GENERALLY APPLICABLE N-PERSON PERCENTILE GAME THEORY FOR CASE OF INDEPENDENTLY CHOSEN STRATEGIES

by

John E. Walsh and Grace J. Kelleher

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ABSTRACT

Considered is discrete N-person game theory where the players choose their strategies separately and independently. Payoff "values" can be of a very general nature and need not be numbers. However, the totality of payoff outcomes (N-dimensional), corresponding to the possible combinations of strategies, can be ranked by each player according to their desirability to that player. A largest level of desirability (associated with one or more outcomes O_i) occurs for the i-th player such that he can assure, with probability at least a given value α_i, that an outcome with at least this desirability level is obtained, and this can be done simultaneously for all the players. This game theory is of a median nature when all the α_i are chosen to the 1/2. A method is given for determining O_i and an optimum (mixed) strategy for every player. Practical aspects of applying this percentile game theory are examined. Application effort can be substantially reduced when the players have relative desirability functions for ranking the outcomes. Some elementary types of relative desirability functions are introduced.

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INTRODUCTION AND DISCUSSION

The case of N players with finite numbers of strategies is considered. Each player selects his strategy separately and independently of the strategies selected by the other players. Mixed strategies are used. That is, a player specifies selection probabilities (sum to unity, with a unit probability possible) for his strategies and randomly chooses the strategy used according to these probabilities.

An N-tuple of payoffs, one to each player, occurs for every possible combination of strategy choice by the N players. These N-tuples are the possible outcomes for the game. The number of possible strategy combinations is

$$\prod_{i=1}^{N} r(i),$$

where $r(i) \geq 2$ is the number of strategies for player i. The payoffs can be of an exceedingly general nature. Some payoffs may not even be numerical (could identify categories, etc.). However, the outcomes are such that they can be ordered, according to relative desirability, separately by each player. Also, all players know the correspondence between outcomes and strategy combinations.

Ordering of outcomes should nearly always be achievable by use of paired comparisons. That is, for each two outcomes, a player expresses his preference (with equal desirability a possibility). An ordering occurs when there is no circularity of definite preference. Frequently, acceptable rules can be imposed that prevent circularity of definite preference. A suitable numerical function of the N payoffs might be used for ordering the outcomes. The amount of application effort can be reduced substantially when each player has a realtive desirability (preference) function for ranking the outcomes. Some methods for developing elementary kinds of preference functions are introduced for the case of numerical payoffs.

It is to be emphasized that an ordering of outcomes not only takes into consideration the payoff to the player doing the ordering but also the corresponding payoffs to the other players. Thus, to each player, his ordering provides the relative desirability of what can occur for the game, including what happens for the other players.

Expression of the payoffs to player i in matrix form is convenient (called the payoff matrix for player i). Here, the rows correspond to the strategies for player i and the columns to the combinations of the strategies for the other players. Let the strategies for player j be denoted as $1, \ldots, r(j)$, where $j = 1, \ldots, N$. For definiteness, the rows of the matrix for player i are numbered $1, \ldots, r(i)$. Also, in the combinations, the strategies for the other player with lowest designation number occur first (listed according to increasing strategy number), those for the other player with the next to lowest designation number occur second, ..., the strategies for the other player with highest designation number occur last.

The material of this paper is an extension of that given in ref. 1 for two players and arbitrary percentiles. The basis for percentile game theory is that each player should want the occurrence of an outcome that has a high level of desirability to him. However, a player only partially controls the outcome choice and needs some meaningful criterion (to guide him in the choice of a mixed strategy) that incorporates his interests and is usable. The class of percentile criteria considered in this paper is virtually always usable and, for each player, should frequently include a criterion that reflects the player's interests.

For player i, let the outcomes be ordered according to increasing desirability to him (i = 1, ..., N). Also, player i specifies a probability α_i that represents the assurance with which he wants to obtain an outcome that has a reasonably high desirability. A largest level of desirability occurs among the outcomes such that player i can assure, with probability at least α_i, that an outcome with at least this desirability occurs. This can be done simultaneously for all players. The outcome, or outcomes, with this largest desirability level is designated by O_i for player i.

