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APOLLO COMMUNICATIONS SYSTEMS
TASK E-59D
TECHNICAL REPORT

TECHNIQUE FOR MEASURING TIME-BASE ERRORS OF
MAGNETIC INSTRUMENTATION RECORDERS/REPRODUCERS

NAS 9-8166

17 May 1971

Prepared for
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
MANNED SPACECRAFT CENTER
HOUSTON, TEXAS

Prepared by
Communication and Sensor Systems Department
Electronic Systems Laboratory

FACILITY FORM 602

N71-27711

(ACCESSION NUMBER) **27**

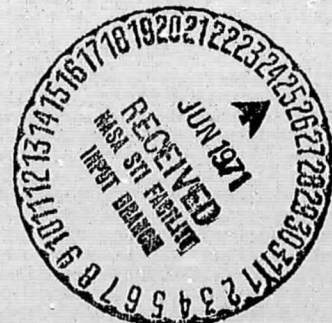
(PAGES) **CR-115029**

(NASA CR OR TMX OR AD NUMBER)

(THRU) **63**

(CODE) **14**

(CATEGORY)



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Prepared by
R. J. Chan

Approved by H. D. Freeman
H. D. Freeman, Manager
Task E-59D

Approved by John DeVillier
J. DeVillier, Manager
Communication and Sensor
Systems Department

ABSTRACT

This technical report is submitted to NASA/MSC by TRW Systems Group in accordance with Task MSC/TRW E-59D, Space Communication Systems Test Analysis, under Contract NAS 9-8166. This document describes a general measurement technique for determining the time-base errors associated with recorded and played back digital data. The technique requires the measurement of tape flutter spectral density, amplitude cumulative probability distribution (ACPD), root-mean-squared (RMS) time-plot, FM discriminator sensitivity, and peak-to-peak percent flutter.

Two analytical expressions are presented to calculate the time-base error or "jitter". One expression determines the time-base error when the tape flutter is known to have Gaussian amplitude statistics and a uniform spectral density. The other expression may be used when the flutter is periodic. An important flutter measurement that is not usually made is the amplitude cumulative probability distribution. This measurement is significant because it makes it possible to assign a statistical confidence to the percent time-base error, thus establishing bounds on time-base errors which manufacturers may specify for their recorders/reproducers.

This technique fulfills a need for a recorder industry standard by which flutter and jitter characteristics and specifications can be uniquely defined.

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1.0 INTRODUCTION

A survey of literature associated with time-base error measurements of recorded/reproduced digital data has revealed a lack of an acceptable standard method to measure time-base errors. This report presents a general technique, which fulfills the need for an industrial standard, for determining time-base errors associated with the recording and playback of digital data by magnetic tape instrumentation recorders/reproducers.

1.1 BACKGROUND

Inherent to the tape recording and playback of digital data by magnetic instrumentation recorders/reproducers is time-base errors or bit jitter. Time-base errors are caused by the variation of tape speed from the uniform tape speed at which the tape is recorded and played back. The variation of tape speed from uniform tape speed is termed flutter. Flutter introduces frequency modulation into any frequency which is recorded and played back. In the case of recording and playing back of digital data, tape flutter introduces time-base errors to the data stream.

1.2 DISCUSSION

The literature survey of tape flutter measurements and time-base error measurements shows a lack of a real good method of measuring time-base errors¹ and a lack of standard flutter measurements that characterizes tape flutter.

The common flutter measurements used to characterize tape flutter are given in References 2 through 10. Basically, a precision oscillator is used to supply a frequency signal to be recorded and played back by the test recorder/reproducer. The amplifier output of the playback head is fed to an FM discriminator whose center frequency is set at the oscillator frequency. The test equipment is calibrated such that the discriminator output is given in terms of percent variation in speed or percent flutter. Then, flutter voltage time records are taken. Another flutter measurement taken is the spectral density which consists of feeding the output of the FM discriminator to a spectrum analyzer and recording the spectral density

with an X-Y plotter.

