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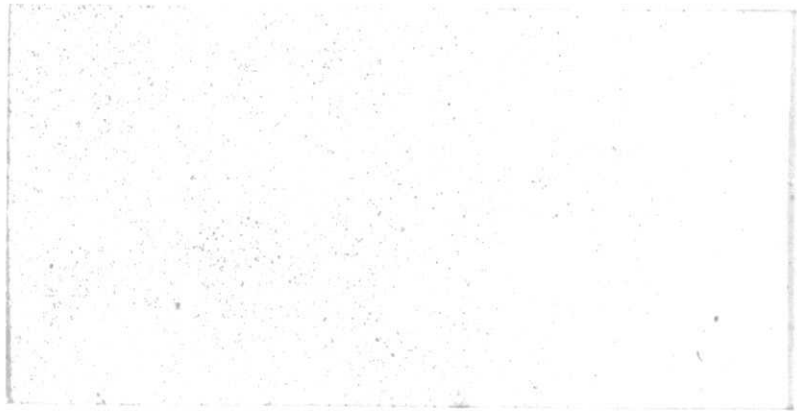

Lockheed

HUNTSVILLE RESEARCH & ENGINEERING CENTER
LOCKHEED MISSILES & SPACE COMPANY
A GROUP DIVISION OF LOCKHEED AIRCRAFT CORPORATION
HUNTSVILLE, ALABAMA

Section 1
INTRODUCTION

During this study a method and associated digital computer programs were developed for evaluating the free undamped vibrational characteristics of complicated linear structural systems mathematically modeled as assemblages of arbitrary substructures. The state of each substructure is represented by a set of generalized displacement functions. Each substructure is mathematically modeled as an assemblage of basic beam and shell elements. Substructure generalized functions include: (1) static functions corresponding to motions of the "juncture nodes" connecting the substructure to other substructures, and (2) arbitrary displacement functions computed subject to specific restraint conditions imposed at the juncture nodes.

The substructure method involves three basic steps:

1. Calculation of substructure generalized functions and the corresponding substructure mass and stiffness matrices;
2. Substructure synthesis, in which the system mass and stiffness matrices are formed on the basis of substructure mass and stiffness matrices, interconnection descriptions, etc.; and
3. Calculation of modes and frequencies of the system.

Two general purpose digital programs were developed for implementing the three steps of the solution procedure. Step 1 is carried out with the Substructure Function Generator program. The Lockheed developed Structural Network Analysis Program, SNAP, and its dynamic analysis counterpart, SNAP/Dynamics, were modified for this purpose. SNAP and SNAP/Dynamics are general purpose programs for performing static and dynamic analyses of structures consisting of various types of finite elements, including beams, triangular and quadrilateral membrane, plate and shell elements. Detailed

accounts of these programs are given in Refs. 1 through 3. Steps 2 and 3 are combined in a single computer program called the Substructure Synthesis Program. Communication between the Function Generator and Synthesis programs is accomplished by means of substructure data files which are created by the Function Generator program and read as input by the Synthesis program. These data files may be stored on magnetic tape, drum or disc units, or punched cards.

The standard mode of operation is to create separate substructure data files for each substructure of the system and then to perform a system analysis using the separate files. Accordingly, this approach is well suited for the types of studies frequently performed early in the design of a system in which it is required to determine the effects on overall system vibration characteristics of design changes in only a few substructures. In such cases, substructure generalized functions, etc., need be recalculated only for the substructures containing the alterations.

Each substructure data file created by the Function Generator Program contains descriptions of a specific set of generalized functions, the number and type of which are controlled by the analyst via input data options. Provisions are included in the synthesis program to use any specified sub-set of these functions as generalized coordinates in the system analysis. Accordingly, the effects on overall system modes and frequencies of different classes of substructure generalized functions can be studied without recreating substructure data files.

Section 2
SYSTEM EQUATIONS OF MOTION

2.1 SYSTEM GENERALIZED COORDINATES

A system is modeled as an array of joints interconnected by arbitrary multi-node substructures. Throughout this report the term "system joint" or "joint" refers to a structural joint in the assembled system and the term "boundary node" or "node" refers to a substructure node that is connected to a system joint either directly or through a rigid offset. Rigid offsets are provided for modeling connections in which boundary nodes do not geometrically coincide with the system joints to which they are connected.

The generalized coordinates used to characterize the motions of a system fall into two categories:

- Motion components of system joints (or coefficients of functions imposing linear relations among various joint motion components), and

- Coefficients of individual substructure generalized functions.

