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MINIMUM WEIGHT DESIGN OF AXIALLY COMPRESSED RING AND STRINGER STIFFENED CYLINDRICAL SHELLS

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16. Abstract

Results of an analytical study to determine desirable stiffener parameters and proportions and design methodology of axially compressed stiffened isotropic cylinders are presented. The investigation examines the panel and general instability of the stiffened shell configuration and from this determines desirable stiffener parameters and proportions. For this analysis classical buckling equations are used which retain important effects of the stiffeners. The study indicates that for both rings and stringers, T-section stiffeners are preferable, and that the stringers should have a stringer area to shell wall area (stringer area parameter) of approximately 0.56 for stringers outside and 0.44 for stringers inside; and that the rings should have a ring area to shell wall area (ring area parameter) of approximately 0.07 for rings inside, stringers outside, and 0.12 for rings inside, stringers outside. Using the desirable stringer and ring parameters, a method for obtaining the final design of the stiffener proportions and the shell thickness is presented by applying the equations governing the shell failure and instability modes. Results of calculations are presented illustrating the design of a stiffened cylinder under an axial compression loading and which show the critical parameters in such a design.

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FOREWORD

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INTRODUCTION

In order to maintain structural integrity of a great variety of vehicles, shell structures which are stiffened with rings (circumferential stiffeners) and stringers (longitudinal stiffeners) are required. The design of these stiffened shells is normally accomplished by judiciously selecting shell material and thickness and stiffener materials and shapes and then calculating the buckling strength for the selected configuration. If the calculated strength is greater than the actual loads the shell will experience, that is, the selected configuration meets its design or mission requirements, then this shell configuration may be used as a possible configuration. This procedure is then repeated for various stiffener shapes and sizes and shell materials, if a minimum weight or optimum structural efficiency is desired. This approach requires that the designer be experienced in choosing the various shapes, sizes, and materials. Thus, the designer of a stiffened shell needs to have a means of determining or to know desirable structural proportions and a procedure of design so that proper selection of stiffening elements and structural materials can be made.

Therefore, the purpose of the present investigation is to determine analytically the desirable design parameters which give a minimum weight configuration for an axially compressed stiffened isotropic cylindrical shell. In addition, the design methodology of the stiffened shell is presented.

A minimum weight investigation may be conducted by using either of two methods of approach. One of the methods of approach is structural synthesis as presented in Reference 1. In using this method, a set of initial design variables is selected and then these design variables are varied until a minimum weight configuration is achieved. This method of minimum weight design, an unconstrained minimization problem, requires a large computer program solution and requires numerical specification of the design variables. Thus, it does not give general results for desirable parameters. However, it does have a distinct advantage whenever more than one loading condition must be considered in the design, since it does not assume simultaneous failure modes.

The other method of approach, the more conventional method (References 2 and 3), and the one employed herein, examines the instability modes of the stiffened shell configuration, and determines minimum weight by equating failure and instability modes. Using this method of approach, the present paper examines the general and panel instability of a stiffened shell and from this analysis determines desirable ring and stringer stiffener parameters and proportions. For this parametric analysis, the classical buckling equations of Reference 4 are used. These equations retain the important effects of the stiffener, such as eccentricity, bending, extensional, and twisting stiffnesses, but allow for simple calculation of the buckling load. Once desirable stiffener parameters are determined, the final design of the stiffener proportions and shell thickness is obtained by applying the equations governing the shell failure and instability modes (buckling and yielding of shell and stiffeners).

SYMBOLS

A	cross-sectional area of stiffener
D	bending stiffness of isotropic plate
E	Young's modulus
F_s	shape factor
G	shear modulus
$\bar{G}_r = \frac{G_r J_r}{D l}$	ring torsional parameter
$\bar{G}_s = \frac{G_s J_s}{D d}$	stringer torsional parameter
I	moment of inertia of stiffener about its centroid
J	torsional constant for stiffener
K	coefficient in plate buckling equation
$K_x = \frac{N_x L^2}{\pi^2 D}$	axial buckling parameter
L	length of cylinder
N_x	axial buckling load
N_{x_A}	axial load applied
R	Radius of cylinder to skin middle surface

$\bar{R} = \frac{E_r A_r}{E t l}$	ring area parameter
$\bar{S} = \frac{E_s A_s}{E t d}$	stringer area parameter
W	total weight of stiffened shell configuration
$Z = \frac{L^2 \sqrt{1-\mu^2}}{R t}$	curvature parameter
c	one-half the total flange width of stiffener
d	stringer spacing
l	ring spacing
h	height of stiffener web
k	ratio of area of flange material in bottom flange of stiffener to total flange area of stiffener
m	number of half waves in cylinder buckle pattern in longitudinal direction
n	number of full waves in cylinder buckle pattern in circumferential direction
p	external pressure
t	thickness of cylinder shell wall or stiffener flange or web
$\bar{t} = t(1 + \frac{\rho_s}{\rho} \bar{S} + \frac{\rho_r}{\rho} \bar{R})$	effective thickness parameter
$\bar{y}$	distance from centroid of stiffener to attachment between stiffener and shell wall
z	distance from centroid of stiffener to middle surface of shell, positive for stiffener on external surface

$\beta = \frac{nL}{m\pi R}$ buckle aspect ratio

γ ratio of material in flanges of stiffener to total material in stiffener, A_f/A

ε prebuckling strain

μ Poisson's ratio

ρ weight density

σ prebuckling stress

σ_Y yield stress

Subscripts:

s,r denote properties of stringers (longitudinal stiffening) and rings (circumferential stiffening), respectively

x,y longitudinal and circumferential directions, respectively

f,w denote the flange and web of stiffener (stringers and rings), respectively

THEORY

The structural efficiency investigation of a stiffened isotropic cylindrical shell is conducted by using a method of approach which examines the general and panel instability of a stiffened cylindrical shell configuration and from this determines desirable ring and stringer parameters and proportions. General instability is defined as the buckling mode in which the rings deform radially, and panel instability is defined as the buckling mode in which the rings have no radial deformation. For the structural efficiency study, classical buckling equations are used. This type of buckling analysis is adequate for predicting trends and desirable proportions and parameters; however, this analysis is only considered fair for predicting the buckling load and may not be used in the design analysis. The choice of buckling equations is discussed in more detail in the Design Analysis Section of the paper.

The isotropic cylinder wall and stiffeners (either rings or stringers) studied are illustrated in Figure 1. By representing the stiffeners as an unsymmetric I section, the stiffener is representative of a large number of cross-sectional stiffener shapes; for example, T section, I section, hat section or modified hat section. All of the stiffener sections may also have tightening holes in the web. By using the stiffener cross-section of Figure 1, the stiffeners may be characterized in the buckling equations by five parameters:

1. The ratio of stiffener modulus and area to shell modulus and thickness and stiffener spacing, either $\bar{S}$ or $\bar{R}$.
2. The ratio of the stiffener torsional stiffness to shell bending stiffness and stiffener spacing, either $\bar{G}_s$ or $\bar{G}_r$.
3. The ratio of the height of the stiffener to thickness of shell, either h_s/t or h_r/t.
4. The ratio of amount of stiffener area in flange to total area of stiffener, either γ_s or γ_r.
5. The ratio of flange area in bottom flange of stiffener to total flange area of stiffener, either k_s or k_r.

Representing the stiffeners in the above manner is similar to the representation of the rings presented in Reference 2.

