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ANALYTICAL PROPERTIES OF NOISE GENERATING MECHANISMS IN A SUPERSONIC JET EXHAUST FLOW

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16. Abstract

Explicit analytic expressions for studying noise generating mechanisms and detailed formulas for numerical computations have been derived, following the generalized supersonic turbulent jet noise theory. In a supersonic jet, both the shear noise and the self noise sources remain equally effective For both types of sources, there are three principal modes of sound radiation, two Mach modes [S.1], [S.2], which follow the U^3-law, and a pure acoustic mode, [S.0], which follows the $U^8[(1-M\cos\theta)^2 + \alpha^2 M^2]^{-2.5}$ dependence. These solutions are markedly different in their analytical structure. In this report, the transition between the U^8-law and the U^3-law, acoustic efficiency for all Mach numbers, source distribution, and the coupled effect of refraction and Doppler shift are discussed in detail. Some preliminary results of noise directivity and spectrum are also given.

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1.0 INTRODUCTION

In a previous study by Pao (Reference 1), a generalized theory of supersonic jet noise production has been formulated. In this theory the analysis is based on the convected wave equation as derived by Phillips (Reference 2). General solutions to the convected wave equation have been obtained in the important range of supersonic and transonic convection speeds. Since the convected wave equation has taken the detailed local mean flow properties of the shear layer into account, the local noise generating mechanisms and far field noise characteristics can be described much more accurately in the generalized theory than in other existing theories. Far reaching understanding of noise generating processes in a supersonic turbulent shear flow can be gained through further developments of this generalized theory.

The general results of Reference 1 can be extended into two areas of immediate interest:

a) The Analytical Implications of the Theory: – The analytical study of the dynamics of wave radiation is only the starting point of a systematic study of jet and rocket noise phenomena. The mean flow parameters, the turbulence structure, and other thermodynamical or aerodynamical effects must be studied simultaneously. The overall studies of jet noise is, therefore, vastly complex. Analytical insight of noise generating mechanisms will be the most valuable guidance for organizing detailed experimental studies, correlation of experimental data, and developments of noise prediction or noise control methods.

b) Application of the Theory to Actual Numerical Calculations: – Since this method is capable of predicting jet noise intensity, spectrum, and directivity through the same equations in a unified manner, it is of great practical benefit to develop the theoretical results into a numerical scheme for noise predictions. According to previous experiences (Reference 5), the noise of low subsonic jets can be predicted analytically to an accuracy of $\pm$ 1 dB. If similar accuracy can be obtained for supersonic jet noise prediction using the generalized theory, the numerical prediction scheme will become a useful tool for defining the noise environments of future rocket engines, estimating the change in acoustical characteristics due to design modifications, and the design and evaluation of various noise control devices or concepts.

In the present report, the major objective is to refine the theoretical results of Reference 1, such that some important questions under area (a) can be resolved. The theory have also been modified such that numerical computational schemes can be programmed as an immediate next step. Via the refinement of the analytical results of the generalized theory, a number of important noise generating mechanisms have been found for the first time on a solid analytical basis. It is found also that the results of the generalized theory correspond precisely with results in the literature.

In either low speed or high speed jets, both the self noise and the shear noise types of sound sources are present, and both of them contribute to the overall sound power in the same order of magnitude. According to the convected wave equation, there are many modes of noise radiation in a supersonic jet. However, their domains of dominance are heavily overlapping. Three representative modes are studied in this report:

a) The Mach modes, [S.1] and [S.2]. Different analytical solutions to the convected wave equation are required for Mach wave radiation in various frequency ranges. The Mach mode wave radiation in the high frequency end, [S.1], and the Mach mode wave radiation in the low frequency end, [S.2], are chosen to cover the entire spectrum. Both the sonic components and the hydrodynamic components in the turbulence are contributing to the Mach mode radiation, with the hydrodynamical source dominating in most cases. The unique character of Mach wave radiation via the hydrodynamical sources is that it depends mainly on the longitudinal wave number component of the turbulent structure in the shear layer.

b) The Acoustical mode, [S.0]. Although both Mach wave and acoustical wave radiations are included in the Mach modes, there is also a purely acoustical mode of noise radiation in a supersonic jet. In this mode, all the sound sources in the turbulent shear flow are "acoustical" in nature, i.e., the sound source is the sonic mode (Reference 36) in the turbulent structure. From an analytical point of view, this mode is also the simplest solution to the convected wave equation. The Lighthill's theory, including the extension to high speed convections given by Ffowcs Williams, can be identified with this acoustical mode, except that the Mach wave radiation as described by Ffowcs Williams would have to be excluded from this correspondence.

Based on the general solutions [S.0], [S.1], and [S.2], the parametric dependence of the sound power on mean flow and turbulent properties has been derived for these modes of wave radiation. By using these parametric equations, comparisons with the Lighthill's theory can be made. It was found that the generalized theory predicts exactly the U^8-law and U^3-law for sound power in the subsonic and supersonic ranges, respectively. However, the detailed mechanisms of noise production for the Lighthill's theory and the generalized theory are subtly different. The transition between the U^8-law and the U^3-law as predicted in the generalized theory occurs early in the transonic range of convection speed, and the acoustic efficiency in this region agrees quite well with previous experimental evidences. It was noted before by Ribner (Reference 6) and Lowson and Pao (Reference 5) that some difficulties in sound power predictions were encountered by using the equations derived via the Lighthill's approach.

The Doppler shift and refraction effects of the shear flow are carried implicitly by the general solutions [S.0], [S.1], and [S.2]. The coupled Doppler shift and refraction effect has produced some important modifications to the analytical properties of noise generation mechanisms, such as:

- The convection effect on the forward propagating waves in the high frequency ranges is reduced by the refraction, and

- The low frequency Mach wave and acoustical wave radiations are strongly favored in the down-stream direction.

The important steps and arguments for developing the analytic results, the characteristics of the convected wave equations and its solutions, and the parametric dependences of the sound power are discussed in Section 2.0. The spectrum, sound source strength distribution, acoustic efficiency, directivity patterns, and the refraction effects are discussed in Section 3.0 and Section 4.0. The detailed formulas and derivations for the principal results of the generalized theory are given in Appendix A. Finally, as an important part of the noise prediction study, the properties of high speed jet exhaust flows are summarized in Appendix B.

2.0 A REFINED ANALYSIS OF THE CONVECTED WAVE EQUATION

For aerodynamical noise rediation problems in the high speed regime, the convection speed in the mean flow is often comparable with the local speed of sound. Aerodynamics and acoustic wave propagation can no longer be treated as separate problems. In order to predict accurately the properties of far field noise radiation, it is necessary to describe fully the complex process of local interactions between the mean flow and the turbulent noise sources. It has long been recognized that the Phillips approach of formulating a convected wave equation provides an accurate description of the fundamental physical properties of noise radiations in a high speed shear flow, (References 2, 3, and 4). However, the Phillips equation was very difficult to solve and only one solution was given by Phillips for shear flows with asymptotically large Mach numbers.

In a previous report by Pao (Reference 1), solutions to the convected wave equation have been constructed which are uniformly valid in both the transonic and low supersonic convected speed ranges. Since these analytical solutions are derived from the convected wave without any approximation, these results are potentially very powerful tools for studying the noise generating mechanisms in supersonic jets and rocket exhausts. Furthermore, the analysis follows an integral equation approach, the convergence of the iteration scheme and the accuracy of numerical computations can be accurately estimated. A critical review of the previous results is presented in Section 2.1.

Although the fundamental solutions to the noise radiation from supersonic turbulent flows have been obtained through the convected wave equation, some important refinements should be made before the results are directly applicable to the study of noise generating mechanisms and numerical calculation. An important analytical detail, which was not encountered in the Phillips' analysis, is the specification of boundary conditions. In the convected wave equation, the instanteneous velocity fluctuation quantities are specified in the source terms. It is well known that the statistical properties of the instanteneous fluctuating quantities are not measureable. Only the mean square values of the fluctuations in a turbulence are statistically measurable. In the asymptotic solution given by Phillips, the far field noise and the instanteneous source term are related via a simple linear relation. The dependence of the mean square sound pressure fluctuation on the mean square turbulence fluctuation can be obtained directly through multiplying the instanteneous solution by its complex conjugate. In the generalized solutions as considered in the present report, such a simple situation does not prevail. A detailed analysis working directly through the boundary conditions is required. A comprehensive discussion of the boundary condition is given in Section 2.2.

In Section 2.3, the magnitude and explicit functional dependence of various terms in the generalized solutions are discussed in detail. In the process of constructing the generalized solutions from the convected wave equation, several transformations of variables have been adopted. By examining the analytical results, their relation to familiar physical quantities, such as Strouhal number, or classical laws, such as Doppler shift, is not obvious at all. The discussion in Section 2.3 will help to clarify the interpretations of the generalized solutions in physical terms.

By including the analytical details as given in Sections 2.2 and 2.3, the generalized solutions are now ready for exact numerical computations. However, they are not in a convenient form for comprehensive discussions of various important supersonic jet noise generating mechanisms. In Section 2.4, a set of approximate solutions to the convected wave equation has been obtained. These approximate solutions are obtained by simplifying the exact generalized solutions. Although these approximate solutions suffer from numerical inaccuracy, their functional dependence on various parameters is preserved. By using these approximate solutions, penetrating insight to the noise generating mechanisms in jet and rocket exhaust flows can be achieved.

2.1 The Fundamental Solutions

The convected wave equation is derived from the momentum equation, the continuity equation, and the equation of state for a perfect gas. The general equation can be given as, (Reference 1)

$$\frac{D^2}{Dt^2} \log\left(\frac{p}{p_0}\right) - \frac{\partial}{\partial x_i}\left\{a^2 \frac{\partial}{\partial x_i} \log\left(\frac{p}{p_0}\right)\right\} = \gamma \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} +$$

$$+ \gamma \frac{D}{Dt}\left(\frac{1}{c_p} \frac{DS}{Dt}\right) - \gamma \frac{\partial}{\partial x_i}\left\{\frac{1}{\rho} \frac{\partial}{\partial x_j} \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij}\right)\right\} \tag{1}$$

where p is the pressure, S is the entropy, and u_i denotes the velocity components. For sound radiation processes in a turbulent flow, both heat conduction and viscosity are likely to be unimportant. Furthermore, if the flow field is shock free, one can probably consider the effect of pressure fluctuation and the effect of entropy fluctuation separately. Under these circumstances, the last two terms on the right of Equation (1) can be neglected.

In order to construct a solution to the convected wave equation, it is necessary to specify the flow field. A parallel shear flow has been chosen such that it has a characteristic thickness of 2L, and that the mean flow properties and the turbulent structure are homogeneous in the x_1 and x_2 directions. The mean flow velocity, $\overline{u}_1$, and the local speed of sound, a, are functions of x_3 only. Equation (1) can be further simplified if small terms are omitted. In a turbulent flow, the fluctuating velocity components are small in comparison with the mean velocity. However, the derivatives of the fluctuating velocities may not be small at all. In the present study, only terms depending on the small velocity fluctuations to the second or higher orders are omitted. Equation (1) then becomes

$$\left\{ \left(\frac{\partial}{\partial t} + u_1 \frac{\partial}{\partial x_1} \right)^2 - \left(\frac{\partial}{\partial t} + u_1 \frac{\partial}{\partial x_1} \right) u_i' \frac{\partial}{\partial x_i} - \frac{\partial}{\partial x_i} a^2 \frac{\partial}{\partial x_i} \right\} \log \left(\frac{p}{p_0} \right) =$$

$$= \gamma \left\{ 2 \frac{\partial u_1}{\partial x_2} \frac{\partial u_2'}{\partial x_1} + \frac{\partial u_i'}{\partial x_j} \frac{\partial u_j'}{\partial x_i} \right\} \tag{2}$$

where γ is the specific heat ratio, and u_i' denotes velocity fluctuations with zero mean.

The resulting Equation (2) is different from the original equation given by Phillips in two respects:

1) On the right-hand side of the equation, there are two source terms. The first term is the shear noise and the second term is the self noise. It is known that in subsonic noise theory that the self noise actually makes a greater contribution to the overall noise power of a jet than the shear noise.

2) An additional term $(\partial/\partial t + \overline{u}_1 \, \partial/\partial x_1) \, u_1' \, \partial/\partial x_i \log (p/p_0)$ appears on the left-hand side. It can be regarded as a dispersion term. Both $\partial/\partial t$ and $\overline{u}_1 \, \partial/\partial x_1$ of the fluctuating velocity are large quantities. Fortunately, their combination represents the evolution of the turbulence in the moving frame of reference. The evolution of turbulence in the moving frame is known to be slow and the value of $\{D/Dt\} \, u'$ is of the same order of magnitude as the acoustic radiation. Hence, the effect produced by this term may be compared to the diffraction of sound by sound, which is a second order effect. After neglecting this term, the resulting equation has the same left-hand side as given by Phillips. It should be noted that such a term is important in the study of second order or nonlinear effects of wave generation processes.

In Reference 1, Equation (2) is further rewritten in a non-dimensional form. Since the parallel shear flow wave radiation problem can be regarded as statistically homogeneous in the x_1, x_2 and t coordinates, Fourier transformation in these coordinates has been taken, and the convected wave equation is reduced to an ordinary differential equation of second order. In the transonic and supersonic convected speed ranges, several types of solutions can be constructed for describing the wave radiation process in the low, intermediate, and high frequency ranges. In References 1 and 2, solutions have been obtained for both the high, and low frequency ranges of wave radiation. The wave radiation in the intermediate frequency range can be approximated by either the high or low frequency solution, and it has estimated that the error thus involved is less than 1.5 dB. In the following, the high frequency "Mach mode" radiation solution, designated as [S.1], and the low frequency "Mach mode" radiation solution, designated as [S.2], are given in their general forms:

[S.1]

$$\phi\,(y, \underset{\sim}{k}, \omega) = q^{-1/2} \left\{\frac{3}{2}\int^{y} q\,(y)\,dy\right\}^{1/6} \eta\,(\xi, \underset{\sim}{k}, \omega)$$

$$\eta\,(\xi, \underset{\sim}{k}, \omega) = H\,(\xi) + \int_{0}^{\xi} R\,(\xi, t) \left[g\,(t)\,H\,(t) + h\,(t)\right] dt \tag{3}$$

where $\phi\,(y, \underset{\sim}{k}, \omega)$ represents the sound pressure fluctuations in the far field, q is a wave number in the y-direction, and ξ is a transformed coordinate related to y. The source function is represented by $h\,(t)$ which includes both the self noise and shear noise sources. $\eta\,(\xi, \underset{\sim}{k}, \omega)$, as well as $H(\xi)$, is a linear function in terms of the Airy's functions $Ai\,(M^{2/3}\,\xi)$ and $Bi\,(M^{2/3}\,\xi)$. The function $R\,(\xi, t)$ is a resolvent kernel, and it is bilinear in terms of the Airy's functions. As a first approximation, the resolvent kernel is equal to the integral kernel of the transformed convected wave equation:

$$K\,(\xi, t) = \frac{\pi}{M^{2/3}} \left\{Bi\,(M^{2/3}\,\xi)\;Ai\,(M^{2/3}\,t) - Ai\,(M^{2/3}\,\xi)\;Bi\,(M^{2/3}\,t)\right\} \tag{4}$$

[S.2]

$$\phi\,(y, \underset{\sim}{k}, \omega) = q^{-1/2} \left\{2\int^{y} q\,(y)\,dy\right\}^{1/4} \eta\,(\xi, \underset{\sim}{k}, \omega)$$

$$\eta\,(\xi, \underset{\sim}{k}, \omega) = H(\xi) + \int_{0}^{\xi} R\,(\xi, t) \left[g\,(t)\,H\,(t) + h\,(t)\right] dt \tag{5}$$

In Equation (5), the definitions of various terms are similar to those in Equation (3). However, $\eta\,(\xi, \underset{\sim}{k}, \omega)$ and $H\,(\xi)$ are linear functions in terms of,

$$F_1\,(M^{1/2}\,\xi) = (1/2\,M\,\xi^2)^{1/4}\;J_{-1/4}\,(1/2\,M\,\xi^2)\,,$$

and

$$F_2\,(M^{1/2}\,\xi) = (1/2\,M\,\xi^2)^{1/4}\;J_{1/4}\,(1/2\,M\,\xi^2)\,. \tag{6}$$

The resolvent kernel $R\,(\xi, t)$ is a bilinear function of $F_1(M^{1/2}\,\xi)$ and $F_2(M^{1/2}\,\xi)$.

In addition to the solutions [S.1] and [S.2], a third solution [S.0] is necessary for a complete description of noise radiations from supersonic jets as well as subsonic jets. The solution [S.0] can be called the pure acoustic mode, or simply "acoustic mode". It is defined as

[S.0]

$$\phi\,(y, \underset{\sim}{k}, \omega) = q^{-1/2}\,\eta\,(\xi, \underset{\sim}{k}, \omega)$$

$$\eta\,(y, \underset{\sim}{k}, \omega) = H(\xi) + \int_0^{\xi} R\,(\xi, t)\; h\,(t)\; dt \tag{7}$$

where $\eta\,(y, \underset{\sim}{k}, \omega)$ and $H\,(\xi)$ are linear in the trigonometric functions $\sin\,(M\xi)$ and $\cos\,(M\xi)$, and the resolvent kernel can be defined as

$$R\,(\xi, t) = M^{-1}\left\{\sin\,(M\xi)\cos\,(Mt) - \cos\,(M\xi)\sin\,(Mt)\right\} \tag{8}$$

The detail expressions for terms in [S.0], [S.1], and [S.2] will be given in Appendix A.

