N71-23238

03346-2-T

NASA CR-117897

THE UNIVERSITY OF MICHIGAN

COLLEGE OF ENGINEERING

High Altitude Engineering Laboratory

Departments of

Aerospace Engineering

Meteorology and Oceanography

Technical Report

PERTURBATIONS TO OBSERVED AMBIENT NEUTRAL DENSITIES DUE TO PRESENCE OF AN ORBITING GEOPHYSICAL OBSERVATORY

B. B. Hinton

R. J. Leite

ORA Project 03346

under contract with:

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

GRANT NO. NGR 23-005-383

WASHINGTON, D. C.

administered through

OFFICE OF RESEARCH ADMINISTRATION, ANN ARBOR

January 1971

03346-2-T

THE UNIVERSITY OF MICHIGAN

COLLEGE OF ENGINEERING

High Altitude Engineering Laboratory

Departments of

Aerospace Engineering
Meteorology and Oceanography

Technical Report

PERTURBATIONS TO OBSERVED AMBIENT NEUTRAL DENSITIES DUE TO PRESENCE OF AN ORBITING GEOPHYSICAL OBSERVATORY

B. B. Hinton
R. J. Leite

ORA Project 03346

under contract with:

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

GRANT NO. NGR 23-005-383

WASHINGTON, D. C.

administered through

OFFICE OF RESEARCH ADMINISTRATION, ANN ARBOR

January 1971

ABSTRACT

A spacecraft perturbs the density of ions and neutral particles of the ambient atmosphere as it moves through its tenuous upper regions. In this report, estimates are given for the resultant perturbations on measured values of ambient neutral particle densities for mass spectrometer experiments mounted in an Orbital Plane Experiment Package (OPEP) of an Orbiting Geophysical Observatory (OGO).

TABLE OF CONTENTS

LIST OF FIGURES

I. INTRODUCTION AND PURPOSE

The neutral and positive ion concentrations obtained by in situ mass spectrometer measurements on an earth-orbiting vehicle must be adjusted to account for the influence of the vehicle.[1] What follows is a description of the work performed in applying the model developed in Reference 2 to the OGO-IV Experiment 15 measurements of neutral-particle concentrations and a presentation of the results of this work. The purpose of this report is to indicate that, while it was significant to perform the calculations, the results indicate that for most measurement considerations the perturbations are small.

The near polar orbit of OGO-IV had a perigee of about 400 km and an apogee of 900 km. In this altitude range, inter-particle collisions may be neglected since the spacecraft dimensions are orders of magnitude smaller than the mean free path. Except for the lightest ambient atoms, spacecraft speed is much greater than their thermal speeds. Therefore, at least for the intermediate and heavier ambient masses, the densities and fluxes are significantly greater on surfaces looking into the velocity vector, and significantly less on surfaces looking away from the velocity. However, the density and flux are determined not only by exposure to the incident particles, but also by reflections of particles from one surface to another. The calculation of this transfer from one surface to another, as well as the directly incident particle densities and fluxes, is described in this report.

The results presented in this report concern only incident ambient particles reflected from spacecraft surfaces and exclude all contributions due to the outgassing and/or leakage of gases from the spacecraft.

II. ASSUMPTIONS AND ELEMENTARY CONSIDERATIONS

The problem is not solvable without certain simplifying assumptions because the interaction of atoms and molecules with surfaces is a poorly understood phenomenon, the surfaces themselves are not well defined, and, in addition, the surface structure may even change with time. Consequently, in order to approximate the real situation by a solvable problem we assumed the following:

(ia) The problem can be approximated for atomic oxygen by a steady state in which the flux of atoms arriving at a surface is equal to the flux of the same species of atoms plus the flux of the recombined diatomic molecules (of the same species of atoms) leaving the surface, the incident molecular oxygen being neglected;

(ib) For other species there is assumed to be no chemical change; hence, the incoming and outgoing fluxes are assumed to be equal.

(ii) The molecules leaving the surface have interacted strongly enough with the surface that they have "forgotten" their direction of arrival and are directed isotropically; or alternatively, we could assume that on the molecular, or atomic scale the surfaces are "rough" and arrive at the same isotropic distribution.

(iii) Intermolecular collisions are neglected.

In order to obtain meaningful results, the solutions of two "elementary" problems were used. The first of these is the calculation of the incident flux of ambient particles on a moving surface and the other is the transfer of particles from one surface to another.

The results of these elementary considerations were applied to the OGO-IV

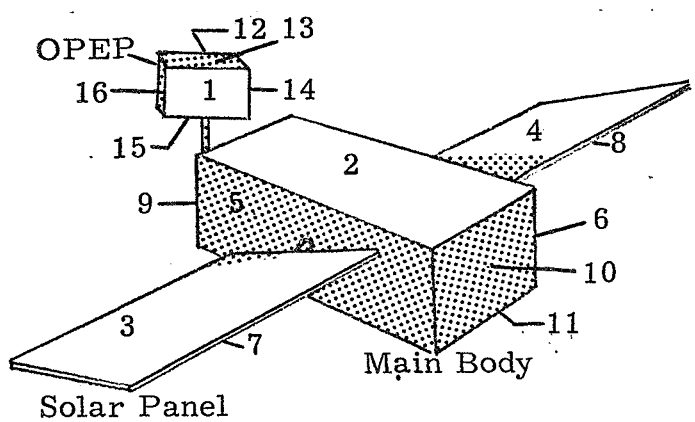

Figure 1. The OGO Spacecraft.

