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SEMI-ANNUAL STATUS REPORT

by

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Contents

1. Introduction

During this reporting period effort has been directed towards studies of seismic velocity models for the moon and the computation of theoretical seismograms to study the lunar seismic reverberation. The possible presence of a low-velocity unconsolidated surface debris layer has implications for the interpretation of lunar active seismic experiments which are planned for traverse geophysics in the future Apollo missions.

We have also concentrated on some aspects of the thermal history of the moon. The thermal history of the moon is vital to understanding the internal stress condition and seismicity, the chemical differentiation history, the electrical and magnetic properties of the interior, the sources of surface lavas and tectonic features and the gravitational field. Recent advances in the theory of natural convection are showing great promise for the understanding of lunar internal temperatures.

2. Lunar Seismic Investigations

A major objective in interpreting lunar seismic data is to obtain a model for the internal structure of the moon. Any description of the bulk properties of the moon is dependent on knowledge of the variation of seismic velocities with depth. It is worth examining the question at this time of our current knowledge of the internal structure of the moon in light of the recently obtained seismic data from the Apollo 11 and 12 missions and the related physical properties of rocks which have been measured on some of the returned lunar samples.

Seismic data are usually combined with data on the physical and chemical properties of rocks, since seldom is a unique determination of material composition made solely from seismic data. Seismic velocity characteristically depends on a large number of factors such as mineralogical composition, mean atomic weight, temperature, pressure, grain size, porosity and the presence of small cracks.

In the moon, however, it is apparent that the overiding parameter controlling the variation of seismic velocity in the upper 20km or so is the presence of cracks. This is a condition unlike that found on earth.

Figure 1 shows the seismic velocities measured as a function of pressure on some of the returned lunar samples. The principal feature to note is the rapid increase in seismic velocity over the first 2 kbar or so, equivalent to a depth of about 40 km in the moon. We can now proceed to examine the reasons for such a variation of seismic velocity. It is a simple matter to estimate the variation of velocity with depth in the moon. Assuming homogeneity this variation can be written as

$$V(r) = V_0(r_0) + \left(\frac{\partial V}{\partial P}\right)_T \Delta P + \left(\frac{\partial V}{\partial T}\right)_P \Delta T$$

$$+ \frac{\partial V}{\partial \phi} \Delta \phi + \left(\frac{\partial V}{\partial P}\right)_{cracks} \Delta P$$

where V = velocity, P = pressure, T = temperature, ϕ = porosity and r = radius. For a self-gravitating sphere of density 3.34 gm/cm^3 in hydrostatic equilibrium we can take pressure to vary with radius as

$$P = 47.0 \; [1-x^2] \text{ kbar}$$

where x is the normalized radius.

For rocks $\left(\frac{\partial V}{\partial P}\right)_T$ is typically of the order of 1×10^{-2} km/sec/kbar once pressures of the order of 10 kbar or so are reached. Temperature acts in opposition to pressure and a reasonable estimate for the temperature coefficient is -3 to -4×10^{-4} km/sec/°C. The thermal gradient in the lunar interior is, or course, unknown but it is difficult to imagine that it could exceed 2 - 4°C/km in the upper 100 km of the Moon. That is, the maximum variation in seismic velocity that could be produced in the outer 50 km of the lunar interior from thermal considerations could not exceed 8×10^{-2} km/sec. It is also a simple matter to show that the contribution to variation in seismic velocity produced by a decrease in spherical porosity is small.

On the other hand, the contribution from the closing of cracks far exceeds any of the above mentioned factors. Data from dry granites suggests the following

0 - 200 bars (∿4 km) ∿ 12 km/sec/kbar

200 bars - 1 kbar (∿20 km) ∿ 1 km/sec/kbar

1 - 3 kbar (∿55 km) ∿ .06 km/sec/kbar

Thus, we see that easily the dominant factor is the closing of cracks.

We can now proceed to test the hypothesis of the closing of cracks being the main contributor to the variation of velocity. Variation is predicted using the pressure variation as measured in a dry granite. This is not to imply that the moon is composed of granite but we have merely utilized the empirical pressure variation. We can see that the predicted velocity variation is close to that measured in the lunar samples so we can anticipate that we will have agreement with the observed travel time data.

Also shown in Figure 1 is a layered velocity model containing a discontinuity at 5 km where the velocity jumps from 2.9 to 4.7 km/sec.

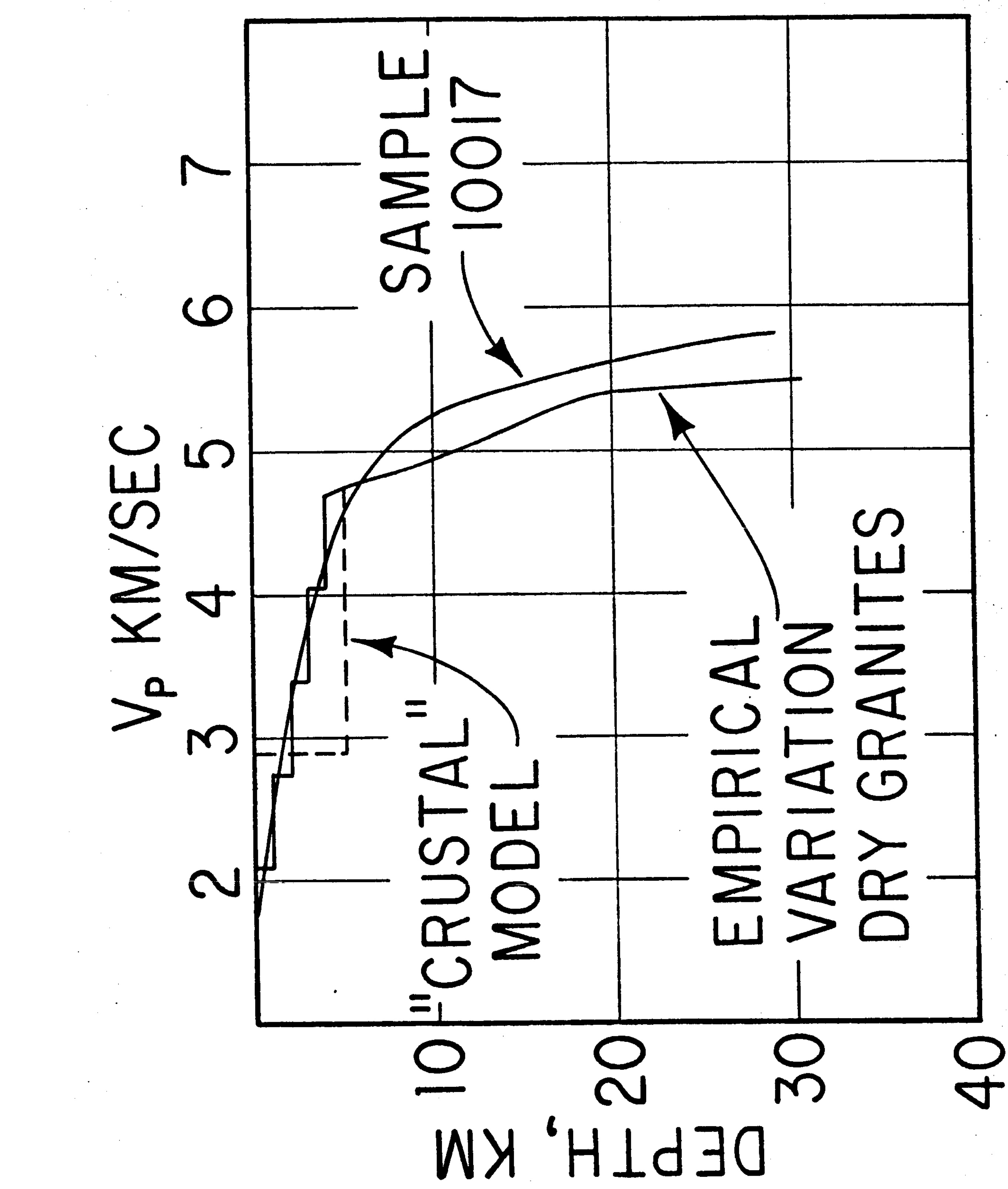
Figure 2 shows the excellent agreement with the limited travel-time data and we can make the following firm conclusions

1. There is a strong velocity gradient or a velocity discontinuity in the upper 5 km or so of the moon reaching to a velocity of 4.7 km/sec at 5 km. Detailed refraction surveys would be needed to delineate between these two models.
2. It is the mechanical state, i.e. the closing of cracks, of the rocks which is controlling the variation of seismic velocity in the outer 20 km or so of the moon and not composition or rock type.
3. Because the average velocity is low - of the order of 2 - 3 km/sec in the upper 5 km of the moon large amounts of permafrost in the maria seem precluded. Pure ice has a velocity of about 3.7 km/sec but permafrost velocities range from about 5.1 km/sec to 5.8 km/sec. In order to preserve the observed travel times large amounts of permafrost are not possible.
4. The seismic velocity approaches 6.5 - 7 km/sec, a value appropriate for ultrabasic racks, at depths of 40 km or so in the moon.

Now what can be said about the lunar seismic reverberation which followed the LM and SIVB impacts. Possible propagation mechanisms fall into two general categories. Dispersion effects where coherent waves propagate at differing group velocities dependent on wave length and scattering effects where the effective path lengths are increased owing to numerous reflections from acoustic boundaries, or a comgination of both. Any explanation must take into account that a surface impact source would generate most of its seismic energy in the form of surface waves - Rayleigh waves.

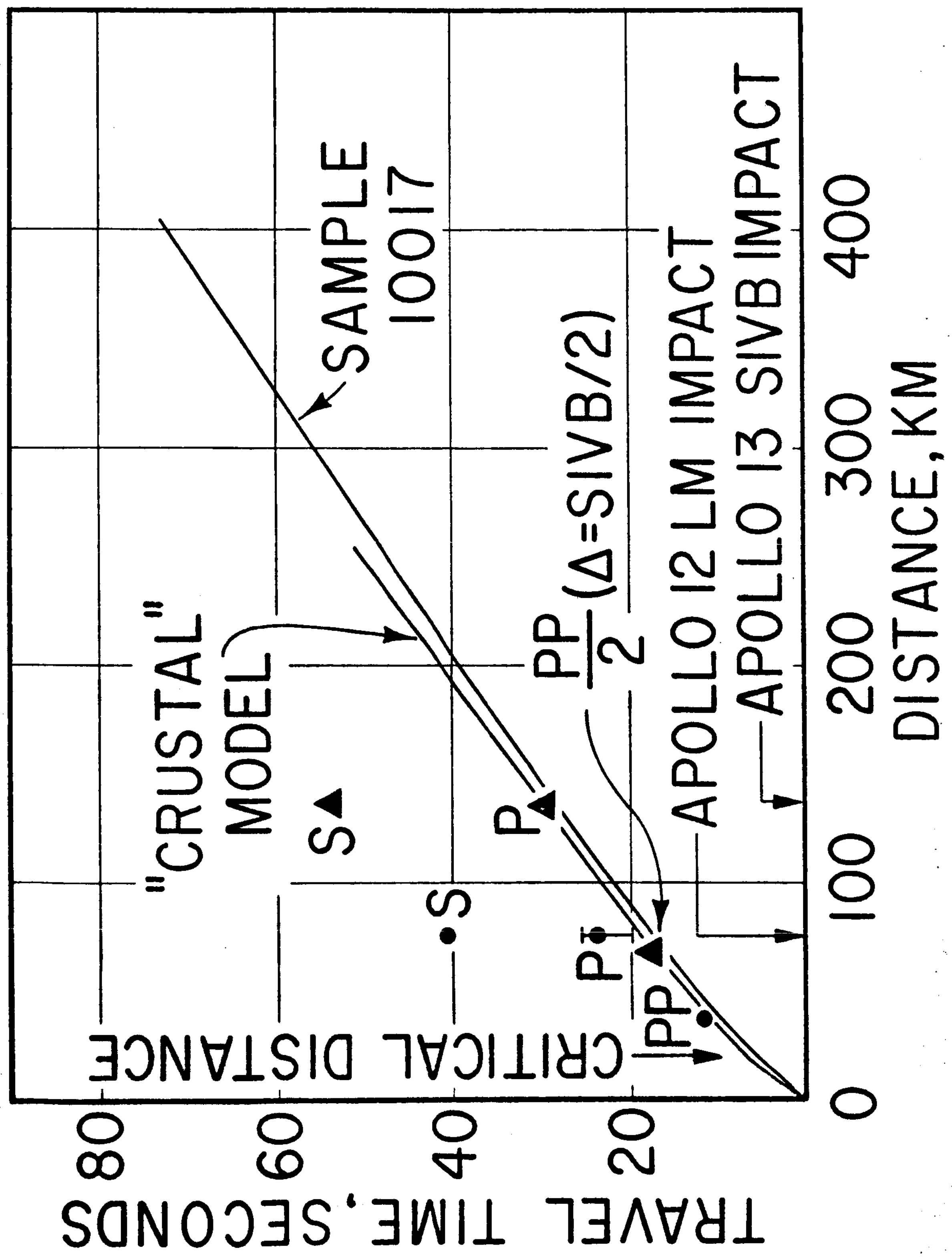
We have examined the possibility of a shallow layered waveguide consisting of the unconsolidated surface debris layer (the lunar regolith) possessing a very low seismic wave velocity and which overlies the more competent although probably fractured basement rocks. Such a model predicts a very steep fundamental mode Rayleigh wave group velocity curve where the group velocity decreases from a value of about 1.6 km/sec to a minimum value of 16 m/sec over the period range from 1.6 to 1.4 seconds (Fig. 3).

