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DURATION DISCRIMINATION OF BRIEF VISUAL OFF-FLASHES

by

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Duration Discrimination of Brief Visual Off-flashes[1]

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Abstract

Two experiments investigated the manner in which human observers discriminate the difference in duration between brief, visual off-flashes. In the first experiment, three observers were run in a two-alternative, single stimulus paradigm, and three in a two-alternative, forced-choice paradigm. In both cases the observer's task was to discriminate between a short (d_0) and a long (d_1) duration for two different values of d_0 and five different incremental durations (Δd) added to d_0. The data indicated that performance increased as a function of Δd and decreased as a function of d_0. Analysis of the data in terms of three models which assume that the observer uses temporal cues to make his judgment, and two which view him as using energy as the cue, revealed that none of the models could account adequately for the results obtained.

The second experiment was designed to investigate the role of memory in the forced-choice situation. One value of d_0, two values of Δd, and four values of the inter-stimulus interval (ISI) were used. The results indicated no decrement in performance as a consequence of increasing ISI.

INTRODUCTION

When an observer is presented with stimuli which differ in duration, what is the mechanism by which he discriminates them? Does he use only the temporal information in the stimuli or does he use some other form of information? How does he discriminate when two stimuli are presented in rapid succession? The present study is an attempt to investigate these problems by an analysis of the performance of human observers on a duration discrimination task in which the stimuli are brief visual off-flashes. To date, three quantitative models have been proposed to account for the performance of observers in a duration discrimination task (Kristofferson, 1965; Creelman, 1962; Allan, Kristofferson and Wiens, in preparation).

Kristofferson's (1965) quantal model postulates an "internal clock" which generates a succession of equally-spaced points in time which are independent of the presentation of an external stimulus. The time points are assumed to occur at the rate of one every q msec., and under normal circumstances the rate is assumed to be constant for each observer. If

$$Xq \leq d_i < (X + 1)q,$$

where X is a non-negative integer and d_i is the duration of the stimulus, then the probability of traversing X time points, P(X), during d_i msec. is

$$P(X) = \frac{(X + 1)q - d_i}{q},$$

and the probability of passing (X + 1) time points, P(X + 1), is 1-P(X). Thus for the two durations, d_0 and d_1, such that

$$Xq \leq d_0 \leq d_1 < (X + 1)q, \qquad (1)$$

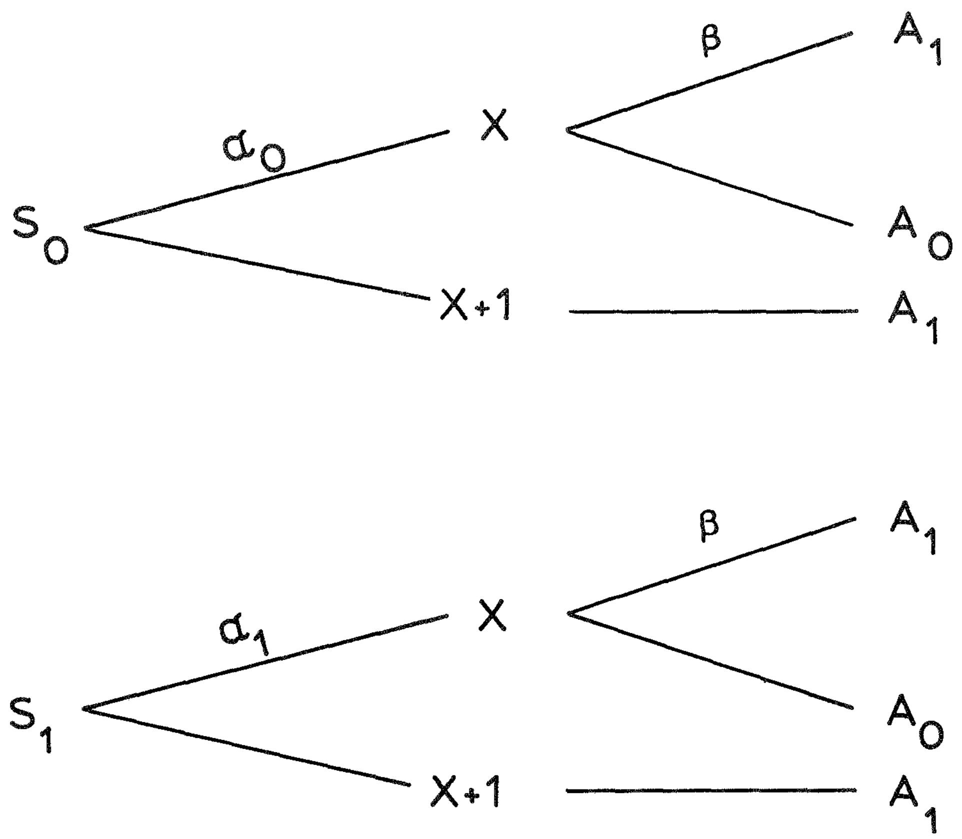
either X or X + 1 time points will be passed given a stimulus of either duration. It is assumed that the observer bases his judgment of the duration of a brief stimulus on the number of time points traversed during the stimulus event.

On each trial of a two-alternative, single stimulus, duration discrimination task (an S-S task) a stimulus is presented for either d_0 msec. (an S_0 stimulus) or for d_1 msec. (an S_1 stimulus), and the observer's task is to decide whether the stimulus was short (an A_0 response) or long (an A_1 response). Thus, the observer should respond A_0 if X time points are passed, and A_1 if X + 1 time points are passed. However, if the difference in duration, Δd, between S_0 and S_1 is small, and if d_0 is not much greater than Xq, most of the stimuli will traverse X time points, and hence appear subjectively short. If the observer is told that S_0 and S_1 will occur with equal frequency, and that he should try to make as many A_1 responses as A_0 responses, he may make an A_1 response on some proportion, β, of the trials on which X time points are traversed. Kristofferson's (1965) model for a two-alternative, single stimulus duration discrimination task is presented schematically in Fig. 1. An estimate of q can be obtained from the observer's performance in the following manner:

$$P_1 = \frac{P(A_1 \mid S_1) - P(A_1 \mid S_0)}{1 - P(A_1 \mid S_0)} = \frac{\Delta d}{(X + 1)q - d_0}. \qquad (2)$$

Eq. 2 shows that the observer's ability to discriminate a difference in duration in the single stimulus situation, denoted as P_1, is a zero-intercept, linear function of Δd.

On each trial of a two-alternative, forced-choice task (an F-C task), two stimuli which differ in duration are presented in succession, and the observer has to indicate whether the first stimulus was the long one, an A_{10} response, or whether the first stimulus was the short one, an A_{01} response. Thus, the observer should make an A_{10} response if the number of time points passed during the first stimulus was greater than the number of time points passed during the second stimulus, and he should make an A_{01} response in the reverse case. If the number of time points passed in each interval are equal, he may be assumed to make an A_{10} response with probability β. If the probabilities of passing X or X + 1 time points given S_0 or S_1 are as represented in Fig. 1, then the F - C situation can be shown schematically as in Fig. 2, where S_{01} symbolizes S_0 followed by S_1 and S_{10} symbolizes S_1 followed by S_0. An estimate of q can be obtained from the observer's performance in the following manner:



where

$$\alpha_0 = \frac{(X+1)q - d_0}{q} ,$$

and

$$\alpha_1 = \frac{(X+1)q - d_1}{q} .$$

Fig.1: Schematic of the Kristofferson (1965) quantal model for a two-alternative, single stimulus duration discrimination task.

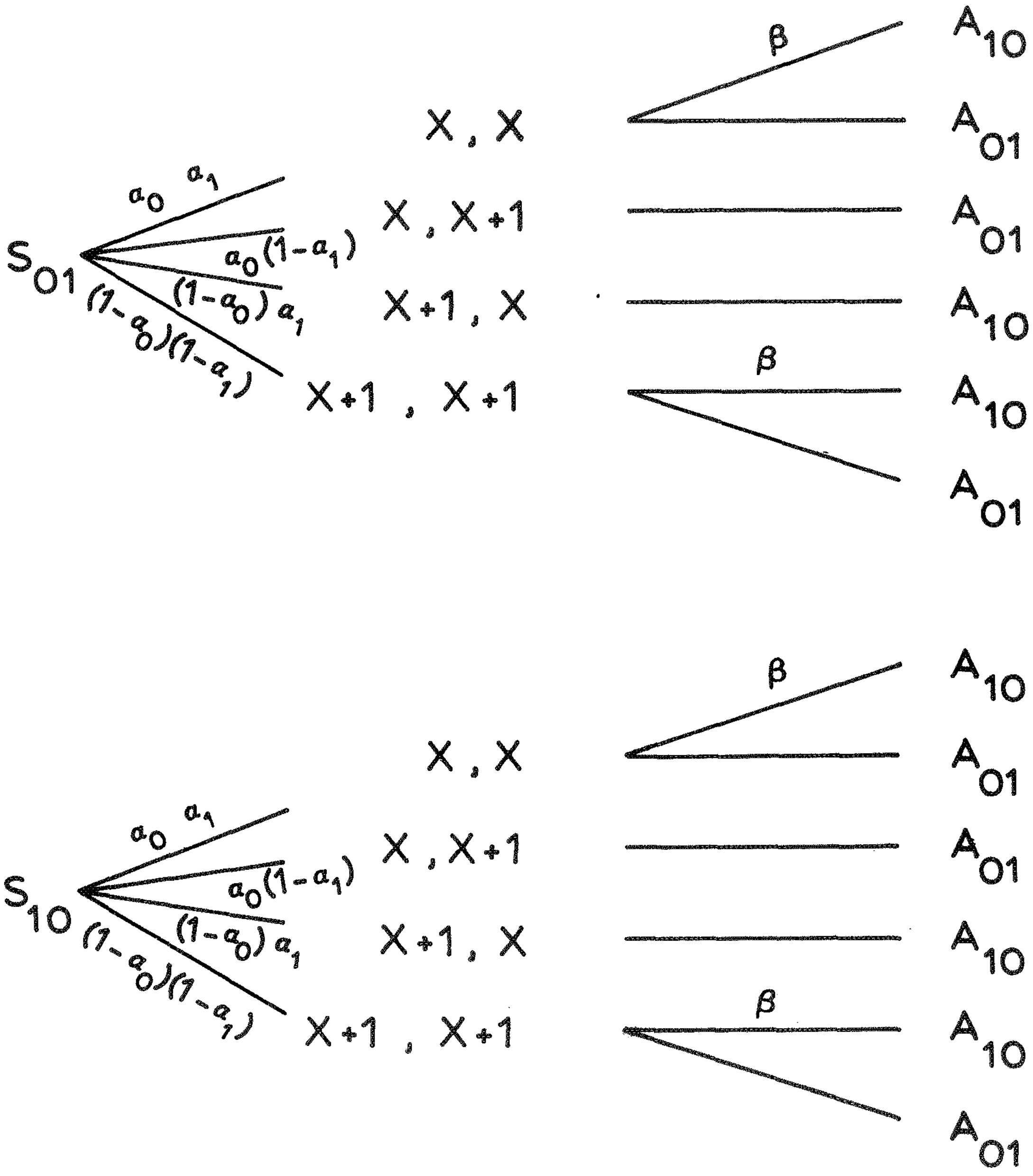


Fig. 2: Schematic of the Kristofferson (1965) quantal model for a two-alternative, forced-choice duration discrimination task.

$$P_2 = P(A_{10} \mid S_{10}) - P(A_{10} \mid S_{01}) = \frac{\Delta d}{q}. \qquad (3)$$

Thus, the observer's ability to discriminate a difference in duration in the F - C case, denoted as P_2, should be a zero-intercept, linear function of Δd, and for a given Δd, P_2 should be independent of the value of d_0.

Creelman's (1962) decision theory model of duration discrimination also assumes that the observer judges duration by the number of pulses occurring during the stimulus interval. These pulses are assumed to come from the firing of a large number of independent elements, each of which has a fixed probability of firing at any given moment. The total number of pulses over a given time interval can be shown to have a Poisson distribution where the probability of n counts, P(n), occurring in d_i msec. can be represented by

$$P(n) = \frac{(\lambda d_i)^n}{n!} e^{-\lambda d_i}, \qquad (4)$$

where λ represents the rate of firing of the pulse source. If λd_i is sufficiently large, this Poisson distribution can be closely approximated by a Gaussian distribution with mean and variance both λd_i.

The observer's decision problem in the S-S case is represented in Fig. 3, which shows two overlapping Gaussian distributions of counts. When S_0 is presented, the number of counts will be distributed as in the left-hand distribution of Fig. 3, and when S_1 is presented, the number

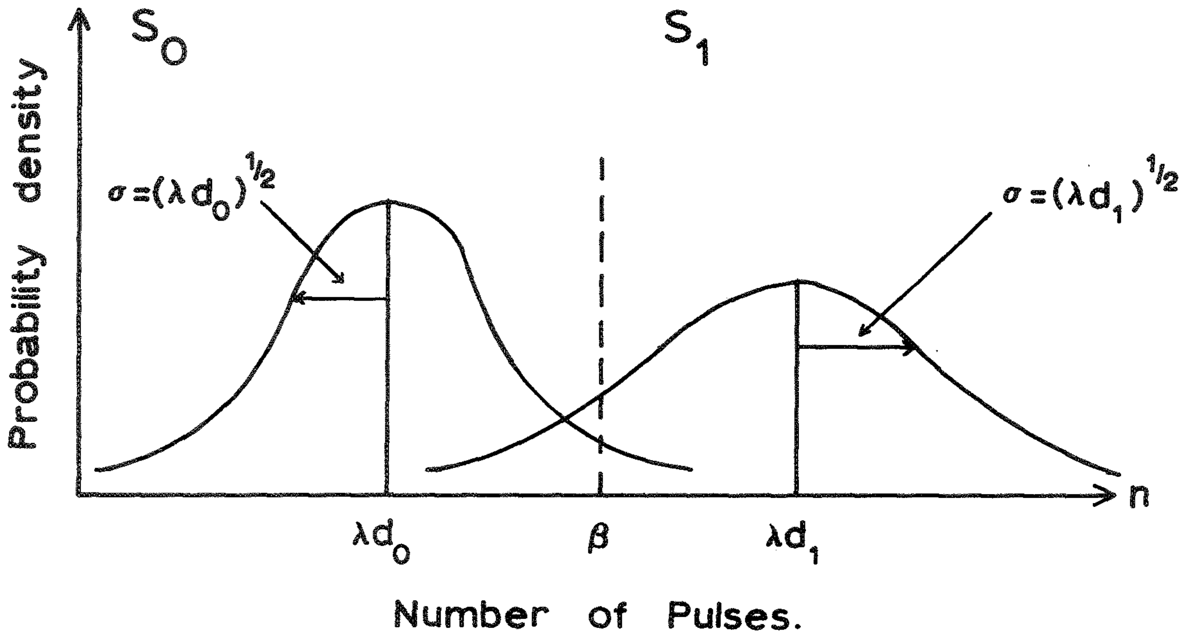


Fig. 3: Schematic of the Creelman (1962) decision theory model for a two-alternative, single stimulus duration discrimination task.

of counts will be distributed as in the right-hand distribution. The means and standard deviations of the distributions are shown in the figure. The observer is assumed to have a criterion number of counts, β. If the number of counts during a stimulus presentation exceeds β, he responds A_1; if not, he responds A_0.