References 2 and 12 describe flutter measurements that determine the total peak-to-peak percent flutter. Both methods assume Gaussian amplitude statistics for the flutter. Reference 2 is the IRIG standards on flutter measurements and Reference 12 is a flutter meter manual that states that "the Micom Model 8300 and 8300-W flutter meters are designed to make measurements only at the frequencies and bandwidths called out in the IRIG standards."

Two additional flutter measurements are described in Section 2.0 which are not widely used. One measurement is the amplitude cumulative probability distribution (ACPD) which defines the amplitude statistics of the flutter. The statistical data removes the difficulty in specifying the peak-to-RMS ratio of tape flutter. A device that measures the ACPD is described in Section 2.2. The other measurement is the RMS time-plot of the flutter which is described in Section 2.3. Both measurements are easy to take.

Thus, the four basic measurements to characterize tape flutter are

1. Spectral density
2. Amplitude cumulative probability distribution
3. RMS time-plot
4. Peak-to-peak percent flutter.

These measurements are necessary to characterize tape flutter and the time-base errors of recorded and played back digital data as it is discussed below.

References 8 and 13 describe the effect of tape flutter on recorded and played back events. Reference 8 gives an analysis of time displacement errors due to sinusoidal and Gaussian flutter. Reference 13 independently developed expressions to calculate the time-base errors for tape flutter that it has characterized. It was found, although the approach of the derivations were different, that the resultant expressions were the same. Reference 13 also describes in detail the measurement techniques that were used to characterize the flight recorders used by the Apollo communication systems.

This report is written to present the general techniques that were developed in analyzing the Apollo flight recorders.

The time-base error analysis is extended to include the calculation of time-base errors for any periodic flutter and has included the general results for Gaussian flutter.

1.3 CONTENTS

Section 2.0 contains descriptions of flutter measurements which includes spectral density, amplitude cumulative probability distribution, root-mean squared (RMS) time-plots, FM discrimination sensitivity, and peak-to-peak percent flutter.

The determination of time-base errors for periodic and Gaussian tape flutter is discussed in Section 3.0.

Section 4.0 contains the conclusions of this study.

An Appendix, which contains the derivation of time-base error expressions, is included to provide the details omitted in the main text of this report.

2.0 FLUTTER MEASUREMENTS

A discussion of the measurements that characterizes tape flutter is presented. These measurements include spectral density, amplitude cumulative probability distribution, RMS time-plots, FM discriminator sensitivity, and peak-to-peak percent flutter.

Figure 1.0 shows a test configuration for making flutter measurements. The test equipment consists of a precision oscillator, frequency counter, recorder/reproducer, bandpass filter, FM discriminator, variable low pass filter, spectrum analyzer with X-Y plotter, RMS voltmeter, amplitude cumulative probability distribution or statistical voltmeter (non-standard), and flutter meter (Micom model 8300/8300-W) or a peak voltage meter.

The tape flutter of a recorder/reproducer is obtained by recording and playing back a sinusoidal frequency signal (1.5 KHz or greater)², which is supplied by the oscillator, through an FM discriminator centered at the frequency of the oscillator. The output of the discriminator is the tape flutter. The frequency counter is used to monitor the frequency supplied by the oscillator and the bandpass filter is used to limit the system noise. The flutter measurements are described in the following sections.

2.1 SPECTRAL DENSITY

The frequency contents of the tape flutter may be determined from the spectral density measurement. The spectral measurement consists of connecting a spectrum analyzer/X-Y plotter to the output of the low pass filter. The low pass filter bandwidth depends on the tape speed and is explicitly specified in the IRIG standards on flutter measurements. Output terminals from the spectrum analyzer are provided to enable an X-Y plotter to be connected to record the spectrum of the input signal to the spectrum analyzer.