A method, which is oriented toward minimum application effort, is given for identifying O_i when α_i is given and for determining an optimum mixed strategy for player i. Given a desirability level for O_i, this method tends to maximize the value of α_i.

A desirability level, represented by O_i, corresponds to each possible value of α_i ($0 < \alpha_i \leq 1$). However, only a finite number of values are achievable for α_i. A value is achievable for α_i when, for the O_i corresponding to α_i, use of a strategy that is optimum for this combination (α_i and O_i) cannot assure an outcome at least as desirable as O_i with probability exceeding α_i. For player i (and the method of solution used), the achievable values of α_i are determined by his ordering for the outcomes and the location of the outcomes in the payoff matrix for player i. Restriction of α_i to achievable values would seem to be advisable. For example, the nearest achievable α_i value that exceeds the stated α_i should be an acceptable choice in many cases.

The application effort for using the method of this paper can be very great. First, N payoffs need to be evaluated for every possible combination of strategies, and the number of combinations can be huge, even when all of N, r(1), ..., r(N) are of moderate size. For example, let N = 10 and r(1) = ... = r(N) = 10. Then, the number of strategy combinations is 10^{10} and the number of payoffs to be evaluated is 10^{11}. Of course, this application difficulty occurs for virtually all possible methods of solution (not just for the percentile method). Second, ordering of the outcomes can require huge effort, although this is substantially reduced when preference functions are available. Third, the solution can require appreciable effort, due to the huge sizes of the payoff matrices for the players. In summary, great application effort can be needed but this is principally due to the massiveness of the number of outcomes (at least for the case where the players have preference functions for ordering the outcomes).

Some material is given for helping to reduce the effort in identifying O_i and determining an optimum mixed strategy for player i. More specifically, for player i, consider all outcomes that are at least as desirable as a given outcome. The locations of these outcomes are marked in the payoff matrix for player i. Depending on the locations, a bound is obtained for the probability with which player i can assure the occurrence of an outcome with at least the desirability level of the given outcome.

It is to be noted that, for given α_i, assuring at least the desirability level of the corresponding O_i is the best that can be "forced" by player i with probability at least α_i.

The next section contains a statement of the method for identifying the O_i that corresponds to a given α_i and of determining an optimum mixed strategy. Some elementary types of preference functions are given in the next to last section. The final section contains some propositions that provide a basis for the method of solution.

METHOD OF SOLUTION

The method used applies to each player and is stated for player i. Results are first stated for the case where the value specified for α_i can be anywhere in the interval $0 < \alpha_i \leq 1$. Then, modifications for the case of achievable α_i are considered. Markings of the outcome locations in the payoff matrix for player i are used in the method of solution. The r(i) rows correspond to the strategies of player i and the

$$c(i) = \prod_{\substack{j=1 \\ j \neq 1}}^{N} r(j)$$

columns correspond to the combinations of strategies for the other players.

The case where the specified α_i is at most 1/2 is considered first. For the initial step, mark the position(s) in the payoff matrix of player i for the outcome(s) with the highest level of desirability to player i. Next, also mark the position(s) of the outcome(s) with the next to highest desirability level. Continue this marking, according to decreasing desirability level, until the first time that marks in all columns can be obtained from a set of rows whose number does not exceed $1/\alpha_i$. If r(i) - s(i) is the smallest number of rows for such a set, player i can assure a marked outcome with probability at least $[r(i) - s(i)]^{-1}$, which is at least α_i, with a probability exceeding $[r(i) - s(i)]^{-1}$ being possible. Next, remove the mark(s) for the outcome(s) that have the smallest desirability level (among the outcomes that received marks). Then, by the following procedure, determine whether some one of the remaining marked outcomes can be assured with probability at least α_i. The procedure is to replace the marked positions by unity and all others by zero. The resulting matrix of ones and zeroes is considered to be the payoff matrix for player i in a zero-sum game with an expected-value basis. Some one of the outcomes corresponding to the marked positions can be assured with probability at least α_i by player i if and only if the value of this game to player i is at least α_i.

