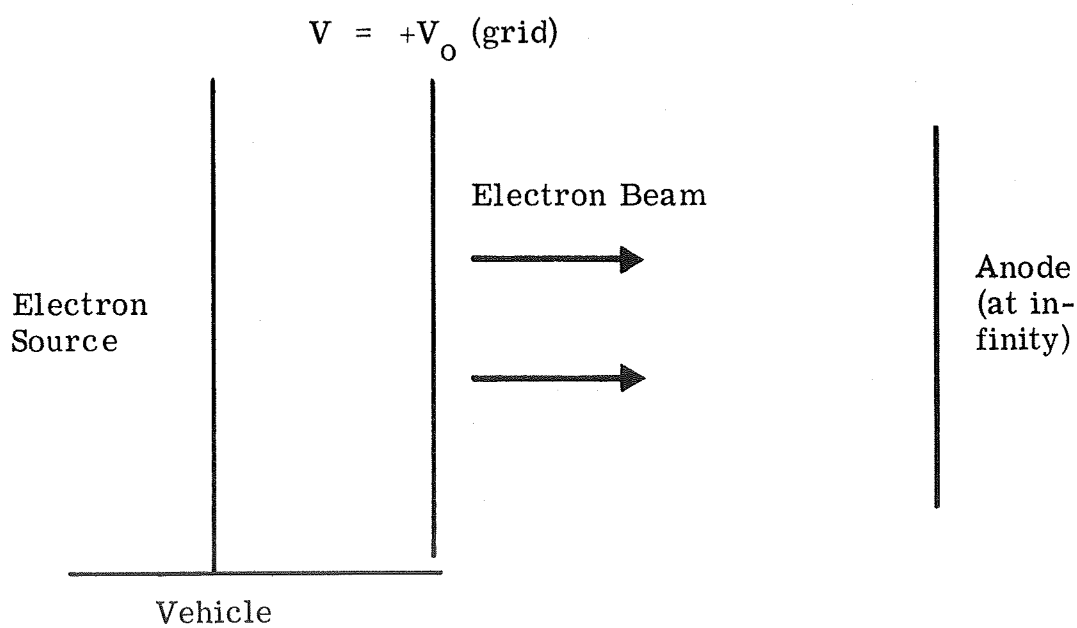

Figure 1.

