# DYNASOR II - A FINITE ELEMENT PROGRAM FOR THE DYNAMIC NONLINEAR ANALYSIS OF SHELLS OF REVOLUTION 

# derospdce engineering oedartment 

## TEXAS A\&M UNIVERSITY

JOE R. TILLERSON AND WALTER E. HAISLER

TEES-RPT-70-19<br>OCTOBER 15. 1970

# Joe R. Tillerson* and Walter E. Haisler** 

October 15, 1970

TEES-RPT-70-19
Texas A\&M University
College Station, Texas

[^0]ABSTRACT

A user's guide for the DYNASOR II (DYnamic Nonlinear Analysis of Shelis $\underline{O f}$ Revolution) computer code is presented in this report. The finite element method of structural analysis is utilized for the analysis with the Houbolt numerical scheme being employed to solve the euqations of motion of the system. Documentation of the analysis techniques is presented along with a section presenting guidelines for utilizing the code and enumerating the limitations placed upon its use. The procedures for preparing the input data are presented and several example problems demonstrate the required input data for specific loadings. The extended appendix provides additional insight into the inner workings of this code. Using the DYNASOR II code, the nonlinear dynamic response of shells of revolution can be obtained in relatively short periods of computer time for a large number of important shell problems.

## ACKNOWLEDGEMENTS

This research was supported under Sandia Contract 82-3683, Sandia Corporation, Albequerque, New Mexico and NASA Grant NGL-44-007 044. Manned Spacecraft Center, Houston, Texas. Appreciation is expressed to Dr. Walter Von Riesemann of Sandia and Dr. Fred Stebbins of the Manned Spacecraft Center who served technical monitors for this research effort.

Appreciation is also expressed to the principal investigator for the research grants, Dr. James A. Stricklin, Professor, Aerospace Engineering Department, Texas A\&M University, for his supervision and guidance in the development of this program.

## SOR - Shell Of Revolution <br> Computer Programs

A family of compatible computer codes for the analysis of the shell of revolution (SOR) structures has been developed by researchers at Texas ARM University. These analyses employ the matrix displacement method of structural analysis utilizing a curved shell element. Geometrically nonitinear static and dynamic analyses can be conducted using these codes. The important natural frequencies and mode shapes can also be determined by employing another of the codes. Efficient programming provides codes capable of performing these desired analyses in relatively small amounts of computer time.

Each of these programs has been extensively tested using problems the solutions to which have been reported by other researchers in order to establish the validity of the codes. In addition, the capabilities of the codes have been demonstrated in a number of publications by prew senting solutions to problems which were unsolved by other researchers.

SAMMSOR II - Stiffness And Mass Matrices for Shells Of Revolution are generated utlizing the first member of this family. This program accepts a description of the structure in terms of the coordinates and slopes of the nodes and the properties of the elements joining the nodes. For shells with simple geometries (such as cylinders, shallow caps, hemispheres, etc.) the shell geometry can be internally generated. Utilizing the element properties, the structural stiffness and mass matrices are generated for as many as twenty harmonics and stored on magnetic tape. Each of the other SOR programs utilizes the output tape generated by SAMMSOR as input
data for the respective analyses. One advantage of creating the stiffo ness and mass matrices in a separate program is that a variety of analyses can be performed on the same shell configuration without having to create the matrices more than once. Obviously, a variety of boundary and loading conditions can be employed without having to create new mass and stiffness matrices for each case.

SNASOR II - The Static Nonlinear Analysis of Shells Of Revolution subjected to arbitrary mechanical and thermal loading is performed using the second computer code. Utilizing the stiffness matrices generated by SAMMSOR and the loading conditions and boundary conditions input to SNASOR II, the equilibrium equations for the structure are generated. The nonlinear strain energy terms result in pseudo generalized forces (as functions of the displacements) which are combined with the applied generalized forces. The resulting set of nonlinear algebraic equilibrium equations is solved by one of several methods: Newton-Raphson type iteration, incremental stiffness method, or a modified incremental stiffness method. In general, the Newton-Raphson procedure is the best and yields accurate results for highly nonlinear oroblems.

DYNASOR II - The third code is used for the DYnamic Nonlinear Analysis of Shells $\underline{O}$ Revolution. The equations of motion of the shell are solved using Houbolt's numerical procedure with the nonlinear terms being moved to the right-hand side of the equilibrium equations and again treated as generalized loads. The displacements and stress resultants can be determined for both symmetrical and asymmetrical loading conditions. Asymm metrical dynamic buckling can be investigated using this program. Solutions can be obtained for highly nonlinear problems in reasonable periods of time on the computer utilizing as many as five of the harmonics generated
in SAMMSOR. A restart capability is incorporated in this code which allows the user to restart the program at a specified time without having to expend the computer time necessary to regenerate the prior respoase FAMSOR - Frequencies And Modes for Shells Of Revolution can be dew termined using the fourth code. Using the stiffness matrix generated by SAMMSOR and a lumped mass representation developed from the consistent mass matrix generated by SAMMSOR, a specified number of natural frequencies (beginning with the lowest or fundamental frequency) are obtained using the inverse iteration method. The mode shapes for each of the frem quencies are also obtained.

## TABLE OF CONTENTS

Page
ABSTRACT ..... ii
ACKNOWLEDGEMENTS ..... iii
SOR - SHELL $\underline{O F}$ REVOLUTION COMPUTER PROGRAMS ..... iv
TABLE OF CONTENTS ..... vii
INTRODUCTION ..... 1
I. METHOD OF ANALYSIS ..... 3
II. USER GUIDELINES AND PROGRAM LIMITATIONS ..... 12
III. PROGRAM INPUT ..... 16
IV. EXAMPLE PROBLEMS ..... 44
REFERENCES ..... 56
APPENDIX

1. Description of Subroutines DYNASOR II ..... 58
2. Significant FORTRAN Variables DYNASOR II ..... 60
3. Subroutine Call Map DYNASOR II ..... 69
4. Flow Chart of Basic Operations DYNASOR II ..... 70
5. Use of the Restart Option ..... 75
6. Load and Temperature Input Discussion ..... 78
7. Program Output ..... 84
8. Modification of Program Capacity ..... 85

## INTRODUCTION

The DYNASOR II (DYnamic Nonlinear Analysis of Shells Of Revolution) code has been developed to determine the time varying response of shells of revolution to a variety of loading conditions. The code utilizes the stiffness and mass matrices created by the SAMMSOR code for selected harmonics, generates generalized forces from a mechanical and thermal load history, and solves the resulting initial value problem. This report is a user's guide for the DYNASOR II code and is divided into four selfcontained sections with an extended appendix.

The first section describes the method of analysis used to obtain the displacements, stresses, and stress resultants for the desired time increments. The formulation of the equations of motion is presented along with the numerical technique employed to obtain the solution to these equations.

A section is then presented to enumerate the limitations of the code and to provide valuable guidelines to aid the user in performing the desired analyses. The limitations result partly from the procedures utilized in the method of analysis and partly from the storage capacity and programming procedures employed.

A description of the input data required by the DYNASOR II code is presented in the third section. Examples are provided in instances where the wording might, at first glance, appear to be unclear or insufficient. The limitations placed upon the input parameters are once again enumerated.

The final section contains selected example problems which are designed to illustrate the wide variety of input variations allowed by the code. A copy of the input data required for each of the cases is presented along with selected values of the output data. A thorough understanding of these example problems is mandatory if the user is to become adept at operating the code.

The extended appendix which follows the main report should prove to be extremely helpful if a thorough understanding of the program is desired. A description of the subroutines and the significant Fortran variables is supported by the presence of the subroutine call map and a flow chart of the basic operations of the code. The sections describing the restart capability and the specification of the loads should prove invaluable to users who desire to obtain optimum performance from the code. A discussion of the program output is then followed by a description of the changes necessary to modify the capacity of the code.

## SECTION I

METHOD OF ANALYSIS

## Introduction

The purpose of this section is to provide theoretical documentation of the equations and procedures employed in the DYNASOR II code to perform the DYnamic Nonlinear Analysis of Shells Of Revolution. The matrix displacement method of a structural analysis is utilized. Since the documentation for the development of the stiffness and mass matrices has been adequately presented in the SAMMSOR II user's manual, ${ }^{1}$ this section will not attempt to duplicate the previous presentation. The dynamic equations of motion are derived and the numerical techniques utilized to effect the solution of these equations are discussed.

## Equations of Motion

The matrix displacement method is an energy formulation and, consequently, the equations of equilibrium for the nonlinear dynamic response are obtained from Lagrange's equation:

$$
\begin{equation*}
\frac{d}{d t}\left(\frac{\partial T}{\partial \dot{q}_{i}^{n}}\right)+\frac{\partial U}{\partial q_{i}^{n}}=Q_{i}^{n} \tag{1}
\end{equation*}
$$

where

$$
\begin{align*}
& q_{i}^{n}=\text { generalized degree of freedom } i \text { of harmonic } n \\
& T=\text { kinetic energy } \\
& U=\text { internal energy }  \tag{2}\\
& Q_{i}^{n}=\text { generalized force for degree of freedom } i \text { of harmonic } n
\end{align*}
$$

Since the internal energy of a structure is a scalar quantity, the expression for this quantity may be separated into various parts. The formulation used in this analysis considers the internal energy as

$$
\begin{equation*}
U=U_{L}+U_{N L}-\left(U_{L}^{t}+U_{N L}^{t}\right) \tag{3}
\end{equation*}
$$

where the superscript, $t$, denotes the inclusion of thermal effects and

$$
\left.\begin{array}{rl}
U_{\mathrm{L}}= & \text { strain energy based upon linear strain } \\
& \text { displacement relations }
\end{array}\right\}
$$

By substituting Eq. 3 into Eq. 1 and taking the nonlinear strain energy terms to the right-hand side, the equations of motion for the nonlinear dynamic analysis of shells of revolution can be written in matrix form as

$$
\begin{equation*}
\left[M^{n}\right]\left\{q^{n}\right\}+\left[K^{n}\right]\left\{q^{n}\right\}=\left\{Q^{n}\right\}+\left\{Q_{t}^{n_{n}}\right\}-\left\{\frac{\partial U_{N L}}{\partial q^{n}}\right\}+\left\{\frac{\partial U_{N L}^{t}}{\partial q^{n}}\right\} \tag{4}
\end{equation*}
$$

The column matrix, $\left\{Q_{t}^{n}\right\}$, of pseudo linear thermal loads is evaluated exactly from $\left\{\partial U_{L}^{t} / \partial q^{n}\right\}$. It should be noted that Eq. 4 is valid for any harmonic $n$ with the coupling between the harmonics appearing on the right-hand side. In this formulation the nonlinear terms are treated as pseudo generalized forces which are applied to the structure. The obvious advantage of this formulation is that a tremendous savings in computer time can be realized since the stiffness matrix does not change as the displacements vary and must, therefore, be calculated only once. With most other formulations for geometric nonlinearities, the stiffness matrix must be updated at each time step.