Functions representing system joint motions are composed of any specified linear combination of explicit joint motion components. Such a function may characterize a single displacement or rotation of a particular joint, or it may characterize several dependent motions of a set of joints. For example, consider a system composed of two substructures connected by a set of joints lying in a circular plane. If the interconnection plane is to remain an undistorted circle, only six generalized functions are required to represent the motions of these system joints, i. e., three displacements and three rotations of the plane as a whole. If radial distortion of the circle is permitted, functions

representing the radial joint displacements associated with several circumferential harmonics in addition to the six functions representing planar motions would satisfactorily characterize the motions of all system joints. Other desired motions of the system joints can be prescribed in a similar manner. This procedure generally results in fewer degrees of freedom than the more conventional approach of using all six motion components of all system joints as generalized coordinates. Consequently, the probability of numerical inaccuracy and coordinate dependence associated with unnecessarily large eigenproblems is reduced.

Functions representing individual substructure motions, in general corresponding to zero boundary node motions, are used to further characterize system motions. The types of functions used fall into the following three categories:

- Undamped free vibrational mode shapes,
- Arbitrary static displacement functions, and
- Uniform acceleration modes.

Vibrational modes and static displacement functions are used to represent substructures for which it is assumed that the predominant part of the total motion is due to the deformation of the substructure itself. However, if relatively small substructures are used, the motions of most of them will be composed primarily of rigid body translation and rotation. Accordingly, uniform acceleration modes are incorporated to accommodate this type of substructure deformation. Consider, for example, the lateral motion of a beam-like structure for which x is a position coordinate directed along the longitudinal axis. The total lateral motion of points along the substructure is

$$u(x) = U + Rx + f(x),$$

where U and R are the displacement and rotation of the origin of the x coordinate axis, and f(x) is the deformation of the substructure. Where

m(x) is the distributed mass intensity, the lateral inertia forces acting on the substructure are proportional to m(x)u(x). If the predominant motion is rigid body (i.e., f(x) is small compared to U and Rx), the distributed lateral inertia forces are approximately proportional to m(x)[U + Rx]. Accordingly, displacement functions produced by lateral loadings corresponding to static lateral force distributions proportional to (1)m(x), and (2) x m(x) are excellent substructure generalized functions. For a general substructure, six functions of this type are used: three functions corresponding to static displacement fields produced by inertia loadings associated with constant rigid-body acceleration in each of three non-parallel directions, and three similar functions produced by inertia loads associated with constant rigid-body angular acceleration about each of the three non-parallel axes.

2.2 SYSTEM ENERGY RELATIONS

The kinetic and potential energies of a system may be written, respectively, as:

$$T = \tfrac{1}{2}\dot{\Phi}^{*} A \dot{\Phi} \text{ , and}$$
$$V = \tfrac{1}{2}\Phi^{*} B \Phi \text{ ,} \qquad (1)$$

where A and B represent the system mass and stiffness matrices and Φ represents a vector of coefficients of the system generalized coordinate functions. In the absence of dissipative effects and externally applied forces, the Lagrange equations yield

$$A\ddot{\Phi} + B\Phi = 0 . \qquad (2)$$

Assuming solutions of the form $\Phi = Z\sin\omega t$ yields the usual linear vibrational eigenproblem

$$\omega^2 AZ - BZ = 0 . \qquad (3)$$

The total system kinetic and potential energies may be expressed as

$$T = \sum_{p=1}^{N} \overline{T}_p, \text{ and}$$

$$V = \sum_{p=1}^{N} \overline{V}_p, \tag{4}$$

where $\overline{T}_p$ and $\overline{V}_p$ represent the kinetic and potential energies of substructure p in terms of the system generalized coordinates, and N is the total number of substructures in the system.

The kinetic and potential energies of substructure p are expressed as

$$T_p = \tfrac{1}{2} \dot{\varphi}_p^* M_p \dot{\varphi}_p, \text{ and}$$

(5)

$$V_p = \tfrac{1}{2} \varphi_p^* K_p \varphi_p,$$

where M_p and K_p are the mass and stiffness matrices of the substructure in terms of the substructure generalized functions, and φ_p is a vector of coefficients of the substructure generalized coordinate functions. In this study substructure generalized functions are assumed to be relative to an intrinsic substructure reference frame located at boundary node 1. For a substructure with n boundary nodes, the set of generalized functions would consist of the following:

- Six independent rigid body motions of the substructure parallel to the intrinsic reference frame axes,

- Functions associated with three unit displacements and three unit rotations in directions parallel to the intrinsic reference frame axes of boundary nodes 2 through n,
- Six uniform acceleration modes, and
- An arbitrary set of static displacement functions and vibrational mode shapes.