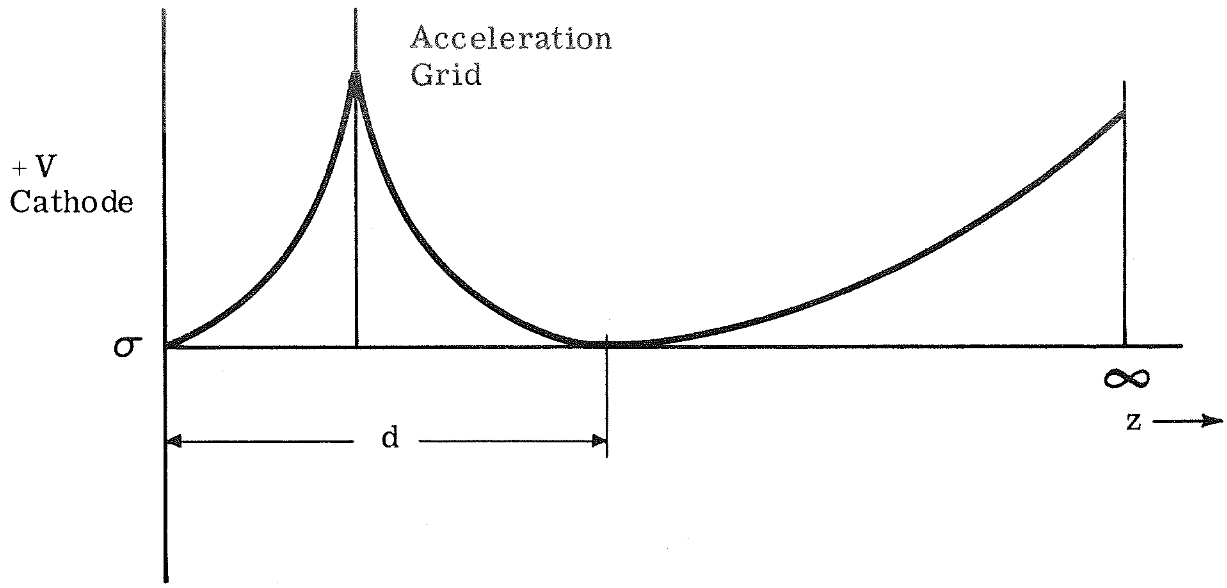

Figure 2.

A sketch representing the model is shown in Figure 1, while the potential distribution including space charge fields is shown in Figure 2.

As shown by Klemperer,(1) due to space charge contributions the potential drops off going away from the acceleration grid, reaching a null at a distance d from the grid. A "virtual cathode" is said to be formed at the null point. The distance d is given by

$$d = 1.53 \times 10^{-3} \frac{V^{3/4}}{J^{1/2}} \qquad (1)$$

in terms of the accelerating potential V and the beam current density J.

If the one dimensional theory is applied to a cylindrical beam, edge effects are ignored. The total current is

$$I = \pi r_b^2 J \qquad (2)$$

for a uniform beam of maximum radius r_b.

Using this in equation (1), we obtain

$$D = \frac{d}{r} = 2.8 \times 10^{-3} \frac{V^{3/4}}{I^{1/2}} \tag{3}$$

where we have introduced the normalized distance D (length normalized to the beam radius).

Now if there were no background plasma or neutral gas, the electron beam could only propagate a distance D as given by equation (3). When a neutral gas is present the beam creates ions from the neutrals. These ions neutralize the beam space charge forces and allow the beam to propagate. The time that it takes for the beam to create an ion density equal to the electron beam density is

$$\Delta t = \frac{1}{n \sigma_i v_e} \tag{4}$$

where

n = neutral density

σ_i = ionization cross section

$v_e = \sqrt{2eV}$ = velocity of beam electrons

The speed of the beam front can be estimated by dividing the turn around distance by the neutralization time

$$\frac{D}{\Delta t} = 2.8 \times 10^{-3} \frac{V^{3/4}}{I^{1/2}} n \sigma_i v_e \tag{5}$$

SECTION 3
TOTAL CURRENT CONSIDERATIONS

As the rocket emits negative charge, return current must be supplied to prevent positive potential build up. In the early portions of the current pulse, this return current must be supplied by the ambient plasma. L. M. Linson[2] has discussed three different theories on the amount of current that is attracted by a charged vehicle immersed in a magnetoactive plasma.

An upper limit to the magnitude of the thermal return current from the ambient plasma can be obtained by neglecting the earth's magnetic field. The maximum current is then given by Langmuir's spherical space charge limited flow between two concentric spherical electrodes. This leads to a current-voltage relation as follows:

$$\frac{I}{I_o} \propto (N_e v a^2)^{-4/7} \phi^{6/7} \tag{6}$$

where

$$I_o \equiv j_o 2\pi a^2 = \frac{\pi}{2} e N_e v a^2 = \text{thermal current} \tag{7}$$

where

j_o = ambient thermal current density
a = vehicle radius
N_e = plasma electron density
v = average thermal speed
ϕ = vehicle potential

Simple orbit calculations show that the effect of the magnetic field is to restrict the flow of electrons to a value given by the following:

$$\frac{I}{I_o} \leq 1 + 4 \sqrt{\frac{4\phi}{\phi_o}} \tag{8}$$

where

$$\phi_o = \frac{m\,\omega_e^2}{2e} = 178\ a^2 \text{ volts}$$

for

$$B = 0.45 \text{ gauss}$$

Linson[2] has developed a theory which indicates that turbulence in the vicinity of the vehicle allows free transport of electrons across magnetic field lines and hence a higher value for the current than that predicted by equation (8).