It can be seen from Fig. 3 that the probability of an A_1 response given an S_1 stimulus, $P(A_1 | S_1)$, is the area under the S_1 distribution to the right of β; similarly, $P(A_1 | S_0)$ is the corresponding area under the S_0 distribution. The observer's ability to discriminate a difference in duration can be specified by the discriminability measure $d'_{C,1}$, where d' is a widely used symbol denoting discriminability in a model which assumes Gaussian distributions of the internal representations of stimulus events, the letter C identifies d' with the Creelman model, and the number 1 is used as a symbol for the S - S case. The term $d'_{C,1}$ represents the distance between the means of the two distributions expressed in standard deviation units of the S_0 distribution. That is,

$$d'_{C,1} = \frac{\lambda^{\frac{1}{2}}\,\Delta d}{d_0{}^{\frac{1}{2}}}. \tag{5}$$

An estimate of $d'_{C,1}$, denoted as $\hat{d}'_{C,1}$, may be obtained from the observer's performance in the following manner:

$$\hat{d}'_{C,1} = Z(A_1 | S_0) - \frac{d_1{}^{\frac{1}{2}}}{d_0{}^{\frac{1}{2}}}\, Z(A_1 | S_1), \tag{6}$$

where $Z(A_1 \mid S_0)$ is that value of a normal deviate which is exceeded with probability $P(A_1 \mid S_0)$ and $Z(A_1 \mid S_1)$ is the value obtained in the same manner from $P(A_1 \mid S_1)$. It is apparent from Eq. 5 that the model predicts that $d'_{C,1}$ should increase as a zero-intercept, linear function of Δd, and that $d'_{C,1}$ should decrease as a power function of d_0.

In the F - C case, it is assumed that the observer subtracts the number of counts produced by the second stimulus from the number produced by the first. Thus, two distributions of differences are generated; an S_{01} distribution when S_0 is presented first, and an S_{10} distribution in the reverse case. The mean of the S_{01} distribution is $-\lambda\Delta d$, the mean of the S_{10} distribution is $\lambda\Delta d$, and the variance of both distributions, σ^2, is the sum of the variances of the S_0 and S_1 distributions. Specifically,

$$\sigma^2 = \lambda d_0 + \lambda d_1 = \lambda(2d_0 + \Delta d).$$

The observer's decision problem in the F - C case is shown in Fig. 4. The discriminability measure, $d'_{C,2}$, where the 2 is the symbol for the F - C case, is defined as the distance between the means of the two difference distributions expressed in standard deviation units of the S_{01} distribution. Thus,

$$d'_{C,2} = \frac{2\ \lambda^{\frac{1}{2}}\ \Delta d}{\sqrt{2d_0 + \Delta d}}.^{\,2} \qquad (7)$$

An estimate of $d'_{C,2}$ may be obtained from the data in a manner analogous

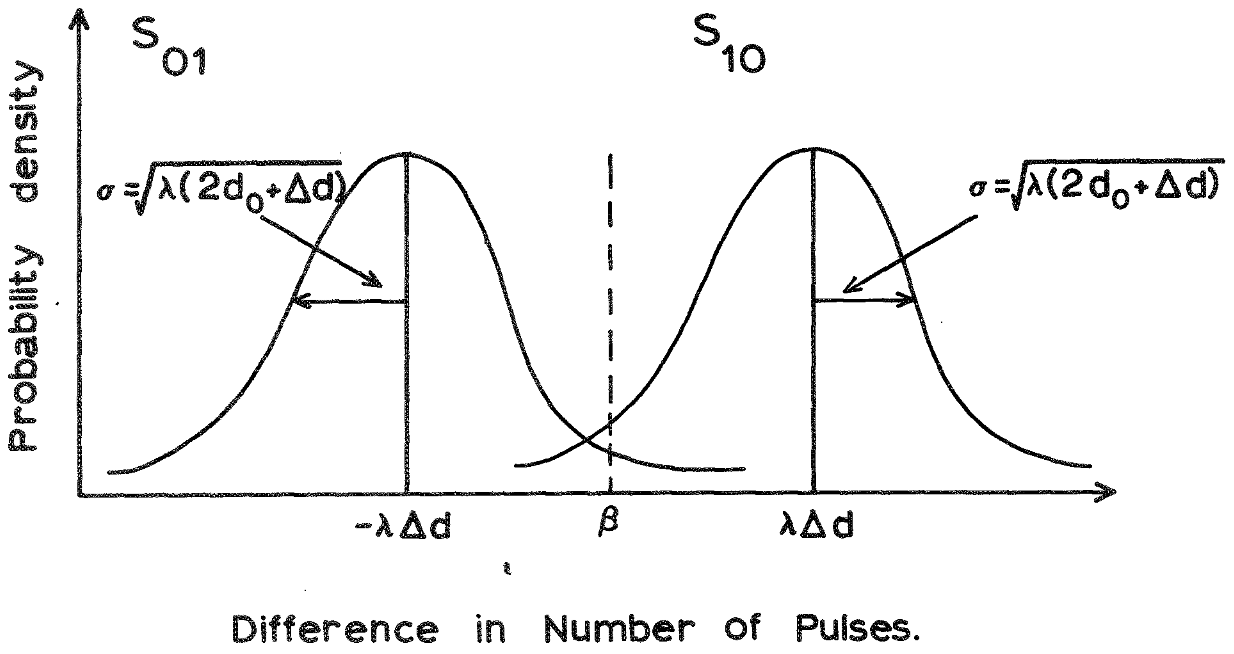


Fig. 4: Schematic of the Creelman (1962) decision theory model for a two-alternative, forced-choice discrimination task.

to that used in the S - S case. Specifically,

$$\hat{d}'_{C,2} = Z(A_{10} \mid S_{01}) - Z(A_{10} \mid S_{10}). \qquad (8)$$

From Eq. 7, it is apparent that the model predicts that $d'_{C,2}$ will increase as a zero-intercept, linear function of the quantity

$\Delta d \;/\; \sqrt{2d_0 + \Delta d}$.

Kristofferson (1965) has presented data from a two-alternative, forced-choice paradigm in which the observer had to compare offset times of a light and a tone. The data indicated some support for a quantal process such as the one described. Creelman (1962) has reported data from experiments in which the stimuli were tones. He also used the F - C paradigm, and his model provided a reasonable account of the data under an extensive set of conditions. Allan, Kristofferson, and Wiens (1970) reported data from an S - S paradigm in which the stimuli were visual on-flashes. That is, the duration to be discriminated was defined by the duration of a positive pulse of light. Analysis in terms of both Kristofferson's and Creelman's models showed that neither model could account adequately for the results. Specifically, analysis in terms of the Creelman model revealed that the variability in the sensory states associated with a particular stimulus did not depend, as the model predicts, on the duration of the stimulus. Furthermore, the ability to discriminate a given difference in duration appeared to be independent of the actual durations used, again contrary to the prediction of the model. An analysis in terms of the quantal model failed to indicate the predicted linearity between P_1 and Δd.

Allan, Kristofferson, and Wiens (in preparation), have proposed a model which seems to provide a reasonable interpretation of their data. The model assumes that at the onset of a d_i msec. stimulus, an internal timing process is activated with a lag time which is uniformly distributed on the interval from zero to q msec. where q is independent of the duration of the stimulus. That is,

$$f_{U_1}(u_1) = \begin{cases} 1/q \text{ for } 0 < u_1 < q \\ 0 \text{ elsewhere} \end{cases},$$

where U_1 is a random variable representing onset times. At the offset of the stimulus, an independent mechanism terminates the activity of the timer with a lag time which is uniformly distributed on the interval from d_i to $d_i + q$ msec. That is,

$$f_{U_2}(u_2) = \begin{cases} 1/q \text{ for } d_i < u_2 < d_i + q, \\ 0 \text{ elsewhere} \end{cases}$$

where U_2 is a random variable representing offset times.

The observer is assumed to measure the duration by a counting process which takes place in the interval, u', between the onset and the offset of the timer. Expressed mathematically,

$$u' = u_2 - u_1,$$

where U' is a random variable representing psychological duration. The distribution of U' can be shown to be triangular, described by the following function (Allan, et al., in preparation):

$$g_{U'}(u') = \begin{cases} \dfrac{q - d_i + u'}{q^2} & \text{for } d_i - q < u' < d_i \\ \dfrac{q + d_i - u'}{q^2} & \text{for } d_i < u' < d_i + q \\ 0 & \text{elsewhere .} \end{cases} \qquad (9)$$

The random variable U' has an expected value of d_i and a variance equal to $q^2/6$. The observer is assumed to make his decision in much the same manner as in the Creelman model. His decision problem in the S - S case may be represented by two overlapping triangular distributions as shown in Fig. 5, where d_0 is the mean of the S_0 distribution, and d_1 is the mean of the S_1 distribution. The discriminability measure, $d_{q,1}$, is defined as the distance between the means of the two distributions expressed in units of q. Therefore,

$$d_{q,1} \doteq \frac{\Delta d}{q}. \qquad (10)$$

Eq. 10 shows that d_q is a zero-intercept, linear function of Δd, and that, for a given Δd, discriminability is independent of the actual durations used. An estimate of $d_{q,1}$, denoted as $\hat{d}_{q,1}$, may be obtained from the observer's performance in the following manner:

$$\hat{d}_{q,1} = Q(A_1 \mid S_0) - Q(A_1 \mid S_1), \qquad (11)$$

where $Q(A_1 \mid S_0)$ is the distance in q units from the mean of the S_0

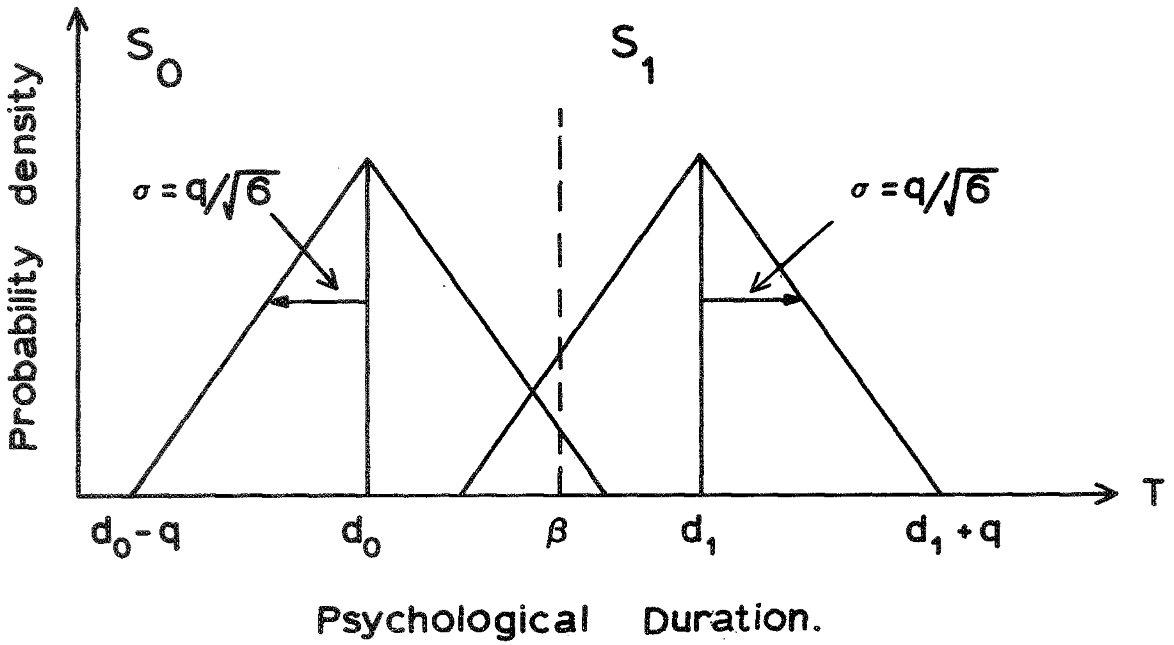


Fig. 5: Schematic of the Allan, et al. (1970) model for a two-alternative, single stimulus duration discrimination task.

distribution to the criterion, and $Q(A_1 \mid S_1)$ is the distance in q units from the mean of the S_1 distribution to the criterion. Thus, $Q(A_1 \mid S_i)$ is that value of a psychological duration, expressed in q units, which is exceeded with probability $P(A_1 \mid S_i)$.

The observer's decision problem in the F - C case is presented in Fig. 6 and is derived from that of the S - S case in the same manner as in the Creelman model. In this case, the subtraction of the psychological durations of the two intervals leads to two distributions of differences, one for each of the S_{10} and S_{01} stimuli, described by the following functions.^{3} For an S_{10}

lus,

$$f_{U''_{10}}(u''_{10}) =$$

$$\begin{cases} \frac{1}{6q^4}\,(2q + u''_{10} - \Delta d)^3 & \text{for } \Delta d - 2q < u''_{10} < \Delta d - q \\ \frac{1}{6q^4}\left[3(\Delta d - u''_{10})^3 - 6q(\Delta d - u''_{10})^2 + 4q^3\right] & \text{for } \Delta d - q < u''_{10} < \Delta d \\ \frac{1}{6q^4}\left[3(u''_{10} - \Delta d)^3 - 6q(u''_{10} - \Delta d)^2 + 4q^3\right] & \text{for } \Delta d < u''_{10} < \Delta d + q \\ \frac{1}{6q^4}\,(2q - u''_{10} + \Delta d)^3 & \text{for } \Delta d + q < u''_{10} < \Delta d + 2q \\ 0 & \text{elsewhere}, \end{cases}$$

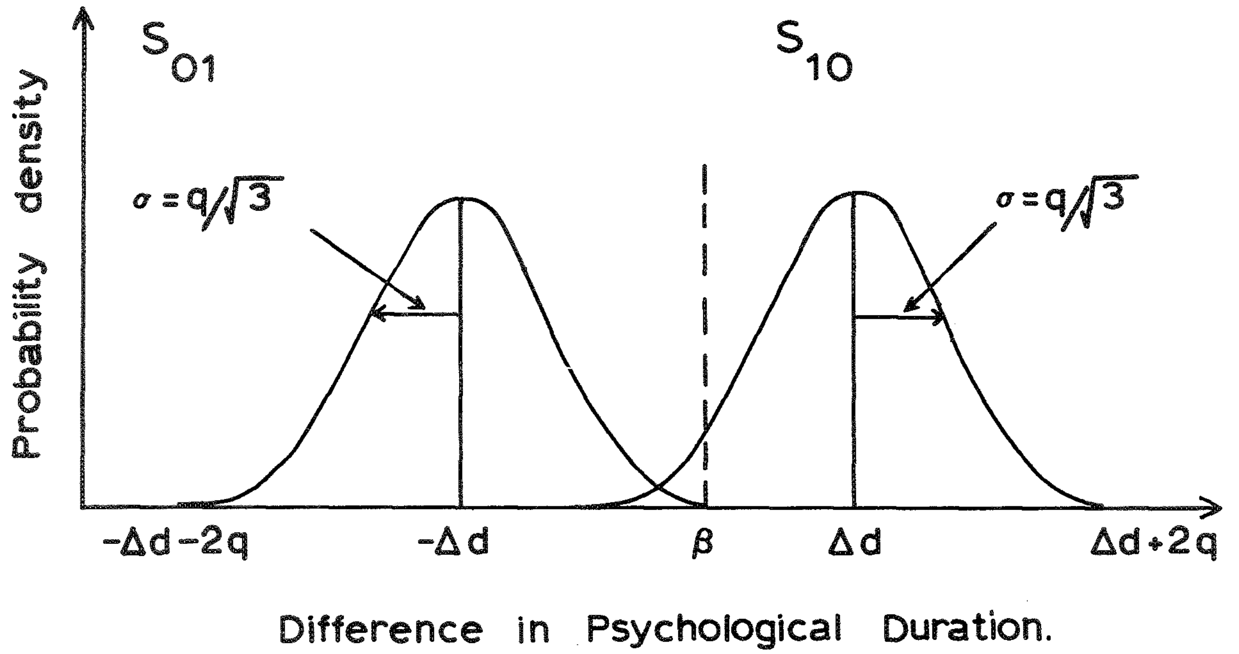


Fig. 6: Schematic of the Allan, et al. (1970) for a two-alternative, forced-choice duration discrimination task.

where U''_{10} is a random variable representing the psychological difference in duration resulting from an S_{10} stimulus. The expected value of U''_{10} is Δd; the variance is $q^2/3$. For an S_{01} stimulus,

$$f_{U''_{01}}(u''_{01}) = \begin{cases} \frac{1}{6q^4}\,(2q - u''_{01} + \Delta d)^3 & \text{for } -\Delta d - 2q < u''_{01} < -\Delta d - q \\[2ex] \frac{1}{6q^4}\left[-3(u''_{01} + \Delta d)^3 - 6q(u''_{01} + \Delta d)^2 + 4q^3\right] & \text{for } -\Delta d - q < u''_{01} < -\Delta d \\[2ex] \frac{1}{6q^4}\left[3(u''_{01} + \Delta d)^3 - 6q(u''_{01} + \Delta d)^2 + 4q^3\right] & \text{for } -\Delta d < u''_{01} < -\Delta d + q \\[2ex] \frac{1}{6q^4}\,(2q - u''_{01} - \Delta d)^3 & \text{for } -\Delta d + q < u''_{01} < -\Delta d + 2q \\[2ex] 0 & \text{elsewhere ,} \end{cases}$$

where U''_{01} is a random variable representing the psychological difference in duration resulting from an S_{01} stimulus. The expected value of U''_{01} is $-\Delta d$; the variance is $q^2/3$.