In the present study, two principle types of noise sources are under consideration; the self noise and the shear noise terms. In a jet, or other shear flows, these two types of sources always appear in parallel, and their contributions to the overall far field noise is of the same order of magnitude. Aerodynamical noise is radiated from these sources in two modes: the "Mach mode" and the "acoustic mode". The "Mach mode" exists only in supersonic jet, while the "acoustic mode" is the dominating noise radiating mechanism in a subsonic jet.

Mathematically, the Mach mode radiations [S.1] and [S.2] are characterized by the process of accelerating the frozen components of pressure fluctuations in the turbulence such that acoustic radiation into the far field is achieved. Along the radiation ray path of solutions [S.1] and [S.2], acoustic mode radiation actually contributes also to the overall noise radiation. However, the Mach mode radiations [S.1] and [S.2] can only reach a limited sector in the first quadrant. Beyond the limits of this sector, the acoustic mode of radiation, [S.0], becomes the only noise generating mechanism. Therefore, the [S.0] acoustic mode remains to be a dominating noise radiation mechanism even in jets with transonic or low supersonic Mach numbers.

In rocket exhaust flows, the Mach mode radiations can practically reach the entire first quadrant since the apparent Mach number is very large. The pure acoustic radiation will dominate the sound field in the second quadrant. Although the contribution to the overall sound power by the [S.0] mode could be small in comparison to those from [S.1] and [S.2], the [S.0] mode of noise radiation remains very important because the rocket vehicle structure is entirely within the second quadrant for most launch and in-flight configurations.

2.2 Boundary Conditions

When the convected wave equation is reduced to an ordinary differential equation through Fourier transformations, two boundary conditions are required to determine the arbitrary constants in the general solution. These boundary conditions are specified by the physical requirements of wave propagations in each of the solutions [S.0], [S.1], and [S.2].

For the acoustic mode, [S.0], the turbulent sound source is the local acoustic component in the turbulent structure itself. Its wave number, frequency, and propagation direction is identical to the corresponding parameters on the radiation ray path as determined by the general solution [S.0]. Both the radiation ray path and the local sonic turbulent source components are vector quantities. Since the radiation ray path has a predetermined direction, the solution [S.0] represents the wave propagating in one way along the ray path. For this simple case, the boundary conditions are

a) The acoustic wave propagates away from the shear layer at positive infinity, and

b) The amplitude of the pressure fluctuation vanishes at negative infinity.

The second condition states that at negative infinity, the radiation path has not yet passed through any source regions, and the wave amplitude is zero.

For the solution [S.1], both acoustic and hydrodynamic source terms are encountered along the radiation path. The hydrodynamical sound sources are located below the so called critical, or transition point. In [S.1], the propagating wave is restricted to the high frequency range, the contribution of hydrodynamic sources to wave radiation decreases rapidly with the distance from the transition point. The statements of the boundary condition ramain the same as those in [S.0]. The continuity of the wave function at the transition point is automatically ensured by using the Airy's functions as the principal solutions.

For the solution [S.2], the physical mechanism of Mach wave radiation is markedly different from [S.1]. Since the wave number is assumed to be small, the subsonic layer which contains the hydrodynamical sources (Reference 1) is mathematically "thin". The hydrodynamic source is responsible for the wave radiations both above and under the shear layer. Continuity of the two radiated waves is maintained through the subsonic layer. In the original Phillips formulation, two separate wave equations are used for the upper branch and the lower branch of the propagation path. Therefore, four boundary conditions are required.

a) The waves are propagating away from the shear layer at positive and negative infinities, and

b) The wave function, as well as its first derivative, is continuous through the subsonic layer.

In view of the generalized analysis, there is no apparent advantage of using two separate wave equations. Same degree of approximation is involved if only one wave equation is used for both the upper branch and the lower branch. If one wave equation is used, the continuity of the wave function at the subsonic layer is automatically satisfied. The only required boundary conditions are the radiation criteria at plus and minus infinities.

In all the analysis carried out so far, only the instantaneous values of the pressure wave as well as the source function has been considered. However, the instantaneous value of the source function in an actual turbulent flow is not a statistically measurable quantity. Only the mean square values of the turbulent fluctuations, or the second order correlation functions, can be defined.

By examining the derivations of the analytical solutions, it can be found that the turbulent source functions is required for the boundary conditions. In a highly simplified notation, the boundary conditions can be written as

$$a_{11}A + a_{12}B = L_1 \qquad (9)$$

$$a_{21}A + a_{22}B = L_2 \qquad (10)$$

where a_{ij} are known constants, A, B are the arbitrary constants in the general solutions of the wave equation, and L_1, L_2 are known integrals of the generalized Fourier transforms of the instantaneous source functions. A parallel wave propagation problem can be posted such that the source function and the wave equation are the complex conjugate to the original mathematical problem. Therefore, the corresponding boundary condition can be written as

$$a^*_{11} A^* + a^*_{12} B^* = L^*_1 \qquad (11)$$

$$a^*_{21} A^* + a^*_{22} B^* = L^*_2 \qquad (12)$$

where the asterisks denote the complex conjugate of the various symbols. By cross multiplying Equations (9), (10), (11), and (12), and taking the statistical averages of the products, a set of four linear equations are obtained.

$$a_{11}a^*_{11} AA^* + a_{12}a^*_{11} A^*B + a_{11}a^*_{12} AB^* + a_{12}a^*_{12} BB^* = \overline{L_1L^*_1} \qquad (13)$$

$$a_{21}a^*_{21} AA^* + a^*_{21}a_{22} A^*B + a_{21}a^*_{22} AB^* + a_{22}a^*_{22} BB^* = \overline{L_2L^*_2} \qquad (14)$$

$$a_{11}a^*_{21} AA^* + a^*_{21}a_{12} A^*B + a_{11}a^*_{22} AB^* + a_{12}a^*_{22} BB^* = 0 \qquad (15)$$

$$a^*_{11}a_{21} AA^* + a^*_{11}a_{22} A^*B + a^*_{12}a_{21} AB^* + a^*_{12}a_{22} BB^* = 0 \qquad (16)$$

The overbar in Equations (13) and (14) denote the ensemble averages of the products of the source integrals. In this set of equations, all the coefficients on the left hand side are known. The products of the arbitrary constants AA*, A*B, AB*, and BB* can be regarded as the four unknown quantities. It can be shown (Appendix A) that the source integrals $\overline{L_1 L_1^*}$ and $\overline{L_2 L_2^*}$ can now be given in terms of the correlation functions of the turbulent fluctuations. The linear equations (13) through (16) can therefore be solved algebraically. It is interesting to note that, mathematically, only the magnitude of the wave amplitudes can be computed. The instantaneous phase of the wave function, will not be known because the values of A, B, A*, and B* can not be solved individually.

2.3 Characteristics of the Wave Functions

Since the convected wave equation is formulated according to the basic equation of aerodynamics, it is valid for shear flows of all speeds. For wave radiations in the acoustic mode, the wave equation is well behaved and the simplest solution, [S.0] can be constructed by methods of wave propagation in a nonhomogeneous medium. If the convection speed in the shear flow exceeds the ambient speed of sound, Mach wave radiation will be also a part of the wave generating mechanisms. Two more important solutions, [S.1] and [S.2], are therefore constructed by means of the WKB method. These representative solutions can cover all the aerodynamical noise generating mechanisms in a subsonic or supersonic shear flow. The domain of validity of these solutions overlap each other, hence, the mathematical problem of matching these solutions does not arise.

In a supersonic jet, the Mach wave radiation can reach a maximum angle of

$$\phi_M = \cos^{-1} (1/M_c) \tag{17}$$

where M_c is the convection Mach number. Beyond this angle, the hydrodynamic noise sources remain partially effective until the radiation angle in the far field reaches a critical angle

$$\phi_{cr} = \cos^{-1} \left\{ \frac{1}{M_c + 1} \right\} \tag{18}$$

For all far field radiation angles greater than ϕ_{cr}, the noise generation mechanism will be purely the acoustic mode. According to Equation (18), the angle ϕ_{cr} is always greater than zero for any positive value of M_c. Therefore, there is always a portion of wave radiation which belongs to the classes of solutions [S.1] and [S.2], even if the shear layer has a subsonic convection speed, (Reference 35).

Since the analysis of the generalized theory follows an approach which is different from the conventional methods employed in acoustics, the solutions [S.0], [S.1], and [S.2] are expressed in unfamiliar terms. However, once the meaning of a few key parameters have been clarified, the physical meaning of these analytical results will not be too difficult to interpret.

All the quantities in the final results are non-dimensional. The free stream Mach number is the ratio of the maximum speed in the shear layer to the ambient speed of sound. All the local convection speed in the shear layer is normalized as V, which has a value between zero and one. For any point in the shear layer, the local convection Mach number, M_c, is given by

$$M_c = M V_c \tag{19}$$

The non-dimensional frequency ω, as defined in the fixed frame of reference, is directly proportional to the Strouhal number, S_t

$$\frac{\omega}{2\pi} = \frac{f L}{U} = S_t \tag{20}$$

According to the definition of terms in the non-dimensional coordinates, the ambient speed of sound has a value of M^{-1}. Therefore, the relation between the frequency ω, and the wave number k, (magnitude of the wave number in the three-dimensional sense), can be written as

$$\omega = \pm \left(\frac{k}{M}\right) \tag{21}$$

It is very important to observe the sign convention for frequency and wave numbers. According to the definitions adopted in this analysis, a wave propagating in the positive direction of time corresponds to a negative value of ω. Hence, if a wave propagates in the positive direction as indicated by a wave number vector $\underset{\sim}{k}$, the frequency, ω, and the magnitude of the wave number, k, will have opposite signs. The quantities ω and k will have the same sign if the wave is propagating in a direction opposite to $\underset{\sim}{k}$. The function q, defined in the generalized theory as

$$q^2(y) = \left\{\frac{\omega + k_1 V(y)}{A(y)}\right\}^2 - \frac{k^2}{M^2} , \tag{22}$$

plays a central role in both the transformation of the wave equation and the final representation of the general solutions. It is directly proportional to the wave number in the y_3-direction:

$$k_3 = M q \tag{23}$$

By knowing q, the chosen values of k_1, k_2, and ω, one can define clearly the local wave propagation direction and phase speed in the three-dimensional space.

The Doppler shift effect, which is explicit in the Lighthill formulation, is implicit in the generalized theory. However, the exact Doppler shift relation can be derived from Equations (21), (22), and (23),

$$\omega = \frac{\omega_0}{1 - M_c \cos \phi} \tag{24}$$

where ω is the radiated wave frequency in the far field, ϕ is the angle between the direction of the shear flow and the direction of the far field wave propagation, and ω_0 is the local wave frequency of the turbulent source in a frame of reference which moves with a convection Mach number M_c.

A striking difference occurs between the Lighthill theory and the generalized theory concerning the practical significance of the Doppler shift effect in the noise generating mechanisms. In the Lighthill or Ffowcs Williams approach, the factor $(1 - M_c \cos \phi)$ in Equation (24) can take very large negative numbers if the convection Mach numbers is sufficiently large. This equation applies equally well to high frequency radiations as well as low frequency radiations. If the local frequency ω_0 equals to zero, it is obvious from Equation (24) that the source term is the frozen component of turbulence. The only possible wave radiation mechanism is Mach wave radiation in which

$$\cos \phi = M_c^{-1} \quad \text{and} \quad 1 - M_c \cos \phi = 0 \quad .$$

However, if $\omega_0 \neq 0$, the nature of the source term becomes very ambiguous because the orientation of the wave number in the turbulent source is unknown. Consequently, the local source fluctuation can be subsonic, sonic, or even the dynamically inadmissible case of locally supersonic. In the absence of refraction effects, one would have to assume that the wave number remains the same in both the source volume and the far field. Such an approach has been tacitly or explicitly in previous studies of jet noise (References 7 and 8). In this case, the source function will be locally subsonic (hydrodynamical) in nature for $|1 - M_c \cos \theta| < 1$; and the source function will be locally supersonic (dynamically inadmissible for infinitesimal waves) for $|1 - M_c \cos \theta| > 1$.

In the generalized theory, the detailed wave generating mechanisms which leads to the Doppler shift effect is clearly demonstrated by the analytical solutions. Two conclusions with practical importance shall be discussed here:

a) For any given supersonic convection Mach number, the convection factor $(1 - M_c \cos \phi)$ is predominately greater than minus one. It may be less than minus one only for wave radiations in the extremely low frequency range, $(k/M) \ll 1$.

b) The local sound source fluctuations are either hydrodynamical (subsonic) or acoustical (sonic) in nature. The correspondence between the far field radiation and the turbulent noise sources can be defined precisely.

In an actual shear layer, the convection and refraction effects are combined. For the solution [S.0], the convection factor is always greater than zero because its domain of application is restricted a region where

$$M_c \cos \phi \;<\; 1 \quad ,$$

in accordance with Equation (18). For the solution [S.1], the sound sources are *contained in two regions. In the subsonic layer between the transition point, the* sources are hydrodynamical. In the region between the upper transition point and the outer edge of the shear layer, the sound source is always sonic because the refraction effect changes the radiation direction continuously such that only the locally acoustic source component can contribute to the far field radiation. According to the definition of the solution [S.1], the region below the lower transition point will not contribute to the far field radiation for waves in the upper branch. If the convection speed of the frozen source layer in the hydrodynamical zone is chosen as M_c, then the convection speeds for layers in the hydrodynamical zone will range from $(M_c + 1)$ to $(M_c - 1)$. Since it is known from previous discussions that $M_c \cos \phi = 1$, therefore, the lowest value the convection factor can take is

$$1 - (M_c + 1) \cos \phi \;=\; - \cos \phi \geq - 1 \qquad (25)$$

Finally, *for the case* [S.2], *where Mach mode radiation in the very low frequency* range is considered, the source region in the shear layer below the lower transition point can also contribute to the far field wave radiation in the upper half space. Only in this case, the convection factor can be less than minus one.

In the expressions of the general solutions, two equivalent coordinates for the axis normal to the shear layer remain in effect, i.e., y and ξ. There is no analytical inconvenience involved because the coordinate y appears only implicitly in the definition of the source function h, and the wave number function q. In practical computations, the correspondence between y and ξ must be specified in order to locate the shear layers and the source functions properly.

The function $g(t)$ represents an error term introduced by the WKB transformation. Generally speaking, $g(t)$ is proportional to the local curvature of the velocity profile near the critical layer. It is generally small, and approaches zero to the order of ξ^{-2} in the far field. The precise definition of $g(t)$ in terms of mean flow parameters will be presented in Appendix A.

2.4 The Parametric Dependence of the Analytical Solution

A very important objective of the development of the generalized theory is identifying the noise generating mechanisms in a supersonic jet. An effective way to achieve such an investigation is to reduce the analytical solution to their essential basic elements by eliminating the quantitative details of the numerical values of various terms. In this section, the derivations of the parametric dependence of the analytical solutions will be discussed in detail. The various implications of the derived results will be discussed in subsequent sections.

In Equations (3), (4), and (5), the source function h (ξ) is related to the actual sound source in the physical coordinates via

$$h(\xi) = - \frac{\gamma M^2 \Gamma(y, \underset{\sim}{k}, \omega)}{\psi'^{3/2} \cdot A^2(y)} \tag{26}$$

where Γ (y, $\underset{\sim}{k}$, ω) is the instantaneous generalized spectrum of the turbulent noise source, M is the free stream Mach number, ψ' is the derivative of ξ with respect to y, and A (y) is the ratio of the local speed of sound to the ambient speed of sound.

The generalized spectral function Γ (y, $\underset{\sim}{k}$, ω) contains both the self noise and the shear noise. According to the definition of the source terms, the self noise is the product of the spatial derivatives of two velocity components. In the Fourier transformed wave number representations, the self noise term contains a factor of k^2 due to the differential operators. Furthermore, if the turbulent velocity fluctuation spectrum is assumed to be normal (Reference 10), the spectrum of $\overline{u_i u_j}$ in the noise production range will be a slowly varying function of k. It can be assumed to be a constant for practical purposes. The shear noise term is the product of the local mean velocity gradient and the derivative of the transverse turbulent velocity component. Since the generalized spectrum for the velocity fluctuation depends on k, the overall shear noise term depends on k^2. Hence, both the self noise and the shear noise source spectrum depends on k^2.

By examining the structures of the general solutions and the boundary conditions, one can easily find that the amplitude of the radiated wave depends mainly on the strength of the source function. The effect of the error term under the integral sign plays only a secondary role as far as the qualitative nature of the solution is concern. In the following derivations, the wave amplitude will be assumed to be directly proportional to the source integral.

