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Technical Memorandum 33-472

Optimization of Space Antenna Structures

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PREFACE

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ABSTRACT

This report develops an optimization scheme which can readily be applied to optimum design of space antennas as well as to the evaluation and comparison of antenna concepts, antenna structural types, and antenna structural materials. The objective function is either cost or weight; the design variables are diameter, weight per unit area, manufacturing precision measure, and sizes of structural elements. With system requirements such as antenna gains, communication frequencies, etc., as constraints, the objective function is minimized with respect to the design variables. Through this optimization process, it can be demonstrated whether the effort of improving a particular technology, such as manufacturing, has advantages under certain operational requirements.

I. INTRODUCTION

In many of its aspects, the evaluation of antenna concepts as well as optimum antenna design is not yet amenable to strictly quantitative procedures. Many decisions in the evaluation and design processes are made by using engineering judgment based on experience, intuition, and the extrapolation of data available from related projects. In this report, it is assumed that, through this or similar considerations on the system level, certain ranges of the system parameters such as antenna gains, communication frequency ranges, weight, etc., have been identified as acceptable or optimum. These parameters are then used as constraints in the optimization problem, where given antenna concepts are optimized with respect to minimum cost, including minimum weight as a special case. The technique developed in this report can readily be applied to (1) detail optimum antenna design including optimum design based on the extrapolation of available information, and to (2) evaluation and comparison of antenna concepts, antenna structural types, and antenna materials (e.g., parabolic concepts vs conical concepts, rib-mesh structures vs truss structures, and conventional materials vs composite materials).

A simplified gain equation developed by Ruze (Ref. 1) is used. The use of the Ruze equation simplifies considerably the computational effort involved in the optimization. However, before a final decision was made on using this equation, a careful evaluation of the general expressions for antenna gain (developed in Ref. 2 and involving numerical double integration) was made to assess the influence of various assumptions on the present optimization problem. These influences can be neglected when compared to other uncertainties involved such as the estimation of antenna loadings. In fact, some observations indicate that the use of Ruze's formula results in a slightly conservative design.

II. OPTIMIZATION OF SPACE ANTENNAS

A. Antenna Gain Vs Tolerances

According to Ruze's formula, the axial gain G can be written as

$$G = \eta\left(\frac{\pi D}{\lambda}\right)^2 \exp\left[-\left(\frac{4\pi\delta}{\lambda}\right)^2\right] \tag{1}$$

in which η, D, λ, and δ represent, respectively, aperture efficiency, antenna diameter, operational wave length and overall rms tolerances.

Although Eq. (1) is derived based on the assumptions that the reflector deviation from the ideal surface is stationary random with Gaussian distribution and that the correlation regions of the reflector deviations are small compared to the antenna diameter, some observations show that Eq. (1) holds in general even if the deformation of the reflector surface is deterministic. In fact, loadings on the space antenna, such as temperature and dynamic loading, are not deterministic and the manufacturing tolerance of the reflector is completely random in nature.

The aperture efficiency can usually be expressed as follows:

$$\eta = \eta_1 \eta_2 \eta_3 \eta_4 \tag{2}$$

in which

η_1 = the blockage efficiency due to support legs of reflector or feed or subreflector itself, etc.

η_2 = spillover efficiency = 1 - S, with S being the fraction of total power in spillover

η_3 = power reflection efficiency = 1 - R, with R being the power reflection coefficient

η_4 = 1 - K, with K being the fractional power absorption due to resistive losses.

With the present state of technology, the aperture efficiency η can be made approximately 0.55 - 0.6 for high-gain parabolic space antennas. Associated with a particular antenna concept as well as a particular structural type, η varies relatively little from antenna to antenna with different size and weight. However, η varies with respect to the antenna concept. For instance, the overall efficiency of the conical Gregorian antenna (Refs. 3 and 4) is different from that of the parabolic antenna with equivalent aperture area. Therefore, with respect to the optimum design of a specific antenna concept and a specific antenna structural type, it is reasonable to assume η a constant (independent of design variables).

The tolerance δ is the total rms deviation of the antenna reflector surface from the ideal shape and, in space antennas, it is reasonably assumed to arise from essentially three sources: (1) the rms deviation δ_m due to manufacturing errors, (2) the rms deviation δ_s due to design, e.g., a reflector mesh stretching between two ribs, thus deviating from an intended parabolic shape, and (3) the residue rms deviation δ_c due to environmental conditions such as temperature change, etc.

B. General Optimization Scheme

The optimization problem considered here can be stated as follows:

Minimize the objective function, "cost"

$$C = C_1(D, b, N) + C_2(D, N) \quad (3)$$

subject to the constraints

$$G_1 = \eta\left(\frac{\pi D}{\lambda}\right)^2 \exp\left\{-\left[\frac{4\pi\delta(D, N, b)}{\lambda}\right]^2\right\} \geq G_{01} \quad (4)$$

$$G_2 = \eta\left(\frac{\pi D}{\lambda + \Delta\lambda}\right)^2 \exp\left\{-\left[\frac{4\pi\delta(D, N, b)}{\lambda + \Delta\lambda}\right]^2\right\} \geq G_{02} \quad (5)$$

$$\lambda \geq \lambda_0 \quad (6)$$

$$\Delta\lambda \geq \Delta\lambda_0 \quad (7)$$

$$W(D, N) \leq W_0 \quad (8)$$

where λ, $\Delta\lambda$, D, b, and N are the design variables, and W(D, N) is the antenna weight.

This formally stated optimization problem together with the symbols used requires some explanations. The objective function "cost" includes two parts: (1) the cost of the fabricated, delivered, and packaged antenna, and (2) the cost of lifting the antenna into outer space and the cost of attitude-control during the mission. The first cost, $C_1(D, N, b)$, is roughly a function of deployed antenna diameter D, weight per unit area N, and a measure of fabrication precision b. The second cost, $C_2(D, N)$, is essentially a function of the total antenna weight, W(D, N), where it is assumed that W is a function of D and N only and is constrained to be less than a given weight W_0 (Eq. 8).

In many cases, operation at more than one frequency or wavelength is required. The above equations reflect simultaneous operation at two different frequencies corresponding to wavelengths λ and $\lambda + \Delta\lambda$, where λ and $\Delta\lambda$ are constrained by overall system considerations to be larger than certain given values λ_0 and $\Delta\lambda_0$, respectively (Eqs. 6 and 7). The gains of the antenna G_1 and G_2 corresponding to λ and $\lambda + \Delta\lambda$ are also constrained by overall system considerations to be larger than certain given values G_{01} and G_{02}, respectively. Optimization for single-frequency operation does not require Eqs. (5) and (7), while optimization at more than two frequencies requires additional equations of the form of Eqs. (5) and (7) for each additional frequency.

