Transmission of Sonic Boom Pressure through a Window Pane

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1. Introduction

In previous studies[1] of the effects of sonic booms on structures, the structural elements were assumed to be subjected to the outside overpressure. Actually, a certain amount of energy is transmitted to the interior of the structure and exterior structural elements are subjected to a net pressure equal to the difference between exterior and interior overpressure.

The computation of the actual net pressure would be a hopeless task since the interior overpressures depend on the multiple reflections and the absorption of sound waves by walls, floors and ceilings. As an approximation of the situation near a window, the case of an elastic plate set in an rigid, infinite baffle and subjected to an N-shaped sonic boom normal to its plane was studied. The plate was assumed square and simply supported along its edges.

Related studies in the literature are by Morse[2] in the study of the transmission of monochromatic vibrations from one acoustic medium to another through a circular membrane set in an infinite baffle and by O'Callahan and Madden[3] in the study of the interaction of a free stream of an ideal gas with a circular membrane set in an infinite rigid wall. The former study was by a variational method while the latter used finite difference equations.

2. Nomenclature

A = area of square plate

a = radius of circular plate

c = speed of sound in air

c_g = speed of sound in plate material

D = flexural rigidity of plate

E = modulus of elasticity of plate

$g_{mn}(t)$ = time factor in normal series expansion

$H(t)$ = unit step function

h = plate thickness

k = wave number in air $2\pi/\lambda = \omega/c$

L = side of square plate

$p(t)$ = free field pressure

p_o = maximum overpressure in $p(t)$

p_{int} = interior pressure

p_{ext} = exterior pressure

q = $p_{ext} - p_{int}$

q_{mn} = coefficient of q in normal series expansion

R = τ/T_{11}

r = distance between 2 points in plate

T_{11} = fundamental period of plate

t = time

w = plate deflection

$w_{mn}(x,y)$ = normal function

x, y = coordinates

Δt = time increment

λ = wave length

ν = Poisson's ratio

Π_i = power transmitted to the interior

Π_o = power impinging on plate

ρ_a = density of air

ρ_g = density of plate material

τ = sonic boom duration

ω = circular frequency

ω_{mn} = circular frequency in m, nth mode

3. Formulation of the Problem

The equation of motion of the plate is taken according to the elementary theory of plates.(4)

$$D\nabla^4 w + \rho_g h \ddot{w} = q \qquad (1)$$

where w is the plate deflection, D is the stiffness of the plate, h its thickness and ρ_g the density of its material and $q = p_{ext} - p_{int}$ the net pressure equal to outside pressure minus inside pressure.

The net pressure is given in terms of the pressure pulse p(t) and the deflection w (x,y,t) through the equation [1]

$$q(x,y,t) = 2\,p(t) - \frac{1}{\pi}\iint_A \frac{\rho_a \ddot{w}(x',y',t-r/c)}{r}\,dx'dy' \qquad (2)$$

where

$$r = \sqrt{(x-x')^2+(y-y')^2} \qquad (3)$$

ρ_a is the density of air, c is the speed of sound, and where the integration is over the area A of the plate. The pressure pulse will be taken as the familiar N wave (Fig. 1)

$$p(t) = P_0\left(1-\frac{t}{\tau}\right)H(\tau-t) \qquad (4)$$

where H(t) is the unit step function

The initial conditions on the plate will be taken as

$$w(x,y,0) = \dot{w}(x,y,0) = 0 \qquad (5)$$

while the boundary conditions are (see fig. 2)

$$w(0,0,t) = w(0,L,t) = w(L,0,t) = w(L,L,t) = 0 \qquad (6)$$

4. Solution

A numerical solution of the problem is obtained by using finite differences on the time variable and the normal mode expansion for the space variable. The solution is taken in the form

$$w(x,y,t) = \sum_m\sum_n W_{mn}(x,y)\,g_{mn}(t) \qquad (7)$$

[1] See Appendix for derivation

where $w_{mn}(x,Y)$ represent the normal mode

$$\sin\frac{m\pi x}{L}\,\sin\frac{n\pi y}{L}$$

with m, n integers

Using the modal representation (7) and the orthoganality of the normal modes, the equation of motion of the plate reduces to the O.D.E.

