TRW®

SYSTEMS GROUP

ONE SPACE PARK • REDONDO BEACH, CALIFORNIA

N71-15965
(ACCESSION NUMBER)
186
(PAGES)
CR-116148
(NASA CR OR TMX OR AD NUMBER)
(THRU)
63
(CODE)
08
(CATEGORY)
FACILITY FORM 602

VOLUME I: FINAL REPORT AND USER'S MANUAL
A COMPUTER PROGRAM TO STUDY THE MOTION
AND APPENDAGE STRESSES OF A SATELLITE DEPLOYING
A NUMBER OF ASYMMETRICAL SEGMENTED APPENDAGES (N-BOOM)

TRW Report No. 13548-6004-R0-00

CONTRACTS NAS 5-11221 AND NAS 5-11258

May 1970

Prepared For

National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland
20771

Prepared For
National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, Maryland
Contracts NAS5-11221 and NAS5-11258

Prepared [signature]
P. B. Grote
Project Manager

Prepared [signature]
I. Tarnove
Mathematical Analyst

Approved [signature]
D. H. Mitchell
Manager,
Dynamics Department

Approved [signature]
P. G. Bhuta
Manager,
Applied Mechanics Laboratory

TRW Systems
One Space Park
Redondo Beach, California
90278

CONTENTS

ILLUSTRATIONS

NOMENCLATURE[1]

$a^k_{0i}, a^k_{1i}, a^k_{2i}, a^k_{3i}$	Kick-off spring parameters associated with kick-off spring k,i.
$\overline{B}^k_i$	Total external torque acting on the ith segment in appendage k (segment k, i)
$\overline{B}_T$	Torque about 0 produced by thrust
$\overline{b}^k_i$	Position vector of the center of mass of the ith segment in appendage k relative to 0
$\overline{b}_M$	Position vector of main body c.m. relative to 0
C	Instantaneous system center of mass
$\overline{C}^k_i$	Position vector of segment k, i center of mass relative to inboard pin
$\frac{D\overline{r}}{Dt}$	Inertial derivative of the vector $\overline{r}$
$\frac{d\overline{r}}{dt}$	Derivative of $\overline{r}$ with respect to an observer fixed in the main body
$\overline{d}_k$	Position vector of first hinge of appendage k relative to 0
$\hat{e}^k_1, \hat{e}^k_2, \hat{e}^k_3$	Unit vector triad fixed to the main body and associated with appendage k
$\overline{F}$	Total external force on the system
$\overline{F}_G$	Gravity force on the main body
$\overline{F}^k_{Gi}$	Gravity force acting on the c.m. of segment k, i
F^k_{si}	The magnitude of the compressive force on kick-off spring, i.k.

[1] Dimensions of quantities input to the program and output by the program are provided in Part II, User's Manual.

$\overline{F}_i^k$	Total external force on segment k, i
$\overline{F}_M$	Total external force on the main body
$F_T(t)$	Thrust magnitude applied to main body as a function of time
$\hat{F}_T$	Unit vector in the direction of the thrust applied to the main body
$\overline{r}_T$	Position vector to a point through which the thrust acts
$\hat{g}$	Unit vector in the direction of 0 from the center of the earth
$\overline{H}_o$	Moment of relative momentum of a body about the moving point 0, defined by $\overline{H}_o = \int_{Body} \overline{r} \times \frac{D\overline{r}}{Dt}\, dm$ where $\overline{r}$ is the position vector of a field point in the body relative to 0
$\overline{h}_i^k$	Position vector of inboard hinge of segment k, i relative to 0
i	Index symbol designating segment number[1]
$\left[I_i^k\right]^*$	Segment k, i inertia matrix in segment coordinates
$\overline{\overline{I}}_i^k$	Segment k, i inertia dyad in main body coordinates
$[J(\overline{a})] = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$	The vector $\overline{a}$ is transformed into a square matrix by the operator J so that $[J(\overline{a})]$ (b) represents $\overline{a} \times \overline{b}$
j	Index symbol designating station number

[1]Although sometimes used for other purposes, the indices k, i, j, ℓ, are generally associated with appendage, segment, station, and vertex numbers, respectively.

$K^k_{i(1)}$, $K^k_{i(2)}$	Spring parameters for the ith hinge of appendage k
ℓ^k_i	Distance between hinge points on the ith segment in appendage k
ℓ	Index symbol designating vertex number
$\overline{M}_o$	Total moment about 0 of external forces and torques
$[M_o]$	Transformation matrix from body fixed coordinates, xyz, to inertial coordinates at initial time
m^k_i	Mass of segment i in appendage k
m_M	Mass of main body
m_T	Total mass of the system
n	Total number of segments in the system
n_a	Total number of appendages in the system
n_k	Total number of elements (segments) in appendage k
n_p	Total number of paddle appendages
n_s	Total number of segmented appendages (does not include paddles) i.e., $n_s = n_a - n_p$
O	Main body fixed origin
O^k_i	Center of mass of the ith segment in appendage k
O_M	Center of mass of the main body
O_N	Origin of the uniformly translating Newtonian frame
$\overline{P}^k_i$	The total force on the inboard hinge of the ith segment in appendage k
$\overline{P}$	Total linear momentum of the system

$\bar{p}_i^k$	Linear momentum of segment k, i
$\bar{Q}_i^k$	Total spring and dashpot torque about the inboard pin of the ith segment in appendage k
$q_{i(1)}^k, q_{i(2)}^k, q_{i(3)}^k, q_{i(4)}^k$	Dashpot parameters for the ith hinge of appendage k
R	Designation of a field point in segment k, i
$\bar{r}_i^k$	Position of attachment point of kick-off spring k,i to the main body
$\bar{r}$	Position vector of a field point in segment k, i from point O
$\bar{s}_i^k$	Position of attachment point of kick-off spring k,i to segment k,i
$\bar{S}$	Position vector of system center of mass
T	Total kinetic energy of the system
T_i^k	Kinetic energy of segment k, i
$\left[T^k\right]$	Transformation matrix which transforms a vector in appendage k coordinates to main body coordinates
T_M	Kinetic energy of the main body
t	Time from beginning of simulation
t_f	Time of thrust termination
t_i	Time at which thrust is initiated
t^k	Release time of first segment of appendage k
$\hat{t}$	Time, $t = \hat{t}$, at the instant of a release or lock-up event
$\hat{t}^-$	Time $t = \hat{t}^-$ immediately preceding a release or lock-up event

$\hat{t}^+$	Time, $t = \hat{t}^+$, immediately following a release or lock-up event
$[U]$	3 x 3 identity matrix
$\bar{v}$	Velocity of main body reference point, O, relative to O_N with respect to an observer fixed in the main body
$(\bar{v})_M$	Means the vector $\bar{v}$ is expressed in a coordinate frame fixed to the main body
$\bar{x}_i^k$	The position of the attachment point of kick-off spring k,i to segment k,i relative to it's main body attachment point.
$x_{i_f}^k$	The length of kick-off spring k,i at which disengagement occurs.
XYZ	Inertial coordinates with origin at O_N
xyz	Main body fixed coordinates with origin at O
$\hat{X}, \hat{Y}, \hat{Z}$	Unit vectors associated with the inertial frame
$\hat{x}, \hat{y}, \hat{z}$	Unit vectors associated with the main body frame
$\bar{Z}_M$	Inertial velocity of the main body
$\bar{Z}_i^k$	Inertial velocity of segment k, i
α_i^k	Angular position of segment k, i relative to the main body
β_i^k	Angular position of segment k, i relative to segment k, i-1 for i > 1. $\beta_1^k = \alpha_1^k$
$\beta_{r_i}^k$	Relative angle of segment k, i-1 (the value of β_{i-1}^k) at which segment k, i is released in appendage k for i > 1
$\beta_{s_i}^k$	Relative angle of segment k, i (the value of β_i^k) at which segment k, i is locked
γ_i^k, θ_i^k	Pre-load angles for ith hinge in appendage k

$\hat{\xi}_i^k, \hat{\eta}_i^k, \hat{\zeta}_i^k$	Unit vector triad associated with the coordinate frame fixed in the ith segment in appendage k
$\overline{\rho}$	Position vector of 0 relative to O_N
$\overline{\sigma}$	Position vector of a field point in segment k, i from the c. m. of segment k, i
σ_i^k	$\dot{\alpha}_i^k$, angular velocity of segment k, i relative to the main body
τ_i^k	$\dot{\beta}_i^k$, angular velocity of segment k, i relative to segment k, i-1
$\psi^k, \theta^k, \varphi^k$	Euler angles defining plane of appendage k with respect to the main body
$\psi^M, \theta^M, \varphi^M$	Euler angles defining the position of the main body in inertial space
$\overline{\omega}$	Angular velocity of main body
$\overline{\omega}_i^k$	Angular velocity of segment k, i

The nomenclature which follows is associated with the stress package, Sections 10 to 14. Since four indices are generally required, a different format is adopted to clarify the indices associated with each quantity.

NOMENCLATURE [1]

Symbol	Index[2] App.k	Seg.i	Station j	Vertex ℓ	Description
$\bar{a}$	✓	✓			The acceleration of the c.m. of segment k,i.
$\bar{a}_j$	✓	✓	✓		The acceleration of the c.m. of subsegment j of segment k,i (subsegment k,i,j).
A	✓	✓	✓		Crossectional area of segment k,i.
$[A]$	✓	✓	✓		Area enclosed by the crossection of segment k,i.
A_ℓ	✓	✓	✓	✓	Crossectional area of plate element associated with vertex ℓ, at station j in segment k,i.
$[A]_\ell$	✓	✓	✓	✓	Portion of enclosed area associated with plate element ℓ, at station j, in segment k,i.
$\bar{C}_j$	✓	✓	✓		The position of the center of mass of subsegment k, i,j.

[1] Dimensions of quantities input to the program are provided in Part II, User's Manual.

[2] Superscripts and subscripts are often deleted in the analysis of segment stresses in cases where no significant ambiguity arises thereby.

NOMENCLATURE (Continued)

Symbol	App.k	Seg.i	Station j	Vertex ℓ	Description
$\left(\frac{1}{C}\right)_i^k$	✓	✓			A constant required to define the pulse shape associated with hinge k,i.
C_ξ, C_ζ	✓	✓	✓		The coordinates of the neutral axis and shear center at station j in segment k,i, if it is noncircular.
$\Delta\overline{C}_j$	✓	✓	✓		The position at the center of mass of the mass element between stations j and j-1 in segment k,i.
d_ℓ	✓	✓	✓	✓	The distance from the η axis of segment k,i to vertex ℓ.
E	✓	✓			Young's modulus.
G	✓	✓			Shear modulus ($G = E/2(1+\nu)$)
g_ℓ	✓	✓	✓	✓	Circumference of a triangular portion of the enclosed area $[A]$.
I	✓	✓	✓		Area moment of inertia of a circular tube segment.

NOMENCLATURE (Continued)

Symbol	App.k	Seg.i	Station j	Vertex ℓ	Description
$I_{\xi\xi}, I_{\xi\eta}, I_{\eta\eta}$	✓	✓	✓		Area moments of inertia of a non-circular section
$\bar{\bar{J}}(\bar{\omega})$					Operator which converts a vector, say $\bar{\omega}$, to a 3 × 3 matrix, that is $\bar{\bar{J}}(\bar{\omega}) = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$
k_1, k_2, k_3	✓	✓	✓		Constants used in calculating the effective moments.
Δm_j	✓	✓	✓		The mass of the portion of segment k,i defined by stations j, and j-1.
$m_j, \bar{\bar{I}}_j$	✓	✓	✓		The mass and mass moment of inertia associated with subsegment k,i,j.

NOMENCLATURE (Continued)

Symbol	App.k	Seg.i	Station j	Vertex ℓ	Description
N	✓	✓			The number stations associated with segment k,i.
$\overline{P}_{e1}, \overline{P}_{e2}$	✓	✓			The force acting on the outboard and inboard hinges, respectively, of segment k,i.
$\overline{P}_i^k$	✓	✓			The force acting on the inboard hinge of segment k,i.
$\overline{P}_j, \overline{Q}_j$	✓	✓			The force and torque, respectively, acting at the inboard end of subsegment k,i,j.
$\left(\hat{P}_{e1}\right)_i^k, \left(\hat{P}_{e2}\right)_i^k$	✓	✓			The impulsive forces acting on the outboard and inboard hinges, respectively, of segment k,i during a lock-up.
$\left(\hat{Q}_{e1}\right)_i^k, \left(\hat{Q}_{e2}\right)_i^k$	✓	✓			The impulsive torque acting on the inboard and outboard hinges of segment k,i during a lock-up.
$\hat{P}_j$	✓	✓	✓		The impulsive force acting on the inboard end of subsegment k,i,j during a lock-up.

NOMENCLATURE (Continued)

Symbol	App.k	Seg.i	Station j	Vertex ℓ	Description
Q_1^*, Q_2^*, Q_3^*	✓	✓	✓		Modified moments acting on the inboard end of subsegment k,i,j.
Q_ξ, Q_ζ	✓	✓	✓	✓	Moments of crossectional areas defined by ξ and ζ, about the neutral axis,(centroid).
$\overline{Q}_{e1}, \overline{Q}_{e2}$	✓	✓			The moments acting on the outboard and inboard hinges, respectively, of segment k,i.
$\left\{ \begin{matrix} \left(P_j\right)_{i,max}^k \\ \left(Q_j\right)_{i,max}^k \end{matrix} \right.$	✓	✓	✓		The maximum, through time, force and torque, respectively, acting on the inboard end of subsegment k,i,j, during a lock-up.
$\hat{Q}_j$	✓	✓	✓		The impulsive torque acting on the inboard end of subsegment k,i,j during a lock-up.
q_t	✓	✓	✓		The shear flow on a non-circular crossection of subsegment k,i,j.
R	✓	✓	✓		The radius of segment k,i if it is circular.
S	✓	✓	✓		$S = 2\pi R^2 t$, a constant required to establish the shear stress on a circular section.

NOMENCLATURE (Continued)

Symbol	App.k	Seg.i	Station j	Vertex ℓ	Description
S_j	✓	✓	✓		$\overline{S}_j = S_j \hat{\eta}^k_i$, a vector defining station j on segment k,i.
t	✓	✓	✓	✓	The thickness of the crossection at station j of segment k,i, vertex ℓ.
$\Delta\overline{V}$	✓	✓			The change in velocity of the center of mass of segment k,i when a lock-up occurs.
ΔV_j	✓	✓	✓		The change in velocity of the center of mass of subsegment k,i,j when a lock-up occurs.
V_0	✓	✓	✓		The value of the distortion energy at a point at stationj, segment k,i
θ					An angle measured from the ξ axis in a c. clockwise sense on circular sections.
θ_ℓ	✓	✓	✓	✓	Angle defining the orientation of plate element ℓ at station j on segment k,i.
υ	✓	✓			Poision's ratio for segment k,i.
$\hat{\xi}^k_i, \hat{\eta}^k_i, \hat{\xi}^k_i$	✓	✓			Segment k,i unit vectors
ξ_ℓ, ζ_ℓ	✓	✓	✓	✓	Coordinates defining vertex k,i,j,ℓ.

NOMENCLATURE (Continued)

Symbol	App.k	Seg.i	Station j	Vertex ℓ	Description
$\bar{\rho}$					Vector defining position of main body origin,0.
$\sigma_{21},\sigma_{23},\sigma_{22}$	✓	✓	✓		Stress components
$\Delta\bar{\omega}$	✓	✓			The change in angular velocity of segment k, i when a lock-up occurs.
$\bar{\omega}$, $\dot{\bar{\omega}}$					Main body angular velocity and aceleration.
$\bar{\omega}_i^k,\dot{\bar{\omega}}_i^k$					Angular velocity and acceleration of segment k,i.

PART I

FINAL REPORT

1. INTRODUCTION

During launch, space vehicles are generally stowed in the confining volume of the launch vehicle nosecone. If the space vehicle has appendages, these must be designed to satisfy this stowability constraint. Appendages may serve any of a number of purposes, such as:

- Experiment package isolation
- Solar cell arrays
- Communications antennas
- Inertia control for spin or gravity gradient stability

In order to be stowable, appendages are often designed as series of hinged elements which can be collapsed to fit within an allowable envelope. Once in orbit, the appendages are deployed under the action of spin of the main body, thrust applied to the main body, or by the action of springs and dashpots which may act about each hinge point.

Deployment in general must be accomplished within constraints of maintaining stability of spacecraft motion and not exceeding the maximum design stresses for the appendages. These constraints are satisfied by selecting a proper combination of design parameters such as release times and lock-up positions of appendages; magnitude, duration and direction of applied thrust; spin rate of the spacecraft; and hinge spring and dashpot parameters.

The first step in determining whether a particular array of design parameters is appropriate is to determine the motion of a prospective system. Having the motion, one can readily determine whether motion constraints are satisfied and estimate maximum stresses.

Since the mathematical formulation describing the motion of such systems is a set of complicated non-linear differential equations, they can only be solved in general by use of numerical techniques. The N-Boom Program, developed by TRW under contract to NASA Goddard, embodies such a system of equations and provides a numerical solution for use in design studies. This report describes the development and use of the program, which calculates the motion of a spinning, accelerating spacecraft deploying a number of asymmetrical segmented appendages with arbitrary hinge torques, and which in addition, estimates appendage segment stresses.

2. SUMMARY

The satellite and its deploying appendages are modeled in the N-Boom Program as a system of rigid bodies. Each appendage is composed of one or more rigid segments, which are hinged together, and attached to the central body at an arbitrary location with an arbitrary orientation. The general features and options provided by the N-Boom Program are the following:

System Configuration

Two general types of appendages are admissable:

- The first, involving one or more segments and constrained to deploy in a plane fixed to the main body.
- The second, involving two segments, with the first constrained to deploy in a plane relative to the main body, while the second segment rotates about the first.
- The center of mass of each body in the system is arbitrarily located.
- The number of segments allowable is dependent on the size of computer memory available. In the case of the IBM 360 Mod 65, the limitation is 20 segments, while for the IBM 360 Mod 91 it may be as large as 100.

Hinge Torques

- Linear or non-linear springs and dashpots may be assumed to act about each segment hinge and, in addition, non-linear, disengaging springs may be assumed to act between arbitrary points on the main body and an arbitrary point on any appendage segment.
- Appendage segments may be released from an initially locked position. The program provides a number of release criteria options:

  a. Each segment may be released at a specified time.

  b. Each segment may be released when any other prescribed segment, which may be in another appendage, has attained a given relative rotation with respect to the segment inboard of it.

- Adjoining segments may be locked together when they have attained a prescribed relative angle. The motion of all the bodies which compose the system is reinitiated whenever a hinge is locked.

<u>Segment Stresses</u>

- The program provides the option of calculating segment stresses which arise in the course of deployment. The stresses are pseudo-dynamic stresses. That is, the internal forces from which the stresses are obtained are calculated by means of the rigid body motion.

- The program calculates stresses and principal stresses at the four points on the crossection lying on segment coordinate axes, and in addition, establishes the most severe stress condition at each station on each segment at times specified by the user and at each lock-up.

- Internal stresses are calculated from the internal forces, obtained above, by strength of materials theory.

- Maximum internal forces during a lock-up are calculated from the impulsive forces and torques which act at each hinge point when a segment locks, and an assumed pulse shape, associated with the locking hinge and specified in input.

<u>Section Properties - Required if Stresses are to be Calculated</u>

- The shear center and neutral axis of each segment are assumed concurrent.

- Two types of segment crossection are admissable:

  a. circular tube

  b. a general polygon crossection having as many as five sides.

  Thus, the user inputs the tube radius and thickness in the first case, and the coordinates of the vertices and the wall thickness between vertices in the case of the second crossection option.

- The above crossection parameters may vary from station to station on the segment. As many as 6 stations are allowed.

- The program calculates all the geometric section properties required for the stress calculation from the above inputs.

- The user inputs mass properties of each portion of segment between stations. The program generates all required mass properties for stress calculation purposes from this input.

Plots

- A plot output option is provided.

3. DISCUSSION

The N-Boom Program is designed to predict the motion of a spinning, accelerating spacecraft and its deploying appendages during the deployment maneuver, and, on the basis of this rigid body motion, estimate segment stresses. The satellite and appendages are modeled as a system of rigid bodies. Two types of appendage models are considered: the first is a series of bodies hinged together end to end and constrained to deploy in a plane fixed to the main body; the second type, simulating a paddle, involves two bodies, the first deploying in a plane fixed to the main body while the second body rotates about the first. The system motion is induced by non-linear springs and dashpots acting at the hinges and/or by external forces and torques. For the prupose of calculating stresses, two types of segment crossections are admissible: (1) circular; (2) polygonal. In addition, appendage segment crossections may vary from station to station along their length.

The external forces and torques, discussed in Section 6, arise from two sources: gravity forces which act on the center of mass of each body in the system, and thrust applied to the main body.

In practice, satellite appendages are initially unreleased, that is, initially the system is one rigid body. Appendages are then released upon command, move out to a fully deployed configuration, and are locked in place. However, a complication is introduced into this sequence of events when appendages and segments are not released simultaneously. The program allows for any hinge in appendage k to be released at time t_i^k, in addition, any hinge in appendage k may be released on the basis of a displacement criteria. That is, segment k,i may be released when segment m,n has attained a prescribed position relative to the segment inboard of it. In the program, motion must be correctly reinitialized whenever a release or lock-up event occurs. The importance of this point and the method by which this is accomplished is discussed in detail in Section 7.

Counting each one of the n appendage segments and the main body, the system consists of a total of n+1 bodies. The main body has 6 degrees of freedom: three rotations, and three translations. The position of each appendage segment can be described by considering one additional degree of freedom relative to the main body for each additional segment in the appendage. Thus, for this system of n+1 bodies, there are a total of n+6 degrees of freedom and to completely describe the motion of the system n+6 dynamical equations of motion are required.

Three equations of motion are contributed by the system moment equation, Equation (4.9) of Section 4. Another moment equation is obtained for the segments outboard of each hinge, Equations (4.16) and (4.17). Finally, three equations are obtained corresponding to the translational motion of the system, Equation (4.3). Thus, a total of n+6 equations are provided to account for the motion of the system.

These equations are later reformulated in a form suitable for solution by standard computer techniques. This form is a matrix equation, Equation (5.1). Solution to problems such as release and lock-up are developed in terms of manipulations of this equation.

Section 8 provides equations for kinetic energy, angular momentum, and linear momentum of the system, Equations (8.11), (8.13), and (8.7), respectively. These equations are not used in the program to calculate motion, but these quantities are calculated from the computed motion. In the check-out phase of the program development, these quantities served as checks on the predicted motion. In addition, the user should find them useful as a check on the results.

In Section 10, Loads, and Section 11, Stresses, the two major steps required to calculate segment stress are presented. In the first of these sections, the means whereby crossection loads are calculated from the general motion or from the motion discontinuities during a lock-up are described. Presentation of this analysis first serves to clarify the most suitable form in which to have dynamic quantities and inertia properties. The crossection loads, derived in Section 10, are in the same format whether or not a segment is locking. Consequently, in Section 11, where the crossection stresses are calculated, it is unnecessary to discriminate whether a segment has locked or not.

In Section 11 the means of calculating crossection stresses is described. Although some complexity is introduced by a consideration of alternate non-circular and circular crossections, no particular difficulty arises from consideration of station-to-station variations.

Sections 12 and 13 translate crossection geometry and mass parameters input by the user into a form admissible to the loads subroutine of Section 10, and the stresses subroutine of Section 11. The input required is in a format most convenient to the user, and consequently, although in some cases the required input may be voluminous, the quantity of input is greatly reduced by the addition of these sections.

Section 14 converts segment motion as calculated by the motion portion N-Boom program into a form suitable for use in the loads calculation.

Appendix A provides definitions of a number of quantities derived during the course of formulation in terms of variables defined in the Nomenclature. The derivations of these is not provided in the report, although in most cases, these are readily apparent.

Part II is the User's Manual. Namelist input quantities are defined in terms of notation used in the formulation and defined in the Nomenclature Section. This part of the report also includes sample load sheets and test cases.

Volume II of this report is the Programmer's Manual. It includes a description of the program, descriptions of subroutines in the program, a flowchart, and a program listing.

4. FORMULATION

The system described in Section 3 involves a total of n + 6 degrees of freedom, where n is the number of degrees of freedom in appendage segments relative to the main body. Therefore, in order to describe the motion of the system, n + 6 dynamical equations are required. These are obtained as follows:

1) Three component equations from Newton's Second Law for the system

2) Three component equations from the system moment equation

3) A moment equation about O for all segments outboard of each hinge

Figure 4-1 introduces some of the notation used in the analysis: $\bar{\rho}$ is the position vector of the reference point O (fixed in the main body) with respect to O_N (fixed in inertial space); the vector $\bar{b}_i^k$ is the position vector of the center of mass of the ith segment in appendage k, O_i^k with respect to O; $\bar{d}^k$ is the position vector of the first hinge in appendage k with respect to O; and $\bar{S}$ is the position vector of the system center of mass C with respect to O.

Figure 4-2 introduces notation associated with a particular appendage segment, the ith segment in appendage k, referred to as segment k, i. Not shown in Figure 4-2 is $\hat{e}_1^k$, a unit vector normal to the plane of deployment of appendage k. The meaning of the geometric quantities is clear; remaining symbols represent forces and torques. The vector $\bar{P}_i^k$ represents the resultant bearing force on the inboard hinge of segment i, and consequently $-\bar{P}_{i+1}^k$ represents the bearing force on the outboard hinge. $\bar{F}_i^k$ is the resultant external force and Q_i^k, $-Q_{i+1}^k$, are the spring and dashpot moments on the inboard and outboard ends of segment i, respectively.

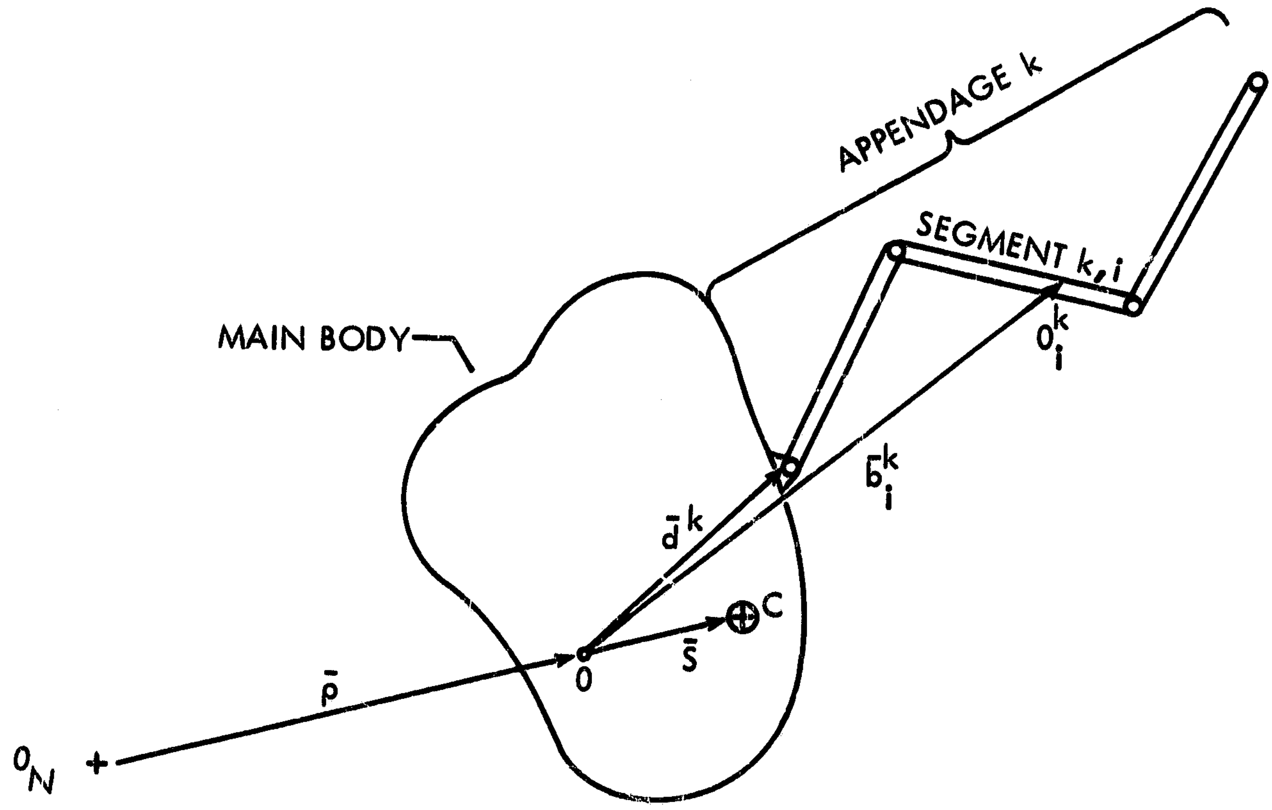


Figure 4-1. Basic System Notation

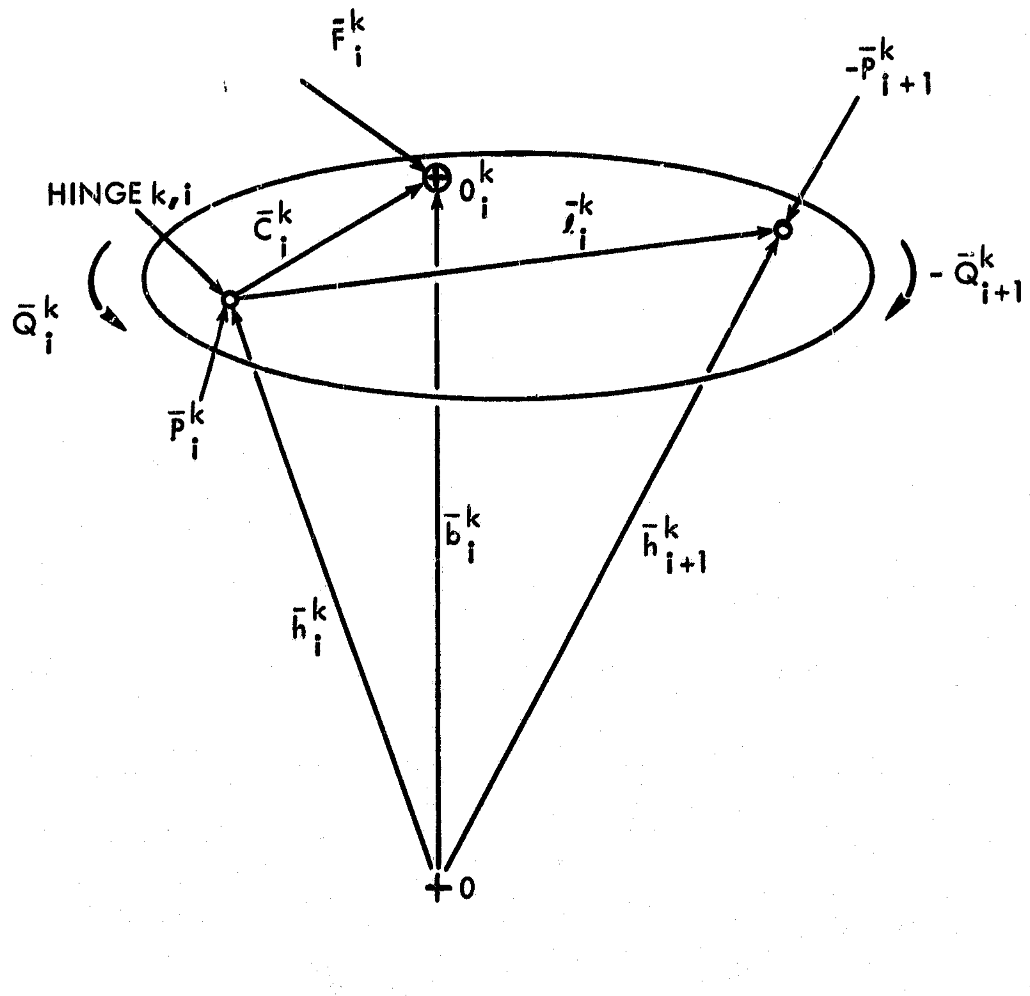


Figure 4-2. Segment i in Appendage k

The notation associated with a paddle appendage is almost identical to that associated with an ordinary appendage. This is shown in Figure 4-3.

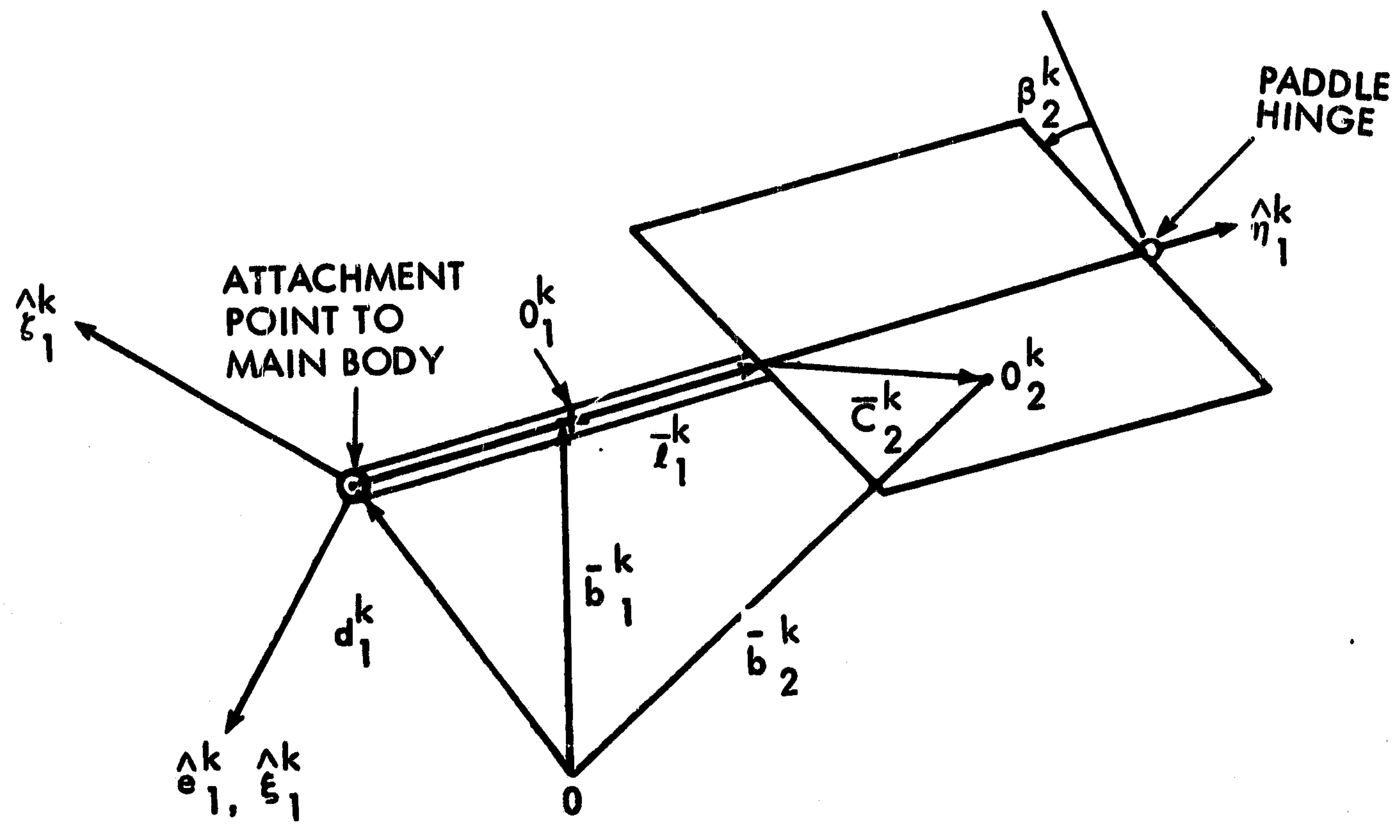


Figure 4-3. Paddle Appendage Coordinates

The vector $\bar{\ell}_1^k$ is to an arbitrary point on the axis of rotation of the paddle. Referring to Figure 4-2, Newton's Second Law for segment k, i is

$$m_i^k \frac{D^2}{Dt^2}(\bar{\rho} + \bar{b}_i^k) = \bar{F}_i^k + \bar{P}_i^k - \bar{P}_{i+1}^k \tag{4.1}$$

where

$$\bar{P}^k_{n_{k+1}} = 0$$

while for the main body, it is

$$m_M \frac{D^2}{Dt^2}(\bar{\rho} + \bar{b}_M) = \bar{F}_M - (\bar{P}_1{}^1 + \bar{P}_1{}^2 + \ldots + \bar{P}_1{}^{n_a}) . \tag{4.2}$$

Summing (4.1) over all appendage segments and combining with (4.2), Newton's Second Law for the system is obtained

$$m_T \frac{D^2}{Dt^2} (\bar{\rho} + \bar{\delta}) = \bar{F} \tag{4.3}$$

where

$$m_T = m_M + \sum_{i,k} m_i^k \tag{4.4}$$

is the total mass of the system,

$$\bar{\delta} = \frac{1}{m_T}\left(m_M \bar{b}_M + \sum_{i,k} m_i^k \bar{b}_i^k\right) \tag{4.5}$$

is the position vector of the instantaneous system center of mass with respect to O, and

$$\bar{F} = \bar{F}_M + \sum_{i,k} \bar{F}_i^k \tag{4.6}$$

is the total external force on the system.

The moment equation for the ith segment of the appendage k is

$$\frac{D\bar{H}_{o_i}^k}{Dt} + m_i^k \bar{b}_i^k \times \frac{D^2\bar{\rho}}{Dt^2} = \bar{B}_i^k + \bar{b}_i^k \times \bar{F}_i^k + \bar{Q}_i^k$$

$$+ \bar{h}_i^k \times \bar{P}_i^k - \left(\bar{Q}_{i+1}^k + \bar{h}_{i+1} \times \bar{P}_{i+1}^k\right) \tag{4.7}$$

where, it is noted that

$$\bar{Q}_{n_{k+1}}^k = 0$$

The corresponding equation for the main body is

$$\frac{D\bar{H}_{oM}}{Dt} + m_M \bar{b}_M \times \frac{D^2\bar{\rho}}{Dt^2} = \bar{B}_M + \bar{b}_M \times \bar{F}_M - \sum_{k=1}^{n_a} \left(\bar{Q}_1^k + \bar{h}_1^k \times \bar{P}_1^k \right) \tag{4.8}$$

In the same manner as Equation (4.3) was obtained, a moment equation for the system is obtained by summing (4.7) over all i and k and adding (4.8). Thus,

$$\frac{D\bar{H}_o}{Dt} + m_T \bar{S} \times \frac{D^2\bar{\rho}}{Dt^2} = \bar{M}_o \tag{4.9}$$

where

$$\bar{H}_o = \bar{H}_{oM} + \sum_{i,k} \bar{H}_{o_i}^k \tag{4.10}$$

is the total moment of relative momentum of the system about point O, and

$$\bar{M}_o = \bar{B}_M + \bar{b}_M \times \bar{F}_M + \sum_{i,k} \left(\bar{B}_j^k + \bar{b}_j^k \times \bar{F}_j^k \right) \tag{4.11}$$

is the total external moment on the system about O.

Equations (4.3) and (4.9) represent 6 component equations out of a total of n + 6 (where n is the total number of appendage segments) required to describe the motion of the system. The additional n equations required are obtained by writing the moment equation about O for all segments outboard of each hinge. This equation is obtained by summing (4.7) over indices corresponding to the ith body outward, to the outboard end of appendage k. Thus,

$$\sum_{j=i}^{n_k} \frac{D\bar{H}^k_{oj}}{Dt} + \sum_{j=i}^{n_k} m^k_j \bar{b}^k_j \times \frac{D^2\bar{\rho}}{Dt^2} = \sum_{j=i}^{n_k} \left(\bar{B}^k_j + \bar{b}^k_j \times \bar{F}^k_j\right) + \bar{Q}^k_i + \bar{h}^k_i \times \bar{P}^k_i \tag{4.12}$$

The interaction forces are eliminated from (4.12) by use of Equation (4.1). Summing (4.1) over j, with $j > i$, in appendage k and then solving for $\bar{P}^k_i$, one obtains

$$\bar{P}^k_i = \sum_{j=i}^{n_k} \left[m^k_j \frac{D^2}{Dt^2}\left(\bar{\rho} + \bar{b}^k_j\right) - \bar{F}^k_j\right] \tag{4.13}$$

Substitution of (4.13) in Equation (4.12) yields

$$\sum_{j=i}^{n_k} \left[\frac{D\bar{H}^k_{oj}}{Dt} + m^k_j\left(\bar{b}^k_j - \bar{h}^k_i\right) \times \frac{D^2\bar{\rho}}{Dt^2} - m^k_j \bar{h}^k_i \times \frac{D^2\bar{b}^k_j}{Dt^2}\right] = \bar{M}^k_i + \bar{Q}^k_i \tag{4.14}$$

where

$$\bar{M}^k_i = \sum_{j=i}^{n_k} \left(\bar{B}^k_j + \bar{b}^k_j \times \bar{F}^k_j\right) \tag{4.15}$$

To arrive at the desired equation, it is necessary to eliminate unknown components of $\bar{Q}^k_i$ from (4.14). In the case of ordinary segments, known components of $\bar{Q}^k_i$ are normal to the plane of deployment. Thus, we obtain

$$\hat{e}_1^k \cdot \sum_{j=1}^{n_k} \left[\frac{D\bar{H}_{oj}^k}{Dt} + m_j^k \left(\bar{b}_j^k - \bar{h}_i^k \right) \times \frac{D^2\bar{\rho}}{Dt^2} - m_j^k \bar{h}_i^k \times \frac{D^2\bar{b}_j^k}{Dt^2} \right]$$

$$= \hat{e}_1^k \cdot \bar{M}_i^k + Q_1^k \qquad (4.16)$$

for each non-paddle segment, that is for $k \leq n_s$ all j, and for $n_s < k \leq n_a$, for $j < n_k$

The corresponding equation for paddle segments is

$$\hat{\eta}_{n_k}^k \cdot \left[\frac{D\bar{H}_{on_k}}{Dt} + m_{n_k}^k \left(\bar{b}_{n_k}^k - \bar{h}_{n_k}^k \right) \times \frac{D^2\bar{\rho}}{Dt^2} - m_{n_k}^k \bar{h}_{n_k}^k \times \frac{D^2\bar{b}_{n_k}^k}{Dt^2} \right]$$

$$= \hat{\eta}_{n_k}^k \cdot \bar{M}_{n_k}^k + Q_{n_k}^k \qquad (4.17)$$

for $n_s < k \leq n_a$ and $j = n_k$.