For the acoustic mode solution [S.0], the coordinate transformation ξ is defined as

$$\xi = \int_0^y q(y)\, dy \quad ,$$

therefore

$$\psi' = \frac{d\xi}{dy} = q(y) \quad . \tag{27}$$

According to Equation (7), the resolvent kernel is a function of (M ξ) and (M t). If the source spectrum Γ (y, $\underset{\sim}{k}$, ω) is assumed to be constant throughout the source region, and the observation point ξ is in the far field, the source integral can be written as

$$\int_0^{\xi} R\,(\xi,t)\,h\,(t)\,dt \;=\; \frac{1}{M^2}\int_0^{\xi} \Big\{ \sin\,(M\,\xi)\,\cos\,(M\,t) - \cos\,(M\xi)\,\sin\,(M\,t) \Big\}\; h\,(Mt)\; d\,(Mt) \qquad (28)$$

Since h (t) is assumed to be constant, therefore, h (t) = h (M t). After this factor is taken out of the integral sign, the remaining integral is a pure mathematical function which is independent of the physical properties of the shear layer. The value of this integral is, therefore, a universal constant. By substituting the result of Equation (28) into Equation (7), the functional dependence of the far field noise radiation can be written as

$$\phi\,(y, \underset{\sim}{k}, \omega) \sim \gamma\; q^{-2} \left(\frac{c_j}{c_0}\right)^{-2} M^2 \left(\frac{k}{M}\right)^2 S_1\,(y, \underset{\sim}{k}, \omega) \qquad (29)$$

for the self noise, and

$$\phi\,(y, \underset{\sim}{k}, \omega) \sim \gamma\; q^{-2} \left(\frac{c_j}{c_0}\right)^{-2} M^2 \left(\frac{k}{M}\right)^2 (a\;\Omega)\; S_2\,(y, \underset{\sim}{k}, \omega) \qquad (30)$$

for the shear noise, where S_1 and S_2 are the generalized spectra for the velocity fluctuation components called for in each case, a is a longitudinal scale of the turbulent structure, and Ω is the shear velocity gradient in the mean flow. Since the shear noise and the self noise have essentially identical functional dependence on the shear flow and turbulence parameters (Reference 5), only the self noise will be discussed in the remainder of this report. Shear noise will not be mentioned separately unless it is necessary.

In the acoustic mode of radiation, the transverse wave number Mq has a value completely independent of the longitudinal wave number k because the orientation of the acoustic noise source orientation is arbitrary. Therefore, the function q in equations (29) and (30) is an arbitrary function, and can be omitted. As will be shown later in this section, q and k are closely related in the Mach modes [S.1] and [S.2] .

The mean square sound pressure fluctuation can be obtained by multiplying Equation (29) by its complex conjugate, and taking an ensemble average:

$$\phi^2\,(y, \underset{\sim}{k}, \omega) \sim \gamma^2\, M^4 \left(\frac{c_j}{c_0}\right)^{-4} \left(\frac{k}{M}\right)^4 \Psi\,(y, \underset{\sim}{k}, \omega) \qquad (31)$$

where $\Psi\,(y, \underset{\sim}{k}, \omega)$ is the spectrum of the mean square turbulent velocity fluctuations of the self noise source. According to the definition of ϕ (Reference 1) and Equation (21), Equation (31) can be written equivalently as

$$\overline{p^2}\,(y, \underset{\sim}{k}, \omega) \sim \gamma^2 p_0^2 \left(\frac{c_j}{c_0}\right)^{-4} M^4 \omega^4 \Psi(y, \underset{\sim}{k}, \omega) \tag{32}$$

For the high frequency Mach mode, [S.1], the coordinate transformation between y and ξ is defined as

$$\xi = \left\{\frac{3}{2}\int^{y} q(y)\,dy\right\}^{2/3} ,$$

and therefore

$$\psi' = \left(\frac{k}{M}\right)^{2/3} (2\,M\,\Omega\cos\theta)^{1/3} \tag{33}$$

in the neighborhood of the hydrodynamical source zone. Since the fluctuation levels of the hydrodynamic sources in a turbulent shear layer is much higher than those of the acoustic source terms, only the contributions from the neighborhood of the hydrodynamical source zone will be considered. By using Equation (3), (4), and the transformation of integration variable similar to the one employed in Equation (28), the contribution of the source integral is proportional to:

$$I \sim \gamma \left(\frac{c_j}{c_0}\right)^{-2} M^{13/6} \left(\frac{k}{M}\right) \Omega^{1/2}\, S_1(y, \underset{\sim}{k}, \omega) \tag{34}$$

According to Equation (3), the transformed solution $\eta(\xi, \underset{\sim}{k}, \omega)$ is a linear function of the Airy's functions Ai $(M^{2/3}\xi)$ and Bi $(M^{2/3}\xi)$. Their coefficients are proportional to the source integral, I, as given by Equation (34). In the far field, the asymptotic magnitude of Ai and Bi is given by

$$\lim_{\xi\to\infty} \mathrm{Ai}\,(M^{2/3}\xi),\ \mathrm{Bi}\,(M^{2/3}\xi) \sim M^{1/6}\left\{\frac{3}{2}\int^{y} q(y)\,dy\right\}^{-1/6} \cos\left\{M\int^{y} q\,dy + \mu\right\} \tag{35}$$

where μ is a constant phase angle. According to Equation (3), the final wave amplitude depends also on $q^{-1/2}$. The function q should take its value in the far field

$$q = \left(\frac{k}{M}\right)\sqrt{M_c^2 - 1} \sim \left(\frac{k}{M}\right)\cdot M \tag{36}$$

The radiated wave pressure fluctuation can now be obtained by combining Equations (3), (34), (35), and (36):

$$\phi\,(y, \underset{\sim}{k}, \omega) \sim \gamma \left(\frac{c_j}{c_0}\right)^{-2} M^{1.5} \left(\frac{k}{M}\right)^{1/2} \Omega^{-1/2} S_1\,(y, \underset{\sim}{k}, \omega) \tag{37}$$

Therefore, the mean square sound pressure fluctuations in the far field is

$$\overline{p^2}\,(y, \underset{\sim}{k}, \omega) \sim \gamma^2\; p_0^2 \left(\frac{c_j}{c_0}\right)^{-4} M^3 \left(\frac{k}{M}\right) \Omega^{-1}\, \Psi\,(y, \underset{\sim}{k}, \omega) \tag{38}$$

By using a similar procedure, the parametric dependence of the sound pressure fluctuations in the far field for the low frequency Mach mode, [S.2], can be obtained:

$$\overline{P^2}\,(y, \underset{\sim}{k}, \omega) \sim \gamma^2\; p_0^2 \left(\frac{c_j}{c_0}\right)^{-4} M^3 \left(\frac{k}{M}\right)^{1.50} \Omega^{-1.50}\, \Psi\,(y, \underset{\sim}{k}, \omega) \tag{39}$$

In Equations (31), (38), and (39), the sound pressure level in the far field is produced by a unit volume of turbulent noise source. The Mach number dependence of the far field sound pressure intensity is M^3. This is different from the $M^{3/2}$ dependence given before by Phillips (Reference 2). This is because that the Mach number dependence of k and q has not been included in the derivations in Reference 2.

3.0 THE OVERALL SOUND POWER AND THE SOUND SOURCE DISTRIBUTION

Equations (31), (38), (39) contain some important clues concerning the dependence of overall radiate sound power and sound source locations on the shear layer parameters such as convection Mach numbers and turbulent structures. The well established laws of U^8, U^3, and $\left\{(1 - M\cos\theta)^2 + \alpha^2 M^2\right\}^{-5/2}$ dependences of jet noise, as given by Lighthill, Ffowcs Williams, and Ribner, can be precisely reproduced in the generalized theory. Beyond these classical laws of dependence, the generalized theory can further explain many hitherto unknown properties of noise generating mechanisms in the transonic and supersonic convection speed ranges. Based on the results obtained in Section 2.0, the overall sound power radiated from a turbulent high speed jet and the sound source distribution along the axis of the jet will be discussed.

3.1 Parametric Dependences of the Overall Sound Power in the Far Field

Analytically, the acoustic mode solution [S.0] of the generalized theory is equivalent to the Lighthill's solution to the aerodynamical noise generation problem. From the parameteric equation (31), the dependence of the far field sound pressure level is proportional to $M^4\omega^4$. In the [S.0] mode of solution, the noise source is always locally sonic. Therefore, the frequency ω is proportional to the natural frequency of the fluctuation of the turbulence itself. In the existing literature, the local fluctuation frequency, ω_0, is often assumed to be proportional to the convection Mach number in the jet (Reference 11). It has also been suggested before (Reference 5) that the fluctuation frequency may be proportional to the local shear strength. In the latter case, the frequency ω_0 is actually proportional to the convection Mach number indirectly. Since the configuration of a round jet remain similar for a wide range of Mach numbers, the mean flow velocity gradient across the jet profile is directly proportional to the maximum speed of the jet. The famous U^8-law of Lighthill for jet noise in the low speed range is therefore, recovered. The far field frequency ω is related to the local sound source frequency ω_0 by the Doppler shift relation as given by Equation (24). Equation (31) represents the constant band width spectrum of the far field noise. If Equation (24) is substituted into Equation (31), and a band width adjustment

$$\frac{d\omega}{d\omega_0} = \frac{1}{1 - M_c\cos\phi}$$

is incorporated, the far field noise spectrum will be

$$\overline{p^2}\,(y, \underset{\sim}{k}, \omega) \sim \gamma^2\, p_0^2 \left(\frac{c_j}{c_0}\right)^{-4} M^4\, \frac{\omega_0^4}{(1 - M_c\cos\phi)^5}\, \Psi(y, \underset{\sim}{k}, \omega_0) \qquad (40)$$

Equation (40) agrees with the Lighthill's solution (Reference 6). It is interesting to note that, in Equation (31), (38), and (39), the factor $(c_j/c_0)^{-4}$ is equivalent to the dependence of jet noise on the density ratio $(\rho_j/\rho_0)^2$.

For the solution [S.1] and [S.2], the sound pressure level depends explicitly on M^3 in both Equations (38) and (39). The factor (k/M) is not a function of M although it is proportional to ω. In the shear layer, the principal noise source term can be related to the frozen turbulence, where the spectral characteristics is a function of k alone. Since k is inversely proportional to the hydrodynamical zone thickness, (1/M), (k/M) will be independent of M. Also, the velocity gradient Ω is a number normalized against a unit free stream velocity, the dependence on M has already been extracted and incorporated into the M^3 term. Hence, the solutions [S.1] and [S.2] represents the U^3-law as predicted by Ffowcs Williams (Reference 9). There are, however, a few important differences.

For the Mach mode radiations, not only that the Mach number dependence has been reduced from M^8 to M^3, but also that the wave number dependence has been reduced from $(k/M)^4$ to $(k/M)^{1.0}$ in [S.1], and to $(k/M)^{1.50}$ in [S.2]. The factor $(k/M)^n$ has an implicit influence on the noise power dependence on Mach number. In the work of Ribner (Reference 6) and also Lowson and Pao (Reference 5), formulas have been derived to express the broadband acoustic intensity. For a spectrum depending of $(k/M)^4$, the broadband intensity will depend on

$$\overline{p^2}\,(y) \sim M^8 \left\{(1 - M\cos\theta)^2 + \alpha^2 M^2\right\}^{-5/2}$$

if a Gaussian turbulent structure is assumed (Reference 5). For the cases of [S.1] and [S.2], the broadband intensity will depend on

$$\overline{p^2}\,(y) \sim M^3 \left\{\frac{(1 - M\cos\theta)^2}{M^2} + \alpha^2\right\}^{-1} \tag{41}$$

and

$$\overline{p^2}\,(y) \sim M^3 \left\{\frac{(1 - M\cos\theta)^2}{M^2} + \alpha^2\right\}^{-5/4} \tag{42}$$

respectively. Figure 1 shows the dependence of acoustic power on Mach number. By using the formulas given by Ribner or Lowson and Pao, a very high noise radiation efficiency can occur near the sonic convection speed of $M_c = 1$. If α^2 were chosen as 0.1, as given in Reference 5 , the deviation form the U^8 law near $M_c = 1$ is greatly reduced. The transition between the U^8 law to the U^3 law, as predicted by the generalized theory, agrees quite well with the existing data, compiled by Powell (Reference 12) on jet and rocket noise (Figure 2).

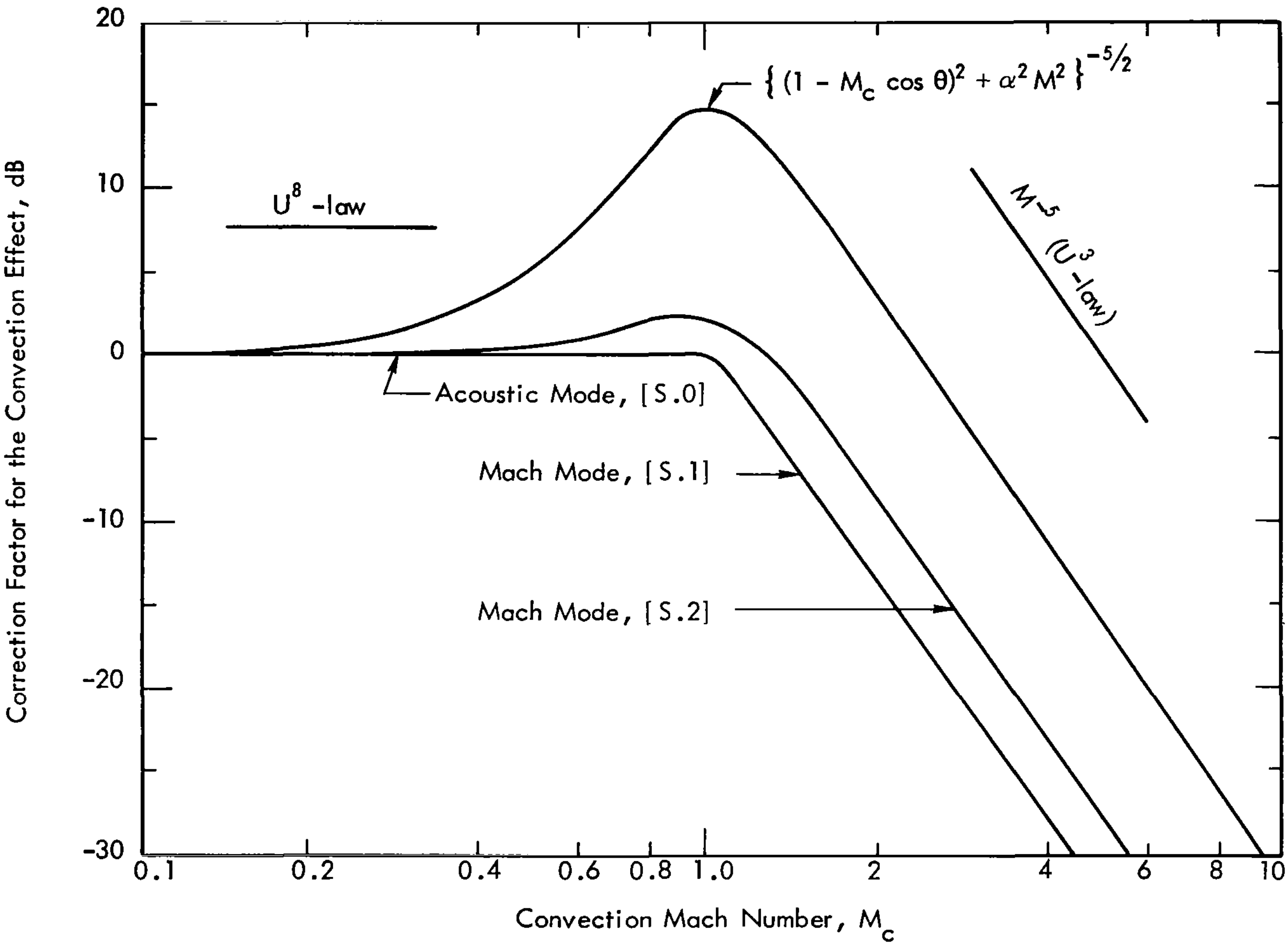

Figure 1. Jet Noise Radiation Efficiency as Predicted by the Lighthill's Theory and the Generalized Theory