Some detail considerations lead to the conclusion that the rms deviations δ_m, δ_s, and δ_c depend mainly on the design variables D, b, and N, and that

$$\left.\begin{aligned} \delta_m &= \delta_m(D, b) \\ \delta_s &= \delta_s(D, N) \\ \delta_c &= \delta_c(D) \end{aligned}\right\} \quad (9)$$

Hence, assuming that the three error contributions are independent, one has

$$\delta^2(D, b, N) = \delta_m^2(D, b) + \delta_s^2(D, N) + \delta_c^2(D) \quad (10)$$

C. Functional Relations

The optimization problem stated by Eqs. (3-10) still requires the explicit specification of the functions C_1, C_2, δ, and W in terms of the design variables. For this purpose, data of existing antennas and antennas currently under design have been used and are fitted and extrapolated with sufficient free parameters so that a broad spectrum of antenna concepts is covered.

The antenna cost C_1 can then be expressed as

$$C_1 = a_1 D^{a_2} N^{a_3} b^{a_{10}} \quad (11)$$

where a_1, a_2, a_3, and a_{10} are parameters.

Equation (11) states that the antenna cost itself is proportional to a_2th power of the diameter D, a_3th power of the weight per unit area N, and a_{10}th power of the precision measure b of manufacture.

The cost C_2 can be expressed as

$$C_2 = a_6 W^{a_4} \quad (12)$$

where a_6 is the cost per unit weight of payload and W can be expressed as

$$W = \frac{\pi}{4} N D^2 \quad (13)$$

Thus, using Eqs. (11), (12), and (13), the objective function, Eq. (3), becomes

$$C = a_1 D^{a_2} N^{a_3} b^{a_{10}} + a_7 N^{a_4} D^{2a_4} \quad (14)$$

It should be mentioned that the power law of the functional relationship for the antenna cost has been used in the literature (Refs. 5 and 6), and it is reasonable and justifiable as indicated by the data of existing antennas.

Available information also indicates that the rms deviation in Eq. (10) can be represented, with sufficient accuracy, in the following form:

$$\delta^2(D, N, b) = \left(\frac{a_8 D^{a_9}}{b}\right)^2 + \left(\frac{a_{11} D}{N^{a_5}}\right)^2 + (a_{12} D)^2 \qquad (15)$$

stating that (1) the rms manufacturing tolerance δ_m is proportional to the a_9th power of the diameter D and inversely proportional to the precision measure b (note that the precision measure b is also reflected in the antenna cost C_1), (2) the rms design tolerance δ_s is proportional to the diameter D and is inversely proportional to the a_5th power of the weight per unit area N, and (3) the rms residue tolerance δ_c is proportional to the diameter D.

Equations (12), (14), and (15) contain twelve parameters, a_1, a_2, $\cdots$, a_{12}, which represent characteristics of antenna concepts. Note that some of these parameters, such as a_5, a_8, a_9, and a_{10}, are variables in the design space defined by D, N, b, λ, and $\Delta\lambda$. Thus the functional forms given by Eqs. (11-15) are general enough to cover a variety of antenna concepts over the entire design space. These parameters can be determined either from the information of existing antennas, past experience, or current technologies, or from the computer program for structural analysis. Methods of determining appropriate parameter values in a particular domain of design space will be discussed later together with the successive approximation technique.

D. Simplification of the Optimization Scheme

From previous discussions, Eqs. (3-15) thus constitute a formal optimization problem. It is noted from these equations that there are four design variables associated with the optimum design of an antenna operating at one single frequency, and one additional design variable is introduced corresponding to one additional frequency operation. However, from the available information, we are sure that $a_9 = 1$, and hence it can be shown from Eqs. (3-15) that, at optimum,

$$\lambda = \lambda_0 \qquad (16)$$

$$\Delta\lambda = \Delta\lambda_0 \qquad (17)$$

In other words, the optimum design is realized when the first operational frequency is as high as allowable, and the second operational frequency is as close as allowable to the first operational frequency. Thus the number of design variables is reduced to 3, i. e., D, N, and b, irrespective of the number of simultaneously operating frequencies.

The optimization scheme can thus be simplified as follows:

Minimize the relative cost $C^* = C/a_7$

$$C^* = Y D^{a_2} N^{a_3} b^{a_{10}} + N^{a_4} D^{2a_4} \qquad (18)$$

subject to the constraints

$$\left.\begin{aligned} G_1 &= \eta\left(\frac{\pi D}{\lambda_0}\right)^2 \exp\left\{-\left[\frac{4\pi\delta(D,N,b)}{\lambda_0}\right]^2\right\} \ge G_{01} \\ G_2 &= \eta\left(\frac{\pi D}{\lambda_0 + \Delta\lambda_0}\right)^2 \\ &\quad \times \exp\left\{-\left[\frac{4\pi\ (D,N,b)}{\lambda_0 + \Delta\lambda_0}\right]^2\right\} \ge G_{02} \\ N D^2 &\le \frac{4W_0}{\pi} \\ \text{where} \\ \delta^2(D,N,b) &= \left(\frac{a_8 D}{b}\right)^2 + \left(\frac{a_{11} D}{N^{a_5}}\right)^2 + (a_{12} D)^2 \end{aligned}\right\} \qquad (19)$$

and

$$Y = \frac{a_1}{a_7} \qquad (20)$$

is a measure of the relative importance of the antenna cost itself to the cost of antenna payload. This parameter varies from mission to mission; its determination depends on the particular mission under consideration.

For ease of reference, let b = 1 represent the precision measure associated with the most advanced manufacturing technology currently available. In other words, the present state of manufacturing technology should be improved in order to achieve a precision measure b greater than 1. Note that a higher value of precision measure b is associated with a higher antenna cost, and vice versa, as can be observed from the cost expression C_1 of the antenna, Eq. (18).

The optimization scheme described in Eqs. (18) and (19) may result in a precision measure b greater than 1, indicating that it is worthwhile to improve the current manufacturing technology. If, however, it is desirable to design an antenna using the available manufacturing technology, additional constraint has to be added to Eq. (19), as follows:

$$b \leq 1 \tag{21a}$$

Furthermore, a large value of gain degradation is sometimes intolerable for space antennas, and hence the dB degradation ΔG_1 associated with λ_0 should be constrainted by the maximum allowable value GD; i.e.,

$$\Delta G_1 = 3.4\left(\frac{4\pi\delta}{\lambda_0}\right)^2 \leq GD \tag{21b}$$

E. Maximum Allowable Gain Constraint

In addition to the manufacturing tolerance δ_m and the design tolerance δ_s, the antenna gain is limited by the inherent residue tolerance δ_c, such as that due to uncontrolled temperature. This tolerance sets an upper bound for the antenna gain which can never be accomplished even if δ_m and δ_s can be eliminated. This upper bound can readily be obtained from Eq. (19) by setting $\delta_m = 0$, $\delta_s = 0$. For instance, for the condition of single-frequency operation, Eq. (19) becomes

$$G_1 = \eta\left(\frac{\pi D}{\lambda_0}\right)^2 \exp\left[-\left(\frac{4\pi D a_{12}}{\lambda_0}\right)^2\right] \tag{22}$$

Setting the derivative of G_1 with respect to D equal to zero, one obtains

$$D = \frac{\lambda_0}{4\pi a_{12}} \tag{23}$$

Hence the upper bound of the maximum achievable gain G_{1max} is

$$G_{1max} = \frac{\eta}{16 a_{12}^2 e} \tag{24}$$

and the gain constraint G_{01} should be less than G_{1max}. If, however, G_{01} is greater than G_{1max}, one can solve a_{12} from Eq. (24) with G_{1max} being replaced by G_{01} and evaluate δ_c to find how much additional temperature control is needed to obtain the required G_{01}.