Suppose that the resulting game value is less than α_i. Then, O_i consists of the outcome(s) with marking(s) removed at this step. Otherwise (game value $\geq \alpha_i$), remove the mark(s) for the outcome(s) with the smallest desirability level among those still having marks. Then, by the procedure just described, determine whether some one of the remaining marked outcomes can be assured with probability at least α_i. If not (game value $< \alpha_i$), the maximum desirability level that can be assured with probability at least α_i is the level corresponding to the outcome(s) with marking(s) removed at this step. If a probability of at least α_i can be assured, continue in the same way until the first time some one of the remaining marked outcomes cannot be assured with probability at least α_i. Then, the maximum desirability level that can be assured with probability at least α_i is the level for the outcome(s) with marking(s) removed at this step.

Now, consider the case where $\alpha_i > 1/2$. Mark the matrix positions of the outcomes according to decreasing desirability until the first time that no less than $(1 - \alpha_i)^{-1}$ columns are needed to obtain unmarked outcomes in all rows. Then, player i can assure some one of the marked outcomes with probability at most α_i, but ordinarily near α_i. When the smallest number of columns needed equals $(1 - \alpha_i)^{-1}$, the possibility exists that a marked outcome can be assured with probability α_i. If this equality occurs, determine the probability with which a marked outcome can be assured by player i. Otherwise, where the smallest number of columns exceeds $(1 - \alpha_i)^{-1}$, also mark the position(s) of the outcome(s) with the highest desirability level among the remaining unmarked positions and determine the probability with which a marked outcome can be assured.

To make the probability determination, for both possibilities, replace the marked positions by unity and the unmarked positions by zero. Consider the resulting matrix of ones and zeroes to be for player i in a zero-sum game with an expected-value basis. Player i can assure an outcome of the marked set with probability α_i or greater if and only if the game value (to him) is at least α_i. When the resulting game value is at least α_i, for either possibility, O_i consists of the marked outcome(s) with the smallest desirability level.

When the game value is less than α_i, also mark the position(s) of the outcome(s) with the highest desirability level amont the outcomes not yet marked. Determine, by the procedure just given, whether an outcome of the marked set can be assured with probability at least α_i. If so, O_i consists of the outcome(s) marked last. Otherwise, continue marking the positions of outcomes according to decreasing desirability level until the first time that an outcome of the marked set can be assured with probability at least α_i. Then, O_i consists of the outcome(s) marked last. A simplification occurs when $\alpha_i > 1 - 1/c(i)$. Then, the marking continues until the first time that a pure strategy occurs that consists of all positions in a row being marked.

Now, consider determination of an optimum strategy for player i. Use the matrix marking of all outcomes whose desirability level is as least as great as that of O_i. Replace the marked positions by unity and the other positions by zero. Treat the resulting matrix as the payoff matrix for player i in a zero-sum game with an expected-value basis. An optimum strategy for player i in this zero-sum game is α_i-optimum for him. Also, the value of this game to player i is an achievable α_i that is the nearest achievable value at least equal to the stated value for α_i.

Next, consider cases where the value wanted for α_i is stated but the requirement of an achievable α_i is imposed. The nearest achievable value at least equal to α_i is determined by the method given for the case of general α_i. When the stated α_i is not achievable, the nearest smaller achievable value is determined by first removing the mark(s) for O_i in the marking that consists of all outcomes at least as desirable as O_i. Then, the remaining marked positions are replaced by unity and the other positions by zero. The value of the resulting zero-sum game to player i is the nearest achievable value that is less than the stated α_i.

The solution method used requires that the positions of all outcomes with equal desirability to player i be simultaneously marked in his payoff matrix. This tends to maximize the probability of assuring at least a given level of desirability for the outcome that occurs and to reduce the amount of application effort. Other ways could be used, however, in which not all outcomes of equal desirability are marked at the same time. In fact, an approach like the preferred sequence method of ref. 2 could be used to mark each outcome separately. These special methods could possibly be useful in some cases but are not considered in this paper.