2.2 AMPLITUDE CUMULATIVE PROBABILITY DISTRIBUTION

The amplitude statistics of the tape flutter is determined by measuring the amplitude cumulative probability distribution (ACPD). The ACPD is the probability that a signal amplitude is equal to or less than a given value. To measure the ACPD, it is necessary to use a statistical voltmeter

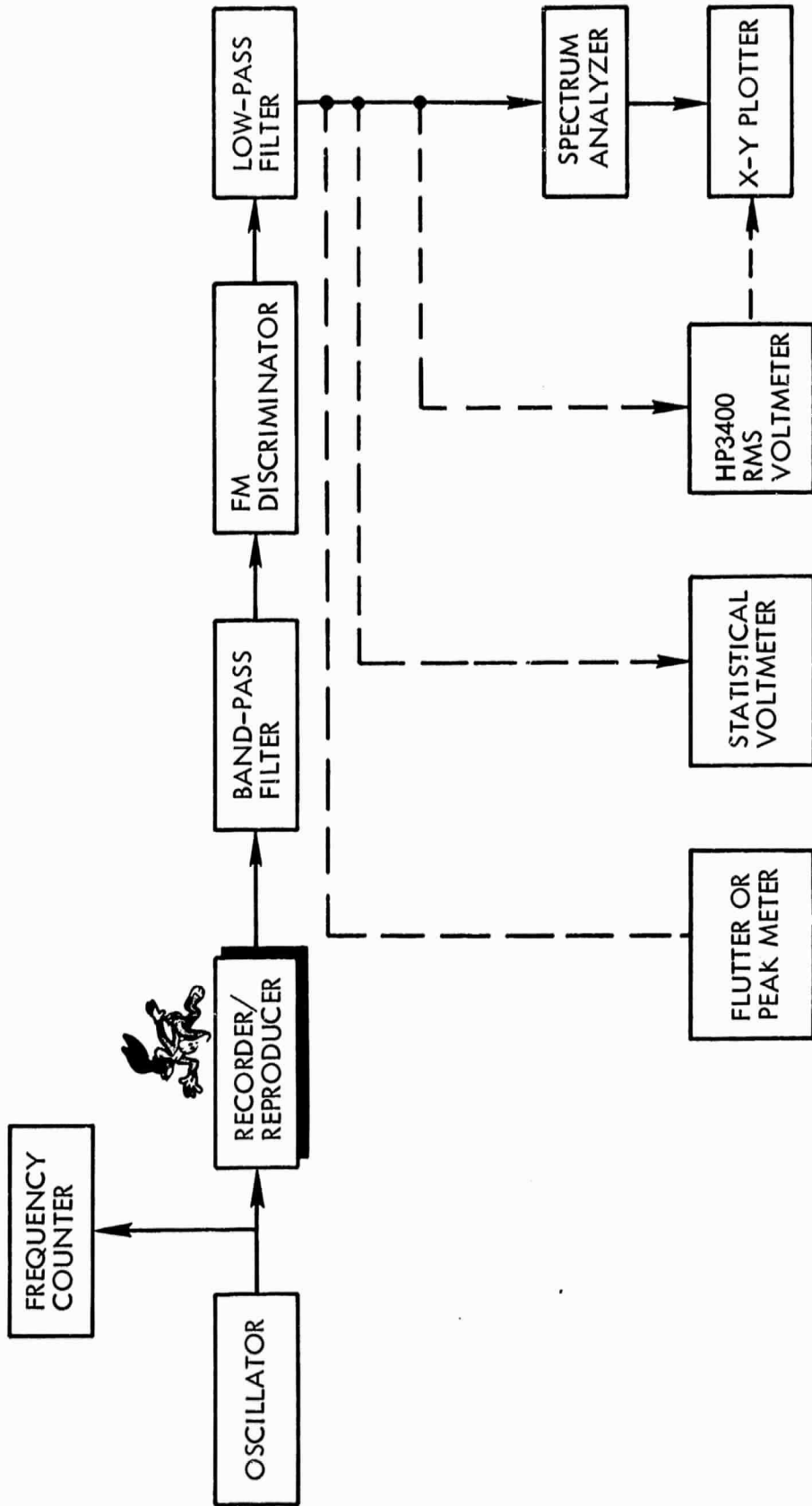


Figure 1. Test Configuration for Flutter Measurements

that operates in the following fashion. The voltmeter should be able to measure the percentage of time that a level is less than a given level. According to probability theory, the measured percentage of time becomes a closer estimate to the true probability as the averaging time increases.