For this model surface waves for the S-IVB impact distance of 135 km would have a predicted duration of 140 minutes. Given such a theoretical dispersion curve it is also possible to compute the impulse response seismogram of the wave media. Figure 4 shows the smoothed predicted envelope for this model compared with the observed envelope for the S-IVB impact signal. The agreement is not satisfactory for the early part of the seismic signal but does show that dispersion could contribute to the observed long duration of the observed seismic signal. Data on possible near surface layering required to resolve this question with certainty will have to await results from the seismic refraction experiments planned for future lunar missions or man-made impacts at closer ranges.



V_P KM/SEC
2
3
4
5
6
7
DEPTH, KM
10
20
30
40
SAMPLE 10017
"CRUSTAL" MODEL
EMPIRICAL VARIATION DRY GRANITES

Fig. 1



TRAVEL TIME, SECONDS
DISTANCE, KM
"CRUSTAL" MODEL
SAMPLE 10017
PP/2 (Δ=SIVB/2)
APOLLO 12 LM IMPACT
APOLLO 13 SIVB IMPACT
CRITICAL DISTANCE
S
P
PP
0
100
200
300
400
20
40
60
80

Fig. 2

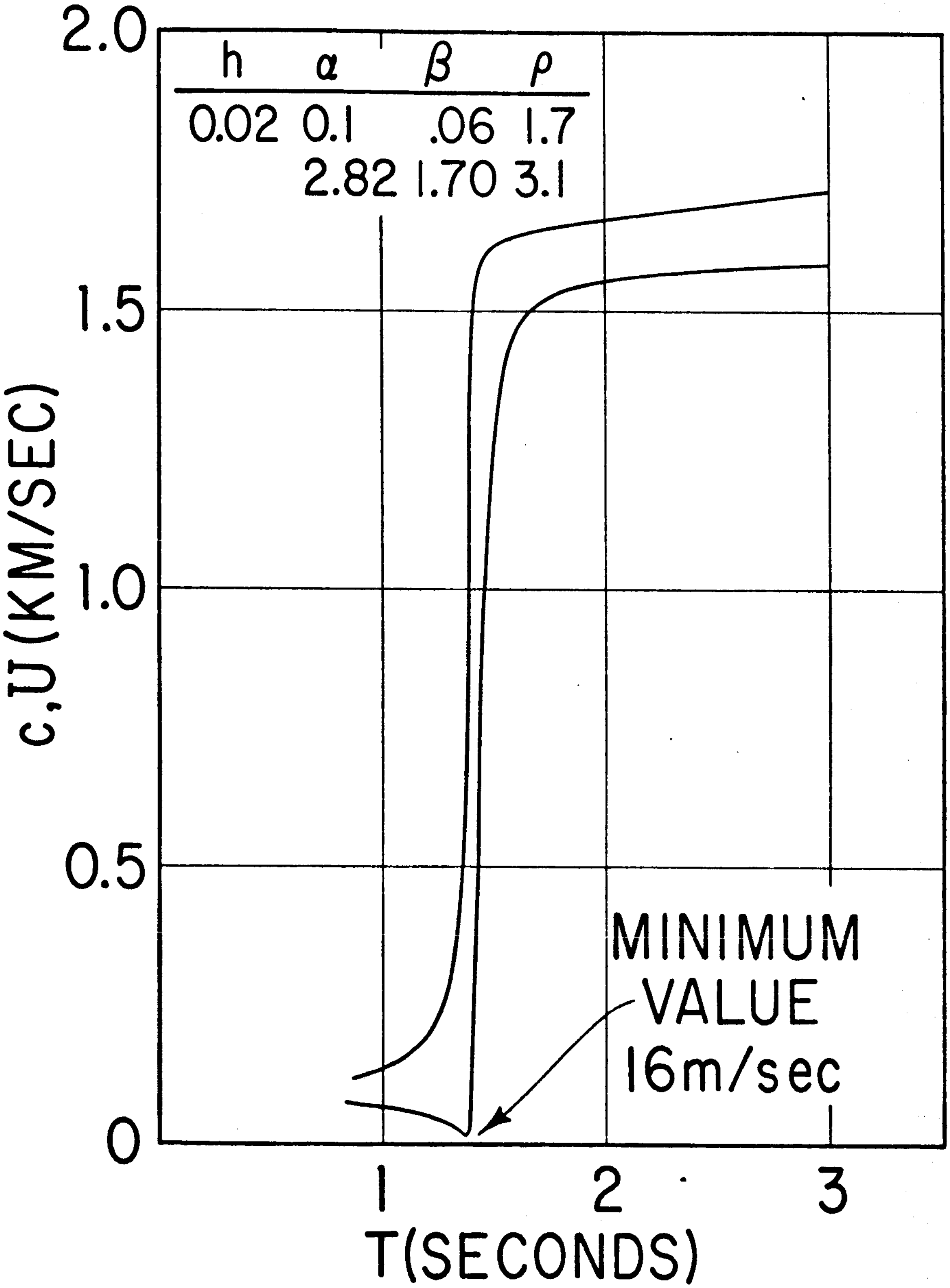

h α β ρ
0.02 0.1 .06 1.7
2.82 1.70 3.1
2.0
1.5
1.0
0.5
0
c,U (KM/SEC)
1
2
3
T(SECONDS)
MINIMUM VALUE 16m/sec

Fig. 3

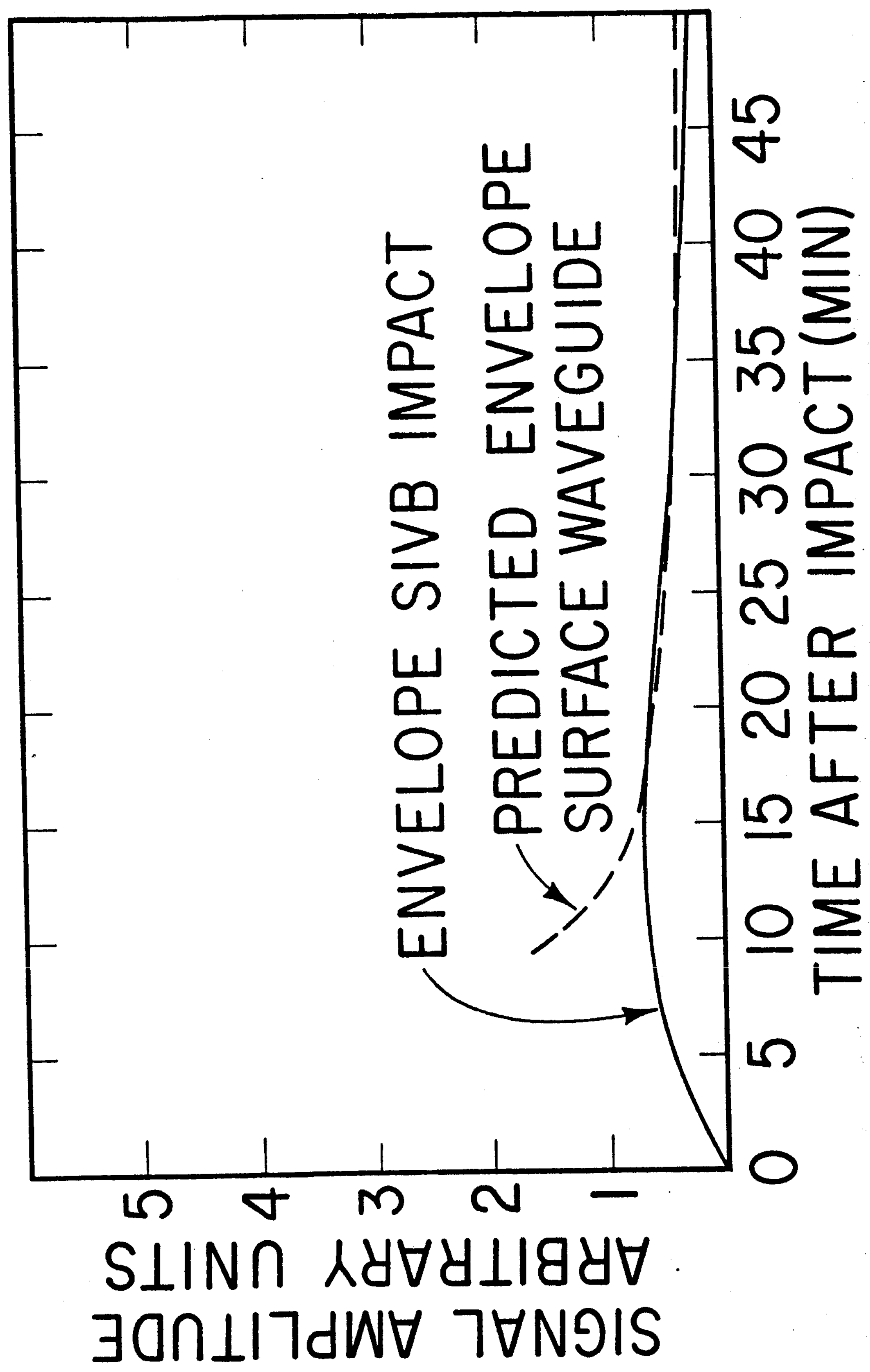

ENVELOPE SIVB IMPACT
PREDICTED ENVELOPE
SURFACE WAVEGUIDE
TIME AFTER IMPACT (MIN)
0
5
10
15
20
25
30
35
40
45
SIGNAL AMPLITUDE
ARBITRARY UNITS
1
2
3
4
5

Fig. 4

3. Lunar Thermal State

Work on the lunar thermal state can be divided into four parts.

(i) An examination of the material properties of the moon which leads almost inevitably to the conclusion that a relatively undifferentiated moon would become thermally unstable within a billion years of its formation. The result would be finite amplitude convection which would strongly control subsequent thermal evolution of the lunar interior.

(ii) Basic theoretical and experimental studies of convection in a material such as the lunar interior which has strongly temperature dependent viscosity. We have shown theoretically that the transition from a stable to an unstable state does not involve an oscillatory situation. We have also experimentally measured the heat flow in a simple convection cell with strong viscosity stratification and find that with an appropriate definition of the Rayleigh number, the heat flow is the same as in a cell with constant viscosity. This very fundamental result permits convective heat transport in planets to be estimated from constant viscosity results.

(iii) A theory of finite amplitude convection in an internally heated self-gravitating sphere has been used to study the internal lunar temperatures. Four points can be made:

a) Rheology rather than heating rate is the most important factor.

b) if Herring-Nabarro creep is applicable, general melting of the lunar interior is unlikely. Central temperatures should range between 1200 and 1800°k.

c) Non-newtonian creep will probably lower these temperatures.

d) A thermal near-catastrophe occurs near the stagnation point above a rising plume. The resultant temperature maximum which is 150-250°k above the average internal temperature can produce a lenticular zone of partial melting at a depth of 200 to 600 km.