spacecraft pictured schematically in Figure 1. The numbers refer to various surfaces. Quantities related to these surfaces were labeled with subscripts corresponding to these numbers. OGO-IV Experiment 15 was mounted in the OPEP* at the surface labeled "1" in the Figure 1. Viewed from this surface, the shaded areas are shielded and play a secondary role in the problem. Surface 10 cannot be seen from any other surface to be considered; hence, in the absence of intermolecular collisions (assumption (iii)) this surface plays no role in the problem. Surface 11 can only exchange flux with the portions of 7 and 8 which cannot exchange this flux with 1, 2, 3, 4, or 9. The flux from 11 can contribute only in a multiple-bounce process, which can be neglected since the incident flux on 11 should itself be negligibly small as the surface is nearly always parallel to the velocity $\vec{V}$ when the spacecraft is functioning properly. Similar reasoning lead to the elimination of 12, 13, 14, 15, and 16. Consequently, the only surfaces which must be considered are 1, 2, 3, 4, 5, 6, 7, 8, and 9. Furthermore, no flux transfer between certain pairs of these is permitted by the geometry.

---

*Orbital Plane Experiment Package

III. EXPANSION OF EXPRESSIONS FOR FLUXES

The notation used in this section, and explicitly defined below, follows that of Appendix A.

Φ_i is the total influx per unit area at a point on the surface S_i from all directions.

Φ_{ij} is the total influx per unit area at a point on a surface S_j from the entire surface S_i, except that when i=o, Φ_{oj} denotes the influx per unit area at a point onto S_j from all "free" space.

ϕ_{ij} is that portion of the flux from a unit area of dS_i falling upon a unit area of dS_j. For i=o, ϕ_{oj} is the flux per unit area falling upon dS_j from "free" space having trajectories within the solid angle $d\Omega$.

$X_{ij} = \frac{1}{2\pi}[(\hat{n}_i \cdot \hat{r}_{ij})(\hat{n}_j \cdot \hat{r}_{ji})/r_{ij}^2]$, from (A6) where $\hat{n}_i$ and $\hat{n}_j$ are the unit surface normals at points on S_i and $\hat{r}_{ij}$ is the vector from the point on S_i to the point on S_j. Note also that $X_{ij} = X_{ji}$.

$\phi_{ij} = X_{ij}\Phi_i$

$\langle v_n \rangle_{ij} = (\hat{n}_j \cdot \hat{r}_{ji})\langle v \rangle_j$ where $\langle v \rangle_j$ is the average speed of particles emitted from S_j.

$\phi_{oj} = (\langle v_n \rangle_o \; N_a)_j$ (from A2, A3)

$\Phi_j = \Sigma\Phi_{ij}$

$\Phi_{ij} = \int \phi_{ij} dS_i$

$\Phi_{oj} = \int_{\Omega_0} \phi_{oj} d\Omega$

The procedure consisted of expanding the Φ's, as sums of other Φ's, and expanding these in turn with the aim of obtaining the ultimate expansion in terms of Φ_{oj}'s since these can be related to the ambient density. Each re-expansion generated terms of higher orders in the X's. Considering the severity of assumption (ii) and the fact that the integrated X's are of the order of 0.2 or less, it was thought reasonable to limit consideration to order two or three in X. For example, if we omit all terms of order three and higher (see Reference 2), then:

$$\Phi_{21} = \int \Phi_{02} X_{12} dS_2 + \int X_{12} \int \Phi_{01} X_{12} dS_1 dS_2 + \int X_{12} \int \Phi_{03} X_{23} dS_3 dS_2 + \int X_{12} \int \Phi_{04} X_{24} dS_4 dS_2$$

$$+ \int X_{12} \int \Phi_{07} X_{27} dS_7 dS_2 + \int X_{12} \int \Phi_{08} X_{28} dS_8 dS_2; \qquad (1)$$

$$\Phi_{31} = \int \Phi_{03} X_{13} dS_3 + \int X_{13} \int \Phi_{01} X_{13} dS_1 dS_3 + \int X_{13} \int \Phi_{05} X_{35} dS_5 dS_3 + \int X_{13} \int \Phi_{02} X_{23} dS_2 dS_3; \qquad (2)$$

$$\Phi_{41} = \int \Phi_{04} X_{14} dS_4 + \int X_{14} \int \Phi_{01} X_{14} dS_1 dS_4 + \int X_{14} \int \Phi_{06} X_{46} dS_6 dS_4 + \int X_{14} \int \Phi_{02} X_{24} dS_2 dS_4; \qquad (3)$$

$$\Phi_{91} = \int \Phi_{09} X_{19} ds_9 + \int X_{19} \int \Phi_{0,16} X_{1,16} dS_{16} dS_9. \qquad (4)$$