SECTION 4
BEAM SPREADING

In a previous section we showed that the electron beam can indeed propagate, with return current being provided initially by the ambient plasma, then later in the pulse by beam generated plasma.

In this section we wish to consider the beam spreading problem. The electron beam as it propagates will tend to spread due to space charge forces. The earth's magnetic field, the self magnetic field and the polarizing of the ambient plasma all work to confine the electron beam.

The repulsive electric field felt by an electron at the beam edge is

$$E = \frac{I\,(1 - f)}{2\pi\, E_o\, v\, r} \tag{9}$$

where v is the velocity of the beam electrons and f is the fractional neutralizations, $f = n+/nb$.

The self-magnetic field which tends to confine the beam has the following magnitude at the beam edge

$$B = \frac{\mu_o I}{2\pi r} \tag{10}$$

The net force felt by an edge electron is outward and is, from (9) and (10), as follows

$$F_r = \frac{m\,d^2 r}{dt^2} = \frac{e\,I\,(1 - f - \beta^2)}{2\pi\, E_o\, v\, r} \tag{11}$$

where $\beta = v/c$.

In the paraxial approximation, this becomes

$$\frac{d^2 r}{dz^2} = \frac{e I}{2\pi E_o m v^3 r} (1 - f - \beta^2) \tag{12}$$

For a 10 KeV beam, this becomes

$$\frac{d^2 r}{d z^2} = \frac{0.1 (1 - f - \beta^2)}{r} \tag{13}$$

This expression neglects the earth's magnetic field. It provides an upper bound to the rate of expansion of an electron beam injected along the field lines. The beam would expand to, at most, a radius equal to the gyro-radius at the beam velocity. The gyro-radius for a 10 KeV electron is

$$r_g = \frac{m v}{e B_e} \cong 6 \text{ m}$$

It would take ~ 100 m according to equation (13) for the beam to reach a radius of 6 m. After this time, the beam would not spread significantly.

The self magnetic field included in equation (13) for a low energy beam is in fact negligible. The ratio of the self magnetic field force to the space charge electric force is

$$\frac{F_B}{F_r} = \frac{\beta^2}{1 - f}$$

where

$$\beta = N/c \text{ and } f = N_i/N_b.$$

Thus unless the beam is nearly neutralized, the self magnetic field is negligible compared to the electric field. In any case, the self magnetic field is small compared to the earth's field.

The spreading of the beam is controlled predominantly by the space charge forces and the earth's magnetic field, as discussed above. In the remainder of this section other phenomena which a priori seemed to be important in causing the beam to spread but which in fact, for the parameters chosen, are negligible, will be discussed.

The presence of an ambient plasma would have put an upper limit to the beam radius had the beam not been confined by the earth's magnetic field. The beam would have spread until each beam electron was separated from its nearest neighboring beam electrons by a Debye length. The Debye length for the ambient plasma ($n_e = 5 \times 10^{10}\ m^{-3}$, $T_e = 2 \times 10^3$) is

$$h = \sqrt{\frac{E_o\, kT}{Ne^2}} = 10^{-2}\ m \tag{14}$$

The electron beam would have to spread until its density was

$$n_b \simeq 10^{+6}\ m^{-3} .$$

or until its radius was

$$r = \sqrt{\frac{I}{e\, n_b\, v\, \pi}} = 180\ m .$$

before beam spreading would be halted by polarization of the ambient plasma. But since the beam is confined to a much smaller radius by the earth's magnetic field, this polarization effect appears to be negligible.

The density of the plasma through which the electron beam moves increases during the 1 second pulse length at a rate

$$N_i = v \sigma_i N_n N_b t \tag{15}$$

where σ_i is the electron neutral ionization cross section, N_n is the neutral density in the propagation region, N_b is the beam density and t is the time measured from passing of the beam front.