ELEMENTARY PREFERENCE FUNCTIONS

Almost complete freedom is available to a player in his ordering of the possible outcomes for the game. This does not imply, however, that any way chosen for doing this ordering is necessarily satisfactory. In fact, great care can be needed in determining a suitable ordering. This great freedom is a valuable asset, but only if used carefully and wisely. Several examples of elementary preference functions are given to illustrate considerations in the development of satisfactory preference functions.

Let the preference function used by player i be denoted by $D_i(p_1, \ldots, p_N)$, where $(p_1, \ldots, p_N)$ is a general outcome. The possible values of $D_i(p_1, \ldots, p_N)$ are real numbers and increasing value represents increasing desirablity to player i (equal value represents equal desirability).

For simplicity, but without great loss of generality, values of p_i are expressed as real numbers, in the same unit, which are such that increasing values of p_i represent nondecreasing (usually increasing) desirability to player i. Also, as a standardization, $D_1(p_1, \ldots, p_N)$ is considered for all the examples. The forms used for $D_1(p_1, \ldots, p_N)$

are always such that, in their use, any differences in the kinds of units used for $p_1, \ldots, p_N$ do not cause difficulties in the statement of $D_1(p_1, \ldots, p_N)$.

The first example involves additive changes in the p_i and the situation is such that an addition of A to p_1 has the same desirability to player 1 as the combination of an addition of $e_i w_i a_i$ to p_i for $i = 2, \ldots, N$. Here, a_i is positive, e_i is 1 or -1 (depending on whether an increase or decrease is to occur), $w_2 + \ldots + w_N = 1$ with all $w_i \geq 0$, and A can be positive or negative. The preference function

$$D_1^{(a)}(p_1, \ldots, p_N) = p_1 + A \sum_{i=2}^{N} e_i p_i / a_i$$

should be suitable, since $D_1^{(a)}(p_1 + A, p_2, \ldots, p_N)$ equals

$$p_1 + A + A \sum_{i=2}^{N} e_i p_i / a_i \equiv p_1 + A \sum_{i=2}^{N} e_i (p_i + e_i w_i a_i)/a_i$$

equals $D_1^{(a)}(p_1, p_2 + e_2 w_2 a_2, \ldots, p_N + e_N w_N a_N)$ for all possible values of $p_1, \ldots, p_N$.

The second example involves multiplicative changes in the p_i and requires that they all have positive values. The situation is such that multiplication of p_1 by the positive factor $(1 + B)$ has the same desirability to player 1 as the combination of multiplying p_i by the factor $(1 + e_i v_i)^{w_i}$ for $i = 2, \ldots, N$. Here, $0 < v_i < 1$, the value of B can be positive or negative, $e_i = 1$ or -1 (depending on whether an increase or decrease is to occur), and $w_2 + \ldots + w_N = 1$, with all $w_i \geq 0$.

The preference function

$$D_1^{(m)}(p_1, \ldots, p_N) = \log_{10}p_1 + \sum_{i=2}^{N} \{[\log_{10}(1 + B)] / [\log_{10}(1 + e_i v_i)]\} \log_{10}p_i$$

should be suitable, since $D_1^{(m)}[(1+B)\, p_1, p_2, \ldots, p_N]$ equals

$$\log_{10}(1 + B)p_1 + \sum_{i=2}^{N} \{[\log_{10}(1 + B)]/[\log_{10}(1 + e_i v_i)]\} \log_{10}p_i$$

$$= \log_{10}p_1 + \sum_{i=2}^{N} \{[\log_{10}(1 + B)]/[\log_{10}(1 + e_i v_i)]\} \log_{10}(1 + e_i v_i)^{w_i} p_i$$

equals $D_1^{(m)}[p_1, (1 + e_2 v_2)^{w_2} p_2, \ldots, (1 + e_N v_N)^{w_N} p_N]$ for all positive values of $p_1, \ldots, p_N$.