(iv) The implications of the lenticular partially melted zone for lunar evolution have been considered. One possible sequence of events is:

a) The moon is formed "cool"

b) Initial radioactivity raises the internal temperature rather uniformly to about 1200°k in about 10^9 years when the rapidly decreasing internal viscosity ensures the onset of convection.

c) The partially melted cap forms over the upwelling of the 1^{st} axisymmetric mode. This is the most unstable mode.

d) The melt percolates to the surface to form the maria taking radioactive elements with it.

e) After several times the cell overturn time of the order of 10^8 years, most radioactivity is removed leading to cell death.

f) As the cell dies its radius decreases so that later surface lavas would have a deeper origin than earlier lavas.

g) The highly assymmetrical nature of the differentiation process shifts the lunar center of mass along the axis of symmetry of the convection cell. Decay of lunar angular momentum by tidal friction will eventually leave the original convection axis aligned with the earth-moon axis.

Besides further basic research on finite amplitude convection, experimental studies of the rheological properties of rocks at moderate confining pressures and very low non-hydrostatic stresses is very desirable. In addition, the establishment of a magnetometer on the lunar farside can give first order information on whether convection was important in lunar evolution.

4. Appendices

Two appendices are included. The first is a paper on the theoretical results described in section 2 (ii) and the second is another paper discussing points 2 (i), (iii) and (iv). The experimental results in 2 (ii) are further described in the second appendix and will be the subject of a future paper.

APPENDIX A

EXCHANGE OF STABILITIES FOR FLUIDS WITH STRONG VISCOSITY STRATIFICATION

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Abstract

The marginally stable state for a fluid with arbitrary viscosity stratification and a linear unstable temperature gradient is stationary in time. However, viscosity stratification may cause an oscillatory instability with a stable temperature gradient. Analogous theorems hold in a sphere heated or cooled within.

1. Introduction

There is now considerable evidence for convective motions in the earth's upper mantle (see papers in Hart, 1969 or Phinney, 1968). Furthermore, thermal instabilities may be important for the internal thermal states and gravitational fields of the Moon and planets (Tozer, 1967; Runcorn, 1967). All the creep laws currently proposed for planetary interiors are strongly temperature and pressure dependent (Turcotte and Oxburgh, 1969; Weertman, 1970). Therefore, it is of considerable interest to consider the thermal instability of a fluid with strong viscosity stratification. The unsettled question of whether this fluid is newtonion for finite amplitude convection (Weertman, 1970) does not arise in this context because at the onset of convection the strain rate is so low that the newtonian assumption is bound to hold.

Schubert, Turcotte and Oxburgh (1969) discuss the stability of a fluid with a strong increase of viscosity with depth heated from below. They assume that the marginally stable state is a stationary pattern of motions, that is, that the principle of the exchange of stabilities holds. In this note, we prove this very fundamental assumption both for the case discussed by Schubert

and for the closely analogous case of a self-gravitating sphere heated within. Schubert's model is probably most useful in the Earth and Venus where high pressures cause a sharp viscosity increase in the lower mantle, while the spherical case is more representative of the Moon and Mars.

For small perturbations with time dependence $e^{\sigma t}$, the principle of exchange of stabilities holds if the imaginary part of σ, σ_i, is zero when the real part, σ_r, is greater than or equal to zero. This is first proved for a constant property fluid heated from below by Pellew and Southwell (1940). They show that the momentum and temperature equations imply

$$\sigma I_1 + I_2 = \sigma^* I_3 + I_4, \qquad (1)$$

where σ^* is the complex conjugate of σ and I_1 to I_4 are positive definite integrals. The imaginary part of (1) is

$$\sigma_i \, (I_1 + I_3) = 0$$

which immediately shows that $\sigma_i = 0$. Allowing the viscosity to vary affects only I_2. Therefore a sufficient condition for the exchange of stabilities to hold in the stratified viscosity case is that I_2 be real.

Davis (1969) has used a general perturbation expansion to prove the exchange of stabilities in situations deviating from Pellew and Southwell's. However, his technique is not very helpful in the geophysical context of very high Prandtl number since its radius of convergence approaches zero. Furthermore, the perturbation operator representing variable viscosity does not belong to the class considered by Davis.

2. Heating from below

Assuming constant properties except for the vertical viscosity gradient and using the Bousinesque approximation, the perturbation equations in an infinite horizontal plane layer with fixed top and bottom boundary temperatures and coordinate system $(\hat{x}, \hat{y}, \hat{z})$ become

$$\sigma \underset{\sim}{q} + \underset{\sim}{\nabla} p = \alpha g \tau \hat{z} + \nu \nabla^2 \underset{\sim}{q} + D\nu [\underset{\sim}{\nabla}(\hat{z} \cdot \underset{\sim}{q}) + \hat{z} \cdot \underset{\sim}{\nabla} \underset{\sim}{q}] \tag{2}$$

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{q} = 0 \tag{3}$$

$$\sigma \tau - \beta(\hat{z} \cdot \underset{\sim}{q}) = \kappa \nabla^2 \tau \tag{4}$$

where $(\underset{\sim}{q}, \tau) = (\underset{\sim}{u}, T) e^{\sigma t}$

and $\underset{\sim}{u}$ and T are the velocity and temperature perturbations. The gravitational acceleration is $-g\hat{z}$; α and κ are the coefficient of expansion and heat diffusivity; p is the pressure perturbation divided by the density; $-\beta$ is the constant super-adiabatic gradient of the basic conduction temperature profile, T_0; β is positive for the unstable situation with heating from below; ν is the viscosity, which through its dependence on the temperature and pressure of the basic equilibrium, is a function only of z; and D = d/dz.

The boundary conditions are q and τ bounded for large x and y and $\hat{z} \cdot \underset{\sim}{q} = 0$, p = 0, $\tau = 0$ on a free boundary and $\underset{\sim}{q} = 0$, $\tau = 0$ on a rigid boundary. Equation (3) further implies

$D(\hat{z}\cdot\underset{\sim}{q}) = 0$ on a rigid boundary while the vanishing of the stress at a free surface implies $D(\hat{x}\cdot\underset{\sim}{q}) = D(\hat{y}\cdot\underset{\sim}{q}) = 0$.

Now take the dot product of (2) with $\underset{\sim}{q}^*$ and multiply the complex conjugate of (4) by τ and integrate both equations over the fluid layer. We obtain

$$\alpha g \int \tau(\hat{z}\cdot\underset{\sim}{q}^*)\,dV = \sigma \int |q|^2 dv$$

$$+ \int\{\underset{\sim}{q}^*\cdot\underset{\sim}{\nabla} p - \nu\underset{\sim}{q}^*\cdot\nabla^2\underset{\sim}{q} - D\nu[\underset{\sim}{q}^*\cdot\underset{\sim}{\nabla}(\hat{z}\cdot\underset{\sim}{q}) + \underset{\sim}{q}^*\cdot(\hat{z}\cdot\underset{\sim}{\nabla}\underset{\sim}{q})]\}dv \quad (5)$$

and

$$\beta \int \tau(\hat{z}\cdot\underset{\sim}{q}^*)dv = \sigma^* \int |\tau|^2\, dv - \kappa \int \tau\nabla^2\tau^* dv. \quad (6)$$

Since α, g, κ and β are positive constants, $\int \tau(\hat{z}\cdot\underset{\sim}{q}^*)dv$ can be eliminated giving equation (1) where I_1 and I_2 are β times the first and second integrals on the right of (5) and I_3 and I_4 are αg times the integrals on the right of (6). Gauss' theorem, $\underset{\sim}{\nabla}\cdot\underset{\sim}{q} = 0$ and either the rigid or free boundary conditions then imply that

$$\int \underset{\sim}{q}^*\cdot\underset{\sim}{\nabla} p\, dv = \int \underset{\sim}{\nabla}\cdot p\underset{\sim}{q}^*\, dv = \int_s p\hat{n}\cdot\underset{\sim}{q}^*\, ds = 0$$

where s is the boundaries and $\hat{n}$ is the outward unit normal.

Likewise

$$\int D\nu\, \underset{\sim}{q}^{*}\cdot\underset{\sim}{\nabla}(\hat{z}\cdot\underset{\sim}{q})\,dv = -\int D^2\nu\,|\hat{z}\cdot\underset{\sim}{q}|^2 dv.$$

A separation of variables of the form

$$[\underset{\sim}{q},\tau] = [\underset{\sim}{Q}(z),\ \theta(z)]\quad f(x,y),$$

where f satisfies (Chandrasekhar, 1961, Ch. 2)

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = -a^2 f,$$

with $a^2>0$ and

$$\int_{(x,y)\,\text{plane}} |f|^2\, ds = 1,$$

gives

$$I_2 = \beta\int[\nu\underset{\sim}{Q}^{*}\cdot(a^2-D^2)\underset{\sim}{Q} + D^2\nu|(z\cdot\underset{\sim}{Q})|^2 - D\nu\underset{\sim}{Q}^{*}\cdot D\underset{\sim}{Q}]\ dz.$$

Integrating the term with $D^2\underset{\sim}{Q}$ by parts and using the boundary conditions, we have

$$- \int \nu \underset{\sim}{Q}^* \cdot D^2 \underset{\sim}{Q} df = \int D(\nu \underset{\sim}{Q}^*) \cdot D\underset{\sim}{Q} \, dz$$

$$= \int [D\nu \underset{\sim}{Q}^* \cdot D\underset{\sim}{Q} + \nu |DQ|^2] dz.$$

Thus I_2 becomes

$$\beta \int [\nu |DQ|^2 + a^2 \nu |Q|^2 + D^2 \nu |(\hat{z} \cdot \underset{\sim}{Q})|^2] dz$$

which is clearly real. Similarly

$$I_4 = \alpha g \kappa \int [|D\theta|^2 + a^2 |\theta|^2] dz,$$

which also real. The exchange of stabilities therefore holds for arbitrary viscosity stratification in a fluid layer heated from below.

Finally it is of interest to consider what happens if $\beta<0$, that is if the temperature increases upwards (stable). Then $I_1<0$ and σ_i can be non-zero. The real part of (1) gives $\sigma_r = (I_4 - I_2)/(I_1 - I_3)$. Thus, if $D^2\nu>0$, we have $I_4<0$ and $\sigma_r<0$ and the fluid is stable. On the other hand, if $D^2\nu < 0$, it is possible to have a positive growth rate. Consequently viscosity stratification may destabilize an otherwise stable equilibrium.

3. Sphere heated within

If β is replaced by βz and g by gz, all the arguments in the preceeding section remain unchanged. Thus the self-gravitating, uniformly heated sphere where $g = \gamma r$ and $\nabla T_0 = -\beta r$ is clearly analogous to the plane layer heated from below. The equations in spherical coordinates $(\hat{r},\hat{\theta},\hat{\phi})$ equivalent to (2) and (4) are

$$\sigma q + \nabla p = \alpha\gamma\tau r + \nu\nabla^2 q$$

$$+ D\nu \left[\hat{r}\cdot\nabla q + \nabla(\hat{r}\cdot q) - \frac{1}{r}\left(q - \hat{r}(q\cdot\hat{r})\right)\right]$$

and $\sigma\tau - \beta(r\cdot q) = \kappa\nabla^2\tau$,

where D now denotes d/dr. The boundary conditions at the surface of the sphere are the same as the layer with $\hat{r}$ replacing $\hat{z}$. At the center of the sphere, q, Dq and τ are finite.

Consider first the expression multiplied by $D\nu$. The volume integral over the sphere of the dot product of q^* and the last two terms is clearly the real quantity

$$- \int (D\nu/r)\left[|q|^2 - |\hat{r}\cdot q|^2\right]dv.$$

Using Gauss' theorem, the second term becomes

$$\int D\nu q^*\cdot\nabla(\hat{r}\cdot q)dv = - \int D^2\nu|\hat{r}\cdot q|^2 dv,$$

and as before, the first term will cancel part of the integral of $\nu q^*\cdot\nabla^2 q$.