From equation (15) we see that the neutralization fraction is, as a function of time, the following:

$$f = \frac{N_i}{N_b} = v N_n \sigma_i t \simeq 240 t \tag{16}$$

for $N_n = 4 \times 10^{15} \text{ m}^{-3}$, $\sigma_i = 10^{-22} \text{ m}^2$.

Thus in $\frac{1}{240}$ sec. the beam will have generated a plasma equal in density to the electron beam density. After 1 sec., the beam generated plasma density will be $N_i \simeq 5 \times 10^{12} \text{ m}^{-3}$, or about two orders of magnitude greater than the ambient. This enhanced plasma density could affect beam spreading. If the magnetic field were not present, the beam would be confined through the well known ion focusing mechanism, wherein secondary electrons created in the ionization of neutrals leave the beam region becuase of their few electron volts of initial energy. The ions being heavier are left behind to neutralize the space charge repulsion of the beam electrons. With a magnetic field present, however, the secondary electrons have a gyro-radius of only a few centimeters and so cannot leave the beam region.

The beam generated plasma would have a Debye length $h \simeq 10^{-3}$ m, assuming a temperature of $T_p = 2 \times 10^3$ °K. Even if the secondary electrons did not leave the beam region, the polarization of the beam generated plasma would then confine the beam radius to about:

$$r_b = 6 \text{ m}$$

However, we have assumed the beam to be confined to the gyro-radius of a 10 KeV electron in order to get the beam-generated plasma density used in the calculation. Furthermore we have used the full duration (1 sec.) of the electron pulse to calculate the plasma density. The overall conclusion is that the beam generated plasma with the earth's magnetic field present is not directly effective in confining the electron beam.

The beam generated plasma might affect beam spreading indirectly through the mechanism of the two stream instability. This possibility is discussed in the next section. It is also important in generating a source of electrons for return current, as discussed in a previous section. In the remainder of this section on beam spreading we discuss some two particle and particle-wave scattering mechanisms.

Collision Effects:

The electron beam can be spread by scattering of the primary electrons off other particles (neutrals, ions, plasma electrons) or by scattering from waves. We can estimate the effect of electron-particle collisions by calculating the mean free path length.

Electron - Neutral:

$$L_{e-n} = \frac{1}{N_n \sigma_{e-n}} \cong \frac{1}{2 \times 10^9 \cdot 1 \times 10^{-16}} \cong 10^7 \text{ cm}$$

$$\cong 10^4 \text{ m}$$

Electron - Ion:

$$L_{e-i} = \frac{N_b}{V_{ei}} = \frac{(N_b)(0.38)(T^{3/2})}{N_p \ln \wedge}$$

where T is the beam energy in °K. Using $N_b = 6 \times 10^7$ m/sec., $T = 10^8$ °K (10^4 ev) and $N_p = 5 \times 10^{10}\ m^{-3}$,

$$L_{e-i} = 2.7 \times 10^6 \ m$$

Thus collisions are important only indirectly; the beam electrons collide with ambient particles so infrequently that no beam spreading can be attributed directly to two particle interactions. However, indirectly collisional ionization increases the background plasma density and leads to anomalous diffusion through unstable growth of waves in the plasma.

The total mean free path for a beam electron is given by

$$\frac{1}{\ell} = \frac{1}{\ell_{collective}} + \frac{1}{\ell_e} + \frac{1}{\ell_i}$$

where $\ell_{collective}$ is the mean free path for electrons-photon interactions. The mean free path for collective interactions is given by

$$\ell_{coll} = v^3/2D\emptyset$$

where

$$D_\emptyset = 1/2 \quad \frac{< \Delta v \ \sin \emptyset \cos \emptyset >^2}{\Delta t}$$

ϕ = $\measuredangle$ through which the electron is scattered, Δv = velocity change in time Δt. In terms of the intensity I of the plasma wave, D_ϕ is given by

$$D_\phi = \left(\frac{I}{kT}\right) \frac{N_e e^4}{8\pi E_o^2 m^2 v} \left\{ \frac{1}{6} \left[\ln 4 + \ln\left(1 - 3\,kT/mv^2\right)\right] \right.$$

$$\left. - \frac{kT}{mv^2} \left[\ln \frac{mv^2}{kT} + \ln\left(1 - 3\,\frac{kT}{mv^2}\right)\right]\right\}$$

where T is the plasma temperature.