As in the single stimulus case, discriminability, here denoted $d_{q,2}$, is defined as the distance between the means of the two distributions expressed in q units. Thus,

$$d_{q,2} = \frac{2\Delta d}{q} . \qquad (12)$$

Again the model predicts that discriminability is a zero-intercept, linear function of Δd and is independent of the value of d_0. An estimate of discriminability may be obtained from the data in the same manner as in the single stimulus case. Thus,

$$\hat{d}_{q,2} = Q(A_{10} \mid S_{01}) - Q(A_{10} \mid S_{10}), \tag{13}$$

where $Q(A_{10} \mid S_{01})$ is the distance in q units from the mean of the S_{01} distribution to the criterion, and $Q(A_{10} \mid S_{10})$ is the distance in q units from the mean of the S_{10} distribution to the criterion.

The first experiment of the present study is an attempt to compare the findings of the on-flash study of Allan, et al. (1970), with those of an experiment in which the stimuli are off-flashes, and the forced-choice as well as the single stimulus paradigm is used. The second experiment is an examination of the effect of varying the interstimulus interval in the forced-choice case.

EXPERIMENT ONE

METHOD

Apparatus

The observer was seated in a chair in a soundproof room with a constant, dim background white light. The light used to present the stimuli came from a glow modulator driven by an Iconix 6195-4 Lamp Driver. The glow modulator was enclosed in a metal box with an aperture 4mm. in diameter (subtending a visual angle of 0^{o} 21') in the centre. The aperture was covered on the inside with a Kodak Wratten No. 96 neutral density 2.00 filter and then translucent milk glass so that the light would be a clearly visible white, yet not so bright as to be uncomfortable. The light was adjusted so that the light coming out of the box was a constant 50 foot-lamberts as measured by a 150-UB photometer (Photo. Research Corp.). The stimulus light was at eye level approximately 66 centimeters in front of the observer. On the right arm of the observer's chair were two buttons. For the S - S observers, the left-hand button was labelled "long" and the right-hand button "short". For the F - C observers, the left-hand button was labelled "1st signal longer" and the right-hand button was labelled "2nd signal longer". The button needed to be pressed only lightly for a response to be recorded. Clearly audible warning tones and feedback were provided through a speaker in each observer's booth. The timing of the stimulus presentations, the recording of responses, and the randomization of the stimulus sequence was performed by a PDP-8S computer.

Observers

There were six observers in this experiment; three were run on an S - S task and three on an F - C task, with two males and one female in each group. All subjects were university students with normal (uncorrected) vision, and all were paid ($2.00 an hour) for their participation.

Procedure

Each observer was run for 30 sessions of approximately 40 min. each. Normally an observer ran only one session each day. Each session consisted of 5 blocks of 100 trials each with a 1-min. rest interval between blocks.

The stimulus light was on at all times except when a signal was being presented. The signal was a brief dark period or off-flash of the stimulus light in front of the observer. The duration (d_0) of the shorter stimulus (S_0) was either 50 or 100 msec. The longer stimulus (S_1) had a duration (d_1) equal to $d_0 + \Delta d$ where Δd was one of 10, 20, 30, 40, or 50 msec. Thus, there were 10 conditions altogether. Each observer ran 3 sessions on each of the 10 conditions, with just one condition being run in each session. The first run on each condition was considered practice and was not included in the final data analysis. In addition, the first 100 trials of each of the remaining 20 sessions were not included in the data analysis.

Thus there were 800 data trials for each observer for each condition. The conditions were run in a random order with the restriction that every condition was used once before any of the 10 conditions was repeated.

Each trial of the S - S case began with a 1.0 sec. warning tone. Exactly 200 msec. after the offset of the tone, the stimulus light went off for a period of either d_0 or d_1 msec. This was followed by a 1.5 sec. response period. At the end of the response period, on a trial in which the stimulus was S_1, feedback was provided by means of a 100 msec. auditory tone. Following this feedback was a 1.0 sec. empty interval followed by the warning tone for the next trial. The response period on a trial in which the stimulus was S_0 was followed immediately by the warning tone for the next trial. Equal numbers of S_0 and S_1 trials were presented within each block of 100 trials, and the order of presentation was randomized.

In the F - C case, each trial began with a 1.0 sec. warning tone, 200 msec. empty interval, and an S_0 or S_1 stimulus as in the S - S case. This was followed by a 500 msec. interstimulus interval followed by S_1 if S_0 was presented first, or S_0 if S_1 was presented first. A 1.5 sec. response period followed the second stimulus. At the end of the response period of an S_{10} trial (that is, a trial in which S_1 preceded S_0), feedback was presented by a 100 msec. auditory tone. Following the feedback was a 1.0 sec. empty period before the warning tone of the next trial.

The next trial began immediately after the response period of an S_{01} trial. Again, there were equal numbers of the two stimulus patterns, S_{10} and S_{01}, in each block of 100 trials, and the order of presentation was randomized.

The observer's task in the S - S case was to indicate on each trial whether he thought the stimulus was short or long by pressing the appropriate pushbutton on the arm of his chair. In the F - C case, the observer was to indicate on each trial whether he thought the first or second signal was the longer of the two by pressing the appropriate button. All observers were told the meaning of the feedback and that they should respond equally on both buttons.

RESULTS

Each observer's performance in each condition for the S-S case may be summarized by the probability of a correct response, P(c), the probability of an A_1 response given an S_1 stimulus, $P(A_1 | S_1)$, and the probability of an A_1 response given an S_0 stimulus, $P(A_1 | S_0)$. These probabilities are presented in Table 1, and P(c) as a function of Δd is shown in Fig. 7. The corresponding probabilities for the F-C case, P(c), $P(A_{10} | S_{10})$ and $P(A_{10} | S_{01})$, are presented in Table 2, and P(c) as a function of Δd is shown in Fig. 8. It is clear from Figs. 7 and 8 that performance in terms of P(c) increases with larger Δd's, and is better for $d_0 = 50$ msec. than for $d_0 = 100$ msec. for all observers.

Many studies involving sequential visual stimuli have found evidence of time order errors (see Woodworth and Schlosberg, 1954). In the experiments they report, the task involved brightness discrimination rather than duration discrimination and most studies found a positive time order error. That is, for two stimuli of equal intensity, the observer's performance indicated that the first stimulus was subjectively brighter than the second. In studies by Stott (1935), Woodrow (1951), and Creelman (1962), in which the task was duration discrimination, the stimuli were auditory, and while there was some evidence of time order errors in some conditions, there were no systematic effects which were consistent from experiment to experiment for any observer.

TABLE 1

Probabilities Summarizing Each Observer's Performance Under Each of the Ten Conditions in the Single Stimulus Case

Observer	d_0	Δd	P(c)	$P(A_1 \mid S_1)$	$P(A_1 \mid S_0)$
RM	50	10	.696	.737	.349
		20	.768	.790	.254
		30	.864	.884	.156
		40	.928	.930	.075
		50	.961	.958	.036
	100	10	.611	.689	.472
		20	.712	.802	.390
		30	.822	.913	.270
		40	.884	.928	.161
		50	.923	.973	.128
SM	50	10	.704	.679	.272
		20	.924	.939	.092
		30	.923	.935	.090
		40	.984	.980	.013
		50	.976	.983	.031
	100	10	.585	.583	.413
		20	.695	.698	.281
		30	.851	.885	.184
		40	.901	.908	.096
		50	.954	.968	.061
LB	50	10	.721	.768	.326
		20	.866	.879	.147
		30	.945	.960	.071
		40	.970	.965	.025
		50	.986	.990	.018
	100	10	.565	.583	.454
		20	.654	.736	.430
		30	.802	.845	.241
		40	.804	.854	.215
		50	.871	.915	.173

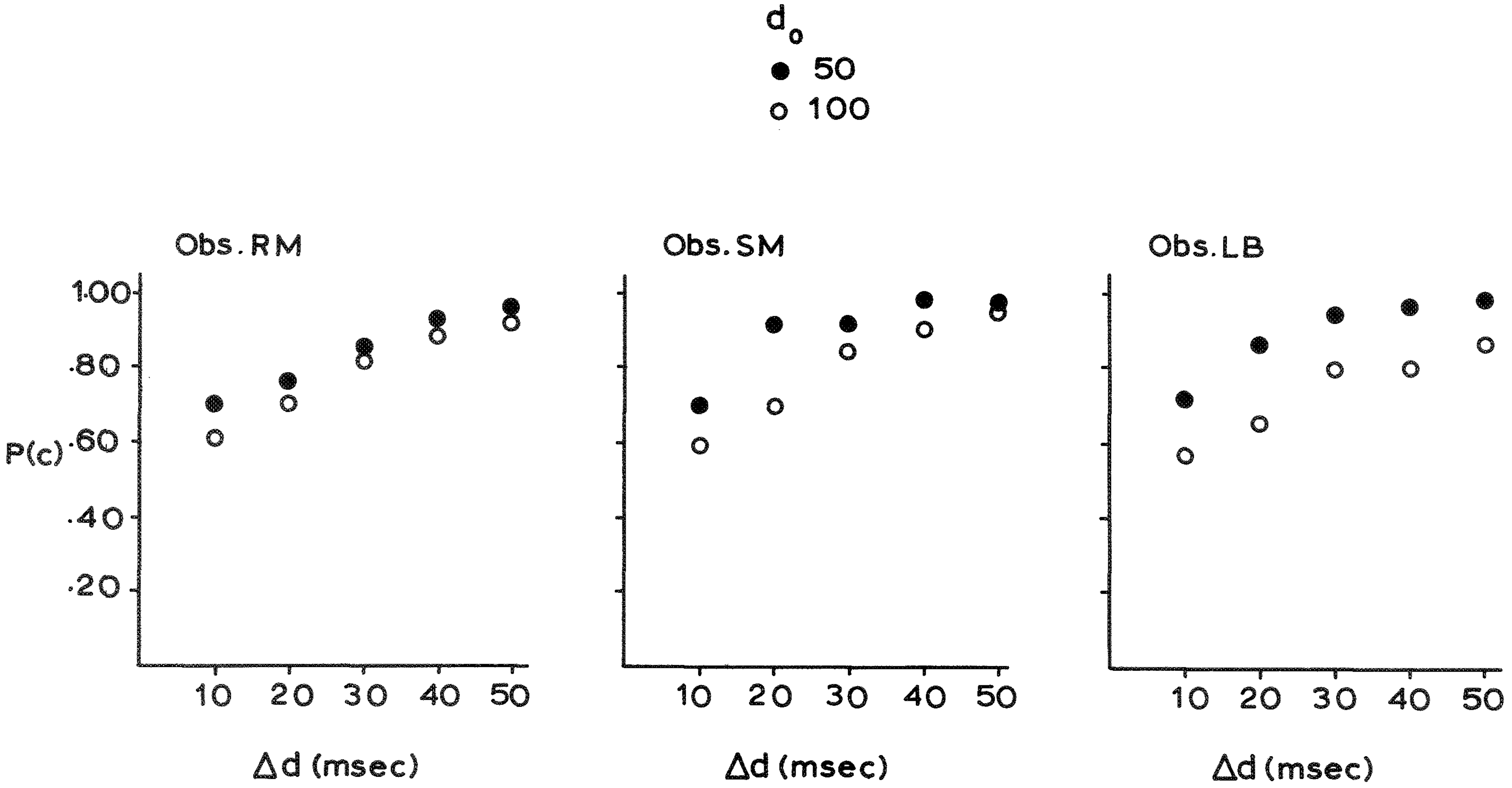


Fig. 7: Probability of a correct response, P(c), as a function of Δd for each single stimulus observer.

TABLE 2

Probabilities Summarizing Each Observer's Performance Under Each of the Ten Conditions in the Forced-Choice Case

Observer	d_0	Δd	P(c)	$P(A_{10} \mid S_{10})$	$P(A_{10} \mid S_{01})$
AJ	50	10	.824	.838	.191
		20	.959	.958	.041
		30	.989	.988	.010
		40	.989	.988	.010
		50	.996	1.000	.010
	100	10	.605	.580	.370
		20	.781	.761	.199
		30	.911	.903	.082
		40	.964	.962	.035
		50	.991	.998	.016
SB	50	10	.736	.681	.252
		20	.796	.835	.245
		30	.906	.922	.110
		40	.899	.894	.096
		50	.966	.960	.028
	100	10	.595	.539	.349
		20	.731	.684	.208
		30	.798	.804	.209
		40	.821	.814	.171
		50	.865	.863	.135
PH	50	10	.738	.655	.180
		20	.867	.852	.119
		30	.855	.829	.121
		40	.921	.922	.080
		50	.935	.920	.052
	100	10	.608	.561	.346
		20	.676	.682	.328
		30	.752	.721	.216
		40	.848	.820	.121
		50	.822	.772	.127

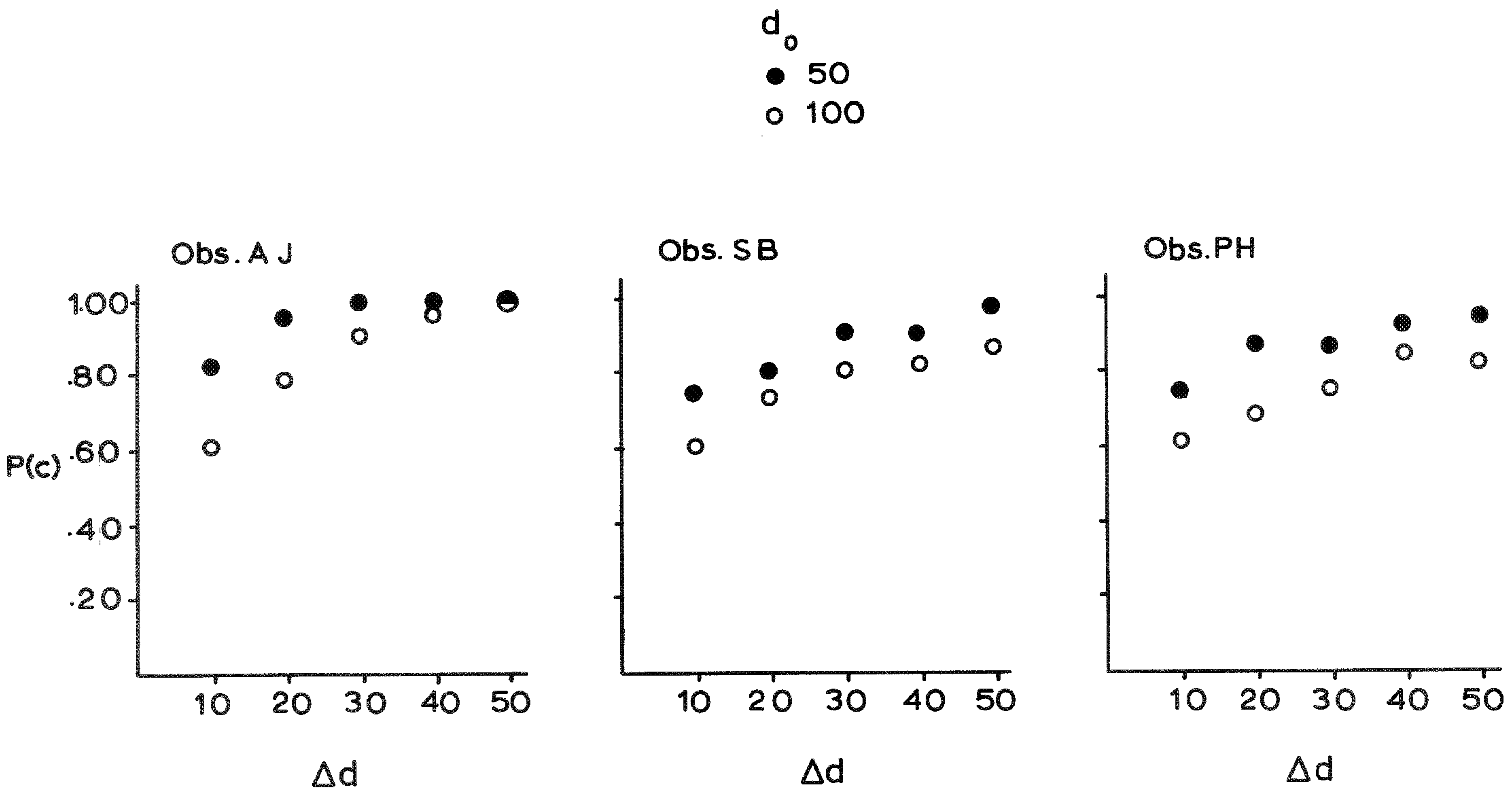


Fig. 8: Probability of a correct response, P(c), as a function of Δd for each forced-choice observer.