Strain Displacement Relations
The DYNASOR II code utilizes the strain displacement equations given by Novozhilov ${ }^{2}$ as restricted to shells of revolution with the additional assumption being made that the only important nonlinear contributions arise from rotations about the shell coordinate axes. The midsurface strain expressions can then be written as

$$
\begin{align*}
& \varepsilon_{s}=\hat{e}_{s}+\frac{1}{2} \hat{e}_{13}^{2} \\
& \varepsilon_{\theta}=\hat{e}+\frac{1}{2} \hat{e}_{23}^{2}  \tag{5}\\
& \varepsilon_{s \theta}=\hat{e}_{s \theta}+\hat{e}_{13} \hat{e}_{23}
\end{align*}
$$

where

$$
\begin{align*}
& \hat{e}_{s}=(\partial u / \partial s)-\phi^{\prime} w \\
& \hat{e}_{\theta}=(1 / r)[(\partial v / \partial \theta)+u \sin \phi+w \cos \phi] \\
& \hat{e}_{s \theta}=(1 / r)(\partial u / \partial \theta)-(v / r) \sin \phi+\partial v / \partial s  \tag{6}\\
& \hat{e}_{13}=(\partial w / \partial s)+u \phi^{\prime} \\
& \hat{e}_{23}=(1 / r)(\partial w / \partial \theta)-(v \cos \phi) / r
\end{align*}
$$

The changes in curvature are those used in linear theory

$$
\begin{align*}
& x_{s}=-\partial \hat{e}_{13} / \partial s \\
& x_{\theta}=-(1 / r)\left(\partial \hat{e}_{23} / \partial \theta\right)-(1 / r) \sin \phi \hat{e}_{13}  \tag{7}\\
& x_{s \theta}=-(1 / r)\left(\partial \hat{e}_{13} / \partial \theta\right)+(\sin \phi / r) \hat{e}_{23}-\partial \hat{e}_{23} / \partial s
\end{align*}
$$

## Pseudo Nonlinear Forces

The nonlinear terms in this analysis are treated in the same way as the generalized forces due to external loading. The generalized forces due to the nonlinearities are evaluated for each element and are then combined at the nodes. A detailed presentation of the procedures utilized in calculating the nonlinear forces has been made in Ref. 3 with an overview of the same material being provided in Ref. 4.

The pseudo forces are obtained by retaining strain energy terms containing the rotations raised to the fourth power. The retention of the fourth order terms has been shown ${ }^{5}$ to be absolutely essential in cases where the nonlinear terms are substantial. The results prew sented in Ref. 6 for static shell analysis did not include the effects of the fourth order terms but results obtained after the incorporation of these terms revealed once again the necessity of retaining these contributions.

The generalized forces due to nonlinearities are evaluated using linear displacement functions in the variables $u, v$, and $w$ and employing strip integration over the length of the eiement. The integrals around the circumference are evaluated in closed form for the particular harmonics chosen. This procedure is simpler than the one employed in Ref. 6 and permits the nonlinear forces to be evaluated without the use of secondary storage on the computer. Detailed justification for this simplified procedure has been made in Ref. 3 so these arguments will not be enumerated again. It will suffice to note that due to the exact evaluation of the integrals in the circumferential
direction, it is reasonable to expect rapid convergence as the number of harmonics is increased. Examples have shown (Ref. 3) that the use of the strip integration over the length of the element produces convergence quite rapidly as the number of elements is increased.

## Thermal Terms

The temperature distribution and the temperature gradients in the normal direction for an element are expanded in a Fourier series in a manner similar to that used for the displacement functions. The temperatures and temperature gradients for an element are assumed constant over each element in the meridional direction with step variations allowed in the circumferential direction. In cases where the step variation in the circumferential direction is not considered accurate enough, the Fourier coefficients may be specified as input information.

The linear and nonlinear contributions are separated with the linear thermal loads for each harmonic being evaluated as

$$
\begin{equation*}
\left\{Q_{t}^{n}\right\}=\left\{\frac{\partial U_{L}^{t}}{\partial q^{n}}\right\} \tag{8}
\end{equation*}
$$

Employing a coordinate transformation to change to partial derivatives with respect to the generalized shell coordinates, the problem reduces to the evaluation of the partial derivatives of $U_{L}^{t}$ with respect to the coefficients $\alpha_{1}, \alpha_{2}, \ldots \alpha_{8}$. These partial derivatives are pre sented in Eqs. 26 of Ref. 3, and the terms of $\left\{0_{t}^{n}\right\}$ are listed in the appendix of the same report.

The nonlinear thermal loads are treated in essentially the same
manner as the generalized forces due to nonlinearities are treated. Utilizing the same approximations as for the nonlinearities due to applied forces, the expression for the nonlinear thermal contribution is given by Eq. 28 of Ref. 3.

## Stress Resultants

In this code, the stress resultants are determined by the use of the assumed displacement functions and finite difference relations at the mid-point of each element.

For orthotropic shells the stress resultants may be written as

$$
\left\{\begin{array}{c}
N_{s}  \tag{9}\\
N_{\theta} \\
N_{s \theta} \\
M_{s} \\
M_{\theta} \\
M_{s \theta}
\end{array}\right\}=\left[\begin{array}{llllll}
C_{1} & v_{s \theta} C_{1} & 0 & 0 & 0 & 0 \\
v_{\theta s} C_{2} & C_{2} & 0 & 0 & 0 & 0 \\
0 & 0 & G_{1} & 0 & 0 & 0 \\
0 & 0 & 0 & D_{1} & v_{s \theta} D_{1} & 0 \\
0 & 0 & 0 & v_{\theta s} D_{2} D_{2} & 0 \\
0 & 0 & 0 & 0 & 0 & G_{2}
\end{array}\right]\left\{\begin{array}{c}
\varepsilon_{s} \\
\varepsilon_{\theta} \\
\varepsilon_{s \theta} \\
x_{s} \\
x_{\theta} \\
x_{s \theta}
\end{array}\right\}
$$

where

$$
\begin{array}{ll}
C_{1}=E_{s} t /\left(1-v_{s \theta} \nu_{\theta s}\right) & C_{2}=E_{\theta} t /\left(1-v_{s \theta} v_{\theta s}\right) \\
G_{1}=G t & G_{2}=G t^{3} / 12 \\
D_{1}=E_{s} t^{3} /\left[12\left(1-v_{s \theta} v_{\theta s}\right)\right] & D_{2}=E_{\theta} t^{3} /\left[12\left(1-v_{s \theta} v_{\theta s}\right)\right]
\end{array}
$$

The shear resultants are determined approximately from the equations of
equilibrium of the undeformed shell as

$$
\begin{align*}
& Q_{s}=\frac{1}{r}\left[\frac{\partial}{\partial s}\left(r M_{s}\right)+\frac{\partial M_{s \theta}}{\partial s}-M_{\theta} \sin \phi\right] \\
& Q_{\theta}=\frac{1}{r}\left[\frac{\partial}{\partial s}\left(r M_{s}\right)+\frac{\partial M_{\theta}}{\partial \theta}+M_{s \theta} \sin \phi\right] \tag{11}
\end{align*}
$$

## Numerical Solution of Equations of Motion

Since a closed-form solution of Eq. 4 is generally not available, a numerical method must be used to determine the solution to the equations of motion. A finite difference procedure developed by Houbolt (Ref. 7) has been selected for use in the DYNASOR II code.

The equations of motion, Eq. 4, can be reduced to a system of equations of the form

$$
\begin{equation*}
[M]\{\ddot{q}\}+[K]\{q\}=\{F(t, q)\} \tag{12}
\end{equation*}
$$

The load matrix, $\{F(t, q)\}$ is equivalent to the right-hand side of Eq. 4. The initial displacements and velocities of the nodes must be specified and can be written as

$$
\begin{align*}
& q_{0}=\{q\}_{0} \\
& \dot{q}_{0}=\{\dot{q}\}_{0} \tag{13}
\end{align*}
$$

Utilizing the Houbolt procedure, the accelerations of the nodes of the shell are approximated by a third-order backwards difference expression

$$
\begin{equation*}
\left\{\ddot{q}_{n+1}\right\}=\frac{1}{(\Delta t)^{2}}\left\{2 q_{n+1}-5 q_{n}+4 q_{n-1}-q_{n-2}\right\} \tag{14}
\end{equation*}
$$

Substitution of Eq. 14 into Eq. 12 yields the following expression which
is utilized to solve for the displacements at the end of each time step, except the first one:

$$
\begin{align*}
\left(2[M]+(\Delta t)^{2}[K]\right)\left\{q_{n+1}\right\}= & (\Delta t)^{2}\left\{F(t, q)_{n+1}\right\}  \tag{15}\\
& +[M]\left\{5 q_{n}-4 q_{n-1}+q_{n-2}\right\}
\end{align*}
$$

To determine the displacements at the end of the first time step, the following equation is employed

$$
\begin{align*}
\left(6[M]+(\Delta t)^{2}[K]\right)\left\{q_{1}\right\}= & (\Delta t)^{2}\left\{F\left(0, q_{0}\right)\right\} \\
& +[M]\left\{2(\Delta t)^{2} \ddot{q}_{0}+6 \Delta t \dot{q}_{0}+6 q_{0}\right\} \tag{16}
\end{align*}
$$

It should be noted that the selection of the Houbolt procedure for inclusion in the code was made only after evaluating the advantages and disadvantages of a number of solution schemes (Ref. 8). The Houbolt procedure proved to be the only method capable of providing stable solutions for highly nonlinear problems while utilizing a reasonably large time increment. The significant observations made in Ref. 8 concerning Houbolt's procedure will now be presented.

It was found that double-precision arithmetic is necessary if the code is utilized for highly nonlinear problems on an IBM 360/65 system (or comparable system). It is believed that if the DYNASOR II code is used on computers which have a longer word length than the $360 / 65$ system (such as the CDC 6600) double-precision arithmetic will not be necessary. Utilizing the Houbolt scheme, it has been shown that the solution converges as the number of elements is increased. Although the Houbolt procedure has been shown to be unconditionally stable for the linear problem, it has found that this is not the case
with the nonlinear formulation. The damping inherent in the Houbolt procedure was noted in some instances, but the savings in computer time resulting from employing this procedure far outweighs this slight drawback. Solutions (without the damping) were obtained, in some instances, in one-eighth (1/8) the amount of time required by other procedures. In all cases which were run, stable, undamped solutions were obtained using larger time increments than could be used with the other methods.

## Extrapolation of Forces

In order to employ Eq. 15, the loads at the end of the $(n+1)$ th time step must be known. These loads, because of the presence of the nonlinear terms, are a function of the displacements to be calculated and therefore cannot be evaluated exactly. The right-hand side of Eq. 12 is, therefore, evaluated using a first-order Taylor's series expanded about the $n^{\text {th }}$ increment:

$$
\begin{equation*}
\left\{F(t, q)_{n+1}\right\}=\left\{F(t, q)_{n}\right\}+\frac{\partial}{\partial t}\left\{F(t, q)_{n}\right\} \Delta t+0(\Delta t)^{2} \tag{17}
\end{equation*}
$$

A second-order extrapolation process has been employed (Ref. 8), but the results indicated that the linear extrapolation procedure was more stable.

## SECTION II

## USER GUIDELINES AND PROGRAM LIMITATIONS

Guidelines for the use of the DYNASOR II code along with the limitations placed upon the analysis are enumerated in this section. Some of these limitations are the result of the procedures used to program the equations while other limitations are inherent in the formulation of the equations. Since most of the limitations are minor in nature, the DYNASOR II code may be used to solve a wide variety of important she11 dynamics problems.