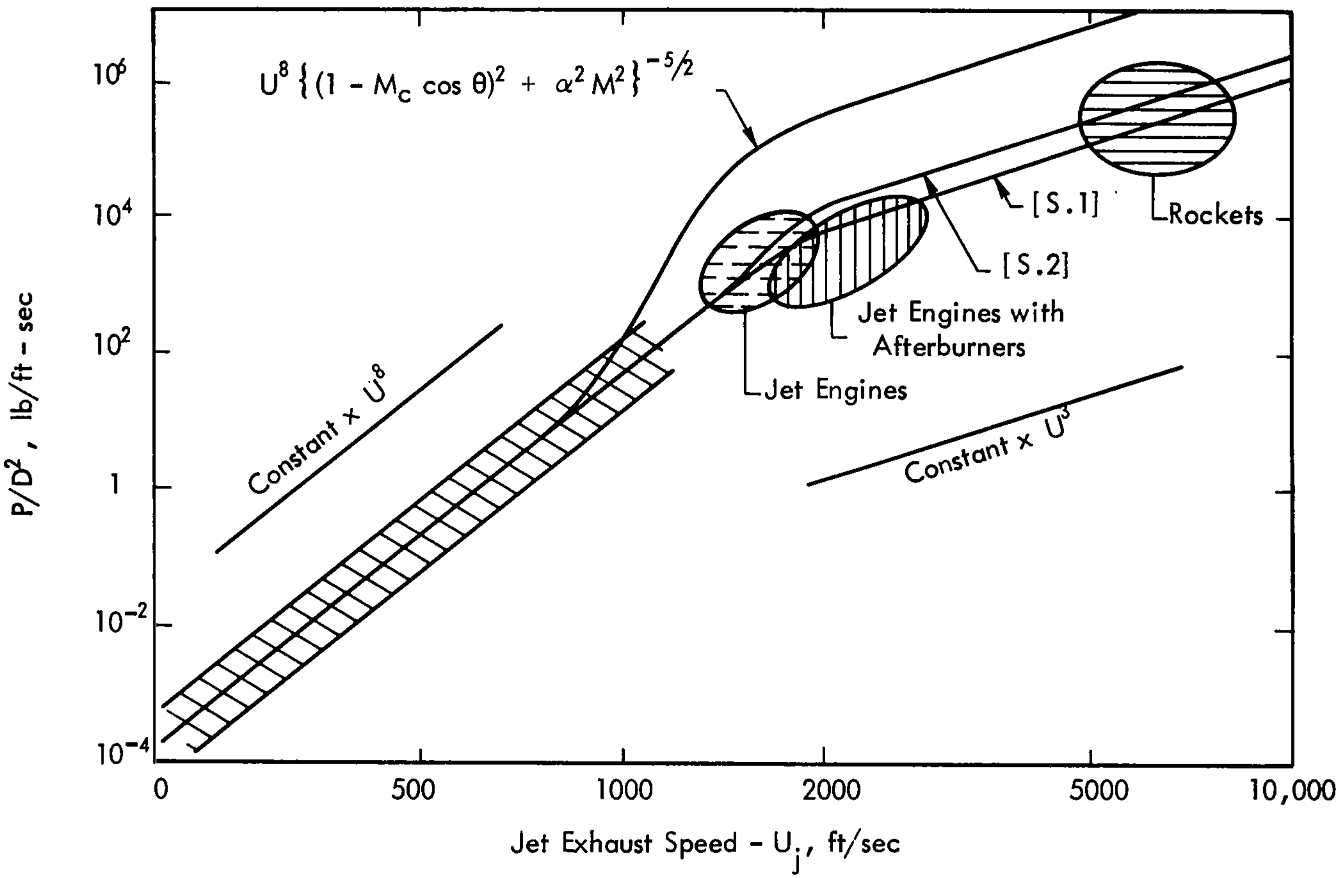

Figure 2. Comparison of Predicted Acoustic Efficiency with Jet Noise Data Obtained by Powell (Reference 12)

Another important mechanism in the Mach mode radiation is the dependence of noise intensity on the velocity gradient, Ω at the center of the mixing layer of the initial zone of a jet, Ω is of the order of unity. Its effect on the noise intensity is small in a cold jet. In the transition zone, however, the most intense turbulence moves toward the center of the jet. The jet turbulence intensity in the transition zone remain more or less the same as the mixing zone, while the shear decreases. According to the solutions [S.1] and [S.2], the radiated noise intensity will be greatly increased*. This mechanism may well account for the increase in noise radiation in the transition zone of a jet, which is commonly observed in experiments.

The temperature of the jet has an indirect influence on the overall intensity of the noise radiation, other than that its effect represented by the factor $(c_j/c_0)^{-4}$. The velocity gradient Ω as given in Equations (38) and (39) is actually a gradient of the local (actual) Mach number in the shear flow. In a high temperature jet, the temperature gradient in the jet reduces the magnitude of Ω significantly. According to preliminary calculations, Ω can be as low as 0.3 to 0.5 in the center of the mixing layer in a rocket exhaust flow. Therefore, the effect of Ω^{-1} in Equation (38) and $\Omega^{-1.50}$ in Equation (39) can produce a 3 dB to 5 dB increase in the overall acoustic power. A conclusive estimate would have to await the results of further numerical computations.

3.2 Sound Source Strength Distribution in a Supersonic Jet

The Equations (31), (38), and (39) represent the far field noise intensity produced by the turbulent noise sources in a unit volume. In physical terms, the wave number k, the frequency ω, and the velocity gradient Ω depend on the characteristic dimension of the jet exhaust flow. *If this characteristic dimension is represented by the integral turbulent* scale, a, then

$$k \sim a^{-1}$$

$$\Omega \sim a^{-1} \qquad (43)$$

$$\omega \sim a^{-1}$$

According to Equation (43), the dimensional forms of Equations (38) and (39) will be independent of a. The dimensional form of Equation (31) will also be independent of a in spite of its dependence on ω^4, because a factor of q^{-4} was omitted from the parametric Equation (31). In order to obtain the broadband noise intensity, Equations (31), (38) and (39) should be integrated with respect to ω or k. A factor of a^{-1} is introduced

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* This does not mean, however, that the Mach wave radiation near the center of the jet, where $\Omega = 0$, becomes infinite. In this case, another WKB solution shall be constructed. This solution is similar to [S.2] except that the dependences on Ω will be replaced by the dependences on Ω', which is the curvature of the local velocity profile.

via the integration. Hence, the broadband noise intensity will depend on a^{-1} for the acoustic mode [S.0], as well as the Mach modes [S.1] and [S.2].

In the initial region of the jet, the turbulent scale, a, can be assumed to be proportional to the distance from the jet nozzle along the axis of the jet, x_1 (Reference 13). The cross section of the mixing zone in the initial region of the jet is an annulus, hence, the volume per unit length of the jet is proportional to x_1. Therefore the sound source strength per unit length of the jet in this region is constant.

For a subsonic jet, this result agrees exactly with the x^0-law as given by Ribner (Reference 14). The greement of the x^0-law with experiment has been verified. In the supersonic regime, the evidences are conflicting. In some detailed jet noise measurements obtained by Eldred, et al., the source distribution in the initial region of the jet seem to follow the x^0-law very well (Reference 15). On the other hand, the unique source distribution experiment given by Potter and Jones (Reference 16), and the recent results by Nagamatsu (Reference 17), indicate that the sound source distribution in the mixing region of the jet increases linearly with the distance from the jet nozzle, i.e., a x^1-law. Such a conflict can not be resolved by using existing results from the theories.

In the transition zone of a supersonic jet, the volume per unit length of the jet as well as the turbulent scale remains constant. The mean velocity of the jet begins to decay. However, the turbulent intensity may still maintain a level close to its intensity in the mixing zone, (Reference 18). Furthermore, a large segment of the transition zone will be supersonic for a rocket exhaust, and Mach mode radiations will remain dominating. Due to the effect of Ω on the far field noise intensity, as discussed in Section 3.1, the noise source per unit length of the jet in the transition zone can actually be greater than the source strength per unit length in the mixing zone. Beyond the so called sonic point of the jet, the exhaust flow is identical with the developed region in a subsonic jet. The noise source dependence will naturally flow the x^{-7} - law of Ribner.

3.3 The Relative Importance of the Acoustic Mode and the Mach Modes

For either the supersonic or the subsonic case, the critical angle θ_{cr} as given by Equation (18) can be used as the dividing line for the domains of application of the acoustic mode and the Mach modes. The Mach modes dominate in the sector of $\phi < \phi_{cr}$ and the acoustic mode dominates in the obtuse sector of $\phi > \phi_{cr}$ which includes the transverse radiation and the upstream radiations.

Although both the Lighthill's theory and the generalized theory predicts the same U^8 and U^3 laws of dependence for various convection speed ranges, the generalized theory provides further insight into the relative importance of the Mach mode and the acoustic mode of radiation in a high speed jet. In the region of $\phi > \phi_{cr}$, the results of Ffowcs Williams (Reference 8) and the generalized theory are identical. For a high speed jet, the convection factor $(1 - M_c \cos \phi)$ is always positive. In upstream propagations, $(1 - M_c \cos \phi)$ is the order M, and therefore the sound power is proportional to U^3 for supersonic jets.

In the region of $\phi < \phi_{cr}$, the Ffowcs Williams' theory does predict both the Mach wave and acoustical wave radiations. However, the Mach wave is only one of the components over the source spectrum, and it does not dominate the far field noise. The change over from U^8 to U^3 is mainly due to the convection factor $(1 - M_c \cos \phi)$ as is the case of $\phi > \phi_{cr}$. Analytically, the transition of sound power from U^8 to U^3 due to the convection effect alone actually occurs at a relatively high supersonic Mach number (Figure 1). On the other hand, the generalized theory predicts that the Mach mode of radiation dominates the entire sector of $\phi < \phi_{cr}$, as discussed in Section 2.3. Since the Mach modes take over the radiation mechanism from the acoustical mode when $M_c \cos \phi \sim 1$, the transition of sound power dependence from U^8 to U^3 will occur at a relative low transonic convection Mach number. This new prediction seems to agree better with experimental results.

4.0 JET NOISE SPECTRUM AND DIRECTIVITY

4.1 A Review of Previous Studies

In practical applications, the knowledge of jet noise spectral and directional characteristics is as important as the overall sound power level. Such informations are commonly obtained via experimental measurements because the prediction of spectrum and directivity in jet noise is far more involved than the prediction of sound power level alone. There have been many previous investigations attempting to uncover the key mechanisms which would relate the actual spectrum and directivity of a jet to its mean flow and turbulent source properties. In this section, several representative apporaches will be discussed.

Shear Noise and Self Noise — The division of the overall turbulent noise source into two principal types was first given by Ribner. The shear noise is generated through the interaction of the turbulence with the mean shear in the jet exhaust flow. The noise source is basically dipole in nature, while its dependence on the convection effect is similar to a quadrupole. The shear noise has a two lobe directivity pattern oriented parallel to the jet axis, and a peak frequency generally lower than the peak frequency of the overall noise. The self noise is produced via the interaction of turbulence with itself. The self noise is basically quadrupole in nature. Since the spectrum of the self noise source is analytically related to the convolution of two turbulent velocity fluctuation spectra, the fundamental frequency of the self noise is one octave above the shear noise. In a stationary coordinate system, the self noise has an omni-directional directivity pattern. Through the distinction of self noise and shear noise, Ribner and his co-workers have explained many of the puzzling features of subsonic jet noise, such as the apparent reverse Doppler shift effect, and have designed various experiments for jet noise studies. Through both theoretical and experimental work (References 5, 6, 19, and 20) it was found that the shear noise and self noise power levels are in the same order of magnitude.

The Convection Factors — The convection factor obtained through the Lighthill theory is a fundamental property of classical acoustics. It remains to be the most important analytical tool for dealing with jet noise directivities. However, other than that some qualitative results concerning the orientation of maximum sound intensity direction, and the correlation of shear noise and self noise predictions with experiments in low speed jets, the effectiveness of this factor in dealing with high speed jet noise is inconclusive. This factor is equally applicable to sound radiation in all frequencies, while experimental evidences have shown that the directivity is frequency dependence, and that only the low frequency sound seems to follow the Lighthill's law of convection, (Reference 21).

Noise Spectrum for Given Turbulence Models – In a few analytical studies by Meecham, the noise spectral characteristics for several given turbulence modes have been predicted via the Lighthill's theory. It is assumed that the noise is produced in the energy containing wave number range of the turbulence. By using some simplying assumptions concerning the inter-relationship between the second order turbulence spectrum and the fourth order spectrum, as required by the jet noise theory, the calculation of the noise spectrum in the higher frequency end is permitted, (References 22 and 23). If one assumes that the energy spectrum is the Kolmogoroff spectrum

$$E(k) \sim k^{-5/3},$$

a power spectrum is obtained for the sound falling off at high frequency end as $\omega^{-4/3}$. On the other hand, if $E(k) \sim k^{-2}$ is used for the energy spectrum of the turbulence, a spectrum sometimes deserved experimentally for low Reynolds number flow, a power spectrum is obtained for the sound falling off as ω^{-2}. These spectra are flatter than some jet noise measurements; a frequency spectrum ω^{-3} would follow from $E(k) \sim k^{-2.5}$ The last spectrum is a realistic one and it has been used before by Lowson and Ollerhead for rotor blade noise predictions.

In some recent studies of Pao and Lowson (References 5, 7), the Lighthill's approach has been used for predicting the spectrum of noise generated by a low speed jet. It was found through these studies that transition zone of the jet is responsible for over 30 percent of the overall sound power. Furthermore, the sound power spectrum produced in this region matches with the overall sound power spectrum near the peak of the spectrum. The model of turbulent structure in this region will have important influences on the accuracy of spectral predictions.

Studies of the Refraction Effect – This effect has been studied both experimentally and analytically by Ribner (Reference 24), Miles (Reference 25), Eldred (Reference 15), Graham and Graham (Reference 26), Grande (Reference 27), and Schubert (Reference 37). In these studies, the classical ray acoustic approach or plane wave analysis is employed. In the analysis of Ribner and Miles, plane wave propagation through a shear layer with high relative speeds (subsonic or supersonic) is studied. The refracted and reflected wave system and the intensity of each wave component are discussed in detail. A puzzling result was obtained through this analysis. If a plane wave propagates against a convected parallel flow such that $-M_c \cos\phi \geq 2$, where M_c is the Mach number and ϕ is the direction of the incident wave, then a wave having an intensity greater than the intensity of the incident wave, will be reflected at the interface between the stationary ambient medium and the convected flow, and a backward propagating wave will be transmitted through the interface.

The Eldred's calculation was performed in connection with a comprehensive study of jet and rocket noise. The ray paths of waves originated at a point in the shear flow was computed. The results was found very useful in correlating acoustic measurements to the turbulent sources in the jet exhaust flow. The refraction of sound originated from a point source in a shear flow was also measured experimentally by Grande. The Graham and Graham analysis is a refined method along the line of the Ribner's analysis. The results can be applied to practical studies of jet noise characteristics. In a recent report by Schubert, the refraction of sound wave in a jet has been studied numerically in great detail.

Each of the above mentioned studies is quite thorough and self content. Yet, the overall jet noise spectral and directional properties are related to all of the properties as reviewed above. In particular, the effects of convection and refraction arc strongly coupled in actual high speed shear flows. Via the generalized theory, these important aspects of the jet noise spectrum and directivity can be examined in a unified manner.

4.2 The Peak Strouhal Number and Spectral Characteristics of Rocket Noise

The noise generated from the mixing zone, the transition zone, and the developed zone of a subsonic jet was found (Reference 6) to dominate the high frequency end, the peak region, and the low frequency end of the spectrum, respectively. Such a subdivision was found also by Lowson and Pao (Reference 5) to be convenient in practical noise prediction calculations. In Section 3.0, it was pointed out that the theoretical source strength distribution along the axis of the flow has essentially the same characteristics for either a subsonic jet or a supersonic jet. Hence, it is realistic to assume that the transition zone in the rocket exhaust flow will generate noise with the same characteristics as those near the peak of the spectrum.

For a supersonic jet flow, such as a rocket exhaust, the Mach mode radiation dominates the power spectrum. The sound frequencies in this regime depends on the longitudinal wave number spectrum of the forzen turbulence component in the jet flow, and not on the time rate of change of the turbulence. In non-dimensional coordinates, the integral spatial scale of the turbulence is proportional to M^{-1}. In other words, the correlation distance in the transverse direction is proportional to the distance between two points where the difference of convection speed is larger than the local speed of sound. There have been very few experimental results regarding turbulent structures in supersonic flows. Phillips (Reference 28) indicated that the ratio of longitudinal integral scale to the transverse turbulence scale can be as large as 18:1. In some measurements in a Mach 3.0 supersonic wake, Demetriades (Reference 29) found that the ratio is 10:1. According to Townsend (Reference 30) this ratio is only 3:1 for low speed shear layers. For noise prediction purposes, it is very important to know this scale ratio. For the above given numbers one can either assume that the ratio is a constant for all supersonic flows, or that the ratio increases with Mach numbers. The latter case is plausible because the potential core length of a supersonic jet does increase linearly with M, (Appendix B).

The spreading rate of the jet exhaust flow in the transition zone is relatively small. The diameter of the transition zone remains approximately constant (Reference 31). The characteristic dimension can be chosen as D, the exit diameter of the jet under perfect expansion conditions. According to the generalized theory, the far field noise frequency,

ω, is related to the longitudinal wave number, k, and the convected Mach number, M_c, via the equation as follows:

$$\omega = k (M_c/M) \tag{44}$$

If the transverse wave number in the turbulence is assumed to be

$$(k_3 A/M) = \pi$$

where A is the local speed of sound ratio, $A \simeq 3.0$, then the longitudinal wave number will be

$$(k_1 A/M) = 0.10\,\pi \tag{45}$$

if the scale ratio is assumed to be 10:1; and

$$(k_1 A/M) = 0.30\,\pi/M\ , \tag{46}$$

if the scale ratio is assumed to be proportional to the Mach number. If Equation (46) were chosen to be the representative model, then the peak Strouhal number will be

$$S_t = \frac{\omega}{2\pi} = \frac{k_1}{2\pi}\left(\frac{M_c}{M}\right) = \frac{1}{2\pi}\cdot\frac{0.30\pi}{A}\cdot\left(\frac{M_c}{M}\right) \simeq 0.05\left(\frac{M_c}{M}\right) \tag{47}$$

If (M_c/M) is assumed to be 0.60, then the peak Strouhal number will be

$$S_t \simeq 0.03.$$

This estimated peak Strouhal number is somewhat higher than the peak Strouhal number commonly measured in rocket noise power spectrum. The discrepancy can be the direct result of an under-estimate of the longitudinal spatial scale of the turbulence. It is more likely, however, that the source volume corresponding to the peak frequency is actually located further downstream. Since both the wave number and the convection speed are lower, the peak Strouhal number for the overall power spectrum will also have a lower estimated value.

