# EXPERIMENTAL $\mathbb{N V E S T I G A T I O N ~ O F ~ P L A N A R ~}$ MOTIONS OF A HUMAN BEING UNDER THE ACTION OF A BODY-FIXED THRUST <br> by <br> J. D. Yatteau and T. R. Kane 

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## January 1971

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Department of Applied Mechanics STANFORD UNIVERSITY

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EXPERIMENTAL INVESTIGATION OF PLANAR

MOTIONS OF A HUMAN BEING UNDER

THE ACTION OF A BODY-FIXED THRUST

## By

J. D. Yatteau and T. R. Kane

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#### Abstract

An experimental investigation of planar motions of a human being subjected to the action of a body-fixed force is described. This work involved use of the Space Operations Simulator of the Martin Marietta Corporation in Denver, Colorado.

The results of the study show that man can perform well controlled planar motions when acted upon by a body-fixed thrust.


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1. Introduction

Manned space exploration requires the ability of astronauts to perform well-controlled rotational and translational motions in space. A number of schemes enabling astronauts to perform such motions have been proposed [1]. The work described in this report deals with one that is simpler than any considered previously, namely the use of a single thruster rigidly mounted on the astronaut's torso, control to be maintained by moving limbs in such a way as to cause the line of action of the thrust vector to be placed suitably. The question to be answered is this: To what extent can motions be controlled by these means?

To find a partial answer to this question, an analytical study of planar motions of a system comprised of two rigid bodies was undertaken [2]. The results of this study revealed that it would be possible to achieve controlled planar motions if an astronaut could perform certain arm motions. Before extending the analytical investigation to threedimensions, it was decided to perform experiments with human subjects. Such experiments, carried out during May and June of 1970 at a facility of the Martin Marietta Corporation in Denver, Coilorado, are the subject of the present report.

### 1.1 Review of Analytical Study

To render the sequel as nearly as possible self-contained, the analysis described in [2] is reviewed briefly.

The system consists of two rigid bodies, $B$ and $B^{\prime}$ (see Fig. 1), connected by a hinge at a point $P$ which is located by a position vector


Fig. 1 Two Hinged Bodies

P relative to a point 0 that is fixed in an inertial reference frame R. The center of mass of $B$, designated $B *$, is located relative to $P$ by a vector $\underline{r}^{\prime}$ of magnitude $r^{\prime}$, The angle between $\underline{r}$ and $\underline{r}^{\prime}$ Is called $\varphi$.

Orthogonal unit vector $\underline{n}_{1}$ and $\underline{n}_{2}$ are fixed in $B$ parallel and perpendicular, respectively, to $\underline{I} ; \underline{N}_{1}, \underline{N}_{2}, \underline{N}_{3}$ are orthogonal unit vectors fixed in $R$; and the angle between $\underline{n}_{1}$ and $N_{1}$ is designated $\theta$.

Body $B$ has a mass $m$ and a moment of inertia $I$ about a line passing through $B^{*}$ and parallel to $\underline{N}_{3}$. Similarly, $B^{\prime}$ has a mass $m^{\prime}$ and a moment of inertia $I^{\prime}$ about a line passing through $B^{\prime \prime} *$ and parallel to $\mathrm{N}_{3}$ 。

A force $E$ is applied to $B$ at a point $S$ which is located relative to $B^{*}$ by a vector $S$.

The following scalar quantities are used in the analysis of the system:

$$
\begin{align*}
& x_{i}=\underline{p} \cdot \underline{N}_{i}, \quad i=1,2  \tag{1,1}\\
& s_{i}=\underline{s} \cdot \underline{n}_{i}, \quad i=1,2  \tag{1,2}\\
& \mathrm{~F}_{\mathrm{i}}=\underline{F} \cdot \underline{n}_{\mathrm{i}} \quad, \quad i=1,2 \tag{1,3}
\end{align*}
$$

Note that the definition of $x_{i}$ involves the inertially fixed unit vector $N_{i}$, whereas $s_{i}$ and $F_{i}$ depend on the vector $\underline{n}_{i}$ fixed in $B$. The Assumption that $s_{i}$ and $F_{i}$ are constants thus implies that both $\underline{s}$ and F are fixed in magnitude and direction relative to $B$.

A dimensionless form of the equation of motion for the system is obtained by introducing the following quantities:

$$
\begin{equation*}
\omega=(g / r)^{1 / 2} \tag{1.4}
\end{equation*}
$$

where $g$ is the acceleration of gravity:

$$
\begin{align*}
& \tau=\omega t  \tag{1.5}\\
& \beta_{1}=r / r^{\prime}, \quad \beta_{2}=m / m^{\prime}  \tag{1.6}\\
& \beta_{3}=s_{1} / r, \quad \beta_{4}=s_{2} / r  \tag{1,7}\\
& \beta_{5}=\frac{I^{\prime}+m^{\prime} r^{\prime} 2}{m^{\prime} r^{\prime 2}}, \quad \beta_{6}=\frac{I+m^{2}}{m^{2}}  \tag{1.8}\\
& a_{1}=\frac{F_{1}}{\left(m+m^{\prime}\right) g}, a_{2}=\frac{F_{2}}{\left(m+m^{\prime}\right) g}  \tag{1.9}\\
& a_{3}=\beta_{1}\left\{\left[1+\left(1+\beta_{2}\right) \beta_{3}\right] a_{2}-\left(1+\beta_{2}\right) \beta_{4} a_{1}\right\}  \tag{1.10}\\
& a_{4}=\left[\left(1+\beta_{2}\right) \beta_{5}-1\right] / \beta_{1} \beta_{2}  \tag{1.11}\\
& a_{5}=a_{4}+\beta_{1}\left[\beta_{6}\left(1+\beta_{2}\right)-\beta_{2}\right] \tag{1.12}
\end{align*}
$$

In terms of these quantities, the equations of motion are

$$
\begin{align*}
& \frac{d}{d \tau}\left[\theta^{\prime}\left(a_{5}-2 \cos \varphi\right)+\varphi^{\prime}\left(a_{4}-\cos \varphi\right)\right] \\
& =\frac{\beta_{2}+1}{\beta_{2}}\left[a_{1} \sin \varphi-a_{2} \cos \varphi+a_{3}\right] \tag{1.13}
\end{align*}
$$

$$
\begin{equation*}
\left(\frac{x_{1}}{r}\right)^{\prime \prime}=a_{1} \cos \theta-a_{2} \sin \theta-\frac{d^{2}}{d_{\tau}^{2}}\left[\frac{\beta_{2}}{1+\beta_{2}} \cos \theta+\frac{\cos (\theta+\varphi)}{\beta_{1}\left(1+\beta_{2}\right)}\right] \tag{1.14}
\end{equation*}
$$

$$
\begin{equation*}
\left(\frac{x_{2}}{x}\right)^{\prime \prime}=a_{1} \sin \theta+a_{2} \cos \theta-\frac{d^{2}}{d \tau^{2}}\left[\frac{\beta_{2}}{I+\beta_{2}} \sin \theta+\frac{\sin (\theta+\varphi)}{\beta_{1}\left(1+\beta_{2}\right)}\right] \tag{1.15}
\end{equation*}
$$

where primes denote differentiation with respect to $T$. Eq. (1.13) governs the rotational motion of the system, and Eqs. (1.14) and (1.15) yield the position of the hingepoint $P$. When Eqs. (1.13) - (1.15) are used to study motions of an astronaut, body $B$ is regarded as comprised of the head, torso and legs of a man in a position of "attention", and $B^{\prime}$ consists of the arms of the subject, these being required to move in unison in planes parallel to the pitch plane. The relevant inertia properties then have the values shown in Table 1.

For the evaluation of the dimensionless parameters in Eqs. (1.4) - (1.12) in connection with specific examples, $F_{1}$ is set equal to zero. This fixes the direction of $\underline{F}$ perpendicular to $\underline{I}$ (see Fig. 1), and it follows from Eqs. (1.9) and (1.10) that $\beta_{4}$ no longer appears in the equations and hence requires no further consideration. The remaining parameters can be expressed in terms of $S_{1}$ and $F_{2}$, and have the values shown in Table 2.

Table 1 Inertia Properties of the Human Body

| symbol | value | units |
| :---: | :---: | :---: |
| $B^{\prime}$ (Arms) |  |  |
| $\mathrm{m}^{\text {8 }}$ | 0.576 | slugs |
| $r^{\prime}$ | 0.903 | ft. |
| $I^{\prime}$ | 0.265 | slug-ft. ${ }^{2}$ |
| $B$ (Head, Torso and Legs in position of Attention) |  |  |
| m | 4.458 | slugs |
| r | 1.481 | ft. |
| I | 8.150 | slug-ft. ${ }^{2}$ |

Table 2 Values of System Parameters

| symbol | $\beta_{1}$ | $\beta_{2}$ | $\beta_{3}$ | $\beta_{5}$ | $\beta_{6}$ | $\omega$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| value | 1.640 | 7.740 | $.675 s_{1}$ | 1.564 | 1.834 | 4.661 <br> $\left(\mathrm{sec}^{-1}\right)$ |
| symbol | $a_{1}$ | $a_{2}$ | $a_{3}$ | $a_{4}$ | $a_{5}$ |  |
| value | 0 | $6.18 \times 10^{-3} \mathrm{~F}_{2}$ | $\left(1.014 \times 10^{-2}+\right.$ <br> $\left.5.979 \times 10^{-2} s_{1}\right) \mathrm{F}_{2}$ | .998 | 14.592 |  |

When the values in Table 2 are substituted into Eqs. (1.13)-(1.15), one obtains

$$
\begin{equation*}
\frac{d}{d \tau}\left[\theta^{-}(14.592-2 \cos \varphi)+\varphi^{\prime}(.998-\cos \varphi)\right]=\left(1.145+6.751 s_{1}-.698 \cos \varphi\right) \frac{\mathrm{F} 2}{100} \tag{1.16}
\end{equation*}
$$

$$
\begin{equation*}
\left(\frac{x_{1}}{r}\right)^{\prime \prime}=-6.18 \times 10^{-3} \mathrm{~F}_{2} \sin \theta-\frac{\mathrm{d}^{2}}{\mathrm{~d} \mathrm{~T}^{2}}[.886 \cos \theta+.0697 \cos (\theta+\varphi)] \tag{1.17}
\end{equation*}
$$

$$
\begin{equation*}
\left(\frac{x_{2}}{r}\right)^{\prime \prime}=6.18 \times 10^{-3} F_{2} \cos \theta-\frac{d^{2}}{d T^{2}}[.886 \sin \theta+.0697 \sin (\theta+\varphi)] \tag{1.18}
\end{equation*}
$$

It can be seen that numerical values must be assigned to $s_{1}$ and $F_{2}$ and that $\varphi$ must be specified as a function of time before numerical results can be obtained. The following expressions for $\varphi$ are considered:

$$
\begin{align*}
& \varphi=\varphi_{e}+\delta, \text { a constant }  \tag{1.19}\\
& \varphi=\varphi_{e}+\delta \cos N \tau  \tag{1.20}\\
& \varphi=\varphi_{e}+c \theta  \tag{1.21}\\
& \varphi=\varphi_{e}+c \theta+d \theta^{\prime} \tag{1.22}
\end{align*}
$$

where $\varphi_{e}, \delta, c$ and $d$ are constant. The quantity $\varphi_{e}$, called an
"equilibrium value" of $\varphi$, is defined to be a value of $\varphi$ for which the line of action of the thrust passes through the center of mass of the system. It follows that, when $\varphi \equiv \varphi_{e}$, the system moves without rotation and the center of mass moves on a straight line.

By proper choice of $s_{1}$, any desired value of $\varphi$ can be made an equilibrium value. Two values of $s_{1}$ are used. One of these results in $\varphi_{e}=\pi / 2$, and the other corresponds to $\varphi_{e}=0$.

In Eqs. (1.19) and (1.20), $\delta$ represents an "initial misalignment" of the arms from their equilibrium position. The value 0.1 rad . is assigned to this quantity.

The symbol $N$ in Eq. (1.20) is a measure of the frequency with which the subject moves his arms. $N$ is chosen so as to make the frequency equal to one cycle per second.

Substituting from Eq. (1.19) in Eqs. (1.16) - (1.18), and using the values in Table 2, one obtains the equations of motion of a rigid human body. Setting $\varphi_{e}=\pi / 2$ implies holding the arms in front of the torso at an angle of $\pi / 2+0.1$ rad. to the legs. This configuration is referred to as "Rigid Body - Arms Up". When $\varphi_{e}=0$, the arms are held in front of the torso at an angle of 0.1 rad . to the legs, and this configuration is called "Rigid Body - Arms Down".

When $\varphi$ in Eqs. (1.16) - (1.18) is replaced in accordance with Eq. (1.20), the resulting equations describe the motion of a man moving his arms in an oscillatory manner about an equilibrium position. With $\varphi_{e}=\pi / 2$, such behavior is referred to as "Arms Oscillating - Arms Up", and for $\varphi_{e}=0$, it is called "Arms Oscillating - Arms Down".

For each of the four configurations just described, two values of $F_{2}$ are used: $F_{2}=\left(m+m^{\prime}\right) g$, which means that the thrust has a magnitude equal to the weight of the subject; and $F_{2}=m+m^{\prime}$ (lbs), which corresponds to a thrust that imparts an acceleration of one foot per second, per second to the mass center of a rigid body of mass $m+m^{\prime}$. These two cases are referred to as "high thrust" and "low thrust", respectively. In all four cases, zero initial values are assigned to the dependent variables and to their first derivatives.

The four configurations described above can be collectively referred to as involving "open loop" behavior, because of the absence of feedback. However, to avoid confusion in what follows, if Eq. (1.19) is used for $\varphi$, the resulting motion is referred to as "rigid body" behavior; and, when $\varphi$ is given by Eq. (1.20), the resulting motion will be called "open loop" behavior. By contrast, Eqs. (1.21) and (1.22) do require feedback; and when these equations are used for $\varphi$, the resulting motion is called "closed loop" behavior.

## "Rigid Body" Behavior

As mentioned earlier, when Eq. (1.19) is substituted into Eqs. (1.16) (1.18), one is faced with solving the differential equations for a rigid body subjected to a misaligned force of constant magnitude. The general solution of the equations can be obtained in terms of Fresnel integrals, and leads to the following conclusion: A rigid body subjected to the action of a thrust of constant magnitude and not passing through the center of mass rotates with an increasing angular velocity. The speed of the mass center approaches a constant value, and the mass center approaches a straight line
oriented at forty-five degrees to the initial direction of the thrust vector. In terms of the human model under consideration, this can be interpreted as follows: Suppose that a man wishes to move on a straight line between two points, using a thruster placed either so that the line of action of the thrust passes through the center of mass when the arms are held at $90^{\circ}$ to the legs $\left(\varphi_{e}=\pi / 2\right)$, or so that this occurs when the arms are held at the sides $\left(\varphi_{e}=0\right)$. In both cases, the thrust vector is perpendicular to the spine, and is directed from back to front, so that the man must position himself with his spine perpendicular to the intended flight path. He then brings his arms as nearly as possible into the desired equilibrium position, and, as his arms are inevitably slightly misaligned from this position, he begins to pitch backward or forward with an increasing angular velocity while his center of mass begins to approach a straight line inclined at forty-five degrees to the intended flight path. The motion turns out to be essentially the same for both equilibrium positions and for both high and low thrusts, the principal difference being in the location of the line approached by the mass center.