The variables may be separated (Chandrasekhar, 1961, Ch 6) with

$$[\underset{\sim}{q},\tau] = [\underset{\sim}{Q}(r),\theta(r)]\ Y_\ell^m\ (\theta,\phi),$$

where the Y_ℓ^m are surface spherical harmonics normalized so that for the spherical surface S,

$$\int_S |Y_\ell^m|^2\ ds = 1$$

Then $\quad r^2\nabla^2 = Dr^2D - \ell(\ell+1)$

and we have

$$\int \nu\underset{\sim}{q}^*\cdot\nabla^2\underset{\sim}{q}dv = \int\nu[\underset{\sim}{Q}^*\cdot Dr^2D\underset{\sim}{Q} - \ell(\ell+1)|Q|^2]dr.$$

Integrating by parts, using the boundary conditions and the boundedness of $\underset{\sim}{Q}^*$ and $D\underset{\sim}{Q}$ as $r\to 0$, we obtain

$$-\int \nu[r^2|DQ|^2 + \ell(\ell+1)|Q|^2] - \int D\nu\underset{\sim}{Q}^*\cdot D\underset{\sim}{Q}\ r^2dr$$

The last term cancels the first term in the expression multiplied by $D\nu$. A straight forward integration by parts of the θ equation then leads to the final results:

$$I_2 = \beta\int \{\nu r^2|DQ|^2 + \ell(\ell+1)\nu|Q|^2 + rD\nu[|\hat{\theta}\cdot\underset{\sim}{Q}|^2 + |\hat{\phi}\cdot\underset{\sim}{Q}|^2]$$

$$+ r^2D^2\nu|\hat{r}\cdot\underset{\sim}{Q}|^2\}dr$$

$$I_4 = \alpha\gamma\kappa \int [r^2|D\theta|^2 + \ell(\ell + 1)|\theta|^2]dr.$$

I_2 and I_4 are clearly real and again the exchange of stabilities holds.

Note that for I_2 to be positive definite, not only must $D^2\nu$ be positive but also $D\nu$. Thus for the stable temperature gradient $\beta<0$, an oscillatory instability may be possible if either $D^2\nu$ or $D\nu$ is negative.

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APPENDIX B

IMPLICATIONS OF LUNAR CONVECTION

by

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A summary of this paper was presented as talk P29 at the A.G.U. Western National Meeting, Jack Tar Hotel, December 9, 1970.

Abstract

An undifferentiated moon is very likely to be thermally unstable. The resultant finite amplitude convection dominates heat flow processes in the lunar interior. The steady state internal temperature of the moon is strongly stabilized by the rheological properties of the lunar material. Present estimates of lunar rheology millitate against melting in an initially cool lunar interior except in a lenticular hot region over an upwelling.This partially melted zone can account for the outpouring of mare material on only one hemisphere of the moon at about one billion years after the moon's formation. The removal of radioactive elements with the partial melt would lead to cell death and a cesession of widespread volcamism several hundred million years later. The assymmetric differentiation process also produces a displacement of the lunar center of mass along the cell axis and spin-orbit coupling and tidal friction will cause the convection axis to align with the earth-moon axis.

I. INTRODUCTION

Convection in planetary interors at temperatures below the melting point has enjoyed a rather bad reputation. This is presumably because of the difficulty of solving the problem of finite amplitude convection. Not only are you dealing with a non-linear system, but you must also contend with such exotic effects as strongly temperature dependent viscosity and perhaps a non-newtonian stress-strain rate relation. However, as we shall see, thermal instabilities seem fairly likely in planetary bodies such as the moon, and considerable insight can be gained into certain restricted aspects of the problem by approximate techniques. One such aspect is the average thermal state of the interior. This was first pointed out by Tozer (1967). The calculations presented in this paper are similar to his. I will discuss both theoretical and experimental justification for his method and further consider basic implications of convection for the moon's thermal history.

II ASSUMPTIONS ABOUT THE MOON'S INTERIOR

I have made several very basic assumptions about the material inside the moon which need discussion. The most important is that from a very long term point of view the solid lunar material behaves like a fluid with a newtonian viscosity. This viscosity is exponentially dependent on temperature in such a way that the material gets softer as it heats up. In particular, I use various diffusion (Herring-Nabarro) creep laws proposed for the earth's mantle.

Five Herring-Nabarro creep laws are shown in Table 1. This type of creep implies a linear stress (σ) - strain rate ($\dot{\varepsilon}$) relation and the concept of viscosity (η) is well defined. Except for model 1, which I believe has an unrealistically large grain size (see Turcotte and Oxburgh 1969a),all the models are plausible. The basic difference between them is the activation energy E which controls the rate of change of viscosity with temperature and the ratio T/T_m (temperature over melting temperature, which is arbitrarily taken to be 1800°K) at which a viscosity of 10^{21} poise is reached. This viscosity value is characteristic of the earth's upper mantle (McConnell, 1968) and is obtained from measurements of the post glacial uplift of such areas as Fennoscandia and Lake Bonneville. As yet no similar measurement exists for the deep interior of the moon. Models 1 and 2 represent a lower bound to the activation energy, while model 5 represents an upper bound (Gordon, 1965).

Herring-Nabarro creep has been strongly attacked recently by Weertman (1970) and something needs to be said in its defense. First of all, Herring-Nabarro creep is justified below some critical stress, σ_c, above which a creep mechanism such as dislocation glide will take over. The strain rates for the two processes are

$$\text{Herring-Nabarro:} \quad \dot{\varepsilon}_H = \left(\frac{\alpha D}{L^2}\right)\left(\frac{\sigma}{KT}\right)$$

TABLE 1.

HERRING-NABARRO CREEP

$$\eta = \frac{L^2kT}{\alpha D\Omega} \qquad D = D_0 \exp(-g\, T_m/T)$$

k = Boltzman const $\Omega = 10^{-23}$ cm^3	L grain size cm	E ev	D_0 Diffusion cm^2/sec	α cm^{-2}	T/T_m $\eta=10^{21}$poise $T_m=1800°K$	σ_c critical stress bars	σ moon $\frac{dT}{dr}\Big\|_a=3°/km$
1. Weertman (1970)	47.	2.9	1	5	1.0	0.03	1.2
2. Weertman (Mod)	2.0	2.9	1	5	0.75	0.75	0.52
3. Tozer (1967)	0.1	4.0	10	20	0.75	11.	0.31
4. Turcotte & Oxburgh (1969a)	0.2	4.5	20	10	0.875	30.	0.44
5. Turcotte & Oxburgh (1969b)	0.1	5.5	20	10	1.03	60.	0.58

Dislocation Glide: $\dot{\varepsilon}_G = \alpha' D \left(\frac{\sigma}{\mu}\right)^2 \left(\frac{\sigma}{KT}\right)$

where D is the diffusion coefficient for vacancies, L is the grain size, Ω is the atomic volume, T is temperature, α is a constant depending on the conditions at the grain boundary, μ is the shear modulus (10^{12} dynes cm^{-12}) and Weertman gives α' $= 2.23 \times 10^{12} cm^{-2}$. At the critical stress, $\dot{\varepsilon}_H = \dot{\varepsilon}_G$. Then

$$\sigma_C = \frac{\mu}{L} \sqrt{\frac{\alpha}{\alpha'}}$$

which is inversely proportional to grain size and is listed in Table 1. Now if we assume Herring-Nabarro creep, we can solve the convection problem and estimate the actual stresses in the cell from the relation

$$\sigma = \frac{U}{R\eta}$$

where U is a typical velocity in the cell and R is the radius. These are listed in the last column of Table 1 for a typical cell. You can see that except for model 1, Herring-Nabarro creep is at least self consistent. That is, the driving stress is less than the critical stress.

My second point is that even if the critical stress is exceeded, Herring-Nabarro creep probably leads to an upper bound for the temperature in the moon's interior. This is because dislocation glide predicts higher creep rates than Herring-Nabarro at stresses above critical. Thus a convection cell in which non-newtonian dislocation glide dominates could be expected to have higher material velocities than the corresponding newtonian Herring-Nabarro cell and heat would be removed from the interior more efficiently.

My second assumption is that the heat sources in the moon are uniformly distributed. This essentially means that the moon is taken to be undifferentiated. This may not be true today but may be reasonable during the early history of the moon. A subsidiary part of this assumption is that the moon started out with an initially uniform temperature. The validity of this assumption is not well understood (R. Reynolds, personal communication) and the problem of the instability of any general initial temperature profile has not yet been solved.

The third assumption is that the elasticity of the moon serves only to ensure a rigid outer boundary condition. Most seismic and volcanic activity in the earth appears to be the result of the breaking up of the cold elastic outer rind and the rubbing together of the various pieces. However, the surface to volume ratio for the moon is much larger than for the earth. Thus for equal heat generation per unit volume, the outer elastic layer is much thicker on the moon than the earth. Lunar convection is confined to the deep interior and there is no seismic or visual evidence for terrestrial type tectonic processes which would accompany the breaking up of the moon's outer shell.

Fourthly, I assume that I can ignore all pressure effects in the moon. This is not strictly valid. However the pressure at the center of the moon corresponds only to a depth of 125 km in the earth. Thus the strong pressure effect on viscosity which greatly stiffens the lower mantle (Turcotte and Oxburgh, 1969a) in the earth is not very important in the moon. The adiabatic temperature rise in the moon roughly cancels the presure effect on viscosity. (Turcotte and Oxburgh, 1969b).

Finally, I assume that all other material properties such as heat diffusivity, thermal expansion coefficient and so on, are constant. The probable variations in these quantities are small and are thus of secondary importance compared to the viscosity. I have adopted the values given by Turcotte and Oxburgh (1969b).

III. THERMAL STABILITY OF THE LUNAR INTERIOR

The thermal stability of a fluid system is governed by a non-dimensional quantity called the Rayleigh number. In an internally heated, self-gravitating sphere with a rigid outer boundary, the Rayleigh number is

$$R_a = \left(\frac{4\pi}{9}\right) \frac{\alpha \rho^2 G Q r^6}{c k^2 \eta}$$

where $\alpha = 3 \times 10^{-5}$ (cgs) is the coefficient of thermal expansion,

$c = 1.34 \times 10^7$ is the specific heat, Q is the volume heating rate, G is the universal constant of gravitation, $k = 10^{-2}$ is the heat diffusivity, η is the viscosity, ρ is the average density, and r is the radius. If the viscosity is constant in the sphere and the Rayleigh number is less than 8040 (Backus, 1955) the system is stable; there is no motion and the radial temperature profile will be the "conduction" solution shown in Figure 2.

Now suppose we calculate the Rayleigh number for various spherical regions concentric with the center of the moon. The viscosity is derived using the conduction solution temperature at the outer boundary of each sphere. This gives a minimum estimate for the actual Rayleigh number of the system. Since the conduction solution is hotter inside the sphere than on its boundary the average viscosity in the region is lower than the viscosity at boundary. Figure 1 shows how the Rayleigh number varies with radius for three different estimates of the heating rate. The curves are actually labeled with the temperature gradient at the surface of the moon because it is essentially an observable which can be estimated from magnetic measurements (C. Sonett, personal communication) and which is linearly related to the heating rate. The range of gradients shown roughly spans those presently considered reasonable. As you can see the Rayleigh number quickly increases to a value greater than critical as we decrease the radius. This is because the exponential drop in viscosity greatly outweighs the decrease in Rayleigh number due to the r^6 dependence. The highly unstable nature of an undifferentiated lunar interior is clearly evident. All the viscosity relations considered lead to the same conclusion. The question seems to be not whether the interior is thermally unstable but how big the unstable zone is.

IV FINITE AMPLITUDE CONVECTION

If the Rayleigh number is significantly greater than the critical value of 8040, finite amplitude convective motions will considerably alter the temperature structure of the cell. The spherically averaged temperature profile for a cell with constant viscosity is also shown in Figure 2. We get an essentially isothermal core and a thin-thermal boundary layer. Note that the outer boundary of the cell is at R'.

The temperature drop across the cell relative to the conduction solution temperature drop is measured by the number M introduced by Roberts (1967). Tozer (1967) estimates M from

experimental results for a fluid with a linear conduction temperature profile. This is probably a reasonable approach in the earth where the convecting layer in the upper mantle is so thin (due to the pressure effect) that the conduction solution gradient and gravity are nearly constant across the layer. In the moon, However, the entire interior of the moon convects and the conduction solution is parabolic. The temperature gradient and the gravitational acceleration both approach zero at the center of the moon. This has a stabilizing effect which will reduce the efficiency compared to the plane Rayleigh-Benard cell. I have therefore used the numerical results by Baldwin (1967) for the first axisymmetric mode in an internally heated, self-gravitating sphere. M is plotted as a function of Rayleigh number in Figure 3. It is 1 up to the onset of instability and then monotonically decreases. The numerical results go up to a Rayleigh number of 1.5×10^{-6} and I have extrapolated them to higher Rayleigh numbers using Baldwin's assymptotic results as a guide.