The maximum number of elements which the program may use is fifty (50). The maximum number of harmonics which may be coupled for the analysis is five (5). It is believed that these limitations will not hinder the user in solving most problems. However, since undoubtedly some users will want to modify the program capacity, instructions for increasing or decreasing the allowable number of elements and/or harmonics are provided in Appendix 8.

In all analyses using the DYNASOR II code, the zero ${ }^{\text {th }}(0)$ harmonic must be specified as one of the input harmonics.

The coefficients of thermal expansion are assumed to be constant in the two principal directions for any given element but may vary from element to element.

The number of nodal restraints must be less than or equal to the maximum number of degrees of freedom for each harmonic (204).

The displacements of the nodes may be calculated for as many as twenty (20) angles around the circumference of the shell element.

While the displacements are calculated at every time increment, it is necessary to calculate the stresses only at time steps where a printout of the stresses is desired. The stresses and stress resul. tants are calculated at the middle of the elements (s-direction) for up to twenty (20) angles in the circumferential direction. The angles at which the stresses are calculated are the same as those at which the displacements are determined. The stresses on both the inner and outer surfaces are determined.

The units used in the program must be consistent with those used in the SAMMSOR code. All calculations in the versions supplied to the users of the code are given inch-pounds-seconds units.

The program accepts the mechanical and thermal load histories by accepting descriptions at discrete points in time. The difference between the times for which loads are specified must, in all cases, be greater than the value of the time increment used in solving the equations of motion. The load variation curve is approximated by assuming a linear variation of the generalized forces between the times at which the loads are specified. It may therefore be necessary to specify the loads and temperatures at a fairly large number of points in time if the loads vary rapidly with time.

If the loads and/or temperatures propagate in any direction (moving loads), it will also be necessary to specify the loads at a fairly large number of points in time.

Pressure loadings, temperatures, and temperature gradients are assumed to be constant over the meridional length of the element but may vary in the circumferential direction. The variation in the
circumferential direction (except for shear loadings) must be symmetric about the meridian corresponding to $0=0$ degrees. These loadings may be input either by specifying the values at a number of circumferential angles for each element or by specifying the values of the Fourier coefficients for each harmonic.

If the program is not being restarted, the loads and temperatures must be specified at time $\mathrm{T}=0.0$. Times at which loads must be specified when restarting the program are noted in Appendix 6.

One of the most important considerations in any dynamic analysis is the selection of the time increment to be used in the analysis. Several criteria have been developed for use in selecting a time increment in analyses utilizing finite difference techniques. Most of these criteria require that the time increment be less than the time required for a signal to travel at the speed of sound from one difference point to the next. These criteria have been found (Ref. 8) inadequate for use in this analysis. A "feel" for the selection of a time increment must be obtained by the user. To facilitate the development of this "feel" the time increments utilized in a number of problems have been carefully documented in Refs. 3 and 4 . In addition, the input data for the example problems should prove helpful.

A restart capability is incorporated in the code to enable the user to calculate the response from a specified point in time without having to recalculate the response prior to this time. A most valuable use of this capability arises if, after evaluation of the results of a run, it is decided to extend the calculations to observe more cycles
of response. If it is desired to employ a different time increment (either smaller or larger), the user should refer to the discussion in Appendix 5. Effective use of the restart capability can result in a substantial savings of computer time. In general, the information necessary for restarting the code should be placed on tape at least every 100-400 time increments to insure that the information will be available if it is deemed desirable to restart the program.

The pseudo loads due to the nonlinearities associated with the initial displacements are neglected when calculating the response at the end of the first time step. However, when restarting the code, the initial increment utilizes both the mechanical and pseudo forces.

An extended effort has been made to check all aspects of the code. Comparisons of the response obtained using DYNASOR II with the results obtained by other researchers are presented in Ref. 3 and 4. These comparisons firmly establish the validity of the code. Although the programming logic and the formulation have been thoroughly checked to insure the correctness of the code, the authors assume no responsibility for the results obtained using the code.

## SECTION III

## PROGRAM INPUT

The DYNASOR II code has been written so that the code can be employed by researchers who are not familiar with the inner workings of the program. Utilizing the guidelines and adhering to the limitations presented in the previous section, it is believed that most users will find it relatively easy to employ the code.

The code is available in the FORTRAN IV language using doubleprecision or single-precision arithmetic. This double-precision version requires a storage space of about 330K bytes on the IBM 360/65 system while the single-precision storage space is about 200K bytes. Efforts have been made to make this code compatible with a large number of computing systems. In particular, adaption of the code for use on a CDC 6600 computer requires only minor changes.

The input data for a run consists of one card.I (card types will be explained on the following pages) followed by a complete set of data (cards II-X) for each case. The set of cards II-X is the input data required to generate the response of a shell for a given number of harmonics due to a particular loading. The cards comprising the data deck for both an initial run and a restart are schematically represented in Fig. 1. The cards specifying the Fourier harmonics, the initial conditions, and the boundary conditions are omitted from the input deck when using the restart mode. If more than one case is to be run, include a set of data for each of the cases. There is no limit on the number of cases which may be included in a run. A card XI must be placed at the end of the data for the final case.

FIG. 1 CONSTITUTION OF DATA DECKS - INITIAL RUN AND RESTART MODES.
I. RUN CONTROL CARD

This card is used to identify the number of cases to be run and the logical unit numbers of the scratch tapes used in the run. (ONLY ONE CARD I IS USED PER RUN.)

Card Type I Format (3I5)

| Columns | Variable | Description |
| :---: | :---: | :---: |
| 1-5 | NCASES | The number of different data sets utilized for this run. |
| 6-10 | ND | Logical unit number of the scratch tape onto which all the data is read at the start of the run. |
| 11-15 | NS | Logical unit number of a second scratch tape used by the program. |

II. CASE IDENTIFICATION CARDS

These cards allow the user to print out comments which identify the problem being run.
A. Control Card (ONE CARD II-A PER DATA SET)

| Card Type II-A Format (215) |  |  |
| :---: | :---: | :---: |
| Columns | Variable | Description |
| 1-5 | NCARDS | Number of comment cards (TYPE II-B) which follow. |
| 6-10 | NT | Logical unit number of the tape (prepared by SAMMSOR) from which the stiffness and mass matrices, element properties, and restart information, if needed, will be read. |

B. Identification Cards - The information punched on these cards is printed as output and should identify the problem being run. These comments should not duplicate those of the SAMMSOR case since the SAMMSOR comments will also appear as output. (IF NCARDS=0, OMIT CARDS II-B, OTHERWISE INCLUDE NCARDS OF TYPE II-B.)

| Card Type | Format (20A4) |  |
| :---: | :---: | :---: |
| Columns | Variable | Description |
| 1-80 | COMENT | Any desired alphanumeric information may be printed on these cards. |

III. CASE CONTROL CARDS
A. Control Constants - Time parameters, restart information, and other miscellaneous control constants are input on this card.
(INCLUDE ONE CARD III-A PER DATA SET.)

| Card Type III-A Format (2F10.0, 4I5) |  |  |
| :---: | :---: | :---: |
| Columns | Variable | Description |
| 1-10 | TOTIME | The maximum time (seconds) for which the cal culations are to be performed. |
| 11-20 | DELTE | Time increment (seconds) used in solving the equations of motion. |
| 21-25 | IRSTRT | Control constant which indicates if the solution is being restarted. If the solution is being restarted set IRSTRT $=1$. If not, set IRSTRT $=0$. |
| 26-30 | INCRST | The number of the time increment at which the solution is to be restarted. INCRST must be an integer multiple of the value of NPRNIT used in the previous run. If IRSTRT= 0 , set INCRST $=0$ 。 |
| 31-35 | NCLOSE | For a closed shell (such as a spherical cap or a hemisphere) where node 1 is at the apex set NCLOSE $=1$. Radial and rotational restraints will then be applied for the zeroth harmonic to aid the numerical stability of the solution. If the shell does not fit the above description, set NCLOSE $=0$. |
| 36-40 | ITELF | If thermal loads are to be applied in the program, set ITELF $=1$. Otherwise, set ITELF $=0$. |

B. Print Control Card - The constants used to control the program output are punched on this card. (INCLUDE ONE CARD III-B PER DATA SET.)

| Card Type | Format (1015) |  |
| :---: | :---: | :---: |
| Columns | Variable | Description |
| 1-5 | NPRNTQ | If the displacements are to be printed, set NPRNTQ $=1$. If not, set NPRNTQ $=0$. |
| 6-10 | IPRINT | If NPRNTQ $=1$, the displacements will be printed every IPRINT time increments beginning with the first time step. If NPRNTQ=0, set IPRINT $=0$. |
| 11-15 | NCLCST | If the stresses and stress resultants are to be calculated, set NCLCST=1. If not, set NCLCST $=0$. |
| 16-20 | NSTRSS | If NCLCST=1, the stress and stress resultants will be calculated and printed every NSTRSS time increments beginning with the first step. If $\operatorname{NCLCST}=0$, set $\mathrm{NSTRSS}=0$. |
| 21-25 | NPRNT | If restart information is to be placed on tape, set NPRNT=1. If not, set NPRNT=0. |
| 26-30 | NPRNIT | If NPRNT=1, the restart information will be written on the output tape every NPRNIT time increments. If NPRNT $=0$, set NPRNIT $=0$. It is suggested that relatively large values of NPRNIT be used, say 200, 400, etc., if the total number of time steps is relatively large. |
| 31-35 | NPRNTL | If a printout of the applied loads is desired, set NPRNTL=?. Otherwise, set NPRNTL=0. |
| 36-40 | NPRNTF | If a printout of the generalized forces is desired, set NPRNTF=1. Otherwise, set NPRNTF=0. |
| 41-45 | NPRNTH | If the Fourier coefficients for the temperature and temperature gradient are to be printed, set NPRNTH=1. Otherwise, set NPRNTH=0. |
| 46-50 | NPRNMS | If the mass and stiffness matrices are to be printed, set NPRNMS=1. If not, set NPRNMS=0. |

## IV. CIRCUMFERENTIAL ANGLE CARDS

The circumferential angles at which the displacements and stresses are to be calculated are read from these cards.
A. Control Card - (ONE CARD IV-A PER DATA SET.)

| Card Type IV-A Format (I5) |  |  |
| :---: | :---: | :---: |
| Columns | Variable | Description |
| 1-5 | NTHETA | The number of circumferential angles at which the displacements and possibly stresses are to be calculated. ( $1 \leq$ NTHETA $\leq 20$ ) |

B. Circumferential Angles - (INCLUDE 1-3 CARDS IV-B PER DATA SET, DEw PENDING UPON THE VALUE OF NTHETA.)

V. FOURIER HARMONICS CARD

This card provides the number of Fourier cosine harmonics to be employed for this analysis and enumerates the specific harmonics to be used, (IF IRSTRT=1, OMIT CARD V. OTHERWISE, INCLUDE ONE CARD V PER DATA SET.)