## "Open Loop" Behavior

Substitution from Eq. (1.20) into Eqs. (1.16) - (1.18) leads to a set of linear differential equations with variable coefficients. An approximate solution of these equations, which agree well with results obtained by numerical integration, predicts that, regardless of the value of $\varphi_{e}$, the subject can reduce the amount of rotation by, at least, a factor of two if, instead of keeping his arms in a fixed posttion, he simply performs small amplitude oscillatory motions of arbitrary frequency about the equilibrium position.

Furthermore, it appears that, once the direction of the thrust has been selected (i,e., once $F_{1}$ and $F_{2}$ have been specified), there exists a special point of application of thrust (i.e., special values of $s_{1}$ and $s_{2}$ or, equivalently, a special value of $\varphi_{e}$ ) for which the undesirable rotation disappears almost completely. In the present situation, this value of $\varphi_{e}$ is $\varphi_{e}=\pi / 2$. Thus one is led to the following conclusion: As a small initial arm misalignment from an equilibrium position is unavoidable, it is best to design the system so that the equilibrium position is $\varphi_{e}=\pi / 2$ (Arms Up). When this is done, the amount of rotation (and, therefore, the amount of divergence from the intended flight path) can be kept small by performing oscillatory arm motions.
"C1osed Loop" Behavior
Even better results can be obtained if a man is capable of performing arm motions more complicated than a harmonic oscillation, namely, motions described by Eqs. (1.21) and (1.22). When $\varphi$ as given in Eq. (1.21) is substituted into Eq. (1.16), it appears that there exist conditions under winich $\theta$ remains arbitrarily small. These conditions depend upon several system parameters and are called "stability conditions". Similarly, use of Eq. (1.22) leads to stability conditions. In some cases, there exist values of $c$ and $d$ for which the pitch motion is damped out completely.

## 1. 2 Need for Experimental Verification

To arrive at the foregoing conclusions, it was necessary to make a number of assumptions about human behavior and capabilities. For example, an angular arm misalignment of 0.1 rad. was assumed to be the cause of thrust misalignment. Further, in dealing with open loop behavior, it was
assumed that a man could perform essentially harmonic oscillations about an equilibrium position, and the analysis concerned with closed loop behavior involved the assumption that the subject could monitor his pitch angle and pitch rate and then move his arms so as to make $\varphi$ a linear combination of these variables. The validity of all of these assumptions can be ascertained only experimentally. In addition, experiments can furnish the answers to the following important questions: If a subject does not know where the equilibrium position of the arms is, can he discover it before losing attitude control? Is there an upper limit on the thrust for which a man can maintain attitude control? Is is easier to maintain control with some thrust levels than with others? To answer these questions, the Space Operations Simulator of the Denver Division of the Martin Marietta Corporation was used in such a way as to replace the assumed human behavior reflected by Eqs. (1.19) - (1.22) with actual human performance.

The Space Operations Simulator is described in Sec. 2. Sec. 3 deals With the test program and procedures. Detailed descriptions of experiments actually performed appear in Sec. 4, and a discussion of results is given in Sec. 5. Finally, conclusions and recommendations are contained in Sec. 6.

The Martin Marietta Space Operations Simulator, designed for rendezvous-docking studies, was later modified for EVA/IVA simulations. In its present configuration the simulator accomodates a test subject in a gimbaled head which is attached to a moving carriage (see Figs. 2 and 3). The motions of the carriage and head are controlled by an analog computer which is programmed to solve the equations of motion of the system under study. The present section of this report contains a detailed description of the simulator and its operation.
2.1 Moving Carriage and Gimbaled Head

The moving carriage and gimbaled head (see Fig. 3) are housed in a 90 by 32 by 24 foot room. This structure is capable of imparting six-degreeof-freedom motion to a subject mounted in the gimbaled head.

Three translational degrees of freedom are provided as follows: the base of the carriage moves along the length of the room on three rails and is driven by four, one horsepower, $A C$ motors which engage two gear tracks mounted on the floor. The vertical pedestal translates laterally relative to the base on rollers and rails and is driven by two, one horsepower, $A C$ servo-motors. The gimbaled head is located on the front face of the vertical pedestal and is supported by a set of negator springs which effectively counterbalance the combined weights of the gimbaled head and test subject. Two one-quarter horsepower DC motors, which engage two vertical gear tracks on the front of the pedesta1, provide power to move the gimbaled head vertically relative to the vertical pedestal.

Three rotational degrees of freedom (yaw, pitch, roll) are associated


Fig. 2 Martin Marietta Space Operations Simulator


Fig. 3 Moving Carriage and Gimbaled Head
with relative angular displacements of parts of the gimbaled head. Motors and gear drives are enclosed in the gimbal structure. Each gimbal is driven by a one-quarter horsepower DC motor.

The performance characteristics of the simulator are given in Table 3. The travel limits shown are the so-called "electrical limits" and were used for this experiment. The actual mechanical limits are somewhat 1arger.

|  | Longitudinal | Lateral | Vertical |
| :--- | :---: | :---: | :---: |
| Travel (ft.) | $\pm 25.0$ | $\pm 6.0$ | $\pm 6.0$ |
| Velocity (ft./sec.) | $\pm 3.0$ | $\pm 3.0$ | $\pm 3.0$ |
| Accel. (ft. $/ \mathrm{sec.}^{2}$ ) | $\pm 6.0$ | $\pm 6.0$ | $\pm 3.0$ |
|  |  |  |  |
|  | Ro11 | Pitch | Yaw |
| Travel (rad.) | $\pm 3.2$ | $\pm 1.0$ | $\pm 3.3$ |
| Velocity (rad. $/ \mathrm{sec}$. ) | $\pm 6.0$ | $\pm 2.0$ | $\pm 2.0$ |
| Accel. (rad./sec. ${ }^{2}$ ) | $\pm 8.0$ | $\pm 8.0$ | $\pm 8.0$ |

Table 3 Performance Characteristics of Simulator

The simulator responds to position, rate and acceleration commands (below the limits specified in Table 3) with approximately 95\% accuracy on or about all axes. The simulator response lags the actual command by .05 sec . for the yaw axis and .16 sec . for the longitudinal axis. The lag times for the remaining axes have values between these two limits.

Because only three degrees of freedom were required for the simulation,
the lateral, yaw, and roll servo drives were not used. Thus the base was free to move along the rails on the floor (see Fig. 3), the gimbaled head could move up and down on the vertical pedestal, and the pitch gimbal was free to rotate relative to the rest of the gimbaled head. In Fig. 4 the pitch gimbal is shown in a rotated state.

### 2.2 Arm Angle Sensor

In order to study actual human performance, it is necessary to monitor the arm angle $\varphi$ and its first derivative. To accomplish this, an arm angle sensor consisting of a single turn precision potentiometer with an extended actuator arm (see Fig. 5) was fabricated from aluminum at the Martin Marietta facility.

The arm angle sensor is attached to the support frame of the gimbaled head in such a way as to permit variation of the hingepoint location. Further, the length of the actuator arm can be adjusted to fit an individual subject's arm length. The combination of the two adjustments permits the subject to manipulate the actuator with his arms fully extended and with minimal resistance. The cross piece of the actuator enables the subject to move both arms in unison, parallel to the pitch plane. The potentiometer is excited by $\pm 10$ volts, and the output is scaled to fit the equations of motion at the computer. From a computational viewpoint (i.e., when one considers the lower threshold voltage of the computer), the potentiometer is capable of registering an angular increment of 0.2 degree.

The determination of the rate of change of the arm angle can be made once the variation in the angle itself is known. The necessary computation is discussed in Appendix A.


Fig. 4 Rotation of Pitch Gimbal


Fig. 5 Arm Angle Sensor

### 2.3 Thrust Simulation

A simulated thruster control (see Fig. 5) consisted of a threeposition switch located on the cross piece of the arm angle sensor. This location permitted the subject to activate the thruster without degradation in the arm angle. The three positions of the switch sent inputs of $+100,0$, and -100 volts, corresponsing to positive, zero, and negative thrust, respectively, to the computer. The actual magnitude of the thrust corresponding to 100 volts depended upon the scale factors in the computer.

### 2.4 Computation and Data Acquisition

As mentioned earlier, the motions of the carriage and gimbaled head are controlled by an analog computer (see Fig. 6) programmed to integrate the equations of motion of the system under study. In the present case, these are Eqs. (1.16) - (1.18). However, before these equations can be programmed, they must be expressed in terms of variables which are compatible with command voltages accepted by the simulator's servo drives, and with real time as the independent variable. The necessary transformation (see Appendix A) leads to the following form of the equations of motion:

$$
\begin{gather*}
\frac{d}{d t}\left[\dot{\theta}_{c}(14.592-2 \cos \varphi)+\dot{\varphi}(.998-\cos \varphi)\right] \\
=\left[.248+1.466 s_{1}-.151 \cos \varphi\right] F_{2}  \tag{2.1}\\
\ddot{x}_{c}=.199 \mathrm{~F}_{2} \cos \theta_{c}+\frac{d^{2}}{d t^{2}}\left[.169 \sin \theta_{c}-.103 \sin \left(\theta_{c}+\varphi\right)\right]  \tag{2.2}\\
\ddot{z}_{c}=-.199 \mathrm{~F}_{2} \sin \theta_{c}+\frac{d^{2}}{d t^{2}}\left[.169 \cos \theta_{c}-.103 \cos \left(\theta_{c}+\varphi\right)\right] \tag{2.3}
\end{gather*}
$$



Fig. 6 Analog Computer and Recording Equipment
where dots denote differentiation with respect to real time $t ; x_{c}$ and $z_{c}$ are respectively the horizontal and vertical coordinates of the center of the pitch gimbal; and $\theta_{c}$ represents the rotation of this gimbal (see Fig. A1, Appendix A). Specification of $s_{1}$ and $F_{2}$ together with the initial values of $\theta_{c}, x_{c}, z_{c}$ and their first derivatives completes the computational requirements for a simulation.

The flow of information governing a simulation can be described as follows (see Fig. 7): The computer integrates the equations of motion while receiving signals for thrust and arm angle from the thrust switch and from the arm angle sensor. The computed values of position and orientation are sent to the servo motors of the simulator, and these drive the carriage and gimbaled head to the proper position and orientation. The command voltages are also sent to recording devices, where they are plotted with appropriate scale factors.

Three $X-Y$ plotters and one strip chart recorder were used to record data during the simulations (see Fig. 6). The $X$ X-Y plotters furnished graphs of $\theta_{c}$ vs. $t, \theta_{c}$ vs. $x_{c}$, and $z_{c}$ vs. $x_{c}$. The strip chart recorder was used to plot the following quantities versus time: $\theta_{c}, \dot{\theta}_{c}, x_{c}, z_{c}, \dot{\varphi}, \varphi_{m}, \varphi_{c}$, and $F_{2}$, where $\varphi_{m}$ is the value of $\varphi$ from the arm angle sensor and $\varphi_{c}$ is the value of $\varphi$ obtained by integrating $\varphi$ as obtained from the differentiation circuit (see Figs. A2 and A3, Appendix A).

In addition to the above data, motion pictures and multiple exposure still photographs were obtained for a number of tests.


Fig. 7 Information Flow Diagram

## Test Program

From the description of the test facility in the preceding section it can be seen that a simulation involves the interaction of many pieces of hardware, some of which were fabricated specifically for the present investigation. To insure proper integration of all components, three weeks were allotted to modifying and testing the simulator (including the computer program), and the actual simulation was scheduled for the fourth week.

### 3.1 Test P1an

Theoretically it is possible for a man to overcome the adverse effects of a thrust misalignment by performing the arm motions discussed under the heading of "open loop" behavior in Sec. 1.1. However, as pointed out in Sec. 1.2, even this apparently easy-to-perform maneuver involves some assumptions requiring experimental verification. Hence, it was decided that the first question to be investigated would be this: Is it possible for a man to overcome the adverse effects of a thrust misalignment by means of "open loop" behavior, that is, by using specified oscillatory arm motions? Next, the subject would be instructed to use visual feedback, that is, he would progress to "closed loop" be" havior. The nature of succeeding tests was to depend on the outcome of these first two. For example, if the data indicated that the subject had difficulty performing the "open loop" maneuver, or if he had been unable to control his pitch motion during the "closed loop" phase, the next series of tests would consist of an attempt to overcome these difficulties through training. The type of training program would depend on the nature $\therefore$ : of the difficulties encountered. For example, the translational motion
could be eliminated from the simulation, thus permitting the subject to concentrate on attitude control alone and, if training failed to improve performance, the remainder of the simulation would be devoted to establishing the nature of the difficulties encountered.

If the subject experienced no serious difficulties during the first phase of the simulation, or if difficulties were overcome by training, it was planned to conduct a series of tests aimed at discovering the limits of man's ability to accomplish various tasks. The following were typical questions to be answered: What are the effects of thrust reversal? For a particular initial thrust misalignment, what is the maximum thrust with which a man can maintain attitude control? Can attitude control be maintained when the direction of the thrust is reversed?

It was to be expected that a time would come at which it would no longer be possible to obtain information by a systematic approach. It was decided to abandon systematic testing in favor of improvisational experiments at that point. Such tasks as approaching a target or retrieving an object, could be studied during this final phase of the simulation.

Fig. 8 shows the test plan in schematic form.

### 3.2 Test Procedure

From the description of the test facility in Sec. 2, it can be seen that a simulation involves the concerted action of a number of men and several pieces of hardware. Specifically, the presence of six men stationed in three rooms is required. The test director and computer operator are stationed in the computer room (see Figs. 2 and 6). Upon receiving


Fig. 8 Simulation Test Plan
the necessary information from the test director, the computer operator readies the computer and recording devices. The test director is responsible for data recording and identification, and he supervises the description of each test in a test log. Furthermore, the test director monitors incoming data, issues instructions to the test subject, and decides when to terminate a particular test run. (A voice communication network permits constant voice contact between all test personnel.)

The output of the computer must be converted into voltages accepted by the servo motors. The necessary conversion equipment is housed in the control room (see Fig. 2), in which two men are stationed, one to monitor the conversion hardware, while the other observes the progress of the moving carriage via closed circuit television. The test observer possesses the capability to cut all power to the moving carriage in the event of an emergency. Finally, the subject and test coordinator are stationed in the test chamber adjacent to the control room (see Fig. 2). A11 other test personnel (including the subject) report to the test coordinator when preparations for a test run have been completed. The coordinator then assumes remote control of the computer and begins the experiment.

A typical experiment proceeds as follows: Having reviewed the data from previous tests and decided which series of experiments to conduct next, the test director provides the computer operator with new parameter values and initial conditions. While the computer operator makes the necessary adjustments, the test director labels strip charts and graph paper in the $X-Y$ plotters and enters a description of the upcoming run in the test log. When preparations have been completed in
the computer room, the new initial conditions are sent to the control room. While the carriage is being driven to the new starting position, the test director issues instructions to the test coordinator and to the subject. Fianlly, the test coordinator assumes control of the computer and, after a final check with all test personnel, places the computer in the operational mode and begins the test. After the test run has been terminated, the test coordinator returns control to the computer operator and preparations begin for the next test. At this time, the test subject relays subjective comments to the test director, who subsequently records them in the test $\log$.