Figure 2 shows a block diagram of a statistical voltmeter. The desired statistics or ACPD is $P(V_1 < V_0)$, where $P(V_1 < V_0)$ is the probability that V_1 is less than V_0 , V_1 is the flutter signal, and V_0 is an arbitrary level. $P(V_1 < V_0)$ is determined from

$$P(V_1 < V_0) = 1 - [\text{percent of time } V_1(t) > V_0]. \quad (1)$$

The selected level, V_0 , is applied to one side of a voltage comparator whose other input is $V_1(t)$. The output of the comparator, which is either zero volts or V_R volts, is routed to an inverter whose output is

$$V_3(t) = V_R - V_2(t). \quad (2)$$

Passing $V_3(t)$ through an averaging filter (a Simpson dc voltmeter will do) yields

$$\begin{aligned} \text{avg}[V_3(t)] &= \text{avg}[V_R - V_2(t)] \\ &= V_R - \text{avg}[V_2(t)], \end{aligned} \quad (3)$$

but

$$\text{avg}[V_2(t)] = V_R[\text{percent of time } V_1(t) > V_0],$$

hence

$$\text{avg}[V_3(t)] = V_R[1 - \text{percent of time } V_1(t) > V_0]. \quad (4)$$

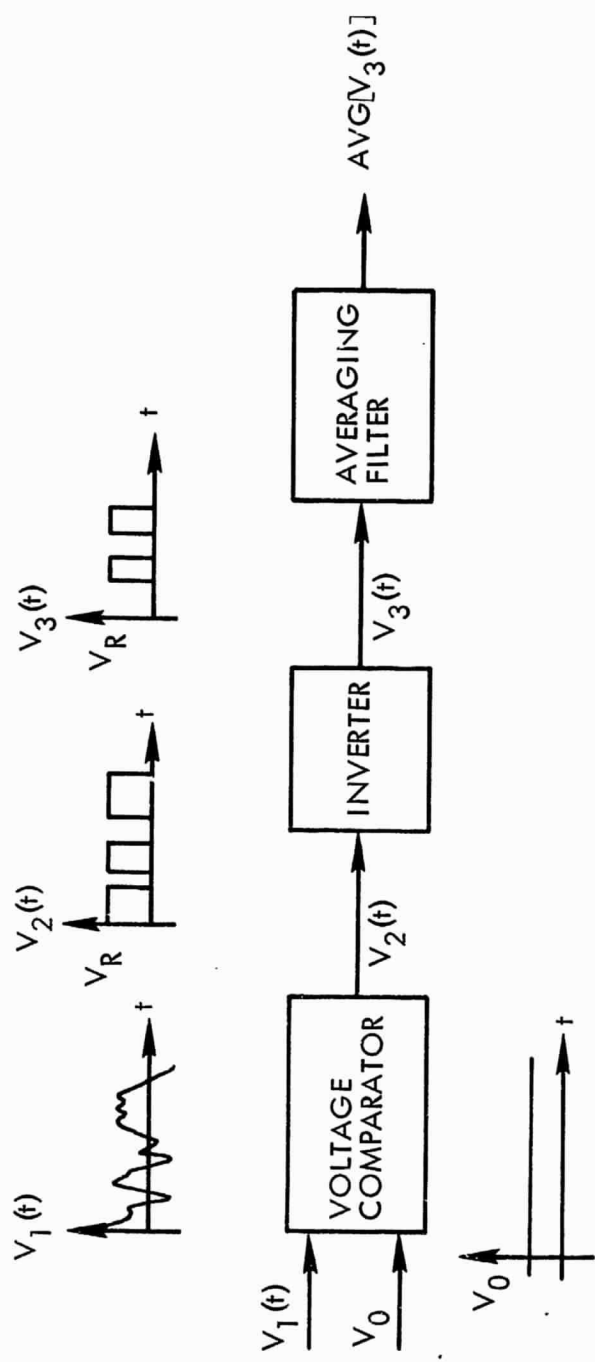


Figure 2. Statistical Voltmeter Block Diagram

By dividing both sides by V_R , we obtain the desired result

$$P[V_1 < V_0] = 1 - [\text{percent of time } V_1(t) > V_0]. \quad (5)$$

Therefore, by connecting the statistical voltmeter to the output of the low pass filter, the ACPD may be measured.