If there were a time order error in the F - C case of the present experiment, then the observer would have had a greater probability of a correct response to one stimulus than to the other. The probability of a correct response on those trials in which the longer stimulus was presented first is $P(A_{10} \mid S_{10})$, and the probability of a correct response when the longer stimulus was presented second is $1 - P(A_{10} \mid S_{01})$. By inspection of Table 1 it may be seen that there is no evidence of a significant time error for any observer. Since observers were told to respond A_{10} and A_{01} equally often, it is possible that the observers shifted their criterion or bias, β , to allow for any time order error that might have occurred. Such an explanation has been suggested for auditory amplitude discrimination by Kinchla and Smyzer (1967). It is also possible that the feedback procedure of the present experiment was responsible for the absence of a time order error.

THEORETICAL ANALYSIS

The following section is devoted to an analysis of the data in terms of each of the models presented in the introduction.

The Quantal Model

Eqs. 2 and 3 specify that P_1 in the S - S case, and P_2 in the F - C case, are zero-intercept, linear functions of Δd. Estimates of P_1, $\hat{P}_1$, obtained from the data according to Eq. 2 and the resultant estimates of q, $\hat{q}$, are presented numerically in Table 3, and P_1 estimates are presented as a function of Δd in Fig. 9. Estimates of P_2, $\hat{P}_2$, obtained from the data according to Eq. 3, and the resultant $\hat{q}$ values are presented in Table 4, and values of P_2 as a function of Δd are presented in Fig. 10.

It is clear that the data are not consistent with the predictions for any of the observers in either the S - S or F - C case. It is obvious that in no case would a zero-intercept, straight line be a good fit to the data. From Tables 3 and 4, it may be seen that, in general, the estimates of q obtained from the data steadily increase with increasing Δd's. It is also clear that, contrary to the predictions of the model, discriminability is superior in all observers for a d_0 of 50 than for a d_0 of 100. Allan et al, (1970), using visual on-flashes and an S - S paradigm, failed to find the predicted linear relation between P_1 and Δd, and they too found steadily increasing estimates of q for larger Δd's. It is interesting, however, that in their study, discriminability appeared to be approximately equal for

TABLE 3

Estimates of P_1 and q for Each Single Stimulus Observer Under Each of the Ten Conditions

Observer	d_0	Δd	$\hat{P}_1$	$\hat{q}$
RM	50	10	.596	33.4
		20	.718	38.9
		30	.863	42.4
		40	.924	46.7
		50	.956	51.2
	100	10	.411	41.4
		20	.675	43.2
		30	.881	44.7
		40	.914	47.9
		50	.969	50.5
SM	50	10	.559	33.9
		20	.933	35.7
		30	.929	41.2
		40	.980	45.4
		50	.982	50.5
	100	10	.290	44.8
		20	.580	44.8
		30	.859	45.0
		40	.898	48.2
		50	.970	50.5
LB	50	10	.656	32.6
		20	.858	36.7
		30	.957	40.7
		40	.964	45.8
		50	.990	50.3
	100	10	.236	47.5
		20	.537	45.8
		30	.796	45.9
		40	.814	49.7
		50	.897	51.9

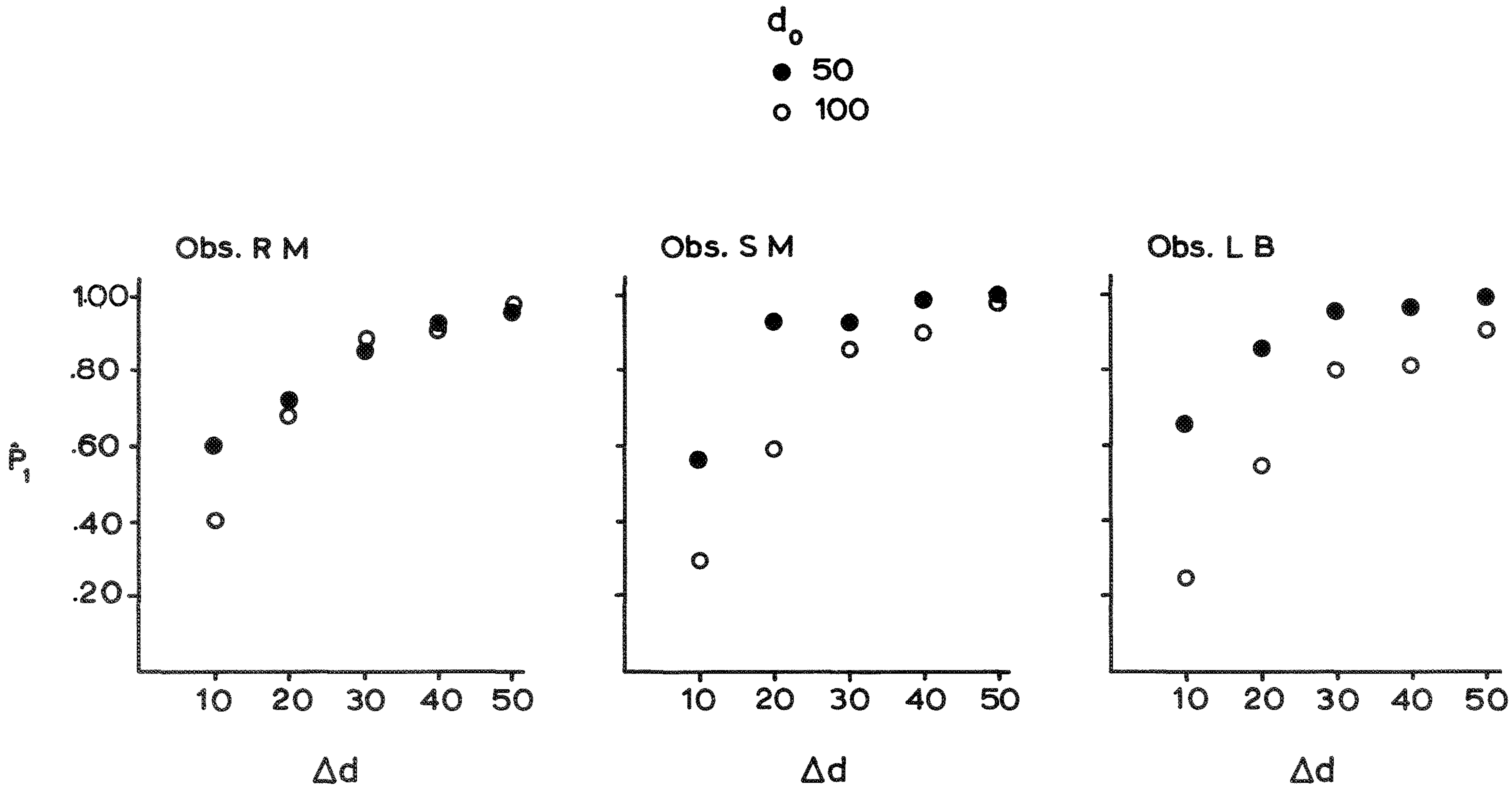


Fig. 9: $\hat{P}_1$ as a function of Δd for each single stimulus observer.

TABLE 4

Estimates of P_2 and q for Each Forced-Choice Observer Under Each of the Ten Conditions

Observer	d_0	Δd	$\hat{P}_2$	$\hat{q}$
AJ	50	10	.648	15.4
		20	.917	21.8
		30	.977	30.7
		40	.977	40.9
		50	.990	50.5
	100	10	.209	47.8
		20	.562	35.6
		30	.822	36.5
		40	.927	43.2
		50	.982	50.9
SB	50	10	.428	23.4
		20	.591	33.9
		30	.811	37.0
		40	.798	50.1
		50	.932	53.7
	100	10	.190	52.6
		20	.476	42.0
		30	.595	50.4
		40	.642	62.3
		50	.728	68.7
PH	50	10	.475	21.1
		20	.733	27.3
		30	.708	42.4
		40	.841	47.5
		50	.868	57.6
	100	10	.215	46.5
		20	.354	56.5
		30	.504	59.5
		40	.699	57.2
		50	.645	77.5

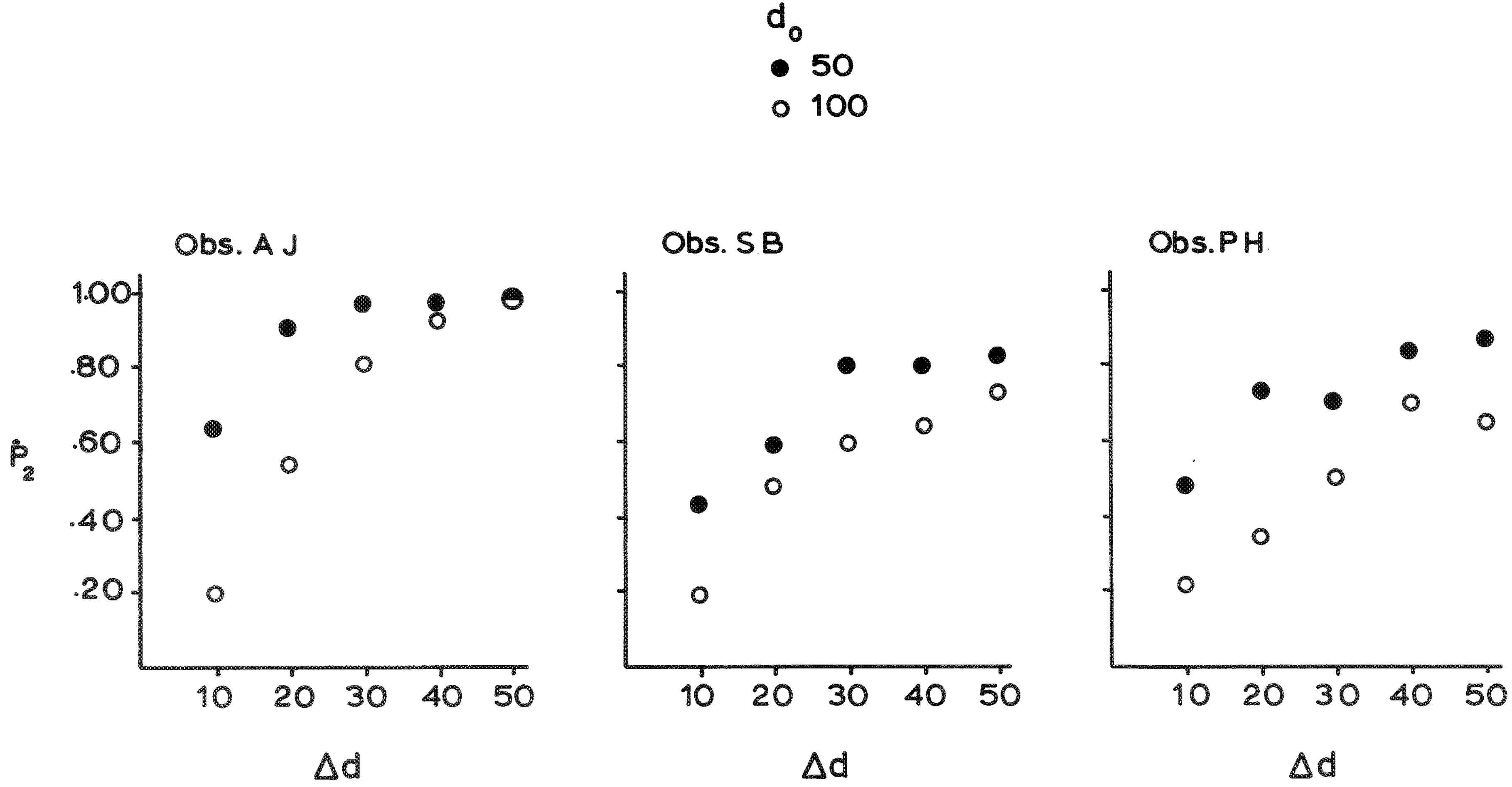


Fig. 10: $\hat{P}_2$ as a function of Δd for each forced-choice observer.

$d_0 = 50$ and $d_0 = 100$ for a given Δd.

It is of interest to note that the model provides no better a fit to the data for the F - C case than for the S - S case. In the latter, the observer is assumed to respond A_1 on some proportion, β, of the trials, even though he only counted X time points. In the F - C case, the observer must sometimes count an equal number of time points during the two stimuli, and so is truly in a state of uncertainty as to which stimulus was the longer. Thus it seems more rational to include the bias parameter, β, in the F - C case, and one might have expected the model to account for the data more adequately than for the S - S case. That it did not is further evidence of the inadequacy of the model for the present experiment.

Kristofferson (1967) has reported data from a number of experimental situations including reaction time and successiveness discrimination experiments and has obtained estimates of q which are very close to 50 msec., although varying somewhat from subject to subject. For the values of d_0 and Δd used in the present experiment, the relationships described in Eq. 1 hold only for q equal to 50 msec. It is possible that equations based on different values of q would provide a better fit to the data of the present experiment.

The Creelman Model

Eq. 5 specified that $d'_{C,1}$ is a zero-intercept, linear function of Δd, and Eq. 7 specified that $d'_{C,2}$ is a zero-intercept, linear function of the quantity $\Delta d / \sqrt{2d_0 + \Delta d}$. Estimates of $d'_{C,1}$ and

$d'_{C,2}$ obtained from the data according to Eqs. 6 and 8 are presented numerically in Tables 5 and 6. Fig. 11 shows $d'_{C,1}$ as a function of Δd and Fig. 12 shows $d'_{C,2}$ as a function of $\Delta d/\sqrt{2d_0 + \Delta d}$. The solid lines in both figures represent the best-fitting zero-intercept straight lines. Note that in the F - C case, the points for $d_0 = 50$ and $d_0 = 100$ lie at different values along the abscissa. This is due to the fact that d_0 is used in the calculation of the abscissa coordinate. There seems to be no consistent deviation from linearity for any of the six observers, although observer SM shows considerable variability, and observer PH shows some suggestion of a systematic deviation for $d_0 = 50$.