The falloff of the power spectrum at the high frequency end is actually determined by the source strength distribution in the mixing zone. Since the characteristic radiating frequency for a small slice of the jet is inverse proportional to the thickness of the mixing layer at that location, the source width per unit frequency falls off as ω^{-2}. Hence, if the mixing zone source strength per unit length of the jet is a constant, the power spectrum at the high frequency end falls off as ω^{-2}. This seems to agree with the rocket noise spectrum given by Cole (Reference 32). If the source strength increases linearly with the distance from the nozzle, then the power spectrum will fall off as ω^{-3}.

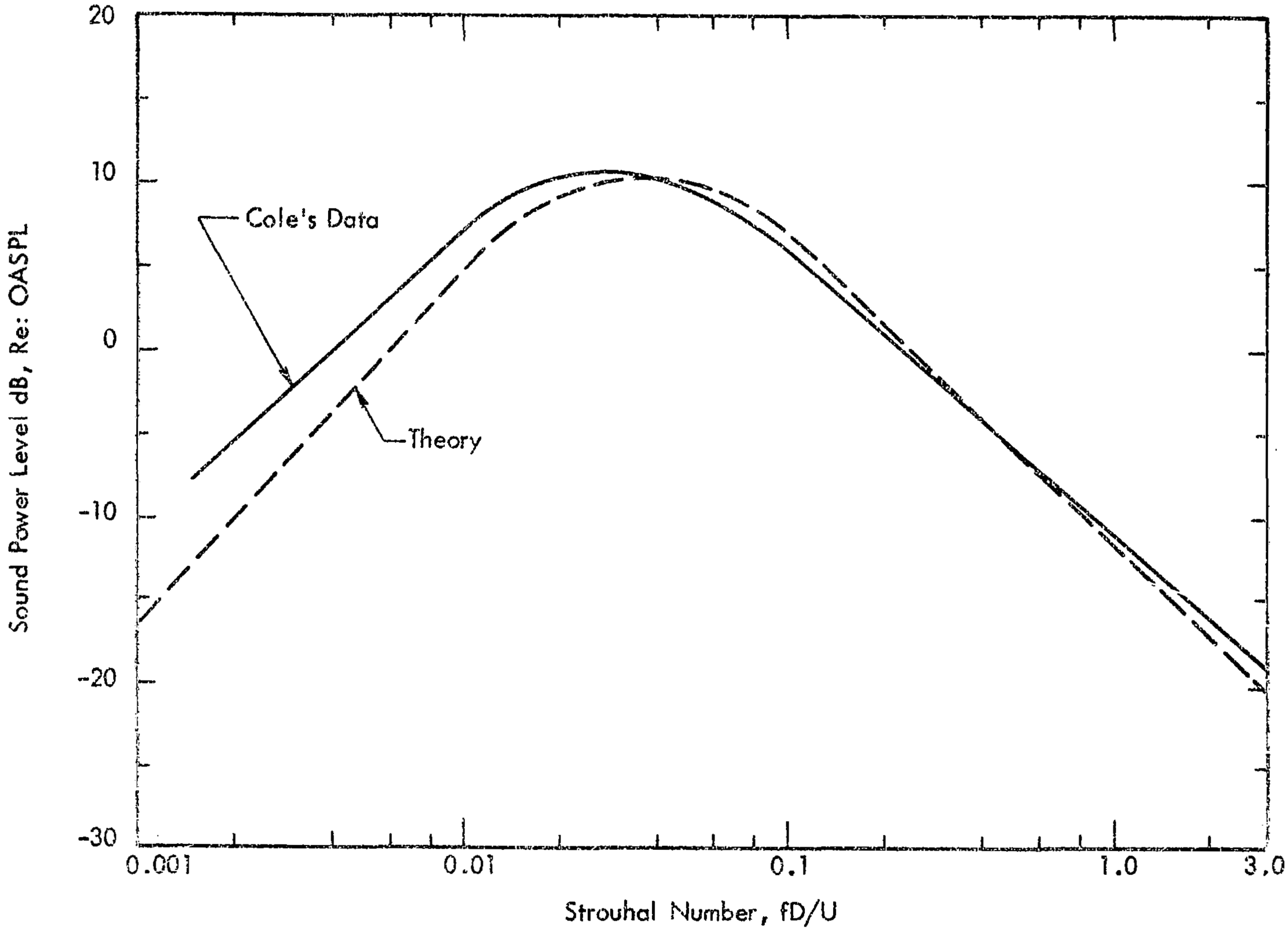

Figure 3. A Preliminary Prediction of Rocket Noise Spectrum, (Constant Band Width, 1 Hz)

The rocket noise spectrum, given by Wilhold, et al., (Reference 33, 34) for near field noise predictions, follows a ω^{-3} law in the high frequency end.

The low frequency of the spectrum corresponds to the noise produced in the developed region of the jet exhaust flow. Since the decreases of the turbulent intensity, the mean flow velocity, and the characteristic wave number are similar to the developed zone of a subsonic jet, the ω^{-2} law as derived by Ribner (Reference 6) should hold. However, such an asymptotic slope may not be reached in the portion of spectrum which is practically important. A preliminary predicted spectrum of rocket noise is shown in Figure 3.

4.3 ᵀ e Coupled Effects of Refraction and Convection

In a turbulent shear flow, the effects of convection and refraction in the noise generating mechanism is coupled. Through this coupling, two important consequences follow:

- For sound radiated locally in the forward direction, the refraction effect can significantly reduce the Doppler shift and intensification of the radiated noise in the far field.

- The high frequency noise and the low frequency noise have distinctively different directivity patterns. Low frequency noise radiation is high favored in the downstream direction.

Analytically, this coupled refraction and convection is described in the convected wave equation via the function q, which in turn depends on the convection velocity, local speed of sound, and the local frequency and longitudinal wave number of the turbulent source.

In the generalized theory, the convection effect is associated with only the longitudinal wave number. In the hydrodynamic pressure fluctuation regime, the convection effect increases the local frequency of the longitudinal wave components until the phase speed finally reaches the local speed of sound. This longitudinal hydrodynamic pressure fluctuation becomes a longitudinal wave component. While the frequency is being increased further by the convection, the transverse wave number, M q, begins to grow. The three-dimensional wave number maintains always a magnitude such that the frequency equals the product of the wave number and the local speed of sound, as required by fundamental fluid dynamical principles. Through this process, the convection factor is weakened. For example, a local wave component with frequency ω_0 in the turbulence, convected at a Mach number M_c parallel to the wave propagation direction, will have a Doppler shift of

$$\omega = \frac{\omega_0}{1 - M_c} \quad ; \qquad M_c < 1 \qquad (48)$$

The subsonic Mach number is chosen mainly for the simplicity of argument, and the generality of the physical phenomenon is not affected.

By including the refraction effect, the Doppler shift will be

$$\omega = \frac{\omega_0}{1 - M_c \cos\phi} = \omega_0 \left\{1 + M_c\right\}$$

with

$$\cos\phi = \left\{1 + M_c\right\}^{-1} \qquad (49)$$

By comparing Equations (48) and (49), it is interesting to note that they are approximately equal for small convection Mach numbers. For the high subsonic Mach numbers, the Doppler shift given by (48) is much larger than the Doppler shift of Equation (49). The leading error term in Equation (48) is proportional to M_c^2, which has the same dimensional dependence as the compressibility effect. The compressibility effect is indeed an underlying mechanism which modifies the classical Doppler shift law.

In addition to the distinction of the self noise and shear noise spectra and directivity pattern, there is another major mechanism which favors the forward radiation of low frequency noise in a high speed jet. For the Mach mode radiations, the far field wave frequency is related to the longitudinal wave number through Equation (44). In a supersonic jet, the longitudinal wave number is determined by the overall mixing process in the jet, and therefore, it will remain approximately the same throughout the thickness of the jet. The peak of the jet noise spectrum is produced in the intense region of turbulence near the half velocity region of the jet, the shear layer near the boundary of the jet is still a strong source region. However, the lower convection speed will result in a lower frequency and a smaller refraction angle.

For the [S.2] mode, the acoustical sources below and above the "hydrodynamic layer" are coupled on the same radiation path. Since the "deep" source, which lies far below the "hydrodynamic layer," obeys the classical Lighthill convection law, these sources will have a predominately low frequency character in the far field for downstream radiations.

$$\omega = \frac{\omega_0}{1 - M_c \cos\phi} \quad ; \quad M_c \gg 1$$

where

$$-\ (1 - M_c \cos\phi) >> 1 \quad \text{for} \quad \phi \simeq 0^\circ \qquad (50)$$

Lowson (Reference 5 , 21) has pointed out repeatedly that the Lighthill's theory is probably obeyed only by the low frequency radiations for high speed jets. His arguments were based on both analytical and experimental evidences. This bewildering postulation is now strongly supported by the generalized theory. Since the far field noise directivity is strongly dependent on the frequency, it is no longer a simple task to define apparent noise sources for rocket noise predictions. For near field noise environment predictions, the major concern is the sound pressure levels on the vehicle structure. According to either the Lighthill's approach or the generalized theory, the characteristic frequency of the upstream noise radiation is expected to be low. In contrast, the acoustic field right angle to the rocket exhaust flow direction may have a significantly higher characteristic frequency. Hence, it is highly disputable to assign only one set of source locations to the same frequencies for noise prediction in different directions.

4.4 Some Preliminary Calculations of Directivity

Based on the results given in Section 2.0, the directivity pattern of overall noise radiation from high speed jet can be computed. Some preliminary calculations are made for the Mach modes to show the trends. General properties of the acoustic mode, [S.0], are essentially the same as the results given previously by Ffowcs Williams, (Reference 8), and therefore, the [S.0] mode is not included in the preliminary study of directivity.

The shear layer is assumed to have a velocity profile defined by

$$V(y) = e^{-2(y-y_0)^2} \qquad (51)$$

where y_0 denotes either the inner edge of the mixing layer in the initial region of the jet exhaust flow, or the center line of the jet in the transition zone. In the latter case, $y_0 = 0$. The temperature in the shear layer is assumed to be a constant, and it equals to the ambient speed of sound. It is further assumed that the sound source intensity across the shear layer is constant.

If only the Mach wave radiation from the hydrodynamic sources are considered, the directivity function has a relative simple from (Reference 2) for the [S.2] mode

$$f(\phi) = \Omega^{1/2} \tan\phi \quad ; \quad \phi = \cos^{-1}\left\{ 1/M_c \right\} \qquad (52)$$

where $f(\phi)$ is the directivity function which is proportional to the sound pressure intensity in the far field. The directivity function for the [S.1] mode can be given in a similar form:

$$f(\phi) = \Omega^{0.67} \tan\phi \quad ; \quad \phi = \cos^{-1}\left\{ 1/M_c \right\} \qquad (53)$$

The directivity pattern for [S.2] mode with various free stream Mach numbers are given in Figure 4. The directivity pattern given for Figure 4 is computed for a plane shear layer. This is representative of the noise radiation pattern for sound originated in the mixing zone. For the transition zone, the shear flow is best represented in a cylindrical coordinate. In this case, the volume of turbulent source is a function of y:

$$d(\mathrm{Vol}) = y\,dy\ .$$

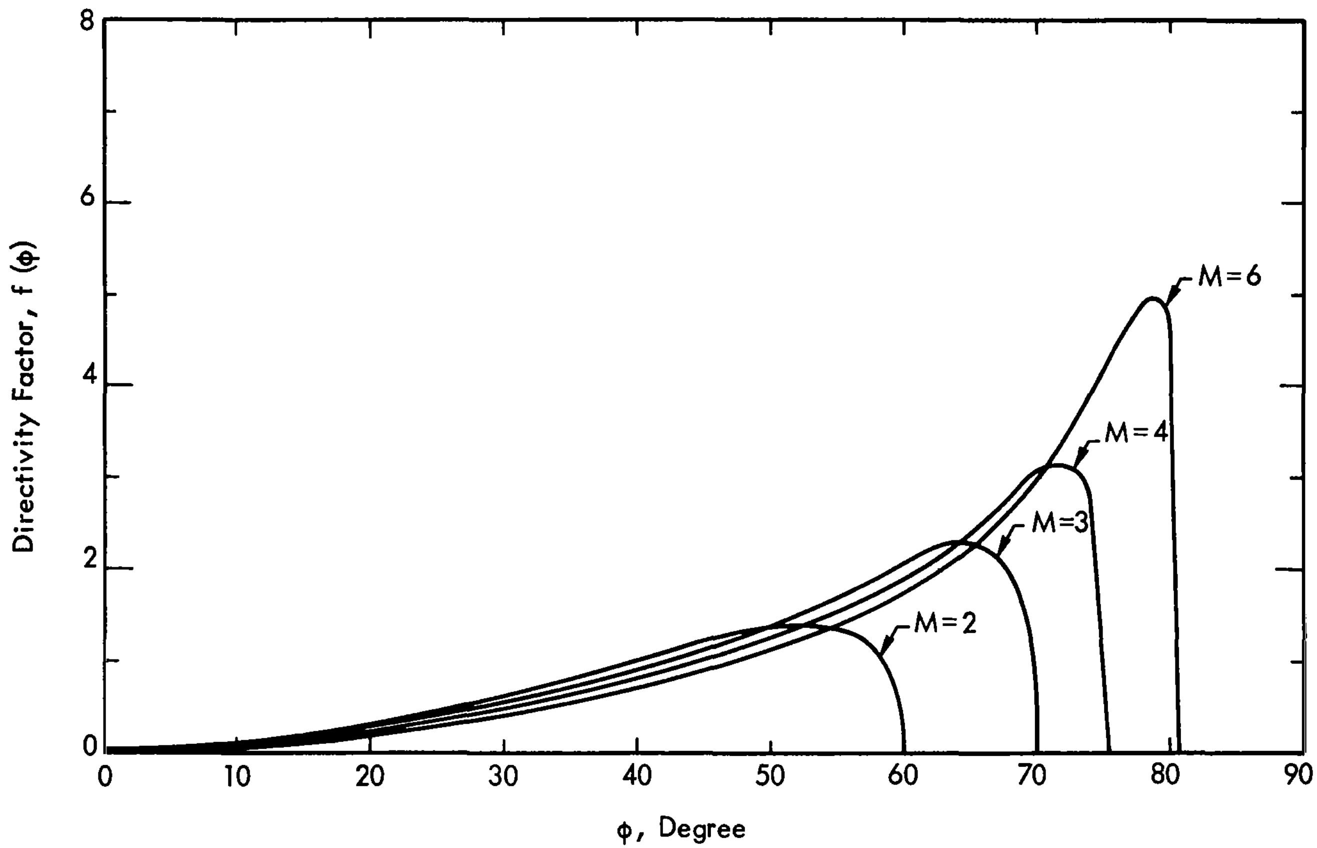

Figure 4. Mach Wave Directivity Factor for a Turbulent Shear Layer. The turbulent source strength is assumed to be constant across the shear layer

Furthermore, the area of the wave front in the far field is a function of ϕ:

$$dA \sim \sin\phi \; d\phi$$

By taking these factors into account, the directivity pattern for noise radiated by a cylindrical turbulent shear layer is shown in Figure 5. The peak noise intensity direction as shown in Figure 5 agrees quite well with the acoustic data commonly obtained for jet engine noise measurements.

For Mach wave radiation from hydrodynamic sources alone, the directivity pattern for the self noise and the shear noise is essentially the same because only the turbulent component parallel to the jet axis is important. The only difference is the magnitude of the sound pressure intensity. If the acoustic sources alone with the Mach mode radiation paths are taken into account, the directivity patterns for the self noise and the shear noise will be different. In Figure 6, a set of computations for a $M = 2.0$ shear flow is shown. For large angles of ϕ, the self noise level is higher than the shear noise. This difference is directly related to the difference in locally directivity patterns for the shear and the self noise sources.

In the moving coordinates, the self noise source directivity is omnidirectional. The shear noise has a two-lobe directivity pattern given by (References 5 and 6)

$$\Psi(\phi) \sim \frac{\cos^2\phi + \cos^4\phi}{2} \tag{54}$$

For a Mach mode radiation, the local direction of propagation, which indicates also the orientation of the sound source component, is given by

$$\cos\phi_s = (M_c - M_s)^{-1} \quad ; \qquad (M_c - M_s) \geq 1 \tag{55}$$

where ϕ_s is the angle of local propagation direction, M_s is the convection Mach number at the layer where the acoustic source is located, and M_c is the convection Mach number at the frozen reference layer in the hydrodynamic zone. According to Equation (54) and (55), the contribution from the shear noise sources decreases rapidly along the radiation path in a high speed shear flow, once the path leaves the hydrodynamic zone.