$$\ddot{g}_{mn}(t) + \omega_{mn}^{2}\, g_{mn}(t) = \frac{q_{mn}(t)}{\rho_g h} \tag{7}$$

where ω_{mn} is the circular frequency associated with $w_{mn}(x,y)$ and

$$q_{mn}(t) = \frac{4}{L^2}\iint w_{mn}(x,y)\, q(x,y,t)\, dx\, dy \tag{8}$$

which, by the use of (2) can be written as:

$$q_{mn} = P_{mn}P(t) -$$

$$\frac{\rho_a}{\pi L^2}\sum_u\sum_s \iiiint_A \sin\frac{m\pi x}{L}\,\sin\frac{n\pi y}{L}\,\sin\frac{u\pi x'}{L}\,\sin\frac{s\pi y'}{L}\,\frac{\ddot{g}_{us}(t-r/c)}{r}\,dx\,dy\,dx'\,dy' \tag{9}$$

where

$$P_{mn} = \frac{32}{\pi^2 mn} \qquad \text{if both } m,n \text{ odd}$$

$$P_{mn} = 0 \qquad \text{otherwise} \tag{10}$$

and where r is given by (3)

In obtaining a numerical solution equation (7) is solved by finite differences and, in using (9) to compute q_{mn}, the summation is truncated while the integration is replaced by a summation.

5. Numerical Results

The case considered was that of a glass pane with $\frac{L}{h} = 250$. The relative densities of glass and air were taken as 2.5 and 0.00129 respectively. Numerical results were obtained for two particular values of $R = \frac{\tau}{T_{11}}$ viz $R = 1$ $R = 3$. In both cases the series was truncated to its first term. In solving the ordinary differential equation (7) by finite differences the time step was taken as $t = \frac{1}{10}\ \frac{L}{c}$ i.e. one tenth of the time required for sound to sweep the plate. The following recurrence formulae were used:[5]

$$\begin{aligned} \dot{g}(t) &= \dot{g}(t-\Delta t) + \tfrac{1}{12}\left[23\ddot{g}(t-\Delta t) - 16\ddot{g}(t-2\Delta t) + 5\ddot{g}(t-2\Delta t)\right] \\ g(t) &= g(t-\Delta t) + \tfrac{1}{12}\left[5\dot{g}(t) + 8\dot{g}(t-\Delta t) - \dot{g}(t-2\Delta t)\right] \end{aligned} \qquad (11)$$

combined with simple two-term formula for the first two steps.

The results are presented in the graphs of Figs. 3 and 4 in which the values of $\frac{q_{11}(t)}{q_{11}(0)}$ and $\frac{q_{11}(t) - P_{11}p(t)}{q_{11}(0)}$ are plotted as functions of $\frac{t}{T_{11}}$. Results were obtained for values of $\frac{t}{T_{11}}$ up to 2.0 since little departure from periodicity is expected beyond this value.

The computations were carried out at the City College Computation Center on a IBM 360-50. The most time-consuming part of the computation was the evaluation of the quadruple integral occurring in equation (9). The total computation time for each case was ten minutes.

6. Discussion

It is clear from the graphs of Fig. 3 that the major contribution to transmitted pressure occurs immediately following the discontinuities in the pressure pulse. A comparison of transmitted pressure due to an N-wave and that due to a step pulse with the same initial pressure rise is shown in Fig. 3; the two curves are hardly distinguishable.