To transform the substructure energy matrices M_p and K_p into the system coordinates a coordinate transformation is performed such that

$$\varphi_p = R_p \Phi . \tag{6}$$

Substituting Eq. (6) into Eqs. (5) yields

$$\begin{aligned} \overline{T}_p &= \dot{\Phi}^* \overline{M}_p \dot{\Phi} , \text{ and} \\ \overline{V}_p &= \Phi^* \overline{K}_p \Phi , \end{aligned} \tag{7}$$

where

$$\begin{aligned} \overline{M}_p &= R_p^* M_p R_p, \text{ and} \\ \overline{K}_p &= R_p^* K_p R_p . \end{aligned} \tag{8}$$

Substitution of Eqs. (7) into Eqs. (4) yield the total system kinetic and potential energies.

The actual transformation from substructure coordinates to system coordinates is much more complicated than is indicated by Eqs. (6) and (8). The transformation matrix R_p includes the effects of the following:

- The orientation of the intrinsic substructure reference frame relative to the system reference frame,
- Rigid link offsets connecting the boundary nodes to system joints, and
- The types of functions used to characterize the motions of system joints.

The transformation indicated by Eq.(8) is performed in a manner ensuring maximum numerical accuracy.

Section 3
COMPUTER PROGRAMS

Two general purpose computer programs were developed for implementing the procedure discussed in the previous section. A substructure synthesis program was developed for forming system mass and stiffness matrices on the basis of input data describing the characteristics of individual substructures and how they are interconnected. A substructure function generator program was developed for calculating generalized functions for substructures modeled as basic finite element networks and for constructing mass and stiffness matrices expressing the kinetic and potential energies of the substructures as quadratic forms in the coefficients of the generalized functions.

Both programs are coded in Fortran V and designed for use on the Univac 1108 Exec II and Exec VIII systems.

3.1 SUBSTRUCTURE SYNTHESIS PROGRAM

For each substructure the following information is supplied to the Synthesis program by means of a substructure data file created by the Substructure Function Generator Program:

1. Mass and stiffness matrices expressing the kinetic and potential energies of the substructure as quadratic forms in the coefficients of its generalized functions,

2. The six motion components associated with each generalized function of all substructure boundary nodes and of all other substructure nodes for which the motion in the assembled system is of interest, and

3. The position coordinates of the boundary nodes relative to a "substructure reference frame" uniquely associated with the substructure.

General system definition data consists of the following:

1. The position and orientation of each substructure reference frame relative to the global system reference frame,

2. The system joint to which each boundary node of each substructure is connected either directly or via rigid links.

3. System joint constraint conditions, so that any motion component of any system joint may be set identically equal to zero,

4. The position coordinates relative to the global reference frame of those system joints to which substructure boundary nodes are connected by rigid links,

5. The identification of functions to be used to represent system joint motions, and

6. The identification of which sub-set of substructure generalized functions are to be read from the substructure data file and used in the system analysis.

Given the above data, the synthesis program forms the mass and stiffness matrices of the system as discussed in the previous section. All matrix transformations are carried out in double precision to alleviate problems associated with numerical roundoff. Dynamic core allocation is utilized throughout the program to enable execution on computers with varying core size by altering only one dimension statement in the main program.

A set of standard full-matrix eigenproblem solution routines are incorporated to perform the system analysis (i.e., compute the modes and frequencies of the $\omega^2 AZ - BZ = 0$ system eigenproblem). The routines will accommodate up to approximately 100 degrees of freedom on a Univac 1108 system. Since an arbitrary selection of generalized coordinate functions is likely to result in a dependent set of system equations of motion, the system analysis routines utilize a solution procedure that eliminates the problem of coordinate dependence by automatically "collapsing" the mass and stiffness matrices of the system. For instance, if during the eigensolution process it is discovered that the N^{th} coordinate function is some linear combination of coordinates 1 through N-1, the N^{th} row and column of the system mass and stiffness matrices are automatically eliminated, thereby reducing the order of the eigenproblem by one.

The solution is then continued, eliminating any subsequently encountered dependence in a similar manner.

Solution information supplied by the synthesis program for each system mode includes the following for each substructure:

- Displacements and rotations relative to the intrinsic substructure reference frame of each boundary node,
- Displacements and rotations relative to the intrinsic reference frame of specified nodes of particular interest, and
- An energy breakdown indicating the contribution of each substructure to the total kinetic and potential energies of the system.