For the S - S observers, Eq. 5 specified that $d'_{C,1}$ decreases as the square root of d_0. Thus, the model predicts that $d'_{C,1}$ for $d_0 = 50$ should be equal to $\sqrt{2}$ times $d'_{C,1}$ for $d_0 = 100$ for each value of Δd. Multiplying the slope of the best-fit lines for $d_0 = 100$ by $\sqrt{2}$, it is possible to obtain predicted lines for $d_0 = 50$. These are shown for each observer by the dotted lines in Fig. 11. It is clear that the data do not conform to the predicted lines. For Observer RM, discriminability for $d_0 = 50$ is considerably worse than predicted; for the other two observers it is considerably better. In the original formulation of the model (1962), Creelman included a parameter, $\sigma_v 2$, to refer to the variance added by uncertainty about when to begin and end the counting process. Including such a parameter, Eq. 5 may be rewritten as

TABLE 5

Estimates of $d'_{C,1}$ for Each Single Stimulus Observer

Under Each of the Ten Conditions

Observer	d_0	Δd	$\hat{d}'_{C,1}$
RM	50	10	1.08
		20	1.62
		30	2.52
		40	3.42
		50	4.24
	100	10	.58
		20	1.22
		30	2.16
		40	2.71
		50	3.49
SM	50	10	.99
		20	3.16
		30	3.24
		40	4.98
		50	4.86
	100	10	.44
		20	1.15
		30	2.29
		40	2.87
		50	3.81
LB	50	10	1.25
		20	2.43
		30	3.68
		40	4.39
		50	5.39
	100	10	.34
		20	.87
		30	1.86
		40	2.03
		50	2.61

TABLE 6

Estimates of $d'_{C,2}$ for Each Forced-Choice Observer Under Each of the Ten Conditions.

Observer	d_0	Δd	$\hat{d}'_{C,2}$
AJ	50	10	1.87
		20	3.50
		30	4.64
		40	4.64
		50	---*
	100	10	.53
		20	1.54
		30	2.68
		40	3.50
		50	4.97
SB	50	10	1.14
		20	1.70
		30	2.63
		40	2.51
		50	3.63
	100	10	.48
		20	1.28
		30	1.64
		40	1.83
		50	2.16
PH	50	10	1.32
		20	2.26
		30	2.12
		40	2.80
		50	3.04
	100	10	.54
		20	.91
		30	1.35
		40	2.09
		50	1.87

*No estimate could be obtained for this condition because $P(A_1 | S_1) = 1.00$.

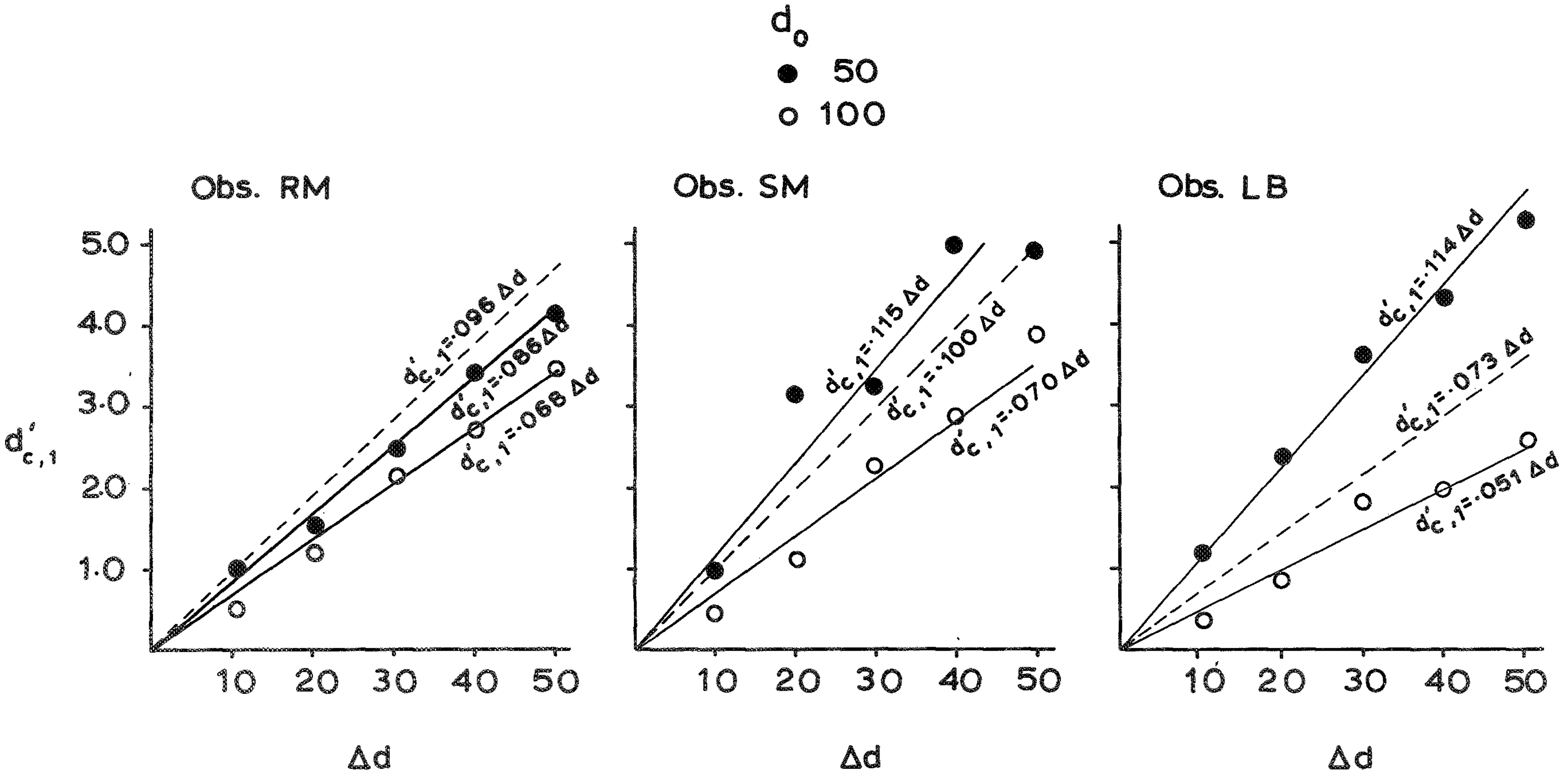


Fig.11: $d'_{c,1}$ as a function of Δd for each single stimulus observer.

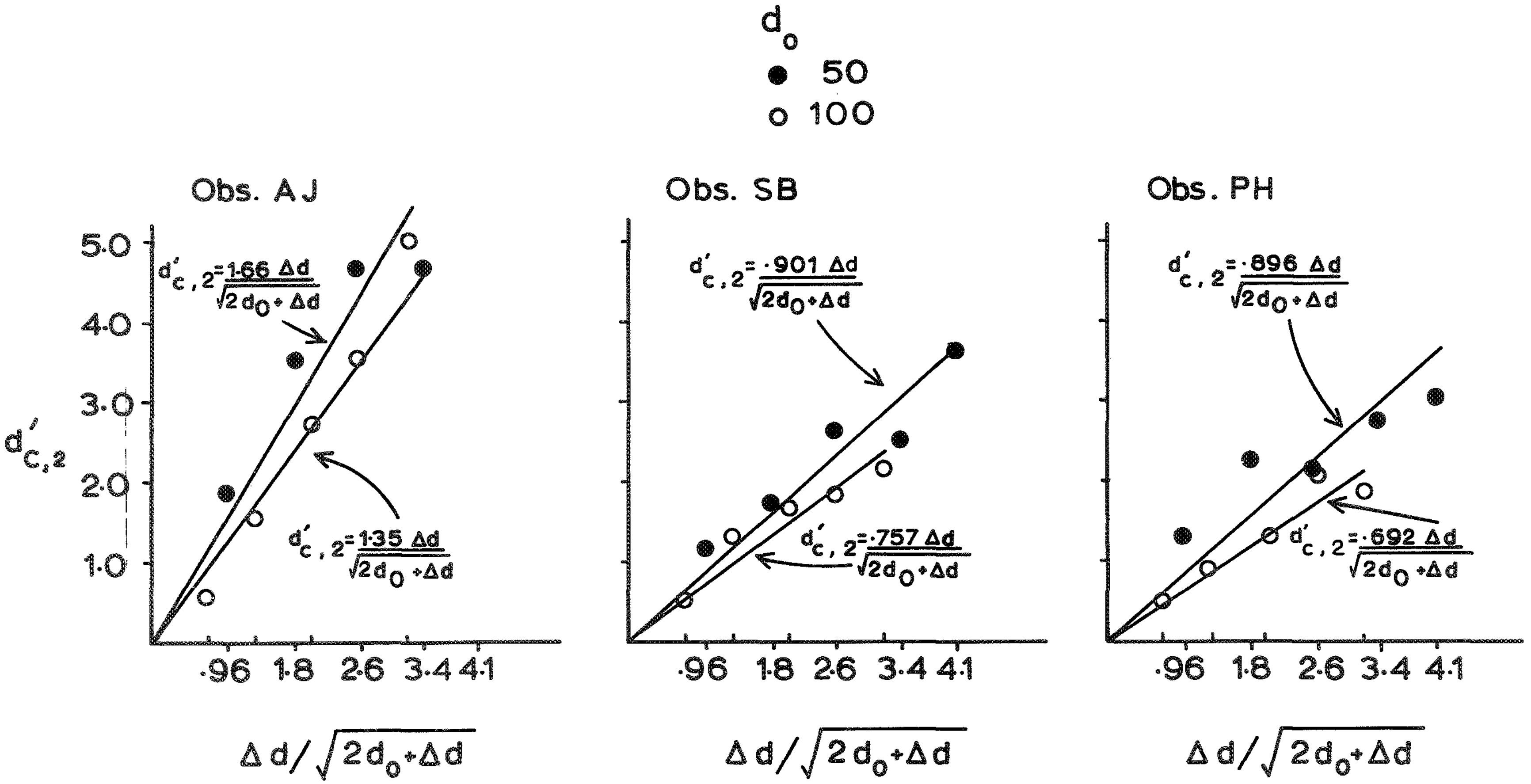


Fig. 12: $d'_{c,2}$ as a function of $\Delta d / \sqrt{2d_0 + \Delta d}$ for each forced-choice observer.

$$d'_{C,1} = \frac{\lambda^{\frac{1}{2}} \Delta d}{\sqrt{d_0 + \sigma_v^2}} .$$

Such a revision in the model may account for the performance of Observer RM by choosing an appropriate value of $\;_v2$; however, it is not possible to account for the performance of Observers SM or LB in this manner.

In the F - C case, it may be seen from Eq. 7 that the model predicts that $d'_{C,2}$ for $d_0 = 50$ should be superior to performance for $d_0 - 100$ by a factor of $\sqrt{200 + \Delta d/}\ \sqrt{100 + \Delta d}$ for each value of Δd. Thus, using obtained values of $d'_{C,2}$ for $d_0 = 100$, it is possible to predict values of $d'_{C,2}$ for $d_0 = 50$. The predicted and obtained values for each observer and each value of Δd are presented in Table 7. It is clear from the table that performance in all cases is better than predicted. The inclusion of an extra parameter as proposed in the S - S case could not account for the results in this case for any of the observers.

The Allan, et al. Model

Equations 10 and 12 specified that discriminability, d_q, is a zero-intercept, linear function of Δd for both the S - S and F - C cases. As in the Kristofferson quantal model, the model predicts constant q values for each observer. Furthermore, it is apparent from Equations 10 and 12 that the model predicts equal discriminability for $d_0 = 50$ and $d_0 - 100$ for a given Δd. Estimates of $d_{q,1}$ and $d_{q,2}$ and q

TABLE 7

Predicted and Obtained Values of $d'_{C,2}$ for $d_0 = 50$ for Each Forced-Choice Observer and Each Value of Δd.

Observer	Δd	Predicted $d'_{C,2}$	Obtained $d'_{C,2}$
AJ	10	.73	1.87
	20	2.09	3.50
	30	3.56	4.64
	40	4.58	4.64
	50	6.41	--
SB	10	.66	1.14
	20	1.73	1.70
	30	2.18	2.63
	40	2.40	2.51
	50	2.16	3.63
PH	10	.75	1.32
	20	1.23	2.26
	30	1.80	2.12
	40	2.74	2.80
	50	2.41	3.04

for each observer for each condition in the S - S and F - C cases respectively are presented in Tables 8 and 9. Figs. 13 and 14 show $d_{q,1}$ and $d_{q,2}$ in that order as a function of Δd. While there seems to be no systematic deviation from linearity, it is obvious that discriminability, d_q, is superior for $d_0 = 50$ than $d_0 = 100$. It may be seen from Tables 8 and 9 that q values shown systematic changes as Δd is increased. Specifically, q increases with Δd for all observers for $d_0 = 50$, and decreases as Δd increases for three of the six observers (Observers SM, LB and AJ) for $d_0 = 100$. The model in its present state cannot account for either the superior discriminability for $d_0 = 50$ or the systematic changes in q.

In the introduction, it was stated that the Allan, et al. model provided an adequate account of the data in an experimental situation much like the present one except that on-flashes were used rather than off-flashes. Yet for the data obtained in the present experiment, the Allan et al. model fails on two accounts to provide a reasonable explanation of the data. One reason why the Creelman model was rejected in the on-flash experiment was that it predicted better performance for $d_0 = 50$ than for $d_0 = 100$ and this was not found to be the case. Although the

Table 8

Estimates of $d_{q,1}$ and q for Each Single Stimulus Observer Under Each of the Ten Conditions

Observer	d_0	Δd	$\hat{d}_{q,1}$	$\hat{q}$
RM	50	10	.43	23.3
		20	.64	31.3
		30	.96	31.3
		40	1.24	32.3
		50	1.44	34.7
	100	10	.24	41.7
		20	.49	40.8
		30	.85	35.3
		40	1.05	38.2
		50	1.26	39.7
SM	50	10	.46	21.7
		20	1.22	16.4
		30	1.22	24.6
		40	1.64	24.4
		50	1.57	31.8
	100	10	.18	55.6
		20	.47	42.6
		30	.91	33.0
		40	1.13	35.4
		50	1.40	35.7
LB	50	10	.51	19.6
		20	.97	20.6
		30	1.34	22.4
		40	1.52	26.3
		50	1.72	29.1
	100	10	.14	71.4
		20	.34	58.8
		30	.75	40.0
		40	.80	50.0
		50	1.00	50.0

TABLE 9

Estimates of $d_{q,2}$ and q for Each Forced-Choice Observer Under Each of the Ten Conditions

Observer	d_0	Δd	$\hat{d}_{q,2}$	$\hat{q}$
AJ	50	10	1.11	18.0
		20	2.01	20.0
		30	2.56	23.4
		40	2.56	31.3
		50	--	
	100	10	.32	62.5
		20	.92	43.5
		30	1.58	38.0
		40	2.07	38.7
		50	2.73	36.7
SB	50	10	.68	29.4
		20	.99	40.4
		30	1.55	38.7
		40	1.50	53.3
		50	2.10	47.6
	100	10	.29	69.0
		20	.77	52.0
		30	.99	60.6
		40	1.09	73.4
		50	1.30	76.9
PH	50	10	.78	25.6
		20	1.01	39.6
		30	.99	60.6
		40	1.36	58.8
		50	1.88	53.3
	100	10	.33	60.6
		20	.55	72.7
		30	.82	73.2
		40	1.23	65.0
		50	1.07	93.5