Card Type V Format (6I5)
Columns Variable Description

1-5 NH The total number of Fourier cosine harmonics to be utilized in this analysis ( $1 \leq \mathrm{NH} \leq 5$ ).
$\begin{array}{rll}6-10 & \text { IHARM(1) } & \text { Specific harmonic numbers to be employed. NH } \\ 11-15 & \text { IHARM(2) } & \text { values must be given and the zero harmonic }\end{array}$ 16-20 IHARM(3) must always be specified as one of the input 21-25 IHARM (4) harmonic numbers. The user should check to be 26-30 IHARM(5) certain that the information for each of these harmonics has been created and stored on tape by the SAMMSOR code.

Example: Consider a case where it is desired to utilize harmonics $0,2,3$ and 4. The input data for card $V$ would then utilize the following values:

$$
\begin{array}{ll}
\text { NH } & =4 \\
\text { IHARM }(1)=0 \quad \text { NOTE: } & \begin{array}{l}
\text { IHARM (1) should always } \\
\text { be set equal to zero. }
\end{array} \\
\text { IHARM }(2)=2 & \\
\operatorname{IHARM}(3)=3 \\
\text { IHARM }(4)=4 &
\end{array}
$$

Columns 26-30 corresponding to IHARM (5) should be left blank for this example since only four harmonics are being run.
VI. NODAL RESTRAINT CARDS (Boundary Conditions)

The displacement constraints applied to the shell are described utilizing these cards. (IF IRSTRT=1, OMIT CARDS VI-A AND VI-B.)
A. Control Card - (ONE CARD VI-A PER DATA SET, UNLESS IRSTRT=1.)

| Card Type | Format (I5) |  |
| :---: | :---: | :---: |
| Columns | Variable | Description |
| 1-5 | NODRES | Total number of displacement constraints to be applied to the shell ( $0 \leq$ NODRES $\leq 204$ ) |

B. Boundary Conditions - (THE NUMBER OF CARDS OF TYPE VI-B MUST EQUAL

NODRES, UNLESS IRSTRT=1. IF NODRES=0, OMIT CARDS VI-B.)

| Card Type VI-B Format (2I5) |  |  |
| :---: | :---: | :---: |
| Columns | Variable | Description |
| 1-5 | NP | Number of the node where the restraint is to be applied. |
| 6-10 | NDIRCT | Key used to indicate the degree of freedom which is restrained. <br> NDIRCT = 1 applies axial restraint <br> NDIRCT $=2$ applies circumferential restraint <br> NDIRCT $=3$ applies radial restraint <br> NDIRCT $=4$ applies rotational restraint |

VII. INITIAL CONDITIONS CARDS

The initial velocities and displacements of the nodes are specified on these cards. (IF IRSTRT=1, OMIT CARDS VII-A, VII-B, AND VII-C.)
A. Control Card - Utilization of this control card greatly simplifies the specification of the initial conditions if either the initial velocities or the initial displacements, or both, are equal to zero. (ONE CARD VII-A PER DATA SET)

| Card Type | Format (2I5) |  |
| :---: | :---: | :---: |
| Columns | Variable | Description |
| 1-5 | IQN | If the initial velocities at all the nodes are zero, set $I Q N=0$. If not, set $I Q N=1$. |
| 6-10 | IQN1 | If the initial displacements at all the nodes are zero, set $\mathrm{I} Q \mathrm{~N} 1=0$. If not, set $\mathrm{IQN}=1$. |

B. Initial Velocities - The initial nodal velocities must be specified for each node of the shell for each harmonic to be run. The logic used to input the nodal velocities is essentially the same as the procedure used to specify the element properties in the SAMMSOR code. The initial velocities for each of the nodes are specified for the first of the input harmonics, then for the second input harmonic, etc. This process is repeated until the nodal velocities for each harmonic have been specified. (IF IQN=0, OMIT CARDS VII-B.)

| Card Type VII-B Format ( $215,4 \mathrm{~F} 10.0$ ) |  |  |
| :---: | :---: | :---: |
| Columns | Variable | Description |
| 1-5 | IN1 | First node to which the velocities specified on this card are applied. |
| 6-10 | IN2 | Last node to which the velocities specified on this card are applied. |
| 11-20 | $\dot{\square}_{1}$ | Initial nodal velocity in the axial direction for a particular harmonic. |
| 21-30 | $\dot{q}_{2}$ | Initial nodal velocity in the circumferential direction for a particular harmonic. |
| 31-40 |  | Initial nodal velocity in the radial direction for a particular harmonic. |
| 41-50 | $\dot{q}_{4}$ | Initial nodal rotational velocity in the meridional direction for a particular harmonic. |
| C. Initial Displacements - In identically the same manner as is utilized for the initial velocities, the initial displacements are specified for each harmonic. (IF IQN1=0, OMIT CARDS VII-C) |  |  |
| Card Type VII-C Format (2I5, 4F10.0) |  |  |
| Columns | Variable | Description |
| 1-5 | IN1 | First node to which the displacements specified on this card are applied. |
| 6-10 | IN2 | Last node to which the displacements specified on this card are applied. |
| 11-20 | $\mathrm{q}_{1}$ | Initial nodal displacement in the axial direction for a particular harmonic. |
| 21-30 | $\mathrm{q}_{2}$ | Initial nodal displacement in the circumferential direction for a particular harmonic. |
| 31-40 | $\mathrm{q}_{3}$ | Initial nodal displacement in the radial direction for a particular harmonic. |
| 41-50 | $q_{4}$ | Initial nodal rotation in the meridional direction for a particular harmonic. |

## VIII. COEFFICIENTS OF THERMAL EXPANSION

If the thermal effects are to be included in the analysis, the coefficients of thermal expansion must be specified using these cards. These coefficients are assumed to be constant for a given element but may vary from element to element. These coefficients are read in the same manner as the element properties in the SAMMSOR code. (THE NUMBER OF CARDS VIII MUST BE $\leq$ NELEMS FOR ANY GIVEN DATA SET. IF ITELF $=0$, OMIT CARDS VIII.)

| Card Type | Format (2I5, 2F10.0) |  |
| :---: | :---: | :---: |
| Columns | Variable | Description |
| 1-5 | IELM1 | Number of the first element to which the properties on this card apply. |
| 6-10 | IELM2 | Number of the last element to which the properties on this card apply. |
| 11-20 | ALSII | Coefficient of thermal expansion in the meridional direction (in/in/deg). |
| 21-30 | ALTII | Coefficient of thermal expansion in the circumferential direction (in/in/deg). |

## IX. APPLIED LOADS, TEMPERATURES, AND TEMPERATURE GRADIENTS

Since the concentrated nodal loads, distributed pressures, temperatures, and temperature gradients may vary in time; it may be necessary to specify these loads at a number of points in time. If these loads and temperatures are input at times $\mathrm{T1}_{\mathbf{j}}$ and $\mathrm{T1}_{\mathbf{i}+1}$, the program will calculate generalized forces due to these loads at each of the input times. A linear variation of the generalized forces is then assumed between the times the loads are input. As soon as the value of the time reaches $T 1_{i+1}$, a new set of loads is read in at $\mathrm{T1}_{\mathbf{i}+2}$ and the process of calculating the generalized forces is repeated. The time increment, DELTE (CARD III-A), used in the solution of the equations of motion must be less than the difference between any two of the times at which the loads are specified. If the loads and/or temperatures propagate in any direction (moving loads), it is advisable to specify the loads at more times than is necessary if they vary in intensity only.

Ring loads can be applied at the nodes and must be input for each of the harmonics. The ring loads utilize the same sign convention employed for the shell nodal displacements.

The pressure loadings, temperatures and temperature gradients are assumed constant over the meridional length of the element but variations in the circumferential direction are allowed. These loadings may be input in one of two ways. Either the Fourier coefficients can be specified for each harmonic or the values of the loads may be specified at a number of circumferential angles around the shell elements. Utilizing this second procedure a step function variation is assumed in the circumferential direction. That $i s$, the load is assumed constant from $\theta_{\mathbf{i}}$ to $\theta_{\mathbf{j}+1}$ with the value of the loads being equal to those specified at $\theta_{i}$. $\operatorname{Sign}$
conventions for the pressure loading are given in Fig. 2.
A control card (Card Type IX-A) containing several key variables is used to guide the reading of the loading conditions. Proper selection of the values of these key variables results in a highly efficient procedure for specifying a wide variety of loading conditions. The key words and their meanings are explained in Fig. 3.

Before attempting to input loads to the code the user is advised to study the guidelines presented in Sec. II, the example problems of Sec. IV, and appendix 6 which presents a thorough discussion of the various procedures necessary for specifying the loads.

(a) Membrane force resultants

(b) Transverse force resultants

(c) Moment resultants

(d) Loads per unit area

FIG 2 POSITIVE DIRECTION OF FORCES, MOMENTS, AND LOADS ON SHELL SEGMENT


Fig. 3 LOAD SPECIFICATION AT TIME T1 ${ }_{i}$
A. Load Control Card

This control card is utilized to direct the input of the loads for a given time. This card indicates the presence or absence of concentrated forces and distributed pressure loadings and indicates the procedure to be utilized for creating the generalized thermal forces.
(ONE CARD IX-A IS NECESSARY FOR EACH TIME AT WHICH THE LOADS ARE BEING INPUT.)

| Card Type IX-A Format (F10.0, 4I5, A8) |  |  |
| :---: | :---: | :---: |
| Columns | Variable | Description |
| 1-10 | T1 | The time for which the loads are being input (sec). |
| 11-15 | NCF | If concentrated ring loads are applied to the structure at time $T 1$, set NCF $=1$. If not, set NCF $=0$. |
| 16-20 | IDELF | If distributed loads are to be applied to the shell at time T1, set IDELF $=1$. If not, set $I D E L F=0$. |
| 21-25 | IDCOE | If the Fourier cosine coefficients for the distributed loadings are to be read in at time T1, set $I D C O E=1$. If not, set $I D C O E=0$. |
| 26-30 | ITCOE | If the Fourier cosine coefficients for the temperatures and temperature gradients are to be read in at time $T 1$, set $I T C O E=1$. If not, set $I T C O E=0$. |
| 31-38 | CONSTF | If the applied loads, temperatures and temperature gradients are constant from time, T1, to the final time, TOTIME (CARD III-A), punch the word CONSTANT-in columns 31-38. If these parameters are not constant, leave columns 31-38 blank. |

B. Concentrated Ring Loads

The concentrated ring loads must be specified for each harmonic.
(IF NCF $=0$, OMIT CARDS IX-B.)

1. Control Card - This card indicates the presence or absence of concentrated ring loads for a particular harmonic. (ONE CARD IX-B-1 FOR EACH HARMONIC.)
Card Type IX-B-1 Format (I5)

| Columns | Variable | Description |
| :---: | :---: | :---: |
| $1-5$ | NCF1 | If there are concentrated ring loads for this <br> particular harmonic, set NCF1=1. If not, set <br> NCF1=0. |

2. Concentrated Ring Loads - For harmonics having ring loads associated with them, the loads are specified using these cards. (IF NCF1=0, OMIT CARDS IX-B-2 FOR THE HARMONIC BEING CONSIDERED.) ONE OR MORE CARDS IX-B-2 MAY BE USED, BUT NEVER UTILIZE MORE THAN 51 PER HARMONIC.

Card Type IX-B-2 Format (2I5, 4F10.0)
Columns Variable Description
1-5 INI First node to which this loading applies.
6-10 IN2 Last node to which this loading applies.