Although Equations (4.3), (4.9), (4.16), and (4.17) form a mathematically complete description of the motion of the system, they are not in a form suitable for obtaining numerical results. In their present form, all terms are implicit functions of the n + 6 unknown parameters (3 translations and 3 rotations for the main body and n relative angular displacements, one for each of the n appendage segments). Furthermore, reducing Equations (4.3), (4.9), (4.16), and (4.17) to equations involving only the n + 6 unknown parameters is not sufficient. In order to be amenable to standard techniques for computer solution, the equations must be reduced to normal form. An intermediate step is to be able to write

$$\begin{bmatrix} \text{[A], a square matrix} \\ \text{whose elements are} \\ \text{composed of geometric} \\ \text{and mass parameters} \\ \text{only (Inertia)} \end{bmatrix} \begin{pmatrix} \text{a column} \\ \text{vector of} \\ \text{unknown} \\ \text{derivatives} \\ \text{(accelerations)} \end{pmatrix} = \begin{pmatrix} \text{a column} \\ \text{vector of} \\ \text{known} \\ \text{quantities} \\ \text{(forces)} \end{pmatrix} \qquad (4.18)$$

Equation (4.18) is shown in detail in Section 5.

Proceeding with the reformulation, the system translation equation, Equation (4.3), is first considered. All terms must be first expressed in terms of the unknown parameters. The inertial acceleration of the reference point O may be expressed relative to an observer fixed in the main body as

$$\frac{D^2\bar{\rho}}{Dt^2} = \left(\frac{d^2\bar{\rho}}{dt^2}\right)_M + \dot{\bar{\omega}} \times \bar{\rho} + 2\omega \times \left(\frac{d\bar{\rho}}{dt}\right)_M + \bar{\omega} \times (\bar{\omega} \times \bar{\rho}) \tag{4.19}$$

and the inertial acceleration of the system center of mass with respect to O in main body coordinates is

$$m_T \frac{d^2\bar{S}}{dt^2} = \frac{d\bar{\omega}}{dt} \times m_T \bar{S} + \sum_{k=1}^{n_s} \sum_{i=1}^{n_k} \bar{\beta}_i^{(1)k} \dot{\sigma}_i^k$$

$$+ \sum_{k=n_s+1}^{n_a} \left[\left(\bar{\beta}_1^{(1)k} + m_2^k \hat{e}_1 \times \bar{C}_2^k\right) \dot{\sigma}_1^k + m_2^k \hat{\eta}_1^k \times \bar{C}_2^k \dot{\sigma}_2^k\right] + \bar{\beta} \tag{4.20}$$

where

$$\bar{\beta}_i^{(1)k} = m_i^k \hat{e}_1^k \times \bar{C}_i^k + \mu_i^k \ell_i^k \hat{\zeta}_i^k \tag{4.21}$$

and

$$\bar{\beta} = \sum_{k=1}^{n_s} \hat{e}_1^k \times \left[\sum_{i=1}^{n_k} \left(\sigma_i^k\right)^2 \bar{\beta}_i^{(1)k} \right] + 2\bar{\omega} \times \sum_{i,k} \sigma_i^k \, \bar{\beta}_i^{(1)k}$$

$$+ \sum_{k=n_s+1}^{n_a} \left\{ \hat{e}_1^k \times \left[\left(\sigma_1^k\right)^2 \bar{\beta}_1^{(1)k} \right] + 2\bar{\omega} \times \sum_{k=n_s+1}^{n_a} \sigma_1^k \, \bar{\beta}_1^{(1)k} \right\}$$

$$+ \sum_{k=n_s+1}^{n_a} m_2^k \left\{ \sigma_1^k \sigma_2^k \hat{\xi}_1^k \times \bar{C}_2^k + \left(\sigma_1^k \hat{e}_1^k + \sigma_2^k \hat{\eta}_1^k \right) \times \left[\left(\sigma_1^k \hat{e}_1^k + \sigma_2^k \hat{\eta}_1^k \right) \times \bar{C}_2^k \right] \right\}$$

$$+ 2\bar{\omega} \times \sum_{k=n_s+1}^{n_a} \left[m_2^k \left(\sigma_1^k \hat{e}_1^k \times \bar{C}_2^k + \sigma_2^k \hat{\eta}_1^k \times \bar{C}_2^k \right) \right] \tag{4.22}$$

Those quantities in the above equations, or yet to be developed equations, which have not been previously defined or are not to be found in the Nomenclature section will be found in Appendix A.

If we define

$$\bar{v} = \left(\frac{d\bar{\rho}}{dt} \right)_M \tag{4.23}$$

then (4.3) can be written

$$m_T \frac{d\bar{v}}{dt} - m_T (\bar{\rho} + \bar{S}) \times \frac{d\bar{\omega}}{dt} + \sum_{i,k}^{n_s} \bar{\beta}_i^{(1)k} \dot{\sigma}_i^k$$

$$+ \sum_{k=n_s+1}^{n_a} \left[\left(\bar{\beta}_1^{(1)k} + m_2^k \hat{e}_1^k \times \bar{C}_2^k \right) \dot{\sigma}_1^k + m_2^k \hat{\eta}_1^k \times \bar{C}_2^k \dot{\sigma}_2^k \right] = \bar{u}_1 \tag{4.24}$$

where

$$\bar{u}_1 = \bar{F} - \bar{\beta} - 2m_T \bar{\omega} \times \bar{v} - m_T \bar{\omega} \times (\bar{\omega} \times \bar{\rho}) \tag{4.25}$$

Equation (4.24) corresponds to the first three rows of Equation (5.1).

In order to reformulate Equation (4.9), it will be necessary to re-express the terms of (4.9) in terms of the parameters of interest. The first term in (4.9) may be written

$$\frac{D\bar{H}_o}{Dt} = \sum_{i,k} \frac{D\bar{H}_{oi}^k}{Dt} + \frac{D\bar{H}_{OM}}{Dt} \tag{4.26}$$

where $\frac{D\bar{H}_{oi}^k}{Dt}$ is the moment of relative momentum of segment k, i about O, and is given by

$$\frac{D\bar{H}_{oi}^k}{Dt} = m_i^k \bar{b}_i^k \times \frac{D^2 \bar{b}_i^k}{Dt^2} + \frac{D\bar{H}_{ci}^k}{Dt} \tag{4.27}$$

where $\bar{H}_{ci}^k$ denotes the relative angular momentum vector of the ith segment of appendage k about its center of mass, and where the first term is given by

$$m_i^k \bar{b}_i^k \times \frac{D^2 \bar{b}_i^k}{Dt^2} = m_i^k \bar{b}_i^k \times \left(\dot{\bar{\omega}} \times \bar{b}_i^k\right) + m_i^k \bar{b}_i^k$$

$$\times \left(\hat{e}_1^k \times \bar{C}_i^k\right) \dot{\sigma}_i^k + m_i^k \bar{b}_i^k \times \sum_{j=1}^{i-1} \ell_j^k \hat{\zeta}^k \dot{\sigma}_j^k + m_i^k \bar{b}_i^k \times \bar{g}_i^k$$

(4.28)

except for paddles, that is, if i = 2, and $n_s < k \leq n_a$, then

$$m_2^k \bar{b}_2^k \times \frac{D^2 \bar{b}_2^k}{Dt^2} = m_2^k \bar{b}_2^k \times \left(\dot{\bar{\omega}} \times \bar{b}_2^k\right) + m_2^k \bar{b}_2^k$$

$$\times \left(\dot{\sigma}_1^k \hat{e}_1^k \times \bar{C}_2^k\right) + \dot{\sigma}_2^k \hat{\eta}_1^k \times \bar{C}_2^k + \bar{b}_1^k \times \dot{\sigma}_1^k \hat{\xi}_1^k + m_2^k \bar{b}_2^k \times \bar{g}_2^k$$

(4.29)

In order to re-write (4.9), the following relations will be of use:

$$\bar{g}^{(1)} = \sum_{i,k} m_i^k \bar{b}_i^k \times \bar{g}_i^k \tag{4.30}$$

where $\bar{g}_i^k$ is defined in Appendix A, and

$$m_M \bar{b}_M \times \frac{D^2 \bar{b}_M}{Dt^2} = m_M \bar{b}_M \times \left(\dot{\bar{\omega}} \times \bar{b}_M\right) + m_M \bar{b}_M \times \left[\bar{\omega} \times \left(\bar{\omega} \times \bar{b}_M\right)\right] \tag{4.31}$$

$$\bar{H}_{ci}^k = \bar{\bar{I}}_i^k \cdot \bar{\omega}_i^k = \bar{\bar{I}}_i^k \cdot \left(\bar{\omega} + \sigma_i^k \hat{e}_1^k\right) \tag{4.32}$$

except for $n_s < k < n_a$ and i=2, in which case we have

$$\bar{H}_{c2}^{k} = \bar{\bar{I}}_2^{k} \cdot \bar{\omega}_2^{k} = \bar{\bar{I}}_2^{k} \cdot \left(\bar{\omega} + \sigma_1^k \hat{e}_1^k + \sigma_2^k \hat{\eta}_1^k\right) \qquad (4.33)$$

the rate of change of the relative angular momentum of each body is

$$\frac{D\bar{H}_{ci}^{k}}{Dt} = \bar{\bar{I}}_i^{k} \cdot \left(\dot{\bar{\omega}} - \sigma_i^k \hat{e}_1^k \times \bar{\omega} + \dot{\sigma}_i^k \hat{e}_1^k\right) + \left(\bar{\omega} + \sigma_i^k \hat{e}_1^k\right) \times \bar{H}_{ci}^{k} \qquad (4.34)$$

except for $n_s < k < n_a$ and i=2, in which case we have

$$\frac{DH_{c2}^{k}}{Dt} = \bar{\bar{I}}_2^{k} \cdot \left[\dot{\bar{\omega}} + \dot{\sigma}_1^k \hat{e}_1^k + \dot{\sigma}_2^k \hat{\eta}_1^k + \sigma_1^k \sigma_2^k \hat{\xi}_1^k - \left(\sigma_1^k \hat{e}_1^k + \sigma_2^k \hat{\eta}_1^k\right) \times \bar{\omega}\right]$$

$$+ \left(\bar{\omega} + \sigma_1^k \hat{e}_1^k + \sigma_2^k \hat{\eta}_1^k\right) \times \bar{H}_{c2}^{k} \qquad (4.35)$$

Using Equations (4.28) through (4.34), Equation (4.9) is written in the form

$$m_T\, S \times \frac{d\bar{v}}{dt} + \left(\sum_{i,k} \bar{\bar{I}}_i^{k} + \bar{\bar{I}}_M\right) \cdot \dot{\bar{\omega}} + m_T\, \bar{S} \times (\dot{\bar{\omega}} \times \bar{\rho})$$

$$+ m_T\, \bar{b}_M \times \left(\dot{\bar{\omega}} \times \bar{b}_M\right) + \sum_{i,k} m_i^k \bar{b}_i^k \times \left(\dot{\bar{\omega}} \times \bar{b}_i^k\right)$$

$$+ \sum_{i,k} \left[\bar{\bar{I}}_i^{k} \cdot \hat{e}_1^k + m_i^k \bar{b}_i^k \times \left(\hat{e}_1^k \times \bar{C}_i^k\right) + \bar{\ell}_i^k \bar{\bar{S}}_i^k \times \hat{\zeta}_i^k\right] \dot{\sigma}_1^k$$

$$+ \sum_{k=n_s+1}^{n_a} \left\{\left[\left(\bar{\bar{I}}_1^{k} + \bar{\bar{I}}_2^{k}\right) \cdot \hat{e}_1^k + m_1^k \bar{b}_1^k \times \left(\hat{e}_1^k \times \bar{C}_1^k\right)\right.\right.$$

$$\left. + m_2^k \bar{b}_2^k \times \left(\hat{e}_1^k \times \bar{C}_2^k\right) + m_2^k \bar{b}_2^k \times \ell_1^k \hat{\zeta}_1^k\right] \dot{\sigma}_1^k$$

$$+\left[\bar{\bar{I}}_2^k \cdot \hat{\eta}_1^k + m_2^k \bar{b}_2^k \times \left(\hat{\eta}_1^k \times \bar{C}_2^k\right)\right] \dot{\sigma}_2^k \Bigg\} = \bar{u}_2 \tag{4.36}$$

where

$$\bar{u}_2 = \bar{M}_o - \bar{g}^{(1)} + \sum_{k,\,i} \bar{\bar{I}}_i^k \cdot \left(\sigma_i^k \hat{e}_1^k \times \bar{\omega}\right) - \sum_{i,\,k} \left(\bar{\omega} + \sigma_i^k \hat{e}_i^k\right) \times \bar{H}_{ci}^k$$

$$- \bar{\omega} \times \bar{H}_{cM} - m_M \bar{b}_M \times \left[\bar{\omega} \times \left(\bar{\omega} \times \bar{b}_M\right)\right] - m_T \bar{S} \times \left[2\bar{\omega} \times \bar{v} + \bar{\omega} \times \left(\bar{\omega} \times \bar{\rho}\right)\right]$$

$$+ \sum_{k=n_s+1}^{n_a} \Bigg\{ \bar{\bar{I}}_1^k \cdot \left(\sigma_1^k \hat{e}_1^k \times \bar{\omega}\right) + \bar{\bar{I}}_2^k \cdot \left[\left(\sigma_1^k \hat{e}_1^k + \sigma_2^k \hat{\eta}_1^k\right) \times \bar{\omega}\right] - \sigma_1^k \sigma_2^k \bar{\bar{I}}_2^k \cdot \hat{\zeta}_1^k$$

$$- \left(\bar{\omega} + \sigma_1^k \hat{e}_1^k\right) \times \bar{H}_{c1}^k - \left(\bar{\omega} + \sigma_1^k \hat{e}_1^k + \sigma_2^k \hat{\eta}_1^k\right) + \bar{H}_{c2}^k \Bigg\} \tag{4.37}$$

Equation (4.36) corresponds to the second three rows of matrix A defined in Equation (5.1).

The re-formulation of Equations (4.16) and (4.17) is more lengthy than that required for the system translational Equation (4.24) or the system moment Equation (4.36). Consequently the derivation will not be presented in detail. The resulting re-formulation of Equation (4.16) is

$$\left(\hat{e}_1^k \times \bar{\beta}_i^{(2)k}\right) \cdot \frac{d\bar{v}}{dt} + \left[\left(\bar{\rho} + \bar{h}_i^k\right) \times \left(\hat{e}_1^k \times \bar{\beta}_i^k\right)\right] \cdot \dot{\bar{\omega}}$$

$$+ \sum_{j=i}^{n_k} \left\{ \hat{e}_1^k \cdot \bar{\bar{I}}_j^k + m_j^k \left(\bar{b}_j^k - \bar{h}_i^k\right) \times \left[\hat{e}_1^k \times \left(\bar{b}_j^k - \bar{h}_i^k\right)\right]\right\} \cdot \dot{\bar{\omega}}$$

$$+ \sum_{j=i}^{n_k} \Bigg\{ \hat{e}_1^k \cdot \bar{\bar{I}}_j^k \cdot \hat{e}_1^k + m_j^k \left[\hat{e}_1^k \times \left(\bar{b}_j^k - \bar{h}_i^k\right)\right] \cdot \left(\hat{e}_1^k \times \bar{C}_j^k\right)$$

$$+ \ell_j^k \hat{e}_1^k \cdot \left(\bar{\beta}_{j+1}^k \times \hat{\xi}_j^k\right)\Bigg\} \dot{\sigma}_j^k + \sum_{j=1}^{i-1} \ell_j^k \bar{\beta}_i^k \times \hat{\zeta}_j^k \dot{\sigma}_j^k = u_i^k \tag{4.38}$$

where

$$u_i^k = \hat{e}_1^k \cdot \left\{ M_i^k - \bar{h}_i^{(2)k} + \sum_{j=i}^{n_k} \left[\sigma_j^k \bar{\bar{I}}_j^k \cdot \left(\hat{e}_1^k \times \bar{\omega} \right) \right. \right.$$

$$\left. \left. - \left(\bar{\omega} + \sigma_j^k \hat{e}_1^k \right) \times \bar{H}_{cj}^k - m_j^k \left(\bar{b}_j^k - \bar{h}_i^k \right) \times \left(\bar{g}_j^k - \bar{h}_i^{(1)k} \right) \right] \right\} + u_{si}^k \qquad (4.39)$$

that is, for non-paddle appendages ($k \le n_s$), and where u_{si}^k is the generalized force corresponding to kick-off springs derived in Section 6.

In the case of paddles, the moment about the inboard hinge is dotted with the normal to the deployment plane, $\hat{e}_1^k$, while for the second segment in this appendage, the paddle segment itself, the moment is dotted with $\hat{\eta}_1^k$. The first of these equations has a different form than the corresponding equation for ordinary 2-segment appendages, but only in that the relative angular velocity of the second body is in the $\hat{\eta}_1^k$ direction. For appendage k, when it is a paddle appendage, the moment equation about the inboard hinge of the first segment in the direction of the normal to the plane of deployment, $\hat{e}_1^k$, is

$$\left(\hat{e}_1^k \times \bar{\beta}_1^{(2)k} \right) \times \frac{d\bar{v}}{dt} + \left\{ \bar{\bar{I}}_1^k \cdot \hat{e}_1^k + \bar{\bar{I}}_2^k \cdot \hat{e}_1^k + \left(\bar{\rho} + \bar{h}_1^k \right) \times \left(\hat{e}_1^k \times \bar{\beta}_1^k \right) \right.$$

$$\left. + m_1^k \left(\bar{b}_1^k - \bar{h}_1^k \right) \times \left[\hat{e}_1^k \times \left(\bar{b}_1^k - \bar{h}_1^k \right) \right] + m_2^k \left(\bar{b}_2^k - \bar{h}_1^k \right) \times \left[\hat{e}_1^k \times \left(\bar{b}_2^k - \bar{h}_1^k \right) \right] \right\} \dot{\bar{\omega}}$$

$$+ \left\{ \hat{e}_1^k \cdot \bar{\bar{I}}_1^k \cdot \hat{e}_1^k + \hat{e}_1^k \cdot \bar{\bar{I}}_2^k \cdot \hat{e}_1^k + m_1^k \left[\hat{e}_1^k \times \left(\bar{b}_1^k - \bar{h}_1^k \right) \right] \cdot \left[\hat{e}_1^k \times \bar{C}_1^k \right] \right.$$

$$\left. + m_2^k \left[\hat{e}_1^k \times \left(\bar{b}_2^k - \bar{h}_1^k \right) \right] \cdot \left[\hat{e}_1^k \times \bar{C}_2^k \right] + \ell_1^k m_2^k \hat{e}_1^k \cdot \left[\bar{b}_2^k - \bar{h}_1^k \right] \times \hat{\zeta}_1^k \right\} \dot{\sigma}_1^k$$

$$+ \left\{ \hat{e}_1^k \cdot \bar{\bar{I}}_2^k \cdot \hat{\eta}_1^k + m_2^k \left[\hat{e}_1^k \times \left(\bar{b}_2^k - \bar{h}_1^k \right) \right] \cdot \left[\hat{\eta}_1^k \times \bar{C}_2^k \right] \right\} \dot{\sigma}_2^k = u_1^k \qquad (4.40)$$

where

$$u_1^k = Q_1^k + \hat{e}_1^k \cdot \Big[M_1^k - \bar{h}_1^{(2)k} + \sigma_1^k \bar{\bar{I}}_1^k \cdot \left(\hat{e}_1^k \times \bar{\omega}\right) + \bar{\bar{I}}_2^k \cdot \left(\sigma_1^k \hat{e}_1^k + \sigma_2^k \hat{\eta}_1^k\right) \times \bar{\omega}$$

$$- \sigma_1^k \sigma_2^k \bar{\bar{I}}_2^k \cdot \hat{\zeta}_1^k - \left(\bar{\omega} + \sigma_1^k \hat{e}_1^k\right) \bar{H}_{c_1}^k - \left(\bar{\omega} + \sigma_1^k \hat{e}_1^k + \sigma_2^k \hat{\eta}_1^k\right) \times \bar{H}_{c_2}^k$$

$$- m_1^k \left(\bar{b}_1^k - \bar{h}_1^k\right) \times \left(\bar{g}_1^k - \bar{h}_1^{(1)k}\right) - m_2^k \left(\bar{b}_2^k - \bar{h}_1^k\right) \times \left(\bar{g}_2^k - \bar{h}_1^{(1)k}\right) \Big] + u_{s1}^k \quad (4.41)$$

for $n_s < k \le n_a$, and u_{s1}^k is the force term corresponding to kick-off springs derived in Section 6.

The moment equation for the second segment in the appendage is obtained from (4.17), where $n_k = 2$ since it is being assumed that paddle appendages consist of two bodies, and where it is to be noted that $\hat{\eta}_1^k = \hat{\eta}_2^k$. Thus,

$$\left(\hat{\eta}_1^k \times \bar{\beta}_2^{(2)k}\right) \cdot \frac{d\bar{v}}{dt} + \Big\{ \hat{\eta}_1^k \cdot \bar{\bar{I}}_2^k + \Big[\left(\bar{\rho} + \bar{h}_2^k\right) \times \left(\hat{\eta}_1^k \times \bar{\beta}_2^{(2)k}\right)\Big]$$

$$+ \Big[\left(\bar{b}_2^k - \bar{h}_2^k\right) \times \left(\hat{\eta}_1^k \times \bar{\beta}_2^{(2)k}\right)\Big]\Big\} \cdot \bar{\omega} + \Big\{ \hat{\eta}_1^k \cdot \bar{\bar{I}}_2^k \cdot \hat{e}_1^k + \left(\hat{\eta}_1^k \times \bar{\beta}_2^{(2)k}\right) \cdot \left(\hat{e}_1^k \times \bar{C}_2^k\right)$$

$$+ \ell_1^k \hat{\eta}_1^k \cdot \left(\bar{\beta}_2^{(2)k} \times \hat{\zeta}_1^k\right)\Big\} \dot{\sigma}_1^k$$

$$+ \Big\{ \hat{\eta}_1^k \cdot \bar{\bar{I}}_2^k \cdot \hat{\eta}_1^k + \left(\hat{\eta}_1^k \times \bar{\beta}_2^{(2)k}\right) \cdot \left(\hat{\eta}_1^k \times \bar{C}_2^k\right)\Big\} \dot{\sigma}_2^k = u_2^k \quad (4.42)$$

where

$$u_2^k = Q_2^k + \hat{\eta}_1^k \cdot \Big\{ \bar{M}_2^k - \bar{\beta}_2^{(2)k} \times \left(\bar{g}_2^k - \bar{h}_2^{(1)k}\right) + \bar{\bar{I}}_2^k \Big[\left(\sigma_1^k \hat{e}_1^k + \sigma_2^k \hat{\eta}_1^k\right)$$

$$\times \bar{\omega} - \sigma_1^k \sigma_2^k \hat{\zeta}_1^k\Big] - \left(\bar{\omega} + \sigma_1^k \hat{e}_1^k + \sigma_2^k \hat{\eta}_1^k\right) \times \bar{H}_{c_2}^k - \bar{h}_2^{(2)k} \Big\} + u_{s2}^k \quad (4.43)$$

for $n_s < k \leq n_a$, and u^k_{s2} is the force term corresponding to kick-off springs derived in Section 6.

Equations (4.24), (4.25), (4.36), (4.37), (4.38), (4.39), (4.40), (4.41), (4.42), and (4.43) are the equations of motion of the system. Symbols in these equations which have not been previously defined are defined in Appendix A. In the following section, these equations will be rewritten in matrix form.

5. EQUATIONS OF MOTION IN NORMAL FORM

As discussed in Section 4, an intermediate step in the numerical solution of the equations of motion of the system is to write an equation in the form of Equation (4.18). The results of the preceding section, Equations (4.24), (4 36), (4.38), (4.40), and (4.42), can now be used to define the coefficient matrix on the left hand side of (4.18), matrix [A]. The [A] matrix has the following structure:

A_{11}	A_{12}	A_{13}	A_{14}	. .	A_{1s}	. .	A_{1P}
A_{21}	A_{22}	A_{23}	A_{24}	. .	A_{2s}	. .	A_{2P}
A_{31}	A_{32}	A_{33}	A_{34}		0		0
A_{41}	A_{42}	A_{43}	A_{44}		0		0
:	:	0	0				:
A_{r1}	A_{r2}	0	0		A_{rs}		0
:	:	:	:				:
A_{P1}	A_{P2}	0	0		0	. .	A_{PP}

So that (4.18) can be written:

$$[A]\,(d)' = (u) \tag{5.1}$$

where the column vector (d), is the column of unknown derivatives:

$$
(d)' = \begin{bmatrix} \dot{\bar{v}} \\ \dot{\bar{\omega}} \\ \hline \dot{\sigma}^1_1 \\ \dot{\sigma}^1_2 \\ \cdot \\ \cdot \\ \cdot \\ \dot{\sigma}^1_{n_1} \\ \hline \dot{\sigma}^2_1 \\ \cdot \\ \dot{\sigma}^2_{n_2} \\ \hline \dot{\sigma}^3_1 \\ \cdot \\ \cdot \\ \cdot \\ \hline \cdot \\ \cdot \\ \hline \dot{\sigma}^{n_a}_1 \\ \cdot \\ \cdot \\ \dot{\sigma}^{n_a}_{n_{n_a}} \end{bmatrix}
\qquad \text{while } (u) = \begin{bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ u_r \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ u_p \end{bmatrix}
$$

$$
\text{and } p = n + 2 = 2 + \sum_{k=1}^{n_a} n_k
$$

The subscripts r and s shown in Equation (5.1) are defined by:

$$r = 2 + i + \sum_{m=1}^{k-1} n_m \tag{5.2}$$

and

$$s = 2 + j + \sum_{m=1}^{k-1} n_m \tag{5.3}$$

The first row of matrices in the partitioned [A] matrix are now to be defined. These have three rows and are designated A_{11}, A_{12}, and A_{1s} where $s = 3, \ldots, 4 + \sum_{m=1}^{n_a} n_m$, (i.e., s = 2 plus the total number of appendage segments in the system). The first row of [A] corresponds to the left hand side of Equation (4.24).

$$A_{11} = m_T\,[U] \text{ where } [U] \text{ is the identity matrix} \tag{5.4}$$

$$A_{12} = -m_T\,[J(\bar{\rho} + \bar{S})] \tag{5.5}$$

$$\begin{aligned} A_{1s} &= \bar{\beta}_j^{(1)k} && \text{for} && k \le n_s \, . \\ &= \bar{\beta}_j^{(1)k} + m_2^k \hat{e}_1^k \times \bar{C}_2^k && j=1 && n_s < k \le n_a \\ &= m_2^k \hat{\eta}_1^k \times \bar{C}_2^k && j=2 && n_s < k \le n_a \end{aligned} \tag{5.6}$$

The next row of sub-matrices in [A] corresponds to the system moment equation, Equation (4.36).

$$A_{21} = m_T\,[J(\bar{S})] \tag{5.7}$$

$$A_{22} = \sum_{i,k} \left|I_i^k\right| + \left|I_M\right| - m_T \left|J(\bar{S})\right| \left|J(\bar{\rho})\right|$$

$$- m_T \left|J(\bar{b}_M)\right| \left|J(\bar{b}_M)\right| - \sum_{i,k} m_i^k \left|J(\bar{b}_i^k)\right| \left|J(\bar{b}_i^k)\right|$$

$$A_{2s} = \bar{\bar{I}}_j^k \cdot \hat{e}_1^k + \ell_j^k \bar{S}_j^k \times \hat{\zeta}_j^k + m_j^k \bar{b}_j^k \times \left[\hat{e}_1^k \times \bar{C}_j^k\right] \qquad k \le n_s$$

$$= \left(\bar{\bar{I}}_1^k + \bar{\bar{I}}_2^k\right) \cdot \hat{e}_1 + \ell_1 m_2^k \bar{b}_2^k \times \hat{\zeta}_1^k \tag{5.9}$$

$$+ m_1^k \bar{b}_1^k \times \left[\hat{e}_1^k \times \bar{C}_1^k\right] + m_2^k \bar{b}_2^k \times \left[\hat{e}_1^k \times \bar{C}_2^k\right] \quad \text{for } j=1 \text{ and } n_s < k \le n_a$$

$$= \bar{\bar{I}}_2^k \cdot \hat{\eta}_1^k + m_2^k \bar{b}_2^k \times \left[\hat{\eta}_1^k \times \bar{C}_2^k\right] \qquad \text{for } j=2 \text{ and } n_s < k \le n_a$$

Equations (4.38), (4.40), and (4.42), correspond to columns of submatrices of dimensions 1 x 3 in the first column, 1 x 3 in the second, and scalar quantities in the sth column.

$$A_{r1} = \hat{e}_1^k \times \bar{\beta}_i^{(2)k} \qquad k \le n_s$$

$$= \hat{e}_1^k \times \bar{\beta}_1^{(2)k} \qquad i=1 \quad n_s < k \le n_a$$

$$= \hat{\eta}_1^k \times \bar{\beta}_2^{(2)k} \qquad i=2 \quad n_s < k \le n_a \tag{5.10}$$

$$A_{r2} = \sum_{j=i}^{n_k} \left\{ m_j^k \left(\bar{b}_j^k - \bar{h}_i^k\right) \times \left[\hat{e}_1^k \times \left(\bar{b}_j^k - \bar{h}_i^k\right)\right] + \bar{\bar{I}}_j^k \cdot e_1^k \right\}$$

$$+ \left[\left(\bar{\rho} + \bar{h}_i^k\right) \times \left(\hat{e}_1^k \times \bar{\beta}_i^{(2)k}\right)\right] \qquad k \le n_s$$

$$= \sum_{j=1}^{2} \left\{ m_j^k \left(\bar{b}_j^k - \bar{h}_1^k\right) \times \left[\hat{e}_1^k \times \left(\bar{b}_j^k - \bar{h}_1^k\right)\right]\right.$$

$$\left. + \bar{\bar{I}}_j^k \cdot \hat{e}_1^k \right\} + \left(\bar{\rho} + \bar{h}_1^k\right) \times \left(\hat{e}_1^k \times \bar{\beta}_1^{(2)k}\right) \qquad \text{for } i=1 \text{ and } n_s < k \le n_a$$

$$= \left(\bar{b}_2^k - \bar{h}_2^k\right) \times \left(\hat{\eta}_1^k \times \bar{\beta}_2^{(2)k}\right) + \bar{\bar{I}}_2^k \cdot \hat{\eta}_1^k$$

$$+ \left(\bar{\rho} + \bar{h}_2^k\right) \times \left(\hat{\eta}_1^k \times \bar{\beta}_2^{(2)k}\right) \qquad \text{for } i=2 \text{ and } n_s < k \le n_a$$

$$A_{rs} = \ell_j^k \hat{e}_1^k \cdot \left(\overline{\beta}_i^{(2)k} \times \hat{\zeta}_j^k\right) \qquad j=1, 2, \ldots, i-1 \text{ and } k \leq n_s$$

$$= \hat{e}_1^k \cdot \overline{\overline{I}}_j^k \cdot \hat{e}_1^k + m_j^k \left[\hat{e}_1^k \times \left(\overline{b}_j^k - \overline{h}_i^k\right)\right]$$

$$\cdot \left|\hat{e}_1^k \times \overline{C}_j^k\right| + \ell_j^k \hat{e}_1^k \cdot \left[\overline{S}_j^k - \mu_j \overline{h}_i^k\right] \times \zeta_j^k \qquad j=1,\ i+1, \ldots,\ n_k \tag{5.11}$$

where for paddle appendages, $n_s < k \leq n_a$, A_{rs} is

$$= \hat{e}_1^k \cdot \overline{\overline{I}}_1^k \cdot \hat{e}_1^k + \hat{e}_1^k \cdot \overline{\overline{I}}_2^k \cdot \hat{e}_1^k$$

$$+ m_1^k \left[\hat{e}_1^k \times \left(\overline{b}_1^k - \overline{h}_1^k\right)\right] \cdot \left[\hat{e}_1^k \times \overline{C}_1^k\right]$$

$$+ m_2^k \left[\hat{e}_1^k \times \left(\overline{b}_2^k - \overline{h}_1^k\right)\right] \cdot \left[\hat{e}_1^k \times \overline{C}_2^k\right]$$

$$+ \ell_1^k m_2^k \hat{e}_1^k \cdot \left[\overline{b}_2^k - \overline{h}_1^k\right] \times \hat{\zeta}_1^k \qquad \text{for } i=1,\ j=1$$

$$A_{rs} = \hat{e}_1^k \cdot \overline{\overline{I}}_2^k \cdot \hat{\eta}_1^k + m_2^k \left[\hat{e}_1^k \times \left(\overline{b}_2^k - \overline{h}_1^k\right)\right]$$

$$\cdot \left[\hat{\eta}_1^k \times \overline{C}_2^k\right] \qquad \text{for } i=1,\ j=2$$

$$= \hat{\eta}_1^k \cdot \overline{\overline{I}}_2^k \cdot \hat{e}_1^k + \left(\hat{\eta}_1^k \times \overline{\beta}_2^{(2)k}\right) \cdot \left(\hat{e}_1^k \times \overline{C}_2^k\right)$$

$$+ \ell_1^k \hat{\eta}_1^k \cdot \left(\overline{\beta}_2^{(2)k} \times \hat{\zeta}_1^k\right) \qquad \text{for } i=2,\ j=1$$

$$= \hat{\eta}_1^k \cdot \overline{\overline{I}}_2^k \cdot \hat{\eta}_1^k + \left(\hat{\eta}_1^k \times \overline{\beta}_2^{(2)k}\right) \cdot \left(\hat{\eta}_1^k \times \overline{C}_2^k\right) \qquad \text{for } i=2,\ j=2 \tag{5.12}$$

The elements of the vector (d) beyond the first six are the angular accelerations of the appendage segments relative to the main body, $\ddot{\alpha}_i^k = \dot{\sigma}_i^k$. In view of the fact that the criteria for lock-up and release are in terms of the relative angles of adjoining segments β_{si}^k, and β_{ri}^k, respectively, it will be found to be of use to write Equation (5.1) in terms of the relative angular accelerations of adjoining segments, $\ddot{\beta}_i^k = \dot{\tau}_i^k$.

For appendage k , the following transformation exists

$$\begin{bmatrix} \sigma_1^k \\ \sigma_2^k \\ \cdot \\ \cdot \\ \cdot \\ \sigma_{nk}^k \end{bmatrix} = [a_k] \begin{bmatrix} \tau_1^k \\ \tau_2^k \\ \cdot \\ \cdot \\ \cdot \\ \tau_{n_k}^k \end{bmatrix}$$

where $[a_k]$ is an n_k x n_k matrix defined by

$$[a_k] = \begin{bmatrix} 1 & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ 1 & 1 & 0 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & 1 & \cdot & \cdot & \cdot & 1 & 1 \end{bmatrix} \text{since for } k \le n_s, \beta_i^k = \alpha_i^k - \alpha_{i-1}^k.$$

$[a_k] = [U]$ for $n_s < k \le n_a$, since $\beta_1^k = \alpha_1^k$, and $\beta_2^k = \alpha_2^k$ for paddle appendages.

Using the transformation defined in (5.13), the column vector (d) can be defined in terms of the relative accelerations of adjoining segments.

$$
\begin{bmatrix} \dot{\bar{v}} \\ \hdashline \dot{\bar{\omega}} \\ \hdashline \dot{\sigma}^{1}_{1} \\ \dot{\sigma}^{1}_{2} \\ \cdot \\ \cdot \\ \dot{\sigma}^{1}_{n_1} \\ \hdashline \dot{\sigma}^{2}_{1} \\ \cdot \\ \cdot \\ \dot{\sigma}^{2}_{n_2} \\ \hdashline \dot{\sigma}^{3}_{1} \\ \cdot \\ \cdot \\ \hdashline \cdot \\ \cdot \\ \hdashline \dot{\sigma}^{n_a}_{1} \\ \cdot \\ \cdot \\ \dot{\sigma}^{n_a}_{n_{n_a}} \end{bmatrix}
=
\begin{bmatrix} [U] & & & & & & \\ & [U] & & & & & \\ & & [a_1] & & & & \\ & & & [a_2] & & & \\ & & & & [a_3] & & \\ & & & & & \ddots & \\ & & & & & & [a_{n_a}] \end{bmatrix}
\begin{bmatrix} \dot{\bar{v}} \\ \hdashline \dot{\bar{\omega}} \\ \hdashline \dot{\tau}^{1}_{1} \\ \cdot \\ \cdot \\ \cdot \\ \dot{\tau}^{1}_{n_1} \\ \hdashline \dot{\tau}^{2}_{1} \\ \cdot \\ \cdot \\ \dot{\tau}^{2}_{n_2} \\ \hdashline \dot{\tau}^{3}_{1} \\ \cdot \\ \cdot \\ \hdashline \cdot \\ \cdot \\ \hdashline \dot{\tau}^{n_a}_{1} \\ \cdot \\ \cdot \\ \dot{\tau}^{n_a}_{n_{n_a}} \end{bmatrix}
\qquad (5.14)
$$

or, more compactly $(d)' = [a]\,(d).$

Using the matrix [a] in (5.14), a new matrix, the B matrix is defined

$$[B] = [A]\,[a] \tag{5.15}$$

Matrix B has a block structure identical to that of A shown in Equation (5.1).

The equations of motion are now written in the form

$$[B]\,(d) = (u) \tag{5.16}$$

The solution of (5.16) provides the values of the derivatives of the variables $\overline{v}$, $\overline{\omega}$, and τ_i^k. The remaining differential equations are

$$\begin{aligned}
\dot{\overline{\rho}} &= \overline{v} \\
\dot{\beta}_i^k &= \tau_i^k \\
\dot{\chi} &= -\frac{1}{2}\overline{\omega} \cdot \overline{K} \\
\dot{\overline{K}} &= \frac{1}{2}\,[\chi\overline{\omega} - \overline{\omega} \times \overline{K}]
\end{aligned} \tag{5.17}$$

The last four equations in (5.17) involve χ, and $\overline{K}$, the Euler parameters. These are used to define the transformation relating inertial and body fixed coordinates at any time. The differential equations involving the Euler parameters are Equations (9.14), and (9.15); the derivations of these are presented in Section 9.

6. FORCES AND TORQUES

In Section 4, equations of motion were derived using rather generally defined forces and torques. It is the purpose of this section to define the internal forces and torques produced by the springs and dashpots that may act about each hinge in the system, the external forces and torques in terms of the applied thrust and the gravity field, and the generalized forces corresponding to the kick-off springs which may act between any point on the main body and any segment.

The internal forces and torques due to the springs and dashpots which act about each segment hinge are in general linear and nonlinear functions of the relative angular positions and relative angular velocities of adjoining segments. The total torque acting about the ith hinge in appendage k is

$$\bar{Q}_i^k = \hat{e}_1^k \Big[q_{i(1)}^k \left(\tau_i^k\right)^2 + q_{i(2)}^k \tau_i^k + q_{i(3)}^k \left(\beta_i^k + \gamma_i^k\right)^2 + q_{i(4)}^k \left(\beta_i^k + \gamma_i^k\right) + K_{i(1)}^k \left(\beta_i^k + \theta_i^k\right)^2 + K_{i(2)}^k \left(\beta_i^k + \theta_i^k\right) \Big] \quad (6.1)$$

The first four terms in (6.1) correspond to the dashpot torque and the last two terms correspond to the spring torque. The constants γ_1^k and θ_1^k are preload angles.

A gravity force is assumed to act on the center of mass of each body in the system. In terms of the local value of g, this is defined by

$$\bar{F}_{Gi}^k = -g\, m_i^k\, \hat{g} \quad (6.2)$$

for the ith segment of appendage k, while for the main body the gravity force is

$$\bar{F}_G = -g\, m_M\, \hat{g} \quad (6.3)$$

The acceleration of gravity, g, is an input quantity and is assumed constant throughout the deployment (any value of g may be used; the lunar value, for example).

Thrust is assumed to act on the main body only and to be represented by a rectangular time pulse, as shown in Figure 6-1.

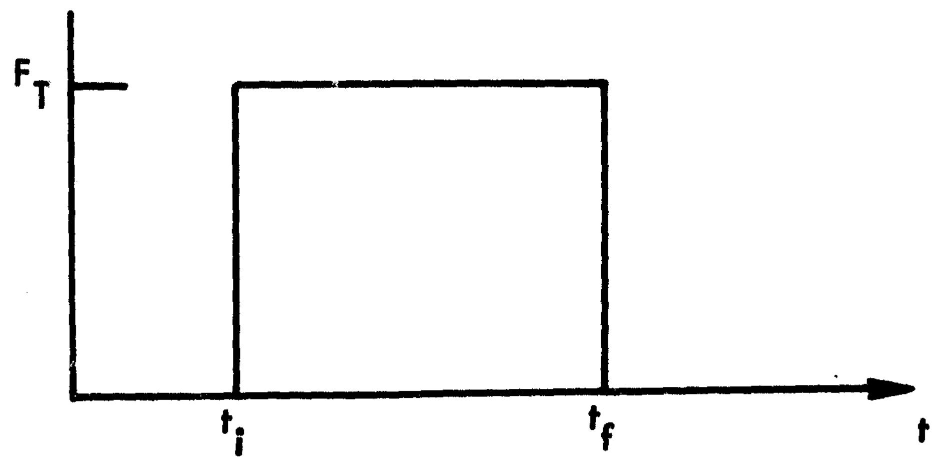


Figure 6-1. Thrust as a Function of Time

Thus, the thrust vector is

$$\bar{F}_T(t) = F_T(t)\,\hat{F}_T \qquad (6.4)$$

where

$$F_T(t) = \begin{cases} F_T & \text{for } t_i \le t \le t_f \\ 0 & \text{for } t < t_i, \text{ and } t > t_f \end{cases}$$

The torque about 0 produced by the thrust is

$$\bar{B}_T(t) = \bar{r}_T \times \bar{F}_T(t) \qquad (6.5)$$

Equations (6.2), (6.3), (6.4) and (6.5) are used to define the terms in (4.6) and (4.11). Thus corresponding to Equation (4.6) we have

$$\bar{F}_M = \bar{F}_T + \bar{F}_G \tag{6.6}$$

$$\bar{F}_i^k = \bar{F}_{G_i}^{\ k}$$

so that (4.6) becomes

$$\bar{F} = \bar{F}_T + \bar{F}_G + \sum_{i,k} \bar{F}_{G_i}^{\ k} \tag{6.7}$$

The terms in Equation (4.11) are defined as follows:

$$\bar{B}_M + \bar{b}_M \times \bar{F}_M = \bar{b}_M \times \bar{F}_G + \bar{f}_T \times \bar{F}_T \tag{6.8}$$

$$\bar{b}_i^{\ k} \times \bar{F}_i^{\ k} = \bar{b}_i^k \times \bar{F}_{G_i}^k$$

Using the relation (6.6), (6.7), and (6.8) in Equations (4.6) and (4.11) defines the external forces and torques on the system.

In Equations (4.39), (4.41), and (4.43), terms are introduced representing the effects of the kick-off springs. These terms, u_{si}^k in Equation (4.39), u_{s1}^k in Equation (4.41), and u_{s2}^k in Equation (4.43), are derived below by means of virtual work considerations. Figure 6.2 serves to introduce the geometrical parameters required in the derivation.