F. Minimum-Weight Design

In many space applications, because of the requirement of the particular mission, the antenna weight may be of primary importance, and it may be necessary to use the available manufacturing technology. Under this circumstance, one can show from the previous formulation that Y = 0 and that at optimum b = 1. Therefore, the number of design variables reduces to two only: N and D. The formulation is then simplified as follows:

Minimize the weight

$$W = \frac{\pi N D^2}{4} \tag{25}$$

subject to the constraints

$$G_1 = \eta\left(\frac{\pi D}{\lambda_0}\right)^2 \exp\left[-\left(\frac{4\pi D}{\lambda_0}\right)^2\left(a_8^2 + a_{12}^2 + \frac{a_{11}^2}{N^{2a_5}}\right)\right] \geq G_{01} \tag{26a}$$

$$G_2 = \eta\left(\frac{\pi D}{\lambda_0 + \Delta\lambda_0}\right)^2 \times \exp\left[-\left(\frac{4\pi D}{\lambda_0 + \Delta\lambda_0}\right)^2\left(a_8^2 + a_{12}^2 + \frac{a_{11}^2}{N^{2a_5}}\right)\right] \geq G_{02} \tag{26b}$$

where

$$\delta_m = a_8 D \tag{27a}$$

$$\delta_s = \frac{a_{11} D}{N^{a_5}} \tag{27b}$$

$$\delta_c = a_{12} D \tag{27c}$$

The solution of the optimization schemes given in Eqs. (18-21) and (26-27) can easily be obtained once the parameter values, $a_1, \cdots, a_{12}$, are given. Hence these optimization schemes are particularly useful for the preliminary design in the evaluation and comparison of the merit of different antenna concepts as well as different possible structural materials. This is because, in the preliminary stage, the parameter values can be estimated in approximation from past experience or extrapolation of available data without detailed structural analysis which usually involves heavy computational effort.

III. OPTIMIZATION PROCEDURES

The optimization procedure employed in this report is a process of successive approximations. Since some of the parameter values, such as a_{10}, a_5, a_8, etc., vary in the design space, the optimization effort also involves a proper determination of these parameter values. Therefore, two optimization procedures, the global optimization procedure and the local optimization procedure, are used alternatively until the optimum design converges. For given parameter values, the procedure of finding the optimum design (i.e., N, D, and b) associated with the optimization scheme given by Eqs. (18) and (19) is called the global optimization procedure. For given N, D and b, the determination of parameter values corresponding to the optimum structural design of antennas is called the local optimization procedure.

The process is started by assuming a parameter vector $\underline{P}_0$ consisting of components (a_1, a_2, $\cdots$, a_{12}). This parameter vector is obtained from past experience, extrapolation of available data, etc. Given $\underline{P}_0$, one can compute the corresponding optimum design vector $\underline{D}_1$ with components N, D, and b from the global optimization procedure, the technique of which will be discussed later. Then, with $\underline{D}_1$ just obtained, another parameter vector $\underline{P}_1$ can be estimated by the local optimization procedure. This iterative process is continued until $\underline{D}_n \simeq \underline{D}_{n+1}$, and $\underline{D}_n$ is the optimum design. Numerical results indicate that $\underline{D}_j$ is insensitive to the variation of $\underline{P}_{j-1}$, and hence the iterative procedure converges rapidly.

A. Global Optimization Technique

The objective function and the constraints given in Eqs. (18) and (19) are nonlinear functions of the design variables D, N, and b. A gradient move technique is employed and is briefly described as follows: Consider a nonlinear objective function C of n design variables A_1, A_2, $\cdots$, A_n and subject to m nonlinear constraints.

Minimize

$$C = C(A_1, A_2, \cdots, A_n) \tag{28}$$

subject to

$$E_j = E_j(A_1, A_2, \cdots, A_n) \le E_{j0}; \qquad j = 1, 2, \cdots, m \tag{29}$$

It is assumed that C is a monotonically increasing function of the design variables A_1, A_2, $\cdots$, A_n so that at optimum at least one of the constraints is active (i.e., the equality sign holds at least for one of the constraints in Eq. 29). This assumption is satisfied in the optimum antenna design, as can be observed from Eq. (18).

Let $\underline{V}$ and $\underline{U}_j$ be the gradients of the objective function C and the constraint E_j, respectively,

$$\underline{V} = \underline{\nabla} C = \sum_{k=1}^{n} \frac{\partial C}{\partial A_k} \underline{i}_k \tag{30}$$

$$\underline{U}_j = \underline{\nabla} E_j = \sum_{k=1}^{n} \frac{\partial E_j}{\partial A_k} \underline{i}_k \tag{31}$$

in which i_k is the unit vector in the direction of the coordinate axis A_k.

1. Phase I: Steepest Descent Modification. A design point B_1 is first chosen arbitrary in the acceptable domain defined by $E_j \leq E_{j0}$ (j=1, 2, ···, m) of the n-dimensional space A_1, A_2, ···, A_n. The design is then modified in the direction $-\underline{V}$ at B_1 by a specified step from B_1 to B_2 with a reduction of C. This process is repeated until a constraint $E_j = E_{j0}$ is reached at point B_0. Note that the direction of design modification $-\underline{V}$ changes from point to point and hence it has to be computed for each step of modification.

2. Phase II: Usable Feasible Direction Modification. Let $\underline{Q}$ be a vector such that, at B_0,

$$\underline{U}_j \cdot \underline{Q} \leq 0 \tag{32}$$

$$\underline{V} \cdot \underline{Q} \leq 0 \tag{33}$$

The direction $\underline{Q}$ defines the so-called usable feasible direction. A systematic scheme for finding $\underline{Q}$ proposed by Zoudendijk (Ref. 7) is used. The design point is modified from B_0 along $\underline{Q}$ in a specified step away from the constraint $E_j = E_{j0}$ into the acceptable domain with a reduction of objective function C as shown by Eqs. (32) and (33). The modification then proceeds along $\underline{Q}$ at B_0 until either one of two cases occurs:

(1) The objective function C starts to increase at the design B_4. Because the objective function C is nonlinear, the continuous modification of the design along $\underline{Q}$ at B_0 does not guarantee the monotonic decrease of C (it is true if C is linear). Should this situation occur, the steepest descent modification described in phase I is then employed at B_4.

(2) A design point B_3 on the constraint $E_i = E_{i0}$ is reached (i may be equal to j). Then another usable feasible direction $\underline{Q}$ at B_3 is computed and the process of phase II is repeated until a design point B^* on the constraint is obtained at which the Kuhn-Tucker optimal condition is satisfied; i.e., $\underline{Q}$ cannot be found at B^* such that at least one of the inequality signs in Eqs. (32) and (33) holds.