11-20 F1 Axial ring load applied at a node (1b).*

[^1]| Columns | Variable | Description |
| :---: | :---: | :---: |
| 21-30 | F2 | Circumferential ring load applied at a node (1b).* |
| 31-40 | F3 | Radial ring load applied at a node (1b).* |
| 41-50 | F4 | Concentrated moment applied at a node (in-1b).* |

Examples:
The use of cards IX-B should become clear after considering the following examples

1. Consider the case where a uniform tensile ring loading of 100 psi is being applied in the axial direction to the first node of a cylinder. The solution for this problem has been presented in Fig. 20 of Ref. 3. The thickness of the cylinder is 0.1 inches with the radius being given as 6 inches. Consider that harmonics 0 and 2 are being run. The total ring load for the zero harmonic will be (100) $\times 2 \pi(6) \times(0.1)=$ 376.9 1b.

Five cards of type IX are required to input these loads assuming they are constant from time $\mathrm{Tl}=0.0$ to TOTIME and assuming 50 elements are used to idealize the structure.

| CARD |  |  | VARIABLE | VALUES |
| :---: | :---: | :---: | :---: | :---: |
| IX-A | $\mathrm{T} 1=0.0$ |  | NCF $=1$ | IDELF $=$ IDCOE $=$ ITCOE $=0$ |
| IX - ${ }^{\text {B }}$ | NCF1 $=1$ |  | (HARMONIC 0) |  |
| IX-C | IN1=1 $\quad$ N $1=1$ | $F 1=-376.9$ | $F 2=F 3=F 4=0$ |  |
| IX-C | IN1=2 IN1=51 | $F 1=F 2=F 3$ | F4=0 |  |
| IX-B | NCF2 $=0$ |  | (HARMONIC 2) |  |

[^2]2. The second example considers a radial ring load of $F \cos 0$ applied to a cylinder of radius $r$.

Performing the integration, one obtains the radial ring load for harmonic 1 as

$$
\begin{aligned}
F 3 & =\int_{0}^{2 \pi}(F \cos \theta) r \cos \theta d \theta \\
& =\pi r F
\end{aligned}
$$

The Fourier coefficients for the other harmonics are zero.
C. Distributed Loads - (IF IDELF $=0$, OMIT CARDS IX-C) The distributed loadings may be input in one of two ways: the Fourier coefficients may be read in for each harmonic or the loadings may be specified at a desired number of circumferential angles ( $\leq 37$ ). If the second option is used, the Fourier coefficients will then be generated internally. The user should note that it is possible to input distributed loads in only one of two ways.

1. Distributed Loads - (Input at various circumferential angles) Since the choice of the displacement functions utilized in this analysis necessitate the presence of loads symmetric about the meridian $\theta=0$, it is necessary to specify the distributed loadings for angles from $0 \rightarrow 180^{\circ}$. The code then assumes that the distribution from $180^{\circ} \rightarrow 360^{\circ}$ is the mirror image of the input distribution. (IF IDCOE $=1$, OMIT CARDS IX-C-1)
a. Control Card - Utilize this card to indicate the number of angles for which the loads will be specified.

| Card Type IX-C-1-a Format (315) |  |  |
| :---: | :---: | :---: |
| Columns | Variable | Description |
| 1-5 | IELM1 | First element to which this distributed loading applies. |
| 6-10 | IELM2 | Last element to which this distributed loading applies. |
| 11-15 | NDP | Number of circumferential angles at which the distributed loads are to be specified ( $1 \leq$ NDP $\leq 37$ ). If the loadings are constant in the circumferential direction set NDP $=1$. |
| b. Distributed Loads at Specified Angles* - This card specifies the angle at which the loads are being input and provides the values of the loads at that angle. (INCLUDE NDP CARDS OF TYPE IX-C-1-b FOR EACH CARD IX-C-1-a:) |  |  |
| Card Type IX-C-1-b Format (4F10.0) |  |  |
| Columns | Variable | Description |
| 1-10 | THETB | Circumferential angle (degrees) for which this data is given. |
| 11-20 | P | Distributed load in the meridional direction (psi). |
| 21-30 | R | Distributed load in the normal direction (psi). |
| $31-40$ | S | Distributed load in the circumferential direction (psi). |

Example: Consider the normal pressure distribution on an element depicted in Fig. 4. To input the pressure on this element requires specification of

[^3]

FIG 4 REPRESENTATIVE VARIATION OF DISTRIBUTED LOADS APPLIED TO A TYPICAL ELEMENT
the pressures for four values of 0 .
THETB R(I)

$120.0 \quad 0.0$
2. Distributed Loads - (Fourier Coefficients) The Fourier coefficients for the distributed loads may be specified using these cards. The coefficients must be specified (even though they may be zero) for each harmonic being employed in the analysis. The coefficients are specified for each harmonic of the first group of elements, then for each harmonic of the second group, etc. until the values have been input for all the elements. (IF IDCOE $=0$, OMIT CARDS IX-C-2)
a. Control Card

| Card Type IX-C-2-a Format (2I5) |  |  |
| :---: | :---: | :---: |
| Columns | Variable | Description |
| 1-5 | IELM1 | First element to which these loads apply. |
| 6-10 | IELM2 | Last element to which these loads apply. |

b. Fourier Coefficients - (NH CARDS OF TYPE IX-C-2-b FOR EACH CARD IX-C-2-a.)

| Card Type IX-C-2-b Format (3F10.0) |  |
| :---: | :---: |
| Columns | Variable |

D. Temperature Distribution and Gradients -

Essentially the same logic is employed for inputting the temperatures and gradients that was used for the specification of the distributed loads. The explanation of this procedure should therefore not need be repeated.

The temperatures are specified for the midsurface of the shell. The temperature gradients (through the thickness) are considered positive if the temperature for the outer surface is greater than the temperature on the inner surface. (IF ITELF $=0$, OMIT CARDS IX-D.)

1. Temperature Distribution and Gradients - (Input at various circumferential angles)

Again, the requirement of symmetry about the meridian $\theta=0$, makes it necessary to specify the temperature distribution and thermal gradients only from $0^{\circ} \rightarrow 180^{\circ}$. The temperature distribution and gradients are input on the same cards for the various angles. (IF ITCOE $=1$, OMIT CARDS IX-D-1.)
a. Control Card - Utilize this card to indicate the number of angles for which the temperature and gradients will be specified.

| Card Type IX-D-1-a Format (3I5) |  |  |
| :---: | :---: | :---: |
| Columns | Variable | Description |
| 1-5 | IELMI | First element to which this data applies. |
| 6-10 | IELM2 | Last element to which this data applies. |
| 11-15 | NDP | Number of circumferential angles at which the temperature distribution and gradient are to be specified ( $1 \leq N D P \leq 37$ ). If the temperature is constant in the circumferential direction, set NDP $=1$. |

b. Temperature and Temperature Gradient at Specified Angles This card specifies the angle at which the temperature and temperature gradient (through the thickness) is being input and provides the value of the temperature at that angle. (INCLUDE NDP CARDS OF TYPE IX-D-1-b FOR EACH CARD IX-D-1-a.)

| Card Type IX-D-1-b Format (3F10.0) |  |  |
| :---: | :---: | :---: |
| Columns | Variable | Description |
| 1-10 | THETB | Circumferential angle for which this temperature and gradient are given. |
| 11-20 | P | Distributed temperature at $\theta=\operatorname{THETB}\left({ }^{\circ} \mathrm{F}\right)$. |
| 21-30 | R | ```Temperature gradient (through the thickness) at 0 = THETB (}\mp@subsup{}{}{\circ}\textrm{F}/\textrm{in}\mathrm{ ).``` |

2. Temperature Distribution and Gradient - (Fourier Coefficients) If the user so desires, the Fourier coefficients for the temperature distribution and gradient may be specified for each of the harmonics being used. Again, the coefficients are specified for all harmonics for the first group of elements, then for the second group, etc., until all the element coefficients have been input. (IF ITCOE $=0$, OMIT CARDS IX-D-2)
a. Control Card

| Card Type IX-D-2-a Format (2I5) |  |  |
| :---: | :---: | :---: |
| Columns | Variable | Description |
| 1-5 | IELM1 | First element to which these properties apply. |
| 6-10 | IELM2 | Last element to which these properties apply. |
| b. Fourier Coefficients - (NH CARDS OF TYPE IX-D-2-b FOR EACH CARD IX-D-2-a.) |  |  |
| Card Type IX-D-2-b Format (2F10.0) |  |  |
| Columns | Variable | Description |
| 1-10 | TH1 | Fourier coefficient of the temperature distribution ( ${ }^{\circ} \mathrm{F}$ ) for a particular harmonic. |
| 11-20 | DTH1 | Fourier coefficient of the temperature gradient ( ${ }^{\circ}$ F/in) for a particular harmonic. |

## X. FINAL DATA CARD FOR A CASE

Place this card after the last card IX of each data set. This signifies the end of the input data for a case. (ONE CARD $X$ PER DATA SET.) Card Type X

| Columns | Punch |
| :---: | :---: |
| $1-11$ | END øF CASE |

## XI. FINAL DATA CARD FOR A RUN

This card must be placed after the card $X$ of the last case to be run. It denotes the end of the input data for a run. (ONE CARD XI PER RUN)

## Card Type XI

| Columns | Punch |
| :---: | :---: |
| $1-10$ | END $\emptyset F$ RUN |

## SECTION IV

## EXAMPLE PROBLEMS

The example problems which follow were chosen to demonstrate the versatility of the code and to further acquaint the users with the procedures for inputting the data to the code. The data presented herein is typical for the problems solved by the code and demonstrates many of the input procedures.

Since the most complex portion of the input data is the specification of the loading conditions, a variety of loadings are demonstrated. Response curves are presented so the user may check his output with the previously obtained curves. The first two example problems utilize the shells described in example problems 1 and 2 of the SAMMSOR user's guide (Ref. 1) while the third example problem demonstrates the two procedures for specifying distributed pressure loadings.

## Example Problem 1

The first example problem was chosen to demonstrate the procedure for inputting a concentrated ring load and to demonstrate the program's capability to solve highly nonlinear problems. For the forty pound load applied in this problem, the static solution shows that the nonlinear displacement is more than four times as large as the linear solution.

The shell to which the load is applied is the shallow spherical cap $(\lambda=6)$ utilized in the first example problem in the SAMMSOR user's guide. The edges of the shell are assumed to be clamped. Since the loading is symmetric, the displacements and stresses will be calculated only along the line $\theta=0$. Only the response for the zero ${ }^{\text {th }}$ harmonic will be determined. A set of input data for this case is presented in Fig. 5 with the displacement response of the apex of the shell being presented in Fig. 6. This response curve should allow the user to check his version of the code.


Fig. 5 INPUT DATA - EXAMPLE PROBLEM 1


## Example Problem 2

The shell described in the second example problem in the SAMMSOR user's guide is now subjected to a 50 psi internal pressure. The loadin is applied at time $T 1=0.0$ and remains constant for the duration of the calculation.

Two sets of input data are provided for this example problem. The first set (Fig. 7) allows the program to calculate the response for the first 300 time steps. The second set of input data (Fig. 8) will restart the code at the end of the 300th time step and will then allow the program to calculate the response for an additional 300 increments.