The transmitted pressure seems to build up almost linearly to its maximum at time $0.7\,\frac{L}{c}$ which is approximately the time required for a sound wave to travel from the corner to the center of the square plate. The magnitude of the maximum transmitted pressure can be roughly checked by assuming that the acceleration remains constant and equal to $\frac{q(0)}{\rho_g h}$ and that the plate moves as a rigid circular disc of radius $\frac{L}{2}$. Application of the integral in (2) yields the expression

$$2p_{int} = 2p - q = \frac{\rho_a}{\rho_g}\frac{L}{h}q(0) \qquad q(0) = 2p_0 \tag{12}$$

For the numerical values used, this is equal to $0.13q(0)$ which is of the same order of magnitude as the value of $0.07q(0)$ obtained from the graph. If instead of assuming that the plate moves as a rigid disc, a cosine law of acceleration variation is assumed with maximum at the center and vanishing at the edges, a value closer to the computed value is obtained viz

$$2p_{int} = \frac{2}{\pi}\frac{\rho_a}{\rho_g}\frac{L}{h}q(0) \qquad q(0) = 2p_0 \tag{13}$$

yielding a value of $0.085q(0)$.

The magnitude of the computed maximum value of the transmitted pressure is quite small $(\frac{0.07}{2} q(0) = 0.070 p_o)$ corresponding to a power transmission loss of about 28 dB. Yet actual measurements[6] of pressures behind plate windows subjected to sonic booms seem to indicate transmitted pressures of the order of $0.5 p_o$ or transmission losses of about 6 dB.

In investigating the reason for the discrepancy, the transmission loss for monochromatic pulses in two models will be compared with the transmission loss of the flexural panel here considered.

The first model shown in Fig. 5 is the one dimensional slab of thickness h with the pressure pulse transmitted from one side to the other. The power ratio Π_i/Π_o is given[7] by the following approximate formula, valid if, as we assume that the wavelength in the slab is large compared to h.

$$\frac{\Pi_i}{\Pi_o} = \frac{1}{1 + \frac{1}{4}\left(\frac{\rho_g h \omega}{\rho_a c}\right)^2} \simeq 4\left(\frac{\rho_a c}{\rho_g h \omega}\right)^2 \tag{14}$$

For frequencies between 20 and 200 Hz and a glass thickness of 1/4 in., the transmission loss according to this formula would vary between 7.6 dB and 27.6 dB.

The second model shown in Fig. 6 is a rigid circular piston of radius a set in plane baffle. Using the asymptotic expressions of the impedance for high and low frequencies[7] the following expressions are found for the power ratio

For $ka = \frac{\omega a}{c} \gg 1$

$$\frac{\Pi_i}{\Pi_o} = \frac{1}{1 + \frac{1}{4}\left(\frac{\rho_g}{\rho_a} kh + \frac{4}{\pi ka}\right)^2} = 4\left(\frac{\rho_a c}{\rho_g h \omega}\right)^2 \tag{15a}$$

For $ka = \frac{\omega a}{c} \ll 1$ (say $\frac{\omega a}{c} < 1.0$)

$$\frac{\Pi_i}{\Pi_o} = \frac{1}{\frac{k^2 a^2}{2} + \frac{1}{2}\left(\frac{\rho_g}{\rho_a}\frac{h}{a} + \frac{16}{3\pi}\right)^2} = 2\left(\frac{\rho_a}{\rho_g}\frac{a}{h}\right)^2 \tag{15b}$$

It is interesting to note that the power ratio for the rigid piston tends to the expression for the one-dimensional slab for high frequencies and to an expression similar to that of Eqs. (12) & (13) for low frequencies. It thus appears that, for low frequencies, the approximate formula (13) and therefore the calculated transmitted pressures are verified. For higher frequencies the transmission loss seems not to depend on the ratio a/h but rather on λ/h.

Thus the lower transmission losses measured at the Edwards Air Force Base tests[6] must be attributed to the likelihood that in the test frame houses much of the energy is transmitted through the roof and wall panels rather than through the windows. An indirect proof of the above statement lies in the almost non-existent correlation between the plate glass window displacement (which were measured by strain gages) and the internal pressure (which was monitored by a microphone) in its immediate vicinity[6] (Fig. G23, G24, G25) subsequent to the passing of pressure wave.