The optimum design thus obtained is a local minimum and the global minimum can usually be obtained by choosing the minimum of local minima obtained from several different starting design points.

B. Local Optimization Procedure

Consider the minimum-weight design given in Eqs. (25-27) where the parameters involved are a_8, a_5, a_{11}, and a_{12}. Corresponding to a design $\underline{D}_j$ with components D and N obtained from the global optimization procedure, the parameter a_8 can be evaluated from Eq. (27a) by considering the most advanced manufacturing technology. The parameter a_{12} (Eq. 27c) can be computed from the thermal deformation of antennas. The parameters a_5 and a_{11} can easily be computed from Eq. (27b) with the aid of the relationship between δ_s and N, such as that given in Fig. 1. Note that Fig. 1 is obtained from the optimum design of rib shapes and rib sizes of a deployable, rib-mesh parabolic space antenna so as to minimize δ_s. As a result, the local optimization procedure is associated with the optimum structural design, e.g., the optimum design of rib sizes. This optimum structural design can only be made with a particular structural type and a particular structural configuration. In this connection, the structural optimization for a deployable, parabolic rib-mesh space antenna has been investigated at the Jet Propulsion Laboratory, and the general computer program is available (Ref. 8).

As the result of separating the optimum antenna design into two procedures, the global optimization procedure can be made on a generalized basis, and thus it can be applied to a broad spectrum of antenna concepts and antenna structural types.

For the optimization scheme given in Eqs. (18) and (19), additional parameters such as a_1, a_2, a_3, a_7, a_4, and a_{10} have to be determined. From past experience, it is reasonable to assume that $a_2 = 2.0$ and $a_4 \simeq 1.0$ to 4.0. Therefore, given a particular design vector $\underline{D}_j$ with components D, N and b obtained from the global optimization procedure, the cost of antenna payload C_2 can be estimated and hence a_7. For b = 1, the same relationship between δ_s and N, used for the determination of a_5 and a_{11}, can be used to estimate the antenna costs and hence the parameters a_1 and a_3. Finally, using the rms manufacturing tolerance δ_m obtained from the global optimization procedure, one can estimate the antenna cost C_1 and determine the parameter a_{10}.

IV. NUMERICAL EXAMPLES

A. Minimum-Weight Design

The minimum-weight design of a deployable, rib-mesh parabolic antenna operating at a single frequency or double frequencies is considered. It is assumed that for a single-frequency operation the operational frequency should be less than f_1; for a double-frequency operation, the first operational frequency should be less than f_1, while the second operational frequency should be less than f_2. The following set of parameter values, which has been determined from an antenna design currently considered at JPL, is employed: $\eta = 0.6$, $a_5 = 3.654$, $a_8 = 0.119 \times 10^{-3}$, $a_{11} = 0.26 \times 10^{-3}$, $a_{12} = 0.6 \times 10^{-4}$.

To understand this set of parameter values, consider, for instance, a 4.267-m-diam antenna. The rms manufacturing tolerance δ_m is 5.08×10^{-2} cm and the rms residue tolerance δ_c, including the rms tolerance due to temperature and deployment, is 2.54×10^{-2} cm. With 30 ribs, the antenna weighs 13.776 kg and has a rms design tolerance $\delta_s = 12.7 \times 10^{-2}$ cm, while the antenna with 48 ribs weighs 17.04 kg and has a rms design tolerance $\delta_s = 5.842$ cm. The rib material is aluminum.

These parameter values are assumed to be constant over the entire design region defined by D and N. The minimum-weight designs associated with different gain constraints G_{01} and G_{02} and two sets of different operational frequencies have been calculated. Results are given in Tables 1a and 1b and plotted in Fig. 2. It is believed that this set of parameter values is very close to the level of current technology for deployable, rib-mesh parabolic space antennas.

Also, the minimum-weight designs associated with the same set of parameter values, except

$$a_8 = 0.3 \times 10^{-4}, \qquad a_{12} = 0.15 \times 10^{-4}$$

are given in Tables 1(c) and 1(d). These values of a_8 and a_{12} require an improvement of the rms manufacturing tolerance δ_m and the rms residue tolerance δ_c by a factor of 4 over the previous example. Therefore, with a 4.267-m-diam antenna, $\delta_m = 1.27 \times 10^{-2}$ cm and $\delta_c = 0.635 \times 10^{-2}$ cm. This level of antenna technology, however, seems unlikely for the time being for deployable, rib-mesh parabolic antennas.

Table 1 shows the optimum design associated with double-frequency operation. The optimum design associated with a single-frequency operation is exactly the same. This is because, for this particular example, the optimum result is not constrained by Eq. (26b).

As observed from Fig. 2 and Table 1, the optimum design associated with high-frequency operation (Tables 1b and 1d) is superior to that associated with low-frequency operation (Tables 1a and 1c). This is compatible with the result of the previous section: the optimum wave lengths are λ_0 and $\lambda_0 + \Delta\lambda_0$. Therefore, from the standpoint of optimum antenna design, high-frequency operation is more desirable.

Comparison of results given in Tables 1a and 1b with those given in Tables 1c and 1d indicates that the effect of improving the rms manufacturing tolerance δ_m and the rms residue tolerance δ_c on the optimum antenna design is negligible when the gain constraint is less than 50 dB. The effect is significant, however, when the gain constraint is higher than 50 dB. This simply indicates that for the low-gain antenna (less than 50 dB), effort spent in decreasing manufacturing and residue tolerances is not necessary, and therefore such effort should be concentrated in decreasing the material density. On the other hand, decreasing manufacturing and residue tolerances are desirable for high-gain antennas (higher than 50 dB).

For the optimum designs given in Tables 1a and 1b, the maximum achievable gain is 58.9 dB, and hence the results associated with 60-dB gain constraint are not given.

B. Minimum-Cost Design

The same problem as in example (1) is considered for the minimum-cost design where additional parameter values are employed, as follows: $\gamma = 0.5$, $a_2 = 2.0$, $a_3 = 1.0$, $a_4 = 1.0$, $a_{10} = 0.5$. These parameter values are assumed to be constant over the entire design region defined by D, N, and b. The optimum designs are listed in Tables 2a-2d. For this particular example, optimum designs associated with the single-frequency operation are the same as those associated with the double-frequency operation, since the results are not constrained by G_{02}. Again, higher-frequency operation results in a lower minimum cost.

For those optimum designs for which $b > 1$, the improvement of the current manufacturing technology is worthwhile. This is particularly true for high-gain antennas, as can be observed from Tables 2a-2d.

V. DISCUSSION

From the optimization scheme discussed previously, it is clear that the parameters such as Y, a_{10}, a_4, a_5, a_8, a_{11}, and a_{12} take different values for different antenna concepts, different structural types, and different materials used. A particular antenna concept as well as its associated optimum design is thus characterized by these parameters. It is interesting, therefore, from the results of the numerical computation, to examine (1) the effect of these parameters on the optimization results, (2) the conditions under which the improvement of certain technologies, such as manufacturing technology and composite material technology, is necessary or worthwhile, and (3) the applicability of a certain antenna concept to high-gain requirements. Although the discussion is essentially for the deployable rib-mesh parabolic antenna, some aspects of the discussion hold, in general, for other antennas as well.