The governing equation for panel and general instability of an axially compressed ring and stringer stiffened isotropic cylinder is given by equation (17) of Reference 4. With the use of the stiffener representation of Figure 1, the reference equation may be written as:

$$K_x = m^2 \left[(1 + \beta^2)^2 + \frac{E_s I_s}{Dd} + \beta^2 (\bar{G}_r + \bar{G}_s) + \beta^4 \frac{E_r I_r}{Dl} \right] + \frac{12Z^2}{m^2 \pi^4} \left[\frac{1 + \bar{S}\Lambda_s + \bar{R}\Lambda_r + \bar{S}\bar{R}\Lambda_{rs}}{\Lambda} \right] \qquad (1)$$

where

$$\Lambda = (1 + \beta^2)^2 + 2\beta^2(1 + \mu)(\bar{R} + \bar{S}) + (1 - \mu^2)\left[\bar{S} + 2\beta^2\bar{R}\bar{S}(1 + \mu) + \beta^4\bar{R}\right]$$

$$\Lambda_s = 1 + 2m^2\pi^2 \frac{h_s}{t} \frac{z_s}{h_s} (\beta^2 - \mu) \sqrt{\frac{1 - \mu^2}{Z}} + m^4\pi^4\left(\frac{h_s}{t}\right)^2\left(\frac{z_s}{h_s}\right)^2 (1 + \beta^2)^2 \frac{(1 - \mu^2)}{Z^2}$$

$$\Lambda_r = 1 + 2m^2\pi^2 \frac{h_r}{t} \frac{z_r}{h_r} \beta^2 (1 - \beta^2\mu) \frac{\sqrt{1-\mu^2}}{Z}$$

$$+ m^4\pi^4 \left(\frac{h_r}{t}\right)^2 \left(\frac{z_r}{h_r}\right)^2 \beta^4(1 + \beta^2)^2 \frac{(1 - \mu^2)}{Z^2}$$

$$\Lambda_{rs} = 1 - \mu^2 + 2m^2\pi^2\beta^2 \frac{(1 - \mu^2)^{3/2}}{Z} \left(\frac{h_s}{t} \frac{z_s}{h_s} + \frac{h_r}{t} \frac{z_r}{h_r}\right)$$

$$+ m^4\pi^4 \frac{(1 - \mu^2)}{Z^2} \beta^2 \left\{\beta^2 \left[1 - \mu^2 + 2\beta^2(1 + \mu)\right] \left(\frac{h_r}{t}\right)^2 \left(\frac{z_r}{h_r}\right)^2\right.$$

$$+ 2\beta^2 (1+ \mu)^2 \frac{h_r}{t} \frac{z_r}{h_r} \frac{h_s}{t} \frac{z_s}{h_s}$$

$$\left.+\left[2(1 + \mu) + \beta^2 (1 - \mu^2)\right] \left(\frac{h_s}{t}\right)^2 \left(\frac{z_s}{h_s}\right)^2\right\}$$

$$K_x = \frac{N_x L^2}{\pi^2 D}$$

$$Z = \frac{L^2\sqrt{1 - \mu^2}}{Rt}$$

$$\overline{S} = \frac{E_s A_s}{Etd}$$

$$\overline{R} = \frac{E_r A_r}{Et\ell}$$

$$\overline{G}_s = \frac{G_s J_s}{Dd}$$

$$\bar{G}_r = \frac{G_r J_r}{Dl}$$

$$\frac{\bar{y}}{h} = \gamma(1 - k) + \frac{1}{2} (1 - \gamma)$$

$$\frac{z}{h} = \pm \left(\frac{\bar{y}}{h} + \frac{1}{2}\frac{t}{h}\right)$$

$$\frac{I}{Ah^2} = \gamma \left[(1 - k) \left(1 - \frac{\bar{y}}{h}\right)^2 + k \left(\frac{\bar{y}}{h}\right)^2\right] + (1 - \gamma) \left[\frac{1}{3} - \frac{\bar{y}}{h} + \left(\frac{\bar{y}}{h}\right)^2\right]$$

$$\frac{E_s I_s}{Dd} = 12(1 - \mu^2) \left(\frac{h_s}{t}\right)^2 \bar{S} \left(\frac{I_s}{A_s h_s^2}\right)$$

$$\frac{E_r I_r}{Dl} = 12(1 - \mu^2) \left(\frac{h_r}{t}\right)^2 \bar{R} \left(\frac{I_r}{A_r h_r^2}\right)$$

Equation (1) above assumes classical simple support boundary conditions and assumes that the stiffeners are "smeared out" over the stiffener spacing and equally spaced. The buckling load from equation (1) is given in terms of the buckling parameter K_x. The minimum buckling load parameter K_x is determined by minimizing the equation on the computer by varying the number of the axial waves m and the buckle aspect ratio β until a minimum is determined. The plus sign in the z/h term applies to external rings or stringers; the negative sign to internal rings or stringers.

Equation (1) may be reduced by minimizing it with respect to the number of circumferential waves n. This procedure results in a buckling equation which must be minimized with respect to the buckle aspect ratio β and when written in terms of weight parameters reduces the following equation.

$$\frac{\bar{t}}{R} = \sqrt{\frac{\frac{N_x}{ER}}{F_s}} \tag{2}$$

where

$$\bar{t} = t\left(1 + \frac{\rho_s}{\rho}\bar{S} + \frac{\rho_r}{\rho}\bar{R}\right)$$

$$F_s = \frac{2\left[\sqrt{A_4(A_1A_2 + A_3^2A_6)} + A_2A_5\right]}{A_2\sqrt{12(1-\mu^2)}\left(1 + \frac{\rho_s}{\rho}\bar{S} + \frac{\rho_r}{\rho}\bar{R}\right)^2}$$

$$A_1 = (1+\beta^2)^2 + \frac{E_sI_s}{Dd} + \beta^4\frac{E_rI_r}{Dl} + \beta^2(\bar{G}_r + \bar{G}_s)$$

$$A_2 = (1+\beta^2)^2 + 2\beta^2(1+\mu)(\bar{R}+\bar{S}) + (1-\mu^2)\left[\bar{S} + 2\beta^2\bar{R}\bar{S}(1+\mu) + \beta^4\bar{R}\right]$$

$$A_3 = \beta^2\sqrt{12(1-\mu^2)}$$

$$A_4 = 1 + \bar{S} + \bar{R} + (1-\mu^2)\bar{R}\bar{S}$$

$$A_5 = \bar{S}\left[\frac{1}{\beta^2}\frac{h_s}{t}(\beta^2-\mu^2)\frac{z_s}{h_s}\right] + \bar{R}\left[\frac{h_r}{t}(1-\beta^2\mu)\frac{z_r}{h_r}\right]$$

$$+ \bar{R}\bar{S}\left\{(1-\mu^2)\left[\frac{h_s}{t}\frac{z_s}{h_s} + \frac{h_r}{t}\frac{z_r}{h_r}\right]\right\}$$

$$A_6 = \overline{S}\left[\frac{(1+\beta^2)^2}{\beta^4}\left(\frac{h_s}{t}\right)^2\left(\frac{z_s}{h_s}\right)^2\right] + \overline{R}\left[(1+\beta^2)^2\left(\frac{h_r}{t}\right)^2\left(\frac{z_r}{h_r}\right)^2\right]$$

$$+ \overline{R}\overline{S}\left\{\left[1 - \mu^2 + 2\beta^2 (1+\mu)\right]\left(\frac{h_r}{t}\right)^2\left(\frac{z_r}{h_r}\right)^2\right.$$

$$+ 2(1+\mu)^2 \frac{h_r}{t}\frac{z_r}{h_r}\frac{h_s}{t}\frac{z_s}{h_s}$$

$$\left. + \left[\frac{2}{\beta^2}(1+\mu) + (1-\mu^2)\right]\left(\frac{h_s}{t}\right)^2\left(\frac{z_s}{h_s}\right)^2\right\}$$