Now of course a convection cell in the moon does not have constant viscosity. However, if the temperature profile in Figure 2 existed in the moon, the temperature dependence of the viscosity would be important only in the boundary layer where fortunately the fluid velocity is small. Therefore I have assumed that the parameter M for the lunar problem can be evaluated from the constant viscosity theory using the value of the viscosity in the isothermal core.

I have examined this assumption experimentally. An oil with a temperature dependent viscosity was placed between horizontal plates and heated from below and cooled from above. This configuration has a thermal boundary layer at each plate and the mainstream temperature is the average between the two plates. The Rayleigh number of the system is based on the viscosity at this average mainstream temperature. For a total viscosity variation in the cell up to at least an order of magnitude and Rayleigh numbers up to at least 10^5, there is no observable difference in heat transport at any given Rayleigh number between variable and constant viscosity cells. This strongly supports the use of the constant viscosity results to estimate the heat transport in a temperature variable viscosity cell.

Finally, we need a criterion to determine the preferred radius of the convection cell. This problem is discussed in some detail by Tozer (1967) who concludes that the absolute temperature of the core should be minimized. This presupposes that a variational principle exists for the problem and as Roberts (1966) points out, a rigorous basis for such a principle is probably unlikely. However, minimizing the core temperature is probably a reasonable first approximation.

V INTERNAL TEMPERATURE

Even with the minimum temperature criterion, calculation of the core temperature is not completely straightforward. If T_c is the conduction solution temperature at the center of the moon and T_0 is the temperature at the outer boundary of the cell, the core temperature is

$$T = T_0 + M(T_c - T_0)$$

We want to find the cell which minimizes T with respect to variations in the cell radius. Differentiating with respect to r gives

$$\frac{\partial T}{\partial r} = (1-M)\frac{\partial T_0}{\partial r} + \frac{\partial M}{\partial r}(T_c - T_0).$$

However, a problem arises in the calculation of

$$\frac{\partial M}{\partial r} = \frac{\partial M}{\partial R_a}\frac{\partial R_a}{\partial r}.$$

If R_a is derived from the viscosity at the core temperature T and the outer radius of the cell, R, it is easy to show that T decreases monotonically with increasing R. Thus the minimum T occurs when R is the radius of the moon. However, because of the extremely large viscosity increase in the thick boundary layer of this cell and the increasing effect that elasticity must have near the lunar surface it seems likely that the radius should be less than the lunar radius. Thus the lunar radius cell gives an absolute minimum estimate for the core temperature. If, on the other hand, R_a is evaluated from the viscosity at the boundary temperature T_0, there is a well defined cell radius with $\frac{\partial T}{\partial r} = 0$. However, since the temperature in the mainstream of this cell will be considerably higher than the boundary temperature at which the viscosity was evaluated, this cell's core temperature represents an extreme upper bound. The compromise between these extremes is to evaluate the Rayleigh number using the viscosity at the core temperature T and the radius of the isothermal core (the cell radius minus the boundary layer thickness).This may be reasonable since the scale of the velocity field should be slightly reduced by the viscosity increase in the boundary layer.

The actual calculation is done by first finding the cell which gives the extreme upper bound described above. A new core temperature is then taken to be the temperature at the outer boundary of this cell (i.e. the original T_0) and the new T_0 is calculated from this new core temperature using the M for the preferred upper bound cell. $\frac{\partial T}{\partial r}$ will then no longer be quite zero. However it can be made zero by an iteration which results in raising the core temperature by about 1%. Except for this final iteration, this calculation is the same as used by Tozer (1967) although the justification is rather different. The three temperature estimates for a fixed heating rate and all five viscosity models are given in Table 2. The most probable temperature turns out to be a rough mean between the extremes. The average terrestrial heating rate used in this table means that the moon has the same radioactive heat generation per unit volume as the earth would have if the observed surface heat flow were due to uniformly distributed sources within the earth.

The probable core temperature versus surface temperature gradient for three of the viscosity models are plotted in Figure 4. The first thing to notice about these results is that the choice of viscosity model has a much larger affect on the core temperature than the heating rate. Doubling the heating rate implies almost doubling the conduction solution temperature

TABLE 2. Internal temperatures (°K) in a convecting Moon with average terrestrial heat generation.

Viscosity Model (Table 1)	Minimum	Probable	Maximum
1	1320	1578	1963
2	1090	1321	1580
3	1152	1387	1589
4	1320	1578	1820
5	1530	1815	2083

at the center of the cell. However it only means about a 100°K change in the convection solution temperature. This powerful stabilization of the convection zone temperature is perhaps the most important result of planetary convection. It is not the heating rate that determines the thermal state; it is the rheology. What happens physically is that as you try to heat up the cell by increasing the heating rate, the viscosity drops, causing an increase in convection efficiency which in turn carries away the extra heat you added. This is probably a very general property of temperature dependent creep which would manifest itself even for a non-newtonian creep mechanism.

The other point I want to make with figure 4 is that all the temperatures are significantly below the solidus. Models 2 and 3 are in fact within the range considered likely from the magnetic results(P. Dyal , personal communication). This millitates against general melting of an initially cold moon. However, local melting may occur as is shown in Figures 5 and 7.

Figure 5 shows the actual radial temperature profile for the first axisymmetric mode along the upwelling axis, the downwelling axis and in the equitorial plane. The equitorial profile is equal to the spherically averaged profile. The temperature drops across the preferred cells are generally about 150 to 250°K. Thus the drop shown in Figure 5 is a rough lower bound. You can see from this figure that within the main stream, the deviations from the isothermal state are small. But at the inner edge of the boundary layer over the upwelling plume, a hot peak forms. This is a general result of finite amplitude convection with internal heat generation. Material in contact with the cold outer boundary becomes cooler as it moves toward the downwelling plume and is at its coldest and densest as it begins its descent into the interior. As soon as the material leaves the immediate vicinity of the boundary, however, its internal heat generation causes it to begin heating up again. As the material rises in the warm plume, its radial velocity continuously decreases. Thus the total heat generated in the material per unit change of radius steadily increases. This causes the temperature to rise rapidly as the material approaches the boundary. There would be a thermal catastophe at the stagnation point above the rising plume if it were not for the increased role of conduction in the boundary layer. The maximum material temperature would be expected to occur at the edge of the boundary layer as is in fact seen in figure 5.

If the temperature in the hot spot rises above the solidus, a lenticular partially melted cap will form over the upwelling plume as is shown schematically in Figure 6. The probability of the formation of a melted cap is somewhat enhanced by the effect of pressure on the solidus as is shown in Figure 7. The solidus in this figure is for the pyrolite moon given by Ringwood (1970). The division between the mainstream and boundary layer in the cells shown occurs almost precisely at the break in slope between the conduction solution and core temperature in this figure. The core temperatures include the adiabatic gradient plus the minimum superadiabatic gradient for marginal stability in the preferred cell.

The rise in temperature in the hot spot relative to the core temperature is essentially limited by the magnitude of the temperature drop from the center to the outer boundary of the cell. Therefore, since viscosity models 2 and 3 have a temperature drop of 200 to 250°K and require hot spots 250 to 300°K warmer than their cores, these models would probably not have a partially melted cap unless the solidus were 100°K lower. On the other hand, it seems fairly certain that model 4 would have a melted cap since its average profile approaches within 100°K of the solidus. Finally, it is clear that model 5, which represents an upper bound, will have a melted zone. However, as figure 5 shows, there is a cold region over the downwelling which will probably pull the temperature there below the solidus. Thus even in model 5 the melting would probably be confined to one hemisphere.

VI Lunar Evolution: Initially Cool Moon

As I remarked earlier, the general time dependent problem of the onset of finite amplitude convection has not been solved. The picture is further complicated in the moon by the fact that the heat production 4.6 billion years ago may have been as much as an order of magnitude larger than at present. This initial heating rate and its early decay depends critically on the relative abundances of k^{40} and U^{235} (Phinney and Anderson, 1967). The present rate probably depends more on U^{238} and Th^{232}.

However a qualitative picture of the early lunar evolution can be constructed and compared with our present knowledge of the moon.

The early development of the time dependent conduction solution involves an almost uniform core temperature with a steep temperature gradient only near the outer boundary. Because of the rapid change of viscosity, the maximum stable temperature gradient drops sharply with increasing temperature. At 1200 - 1500°k, which should be reached in about one billion years (Phinney and Anderson, 1967), the viscosity can only stabilize a temperature drop of less than a few degrees in a cell with radius greater than 1000 km. This is clear because the operating Rayleigh numbers of the preferred cells discussed in the last section are orders of magnitude larger than the critical value of 8040 (see Figure 8). The outer part of the moon where the thermal gradient is highest would become unstable first but the instability would quickly spread to the entire interior.

Once convection begins it will dominate all other heat flow processes and the system will evolve towards a stable convection solution. If this is accomplished without any melting, differentiation in the lunar interior would be by the slow process of diffusion. The convection cell would still be operating although not necessily in the first mode and might support the moon's dynamical ellipticity as proposed by Runcorn (1967); The initial conditions would be long forgotten; the present thermal state would be controlled by the rheology and such features as the maria would have to be the result of external heating.

However geochemical evidence points towards partial melting at some depth inside the moon (Ringwood, 1970). Furthermore, at the time of convection onset, the heat production would still be two or three times the present rate. Thus the initial convective equilibrium temperature may be more than a hundred degrees higher than discussed in the last section. This would greatly enhance the chances of forming a partially melted zone unless non-newtonian effects and small grain size and activation energy substantially increase convective heat transport compared to the models considered here. General melting would still be unlikely although model 5 in Figure 7 could be expected to partially melt much of the moon at shallow to intermediate depths.

The formation of the partially melted zone alone would probably have little affect on the convection cell. However, if the melt can percolate to the lunar surface through fissures such as might be produced by the large impacts which created the ringed maria, the situation may be quite different. To begin with, radioactivity would be preferrentially removed with the melt. This means that the heat source in the cell would decay. This would presumably continue until the hot spot no longer penetrated the solidus and would perhaps eventually lead to cell death. The time scale for this process is likely to be several times the cell overturn time. The velocity of the center of typical cell is plotted versus surface gradient in Figure 8.

At 4°/km, the velocities are considerably less than discussed by Turcotte and Oxburgh (1970) and imply a cell overturn time $\tau = \pi R/U$ of about 5×10^7 years. Thus the time of strong convection and the partially melted zone - several hundred million years after onset - agrees roughly with the age of the maria.

Another interesting point is that as the heating rate decreases, partial melt over the downwelling will disappear and the depth to the partially melted zone over the upwelling will increase. This latter point is also illustrated in Figure 8. This implies an age for mare on the lunar farside similar to the oldest mare on the near side and a significant increase with time of source depth for nearside lavas. This probably would have geochemical implications which could be tested.

I would also like to point out that the first axisymmetric mode has the highest growth rate and is therefore the most likely to exist initially. Consequently differentiation would essentially be a one hemisphere process. This would permanently shift the center of mass of the moon along the axis of upwelling. This shift would persist after the cell had decayed or died and the upwelling axis would become the axis of minimum moment of inertia. Decay of the lunar angular momentum by tidal friction and spin-orbit coupling would then eventually leave the moon with the convection cell axis aligned with the earth-moon axis.

Finally, the question of whether a convection cell still exists in the lunar interior is very difficult to answer. The structure of the lunar gravitational field seems the most useful tool at present. Heat flow and electrical conductivity structure are theoretically useful. But it is unlikely that measurements in the near future will be good enough to distinguish the subtle differences between continuing convection and the asymmetric heat flow created by the asymmetric differentiation. On the other hand, measurement of the heat flow through the lunar farside either directly or by indirect electro-magnetic methods would be a very useful first order check of the convection picture of lunar evolution.