A. The Effect of Y

The parameter Y is an indicator representing the relative importance of antenna cost to the cost of payload. This value also depends on a particular mission. When Y is small, the antenna weight is much more important than the antenna cost. Since the weight is not a function of the precision measure b, the optimum design will result in a higher value of b, and hence a smaller rms tolerance is allocated to the manufacture. This can be shown from the cost function of Eq. (18). As Y increases, the antenna cost becomes more important, so that a smaller value of b is obtained for the optimum design. It is observed, however, that the optimum design is, in general, less sensitive to the variation of Y.

B. The Effect of a_{10}

The parameter a_{10} is a measure of the increment (or decrement) of the antenna cost associated with the increment (or decrement) of the precision of manufacture. When the value of a_{10} is high, the cost of the antenna increases rapidly with respect to the manufacturing precision. Hence, in the optimum design, a higher value of a_{10} results in a lower value of b, and thus a larger portion of the entire rms tolerance is allocated to the manufacture.

C. The Effect of a_8, a_{12}, a_{11}, and a_5

The parameters a_8 and a_{12} represent, respectively, the level of manufacturing technology and the thermal coefficient of expansion of material used. Smaller values of a_8 or a_{12} will result in a less costly or lightweight optimum design. This effect can easily be observed from the formulation. The parameters a_5 and a_{11} relate the rms design tolerance to the antenna weight and characterize a specific antenna design. Smaller values of a_5 and a_{11} will result in a lighter optimum antenna.

A number of numerical results indicate that the optimum antenna diameter is very insensitive to the change of parameter values, e.g., a_4, a_5, a_8, a_{10}, a_{11}, and a_{12}. Optimum diameter is, however, very sensitive to the change of maximum allowable operational frequencies (or the minimum allowable wave lengths λ_0 and $\Delta\lambda_0$). Once λ_0 and $\Delta\lambda_0$ are given or identified, the change of the optimum antenna diameter due to the variation of parameter values is very small. It is because of this characteristic that the successive approximation technique described previously converges rapidly.

For the particular antenna discussed in the numerical examples, the antenna weight increases rapidly as the gain constraint increases beyond 50 dB, and it soon becomes intolerable. This indicates that in a straightforward application of such an antenna technology for high-gain performance (e.g., higher than 52 dB), the following efforts should be made: (1) The density of rib material should be reduced, e.g., by the use of low-density composite material. (2) The rms manufacturing tolerance occupies a large portion of the entire rms deformation, which indicates the necessity of improving the manufacturing technology. Another solution may be the use of another type of structure, such as the Convair truss-mesh antenna (Ref. 9). (3) The use of composite materials with low coefficient of thermal expansion is important in order to lessen antenna weight as well as gain degradation.

The significance of the effort made in items (2) and (3) can be observed by comparing Tables 1a and 1b to Tables 1c and 1d. For the low-gain antenna (less than 50 dB), however, the above efforts appear to be unnecessary as indicated by the numerical results. Similar observations also hold for the minimum-cost design.

Table 1a. Minimum-weight design (f_1 = 8.46 GHz; f_2 = 7.46 GHz; $a_8 = 0.119 \times 10^{-3}$; $a_{12} = 0.6 \times 10^{-4}$)

Parameter	Value					
Gain constraint, dB G_{01}	40	45	50	52.5	55	57.5
G_{02}	38	43	48	50.5	53	55.5
Efficiency η	0.6	0.6	0.6	0.6	0.6	0.6
Diameter D, m	1.551	2.774	5.03	6.83	9.51	14.1
Weight per unit area N, kg/m^2	0.786	0.923	1.084	1.196	1.328	1.552
Manufacturing tolerance δ_m (rms), 10^{-3} m	0.1854	0.330	0.597	0.813	1.13	1.68
Design tolerance δ_s (rms), 10^{-3} m	0.965	0.965	0.965	0.927	0.879	0.732
Residue tolerance δ_c (rms), 10^{-3} m	0.0932	0.1651	0.302	0.409	0.569	0.846
Total tolerance δ (rms), 10^{-3} m	0.996	1.034	1.176	1.290	1.542	2.019
Gain, dB G_1	40	45	50	52.5	55	57.5
G_2	39	44	49.08	51.61	54.19	56.9
Gain degradation, dB ΔG_1	0.542	0.582	0.755	0.908	1.3	2.23
ΔG_2	0.421	0.453	0.587	0.706	1.0	1.73
Weight W, kg	1.48	5.58	21.56	43.82	94.35	243.13

Table 1b. Minimum-weight design (f_1 = 12 GHz; f_2 = 10 GHz; $a_8 = 0.119 \times 10^{-3}$; $a_{12} = 0.6 \times 10^{-4}$)

Parameter	Value					
Gain constraint, dB G_{01}	40	45	50	52.5	55	57.5
G_{02}	38	43	48	50.5	53	55.5
Efficiency η	0.6	0.6	0.6	0.6	0.6	0.6
Diameter D, m	1.091	1.948	3.548	4.812	6.700	9.955
Weight per unit area N, kg/m^2	0.7958	0.9326	1.0888	1.2011	1.333	1.5575
Manufacturing tolerance δ_m (rms), 10^{-3} m	0.1298	0.2316	0.422	0.572	0.797	1.185
Design tolerance δ_s (rms), 10^{-3} m	0.659	0.658	0.673	0.638	0.609	0.511
Residue tolerance δ_c (rms), 10^{-3} m	0.0655	0.1168	0.2134	0.289	0.402	0.597
Total tolerance δ (rms), 10^{-3} m	0.676	0.708	0.823	0.905	1.081	1.422
Gain, dB G_1	40	45	50	52.5	55	57.5
G_2	38.59	43.59	48.66	51.2	53.8	56.6
Gain degradation, dB ΔG_1	0.5	0.55	0.744	0.90	1.282	2.22
ΔG_2	0.348	0.382	0.517	0.625	0.891	1.54
Weight W, kg	0.742	2.77	10.76	21.85	46.99	121.25

Table 1c. Minimum-weight design (f_1 = 8.46 GHz; f_2 = 7.46 GHz; $a_8 = 0.3 \times 10^{-4}$; $a_{12} = 0.15 \times 10^{-4}$)