In actual jet noise predictions, the distribution of source strength in the shear layer must be taken into account. Since the self noise depends on the turbulent intensity to the fourth power, the effect of source intensity distribution on directivity patterns will be important in the generalized approach. Further results would have to await the development of a comprehensive computer program for the generalized jet noise prediction theory.

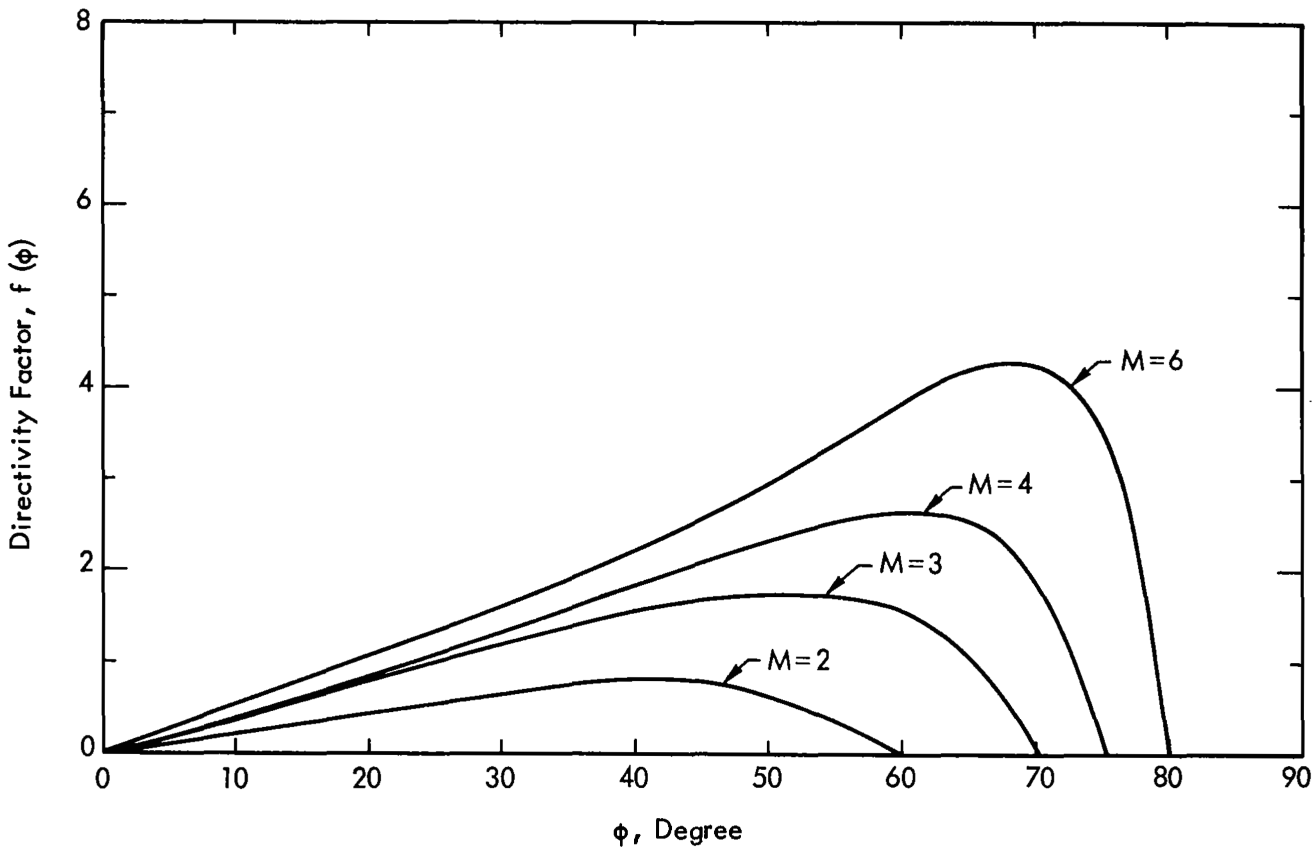

Figure 5. Mach Wave Directivity Factor for a Cylindrical Turbulent Shear Flow. The turbulent source strength distribution in the transverse direction is assumed constant

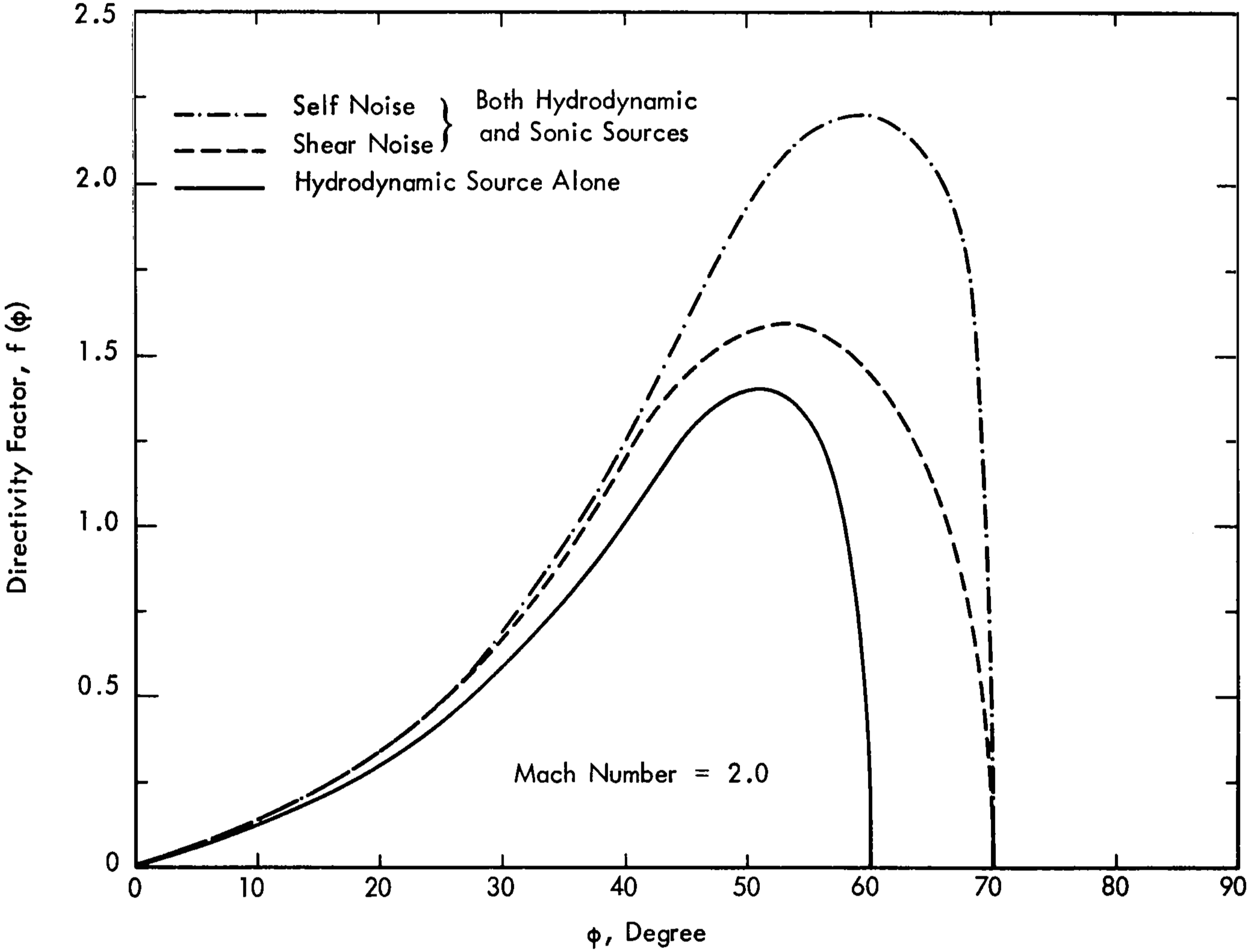

Figure 6. Noise Radiation Directivity Pattern for the Mach Modes [S.1] and [S.2]. The turbulent source strength is assumed to be constant across the shear layer

REFERENCES

1. Pao, S.P., "A Generalized Theory on the Radiation of Noise from Supersonic Shear Layers," Wyle Research Staff WR 70-1, 1970.

2. Phillips, O.M., "On the Generation of Sound by Supersonic Turbulent Shear Layer," Journal of Fluid Mechanics, Vol. 9, pp. 1-28, 1960.

3. Laufer, J., Ffowcs-Williams, J.E., and Childress, S., "Mechanism of Noise Generation in the Turbulent Boundary Layer," Agardograph No. 90, NATO (1964).

4. Ffowcs Williams, J.E., "Hydrodynamic Noise," article in Annual Review of Fluid Mechanics (Ed. W.R. Sears) Vol. 1, pp 197-222, Annual Review Inc., Palo Alto, (1969).

5. Lowson, M.V., Pao, S.P., "Some Applications of Jet Noise Theory," Wyle Laboratories Research Report WR 70-4, 1970.

6. Ribner, H.S., "The Generation of Sound by Turbulent Jets," Advances in Applied Mechanics, Vol. 8, Academic Press, New York, 1964.

7. Pao, S.P., Lowson, M.V., "Spectral Technique in Jet Noise Theory," Wyle Laboratories Research Report WR 68-21, (1969).

8. Ffowcs Williams, J.E., "The Noise from Turbulence Convected at High Speed," Phil. Trans. Roy. Soc. London, Vol. A255, p. 469, (1963).

9. Lighthill, M.J., "On Sound Generated Aerodynamically. II. Turbulence as a Source of Sound," Proceedings of the Royal Society, Vol. A222, 1954, pp. 1-21.

10. Batchelor, G.K., The Theory of Homogeneous Turbulence, Cambridge University Press (1953).

11. Davies, P.O.A.L., et al., "The Characteristics of the Turbulence in the Mixing Region of a Round Jet," J. Fluid Mech., Vol. 15, pp. 337-367, Corrigendum, p. 559, 1963.

12. Powell, A., "On the Generation of Noise by Turbulent Jets," Amer. Soc. Mech. Eng. ASME Paper 59-AV-53 (1959).

13. Laurence, J.C., "Intensity, Scale, and Spectra of Turbulence in Mixing Region of Free Subsonic Jet," NACA Report 1292, Lewis Center Flight Propulsion Laboratory (1956).

14. Ribner, H.S., "On the Strength Distribution of Noise Sources Along a Jet," University of Toronto, Inst. of Aerophysics, UTIA Report No. 51 (1958).

15. Eldred, K.M., et al., "Suppression of Jet Noise with Emphasis on the Near Field," TR ASD-TDR-62-578 Flight Dynamics Laboratory, Wright-Patterson AFB, Ohio, 1963.

16. Potter, R.C., and Jones, J.H., "An Experiment to Locate the Acoustic Sources in a High Speed Jet Exhaust Stream," Wyle Laboratories Report, 1967.

17. Nagamatsu, H.T., and Horvay, G., "Supersonic Jet Noise," General Electric Research and Development Center, Mechanical Engineering Laboratory, Report No. 69-C-161, April 1969.

18. Tu, B.J., "On the Measurement of Turbulent Velocities u_0, u_{45}, u_{90}, u_{135}, and w in a Circular Jet," Wyle Research Staff WR 70-2, 1970.

19. Ribner, H.S., "New Theory of Jet-Noise Generation, Directionality, and Spectra," Journal of the Acoustic Society of America, Vol. 31, No. 2, 1959, pp. 245-246.

20. Ribner, H.S. and MacGregor, G.R., "The Elusive Doppler Shift in Jet Noise," The Sixth International Congress on Acoustics, Paper No. F-3-8, Tokyo, Japan (1968).

21. Lowson, M.V., et al., Private Communication, 1970.

22. Meecham, W.C., "Acoustic Spectra from Turbulent Jets," Basic Aerodynamic Noise Research - NASA SP-207, July 1969.

23. Meecham, W.C., "Acoustic Spectra from Turbulent Jets," Journal Acoust. Soc. Am., Vol. 49, pp. 334-338 (1970).

24. Ribner, H.S., J. Acoust. Soc. Amer., 29, 435-441 (1957).

25. Miles, J.W., J. Acoust. Soc. Amer., 29, 226-228 (1957).

26. Graham, E.W., and Graham, B.B., "Effect of a Shear Layer on Plane Waves of Sound in a Fluid," The Journal of Acoust. Soc. Am., Vol. 46, p. 169 (1969).

27. Grande, E., "Refraction of Sound by Jet Flow and Jet Temperature, II," University of Toronto, Inst. for Aerospace Studies, UTIAS Report No. 110 (1966).

28. Phillips, O.M., "Shear Flow Turbulence," article in Annual Review of Fluid Mechanics, (Ed. W.R. Sears) Vol. 1, pp 245-264, Annual Review Inc., Palo Alto, (1969).

29. Demetriades, A., "Turbulence Measurements in an Axisymmetric Compressible Wake," Phys. of Fluids, Vol. 11, p. 1841, 1968.

30. Townsend, A.A., The Structure of Turbulent Shear Flow, Cambridge Univ. Press. 1956.

31. Donaldson, C. duP., and Gray, K.E., "Theoretical and Experimental Investigation of the Compressible Free Mixing of Two Dissimilar Gases," AIAA J., Vol. 4, pp. 2017-2025, 1966.

32. Cole, J.N., et al., "Noise Radiation from Fourteen Types of Rockets in the 1000 to 130,000 Pound Thrust Range," WADC Technical Report TR 57-354 (1957).

33. Wilhold, G.A., et al., "A Technique for Predicting Far Field Acoustic Environments Due to Moving Rocket Sound Source," NASA TN D-1832, 1963.

34. Wilhold, G.A., et al., Private Communication.

35. Powell, A., "Concerning the Noise of Turbulent Jets," J. Acoust. Soc., Am., Vol. 32, p. 1609 (1960).

36. Kovasznay, L.S.G., "Turbulence in Supersonic Flow," Journal of the Aeronautical Sciences, Vol. 20, pp 657-682 (1953).

37. Schubert, L.K., "Refraction of Sound by a Jet, A Numerical Study," Institute for Aerospace Studies, University of Toronto, UTIAS Report No. 144, (1969).

APPENDIX A

APPENDIX A

DETAILS OF THE MATHEMATICAL ANALYSIS

The Derivation of the Acoustic Mode Solution, [S.0]

In Reference 1, the convected wave equation in non-dimensional coordinates is

$$\frac{d^2\phi}{d\,y^2} + M^2 q^2 \phi = - \frac{M^2 \cdot \Gamma\,(y, \underset{\sim}{k}, \omega)}{A^2} \tag{A-1}$$

where

$$q^2(y) = \left\{ \frac{\omega + k_1 V(y)}{A\,(y)} \right\}^2 - \frac{k^2}{M^2} \quad .$$

For the acoustic mode of radiation, the function $q^2(y)$ is always positive. Equation (A-1) can be transformed into a simple wave equation

$$\frac{d^2\eta}{d\,\xi^2} + M^2\eta = - \frac{M^2 \;\; \Gamma\,(y, \underset{\sim}{k}, \omega)}{\psi'^{\,3/2} \;\; A^2} \tag{A-2}$$

by using the transformations

$$\eta\;(\xi) = q^{1/2}\;\phi\,(y)$$

and

$$\xi = \psi(y) = \int^{y} q\,(y)\,dy \quad . \tag{A-3}$$

The general solution of Equation (A-2) can be written as

$$\eta\,(\xi) = A \cos(M\xi) + B \sin(M\xi) - \int^{\xi} R\,(\xi, t) \;\; \frac{M^2\;\Gamma(y, \underset{\sim}{k}, \omega)}{\psi'^{\,3/2}\;A^2} \;\; dt \tag{A-4}$$

with

$$R\,(\xi, t) = \frac{1}{M} \left\{ \sin(M\xi)\,\cos(Mt) - \cos(M\xi)\,\sin(Mt) \right\} \tag{A-5}$$

as a first approximation. The error involved in this case is small. This acoustic mode solution is applicable to acoustic wave propagations along ray paths which are nowhere parallel to the shear flow direction. According to Equation (A-4), the local sound sources, for given ω and $\underset{\sim}{k}$, will necessarily be the sonic component of the local turbulence structure. The integral on the right hand side of Equation (A-4) can be interpreted as a "local" Fourier transformation which selects a component, with a wave number approximately the value of Mq, from the source function, Γ. Through some algebraic manipulations, it can be shown that the product of the local speed of sound and the combined magnitude of k and Mq, is equal to the local frequency of turbulent fluctuations.

The Boundary Conditions

Two types of boundary conditions can be prescribed for the convected wave equation. The first type is a radiation condition at infinity. In the y-, and t-(time) coordinates, the wave velocity component on the y-axis is given by (Reference 2)

$$W = -i\,\omega\,\phi\,(y) \bigg/ \left\{ \frac{\partial\,\phi\,(y)}{\partial\,y} \right\} \qquad \text{(A-6)}$$

For the present case of a jet exhaust flow, W is required to be positive in the positive y-direction, and negative in the negative y-direction. The second type of boundary condition requires simply that the wave function, ϕ, vanishes at minus infinite. It states that the wave amplitude is zero at the beginning of the wave radiation path,

$$\lim_{y \to -\infty} \phi\,(y) = 0 \qquad \text{(A-7)}$$

In all three modes of solutions, [S.0], [S.1], and [S.2], the wave function can be reduced to trigonometric functions with constant coefficients at positive infinity. The radiation boundary condition can be obtained via Equation (A-6) as

$$a_{11} A + a_{12} B = L_1 \qquad \text{(A-8)}$$

where a_{11} and a_{12} are known constants, A, B, are the arbitrary constants in the general solution, and L_1 is given by the source integral

$$L_1 = \int_0^{\infty} R\,(\xi, t)\,h\,(t)\,dt\ ; \quad \xi \to \infty \qquad \text{(A-9)}$$

The second boundary condition, which is either a radiation condition or an initial condition, can be written as

$$a_{21} A + a_{22} B = L_2 \qquad \text{(A-10)}$$

where L_2 is also a source integral with a definition formally identical to Equation (A-9). For the acoustic mode [S.0], the value of L_2 is zero.