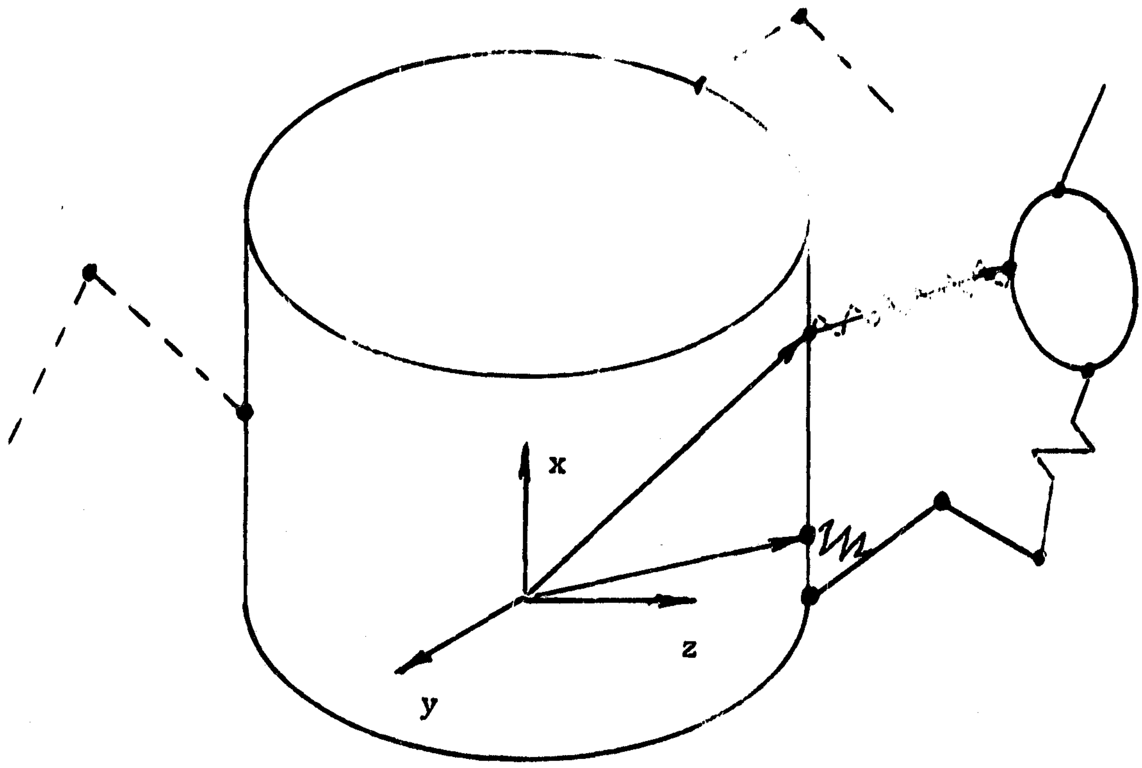


Figure 6.2 Geometry Parameters Associated with Kick-off Springs

The position of the attachment point of kick-off spring i, k relative to the main body is

$$\bar{x}_i^k = \bar{h}_i^k + \bar{s}_i^k - \bar{r}_i^k \tag{6.9}$$

The magnitude of the compressive force in the kick-off spring is expressible as a non-linear discontinuous function of its length. Thus,

$$F_{s_i}^k = a_{0i}^k + a_{1i}^k\,|\bar{x}_i^k| + a_{2i}^k\,|\bar{x}_i^k|^2 + a_{3i}^k\,|x_i^k|^3 \qquad \text{for } |\bar{x}_i^k| \le |\bar{x}_i^k|_f$$

$$F_{s_i}^k = 0 \text{ for } |\bar{x}_i^k| > |\bar{x}_i^k|_f \tag{6.10}$$

The additional force terms to be included in the equations of motion may be obtained by means of virtual work considerations. A virtual change in length of the spring results in a virtual displacement expressed in vector notation

$$\delta \bar{x}_i^k = \delta \bar{h}_i^k + \delta \bar{r}_i^k \tag{6.11}$$

The virtual displacement of the attachment point on segment k,i relative to the segment k,i hinge may be written

$$\delta \bar{s}_i^k = \delta \alpha_i^k \hat{e}_1^k \times \bar{s}_i^k + \delta \bar{s}_{m_i}^k \tag{6.12}$$

where $\delta \bar{s}_{m_i}^k$ is the component of $\delta \bar{s}_i^k$ corresponding to virtual displacements and rotations of the main body. The virtual displacement of hinge k, i may be expressed

$$\delta \bar{h}_i^k = + \sum_{j=1}^{i-1} \alpha_j^k \hat{e}_1^k \times \bar{\ell}_j^k + \delta \bar{h}_{m_i}^k \tag{6.13}$$

where $\delta \bar{h}_{m_i}^k$ is the component of $\delta \bar{h}_i^k$ corresponding to virtual displacements and rotations of the main body.

The virtual work performed by kick-off spring k,i is

$$\bar{F}_{s_i}^k \cdot \delta \bar{x}_i^k = \bar{F}_{s_i}^k \cdot (\delta \bar{h}_i^k + \delta \bar{s}_i^k - \delta \bar{r}_i^k) \tag{6.14}$$

where

$$\bar{F}_{s_i}^k = \frac{F_{s_i}^k \bar{x}_i^k}{|x_i^k|}$$

Substitution of Equations (6.12) and (6.13) into Equation (6.14) yields

$$\overline{F}^k_{s_i} \cdot \delta\overline{x}^k_i = \overline{F}^k_{s_i} \cdot \sum_{j=1}^{i-1} (\hat{e}^k_1 \times \overline{\ell}^k_j)\, \delta\alpha^k_j$$

$$+ \overline{F}^k_{s_i} \cdot (\hat{e}^k_1 \times \overline{s}^k_i)\, \delta\alpha^k_i \tag{6.15}$$

$$+ \overline{F}^k_{s_i} \cdot (\delta\overline{h}^k_{m_i} + \delta\overline{s}^k_{m_i} - \delta\overline{r}^k_i)$$

The virtual displacement appearing in the last term of Equation (6.15) represents the component of the virtual change in length of kick-off spring k,i arising from virtual displacements and virtual rotations of the main body. This virtual displacement is zero since virtual rotations and displacements of the main body do not result in a virtual change in length of the kick-off springs.

Consequently, the total virtual work performed on arbitrary virtual displacements of regular segments is

$$\delta W = \sum_{k=1}^{n_s} \sum_{i=1}^{n_k} \left\{ \overline{F}^k_{s_i} \cdot \sum_{j=1}^{i-1} (\hat{e}^k_1 \times \overline{\ell}^k_j)\, \delta\alpha^k_j \right.$$

$$\left. + \overline{F}_{s_i} \cdot (\hat{e}^k_1 \times \overline{s}^k_i)\, \delta\alpha^k_i \right\}$$

which becomes

$$\delta W = \sum_{k=1}^{n_s} \sum_{j=1}^{n_k} \left\{ \overline{F}^k_{s_j} \cdot (\hat{e}^k_1 \times \overline{s}^k_j) \right.$$

$$\left. + (\hat{e}^k_1 \times \overline{\ell}^k_j) \cdot \sum_{i=j+1}^{n_k} \overline{F}^k_{s_i} \right\} \delta\alpha^k_j \tag{6.16}$$

From Equation (6.16) it is clear that $u^k_{s_i}$ in Equation (4.39) is given by

$$u^k_{s_i} = \bar{F}^k_{s_i} \cdot (\hat{e}^k_1 \times \bar{s}^k_i) + (\hat{e}^k_1 \times \bar{\ell}^k_i) \cdot \sum_{j=i+1}^{n_k} \bar{F}^k_{s_j}$$

$$\text{for } k \leq n_s \tag{6.17}$$

If appendage k is a paddle appendage, the virtual work corresponding to the kick-off springs acting on the appendage is

$$\delta W^k = \bar{F}^k_{s_1} \cdot (\hat{e}^k_1 \times \bar{s}^k_1)\, \delta\alpha^k_1$$

$$+ \bar{F}^k_{s_2} \cdot ((\hat{e}^k_1 \times \bar{\ell}^k_1)\, \delta\alpha^k_1 + (\hat{n}^k_2 \times \bar{s}^k_2)\, \delta\alpha^k_2) \tag{6.18}$$

Rearranging Equation (6.18), we obtain

$$\delta W^k = \left\{ \bar{F}^k_{s_2} \cdot (\hat{e}^k_1 \times \bar{\ell}^k_1) + \bar{F}^k_{s_1} \cdot (\hat{e}^k_1 \times \bar{s}^k_1) \right\} \delta\alpha^k_1$$

$$+ \bar{F}^k_{s_2} \cdot (\hat{n}^k_2 \times \bar{s}^k_2)\, \delta\alpha^k_2 \qquad n_s \quad k \quad n_a \tag{6.19}$$

From Equation (6.19), it is clear that the generalized force components, $u^k_{s_1}$, and $u^k_{s_2}$, are given as follows

$$u^k_{s_1} \quad \cdot (\hat{e}^k_1 \times \bar{s}^k_1) + \bar{F}^k_{s_2} \cdot (\hat{e}^k_1 \times \bar{\ell}^k_1)$$

$$u^k_{s_2} = \bar{F}^k_{s_2} \cdot (\hat{n}^k_2 \times \bar{s}^k_2) \tag{6.20}$$

$$\text{for } n_s \leq k \leq n_a$$

Equations (6.17) and (6.20) provide definitions of $u_{s_i}^k$, $u_{s_1}^k$, and $u_{s_2}^k$. These are the additional terms required in Equations (4.39), (4.41), and (4.43), respectively, and represent the effects of the kick-off springs.

7. RELEASE AND LOCK-UP OF HINGES

An important aspect of the N-Boom appendage deployment model is that in general the hinge associated with an appendage segment is in any one of three possible states: (1) not yet released, (2) free, allowing relative motion between the segments it interconnects, and (3) locked. Thus, for the system as a whole at a particular time, some segments are unreleased, some are moving relative to adjoining segments, while others may be locked. The formulation admits this general case.

The equations of motion as given in Section 5 provide for the case in which all segments of the system are in state (2). The method by which release and lock-up of hinges are accounted for in the equations of motion is most clearly developed in terms of modifications to the equations of motion as given in Equation (5.16) and (5.17).

There are two admissable criteria for releasing segment hinges: (1) hinge k,i may be released at a specified time, $t = t_i^k$, or (2) hinge k,i may be released when another segment, segment ℓ,m, has attained a prescribed position relative to the segment inboard of it, that is, hinge k,i is released when β_m^ℓ reaches a prescribed value. In the case of the second option, it is to be noted that release may be represented as dependent on segments in other appendages.

If the ith segment of appendage k is not yet released, the matrix [B] is modified by setting all the elements in the row r equal to zero except for the element in column r, which is set equal to unity. In addition, u_r and τ_i^k in (5.17) are set equal to zero. Thus, the differential equations corresponding to this segment result in zero relative velocity and acceleration of the segment so that the relative angle is constant.

The criteria for locking a particular hinge, hinge i, is that the relative angular displacement, β_i^k, of segment i has attained a prescribed value, $\beta_{s_i}^k$. After lock-up, the angular velocity $\dot{\beta}_i^k = \tau_i^k = 0$, while in general it is not zero preceding lock-up. This requires providing for an impulsive internal torque about hinge i, which reduces $\dot{\beta}_i^k$ to zero and introduces discontinuities into all of the angular velocities of the system. However, displacements and the velocity of the system center of mass in inertial space remain unchanged.

As in the case of release, lock-up is accounted for by manipulating Equation (5.16). Assume that Equation (5.16) has been integrated over a short period of time Δt, including the lock-up of the rth hinge, (r is defined in (5.2)), Equation (5.16) becomes

$$[B]\,(\Delta d) = (I) \qquad (7.1)$$

where all the elements of (I) are zero except the rth element, which is the unknown impulsive locking torque on hinge i.

The elements of the vector (Δd) are the changes in the velocities. Only one element of this vector is known, the rth element. In this case, we have

$$\Delta d_r = -\tau_i^k \qquad (7.2)$$

where τ_i^k is the relative velocity of segment i immediately preceding lock-up, that is at time $t = \hat{t}^-$.

Thus, the system of p equations, Equation (7.1), may be solved for the (p-1) unknown velocity changes and the one impulsive locking torque.

In order to solve for the unknown discontinuities in the velocities and implusive locking torque, Equation (7.1) is first partitioned as follows

$$\left[[B'] \,\vdots\, (B_r) \,\vdots\, [B''] \right] \begin{Bmatrix} (\Delta d)' \\ \hline -\tau_i^k \\ \hline (\Delta d)'' \end{Bmatrix} = \begin{Bmatrix} (0) \\ \hline \hat{I}_r \\ \hline (0) \end{Bmatrix} \qquad (7.3)$$

where

$[B']$ = a p × (r-1) matrix

$\{B_r\}$ = the rth column of $[B]$

$[B'']$ = a p × (p-r) matrix

Multiplying, we obtain

$$[B'] \left(\Delta d\right)' - \tau_i^k \left(B_r\right) + [B''] \left(\Delta d\right)'' = \begin{Bmatrix} (0) \\ --- \\ \hat{I}_r \\ --- \\ (0) \end{Bmatrix} \qquad (7.4)$$

Rearranging (7.4), we obtain

$$[B'] \left(\Delta d\right)' - \begin{Bmatrix} (0) \\ --- \\ \hat{I}_r \\ --- \\ (0) \end{Bmatrix} + [B''] \left(\Delta d\right)'' = \tau_i^k \left(B_r\right) \qquad (7.5)$$

Equation (7.5) is rewritten in the form

$$\left[[B'] \,\vdots\, -\left(e_r\right) \,\vdots\, [B''] \right] \begin{Bmatrix} (\Delta d)' \\ --- \\ \hat{I}_r \\ ---- \\ (\Delta d)'' \end{Bmatrix} = \tau_i^k \left(B_r\right) \qquad (7.6)$$

where $\left(e_r\right)$ is the rth column of the p × p identity matrix.

The result yields the changes in the derivatives for the unlocked segments as well as the impulsive torque applied by the locking mechanism. Hence, typically, for the unlocked quantities

$$\left\{d\left(\hat{t}^+\right)\right\} = \left\{d\left(\hat{t}^-\right)\right\} + \left\{\Delta d\right\}$$

We thus have the values of all the variables and their derivatives after lock-up with which we may continue the solution.

8. SYSTEM KINETIC ENERGY AND MOMENTUM

In addition to the details of the motion of each body, an indication of the system motion is provided through computation of the total kinetic energy, T, linear momentum, $\bar{P}$, and moment of momentum about the system center of mass, $\bar{H}_c$. As is well known, in the absence of external forces and torques, both $\bar{P}$ and $\bar{H}_c$ are conserved. If, in addition, there are no springs or dashpots, T is also conserved. When conserved, these quantities serve as a check on the accuracy of the computed results. The nutation angle, θ, i.e., the angle between $\bar{H}_c$ and $\hat{x}$, is provided since it is likewise of interest when $\bar{H}_c$ is conserved. In this section we derive the expressions for T, $\bar{H}_c$, and $\bar{P}$ incorporated in the program.

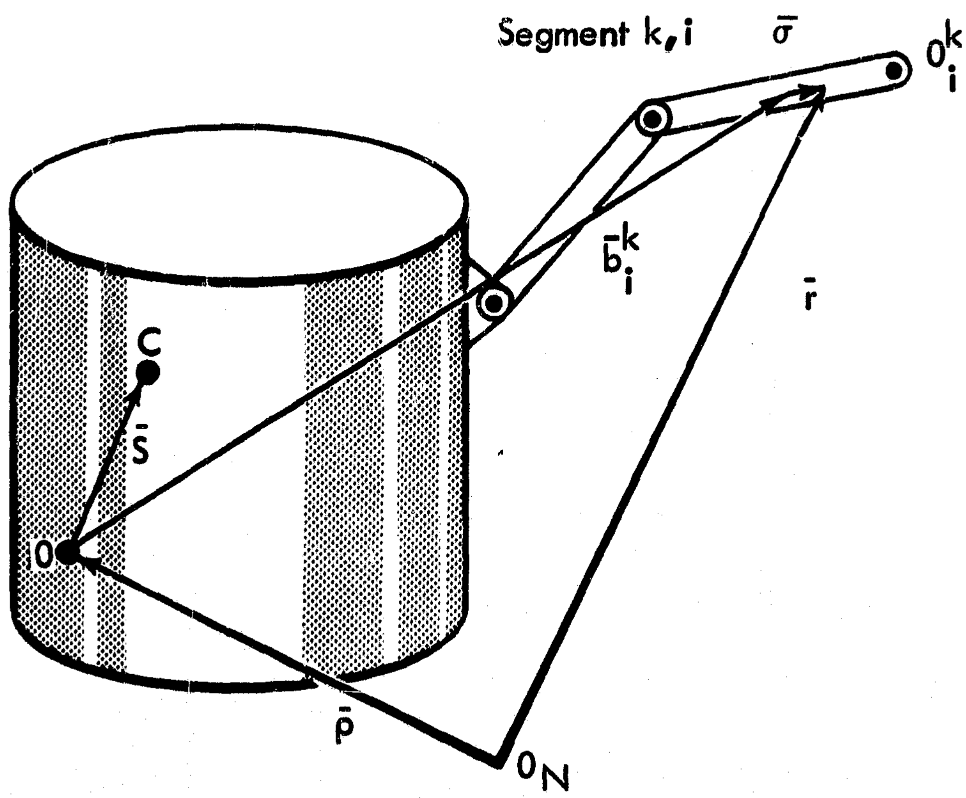


Figure 8-1. Fundamental Position Vectors

In Figure 8-1, 0_N denotes the origin of the uniformly translating inertial frame. An arbitrary point R on the ith segment of appendage k is specified by its position vector relative to 0_N, namely,

$$\bar{r} = \bar{\rho} + \bar{b}_i^k + \bar{\sigma} \tag{8.1}$$

where $\bar{\sigma}$ is a vector from the center of mass of segment k, i.

$$\bar{p}_i^k = \int \frac{D\bar{r}}{Dt} \, dm \; . \tag{8.2}$$

Since

$$\frac{D\bar{r}}{Dt} = \frac{D\bar{\rho}}{Dt} + \frac{D\bar{b}_i^k}{Dt} + \frac{D\bar{\sigma}}{Dt} \tag{8.3}$$

and

$$\frac{D\bar{\sigma}}{Dt} = \bar{\omega}_i^k \times \bar{\sigma} \quad , \tag{8.4}$$

setting

$$\bar{Z}_i^k = \frac{D\bar{\rho}}{Dt} + \frac{D\bar{b}_i^k}{Dt} \; , \tag{8.5}$$

we have

$$\bar{P}_i^k = \int \left(\bar{Z}_i^k + \bar{\omega}_i^k \times \bar{\sigma} \right) dm = m_i^k \, \bar{Z}_i^k \; . \tag{8.6}$$

Similarly, for the main body, $\bar{P}_M = m_M \bar{Z}_M$, so that the system linear momentum

$$\bar{P} = m_M \bar{Z}_m + \sum_{i,k} m_i^k \, \bar{Z}_i^k \; . \tag{8.7}$$

Note that we have

$$\overline{P} = m_T \frac{D}{Dt} (\overline{\rho} + \overline{S}) .$$

The kinetic energy of segment k, i is defined by

$$T_i^k = \frac{1}{2} \int \frac{D\overline{r}}{Dt} \cdot \frac{D\overline{r}}{Dt} \, dm \ . \tag{8.8}$$

We have

$$T_i^k = \frac{1}{2} \int \left[\overline{Z}_i^k \cdot \overline{Z}_i^k + 2\overline{Z}_i^k \cdot (\overline{\omega} \times \overline{\sigma}) + \left(\overline{\omega}_i^k \times \overline{\sigma}\right) \cdot \left(\overline{\omega}_i^k \times \overline{\sigma}\right) \right] dm$$

$$T_i^k = \frac{1}{2} m_i^k \, \overline{Z}_i^k \cdot \overline{Z}_i^k + \frac{1}{2} \overline{\omega}_i^k \cdot \overline{\overline{I}}_i^k \cdot \overline{\omega}_i^k \ . \tag{8.9}$$

Similarly, for the main body,

$$T_M = \frac{1}{2} m_M \overline{Z}_M \cdot Z_M + \frac{1}{2} \overline{\omega} \cdot \overline{\overline{I}}_{C_M} \cdot \overline{\omega} \ , \tag{8.10}$$

so that

$$T = T_M + \sum_{i,k} T_i^k \ . \tag{8.11}$$

The moment of momentum about the system center of mass and about the reference point, $\overline{H}_c$ and $\overline{H}_o$, respectively, are related by the equation

$$\overline{H}_C = \overline{H}_o - m_T \overline{S} \times \frac{D\overline{S}}{Dt} \tag{8.12}$$

where

$$\overline{H}_o = \overline{H}_{o_M} + \sum_{i,k} \overline{H}_{o_i}^{\ k} \tag{8.13}$$

$$\bar{H}^k_{o_i} = \int_{B^k_i} \left(\bar{b}^k_i - \bar{\sigma}\right) \times \frac{D}{Dt}\left(\bar{b}^k_i + \bar{\sigma}\right) \, dm$$

$$= m^k_i \bar{b}^k_i \times \frac{D\bar{b}^k_i}{Dt} + \bar{\bar{I}}^k_i \cdot \bar{\omega}^k_i$$

and

$$\bar{H}_{o_M} = m_M \bar{b}_m \times \frac{D\bar{b}_m}{Dt} + \bar{\bar{I}}_M \cdot \bar{\omega}$$

$$= m_M \bar{b}_M \times (\bar{\omega} \times \bar{b}_M) + \bar{\bar{I}}_M \cdot \bar{\omega} .$$

It remains to specify explicit expressions for $\bar{\omega}^k_i$, $\frac{Db^k_i}{Dt}$, $\bar{Z}^k_i$ and $\frac{D\bar{S}}{Dt}$.

We have

$$\bar{\omega}^k_i = \bar{\omega} + \bar{\Omega}^k_i \tag{8.14}$$

where $\bar{\Omega}^k_i$ is the angular velocity of segment k, i relative to the main body, namely,

$$\bar{\Omega}^k_i = \begin{cases} \sigma^k_i \, \hat{e}^k_1 & \text{for } k \le n_s \\ \sigma^k_1 \, \hat{e}^k_1 + \sigma^k_2 \, \hat{\eta}^k_1 & \text{for } n_s < k \le n_a \text{ and } i = 2 \end{cases} \tag{8.15}$$

From

$$\bar{b}^k_i = \bar{a}^k + \bar{C}^k_i + \sum_{j=1}^{i-1} \ell^k_j \, \hat{\eta}^k_j \tag{8.16}$$

we find that

$$\left(\frac{d\bar{b}_i^k}{dt}\right)_M = \bar{\Omega}_i^k \times \bar{C}_i^k + \sum_{j=1}^{i-1} \ell_j^k \hat{\zeta}_j^k \sigma_j^k \quad (8.17)$$

and

$$\frac{D\bar{b}_i^k}{Dt} = \left(\frac{d\bar{b}_i^k}{dt}\right)_M + \bar{\omega} \times \bar{b}_i^k . \quad (8.18)$$

Thus

$$\bar{Z}_i^k = \frac{D\bar{\rho}}{Dt} + \frac{D\bar{b}_i^k}{Dt} = \bar{v} + \bar{\omega}\left(\bar{\rho} + \bar{b}_i^k\right) + \left(\frac{d\bar{b}_i^k}{dt}\right)_M \quad (8.19)$$

and, finally, since

$$m_T\bar{S} = m_M\bar{b}_m + \sum_{k,i} m_i^k \bar{b}_i^k$$

we have

$$m_T \frac{D\bar{S}}{Dt} = m_T\bar{\omega} \times \bar{S} + \sum_{i,k} m_i^k \left(\frac{d\bar{b}_i^k}{dt}\right)_M \quad (8.20)$$

9. COORDINATES

There are four types of coordinate systems associated with the system: an inertial coordinate system uniformly translating with the initial velocity of the system, a main body fixed system with its origin at 0, n_a main body fixed systems fixed to the main body at the point of attachment of each appendage, and n coordinate systems associated with the n segments which compose the appendages.

The equations of motion as given by Equation (5.1) are expressed in one coordinate system: the main body fixed system with its origin at 0. Therefore, quantities referred to coordinate systems other than the main body system must be transformed into main body coordinates.

Figure 9-1 illustrates the coordinate systems associated with appendage k, where in this case it is not a paddle appendage.

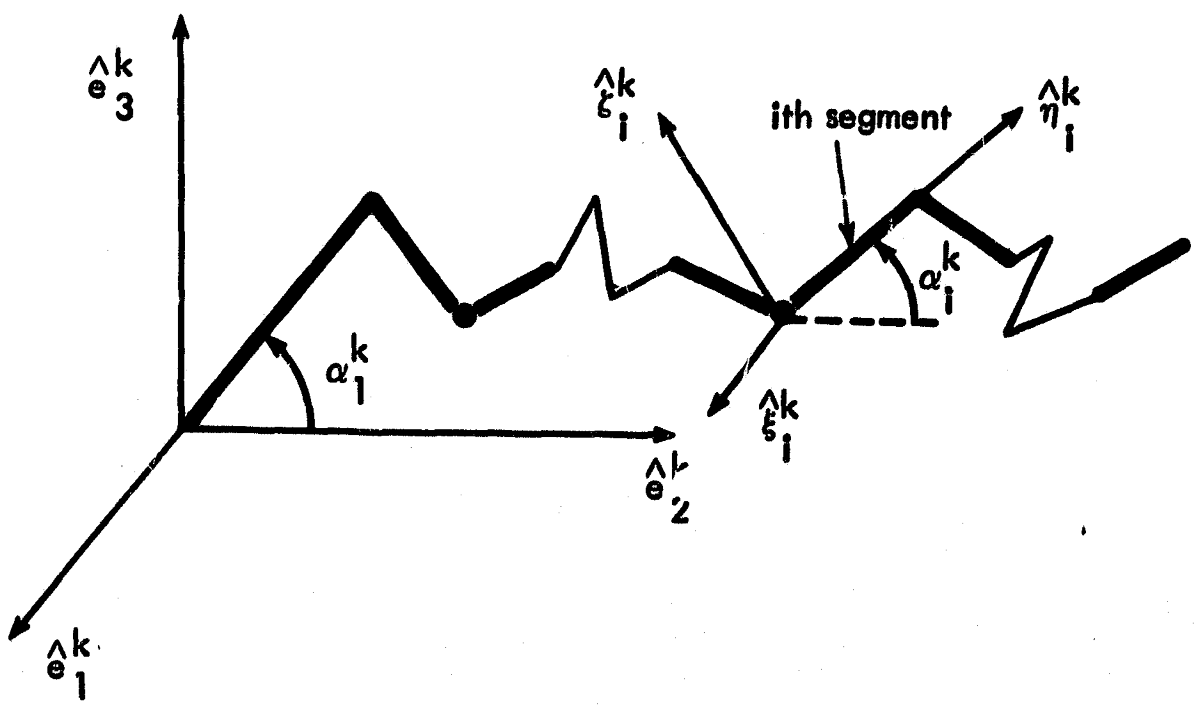


Figure 9-1. Coordinates Associated with Appendage k

The mass properties associated with the segment k, i are input in the $\hat{\xi}_i^k$, $\hat{\eta}_i^k$, $\hat{\zeta}_i^k$ coordinate system, whereas in the course of computation the program refers these to main body coordinates. The transformation from segment to appendage coordinates is

$$\hat{\xi}_i^k = \hat{e}_1^k$$

$$\hat{\eta}_i^k = \cos \alpha_i^k \hat{e}_2^k + \sin \alpha_i^k \hat{e}_3^k$$

$$\hat{\zeta}_i^k = -\sin \alpha_i^k \hat{e}_2^k + \cos \alpha_i^k \hat{e}_3^k \qquad (9.1)$$

Paddle appendages are two segment appendages in which the second rotates about the first, that is, $\bar{\eta}_2^k = \hat{\eta}_1^k$ in this case. In these appendages α_2^k is defined differently (with this change in definition, changes in the equations of motion from the case of ordinary appendages to paddle appendages are minimized) as shown in Figure 9-2.

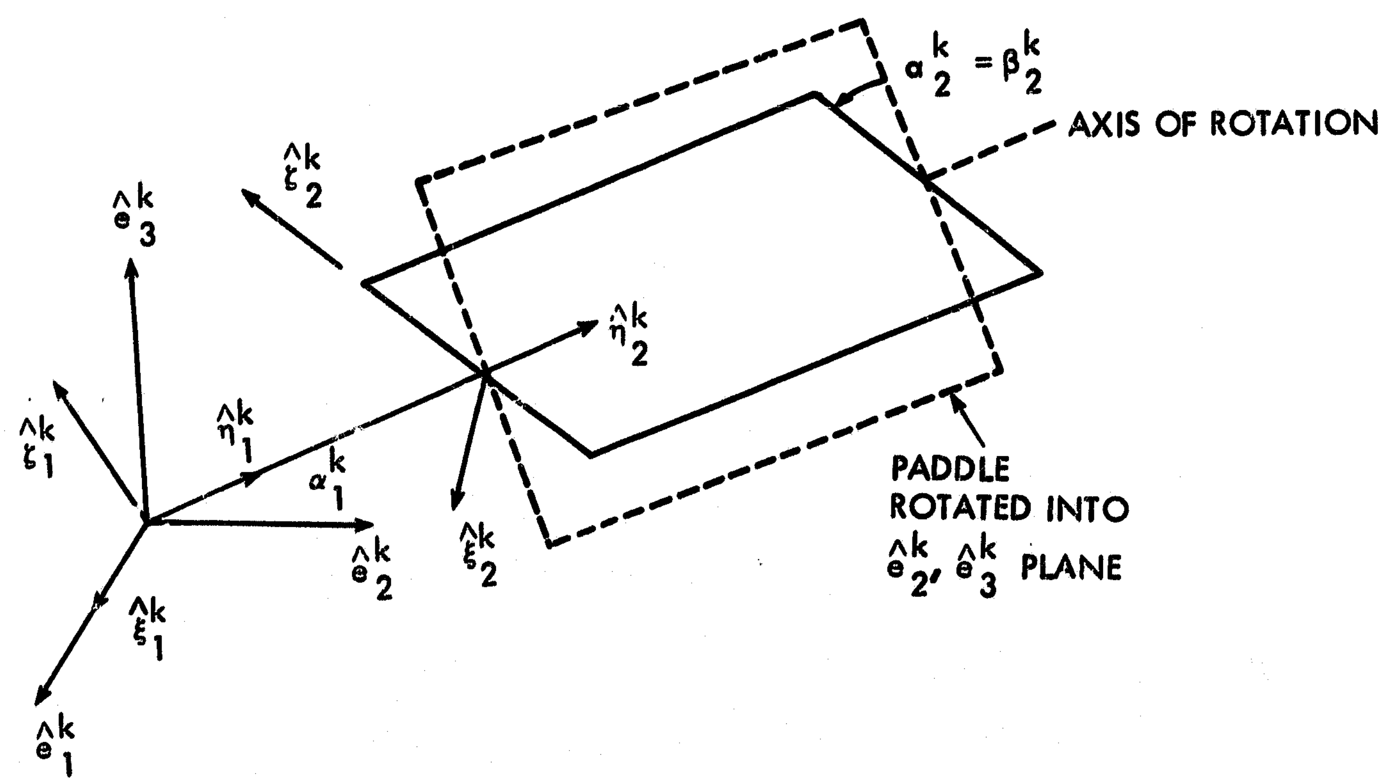


Figure 9-2. Coordinates Associated with Appendage k if the Second Element is a Paddle

As noted in Equation (5.13), $\alpha_2^k = \beta_2^k$ in the case of a paddle appendage.

The transformation to segment 1 coordinates from the paddle segment coordinates is

$$\hat{\xi}_2^k = \cos \alpha_2^k \hat{e}_1^k - \sin \alpha_2^k \hat{\zeta}_1^k$$

$$\hat{\eta}_2^k = \hat{\eta}_1^k$$

$$\hat{\zeta}_2^k = \sin \alpha_2^k \hat{e}_1^k + \cos \alpha_2^k \hat{\zeta}_1^k$$

where

$$n_s < k \leq n_a \tag{9.2}$$

The transformation to appendage coordinates from segment 1 coordinates is obtained from (9.1).

By use of Equations (9.1) and (9.2), all appendage properties at any time, mass properties, angular rates, forces, can be referred to the corresponding appendage coordinate system. These quantities must now be transformed into a common main body fixed coordinate system. Since the appendage coordinate system, $\hat{e}_1^k$, $\hat{e}_2^k$, $\hat{e}_3^k$ is also fixed to the main body, the transformations from appendage coordinates to main body coordinates are constant through time.

The transformations are achieved by use of a set of Euler angles corresponding to each appendage, ψ^k, θ^k, and Φ^k. Beginning in main body coordinates, the first rotation is through ψ^k about $\hat{x}$ in the positive direction as shown in Figure 9-3.

The transformation from $\hat{x}$, $\hat{y}$, $\hat{z}$ to $\hat{x}'$, $\hat{y}'$, $\hat{z}'$ is

$$\hat{x}' = \hat{x}$$

$$\hat{y}' = \cos \psi^k \hat{y} - \sin \psi^k \hat{z}$$

$$\hat{z}' = \sin \psi^k \hat{y} + \cos \psi^k \hat{z} \tag{9.3}$$

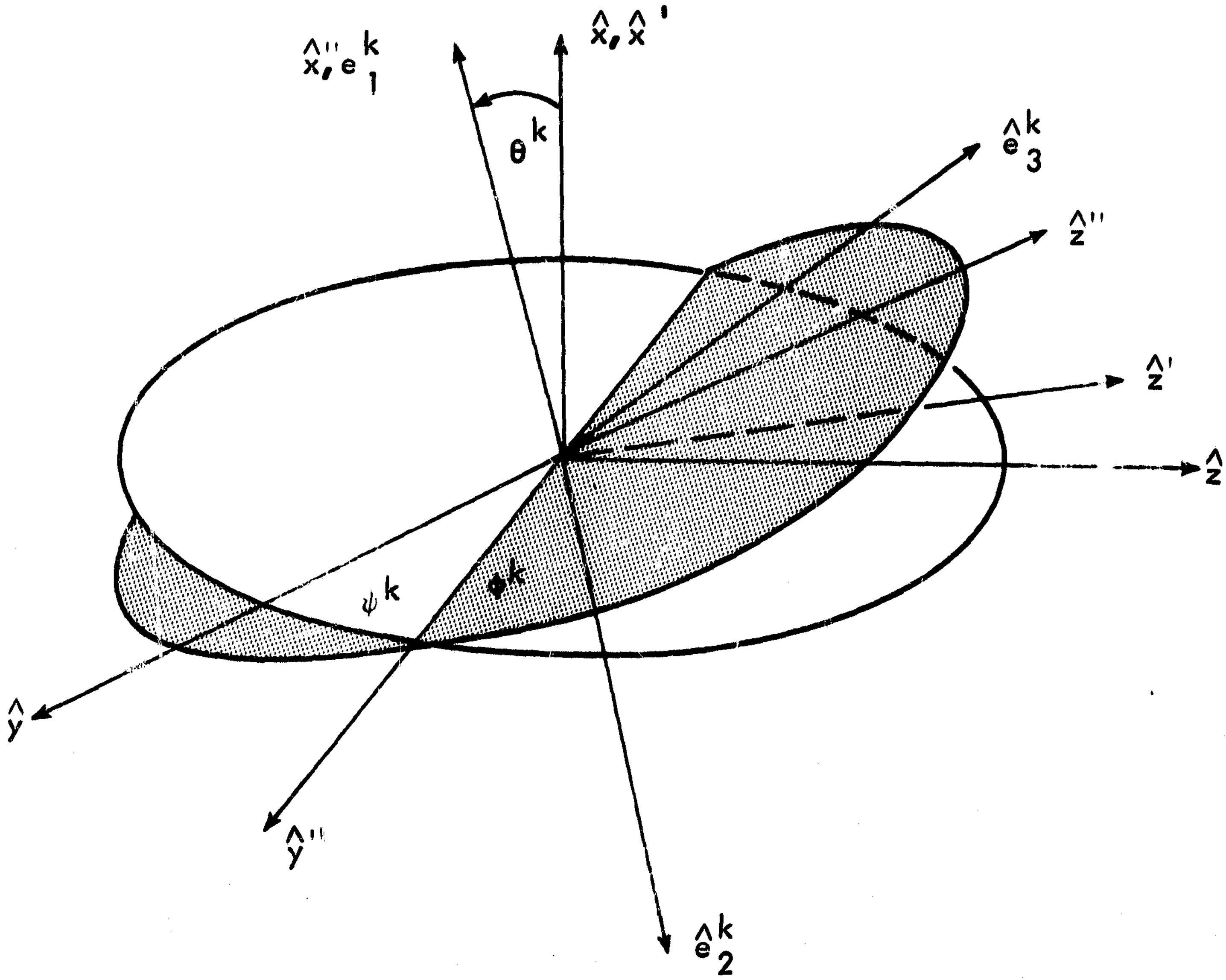


Figure 9-3. Transformation from $\hat{x}$, $\hat{y}$, $\hat{z}$ to $\hat{e}_1^k$, $\hat{e}_2^k$, $\hat{e}_3^k$ Coordinates

The second rotation is through θ^k about $\hat{y}'$ in the positive sense. The transformation from $\hat{x}'$, $\hat{y}'$, $\hat{z}'$ to $\hat{x}''$, $\hat{y}''$, $\hat{z}''$ is

$$\hat{x}'' = \cos\theta^k \hat{x}' + \sin\theta^k \hat{z}'$$

$$\hat{y}'' = \hat{y}'$$

$$\hat{z}'' = -\sin\theta^k \hat{x}' + \cos\theta^k \hat{z}' \qquad (9.4)$$

The final rotation, also shown in Figure 9.3, rotates $\hat{x}''$, $\hat{y}''$, $\hat{z}''$ into the $\hat{e}_1^k$, $\hat{e}_2^k$, $\hat{e}_3^k$ triad. This transformation, rotation through φ^k about $\hat{x}''$ is

$$\begin{aligned} \hat{e}_1^k &= \hat{x}'' \\ \hat{e}_2^k &= \cos\varphi^k \hat{y}'' + \sin\varphi^k \hat{z}'' \\ \hat{e}_3^k &= -\sin\varphi^k \hat{y}'' + \cos\varphi^k \hat{z}'' \end{aligned} \tag{9.5}$$

Combining (9.3), (9.4) and (9.5), the $\hat{e}_1^k$, $\hat{e}_2^k$, $\hat{e}_3^k$ triad is expressed in main body coordinates as follows:

$$\hat{e}_1^k = \begin{pmatrix} \cos\theta^k \\ \sin\psi^k \sin\theta^k \\ -\cos\psi^k \sin\theta^k \end{pmatrix}$$

$$\hat{e}_2^k = \begin{pmatrix} \sin\theta^k \sin\varphi^k \\ \cos\psi^k \cos\varphi^k - \sin\psi^k \cos\theta^k \sin\varphi^k \\ \sin\psi^k \cos\varphi^k + \cos\psi^k \cos\theta^k \sin\varphi^k \end{pmatrix}$$

$$\hat{e}_3^k = \begin{pmatrix} \sin\theta^k \cos\varphi^k \\ -\cos\psi^k \sin\varphi^k - \sin\psi^k \cos\theta^k \cos\varphi^k \\ -\sin\psi^k \sin\varphi^k + \cos\psi^k \cos\theta^k \cos\varphi^k \end{pmatrix}$$

That is, the transformation matrix is

$$\left[T^k\right] = \left[\hat{e}_1^k, \hat{e}_2^k, \hat{e}_3^k\right] \tag{9.6}$$

By use of (9.6), appendage properties can be expressed in main body fixed coordinates, x y z at any time.

The remaining coordinate transformation to be defined is the transformation from main body unit vectors $\hat{x}$, $\hat{y}$, $\hat{z}$ to the inertial unit vectors $\hat{X}$, $\hat{Y}$, $\hat{Z}$. This transformation at time t = 0 is of the same form as (9.6), ψ_M, θ_M, and ϕ_M replacing ψ^k, θ^k, and ϕ^k, respectively. The transformation is

$$\hat{x} = \begin{pmatrix} \cos\theta_M \\ \sin\psi_M \sin\theta_M \\ -\cos\psi_M \sin\theta_M \end{pmatrix}$$

$$\hat{y} = \begin{pmatrix} \sin\theta_M \sin\varphi_M \\ \cos\psi_M \cos\varphi_M - \sin\psi_M \cos\theta_M \sin\varphi_M \\ \sin\psi_M \cos\varphi_M + \cos\psi_M \cos\theta_M \sin\varphi_M \end{pmatrix}$$

$$\hat{z} = \begin{pmatrix} \sin\theta_M \cos\varphi_M \\ -\cos\psi_M \sin\varphi_M - \sin\psi_M \cos\theta_M \cos\varphi_M \\ -\sin\psi_M \sin\varphi_M + \cos\psi_M \cos\theta_M \cos\varphi_M \end{pmatrix} \quad (9.7)$$

Thus, the matrix M_o is defined by

$$\left[M_o\right] = \left[\hat{x}, \hat{y}, \hat{z}\right] \quad (9.8)$$

that is, the above vectors form the columns of M_o.

The matrix M_o defined in (9.8) is the initial value of the transformation matrix M, which at time t = 0 defines the orientation of the main body in inertial space. The columns of M are the body-fixed unit vectors $\hat{x}$, $\hat{y}$, $\hat{z}$ resolved in the inertial frame. The problem now to be considered is to determine the means of finding M at any time t.

In the course of the motion the orientation varies in accordance with the differential equation

$$[\dot{M}] = [M]\,[J(\overline{\omega})] \qquad (9.9)$$

where ω_1, ω_2, ω_3 are the components of $\overline{\omega}$ resolved in the body-fixed frame, i.e., $\overline{\omega} = \omega_1\hat{x} + \omega_2\hat{y} + \omega_3\hat{z}$.

The initial orientation is obtained by use of $[M(0)] = [M_o]$, which is specified by initial values of the Euler angles ψ_M, θ_M, ϕ_M as indicated in Equations (9.7) and (9.8). The matrix $[M]$ can be written in terms of $[M_o]$, a constant, and a matrix $[C]$, a function of time. We write

$$[M(t)] = [M_o]\,[C(t)] \qquad (9.10)$$

and note that C(t) also satisfies the same differential equation as $[M]$, i.e., $[\dot{C}] = [C]\,[J(\overline{\omega})]$.

Instead of solving the above matrix differential equation for $[C]$, we represent $[C]$ in terms of the four Euler parameters consisting of a scalar χ and a vector $\overline{K}$ and solve only four scalar differential equations.

To introduce the Euler parameters, we note that any orientation of a body may be achieved by a counterclockwise rotation about an appropriate axis $\hat{a}$ through an angle Θ.