2.3 RMS TIME-PLOT

The RMS time-plot can be determined by connecting an X-Y plotter to the DC output of a HP3400 RMS voltmeter. The time-plot will show the time variation of the RMS flutter voltage. The time scale of the plot can be adjusted to record a given portion of the tape or the entire tape.

2.4 FREQUENCY SENSITIVITY CONSTANT

The frequency sensitivity is the FM discriminator constant expressed in percent deviation from center frequency per volt which is equivalent to percent speed variation per volt. The constant is measured by calibrating the FM discriminator to obtain its characteristic curve, that is, percent deviation versus output voltage. The slope of the characteristic curve is the frequency sensitivity constant which is needed to compute the peak-to-peak percent flutter and the time-base errors of recorded digital data.

2.5 PEAK-TO-PEAK PERCENT FLUTTER

There are several methods for determining the peak-to-peak percent flutter. One method is to calculate the percent flutter from the frequency sensitivity, ACPD, and RMS time-plot data. The peak-to-peak percent flutter is the frequency sensitivity constant multiplied by k times the average RMS (sigma) voltage of the flutter, where k is a number determined from the ACPD plot that yields whatever probability that may be assigned to the peak-to-peak percent flutter amplitude. For example, the peak-to-peak flutter may be equal to two sigma with 0.95 probability. If the flutter has Gaussian statistics, the above statement is exactly true.

If a Micom flutter meter (model 8300/8300w) is available, the peak-to-peak percent flutter for Gaussian flutter can be measured directly. The peak-to-peak flutter can also be measured with a peak reading meter that indicates the largest positive and negative peaks. Then the peak-to-peak percent flutter can be calculated by multiplying the peak meter readings with the frequency sensitivity constant.

2.6 GENERAL COMMENTS

In general, the flutter data will not vary from one portion of the tape to another, assuming that a long enough sample of the tape is used. It is recommended that the recording of the test frequency be long enough so that flutter data can be taken at the beginning, middle, and end of the tape. If there is a slight difference in the data tape, then the sets of data may be averaged. It is suggested that several sets of data be taken for each portion (beginning, middle, and end) of the tape. If there is a significant difference, then determine if the data is repeatable.

3.0 TIME-BASE ERRORS

Time-base errors of recorded and played back digital data are given for periodic and Gaussian flutter.

The relative time-base error or jitter between two recorded digital events, separated by time τ , due to tape flutter is given by the normalized standard deviation σ_s of the sample distance of the two recorded events. The sample distance is obtained by integrating the tape flutter over time τ , i.e.,

$$s(t_i, \tau) = KV_0 \int_{t_i}^{t_i+\tau} f(t) dt \quad (6)$$

where

$f(t)$ = tape flutter function defining speed variation from the uniform tape speed v_0 .

t_i = arbitrary time

K = $K_1 K_2$

K_1 = FM sensitivity constant (% speed/volt)

K_2 = ratio of flutter peak-to-RMS or factor to assign the probability of occurrence of the flutter or the time-base error,

and the standard deviation of the sample distance is computed from the variance of $s(t_i, \tau)$, which is given by

$$\sigma_s^2 = \overline{[s(t_i, \tau)]^2} - \overline{s(t_i, \tau)}^2 \quad (7)$$

where

$$\overline{[s(t_i, \tau)]^2} = \text{mean square of the sample distance}$$

$\overline{s(t_i, \tau)}$ = mean of the sample distance.