In order to maintain confidence in the data being accumulated, it is necessary to periodically check the accuracy with which the computer integrates the equations of motion. This is accomplished by comparing the output of the computer with results obtained analytically for two special cases. Such checks were performed twice daily. The check cases are described in detail in Sec. A. 4 of Appendix A.

After approximately three weeks of preparation, experiments were conducted for five days. This work was carried out in three phases and involved two test subjects in a total of 103 tests.

To supplement the results of the Martin simulation, an experiment dealing with static arm misalignments was conducted at Stanford University. In the present section, experiments are described in chronological order. A summary of all tests appears at the end of the section, and detailed discussion of test results is presented in Sec. 5.

### 4.1 Preliminary Tests

Three objectives were to be accomplished during the preliminary tests. The first was to obtain a measure of a man's ability to perform "open loop" behavior and thus to establish a lower bound on man's ability to overcome the adverse effects of a thrust misalignment. The second abjective was to determine whether a man, without the benefit of training or experience, could improve upon "open loop" results by using visual feedback. The third objective was to obtain a motion picture comparison of "open loop", closed loop", and "rigid body" behavior.

It is possible, by using the analysis presented in Sec. 1.1, to estimate the difference in the pitch motions resulting from "rigid body" and "open loop" behavior. When this is done, one finds that, for the assumed static arm misalignment ( $\delta=.1$ rad, $)$, the "net" amount of pitch motion resulting from "rigid body" and "open loop" behavior does not differ appreciably in the short travel distance available (approx. $50 \mathrm{ft}$. ) and, therefore, cannot be detected in the motion picture comparison of the two cases. To overcome this difficulty, it was decided to use an
exaggerated initial arm misalignment (i.e.s take $\delta>.1$ rad.). With such values of $\delta$, the difference between the "rigid body" and "open loop" results at the end of the available travel length are not only apparent in a motion picture, but the test data remain valid in analyzing man's ability to perform oscillatory arm motions about a specified mean position. Thus, the first tests of the simulation were conducted as follows: After setting $\varphi_{e}$ equal to $66^{\circ}$, the subject was instructed to position his arms at $90^{\circ}$, turn on the thruster, and hold his arms stationary for the duration of the run ("rigid body" behavior). Next, the subject was blindfolded and instructed to position his arms at $90^{\circ}$. He was then told to perform oscillatory arm motions about the $66^{\circ}$ position, immediately after initiating thrust ("open loop" behavior). Finally, with the blindfold removed and the arms once again at the $90^{\circ}$ position, the subject was instructed to turn on the thruster and, once he detected some pitch motion, to begin arm motions which, in his judgment, would nullify the effects of the initial thrust misalignment ("closed loop" behavior). These three tests were then repeated using an equilibrium position of $156^{\circ}$. In all six cases, a thrust level of 0.1 lb . was used.

Preliminary review of the data indicated that a man could reduce the effects of a thrust misalignment appreciably by performing oscillatory arm motions. The "closed loop" results were inconclusive because pitch motion resulting from the subject's arm motion took place so slowly that only one correction could be made before the travel limit of the simulator was reached. In an attempt to obtain more conclusive "closed loop ${ }^{\text {it }}$ results, three additional experiments were conducted. In these,
the equilibrium position as well as the subject's initial arm position was $90^{\circ}$. The subject was instructed to initiate thrust and then deliberately induce a small amount of pitch motion by raising or lowering his arms. He would then attempt to reduce the pitch angle to zero with appropriate arm motions. In all three cases, the subject, preoccupied with keeping the vertical travel within the limits of the simulator, reached the pitch travel limits, which necessitated termination of the test.

The conclusion drawn from the last three tests was that the subject (without benefit of training or experience) did not have sufficient control of pitch motion to keep the translational motion within the performance limits of the simulator. For this reason, it was decided to omit translation from the next phase of the simulation; that is, to perform a series of tests involving pitch motion alone. It was hoped that this would enable the subject to become more effective in attitude control and, later, to use his improved ability to perform translational motions within the limits of the simulator.

### 4.2 Pitch Motion Tests

In addition to providing the subjects with experience, the experiments conducted during this phase of the simulation were intended to provide quantitative information about the subject's ability to control the pitch motion.

The first question to be answered was this: What is the maximum thrust level at which a man can maintain attitude control?

Two subjects participated in these tests. In each case, the subject was instructed to position his arms at the $90^{\circ}$ equilibrium position,
turn on the thruster, and, using visual feedback, keep the pitch angle, $\theta$, as small as possible. The thrust level would be increased for the next test and this would continue until a level was reached at which the subject could no longer keep $\theta$ small. Ten such tests were conducted, the thrust level varying from .1 to 30 lbs .

The next series of tests was similar to those just described, the difference being that, after demonstrating his ability to keep $\theta$ small, the subject was instructed to attempt to attain a particular pitch angle other than zero, hold it for a, specified time, and then attempt to attain a different pitch angle. This would continue until the subject's arms became tired, at which time the test would be terminated. Again, the thrust leve1 was increased between successive tests. Both subjects participated in a total of thirteen of these tests, thrust level ranging from .1 to 100 lbs.

Not only improper arm position, but also errors in thruster placement can cause thrust misalignments. In fact, a relatively small error in thruster placement tends to result in a relatively large change in the equilibrium configuration. Hence it is important to consider the following question: Assuming that there is an error in thruster location, can the subject discover the equilibrium position and stabilize the pitch motion before losing attitude control? In this context, loss of attitude control is said to have occurred when the pitch angle reaches the simulator performance limits ( $\pm 1 \mathrm{rad}$ ). To answer the question and to gain information about the subjects ${ }^{\wedge}$ ability to control the pitch angle when the equilibrium value of $\varphi$ differed from $90^{\circ}$, subjects were instructed to place their arms in the $90^{\circ}$ position, turn on the thruster,
and then attempt to locate the equilibrium position. (Prior to each run, an equilibrium value other than $90^{\circ}$ and not known to the subject was programmed in the computer.) If the subject was successful in this first task, he would be given instructions to attain a number of pitch angles, thus demonstrating his ability to control the pitch motion with $\varphi \neq 90^{\circ}$. Thrust levels ranged from .1 to 50 lbs ., and equilibrium positions of $45^{\circ}, 60^{\circ}, 80^{\circ}, 100^{\circ}, 120^{\circ}, 135^{\circ}$ were used.

The final series of tests in this phase of the simulation concerned the subjects' ability to control the pitch motion when the direction of the thrust was reversed. The subject would place his arms at the equilibrium position ( $90^{\circ}$ in all cases) and turn on the thruster. Then, as in the preceeding tests, the subject was instructed to maneuver into various attitudes by using arm motuons to control the torque.

### 4.3 Pitch and Translation

To gain some idea of the precision with which subjects could maneuver, a number of improvisational tests were conducted with a thrust level of 0.5 lb . and equilibrium position at $90^{\circ}$.

The first series of tests dealt with the subjects' ability to fly along a straight line, and, by reversing the thrust, come to rest at a particular point on the line. In one of these tests, the subject was given the task of retrieving an object from the far end of the room. Next, the ability to attain a desired vertical coordinate and thereby reach a target not on the original line of flight was tested by instructing subjects to hold their arms above the equilibrium position in order to develop a positive pitch angle, and an associated vertical component of the thrust, and then to attempt to develop a negative pitch
turn on the thruster, and, using visual feedback, keep the pitch angle, $\theta$, as small as possible. The thrust level would be increased for the next test and this would continue until a level was reached at which the subject could no longer keep $\theta$ small. Ten such tests were conducted, the thrust level varying from .1 to 30 1bs.

The next series of tests was similar to those just described, the difference being that, after demonstrating his ability to keep $\theta$ small, the subject was instructed to attempt to attain a particular pitch angle other than zero, hold it for aspecified time, and then attempt to attain a different pitch angle. This would continue until the subject's arms became tired, at which time the test would be terminated. Again, the thrust level was increased between successive tests. Both subjects participated in a total of thirteen of these tests, thrust level ranging from .1 to 100 lbs .

Not only improper arm position, but also errors in thruster placement can cause thrust misalignments. In fact, a relatively small error in thruster placement tends to result in a relatively large change in the equilibrium configuration. Hence it is important to consider the following question: Assuming that there is an error in thruster location, can the subject discover the equilibrium position and stabilize the pitch motion before losing attitude control? In this context, loss of attitude control is said to have occurred when the pitch angle reaches the simulator performance limits ( $\pm 1 \mathrm{rad}$ ). To answer the question and to gain information about the subjects' ability to control the pitch angle when the equilibrium value of $\varphi$ differed from $90^{\circ}$, subjects were instructed to place their arms in the $90^{\circ}$ position, turn on the thruster,
and then attempt to locate the equilibrium position. (Prior to each run, an equilibrium value other than $90^{\circ}$ and not known to the subject was programmed in the computer.) If the subject was successful in this first task, he would be given instructions to attain a number of pitch angles, thus demonstrating his ability to control the pitch motion with $\varphi \neq 90^{\circ}$. Thrust levels ranged from . 1 to 50 lbs., and equilibrium positions of $45^{\circ}, 60^{\circ}, 80^{\circ}, 100^{\circ}, 120^{\circ}, 135^{\circ}$ were used.

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angle, thus reversing the vertical component of the thrust and coming to rest somewhere above the initial vertical coordinate.

Finally, in order to arrive at a technique for precision maneuvering, some simulation time was allotted to training the subject in a maneuver that would alter the pitch angle without requiring thrust,

### 4.4 Static Arm Misalignment Tests

In order to establish the efficacy of "open loop" and "closed loop" behavior, one must know what motion results when a subject attempts to place his arms in the equilibrium position for the duration of the flight. As solutions for this "rigid body" behavior are available, it is possible to predict the resulting motion if the magnitude of the thrust misalignment is known. Since the thrust misalignment depends on the location of the arms relative to the equilibrium position, one is faced with the following question: When a man is performing "rigid body" behavior, what is the magnitude of the static arm misalignment from the equilibrium position? Prior to the simulation, the magnitude was assumed to be 0.1 rad. The data from the simulation furnished only limited information about static arm misalignments. To supplement these data, an experiment was conducted at Stanford University.

The device shown in Fig. 9 was used to measure arm angles during static arm misalignment tests. This instrument is essentially the same as the arm angle sensor described in Sec. 2.2 , except that arm angles are indicated by a protractor rather than by a potentiometer (see Fig. 9a). The subject stands adjacent to the vertical post, and the pivot point of the device is positioned opposite the subject's arm socket (see Fig. 9b). The length of the moving arm is then adjusted to the length of the subject's


Fig. 9a Pivot Point and Arm Angle Indicator


Fig. 9b Subject Responding to Arm Angle Command

Fig. 9 Arm Angle Measuring Device
arms. These adjustments result in the vertical post and moving arm being aligned with the subjects legs and arms, respectively. When this is the case, the protractor indicates the position of a subject's arms relative to his torso and legs.

Four subjects participated in these tests. Each was given a sequence of 28 position (arm angle) commands. In each case, the subject would respond by placing his arms in the position he identified with the arm angle command. Without informing the subject of his true response, the test monitor would record the angle indicated by the protractor, after which the subject would return his arms to his sides. The sequence of position commands involved seven different angles, each repeated four times. The order of the commands was arranged so that a particular angle would not occur twice in succession. Prior to the sequence of commands, each subject was shown the position of his arms that would result in a protractor reading of $90^{\circ}$. After acknowledging the $90^{\circ}$ reference position, the subject would drop his arms to his sides to await the first of the position commands.

Throughout the tests, the subjects were cautioned to avoid using the physical features of the room in responding to a particular command. In some cases, the arm angle device was rotated (so that the subject faced a new direction) prior to repeating the basic sequence of seven angles. As a further precaution, two of the subjects repeated the tests with their eyes closed.

All of the experiments described in this section are summarized in Table 4. The results of the tests are discussed in Sec. 5.

| Test <br> Sequence | Purpose | No. of <br> Subjects | Total No, <br> of Tests |
| :--- | :--- | :---: | :---: |
| Preliminary <br> Test <br> (see Sec. 4.1) | Comparison of "rigid body", <br> "open loop", and "closed <br> loop" behavior. | 1 | 20 |
| Pitch Motion <br> Only <br> (see Sec. 4.2) | Provide "closed loop" exper- <br> ience for subjects; determine <br> limits of subjects' pitch <br> control ability. | 2 | 55 |
| Pitch and <br> Translation <br> (see Sec. 4.3) | Determine ability to control <br> vertical translation; pre- <br> cision maneuvers using "self- <br> induced rotations". | 2 | 28 |
| Static Arm <br> Misalignment | Determine ability to stati- <br> (see Sec. 4.4) | equilibrium align arms with the | 4 |

Table 4 Summary of Experiments
*
Static arm misalignment tests were conducted at Stanford University.

## 5. Results

Although the static arm misalignment tests were performed last in chronological order it is convenient to discuss these tests first, for they have the most direct bearing on "rigid body" behavior (see Fig. 10), and this, in turn plays a role in the discussion of "open loop" and "closed 1oop" behavior.