One last remark may be made concerning Eq. 12: It might at first seem strange that the stiffness of the plate does not enter this formula. If, however, it is remembered that the derivation of the formula assumed a constant acceleration $\frac{P_o}{\rho_g h}$ during the time it takes the sound to travel the span L, then it is seen that the stiffness will not affect the result provided that the fundamental period of the pane T_{11} is large compared to $\frac{c}{L}$. This seems to be the case for ordinary window panes since it reduces, after some manipulation, to

$$\frac{\sqrt{12(1-\nu^2)}}{\pi} \frac{L}{h} \frac{c}{c_g} \gg 1 \tag{16}$$

where $$c_g = \sqrt{\frac{E}{\rho_g}}$$

or, substituting numerical values for glass:

$$\frac{L}{h} \gg 14.5 \tag{17}$$

7. Conclusions

The pressure transmitted through a square window for a normal N-wave has been computed under the assumption that the window is set in a rigid baffle. For usual dimensions the pressure transmitted has small magnitude (about $0.035q_1(0)$).

A simple approximate formula was developed (Eq. 13) which can be used in most cases commonly encountered in practice.

The computed transmitted pressure are much less than the internal pressure measured in frame houses during sonic boom experiments. It is believed that the discrepancy is due to transmission of pressure through the areas of wall and roof as well as windows.

8. References

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Appendix

Derivation of Equation (2)

Let the Fourier transform of a time-dependent quantity $x(t)$ be denoted by

$$\hat{x}(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-i\omega t}\, dt$$

Then, in the frequency domain ω, the transmission problem shown in Fig. 2 reduces to finding $p_{ext}\,(x,y,z,\omega)$ defined for $z \leq 0$, $p_{int}\,(x,y,z,\omega)$ defined for $z \geq 0$, which satisfy:

A - The reduced wave equation:

$$\nabla^2 p + k^2 p = 0 \qquad k = \frac{\omega}{c} \tag{18}$$

B - The following boundary condition for $z = 0$.

$$\frac{\partial \hat{p}_{ext}}{\partial z} = \frac{\partial \hat{p}_{int}}{\partial z} = \rho\,\omega^2 \hat{w} \quad \text{if} \quad \begin{array}{l} 0 \leq x \leq L \\ 0 \leq y \leq L \end{array} \tag{19}$$

$$\frac{\partial \hat{p}_{ext}}{\partial z} = \frac{\partial \hat{p}_{int}}{\partial z} = 0 \qquad \text{otherwise}$$

C - The following radiation conditions: The solution $\hat{p}_{int}$ corresponds to an outgoing wave while the solution $\hat{p}_{ext}$ contains the wave $\hat{p}(\omega)e^{-ikz}$ travelling along the positive z direction, a wave travelling along the negative z direction, and an outgoing wave.

Using the properties of the Green function for a plane[2] it can be seen that all conditions A,B,C are satisfied by

$$\hat{p}_{ext} = \hat{p}(\omega)e^{-ikz} + \hat{p}(\omega)e^{ikz} + \frac{\rho\omega^2}{2\pi}\iint_A \frac{\hat{w}(x',y',\omega)\,e^{-i\frac{\omega}{c}R}}{R}\,dx'dy' \tag{20}$$

$$\hat{p}_{int} = -\frac{\rho\omega^2}{2\pi}\iint_A \frac{\hat{w}(x',y',\omega)\,e^{-i\frac{\omega}{c}R}}{R}\,dx'dy'$$

where $R = \sqrt{(x-x')^2+(y-y')^2+z^2}$ and A is the area of the square plate

Since the conditions A, B, C define a problem having a unique solution, the above formulas solve the transmission problem in the frequency domain in terms of $\hat{w}$.

By the definition of q one finds from (20)

$$\begin{aligned} \hat{q}(x,y,\omega) &= \hat{P}_{ext}(x,y,0,\omega) - \hat{P}_{int}(x,y,0,\omega) \\ &= 2\hat{p}(\omega) + \frac{\rho\omega^2}{\pi}\iint_A \frac{\hat{w}(x',y',\omega)e^{-\frac{i\omega}{c}r}}{r}\,dx'dy' \end{aligned} \tag{21}$$

From the well-known differentiation and shifting properties of the Fourier transform, eq. (2) follows from eq. (21).