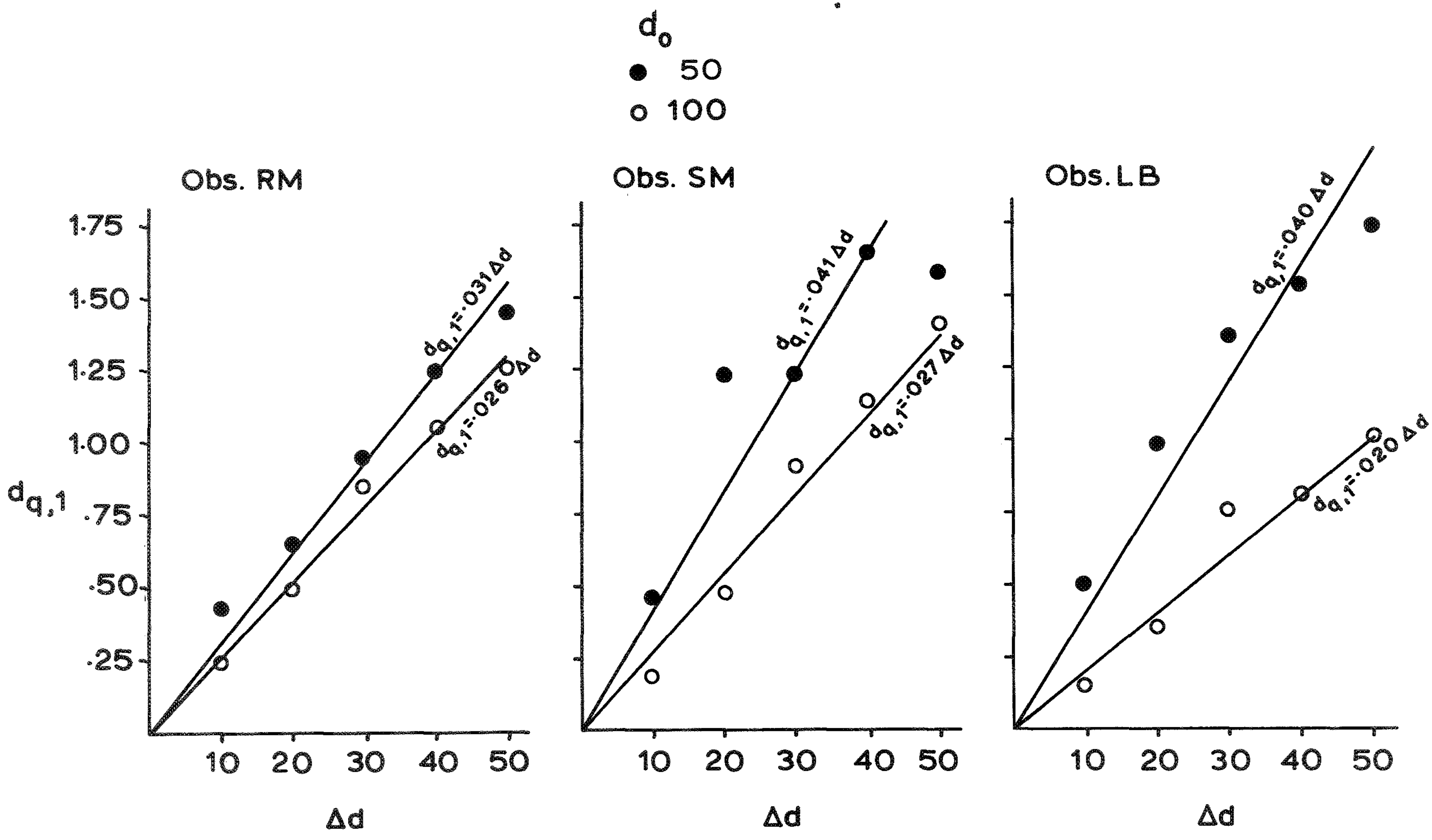


Fig.13: $d_{q,1}$ as a function of Δd for each single stimulus observer.

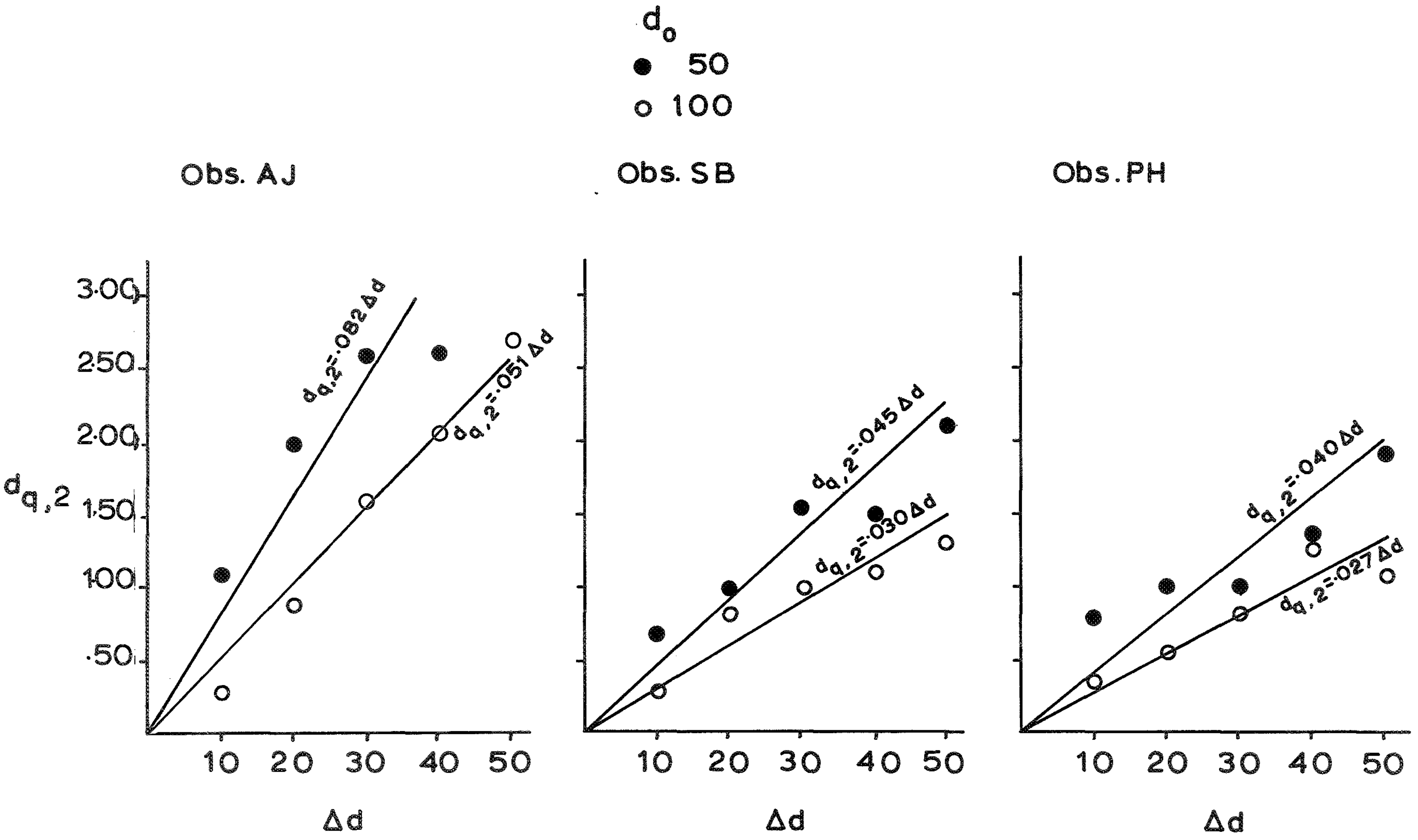


Fig.14: $d_{q,2}$ as a function of Δd for each forced-choice observer.

exact amount by which performance was found to be better for $d_0 = 50$ was not as predicted, discriminability was superior for $d_0 = 50$ for every every observer in the present experiment. Thus it appears that the observer handles these seemingly similar tasks in very different ways. One observer in the present experiment (Observer AJ), ran earlier in the on-flash experiment (Observer 4 in the Allan, et al. 1970 study). In that experiment, his ability to discriminate, $d'_{C,1}$, in an S - S task was almost identical for $d_0 = 50$ and $d_0 = 100$. (In fact, he did slightly better for $d_0 = 100$). Yet in the present experiment, his discriminability is clearly better for $d_0 = 50$. Thus for this observer at least, there is evidence that the tasks are dealt with differently in the two experiments.

The models presented thus far have been based on the assumption that the observer is basing his judgment on the temporal information available in the stimuli rather than some other cue such as total energy. Creelman, using auditory stimuli, compared the effect of signal voltage on both duration discrimination and amplitude discrimination, and stated that, "it seems reasonable to conclude that duration discrimination is not treated by human observers simply as a signal detection task, but that some other explanation is necessary [1962, p. 585]." In the Allan, et al. (1970) study using visual on-flashes as stimuli, an experiment was run to determine if changes in the luminance of the longer flash affected the observer's ability to discriminate a difference in duration, and it was found that it did not. They concluded,

> "in general it appears that when observers are asked to compare flashes of different durations, for durations within which Bloch's law has been shown to hold, their comparisons are made on the temporal information available in the two stimuli, and not on their apparent brightnesses [p. 19] ."

Yet in view of the fact that none of the models presented thus far, all of which assume the observer to be basing his judgment on temporal cues, can provide an adequate account of the data, it might prove worthwhile to investigate models which assume the observer to be using some other cue, and energy is an obvious possibility. Two models, both based on the assumption that the observer compares amounts of "residual energy" in the off-flash duration discrimination experiment, will be presented. In both models, a value of an external stimulus, in this case light, is assumed to be imposed upon a variable, normally distributed noise background, with a mean of zero and a variance of one. The light is assumed to build up internal excitation rapidly to an asymptotic level which is greater than its initial level by a constant amount, k. At the offset of the light (that is, the onset of a stimulus), the amount of excitation present at the moment of the offset is assumed to begin to decay to its initial level. The decay process continues until the light is restored.

The Linear Decay Model

The first model assumes that the decay process, triggered by the offset of the light, proceeds in a linear fashion at a constant rate, c. The residual excitation at the end of the stimulus duration, d_i, is thus a normally distributed random variable with mean $k - cd_i$ and a variance of one. Note that the larger the value of d_i, the <u>less</u> the expected value of the residual excitation. The discriminability measure, here denoted $d'_{L,1}$ for the S - S case, and $d'_{L,2}$ for the F - C case, is defined in the usual manner, as the distance between the means of the S_0 and S_1 distributions, expressed in terms of the standard deviation of the S_0 distribution. Thus,

$$d'_{L,1} = \frac{(k - cd_0) - (k - cd_1)}{(\mathrm{Var}\ S_0)^{\frac{1}{2}}} \cdot$$

$$= c\Delta d \quad . \tag{14}$$

An estimate of $d'_{L,1}$, denoted as $\hat{d}'_{L,1}$, may be obtained from the data in the following manner:

$$\hat{d}'_{L,1} = Z(A_1 \mid S_1) - Z(A_1 \mid S_0). \tag{15}$$

In the F - C case,

$$d'_{L,2} = \frac{c\Delta d - (-c\Delta d)}{\sqrt{\mathrm{Var}\ S_0 + \mathrm{Var}\ S_1}}$$

$$= \sqrt{2}\, c\Delta d. \tag{16}$$

and an estimate of $d'_{L,2}$, denoted as $\hat{d}'_{L,2}$ may be obtained from the data in the following manner:

$$\hat{d}'_{L,2} = Z(A_{10} \mid S_{10}) - Z(A_{10} \mid S_{01}). \tag{17}$$

Note that in equations 15 and 17, the subtraction is performed in the reverse order from the usual. This is due to the fact that the longer the stimulus, the more the decay and thus the less the residual excitation. Note however, that the discriminability measure which is obtained is equal to the absolute distance between the means of the distributions and is independent of the order of subtraction. Taking this fact into account, it may be observed from an examination of Eqs. 8 and 17 that the estimates of discriminability for the F - C case are the same for the Creelman and linear decay models. Also, examination of Eqs. 14 and 16 reveals that for both the S - S and the F - C cases, the linear decay model predicts that discriminability is a zero-intercept, linear function of Δd and is independent of the value of d_0.

In Table 10 are the estimates of $d'_{L,1}$ obtained from the data according to Eq. 15, and in Fig. 15 $d'_{L,1}$ is plotted as a function of Δd. The estimated discriminability measures for the F - C case, $d'_{L,2}$, are the same as the $d'_{C,2}$ values for the Creelman model shown in Table 6. These values are plotted as a function of Δd in Fig. 16. While the prediction of a linear relation between discriminability and Δd receives support from the data, it is clear that performance is superior for all six observers for d_0 - 50. The model cannot account for such a finding.

The Exponential Decay Model

The second energy decay model is similar in all respects to

Table 10

Estimates of $d'_{L,1}$ for Each Single Stimulus Observer Under Each of the Ten Conditions

Observer	d_0	Δd	$\hat{d}'_{L,1}$
RM	50	10	1.02
		20	1.48
		30	2.16
		40	2.87
		50	3.50
	100	10	.58
		20	1.12
		30	1.95
		40	2.46
		50	3.01
SM	50	10	1.08
		20	2.89
		30	2.89
		40	4.37
		50	3.93
	100	10	.43
		20	1.10
		30	2.14
		40	2.68
		50	3.43
LB	50	10	1.18
		20	2.22
		30	3.22
		40	3.76
		50	4.37
	100	10	.33
		20	.82
		30	1.74
		40	1.81
		50	2.35

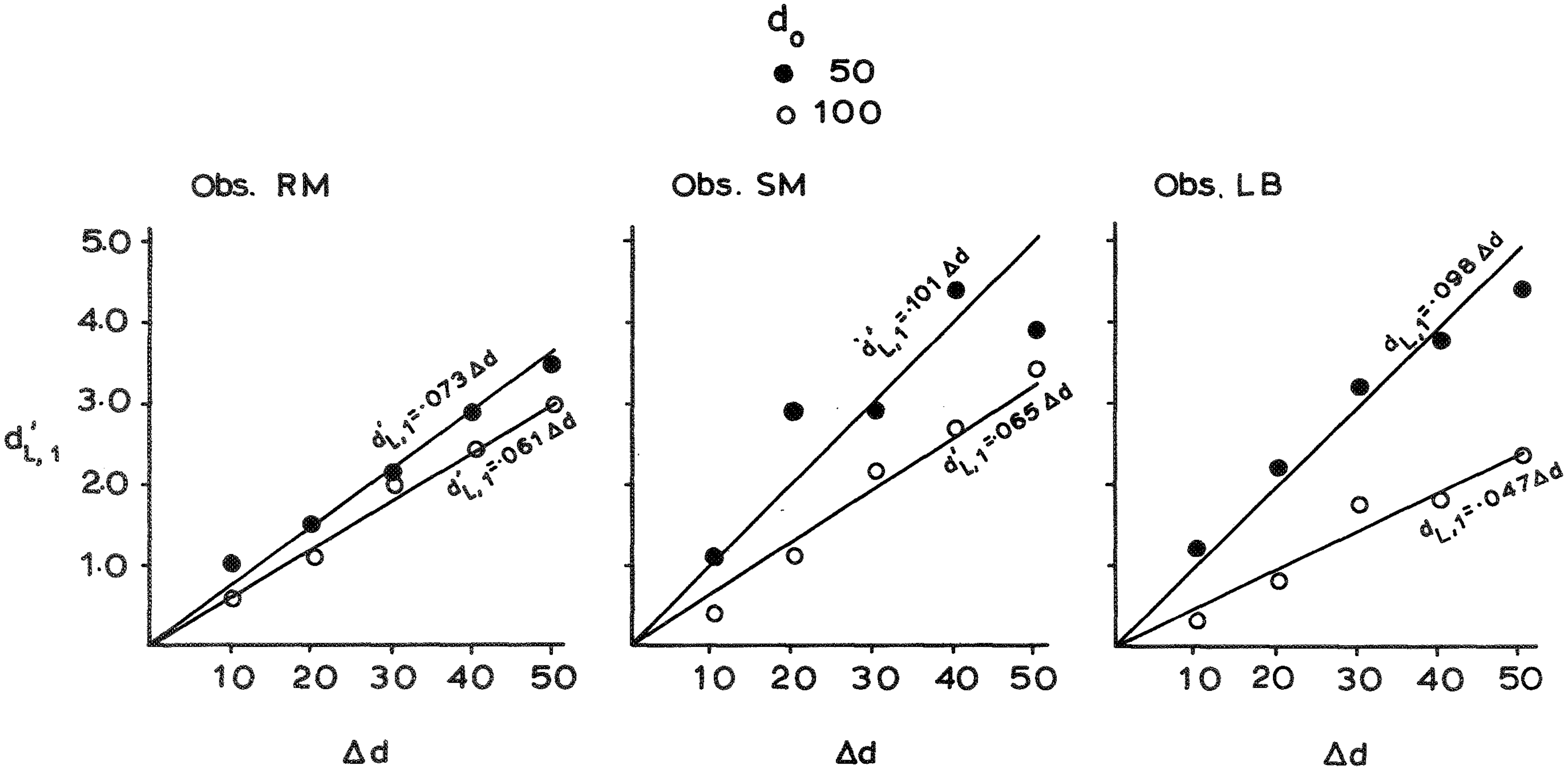


Fig.15: $d'_{L,1}$ as a function of Δd for each single stimulus observer.