Accordingly, C has the representation

$$[C] = [U] + \sin\Theta\,[J(\hat{a})] + (1 - \cos\Theta)\,[J(\hat{a})]^2 \qquad (9.11)$$

The Euler parameters are defined in terms of $\hat{a}$ and Θ by

$$\chi = \cos\frac{\Theta}{2}$$

$$\overline{K} = \sin\frac{\Theta}{2}\,\hat{a} \qquad (9.12)$$

Then

$$C = [U] + 2\chi[J(\overline{K})] + 2[J(\overline{K})]^2 \tag{9.13}$$

The above differential equation for $[C]$ leads to the corresponding differential equations for χ and $\overline{K}$, namely

$$\frac{d\chi}{dt} = -\frac{1}{2}\,\overline{\omega}\cdot\overline{K}$$

$$\frac{d\overline{K}}{dt} = \frac{1}{2}\chi\,\overline{\omega} - \overline{\omega}\times\overline{K} \,. \tag{9.15}$$

We note that by definition $\chi^2 + \overline{K}\cdot\overline{K}$ is equal to one and indeed this function is an integral of (9.14) and (9.15). This fact may be used to provide a check on the computation.

As seen from (9.10), the initial value of $[C]$ is the identity matrix. Accordingly, we take $\chi(0) = 1$ and $\overline{K}(0) = 0$ so that (9.13) yields $[C(0)] = [U]$.

At any time t, (9.14) and (9.15) are solved along with the equations of motion (5.16) and (5.17). The solutions of (9.14) and (9.15) are used in (9.13) to obtain $[C(t)]$, which when substituted into (9.10) yields the required transformation matrix, $[M]$.

10. CROSSECTION LOADS

This section provides a discussion of the method used to obtain the loads acting on each section on which stresses are desired. The motion of segment k,i at any time is given by the existing N-Boom program. The motion is specified by three quantities which must be referred to segment k,i coordinates; segment k,i angular velocity, $\bar{\omega}_i^k$, angular acceleration, $\dot{\bar{\omega}}_i^k$, and the acceleration of its c.g., $\frac{D^2}{Dt^2}(\bar{\rho} + \bar{b}_i^k)$.

These quantities in conjunction with the inertia properties of subsections of segment k,i are sufficient to establish the loads at each station.

The derivation proceeds from the assumption that all vector quantities are referred to segment k,i coordinates. The superscript designating the appendage number, and the subscript designating the segment number are dropped for notational convenience and to minimize confusion. Thus, while the subscripting in the analysis to follow will refer to stations and other quantities associated with a particular segment, it is implied that the process is carried out for every segment k,i.

Figure 10.1 shows quantities to be associated with segment k,i (subscript and superscript dropped).

Equation (4.1) of Reference 1 relates the motion of segment k,i to the forces acting at the hinges as follows:

$$M_i^k \frac{D^2}{Dt^2}(\bar{\rho} + \bar{b}_i^k) = \bar{P}_i^k - \bar{P}_{i+1}^k \tag{10.1}$$

Dropping the k,i notation for convenience, we have

$$\bar{P}_{e_2} = m\bar{a} - \bar{P}_{e1} \tag{10.2}$$

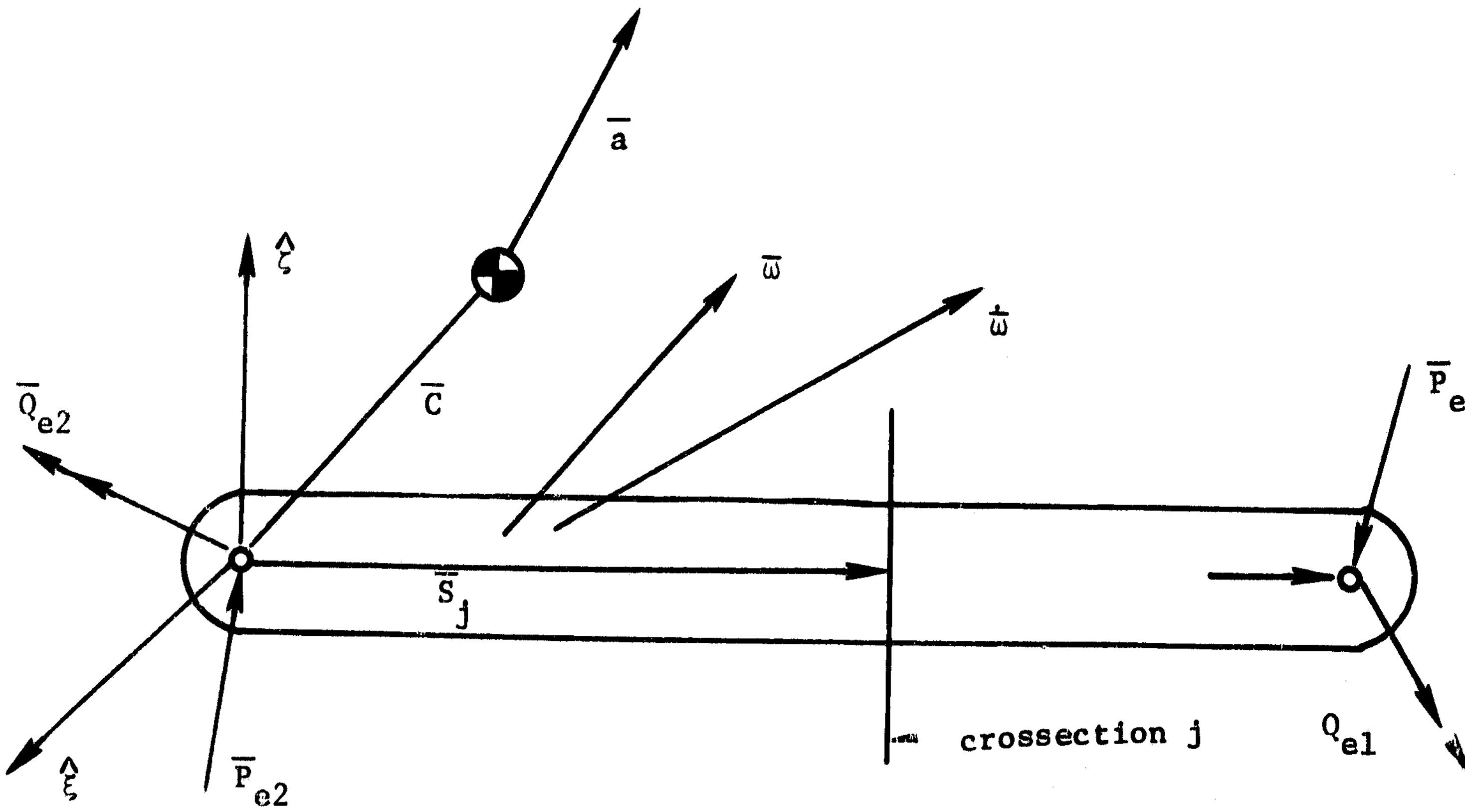


Figure 10.1 Quantities Required in the Loads Calculation Associated with Segment k,i.

where we have defined

$$\bar{P}_{e2} = \bar{P}_i^k$$

$$\bar{P}_{e1} = - \bar{P}_{i+1}^k = - (\bar{P}_{e2})_{i+1}^k$$

$$(P_{e1})_{n_k}^k = 0$$

$$m = M_i^k$$

$$\bar{a} = \frac{D^2}{Dt^2} (\bar{\rho} + \bar{b}_i^k)$$

It is clear that by means of Equation (3.2) all the forces acting on hinge points of the segments of an appendage can be found.

As shown in Figure 3.2, a station defines a crossection through a segment normal to the η-axis and the position of the j^{th} station in segment k,i is $\bar{S}_j$. Thus,

$$\bar{S}_j = S_j \hat{\eta} \tag{10.3}$$

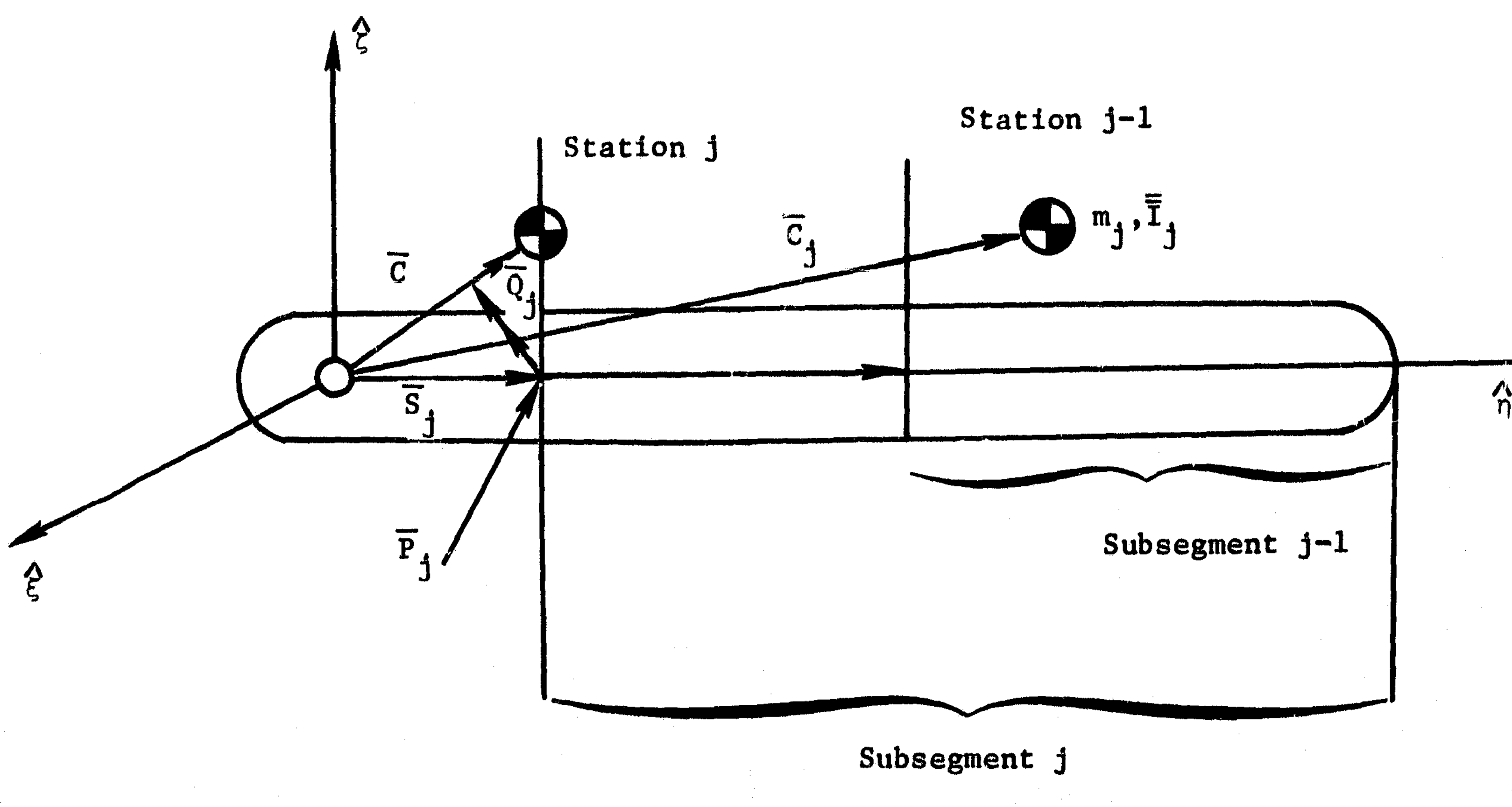


Figure 10.2 Subsegment Forces and Properties

Station j-1 is outboard of station j in segment k,i.

Mass and inertial properties $(m_j, \bar{\bar{I}}_j)$ are associated with the subsegments defined by each station S_j and the center of mass of the subsegment of segment k,i outboard of station j is given by $\bar{C}_j$.

The acceleration in inertial space of the c.g. of subsegment j is $\bar{a}_j$, and is given by

$$\bar{a}_j = \bar{a} + \bar{\omega} \times (\bar{\omega} \times (\bar{C}_j - \bar{C})) + \dot{\bar{\omega}} \times (\bar{C}_j - \bar{C}) \qquad (10.4)$$

Thus the force on the crossection at station j is

$$\bar{P}_j = m_j \, \bar{a}_j - \bar{P}_{e1} \qquad (10.5)$$

where $\bar{a}_j$ is defined by Equation (10.4).

The moment on crossection j in segment k,i is obtained from

$$\bar{\bar{J}}(\bar{\omega}) \; \bar{\bar{I}}_j \cdot \bar{\omega} + \bar{\bar{I}}_j \cdot \dot{\bar{\omega}}$$

$$= \bar{Q}_j + \bar{Q}_{e1} + (\bar{l} - \bar{C}_j) \times \bar{P}_{e1} + (\bar{S}_j - \bar{C}_j) \times \bar{P}_j$$

where $\bar{Q}_{e1}$, the total torque acting on the outboard end of segment k,i, is the negative of $\bar{Q}_{e2}$ for segment k, i+1, and $Q_{e1} = 0$ for $i = n_k$. Thus, for segment k,i

$$Q_{e2} = Q_N$$

where N designates the last station at which Q_j is computed on the segment, and for segment k,i - 1.

$$\left(Q_{e1}\right)^k_{i-1} = - \left(Q_{e2}\right)^k_i$$

That is

$$\overline{Q}_j = \overline{\overline{J}}(\overline{\omega})\ I_j \cdot \overline{\omega} + \overline{\overline{I}}_j \cdot \dot{\overline{\omega}}$$

$$-\overline{Q}_{e1} + (C_j - \overline{\ell}) \times \overline{P}_{e1} + (\overline{C}_j - \overline{S}_j) \times \overline{P}_j \qquad (10.6)$$

Equations (10.5) and (10.6) define the forces on crossection j.

Stresses will also be required whenever a hinge has locked. If, for example, hinge m, n locks the angular velocity and the velocity of the center of mass of each segment k,i will change instantaneously. The change in the motion of each segment, which is provided by the original N-Boom Program, may be used to calculate the impulsive forces and torques acting at each hinge k,i, as hinge m,n locks. If a pulse shape is associated with hinge m,n, this shape may be used to calculate the maximum forces and torques acting at each station j, in each segment k,i. The forces thus obtained are then used in the stress subroutine in the same manner as the forces that act at any other time in the course of motion.

The calculation of the impulsive forces closely parallels that of the forces previously derived. Thus, for the last segment in appendage k, segment k,n_k.

$$\left(\hat{P}_{e_1}\right)^k_{n_k} = \left(\hat{Q}_{e_1}\right)^k_{n_k} = 0$$

$$\left(\hat{P}_{e_2}\right)^k_{n_k} = m^k_{n_k}\ \Delta\overline{V}^k_{n_k} \qquad (10.7)$$

where the symbol ($\wedge$) indicates the vector is an impulse.

In addition, as in the case of forces, the impulse on the outboard end of segment k,i, $\left(\hat{P}_{e1}\right)_i^k$ is equal to the negative of the impulse on the inboard end of segment k,i + 1.

The change in velocity of subsegment j of segment k,i is given by

$$\Delta\overline{V}_j = \Delta\overline{V} + \Delta\overline{\omega} \times (\overline{C}_j - \overline{C}) \qquad (10.8)$$

The impulsive force applied to subsegment j is then

$$\hat{P}_j = m_j \Delta\overline{V}_j - \hat{P}_{e_1} \qquad (10.9)$$

The change in the moment of momentum about the center of mass of the segment is equal to the moment of the impulsive forces about the center of mass of segment k,i.

$$\overline{\overline{I}} \cdot \Delta\omega = -C \times \hat{P}_{e2} + (\overline{\ell} - \overline{C}) \times \hat{P}_{e1}$$

$$+ \hat{Q}_{e1} + \hat{Q}_{e2} \qquad (10.10)$$

Thus, for segment k,i

$$\left(\hat{Q}_{e1}\right)_i^k = -\left(\hat{Q}_{e2}\right)_{i+1}^k$$

$$\hat{Q}_{e_2} = \overline{\overline{I}}\,\Delta\omega + (\overline{C}_j - \overline{S}_j) \times \hat{P}_j + (C_j - \ell) \times \hat{P}_{e1} - \hat{Q}_{e1} \qquad (10.11)$$

For subsegment j of segment k, ℓ we then have

$$\hat{Q}_j = \overline{\overline{I}}_j\ \Delta\overline{\omega} + (\overline{S}_j \times \hat{P}_j + (\overline{C}_j - \overline{\ell}) \times \hat{P}_{e1} - \hat{Q}_{e1} \qquad (10.12)$$

The equation for $\hat{Q}_{e_2}$, Equation (10.11), is obtained from Equation (10.12) by defining the last station on segment k,i to be at the inboard hinge, i.e., for $j = N, C_j = C$, $\hat{P}_N = \hat{P}_{e_2}$.

The above expressions define the impulsive forces and moments acting on station j of each segment k,i. The maximum forces acting on each crossection of the segment are obtained from the above relations. It will be assumed that a pulse shape, as shown below, can be associated with the locking of each hinge, m,n.

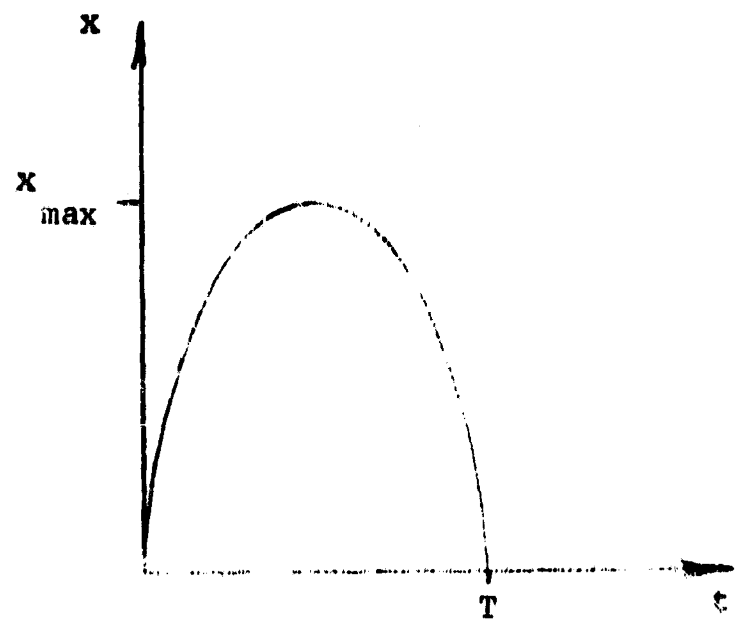


Figure 10.3. Pulse Shape Associated with the Locking of Hinge m,n.

The pulse shown is of unit area. That is,

$$\int_0^T x(t)dt = 1$$

or

$$cx_{max} = 1$$

$$x_{max} = 1/c \qquad (10.13)$$

For a given shape the maximum force in the course of the pulse is then

$$\left(\bar{P}_j\right)^k_{i_{max}} = \left(\frac{1}{c}\right)^m_n \left(\hat{P}_j\right)^k_i$$

$$\left(\bar{Q}_j\right)^k_{i_{max}} = \left(\frac{1}{c}\right)^m_n \left(\hat{Q}_j\right)^k_i \qquad (10.14)$$

A constant 1/c is associated with the hinge that has just locked, m,n. The forces obtained from the above are used in the stress subroutine.

11. SEGMENT STRESSES

Circular Tube Section

The loads on the circular tube section at station j are shown in Figure 11.1.

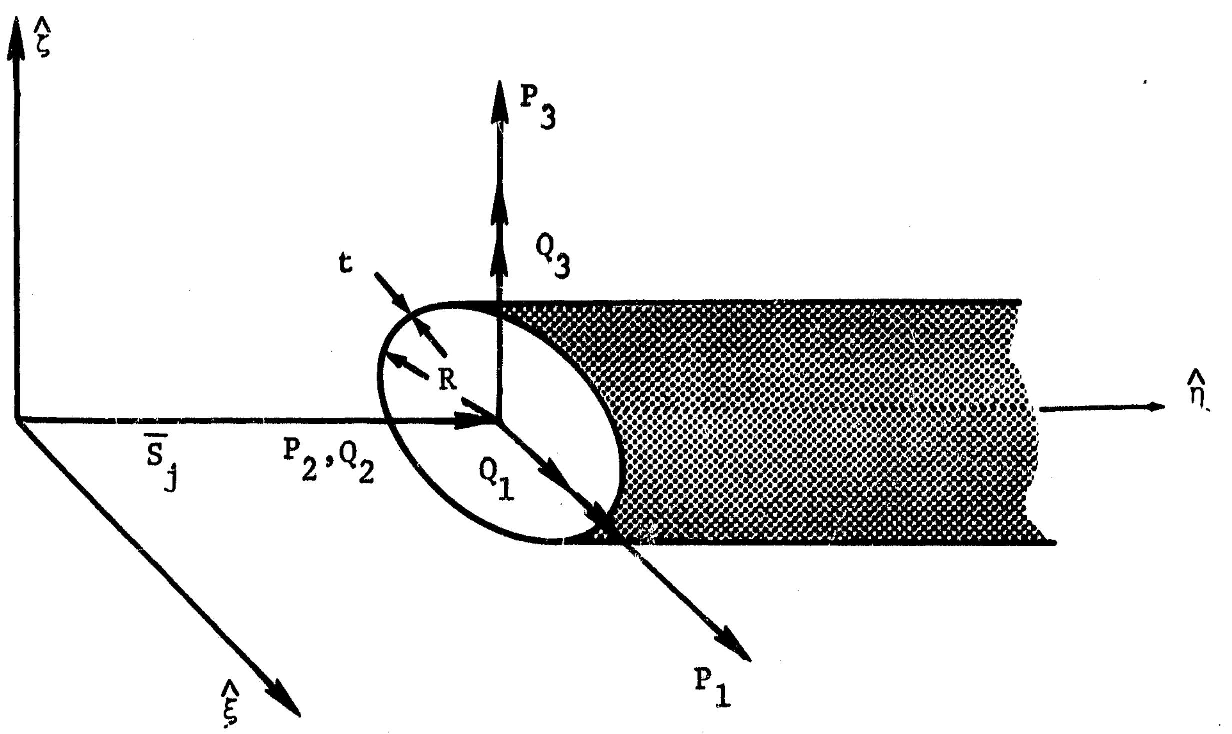


Figure 11.1 Circular Tube Crossection Loads.

The stress for each crossection load will first be determined. The total stress at any point on the crossection is then the sum of the stresses arising from these various effects.

All the stresses on the crossection can be expressed as functions of ξ and ζ as shown in Figure 11.2. ($-R \leq \xi \leq R$, and $-R \leq \zeta \leq R$)

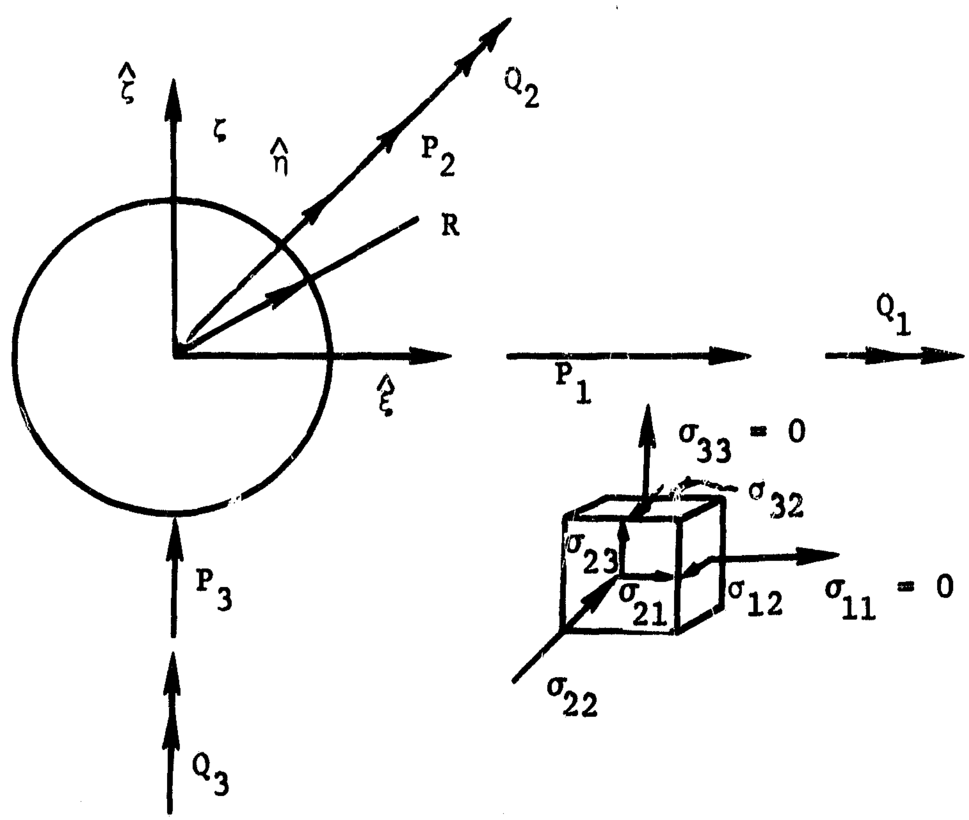


Figure 11.2 Forces and Stresses on a Circular Crossection.

The shear stress arising from P_3 may be written

$$\sigma'_{23} = \frac{2P_3}{A}\left|\frac{\xi}{R}\right| \tag{11.1}$$

and similarly for P_1

$$\sigma'_{21} = \frac{2P_1}{A}\left|\frac{\zeta}{R}\right| \tag{11.2}$$

The normal stress σ_{22}, arises from bending and axial stresses

$$\sigma_{22} = \frac{P_2}{A} + \frac{Q_3\,\xi}{I} - \frac{Q_1\,\zeta}{I} \tag{11.3}$$

where

$$A = 2\pi Rt$$

$$I = \pi R^3 t$$

The torsional shear stress is constant around the crossection

$$\tau = \frac{Q_2}{S}$$

where $S = 2\pi R^2 t$

The torsional shear stress, τ, contributes to both σ_{21} and σ_{23}. Thus, these stresses are as follows:

$$\sigma_{21} = \frac{2P_1}{A} \left| \frac{\xi}{R} \right| + \frac{Q_2}{S} \frac{\xi}{R}$$

$$\sigma_{23} = \frac{2P_3}{A} \left| \frac{\xi}{R} \right| - \frac{Q_2}{S} \frac{\xi}{R} \tag{11.4}$$

The maximum distortion energy is not only the most appropriate criteria for establishing the severity of a stress condition, but also, is the most convenient. The distortion energy (Timoshenko, 1951) is given by

$$V_o = \frac{1+\upsilon}{6E} \left((\sigma_x-\sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 \right)$$

$$+ \frac{1}{2G} (\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2) \tag{11.5}$$

where

$$G = \frac{E}{2(1+\nu)}$$

Since σ_{13}, σ_{11}, and σ_{33} are zero, the distortion energy in the present notation is

$$V_o = \frac{(1+\upsilon)}{3E} \sigma_{22}^2 + \frac{1}{2G} (\sigma_{21}^2 + \sigma_{23}^2) \tag{11.6}$$

where σ_{22}, and σ_{21} and σ_{23} are defined by Equations (11.3) and (11.4), respectively.

Stresses in Non-Circular Sections

The loads acting on the tube crossections have been defined at points on the η axis. However, it will be assumed that the neutral axis is not necessarily on the η axis in general, but at the point (C_ξ, S_j, C_ζ) on the j^{th} crossection of segment k,i. It will be assumed that the neutral axis and the shear center are coincident.

On the basis of the above considerations, it is clear that the moments must be recalculated about the neutral axis. The moments will be designated Q_1^*, Q_2^*, and Q_3^*.

These moments may be written in terms of the forces and moments previously defined. Thus,

$$Q_1^* = Q_1 + C_\zeta P_2$$

$$Q_2^* = Q_2 - C_\zeta P_1 + C_\xi P_2$$

$$Q_3^* = Q_3 - C_\xi P_2 \tag{11.7}$$

We wish to find the stresses at a point ξ,ζ on the crossection as indicated in Figure 11.3.

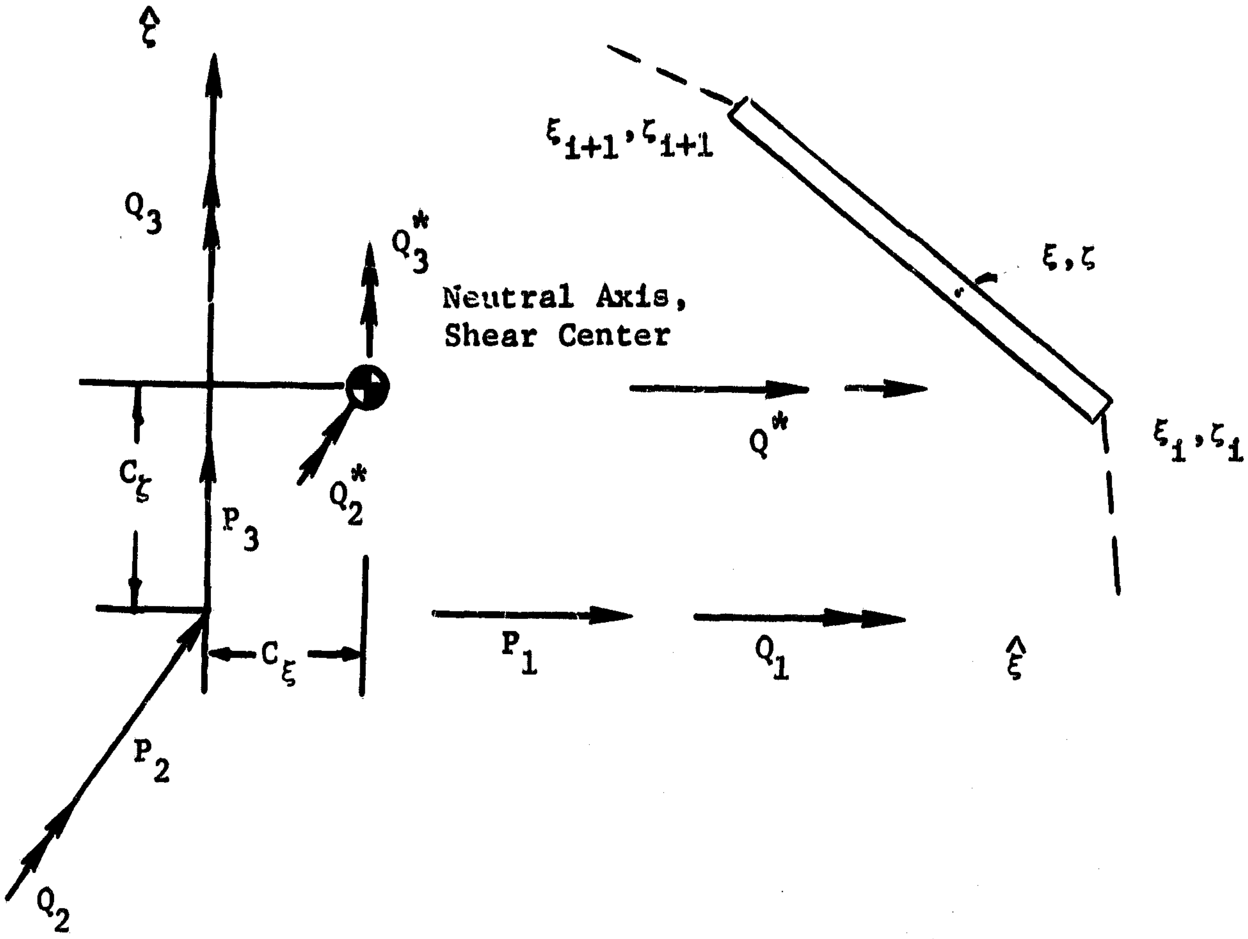


Figure 11.3. Crossection Loads and Stresses

Bending Stress

In general, the asymmetry of the non-circular crossection will result in combined bending. Thus, a moment about one axis results in bending about both axes. This well-known effect can be accounted for by establishing effective bending moments. For convenience, we first define three constants.

$$k_1 = I_{\xi\zeta}/I_{\zeta\zeta}$$

$$k_2 = I_{\xi\zeta}/I_{\xi\xi}$$

$$k_3 = 1 - k_1/k_2$$

Using the above, the effective bending moments on the cross-sections are

$$Q_1^{*'} = \frac{Q_1^* + k_1 Q_3^*}{k_3}$$

$$Q_3^{*'} = \frac{Q_3^* + k_2 Q_1^*}{k_3} \qquad (11.8)$$

At a particular point ξ, ζ the bending stress is then

$$\sigma_{22}^b = \frac{Q_1^{*'}(\zeta - C_\zeta)}{I_{\xi\xi}} - \frac{Q_3^{*'}(\xi - C_\xi)}{I_{\zeta\zeta}} \qquad (11.9)$$

The total compressive stress is found by combining the compressive stress arising from bending, σ_{22}^b, with the compressive stress arising from the force P_2. Thus,

$$\sigma_{22} = \frac{P_2}{A} + \frac{Q_1^{*'}(\zeta - C_\zeta)}{I_{\xi\xi}} - \frac{Q_3^{*'}(\xi - C_\xi)}{I_{\zeta\zeta}} \qquad (11.10)$$

Torsional Stress

The torsional shear flow at the point (ξ,ζ) will be designated q_t. The shear flow is found readily from the torsional moment Q_3^* and the enclosed area, $[A]$. Thus

$$q_t = \frac{Q_2^*}{2[A]} \qquad (11.11)$$

The shear flow, q_t, contributes to the shear stresses in two directions.

$$\sigma_{21}^t = \frac{-q_t \cos\theta}{t_1}$$

$$\sigma_{23}^t = \frac{-q_t \sin\theta}{t_1} \qquad (11.12)$$

Using Equations (11.11) and (11.8) in Equations (11.12) we obtain

$$\sigma_{21}^{t} = \frac{-Q_2^*}{2[A]} \quad \frac{\xi_{i+1} - \xi_i}{A_i}$$

$$\sigma_{23}^{t} = \frac{-Q_2^*}{2[A]} \quad \frac{\zeta_{i+1} - \zeta_i}{A_i} \tag{11.13}$$

The shearing forces P_1 and P_3 contribute to the shear stresses σ_{21} and σ_{23}. Using parameters derived in Section 5, we have

$$\sigma_{23}^{s} = \frac{P_3 Q_\zeta}{I_{\xi\xi}} \left(\frac{A_i}{\xi_{i+1} - \xi_i} + \frac{A_{i'}}{\xi_{i'+1} - \xi_{i'}} \right)$$

$$\sigma_{21}^{s} = -\frac{P_1 Q_\xi}{I_{\zeta\zeta}} \left(\frac{A_i}{\zeta_{i+1} - \zeta_i} + \frac{A_{i'}}{\zeta_{i'+1} - \zeta_{i'}} \right) \tag{11.14}$$

where the superscript s indicates that these are the shear stresses arising from the shearing forces P_1, and P_3 alone, and where the coefficients in parantheses in the denomenator represent the crossectional width cut by lines normal to the ζ and ξ-axes, respectively, at the point at which the stress is being calculated.

Equations (11.14) and (11.15) are combined to establish the total shear stress. Thus,

$$\sigma_{21} = \frac{-Q_2^* (\xi_{i+1} - \xi_i)}{2 [A] A_i} + \frac{P_1 Q_\xi}{I_{\zeta\zeta}\left(\frac{A_i}{\zeta_{i+1} - \zeta_i} + \frac{A_{i'}}{\zeta_{i'+1} - \zeta_{i'}}\right)}$$

$$\sigma_{23} = \frac{-Q_2^* (\zeta_{i+1} - \zeta_i)}{2 [A] A_i} + \frac{P_3 Q_\zeta}{I_{\xi\xi}\left(\frac{A_i}{\xi_{i+1} - \xi_i} + \frac{A_{i'}}{\xi_{i'+1} - \xi_{i'}}\right)} \tag{11.15}$$

Since, as before, σ_{13}, σ_{11}, and σ_{33} are zero on the crossection, the distortion energy on an arbitrary point of crossection segment k,i is of the same form as given in Equation (11.6).

$$V_o = \frac{(1 + \nu)}{3E} \sigma_{22}^2 + \frac{1}{2G}\left(\sigma_{21}^2 + \sigma_{23}^2\right) \tag{11.16}$$

The stress σ_{22} varies linearly on a crossection element while σ_{21}, and σ_{22} very quadratically. Consequently, V_o varies as a fourth order polynomial on the element. The point at which the maximum value of this polynomial occurs is the point at which the crossection element is most severely stressed. A maximum value of V is established in this manner for each plate element. The location of the maximum value among all the elements is the location of the most severe stress condition on the crossection.

12. SECTION PROPERTIES

Circular Tube Section

In this section the properties required to establish the stresses on a crossection are developed. For the circular tube crossection, Figure 12.1, these are readily obtainable as follows:

$$I = \pi R^3 t, \quad \text{the area moment of inertia for the crossection}$$

$$A = 2\pi R t, \quad \text{the crossectional area} \qquad (12.1)$$

The shear stress on a circular tube is proportional to the static moment of the area above the point at which the stress is desired and inversely proportional to the moment of inertia and tube thickness. For shear forces in the ζ-direction the proportionality factor is $\frac{2\cos\theta}{A}$, and in the ξ-direction, $\frac{2\sin\theta}{A}$.

Non-Circular Section

The non-circular section is a general n-sided polygon. The polygonal crossection is specified by the coordinates of the vertices in segment coordinates (at each station if the section is non-uniform) and the thickness of the intervening plate elements as shown in Figure 12.1.

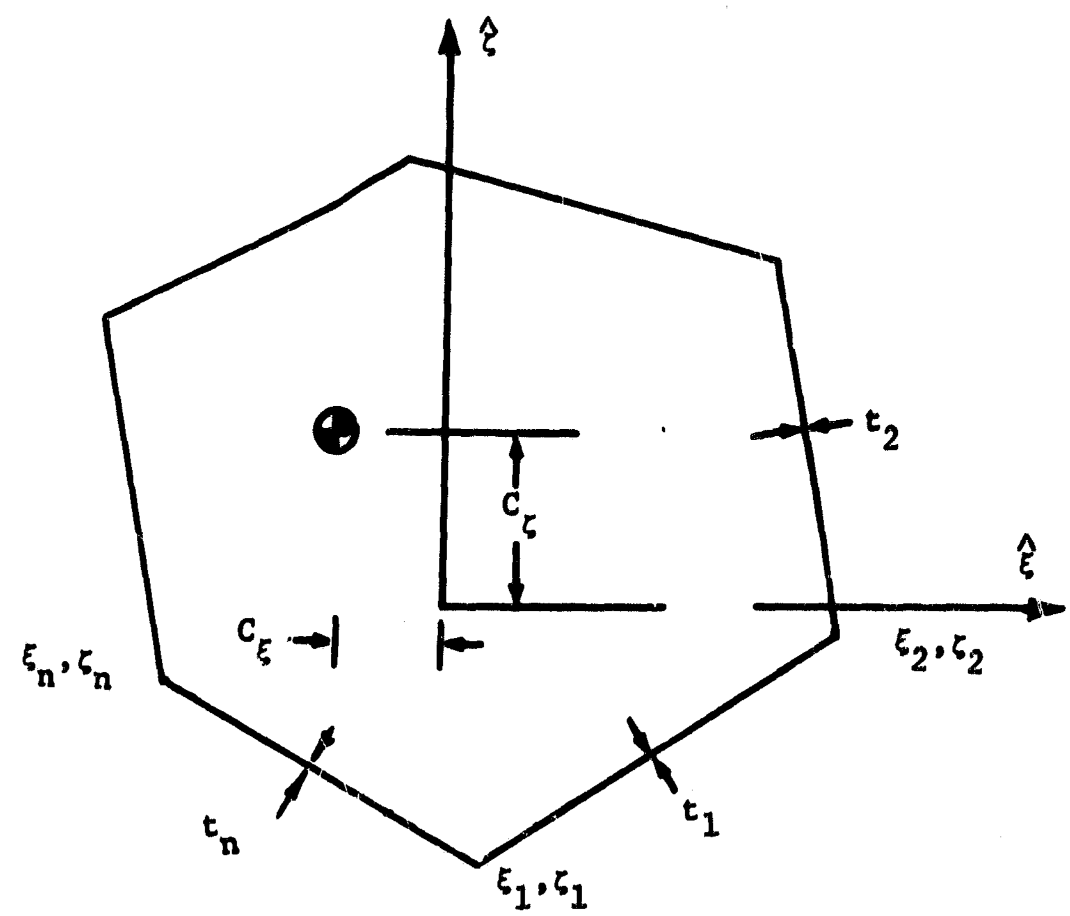


Figure 12.1. Polygonal Crossection Geometry

The area of the crossection is given by

$$A = \sum_{i=1}^{n} A_i \tag{12.2}$$

where

$$A_i = t_i \sqrt{(\xi_{i+1} - \xi_i)^2 + (\zeta_{i+1} - \zeta_i)^2}$$

and

$$\xi_{n+1} = \xi_1, \ \zeta_{n+1} = \zeta_1$$

The centroidal distances C_ξ, and C_ζ are obtained from

$$C_\xi = \frac{1}{A} \sum_{i=1}^{n} \frac{(\xi_{i+1} + \xi_i) A_i}{2} \tag{12.3}$$

and

$$C_\zeta = \frac{1}{A} \sum_{i=1}^{n} \frac{(\zeta_{i+1} + \zeta_i) A_i}{2}$$

The centroid is the neutral axis and will be assumed to be the shear center as well.

The area moments of inertia are obtained from the area properties defined above in combination with the area moments of the individual plate elements. Consider the ith plate element at station j in segment k, i as shown in Figure 12.2.