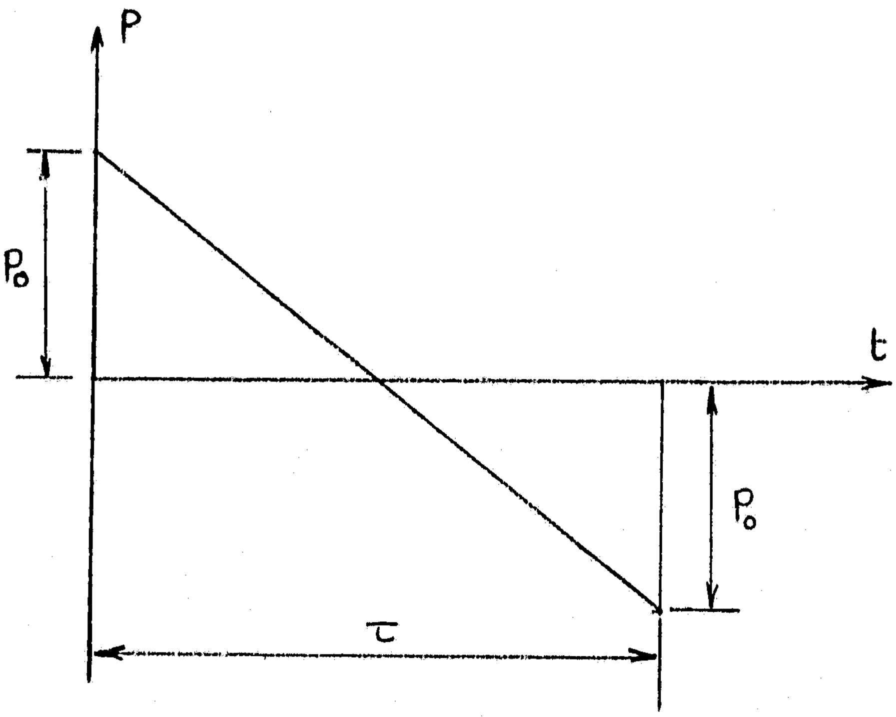

Fig. 1 N - wave

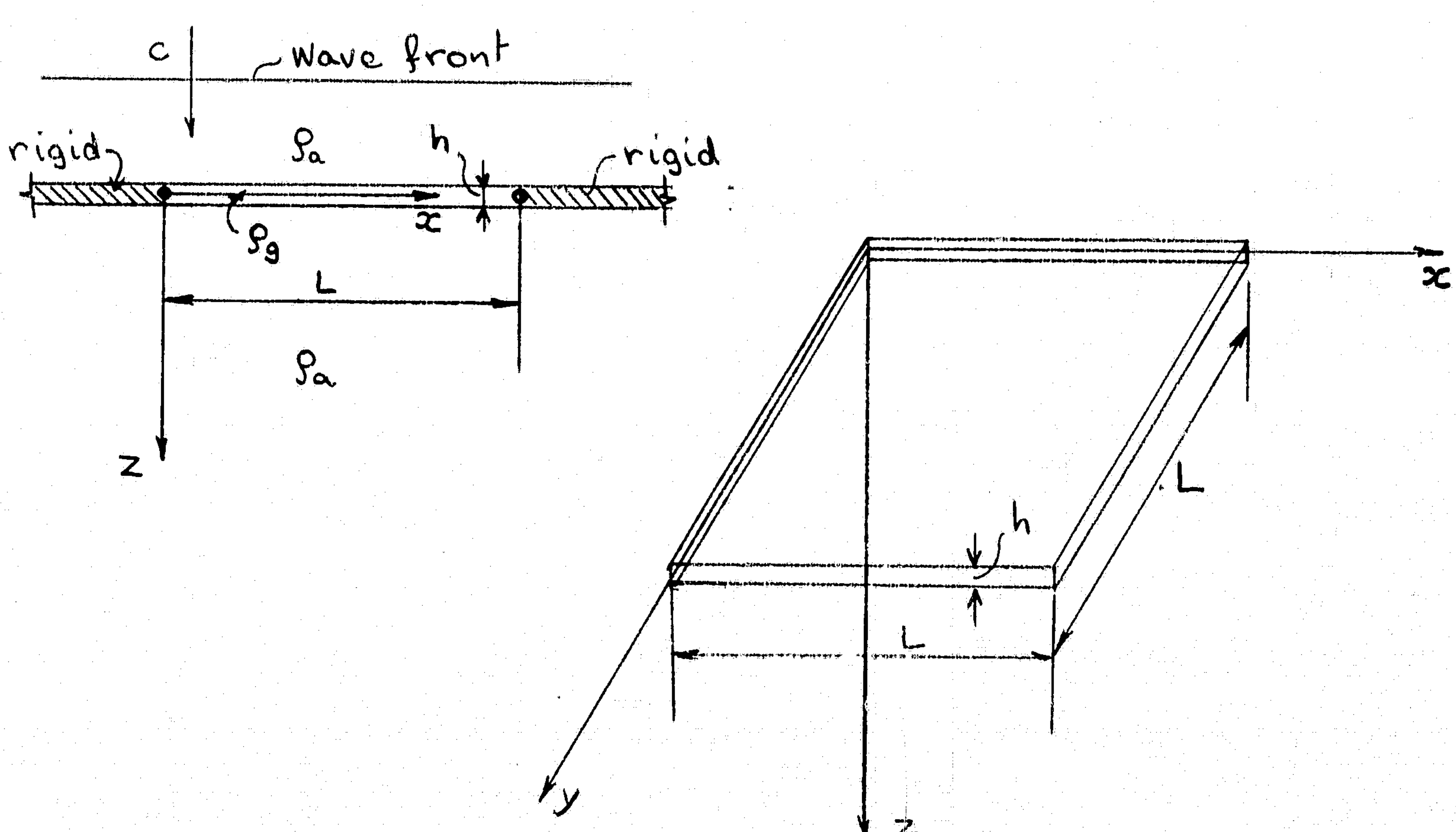

Fig. 2 Square Window Pane in Rigid Baffle