From equation (2) it can be seen that a maximum value of F_s will give minimum weight. Therefore, F_s can be considered a shape factor for the problem, i.e., for panel and general instability buckling modes. In a given design, the shape factor F_s is made as large as possible for a given loading N_x/ER by varying the cylindrical wall thickness and the stiffener geometry. Note that F_s depends upon β; thus, the minimization of the buckling load with respect to β must be performed. For this reason, the computer program which was used determined a minimum K_x for a given set of stiffener parameters; $\overline{G}$, $\overline{R}$ or $\overline{S}$, γ, k, h/t; and curvature parameter Z. From the minimum K_x, the F_s value was calculated using the following equation.

$$F_s = \frac{K_x \pi^2}{12\sqrt{1-\mu^2}\, Z\left(1 + \frac{\rho_s}{\rho}\overline{S} + \frac{\rho_r}{\rho}\overline{R}\right)^2} \tag{3}$$

Equation (3) is obtained by rewriting equation (2) in terms of the parameters and Z.

Note the variables used in the computation of the shape factor are $\overline{G}_s$, $\overline{G}_r$, $\overline{S}$, $\overline{R}$, γ_r, γ_s, k_r, k_s, h_s/t, h_r/t and Z. The variables γ_r, γ_s, k_r and k_s are associated with the shape of the stiffeners; $\overline{G}_s$, $\overline{G}_r$, $\overline{S}$, $\overline{R}$, h_s/t, and h_r/t with the ratios of stiffener properties to cylinder wall properties; and Z with the cylinder geometry. Each of these parameters is discussed in the next section where results of computations for desirable stiffener proportions and parameters are presented.

PARAMETRIC STUDY RESULTS

In this section, results are presented of parametric studies to determine desirable stiffener (ring and stringer) proportions and parameters. These parametric studies were accomplished by making numerous computer computations. The computations were made by fixing the value of Z, the cylinder curvature parameter, the location of the stiffeners, either inside or outside, and making the stiffeners torsional constant zero.

The effect of the stiffener torsional constant $\overline{G}$ on the parametric study results was checked and found to be negligible (see Table 1). However, the effect of the torsional constant should be included when calculating buckling loads for the cylinder design analysis. With the above four parameters fixed, the other eight stiffener parameters, $\overline{S}$, $\overline{R}$, γ_s, γ_r, h_s/t, h_r/t, k_s, and k_r, were varied.

DESIRABLE STRINGER PROPORTIONS AND PARAMETERS

The results of the parametric studies for stringer stiffened cylinders are presented in Figures 2 through 5 and Table 1. Note that the results for stringer stiffened cylinders also apply to panel instability modes of buckling. The results shown on Figures 2 through 5 are for stringers outside.

Figures 2 and 3 show the variation of F_s, the shape factor, with respect to the stringer parameter k_s, the amount of bottom flange area, for selected values of $\overline{S}$, γ_s, and h_s/t, and for a cylinder curvature parameter Z of 10 and 1,000, respectively.

For a cylinder with constant radius and thickness, a curvature parameter Z value of 10 corresponds to a short-length cylinder, while a value of Z of 1,000 corresponds to a long-length cylinder. Figures 2 and 3 show that the maximum shape factor, i.e., maximum structural efficiency, occurs for a value of k_s equal to zero. The k_s value of zero implies T-shaped stringers are most efficient. Note also in the figures the variation of F_s with respect to γ_s and h_s/t.

Figures 4 and 5 show the variation of the shape factor with respect to the stringer area parameter $\overline{S}$ for selected values of other parameters and for Z values of 10 and 1,000, respectively. Figures 4 and 5 show that the maximum F_s occurs for $\overline{S}$ value of 0.2 to 0.6.

The shape of the curves shown in Figures 2 through 5 have been typical of all cases run, including the cases for inside stringers. Note also from Figures 2 through 5 that the maximum shape factor occurs at the largest values of h_s/t and γ_s.

Presented in Table 1 is a tabulation of the results for stringer stiffened cylinders. Table 1 shows for each Z value, and various h_s/t and γ_s values, the stiffener area parameter $\overline{S}$ which gives the maximum value of the shape factor F_s. The $\overline{S}$ values of Table 1 are for stringer torsional parameters $\overline{G}_s$ of 0.and 0.5, thus illustrating why $\overline{G}$ may be neglected in the parametric studies. The $\overline{S}$ values of Table 1 vary as a function of Z, h_s/t, γ_s, and stringer location, and are in the same range with similar values presented in Reference 5.

For design it would be desirable to have a single $\bar{S}$ value. This may be accomplished by determining the mean of the data of Table 1. Considering the $\bar{S}$ values from Z equal 1 up to and including 1,000, the mean of the $\bar{S}$ values is 0.56 for stringers outside and 0.44 for stringers inside with standard deviations of 0.14 and 0.19 for stringers outside and inside, respectively.

DESIRABLE RING PROPORTIONS AND PARAMETERS

The results of the parametric studies for ring stiffeners are presented in Figures 6 through 9 and in Tables 2 through 10. The ring results in Figures 6 through 9 are for selected stringer parameters and for Z values of 500 and 10,000. Due to practical considerations, only the case of inside rings was considered.

Figures 6 and 7 show the variation of the shape factor with respect to the amount of ring bottom flange area k_r. As in the case of stringer stiffened cylinders, Figures 6 and 7 show that the maximum shape factor occurs for a value of k_r equal to zero, which implies T-shaped rings are most efficient. This result differs from the results of Reference 2 which considered ring stiffened corrugated cylinders and determined a k_r value of 0.8. Again the values of γ_r and h_r/t want to be as large as possible. In an actual design, the values of γ and h/t for either rings or stringers have a maximum value normally determined from stiffener fabrication or buckling constraints. Figures 8 and 9 present the variation of the shape factor with respect to the ring area parameter $\overline{R}$. Figures 8 and 9 show that the maximum F_s occurs for $\overline{R}$ values of 0.05 to 0.2. Values of the ring area parameter of 0.2 to 0.5 for maximum structural efficiency and for bar type stiffeners are presented in Reference 5.

Again as in the case of the stringer stiffeners, the shape of the curves of Figures 6 through 9 were typical of all cases.

A tabulation of the results for ring and stringer stiffened cylinders is presented in Tables 2 through 10. These tables show the ring stiffener area parameter $\overline{R}$ which gives the maximum shape factor F_s. Tables 2 through 10 are required to present the results since the ring area parameter $\overline{R}$ varies with respect to the stringer parameters $\overline{S}$, γ_s, and h_s/t as well as the ring parameters γ_r and h_r/t. For all the tables, $\overline{G}_s$, $\overline{G}_r$, k_s and k_r are taken to be zero. Again as in the case of stringers, it is desirable to condense the ring area parameter results by determining the mean and standard deviation. Therefore, the means of the $\overline{R}$ values of Tables 2 through 10 are 0.07 for rings inside, stringers outside, and 0.12 for rings inside, stringers inside, with standard deviations of 0.05 and 0.07, respectively.

The use of the desirable ring and stringer parameters and proportions is illustrated in the next section where the design methodology is presented.

DESIGN ANALYSIS THEORY

The design of the stiffeners and of the shell wall for a minimum weight stiffened cylindrical shell may be accomplished by employing the results of the parametric studies of the previous section and the equations governing the different shell failure modes. The failure modes considered are: general and panel instability, local stiffener and skin buckling and stiffener and skin yielding.