The one sigma percent displacement error is

$$\text{Percent displacement error} = \frac{\sigma_s}{S_0} \quad (8)$$

where

$S_0 = v_0 \tau$ = uniform distance of the recorded events corresponding to the uniform velocity v_0 of the tape and time τ .

Equation (8) is the general expression for time-base error calculations and it is used to determine the time-base errors for periodic and Gaussian tape flutter.

3.1 PERIODIC FLUTTER

The relative time-base error for periodic flutter is derived in Appendix A and is given by

$$\sigma_{sn} = K \sigma_f \quad (9)$$

where

$$K = K_1 K_2$$

K_1 = FM discriminator sensitivity constant

K_2 = ratio of peak to RMS

σ_f = RMS value of the periodic flutter
(assuming the mean is zero)

Equation (9) assumes that the separation of recorded events is much less than $n\omega_0$, where n is the highest significant harmonic of the periodic flutter and ω_0 is the fundamental angular frequency.

If the separation of recorded events is not less than $n\omega_0$, then the relative time-base error is given by

$$\sigma_{sn} = K \left[\sum_{n=-\infty}^{\infty} \alpha_n^2 \left(\frac{\sin \eta \omega_0 \tau / 2}{\eta \omega_0 \tau / 2} \right)^2 - \alpha_0^2 \right]^{1/2} \quad (10)$$

where

α_0 = mean value of the flutter

α_n = Fourier coefficient (see Appendix A, Equation A-5)

In general, Equation (9) and (10) gives the time-base error in terms of percent displacement referenced to the recorded distance τ . The peak to RMS ratio of the time-base error is the same as the peak-to-RMS ratio of the flutter so that the peak or peak-to-peak percent time-base error can be calculated and that the probability that the peak-to-peak time-base error is within the calculated value is one.

3.2 GAUSSIAN FLUTTER

The relative time-base error for Gaussian flutter is derived in Appendix A and is given by

$$\sigma_{sn} = \frac{K\sigma_f}{\sqrt{B\tau}} \quad (11)$$

where

$K\sigma_f$ = peak-to-peak percent flutter/k

k = 2(1 sigma, .68 probability), 4(2 sigma,
.95 probability)

B = flutter measurement bandwidth

τ = time between recorded events.

For example, suppose that $B\tau = 1$ and the peak-to-peak flutter is measured as 2 per cent. The RMS flutter is 0.5 percent, that is, assuming that the flutter peaks are exceeded only 5 percent of the time. The expected time-base error will be 0.50 percent or less for 68 percent of the time, 1 percent or less for 95.45 percent of the time or 1.5 percent or less for 99.73 percent of the time.

4.0 CONCLUSIONS

A general technique for determining time-base errors of recorded and played back digital data has been presented.

The technique includes the use of established flutter measurement techniques and two flutter measurements not commonly taken. The established flutter measurements include spectral density and peak-to-peak percent flutter. The uncommon flutter measurements include the amplitude cumulative probability distribution and the RMS time-plot.

The flutter measurements described in this report should be the minimum measurement requirements to characterize tape flutter.

It is shown that the minimum measurement requirements can be used to calculate the time-base errors of recorded/reproduced digital data for periodic and Gaussian flutter.

APPENDIX A

DERIVATION OF TIME-BASE ERROR EXPRESSIONS

The relative time-base error between two recorded events, separated by time τ , due to tape flutter is given by the normalized standard deviation, σ_{sn} , of the sample distance of two recorded events. The sample distance is obtained by integrating the tape flutter over the time τ , i.e.,

$$s(t_i, \tau) = KV_0 \int_{t_i}^{t_i+\tau} f(t) dt \quad (A-1)$$

where

$f(t)$ = tape flutter function defining speed variation from the uniform tape speed V_0 .

t_i = arbitrary time

K = percent peak deviation of instantaneous speed

and the standard deviation of the sample distance is determined from the variance of $s(t_i, \tau)$ which is given by

$$\sigma_s^2 = \overline{s^2(t_i, \tau)} - \left[\overline{s(t_i, \tau)} \right]^2 \quad (A-2)$$

where

$\overline{s^2(t_i, \tau)}$ = mean square of the sample distance

$\overline{s(t_i, \tau)}$ = mean of the sample distance.