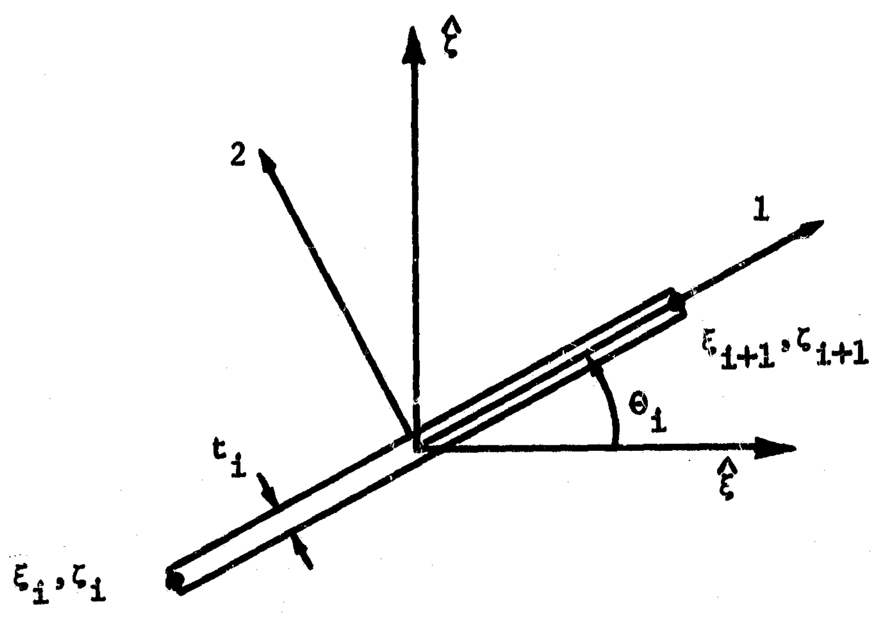


Figure 12.2. The ith Plate Element at Station j in Segment k,i

The area moments of inertia of the plate crossection are derived first about the 1-2 axes through the centroid of the plate element and are subsequently transformed into segment coordinates. Thus, we have

$$I_{1i} = \frac{A_i t_i^2}{12}$$

$$I_{2i} = \frac{A_i^3}{12 t_i^2}$$

These area moments of inertia are transformed to segment coordinates as follows:

$$I_{\xi\xi_i} = \frac{I_{1i} + I_{2i}}{2} + \frac{I_{1i} - I_{2i}}{2} \cos 2\theta_i$$

$$I_{\xi\zeta_i} = \left(\frac{I_{2i} - I_{1i}}{2}\right) \sin 2\theta_i$$

$$I_{\zeta\zeta_i} = \frac{I_{1i} + I_{2i}}{2} + \frac{I_{2i} - I_{1i}}{2} \cos 2\theta_i \qquad (12.4)$$

where

$$\sin\theta_i = \left(\frac{\zeta_{i+1} - \zeta_i}{A_i}\right) t_i$$

$$\cos\theta_i = \left(\frac{\xi_{i+1} - \xi_i}{A_i}\right) t_i$$

$$\sin 2\theta_i = \frac{2t_i^2}{A_i^2} (\zeta_{i+1} - \zeta_i)(\xi_{i+1} - \xi_i)$$

$$\cos 2\theta_i = \frac{t_i^2}{A_i^2} ((\xi_{i+1} - \xi_i)^2 - (\zeta_{i+1} - \zeta_i)^2)$$

Substitution of the expressions for the trigonmetric functions and Equations (11.3) into (11.4) yields

$$I_{\xi\xi_i} = \frac{A_i}{t_i^2} \frac{(t_i^4 + A_i^2)}{24} + \frac{(t_i^4 - A_i^2)}{24A_i}\left((\xi_{i+1} - \xi_i)^2 - (\zeta_{i+1} - \zeta_i)^2\right)$$

$$I_{\xi\zeta_i} = \frac{A_i^2 - t_i^4}{12A_i} (\zeta_{i+1} - \zeta_i)(\xi_{i+1} - \xi_i)$$

$$I_{\zeta\zeta_i} = \frac{A_i(t_i^4 + A_i^2)}{24t_i^2} + \frac{(A_i^2 - t_i^4)}{24A_i}\left((\xi_{i+1} - \xi_i)^2 - (\zeta_{i+1} - \zeta_i)^2\right) \quad (12.5)$$

The area moments of inertia of the section are obtained from Equations (12.5) and the moments of the areas of the plate elements about the centroid of the section. Thus, we have

$$I_{\xi\xi} = \sum_{i=1}^{n} I_{\xi\xi_i}$$

$$+ \sum_{i=1}^{n} \left(\frac{\zeta_{i+1} + \zeta_i}{2} - c_\zeta\right)^2 A_i$$

$$I_{\xi\zeta} = \sum_{i=1}^{n} I_{\xi\zeta_i}$$

$$+ \sum_{i=1}^{n} \left(\frac{\xi_{i+1} + \xi_i}{2} - c_\xi\right)\left(\frac{\zeta_{i+1} + \zeta_i}{2} - c_\zeta\right) A_i$$

$$I_{\zeta\zeta} = \sum_{i=1}^{n} I_{\zeta\zeta_i} + \sum_{i=1}^{n} \left(\frac{\xi_{i+1} + \xi_i}{2} - C_\xi \right)^2 A_i \qquad (12.6)$$

In Figure 12.3, the area above ζ is shown crosshatched. The horizontal line defined by ζ cuts the crossection between the vertices i and i + 1, and between j and j + 1. Q_ζ is the absolute value of the moment of the area above ζ about the centroid.

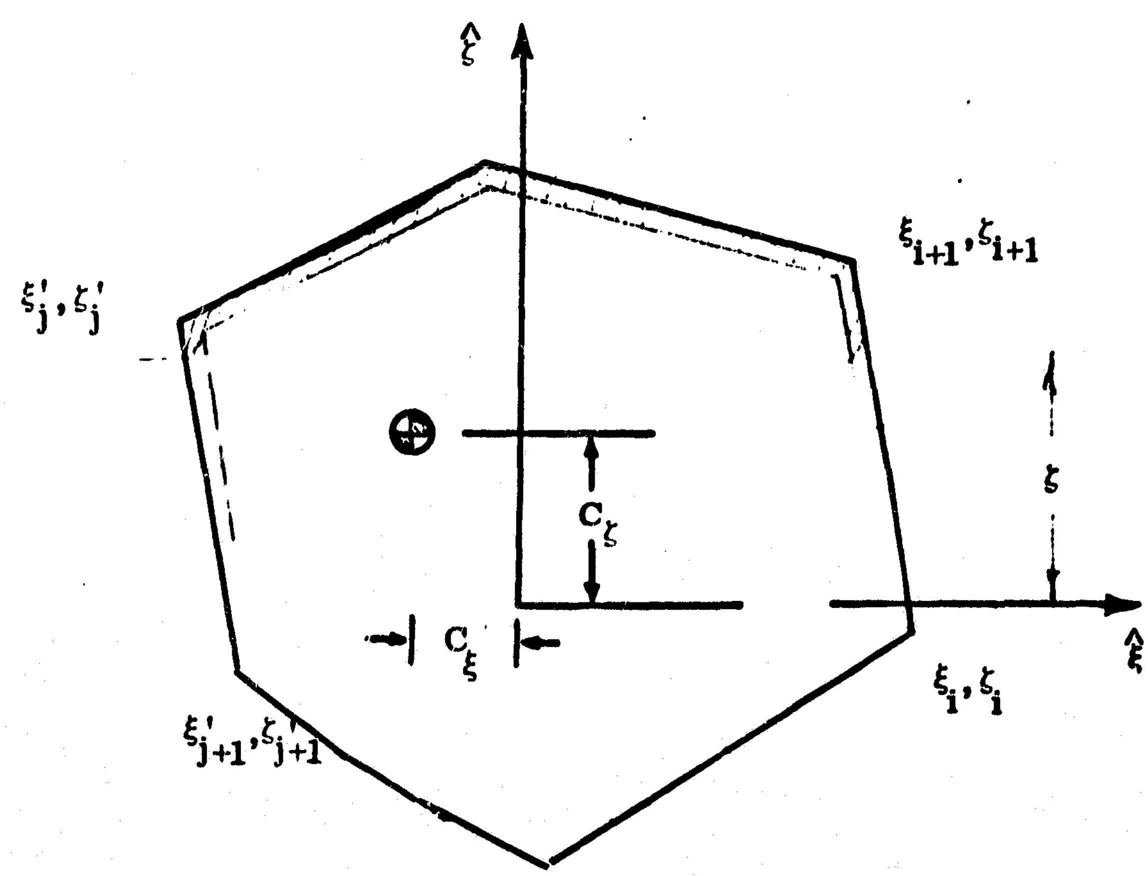


Figure 12.3. The Definition of Q_ζ

The value of Q_ζ is obtained by interpolation from a table of values of H_ζ, the integral of the moment of the area about ξ axis from one fixed point on the crossection. Thus, if we already have the value of H_ζ at vertex i, we obtain H_ζ at an arbitrary point on plate element i as follows:

$$H_\zeta = H_{\zeta_i} + \int_{\zeta i}^{\zeta} \frac{(\zeta - C_\zeta)\, A_i}{\zeta_{i+1} - \zeta_i}\, d\zeta$$

$$\text{for} \quad \zeta_i \le \zeta \le \zeta_{i+1}$$

Using the above, the value of H_ζ can be found at any point on plate element i, or H_ζ, may be found at the same value of ζ on element i'. Q_ζ is then found from

$$Q_\zeta = | H_\zeta - H_{\zeta'} | = | H_{\zeta'} - H_\zeta | \tag{12.7}$$

In the same manner as above,

$$H_\xi = H_{\xi_i} + \int_{\xi}^{\xi} \frac{(\xi - C_\xi)\, A_i}{\xi_{i+1} - \xi_i}\, d\xi$$

$$\text{for} \quad \xi_i \le \xi \le \xi_{i+1}$$

and,

$$Q_\xi = | H_\xi - H_{\xi'} | = | H_{\xi'} - H_\xi | \tag{12.8}$$

In order to calculate the shear flow arising from torsion, the area inclosed by the section, [A] will be required.

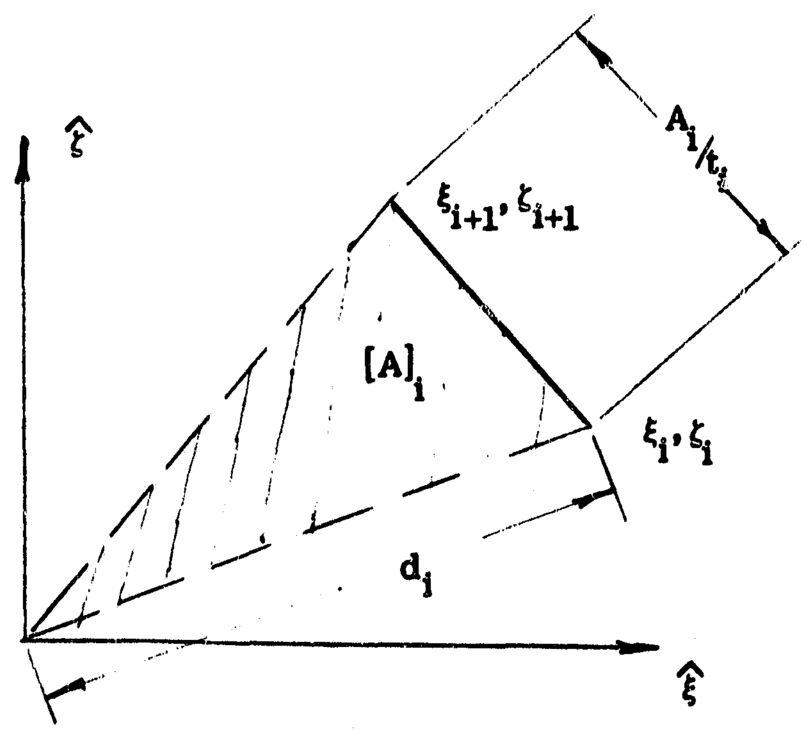


Figure 12.4. The Portion of the Inclosed Area Corresponding to Plate Element i.

let $g_i = d_i + d_{i+1} + \frac{A_i}{t_i}$ where $d_i = \sqrt{\xi_i^2 + \zeta_i^2}$

then $\left[A\right]_i = \sqrt{g_i(g_i-d_i)(g_i-d_{i+1})(g_i - \frac{A_i}{t_i})}$

and

$$[A] = \sum_{i=1}^{n} [A]_i \qquad (12.9)$$

13. CALCULATION OF SUBSEGMENT MASS PROPERTIES FROM THE MASS PROPERTIES OF THE ELEMENTS BETWEEN STATIONS

In this section, the mass properties required in the loads calculation will be derived from input mass properties. These include the total mass between station j and j-1, Δm_j. This mass is the total of the mass of the crossection and any lumped mass, such as wire bundles, and instruments between stations j and j-1. Similarly, $\Delta\bar{\bar{I}}_j$ designates the mass moment of inertia of the portion of the segment between stations j and j-1. The remaining quantity required is $\Delta\bar{C}_j$, the position vector, $\Delta\bar{C}_j$, of the center of mass of $\Delta\bar{m}_j$.

In Section 3, the loads at station j were calculated from $\bar{C}_j$, m_j, and $\bar{\bar{I}}_j$, the mass properties of the subsegment defined by station j, and the motion as given by the present N-Boom program. Given the mass properties of the element between stations j-1 and j, $\Delta m_j, \Delta\bar{C}_j$, and the mass properties for subsegment j-1, we can establish the required mass properties for subsegment j. Thus, we have

$$m_j = m_{j-1} + \Delta m_j$$

$$\bar{C}_j = \frac{m_{j-1}\bar{C}_{j-1} + \Delta m_j \; \Delta\bar{C}_j}{m_j} \qquad (13.1)$$

for

$j = 2,3,\ldots N$ on segment k,i, and $m_1 = \Delta m_1$, $\bar{C}_1 = \Delta\bar{C}_1$

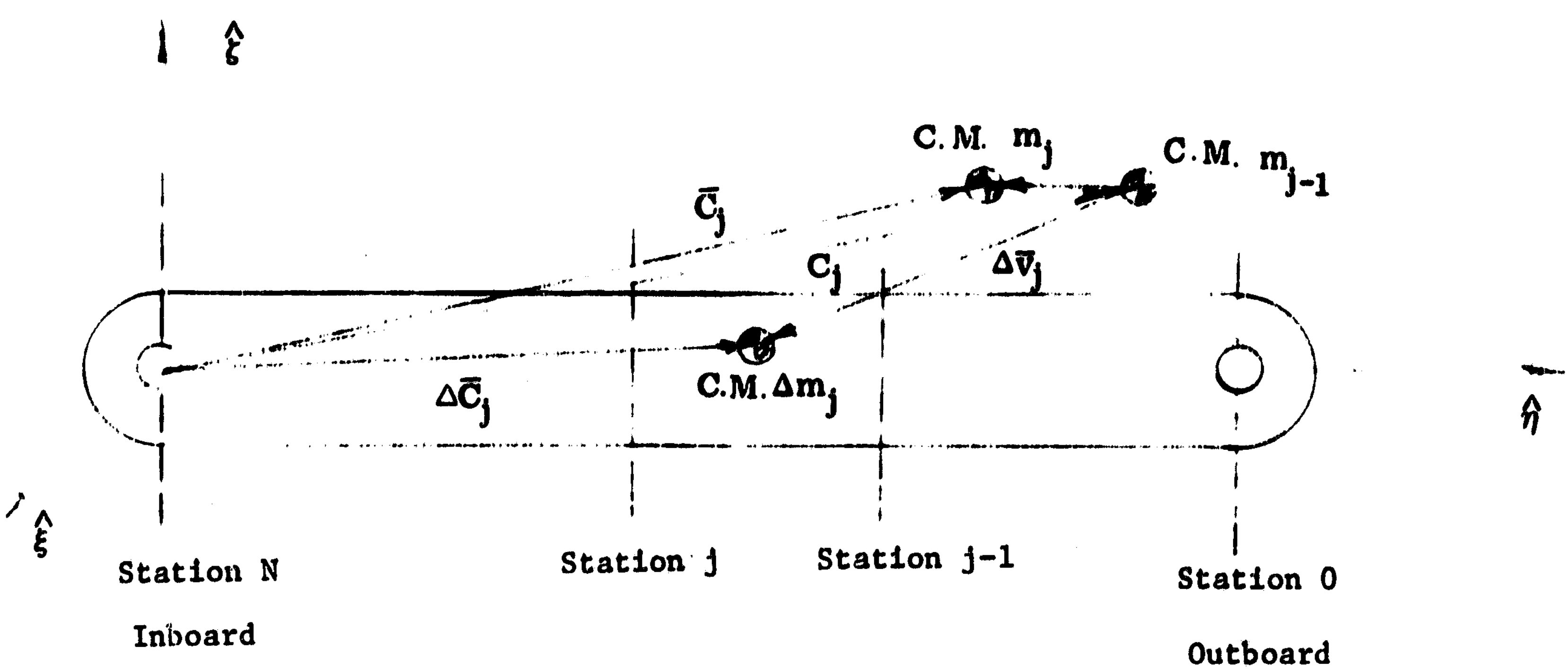


Figure 13.1. Subsegment Parameters

The calculation of the moment of inertia for subsegment j from the mass properties of subsegment j-1 and the element defined by stations j and j-1 is slightly more complex. Referring to Figure 13.1,

$$\bar{r}_{j-1} = \bar{c}_j - \bar{c}_{j-1}$$

$$\Delta\bar{r}_j = \Delta\bar{C}_j - \bar{C}_{j-1} \qquad (13.2)$$

The moment of inertia of subsegment j about its center of mass is obtained from

$$\bar{\bar{I}}_j = \bar{\bar{I}}_{j-1} + \Delta\bar{\bar{I}}_j - m_{j-1}\,\bar{\bar{J}}\,(\bar{r}_{j-1})\,\bar{\bar{J}}\,(r_{j-1})$$

$$- \Delta m_j\,\bar{\bar{J}}\,(\Delta\bar{r}_j)\,\bar{\bar{J}}\,(\Delta\bar{r}_j) \qquad (13.3)$$

for j = 2,3, ...N on segment k,i, and $\bar{\bar{I}}_1 = \Delta\bar{\bar{I}}_1$.

14. MOTION QUANTITIES REQUIRED AS INPUT TO THE STRESS SUBROUTINE

Vector quantities obtained from the main flow of the program are generally expressed in main body coordinates. In this section, vector expressions are written for the vector quantities required as input to the loads subroutine. At the end of the section the appropriate transformations are defined for expressing all vector quantities in segment k, i coordinates.

When no lock-ups are occuring, $\bar{\omega}_i^k$, $\dot{\bar{\omega}}_1^k$, and $\bar{a}$ are required in the loads subroutine. $\bar{\omega}_i^k$ is defined in Equation (8.14) as follows:

$$\bar{\omega}_i^k = \bar{\omega} + \bar{\Omega}_i^k \tag{14.1}$$

where $\bar{\Omega}_i^k = \sigma_i^k = \sigma_i^k \, \hat{e}_1^k$ for $k \le n_s$, $i = 1,2,\ldots n_k$

$= \sigma_1^k \, \hat{e}_1^k + \sigma_2^k \, \hat{\eta}_1^k$ for $n_s < k \le n_a$, $i = 2$

The angular acceleration of segment k, i is obtained by differentiation of Equation (14.1) and is as follows:

$$\dot{\bar{\omega}}_1^k = \dot{\bar{\omega}} + \dot{\sigma}_1^k \, \hat{e}_1^k + \dot{\sigma}_1^k \, (\bar{\omega} \times \hat{e}_1^k) \qquad \text{for non-paddle segments}$$

$$\dot{\bar{\omega}}_2^k = \dot{\bar{\omega}} + \dot{\sigma}_1^k \, \hat{e}_1^k + \dot{\sigma}_2^k \, \hat{\eta}_2^k$$

$$+ \sigma_1^k \, (\bar{\omega} \times \hat{e}_1^k) + \sigma_2^k \, (\bar{\omega} \times \hat{\eta}_2^k) \tag{14.2}$$

The acceleration $\bar{a}_i^k$ of the center of mass of segment k, i is given by

$$\bar{a}_i^k = \frac{D^2}{Dt^2} (\bar{\rho} + \bar{b}_i^k) \qquad (14.3)$$

The first term of Equation (7.3) is defined in Equation (14.19) as follows:

$$\frac{D^2\bar{\rho}}{dt^2} = \left(\frac{d^2\rho}{dt^2}\right)_M + \dot{\bar{\omega}} \times \bar{\rho} + 2\bar{\omega} \times \left(\frac{d\bar{\rho}}{dt}\right)_M$$

$$+ \bar{\omega} \times (\bar{\omega} \times \bar{\rho}) \qquad (14.4)$$

Using the following definitions,

$$\bar{v} = \left(\frac{d\bar{\rho}}{dt}\right)_M, \text{ and } \dot{\bar{v}} = \left(\frac{d^2\rho}{dt^2}\right)_M \qquad (14.5)$$

Equation (7.4) may be written as follows

$$\frac{d^2\bar{\rho}}{dt^2} = \dot{v} + \dot{\bar{\omega}} \times \bar{\rho} + 2\bar{\omega} \times \bar{v}$$

$$+ \bar{\omega} \times (\bar{\omega} \times \bar{\rho}) \qquad (14.6)$$

The second term in Equation (14.3) is somewhat more complex. For non-paddle appendages and the first segments in paddle appendages, that is, that is, for $k \le n_s$ $1 \le i \le n_k$, and $n_s < k \le n_a$, $i = 1$, we have

$$\frac{D^2\bar{b}_i^k}{Dt^2} = \dot{\bar{\omega}} \times \bar{b}_i^k + (\hat{e}_1^k \times \bar{C}_i^k)\,\dot{\sigma}_i^k + \sum_{j=1}^{i-1} \ell_j^k\,\hat{\zeta}_j^k\,\dot{\sigma}_j^k + \bar{g}_i^k \tag{14.7}$$

While for paddle appendages, that is, $n_s \leq k \leq n_a$ and $i = 2$

$$\frac{D^2\bar{b}_2}{Dt^2} = \dot{\bar{\omega}} \times \bar{b}_2^k + \dot{\sigma}_1^k\,\hat{e}_1^k \times \bar{C}_2^k + \dot{\sigma}_2^k\,\hat{\eta}_1^k \times \bar{C}_2^k + \bar{b}_1^k \times \dot{\sigma}_1^k\,\hat{e}_1^k + \bar{g}_2^k \tag{14.8}$$

In the event of a lock-up, all angular velocity and velocity terms change instantaneously, while all positions remain fixed. Thus, from Equation (7.1) we have

$$\Delta\bar{\omega}_i^k = \Delta\bar{\omega} + \Delta\sigma_i^k\,\hat{e}_1^k \quad \text{for non-paddles}$$

and

$$\Delta\bar{\omega}_2^k = \Delta\bar{\omega} + \Delta\sigma_1^k\,\hat{e}_1^k + \Delta\sigma_1^k\,\hat{\eta}_1^k \quad \text{for paddle segments} \tag{14.9}$$

The change in the velocity of the center of mass of segment k, i is found from Equation (8.19) to be

$$\Delta V_i^k = \Delta v + \Delta\bar{\omega} \times (\bar{\rho} + \bar{b}_i^k) + \Delta\bar{\Omega}_i^k \times \bar{C}_i^k + \sum_{j=1}^{i-1} \ell_j^k\,\Delta\sigma_j^k\,\hat{\zeta}_j^k \tag{14.10}$$

where

$$\Delta\vec{\Omega}_1^k = \Delta\sigma_1^k \, \hat{e}_1^k \qquad \text{for non-paddle segments}$$

$$\Delta\vec{\Omega}_2^k = \Delta\sigma_1^k \, \hat{e}_1^k + \Delta\sigma_2^k \, \hat{\eta}_1^k \qquad \text{for paddle segments}$$

Coordinate Transformations

Generally two types of coordinate transformations will be required: 1) transformation of forces and torques expressed in segment k, i + 1 coordinates to segment k, i coordinates, and (2) transformation of quantitites expressed in main body coordinates to segment k, i coordinates.

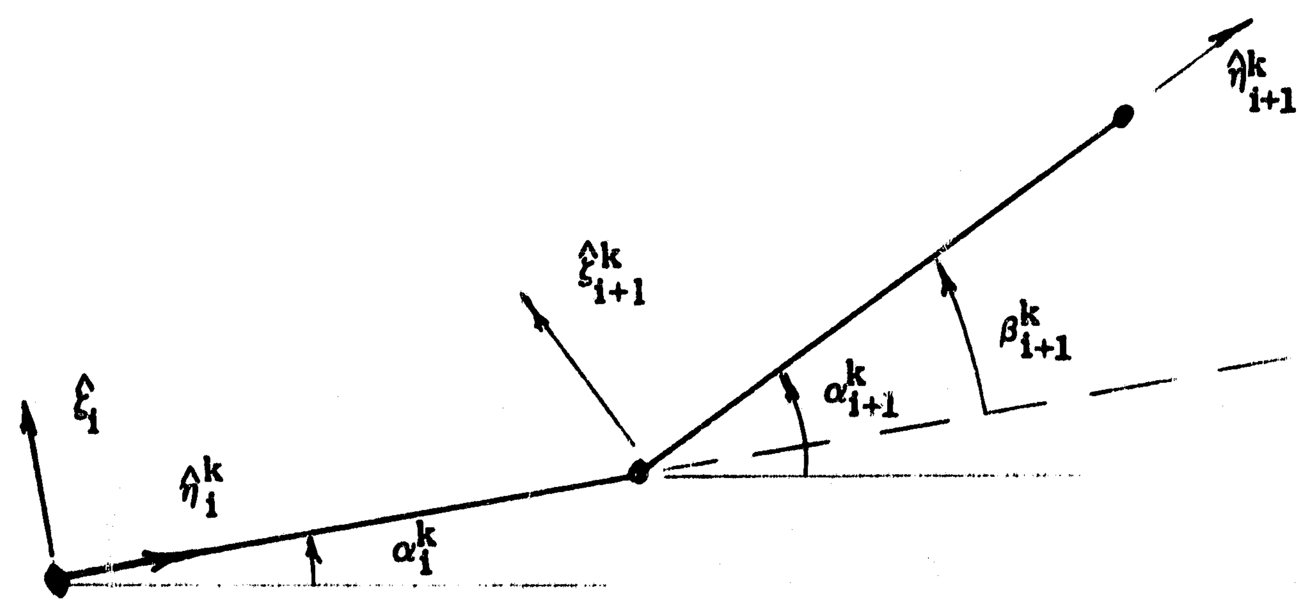


Figure 14.1. Quantities Defining Segment Position

Consider adjoining segments, segments k, i, and k, i + 1 as shown in Figure 14.1.

Segment k, i + 1 unit vectors may be expressed in terms of segment k, i unit vectors by means of the expressions

$$\hat{\xi}^k_{i+1} = \hat{\xi}^k_i = \hat{e}^k_1$$

$$\hat{\eta}^k_{i+1} = \cos \beta^k_{i+1} \, \hat{\eta}^k_i + \sin \beta^k_{i+1} \, \hat{\zeta}^k_i$$

$$\hat{\zeta}^k_{i+1} = -\sin \beta^k_{i+1} \, \hat{\eta}^k_i + \cos \beta^k_{i+1} \, \hat{\zeta}^k_i \tag{14.11}$$

Thus, for example, if the vector $\bar{r}$ is expressed in body k, i + 1 coordinates, it may be written

$$\bar{r} = r_1 \, \hat{e}^k_1 + r_2 \, \hat{\eta}^k_{i+1} + r_3 \, \hat{\zeta}^k_{i+1} \tag{14.12}$$

Then by means of Equation (7.11), Equation (7.12) becomes

$$\bar{r} = r_1 \hat{e}^k_1 + (r_2 \cos\beta^k_{i+1} - r_3 \sin\beta^k_{i+1}) \, \hat{\eta}^k_i$$

$$+ (r_2 \sin\beta^k_{i+1} + r_3 \cos\beta^k_{i+1}) \, \hat{\zeta}^k_i \tag{14.13}$$

The remaining transformation, the transformation from main body to segment k, i coordinates is the reverse of the ordinary transformation performed. The main body unit vectors may be defined as the columns of a 3 × 3 matrix G^k_i as follows

$$\left[\hat{x}, \hat{y}, \hat{z}\right] = \left[G^k_i\right] \tag{14.14}$$

where

$$\left[G_i^k\right] = \left[T^k\right]^T \left[D_i^k\right]^T \tag{14.15}$$

$\left[T^k\right]$ is defined in Equation (9.6), and

$$\left[D_i^k\right] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_i^k & \sin\alpha_i^k \\ 0 & -\sin\alpha_i^k & \cos\alpha_i^k \end{bmatrix} \tag{14.16}$$

PART II

USER'S MANUAL

15. INTRODUCTION TO USER'S MANUAL

The four sections included in Part II of this volume comprise the User's Manual for the program. The Nomenclature, Sections 1, 2, 3, 9, 10, 11, 12, and 14 of Part I in conjunction with the four sections of Part II should provide sufficient information to the prospective user to use the program.

It is the purpose of the User's Manual to describe the quantities which may be input, restrictions on input format, and by example, to illustrate the output the program produces.

Thus, Section 16 is a glossary of program input symbols, Section 17 describes how input is prepared for a test case, and Section 18 discusses the output obtained from the test case, including graphical output as well as printed output.

16. GLOSSARY OF INPUT SYMBOLS

The second column of Table 16.1 is a complete list of input symbols for the program. The left column defines the input quantities in terms of quantities defined in the Nomenclature, Part I, and subsequently used in the derivations. The right hand column provides a brief description of the meaning of the input symbol and information as to dimensions. It is to be noted that input quantities may be input in any self-consistent units, the only restriction being that some quantities having angular dimensions are input in degrees, while others are input in radians. Dimension symbols, in parenthesis in the right column, have the following meaning: D = degrees, R = radians, F = force, L = length, and T = time. It should be noted that although the program output is correct in any self-consistent units, the output is labeled FT, LB, and SEC.

Table 16.1 does not provide sufficient information for the user to understand the meaning of all the input quantities. Referral to the Nomenclature, Part I, will clarify definitions. The coordinate frames to which the various vectors and inertia matrices are referred are described in detail in Section 9, Part I.

The table indicates that the data is input in five groups; these are: the data that appears between &DIM and &END, &NXPUT and &END, &DYNSTA and & END, & DYNSTB and & END, and & ROCK and & END. The data groups must be input in the order shown although the data within a group may be input in any order in that group.

The means of inputting data to the program is described in further detail, by example, in Section 17.

General Input Notes

The flag INSTR which controls the calling of the stress package has the following meaning when it is nonzero:

INSTR = 1 means the stress package is to be called at every time to print the output.

INSTR = N means the stress package is to be called at every nth time to print.

It should be noted that each segment can have a maximum of 5 stations and each station can have a maximum of 6 vertices. Stations are numbered from 1 to 5 starting from the outboard end of the segment to the inboard end.

Input names which include segment and station number are assigned as follows:

For example ξ_ℓ data for segment 9 is entered by giving the name XI9 = followed by the data for the 6 x 5 matrix. Data for segment 11 would have the name XI11.

ΔI_j data for segment 3 and station 5 would be entered using DLI35 followed by the data for the 3x3 matrix. Data for segment 15 and station 4 would have the name DLI154.

Table 16.1. List of Input Symbols

Mathematical Symbol	Input-Program Symbol	Format I - integer F - floating point	Descriptions and Dimensions
	& DIM		Control symbol
	IXK	I	Total number of segments
	IXYZ	I	Flag .. (ψ_M, θ_M, ϕ_M)
			0 ~ Angles
			1 ~ X, Y, Z
	IAB	I	Flag -- (α, β, τ, σ)
			$\neq$ 0 - α and σ
			0 -- β and τ
n_p	NP	I	Number of paddles
n_a	NA	I	Number of appendages
	NØPLT	I	= 1, no plot desired, $\neq$ 1, plot output desired.
	& END		End of data for read Step 1.
PROGRAM PARAMETERS			
	& NXPUT		Control symbol
	INSTR	I	Flag to call stress package, if INSTR = 0, no stresses are calculated, if $1 \leq$ INSTR ≤ 1000, stresses are calculated.

Table 16.1. List of Input Symbols (Continued)

Mathematical Symbol	Input-Program Symbol	Format I - integer F - floating point	Descriptions and Dimensions
β_i^k	BETAIK(N)	F	Angular position of segment[1] (D) (relative to preceding segment)
σ_i^k	SIGIK (N)	F	Angular velocity of segment, α_i^k (D T^{-1})
α_i^k	ALIK (N)	F	Angular position of segment relative to the main body (D)
τ_i^k	TAUIK (N)	F	Angular velocity of segment relative to preceding segment (DT^{-1})
ψ^M, Θ^M, ϕ^M	FTSM	F	Euler angles of main body by angles (D)
M_o	EMO	F	Initial value of the matrix M
t^k	TSK (NA)	F	Release time of segment 1 of kth appendage (T)

1. $N = i + \sum_{m=1}^{K-1} n_m$, where N is the segment index (N = 1,2,..., IXK) and where K = 1, ..., NA.

Table 16.1. List of Input Symbols (Continued)

Mathematical Symbol	Input-Program Symbol	Format I - integer F - floating point	Descriptions and Dimensions
$\beta^k_{r_i}$	IREL(N)	I	Release option parameter: a segment number or zero. 0 if segment N is to be released at a specified time, or J if segment N is to be released when segment J reaches a prescribed relative angle.
$\beta^k_{s_i}$	RELTB(N)	F	The time of release of segment N if IREL(N)=0; or the relative angle of segment IREL(N) at which segment N is to be released.
$\beta^k_{s_i}$	BSIK	F	Relative angle of segment i-1 at which segment i is released (D)
$q^k_{i(J)}$	QNIK (N,J)	F	Dashpot parameters, J= 1 ($FLT^2 R^{-2}$)
$q^k_{i(J)}$	QNIK (N,J)	F	Dashpot parameters, J=2 ($FLT R^{-1}$)
$q^k_{i(J)}$	QNIK (N,J)	F	Dashpot parameters, J=3 ($FL R^{-2}$)
$q^k_{i(J)}$	QNIK (N,J)	F	Dashpot parameters, J=4 (FLR^{-1})

Table 16.1. List of Input Symbols (Continued)

Mathematical Symbol	Input-Program Symbol	Format I - integer F - floating point	Descriptions and Dimensions
$K^k_{i(1)}$	EKNIK (N,J)	F	Spring parameters, J=1 (FLR^{-2})
$K^k_{i(2)}$	EKNIK (N,J)	F	Spring parameters, J=2 (FLR^{-1})
γ^k_i	GAMAIK (N)	F	Dashpot preload angle (D)
θ^k_i	THETIK (N)	F	Spring preload angle (D)
$\bar{r}^k_i$	RIK1(3,N)	F	Main body attachement point of kick-off springs (L).
$\bar{S}^k_i$	SIK1(3,N)	F	Segment attachment point of kick-off springs (L).
$a^k_{o,i}, a^k_{1_i}, a^k_{2_i}, a^k_{3_i}$	AES(4,N)	F	Spring constants for kick-off springs.
$x^k_{f,i}$	XIKF(N)	F	Disengagement length of kick-off springs (L).
$\bar{\rho}$	RHØ	F	Position vector of main body reference point. Always input as (0,0,0), (L)
$\bar{v}$	VBAR	F	Velocity of main body reference point. Always input as (0,0,0)

Table 16.1. List of Input Symbols (Continued)

Mathematical Symbol	Input-Program Symbol	Format I - integer F - floating point	Descriptions and Dimensions
$\bar{\omega}$	ØMEG	F	Angular velocity of main body ($R\ T^{-1}$)
n_k	NK	I	Number of segments in kth appendage, (Integer)
$\bar{d}_k$	DBK	F	Position of attachment point of kth appendage in main body coordinates (L)
ψ^k, θ^k, ϕ^k	FTSIK (J,K)	F	Euler angles defining plane of deployment of kth appendage. J=1 for ψ^k, 2 for θ^k, and 3 for ϕ^k (D)
$\bar{\ell}_i^k$	ELIK (N)	F	Distance between pins of segment k,i (L)
m_i^k	EMIK (N)	F	Mass of segment k,i ($FL^{-1}T^2$)
$\left[I_i^k\right]^*$	HIIK (3,3,N)	F	Inertia matrix of segment k,i about center of mass of segment k,i in segment k,i coordinates (FLT^2)
$C_{i(J)}^k$	CNIK (N,J)	F	Components of the position vector of segment k, i center of mass in segment coordinates, where J=1,2,3 corresponds to the $\hat{\xi}$, $\hat{\eta}$, $\hat{\zeta}$ components, respectively

Table 16.1. List of Input Symbols (Continued)

Mathematical Symbol	Input-Program Symbol	Format I - integer F - floating point	Descriptions and Dimensions
m_M	EMM	F	Mass of main body ($FL^{-1}T^2$)
$[I]_M$	HM	F	Main body inertia matrix in main body coordinates (FLT^2)
$\bar{b}_M$	BBM	F	Center of mass of main body in main body coordinates
χ	X	F	Euler parameter (None)
$\bar{K}$	EKBAR	F	3 Euler parameters (None)
F_T	FT	F	Thrust magnitude (F)
$\hat{F}_T$	FTHAT	F	Unit vector in the direction of thrust (None)
$\bar{f}_T$	FTBAR	F	Position vector of the point of thrust application (L)
t_i	TINT	F	Time of thrust initiation (T)
t_f	TFIN	F	Time of thrust termination (T)
$\hat{g}$	HS	I	Unit vector in the direction of gravity (positive from the center of the gravity field). $\hat{g}$ is expressed in inertial coordinates (None)

Table 16.1. List of INput Symbols (Continued)

INPUT FOR STRESS CALCULATION[1]

Mathematical Symbol	Input-Program Symbol	Format I - integer F - floating point	Descriptions and Dimensions
g	G		Gravity constant (LT^{-2})
	& END & DYNSTA		Control symbol
$\Delta\overline{C}_j$	DELCi	F	Contains up to 20, $1 \le i \le 20$, 3×5 matrices (one for each segment). Each matrix represents the vector $\Delta\overline{C}_j$ at up to 5 stations. Defines the position of the center of mass of Δm_j. (L)
R	R	F	The row designates station and the column designates segment. The radius of the segment at station j if it is a circular segment. (L)
t_ℓ	Ti	F	Contains up to 20, $1 \le i \le 20$ 6×5 matrices, one for each segment. The matrix row corresponds to the vertex and the column to the station. The thickness of the crossectional plate element. (L)

[1]As many as 5 stations, and 6 vertices at each station are allowed.

Table 16.1. List of Input Symbols (Continued)

Mathematical Symbol	Input-Program Symbol	Format I - integer F - floating point	Descriptions and Dimensions
ξ_ℓ	XIi	F	Segment coordinate defining a vertex at a particular station. Input in the same order as values of t_ℓ. (L)
ζ_ℓ	ZETAi	F	Input name is ZETAi ($1 \le i \le 20$). Segment coordinate defining a vertex at a particular station. Input in the same order and manner as values of t_ℓ(L).
Δm_j	DELM	F	The mass of that portion of a segment defined by stations j and J-1. Input in the same order as values of R. ($FL^{-1}T^2$)
C_n^m	CMN	F	A parameter describing the lock-up pulse shape. 1 entry for each segment. (T)
ν	XNU	F	One entry for each segment. Poisson's ratio.
E	E	F	Young's modulus for segment N. (FL^{-2})
	NSTA	I	Number of stations present in each segment (1 to 5).
	NVER	I	Number of vertices for each station at each segment (1 to 6). The row designates station and the column designate segment.

Table 16.1. List of Input Symbols (Continued)

Mathematical Symbol	Input-Program Symbol	Format I - integer F - floating point	Descriptions and Dimensions
	ICIR	I	One entry for each segment flag; = 0 - circular segment, ≠ 0 - non-circular segment.
S_j	S	F	Station j position from segment measured from outboard end. One entry for each station. At each segment the row designates station and the column designates segment (L).
& END			
& DYNSTB			
ΔI_j	DLIij	F	Contains a 3 × 3 matrix for each station j and segment i. The mass moment of inertia of that portion of a segment defined by station j and j-1. (FLT^2)
INTEGRATION PARAMETERS			
	&RØCK		Control symbol
h_o	HZO	F	RKAM input
$\overline{E}$	EU1	F	Vector of upper bounds of predictor corrector of RKAM

Table 16.1 List of Input Symbols (Continued)

Mathematical Symbol	Input-Program Symbol	Format I - integer F - floating point	Descriptions and Dimensions
E	EL1	F	Vector of lower bounds of predictor corrector of RKAM
$\bar{h}_{min}$	HMIN	F	Minimum allowable step size (T)
h_{max}	HMAX	F	Maximum allowable step (T)
t_o	TZO	F	Initial time (T)
t_{final}	TF	F	Maximum real time to run job (T)
	LSTEP	I	Maximum number of steps to be taken
	IAX	I	Number of integration steps to be taken between printouts after the 20th step
	SEGSTT(N)	F	Segment state vector 0,1,2, means segment N is initially unrealeased, released, or locked, respectively.
	&END		Control symbol

17. TEST CASE INPUT DATA

The problem considered for the test case described below is that of a satellite with two appendages, as shown in Figure 17.1. The first appendage is a two-segment regular appendage, and the second is a paddle appendage. Springs and dashpots act about each hinge point and no external forces are applied. In addition, kick-off springs are attached to each segment. In the following pages, a physical description of the problem is given first and the pertinent data are then transcribed onto load sheets.

The geometric and mass data and initial conditions associated with the main body is developed first, and this will be followed by the segment data. The position vectors of the points of attachment of the hinges are:

$$\bar{d}_1 = 10\,\hat{x} + 0\hat{y} + 20\,\hat{z}$$

$$\bar{d}_2 = -10\,\hat{x} + 20\,\hat{y} + 0\,\hat{z} \qquad (17.1)$$

The deployment planes in this case are chosen to be radial planes. The coordinate system fixed in the main body at each appendage attachment point, appendage coordinates, are defined by the Euler angles for each appendage. These angles, expressed in degrees, are defined as follows:

$$\psi^1 = 90\ ,\ \theta^1 = 90,\ \phi^1 = 0$$

$$\psi^2 = 0\ ,\ \theta^2 = 90,\ \phi^2 = 0 \qquad (17.2)$$

The main body frame, $\hat{x}$, $\hat{y}$, and $\hat{z}$, fixed at 0 is assumed to be initially coincident with the inertial frame, thus

$$\psi_M = 0\ ,\ \theta_M = 0,\ \phi_M = 0 \qquad (17.3)$$

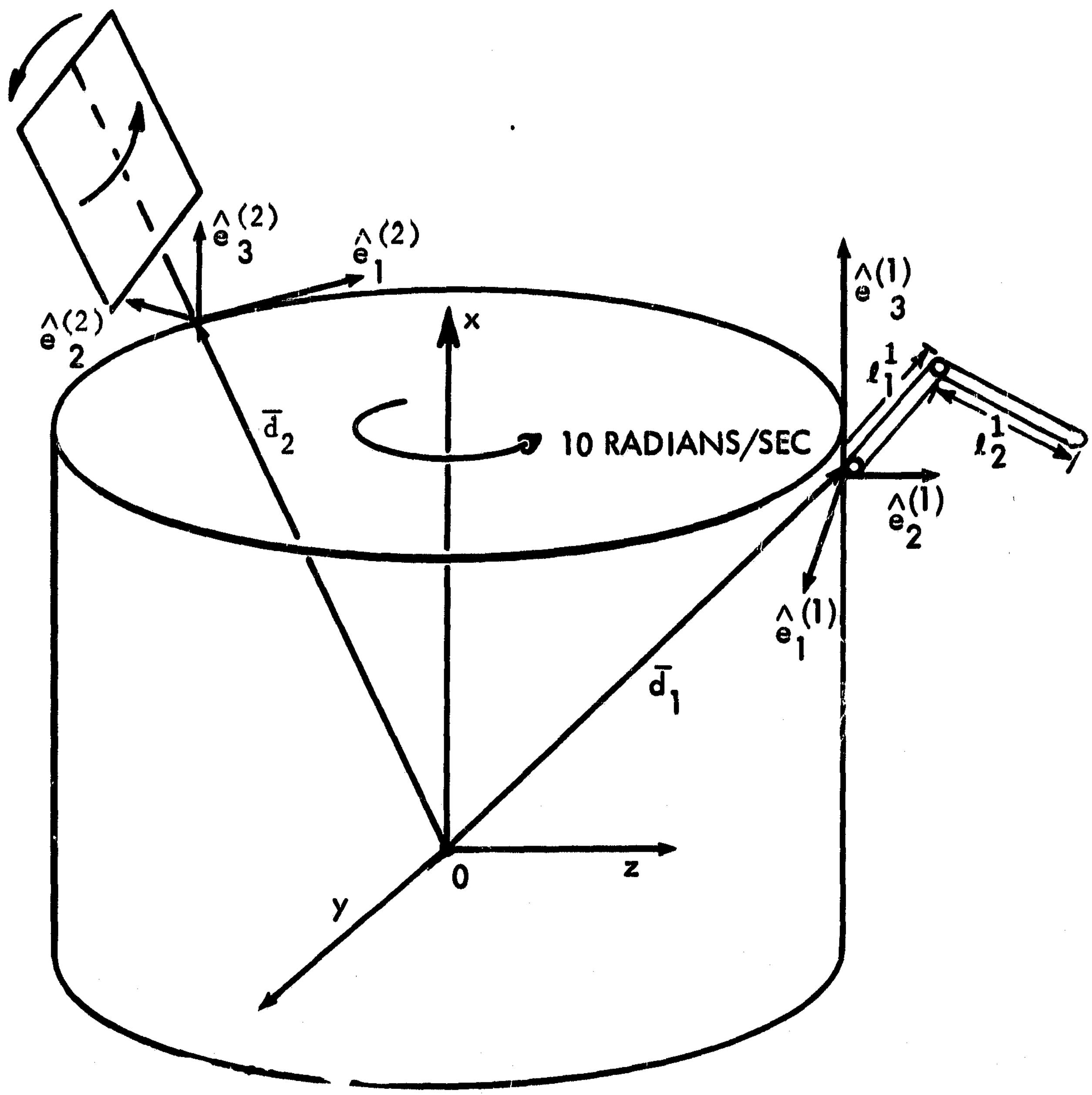


Figure 17.1. **Test Case: Satellite with Partially Deployed Appendages**

Point 0 is chosen to be fixed at the main body center of mass, thus,

$$\overline{b}_M = 0 \tag{17.4}$$

The initial angular velocity (degrees/sec) is exclusively about the spin axis, $\hat{x}$,

$$\overline{\omega} = 572.9577\ \hat{x} + 0\ \hat{y} + \hat{z} \tag{17.5}$$

Both segments of appendage 1 will be released at specified times, namely, segment 1,1 at .007 sec. and segment 1,2 at .028 sec. The first segment of appendage 2 is to be released at .01 sec. whereas the second segment of appendage 2 is to be released when segment 2 of appendage 1 reaches 86.5 degrees.