General and Panel Instability - The buckling modes of general and panel instability are governed by equation (1). Panel instability is taken to be the buckling mode of a stringer stiffened cylinder of the length of the ring spacing. This method assumes that the modes of failure for panel and general instability differ appreciably; thus, there is little or no action between the two failure modes. As mentioned previously, equation (1) can only be considered fair in predicting buckling loads. It is the author's opinion that for an actual design, the buckling equation should consider both theory and experimental results, i.e., an analysis in which theory is combined with experiment by using probability considerations (see Reference 6 for example).

Local Stiffener and Skin Buckling - The local buckling of the stiffeners and the skin is governed by buckling of a flat plate (the cylinder curvature is neglected for skin buckling) and is given by the following equation (Reference 7).

$$\sigma_{cr} = \frac{K\pi^2 E(t/d)^2}{12(1-\mu^2)} \quad (4)$$

where t = thickness of stiffener web, thickness of stiffener flange or shell wall thickness

d = height of stiffener web, width of stiffener flange, or stiffener spacing

and K = 5.42 for stiffener webs

= 1.27 for stiffener flanges

= 4.0 for local skin buckling

The K factor in equation (4) represents the different boundary conditions, that is, for stiffener webs one edge fixed the other simply supported; for stiffener flanges one edge fixed the other free; and for the shell wall both edges simply supported.

Stiffener and Skin Yielding - The yielding of the shell skin and the stiffeners is governed by the distortion energy yield criterion which may be formulated as follows. From Reference 8, the prebuckling strains (assuming a membrane force distribution) for a stiffened isotropic cylinder may be written as

$$\varepsilon_y = \frac{1}{Et}\left(\frac{\mu N - pR[1 + \overline{S}(1 - \mu^2)]}{1 + \overline{S} + \overline{R}[1 + \overline{S}(1 - \mu^2)]}\right) \quad (5)$$

$$\varepsilon_x = \frac{1}{Et}\left(\frac{-N(1 - \mu^2) - \mu Et\,\varepsilon_y}{1 + \overline{S}(1 - \mu^2)}\right) \quad (6)$$

where the minus sign denotes a compressive strain. The prebuckling stresses in the skin then are

$$\sigma_x = \frac{E}{1 - \mu^2} (\varepsilon_x + \mu \varepsilon_y) \tag{7}$$

$$\sigma_y = \frac{E}{1 - \mu^2} (\varepsilon_y + \mu \varepsilon_x) \tag{8}$$

and in the stiffeners are

$$\begin{aligned} \sigma_s &= E_s \varepsilon_x \\ \sigma_r &= E_r \varepsilon_y \end{aligned} \tag{9}$$

for a stringer and a ring, respectively. In equations (7) through (9), ε_x and ε_y are given by equations (5) and (6).

Finally, the cylinder skin yielding is given by

$$\sigma_Y^2 = \sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 \tag{10}$$

and the stringer and ring yielding are given by

$$\sigma_{Ys} = \sigma_s \tag{11}$$

$$\sigma_{Yr} = \sigma_r \tag{12}$$

where

σ_Y is the yield stress.

The next paragraphs explain how the shell failure mode equations above are applied.

In the design of the stiffened shell configuration, the designer is assumed to know the following quantities.

1. The applied axial load on the shell, N_{x_A} .
2. The radius, R, and length, L, of the shell.
3. The shell and stiffener materials. Thus, the modulus of elasticity, E, the yield stress, σ_Y, Poisson's ratios, μ, and the material weight, ρ, are known for the shell and stiffeners.
4. The location of the stiffeners--either inside or outside.
5. The ratio of the stiffeners flange thickness to web thickness, t_f/t_w.

Therefore, the design variables to be determined are the skin thickness and ring and stringer stiffener shapes and spacing. The design is accomplished in the following manner.

1. From the parametric study results of the previous section, the stiffeners are taken to be T shaped (k_s and k_r are made zero), and values of $\bar{S}$ and $\bar{R}$ are selected. For the initial choice, the $\bar{S}$ and $\bar{R}$ mean values of 0.56 and 0.07, respectively, for outside stringers and 0.44 and 0.12, respectively, for inside stringers may be chosen.
2. Using the yielding equations (10), (11), and (12), three values of the skin thickness are calculated from the following.

$$t_1 \geq \sqrt{\frac{(\overline{E}x + \mu\overline{E}y)^2 - (\overline{E}x + \mu\overline{E}y)\ (\overline{E}y + \mu\overline{E}x) + (\overline{E}y + \mu\overline{E}x)^2}{(1 - \mu^2)^2 \sigma_Y^{\ 2}}} \quad (13)$$

$$t_2 \geq \frac{E_s \overline{E}x}{E\sigma_{Ys}} \quad (14)$$

$$t_3 \geq \frac{E_r \overline{E}y}{E\sigma_{Yr}} \quad (15)$$

where

$$\overline{E}y = \frac{\mu N_{x_A} - PR[1 + \overline{S}(1 - \mu^2)]}{1 + \overline{S} + \overline{R}[1 + \overline{S}(1 - \mu^2)]}$$

$$\overline{E}x = \frac{-N_{x_A}(1 - \mu^2) - \mu\overline{E}y}{1 + \overline{S}(1 - \mu^2)}$$

The maximum of the three values given by equations (13), (14), and (15) is chosen as the skin thickness t. This value of skin thickness t is a lower bound or minimum value. At this point it should be mentioned that the maximum value of t determined from equations (13), (14), and (15) may not be large enough to give the desired panel or general instability buckling load and, thus, this value of t would have to be increased by some factor. Note that an upper bound on t is given by the classical buckling equation for an unstiffened cylinder or

$$t_u \leq \sqrt{\frac{N_{x_A} R}{.6E}} \quad (16)$$

3. Next, equation (4) is used for local skin buckling to determine the stringer spacing. By rewriting equation (4) in the following manner, the maximum value of the stringer spacing d is

$$d \leq \sqrt{\frac{\pi^2 Et^3}{3(1 - \mu^2)N_{x_A}}} \qquad (17)$$

where the skin thickness t is the value determined from equations (13), (14), or (15). The value of d calculated from equation (17) is taken as the stringer spacing.

4. Knowing the selected value of $\bar{S}$ (step 1), the shell thickness t (step 2), the stringer spacing (step 3), and the material, the cross-sectional area of the stringer A_s is

$$A_s = \frac{\bar{S}\ Etd}{E_s} \qquad (18)$$

5. The shape of the stringer is then calculated as follows. Writing equation (4) in the following manner for the stringer web

$$\frac{h_s}{t_{w_s}} = \sqrt{\frac{5.42\ \pi^2\ E_s}{12(1 - \mu_s^2)\ \sigma_s}} \qquad (19)$$

and for the stringer flange

$$\frac{c_{f_s}}{t_{f_s}} = \sqrt{\frac{1.27\ \pi^2\ E_s}{12(1 - \mu_s^2)\ \sigma_s}} \qquad (20)$$

the height-to-thickness ratio and width-to-thickness ratio of the stringer are calculated. The equation for the area of a T-shaped stringer is

$$A_s = 2\, t_{f_s}\, c_{f_s} + t_{w_s}\, h_s \tag{21}$$

Rewriting equation (21) in terms of the known stringer ratios h_s/t_{w_s}, c_{f_s}/t_{f_s}, and t_{f_s}/t_{w_s}, and solving for thickness of the web gives

$$t_{w_s} = \sqrt{\frac{A_s}{2\left(\frac{t_{f_s}}{t_{w_s}}\right)^2 \frac{c_{f_s}}{t_{f_s}} + \frac{h_s}{t_{w_s}}}} \tag{22}$$

Knowing the web thickness; the flange thickness, web height, and flange width are calculated from the stringer ratios. Thus, the stringer shape is known. At this point the stringer parameters needed for the general and panel instability buckling equations may also be calculated.