### 5.1 Static Arm Misalignments

Four subjects participated in the static arm misalignment tests. Each subject made four attempts to position his arms at each of seven different angles. The same position commands were used for all the subjects. In what follows, each position command is considered as a particular value of the variable $\varphi_{c}$. Thus, corresponding to each value of $\varphi_{c}$ there are 16 responses, four from each subject. The $j^{\text {th }}$ response of the $i^{\text {th }}$ subject is called $\varphi_{i j}$. In order to facilitate discussion of the data, the following quantities are defined for each value of $\varphi_{c}$ :

$$
\begin{align*}
& \Delta_{i j}=\varphi_{i j}-\varphi_{c} \quad i, j=1,2,3,4  \tag{5.1}\\
& \bar{\Delta}_{i}=\frac{1}{4} \sum_{j=1}^{4}\left|\Delta_{i j}\right|  \tag{5,2}\\
& \bar{\Delta}_{i}=\frac{1}{4} \sum_{j=1}^{4} \Delta_{i j}  \tag{5.3}\\
& \bar{\varphi}_{i}=\frac{1}{4} \sum_{j=1}^{4} \varphi_{i j}  \tag{5.4}\\
& \delta_{i j}=\varphi_{i j} \bar{\varphi}_{i}  \tag{5.5}\\
& \bar{\delta}_{i}^{*}=\frac{1}{4} \sum_{j=1}^{4}\left|\delta_{i j}\right| \tag{5,6}
\end{align*}
$$



Fig. 10
Typical Motion Resulting from "Rigid Body" Behavior

From (5.1), it can be seen that $\Delta_{i j}$ is the misalignment occurring on the $j^{\text {th }}$ attempt of $i^{\text {th }}$ subject to position his arms at the angle $\varphi_{c}$. In (5.2), $\bar{\Delta}_{\mathcal{L}}^{\prime}$ is the average magnitude of the misalignments resulting from the $i^{\text {th }}$ subject's four attempts to position his arms at $\varphi_{c}$, and, from (5.3) it follows that $\bar{\triangle}_{i}$ is the algebraic average of the same four misalignments. From (5.4), $\bar{\varphi}_{i}$ is seen to be the average of the $i{ }^{\text {th }}$ subject's four responses to $\varphi_{c}$, and it is apparent from (5.5) that $\delta_{i j}$ is the misalignment, from $\bar{\varphi}_{i}$, occurring on the $i^{\text {th }}$ subject's $j^{\text {th }}$ response to $\varphi_{c}$. Finally, $\bar{\delta}_{i}^{\prime}$ (eq. $(5.6)$ ) is the average magnitude of the $\delta_{i j}$ corresponding to the $i^{\text {th }}$ subject's four responses to $\varphi_{c}$. It follows that a small value of $\bar{\delta}_{i}^{\prime}$ indicates that the $i^{\text {th }}$ subject's four responses to a particular position command are approximately the same, whereas a large value implies that these same four responses were not similiar. To avoid confusion, it may help to note that quantities represented by a " $\triangle$ " refer to misalignments from $\varphi_{c}$, and those involving a " $\delta$ " deal with misalignments from $\bar{\varphi}_{i}$. The test data, in terms of $\bar{\Delta}_{i}, \bar{\Delta}_{i}$, and $\bar{\delta}_{i}$, are shown in Table 5. From these data it can be seen that subject's responses to a particular position command are (with a few exceptions) approximately the same. It is, therefore, reasonable to speak of a "representative" behavior for the group, and, from this to form conclusions about "rigid body" behavior. The characteristic behavior of the group in specified by the following quantities:

$$
\begin{equation*}
\overline{\bar{\Delta}}^{\prime}=\frac{1}{4} \sum_{i=1}^{4} \bar{\Delta}_{i} \tag{5.7}
\end{equation*}
$$

| $\bar{\varphi}_{c}$ | $30^{\circ}$ | $45^{\circ}$ | $60^{\circ}$ | $90^{\circ}$ | $120^{\circ}$ | $135^{\circ}$ | $150^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bar{\Delta}_{1}$ | 17.8 | 10.0 | 8.3 | 1.3 | 2.5 | 3.3 | 14.8 |
| $\bar{\Delta}_{2}^{\prime}$ | 6.5 | 20.0 | 16.8 | 4.5 | 2.5 | 5.5 | 8.3 |
| $\bar{\Delta}_{3}^{*}$ | 15.8 | 17.8 | 11.0 | 1.3 | 2.3 | 2.3 | 4.8 |
| $\bar{\Delta}_{4}$ | 13.3 | 13.8 | 14.3 | 0.8 | 3.0 | 8.0 | 14.3 |
| $\bar{\Delta}_{1}$ | 17.8 | 10.0 | 8.3 | -0.8 | 2.5 | -3.3 | -14.8 |
| $\bar{\Delta}_{2}$ | 6.5 | 20.0 | 16.8 | 4.5 | $-2.5$ | -5.5 | $-8.3$ |
| $\bar{\Delta}_{3}$ | 15.8 | 17.8 | 11.0 | 0.3 | -1.8 | -2.3 | 2.3 |
| $\bar{\Delta}_{4}$ | 13.3 | 13.8 | 14.3 | -0.8 | -3.0 | -8.0 | -14.3 |
| $\delta_{1}$ | 2.3 | 2.5 | 1.8 | 1.3 | 2.0 | 1.8 | 0.4 |
| $\delta_{2}$ | 2.3 | 5.5 | 2.3 | 1.5 | 1.5 | 3.0 | 4.4 |
| $\delta_{3}$ | 4.9 | 2.9 | 2.0 | 1.1 | 2.3 | 2.3 | 3.6 |
| ${ }_{4}$ | 1.8 | 3.3 | 1.1 | 1.1 | 3.0 | 2.5 | 2.8 |

Table 5 Data From Static Arm Misalignment
Tests (all quantities in degree)

$$
\begin{align*}
& \overline{\bar{\Delta}}=\frac{1}{4} \sum_{i=1}^{4} \bar{\Delta}_{i}  \tag{5.8}\\
& \bar{\delta}^{\prime}=\frac{1}{4} \sum_{i=1}^{4} \bar{\delta}_{i}^{\prime} \tag{5.9}
\end{align*}
$$

that is, $\overline{\bar{\Delta}}, \overline{\bar{\Delta}}$, and $\overline{\bar{\delta}}^{\circ}$ are the average values of the quantities in Table 5; corresponding to a particular value of $\varphi_{c}$. The average behavior of the group is shown in Fig. 11.

It is apparent from the graph of $\overline{\bar{\Delta}}^{\prime}$ (average magnitude of misalignments) that the group responds more accurately to the $90^{\circ}$ position command than to any other. This is reasonable in light of the fact that, prior to the test sequence, all subjects were shown the $90^{\circ}$ position as a reference. Further, from the values of $\overline{\bar{\Delta}}$ (algebraic average of misalignments) it can be concluded that the group has a tendency to overestimate small angles and to underestimate the larger angles. The fact that $\overline{\bar{\delta}}$. is less than $5^{\circ}$ for all values of $\varphi_{c}$ (see Fig.11) implies that the individual subjects, on each of their four responses to the same position command, placed their arms at approximately the same angle, $\bar{\varphi}_{i}$. In the neighborhood of $90^{\circ}$ (where $\overline{\bar{\delta}}^{\prime} \approx \overline{\bar{\Delta}}^{\prime}$ ) this angle corresponds to the position command; but, away from $90^{\circ}$, $\bar{\varphi}_{i}$ differs by as much as $15^{\circ}$ from $\varphi_{c}$. Once again, the fact that the subjects were assisted in positioning their arms at exactly $90^{\circ}$ prior to the tests would explain the proximity of $\bar{\varphi}_{i}$ and $\varphi_{c}$ in the neighborhood of $90^{\circ}$.

From the similarity of responses described above, one can conclude that the subjects could be trained to position their arms within $\pm 5^{\circ}$ of $\varphi_{c}$ for any value of $\varphi_{c}$. The reasoning behind this conclusion is an follows:


Fig. 11 Average Behavior of Four Subjects

The subjects are able to position their arms within $\pm 5^{\circ}$ of $\bar{\varphi}_{i}$ for all values of $\varphi_{c}$; for values of $\varphi_{c}$ in the neighborhood of $90^{\circ}, \bar{\varphi}_{i}$ corre sponds to $\varphi_{c}$; and finally, the subjects had been effectively "trained" to position their arms in the neighborhood of $90^{\circ}$ when they were shown the $90^{\circ}$ position prior to the test sequence. Thus, it can be concluded that, had an angle other than $90^{\circ}$ been used as a reference, $\bar{\varphi}_{i}$ would have corresponded to $\varphi_{c}$ in the neighborhood of this new reference position. Therefore, by proper choice of the reference position, the subjects would be able to position their arms within $\pm 5^{\circ}$ of any value of $\varphi_{c}$.

If the subjects were making use of visual cues from their immediate surroundings (i.e., the physical features of the room) then the foregoing results would not necessarily apply in a space environment. To determine whether or not use of visual cues was a significant factor, two of the subjects repeated the entire series of tests with their eyes closed. As before, the reference position was $90^{\circ}$; however, in this case, because their eyes were closed, the test conductor placed the subjects arms at $90^{\circ}$. The data from these tests were treated as before, and the average behavior of the two subjects is shown in Fig. 12.

By comparing Fig, 11 and Fig. 12, it can be seen that the average responses of the subjects with their eyes closed are quite similiar to those with their eyes open. That is, in both cases, the subjects overestimate small angles, underestimate large angles, and maintain $\overline{\bar{\delta}}$ less than $5^{\circ}$ for all values of $\varphi_{c}$. Finally, in both cases, $\overline{\bar{\delta}}$, is approximately equal to $\overline{\bar{\Delta}}$ in the neighborhood of $90^{\circ}$. From the similarity of the two cases, one can conclude that visual cues are not a factor in the subject's


Fig. 12 Average Behavior of Two Subjects With Eyes Closed
response characteristics and that the conclusions drawn earlier are thus applicable in a space environment.

There is still another aspect of the subjects responses that deserves comment, namely response to the $90^{\circ}$ position command. It follows from (5.2) and (5.3) that, if for a particular value of $\varphi_{c},\left|\bar{\Delta}_{i}\right|$, is less than $\bar{\Delta}_{i}$, then the subject both overestimates and underestimates $\varphi_{c}$ on different attempts. Inspection of Table 5 indicates that this is the case with the first subject for $\varphi_{c}$ equal to $90^{\circ}$ and with the third subject for $\varphi_{c}$ equal to $90^{\circ}$ and $120^{\circ}$. It is felt that, if more tests were conducted, this aspect of subject behavior would be more in evidence. That is, from Table 5, it is seen that, on different responses to the same position command, all of the subjects overestimate angles less than $90^{\circ}$ and, in most cases, underestimate angles greater than $90^{\circ}$. It follows that, for $\varphi_{c}$ in the neighborhood of $90^{\circ}$, a subject will both overestimate and underestimate a particular angle on successive attempts.

From the foregoing results of the static arm misalignment tests it is now possible to present a more detailed description of "rigid body" behavior. In so doing, two subjects will be considered. First, an "untrained" sub. ject that is, one who has not been shown the equilibrium position, but who has been told the location of this position, will be considered. A man attempting rigid body behavior for the first time is such a subject. Second, a "trained" subject, that is, one who, prior to embarking on a "rigid body" flight, has had the benefit of seeing his arms in the equilibrium position, will be discussed. This subject might be a man with previous flight experience or one who has experimented with the thrust device prior to attempting flight.

In both cases it is assumed that the subjects attempt to place their arms at the equilibrium position, turn on the thruster, and maintain the rigid body configuration for the duration of the flight.

From the results of the static arm misalignment tests, it can be concluded that both subjects' arms will be misaligned from the equilibrium position. If the misalignment of the untrained subject is called $\delta u$, and that of the trained subject $\delta_{T}$, then, in assigning a value to $\delta u$, the average of all the values of $\bar{\Delta}_{i}$ in Table 5 , excluding those corresponding to $\varphi_{c}$ equal to $90^{\circ}$ and $120^{\circ}$ (i.e., the neighborhood of the reference position), must be used; and for $\delta_{T}$, we use the average of all the values of $\vec{\delta}_{i}$ in Table 5 (it is recalled that, in light of the results shown in Fig. $12, \bar{\delta}_{i}$ is an indication of a trained subject's response capability). Thus it can be concluded that, on the average,

$$
\begin{align*}
& \delta_{u} \approx 12^{\circ}  \tag{5.10}\\
& \delta_{T} \approx 3^{\circ} \tag{5.11}
\end{align*}
$$

In addition, as mentioned earlier, $\delta_{T}$ will probably be positive on one attempt at "rigid body" behavior and negative on another attempt at the same configuration; whereas $\delta u$ is likely to be either positive or negative (but not both) on all attempts, depending on whether the equilibrium position is small or large angle. These last statements will become more significant during the discussion of "open loop" behavior in the next section.

## 5.2 "Open Loop" Behavior

During the preliminary tests, a single subject participated in nine tests involving "open loop" behavior. A multiple exposure photograph of
an "open loop" test is shown in Fig. 13. In this test the subject reversed the thrust, in order to come to rest before reaching the horizontal travel limit. In what follows, the results of the "open loop" tests will first be compared to "rigid body" behavior in order to es. tablish the efficacy of using "open loop" behavior. Then, in an effort to determine whether or not the implications of the comparison can be extended to situations not included in the simulation, an analysis will be made of the subject's arm motions.

Because the "open loop" tests were among the first to be conducted, and because the subject had not been shown the equilibrium position as a reference, the data from these tests will be considered or resulting from the behavior of an "untrained" subject (see sec. 5.1). The pitch motion resulting from this behavior is called $\theta_{\text {OL }}$, and it will be compared to that which would result if the subject were to attempt the same flight using "rigid body" behavior. The rigid body rotation is called $\theta_{R B}$, and it will be computed using the results stated in sec. 5.1.

Throughout the simulation, $\theta_{0 L}$ was recorded as a function of time, and a sample of these data is shown in Fig. 14. During all "open 10op" runs (including those for which $\varphi_{e}=90^{\circ}$ ), there was, a net increase or decrease in the pitch angle. This change $\theta_{O L}$ is called $\Delta \theta_{O L}$, and it is shown in Fig. 14. For the tests involving both positive and negative thrust, two values of $\Delta \theta_{0 L}$ are measured. The first value is the change in $\theta_{0 L}$ occurring during the positive thrust portion of the test, and the second is the change in $\theta_{O L}$ taking place during the negative thrust, over and above that caused by the pitch rate developed during the positive


Fig. 13 "Open Loop" Behavior with Thrust Reversal


Fig. 14 Pitch Motion During "Open Loop" Behavior
thrust portion. These net changes in $\theta_{\mathrm{OL}}$ could be expected, for, if one assumes that a man cannot statically place his arms at the equilibrium position, it follows that the man will commit errors when trying to perform oscillatory motions about the same position. However, there remains a possibility that $\theta_{0 L}$ will be less than $\theta_{R B}$ due to a cancellation of errors over many cycles of oscillation.

In order to compare $\theta_{O L}$ with $\theta_{R B}$ in a way that that is independent of thrust and equilibrium position, an effective static arm misalignment is computed for each test. This quantity, called $\delta_{\text {eff }}$, is determined in the following manner: First, it is assumed that $\Delta \theta$ OL is caused by a static arm misalignment. Then, using the solution for rigid body pitch motion, it is a simple matter to find the angle at which the arms must be held in order to cause a rotation of $\Delta \theta_{\text {OL }}$. The difference between this calculated angle and the equilibrium position is the effective static arm misalignment, $\delta_{\text {eff }}$. Thus, setting $\dot{\varphi}=0$ and $\theta_{c}=\theta_{R B}$ in (2.1), one obtains the following expression for the rigid body rotation:

$$
\begin{equation*}
\theta_{\mathrm{RB}}=\left[\frac{.248+1.466 s_{1}-.151 \cos \varphi}{14.592-2 \cos \varphi}\right] \mathrm{F}_{2} \frac{t^{2}}{2} \tag{5.14}
\end{equation*}
$$

where it has been assumed that $\dot{\theta}_{R B}(0)=\theta_{R B}(0)=0$. Now when the arms are at the equilibrium position (i.e., when $\varphi=\varphi_{e}$ ), there is no rotation. Hence it follows from (5.14) that

$$
\begin{equation*}
.248+1.466 s_{1}=.151 \cos \varphi_{e} \tag{5.15}
\end{equation*}
$$

when (5.15) is substituted into (5.14) one is left with the following
expression for $\theta_{\mathrm{RB}}$ :

$$
\begin{equation*}
\theta_{R B}=\left[\frac{\cos \varphi_{e}-\cos \varphi}{14.592-2 \cos \varphi}\right] \cdot 151 F_{2} \frac{t^{2}}{2} \tag{5.16}
\end{equation*}
$$

Finally, if $\theta_{R R}$ and $\varphi$ are replaced by $\Delta \theta_{\mathrm{OL}}$ and $\varphi_{\mathrm{e}}+\delta_{\text {eff }}$, respectively, the resulting expression can be solved for $\delta_{\text {eff }}$, leaving

$$
\delta_{e f f}=\cos ^{-1}\left[\frac{.151 F_{2} t_{1}{ }^{2} \cos \varphi_{e}-29.184 \Delta \theta_{O L}}{.151 F_{2} t_{1}{ }^{2}-4 \Delta \theta_{O L}}\right]-\varphi_{e}
$$

where $t_{I}$ is the time over which $\Delta \theta_{O L}$ was recored (see Fig. 13).
The average magnitude of all the $\delta_{\text {eff }}$ values is used to sharacterize the effectiveness of "open loop" behavior. To the nearest degree, this quantity, called $\bar{\delta}_{\text {eff }}$, is $4^{\circ}$; thus, it follows from (5.10) that

$$
\begin{equation*}
\frac{\delta_{\mathrm{u}}}{\bar{\delta}_{\text {eff }}}=3 \tag{5.18}
\end{equation*}
$$

It is therefore concluded that, by performing oscillatory arm motions, one is able to reduce the effective static arm misaiignome by a factor of three.