3.2 SUBSTRUCTURE FUNCTION GENERATOR PROGRAM

During the study a digital program was developed for calculating generalized coordinate functions for substructures modeled as networks of basic finite elements. The program, which is a modification of the Lockheed developed Structural Network Analysis Program (SNAP, Ref. 3) and its counterpart for computing the modes and frequencies of finite element networks (SNAP/Dynamics), provides an extremely fast and accurate means of computing any desired set of static or dynamic generalized functions for a substructure composed of an assemblage of two, three, or four-node finite elements. The program constructs mass and stiffness matrices expressing the kinetic and potential energies of a substructure as quadratic forms in the coefficients of the generalized coordinate functions. The substructure functions generated by the program are relative to an intrinsic reference frame located at the substructure boundary node 1. The program contains provisions for automatically computing the types of substructure functions discussed in Section 2.2.

For substructure modeling the program contains the following finite element formulations:

- General Timoshenko beam element including shear and torsional effects,

- Aeolotropic constant strain triangular membrane,
- Aeolotropic quadrilateral membrane formulated on the basis of hybrid variational procedures,
- Quadrilateral shear panel,
- Orthotropic triangular bending element, and
- Isotropic and aeolotropic quadrilateral bending elements.

The Function Generator program creates the substructure data file used by the Synthesis program. The data file is stored on magnetic tape, drum or disc units, or punched cards.

Section 4
RESULTS

Results are presented for two example solutions computed with the programs described in the previous section. Each solution is compared with an analogous solution computed with the SNAP/Dynamics program.

During solution of these examples two basic problem areas associated with substructuring methods were identified. These were: (1) random selection of substructure generalized functions often leads to system coordinate dependence; and (2) numerical inaccuracies are often encountered during transformation from substructure coordinates to system coordinates. Identification of these problem areas lead to the development of the following program features:

- Automatic collapsing of the system mass and stiffness matrices upon encountering a coordinate dependence during the eigensolution,
- Performing all matrix transformations in double precision,
- Characterizing joint motions with specified functions instead of using explicit motion components of all system joints as coordinates, and
- Normalizing all substructure generalized functions to a constant value of kinetic energy.

A study was conducted to evaluate the relative merits of various classes of generalized substructure displacement functions. Examples were executed to compare static displacement functions, natural vibrational mode shapes, and uniform acceleration modes. No general conclusions could be drawn from the results except that generalized function selection should be governed by the anticipated motion of the substructure in the assembled system. It was determined, however, that in most applications an adequate set of substructure coordinates would be functions associated with boundary node motions along

with uniform acceleration modes. In almost all comparisons uniform acceleration modes represented substructure behavior as well as, if not better than, natural vibrational modes. Since uniform acceleration modes can be generated at a much lower cost than natural modes, they should generally be included in the substructure generalized function repertoire.

4.1 PLANE FRAME EXAMPLE

The plane frame example is illustrated on Fig. 1. The system is composed of two identical substructures connected through system joints 1 and 2. One static displacement function corresponding to in-plane point loads applied at the corner joints with the boundary nodes constrained was used to represent the individual motion of each substructure. Three identical solutions were computed for the frame using different types of functions to represent the system joint motions. The three types of functions were:

- Six explicit joint motion components representing the three planar motions of each joint,
- Six independent functions representing relative motions between the joints, and
- A combination of four relative motion functions and two explicit joint motion components.

Solutions of the first three modes of the system were also computed with the SNAP/Dynamics program. A comparison of the frequencies obtained with the two independent analyses is presented in Table 1.

Table 1

FREQUENCY COMPARISON, EXAMPLE 1

Elastic Mode	Substructure Frequency	SNAP/Dynamics Frequency
1	16.05 cps	15.87 cps
2	27.18 cps	25.26 cps
3	43.29 cps	42.28 cps