VII Other Initial Conditions

The minor, but distinct presence of anorthosite in lunar samples (Wood, et al., 1970) has lead to the assumption that the lunar highlands represent an anorthositic lunar crust (i.e. Ringwood, 1970). The fact that the crater density is higher for highlands than the mare is interpreted to indicate a greater age for the highlands. In fact this anorthositic crust may be the result of high temperatures and consequent melting at the lunar surface at the time of the moon's formation. These high temperatures may either extend into the lunar interior or decrease with increasing depth.

If the moon is completely molten, the viscosity will be so low that the sharp temperature gradient which will form at the outer boundary will become unstable. This instability will quickly propogate to the interior. For a viscosity of the order of 10^{10} poise (as compared to 10^{21} poise for the solid convection in this paper), velocities of the order of 10^{-2} cm/sec are not unreasonable. The internal temperature will drop very rapidly at first and then more slowly as the outer part of the moon, the deep interior and finally intermediate depths solidify. The partially melted cap region over an upwelling will be the last melt and from then on the evolution of the lunar interior will be indistinguishable from an initially cold moon.

If initially very high temperatures exist only near the lunar surface, convective instability limited to this zone will cause it to freeze. Convection will not start in the deeper interior until radiogenic heat has raised the internal temperature nearly to the solidus and locally reversed the initial stable temperature gradient. Again instability will begin at shallow to intermediate depths. Propogation of the instability into the deep interior will be slow because it is necessary to wait until radioactive heating has reduced the interior viscosity sufficiently. If the initial central temperature is the same as the initial temperature of a cold moon, convection throughout the lunar interior occurs only slightly earlier than the time of convection onset in the initially cool moon.

It should be clear from this discussion, that from the point of view of the present thermal state of the lunar interior and its evolution for the past three billion years, the question of whether the moon was initially hot or cold is largely irrelevant. Convectively unstable systems forget their initial condition.

The geochemisty and age of the lunar crust provides the appropriate data for determining the initial state of the moon.

VIII Concluding Remarks

Because of the approximate nature of the assumptions and calculations in this paper, the specific results should be considered very preliminary and treated with an appropriate amount of caution. In order to improve this situation, high priority should be given to:

1. Experimentally investigation of the creep of rocks at pressures of 10 to 50 kb, non-hydrostatic stresses from 0.1 to 100 bars, and temperatures greater than 750°k.

2. Evaluation of the effects of non-newtonian flow on convective heat transport.

3. Theoretical studies of the time dependent onset of convection.

4. Determination of heat flow through the surface of the earthside and farside of the moon. Barring a manned expedition to the lunar farside, considerable effort should be made to establish a surface magnetometer near the center of the farside and a series of magnetometer bearing lunar orbiters.

Finally, I want to reiterate the main point illustrated in this paper: the thermal state of an unstable planetary interior is strongly dependent on the rheological properties of the planetary material.

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Captions

Fig. 1 The Rayleigh number versus the radius of a spherical cell concentric with center of the moon. The curves are labelled with the surface temperature gradient of three possible conduction temperature profiles. This gradient is linearly proportional to the internal heating rate. The viscosity is assumed constant inside the cell and is evaluated at the conduction solution temperature at the cell's outer boundary. The viscosity model is number 4 in Table 1.

Fig. 2 The spherically averaged temperature profile in a convecting moon of radius a. R' is the radius of the convecting zone and R'-R is the thermal boundary layer thickness. The solidus is arbitrary

Fig. 3 The convective efficiency parameter M versus Rayleigh number after Baldwin (1967).

Fig. 4 The steady state core temperature in a convecting moon versus the surface temperature gradient. This gradient is linearly proportional to the internal heating rate. The numbers labelling the curves correspond to the viscosity models in Table 1.

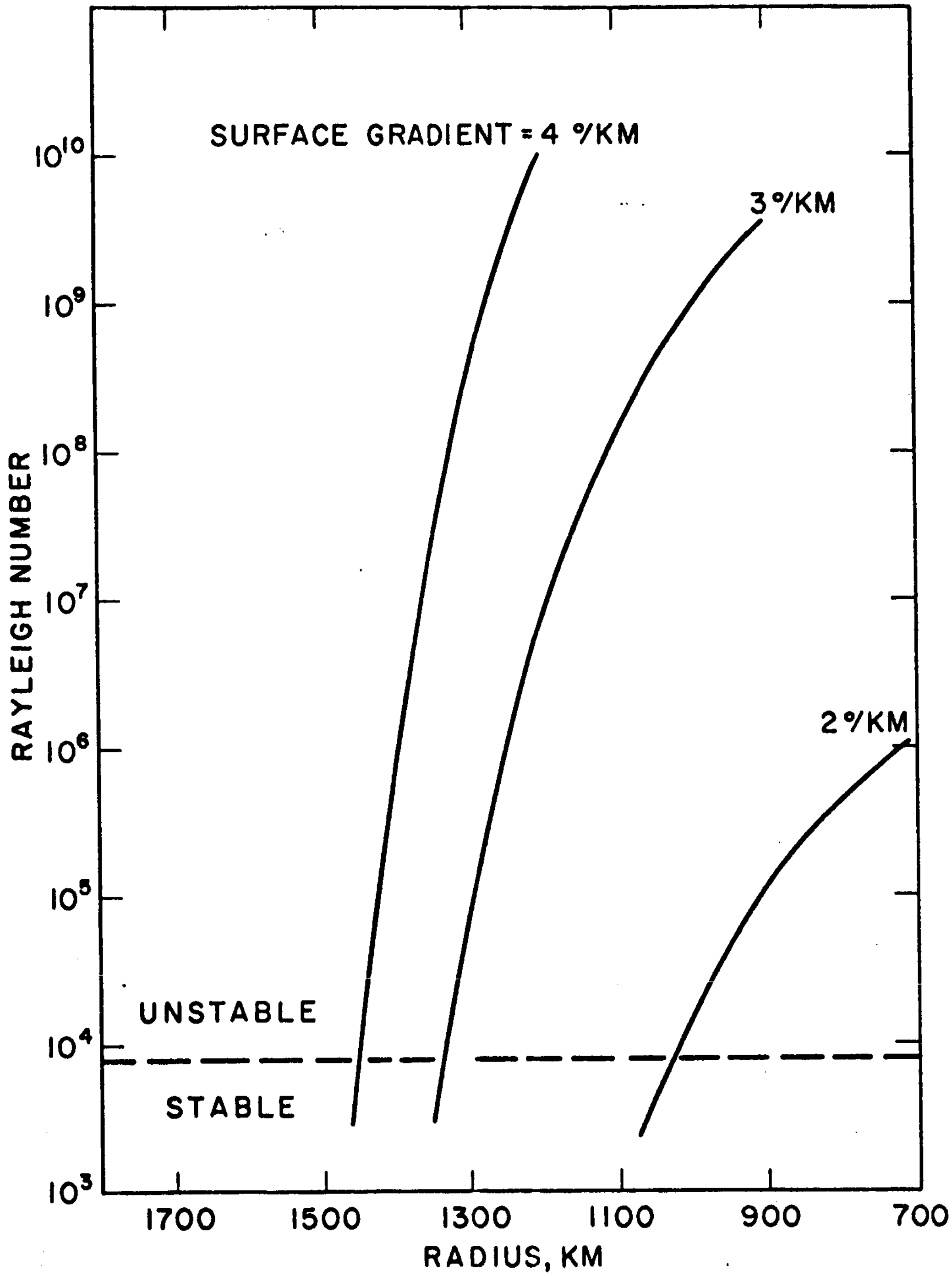
Fig. 5 The actual radial temperature profiles along the upwelling axis ($\theta = 0$), the downwelling axis ($\theta = \pi$) and in the equitorial plane ($\theta = \pi/2$) for the first axisymmetric convective mode in a sphere. The profile locations appear as O-A, O-A' and O-B respectively in Figure 6. The Rayleigh number is 1.5×10^6. The temperature scale is arbitrary and can be stretched. The small arrows indicate the theoretical edge of boundary layer.

Fig. 6 Schematic picture of the first axisymmetric convection cell in the moon showing the outer rigid shell, the temperature boundary layer, the possible zone of partial melt and the approximate stream lines.

Fig. 7 The spherically averaged temperature profile in the moon for four viscosity models and the average terrestrical heating rate. The labels on the curves correspond to the viscosity models in Table 1. The knee in the curve between the conduction solution and the convective core temperature occurs at the inner edge of the thermal boundary layer which corresponds to the radius R in Figure 2. The adiabatic gradient and the minimum superadiabatic gradient for marginal stability have been added to the core temperature. The solidus is for the

Ringwood's (1970) pyrolite moon. The change in temperatures at the knee of the curve with heating rate is shown in Figure 4.

Fig. 8 The velocity, U at the center of the moon, the Rayleigh number, R_a, and the thickness of the outer shell, D, (here defined as the rigid shell plus the thermal boundary layer) versus heating rate for viscosity model 3 in Table 1.



SURFACE GRADIENT = 4 °/KM
3 °/KM
2 °/KM
UNSTABLE
STABLE
RAYLEIGH NUMBER
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
1700
1500
1300
1100
900
700
RADIUS, KM

Fig. 1

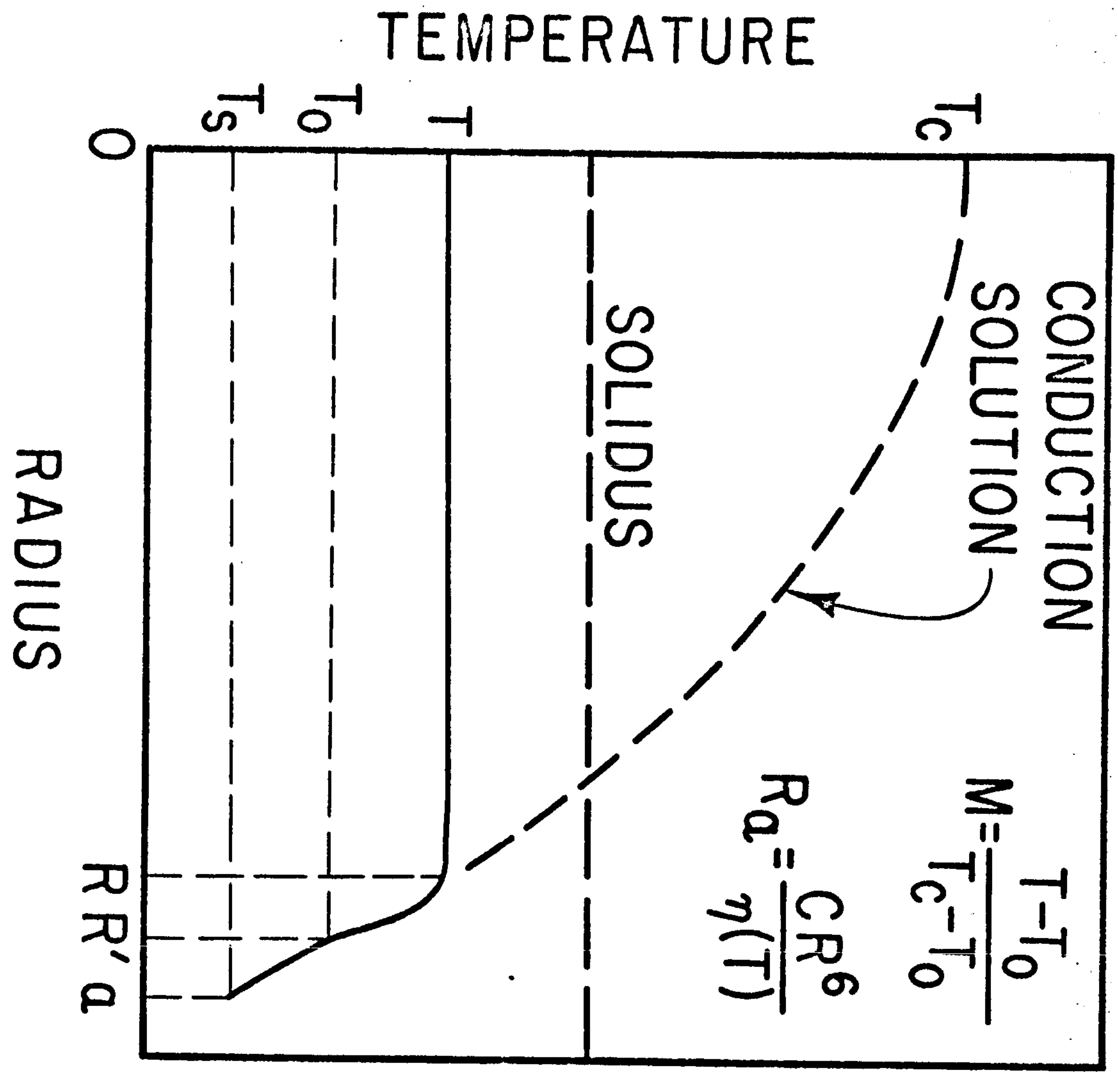

TEMPERATURE
RADIUS
CONDUCTION SOLUTION
SOLIDUS
T_c
T
T_o
T_s
0
R R' a
M=(T-T_o)/(T_c-T_o)
R_a=CR^6/η(T)