In order to determine expressions for $\theta_{\mathrm{RB}}$ and ${ }^{\theta_{C L}}$ that reflect the experimental results, one must substitute $\varphi_{e}+\delta_{u}$ and $\varphi_{e}+\delta_{\text {eff }}$, respectively, for $\varphi$ in (5.16) . The following equation is then obtained for $\theta_{\mathrm{RB}} / \theta_{\text {of. }}$ :

Now, if $(5.10),(5.18)$, and (5.19) ace combined and if $\varphi_{e}$ is assumed be $90^{\circ}$, the following result be obtained:

$$
\begin{equation*}
\frac{\theta_{\mathrm{RB}}}{\vartheta_{\mathrm{OL}}}=2.96 \approx 3 \tag{5.20}
\end{equation*}
$$

It can be concluded that, by performing oscillatory ammotions, an "unstrained" subject will, on the average, experience three times less rotation than would result if he attempted to hold his arms at the equi-. librium position.

The results given by (5.18) and (5.20) deal with an "untrained" subject. As yet, nothing has been said about "trained" subjects. Unfortunately, at the time of the simulation, it was not known that "training" required nothing more than showing the subject the equilibrium position. For this teason, there were no tests involving a "trained" subject perEorming "open loop" behavior.

In lieu of test data for "trained" subject's, the untrained subject's acm uotions are examined in an effort to determine whether or not (5.20) can be applied to a "trained" subject.

A sample $\quad$ rip chart indicating the arm angie as a function of time is shown in Fig. 15. It can be seen that the autject's arm motions are oscillatory in nature. However, the mean value, amplitude, and frequency of the oscillations all change with tine.


Fig. 15 Arm Motions during "Open Loop" Behavior

In order to measure the change in amplitude, two curves are drawn connecting the extreme values of $\varphi$. These curves, shown in Fig. 15, are called $\varphi_{\max }(t)$ and $\varphi_{\min }{ }^{(t)}$. The amplitude, $A(t)$, is then given by

$$
\begin{equation*}
A(t)=\frac{\varphi_{\max }(t)-\varphi_{\min }(t)}{2} \tag{5.21}
\end{equation*}
$$

To study the change in frequency, the period, $T(t)$, is first measured as shown in Fig. 15. The frequency, $N(t)$, is then given by

$$
\begin{equation*}
N(t)=\frac{2 \pi}{T(t)} \tag{5,22}
\end{equation*}
$$

Finally, the mean value of the oscillation, $\varphi_{M}(t)$, is assumed to be

$$
\begin{equation*}
\varphi_{M}(t)=\varphi_{\min }(t)+\frac{A(t)}{2} \tag{5,23}
\end{equation*}
$$

It follows from $(5,21)-(5,23)$ that "open loop" behavior can be regarded as taking place in accordance with the relationship

$$
\begin{equation*}
\varphi(t)=\varphi_{M}(t)+A(t) \cos [N(t) t] \tag{5.24}
\end{equation*}
$$

Because specific amplitude and frequency instructions were not given, the average values of these two quantities, given by

$$
\begin{equation*}
\bar{A}=\frac{1}{t_{1}} \int_{0}^{t_{1}} A(t) d t \tag{5.25}
\end{equation*}
$$

and

$$
\begin{equation*}
\overline{\mathbb{N}}=\frac{1}{t_{1}} \int_{0}^{t_{1}} N(t) d t \tag{5.26}
\end{equation*}
$$

are assumed to be the subject's intended amplitude and frequency, respectively ( $\mathrm{t}_{1}$ is the duration of the test). The change in amplitude and frequency is then given by the following quantities.

$$
\begin{align*}
& \delta_{A}^{\prime}(t)=|A(t)-\bar{A}|  \tag{5.27}\\
& \delta_{N}^{*}(t)=|N(t)-\bar{N}| \tag{5.28}
\end{align*}
$$

To characterize the subject's ability to maintain a constant amplitude and frequency over a particular run, the following quantities are computed for
each test:

$$
\begin{align*}
& \bar{\delta}_{A}^{\circ}=\frac{1}{t_{1}} \int_{0}^{t_{1}} \delta_{A}^{-}(t) d t  \tag{5.29}\\
& \bar{\delta}_{\hat{N}}^{\sim}=\frac{1}{t_{1}} \int_{0}^{t_{1}} \delta_{N}^{-}(t) d t \tag{5.30}
\end{align*}
$$

The values of $\bar{A}, \bar{N}, \bar{\delta}_{\dot{A}}$, and $\bar{\delta}_{N}$, for each test, are shown in Table 6.

| Test No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\bar{A}$ (Deg.) | 21 | 18 | 19 | 19 | 20 | 24 | 24 | 22 | 23 |
| $\bar{\delta}_{A}^{\prime}$ (Deg.) | 2 | 1 | 2 | 1 | 2 | 2 | 2 | 1 | 1 |
| $\bar{N}^{\prime}$ (cyc./sec.) | .9 | .9 | .6 | .5 | .5 | .4 | .4 | .8 | .8 |
| $\bar{\delta}_{\mathrm{N}}^{\prime}$ (cyc./sec.) | .02 | .05 | .03 | .05 | .04 | .06 | .03 | .03 | .03 |

Table 6 Amplitude and Frequency Data for "Open Loop" Tests

It is seen from Table 6 that $\bar{A}$ and $\bar{\delta}_{A}^{\prime}$ did not change significantly from test to test. Therefore the average of all the $\bar{\delta}_{A}^{\prime}$ values in Table 5 is used to characterize the subject's ability to maintain a constant amplitude. This quantity, called $\overline{\bar{\delta}}_{\mathrm{A}}^{\prime}$, is, to the nearest degree

$$
\begin{equation*}
\overline{\bar{\delta}}_{A}^{+}=2^{\circ} \tag{5.31}
\end{equation*}
$$

On the other hand the average frequency (see Table 6) is seen to change significantly. In this case, the ratio

$$
\begin{equation*}
R_{\mathrm{N}}=\frac{\bar{\delta}_{\mathrm{N}}}{\overline{\mathrm{~N}}} \tag{5.32}
\end{equation*}
$$

Is computed for each test, and the average value of these ratios, $\bar{R}_{\mathrm{N}}$, is then used to reflect the subject's ability ot maintain a constant frequency. This last quantity turns out to be

$$
\begin{equation*}
\bar{R}_{N}=.07 \tag{5,33}
\end{equation*}
$$

From (5.31) and (5.33) it is concluded that, when the subject oscillates his arms in the frequency range considered, the amplitude will on the average, differ by $\pm 2^{\circ}$ from the intended amplitude, and the frequency will, on the average, differ by $\pm 6 \%$ from the intended frequency. It is felt that such small changes in amplitude and frequency will have very little effect on the pitch motion, and therefore, for all practical purposes, the subject is able to oscillate his arms with a constant amplitude and frequency.

In light of the foregoing results, it is concluded that the drift in $\theta_{\text {OL }}$ (see Fig. 14) is due, primarily, to the misalignment of the mean value of the oscillation (see (5.23)) from the equilibrium position. In order to study the variation of the mean value, the following quantities are defined:

$$
\begin{align*}
& \bar{\Delta}_{M}=\frac{1}{t_{1}} \int_{0}^{t_{1}}\left[\varphi_{M}(t)-\varphi_{e}\right] d t  \tag{5,34}\\
& \bar{\Lambda}_{M}^{\prime}=\frac{1}{t_{1}} \int_{0}^{t_{1}}\left|\varphi_{M}(t)-\varphi_{e}\right| d t  \tag{5.35}\\
& \bar{\varphi}_{M}=\frac{1}{t_{1}} \int_{0}^{t_{1}} \varphi_{M}(t) d t  \tag{5,36}\\
& \bar{\delta}_{M}^{\prime}=\frac{1}{t_{1}} \int_{0}^{t_{1}}\left|\varphi_{M}(t)-\bar{\varphi}_{M}\right| d t \tag{5,37}
\end{align*}
$$

It follows that $\bar{\triangle}_{M}$ is the average misalignment of the mean value from the equilibrium position for a particular test, and $\bar{\Delta}_{\mathrm{M}}$ is the average magnitude of this misalignment. From (5.36) and (5.37) it can be seen
that $\bar{\delta}_{M}^{\prime}$ is the average magnitude of the misalignment of $\varphi_{M}$ from the average mean value， $\bar{\varphi}_{M}$ ．It follows that $\bar{\delta}_{M}^{\prime}$ is a measure of the sub－ ject＇s ability to maintain a constant mean value for the duration of a test． The values of $\bar{\Lambda}_{M}, \bar{\Lambda}_{M}$, and $\bar{\delta}_{M}^{\prime}$ ，for each test are shown in Table 7 ．

| Test No． | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\bar{A}_{M}$（Deg。） | 7 | -3 | -6 | -1 | -8 | 2 | -4 | 10 | 5 |
| $\bar{\Delta}_{M}^{\prime}$（Deg。） | 7 | 12 | 7 | 2 | 9 | 5 | 5 | 10 | 5 |
| $\bar{C}_{M}^{\prime}$（Deg。） | 2 | 11 | 4 | 2 | 4 | 4 | 3 | 3 | 2 |

Table 7 Variation of Mean Value During ＂Open Loop＂Behavior

Although the nine＂open loop＂runs above involve equilibrium positions of $66^{\circ}, 90^{\circ}$ ，and $156^{\circ}$ ，there appears to be no correlation of these data with $\varphi_{e}$ ．It is felt that this is primarily due to the fact that the subject was equally＂untrained＂for all the tests；however，even if this were not the case，it would be difficult to establish such a correlation with only nine runs．

It is seen in Table 7 that，for six of the nine tests，the magnitude of $\bar{\Delta}_{M}$ is less than $\overline{\mathrm{A}}_{\mathrm{M}}$ ．It follows that，for these runs，the mean value of the oscillation was，at different times，both greater than and less than $\varphi_{e}$ ．Further，it is seen that，in general， $\bar{\delta}_{M}^{\prime}$ is less than $\bar{\Lambda}_{M}^{\prime}$ ，from which it can be concluded that $\varphi_{M}$ remains measurably closer to $\bar{\varphi}_{M}$ than to $\varphi_{e}$ ．This last characteristic is reflected by the averages of the $\overline{4}_{M}^{\prime}$
and $\bar{\delta}_{M}^{\prime}$ values in Table 7. To the nearest degree, these quantities, called $\overline{\mathrm{A}}_{\mathrm{M}}$ and $\overline{\bar{\delta}}_{\mathrm{M}}^{\prime}$ respectively, are

$$
\begin{align*}
& \overline{\overline{Q_{M}^{\prime}}}=7^{\circ}  \tag{5.38}\\
& \overline{\bar{\delta}}_{M}^{\prime}=4^{\circ} \tag{5.39}
\end{align*}
$$

From (5.38) and (5.39), it can be concluded that, on the average, the mean value of the oscillation will be misaligned by $\pm 7^{\circ}$ from $\varphi_{e}$, and by $\pm 4^{\circ}$ from $\bar{\varphi}_{M}$ 。

Six of the nine tests were conducted with the subject either blindfolded or in a dark room (i.e., subject could not use visual cues). From the similarity of the results of these tests with those during which the subject could see, it is concluded that visual cues are not an important factor in the above results.

From the foregoing analysis of the subject's arm motions, it is possible to conclude that, for a "trained" subject (see sec 5.1), the rotational motion resulting from "open loop" behavior will be less than that resulting from "rigid body" behavior. This conclusion is based on the following interpretation of the results from the static arm misalignment and "open 1oop" tests: First, it is assumed that (5.10) and (5.11) reflect the ability of an "untrained" and "trained" subject, respectively, to estimate a particular angle. Next, it is assumed that the location of the mean value of the arm motions during "open loop" behavior is the result of continually estimating the location of the equilibrium position. Finally, the amplitude of the arm oscillations are simply successive estimates of the same angle, $\overline{\mathbb{A}}$.

In other words, "open loop" behavior is interpreted as consisting of many repetitions of the same static arm misalignment test. In order to check the validity of this interpretation, the results of the static arm misalignment tests are used to predict the nature of the arm motions that would result when an "untrained" subject performs "open loop" behavior. First, since it is assumed that $\varphi_{M}(t)$ is the subject's estimation of $\varphi_{e}$, one would expect, in light of (5.10), that $\overline{\mathrm{A}}_{\mathrm{M}}$ (see (5.35)) would, on the average, be $12^{\circ}$. The average value of $\bar{\Lambda}_{M}^{\prime}$ was found to be $7^{\circ}$ (see (5.38)). Second, as $\delta_{T}$ in (5.11) indicates a subject's ability to estimate an angle he has seen at least once, it must also be an indication of a subject's ability to re-estimate the same angle repeatedly. Thus one would expect $\varphi_{M}$ (since $\varphi_{M}(t)$ represents a continuous estimate of $\varphi_{e}$ ) to be misaligned, on the average, by $3^{0}$ from $\bar{\varphi}_{M}$. For the same reason, one would expect $A(t)$ to differ from $\bar{A}$ by $3^{0}$, on the average. The average misalignment of $\varphi_{M}(t)$ from $\bar{\varphi}_{M}$ was found to be $4^{0}$ and $A(t)$ was found to differ by $2^{\circ}$, on the average, from $\bar{A}$. Thus it is concluded that the predicted results and the actual results of the "open loop" tests agree reasonably well and that, therefore, the results of the static arm misalignment tests can be used to predict the results of "open loop" behavior performed by a "trained" subject. When the subject has been shown the location of the equilibrium position, his estimate of this angle will be incorrect, on the average, by $3^{\circ}$. Therefore, it is concluded that, for a "trained" subject, the mean value of the oscillations will generally be misaligned by $3^{0}$ from the equilibrium position. Further, it was mentioned at the end sec. 5.1 that the static arm misalignment could
be either positive and negative on successive attempts. From this it is concluded that during "open loop" behavior of a "trained" subject the mean value of the oscillations will oscillate about the equilibrium position. When this is the case, a subject's actual "open loop" behavior will approach "open loop" behavior as defined by (1.20). It follows that the ratio of $\theta_{\mathrm{RB}} / \theta_{\mathrm{OL}}$ would then be much larger than that given by (5.20). Thus it is concluded that

$$
\begin{equation*}
\frac{\theta_{\mathrm{RB}}}{\theta_{\mathrm{OL}}} \geq 3 \tag{5.40}
\end{equation*}
$$

where the equality sign applies to an "untrained" subject and the inequality refers to a "trained"subject. In other words, when a subject attempts to perform oscillatory arm motions about the equilibrium position, he effectively reduces the thrust misalignment and thus achieves at least a threefold reduction in the pitch motion. In the next section, the results of the second and third phases (see sec. 4) of the simulation will be evaluated in an effort to determine whether or not the subjects can improve upon the foregoing results by using "closed loop" behavior.