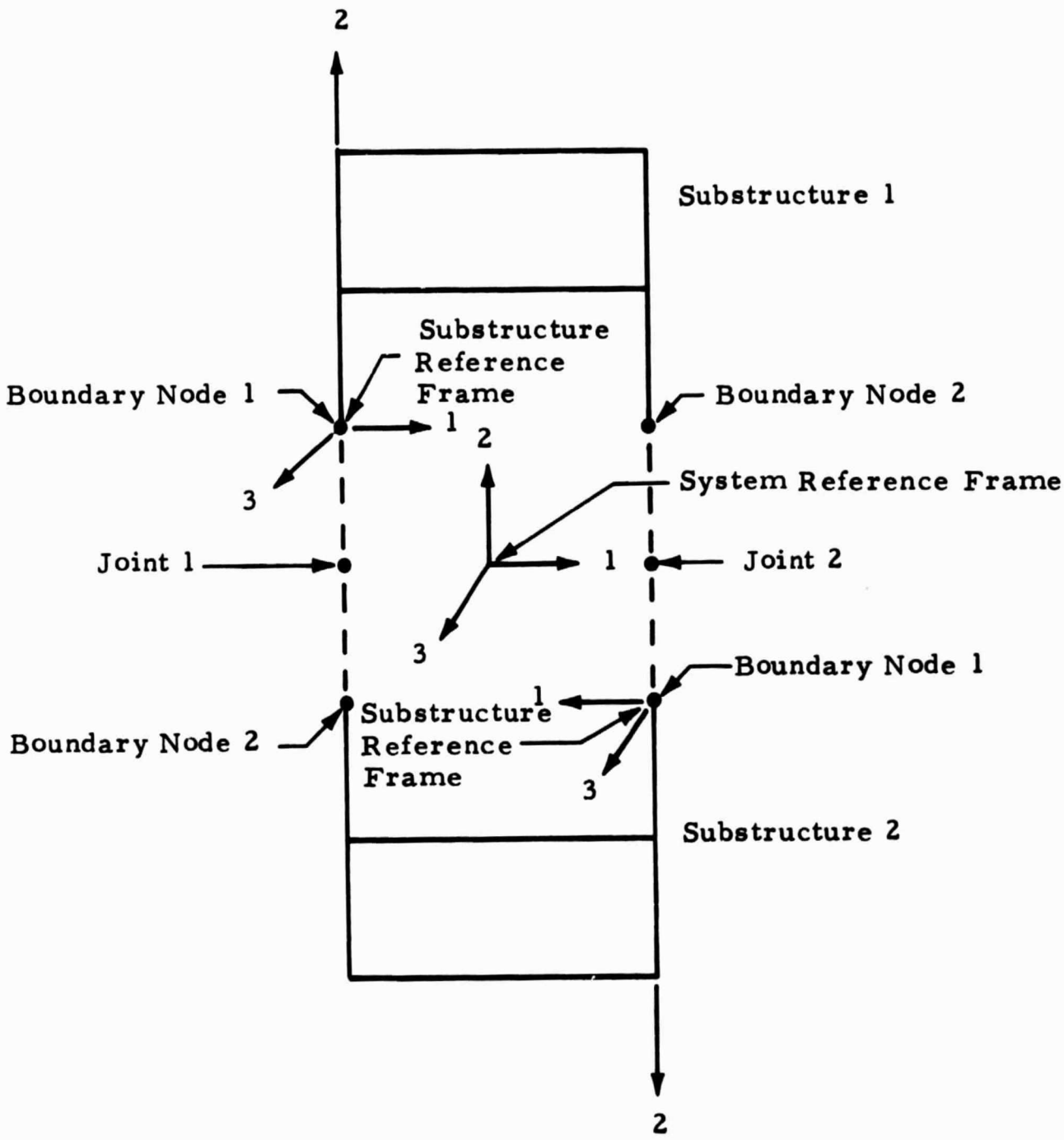

Fig. 1 - Plane Frame Example

By utilizing a larger number of individual substructure generalized displacement functions, the results would compare even more favorably with those obtained with the essentially exact solutions computed by the SNAP/ Dynamics program.

4.2 SPACE SHUTTLE LAUNCH CONFIGURATION

The Space Shuttle launch configuration example is illustrated on Fig. 2. Each vehicle was used as a substructure in the system model. Two system joints interconnected the two substructures. The forward joint lies on the symmetry plane, and the aft joint lies off the symmetry plane. A half-model on one side of the symmetry plane was used to obtain the symmetric modes of the system. The generalized coordinate functions used to characterize system motion were:

- Three functions representing the explicit symmetric motions of the forward system joint,
- Six functions representing the explicit motions of the aft system joint, and
- The first three symmetric vibrational modes of each substructure, corresponding to zero boundary node motion.

Table 2 presents a comparison of the frequencies of the first five symmetric elastic modes obtained with the substructure program with results obtained with the SNAP/Dynamics program.