For reference, each of the additive terms in (1) through (4) was given a symbolic name by extending the Φ notation already introduced, namely:

$$\begin{array}{llll}
\int X_{12} \Phi_{02} dS_2 = \Phi_{021}; & & \int X_{12} \int X_{21} \Phi_{01} dS_1 dS_2 = \Phi_{0121}; & \\
\int X_{12} \int X_{23} \Phi_{03} dS_3 dS_2 = \Phi_{0321}; & & \int X_{12} \int X_{24} \Phi_{04} dS_4 dS_2 = \Phi_{0421}; & \\
\int X_{12} \int X_{27} \Phi_{07} dS_7 dS_2 = \Phi_{0721}; & & \int X_{12} \int X_{28} \Phi_{08} dS_8 dS_2 = \Phi_{0821}; & \\
\int X_{13} \Phi_{03} dS_3 = \Phi_{031}; & & \int X_{13} \int X_{31} \Phi_{01} dS_1 dS_3 = \Phi_{0131}; & \\
\int X_{13} \int X_{35} \Phi_{05} dS_3 dS_3 = \Phi_{0531}; & & \int X_{13} \int X_{23} \Phi_{02} dS_2 dS_3 = \Phi_{0231}; & (5) \\
\int X_{14} \Phi_{04} dS_4 = \Phi_{041}; & & \int X_{14} \int X_{41} \Phi_{01} dS_1 dS_4 = \Phi_{0141}; & \\
\int X_{14} \int X_{46} \Phi_{06} dS_6 dS_4 = \Phi_{0641}; & & \int X_{14} \int X_{24} \Phi_{02} dS_2 dS_4 = \Phi_{0241}; & \\
\int X_{19} \Phi_{09} dS_9 = \Phi_{091}; & & \int X_{19} \int X_{1,16} \Phi_{0,16} dS_{16} dS_9 = \Phi_{0,16,91}. &
\end{array}$$

A program was written to calculate the above flux components. The input required was the set of unit vectors describing the relative orientation of the various spacecraft surfaces, the mass number of the constituent being considered, and ambient atmospheric temperature as well as the incident stream velocity vector.

The Φ_{oj}'s were calculated using "unit" ambient density in an expression for (A1) obtained by carrying out the integration. Next, the X_{ij}'s were calculated using the $\hat{n}_i$'s and the $\hat{r}_{ij} = \hat{r}_j - \hat{r}_i$. The $\hat{r}_i$ representing the various points on the $i^{\underline{th}}$ surface. The integrations are approximated by considering $\hat{r}_i$ on lattice points only (separated by a distance Δ, replacing dS_i by Δ^2, and performing integration by summation). Effectively, each summation is confined to its proper range of values by multiplying each "integrand" by one or more "switch" functions. These switches are equal to either zero or one depending on whether a point $\hat{r}_j$ on surface S_j can be seen from $\hat{r}_i$ on S_i. The switches then are functions of $\hat{r}_{ij}$, $\hat{n}_i$, and $\hat{n}_j$.

To interpret the results, it is useful to introduce the spacecraft coordinates illustrated below in Figure 2. The relationships can also be described in terms of the solar array angle, ϕ_p, and the OPEP angle, ψ_e. By definition, ϕ_p is the angle between $+\hat{Y}_b$ and $+\hat{Y}_p$, while ψ_e is the angle between $+\hat{X}_b$ and $+\hat{X}_e$.

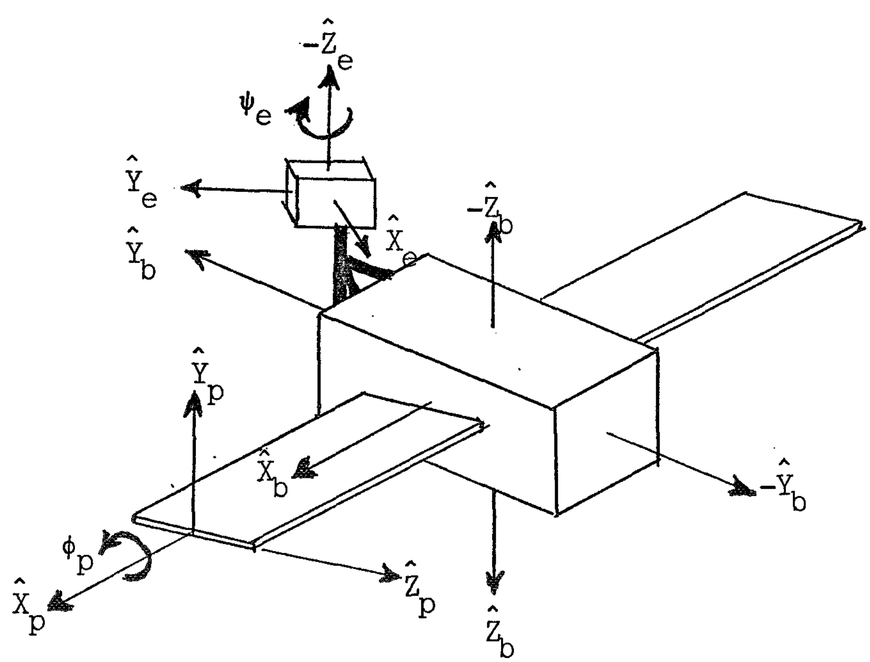

Figure 2. Spacecraft Coordinate Axes

SUMMARY OF RESULTS

The largest contributions, as expected, come from the fluxes incident on the solar array reflected directly to the OPEP, and those incident on the top of the main body and reflected directly to the OPEP. In certain orientations, there is also a noticeable contribution from the flux incident upon the side of the main body, reflected to the solar array and then to the OPEP. Thus, the more important fluxes are: Φ_{021}, Φ_{031}, and Φ_{041}; while Φ_{531} and Φ_{641} are also significant. All other fluxes are quite negligible. In Figure 3, an attempt has been made to summarize all the results (for M=16amu, Ta=1000°K) by showing the envelope which contains the results for all ψ_e and ϕ_p taken in 15° increments. The azimuth angle in this plot is ψ_e while dashed lines indicate the nature of the results for various ϕ_p. The leporine (hare-like) shape arises in the following way:

The "ears" are the result of Φ_{031} and Φ_{041} contributions from the solar array. The "face" results mostly from Φ_{021} with some contribution from Φ_{531} and Φ_{641}, which form the bulging "cheeks". The circle outlining the figure represents approximately a 1% perturbation to the flux due to reflections. For number densities, the reflected contributions would be of the order of $\sqrt{Ta}/\sqrt{Ts}$ times greater where Ts is the characteristic, or "mean" surface temperature of the spacecraft, and Ta is the ambient temperature.* Thus, the number density perturbations resemble Figure 3, with the bounding circle representing approximately 1.8% for Ta$\approx$1000, Ts$\approx$300°K.

Variations of the reflected flux with ψ_e and ϕ_p can be seen in Figure 4. The greatest excursions of these data were used collectively to construct Figure 3.

*See Appendix B.

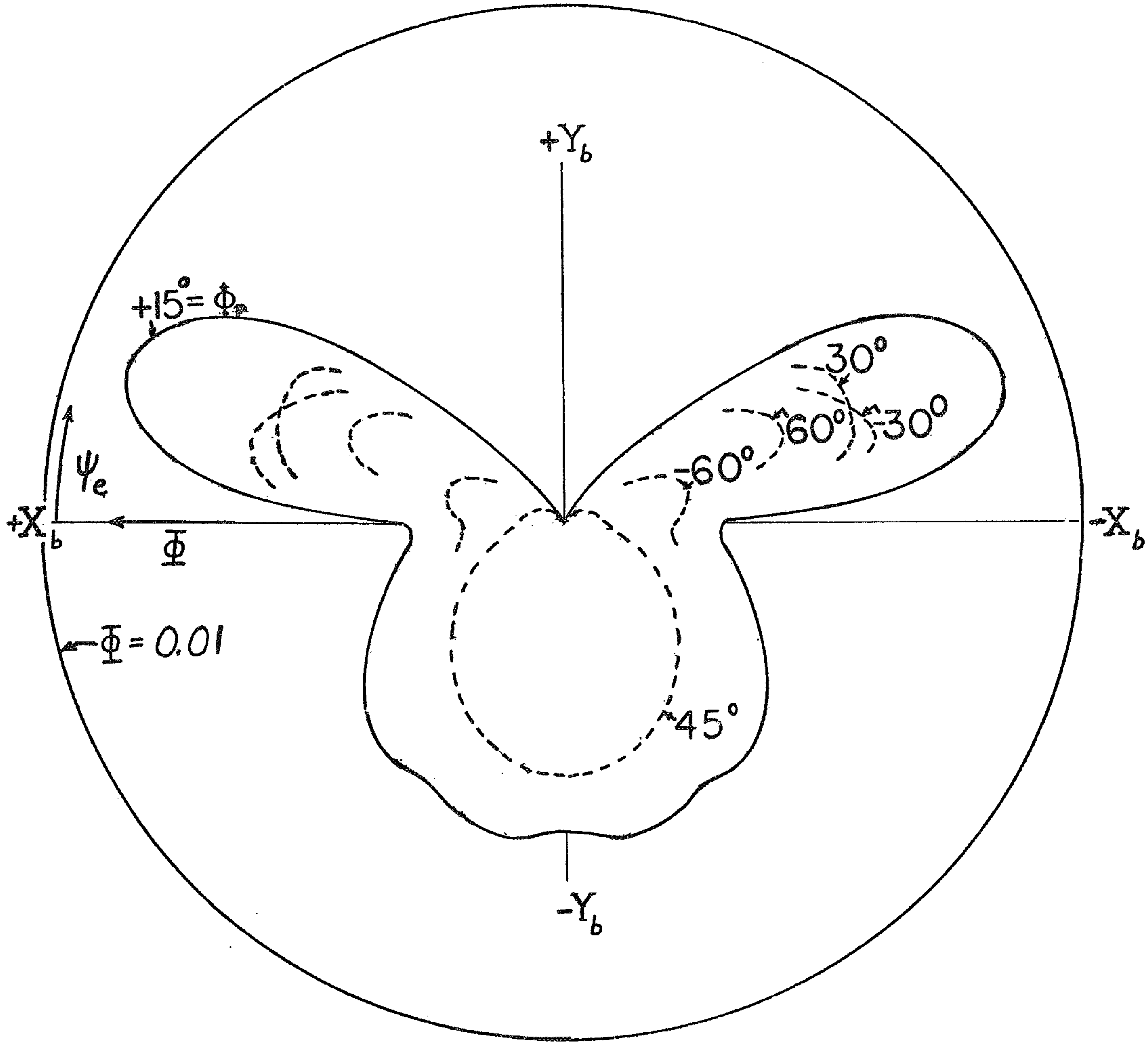

Figure 3. Flux Perturbations Due to Reflected Particles
(M = 16 amu, Ta = 1000°K, Ts = 300°K)