## 5.3 "Closed Loop" Behavior

During the "closed Loop" portion of the preliminary test, the subject attempted to change the direction of his line of flight and then return to the initial direction. As mentioned in sec. 4.1 , the subject was not able to accomplish this task before reaching one of the travel limits of the simulator. The conclusion drawn from these tests was that the subject did not have the necessary ability to control pitch. In order


#### Abstract

to provide the subjects with pitch control experience and, at the same time, learn something about man's "closed loop" capabilities, a number of tests were conducted involving pitch motion only. Following these tests, the translational motion was, once again, introduced into the simulation in order to determine whether or not the added experience in pitch control resulted in an increased ability to control changes in direction of the line of flight.


## Subjects ${ }^{\text {' Ability of Control Pitch Motion }}$

To determine the maximum thrust level at which attitude control could be maintained, subjects were instructed to place their arms at the equilibrium position $\left(90^{\circ}\right)$, turn on the thruster, and maintain the pitch angle or small as possible. Throughout these tests, the pitch angle, called $\theta_{C L}$, was plotted as a function of time. One of these graphs is shown in Fig. 16.

To characterize the subject ${ }^{8}$ s ability to control the pitch motion, the maximum amplitude of the variations in $\theta_{C L}$ is determined for each test. This quantity, called $\Delta \theta_{\text {CLM }}$, is shown in Fig. 16 , and is tabulated in Table 8.

Inspection of Table 8 indicates that the subject did not encounter any difficulty in maintaining attitude control for thrusts up to 30 lbs . In other words, the maximum thrust at which the subjects could maintain $\theta_{C L}$ within reasonable bounds (when their arms are initially in the neighborhood of the $90^{\circ}$ equilibrium position) may be expected to be considerably greater than 30 1bs. The experiments were therefore, modified so as to provide a more severe test of the subjects" pitch control ability. It was decided

rig 16 Pitch Motion During Max. Thrust Determination Test

| Test No. <br> (Day/Test) | Subject | Thrust <br> (1bs.) | ${ }^{\prime}{ }_{\text {CLM }}$ <br> (Rad.) | $\mathrm{t}_{1}$ <br> (Sec.) |
| :---: | :---: | :---: | :---: | :---: |
| $1 / 10$ | 1 | .5 | .26 | 175 |
| $1 / 11$ | 1 | .5 | .14 | 225 |
| $1 / 12$ | 1 | 1.0 | .06 | 150 |
| $1 / 13$ | 1 | 1.0 | .12 | 100 |
| $1 / 14$ | 1 | 10.0 | .14 | 150 |
| $1 / 15$ | 1 | 30.0 | .16 | 100 |
| $1 / 16$ | 2 | 5.0 | .10 | 150 |
| $1 / 17$ | 2 | 10.0 | .20 | 175 |
| $1 / 18$ | 2 | 30.0 | .10 | 75 |

Table 8 Results of Max. Thrust Determination
that, after demonstrating an ability to keep $\theta_{\mathrm{CL}}$ bounded about $\theta_{\mathrm{CL}}=0$, the subjects would attempt to maintain pitch angles other than zero. A sample of the data thus obtained is shown in Fig. 17 where, as before, $\Delta \theta_{\text {CLM }}$ is measured to characterize the subjects' pitch control ability, In this case, values of $\Delta \theta_{\text {CLM }}$ corresponding to each pitch angle command are determined for each test. In tabulating the results of these tests, the average of the $\Delta \theta_{\text {CLM }}$ values (called $\bar{\Delta} \theta_{\text {CLM }}$ ) for the non-zero pitch commands is compared to the $\Delta \theta_{C L M}$ value corresponding to $\theta_{C L}=0$. This comparison is shown in Table 9.

From the values of $\Delta \theta_{\text {CLM }}$ in Table 9 it can be seen that the subjects' ability to maintain non-zero pitch angles is essentially the same as that for $\theta_{\text {CL }}=0$. Further, it is seen that this ability to control the pitch motion remains essentially unaltered for thrust levels from. 1 lb . to 100 lbs . Once again, therefore, it was decided to devise a new series of tests that would require even greater pitch control ability.

The third series of tests was designed to test the subjects' ability to locate an unknown equilibrium position without losing attitude control. If the subjects were able to find the equilibrium position and then stabilize the pitch motion, they would next be required to maintain various pitch angles. A plot of $\theta_{\text {CL }}$ from one of these tests in shown in Fig. 18. In this case, the maximum value of $\theta_{C L}$ (called $\theta_{C L M}$ ) that occurs before the pitch motion was stabilized is determined for each test. In addicion, values of $\Delta \theta_{\text {CLM }}$ corresponding to the subsequent pitch commands are determined, and the average of these quantities is computed for each test (the average is computed because the results in Table 9 indicate that, for a particular equilibrium

(1) Subject maintains ${ }^{\theta}{ }_{\mathrm{CL}} \approx 0$.
(2) Subject instructed to maintain $\theta_{\mathrm{CL}} \approx-15^{\circ}$.
(3) Response to $-15^{\circ}$ instruction.
(4) Subject instructed to maintain ${ }^{\theta} \mathrm{CL} \approx+15^{\circ}$.
(5) Response to $+15^{\circ}$ instruction.
(6) Subject instructed to maintain ${ }^{\theta}{ }_{\mathrm{CL}} \approx 0^{\circ}$.
(7) Response to $0^{\circ}$ instruction.

Fig. 17 Pitch Control With $\varphi_{e}=90^{\circ}$

| $\begin{aligned} & \text { Test No. } \\ & \text { (Day/Test) } \end{aligned}$ | Subject | Thrust <br> (lbs.) | ${ }^{\Delta \theta_{\mathrm{CLM}}} \mid \theta_{\mathrm{CL}} \approx 0$ <br> (Rad.) | $\begin{gathered} \overline{\Delta \theta} \operatorname{CLM}^{\mid \theta} \mathrm{CL} \neq 0 \\ \text { (Rad.) } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| 2/8 | 1 | 1 | . 05 | . 10 |
| 2/9 | 1 | 5 | . 03 | . 08 |
| 2/10 | 1 | 10 | . 04 | . 05 |
| 2/11 | 1 | 50 | . 08 | . 03 |
| 2/12 | 1 | 75 | . 08 | . 05 |
| 2/13 | 1 | 100 | . 06 | . 03 |
| 2/1 | 2 | . 1 | . 08 | . 06 |
| 2/2 | 2 | . 5 | . 04 | . 06 |
| 2/3 | 2 | 1.0 | . 03 | . 04 |
| 2/4 | 2 | 1.0 | . 05 | . 05 |
| 2/5 | 2 | 5 | . 04 | . 07 |
| 2/6 | 2 | 10 | . 08 | . 08 |

Table 9 Results of Pitch Control Tests with $\varphi_{e}=90^{\circ}$

(1) After locating equilibrium position, subject maintains $\theta_{\mathrm{CL}} \approx 0$.
(2) Subject instructed to maintain $\theta_{\mathrm{CL}} \approx-20^{\circ}$.
(3) Response to $-20^{\circ}$ instruction.
(4) Subject instructed to maintain $\theta_{\mathrm{CL}} \approx 0^{\circ}$.
(5) Response to $0^{\circ}$ instruction.

$$
\text { Fig. } 18 \text { Pitch Control With } \varphi_{e} \neq 90^{\circ}
$$

position, $\Delta \theta_{\text {CLM }}$ is independent of $\theta_{C L}$ ). The average $\Delta \theta_{C L M}$ value (called $\bar{\Delta} \theta_{C L M}$ ) and the value of $\theta_{\text {CLM }}$, for each test, are given in Table 10 . From Table 10, it can be seen that the subjects were able to locate the equilibrium position and stabilize the pitch motion before losing attitude control (i.e., in all cases, $\theta_{\text {CLM }}<1$ rad.). Further, from the values of $\overline{\Delta \theta}_{\text {CLM }}$ in Tables 9 and 10 , it follows that the subjects' ability to control the pitch motion is independent of the equilibrium position. Finally, because thrust levels up to $50 \mathrm{lbs}$. . were used in these tests, and up to 100 lbs. were used in previous tests, it is concluded that there is no "practical" upper bound on the thrust level with which the subjects can control the pitch motion. This last statement is made in light of the fact that thrust levels much greater than 10 lbs . would be impractical because of weight, fuel requirements, etc.

The above conclusions deal with the subjects' ability to control the attitude motion while acted upon by a positive thrust (i.e., one directed from back to front). In order to determine whether or not these results apply when the thrust is reversed, some of the tests were repeated with negative thrusts.

The first of the negative thrust tests were simply repetitions of some of the tests covered in Table 9 (with the exception of thrust direction). The results of these tests are tabulated in Table 11 , where $\overline{\Delta \theta}_{\text {CLM }}$ is the average of $\Delta \theta_{C L M}$ values corresponding to the various pitch commands (including $\theta_{C L}=0$ ), Comparison of the $\overline{\Delta \theta}_{C L M}$ values in Table 11 with those in Tables 9 and 10 indicates that the subjects ability to control pitch motion is independent of thrust direction.

| Test No. <br> (Day/Test) | Subject | $\begin{gathered} \varphi_{e} \\ \text { (Deg.) } \end{gathered}$ | Thrust <br> (1bs.) | $\begin{gathered} { }^{\theta} \mathrm{CLM} \\ \text { (Rad.) } \end{gathered}$ | $\begin{aligned} & \overline{\Delta \bar{\theta}} \mathrm{CLM} \\ & \text { (Rad.) } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2/14 | 1 | 120 | . 1 | . 40 | . 03 |
| 2/15 | 1 | 60 | . 1 | . 20 | . 03 |
| 2/16 | 1 | 80 | . 5 | . 07 | . 02 |
| 2/17 | 1 | 60 | . 5 | . 22 | . 03 |
| 2/18 | 1 | 135 | 1 | . 50 | . 05 |
| 2/32 | 1 | 120 | 5 | . 26 | . 03 |
| 2/33 | 1 | 100 | 10 | . 08 | . 03 |
| 2/34 | 1 | 60 | 10 | . 30 | . 03 |
| 2/20 | 2 | 60 | . 1 | . 14 | . 02 |
| 2/21 | 2 | 135 | . 5 | . 20 | . 02 |
| 2/22 | 2 | 100 | . 5 | . 16 | . 04 |
| 2/23 | 2 | 80 | 1.0 | . 06 | . 02 |
| 2/24 | 2 | 45 | 1.0 | . 26 | . 03 |
| 2/25 | 2 | 45 | 5.0 | . 36 | . 03 |
| 2/26 | 2 | 100 | 5.0 | . 12 | . 03 |
| 2/27 | 2 | 120 | 10 | . 30 | . 03 |
| 2/28 | 2 | 60 | 10 | . 24 | . 05 |
| 2/30 | 2 | 60 | 50 | . 26 | . 04 |

Table $10 \cdot$ Results of Pitch Control Tests with $\varphi_{e} \neq 90^{\circ}$

| Run No. | Subject | Thrust <br> (Lbs.) | $\overline{\Delta \theta}$ <br> (radM |
| :---: | :---: | :---: | :---: |
| $3 / 1$ | 1 | -1.0 | .09 |
| $3 / 2$ | 1 | -1.0 | .03 |
| $3 / 3$ | 1 | -5.0 | .03 |
| $3 / 4$ | 1 | -5.0 | .04 |
| $3 / 5$ | 1 | -10.0 | .03 |
| $3 / 6$ | -10.0 | .03 |  |

Table 11 Pitch Control With Negative Thrust

The final series of tests in the second phase of the simulation dealt With the subjects' ability to maintain attitude control when acted upon alternately by positive and negative thrusts. These tests were conducted like those providing the data for Table 9, with the exception that, between various pitch commands, the subject was instructed to reverse the direction of the thrust. A sample of the data is shown in Fig. 19, and the average $\Delta \theta_{\text {CLM }}$ values are tabulated in Table 12 . For each test, the $\overline{\Delta \theta}_{\text {CLM }}$ value resulting from the positive thrust portions of the tests is compared to $\overline{\Delta \theta}_{C L M}$ resulting from the responses made while using the negative thrust. From this comparison, it appears that there is no effective degradation in pitch control ability when the thrust is reversed a number of times during the same test.

(1) Subject maintains ${ }_{\mathrm{CL}} \approx 0^{\circ}$.
(2) Subject instructed to maintain $\theta_{\mathrm{CL}} \approx-10^{\circ}$.
(3) Response to $-10^{\circ}$ instruction.
(4) Subject instructed to maintain $\theta_{\mathrm{CL}} \approx 0^{\circ}$.
(5) Response to $0^{\circ}$ instruction.
(6) Subject instructed to reverse thrust and maintain $\theta_{\mathrm{CL}} \approx 0^{\circ}$.
(7) Subject continues to maintain $\theta_{\mathrm{CL}} \approx 0^{\circ}$.
(8) Subject instructed to maintain ${ }^{\theta} \mathrm{CL} \approx 20^{\circ}$.
(9) Response to $20^{\circ}$ instruction.
(10) Subject instructed to maintain $\theta_{\mathrm{CL}} \approx 0^{\circ}$.
(11) Response to $0^{\circ}$ instruction.