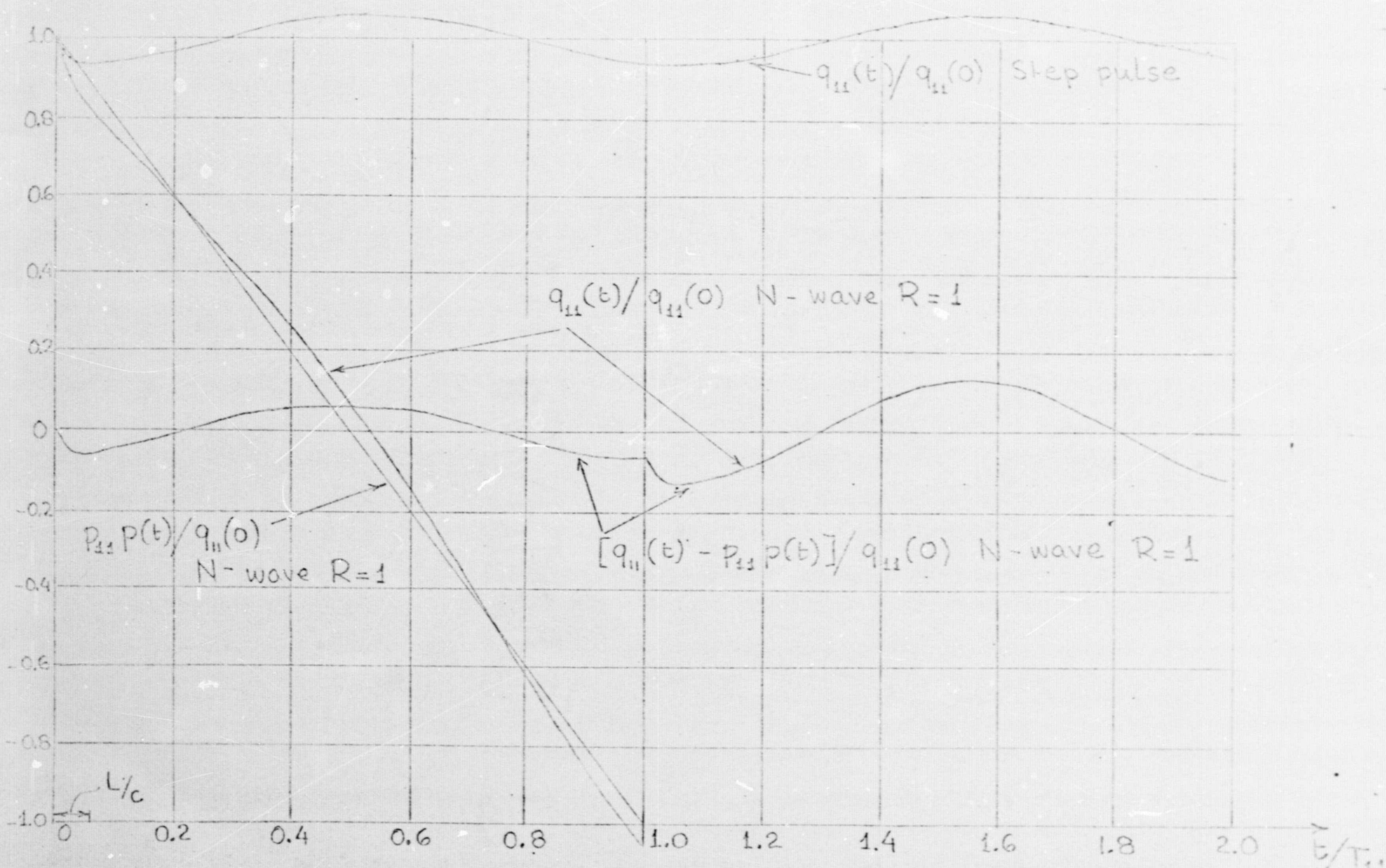

Fig. 3. Net and Transmitted Pressures for N-waves and Step Pulse