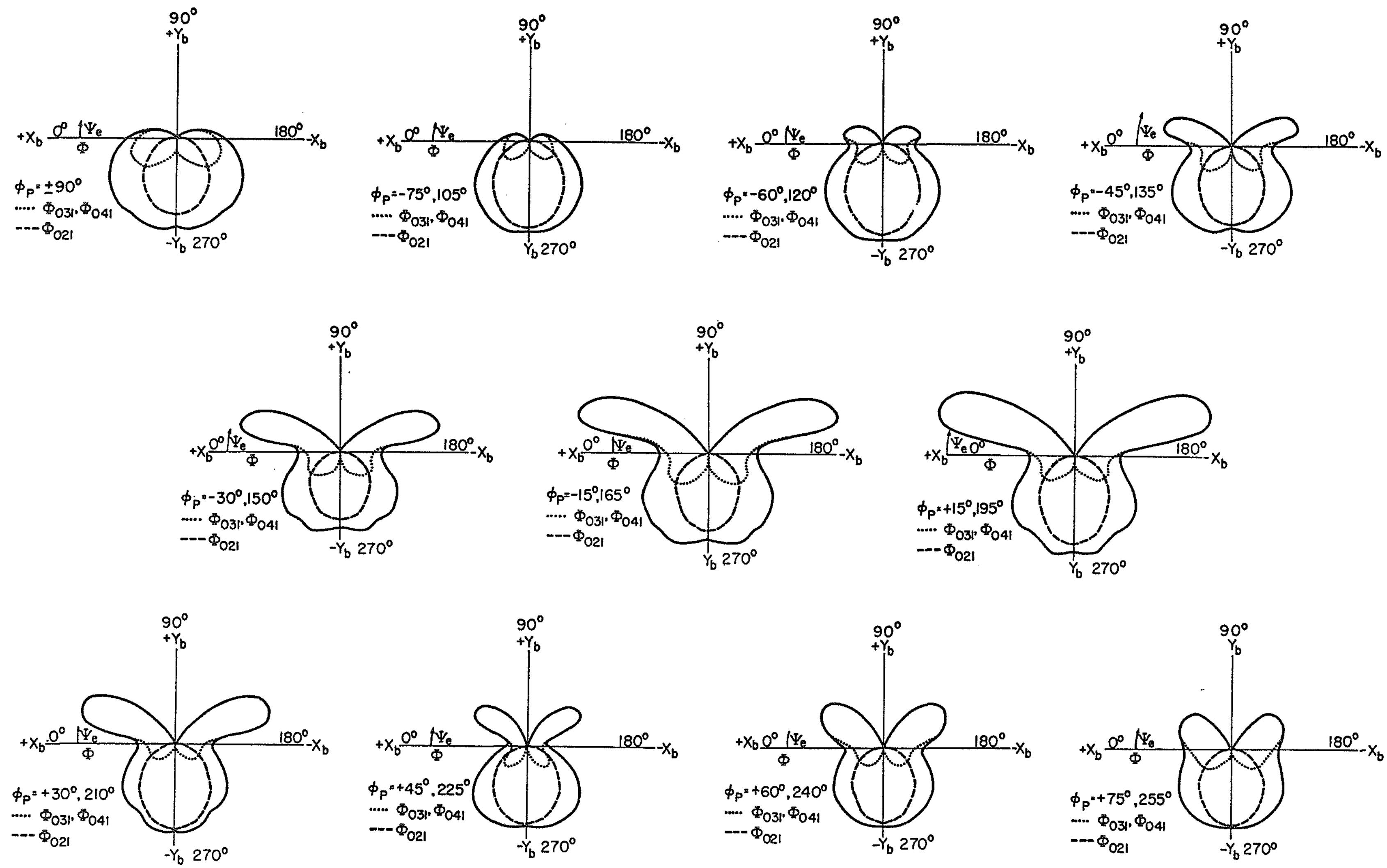

Figure 4. Reflected Ambient Particle Flux Perturbations for Varying ϕ_p.

IV. CONCLUSIONS

The introduction of simplifying assumptions, permitted an estimate to be obtained for the influence of the OGO spacecraft upon "in situ" atmospheric density measurements made by an "open source" mass spectrometer located in OPEP-2. Solutions for important orientations of the solar panels, OPEP, and main body, with respect to the spacecraft velocity, were obtained using a Digital Equipment Corporation PDP-8 computer. These solutions indicate that the influence of the spacecraft produces errors in measured number densities, or fluxes of the order of one percent.

REFERENCES

(1) "A Sweeping Neutral and Positive Ion Mass Spectrometer for Atmospheric Composition at Satellite Altitudes," B. B. Hinton, R. D. Kistler, R. J. Leite, and C. J. Mason, IEEE Trans. on Geoscience Electronics GE-7, No. 2, pp. 107-114, April 1969.

(2) "Theoretical Model for Conversion of Observed Neutral and Ion Densities to Ambient Densities for Orbiting Geophysical Observatories," B. B. Hinton, R. J. Leite, and C. J. Mason, Report No. 03346-1-T, High Altitude Engineering Laboratory, The University of Michigan, Ann Arbor, Michigan, January 1970.