The mass properties for the main body are

$$M_M = 25$$

$$I_M = \begin{vmatrix} 4000 & 0 & 0 \\ 0 & 2000 & 0 \\ 0 & 0 & 2000 \end{vmatrix} \tag{17.6}$$

Several integers are input to specify the number and type of appendages, and the number of segments in each appendage. These are in this case

n_a = 2, two appendages
n_p = 1, one paddle appendage
n_1 = 2, 2 segments in appendage 1
n_2 = 2, 2 segments in appendage 2 (17.7)

The segment data required as input to the stress routine will be described segment by segment.

Segment 1, 1, or Segment 1

Segment 1, 1 is a uniform circular tube for which only one station has been defined. The subscripts for the quantities stated below represent the station number.

$$\Delta\bar{C}_1 = 10\,\hat{\eta}$$

$$\Delta m_1 = m_1^1 = .5$$

$$\Delta\bar{\bar{I}}_1 = \left[I_1^1\right]^* = \begin{vmatrix} 10 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 10 \end{vmatrix}$$

$$S_1 = 0$$

$$R_1 = 2$$

$$t_1 = .05 \qquad (17.8)$$

Segment 1,2, or segment 2

Segment 1,2 is a non-uniform non-circular segment with a pentagonal crossection at the two ends and a triangular section in the center.

The positions of the stations are given by

$$S_1 = 20$$

$$S_2 = 10$$

$$S_3 = 0 \qquad (17.9)$$

The centers of mass and mass of the elements outboard of each station on segment 1,2 are given by

$$\Delta\bar{C}_1 = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix}, \quad \Delta m_1 = 0$$

$$\Delta\bar{C}_2 = \begin{pmatrix} 0 \\ 15 \\ 0 \end{pmatrix}, \quad \Delta m_2 = .2$$

$$\Delta\bar{C}_3 = \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix}, \quad \Delta m_3 = .2, \qquad (17.10)$$

and the corresponding inertias are

$$\Delta\bar{\bar{I}}_1 = 0 \,, \; \Delta\bar{\bar{I}}_2 = \begin{vmatrix} 10 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 30 \end{vmatrix}, \; \Delta\bar{\bar{I}}_3 = \begin{vmatrix} 10 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 30 \end{vmatrix} \qquad (17.11)$$

From Equations (17.10) and (17.11) it is clear that the over-all inertia matrix for segment 1,2 must be

$$\left[I_2^1\right]^* = \Delta\bar{\bar{I}}_2 + \Delta\bar{\bar{I}}_3 + \begin{vmatrix} 25 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 25 \end{vmatrix}$$

$$= \begin{vmatrix} 45 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 85 \end{vmatrix} \qquad (17.12)$$

The crossections at stations 1, and 3, the two ends of the segment, are assumed to be identical pentagons 2.52 inches on a side and are defined as follows:

ℓ	ξ_ℓ	ζ_ℓ	$t_\ell = .15$
1	2.15	0	
2	.67	2.04	
3	-1.74	1.26	
4	-1.74	-1.26	
5	.67	-2.04	

(17.13)

While at station 2 on segment 1, 2 the crossection is an equilateral triangle 10 inches on a side and is defined as follows:

ℓ	ξ_ℓ	ζ_ℓ	$t_\ell = .05$
1	4.33	0	
2	-4.33	5	
3	-4.33	-5	

(17.14)

Segment 2,1, or Segment 3

The first segment of the paddle appendage, segment 2,1, is a uniform circular tube segment of radius 2 inches, .05 inches thick, and 20 inches long. The centers of mass and mass of the elements outborad of each station on segment 2,1 are given by

$$\Delta\bar{C}_1 = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix} \qquad \Delta m_1 = 0 \qquad S_1 = 20$$

$$\Delta\bar{C}_2 = \begin{pmatrix} 0 \\ 10 \\ 0 \end{pmatrix} \qquad \Delta m_2 = .5 \qquad S_2 = 0$$

while

$$\Delta\bar{\bar{I}}_1 = [0] \ , \quad \text{and } \Delta\bar{\bar{I}}_2 = \left[I_1^2\right]^* = \begin{vmatrix} 10 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 10 \end{vmatrix}$$

and, $R_1 = R_2 = 2$, $t_1 = t_2 = .05$ (17.15)

Segment 2,2, or Segment 4

The paddle segment, segment 2,2, is rectangular in crossection. One station is defined at the inboard end. For segment 2,2 we define

$$\Delta\bar{C}_1 = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix} \qquad \Delta m_1 = .2 \qquad s_1 = 0$$

$$\Delta\bar{\bar{I}}_1 = \left[I_2^2\right]^* = \begin{vmatrix} 20 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 20 \end{vmatrix} \tag{17.16}$$

and crossection parameters as follows:

ℓ	ξ_ℓ	ζ_ℓ	t_ℓ
1	20	1	.05
2	-20	1	.1
3	-20	-1	.05
4	20	-1	.1

(17.17)

Finally, for all segments

$$C_n^m = .1, \ \nu_i^k = .3, \text{ and } E_i^k = 30 \ \ 10^6 \text{ lb/in}^2 \tag{17.18}$$

Table 17.1 and equations following present the segment data that is required if motion alone is required, or if motion and segment stress, are both required.

Table 17.1. Additional Segment Properties (Segment i, Appendage k)

Input Quantity	k=1 i=1	k=1 i=2	k=2 i=1	k=2 i=1
The length of segment k,i ℓ^k_i (in.)	20	20	20	40
Component of segment center of mass location, $C^k_{i(2)}$ (in.)	10	10	10	20
The mass of segment k,i, m^k_i (lb/sec^2/in.)	.5	.4	.5	.5
The dashpot parameter of the dashpot acting about hinge k,i, $q^k_{i(2)}$	-.001	-.001	-.001	-.001
The stiffness of the spring acting about hinge k,i, $K^k_{i(2)}$ (lb in/radian)	-.4	-.1	-.4	-.4
Spring pre-load angle θ^k_i (degrees)	-20	-320	-20	-150
Release option IREL	0	0	0	2
RELTB	.007	.028	.01	86.5
$\beta^k_{r_i}$ (degrees)				
Lock-up angle of segment k,i $\beta^k_{s_i}$ (degrees)	0	360	0	180
Initial segment position, α^k_i, (degrees)	90	270	90	90
Segment number (N=)	1	2	3	4

The moments of inertia of the segments is expressed in segment coordinates about the segment center of mass. These parameters must be consistent with the mass properties input to the stress package. Thus,

$$\left[I_1^1\right]^* = \begin{vmatrix} 10 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 10 \end{vmatrix}$$

$$\left[I_2^1\right]^* = \begin{vmatrix} 45 & 0 & 0 \\ 0 & 40 & 0 \\ 0 & 0 & 85 \end{vmatrix}$$

$$\left[I_1^2\right]^* = \begin{vmatrix} 10 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 10 \end{vmatrix}$$

$$\left[I_2^2\right]^* = \begin{vmatrix} 20 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 20 \end{vmatrix} \qquad (17.19)$$

The data as presented in the preceding pages is shown entered on load sheets on the following pages. All symbols which designate various sections, such as & DIM, & END, etc., must appear on separate cards with the & symbol appearing in card column 2. Other input quantities describing the various parameters of the system may appear in any order with one or more such input quantities on one card.

NL

Date .... Page ..... of .....

COMPUTATION AND DATA REDUCTION CENTER

Name .... Priority ....

Problem No. .... **NAMELIST INPUT FORM** Keypunched by ....

No. of Cards .... 1 Verified by ....

2 TITLE: 80 COMMENTS

```
+ DIM
NA=2,IXYZ=0,IAB=1, NP=1,NØPLT=0,IXK=4,
+ END
+NXPUT
INSTR=3,
BETAIK=4*0.0,
SIGIK=4*0.0,
ALIK=90.0,270.0,90.0,90.0,
TAUIK=4*0.0,
FTSM=3*0.0,
BSIK=0.0,360.0,0.0,180.0,
QNIK (1,2)=-.001,QNIK(2,2)=-.001,QNIK(3,2)=-.001,QNIK(4,2)=-.001,
EKNIK (1,2)=-.4,EKNIK(2,2)=-.1,EKNIK(3,2)=-.4,EKNIK(4,2)=-.4,
GAMAIK=4*0.0,
THETIK=-20.0,-320.0,-20.0,-150.0,
RHØ=3*0.0,
VBAR=3*0.0,
ØMEG=572.9577,0.0,0.0,
NK=2,2,
DBK=10.0,0.0,20.0,-10.0,20.0,0.0,
FTSIK=90.0,90.0,0.0,0.0,90.0,0.0,
ELIK=20.0,20.0,20.0,40.0,
EMIK=.5,.4,.5,.5,
HIIK = 10.0,3*0,0.5.0,3*0,10.0,
```

NL

SYSTEMS 2884 REV. 8-65

NL

Date

Page of

COMPUTATION AND DATA REDUCTION CENTER

Name

Priority

Problem No.

NAMELIST INPUT FORM

Keypunched by

No. of Cards

1

Verified by

TITLE: 2 80 COMMENTS

```
45.0,3*0.0,40.0,3*0.0,85.0,10.0,3*0.0,5.0,3*0.0,10.0,20.0,3*0.0,10.0,3*0.0,
20.0,
CNIK (1,2)=10.0,CNIK(2,2)=10.0,CNIK(3,2)=10.0,CNIK(4,2)=20.0,
EMM=25.0,
HM=4000.0,3*0.0,2000.0,3*0.0,2000.0,
BBM=3*0.0,
X=1.0,
EKBAR=3*0.0,
FTBAR=3*0.0,FTHAT=3*0.0,FT=0.0,
G=0.0,
RIK1=20.0,0.0,10.0,10.0,0.0,10.0,10.0,18.0,0.0,10.0,18.0,-5.0,
SIK1=0.0,10.0,2.0,0.0,20.0,0.0,0.0,20.0,0.0,0.0,0.0,5.0,
AES=10.0,-.2,0.0,0.0,4.0,-.5,.05,0.0,1.,-.02,0.0,0.0,20.0,-.6,0.0,0.0,
XIKF=9.0,11.0,50.0,3.0,
IREL=0,1,1,2,RELTB=0.007,89.5,89.51,181.0,
+END
+DYNSTA
DELC1=0.0,10.0,0.0,DELC2=0.0,20.0,0.0,0.0,15.0,0.0,0.0,5.0,0.0,DELC3=0.0,20.0,
0.0,
0.0,10.0,0.0,DELC4=0.0, 20.0,0.0,
R (1,1)=2.0,R(1,3)=2.0,R(2,3)=2.0,
DELM(1,1) =.5,DELM(1,2)=0.0,DELM(2,2)=.2,DELM(3,2)=.2,DELM(1,3)=0.0,
DELM(2,3)=.5,
DELM(1,4)=.2,
```

NL

SYSTEMS 2554 REV. 8-65

NL

Date ______ Page ____ of ____

COMPUTATION AND DATA REDUCTION CENTER

Name ______ Priority ______

Problem No. ______ **NAMELIST INPUT FORM** Keypunched by ______

No. of Cards ______ **1** Verified by ______

2 TITLE: 80 COMMENTS

```
DELM(1,2)=.001,DELM(1,3)=.001,
T1=.05,T2=5*.15,0.0,3*.05,3*0.0,5*.15,T3=.05,5*0.0,.05,5*0.0,T4=.05,.1,.05,.1,
2*0.0,
X12=2.15,.67,-1.74,-1.74,.67.0.0,4.33,-4.33,-4.33,3*0.0,2.15,.67,-1.74,-1.74,
.67,0.0,
XI4=20.0,-20.0,-20.0,20.0,
ZETA2=0.0,2.04,1.26,-1.26,-2.04,0.0,0.0,5.0,-5.0,3*0.0,0.0,2.04,1.26,-1.26,
-2.04,
ZETA4=1.0,1.0,-1.0,-1.0,
CMN=4*.1,
XNU=4*.3,
E=4*30.0E6
NSTA=1,3,2,1,
NVER(1,2)=5,NVER(2,2)=3,NVER(3,2)=5,NVER(1,4)=4,NVER(1,1)=1,NVER(1,3)=1,
NVER(2,3)=1,
ICIR=0,1,0,1,
S(1,1)=0.0,S(1,2)=20.0,S(2,2)=10.0,S(3,2)=0.0,S(1,3)=20.0,S(2,3)=0.0,
S(1,4)=0.0,
+END
+DYNSTB
DLI11=10.0,3*0.0,5.0,3*0.0,10.0,DLI21=9*0.0,DLI22=10.0,3*0.0,20.0,3*0.0,30.0,
DLI23=10.0,3*0.0,20.0,3*0.0,30.0,DLI31=9*0,0,DLI32=10.0,3*0.0,5.0,3*0.0,10.0,
DLI41=20.0,3*0.0,10.0,3*0.0,20.0,
+END
```

NL

SYSTEMS 2554 REV. 8-65

NL

Date ______________ Page ____ of ____

COMPUTATION AND DATA REDUCTION CENTER

Name ______________ Priority ______________

NAMELIST INPUT FORM

Problem No. ______________ Keypunched by ______________

No. of Cards ______________ **1** Verified by ______________

2 TITLE: 80 COMMENTS

```
+RØCK
HZO=.001,EU1=0.5E-3,EL1=1.0E-5,HMIN=.001,HMAX=0.1E6,TZO=0.0,
TF=1.0,LSTEP=150,SEGSTT=7*0.0,
IAX=2,
+END
```

NL

SYSTEMS 2554 REV. 8-65

The elements of an array may be input in order, e.g., the elements of ALIK may be input in the order of increasing N: α_1^1, α_2^1, ... $\alpha_{n_1}^1$, α_1^2, α_2^2,... $\alpha_{n_2}^2$, ..., $\alpha_1^{n_a}$, $\alpha_2^{n_a}$, ... $\alpha_{n_{n_a}}^{n_a}$.

Alternatively, elements of an array may be input individually, e.g., ALIK(2) = α_2^1, ALIK(4) = α_2^2, etc. The inertia matrices of the segments may be input in the same manner except that each element of the array now consists of a series of nine numbers. The elements of the inertia matrix are read by rows.

Other properties of NAMELIST input are that input quantities and symbols are separated by commas, each line of data is terminated by a comma, a series of equal quantities may be input by use of the symbol *, e.g., 4*X means X,X,X,X, and symbols beginning with the letters I, J, K, L, M, N must be input as integers while other quantities are input in floating point format.

18. TEST CASE OUTPUT

The following pages are a portion of the output of the program generated by the load sheets developed in the last section. The output includes: (1) an output of the input quantities (2) printout of the results; and (3) graphical output. All of the output in (1) and (3) are included in this report, as well as a representative sample of (2).

The output of the input quantities is generally self-explanatory. The numbers in each array are output in the order in which they are input.

The parameters EU and EL are used to control the accuracy with which the RKAM subroutine obtains the numerical solution of the differential equations. Specifically, these parameters govern the halving and doubling of the integration step-size while in the Adams-Moulton mode. If the absolute value of the difference between the predicted and corrected values of all of the variables being integrated is less than EL, the step-size is doubled, whereas, if any of these differences is greater than EU the step-size is halved.

T is the time at which the simulation is initiated, H is the nominal step-size, HMAX is the maximum step-size allowed, and HMIN is the minimum step-size allowed.

The position and motion of elements of the system are output every time step, up to the twentieth time step. After the twentieth time step this information is output every IAX time steps. Table 18-1 provides a description of output designations in terms of previously defined parameters.

Table 18.1. List of Output Symbols

Output Symbol	Description or Meaning in Terms of Symbols Defined in The Nomenclature
TIME	Current value of time in the simulation
NUMBER OF STEPS	Number of time steps taken since initial time
TIME STEP	The next increment in simulated time to be attempted by the numerical routine
APP	Appendage number (K)
SEG	Segment number (I)
BETA	The current value of β_i^k
BETA DOT	The current value of $\dot{\beta}_i^k$
DETA DDOT	The current value of $\ddot{\beta}_i^k$
ALPHA	The current value of α_i^k
ALPHA DOT	The current value of $\dot{\alpha}_i^k$
ALPHA DDOT	The current value of $\ddot{\alpha}_i^k$
RELEASE LOCK-UP STATE VECTOR	-1, 0, 1 means hinge k, i is unreleased, in motion, or locked, respectively
SPRING DASHPOT TORQUE	The current value of Q_i^k
KICK-OFF SPRING FORCE	The current value of $F_{s_i}^k$
EXTENSION OF KICK-OFF SPRING	The current value of $\left\| x_i^k \right\|$
IN MAIN BODY COORDINATES COMP X, Y, Z	Designates the x, y, and z components of vectors referred to main body coordinates

Table 18-1. Description of Symbols Defined in Nomenclature (Continued)

Output Symbol	Description or Meaning in Terms of Symbols Defined in the Nomenclature
OMEGA	The current value of $\overline{\omega}$
OMEGA DOT	The current value of $\dot{\overline{\omega}}$
MASS CTR POSITION	The current value of $\overline{S}$
IN INTERTIAL COORDINATES COMP X, Y, Z	Designates the X, Y, Z, components of vectors referred to the inertial coordinate system
REFERENCE PT POSITION	The current value of $\overline{\rho}$
MASS CTR POSITION	The current value of $\overline{\rho} + \overline{S}$
MASS CTR VELOCITY	The current value of $\overline{P}/M_T$
MASS CTR ACCELERATION	The current value of $\overline{F}/M_T$
ANGULAR MOMENTUM	The current value of $\overline{H}_o$
LINEAR MOMENTUM	The current value of $\overline{P}$
KINETIC ENERGY	The current value of T
NUTATION ANGLE	The current value of $\cos^{-1}\left[\overline{H}_o(o) \cdot \hat{x}/\lvert\overline{H}_o\rvert\right]$
EULER ANGLES OF MAIN BODY	The current value of ψ_M, θ_M, ϕ_M
MAIN BODY FIXED UNIT VECTORS IN INERTIAL COORDINATES	The current value of the direction cosines of $\hat{x}$, $\hat{y}$, $\hat{z}$.
N-BOOM DYNAMIC STRESSES	Title on each page of output from the stress package

Table 18.1. Description of Symbols Defined in Nomenclature (Continued)

Output Symbol	Description or Meaning in Terms of Symbols Defined in the Nomenclature
APP k SEG i CIRCULAR OR NON-CIRCULAR	Sub-title preceding stress and loads data for each segment indicating segment number and shape
STATION j	Heading preceding block of output for station j of segment k,i
ANG	The angle, in 90° increments, measured from the segment ξ-axis to which the stress output corresponds.
XI-Q	The ξ - coordinate at a 90° point on the segment crossection
ZETA-Q	The ζ - coordinate at a 90° point on the segment crossection
SIGMA22, SIGMA21, AND SIGM23	The stresses in the η, ξ and ζ directions, respectively, on a plane normal to the η-axis at a 90° point.
PRINCIPAL STRESS SIGMA-11, SIGMA-22, SIGMA-33	The eigenvalues of the stress tensor defined by σ_{21}, σ_{23}, σ_{22}, above, at each 90° point
Q-BAR	The ξ, η and ζ - components of the moment at this station.
P-BAR	The ξ, η and ζ - components of the force at this station.
THETA-MAX	The angular position measured from ξ to the most severe combined stress condition if this is a circular segment.
SIGMA 22-MAX, SIGMA 21-MAX, and SIGMA 23-MAX	The stress components at the position of the most severe combined stress condition on the crossection.
XI-MAX, ZETA-MAX	The coordinates, ξ,ζ, of the position of the most severe combined stress condition on a non-circular section.

Stress Package Error Processing

Since there are no iterative calculations where a non-convergence problem might occur and the input data is relatively simple to enter, only two types of errors are checked in the stress package. A test is made for the correct entry of key counters in the input data and a test is made for a divisor to be zero before any division is performed. An error in the input counters terminates the processing of the case. If a divide check occurs, the result in question is set to 0 and processing continues. In either situation an error message is printed out describing the problem. The error messages for input errors immediately follow the printing of the input data. The error messages for divide checks are interspersed with the data output.

A. INPUT ERROR CHECKS

All input checking and printing of messages is completed before processing of the case is discontinued.

1. If the number of stations in NSTA for a segment has not been entered or exceeds 5, then an error message is printed in the format:

   NSTA-n IS NØT FILLED IN ØR IS TØØ LARGE

   where n is the segment number.

2. If, the number of vertices for a station within a segment has not been entered or exceeds 6, then an error message is printed in the format:

   NVER-n,m IS NØT FILLED IN ØR IS TØØ LARGE

   where n is the segment number and m is the station number.

B. COMPUTATIONAL CHECKS

A test for possible divide checks is made in all computational sections. The section where the divide check could occur is identified as well as the name of the quantity being computed. There is a different message for each section. In a case where this occurs, an error message in one of the following formats is printed:

1. DIVIDE CHECK IN MASS AND AREA RØUTINE-CØMPUTING _ _ _ _ _ _ SEGMENT _ _ STATIØN _
2. DIVIDE CHECK IN LØADS RØUTINE-CØMPUTING _ _ _ _ _ _ SEGMENT _ _ STATIØN _
3. DIVIDE CHECK IN CIRCULAR SEG STRESS RØUTINE-CØMPUTING _ _ _ _ _ _ SEGMENT _ _ STATIØN _
4. DIVIDE CHECK IN NØN-CIRC SEG STRESS RØUTINE-CØMPUTING _ _ _ _ _ _ _ SEGMENT _ _ STATIØN _

The six blanks immediately following the word computing are filled with the program name of the quantity being computed For example, SG2290 for σ_{2290_n} would be printed if either A or I were 0. The current segment and station numbers are printed as shown.

See the Table below for a list of engineering symbols versus programming symbol and equation number where quantity is computed

Prog. Symbol	Eng. Symbol	Equation No.
AMI	I	12.5
CXZ	C_ξ	12.3
D	For circular seg.	Not in manual (dependent on maximum V_o)
PBAR	$\overline{P}_j$	10.14
QBAR	$\overline{Q}_j$	10.14

Prog. Symbol	Eng. Symbol	Equation No.
CMBAR	$\overline{C}_j$	13.1
K	K_1,K_2,K_3	Page 11-5
QSTAR	Q_1^*, Q_3^*	11.9
D	For noncir. Seg.	Not in manual (dependent on maximum V_o within a plate element)
XIM-ZETA	For noncir. Seg.	Calculation of max ξ and ζ
A	$[A]_i$	12.8
SIGMA22	σ_{22}	11.3 or 11.11
SIG21-23	σ_{21},σ_{23}	11.4 or 11.14
VSUBØ	V_o	11.6 or 11.15

C. TOO MANY ARITHMETIC ERRORS

If more than a predetermined number of arithmetic errors occur, the following message is printed and the case is discontinued:
MAXIMUM NUMBER ØF ARITHMETIC ERRØRS. CASE IS DISCØNTINUED

D. TABLE SEARCH ERRORS

The chance of any of these errors occurring should be minimal. This would arise only if the input data describes meaningless geometric figures in computing the area and mass properties.

If one of these errors occurs the program stops and an identification digit is displayed. The following is a list of these errors, their meaning and associated display digit:

	Display	Meaning
1.	1	Given ξ_{Q_k} , ξ_{Q_m} and $\xi_{Q_{m+1}}$ cannot be found within the ξ_Q table such that $\xi_{Q_m} \leq \xi_{Q_k} \leq \xi_{Q_{m+1}}$ or $\xi_{Q_m} \geq \xi_{Q_k} \geq \xi_{Q_{m+1}}$. The same situation could occur for the ζ_Q table also.
2.	2	Each entry in the ξ_Q table is equal to each other thus defining the cross-section as a straight line. Same thing could occur in the ζ_Q table.
3.	10	A ζ_ℓ and $\zeta_{\ell+1}$ cannot be found in this ζ table such that ζ_ℓ is $\leq C_\zeta$ and $\zeta_{\ell+1}$ is C_ζ. This can also occur in the ξ table.
4.	11	A ζ_ℓ and $\zeta_{\ell+1}$ cannot be found in the ζ table such that ζ_ℓ is $\geq C_\zeta$ and $\zeta_{\ell+1}$ is $> C_\zeta$. This can occur in the ξ table also.

```
                                        INPUT QUANTITIES
THE TOTAL NUMBER OF APPENDAGES #      2
NUMBER OF PADDLE APPENDAGES #      1
TOTAL NUMBER OF LINKS #      4
LINKS PER APPENDAGE #      2      2
THE MASS OF THE MAIN BODY #   0.25000000E 02


 VECTOR TO ORIGIN OF APPENDAGE COORDINATES (FT) FROM ORIGIN OF MAIN BODY COORDINATES

THE      D BAR K           ARRAY
  0.10000000E 02 -0.10000000E 02  0.0
  0.20000000E 02  0.20000000E 02  0.0


 EULER ANGLES USED TO EXPRESS APPENDAGE UNIT VECTORS IN MAIN BODY COORDINATES (DEG)

THE      PSI, THETA, PHI ARRAY
  0.90000000E 02  0.0             0.90000000E 02
  0.90000000E 02  0.0             0.0


 LENGTHS OF APPENDAGE SEGMENTS (FT)

THE      L SUB I,K         ARRAY
  0.20000000E 02  0.20000000E 02  0.20000000E 02
  0.40000000E 02


 MASS OF APPENDAGE SEGMENTS (SLUGS)

THE      M SUB I,K         ARRAY
  0.50000000E 00  0.39999998E 00  0.50000000E 00
  0.50000000E 00


 INERTIA MATRICES OF APPENDAGE SEGMENTS (SLUG-FT SQ) INSEGMENT COORDINATES

THE      I PRIME           ARRAY
  0.10000000E 02  0.0             0.0
  0.0             0.50000000E 01  0.0
  0.0             0.0             0.10000000E 02

THE      I PRIME           ARRAY
  0.45000000E 02  0.0             0.0
  0.0             0.40000000E 02  0.0
  0.0             0.0             0.85000000E 02

THE      I PRIME           ARRAY
  0.10000000E 02  0.0             0.0
  0.0             0.50000000E 01  0.0
  0.0             0.0             0.10000000E 02

THE      I PRIME           ARRAY
  0.20000000E 02  0.0             0.0
  0.0             0.10000000E 02  0.0
  0.0             0.0             0.20000000E 02


 VECTOR TO CENTER OF MASS (FT) OF APPENDAGE SEGMENTS

THE      C SUB I,K         ARRAY
  0.0             0.10000000E 02  0.0
```

```
 0.0              0.10000000E 02  0.0
 0.0              0.10000000E 02  0.0
 0.0              0.20000000E 02  0.0
```

INERTIA MATRIX OF MAIN BODY (SLUG-FT SQ) IN MAIN BODY COORDINATES

```
THE       I SUB M         ARRAY
 0.40000000E 04  0.0             0.0
 0.0             0.20000000E 04  0.0
 0.0             0.0             0.20000000E 04
```

CENTER OF MASS OF MAIN BODY (FT) IN MAIN BODY COORDINATES

```
THE      B BAR M          ARRAY
 0.0             0.0             0.0
```

DASH POT PARAMETERS (FT LB SEC,SQ , FT LB SEC , FT LB , FT LB)

```
THE       Q SUB I,K       ARRAY
 0.0            -0.99999993E-03  0.0
 0.0             0.0            -0.99999993E-03
 0.0             0.0             0.0
-0.99999993E-03  0.0             0.0
 0.0            -0.99999993E-03  0.0
 0.0
```

SPRING PARAMETERS (FT LB , FT LB)

```
THE       K SUB I,K       ARRAY
 0.0            -0.39999998E 00  0.0
-0.99999964E-01  0.0            -0.39999998E 00
 0.0            -0.39999998E 00
```

MAIN BODY ATTACHMENT POINT OF KICKOFF SPRING I

```
THE      R1IK             R2IK             R3IK ARRAY
 0.20000000E 02  0.0             0.10000000E 02
 0.10000000E 02  0.0             0.10000000E 02
 0.10000000E 02  0.18000000E 02  0.0
 0.10000000E 02  0.18000000E 02 -0.50000000E 01
```

SEGMENT ATTACHMENT POINT OF KICKOFF SPRING I

```
THE    S1IK              S2IK             S3IK ARRAY
 0.0             0.10000000E 02  0.20000000E 01
 0.0             0.20000000E 02  0.0
 0.0             0.20000000E 02  0.0
 0.0             0.0             0.50000000E 01
```

SPRING CONSTANTS FOR KICKOFF SPRING I

```
THE      AES              ARRAY
 0.10000000E 02 -0.19999999E 00  0.0             0.0
 0.40000000E 01 -0.50000000E 00  0.49999997E-01  0.0
 0.10000000E 01 -0.20000000E-01  0.0             0.0
 0.20000000E 02 -0.59999996E 00  0.0             0.0
```

DISENGAGEMENT LENGTH OF KICKOFF SPRING I

```
THE      X SUB F IK        ARRAY
  0.90000000E 01  0.11000000E 02  0.50000000E 02
  0.30000000E 01

 DASH POT PRE LOAD ANGLES (RADIANS)

THE       GAMMA SUB I,K  ARRAY
  0.0            0.0              0.0
  0.0

 SPRING PRE LOAD ANGLES (RADIANS)

THE       THETA SUB I,K  ARRAY
 -0.20000000E 02 -0.32000000E 03 -0.20000000E 02
 -0.15000000E 03

 INITIAL MAIN BODY ANGULAR RATES (DEG/SEC)

THE       OMEGA          ARRAY
  0.57295752E 03  0.0             0.0

 INITIAL SEGMENT POSITIONS (DEG)

THE       ALPHA SUB I,K  ARRAY
  0.90000000E 02  0.27000000E 03  0.90000000E 02
  0.90000000E 02

THE       BETA SUB I,K   ARRAY
  0.0            0.0              0.0
  0.0

 INITIAL SEGMENT ANGULAR RATES (DEG/SEC)

THE       SIGMA SUB I,K  ARRAY
  0.0            0.0              0.0
  0.0

THE       TAU SUB I,K    ARRAY
  0.0            0.0              0.0
  0.0

 EULER ANGLES USED TO EXPRESS INERTIAL COORDINATE UNIT VECTORS IN MAIN BODY COORDINATES (DEG)

THE     PSI,THETA,PHI    ARRAY
  0.0            0.0              0.0
```

RELEASE SEGMENT NUMBER	TIME/SEG. NO RELEASE OPTION	TIME(SEC) OR ANGLE(DEG)
1	0	0.69999993E-02
2	1	0.89500000E 02
3	1	0.89509995E 02
4	2	0.18100000E 03

```
 LOCK UP ANGLE (BETA(I,K)) AT WHICH HINGE (I,K) IS LOCKED (DEG)

THE     BETA S SUB I,K  ARRAY
  0.0            0.36000000E 03  0.0
```

```
0.18000000E 03

ACCELERATION OF GRAVITY (FT/SEC SQ)
   G
    0.0

UNIT VECTOR FROM THE DIRECTION OF THE CENTER OF THE EARTH TO SPACECRAFT IN INERTIAL COORDINATES
   THE DIRECTION OF G
    0.0                 0.0                 0.0

THRUST MAGNITUDE (LB)
   THE FT
    0.0

THRUST DIRECTION IN MAIN BODY COORDINATES
   THE FT UNIT VECTOR
    0.0                 0.0                 0.0

VECTOR TO POINT OF APPLICATION OF THRUST (FT) IN MAIN BODY COORDINATES
   THE POSITION OF THRUST
    0.0                 0.0                 0.0

THE TIME OF THRUST INITIATION (SEC)
   THE T INITIAL
    0.0

THE TIME OF THRUST TERMINATION (SEC)
   THE T FINAL
    0.0
```

INPUT FOR STRESS CALCULATIONS

MATRIX - CMN

I	J	1
1	1	0.99999964E-01
2	1	0.99999964E-01
3	1	0.99999964E-01
4	1	0.99999964E-01

MATRIX - XNU

I	J	1
1	1	0.29999995E 00
2	1	0.29999995E 00
3	1	0.29999995E 00
4	1	0.29999995E 00

MATRIX - E

I	J	1
1	1	0.30000000E 08
2	1	0.30000000E 08
3	1	0.30000000E 08
4	1	0.30000000E 08

MATRIX - NSTA

I	J	1
1	1	1
2	1	3
3	1	2
4	1	1

MATRIX - ICIR

I	J	1
1	1	0
2	1	1
3	1	0
4	1	1

MATRIX - R

I	J	1	2	3	4
1	1	0.20000000E 01	0.0	0.20000000E 01	0.0
2	1	0.0	0.0	0.20000000E 01	0.0
3	1	0.0	0.0	0.0	0.0
4	1	0.0	0.0	0.0	0.0
5	1	0.0	0.0	0.0	0.0

MATRIX - DELM

I	J	1	2	3	4
1	1	0.50000000E 00	0.99999993E-03	0.99999993E-03	0.19999999E 00
2	1	0.0	0.19999999E 00	0.50000000E 00	0.0
3	1	0.0	0.19999999E 00	0.0	0.0
4	1	0.0	0.0	0.0	0.0
5	1	0.0	0.0	0.0	0.0

MATRIX - NVER

I	J	1	2	3	4
1	1	1	5	1	4
2	1	0	3	1	0
3	1	0	5	0	0
4	1	0	0	0	0
5	1	0	0	0	0

MATRIX - S

I	J	1	2	3	4
1	1	0.0	0.20000000E 02	0.20000000E 02	0.0
2	1	0.0	0.10000000E 02	0.0	0.0
3	1	0.0	0.0	0.0	0.0
4	1	0.0	0.0	0.0	0.0
5	1	0.0	0.0	0.0	0.0

MATRIX - DELC1

I	J	1
1	1	0.0
2	1	0.10000000E 02
3	1	0.0

MATRIX - T1

I	J	1
1	1	0.49999997E-01
2	1	0.0
3	1	0.0
4	1	0.0
5	1	0.0
6	1	0.0

MATRIX - XI1

I	J	1
1	1	0.0
2	1	0.0
3	1	0.0
4	1	0.0
5	1	0.0
6	1	0.0

MATRIX - ZETA1

I	J	1
1	1	0.0
2	1	0.0
3	1	0.0
4	1	0.0
5	1	0.0
6	1	0.0

MATRIX - DLI11

I	J	1	2	3
1	1	0.10000000E 02	0.0	0.0
2	1	0.0	0.50000000E 01	0.0
3	1	0.0	0.0	0.10000000E 02

MATRIX - DELC2

I	J	1	2	3
1	1	0.0	0.0	0.0
2	1	0.20000000E 02	0.15000000E 02	0.50000000E 01
3	1	0.0	0.0	0.0

MATRIX - T2

I	J	1	2	3
1	1	0.14999998E 00	0.49999997E-01	0.14999998E 00
2	1	0.14999998E 00	0.49999997E-01	0.14999998E 00
3	1	0.14999998E 00	0.49999997E-01	0.14999998E 00
4	1	0.14999998E 00	0.0	0.14999998E 00
5	1	0.14999998E 00	0.0	0.14999998E 00
6	1	0.0	0.0	0.0

MATRIX - XI2

I	J	1	2	3
1	1	0.21499996E 01	0.43299999E 01	0.21499996E 01
2	1	0.66999996E 00	-0.43299999E 01	0.66999996E 00
3	1	-0.17399998E 01	-0.43299999E 01	-0.17399998E 01
4	1	-0.17399998E 01	0.0	-0.17399998E 01
5	1	0.66999996E 00	0.0	0.66999996E 00
6	1	0.0	0.0	0.0

MATRIX - ZETA2

I	J	1	2	3
1	1	0.0	0.0	0.0
2	1	0.20400000E 01	0.50000000E 01	0.20400000E 01
3	1	0.12599993E 01	-0.50000000E 01	0.12599993E 01
4	1	-0.12599993E 01	0.0	-0.12599993E 01
5	1	-0.20400000E 01	0.0	-0.20400000E 01
6	1	0.0	0.0	0.0

MATRIX - DLI21

I	J	1	2	3
1	1	0.0	0.0	0.0
2	1	0.0	0.0	0.0
3	1	0.0	0.0	0.0

MATRIX - DLI22

I	J	1	2	3
1	1	0.10000000E 02	0.0	0.0
2	1	0.0	0.20000000E 02	0.0
3	1	0.0	0.0	0.30000000E 02

MATRIX - DLI23

I	J	1	2	3
1	1	0.10000000E 02	0.0	0.0
2	1	0.0	0.20000000E 02	0.0
3	1	0.0	0.0	0.30000000E 02

MATRIX - DELC3

I	J	1	2
1	1	0.0	0.0
2	1	0.20000000E 02	0.10000000E 02
3	1	0.0	0.0

MATRIX - T3

I	J	1	2
1	1	0.49999997E-01	0.49999997E-01
2	1	0.0	0.0
3	1	0.0	0.0
4	1	0.0	0.0
5	1	0.0	0.0
6	1	0.0	0.0

MATRIX - XI3

I	J	1	2
1	1	0.0	0.0
2	1	0.0	0.0
3	1	0.0	0.0
4	1	0.0	0.0
5	1	0.0	0.0
6	1	0.0	0.0

MATRIX - ZETA3

I	J	1	2
1	1	0.0	0.0
2	1	0.0	0.0
3	1	0.0	0.0
4	1	0.0	0.0
5	1	0.0	0.0
6	1	0.0	0.0

MATRIX - DLI31

I	J	1	2	3
1	1	0.0	0.0	0.0
2	1	0.0	0.0	0.0
3	1	0.0	0.0	0.0

MATRIX - DLI32

I	J	1	2	3
1	1	0.10000000E 02	0.0	0.0
2	1	0.0	0.50000000E 01	0.0
3	1	0.0	0.0	0.10000000E 02

MATRIX - DELC4

I	J	1
1	1	0.0
2	1	0.20000000E 02
3	1	0.0

MATRIX - T4

I	J	1
1	1	0.49999997E-01
2	1	0.99999964E-01
3	1	0.49999997E-01
4	1	0.99999964E-01
5	1	0.0
6	1	0.0

MATRIX - XI4

I	J	1
1	1	0.20000000E 02
2	1	-0.20000000E 02
3	1	-0.20000000E 02
4	1	0.20000000E 02
5	1	0.0
6	1	0.0

MATRIX - ZETA4

I	J	1
1	1	0.10000000E 01
2	1	0.10000000E 01
3	1	-0.10000000E 01
4	1	-0.10000000E 01
5	1	0.0
6	1	0.0

MATRIX - DLI41

I	J	1	2	3
1	1	0.20000000E 02	0.0	0.0
2	1	0.0	0.10000000E 02	0.0
3	1	0.0	0.0	0.[illegible]0000000E 02

TIME 0.0 NUMBER OF STEPS 0 TIME STEP 0.9999999E-03

APPENDAGE MOTION

APP	SEG	BETA (DEG)	BETA DOT (DEG/SEC)	BETA DDOT (DEG/SEC SQ)	ALPHA (DEG)	ALPHA DOT (DEG/SEC)	ALPHA DDOT (DEG/SEC SQ)
1	1	9.0000E 01	0.0	0.0	9.0000E 01	0.0	0.0
1	2	1.8000E 02	0.0	0.0	2.7000E 02	0.0	0.0
2	1	9.0000E 01	0.0	0.0	9.0000E 01	0.0	0.0
2	2	9.0000E 01	0.0	0.0	9.0000E 01	0.0	0.0

APP	SEG	RELEASE LOCK-UP STATE VECTOR	SPRING DASHPOT TORQUE FT LB	KICKOFF SPRING FORCE LBS	EXTENSION OF KICKOFF SPRING
1	1	-1	-4.8869E-01	8.4000E 00	8.0000E 00
1	2	-1	2.4435E-01	4.0000E 00	1.0000E 01
2	1	-1	-4.8869E-01	9.6000E-01	2.0000E 00
2	2	-1	4.1888E-01	1.0800E 01	2.0000E 00

SYSTEM MOTION

IN MAIN BODY COORDINATES

COMP	OMEGA (RAD/SEC)	OMEGA DOT (RAD/SEC SQ)	MASS CTR POSITION (FT)
X	1.0000E 01	-1.1555E-02	1.2268E 00
Y	0.0	-1.0588E 01	7.4349E-01
Z	0.0	0.4670E 00	6.6914E-01

IN INERTIAL COORDINATES

COMP	REFERENCE PT POSITION (FT)	MASS CTR POSITION (FT)	MASS CTR VELOCITY (FT/SEC)	MASS CTR ACCELERATION (FT/SEC SQ)	ANGULAR MOMENTUM (LB SEC FT)	LINEAR MOMENTUM (LB SEC)
X	0.0	1.2268E 00	0.0	0.0	4.8031E 04	0.0
Y	0.0	7.4349E-01	-6.6915E 00	0.0	-2.7546E 03	-1.8000E 02
Z	0.0	6.6914E-01	7.4349E 00	0.0	-3.3792E 03	2.0000E 02