Since this problem is only moderately nonlinear, it is interesting to note that a much larger time step can be used for this problem than was employed in the previous example problem. The displacement response obtained for this problem is presented in Fig. 9.
NCASE= 2


Fig. 7 INPUT DATA - EXAMPLE PROBLEM 2


Fig. 8 INPUT DATA - EXAMPLE PROBLEM 2 - RESTART MODE

PRESSURE

Example Problem 3
This example problem was selected to demonstrate the procedures for inputting the distributed loadings on a shell. A cylindrical shell (Fig 10) is subjected to a half cosine loading which is symmetric about the meridian $=0$. This load is applied along the entire length of the shell. The pressure loading may be specified in one of two ways:

1) The Fourier coefficients may be input for each harmonic.
2) The pressure may be specified at various circumferential angles with the Fourier coefficients then being internally generated.

The first set of input data (Fig. 11) utilizes the first of the above procedures and inputs the Fourier coefficients. The input data presented in Fig. 12 describes the loading by specifying the value of the pressure at the various angles. The same procedure is employed to describe the temperature and temperature gradient distributions.

Considering the symmetry of the loading and the boundary conditions applied to this shell, it can easily be recognized that the displacements and stresses will be symmetric abour the center of this cylindrical tube. Therefore, only one-half of the shell needs to be analyzed. The plane of symmetry is assured by applying an axial and a rotational restraint at node one (1).


FIG 10 CYLINDRICAL SHELL SUBJECTED TO HALF cosine pressure loading


NCASE $=4$
PRINTOUT OF INPUT DATA


Fig. 12 INPUT DATA - (SET \#2) - EXAMPLE PROBLEM 3

## REFERENCES

1. Tillerson, J. R., and Haisler, W. E., "SAMMSOR II - A Program for the Calculation of Stiffness and Mass Matrices of Shells of Revolution," TEES-RPT-70-18, Texas A\&M University, Oct. 1970.
2. Novozhilov, V. V., Foundations of the Nonlinear Theory of Elasticity, Graylock Press, Rochester, N.Y., 1956.
3. Stricklin, J. A., Martinez, J. E., Tillerson, J. R., Hong, J. H., and Haisler, W. E., "Nonlinear Dynamic Analysis of Shells of Revolution by Matrix Displacement Method," Sandia Corporation Contractor Report SC-CR-70-6070, Feb. 1970.
4. Stricklin, J. A., Martinez, J. E., Tillerson, J. R., Hong, J. H., and Haisler, W. E., "Nonlinear Dynamic Analysis of Shells of Revolution by Matrix Displacement Method," AIAA/ASME 11 th Structures, Structural Dynamics, and Materials Conference, Denver, Colorado, Apri1, 1970.
5. Stricklin, J. A. and Martinez, J. E., "Dynamic Buckling of Clamped Spherical Caps Under Step Pressure Loadings," AIAA Journal, Vol. 7, No. 6, June 1969, pp. 1212-1213.
6. Stricklin, J. A., Haisler, W. E., MacDougall, H. R., and Stebbins, F. J., "Nonlinear Analysis of Shells of Revolution by the Matrix Displacement Method," AIAA Journal, Vol. 6, No. 12, Dec. 1968, pp. 2306-2312.
7. Houbolt, J. C., "A Recurrence Matrix Solution for the Dynamic Response of Elastic Aircraft," Journal of Aeronautical Science, Vol. 17, Sept. 1950, pp. 540-550.
8. Tillerson, J. R. and Stricklin, J. A., "Numerical Methods of Integration Applied in the Nonlinear Dynamic Analysis of Shells of Revolution," TEES Report 70-23, Texas A\&M University, July 1970.

APPENDIX

## Appendix 1 - Description of Subroutines DYNASOR II

| Subroutine | Description |
| :---: | :---: |
| 1. MAIN | Controls flow of program by incrementing the time and calling subroutines 2-15 to obtain the displacements, stress resultants, and upper and lower surface stresses. |
| 2. INPUT | Reads stiffness and mass matrices from tape. Reads program control parameters. Prints stiffness and mass matrices, shell properties, loads, restart information, etc. |
| 3. SETUP | Calls subroutines to solve equations of motion and checks the stability of numerical solution. |
| 4. NLTERM | Computes various terms necessary for calculating the nonlinear loads. Adds generalized loads due to external loads and temperatures to nonlinear loads. Controls calculation of stress resultants. |
| 5. QPRIME | Calculates generalized nonlinear loads. |
| 6. HOUBQ1 | Set up the equations for calculating the displacements for the first time increment and prepares coefficient matrices for use in all subsequent steps. |
| 7. HOUBQN | Sets up equations for calculating the displacements for each time increment except the first one. |
| 8. NRESTR | Applies the desired boundary conditions to the left side of the equations of motion. |
| 9. MATMUT | Multiplies two specialized matrices. |
| 10. SOLVEQ | Utilizing a modified Gaussian elimination technique which considers the diagonal symmetry of the matrix equations the solution to a set of simultaneous linear algebraic equations is obtained. |

Appendix 1 - Continued

| Subroutine | Description |
| :---: | :---: |
| 11. STRESS | Calculates and prints stress resultants and stresses at upper and lower faces. |
| 12. FRCES | Reads distributed loads and calculates linear generalized forces. |
| 13. THCOE | Reads temperatures and temperature gradients and calculates thermal Fourier coefficients. |
| 14. TFORCE | Calculates linear thermal loads. |
| 15. TRI 40 R | Calculates various trigonometric integrals required in the calculations made in subroutine QPRIME. |

## Appendix 2 - Significant FORTRAN Variables DYNASOR II

| Variable | Subroutine where variable is defined or calculated | ined Description |
| :---: | :---: | :---: |
| ALS | INPUT | Matrix of coefficients of thermal expansion in the meridional direction for the elements. |
| ALT | INPUT | Matrix of coefficients of thermal expansion in the circumferential direction for the elements. |
| ARCL | INPUT | Matrix of the arc lengths of the elements (sdirection). |
| CCC | TRI40R | Matrix containing integrals from 0 to $2 \pi$ of cosine $i \theta$ cosine $j \theta$ cosine $k \theta$. |
| CCCC | TRI40R | Matrix containing integrals from 0 to $2 \pi$ of cosine $i \theta$ cosine $j \theta$ cosine $k \theta$ cosine $1 \theta$. |
| CONSTF | INPUT | Alphameric constant which controls input of loads. |
| COSINE | INPUT | Matrix whose elements are the cosine of $\phi$ at each of the nodes. |
| COSM | INPUT | Matrix whose elements are the cosine of $\phi$ at the middle of each element. |
| DELTE | INPUT | Time increment used in solving the equations of motion of the shell. |
| DELTEP | INPUT | Used in restarting the solution, this variable is the time increment used in the previous run. |
| DTH | INPUT | Matrix of Fourier coefficients for the circumferential temperature gradient distribution. |
| DT2 | INPUT | The square of the time increment. |

## Appendix 2 - Continued

| Variable | Subroutine where variable is defin or calculated | ned Description |
| :---: | :---: | :---: |
| ES | QPRIME Mat | Matrix of the linear strains, $\hat{e}_{s}$, used in calculating $\left\{\partial U_{\mathrm{NL}} / \partial q\right\}$ for each harmonic. |
| EST | QPRIME Ma | Matrix of the linear strains, $e_{s e}$, used in calculating $\left\{\partial U_{N L} / \partial q\right\}$ for each firmonic. |
| ET | QPRIME Mat | Matrix of the linear strains, $e_{\theta}$, used in calculating $\left\{\partial U_{N L} / \partial q\right\}$ for each harmonic. |
| E1 | INPUT Ma | Matrix containing the Young's modulus in the meridional direction, $\mathrm{E}_{\mathrm{S}}$, for each element. |
| E2 | INPUT | Matrix containing the Young's modulus in circumferential direction, $E_{\theta}$, for each element. |
| E13 | QPRIME Ma | Matrix of the nonlinear strains, $e_{13}$, used in calculating $\left\{\partial U_{N L} / \partial q\right\}$ for each harmonic. |
| E23 | QPRIME Ma | Matrix of the nonlinear strains, $e_{23}$, used in calculating $\left\{\partial \mathrm{U}_{\mathrm{NL}} / \partial q\right\}$ for each harmonic. |
| FNU1 | INPUT Ma | Matrix containing the values of Poisson's ratio, $\nu_{S_{\theta}}$, for each element. |
| FNU2 | INPUT Ma | Matrix containing the values of Poisson's ratio, $\nu_{\theta S}$, for each element. |
| FORCE | INPUT <br> TFORCE FRCES | Matrix of generalized forces due to external loads and temperatures. |
| F1 | INPUT Ax | Axial ring load applied at a specified node. |
| F2 | INPUT $\quad$ C | Circumferential ring load applied at a specified node. |
| F3 | INPUT Rad | Radial ring load applied at a specified node. |

Appendix 2 - Continued

| Variable | Subroutine where variable is defined or calculated | ined Description |
| :---: | :---: | :---: |
| F4 | INPUT | Concentrated moment applied at a specified node. |
| G | INPUT | Shear modulus, $G$ [for an isotropic material $G=E / 2(1+v)]$. |
| IDCOE | FRCES | Input constant which indicates whether the Fourier coefficients will be read as input data or will be generated internally. |
| IDELF | INPUT | Control parameter used to indicate the presence or absence of distributed pressure loads at a given time. |
| IHARM | INPUT | Matrix of harmonic numbers for which displacements and/or stresses will be calculated. |
| INCRST | INPUT | The time increment at which the program will be restarted. |
| IPRINT | INPUT | Input constant which controls printing of displacements. |
| IQN | INPUT | Input constant which controls the reading of the nodal initial velocities. |
| IQN1 | INPUT | Input constant which controls the reading of the nodal initial displacements. |
| IRSTRT | INPUT | Input constant which indicates if the program is being restarted. |
| ITAM | MAIN | Number of time cycle. |
| ITCOE | INPUT | Input constant which indicates whether the Fourier thermal load coefficients are to be read in or to be internally calculated. |

Appendix 2 - Continued

| Variable | Subroutine where variable is defined or calculated | ined Description |
| :---: | :---: | :---: |
| ITELF | INPUT | Input constant which indicates the inclusion or exclusion of thermal loads. |
| ITP | INPUT | Number of the time cycle at which restart data has been written on tape. |
| Junk | INPUT | Dummy variable used for reading and writing on tape without violating the minimum record length allowed. |
| KKP2 | MAIN | Largest value which ITAM can assume for a given case. |
| LARGE | SETUP | Constant which controls termination of problem if displacements become excessive. |
| LK | INPUT | Matrix indicating the nodal restraints which are applied on the shell. |
| NCF | INPUT | Input constant which indicates the presence or absence of concentrated nodal loads at a specified time. |
| NCLCST | INPUT | Input constant used to indicate if the stresses and stress resultants are to be calculated. |
| NCLOSE | INPUT | Constant used to indicate the presence of a singularity at the first node of a closed shell |
| ND | MAIN | Logical unit number of the scratch tape on which the input data is stored. |
| NDIRCT | INPUT | Constant indicating the direction of the restraint applied at a node ( $1 \leq$ NDIRCT $\leq 4$ ). |
| NDP | $\begin{aligned} & \text { FRCES } \\ & \text { THCOE } \end{aligned}$ | Number of circumferential angles at which distributed loads or temperatures are to be specified. |