APPENDIX A

In Figure A1, consider the surface S, which is non-concave; that is, the solid angle subtended by S at any point on the surface is zero. Other surfaces S' may subtend non-zero solid angles when viewed from points on S however. The velocity of the surface is $\vec{V}$. The Maxwellian velocity distribution of the ambient particles relative to S is $f_3(\vec{v}_m) = f(\vec{V}-\hat{r}v)$ where $\vec{v}_m$ is the Maxwellian velocity of a particle, f_3 the three dimensional distribution, and $\vec{v}$ the relative velocity of the spacecraft with respect to the particles ($\vec{v} + \vec{v}_m = \vec{V}$). The particles

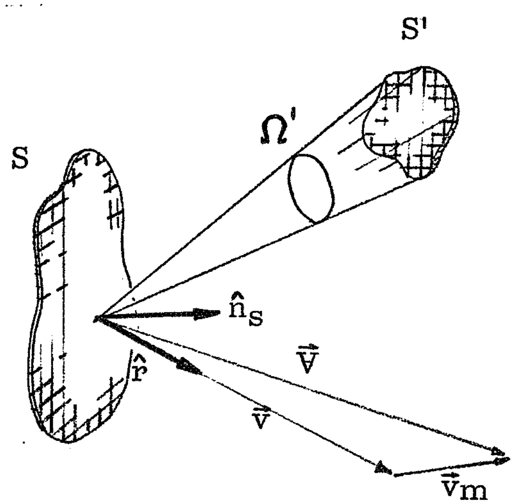

Figure A1. Incident particle.

approaching a point on S from the direction r are those for which $v(\hat{r} \cdot \hat{n}_s) \geq 0$, where n_s is the unit normal at the point; thus Φ_{in}, the flux per unit area at the point on S, is given by (1) where f_1 is the one-dimensional Maxwellian distribution and N_{in}, the number density of incoming particles, is defined below.

$$\Phi_{in} = N_{in} \int_{\Omega_0} \int_{\vec{v} \cdot \hat{n}_s \geq 0}^{\infty} f_1 (\hat{r} \cdot (\vec{V} - \hat{r}v) \, v\hat{r} \cdot \hat{n}_s dv d\Omega \, . \qquad \text{(A1)}$$

The integration in solid angle is to be carried out over the entire hemisphere less Ω' the solid angle shielded from the in-coming flux by other surfaces. The vector $\hat{r}$ in (A1) is a unit vector in the direction of $\vec{v}$. Elsewhere in this discussion, use will be made of an abbreviated form of (A1), as in (A3) using

$$\langle v_n \rangle_o = \int_\Omega \int_{\vec{V}\cdot\hat{n}_s \geq o}^{\infty} f_1(\hat{r}\cdot(\vec{V}-\hat{r}v))\, v\hat{r}\cdot\hat{n}_s\, dv\, d\Omega . \qquad \text{(A2)}$$

Thus we have

$$\Phi_{in} = N_a \langle v_n \rangle_o . \qquad \text{(A3)}$$

To develop a notation parallel with that used for the particle transfer between two surfaces, it was desirable to have a differential form of (A1):

$$\frac{d\,\Phi_{in}}{d\,\Omega} = \varphi_{in} = N_{\bar{a}} \int_{\hat{r}\cdot\hat{n}_s \geq o}^{\infty} f_1(\hat{r}\cdot(\bar{V}-\hat{r}v))\, v\hat{r}\cdot\hat{n}_s\, dv. \qquad \text{(A4)}$$

In (A4) ϕ_{in} is the flux per unit area onto S per unit solid angle in a direction $\hat{r}$.

The second elementary problem is the transfer of particles to a surface S_j from a second surface S_i, at rest with respect to S_j. From previous assumptions, the element of flux emitted in the direction $\hat{r}_{ij}$ (unit vector from a point p_i on S_i to a point p_j on S_j) by an element of area dS_i into an element of solid angle $d\Omega_j$ is,

$$(\Phi_i/2\pi)dS_i\ (\hat{r}_{ij}\cdot\hat{n}_i)d\Omega_j$$

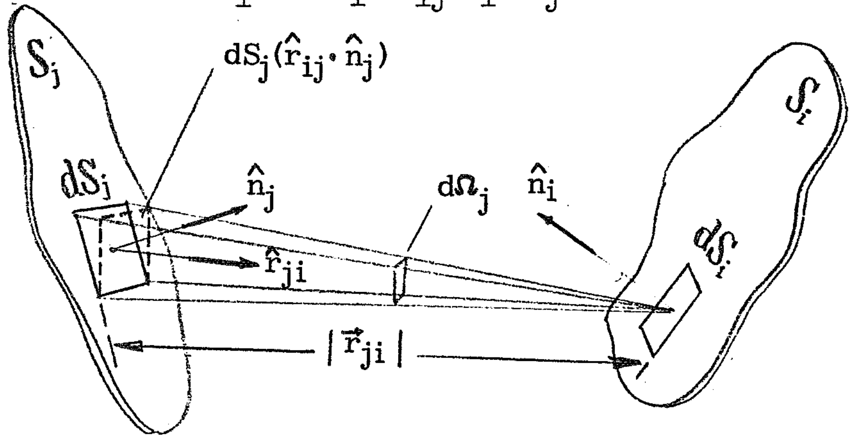

Figure A2. The Solid Angle Element

where Φ_i is the efflux per unit area of dS_i into the entire hemisphere. The portion of this flux arriving at a unit area of dS_j, ϕ_{ij}, may be found by observing that $d\Omega_j = dS_j\ (\hat{n}_j \cdot \hat{r}_{ji})\ /r_{ji}^2$ in Figure (A2) and substituting for $d\Omega_j$ in the above equation to obtain,

$$\frac{d}{dS_j}\ \frac{d\Phi}{d\Omega_j} = \varphi_{ij} = \Phi_i(\hat{n}_i \cdot \hat{r}_{ij})(\hat{n}_j \cdot \hat{r}_{ji})/2\pi r_{ji}^2\ . \tag{A5}$$