Fig. 19 Pitch Control with Both Positive and Negative Thrust

| Test No. <br> (Day/Test) | Subject | Thrust <br> (lbs.) | $\overline{\Delta \bar{\theta}}$ CLM <br> (Pos. Thrust) <br> (Rad.) | $\overline{\Delta \theta}$ CLM <br> (Nag. Thrust) <br> (Rad.) |
| :---: | :---: | :---: | :---: | :---: |
| $3 / 7$ | 1 | $\pm 1$ | .02 | .04 |
| $3 / 8$ | 1 | $\pm 5$ | .03 | .05 |
| $3 / 9$ | 1 | $\pm 5$ | .03 | .04 |
| $3 / 10$ | 1 | $\pm 5$ | .03 | .02 |
| $3 / 11$ | 1 | $\pm 5$ | .02 | .03 |

Table 12 Results of Pitch Control Tests with Positive and Negative Thrust

Although the tests involving negative thrust were concerned with an equilibrium position of $90^{\circ}$ ，it is felt that the results stated above can be applied to situations which involve equilibrium positions other than $90^{\circ}$ ． The reasons for this are as follows：First，the results of the negative thrust tests are quite similar to the corresponding positive thrust tests（cf．，Tables 9， 11 and 12）；second，the results of the positive thrust tests involving a $90^{\circ}$ equilibrium position are approximately the same as those involving other equilibrium positions（cf．，Tables 9 and 10）；finally，the necessary arm motions required to achieve a desired torque，when the subject is acted upon by a negative thrust，are simply the opposites of those necessary for a posi－ tive thrust．One can therefore expect the similarity of positive and negative thrust results for the $90^{\circ}$ equilibrium position to extend to other equilibrium positions．From this，it follows that the negative thrust results for $\varphi_{e}=$ $90^{\circ}$ apply to cases for which $\varphi_{e} \neq 90^{\circ}$ ．

It was concluded earlier that there is no＂practical＂upper bound on the thrust level with which a subject can maintain attitude control．Further examination of the results in Tables $8-12$ indicates that pitch control ability is independent of thrust level．In order to check this，the thrust levels used during the tests were divided into three groups：Low Thrust（ $\left|\mathrm{F}_{2}\right|<1 \mathrm{lb}$ 。）， Medium Thrust（ $1 \mathrm{lb} . \leq\left|\mathrm{F}_{2}\right| \leq 10 \mathrm{lbs}$ 。），and High Thrust（ $\left|\mathrm{F}_{2}\right|>10 \mathrm{lbs}$ ）。 Next， the average of all the $\Delta \theta_{\text {CLM }}$ values（called $\overline{\bar{\Delta}} \theta_{C L M}$ ）corresponding to each thrust range was determined for each subject．The results of this calculation are shown in Table 13.

| $\overline{\Delta \theta}_{\text {CLM }}$ (Rad.) |  |  |
| :---: | :---: | :---: |
|  | Subject 非1 | Subject 非2 |
| Low Thrust | .07 | .04 |
| Med. Thrust | .05 | .06 |
| High Thrust | .03 | .07 |

Table 13 Pitch Control vs: Thrust Level

It can be seen from the values of $\overline{\overline{\Delta \theta}}_{\text {CLM }}$ in Table 13 that ability to control the pitch motion does not change significantly with thrust. However, although the subjects are able to maintain approximately the same degree of attitude control over a wide range of thrust levels, one should not conclude that there is no preferred range of thrusts. Both subjects reported that it was easiest to control the pitch motion when acted upon by thrusts in the medium range ( $1-10 \mathrm{lbs}$.). The relative ease with which the subjects were able to maintain attitude control can be seen in the type of arm motions used. Time histories for the arm motions of one of the subjects for the thrust levels are shown in Fig. 20. It is seen that the arm motions are quite different for the three thrust levels. The amplitude of arm motions decreases as the thrust level increases, and the arms are held in a particular position for longer times at low thrust than at high thrust. In other words, the system as a whole is much more responsive to arm motions for high thrusts than for low thrusts.


Fig. 20 Arm Motions During Low, Medium, and High Thrust Tests

Thus, if the thrust level is too low, the slow response of the system necessitates large amplitude arm motions which tend to be tiring and if the thrust level is too high, the pitch motion becomes so sensitive to arm motions that excessive concentration may be required of the subject. For these reasons, it is concluded that medium thrust levels ( $1-10$ 1bs.) are preferable to both low and high thrust levels.

Because it has been found that the subjects' ability to control the pitch motion is largely independent both of the thrust level (.1 1 bs $\leq$ $\mathrm{F}_{2} \leq 100 \mathrm{lbs}$.) and of the location of the equilibrium position, it is possible to use the values of $\Delta \theta_{\text {CLM }}$ in Tables 8-12 to assess the effect of experience on pitch control ability. For each subject, the average value of $\Delta \theta_{\text {CLM }}$ corresponding to each day of the simulation is computed from Tables $8-12$, and these values are plotted Fig. 21.


Fig. 21 Improvement in Pitch Control Ability With Experience

Inspection of Fig. 21 shows that the pitch control ability of both subjects improved significantly as a result of experience gained during the first day of the simulation, During the first day of the simulation, the subjects spent less than two hours each actually participating in tests. It is therefore concluded that a few hours training will enable a subject to maintain

$$
\begin{equation*}
\left|\theta_{\mathrm{CL}}\right| \leq .05 \mathrm{Rad} \tag{5.41}
\end{equation*}
$$

regardless of the location of the equilibrium position $\left(0^{\circ} \leq \varphi_{e} \leq 180^{\circ}\right.$ ) or of the magnitude and direction (positive or negative) of the thrust.

Before evaluating the results of the third phase of the simulation (pitch and translation), the subjects' behavior during the tests covered in Table 10 will be studied in an effort to determine the subjects" ability to detect pitch motions.

Ability of Detect Pitch Motion
To test the ability to locate an unknown equilibrium position, a subject held his arms at $90^{\circ}$ until he detected some pitch motion. As soon as the subject identified the pitch motion, he moved his arms in order to reverse the sign of the torque and eventually stabilize the motion and locate the equilibrium position. Thus, until the arms were first moved, the subject was acted upon by a constant torque) the magnitude of this torque being determined by the equilibrium position and by the magnitude of the thrust. From the strip chart data, it is possible to determine, quite accurately, the time at which the subject fixst moves his arms in response to the pitch motion. The response time, called $t_{R}$, and the torque (about
the subject's center of mass), $T$, for the tests in Table 10 are tabulated in Table 14.

The response time $t_{R}$ can be divided into three parts. First, there is the time required to detect and identify the pitch motion; second, the subject spends some time in deciding which way to move his arms; and finally, he must actually move them. Because of the particularly simple relationship between arm motions and the sign of the torque, it is felt that the last two contributions to $t_{R}$ are neglegible compared to the first. Consequently, $t_{R}$ is interpreted as the time required by the subject to detect the pitch motion. With this in mind, it is possible (using the solution for rigid body motion) to compute the pitch angle, $\theta_{R}$, and the pitch rate, $\dot{\theta}_{R}$, existing at time, $t_{R}$. The results of this computation are plotted Fig. 22. From Fig, 22 it is apparent that, for a wide range of torque magnitudes, pitch angles never differ by more than $2^{\circ}$. Because of this similarity in $\theta_{R}$ values, it is reasonable to conclude that the subjects are sensing pitch angle rather than the pitch rate, and further, that the minimum angle which can be detected is given by the average of the $\theta_{R}$ values in Fig. 22. This value is

$$
\begin{equation*}
\overline{\left|\theta_{R}\right|}=1.3^{0} \tag{5.42}
\end{equation*}
$$

The validity of these conclusions can be tested by calculating $t_{R}$ and $\theta_{R}$ for $\theta_{R}=1.3^{\circ}$ and for various torque levels, using rigid body theory. A comparison of the results of such calculations with the experimentally obtained values of $t_{R}$ and $\dot{\theta}_{R}$ is shown in Fig. 23.

| Test No. | Subject | $\mathrm{T}\left(\frac{\mathrm{Ft}_{\mathrm{o}}-1 \mathrm{lbs}_{\mathrm{o}}}{100}\right)$ | $\mathrm{t}_{\mathrm{R}}$ (Sec.) |
| :---: | :---: | :---: | :---: |
| 2/1/4 | 1 | - . 5 | 13.2 |
| 2/15 | 1 | $+\quad .5$ | 10.3 |
| 2/16 | 1 | $+\quad .9$ | 7.3 |
| 2/17 | 1 | + 2.5 | 4.5 |
| 2/18 | 1 | $-7.3$ | 3.3 |
| 2/19 | 1 | $+7.3$ | 3.2 |
| 2/31 | 1 | $+25$ | 1.4 |
| 2/32 | 1 | - 25 | 1.1 |
| 2/33 | 1 | $+18$ | 1.1 |
| 2/34 | 1 | $+50$ | 1.0 |
| 2/20 | 2 | + . 5 | 8.6 |
| 2/21 | 2 | - 3.7 | 2.6 |
| 2/22 | 2 | - . 9 | 5.4 |
| 2/23 | 2 | + 1.8 | 3.9 |
| 2/24 | 2 | + 7.3 | 2.4 |
| 2/25 | 2 | $+37$ | 1.4 |
| 2/26 | 2 | - 9 | 2.4 |
| 2/27 | 2 | - 50 | 1.2 |
| 2/28 | 2 | $\pm 50$ | 0.8 |
| 2/30 | 2 | +250 | 0.4 |

Table 14 Response Times for Various Torques



Fig. 22 Ability to Detect Pitch Rates and Angles for Various Torques


Fis. 23 Comparison of Calculated Response Times and Pitch Rates with Test Data

The agreement between the calculated values of $t_{R}$ and $\dot{\theta}_{R}$ and the test data is seen to be such as to give strong support to the conclusions that the minimum pitch angle the subjects are able to detect (when visual cues from a stationery background are available) is approximately $1.3^{\circ}$ and that this angle is independent of the pitch rate. Subjects' Ability To Control Pitch And Translation

To determine how far a subject can travel in the horizontal direction before reaching either the pitch or the vertical travel limits of the simulator, one can eliminate the actual horizontal translation of the moving carriage by sending the horizontal position commands to the recording devices and bypassing the horizontal servo drives. In so doing, one obtains a record of the motion that would occur if the horizontal motion were included in the test. Accordingly, the subject was instructed to place his arms at the $90^{\circ}$ equilibrium position, turn on the thruster, and, by using appropriate arm motions, avoid reaching the pitch and vertical travel limits. The resulting trajectory of the subject is shown in Fig. 24 (see Fig. Al for simulator coordinates).

From Fig. 24, one can conclude that, when visual cues are available, the subject is able to fly at least 450 ft . while remaining within $\pm 2 \mathrm{ft}$. of his intended flight path. Whether or not this result is applicable to situations in which the subject cannot monitor the vertical translation can only be determined by further experimentation or by discovery of the feedback law used by the subject when he monitors solely the pitch motion.

In addition to being able to fly along a straight line, the subject should be able to brake to a stop at any desired point on his line of


Fig. 24 Subject's Ability to Fly Along a Straight Line


Fig. 25 "Straight Line" Flight Using Negative Thrust to Brake
flight. For this reason, the horizontal translation was reintroduced into the simulation and a number of tests were devoted to straight lineflights during which the subjects used the reverse thruster to brake to a stop. A multiple exposure photograph mode during one such test in this series is shown in Fig. 25. In order to facilitate interpretation of the photographic data, two lights were mounted on the gimbal ring. The shoulder lights indicate the direction of the thrust while the light on the gimbal ring, being on throughout the test, produces a trace of the subject's trajectory. Because the images in Fig, 25 are separated by equal time intervals, it can be deduced from the trace of light thr ough the subjects shoulder that positive thrust was used initially (first two images in Fig. 25); the thruster was then turned off and the subject coasted (second two images); and, finally, the negative thruster was fired, bringing the subjest to a stop. The trace of light through the gimbal ring indicates that the subject was able to fly in a very nearly straight line for the duration of the run. During one of the tests, the subject was required to retrieve an object located on his line of flight; no difficulty was encountered in accomplishing this task. "Rigid Body" behavior was used during all of these tests as very little rotation occurs during the short time that the thruster is on (e.g., see Fig. 25). Thus it can be concluded that "rigid body" behavior is quite satisfactory when the subject pulses the thruster, coasting most of the way to the target. After demonstrating an ability to fly along a straight line for both long and short distances, as well as the ability to stop by using reversed thrust, the subject made several attempts to effect controlled
changes in the vertical coordinate. During these tests, the subject would deliberately develop a positive pitch angle and hence a vertical velocity. On successive tests, the subject then tried different maneuvers to stabilize the vertical motion (including the use of negative thrust). Without fail, the subject reached either the pitch or the vertical travel limit before the vertical motion was stabilized. From this, it is concluded that the subject's control of the pitch motion is not good enough to permit precise changes in the line of flight.

In an effort to arrive at a technique for precision maneuvering, some simulation time was allotted to training the subject in maneuvers that would alter his pitch angle without requiring thrust. These maneuvers, resulting in so-called "self-induced rotations," consisted of moving the arms on an arc between 0 and $180^{\circ}$, causing the rest of the body to rotate in the opposite direction. The maneuver, shown in Fig, 26, begins with the arms at the equilibrium position. The arms are first lowered to the sides, which produces a positive increment in the pitch angle. Keeping them in the roll plane, the arms are then raised above the head without affecting the pitch angle $\theta$. Finally, the arms are lowered (again increasing $\theta$ ) to the equilibrium position, this being a convenient point at which to initiate thrust or, if a larger pitch angle is desired, to begin another cycle of the pitch maneuver. A detailed discussion of "self-induced rotation" appears in Ref. [3]. The particular maneuver described was dictated by the construction of the arm angle sensing device (see Fig, 5).

$\infty$

1. From equillibrium position, lower arms to sides: $\Delta \boldsymbol{\theta} \boldsymbol{0}$
2. Maintaining arms in roll plane, raise them above head: $\Delta 0=0$
3. Lower arms to equilibrium position: $\Delta \theta>0$
4. Ready to fly or repeat maneuver

The final tests in the simulation combined the "self-induced rotations" described above with the subjects ability to fly in a straight line while employing both positive and negative thrust. Starting with his arms at the $90^{\circ}$ equilibrium position, the subject developed a positive pitch angle using the "self-induced rotation" maneuver. With his arms once again in the equilibrium position, the subject turned on positive thrust and maintained this until he had developed a comfortable velocity along a line inclined to the horizontal. He then turned off the thruster, coasted upward toward the desired vertical coordinate, and pulsing the thruster negatively, slowed to a stop. Finally, by performing the reverse of the previous "selfinduced rotation" maneuver, the subject was able to reduce the pitch angle to zero, thus completing the test. Thus it appears that, by combining the abilities to fly along a straight line, to use reversal of thrust, and to perform "self-induced" rotation, a subject can translate between any two points.

Both a summary of the results and the conclusions drawn from the foregoing experiments are discussed in the next section.

It was the purpose of this study to investigate the possibility of providing a "weightless" astronaut with a particulary simple maneuvering device, namely, a single thruster rigidly attached to one part of the body. The experiments described in this report were conducted to determine whether a man, when acted upon by a body-fixed thrust, could achieve controlled planar motions.

### 6.1 Pitch Motion

In attempting to control pitch motions, the subjects employed three types of arm motions. In all three cases, the arms were fully extended and moved in unison in planes parallel to the pitch plane. The remainder of the body was held rigidly in a position of "attention".

The simplest way to control the pitch motion would be to hold the arms in such a position that the line of action of the thrust passes through the system center of mass. When this is the case, there will be no rotation, and the subject will translate along a straight line. This ideal arm location is called the equilibrium position, and the subject's behavior, when he attempts to fly in this manner is referred to a "rigid body" behavior. However, it was felt that, when a subject uses "rigid body" behavior, he would not be able to align his arms exactly with the equilibrium position. If an error in arm position does occur, the system will be acted upon by an external torque, resulting in rotational motion. The magnitude of the torque, and therefore the amount of rotation, depend on the amount of misalignment between the equilibrium position and the actual position of the arms.

The object of the static arm misalignment tests discussed in Secs.
4.9 and 5.1 was to determine how well a man could place his arms at the equilibrium position. The results of these tests are as follows: If a man has simply been told the location of the equilibrium position and subsequently attempts to place his arms in this position, there will be, on the average, a $12^{\circ}$ misalignment between the equilibrium position and the actual position of his arms. On the other hand, if the subject has previously seen his arms in the equilibrium position, he will be able to return his arms to within $\pm 3^{\circ}$ of that position. Thus, "rigid body" behavior of an "untrained" subject results in a $12^{\circ}$ misalignment, and that of a "trained" subject results in a $3^{\circ}$ misalignment. With knowledge of the arm misalignment, it is possible to predict the rotational motion resulting from "rigid body" behavior.