Appendix 2 - Continued

| Variable | Subro varia or ca | ined | Description |
| :---: | :---: | :---: | :---: |
| NELEMS | INPUT | Tota stru | ements used to |
| NEQ | INPUT | Numb | ium equation |
| NEQT | INPUT | Tot har | uilibrium equ |
| NH | INPUT | Tota ana | rmonics used |
| NHNS | INPUT | Leng for | 1 stiffness stored in vec |
| NHP | INPUT |  | that are st by SAMMSOR. |
| NNODES | INPUT | Tota | des, equal to |
| NODRES | INPUT | Numb the | ment constrai |
| NOIT | MAIN | Numb give | rements to be |
| NPRNIT | INPUT | Inpu infor | oting how oft be placed on |
| NPRNMS | INPUT | Inpu and outp | ch determines ices are to |
| NPRNT | INPUT | Inpu info | ch determines be placed on |

## Appendix 2 - Continued

| Variable | Subroutine where variable is defined or calculated |  | Description |
| :---: | :---: | :---: | :---: |
| NPRNTF | INPUT | Inpu eral info | ch determines to be print |
| NPRNTH | INPUT | Inpu the temp | do control cients for te t. |
| NPRNTL | INPUT | Inpu not | ch determines ads will be p |
| NPRNTQ | INPUT | Inpu disp | ting whether be printed. |
| NS | MAIN | Logi <br> tape | of one of the |
| NSIZE | INPUT | The ness part | in the stru (in vector c. |
| NSTRSS | INPUT | Inpu stre ed. | ch determines and stresses |
| NT | INPUT | Logi <br> the and be | of the tape ness matrices restart info |
| NTHETA | INPUT | The disp cula | umferential a or stresses a |
| P | FRCES | Dist | the meridion |
| $p$ | THCOE | Elem ent | which may va |

Appendix 2 - Continued

| Variable | Subroutine where <br> variable is defined <br> Description or calculated |  |
| :---: | :---: | :---: |
| PH | INPUT | Slope at the middle of an element. |
| PHP | INPUT | $d \phi / d s$ at the middle of an element. |
| QLOAD | HOUBQ1 <br> HOUBQN | Right-hand-side of the dynamic equations of motion before calling SOLVEQ. |
| QN | INPUT | Initial nodal velocities. |
| QN | HOUBQN | Displacements at time increment ( $n-1$ ) up to statement 11. After statement 13 this matrix has been changed the displacements at time step ( $n$ ). |
| QN1 | INPUT | Initial nodal displacements. |
| QN1 | HOUBQN | Displacements at time increment ( $n-2$ ) before statement 10 and at time increment ( $n-1$ ) after statement 11. |
| QN2 | HOUBQN | Displacements at time increment ( $n-3$ ) before statement 10 and at time step ( $n-2$ ) after statement 11. |
| QP | NLTERM | $\{Q\}-\left\{\partial U_{N L} / \partial q\right\} \text { at time step }(n-1)$ |
| QP1 | HOUBQN | $\{Q\}-\left\{\partial U_{N L} / \partial q\right\}$ at time step (n-2) |
| QPR | QPRIME | $-\left\{\partial U_{N L} / \partial q\right\}$ |
| R | FRCES | Distributed load in the normal direction. |
| R | INPUT | Matrix whose terms are the radius to the middle of each element. |

## Appendix 2 - Continued

| Variable | Subroutine where <br> variable is defined <br> Descriptio <br> or calculated |  |
| :---: | :---: | :---: |
| R | THCOE | Temperature gradient (through the thickness) which may vary circumferentially for an ele* ment. |
| S | FRCES | Distributed load in the circumferential direction. |
| SINE | INPUT | Sine of $\phi$ at the nodes of the structure. |
| SINM | INPUT | Sine of $\phi$ at the middle of the elements. |
| SSC | TRI40R | Matrix containing integrals from 0 to $2 \pi$ of sine $i \theta$ sine $j \theta$ cosine $k \theta$. |
| SSCC | TRI40R | Matrix containing integrals from 0 to $2 \pi$ of sine $i \theta$ sine $j \theta$ cosine $k \theta$ cosine $1 \theta$. |
| SSSS | TRI 40R | Matrix containing integrals from 0 to $2 \pi$ of sine $i \theta$ sine $j \theta$ sine $k \theta$ sine $1 \theta$. |
| T | INPUT | Matrix of element thicknesses. |
| TH | INPUT | Matrix whose elements are the Fourier coeffi. cient of the circumferential temperature distribution. |
| THETA | INPUT | Matrix containing circumferential angles at which stresses and/or displacements are to be calculated. |
| THETB | FRCES THCOE | Circumferential angle at which distributed loads or temperatures are to be input. |
| TIMEP | INPUT | Time at which restart data has been written on tape for future use. |
| TOTIME | INPUT | Total time, in seconds, for which the analysis is to be performed. |

## Appendix 2-Continued

| Variable | Subroutine where <br> variable is defined <br> Description or calculated |  |
| :---: | :---: | :---: |
| XN | INPUT | Structural stiffness matrix as read from input tape. After the first time step, this matrix is replaced by a combination of the mass and stiffness matrices $\left\{(\Delta t)^{2}[K]+2[M]\right\}$. |
| XP | INPUT | Structural mass matrix as read from input tape. |

APPENDIX 3 - SUBROUTINE CALL MAP DYNASOR II


## APPENDIX 4 - Flow Chart of Basic Operations DYNASOR II



APPENDIX 4 - Continued


Segment $C-C^{\prime}$ is flow-charted at


APPENDIX 4 Continued


## APPENDIX 4 - Continued

Flow Chart $C-C^{\prime}$


Appendix 4 - Continued


Appendix 5 - Use of the Restart Option

In order for efficient use to be made of the DYNASOR II code, the user should become familiar with the option provided for restarting the program. Through effective use of this option the dynamic response studies can be completed using a minimum amount of computer time.

Use of the restart option may prove invaluable in a number of situations. Abnormal termination of the program may occur if a numerical instability is noted in the response. If this occurs, the restart option can be used with a different value of the time increment. Another important use of the restart option arises when the user is satisfied with the results previously obtained but desires to extend the response data to a further point in time. In such a case the program is restarted at the last time step for which the restart information was placed on tape. A most effective use of this option can be made when conducting dynamic stability analyses where it is desirable to evaluate the response to see if buckling has occurred. If it has not, the decision can then be made to extend the run to further points in time.

Utilizing large time steps can result in a damping effect upon the solution so it is advisable to run the problem for a couple of oscillations, check to see if the solution is significantly damped, and then run the problem for the desired number of oscillations. If an evaluation of the initial results indicates that a smaller or larger time step should be used, the restart facility might be used to keep from having to repeat the initial calculations.

The displacements, velocities, and forces should be written on tape for almost all of the cases to insure that the restart information will be available if an evaluation of the calculated response indicates that the program should be restarted. The time required to write the restart information on tape is negligible when compared with the amount of time required to obtain the total response.

If it is desirable to decrease the time increment when restarting the program, the user should exercise care in selecting the increment (INCRST) at which the program will be restarted. The decision to decrease the size of the time step will usually be based upon the observation that the solution has become unstable or that significant damping is present in the response. To restart the program the user must be sure that the increment (INCRST) has been selected small enough to insure that the inaccuracies created by the larger time step can be neglected.

On the other hand, if the results from a previous run indicate that it is possible to increase the size of the time step for the remaining calculations, then care must also be taken in the selection of INCRST. For the numerical extrapolation procedure to produce accurate sets of displacements, it is recommended that the solution be restarted on a relatively straight portion of the displacement response curve. Considering the curve presented in Fig. 6 , it would be recommended that the program be restarted at 500 microseconds rather than at 600 microseconds because of the extrapolation procedure being utilized (i.e. the curve is smoother at 500 microseconds).

When using the restart option, it is possible to specify different values for a number of the control constants and input parameters. The data on cards I-IV may be changed, but the same Fourier harmonics and boundary conditions must be used. It is also required that the coefficients of thermal expansion remain the same when restarting the program. These requirements allow the user to omit card types V, VI, and VII when preparing data for restart operations. The considerations effecting the input of the loads for restart operations are presented in Appendix 6.

Appendix 6 - Load and Temperature Input Discussion

Since the DYNASOR II program accepts time varying loading and temperature conditions, the logic required to input these conditions is of necessity more complex than the logic required to input the other parameters. A discussion of the procedures for inputting these loading conditions is contained in this section. In this appendix the term loads refers to all distributed and concentrated forces while the term temperatures refers to both the temperature and temperature gradient distributions.

If there are no loads or no temperatures, it should be noted that a proper selection of the input constants allows omission of the input cards pertaining to the missing terms. In other words, the user selects the proper values for input keys and the proper read statements are automatically skipped.

To illustrate the procedure for inputting time varying loads and temperatures the information presented in Fig. A6-1 is utilized. The load-time and temperature-time curves are approximated as a series of linear segments by specifying values of both the loads and temperatures at discrete points in time and then assuming linear variations between the times. In order to specify the loads and temperatures in Fig. A6-1, it is necessary to specify both the loads and the corresponding temperatures at times $\mathrm{T1}_{1}, \mathrm{T1}_{2}$, and $\mathrm{T1}_{3}$. Both the applied loads and temperatures are constant from time $\mathrm{T1}_{3}$ to the selected TOTIME so the value of CONSTF should be set equal to CØNSTANT at time $\mathrm{T1}_{3}$. Obviously, if the loads or

time


FIG A6-1 MECHANICAL AND THERMAL LOAD HISTORY FOR AN ELEMENT
temperatures vary rapidly with time, it may be necessary to specify these conditions at a large number of times in order for the linear variation to be an accurate representation of the load-time and temperature-time curves.

The logic for the load and temperature input is now discussed for each of the two program start conditions, namely:

$$
\begin{array}{ll}
\text { IRSTRT }=0 & \text { Calculation begins at time increment }=0 \\
\text { IRSTRT }=1 & \text { Calculation begins at time increment }=\text { INCRST }
\end{array}
$$

Although the considerations for inputting the loads and temperatures differ in many respects for the two modes of starting the programs, several considerations are the same for both modes:

1. The loads and the temperatures must both be input at each time T1 at which the loads or temperatures vary. In other words, the loads cannot be input at one time and the temperatures at another.
2. The difference between successive times at which the loads and temperatures are input ( $\mathrm{T} \mathrm{l}_{\mathrm{i}+1}-\mathrm{T} 1_{\mathrm{j}}$ ) must always be greater than the time increment (DELTE) specified for solving the equations of motion.
$\underline{\text { IRSTRT }}=0$
The cases which may arise when considering the loads and temperatures and the input logic required to describe these situations are as follows when the program is making an initial run on a problem:
3. Loads and temperatures are con- Input only one set of loads and stant (in time) on each element. temperatures. These must be speciNote, however, that variations from element to element are fied at time $T 1=0.0$ and the value allowed.
4. Loads or temperatures (or both) vary with time.

Input, in order, the loads and temperatures at times $\mathrm{Tl}_{1}$ (must be equal to 0.0 ), $\mathrm{T1}_{2}, \mathrm{Tl}_{3}, \ldots$ until the value of $T 1_{1}$ reaches or exceeds the value of TOTIME (total time for the case). The columns for CONSTF should be left blank.