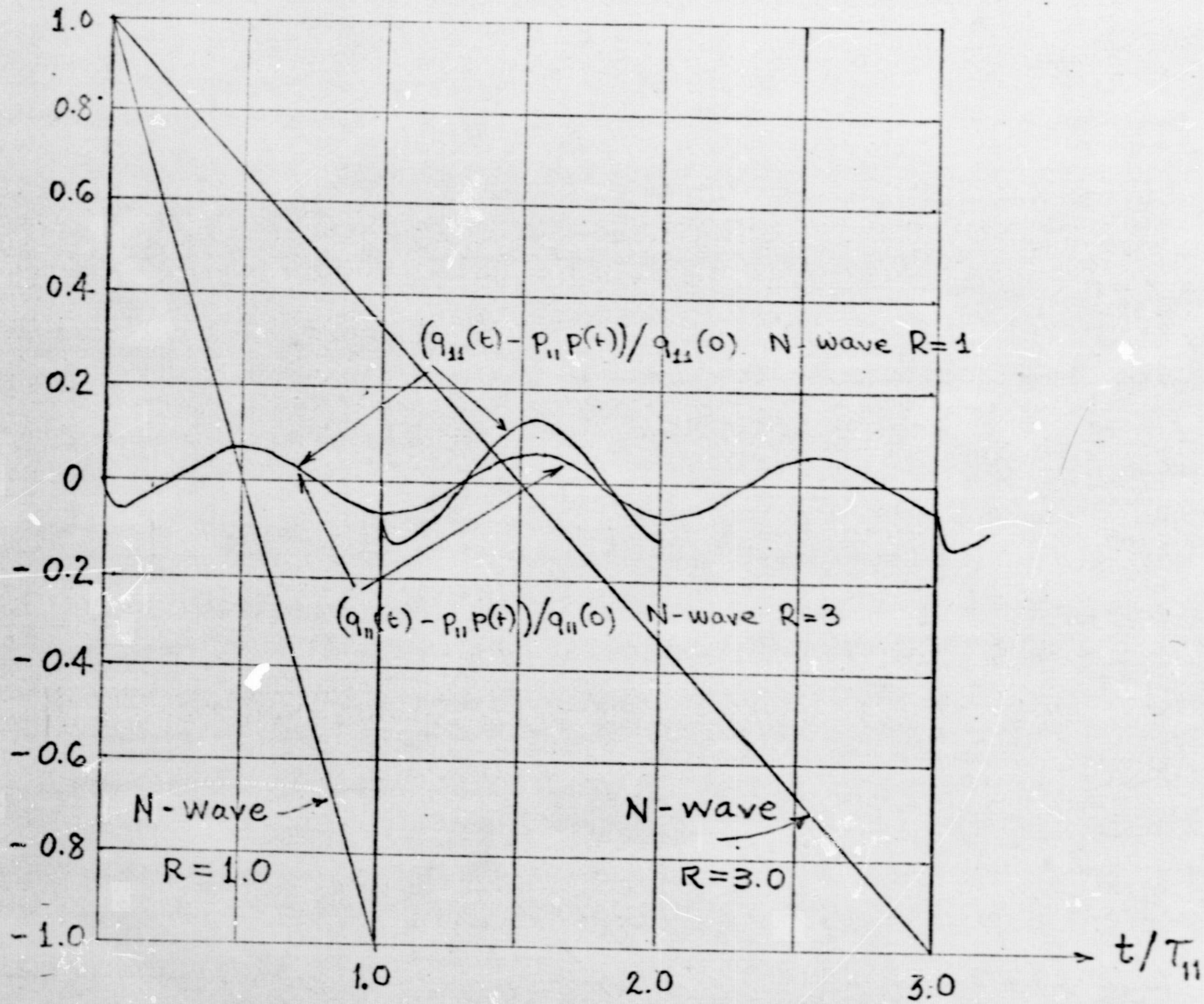

Fig. 4. Transmitted Pressures for N-waves.

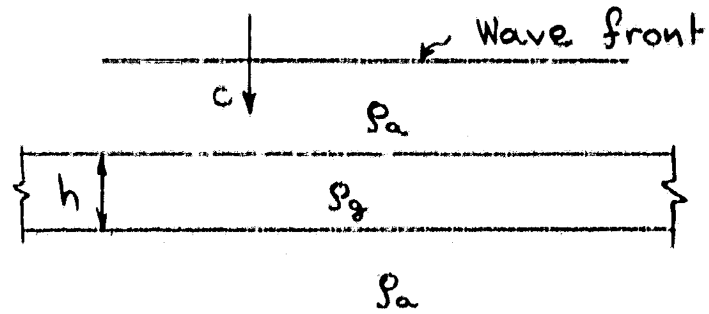

Fig. 5 One-dimensional Slab in Acoustic Medium

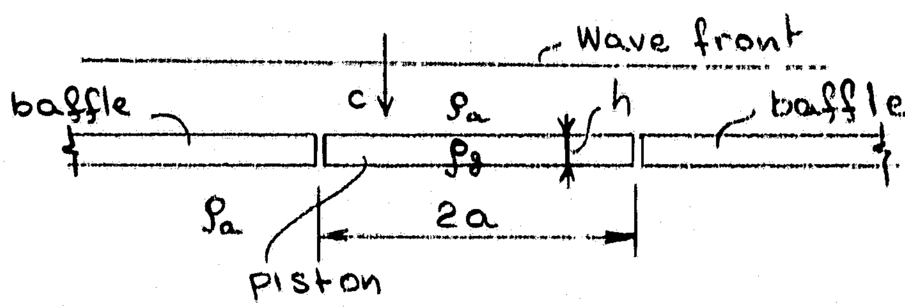

Fig. 6 Rigid Circular Piston in Rigid Baffle