6. Next, the panel instability mode is considered by using equation (1) and making the ring terms zero. From the panel instability mode calculations, the ring spacing, and, thus, the number of rings, can be determined by using an iterative process which increases the number of rings and thus decreases the length of stringer stiffened cylinder until panel instability buckling load value N_x is determined which is greater than the applied load, N_{x_A}. For this analysis, the number of rings is taken to be a whole number.

7. Finally, the general instability calculations are performed to determine the value of the ring height to shell thickness h_r/t. The h_r/t value can be determined by an iterative process in which a value of h_r/t is specified and then the general instability buckling load N_x is calculated from equation (1). If the general instability buckling load N_x is less than the applied load N_{x_A}, the h_r/t value is increased and a new buckling load N_x is calculated. This process is repeated until N_x is greater than N_{x_A}. Note that once h_r/t is specified or known, the ring shape is entirely known and can be calculated from equations (18), (19), (20), and (21) wherein these equations the stringer subscript s is replaced by ring subscript r, $\overline{S}$ is replaced by $\overline{R}$, and the stringer spacing d is replaced by ring spacing l. The general instability calculations should take into account the torsional stiffness of the rings.

8. The total weight of the stiffened shell is given by

$$W = 2\pi RLt\rho \left[1 + \frac{\rho_s}{\rho} \overline{S} \frac{E}{E_s} + \frac{\rho_r}{\rho} \overline{R} \frac{E}{E_r} \left(1 - \frac{l}{L}\right) \right] \tag{23}$$

In the next section, results are presented illustrating the application of the above theory and equations to design a stiffened shell.

DESIGN ANALYSIS RESULTS

Several calculations which make use of the design analysis of the previous section are discussed in this section of the paper in order to illustrate minimum weight design of an axially compressed stiffened cylinder. The calculations presented in this section were performed on the computer.

Simple calculations were made using the cylinder configuration of Case 6-I' of Reference 1. The results of these calculations are presented in Figures 10 and 11 and Tables 11 and 12. For this cylinder, the given design quantities are presented in the upper right hand corner and the legend of the figures and tables. Both the rings and stringers are inside.

Figure 10 shows the variation of the total weight of the shell with respect to the stringer area parameter $\overline{S}$. In Figure 10, the solid curves present the variation of the cylinder weight when the ring area parameter $\overline{R}$ is held constant; and the dashed curves present the variation of the cylinder weight when the ratio of the ring web height to ring web thickness h_r/t_{w_r} is held constant. The results of Figure 10 are for ring and stringer flange thickness to web thickness ratios of 1. For the case where the ring is not a direct load-carrying member, the value of the ratio of ring web height to web thickness h_r/t_{w_r} is the critical parameter. The maximum value of the h_r/t_{w_r} ratio should be probably no greater than 80 on the premise that unfavorable fabrication stresses may buckle the webs (References 2 and 9). Thus, from Figure 10, the minimum weight design occurs at the minimum point on the h_r/t_{w_r} equal to 80 curve and at a stringer area parameter $\overline{S}$ of 0.48 and a ring area parameter $\overline{R}$ of 0.05. Note these values are very close to the mean values of $\overline{S}$ and $\overline{R}$ (0.44 and 0.12) determined in the parametric study section. These values give a total shell weight of 3.78 pounds. This minimum weight is slightly less than the minimum weight determined in Reference 1; however, there are differences in the theory of Reference 1 and the theory herein which account for some unknown portion of the weight difference.

Presented in Table 11 are the final calculated design values for the minimum weight point of Figure 10. Also presented in Table 11 are the ratios of the applied loads to the calculated or critical loads for the various shell failure modes.

For the above stiffened cylinder, a set of additional calculations was made which varied the ratio of the ring and stringer outer flange thickness to web thickness. The results of these calculations are presented in Figure 11 and Table 12. Shown in Figure 11 is the variation of the minimum cylinder weight with respect to the ratio of ring and stringer flange thickness to web thickness. In Figure 11, t_f/t_w equal to zero is the case where both the stiffeners are bar-shaped and t_f/t_w equal to 1 is the case presented in Figure 10 and Table 11. From Figure 11, the minimum weight configuration occurs for unsymmetrical T sections with the outer flange thickness less than the web thickness ($t_f/t_w < 1$). Presented in Table 12 are the critical stiffener parameters for the results of Figure 11. Note in Table 12 that there is an interplay between the two stiffener parameters h/t and γ. That is, γ achieves its maximum value at large values of t_f/t_w, while h/t has its maximum value at small values of t_f/t_w. This interplay is reflected in the results of Figure 11. In an actual design the value of t_f/t_w ratio will probably be governed by manufacturing constraints.

Results of calculations for a large diameter cylinder configuration similar to the one in Reference 4 and Case 4 of Reference 1 are shown in Figures 12 through 14 and Tables 13 through 15. For this cylinder the given design quantities are again presented in the upper right hand corner and the legend of the figures and the tables.

Figures 12 through 14 are the same type of plot as Figure 10 and show the variation of the total shell weight with respect to the stringer area parameter, the ring area parameter, and the h_r/t_{w_r} ratio. The variation in the figures and tables is that three different values of the applied load N_{x_A} act on the shell.

Figure 12 is for a heavily loaded cylinder ($N_{x_A}/R = 0.5$ ksi) and shows that ring stiffeners are not required. ($\overline{R} = 0$ is the minimum curve since h_r/t_{w_r} is well below the acceptable value.) Note also that the stiffener area parameter $\overline{S}$ does not affect the total weight of the shell since the shell thickness is reduced as $\overline{S}$ is increased. However, the maximum panel instability buckling load occurs at an $\overline{S}$ value of 0.8; thus, this value should be chosen as the design point. The final design values for this case are presented in Table 13.

Figure 13 is for a cylinder loaded with $N_{x_A}/R = 0.05$ ksi (medium loading). For this cylinder and loading, the minimum weight occurs at the minimum point of h_r/t_{w_r} equal to 80 curve. The final design values for this case are given in Table 14.

The final figure, Figure 14, is for a lightly loaded cylinder (N_{x_A}/R = 0.005 ksi). Again the minimum weight occurs at the minimum point of h_r/t_{w_r} equal to 80 curve. However, for this cylinder the skin thickness given by yielding (equations (13), (14) or (15)) was not large enough to give the desired general instability load and had to be increased. Thus, the skin thickness determined by the yielding equations was increased by a factor of 2.5, the minimum value which would give the desired general instability buckling load, to give the final results of Figure 14 and Table 15.

CONCLUDING REMARKS

In conclusion, the results of this investigation present desirable structural parameters and proportions, and a procedure of designing axially compressed stiffened isotropic cylinders for minimum weight. The results of the parametric studies revealed the following trends concerning desirable parameters for ring and stringer stiffeners.

1. The values of the amount of bottom flange area in the stiffener cross-section (k_s and k_r) are zero. This implies T-shaped stiffeners are most desirable.

2. The values of stiffener height to skin thickness, h/t, and of the amount of stiffener area in flange to total area of stiffener, γ, should be made as large as possible.