Parameter		Value						
Gain constraint, dB	G_{01}	40	45	50	52.5	55	57.5	60
	G_{02}	38	43	48	50.5	53	55.5	58
Efficiency η		0.6	0.6	0.6	0.6	0.6	0.6	0.6
Diameter D, m		1.548	2.752	4.907	6.538	8.748	11.71	15.71
Weight per unit area N, kg/m^2		0.7861	0.9277	1.0741	1.167	1.2597	1.3622	1.489
Manufacturing tolerance δ_m (rms), 10^{-3} m		0.0465	0.0826	0.1471	0.1961	0.262	0.351	0.471
Design tolerance δ_s (rms), 10^{-3} m		0.973	0.960	0.984	0.968	0.970	0.975	0.955
Residue tolerance δ_c (rms), 10^{-3} m		0.0231	0.0414	0.0737	0.0965	0.1311	0.1753	0.2357
Total tolerance δ (rms), 10^{-3} m		0.975	0.965	0.998	0.993	1.024	1.051	1.091
Gain, dB	G_1	40	45	50	52.5	55	57.5	60
	G_2	39.03	44.03	49.03	51.53	54.03	56.54	59.06
Gain degradation, dB	ΔG_1	0.518	0.508	0.5435	0.537	0.561	0.603	0.65
	ΔG_2	0.403	0.395	0.4195	0.418	0.436	0.469	0.505
Weight W, kg		1.48	5.49	20.32	39.19	75.66	147.01	288.53

Table 1d. Minimum-weight design (f_1 = 12 GHz; f_2 = 10 GHz; $a_8 = 0.3 \times 10^{-4}$; $a_{12} = 0.15 \times 10^{-4}$)

Parameter		Value						
Gain constraint, dB	G_{01}	40	45	50	52.5	55	57.5	60
	G_{02}	38	43	48	50.5	53	55.5	58
Efficiency η		0.6	0.6	0.6	0.6	0.6	0.6	0.6
Diameter D, m		1.091	1.945	3.456	4.621	6.163	8.230	10.994
Weight per unit area N, kg/m^2		0.796	0.923	1.074	1.162	1.262	1.372	1.518
Manufacturing tolerance δ_m (rms), 10^{-3} m		0.0328	0.0582	0.1036	0.1387	0.1849	0.2471	0.330
Design tolerance δ_s (rms), 10^{-3} m		0.660	0.683	0.692	0.693	0.685	0.678	0.620
Residue tolerance δ_c (rms), 10^{-3} m		0.0165	0.0292	0.0518	0.0693	0.0925	0.1237	0.1648
Total tolerance δ (rms), 10^{-3} m		0.660	0.686	0.701	0.710	0.715	0.732	0.721
Gain, dB	G_1	40	45	50	52.5	55	57.5	60
	G_2	38.62	43.6	48.58	51.09	53.59	56.1	58.61
Gain degradation, dB	ΔG_1	0.48	0.518	0.54	0.554	0.561	0.5875	0.571
	ΔG_2	0.33	0.36	0.375	0.385	0.39	0.408	0.3963
Weight W, kg		0.744	2.735	10.08	19.48	37.65	73.09	144.15

Table 2a. Minimum-cost design (f_1 = 8.46 GHz; f_2 = 7.46 GHz; $a_8 = 0.119 \times 10^{-3}$; $a_{12} = 0.6 \times 10^{-4}$)

Parameter		Value						
Gain constraint, dB	G_{01}	40	45	50	52.5	55	57.5	60
	G_{02}	38	43	48	50.5	53	55.5	58
Efficiency η		0.6	0.6	0.6	0.6	0.6	0.6	0.6
Diameter D, m		1.570	2.807	5.060	6.797	9.083	12.405	17.008
Weight per unit area N, kg/m^2		0.806	0.933	1.103	1.191	1.294	1.416	1.562
Manufacturing tolerance δ_m (rms), 10^{-3} m		0.587	0.599	0.683	0.709	0.721	0.792	0.813
Design tolerance δ_s (rms), 10^{-3} m		0.899	0.942	0.927	0.942	0.919	0.907	0.866
Residue tolerance δ_c (rms), 10^{-3} m		0.0940	0.1684	0.302	0.406	0.546	0.743	1.020
Total tolerance δ (rms), 10^{-3} m		1.080	1.130	1.194	1.247	1.290	1.415	1.567
Gain, dB	G_1	40	45	50	52.5	55.0	57.5	60.0
	G_2	39.1	44.1	49.1	51.6	54.1	56.7	59.2
Gain degradation, dB	ΔG_1	0.636	0.697	0.77	0.848	0.908	1.095	1.34
	ΔG_2	0.495	0.542	0.632	0.66	0.71	0.85	1.04
Weight W, kg		1.556	5.77	22.14	42.77	84.14	170.51	354.30
Precision measure b		0.318	0.557	0.876	1.138	1.50	1.86	2.49

Table 2b. Minimum-cost design (f_1 = 12 GHz; f_2 = 10 GHz; $a_8 = 0.119 \times 10^{-3}$; $a_{12} = 0.6 \times 10^{-4}$)

Parameter		Value						
Gain constraint, dB	G_{01}	40	45	50	52.5	55	57.5	60
	G_{02}	38	43	48	50.5	53	55.5	58
Efficiency η		0.6	0.6	0.6	0.6	0.6	0.6	0.6
Diameter D, m		1.116	1.990	3.566	4.801	6.462	8.729	12.024
Weight per unit area N, kg/m^2		0.801	0.933	1.094	1.191	1.299	1.421	1.562
Manufacturing tolerance δ_m (rms), 10^{-3} m		0.455	0.468	0.505	0.516	0.527	0.567	0.588
Design tolerance δ_s (rms), 10^{-3} m		0.658	0.6622	0.6617	0.655	0.645	0.627	0.614
Residue tolerance δ_c (rms), 10^{-3} m		0.0668	0.1194	0.2139	0.288	0.388	0.524	0.721
Total tolerance δ (rms), 10^{-3} m		0.803	0.820	0.860	0.893	0.919	0.994	1.115
Gain, dB	G_1	40	45	50	52.5	55	57.5	60
	G_2	38.63	43.64	48.67	51.2	53.75	56.25	58.83
Gain degradation, dB	ΔG_1	0.71	0.738	0.812	0.896	0.927	1.086	1.365
	ΔG_2	0.5	0.512	0.564	0.622	0.644	0.754	0.948
Weight W, kg		0.780	2.90	10.95	21.59	42.59	85.05	176.99
Precision measure b		0.291	0.505	0.84	1.137	1.46	1.83	2.435

Table 2c. Minimum-cost design (f_1 = 8.46 GHz; f_2 = 7.46 GHz; a_8 = 0.3×10^{-4}; a_{12} = 0.15×10^{-4})

Parameter		Value						
Gain constraint, dB	G_{01}	40	45	50	52.5	55	57.5	60
	G_{02}	38	43	48	50.5	53	55.5	58
Efficiency η		0.6	0.6	0.6	0.6	0.6	0.6	0.6
Diameter D, m		1.567	2.798	4.999	6.645	8.900	11.887	15.941
Weight per unit area N, kg/m^2		0.791	0.928	1.074	1.177	1.269	1.387	1.494
Manufacturing tolerance δ_m (rms), 10^{-3} m		0.487	0.550	0.577	0.587	0.641	0.654	0.683
Design tolerance δ_s (rms), 10^{-3} m		0.956	0.959	0.965	0.955	0.945	0.942	0.940
Residue tolerance δ_c (rms), 10^{-3} m		0.0235	0.0419	0.0749	0.0996	0.1336	0.1783	0.2388
Total tolerance δ (rms), 10^{-3} m		1.073	1.113	1.125	1.133	1.148	1.161	1.191
Gain, dB	G_1	40	45	50	52.5	55	57.5	60
	G_2	39.05	44.05	49.06	51.55	54.1	56.6	59.1
Gain degradation, dB	ΔG_1	0.629	0.663	0.7	0.69	0.72	0.735	0.775
	ΔG_2	0.49	0.515	0.54	0.535	0.56	0.571	0.602
Weight W, kg		1.526	5.702	21.20	41.06	79.51	153.7	298.3
Precision measure b		0.095	0.153	0.26	0.34	0.42	0.545	0.7