The third example involves both addition and multiplication, where changes in $p_1, \ldots, p_J$ are by addition and changes in $p_{J+1}, \ldots, p_N$ are by multiplication (with $p_{J+1}, \ldots, p_N$ all positive). The situation is such that an addition of A to p_1 has the same desirability to player 1 as the combination of an addition of $e_j w_j a_j$ to p_j for $j = 2, \ldots, J$, and multiplication of p_j by $(1 + e_j v_j)^{w_j}$ for $j = J + 1, \ldots, N$. Here, $w_2 + \ldots + w_N = 1$ with all $w_j \geq 0$, the value of A can be positive or negative, and the e_j, $_j$, v_j have the same properties as for the first and second examples. The preference function

$$D_1^{(am)}(p_1, \ldots, p_N) = p_1 + A \sum_{j=2}^{N} e_j p_j / a_j + A \sum_{j=J+1}^{N} [\log_{10}(1 + e_j v_j)]^{-1} \log_{10}p_j$$

should be suitable, since $D_1^{(am)}(p_1 + A, p_2, \ldots, p_N)$ equals

$$p_1 + A + A \sum_{j=2}^{J} e_j p_j / a_j + A \sum_{j=J+1}^{N} [\log_{10}(1 + e_j v_j)]^{-1} \log_{10} p_j$$

$$\equiv p_1 + A \sum_{j=2}^{N} e_j (p_j + e_j w_j a_j) / a_j + A \sum_{j=J+1}^{N} [\log_{10}(1 + e_j v_j)]^{-1} \log_{10} (1 + e_j v_j)^{w_j} p_j$$

equals $D_1^{(am)}[p_1, p_2 + e_2 w_2 a_2, \ldots, p_J + e_J w_J a_J, (1 + e_{J+1} v_{J+1})^{w_{J+1}} p_{J+1}, \ldots,$ $(1 + e_N v_N)^{w_N} p_N]$ for all permissible values of $p_1, \ldots, p_N$.

The final example also involves both addition and multiplication, but p_1 changes by multplication. Again, as a standardization, the changes in $p_2, \ldots, p_J$ are by addition and the changes in $p_{J+1}, \ldots, p_N$ are by multiplication (with $p_1, p_{J+1}, \ldots, p_N$ all positive for this case). The situation is such that multiplication of p_1 by the posit[illegible] factor $(1 + B)$ has the same desirablity to player 1 as the combination of an addition of $e_j w_j a_j$ to p_j for $j = 2, \ldots, J$, and multiplication of p_j by $(1 + e_j v_j)^{w_j}$ for $j = J + 1, \ldots, N$. Here, $w_2 + \ldots + w_N = 1$ with all $w_j \geq 0$, the value of B can be positive or negative, and the e_j, a_j, v_j have the same properties as for the first two examples. The preference function

$$D_1^{(ma)}(p_1, \ldots, p_N) = \log_{10} p_1 + [\log_{10}(1 + B)] \sum_{j=2}^{J} e_j p_j / a_j$$

$$+ \sum_{j=J+1}^{N} \{[\log_{10}(1 + B)]/[\log_{10}(1 + e_j v_j)]\} \log_{10} p_j$$

should be suitable, since $D_1^{(ma)}[(1 + B)p_1, p_2, \ldots, p_N]$ equals

$$\log_{10}(1 + B)p_1 + [\log_{10}(1 + B)] \sum_{j=2}^{J} e_j p_j / a_j$$

$$+ \sum_{j=J+1}^{N} \{[\log_{10}(1 + B)]/[\log_{10}(1 + e_j v_j)]\} \log_{10} p_j$$

$$\equiv \log_{10} p_1 + [\log_{10}(1 + B)] \sum_{j=2}^{J} e_j (p_j + e_j w_j a_j)/a_j$$

$$+ \sum_{j=J+1}^{N} \{[\log_{10}(1 + B)]/[\log_{10}(1 + e_j v_j)]\} \log_{10} (1 + e_j v_j)^{w_j} p_j$$

equals $D_1^{(ma)}[p_1, p_2 + e_2 w_2 a_2, \ldots, p_J + e_J w_J a_J, (1 + e_{J+1} v_{J+1})^{w_{J+1}} p_{J+1}, \ldots, (1 + e_N v_N)^{w_N} p_N]$ for all permissible values of $p_1, \ldots, p_N$.