The relative time-base error is given by

$$\text{Percent time-base error} = \frac{\sigma_s}{S_0} = \sigma_{sn} \quad (\text{A-3})$$

where

$S_0 = V_0 \tau$ = uniform distance of the two recorded events corresponding to the uniform velocity V_0 of the tape and time τ .

σ_{sn} = normalized standard deviation of the sample distance or time-base error.

Equations (A-1) to (A-3) will be used to derive general time-base error expressions for periodic and Gaussian flutter.

A.1 PERIODIC FLUTTER

Periodic functions can be represented by a Fourier series¹⁴

$$f(t) = \sum_{n=-\infty}^{\infty} \alpha_n e^{jn\omega_0 t} \quad (\text{A-4})$$

where

$$\alpha_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-jn\omega_0 t} dt = \text{Fourier coefficients} \quad (\text{A-5})$$

$$\omega_0 = \frac{2\pi}{T} = \text{fundamental angular frequency}$$

T = period.

If the tape flutter is periodic, the sample distance of two recorded events separated by distance τ is found by substituting Equation (A-4) into Equation (A-1), i.e.,

$$s(t_i, \tau) = KV_0 \int_{t_i}^{t_i + \tau} \sum_{n=-\infty}^{\infty} \alpha_n e^{jn\omega_0 t} dt \quad (A-6)$$

$$= KV_0 \tau \sum_{n=-\infty}^{\infty} \alpha_n \left(\frac{\sin n\omega_0 \tau / 2}{n\omega_0 \tau / 2} \right) e^{jn\omega_0 (t_i + \tau / 2)} \quad (A-7)$$

Observe that Equation (A-7) is in the form of the Fourier series of the function $s(t_i, \tau)$ where the Fourier coefficient is given by

$$F(n) = KV_0 \tau \alpha_n \left(\frac{\sin n\omega_0 \tau / 2}{n\omega_0 \tau / 2} \right). \quad (A-8)$$

The power spectrum for a periodic function is given by¹⁵

$$\Phi(n) = |F(n)|^2 \quad (A-9)$$

or in terms of ω

$$\Phi(\omega) = \Phi(n\omega_0 \rightarrow \omega) 2\pi \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0), \quad (A-10)$$

and the total power or mean square sample distance is given by

$$\overline{s^2(t_i, \tau)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi(\omega) d\omega. \quad (A-11)$$

Substituting Equation (A-10) into Equation (A-11) yields

$$\begin{aligned} \overline{s^2(t_i, \tau)} &= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} (KV_0\tau)^2 \alpha_n^2 (n\omega_0 \rightarrow \omega) \left(\frac{\sin n\omega_0\tau/2}{n\omega_0\tau/2} \right)^2 \delta(\omega - n\omega_0) d\omega \\ &= \sum_{n=-\infty}^{\infty} (KV_0\tau)^2 \alpha_n^2 \left(\frac{\sin n\omega_0\tau/2}{n\omega_0\tau/2} \right)^2. \end{aligned} \quad (A-12)$$

The average value of $s(t_i, \tau)$ (Equation A-7) is given by

$$\overline{s(t_i, \tau)} = KV_0\tau\alpha_0 \quad (A-13)$$

so that the sample distance variance is

$$\sigma_s^2 = \sum_{n=-\infty}^{\infty} (KV_0\tau)^2 \alpha_n^2 \left(\frac{\sin n\omega_0\tau/2}{n\omega_0\tau/2} \right)^2 - \left(KV_0\tau\alpha_0 \right)^2. \quad (A-14)$$