If the source function is replaced by its complex conjugate, a new wave radiation problem will result. The boundary conditions for this complex conjugate problem are:

$$a^*_{11} A^* + a^*_{12} B^* = L^*_1 \tag{A-11}$$

$$a^*_{21} A^* + a^*_{22} B^* = L^*_2 \tag{A-12}$$

By cross multiplying Equations (A-8), (A-10), (A-11), and (A-12), and taking the ensemble average of each product, a set of four equations will result

$$a_{11}a^*_{11} AA^* + a_{12}a^*_{11} A^*B + a_{11}a^*_{12} AB^* + a_{12}a^*_{12} BB^* = \overline{L_1 L^*_1} \tag{A-13}$$

$$a_{21}a^*_{21} AA^* + a^*_{21}a_{22} A^*B + a_{21}a^*_{22} AB^* + a_{22}a^*_{22} BB^* = \overline{L_2 L^*_2} \tag{A-14}$$

$$a_{11}a^*_{21} AA^* + a^*_{21}a_{12} A^*B + a_{11}a^*_{22} AB^* + a_{12}a^*_{22} BB^* = 0 \tag{A-15}$$

$$a^*_{11}a_{21} AA^* + a^*_{11}a_{22} A^*B + a^*_{12}a_{21} AB^* + a^*_{12}a_{22} BB^* = 0 \tag{A-16}$$

Since the source terms in the integrals L_1 and L_2 are separated by a convection speed of more than the speed of sound, these source integrals are not correlated. Therefore, the right-hand sides of (A-15) and (A-16) have zero ensemble averages. The definitions of $\overline{L_1 L^*_1}$ and $\overline{L_2 L^*_2}$ are similar in form:

$$L_1 L^*_1 = \lim_{\xi \to \infty} \iint_0^{\xi} R\ (\xi, s)\ R^*\ (\xi, t)\ h(s)\ h^*(t)\ ds\ dt \tag{A-17}$$

If the ensemble average and the integral signs are assumed to be exchangable in position, then

$$\overline{L_1 L^*_1} = \lim_{\xi \to \infty} \iint_0^{\xi} R\ (\xi, s)\ R^*\ (\xi, t)\ \Psi\ (s, t;\ \underset{\sim}{k},\ \omega)\ ds\ dt \tag{A-18}$$

with

$$\Psi (s, t;\ \underset{\sim}{k},\ \omega) = \overline{h\ (s)\ h^*\ (t)} \tag{A-19}$$

where the parameters $\underset{\sim}{k}$, and ω , as given in Ψ, have been omitted so far from h (s) and h (t) for simplicity; and Ψ is the second order correlation function for the turbulent sources (Reference 10 in main text).

In the case where the turbulence is homogeneous, the correlation function will depend only on (s - t). A convolution kernel (second order) can be constructed, and Equation (A-18) can be reduced to a single integration.

The solution to Equations (A-13) - (A-16) is given below:

$$AA^* = \frac{1}{D_2}\left\{ |a_{22}|^2 \{1 + E_1\} \overline{L_1 L_1^*} - |a_{12}|^2 \{1 + E_2\} \overline{L_2 L_2^*} \right\}$$

$$BB^* = \frac{1}{D_2}\left\{ |a_{11}|^2 \{1 + E_1\} \overline{L_2 L_2^*} - |a_{21}|^2 \{1 + E_2\} \overline{L_1 L_1^*} \right\}$$

$$A^*B = \frac{1}{D_1}\left\{ e_1 \; AA^* + e_2 \; BB^* \right\}$$

$$AB^* = \frac{1}{D_1}\left\{ e_1^* \; AA^* + e_2^* \; BB^* \right\} \qquad \text{(A-20)}$$

where

$$e_1 = a_{11}a_{12}^* \; |a_{21}|^2 - a_{21}a_{22}^* \; |a_{11}|^2$$

$$e_2 = a_{11}a_{12}^* \; |a_{12}|^2 - a_{21}a_{22}^* \; |a_{22}|^2$$

$$D_1 = |a_{11}a_{22}|^2 - |a_{21}a_{12}|^2$$

$$D_2 = |a_{11}a_{22}|^2 \{1 + E_1\}^2 - |a_{12}a_{21}|^2 \{1 + E_2\}^2$$

$$E_1 = \frac{1}{D_1}\left\{ 2 \; |a_{12}a_{21}|^2 - (a_{11}a_{22}a_{12}^*a_{21}^* + a_{11}^*a_{22}^*a_{12}a_{21}) \right\}$$

$$E_2 = \frac{1}{D_1}\left\{ (a_{11}a_{22}a_{12}^*a_{21}^* + a_{11}^*a_{22}^*a_{12}a_{21}) - 2 \; |a_{11}a_{22}|^2 \right\}$$

The Error Term g and the Transformation Scale ψ'

The function $g(y)$ is a function which indicates the error introduced by the WKB approximation. It is defined as

$$g(y) = \frac{1}{2\,\psi'^2}\left\{ \frac{\psi'''}{\psi'} - \frac{3}{2}\left(\frac{\psi''}{\psi'}\right)^2 \right\} \qquad \text{(A-21)}$$

with

$$\psi' = \frac{d\xi}{dy}$$

This function can be written explicitly as a function of ξ and other known quantities in the mean flow.

$$[S.0] \qquad g(\xi) = \frac{1}{4q^4}\left\{\Phi' - \frac{5}{4}\frac{\Phi^2}{q^2}\right\}$$

$$[S.1] \qquad g(\xi) = \frac{1}{4q^4}\left\{\Phi' - \frac{5}{4}\frac{\Phi^2}{q^2}\right\} + \frac{5}{16}\frac{1}{\xi^2} \qquad \text{(A-22)}$$

$$[S.2] \qquad g(\xi) = \frac{1}{4q^4}\left\{\Phi' - \frac{5}{4}\frac{\Phi^2}{q^2}\right\} + \frac{5}{4}\frac{1}{\xi^2} \qquad \text{(A-23)}$$

where $$\Phi = 2\left(\frac{k}{M}\right)^2 \left\{\frac{M\cos\theta\,(V - V_c)}{A}\right\} \cdot M\,\Omega\cos\theta$$

and $$\Omega = \frac{d}{dy}\left\{\frac{V - V_c}{A}\right\}$$

The first term in the error function is proportional to $(k/M)^{-2}$ in the shear layer, and it vanishes outside the shear layer. In the far field, $g(\xi)$ approaches zero as ξ^{-2} for [S.1] and [S.2], and it is identically zero for the [S.0] mode. The value of $g(\xi)$ remains finite as $\xi \to 0$, because the singularities in the first and the second terms cancel each other.

The limiting value of ψ' as $\xi \longrightarrow 0$ in cases [S.1] and [S.2] are very important for the computation of noise radiation intensity. The values for ψ' can be given as

$$[S.1] \qquad \lim_{\xi\to\infty} \psi'(y) = \left(\frac{k}{M}\right)^{2/3} \left\{2\,M\,\Omega\cos\theta\right\}^{1/3} \qquad \text{(A-24)}$$

and

$$[S.2] \qquad \lim_{\xi\to\infty} \psi'(y) = \left(\frac{k}{M}\right)^{1/2} \left\{M\,\Omega\cos\theta\right\}^{1/2} \qquad \text{(A-25)}$$

The values of $\psi'(y)$ for $\xi \neq 0$ are given for [S.0], [S.1], and [S.2] as

$$[S.0] \qquad \psi'(y) = q(y)$$

$$[S.1] \qquad \psi'(y) = q(y)/\{\xi(y)\}^{0.5}$$

$$[S.2] \qquad \psi'(y) = q(y)/\xi(y) \qquad \text{(A-26)}$$

The value of these expressions can be computed numerically.

The Numerical Scheme for Computing the Iterated Kernel

The kernel for the convection wave equation can be written in a general form as

$$K(\xi, t) = \left\{ F_2(\xi) F_1(t) - F_1(\xi) F_2(t) \right\} \qquad \text{(A-27)}$$

where $F_1(\xi)$ and $F_2(\xi)$ are the normalized independent solutions to the homogeneous wave equation. The iterated kernels are defined as

$$K^n(\xi, t) = \int_t^{\xi} K(\xi, r)\, g(r)\, K^{n-1}(r, t)\, dr \qquad n = 1, 2, \ldots \qquad \text{(A-28)}$$

with

$$K^1(\xi, t) = K(\xi, t)$$

In the numerical calculations, the functional dependence of the wave function, $\eta(\xi)$, on the independent solutions $F_1(\xi)$, $F_2(\xi)$ must remain explicit such that boundary conditions can be established. Hence, the functions $F_1(\xi)$, $F_2(\xi)$ must be separated out from the numerical computations of the iterated kernels.

The kernels can actually be written in matrix notations:

$$K^n(\xi, t) = F_i(\xi)\, G^n_{ij}(\xi, t)\, F_j(t) \qquad n = 1, 2, \ldots \qquad \text{(A-29)}$$

Furthermore, if $G^{(n)}_{ij}(\xi, t)$ is defined as

$$G^{(n)}_{ij}(\xi, t) = \sum_{\alpha = 1, n} G^{\alpha}_{ij}(\xi, t), \qquad \text{(A-30)}$$

then the resolvent kernel can be written as

$$R(\xi, t) = \lim_{n \to \infty} \sum K^n(\xi, t) = \lim_{n \to \infty} F_i(\xi)\, G^{(n)}_{ij}(\xi, t)\, F_j(t) \qquad \text{(A-31)}$$

By using this scheme, only the value of $G^n_{ij}(\xi, t)$ needs to be computed. The functional dependence of the resolvent kernel on $F_1(\xi)$ and $F_2(\xi)$ is preserved explicitly.

By definition given in (A-27) the value $G^1_{ij}(\xi, t)$ is a constant matrix

$$G^1_{ij}(\xi, t) = \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} \qquad \text{(A-32)}$$

and, by Equation (A-28),

$$G^n_{ij}(\xi, t) = \int_t^{\xi} G^1_{ik}(\xi, r)\, F_k(r)\, g(r)\, F_\ell(r)\, G^{n-1}_{\ell j}(r, t)\, dr \qquad \text{(A-33)}$$

Since $g(\xi)$ approaches zero as ξ^{-2} for $\xi \to \infty$, the iteration is expected to converge rapidly.

It is also important to note that

$$\frac{\partial G^n(\xi, t)}{\partial \xi} = G^1_{ik}(\xi, \xi)\, F_k(\xi)\, G^{n-1}_{\ell j}(\xi, t)\, F_\ell(\xi)\, g(\xi)$$

and

$$\lim_{\xi \to \infty} \frac{\partial G^n(\xi, t)}{\partial \xi} = 0 \quad , \quad \text{since } g(\xi) \longrightarrow 0. \qquad \text{(A-34)}$$

In the radiation boundary conditions, it is necessary to know the functional dependence of

$$\lim_{\xi \to \infty} \frac{\partial R(\xi, t)}{\partial \xi} = \lim_{\xi \to \infty} \frac{\partial}{\partial \xi} \left\{ F_i(\xi)\, G^{(n)}_{ij}(\xi, t)\, F_j(t) \right\}$$

$$= \lim_{\xi \to \infty} \left\{ \frac{\partial F_i(\xi)}{\partial \xi} \cdot G^{(n)}_{ij}(\xi, t) + F_i(\xi) \frac{\partial}{\partial \xi} G^{(n)}_{ij}(\xi, t) \right\} F_j(t) \qquad \text{(A-35)}$$

According to Equations (A-30) and (A-34), the second term in the bracket on the right hand side of (A-35) is zero. Therefore, only the numerical value of $G^n_{ij}(\xi, t)$ is required, and it is not necessary to know the functional dependence of $G^n_{ij}(\xi, t)$ in the calculations.

APPENDIX B

APPENDIX B

MEAN FLOW STRUCTURES IN A HIGH SPEED JET

As a jet exhaust flow initially enters the ambient air stream through a nozzle, the exhaust flow is generally laminar with only a small disturbance in velocity, temperature, and density. The shear stress between the jet exhaust and the ambient gas immediately facilitates the mixing process, where the development of the jet exhaust flow downstream of the nozzle begins. The development contains three stages. In the initial segment, or the mixing zone, a turbulent flow layer grows in between the jet exhaust and the ambient gas; the initial laminar jet flow is entrained into the mixing layer gradually, and vanishes at the end of the mixing zone. The next segment of the jet exhasut enters a transition stage, entrainment of ambinet air will continue, while the major event is the re-adjustment of mean flow and turbulent structure within the jet itself. The length of this segment is about the same as the mixing zone. In the final stage, the jet flow has reached similarity profile. Both the turbulence and the mean flow decay asymptotically.

The noise generation characteristics of a jet are directly related to the structure of the mean flow and the mixing parameters. Since most of the acoustic power comes from the mixing region and the transition region, the aerodynamical properties in these two regions will be the most important concern. In subsonic jets with constant density, the jet flow structure has been studied extensively. The mean flow properties and the turbulence structures are well-defined. In supersonic jets, the task of defining the precise jet flow structure becomes immensely more difficult. There are a large number of variable parameters, and both theoretical analysis and experimental flow measurement are much more involved. Nevertheless, knowledge in this field has been advanced significantly in the past few decades, and these results provide a firm basis for further refined studies.

The basic properties of mean flow developments in a high speed incompressible jet have been known for some time. There are theories (References B1-B3), experiments (References B4-B6), and comprehensive texts (References B7-B8) which describe the jet flow properties in great detail. The main concern in this study is, however, the flow properties of compressible, high temperature jets.

A typical jet exhaust flow configuration is shown here in Figure B1. The key dimensions for defining a jet configuration are the laminar core length, the spreading rate of the jet boundary, the mixing layer thickness, and the location in the radial direction of the point where the flow velocity is half of that at the center line. There are essentially

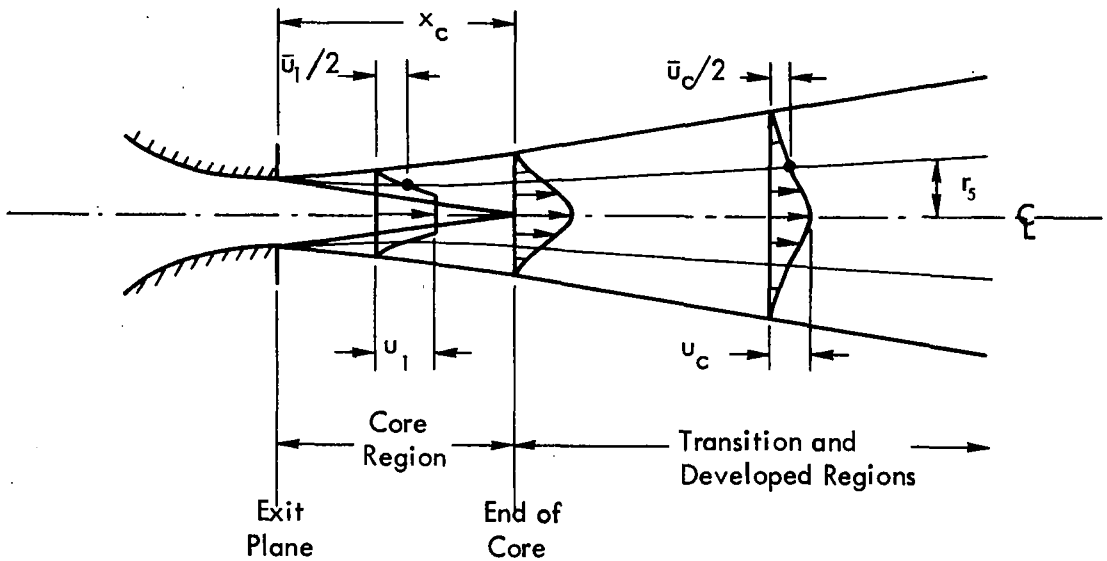

Figure B1. Idealized Jet Structure

three methods in use today by which the decay of free jets and wakes are studied. These consist of two methods that have been in use for some time, and a method that has become available within the last few years as a result of the rapid development and use of high-speed computers. The three methods are:

- The simple momentum integral method
- Solution of the equations of motion when certain assumptions are made which render these equations tractable to existing analytical techniques
- Step-by-step numerical solution of the equations on high-speed digital computers

All of these methods depend ultimately on experimental turbulent mixing data since an essential part of all these techniques is the specification on the local eddy viscosity of the particular turbulent flow in question at one or more general location in the mixing region; and, this is a task quite beyond the power of existing theory.