## $\underline{\text { IRSTRT }=1}$

The program may be restarted utilizing a new value for TOTIME which may be less than, equal to, or greater than the value which was utilized in the previous run which created and stored the restart information for use in this run. The previous value of TOTIME will be referred to as

TOTIMEP. The input logic varies according to the relative values of T0TIME and TOTIMEP so each possible combination will be discussed separately.

Procedures which may not be utilized in the restart mode are:

1. If the program was originally run as case A with IRSTRT $=0$, it is not possible to input loads and temperatures at any time until the value of TOTIMEP has been exceeded.
2. Consider that the program is being restarted at a time which is within the interval $\mathrm{T1}_{\dot{j}}-\mathrm{Tl}_{\mathrm{i}+1}$. The loads and temperatures were input in the previous run for times $T 1_{i}$ and $T 1_{i+1}$. The first value of T 1 for which the loads and temperatures may be specified in the restart mode must be greater than the time $T 1_{i+1}$ which was utilized in the previous run.

Consideration will first be given to the cases where the new value of the maximum time is less than or equal to the one previously used.

$$
\text { TOTIME } \leq \text { TOTIMEP }
$$

CASE
INPUT LOGIC

1. Both the loads and the temperatures are constant (in time) and are equal to the values specified for $\operatorname{IRSTRT}=0$, Case 1.

No loads or temperatures are input.
2. Both the loads and temperatures are constant (in time) but are different from the values specified for IRSTRT $=0$, Case 1 .

This problem is not allowed by the program. If the user desires to run this case, it is suggested that the problem be rerun beginning at time $=0.0$.
3. Loads or temperatures vary with

Input loads and temperatures at time. (This cannot be a restart of Case 1 , IRSTRT $=0$.)
times $\mathrm{Tl}_{1}, \mathrm{T1}_{2}, \ldots$ until the value $T 1_{n}$ reaches or exceeds the value of TOTIME. The value of $\mathrm{T} 1_{1}$ must be greater than the value of $\mathrm{Tl}_{\mathrm{i}+1}$ of the previous run.

The possible cases which may arise if the value of TOTIME is greater than TOTIMEP are now presented. It should be noted that cases differ only slightly from those previously discussed.

1. Both the loads and temperatures are constant (in time) and are equal to the values specified for $\operatorname{IRSTRT}=0$, Case 1 .

The loads and temperatures must be input for T1 = TOTIMEP and the value of CONSTF is set as CØNSTANT. The specified loads and temperatures must be identical with those read for the previous run (IRSTRT=0).
2. Both the loads and temperatures are constant (in time) but are different from the values specified for IRSTRT $=0$, Case 1 .
3. Loads at temperatures (or both) vary with time.
3. Loads (or both)

The new loads will not be applied until TOTIMEP is reached. The logic for Case 1, above, is then applied.

The loads and temperatures must be input at times $T 1_{1}, T 1_{2}, \ldots$ until the value $T 1_{n}$ reaches or exceeds the value of TOTIME.

Appendix 7 - Program Output

The output of the DYNASOR II code consists basically of seven parts.

1. The input data for all cases comprising the run
2. Alphanumeric case identification data
3. Case control constants, boundary conditions, and initial conditions
4. Output generated in the SAMMSOR code
5. Loading and temperature information
6. Nodal displacements
7. Stresses and stress resultants

The output of part 1 is helpful since the input data for all cases is printed at the start of a run. If an error occurs in the execution of the program, a glance at the listing of the input data may in many cases reveal the source of the error.

A description of the particular case being run may be printed by the user utilizing the case identification cards. In addition, the descriptive comments prepared by the SAMMSOR code to identify the shell are printed.

At the start of each case a number of the control constants are printed. In addition, the initial and boundary conditions are specified. A printout of these constants allows the user to check to see if the desired options of the program are being properly utilized.

If desired, the stiffness and mass matrices for each of the specified harmonics appears as output for the DYNASOR II code. The printout of the stiffness and mass matrices may insure that the tape is being properly read if a check with the SAMMSOR output is made (identical formats are used). A table of important element properties is always printed.

If desired, a description of the loading, temperatures, and temperature distribution is printed at each time for which the loads are input. The generalized forces resulting from these contributions may also appear as output.

The nodal displacements for each harmonic may be output at each selected time increment. In order to limit the amount of output, a control parameter is utilized to indicate how of ten the displacements will be printed. The generalized displacements (in cylindrical coordinates) are printed at up to twenty angles for all of the harmonics.

For each desired time step the stress resultants and stresses are printed. The force, moment, and shear resultants for each element are underscored by the stresses at the upper and lower surfaces. Once again a control parameter regulates how often these stresses will be calculated and printed.

## Appendix 8 - Modification of Program Capacity

The procedure for modifying the maximum number of elements and cosine harmonics which can be utilized in the DYNASOR II code is presented in this section. The modifications consist of changing only DIMENSIØN and CØMMØN statements. The following terms are defined to facilitate the modifications:

$$
\begin{aligned}
& \text { NE }=\text { Maximum number of elements } \\
& \text { NHM }=\text { Maximum number of harmonics } \\
& \text { NN }=\text { NE }+1 \\
& \text { NSZ }=26 * \text { NE }+10 \\
& \text { NSZ1 }=\text { NHM } * \text { NSZ } \\
& \text { NR }=4 * \text { NN } \\
& \text { NS }=\text { NR } * \text { NHM } \\
& \text { NS2 }=2 * \text { NS } \\
& \text { NL }=6 * \text { NE } \\
& \text { NSC3 }=\text { Value from table A8-1 } \\
& \text { NSC4 }=\text { VAlue from table A8-1 } \\
& \text { NMAX }=\text { Maximum of NS2 and NSZ }
\end{aligned}
$$

Having calculated these constants for the desired maximum number of harmonics and elements, the following cards in the specified subroutines must be changed by substituting the values of the above constants.

```
Main Program
C\emptysetMM\emptysetN /QS/QN(NS), QN1(NS), F\emptysetRCE(NMAX), QP(NS), QP1(NS), QN2(NS)
CØMMФN /HARM/NHP, IHARM(NHM)
Subroutine INPUT
CØMM@N /SLVEEQ/ XN(NSZ1), QLØAD(NR)
*CDMMDN /QS/
CÖMMøNN /RSTRNT/ NODRES, NCLOSE, LK(NR)
CØMMDN /PS/XP(NSZ1)
CØMM\N /GEDM/ FNU1(NE), FNU2(NE), E1(NE), E2(NE),G(NE), T(NE), SINE(NN),
COSINE(NN),SINM(NE), COSM(NE), R(NE), PH(NE), PHP(NE), ARCL(NE)
*CDMMDNN /HARM/
CØMM\emptysetN /THER/ TH(NE,NHM,2), DTH(NE, NHM, 2), ALS(NE), ALT(NE)
```

[^4]```
DIMENSION QQN2(NS), QQN1(NS), QQN(NS)
DIMENSION COMENT(20), JUNK(20), TH1(NHM), DTH1(NHM)
DIMENSION DUM(NSZ)
```

Subroutine SETUP
*CDMMON /SLVEEQ/
*CDMMDN /QS/
*CDMMON /HARM/

Subroutine NLTERM

```
*COMMON /QS/
*CDMMON /GEOM/
    CØMMQN /EES/ ES(NHM), ET(NHM), EST(NHM), E13(NHM, E23(NHM)
    CDMMON /NLTRMS/ QPR(8, NHM)
```

Subroutine QPRIME

```
**CØMMQN /CS/ CC(NSC3), SSC(NSC3), CSS(NSC3)
***C\MM9N /CS4/ CCC(NSC4), SSSS(NSC4), SSCC(NSC4), SCCS(NSC4)
    *CØMM@N /EES/
    *CØMMON /NLTRMS/
    *CDMMON /GEOM/
    *CDMMON /HARM/
    *CØMMQN /THER/
    *COMMON /QS/
```

    DIMENSIดN E23Q1(NHM), E23Q3(NHM), E23Q5(NHM), E23Q7(NHM), ESTQ1(NHM),
        ESTQ3(NHM), ESTQ5(NHM), ESTQ7(NHM), ETQ2(NHM), ETQ6(NHM)
    Subroutine HOUBQ1
*CQMMON /SLVEEQ/
*СФMMON /QS/
*CØMMФN /RSTRNT/
*CØMM $\ddagger$ N /PS/
DIMENSION QLOADT(NS)
Subroutine HOUBQN
*CDMMUN /SLVEEQ/
*CØMMON /QS/
*CDMMØ゙ /RSTRNT/
*CQMMळN /PS/
Subroutine NRESTR
*CØMMØN /SLVEEQ/
*CDMMON /RSTRNT/
** Value of NSC3 should be taken from table A8-1
*** Value of NSC4 should be taken from table A8-1
Subroutine MATMUT
DIMENSION A(NR), STIFM(NSZ1), FORCE(NMAX)
Subroutine SØLVEQ
*CØMMON /SLVEEQ/
DIMENSI甲N A(NSZ), R(NR)
Subroutine STRESS
*CØMM9N /EES/
*CDMMON ..... /GEOM/
*CØMM@N ..... /HARM/
*СФMM@N ..... /QS/
*CDMMDN ..... /THER/
Subroutine FRCES
*CØMM@N ..... /QS/
*CDMMON /HARM/
Subroutine THCDE
*CØMMON /THER/
*CØMMФN ..... /HARM/
Subroutine TFORCE
*CØMM@N /GEOM/
*CФMM@N /THER/
*CDMMON /HARM/
*CØMMФN ..... /QS/
Subroutine TRI40R
*CDMMON ..... /CS/
*СФММФN ..... /CS4/
*CøMMФN ..... /HARM/

Table A8-1
Number of Elements in Sine and Cosine Integration Arrays

| Maximum No. of <br> Harmonics <br> NHM | No. of Terms 3rd <br> Order Arrays <br> NSC3 | No. of Terms 4th <br> Order Arrays <br> NSC4 |
| :---: | :---: | :---: |
| 1 | 1 | 1 |
| 2 | 4 | 8 |
| 3 | 10 | 33 |
| 4 | 19 | 92 |
| 5 | 31 | 201 |
| 6 | 46 | 376 |
| 7 | 64 | 633 |
| 8 | 85 | 988 |
| 10 | 109 | 1457 |


[^0]:    * Research Assistant, Aerospace Engineering Department
    ** Assistant Professor, Aerospace Engineering Department

[^1]:    * The total value of the ring load for each harmonic is input, not the load per unit length of circumference. For complicated ring loads the value of the load input for each harmonic is obtained by integrating the product of the load and the corresponding displacement function around the circumference.

[^2]:    * The total value of the ring load for each harmonic is input, not the load per unit length of circumference. For complicated ring loads the value of the load input for each harmonic is obtained by integrating the product of the load and the corresponding displacement function around the circumference.

[^3]:    *NOTE: The first loading must always be given for $\theta=0^{\circ}$. The next loading is given at the angle where the load changes in value. If the load is constant with respect to $\theta$, only one card will be necessary to input the load. Do not input values for the loads at $\theta=180^{\circ}$ since the load at that angle will be equal in all cases to the load input at the previous value of THETAB.

[^4]:    * Elements in this block are previously shown