Table 2

FREQUENCY COMPARISON, EXAMPLE 2

Elastic Mode	Substructure Frequency	SNAP/Dynamics Frequency
1	2.3179 cps	2.3329 cps
2	2.6994 cps	2.6488 cps
3	4.1891 cps	3.8209 cps
4	4.4330 cps	4.2454 cps
5	6.9777 cps	6.2128 cps

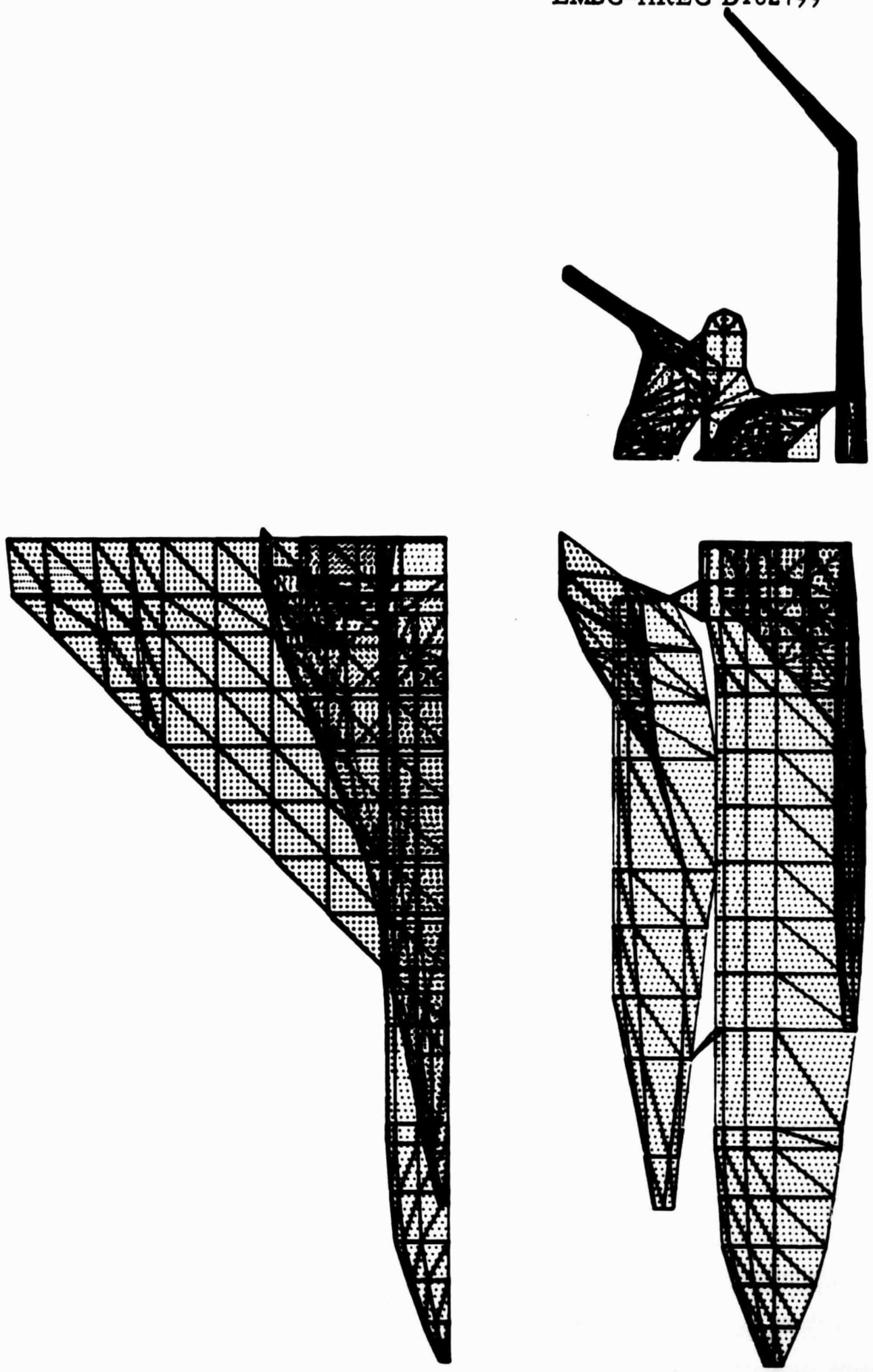

Fig. 2 - Space Shuttle Launch Configuration

The fact that the first mode computed by the substructure program is slightly lower than that computed by the SNAP/Dynamics program is attributed to some small differences in the basic finite element nets employed in the two analyses.

Section 5

REFERENCES

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