Besides increased diffusion, the electron plasma wave interaction leads to energy loss by the electron.

$$G = \text{energy loss/unit length}$$

$$= \frac{-mv}{kT} D_v$$

where

$$D_v = \frac{1}{2} \frac{< \Delta v \cos \phi >^2}{\Delta T}$$

$$= \left(\frac{I}{kT}\right) \frac{N_e e^4}{8\pi E_o^2 m^2 v_b} \left(\frac{1}{mv_b^2}\right)$$

$$\left[\ln \frac{mv_b^2}{kT} + \ln\left(1 - 3\,\frac{kT}{mv_b^2}\right)\right]$$

where I is the intensity of the plasma wave. Unless I exceeds kT by many orders of magnitude, D_ϕ and D_v will both be negligibly small. That is, the electromagnetic wave energy (energy density) must be much larger than the thermal energy (energy density). In the next section it will be shown that there is possible anomalous growth of electromagnetic disturbances occur along the beam axis (two stream instability)

SECTION 5

BEAM - PLASMA INSTABILITIES

In the previous sections it was shown that the electron beam initially propagates outward from the accelerator under the assumed conditions. It confines itself to at most a radius equal to the gyro-radius.

Later on in the pulse, the plasma density in the beam builds up because of ionization of the neutral gases due to the primary beam current, as well as possibly the slower moving (and therefore more efficient ionizing) back streaming secondary electrons.

The constantly increasing density of the beam generated plasma up to the beam density means that there is the possibility of unstable wave growth along the beam, through interaction between space charge waves on the beam and space charge waves in the plasma.

The existence of unstable growth of plasma oscillations lends to the following:

(1) Increased diffusion of primary electrons.

(2) Energy loss of the primary electrons.

If the plasma oscillations increased in amplitude without bound, they could lead to break up of the beam. However, nonlinear phenomena prohibit the growth beyond a certain point; however (1) and (2) could be important for attainable amplitudes of plasma oscillation.

Growth of Plasma Oscillations

The electron beam excites growing waves in the beam generated plasma, whose frequencies and wave numbers are determined by the parameters ω_c, ω_p, and ω_b. The dominant growth mechanism is the two-stream instability.

The dispersion relation for an electron beam of plasma frequency ω_b penetrating an electron plasma of the same density and therefore frequently is[4] (wave $\sim e^{i(\omega t + kz)}$)

$$\omega_b^2 \left[\frac{1}{(kv - \omega)^2} + \frac{1}{(kv + \omega)^2} \right] = 1 \qquad (17)$$

Solving for ω^2, we get

$$\omega^2 = k^2 v^2 + \omega_b^2 \left[1 \pm \sqrt{1 + 4 k^2 v^2 / \omega_b^2} \right] \qquad (18)$$

This will yield four solutions for ω; we note that if $|k| > \sqrt{2}\, \omega_b/v$, all four roots are real and we have oscillatory solutions. However, for long waves $|k| < \sqrt{2}\, \omega_b/v$ the pair of solutions corresponding to taking the - sign in (18) are imaginary, yielding an exponentially decaying and a growing wave.

The maximum growth rate occurs for the longest wavelength (smallest wave number) and is, from equation (18),

$$\omega = i\, \omega_b/2 \qquad (19)$$

independent of v.