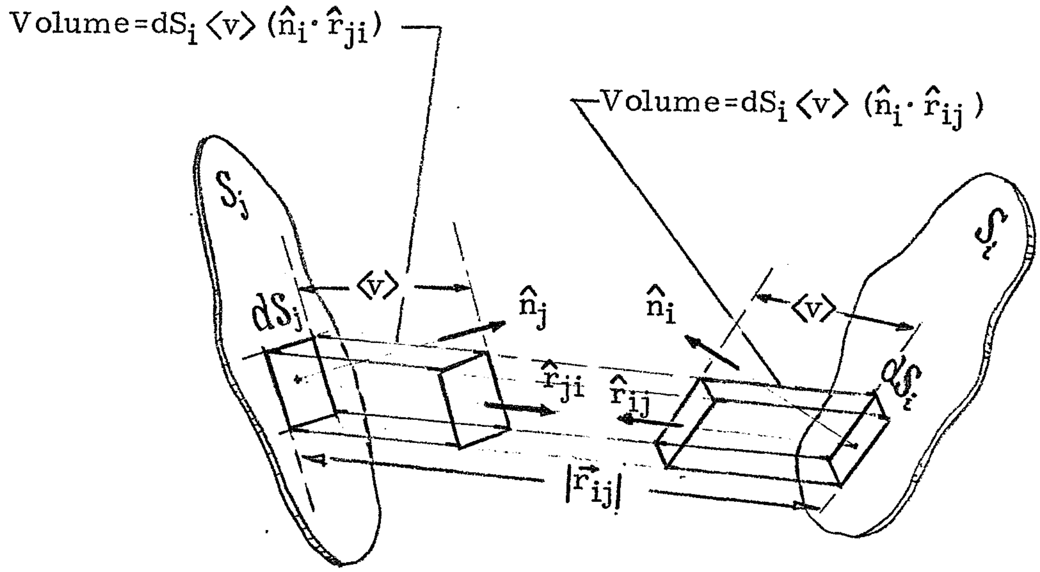

Figure A3. Number densities.

From Figure A3, it can be seen that the flux per unit area leaving dS_i makes a contribution to the total number density just outside dS_i and its magnitude is determined by considering the number of particles contained in an elemental volume $\langle v\rangle\ dS_i\ \hat{n}_i \cdot \hat{r}_{ij}$:

$$(dN_i)_{out} = \Phi_i dS_i/2\pi\ (\hat{n}_i \cdot \hat{r}_{ij})\ \langle v\rangle \tag{A6}$$

The cross sectional area of the volume is $dS_i\ \hat{n}_i \cdot \hat{r}_{ij}$, $\langle v\rangle$ is the average particle speed in the $\hat{r}_{ij}$ direction and represents the length of the volume, and $\Phi_i/2\pi$ is the number of particles leaving per unit time. Referring to the form of (A2), this can be expressed as:

$$(dN_i)_{out} = \Phi_i dS_i/2\pi \langle v_n\rangle \tag{A7}$$

Similarly, the influx per unit area of dS_j from dS_i contributes

$$(dN_j)_{in} = \phi_{ij} dS_i / (\hat{n}_j \cdot \hat{r}_{ji}) \langle v \rangle = \phi_{ij} dS_i / \langle v_n \rangle \qquad \text{(A8)}$$

to the number density just outside dS_j.

The use of $\langle v \rangle$ in (A7) and (A8) is justified by assumption (ii) as well as the fact that Φ_i in (A7) refers only to the flux <u>leaving</u> S_i and ϕ_{ij} refers only to flux incident upon S_j.

APPENDIX B

For "open source" instruments the number density contributions of the fluxes must be determined. In order to do this by means of relations similar to (A7) and (A8), it was necessary to make an assumption about the relation of $<v_n>$ before a surface collision with $<v_n>$ after this collision. The simplest assumption was that the particles are fully accommodated. That is, $<v_n>$ may be calculated from the Maxwellian distribution characteristic of the temperature of the last surface touched by the particles, or in the case of those incident from "free space," $<v_n>_o$ is to be calculated from $f(\vec{V}-\vec{v})$ with a temperature corresponding to the ambient atmospheric temperature. Unfortunately, full accommodation is probably a poor assumption. For this reason, formulae were derived for the number densities which allowed the assumption about $<v_n>$ to be made later. Let $<v_n>_{o,i_1 \ldots i_k}$ be the average speed of particles incident from space onto S_i, and subsequently transferred to $S_{i_2}, \ldots$ to S_{i_k}. Note $<v_n>_{o,i_1 \ldots i_k}$ is the mean normal speed <u>after</u> interaction with S_{i_k}, not before interaction with S_{i_k}.