3. The stiffener area to shell wall area (stiffener area parameter) depends upon other parameters for its most desirable value. However, stringers have mean values of stringer area paramters $\bar{S}$ of 0.56 for stringers outside and 0.44 for stringers inside; and rings have mean values of ring area parameters $\bar{R}$ of 0.07 for rings inside and stringers outside and of 0.12 for rings inside and stringers inside.

Sample calculations are presented which illustrate the design of a stiffened cylinder. These calculations make use of the above parameters and show that for most designs the critical parameter is the ring web height to web thickness. The calculations also show that a slight weight advantage may be received by making the stiffener outer flange thickness less than the stiffener web thickness.

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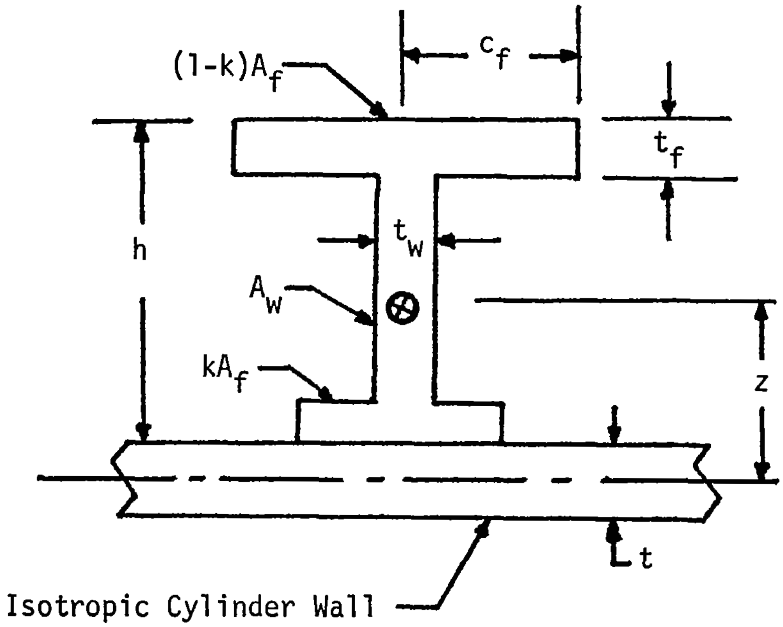

$\gamma = A_f/(A_w+A_f)$

Figure 1. - Ring and stringer cross section representation.

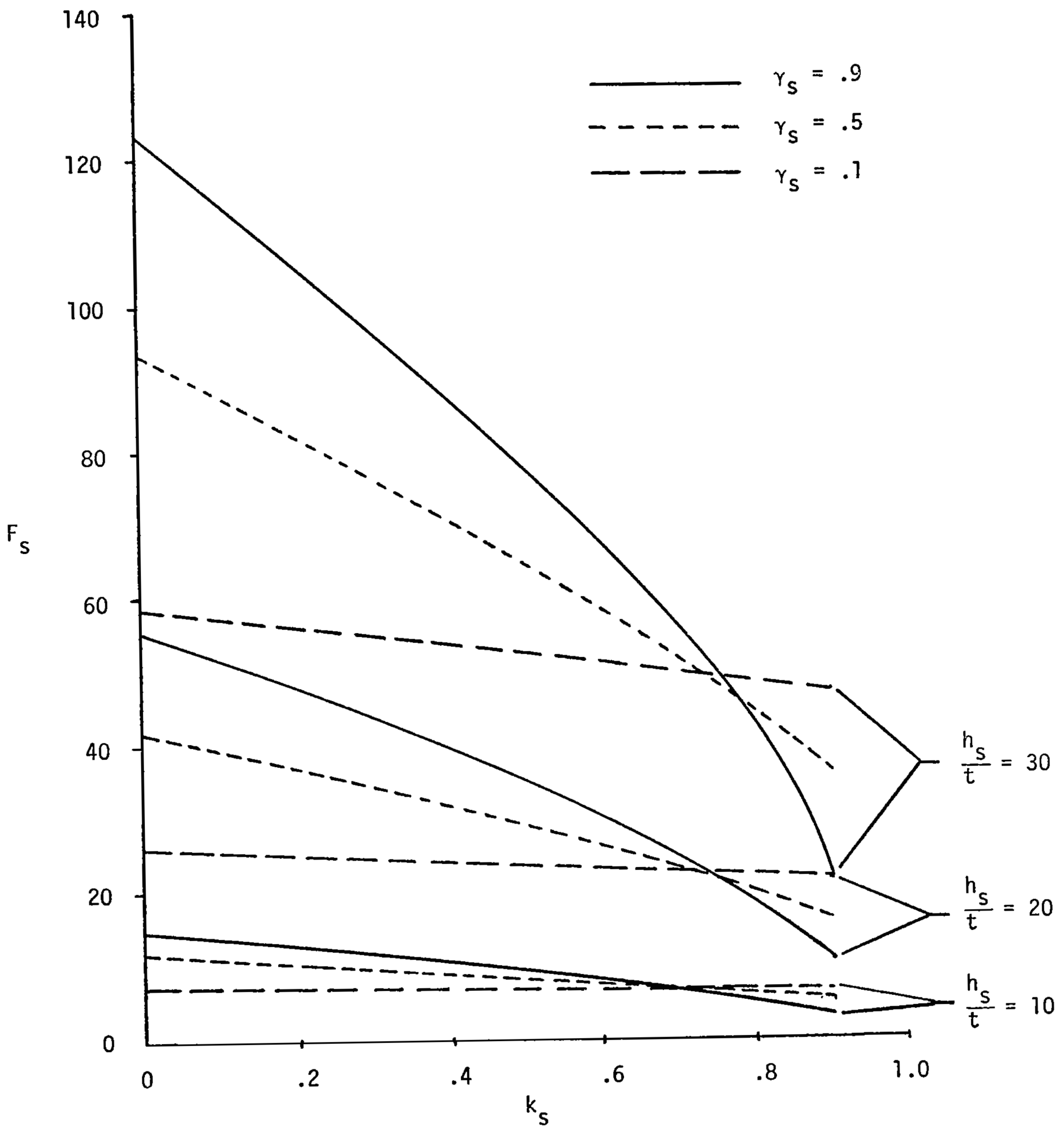

Figure 2. - Calculations to determine shape of stringer stiffener. $Z = 10$; $\bar{S} = 0.5$; $\bar{G}_s = 0$; stringers outside.