KINETIC ENERGY= 2.4190E 05

NUTATION ANGLE (DEG) 5.1863E 00

EULER ANGLES (DEG)	PSI	THETA	PHI
	0.0	0.0	0.0

MAIN BODY FIXED UNIT VECTORS

X	Y	Z
1.00000E 00	0.0	0.0
0.0	1.00000E 00	0.0
0.0	0.0	1.00000E 00

TIME 0.6999999E-02 NUMBER OF STEPS 6 TIME STEP 0.9999997E-03

APPENDAGE MOTION

APP	SEG	BETA (DEG)	BETA DOT (DEG/SEC)	BETA DDOT (DEG/SEC SQ)	ALPHA (DEG)	ALPHA DOT (DEG/SEC)	ALPHA DDOT (DEG/SEC SQ)
1	1	9.0000E 01	0.0	0.0	9.0000E 01	0.0	0.0
1	2	1.8000E 02	0.0	0.0	2.7000E 02	0.0	0.0
2	1	9.0000E 01	0.0	0.0	9.0000E 01	0.0	0.0
2	2	9.0000E 01	0.0	0.0	9.0000E 01	0.0	0.0

APP	SEG	RELEASE LOCK-UP STATE VECTOR	SPRING DASHPOT TORQUE FT LB	KICKOFF SPRING FORCE LBS	EXTENSION OF KICKOFF SPRING
1	1	0	-4.8869E-01	8.4000E 00	8.0000E 00
1	2	-1	2.4435E-01	4.0000E 00	1.0000E 01
2	1	-1	-4.8869E-01	9.6000E-01	2.0000E 00
2	2	-1	4.1888E-01	1.8000E 01	2.0000E 00

SYSTEM MOTION

IN MAIN BODY COORDINATES

COMP	OMEGA (RAD/SEC)	OMEGA DOT (RAD/SEC SQ)	MASS CTR POSITION (FT)
X	9.9995E 00	-1.4112E-01	1.2268E 00
Y	-7.5137E-02	-1.0877E 01	7.4349E-01
Z	5.7934E-02	8.0837E 00	6.6914E-01

IN INERTIAL COORDINATES

COMP	REFERENCE PT POSITION (FT)	MASS CTR POSITION (FT)	MASS CTR VELOCITY (FT/SEC)	MASS CTR ACCELERATION (FT/SEC SQ)	ANGULAR MOMENTUM (LB SEC FT)	LINEAR MOMENTUM (LB SEC)
X	3.2854E-04	1.2269E 00	-5.9272E-08	0.0	4.8031E 04	-1.5944E-06
Y	1.5597E-03	7.0377E-01	-6.6914E 00	0.0	-2.7546E 03	-1.8000E 02
Z	1.3342E-03	7.1410E-01	7.4349E 00	0.0	-3.3792E 03	2.0000E 02

KINETIC ENERGY= 2.4190E 05

NUTATION ANGLE (DEG) 5.2003E 00

EULER ANGLES (DEG)	PSI	THETA	PHI
	1.4422E 02	1.3978E-02	-1.4079E 02

MAIN BODY FIXED UNIT VECTORS

X	Y	Z
1.00000E 00	-1.54240E-04	-1.89019E-04
1.42628E-04	9.98201E-01	-5.99631E-02
1.97927E-04	5.99631E-02	9.98201E-01

ACC. CHECK # 0.54420570E-06

DELTA # 0.40279701E-07

TOTAL KINETIC ENERGY # 0.24149963E 06

LINEAR MOMENTUM # -0.78976154E-06 -0.17999997E 03 0.19999983E 03

TIME 0.1902837E-01 NUMBER OF STEPS 13 TIME STEP 0.9999999E-03

APPENDAGE MOTION

APP	SEG	BETA (DEG)	BETA DOT (DEG/SEC)	BETA DDOT (DEG/SEC SQ)	ALPHA (DEG)	ALPHA DOT (DEG/SEC)	ALPHA DDOT (DEG/SEC SQ)
1	1	8.9510E 01	-8.1184E 01	-6.6954E 03	8.9510E 01	-8.1184E 01	-6.6954E 03
1	2	1.8000E 02	0.0	0.0	2.6951E 02	-8.1184E 01	-6.6954E 03
2	1	9.0000E 01	0.0	-3.3165E 03	9.0000E 01	0.0	-3.3165E 03
2	2	9.0000E 01	0.0	0.0	9.0000E 01	0.0	-3.3165E 03

APP	SEG	RELEASE LOCK-UP STATE VECTOR	SPRING DASHPOT TORQUE FT LB	KICKOFF SPRING FORCE LBS	EXTENSION OF KICKOFF SPRING
1	1	0	-4.6385E-01	8.3829E 00	8.0856E 00
1	2	-1	2.4435E-01	4.0000E 00	1.0000E 01
2	1	0	-4.6869E-01	9.6000E-01	2.0000E 00
2	2	-1	4.1880E-01	1.8800E 01	2.0000E 00

SYSTEM MOTION

IN MAIN BODY COORDINATES

COMP	OMEGA (RAD/SEC)	OMEGA DOT (RAD/SEC SQ)	MASS CTR POSITION (FT)
X	9.9993E 00	-1.4807E 00	1.2269E 00
Y	-1.0931E-01	-3.1756E 00	7.4349E-01
Z	1.6557E-01	-7.6509E-01	6.7201E-01

IN INERTIAL COORDINATES

COMP	REFERENCE PT POSITION (FT)	MASS CTR POSITION (FT)	MASS CTR VELOCITY (FT/SEC)	MASS CTR ACCELERATION (FT/SEC SQ)	ANGULAR MOMENTUM (LB SEC FT)	LINEAR MOMENTUM (LB SEC)
X	2.0710E-03	1.2268E 00	1.0309E-05	0.0	4.8031E 04	2.7716E-04
Y	1.1540E-02	6.1617E-01	-6.6914E 00	0.0	-2.7946E 03	-1.8000E 02
Z	8.2249E-03	8.1062E-01	7.4349E 00	0.0	-3.3792E 03	2.0000E 02

KINETIC ENERGY= 2.4190E 05

NUTATION ANGLE (DEG) 5.3042E 00

EULER ANGLES (DEG) PSI THETA PHI
1.3877E 02 1.1787E-01 -1.2787E 02

MAIN BODY FIXED UNIT VECTORS

X	Y	Z
9.99998E-01	-1.62413E-03	-1.26278E-03
1.35597E-03	9.81949E-01	-1.89142E-01
1.54717E-03	1.89139E-01	9.81949E-01

DELTA # 0.52154064E-07

TOTAL KINETIC ENERGY # 0.24150075E 06

LINEAR MOMENTUM # 0.27716160E-03 -0.17999905E 03 0.19999863E 03

TIME 0.1914658E-01 NUMBER OF STEPS 14 TIME STEP 0.9999959E-03

APPENDAGE MOTION

APP	SEG	BETA (DEG)	BETA DOT (DEG/SEC)	BETA DDOT (DEG/SEC SQ)	ALPHA (DEG)	ALPHA DOT (DEG/SEC)	ALPHA DDOT (DEG/SEC SQ)
1	1	8.9500E 01	-8.1977E 01	-7.0896E 03	8.9500E 01	-8.1977E 01	-7.0896E 03
1	2	1.8000E 02	0.0	5.7608E 03	2.6950E 02	-8.1977E 01	-1.3289E 03
2	1	9.0000E 01	-3.9169E-01	-3.3151E 03	9.0000E 01	-3.9169E-01	-3.3151E 03
2	2	9.0000E 01	0.0	0.0	9.0000E 01	0.0	-3.3151E 03

APP	SEG	RELEASE LOCK-UP STATE VECTOR	SPRING DASHPOT TORQUE FT LB	KICKOFF SPRING FORCE LBS	EXTENSION OF KICKOFF SPRING
1	1	0	-4.6377E-01	8.3825E 00	8.0873E 00
1	2	0	2.4435E-01	4.0000E 00	1.0000E 01
2	1	0	-4.8860E-01	9.6000E-01	2.0001E 00
2	2	-1	4.1888E-01	1.8800E 01	2.0001E 00

SYSTEM MOTION

IN MAIN BODY COORDINATES

COMP	OMEGA (RAD/SEC)	OMEGA DOT (RAD/SEC SQ)	MASS CTR POSITION (FT)
X	9.9991E 00	-1.3455E 00	1.2268E 00
Y	-1.0967E-01	-1.6874E 00	7.4350E-01
Z	1.6549E-01	-7.3515E-01	6.7296E-01

IN INERTIAL COORDINATES

COMP	REFERENCE PT POSITION (FT)	MASS CTR POSITION (FT)	MASS CTR VELOCITY (FT/SEC)	MASS CTR ACCELERATION (FT/SEC SQ)	ANGULAR MOMENTUM (LB SEC FT)	LINEAR MOMENTUM (LB SEC)
X	2.0949E-03	1.2268E 00	1.7678E-05	0.0	4.8031E 04	4.7553E-04
Y	1.1687E-02	6.1539E-01	-6.6914E 00	0.0	-2.7546E 03	-1.0000E 02
Z	8.3163E-03	8.1150E-01	7.4349E 00	0.0	-3.3791E 03	2.0000E 02

KINETIC ENERGY= 2.4150E 05

NUTATION ANGLE (DEG) 5.3054E 00

```
EULER ANGLES (DEG)     PSI          THETA          PHI
                    1.3872E 02   1.1922E-01   -1.2775E 02


MAIN BODY FIXED UNIT VECTORS
      X                          Y                          Z
   9.99998E-01              -1.64516E-03               -1.27395E-03
   1.37266E-03               9.81725E-01               -1.90302E-01
   1.56374E-03               1.90300E-01                9.81725E-01

DELTA #    0.89406967E-07

TOTAL KINETIC ENERGY #   0.24150050E 06

LINEAR MOMENTUM #  0.47552586E-03 -0.17999806E 03  0.19999767E 03
```

TIME 0.3756469E-01 NUMBER OF STEPS 25 TIME STEP 0.9999999E-03

APPENDAGE MOTION

APP	SEG	BETA (DEG)	BETA DOT (DEG/SEC)	BETA DDOT (DEG/SEC SQ)	ALPHA (DEG)	ALPHA DOT (DEG/SEC)	ALPHA DDOT (DEG/SEC SQ)
1	1	8.6734E 01	-2.1908E 02	-7.8127E 03	8.6734E 01	-2.1908E 02	-7.8127E 03
1	2	1.8100E 02	1.0012E 02	6.0060E 03	2.6773E 02	-1.1096E 02	-1.8967E 03
2	1	8.9434E 01	-5.9912E 01	-3.1499E 03	8.9434E 01	-5.9912E 01	-3.1499E 03
2	2	9.0000E 01	0.0	3.0674E 02	9.0000E 01	0.0	-2.9432E 03

APP	SEG	RELEASE LOCK-UP STATE VECTOR	SPRING DASHPOT TORQUE FT LB	KICKOFF SPRING FORCE LBS	EXTENSION OF KICKOFF SPRING
1	1	0	-4.6207E-01	8.2853E 00	8.5735E 00
1	2	0	2.4071E-01	4.1804E 00	1.0349E 01
2	1	0	-4.8369E-01	9.5605E-01	2.1977E 00
2	2	0	4.1808E-01	1.8681E 01	2.1977E 00

SYSTEM MOTION

IN MAIN BODY COORDINATES

COMP	OMEGA (RAD/SEC)	OMEGA DOT (RAD/SEC SQ)	MASS CTR POSITION (FT)
X	9.9381E 00	-5.2833E 00	1.2261E 00
Y	-9.1392E-02	3.7265E 00	7.5268E-01
Z	1.8411E-01	2.7871E 00	6.9080E-01

IN INERTIAL COORDINATES

COMP	REFERENCE PT POSITION (FT)	MASS CTR POSITION (FT)	MASS CTR VELOCITY (FT/SEC)	MASS CTR ACCELERATION (FT/SEC SQ)	ANGULAR MOMENTUM (LB SEC FT)	LINEAR MOMENTUM (LB SEC)
X	6.4612E-03	1.2268E 00	1.1801E-03	0.0	4.8031E 04	3.1745E-02
Y	4.0476E-02	4.3233E-01	-6.6502E 00	0.0	-2.7537E 03	-1.7997E 02
Z	2.4626E-02	9.4852E-01	7.4336E 00	0.0	-3.3783E 03	1.9996E 02

KINETIC ENERGY= 2.4150E 05

NUTATION ANGLE (DEG) 5.8165E 00

```
EULER  ANGLES (DEG)     PSI          THETA            PHI
                   1.3887E 02   3.3187E-01   -1.1738E 02


MAIN BODY FIXED UNIT VECTORS
      X                           Y                            Z
   9.99983E-01               -5.14353E-03                 -2.66356E-03
   3.80974E-03                9.30433E-01                 -3.66441E-01
   4.36306E-03                3.66425E-01                  9.30437E-01

DELTA #    0.13783574E-06

TOTAL KINETIC ENERGY #    0.24149800E 06

LINEAR MOMENTUM #  0.31744838E-01 -0.17996631E 03  0.19996410E 03
```

TIME 3.1667750E 00 NUMBER OF STEPS 72 TIME STEP 0.9999999E-03

APPENDAGE MOTION

APP	SEG	BETA (DEG)	BETA DOT (DEG/SEC)	BETA DDOT (DEG/SEC SQ)	ALPHA (DEG)	ALPHA DOT (DEG/SEC)	ALPHA DDOT (DEG/SEC SQ)
1	1	2.1344E-07	0.0	0.0	2.1344E-07	0.0	0.0
1	2	2.6770E 02	7.4608E 02	6.2189E 03	2.6700E 02	7.4608E 02	6.2189E 03
2	1	5.7329E 01	-4.5778E 02	-4.9089E 03	5.7329E 01	-4.5778E 02	-4.9089E 03
2	2	9.7685E 01	2.3213E 02	3.0308E 03	9.7685E 01	2.3213E 02	-1.8781E 03

APP	SEG	RELEASE LOCK-UP STATE VECTOR	SPRING DASHPOT TORQUE FT LB	KICKOFF SPRING FORCE LBS	EXTENSION OF KICKOFF SPRING
1	1	1	1.3963E-01	0.0	2.1541E 01
1	2	0	7.9483E-02	0.0	3.5173E 01
2	1	0	-2.5261E-01	7.3636E-01	1.3182E 01
2	2	0	3.6117E-01	0.0	1.3917E 01

SYSTEM MOTION

IN MAIN BODY COORDINATES

COMP	OMEGA (RAD/SEC)	OMEGA DOT (RAD/SEC SQ)	MASS CTR POSITION (FT)
X	7.4550E 00	-1.0566E 01	5.9665E-01
Y	-9.0370E-01	-4.1716E 00	1.2452E 00
Z	5.5823E-01	-3.4570E 01	1.1446E 00

IN INERTIAL COORDINATES

COMP	REFERENCE PT POSITION (FT)	MASS CTR POSITION (FT)	MASS CTR VELOCITY (FT/SEC)	MASS CTR ACCELERATION (FT/SEC SQ)	ANGULAR MOMENTUM (LB SEC FT)	LINEAR MOMENTUM (LB SEC)
X	5.6287E-01	1.2270E 00	-4.1600E-03	0.0	4.8045E 04	-1.1244E-01
Y	6.5534E-01	-3.7212E-01	-6.6930E 00	0.0	-2.7717E 03	-1.8004E 02
Z	5.9765E-01	1.9091E 00	7.4326E 00	0.0	-3.3646E 03	1.9994E 02

KINETIC ENERGY= 2.1903E 05

NUTATION ANGLE (DEG) 1.1959E 01

EULER ANGLES (DEG)	PSI	THETA	PHI
	1.1069E 02	7.1822E 00	-2.2642E 01

MAIN BODY FIXED UNIT VECTORS

X	Y	Z
9.92154E-01	-4.81317E-02	1.15388E-01
1.16963E-01	3.12917E-02	-9.92643E-01
4.41670E-02	9.98351E-01	3.66758E-02

```
                                   N-BOOM DYNAMIC STRESSES

                APPENDAGE 1                       SEGMENT  2                       NON-CIRCULAR

 STATION 1                                                                                PRINCIPLE STRESS
 ANG      XI-Q          ZETA-Q         SIGMA22        SIGMA21        SIGMA23        SIGMA-11       SIGMA-22       SIGMA-33
  0   0.21500E 01 -0.34617E-06 -0.16288E 01 -0.22343E-03 -0.12860E 00  0.10291E-01 -0.16389E 01  0.0
 90   0.95788E-03  0.18235E 01 -0.19046E 01 -0.24130E 01 -0.13912E-01  0.16419E 01 -0.35464E 01  0.0
180  -0.17400E 01 -0.34617E-06 -0.16285E 01 -0.78889E-03 -0.48566E-01  0.14472E-02 -0.16299E 01  0.0
270   0.95788E-03 -0.18235E 01 -0.11794E 01 -0.24117E 01 -0.14839E-01  0.18931E 01 -0.30725E 01  0.0
                    Q-BAR          P-BAR          XI-MAX        ZETA-MAX     SIGMA22-MAX   SIGMA21-MAX   SIGMA23-MAX
                 0.0           -0.26526E 01   0.67000E 00   0.20400E 01 -0.19046E 01 -0.27208E 01 -0.25595E-01
                 0.0           -0.28484E 01
                 0.0           -0.12863E 00

 STATION 2                                                                                PRINCIPLE STRESS
 ANG      XI-Q          ZETA-Q         SIGMA22        SIGMA21        SIGMA23        SIGMA-11       SIGMA-22       SIGMA-33
  0   0.43300E 01  0.0             0.94066E 02 -0.21048E 03 -0.29322E 03  0.41103E 03 -0.21696E 03  0.0
 90  -0.14434E 01  0.33333E 01 -0.13472E 04 -0.23140E 03 -0.17876E 03  0.60726E 02 -0.14079E 04  0.0
180  -0.43300E 01  0.0            -0.13472E 04  0.21048E 03 -0.12152E 03  0.42504E 02 -0.13897E 04  0.0
270  -0.14434E 01 -0.33333E 01  0.40181E 03 -0.44188E 03  0.18581E 03  0.72065E 03 -0.31885E 03  0.0
                    Q-BAR          P-BAR          XI-MAX        ZETA-MAX     SIGMA22-MAX   SIGMA21-MAX   SIGMA23-MAX
                 0.38468E 03 -0.37875E 03  0.43300E 01   0.0          -0.13472E 04 -0.26296E 03 -0.20737E 03
                 0.42798E 03 -0.42566E 03
                 0.29167E 04 -0.12676E 03

 STATION 3                                                                                PRINCIPLE STRESS
 ANG      XI-Q          ZETA-Q         SIGMA22        SIGMA21        SIGMA23        SIGMA-11       SIGMA-22       SIGMA-33
  0   0.21500E 01 -0.34617E-06 -0.15187E 06  0.70701E 02 -0.68083E 03  0.30625E 01 -0.15187E 06  0.0
 90   0.95788E-03  0.18235E 01 -0.11826E 07 -0.24947E 03 -0.26779E 03  0.0           -0.11826E 07  0.0
180  -0.17400E 01 -0.34617E-06 -0.58487E 06  0.25303E 03 -0.92093E 02  0.93750E-01 -0.58487E 06  0.0
270   0.95788E-03 -0.18235E 01  0.10912E 07 -0.65868E 03  0.29374E 02  0.10912E 07  0.0            0.0
                    Q-BAR          P-BAR          XI-MAX        ZETA-MAX     SIGMA22-MAX   SIGMA21-MAX   SIGMA23-MAX
                -0.15463E 04 -0.44601E 03  0.21500E 01   0.0          -0.11826E 07 -0.17917E 02 -0.60151E 03
                 0.87597E 03 -0.55473E 03
                 0.80502E 04 -0.46326E 0.
```

N-BOOM DYNAMIC STRESSES

APPENDAGE 1 SEGMENT 1 CIRCULAR

STATION 1

ANG	XI-Q	ZETA-Q	SIGMA22	SIGMA21	SIGMA23	PRINCIPLE STRESS SIGMA-11	SIGMA-22	SIGMA-33
0			0.47377E 05	0.0	-0.19900E 04	0.47461E 05	-0.83438E 02	0.0
90			0.31870E 05	-0.47638E 03	-0.62481E-03	0.31877E 05	-0.71172E 01	0.0
180			-0.51729E 05	-0.29915E-03	-0.87093E 03	0.14656E 02	-0.51744E 05	0.0
270			-0.36221E 05	-0.15954E 04	-0.82020E-03	0.70135E 02	-0.36292E 05	0.0

Q-BAR	P-BAR	THETA-MAX
-0.10696E 05	-0.32544E 03	0.41375E 01
0.70312E 03	-0.13671E 04	
0.15569E 05	-0.44939E 03	

SIGMA22-MAX	SIGMA21-MAX	SIGMA23-MAX
-0.57692E 05	-0.13390E 04	-0.47353E 03

N-BOOM DYNAMIC STRESSES

APPENDAGE 2 SEGMENT 2 NON-CIRCULAR

STATION 1

ANG	XI-Q	ZETA-Q	SIGMA22	SIGMA21	SIGMA23	SIGMA-11 (PRINCIPLE STRESS)	SIGMA-22 (PRINCIPLE STRESS)	SIGMA-33 (PRINCIPLE STRESS)
0	0.20000E 02	0.0	0.99986E 03	-0.25563E 02	-0.22377E 03	0.10483E 04	-0.48392E 02	0.0
90	0.0	0.10000E 01	0.85268E 03	0.12486E 03	-0.63908E 01	0.87063E 03	-0.17953E 02	0.0
180	-0.20000E 02	0.0	-0.86418E 03	0.25563E 02	-0.22377E 03	0.55177E 02	-0.91936E 03	0.0
270	0.0	-0.10000E 01	-0.71700E 03	-0.29568E 01	0.12782E 02	0.23999E 00	-0.71724E 03	0.0

Q-BAR	P-BAR	XI-MAX	ZETA-MAX	SIGMA22-MAX	SIGMA21-MAX	SIGMA23-MAX
-0.35487E 04	-0.51250E 02	-0.20000E 02	-0.10000E 01	0.99986E 03	-0.43304E 02	0.0
0.20453E 03	0.29850E 03					
0.25512E 04	-0.92507E 02					

```
                                          N-BCOM DYNAMIC STRESSES

                    APPENDAGE 2                     SEGMENT  1                       CIRCULAR

STATION 1                                                                                  PRINCIPLE STRESS
 ANG      XI-Q         ZETA-Q      SIGMA22       SIGMA21       SIGMA23       SIGMA-11      SIGMA-22      SIGMA-33
  0                             0.53093E 04  0.0           0.22632E 04  0.61431E 04 -0.83380E 03  0.0
 90                             0.11674E 05 -0.17882E 04  0.71060E-03  0.11941E 05 -0.26779E 03  0.0
180                            -0.45541E 04 -0.11229E-02 -0.97897E 03  0.20153E 03 -0.47556E 04  0.0
270                            -0.12918E 05  0.14539E 04 -0.92195E-03  0.19030E 03 -0.11109E 05  0.0
                      Q-BAR         P-BAR      THETA-MAX                 SIGMA22-MAX  SIGMA21-MAX  SIGMA23-MAX
                 -0.35497E 04 -0.52510E 02  0.78540E 00                 0.11852E 05 -0.12645E 04  0.16003E 04
                 -0.20371E 04  0.23726E 03
                  0.15493E 04  0.20173E 03

STATION 2                                                                                  PRINCIPLE STRESS
 ANG      XI-Q         ZETA-Q      SIGMA22       SIGMA21       SIGMA23       SIGMA-11      SIGMA-22      SIGMA-33
  0                             0.15999E 05  0.0           0.34957E 04  0.16730E 05 -0.73042E 03  0.0
 90                            -0.10649E 05 -0.26105E 04  0.10976E-02  0.60552E 03 -0.11255E 05  0.0
180                            -0.16495E 05 -0.16393E-02 -0.20324E 03  0.25000E 01 -0.16497E 05  0.0
270                             0.10154E 05  0.10884E 04 -0.19141E-03  0.10269E 05 -0.11536E 03  0.0
                      Q-BAR         P-BAR      THETA-MAX                 SIGMA22-MAX  SIGMA21-MAX  SIGMA23-MAX
                  0.32677E 04 -0.23910E 03  0.23562E 01                -0.19091E 05 -0.18459E 04 -0.14372E 03
                 -0.23241E 04 -0.15566E 03
                  0.51042E 04  0.51716E 03

ACC. CHECK #  0.55313958E-05

ACC. CHECK #  0.55320979E-05

ACC. CHECK #  0.49457731E-05

ACC. CHECK #  0.49388018E-05

DELTA #    0.35762787E-06

TOTAL KINETIC ENERGY #    0.21904694E 06

LINEAR MOMENTUM # -0.11233521E 00 -0.18004086E 03  0.19993771E 03
```

TIME 0.3281157E 00   NUMBER OF STEPS 113   TIME STEP 0.9959999E-03

APPENDAGE MOTION

APP	SEG	BETA (DEG)	BETA DOT (DEG/SEC)	BETA DDOT (DEG/SEC SQ)	ALPHA (DEG)	ALPHA DOT (DEG/SEC)	ALPHA DDOT (DEG/SEC SQ)
1	1	2.1344E-07	0.0	0.0	2.1344E-07	0.0	0.0
1	2	3.6000E 02	0.0	0.0	3.6000E 02	0.0	0.0
2	1	1.3340E-08	0.0	0.0	1.3340E-08	0.0	0.0
2	2	1.8000E 02	0.0	0.0	1.8000E 02	0.0	0.0

APP	SEG	RELEASE LOCK-UP STATE VECTOR	SPRING DASHPOT TORQUE FT LB	KICKOFF SPRING FORCE LBS	EXTENSION OF KICKOFF SPRING
1	1	1	1.3963E-01	0.0	2.1541E 01
1	2	1	-6.9813E-02	0.0	5.0000E 01
2	1	1	1.3963E-01	4.0536E-01	2.9732E 01
2	2	1	-2.0944E-01	0.0	3.3675E 01

SYSTEM MOTION

IN MAIN BODY COORDINATES

COMP	OMEGA (RAD/SEC)	OMEGA DOT (RAD/SEC SQ)	MASS CTR POSITION (FT)
X	6.3492E 00	4.0537E-01	-3.7175E-02
Y	-7.6055E-01	-9.4022E 00	1.6729E 00
Z	1.0141E 00	-8.2741E 00	1.3011E 00

IN INERTIAL COORDINATES

COMP	REFERENCE PT POSITION (FT)	MASS CTR POSITION (FT)	MASS CTR VELOCITY (FT/SEC)	MASS CTR ACCELERATION (FT/SEC SQ)	ANGULAR MOMENTUM (LB SEC FT)	LINEAR MOMENTUM (LB SEC)
X	1.3367E 00	1.2263E 00	-5.7848E-03	0.0	4.8061E 04	-1.5561E-01
Y	6.5444E-01	-1.4513E 00	-6.6516E 00	0.0	-2.7820E 03	-1.8000E 02
Z	3.3231E 00	3.1081E 00	7.4310E 00	0.0	-3.5116E 03	1.9989E 02

KINETIC ENERGY= 1.5550E 05

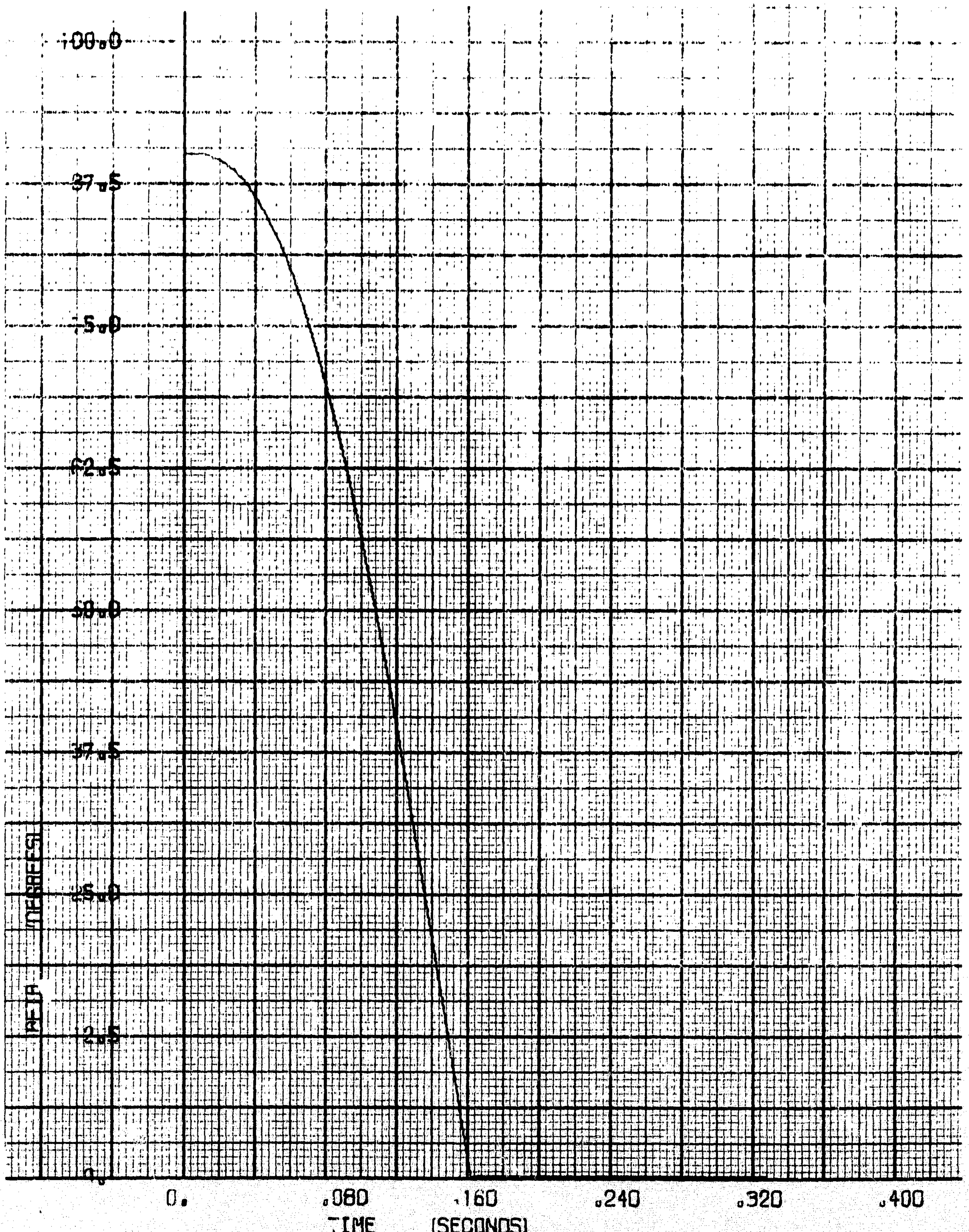
NUTATION ANGLE (DEG) 2.6484E 00

EULER ANGLES (DEG)	PSI	THETA	PHI
	-4.1699E 01	2.6861E 00	-1.7028E 02

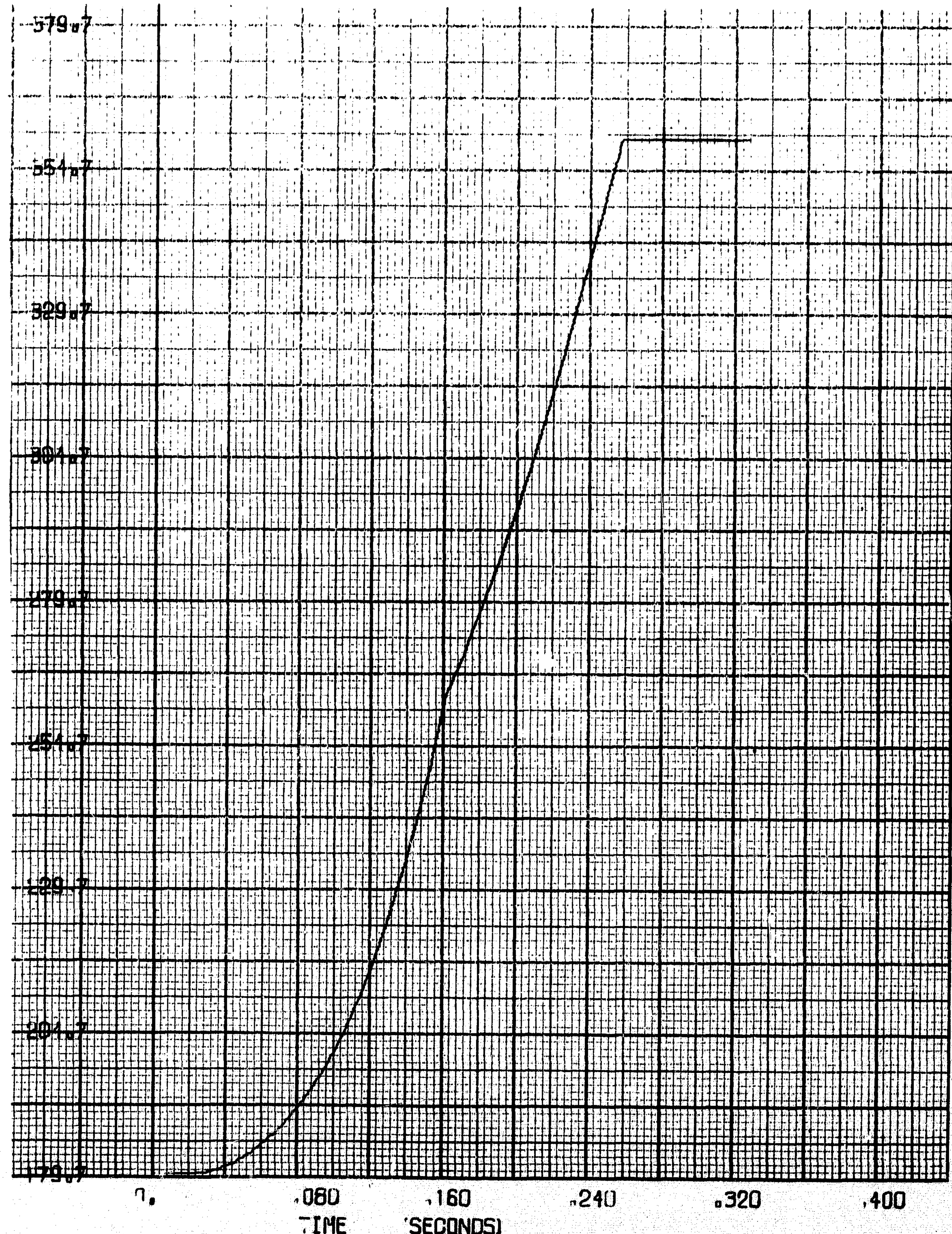
MAIN BODY FIXED UNIT VECTORS

X	Y	Z
9.98901E-01	-7.91016E-03	-4.61914E-02
-3.11746E-02	-8.48095E-01	-5.28926E-01
-3.49908E-02	5.29784E-01	-8.47409E-01

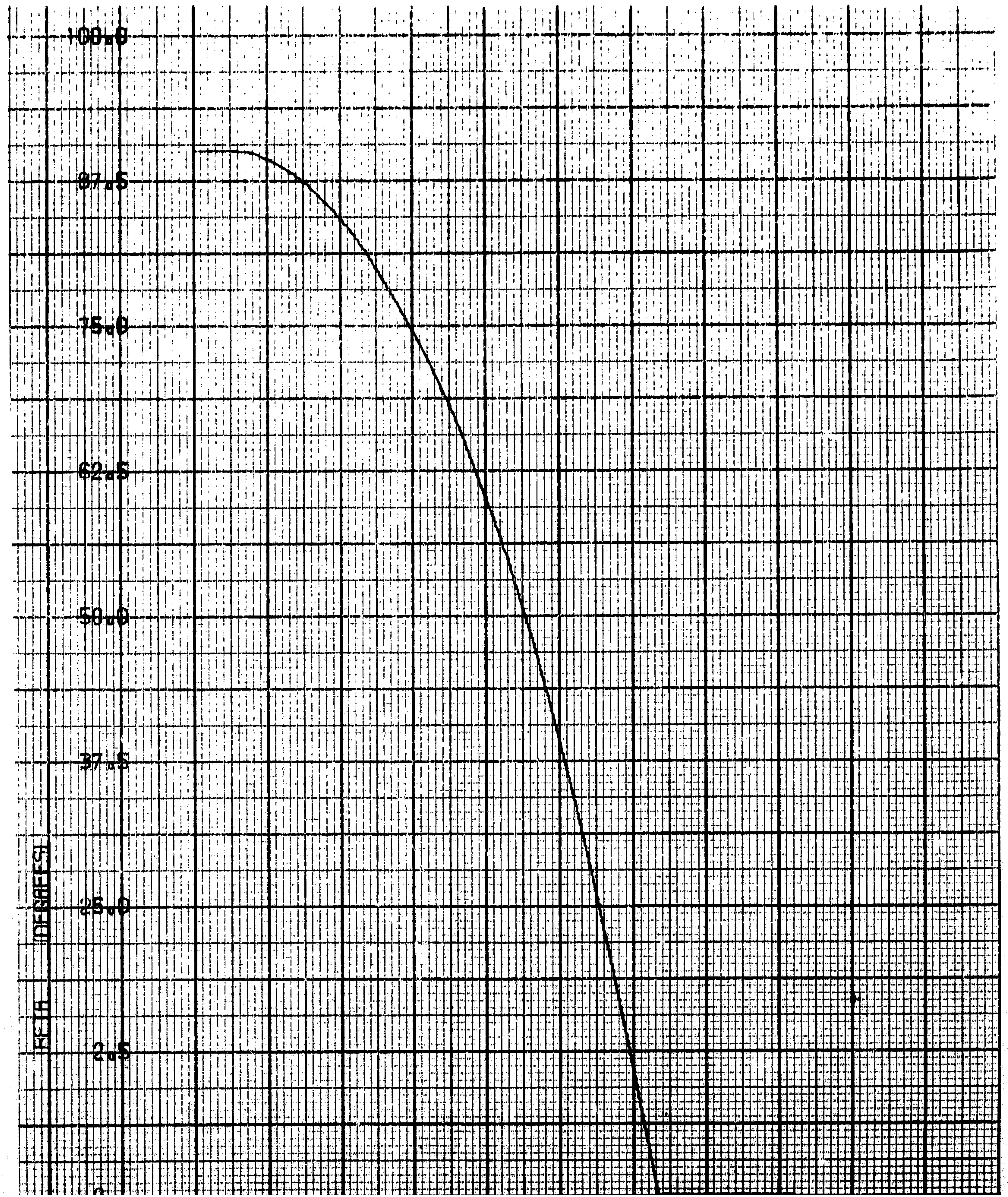
DEPLOYMENT ANGLE BETA, SEGMENT 1, APPENDAGE 1



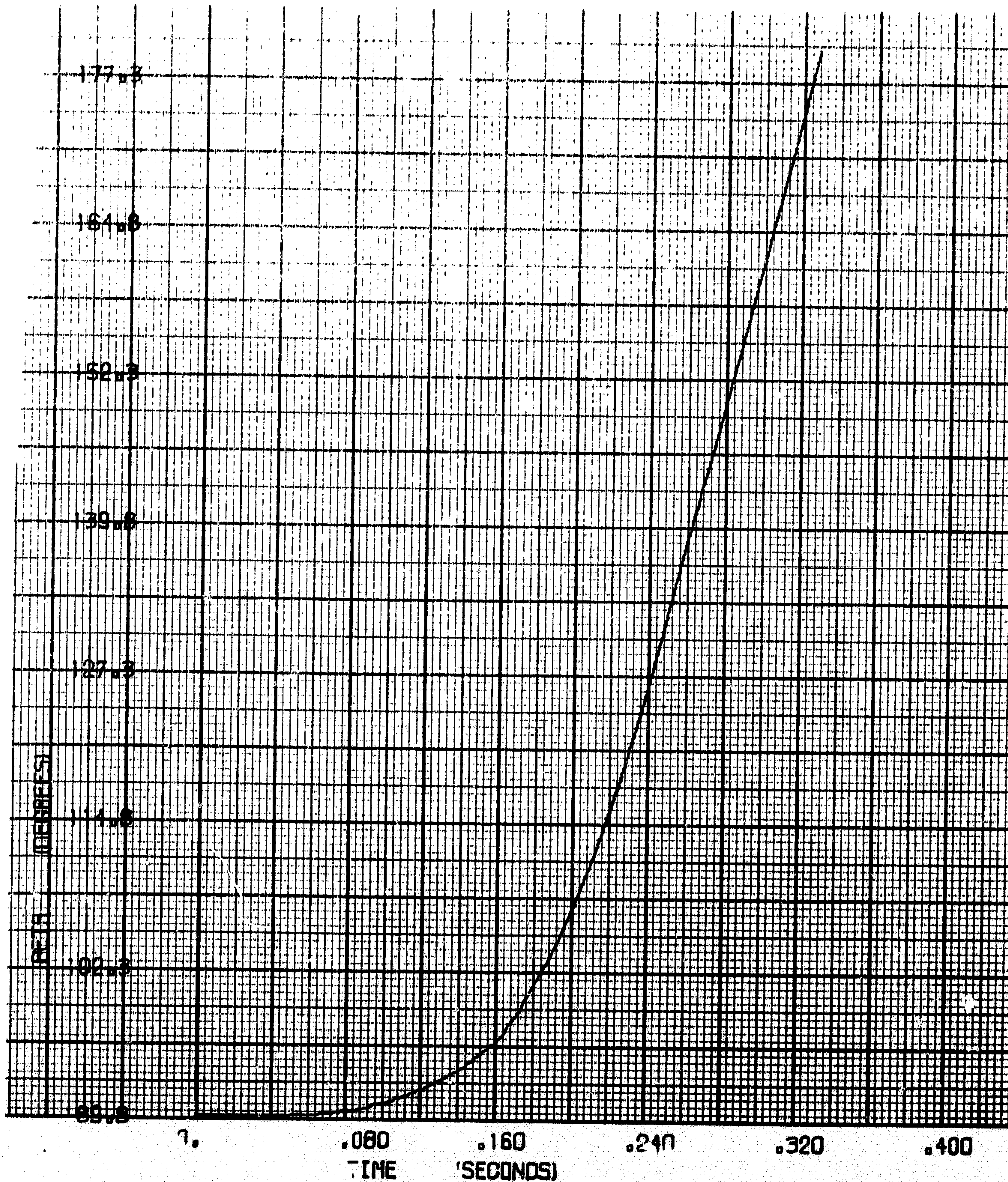
DEPLOYMENT ANGLE BETA, SEGMENT 2, APPENDAGE 1