Table 2d. Minimum-cost design (f_1 = 12 GHz; f_2 = 10 GHz; a_8 = 0.3×10^{-4}; a_{12} = 0.15×10^{-4})

Parameter		Value						
Gain constraint, dB	G_{01}	40	45	50	52.5	55	57.5	60
	G_{02}	38	43	48	50.5	53	55.5	58
Efficiency η		0.6	0.6	0.6	0.6	0.6	0.6	0.6
Diameter D, m		1.100	1.963	3.527	4.709	6.288	8.397	11.247
Weight per unit area N, kg/m^2		0.796	0.928	1.080	1.172	1.269	1.382	1.499
Manufacturing tolerance δ_m (rms), 10^{-3} m		0.329	0.368	0.416	0.434	0.450	0.472	0.502
Design tolerance δ_s (rms), 10^{-3} m		0.655	0.663	0.691	0.684	0.678	0.665	0.663
Residue tolerance δ_c (rms), 10^{-3} m		0.0165	0.0295	0.0533	0.0706	0.0942	0.1260	0.1689
Total tolerance δ (rms), 10^{-3} m		0.734	0.757	0.808	0.813	0.818	0.826	0.848
Gain, dB	G_1	40	45	50	52.5	55	57.5	60
	G_2	38.6	43.6	48.64	51.14	53.64	56.15	58.66
Gain degradation, dB	ΔG_1	0.59	0.63	0.717	0.73	0.736	0.75	0.79
	ΔG_2	0.41	0.44	0.498	0.51	0.51	0.521	0.55
Weight W, kg		0.757	2.826	10.56	20.41	39.46	76.61	149.2
Precision measure b		0.1	0.16	0.254	0.33	0.42	0.533	0.671