The wave number for which this maximum growth occurs is

$$k = \sqrt{3/2}\, \omega_b/v \qquad (20)$$

For $\omega_b = 2 \times 10^6$, $v = 6 \times 10^7$ m/sec,

$$k = \frac{\sqrt{3}}{2} \, \frac{2.10^6}{6.10^7} = \frac{\sqrt{3}}{6} \times 10^{-1}\ \mathrm{m}^{-1}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\sqrt{3}}\ 6 \times 10 = 218\ \mathrm{m}$$

For the case of an electron beam of plasma frequency ω_b penetrating a plasma of frequency ω_{pe} in the presence of an axial magnetic field, the dispersion relation is more complicated. Simpson and Dunn[5] have studied this case numerically. The dispersion relation is

$$\omega_{pe}^2/\omega^2 + \frac{\omega_b^2}{(\omega \cdot kv)^2} - \left(\frac{k_{\perp}}{k}\right)^2 \left[1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2}\right]$$

$$+ \frac{\omega_b^2}{(\omega - kv)^2 - \omega_{ce}^2} = 1 \qquad (21)$$

where

ω_{pe} = plasma frequency

ω_b = beam frequency

ω_{ce} = gyro-frequency for plasma electrons

Finally the perpendicular wavenumber k is related to k by the following equation, which comes from matching fields at the beam boundary:

$$(-E_{zz}\, E_{rr})^{1/2} \frac{J_1\,(k_{\perp} r_b)}{J_o\,(k_{\perp} r_b)} = \frac{K_1\,(k r_b)}{K_o\,(k r_b)} \qquad (22)$$

where E_{zz} and E_{rr} are the diagonal elements of the dielectric tensor and standard notation has been used for ordinary and modified Bessel functions of complex argument.

The numerical study of the pair (21) and (22) reveals that essentially two types of waves on the plasma can grow unstably, taking their energy from the electron beam. The first, and the dominant, wave arises from coupling between the slow space-charge wave on the beam and the low frequency plasma wave. The other arises from coupling between the slow space charge wave on the beam and, for high density plasmas, a cyclotron wave.

According to J. R. Apel's[6] interpretation the low frequency waves are convectively (growing in space) rather than nonconvectively (growing in time) unstable. Hence, in looking for growth rates one should assume real ω and solve for complex k.

The results of Simpson and Dunn are shown in Figure 3 for the following set of parameters:

$$\frac{\omega_c}{\omega_p} = 1.65 \qquad \frac{r_b \, \omega_p}{v_b} = 1.46$$

$$\frac{\omega_b}{\omega_p} = 0.21 \qquad \frac{V_c}{\omega_p} = 1.5 \times 10^{-3}$$

For the assumed magnetic field strength, we have $\omega_{ce} = 9 \times 10^6$. The other parameters have the same ratios as those used in the calculation of Simpson and Dunn if the following values are taken: $r_b = 1.5$ m, $v_b = 5.5 \times 10^6$ m/sec. Now, the value for v_b is lower than the velocity of a 10 kV electron, but the electrons move at slower velocities due to the formation of the virtual cathode discussed in the beam propagation section. Then, according to and Dunn, the maximum growth rate is given by (see curve marked SSCW - Figure

$$\frac{k \, v_b}{\omega_p} = 0.22 \tag{23}$$

or Im k $\cong$ 0.22. Thus for the parameters chosen the growth rate in space of a wave is substantial. This will lead to increased diffusion and subsequent beam spreading; the beam will stabilize at some larger radius.