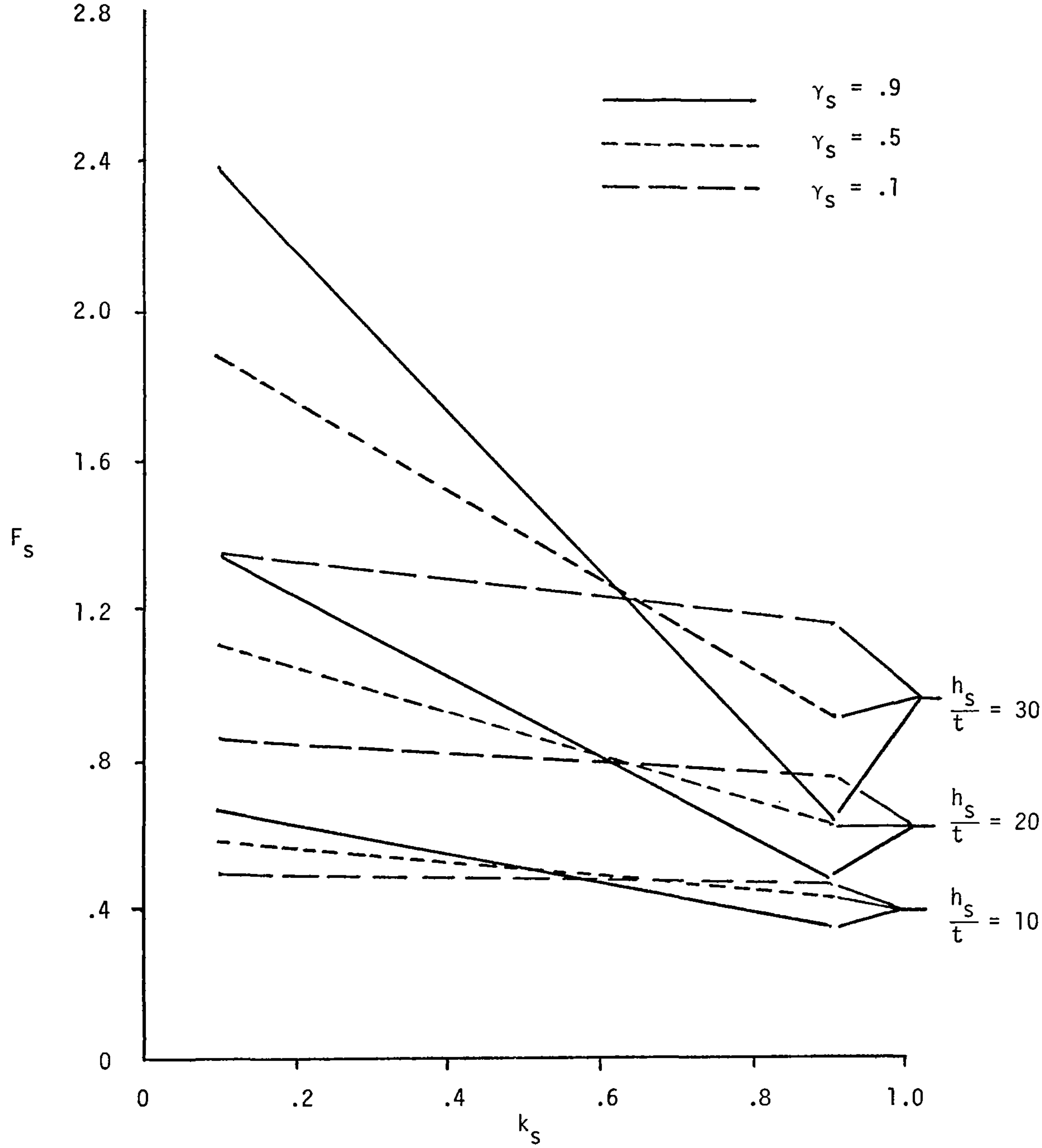

Figure 3. - Calculations to determine shape of stringer stiffener.
$Z = 1{,}000$; $\overline{S} = 0.6$; $\overline{G}_s = 0$; stringers outside.

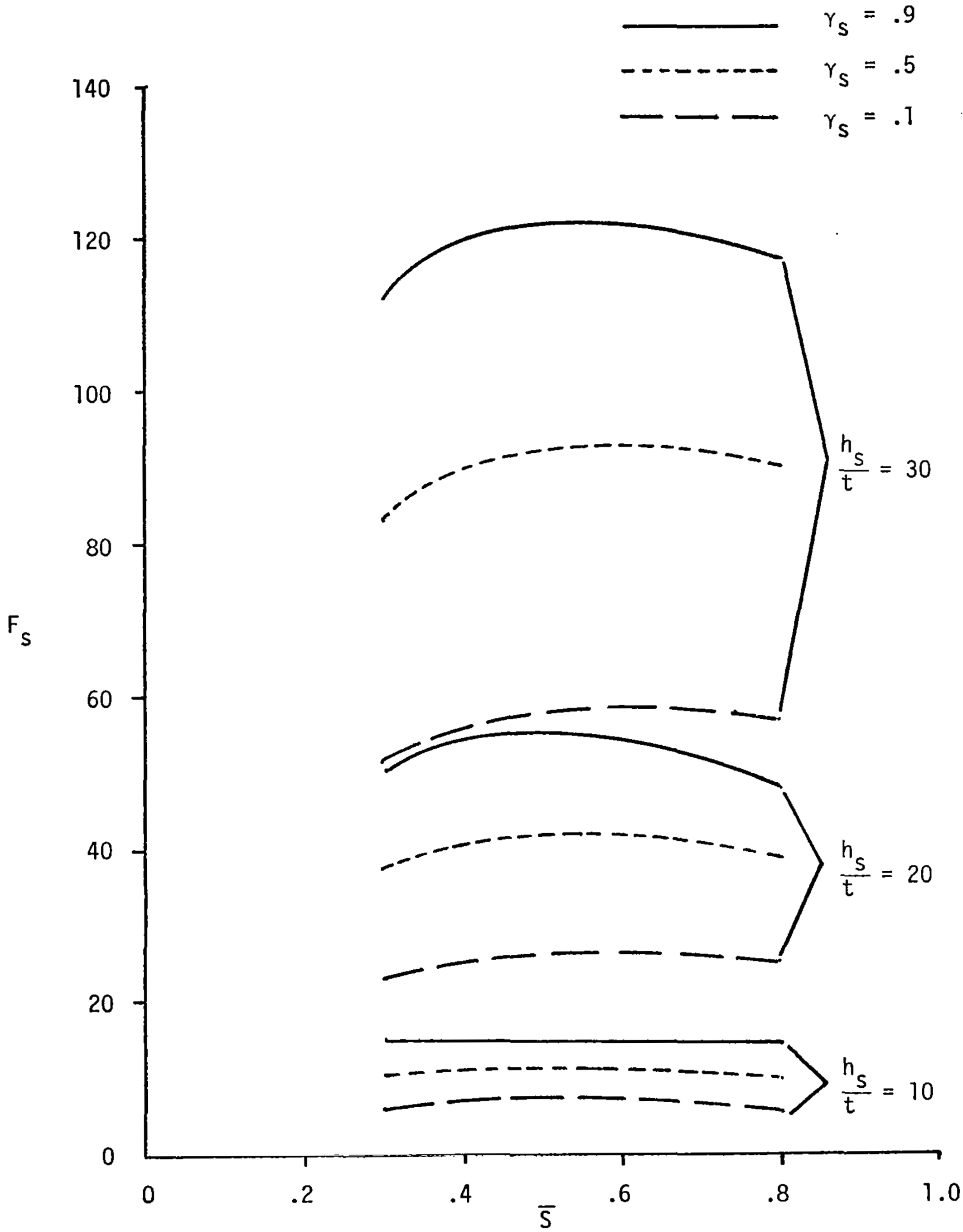

Figure 4. - Calculations to determine desirable stringer area parameter. $Z = 10$; $k_s = 0$; $\bar{G}_s = 0$; stringers outside.