In an effort to reduce the rotational motion resulting from "rigid body" behavior, the effects of oscillating the arms about the equilibrium position was investigated. From the analytical study, it was found that, if the subject could oscillate his arms harmonically about the equilibrium position, the resulting pitch motion would be consistently less than that resulting from "rigid body" behavior. Such behavior on the part of the subject is referred to as "open loop" behavior because these arm motions do not involve the use of feedback. As with "rigid body" behavior, a man's ability to perform "open loop" behavior was not known. If it is assumed that a man cannot perfectly align his arms with the equilibrium position, it must be assumed that he will commit errors in oscillating his arms about the same position. However, it was felt that errors committed during "open loop" behavior would tend to cancel one another, thereby resulting in less pitch motion than that from "rigid body" behavior.

The object of the "open loop" tests described in Secs. 4.1 and 5.2 was to determine a man's ability to perform harmonic oscillations about the equilibrium position and to determine the efficacy of this behavior in reducing the rotational motion. The results of the tests are as follows: It was found that the subject is able to maintain a fairly constant amplitude and frequency. On the average, the amplitude differs by $\pm 2^{\circ}$ from the intended amplitude and the frequency remains within $\pm 6 \%$ of a constant value. However, as expected, the mean value of the oscillations is misaligned from the equilibrium position. For an "untrained" subject (i.e., one who has not been shown the equilibrium position) the mean value of the oscillation will, on the average, differ by $\pm 7^{\circ}$ from the equilibrium position. For a "trained" subject, the mean value of the oscillations will, itself, oscillate about the equilibrium position with the average magnitude of the misalignment being 4\%. It is this variation in the misalignment of the mean value as opposed to the constant misalignment during "rigid body" behavior that accounts for the reduced pitch motion during "open loop" behavior. If an "untrained" subject uses "open loop" behavior, the resulting pitch motion will be approximately three times less than that which would occur if he used "rigid body" behavior. As for "trained" subjects, it can be said that the reduction will be greater than a factor of three. Finally, in an effort to improve upon the "open loop" results, the subjects used "closed loop" behavior. In this case, the subjects act as a feedback control element in monitoring the pitch motion and responding with appropriate arm motions. Analysis had shown that, if the subjects were capable of moving their arms according to certain feedback laws (see Eqs. 1. 21 and 1.22), pitch angles could be kept arbitrarily small.

However, there is no reason to believe that man can duplicate these feedback laws and, therefore, a number of questions arise. For example, is a man capable of any kind of feedback that results in less rotation than "open loop" behavior? If so, is there a maximum thrust level with which a man can still control the pitch motion? Does the ability to control the pitch motion depend on the location of the equilibrium position? To answer these and other questions, the "closed loop" tests discussed in Secs. 4.2 and 5.3 were conducted.

Rather than attempt to discover the actual feedback law, it was decided to simply evaluate the effectiveness of "closed loop" behavior in reducing the pitch motion. It was found that, with a few hours experience, the subject could maintain pitch angles less than or equal to .05 rad . for long periods of time (up to 250 sec. ). Further, it was found that this ability is independent of the equilibrium position and the magnitude of the thrust (from . 1 lb . to 50 lbs ). In addition, subjects performed equally well when the direction of the thrust was reversed (i.e., front to back). Although the ability to control the pitch motion was independent of thrust level, it was found that the subjects perferred medium thrust levels ( $1-10 \mathrm{lbs}$.$) to both low and high thrust levels.$ In order to simulate an error in thrust placement, the subjects were given the task of locating unknown equilibrium positions when acted upon by thrusts of various intensities. In all cases, the subjects were able to locate the equilibrium position and stabilize the pitch motion before rotating through 1 radian. Finally, by analyzing the response times of the subjects in these tests, it was found that the subjects were capable of detecting a pitch angle of $1.3^{\circ}$ and the detection ability is independent of the pitch rate.

Using the foregoing results, one is able to compare the pitch motion resulting from "rigid body", "open loop", and "closed loop" behavior. Such a comparison for a "trained" subject using a .51 b . thrust is shown in Fig. 27. It can be seen that, for short times, the pitch motion resulting from the three types of bahavior is small. Therefore it is concluded that, because it is easiest to perform, "rigid body" behavior should be used for flights involving short firings of the thruster. For longer thrust periods, it is evident from Fig. 27 that one should use "closed loop" behavior. Finally, for intermediate thrust durations, the subject should use "open loop" behavior. The exact range over which each type of behavior should be used will depend on the individual.


Fig. 27 Comparison of Pitch Motion

### 6.2 Translation

The ultimate measure of a man's ability to control the attitude motion is whether such control enables the man to achieve controlled translational motions. In order to determine whether the subject's pitch control ability satisfies this criterion, several tests involving translation were conducted in the final phase of the simulation (see Secs. 4.3 and 5.3). The results of these tests can be summarized as follows: When visual cues are available (i.e., when motion, perpendicular to the line of flight, can be detected), the subject's pitch control ability enables him to fly long distances (at least $450 \mathrm{ft}$. ) without significant deviation from the intended flight path (within $\pm 2 \mathrm{ft}$. of the intended flight path). For shorter distances (less than 50 ft .), the subject is able to reach a target using "rigid body" behavior. During these flights, the subject fires the positive thruster at the beginning of the run until be achieves a comfortable velocity. At this point, thrust is terminated, and the subject coasts toward his destination. In the vicinity of the target, the subject pulses the reverse thruster, slowing down and eventually coming to rest. From these results, it is concluded that the subject can effectively fly along a straight line and, by using the reverse thruster, come to a stop at any desired point on the line.

During the last series of tests, the subject demonstrated the ability to alter the pitch angle without the use of thrust. When these so-called "self-induced rotations" are combined with the ability to fly along a straight line, controlled translation between any two points in the plane of motion becomes possible.

### 6.3 Conclusion

The results summarized above show that a man can perform well-controlled planar translational motions by using arm motions when acted upon by a body-fixed thrust. In the light of these results, it would appear desirable to extend the investigation by removing the restriction to planar motions.

## Appendix A Analog Computer Programming Considerations

## A. 1 Coordinate Transformation

When the system shown in Fig. 1 is used to study the motion of a human body, the equations of motion of the system become (see Sec. 1.2)

$$
\begin{gather*}
\frac{d}{d \tau}\left[\theta^{\prime}(14.592-2 \cos \varphi)+\varphi^{r}(.998-\cos \varphi)\right] \\
=\left(1.145+6.75 s_{1}-.698 \cos \varphi\right) \frac{F_{2}}{100}  \tag{A.1}\\
\left(\frac{x_{1}}{r}\right)^{\prime \prime}=-6.18 \times 10^{-3} F_{2} \sin \theta-\frac{d^{2}}{d \tau^{2}}[.886 \cos \theta+.0697 \cos (\theta+\varphi)]  \tag{A.2}\\
\left(\frac{x_{2}}{r}\right)^{\prime \prime}=6.18 \times 10^{-3} F_{2} \cos \theta-\frac{d^{2}}{d \tau^{2}}[.886 \sin \theta+.0697 \sin (\theta+\varphi)] \tag{A.3}
\end{gather*}
$$

where the primes denote differentiation with respect to $\tau$ and

$$
\begin{equation*}
\tau=4.661 t \tag{A.4}
\end{equation*}
$$

The remaining symbols in Eqs. (A.1) - (A.2) are defined in Sec. 1. 2 and are shown in Fig. Al.

It was noted in Sec. 2.3 that the variables appearing in Eqs. (A.1) (A.3) are not compatible with the command voltages accepted by the simu1ator. That is, the vertical and horizontal position commands accepted by the servo motors correspond to the position of the center of the pitch gimbal ( $B^{*}$ is assumed to occupy the center of the pitch gimbal) and are called $z_{c}$ and $x_{c}$, respectively (see Fig. Ai). The servo drive for the pitch gimbal accepts a command voltage, $\theta_{c}$, corresponding to the inclination of the pitch gimbal to the horizontal. Thus Eqs. (A.2) (A.3) deal with the translation of the hingepoint $P$, but the simulator requires command voltages for the translation of the center of the pitch gimbol $\mathrm{B}^{*}$. The discrepancy can be eliminated by writing the


Analytical Coordinates

Fig. A1 Comparison of Coordinates

$$
\begin{gather*}
\theta=\theta_{c}  \tag{A.5}\\
x_{1}=z_{c}-1.481 \cos \theta_{c}  \tag{A.6}\\
x_{2}=x_{c}-1.481 \sin \theta_{c} \tag{A.7}
\end{gather*}
$$

When Eqs. (A.4) - (A.7) are substituted into Eqs. (A.1) - (A.3), one is left with

$$
\begin{gather*}
\frac{d}{d t}\left[\dot{\theta}_{c}(14.592-2 \cos \varphi)+\dot{\varphi}(.998-\cos \varphi)\right]  \tag{A.8}\\
=\left[.248+1.466 s_{1}-.151 \cos \varphi\right] F_{2} \\
\ddot{x}_{c}=.199 \mathrm{~F}_{2} \cos \theta_{c}+\frac{d^{2}}{d t^{2}}\left[.169 \sin \theta_{c}-.103 \sin \left(\theta_{c}+\varphi\right)\right]  \tag{A.9}\\
\ddot{z}_{c}=-.199 \mathrm{~F}_{2} \sin \theta_{c}+\frac{d^{2}}{d t^{2}}\left[.169 \cos \theta_{c}-.103 \cos \left(\theta_{c}+\varphi\right)\right] \tag{A.1.0}
\end{gather*}
$$

where the dots denote differentiation with respect to $t$. Eqs. (A.8) (A.10) are identical to Eqs. (2.1) - (2.3) in Sec. 2.3

If one were to perform the indicated differentiations in Eqs. (A.8) (A. 10), it would be necessary to determine $\ddot{\varphi}$ in addition to $\dot{\varphi}$. Such a determination would require a more complicated differentiation circuit and the associated filters would cause greater inaccuracies (see Sec. A.2). To avoid these difficulties, the above equations are integrated twice, which yields the following expressions:

$$
\left.\begin{array}{rl}
\theta_{c} & =\int \tag{A.11}
\end{array} \frac{\int\left[0.248+1.466 s_{1}-0.151 \cos \varphi\right] \mathrm{F}_{2} \mathrm{dt}}{14.592-2 \cos \varphi}\right\} d t
$$

$$
\begin{align*}
x_{c}= & x_{0}+0.199 F_{2} \iint \cos \theta_{c} d t d t  \tag{A.12}\\
& +.169 \sin \theta_{c}-0.103\left[\sin \left(\theta_{c}+\varphi\right)-\sin \varphi_{o}\right] \\
z_{c}= & z_{o}-0.199 F_{2} \iint \sin \theta_{c} d t d t  \tag{A.13}\\
& +0.169\left(\cos \theta_{c}-1.0\right)-0.103\left[\cos \left(\theta_{c}+\varphi\right)-\cos \varphi_{0}\right]
\end{align*}
$$

In Eqs. (A.11) - (A.13), $x_{0}, z_{0}, \varphi_{0}$ refer to the initial values of $x_{c}, z_{c}$, and $\varphi$, respectively; and it has been assumed that $\theta_{c}, \dot{\theta}_{c}$ along with $\dot{x}_{c}, \dot{z}_{c}$ and $\dot{\varphi}$ are all initially equal to zero.

## A. 2 Differentiation Circuit

It can be seen from Eq. (A.8) that both $\dot{\varphi}$ and $\varphi$ must be monitored in order to simulate the desired motion. To meet this requirement, a portion of the analog computer was used as a filtered differentiation circuit (see Fig. A2). The filter, necessary to remove the noise present in the circuit, reduced the non-steady state amplitude of $\dot{\varphi}$ by four percent and had a time constant of .05 sec . To serve as a check on the circuit, $\varphi$ measured (from arm angle sensor) was compared with $\varphi$ calculated (result of integrating $\dot{\varphi}$ ) for the duration of the experiment. Such a comparison is shown in Fig. A3.


Fig. A2 Arm Angle Differentiator and Filter


## A. 3 Check Cases

Because the moving carriage and pitch gimbal are driven by commands from the analog computer, the validity of the data from this simulation depends upon the accuracy with which the computer integrates the equations of motion. One way to check the accuracy is to have the computer integrate the equations of motion for cases which have known solutions. One such case is that of the rigid body obtained by setting $\varphi$ equal to a constant in Eqs. (2.1) - (2.3). The rigid body solution is described in detail in [2] and therefore will not be presented here. It should be noted, however, that solutions for a number of combinations of $s_{1}, F_{2}$, and $\varphi$ were plotted and used to check the accuracy of the computer.

The form of the equations of motion as they were programmed on the computer is given by Eqs. (A.11) - (A.13). It can be seen from Eq. (A.11) that the pitch motion is the result of two contributions. The first integral in Eq. (A.11) is the contribution due the misaligned thrust while the second provides the inertial effects of arm motion. As expected, the "rigid body" cases fail to provide a check on the contribution due to arm motions. However, the equations for $x_{c}$ and $z_{c}$ do not contain $\dot{\varphi}$ explicitly and therefore the "rigid body" solutions do provide a check on all the terms in Eqs. (A.12) and (A.13). What is needed, then is another known solution in which $\dot{\varphi}$ is not identically equal to zero.

A case that meets the above requirement is that in which $\varphi$ is given by Eq. (1.20), i.e., when the arms are assumed to oscillate harmonically. Although the general solution for this case could not be obtained, any particular solution is readily available from numerical integration on a digital computer (see Ref. [2]). Further, it was felt that harmonic
oscillations for $\varphi$ could easily be produced in the analog computer with the aid of a sine wave generator. As it turned out, this was not to be the case. The difficulty lay in not being able to control the initial conditions for each integration. Instead, the initial values of $\varphi$ and $\dot{\varphi}$ would differ from zero depending on which values the sine wave generator was sending at the instant the computer began to integrate the equation. The result of this inconsistency was that a random initial angular momentum was sent to the computer. It was therefore decided to discard this possibility for a check case in favor of another for which a solution can be written in closed form, namely, torque-free motion.

When $F_{2}$ is set equal to zero, Eq. (A.8) can be integrated once to yield

$$
\begin{equation*}
\dot{\theta}_{c}=-\left(\frac{.998-\cos \varphi}{14.592-2 \cos \varphi}\right) \dot{\varphi} \tag{A.14}
\end{equation*}
$$

where it has been assumed that $\dot{\theta}_{c}$ and $\dot{\varphi}$ are initially zero. Eq. (A.14) can be rewritten with $\varphi$ as the independent variable and, after integration, one is left with

$$
\begin{equation*}
\theta_{c}-\theta_{c o}=-\left[f(\varphi)-f\left(\varphi_{o}\right)\right] \tag{A.15}
\end{equation*}
$$

where $\theta_{c o}$ and $\varphi_{o}$ are the initial values of $\theta_{c}$ and $\varphi$, respectively, and $f(\varphi)$ is given by

$$
\begin{equation*}
f(\varphi)=\frac{\varphi}{2}-.871 \tan ^{-1}\left(1.148 \tan \frac{\varphi}{2}\right) \tag{A.16}
\end{equation*}
$$

Hence, when the arms are moved between any two angles $\varphi$ and $\varphi_{0}$, Eq. (A.15) affords the means to calculate the resulting pitch motion of the head torso and legs.

Thus, the "rigid body" solution provides a check on Eqs. (A.12) and (A.13) and the first term of Eq. (A.11) while the torque-free solution furnished a check on the second term in Eq. (A.11).

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