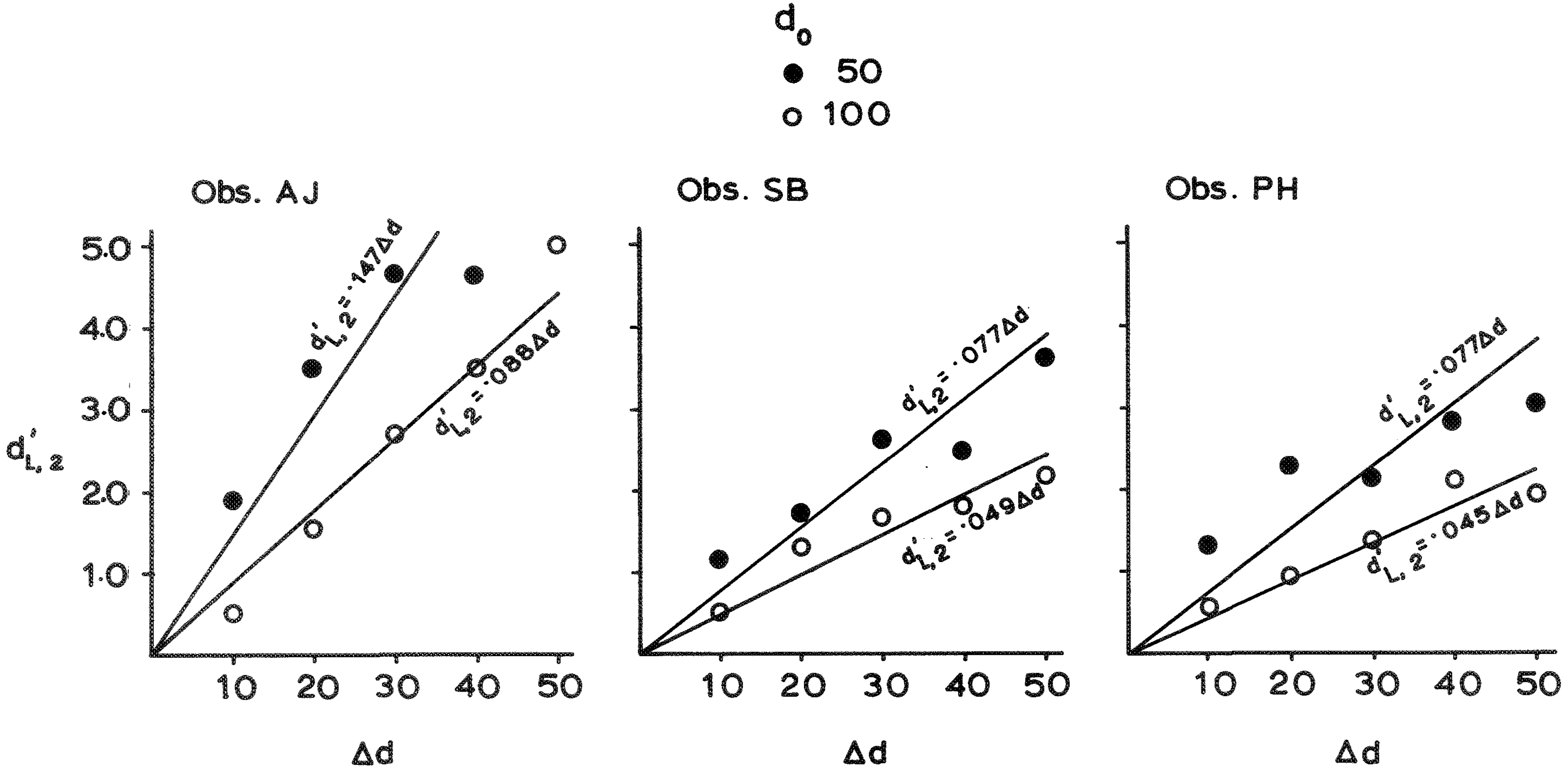


Fig. 16. $d'_{L,2}$ as a function of Δd for each forced-choice observer.

the linear decay model except that the decay process is assumed to take place in an exponential, rather than linear, fashion. In this model then, the residual excitation at the end of the stimulus duration d_i is a normally distributed random variable with mean $k(e^{-cd_i})$ and variance one, where c again refers to the rate of decay. The discriminability measure in the S - S case, here denoted $d'_{E,1}$, may be defined as follows:

$$\begin{aligned} d'_{E,1} &= k(e^{-cd_0} - e^{-cd_1}) \,/\, (\text{Var } S_0)^{\frac{1}{2}} \\ &= ke^{-cd_0}(1 - e^{c\Delta d}), \end{aligned} \tag{18}$$

and an estimate of $d'_{E,1}$, denoted as $\hat{d}'_{E,1}$, may be obtained from the data in the following manner:

$$\hat{d}'_{E,1} = Z(A_1 \mid S_1) - Z(A_1 \mid S_0). \tag{19}$$

Inspection of Eqs. 15 and 19 reveals that the estimates of discriminability for the S - S case of the exponential and linear decay models are the same.

In the F - C case, the discriminability measure, $d'_{E,2}$, may be defined as follows:

$$\begin{aligned} d'_{E,2} &= \frac{ke^{-cd_0}(1 - e^{-c\Delta d}) - \left[ke^{-cd_0}(1 - e^{-c\Delta d})\right]}{\sqrt{\text{Var } S_0 + \text{Var } S_1}} \\ &= 2ke^{-cd_0}(1 - e^{-c\Delta d}). \end{aligned} \tag{20}$$

The estimate of $d'_{E,2}$, denoted as $\hat{d}'_{E,2}$, obtained from the data,

$$\hat{d}'_{E,2} = Z(A_{10} \mid S_{10}) - Z(A_{10} \mid S_{01}), \tag{21}$$

may be seen to be identical to that obtained in the F - C case of the linear decay and Creelman models. If, in Eqs. 18 and 20 the natural logarithm is taken on both sides of each equation, the result is

$$\ln d'_{E,1} = \ln k + \ln(1 - e^{-c\Delta d}) - cd_0$$

for the S - S case, and

$$\ln d'_{E,2} = \ln \sqrt{2} + \ln k + \ln(1 - e^{-c\Delta d}) - cd_0$$

for the F - C case. Thus, in both cases, the model predicts that the function relating the logarithm of discriminability and d_0 is linear with slope -c. Furthermore, since c represents the rate of decay, it should be constant over all possible values of Δd.

The estimates of discriminability obtained from the data are the same as those obtained in the linear decay model. Thus, the estimates for the S - S case, $\hat{d}'_{E,1}$, are the same as those presented in Table 10, and the estimates for the F - C case, $\hat{d}'_{E,2}$, are the same as those presented in Table 6. Since only two values of d_0 were used, the prediction of a linear relation between $\ln d'_{E,1}$ or $\ln d'_{E,2}$ and d_0 cannot be tested directly. However, the function is assumed to have a slope of -c and c should be constant for each observer for all values of Δd. Values of c may be obtained for each value of Δd by determining the slope of the line relating ln $d'_{E,1}$ or ln $d'_{E,2}$ and d_0. These values are shown numerically in Tables 11 and graphically in Figs. 17 and 18 for the S - S and F - C cases respectively. While there is considerable variability, it is clear that c decreases with increasing Δd's. Thus the model is not supported by the data.

Table 11

Estimates of c for Every Observer for Each Value of Δd.

Observer	Δd	$\hat{c}$	Mean $\hat{c}$
RM	10	.011	.005
	20	.006	
	30	.002	
	40	.003	
	50	.005	
SM	10	.018	.011
	20	.019	
	30	.006	
2	40	.010	
	50	.003	
LB	10	.026	.017
	20	.020	
	30	.012	
	40	.015	
	50	.012	
AJ	10	.025	.014
	20	.016	
	30	.011	
	40	.006	
	50	--	
SB	10	.017	.010
	20	.006	
	30	.009	
	40	.006	
	50	.010	
PH	10	.018	.012
	20	.018	
	30	.009	
	40	.006	
	50	.010	

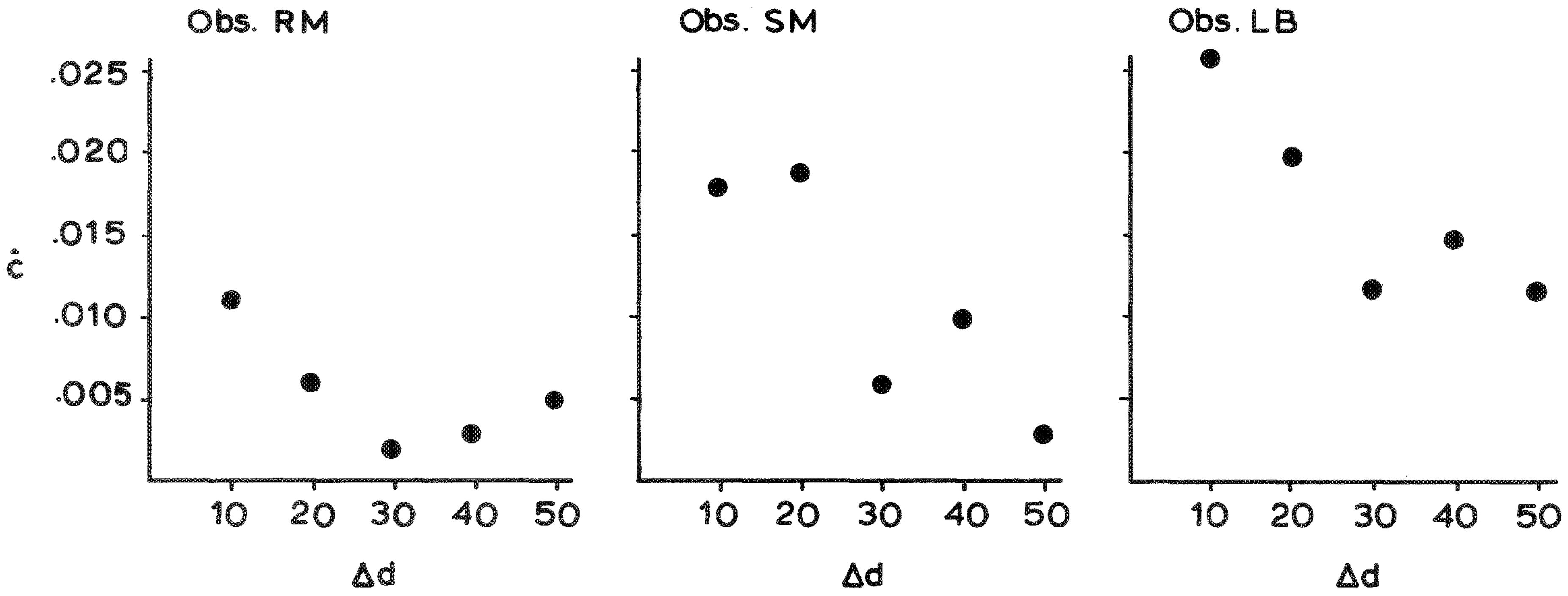


Fig. 17: $\hat{c}$ as a function of Δd for each single stimulus observer.

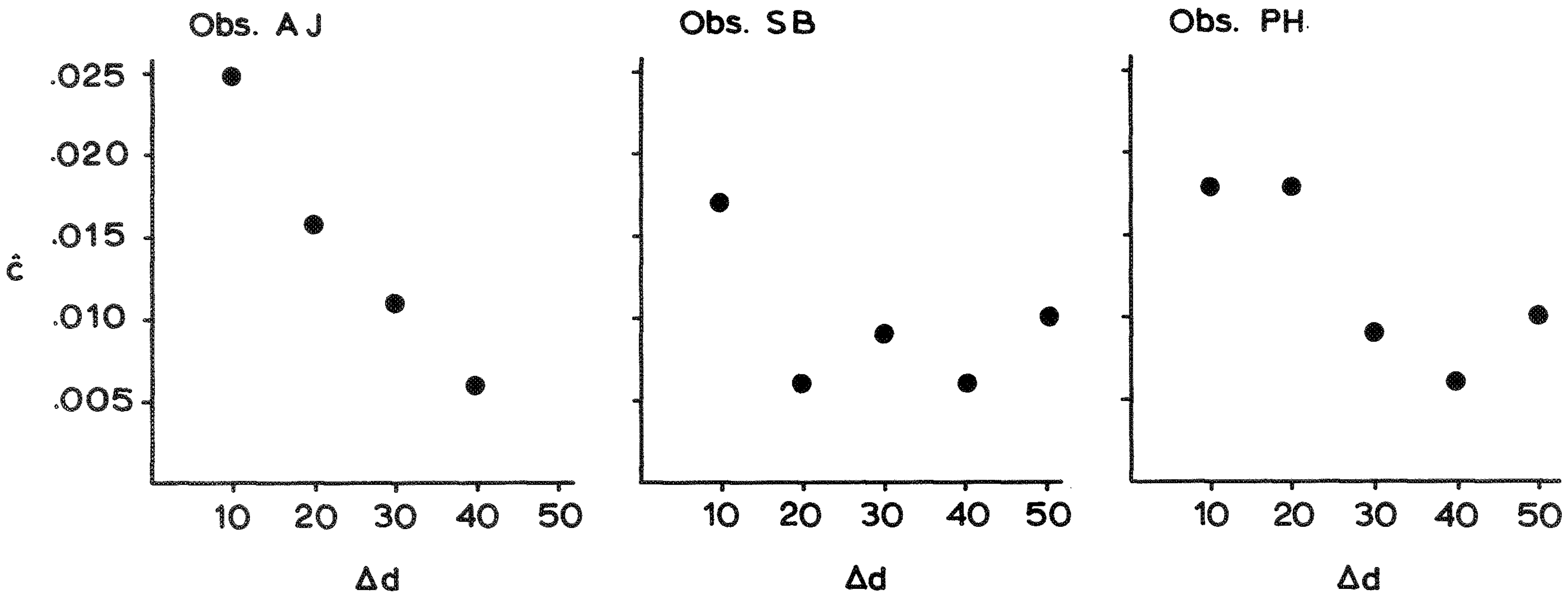


Fig. 18: $\hat{c}$ as a function of Δd for each forced-choice observer.

CONCLUSIONS

In summary, it appears that none of the models does a completely adequate job of accounting for the results of the present experiment, but that some models do better than others. The four models which assume either Gaussian or triangular distributions of sensory values do a more adequate job of accounting for the data than the quantal state model. The Creelman and the exponential decay models could account for the finding of superior performance for d_0 = 50 msec. over d_0 = 100 msec., although neither could predict the amount by which discriminability would improve by the doubling of d_0.

Since neither the models which assume the observer is using the temporal information in the stimuli nor the models which assume the observer is using energy as the cue upon which to base his decision provides an adequate interpretation of the data, there is no reason to accept or reject either hypothesis on the basis of the present experiment.

A comparison of the results of the present experiment with those of the Allan, et al. (1970) study reveals the difference between dark flashes and light flashes in terms of the observer's performance. While a temporal model works well in the latter case, it does not in the former, suggesting that whether a temporal model can be applied may depend upon the stimulus used. Hopefully future research will clarify the problem.