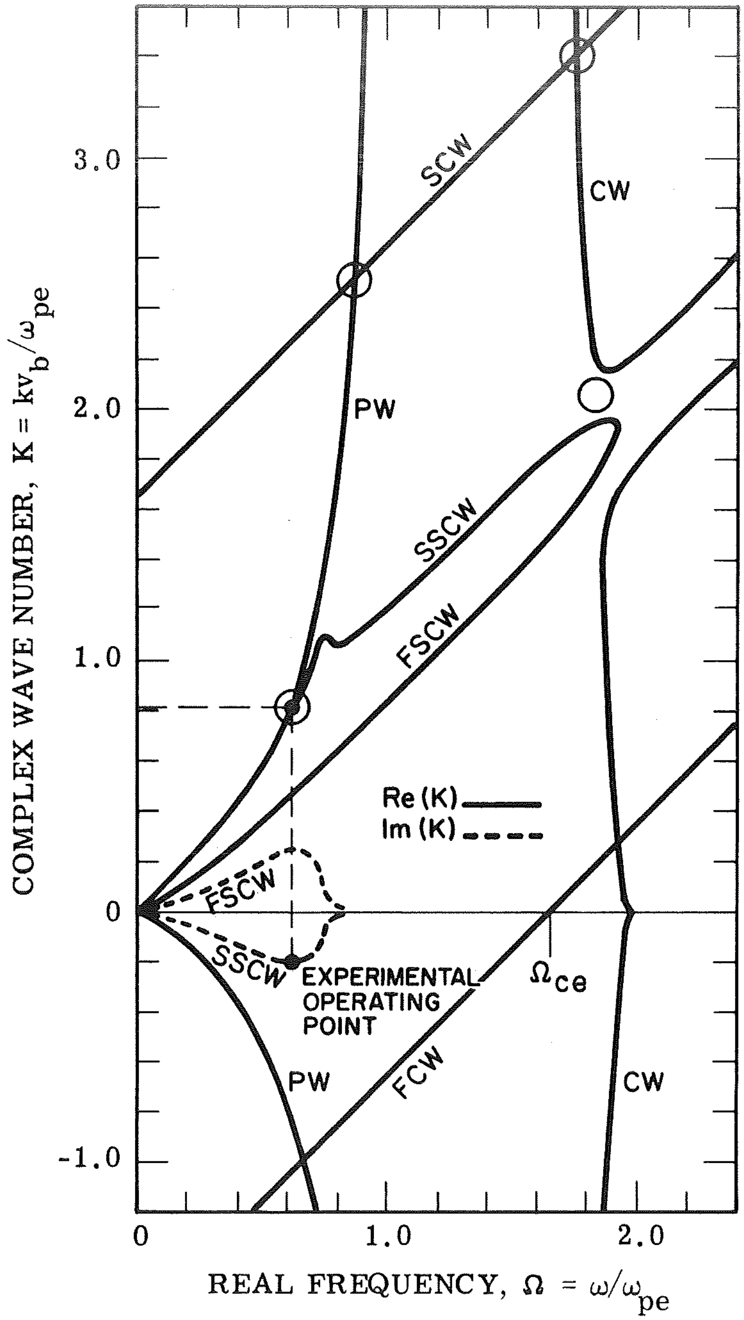

Figure 3. Dispersion diagram for beam-plasma instabilities in a magnetic field, showing complex-k solutions for real ω and nonzero collision frequency. While growing waves may occur in the vicinity of the four open circles, only the convective instability near the region marked "Experimental operating point" has been observed. PW, plasma wave; CW, cyclotron wave; SSCW, FSCW, slow and fast space-charge wave on beam; SCW, FCW, slow and fast cyclotron wave on beam; $\omega_{ce}/\omega_{pe} = 1.65$; $\omega_b/\omega_{pe} = 0.21$, $a\,\omega_{pe}/v_b = 1.46$; $v_e/\omega_{pe} = 1.5 \times 10^{-3}$.

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SECTION 6
CONCLUSIONS AND RECOMMENDATIONS

The dynamic behavior of an electron beam ejected from a space vehicle at high altitudes depends largely on collective effects rather than two particle interactions. These effects become very complex and interdependent on particle motion.

For the region near the vehicle, a series of computer experiments similar to those used for ion engine and diode studies(7) would be most informative. In such experiments, equations of motion for beam particles are solved simultaneously with electromagnetic fields generated by the particles themselves. In this way two stream wave growth and concmitant diffusion, as well as space charge forces, are considered explicitly. Beam behavior can be studied as a function of time and space, and the effects of initial conditions (velocity distribution, divergence, and current density profile) on beam propagation can be examined.

It is felt that for the region near the vehicle these deterministic computer experiments would be more meaningful than a purely statistical (Monte Carlo) procedure, which is now being used for the portion of the beam that penetrates into the atmosphere.

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