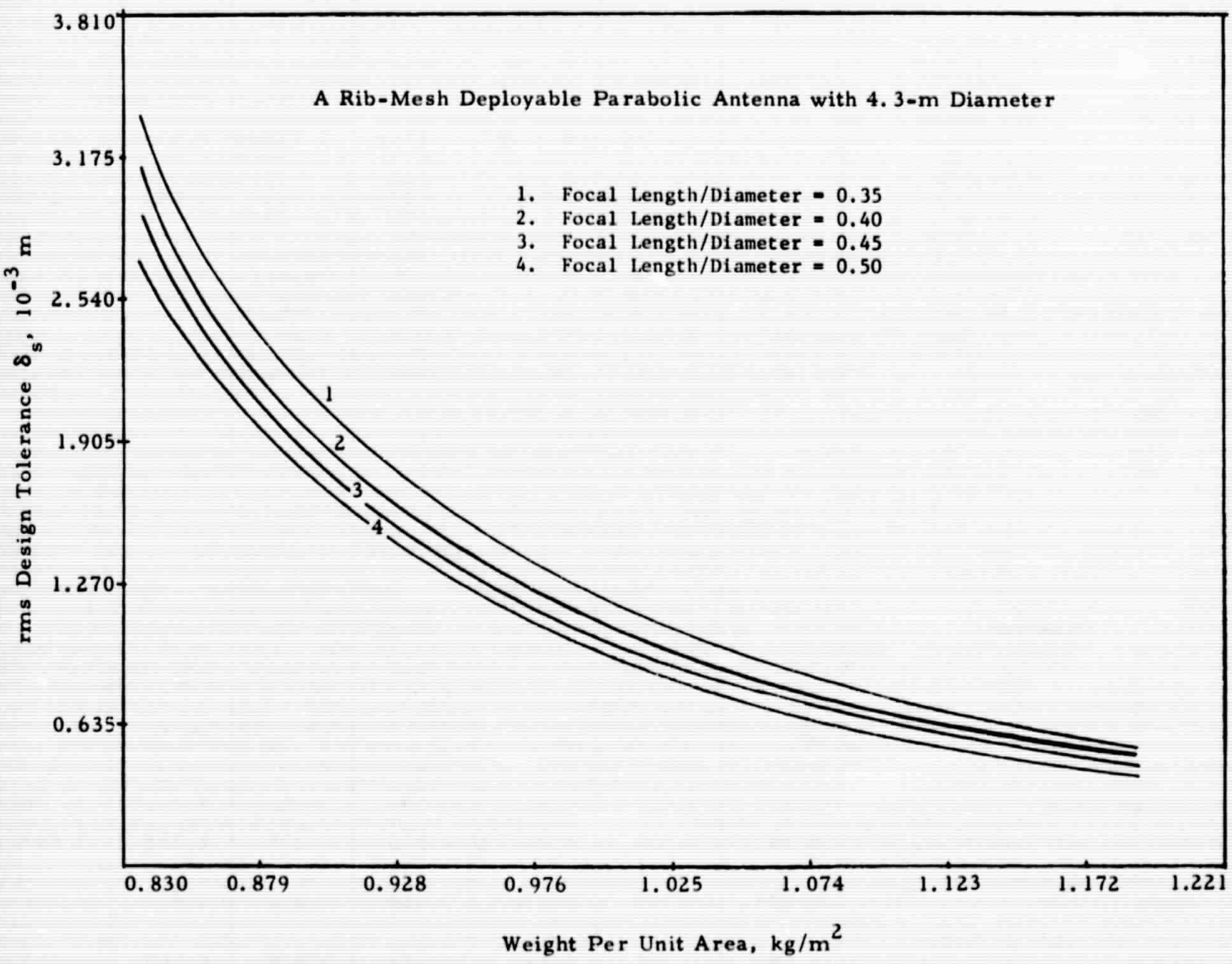

Fig. 1. Design tolerance δ_s vs weight per unit area

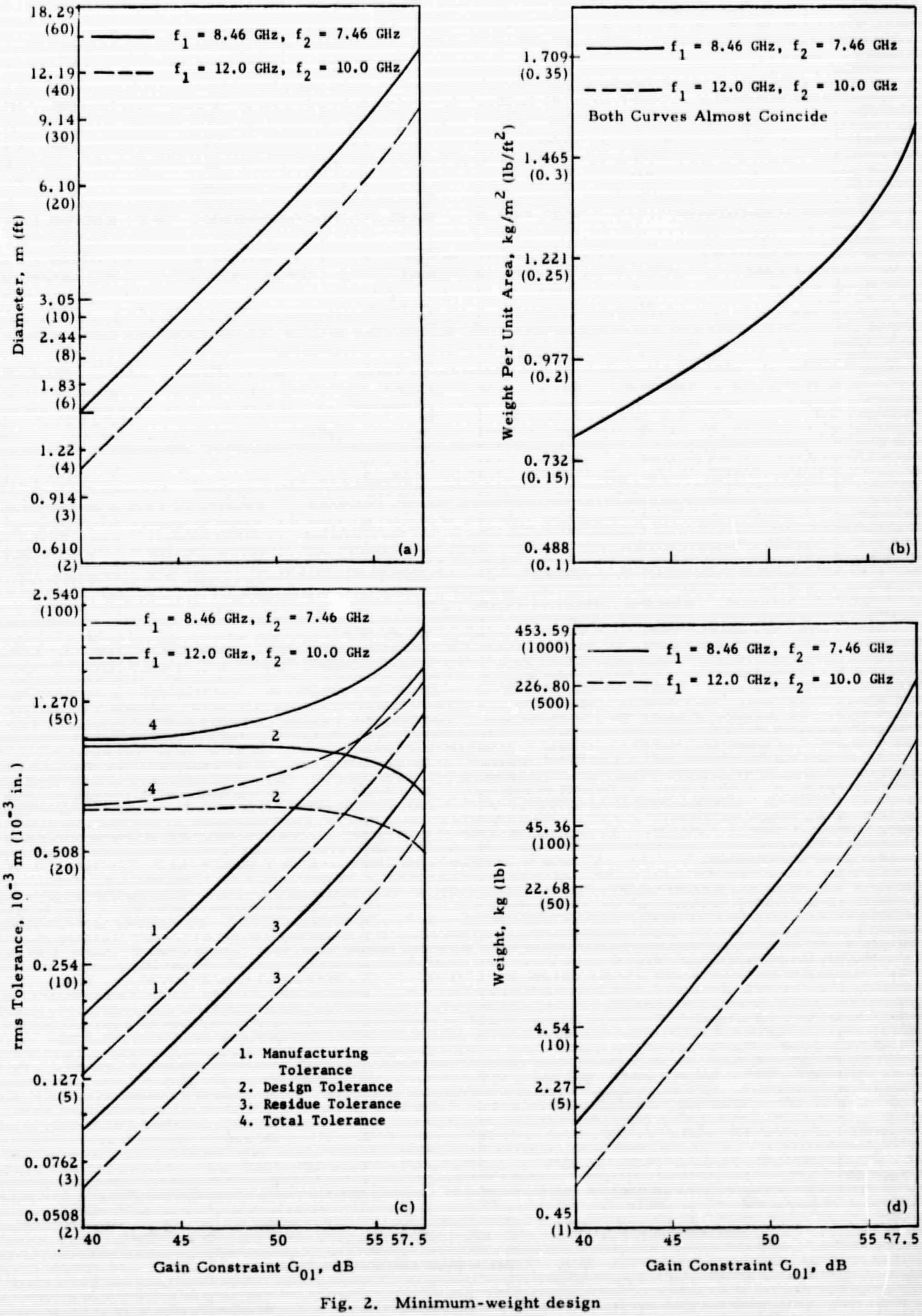

Fig. 2. Minimum-weight design

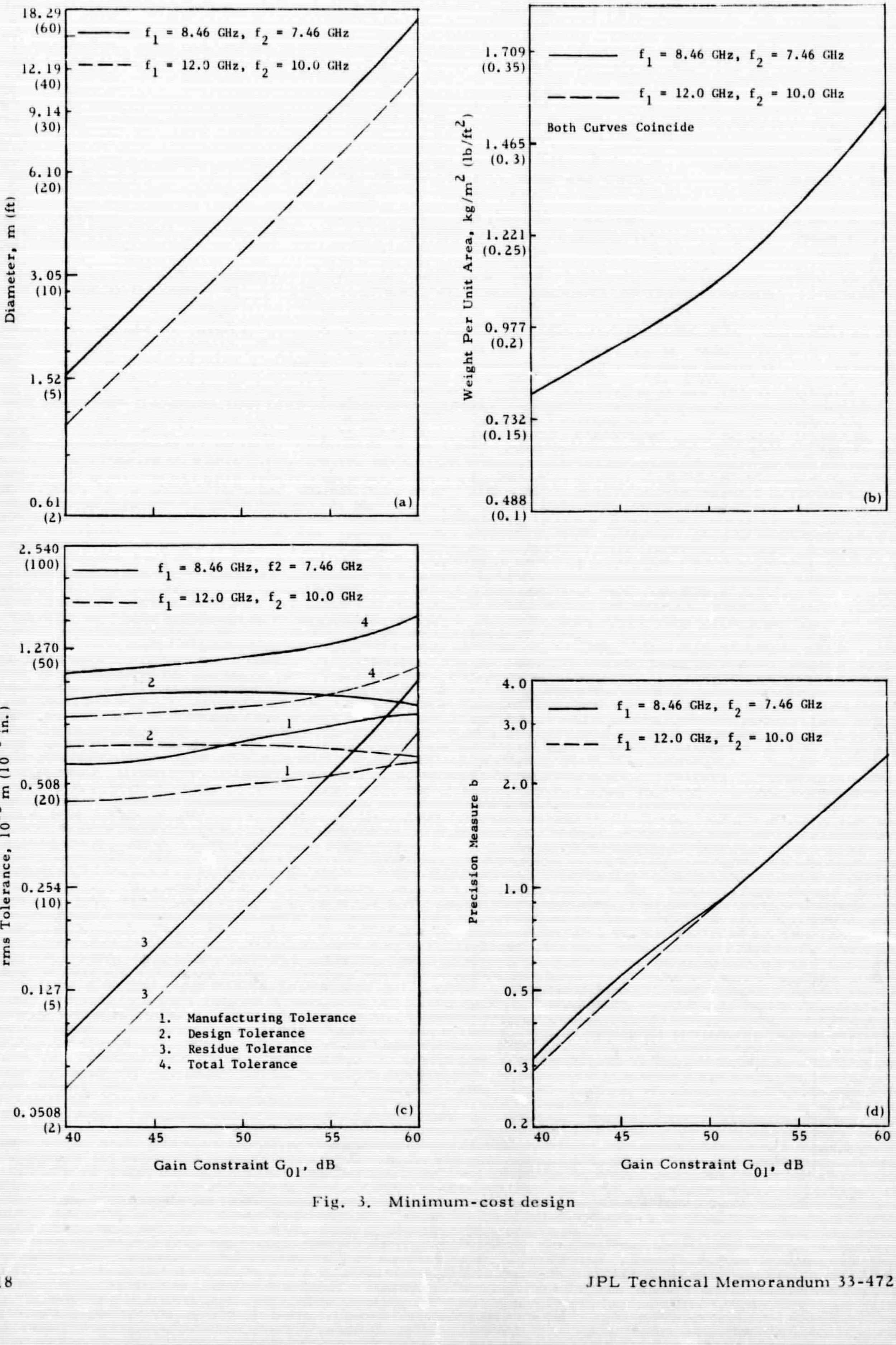

Fig. 3. Minimum-cost design

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