Of course, any strictly increasing function of a preference function provides an equivalent preference function.

SOME PROPOSITIONS

The statements about the probability inequalities when marks in all columns can be obtained from $r(i) - s(i)$ rows, and about unmarks in all rows from no less than $(1 - \alpha_i)^{-1}$ columns, follow from

THEOREM 1. When the marked positions of outcomes in the matrix for player i are such that marks in all columns are obtained from $r(i) - s(i)$ rows, player i can assure occurrence of a marked outcome with probability at least $[r(i) - s(i)]^{-1}$, or at least α_i when $r(i) - s(i) \leq 1/\alpha_i$.

COROLLARY. When the unmarked positions of outcomes in the matrix for player i are such that unmarked positions in all rows are obtained from $c(i) - t(i)$ columns, the combination of other players, which have the $c(i)$ columns are strategies, can assure an unmarked outcome with probability at least $[c(i) - t(i)]^{-1}$. Thus under these circumstances, player i *can assure a marked outcome with probability at most* $1 - [c(i) - t(i)]^{-1}$, *or at most* α_i *when* $c(i) - t(i) \geq (1 - \alpha_i)^{-1}$.

Proof of Theorem 1. When $r(i) - s(i) = 1$, so that some row is fully marked, the probability is unity that a marked outcome can be assured by player i.

Now suppose that $r(i) - s(i) \geq 2$. Let $P_1, \ldots, P_{r(i)}$ and $q_1, \ldots, q_{c(i)}$ be the mixed strategies used. The probability of the occurrence of a marked outcome is

$$\sum_{k=1}^{r(i)} P_k Q_k,$$

where Q_k is the sum of the q's for the columns that have marked outcomes in the k-th row. The largest value of this probability that player i can assure, through choice of $P_1, \ldots, P_{r(i)}$, is

$$G = \min_{q_1, \ldots, q_{c(i)}} (\max_k Q_k).$$

Let $k[1], \ldots, k[r(i) - s(i)]$ be $r(i) - s(i)$ rows that together contain marked positions in all columns. For any minimizing choice of the values for $q_1, \ldots, q_{c(i)}$, all of $Q_{k[1]}, \ldots, Q_{k[r(i) - s(i)]}$ are at most G. Hence,

$$[r(i) - s(i)]G \geq Q_{k[1]} + \ldots + Q_{k[r(i) - s(i)]} \geq 1,$$

so that a probability of at least $[r(i) - s(i)]^{-1}$ can be assured by player i.

The remaining part of the method of solution has as its basis

THEOREM 2. A sharp lower bound on the probability with which player i can assure occurrence of an outcome of a specified set whose positions are marked in his payoff matrix, and identification of one or more optimum strategies for him in accomplishing this, can be obtained from solution for the value to player i of a zero-sum game with an expected-value basis. The payoff matrix for player i in this game has the value unity at all marked positions (in the original matrix for player i) and the value zero at all other positions.

Proof. Let arbitrary but specified mixed strategies be used for the rows and for the columns. With the matrix considered for the zero-sum game, the expression for the expected payoff to player i is identically equal to the expression for the probability that a marked outcome occurs.

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13. ABSTRACT

Considered is discrete N-person game theory where the players choose their strategies separately and independently. Payoff "values" can be of a very general nature and need not be numbers. However, the totality of payoff outcomes (N-dimensional), corresponding to the possible combinations of strategies, can be ranked by each player according to their desirability to that player. A largest level of desirability (associated with one or more outcomes O_i) occurs for the i-th player such that he can assure, with probability at least a given value α_i, that an outcome with at least this desirability level is obtained, and this can be done simultaneously for all the players. This game theory is of a median nature when all the α_i are chosen to the 1/2. A method is given for determining O_i and an optimum (mixed) strategy for every player. Pr ctical aspects of applying this percentile game theory are examined. Application effort can be substantially reduced when the players have relative desirability functions for ranking the outcomes. Some elementary types of relative desirability functions are introduced.

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