Each term in (5) contributes twice to the number density just outside a point on S_1, once as an incident density as in (A7) and once as a reflected density as in (A8). Let N_{021} be the sum of the incident and reflected contributions of Φ_{021}. Then,

$$N_{021} = \Phi_{021}\left(\frac{1}{<v_n>_{02}} + \frac{1}{<v_n>_{021}}\right)$$

and similar relations hold for the other contributions in (5), a complete list of which is given by,

$$N_{021} = \Phi_{021}\left(\frac{1}{<v_n>_{02}} + \frac{1}{<v_n>_{021}}\right); \qquad N_{0121} = \Phi_{0121}\left(\frac{1}{<v_n>_{012}} + \frac{1}{<v_n>_{0121}}\right);$$

$$N_{0321} = \Phi_{0321}\left(\frac{1}{<v_n>_{032}} + \frac{1}{<v_n>_{0321}}\right); \qquad N_{0421} = \Phi_{0421}\left(\frac{1}{<v_n>_{042}} + \frac{1}{<v_n>_{0421}}\right); \qquad \text{(B1)}$$

$$N_{0721} = \Phi_{0721}\left(\frac{1}{<v_n>_{072}} \quad \frac{1}{<v_n>_{0721}}\right); \qquad N_{0821} = \Phi_{0821}\left(\frac{1}{<v_n>_{082}} + \frac{1}{<v_n>_{0821}}\right);$$

$$N_{031}=\Phi_{0531}\left(\frac{1}{\langle v_n\rangle_{032}}+\frac{1}{\langle v_n\rangle_{031}}\right);\qquad N_{0131}=\Phi_{0131}\left(\frac{1}{\langle v_n\rangle_{013}}+\frac{1}{\langle v_n\rangle_{0131}}\right);$$

$$N_{0531}=\Phi_{0531}\left(\frac{1}{\langle v_n\rangle_{053}}+\frac{1}{\langle v_n\rangle_{0531}}\right);\qquad N_{0231}=\Phi_{0231}\left(\frac{1}{\langle v_n\rangle_{023}}+\frac{1}{\langle v_n\rangle_{0231}}\right);$$

$$N_{041}=\Phi_{041}\left(\frac{1}{\langle v_n\rangle_{04}}+\frac{1}{\langle v_n\rangle_{041}}\right);\qquad N_{0141}=\Phi_{0141}\left(\frac{1}{\langle v_n\rangle_{014}}+\frac{1}{\langle v_n\rangle_{0141}}\right); \tag{B1}$$

$$N_{0641}=\Phi_{0641}\left(\frac{1}{\langle v_n\rangle_{064}}+\frac{1}{\langle v_n\rangle_{0641}}\right);\qquad N_{0241}=\Phi_{0241}\left(\frac{1}{\langle v_n\rangle_{024}}+\frac{1}{\langle v_n\rangle_{0241}}\right);$$

$$N_{091}=\Phi_{091}\left(\frac{1}{\langle v_n\rangle_{09}}+\frac{1}{\langle v_n\rangle_{091}}\right);\qquad N_{0,16,9,1}=\Phi_{0,16,9,1}\left(\frac{1}{\langle v_n\rangle_{0,16,9}}+\frac{1}{\langle v_n\rangle_{0,16,9,1}}\right);$$

The above are to be compared, along with $(\Phi_{01}-\Phi'_{01})\left(\frac{1}{\langle v_n\rangle_{01}}+\frac{1}{\langle v_n\rangle_0}\right)$* to $\Phi'_{01}\left(\frac{1}{\langle v_n\rangle_{01}}+\frac{1}{\langle v_n\rangle_0}\right)$. If N_{21}, N_{31}, N_{41}, N_{91}, N_{01}, and N'_{01} are defined as in (B2), the error in considering only the front plate of the OPEP instead of the entire spacecraft is given by (B3). The number density contributions are:

$$\begin{aligned}
&N_{01}=\Phi_{01}\left(\frac{1}{\langle v_n\rangle_{01}}+\frac{1}{\langle v_n\rangle_0}\right);\\
&N'_{01}=\Phi'_{01}\left(\frac{1}{\langle v_n\rangle_{01}}+\frac{1}{\langle v_n\rangle_0}\right);\\
&N_{21}=N_{021}+N_{0121}+N_{0321}+N_{0421}+N_{0721}+N_{0821};\\
&N_{31}=N_{031}+N_{0131}+N_{0531}+N_{0231}:\\
&N_{41}=N_{041}+N_{0141}+N_{0641}+N_{0241}:\\
&N_{91}=N_{091}+N_{0,16,9,1}:
\end{aligned} \tag{B2}$$

The relative error is:

$$\frac{\Delta N}{N}=\frac{N_{01}-N'_{01}+N_{21}+N_{31}+N_{41}+N_{91}}{N_{01}} \tag{B3}$$

---

* Φ_{01} is the directly incident flux calculated by (A1) and Φ'_{01} is the flux one would obtain if the entire hemisphere were free of obstacles to the incoming flux.

For (B1) to be correct, it is necessary that the $<v_n>$ not be a function of position on the various surfaces. The legitimacy of this was examined. In the first place, it was effectively postulated that $<v_n>_{o,i_1, i_2 \ldots i_k}$ was a function of $<v_n>_o$, $T_{i_1} \ldots T_{i_k}$, and it is evident from (A2) that $<v_n>_o$ is a function of position on a surface since the limits of integration (schematically written Ω_o in (A2)) are functions of position on a surface. The particles contributing to the N's in (B1) undergo one, two or three bounces and each interaction tends to "erase" more and more of the identity of the incident velocity $<v_n>_o$. Since the T_i are essentially constant on S_i and T_i is approximately equal to T_j, it was reasonable to assume $<v_n>$ to be constant across a given surface which permits the use of (B1).