Fig. 2

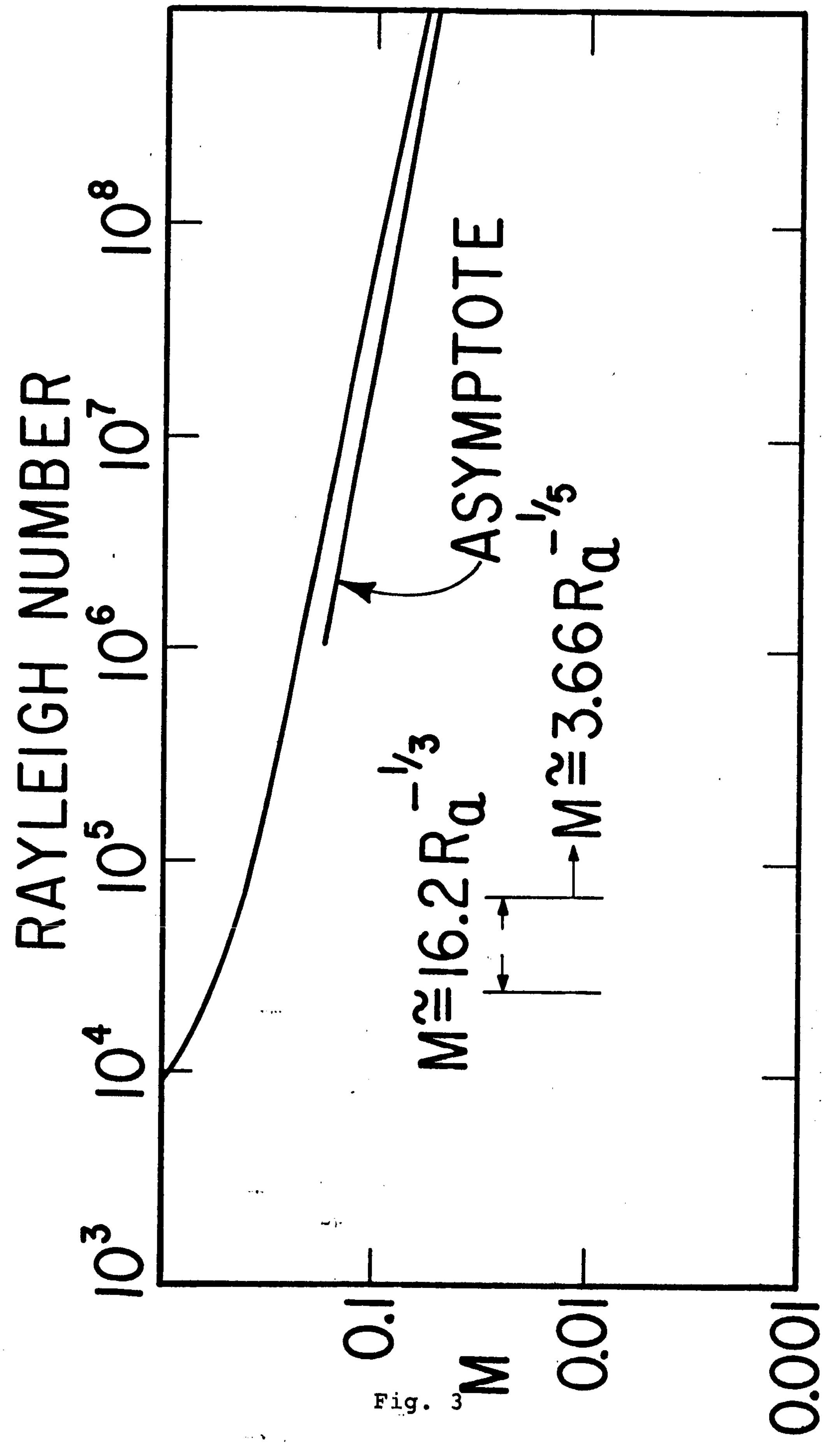

RAYLEIGH NUMBER
10³
10⁴
10⁵
10⁶
10⁷
10⁸
ASYMPTOTE
M ≅ 16.2 Ra^-1/3
M ≅ 3.66 Ra^-1/5
M
0.1
0.01
0.001

Fig. 3

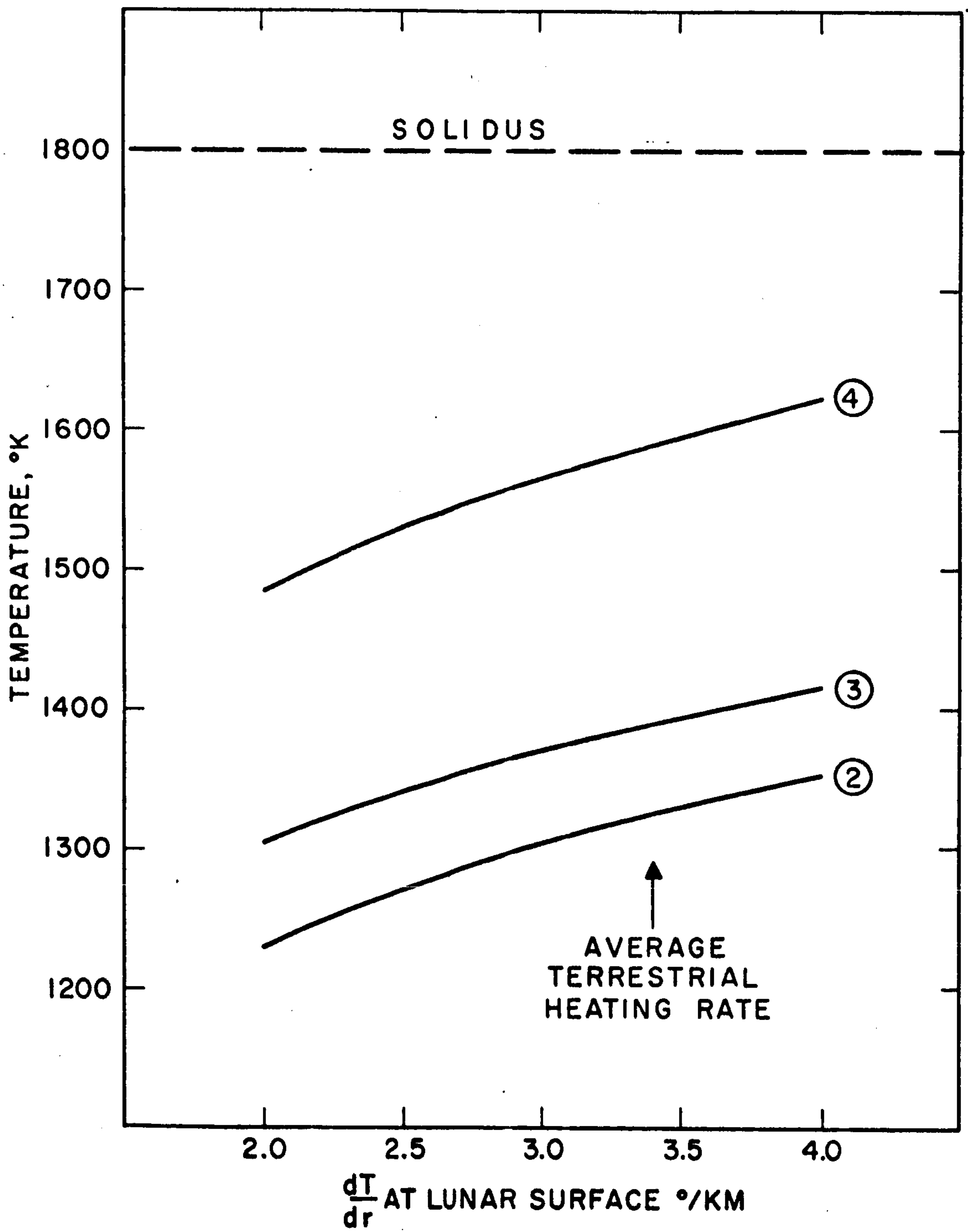
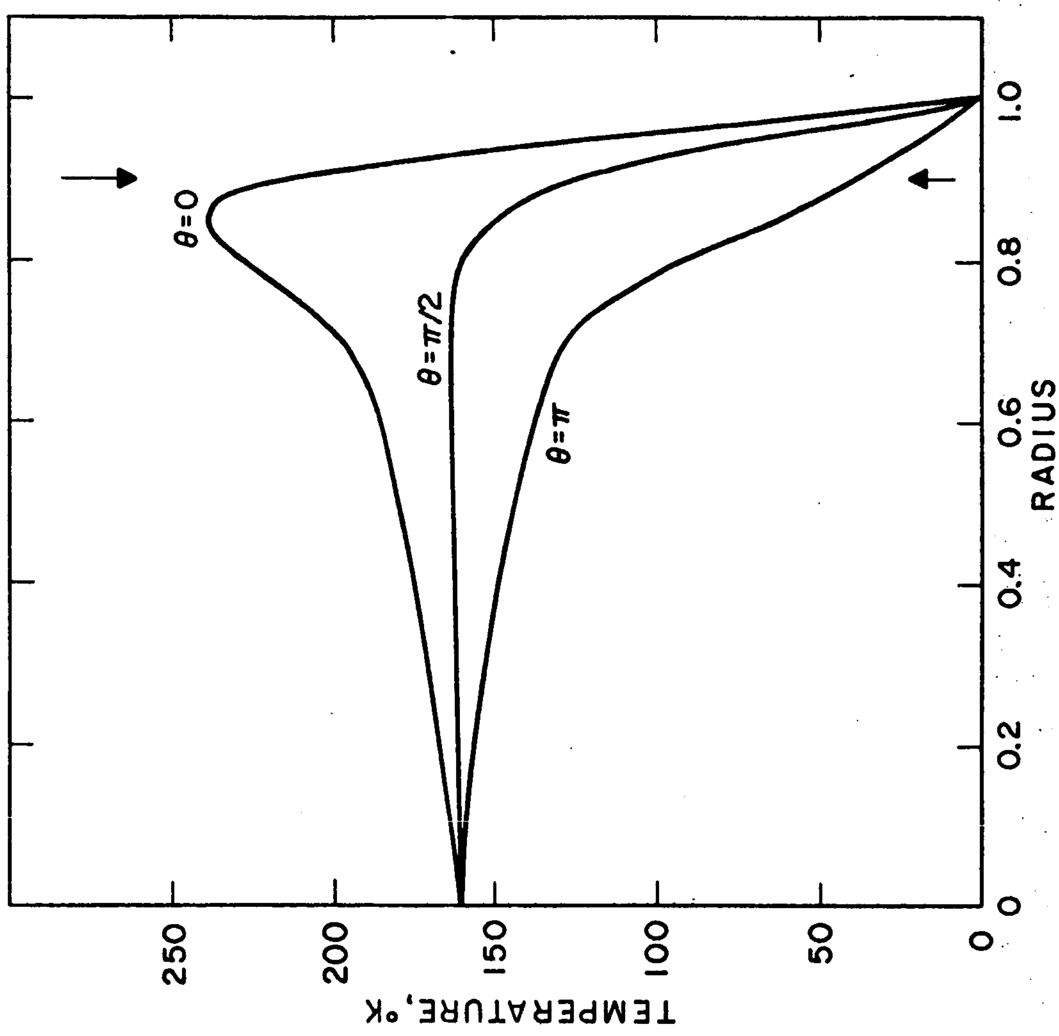