The variance of the periodic flutter represented by Equation (A-4) is given by

$$\sigma_f^2 = \sum_{n=-\infty}^{\infty} \alpha_n^2 - \alpha_0^2. \quad (A-15)$$

If the distance τ between the recorded events is small compared to $n\omega_0$ such that

$$\frac{\sin n\omega_0\tau/2}{n\omega_0\tau/2} \approx 1, \quad (A-16)$$

then Equation (A-14) can be written in terms of the flutter variance, i.e.,

$$\sigma_s^2 = (KV_0\tau)^2 \sigma_f^2. \quad (A-17)$$

The relative time-base error is given by

$$\sigma_{sn} = \frac{\sigma_s}{S_0} = K\sigma_f. \quad (A-18)$$

A.2 GAUSSIAN FLUTTER

Now consider the flutter to be Gaussian with a period $T = \frac{\omega_0}{2\pi}$ that is long compared with all periods occurring in the system. The random flutter can be written in a Fourier series similarly to Equation (A-4) except that the Fourier coefficients (α_n) are Gaussian distributed and that the power spectral density of the flutter is given by¹⁴

$$S_f(\omega) = \lim_{T \rightarrow \infty} \frac{E\{|\alpha_n|^2\}}{2T} = \frac{W_0}{2} \quad (A-19)$$

where

$$\frac{W_0}{2} = \text{uniform density function of the random Gaussian process.}$$

The sample distance as defined by Equation (A-1) when the flutter is random and represented by Equation (A-4) where α_n 's are Gaussian distributed is given by Equation (A-7). The modified Fourier coefficients are given by Equation (A-8).

The power spectral density, which is determined from the Fourier coefficients of the sample distance is given by

$$\begin{aligned}
S(\omega) &= \lim_{T \rightarrow \infty} \frac{E\{|F(n)|^2\}}{2T} \\
&= \lim_{T \rightarrow \infty} \frac{(KV_0\tau)^2 E\{|\alpha_n|^2\}}{2T} \left(\frac{\sin n\omega_0\tau/2}{n\omega_0\tau/2} \right)^2 \\
&= (KV_0\tau)^2 \frac{W_0}{2} \left(\frac{\sin \omega\tau/2}{\omega\tau/2} \right)^2.
\end{aligned} \tag{A-20}$$

The variance of the sample distance is given by

$$\begin{aligned}
\sigma_s^2 &= \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) d\omega \\
&= \frac{1}{2\pi} (KV_0\tau)^2 \frac{W_0}{2} \int_{-\infty}^{\infty} \left(\frac{\sin \omega\tau/2}{\omega\tau/2} \right)^2 d\omega \\
&= \frac{(KV_0\tau)^2 W_0\tau}{2}
\end{aligned} \tag{A-21}$$

But, the variance or mean square (the average value is assumed to be zero) of the flutter in a flutter measurement bandwidth B is given by

$$\begin{aligned}
\overline{f^2(t)} = \sigma_f^2 &= \frac{1}{2\pi} \int_{-2\pi B}^{2\pi B} S(\omega) d\omega \\
&= W_0 B
\end{aligned} \tag{A-22}$$

Thus, the sample variance or time base-error in terms of the variance of the random flutter is given by

$$\sigma_s^2 = \frac{(KV_0)^2 \sigma_f^2 \tau}{2B} . \quad (\text{A-23})$$

Equation (A-23) represents the variance of the sample distance resulting from the recording of two events separated by τ . An identical expression for the variance is obtained when the recorded events are played back so the total time-base error is given by the sum of the variances, i.e.,

$$\sigma_{st}^2 = \frac{(KV_0)^2 \sigma_f^2 \tau}{B} . \quad (\text{A-24})$$

The relative time-base error is given by

$$\sigma_{sn} = \frac{\sigma_{st}}{S_0} = \sqrt{\frac{(KV_0)^2 \sigma_f^2 \tau}{S_0^2 B}} = \frac{K\sigma_f}{\sqrt{B\tau}}$$

where

- σ_{sn} = normalized standard deviation of the sample distance or time-base error
- σ_f = RMS tape flutter
- B = flutter measurement bandwidth
- τ = time between recorded events
- K = percent peak deviation of flutter.

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