EXPERIMENT TWO

METHOD

Apparatus

The apparatus used in this experiment was the same as that used in Experiment 1.

Observers

Three observers from the previous experiment were run again in the present experiment. Observers AJ and SB had run in the F - C case of the first experiment; Observer SM had run in the S - S case.

Procedure

The procedure was the same as that used in the F - C case of the previous experiment, with a few modifications. Only two values of Δd, 10 and 30 were used, and d_0 was 50 msec. for all conditions. The interstimulus interval (ISI), which was kept constant at 500 msec. in Experiment 1, was varied in the present experiment. Four values, 500, 1000, 1500 and 2000 msec., were used. Thus there were eight conditions, four for each value of Δd. Since all observers had participated in the previous experiment, they were given only one practice session with a long (2000 msec.) ISI to acquaint them with the new task. In addition, Observer SM was given an extra session prior to this to familiarize her with the forced-choice procedure. All observers were then given two sessions on each experimental condition. They were run in random order with the restriction that each condition was run once before any condition was repeated. The instructions were the same as in the F - C case of the previous experiment except that the observers were told that the time between the two stimuli would vary from session to session.

RESULTS AND DISCUSSION

For each observer and each condition, P(c), $P(A_{10} | S_{10})$, and $P(A_{10} | S_{01})$ are shown in Table 12. In this experiment, as in the last, there is no evidence of a time order error. If there had been an effect too small to be observed for an ISI of 500, one might have expected it to have magnified for large values of ISI, but there is no evidence that that is the case. In Fig. 19, P(c) is shown as a function of ISI for each observer. The lines represent average performance in terms of P(c). The data very clearly indicate that performance does not change with increasing values of ISI. Such a finding is surprising; one would have expected a decrement in performance for larger values of ISI. Tanner (1956), using an amplitude discrimination task, and Kinchla and Allan (1969) using a task involving visual movement perception, found evidence that memory over the interstimulus interval was not perfect. Kinchla and Smyzer (1967) developed a mathematical model to account for decreased performance with increased ISI, and reported data from two experiments which supported their model. While they used visual position discrimination and auditory amplitude discrimination tasks, they presented the model as being applicable to any situation in which an observer compares two consecutively observed stimuli. The results of the present experiment indicate no need for a memory parameter in a duration discrimination task using visual stimuli. Creelman (1962) included a memory parameter

Table 12

Summary of Results for Experiment Two

Observer	Δd	ISI	P(c)	$P(A_{10} \mid S_{01})$	$P(A_{10} \mid S_{10})$
AJ	10	500	.797	.228	.817
		1000	.888	.194	.771
		1500	.812	.225	.844
		2000	.800	.192	.794
	30	500	.991	.013	.995
		1000	.983	.017	.993
		1500	.887	.020	.993
		2000	.984	.023	.990
SB	10	500	.694	.310	.698
		1000	.718	.267	.706
		1500	.645	.279	.561
		2000	.707	.197	.613
	30	500	.866	.202	.932
		1000	.919	.080	.917
		1500	.919	.062	.898
		2000	.914	.041	.890
SM	10	500	.765	.225	.755
		1000	.702	.324	.724
		1500	.765	.273	.798
		2000	.680	.401	.751
	30	500	.961	.041	.963
		1000	.910	.117	.935
		1500	.962	.042	.965
		2000	.941	.067	.948

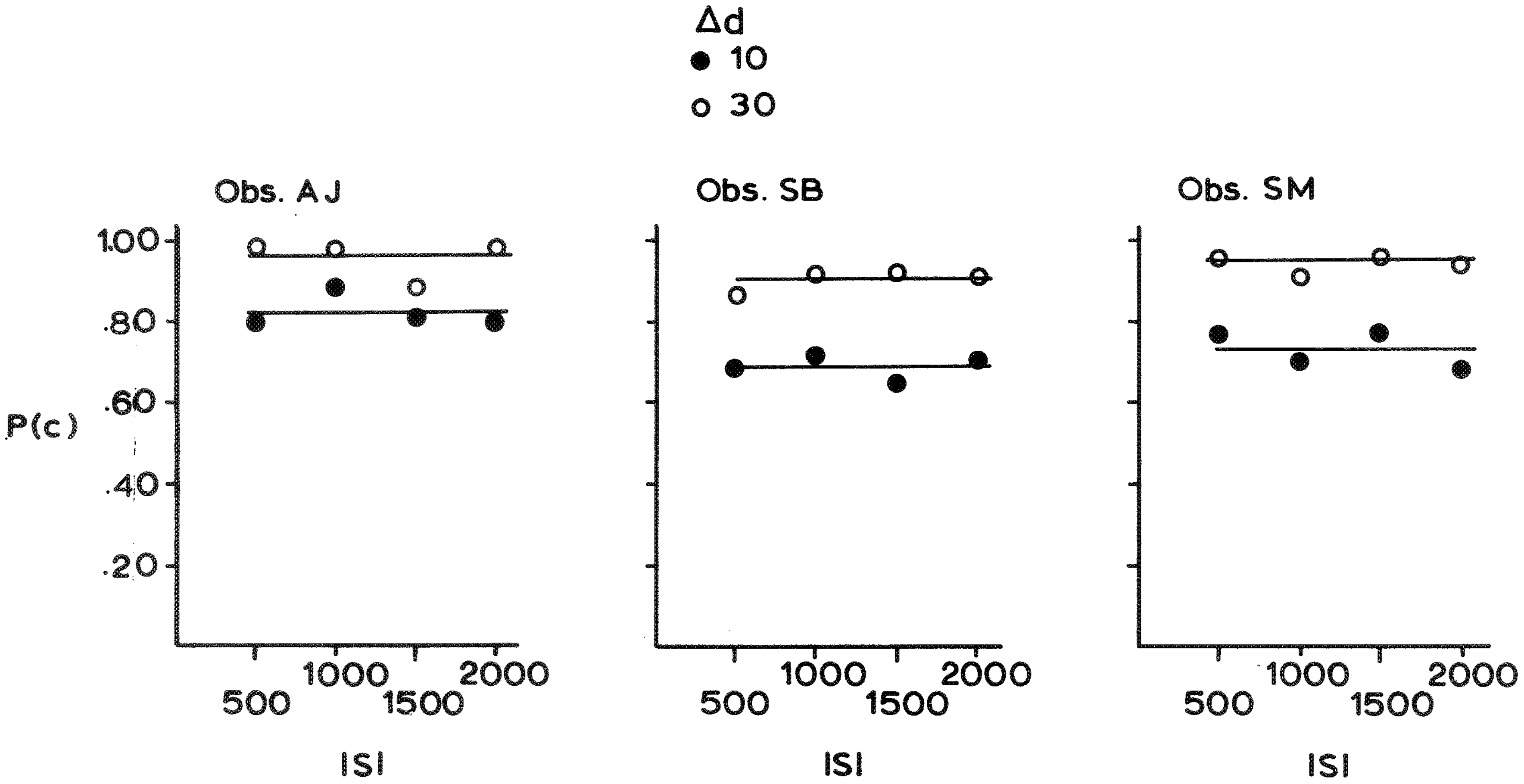


Fig.19: P(c) as a function of ISI in Experiment Two.

in his model of duration discrimination, and while he did not vary ISI to test the validity of including such a parameter, he obtains good fits to his auditory data by postulating a memory process.

CONCLUSIONS

The finding that performance does not change as a function of ISI may be interpreted in two ways. On the one hand, it is possible that the observer has perfect memory over the interstimulus interval. On the other hand, it may be that the observer ignores one stimulus completely and makes his decision as if he saw only one of the two signals. An experiment in which the same subjects ran in both a single stimulus and a forced-choice task would be useful in deciding between these two hypotheses.

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Footnotes

1. This research was supported by grants APA - 0112 and APA - 0175 from the National Research Council of Canada, and by grant NGR - 52-059-001 from the National Aeronautics and Space Administration.

2. In the original Creelman paper (1962), this formula is given as

$$d'_{C,2} = \frac{2\lambda^{\frac{1}{2}}\Delta d}{\sqrt{2d_0 + \Delta d}} .$$

Creelman (1970, personal communication), has since realized that this was an error and that the formula should read as in Eq. 7.

3. The derivation of the functions is presented in Appendix A.

APPENDIX A

The following section includes a derivation of the distribution of differences function for the forced-choice case of the Allan, et al. model.

In the S-S case,

$$g_{U'}(u') = \begin{cases} \dfrac{q - d_i + u'}{q^2} & \text{for } d_i - q < u' < d_i \\[2ex] \dfrac{q + d_i - u'}{q^2} & \text{for } d_i < u' < d_i + q \\[2ex] 0 & \text{elsewhere} \end{cases} \qquad (9)$$

where $U' = U_2 - U_1$ and U_1 and U_2 are independent random variables representing onset and offset times respectively, of an internal timing process, and d_i is the duration of the stimulus. Also,

$$E(U') = d_i$$

and

$$\text{Var }(U') = q^2/6.$$

In the F-C case, two stimuli are presented on each trial; one has a duration d_0 (an S_0 stimulus), and the other has a duration

$$d_1 = d_0 + \Delta d$$

(an S_1 stimulus). Let the psychological duration of S_0 be represented by

$$U'_0 = U_2 - U_1,$$

and let the psychological duration of S_1 be represented by

$$U'_1 = U_4 - U_3.$$

The observer in the F-C case is assumed to subtract the psychological duration of the second stimulus from that of the first. Thus, when presented with an S_{01} stimulus, the psychological difference in duration may be represented by a random variable U''_{01} where

$$\begin{aligned} U''_{01} &= U'_0 - U'_1 \\ &= (U_2 - U_1) - (U_4 - U_3). \end{aligned} \tag{22}$$

Also,

$$\begin{aligned} E(U''_{01}) &= E(U'_0) - E(U'_1) \\ &= d_0 - d_1 \\ &= -\Delta d , \end{aligned}$$

and

$$\begin{aligned} \text{Var } (U''_{01}) &= \text{Var } (U'_0) + \text{Var } (U'_1) \\ &= q^2/3 . \end{aligned}$$

When presented with an S_{10} stimulus, the psychological difference in duration may be represented by a random variable U''_{10} where

$$\begin{aligned} U''_{10} &= U'_1 - U'_0 \\ &= (U_4 - U_3) - (U_2 - U_1) . \end{aligned}$$

Also,

$$\begin{aligned} E(U''_{10}) &= E(U'_1) - E(U'_0) \\ &= d_1 - d_0 \\ &= \Delta d , \end{aligned}$$

and,

$$\begin{aligned} \text{Var } U''_{10} &= \text{Var } (U'_1) + \text{Var } (U'_0) \\ &= q^2/3 . \end{aligned}$$

The probability density functions of U''_{01} and U''_{10} may be obtained by the use of convolution integrals (see Freund, 1962). In order to simplify the following derivation, it will be assumed that

$$d_0 = d_1 = d_i.$$

Thus, the probability density functions of U''_{01} and U''_{10} are congruent, and only the derivation of the probability density function of U''_{01} (henceforth called U'') need be presented here.

From Eq. 22,

$$U'' = U' - U_4 + U_3, \tag{23}$$

where U' represents U'_0. The probability density function of U' is given in Eq. 9 and

$$f_{U_4}(u_4) = \begin{cases} 1/q \text{ for } d_i < u_4 < d_i + q \\ 0 \text{ elsewhere} \end{cases},$$

$$f_{U_3}(u_3) = \begin{cases} 1/q \text{ for } 0 < u_3 < q \\ 0 \text{ elsewhere} \end{cases},$$

where U', U_4 and U_3 are independent random variables. Let

$$Y = U' - U_4.$$

Then,

$$U_4 = U' - Y,$$

and

$$f_Y(y) = \int_{-\infty}^{\infty} f_{U'}(u') f_{U_4}(u' - u_4)\,du'$$

$$\text{for } -2q < y < q$$

Let $-2q < y < -q$. Then,

$$f_Y(y) = \int_{d_i - q}^{d_i + q + y} \left[\frac{q - d_i + u'}{q^2}\right] \left[\frac{1}{q}\right] du'$$

$$= 1/q^3 \; (2q^2 + 2qy + y^2/2).$$

Let $-q < y < 0$. Then,

$$f_Y(y) = \int_{d_i}^{d_i + q + y} \left[\frac{q + d_i - u'}{q^2}\right] \left[\frac{1}{q}\right] du' \quad +$$

$$\int_{d_i + y}^{d_i} \left[\frac{q - d_i + u'}{q^2}\right] \left[\frac{1}{q}\right] du'$$

$$= 1/q^3 \; (q^2/2 - qy - y^2).$$

Let $0 < y < q$. Then,

$$f_Y(y) = \int_{d_i + y}^{d_i + q} \left[\frac{q + d_i - u'}{q^2} \right] \frac{1}{q} du'$$

$$= 1/q^3 \ (q^2/2 - qy + y^2/2).$$

Thus,

$$f_Y(y) = \begin{cases} 1/q^3 \ (2q^2 + 2qy + y^2/2) & \text{for } -2q < y < -q \\ 1/q^3 \ (q^2/2 - qy - y^2) & \text{for } -q < y < 0 \\ 1/q^3 \ (q^2/2 - qy + y^2/2) & \text{for } 0 < y < q \\ 0 & \text{elsewhere .} \end{cases}$$

Now, $U'' = Y + U_3$. Thus, $U_3 = U'' - Y$

and

$$f_{U''}(u'') = \int_{-\infty}^{\infty} f_Y(y) f_{U_3}(u'' - y) dy$$

$$\text{for } -2q < u'' < 2q .$$

Let $-2q < u'' < -q$.

Then,

$$f_{U''}(u'') = \int_{-2q}^{u''} \left[\frac{1}{q^3} (2q^2 + 2qy + y^2/2) \quad (\frac{1}{q}) dy \right]$$

$$= \frac{(2q + u'')^3}{6q^4}.$$

Let $-q < u'' < 0$.

Then,

$$f_{U''}(u'') = \int_{u''-q}^{-q} \left[\frac{1}{q^3}(2q^2 + 2qy + y^2/2)\right] (\tfrac{1}{q})\,dy \; +$$

$$\int_{-q}^{u''} \left[\frac{1}{q^3}(q^2/2 - qy - y^2)\right] (\tfrac{1}{q})\,dy$$

$$= \frac{4q^3 - 6q(u'')^2 - 3(u'')^3}{6q^4}.$$

Let $0 < u'' < q$.

Then,

$$f_{U''}(u'') = \int_{u''-q}^{0} \left[\frac{1}{q^3}(q^2/2 - qy - y^2)\right] (\tfrac{1}{q})\,dy \; +$$

$$\int_{0}^{u''} \left[\frac{1}{q^3}(q^2/2 + y^2/2 - qv)\right] (\tfrac{1}{q})\,dy$$

$$= \frac{4q^3 - 6q(u'')^2 + 3(u'')^3}{6q^4}.$$

Let $q < u'' < 2q$

Then,

$$f_{U''}(u'') = \int_{u''-q}^{q} \left[\frac{1}{q^3}(q^2/2 - qy + y^2/2)\right] (\frac{1}{q})dy$$

$$= \frac{(2q - u'')^3}{6q^4} .$$

Thus,

$$f_{U''}(u'') = \begin{cases} \dfrac{(2q + u'')^3}{6q^4} & \text{for } -2q < u'' < -q \\[2ex] \dfrac{4q^3 - 6q(u'')^2 - 3(u'')^3}{6q^4} & \text{for } -q < u'' < 0 \\[2ex] \dfrac{4q^3 - 6q(u'')^2 + 3(u'')^3}{6q^4} & \text{for } 0 < u'' < q \\[2ex] \dfrac{(2q - u'')^3}{6q^4} & \text{for } q < 0 < 2q \\[2ex] 0 \quad \text{elsewhere .} \end{cases} \qquad (24)$$

For $d_o \neq d_1$, the derivation must be performed separately for an S_{01} and S_{10} stimulus. However, the reader may verify that the functions have the same shape as that described in Expression 24; the exact formulations are given in the introduction.