SOLIDUS
1800
1700
1600
1500
1400
1300
1200
TEMPERATURE, °K
4
3
2
AVERAGE
TERRESTRIAL
HEATING RATE
2.0
2.5
3.0
3.5
4.0
dT/dr AT LUNAR SURFACE °/KM

Fig. 4


TEMPERATURE, °K
250
200
150
100
50
0
RADIUS
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0.2
0.4
0.6
0.8
1.0
θ=0
θ=π/2
θ=π

Fig. 5

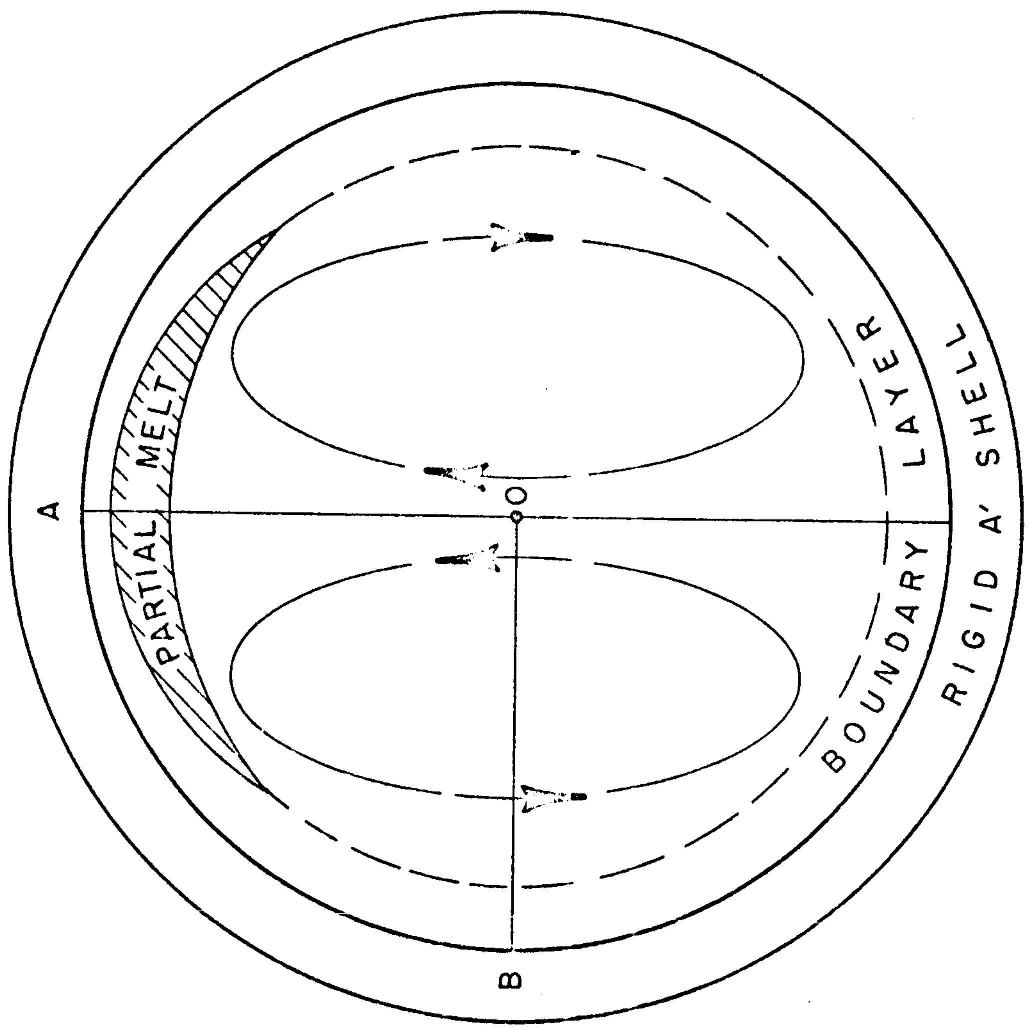

Fig. 6

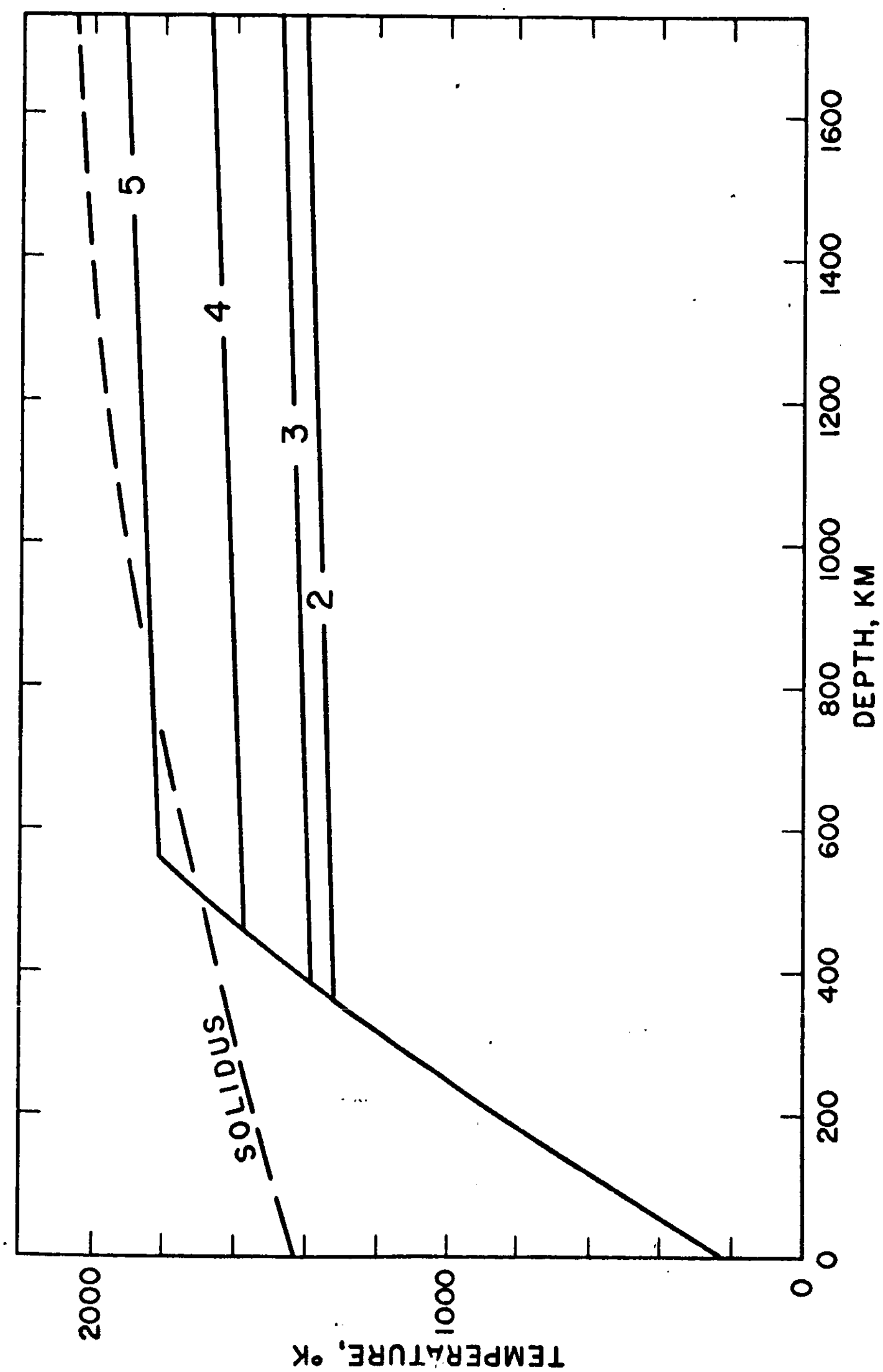
SOLIDUS
5
4
3
2
2000
1000
0
TEMPERATURE, °K
0
200
400
600
800
1000
1200
1400
1600
DEPTH, KM

Fig. 7

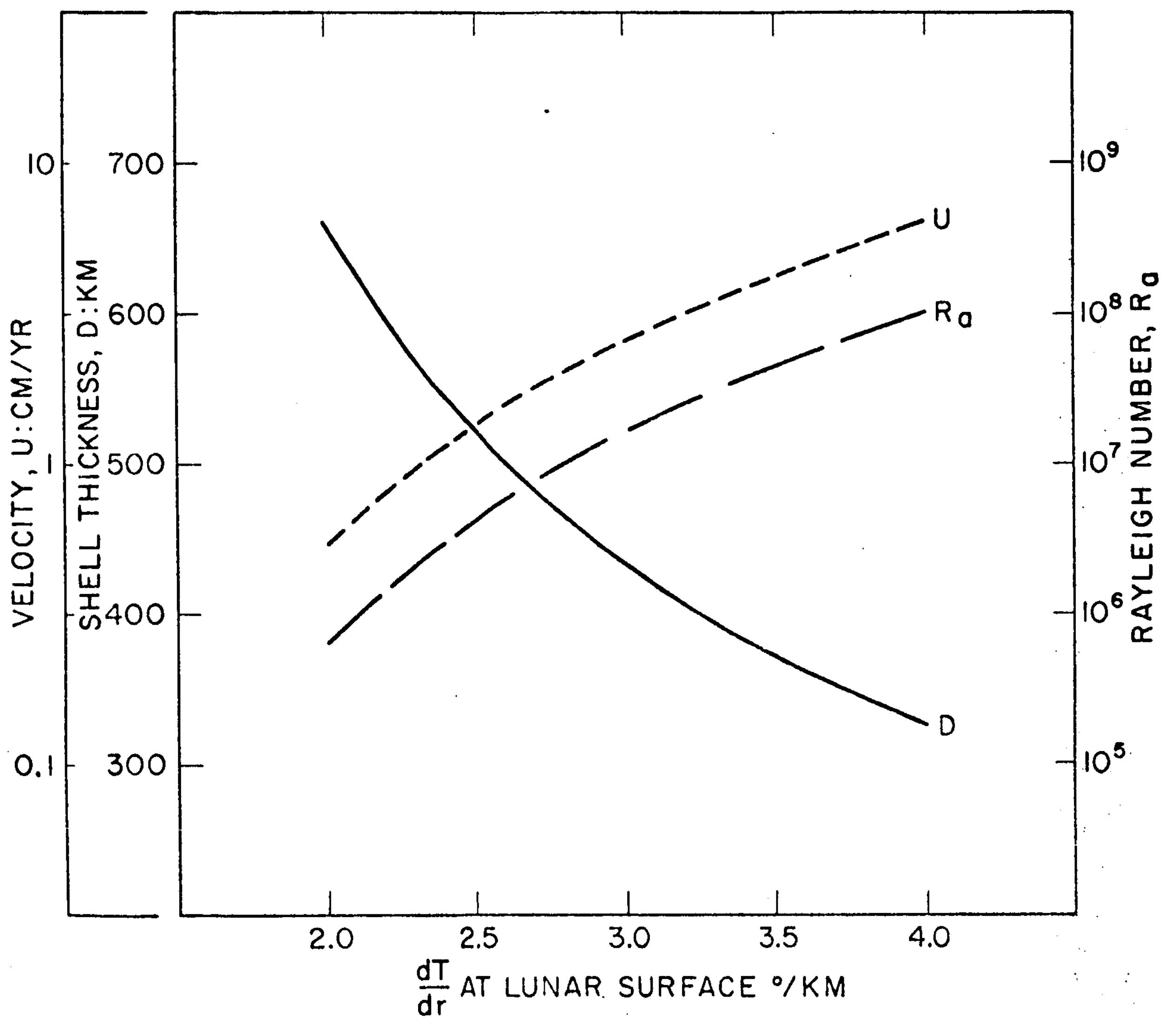
10
700
600
1
500
400
0.1
300
VELOCITY, U:CM/YR
SHELL THICKNESS, D:KM
U
Ra
D
10^9
10^8
10^7
10^6
10^5
RAYLEIGH NUMBER, Ra
2.0
2.5
3.0
3.5
4.0
dT/dr AT LUNAR SURFACE °/KM

Fig. 8