A significant amount of work on the turbulent mixing process in jets and wakes has been done in recent years. Among the important publications, three references should be mentioned here specifically. The first, by Warren (Reference B1), presents the result of a number of experiments on the mixing of properly expanded, heated jets in quiescent air. In these experiments the initial Mach number of the air in the jets studied was varied between approximately 0.7 and 2.6. These experimental results were analyzed by means of the momentum integral technique and the general character or mixing rate of the jets studied which were presented in terms of a mixing-rate parameter that was a function of Mach number. It is typical of the analysis or description of mixing phenomena by means of the momentum integral technique that only general characteristic trends of the motions involved can be described. Some applications of this method can be found in Eldred, et al. (Reference B9).

The second paper that should be mentioned is one by Libby (Reference B2). In this paper, an attempt is made to go beyond the momentum integral technique and look for more complete information concerning the nature of jet mixing by means of analytical solutions of approximate equations of motion. The method presented by Libby was developed in connection with the mixing of two coaxial streams of dissimilar gases, the outer being of infinite extent. This paper is important because the analysis developed forms the basis for subsequent research on turbulent mixing.

The third and most recent is a paper given by Donaldson and Gray (Reference B3). In this paper, an extension of the Warren's method was developed for predicting free jet decay. The relation between the jet mixing process and various flow parameters such as jet molecular weight, temperature, both subsonic and supersonic Mach numbers, and jet expansion conditions, has been investigated by using this extended theory and a series of experiments. The key parameter investigated in this study is the turbulent mixing coefficient. If this coefficient is assumed known, then the jet structure for given Mach number, temperature, and density can be predicted. In this study, the mixing coefficient is found to depend only on a properly defined local Mach number (Figure B2), and within the accuracy of the experiments, this general relationship is independent of the physical property or the thermodynamic state of the mixing gas. The dependence of jet flow decay on temperature, molecular weight and other physical and thermodynamic state of the jet, is encompassed by the extended Warren's theory. The resulting theoretical prediction of mean flow development of a jet was found to agree with experimental results over a Mach number range from 0.75 to 2.20, and for gases in the jet with molecular weight ranging from 4 to 88. By using any one of the above three methods, a sufficiently accurate decay profile of a high speed jet can be obtained.

The dependence of the laminar core length on Mach number is shown in Figure B3. For a subsonic jet, the core length remains more or less constant. The length is about 5.5 jet exit diameters. The core length becomes larger when supersonic Mach numbers are reached. For very high Mach numbers, the core length varies almost linearly with the jet exit Mach number. Inside the laminar core, the velocity at the center line of the jet remains constant. Beyond the mixing region, the velocity at the centerline of the jet begins to decay. The typical velocity profiles for an isothermal air jet at various Mach numbers are shown in Figure B4. The growth of thickness of the jet is also a function of Mach number. As Mach number increases, the mixing layer becomes thinner if the temperature of the jet remains the same (Figure B5). In a supersonic jet the growth rate of jet thickness in the initial mixing region is a constant. The thickness of the jet remains approximately constant through the transition zone. Growth of the jet thickness resume in the developed region. The growth rate in the developed region is strongly influenced by the temperature of the jet.

In the theories of jet structures, the profiles across a section of the jet of quantities such as temperature, velocity, density, and enthalpy are usually studied separately. In the mixing region, the velocity profile can be described by the so-called Schlichting's formula

$$\overline{u}_0 - \overline{u} = u_0 \left(1 - \eta^{3/2}\right)^2 \qquad (8)$$

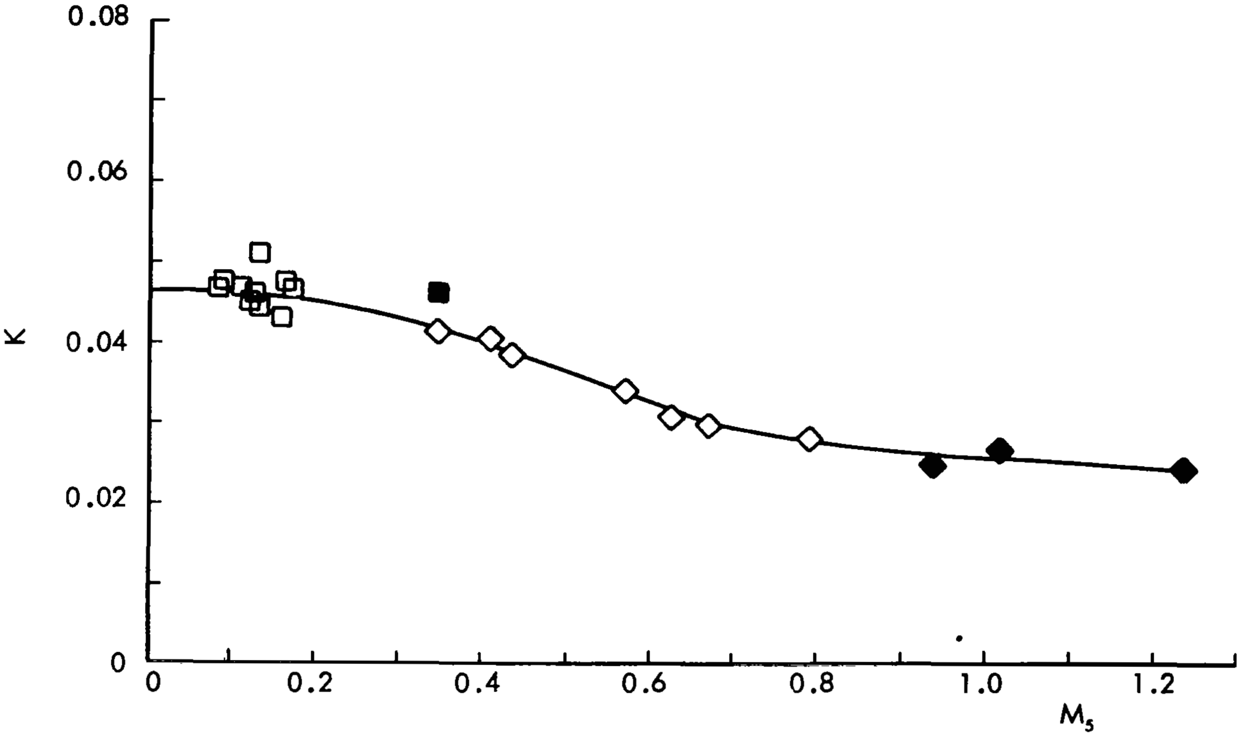

Figure B2. The Dependence of the Turbulent Mixing Coefficient, K, on the Local Mach Number at the Center of the Shear Layer (Ref. B3)

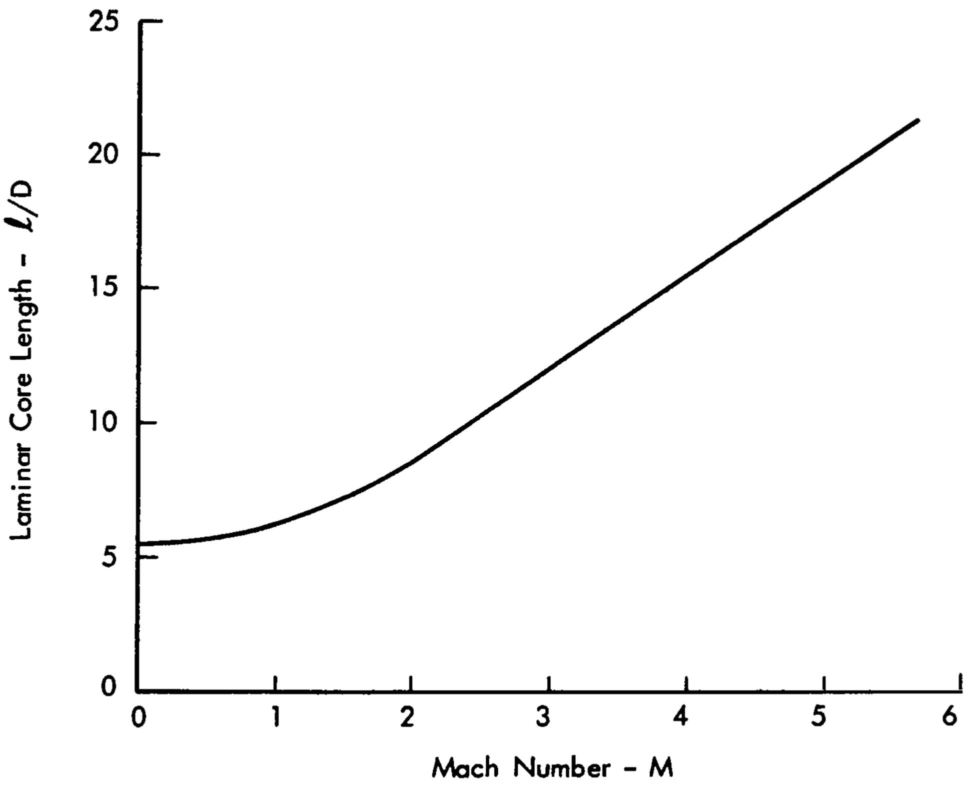

Figure B3. The Laminar Core Length as a Function of Jet Exahust Flow Mach Number

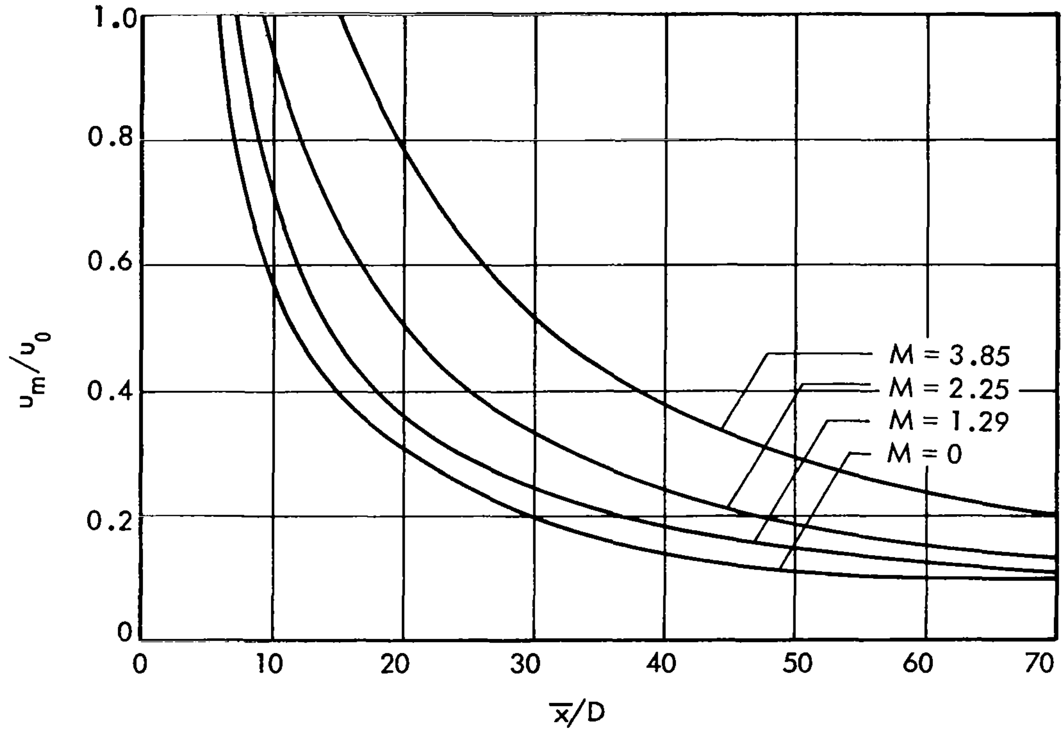

Figure B4. Variation in Velocity Along Axis of Axially Symmetric Supersonic Gas Jet In Stationary Medium

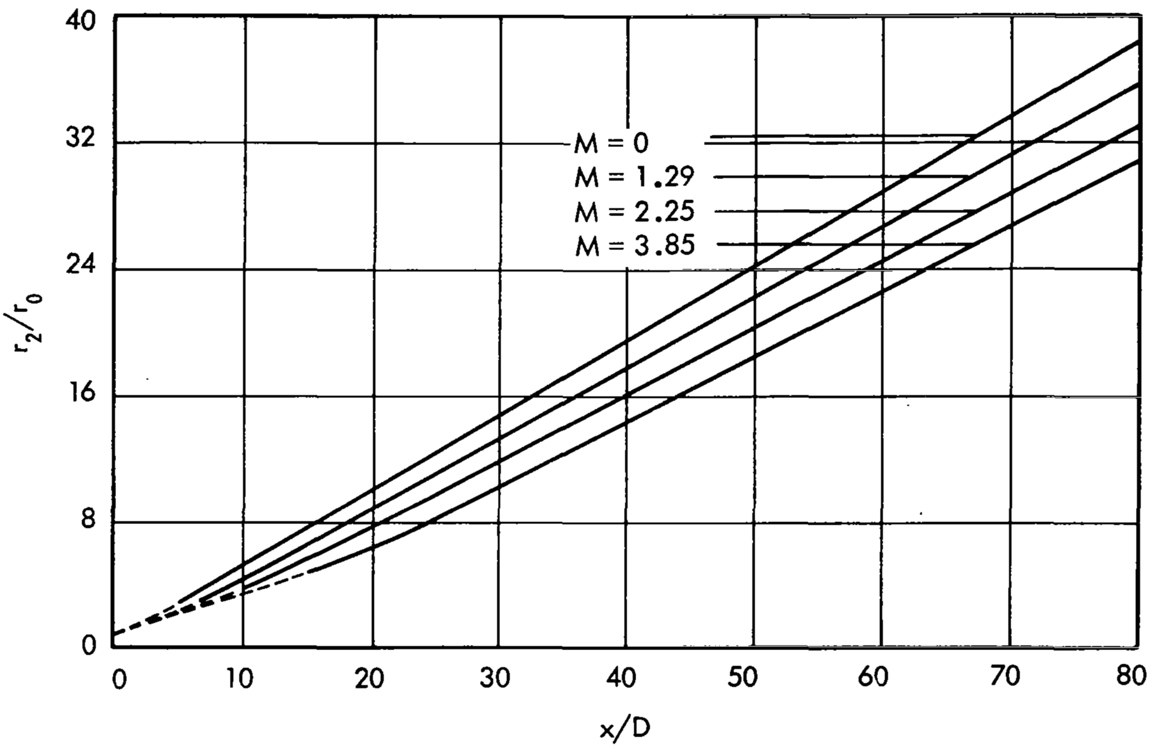

Figure B5. Growth of Thickness Along the Axis of Axially Symmetric Supersonic Gas Jet In Stationary Medium where $\overline{U}_0$ is the jet exit velocity, $\overline{U}$ is the velocity at any point in the mixing layer, and η is a nondimensional coordinate. A similar formula is used for defining the velocity profile in the transition region. In the developed region of the jet, the velocity profile approaches a Gaussian distribution.

The temperature profile of a heated supersonic jet has been measured before (Reference B7). In the mixing region, it can be closely approximated by a straight line. For heated jet exhaust streams, both the enthalpy and the local Mach number profiles are difficult to obtain because very complicated heat transfer properties are involved. The local Mach number across the profile of a jet is readily measurable because it is simply a function of the ratio of the local pressure and the local total pressure in the flow field.

The detailed development of the jet exhaust along the axial direction and profiles of various physical and thermodynamic quantities are of particular importance in the advanced supersonic jet noise theory. These structures affect directly the intensity, directivity, and the source location of the radiated noise.

REFERENCES - APPENDIX B

B1 Warren, W.R., "The Mixing of an Axially Symmetric Compressible Jet with Quiescent Air," Aero. Eng. Lab. Report No. 252, Princeton University, September 1953.

B2 Libby, P.A., "Theoretical Analysis of Turbulent Mixing of Reactive Gases with Application to Supersonic Combustion of Hydrogen," J. Am. Rocket Soc. 32, pp. 388-396, (1963).

B3 Donaldson, C. duP., and Gray, K.E., "Theoretical and Experimental Investigation of the Compressible Free Mixing of Two Dissimilar Gases," AIAA J., Vol. 4, pp. 2017-2025, 1966.

B4 Davies, P.O.A.L., et al., "The Characteristics of the Turbulence in the Mixing Region of a Round Jet," J. Fluid Mech., Vol. 15, pp. 337-367, Corrigendum, p. 559, 1963.

B5 Tu, B.J., "On the Measurement of Turbulent Velocities u_0, u_{45}, u_{90}, u_{135}, and w in a Circular Jet," Wyle Research Staff WR 70-2, 1970.

B6 Chu, W.T., "Turbulence Measurements Relevant to Jet Noise," University of Toronto, Inst. for Aerospace Studies, UTIAS Report No. 119, 1966.

B7 Abramovich, G.N., The Theory of Turbulent Jets, MIT Press, Cambridge, Mass., 1963.

B8 Townsend, A.A., The Structure of Turbulent Shear Flow, Cambridge Univ., Press, 1956.

B9 Eldred, K.M., et al., "Suppression of Jet Noise with Emphasis on the Near Field," TR ASD-TDR-62-578 Flight Dynamics Laboratory, Wright-Patterson AFB, Ohio, 1963.

NASA-Langley, 1971 —— 1