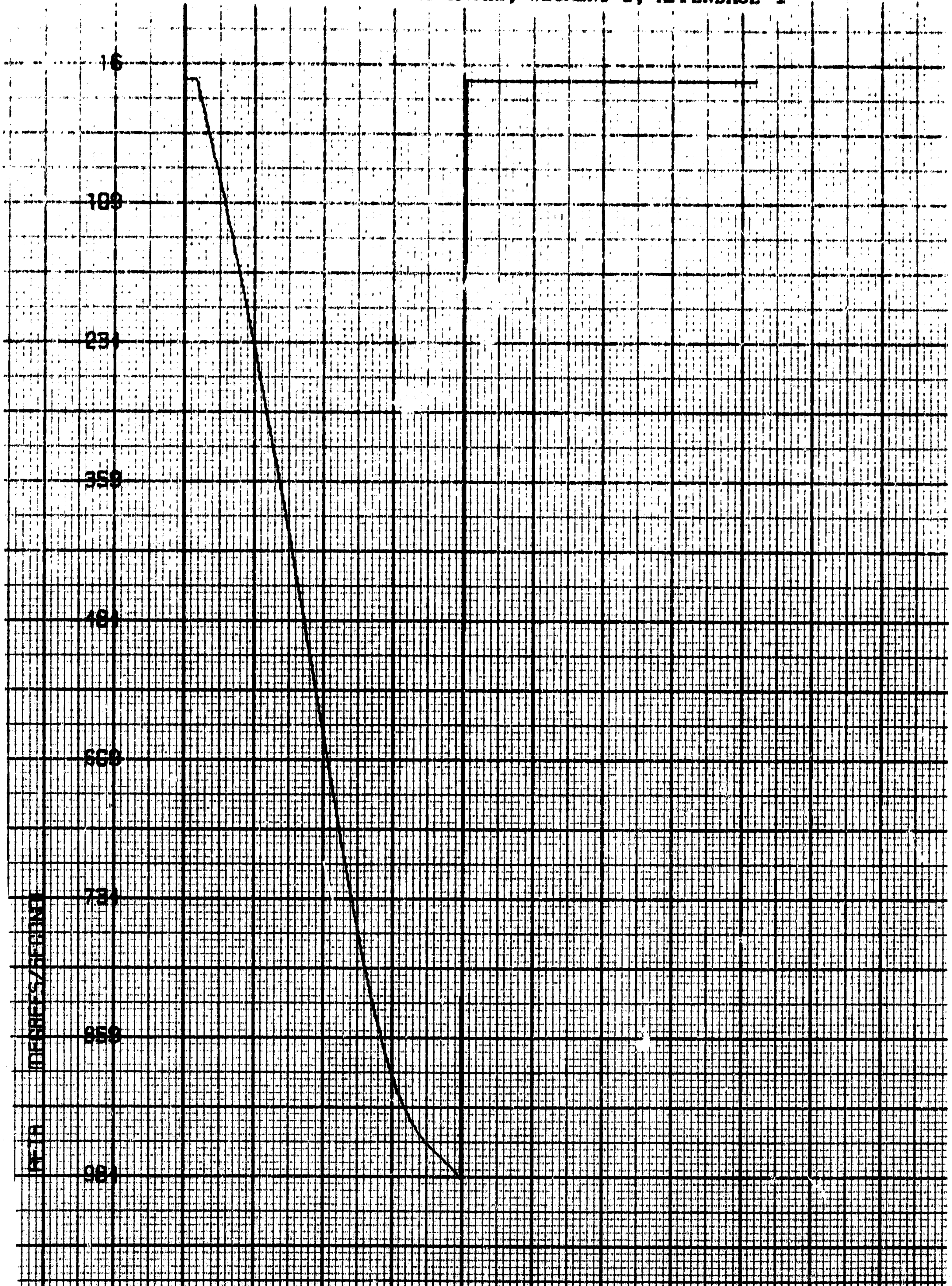
DEPLOYMENT ANGLE BETA, SEGMENT 1, APPENDAGE 2



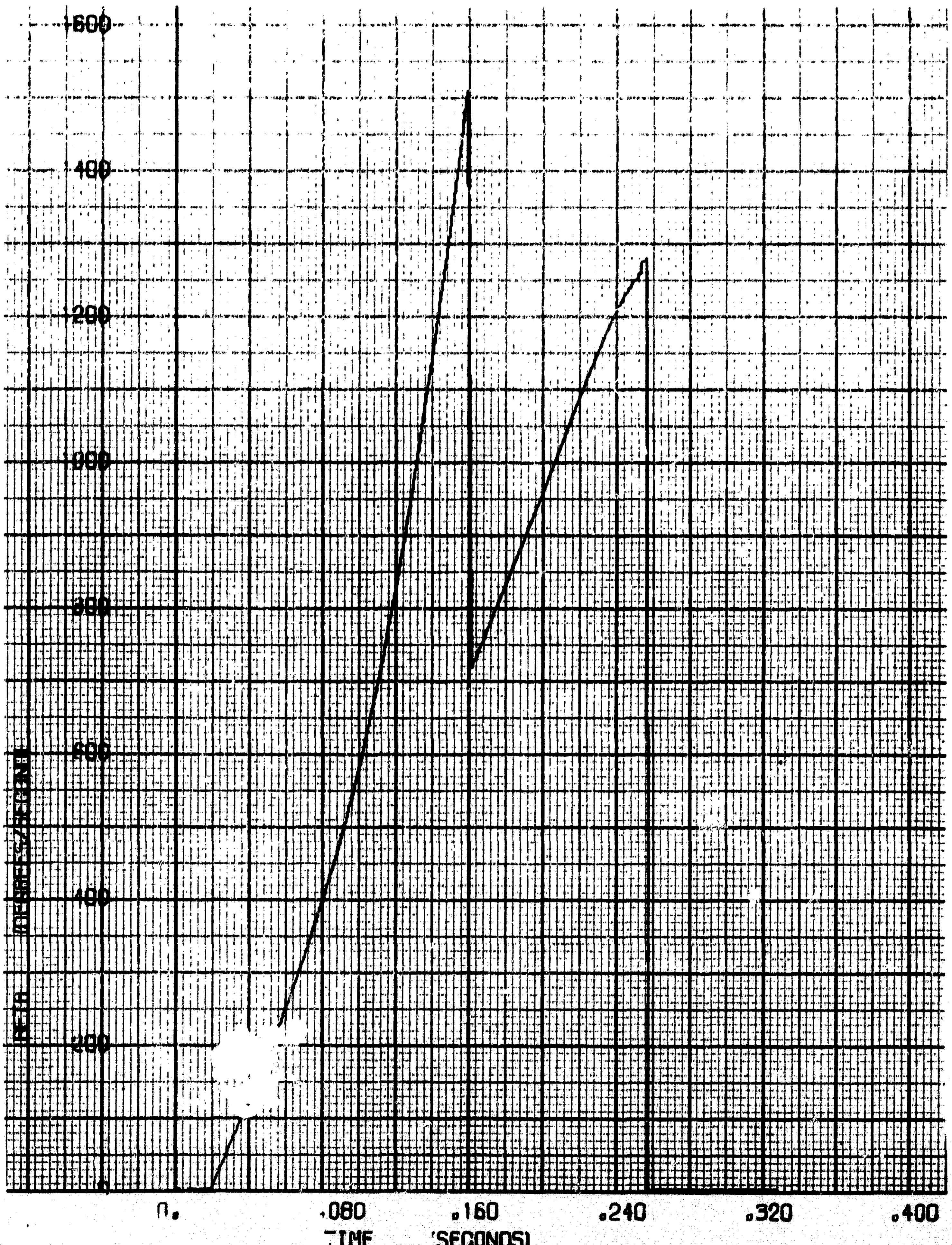
DEPLOYMENT ANGLE BETA, SEGMENT 2, APPENDAGE 2



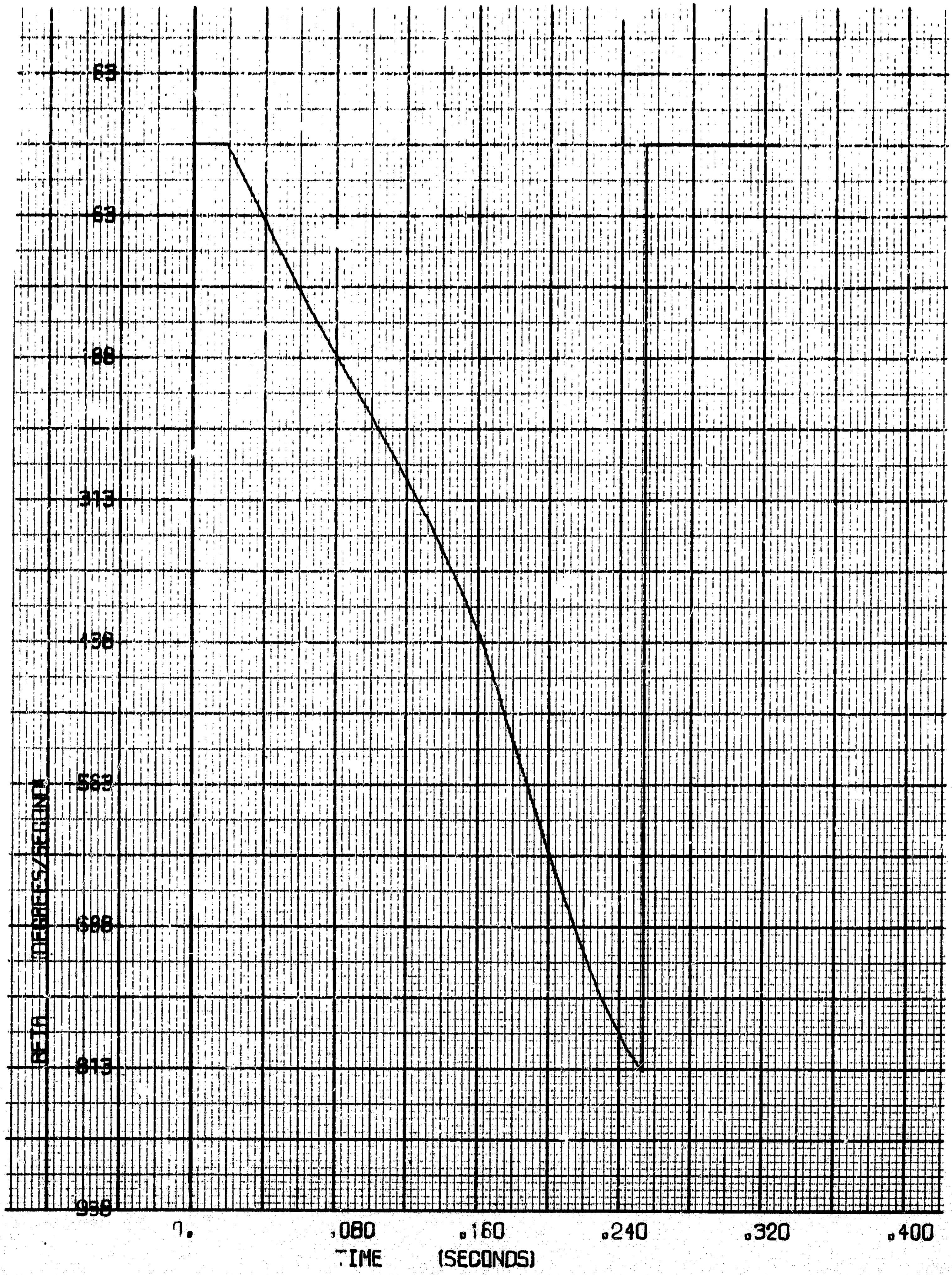
RATE OF CHANGE OF DEPLOYMENT ANGLE, SEGMENT 1, APPENDAGE 1



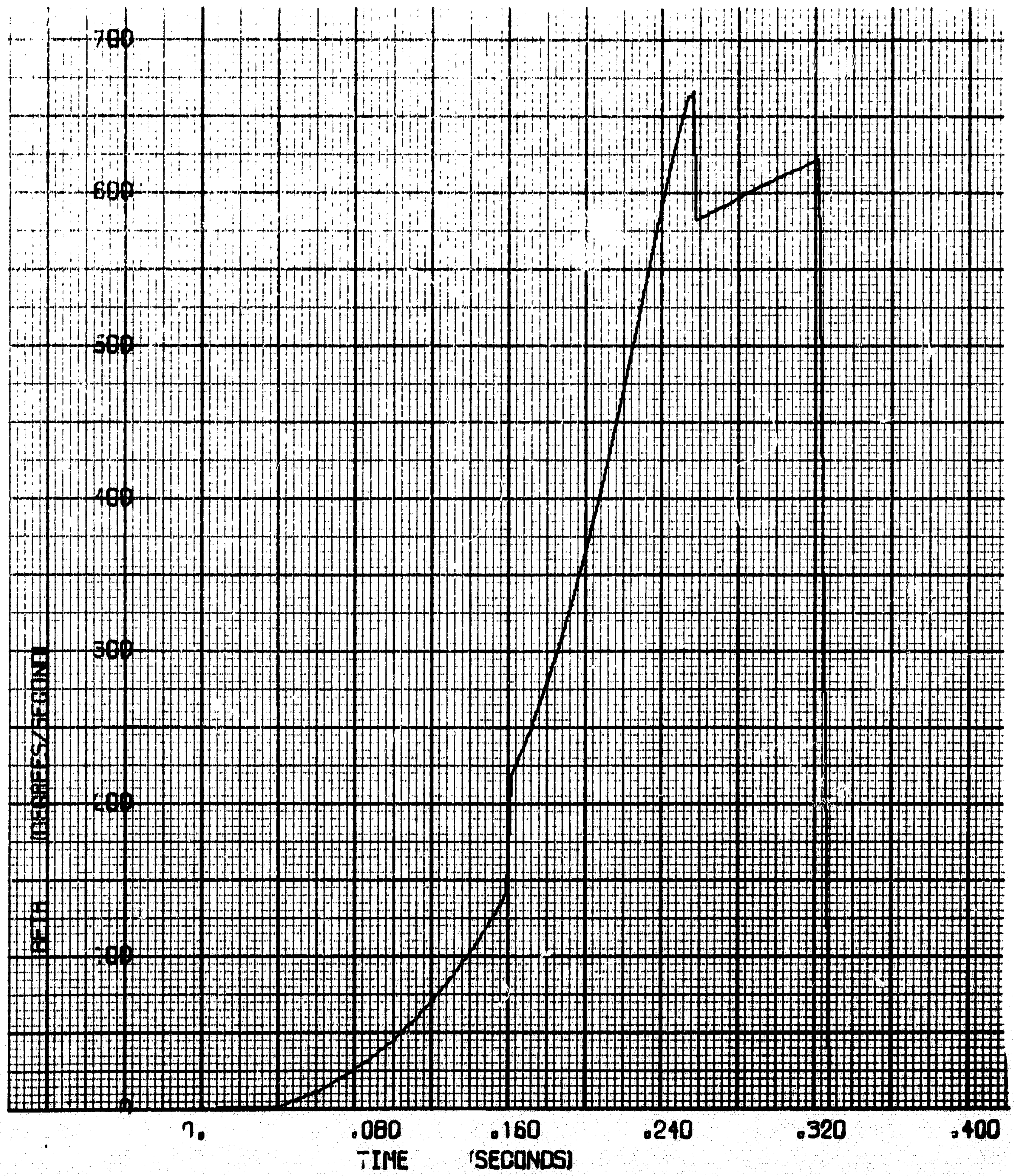
RATE OF CHANGE DEPLOYMENT ANGLE BETA, SEGMENT 2, APPENDAGE 1



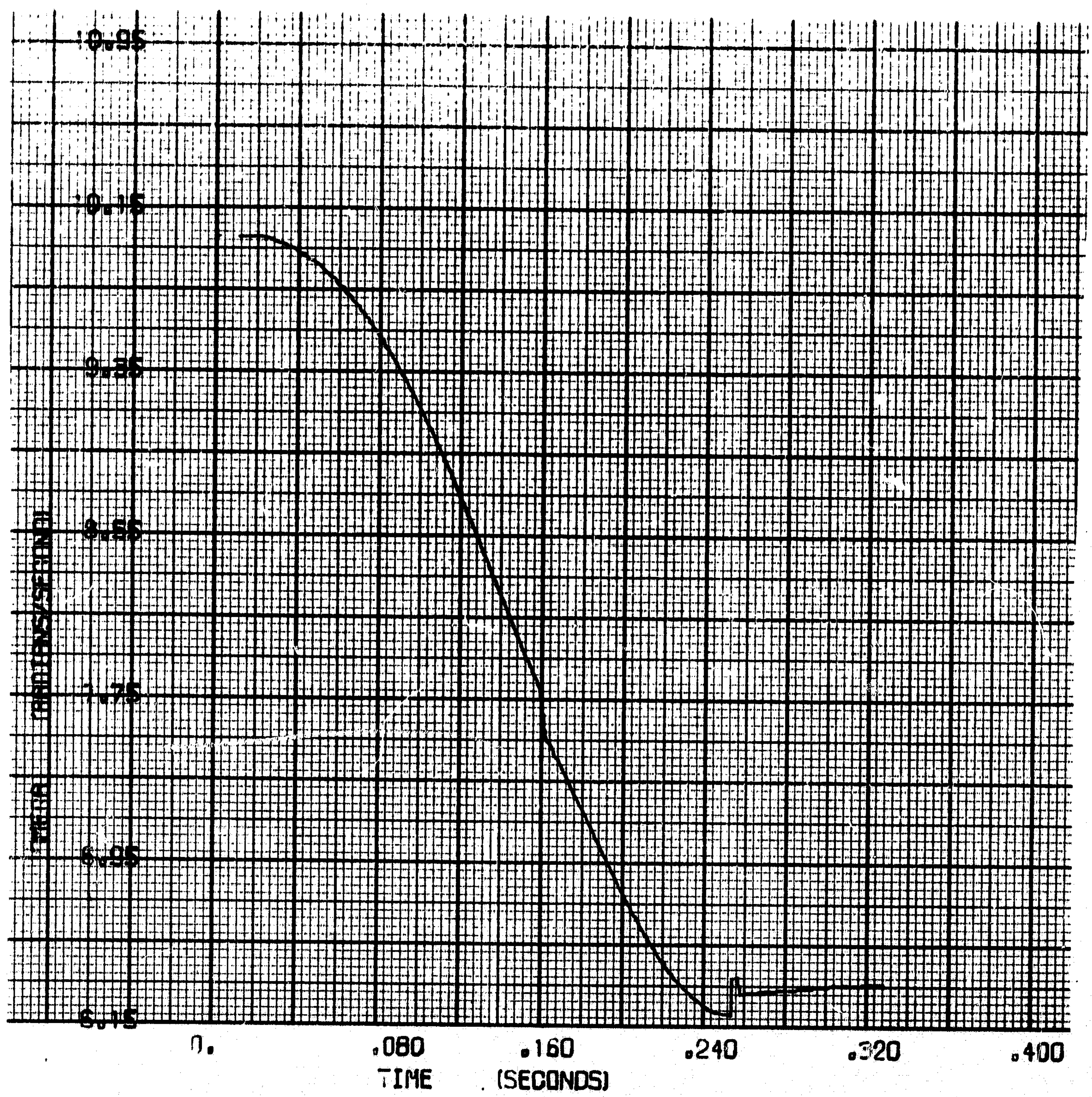
RATE OF CHANGE DEPLOYMENT ANGLE BETA, SEGMENT 1, APPENDAGE 2



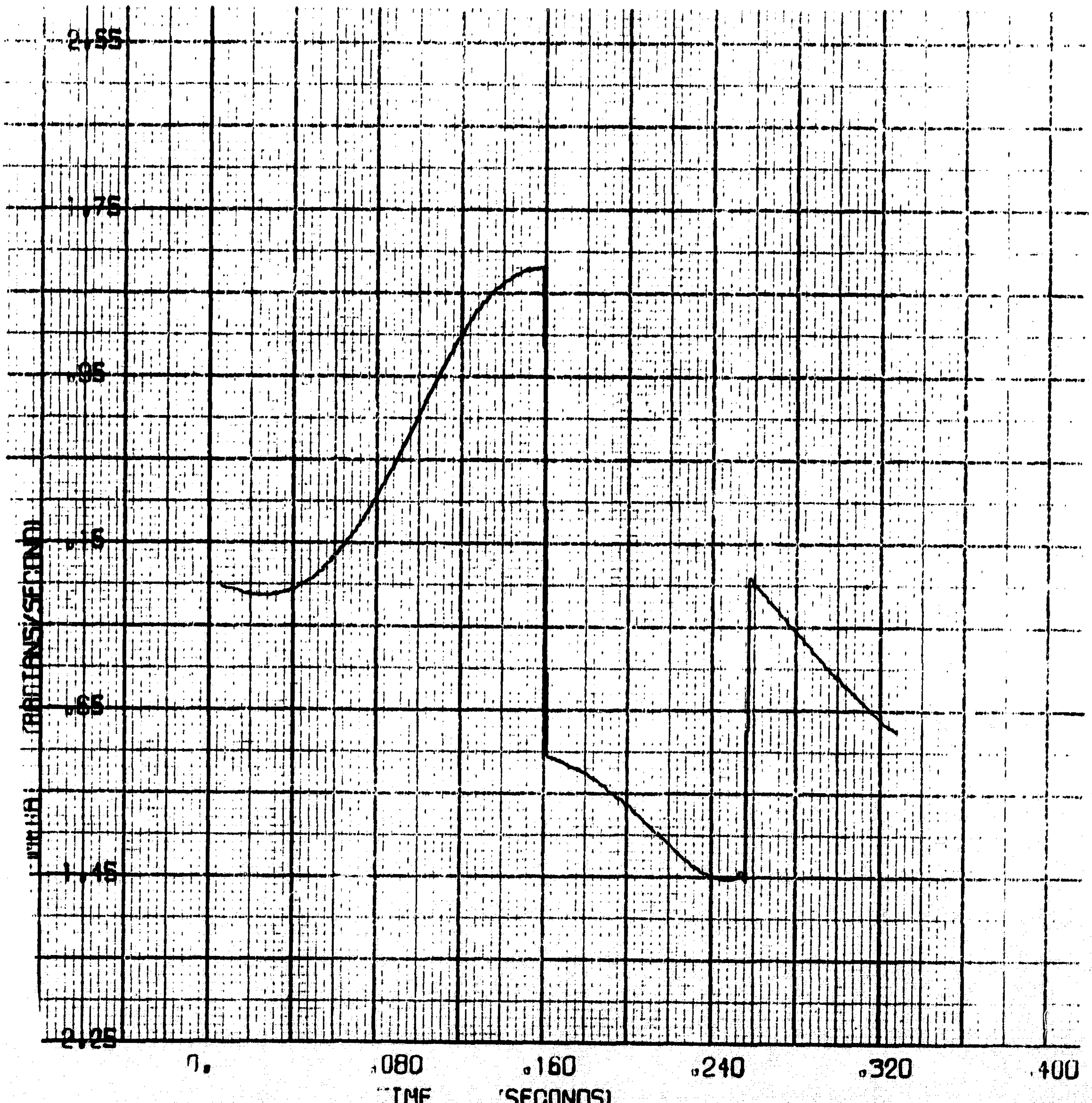
RATE OF CHANGE DEPLOYMENT ANGLE BETA, SEGMENT 2, APPENDAGE 2



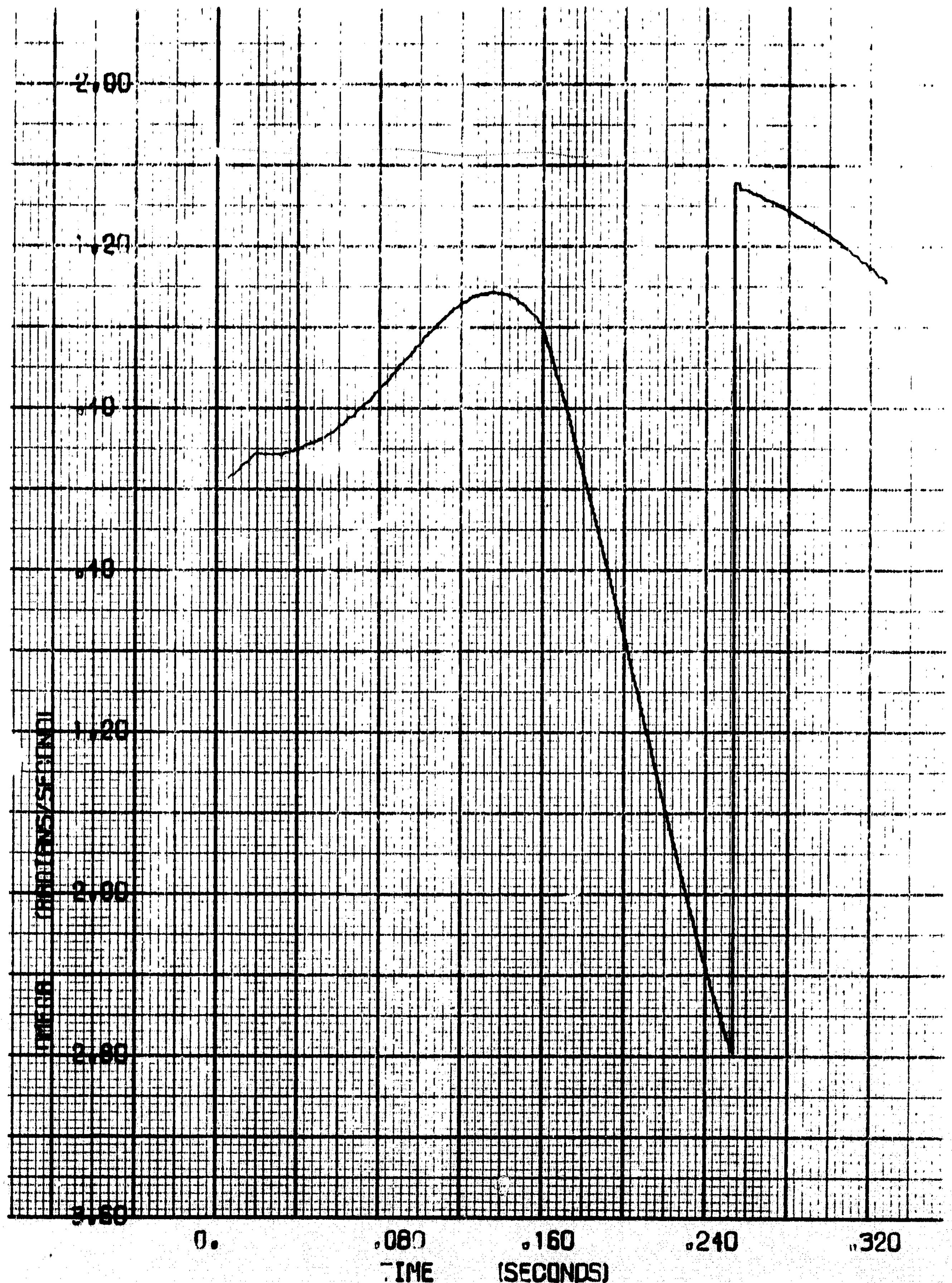
BODY ANGULAR RATE OMEGA 1, AS A FUNCTION OF TIME



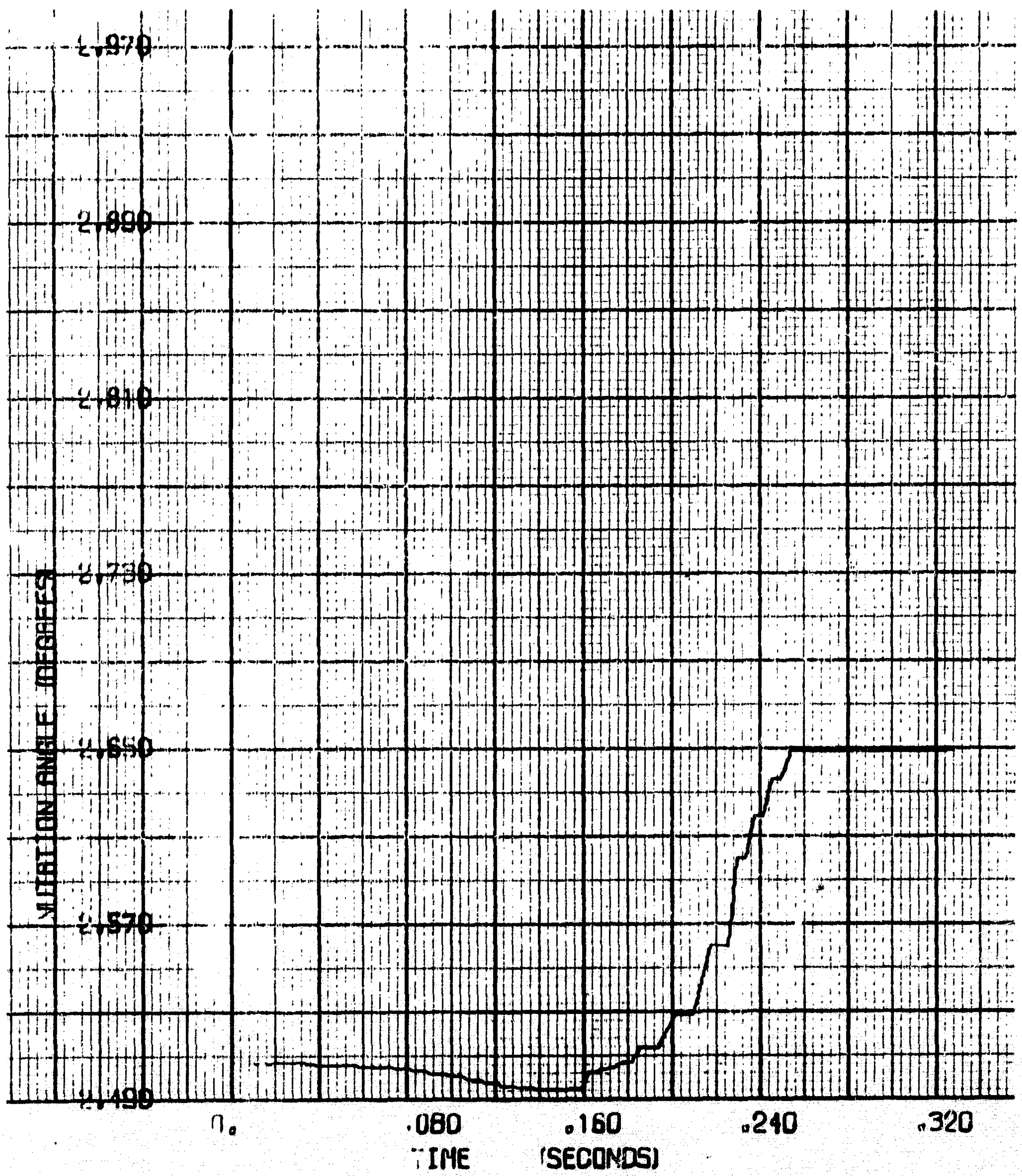
BODY ANGULAR RATE OMEGA 2, AS A FUNCTION OF TIME



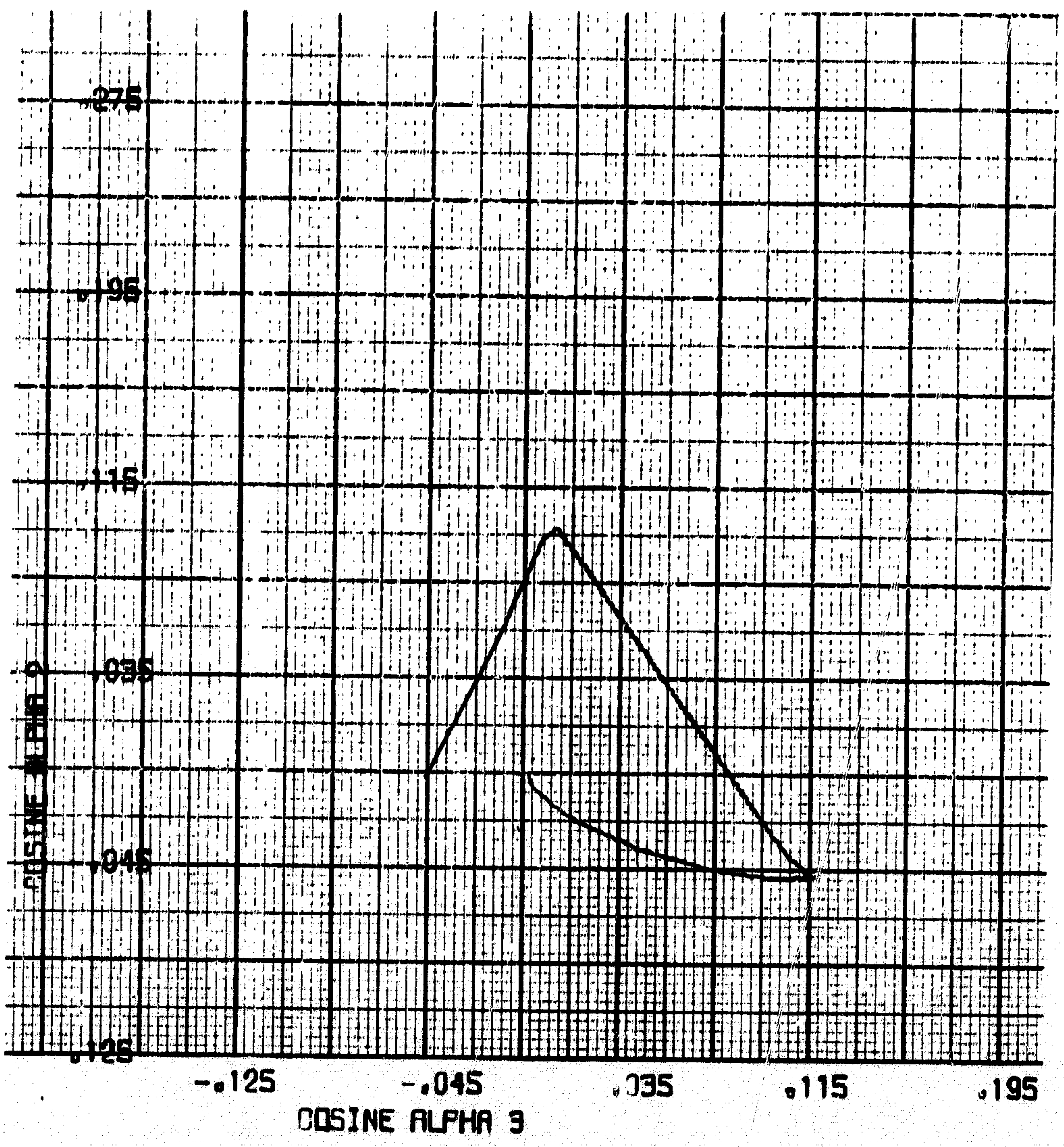
BODY ANGULAR RATE OMEGA 3, AS A FUNCTION OF TIME



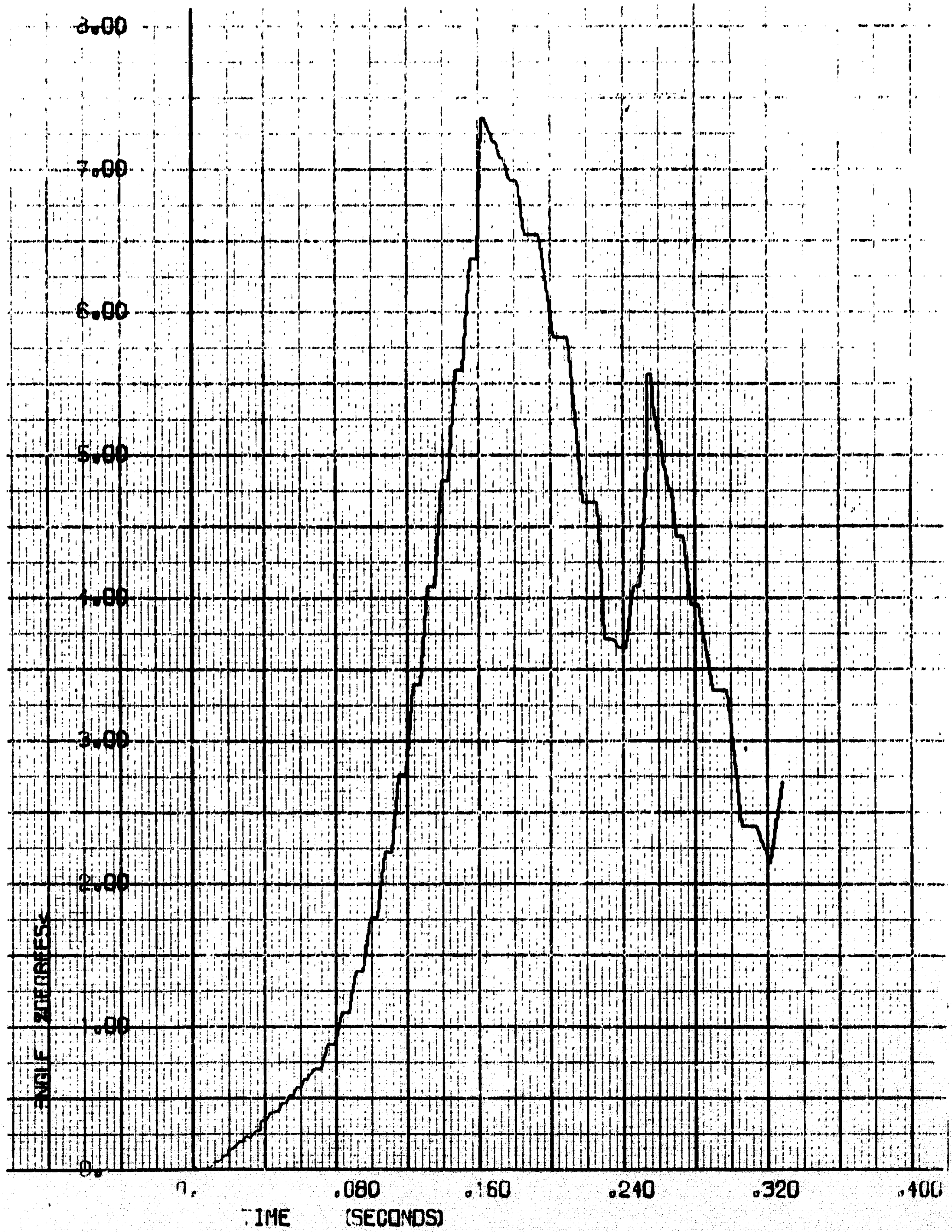
NUTATION ANGLE THETA, AS A FUNCTION OF TIME



SPIN AXIS TRACE IN EQUATORIAL PLANE



ANGLE FROM THE PERPENDICULAR TO EQUATORIAL PLANE TO SPIN AXIS



APPENDIX A

Some of the quantities used in Equations (4.24), (4.36), (4.38), (4.40), and (4.42) were not defined in the Nomenclature. These quantities are functions of variables defined in the Nomenclature which are used to write the equations in Section 4 in a more compact form. Since the derivation of these quantities is straightforward, they will be only defined, not derived, in this Appendix.

The first eight relations are associated with appendage geometry

$$\beta_1^k = \alpha_1^k \tag{A-1}$$

$$\beta_i^k = \alpha_i^k - \alpha_{i-1}^k \tag{A-2}$$

$$\tau_1^k = \sigma_1^k \tag{A-3}$$

$$\tau_i^k = \sigma_i^k - \sigma_{i-1}^k \tag{A-4}$$

$$\bar{h}_1^k = \bar{d}^k \tag{A-5}$$

$$\bar{h}_i^k = \bar{h}_{i-1}^k + \ell_{i-1}^k \, \hat{\eta}_{i-1}^k \tag{A-6}$$

$$\bar{C}_i^k = C_{i(1)}^k \, \hat{e}_1^k + C_{i(2)}^k \, \hat{\eta}_i^k + C_{i(3)}^k \, \hat{\zeta}_i^k \tag{A-7}$$

$$\bar{b}_i^k = \bar{h}_i^k + \bar{C}_i^k \tag{A-8}$$

Reference to the Nomenclature should clarify the meaning of these relations.

The following relations have been derived to make the equations in Section 4 more compact:

$$\overline{S}^k_{n_k} = 0 \tag{A-9}$$

$$\overline{S}^k_i = \overline{S}^k_{i+1} + m^k_{i+1}\,\overline{b}^k_{i+1} \quad i=n_k-1,\ldots,1 \tag{A-10}$$

(A-9) and (A-10) are related to the system center of mass

$$\overline{S} = \left[m_M\,\overline{b}_M + \sum_{k=1}^{n_a} \overline{S}^k\right]/m_T \tag{A-11}$$

where

$$\overline{S}^k = \overline{S}^k_1 + m^k_1\,b^k_1$$

The following four relations define mass parameters

$$\mu^k_{n_k} = 0 \tag{A-12}$$

$$\mu^k_i = \mu^k_{i+1} + m^k_{i+1} \tag{A-13}$$

$$m^k = \left(\mu^k_o\right) = \mu^k_1 + m^k_1 \tag{A-14}$$

$$m_T = m_M + \sum_{k=1}^{n_a} m^k \tag{A-15}$$

Other derived quantities are

$$\overline{g}^k_i = \overline{\omega} \times \left(\overline{\omega} \times \overline{b}^k_i\right) + 2\overline{\omega} \times \overline{b}_i^{(1)k} + \left(\sigma^k_i\right)^2 \hat{e}^k_1 \times \left(\hat{e}^k_1 \times \overline{C}^k_i\right)$$

$$-\sum_{j=1}^{i-1} \ell^k_j\,\hat{\eta}^k_j\,\left(\sigma^k_j\right)^2 \tag{A-16}$$

$$\bar{b}_i^{(1)k} = \hat{e}_1 \times \bar{C}_i^k \sigma_i^k + \sum_{j=1}^{i-1} \ell_j^k \hat{\zeta}_j^k \sigma_j^k \qquad \text{(A-17)}$$

$$\bar{H}_{c_i}^k = \bar{\bar{I}}_i^k \cdot \left(\bar{\omega} + \sigma_i^k \hat{e}_1^k\right) \qquad \text{(A-18)}$$

while

$$\bar{H}_{c_2}^k = \bar{\bar{I}}_2^k \cdot \left(\bar{\omega} + \sigma_1^k \hat{e}_1^k + \sigma_2^k \hat{\eta}_1^k\right) \quad \text{for } n_s < k \le n_a \text{ and } i = 2 \qquad \text{(A-19)}$$

$$\bar{H}_{cM} = \bar{\bar{I}}_M \cdot \bar{\omega} \qquad \text{(A-20)}$$

$$\bar{\beta}_i^{(2)k} = \bar{S}_{i-1}^k - \mu_{i-1}^k \bar{h}_i^k \qquad \text{(A-21)}$$

$$\bar{h}_i^{(1)k} = \bar{\omega} \times \left(\bar{\omega} \times \bar{h}_i^k\right) - \sum_{j=1}^{i-1} \ell_j^k \left(\sigma_j^k\right)^2 \hat{\eta}_j^k + 2\bar{\omega} \times \sum_{j=1}^{i-1} \ell_j^k \sigma_j^k \hat{\zeta}_j^k \qquad \text{(A-22)}$$

$$h_i^{(2)k} = \bar{\beta}_i^{(2)k} \times \left[\bar{h}_i^{(1)k} + 2\bar{\omega} \times \bar{v} + \bar{\omega} \times (\bar{\omega} \times \bar{\rho})\right] \qquad \text{(A-23)}$$