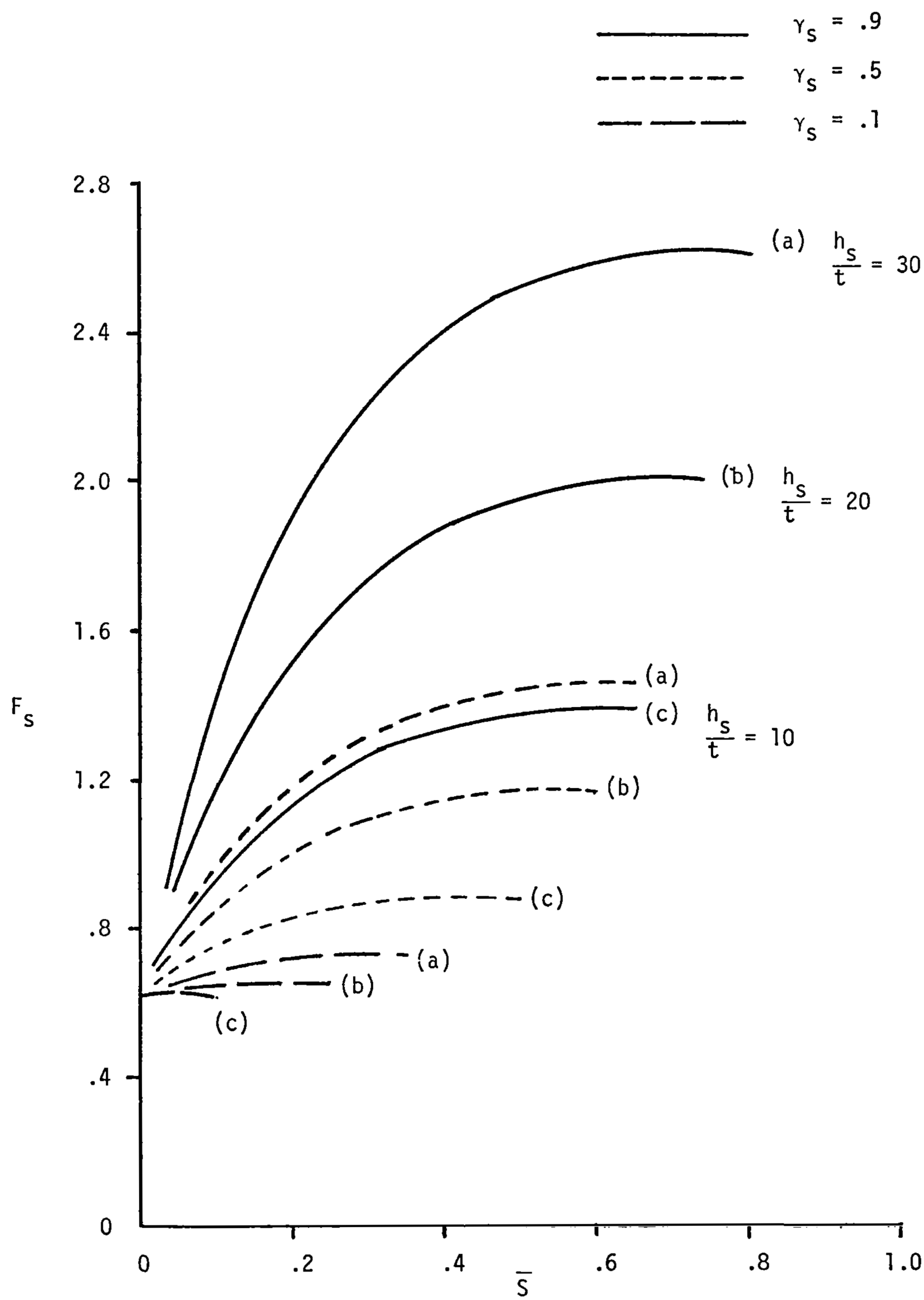

Figure 5. - Calculations to determine desirable stringer area parameter. Z = 1,000; k_s = 0; $\overline{G}_s$ = 0; stringers outside.

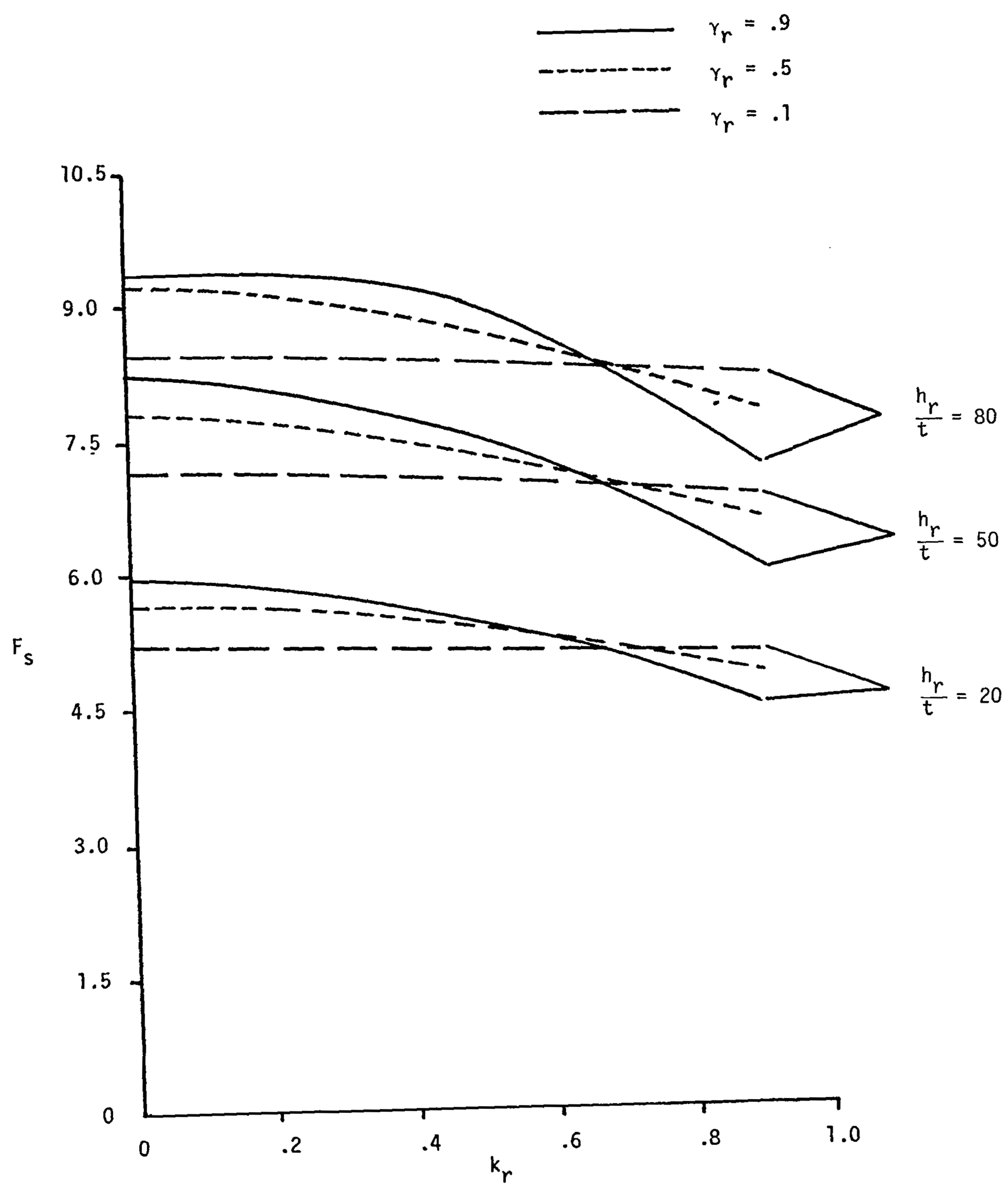

Figure 6. - Calculations to determine shape of ring stiffener. $Z = 500$; $\bar{S} = 0.6$; $k_s = 0$; $\gamma_s = 0.5$; $h_s/t = 30$; $\bar{G}_s = 0$; stringers outside; $\bar{R} = 0.1$; $\bar{G}_r = 0$; rings inside.

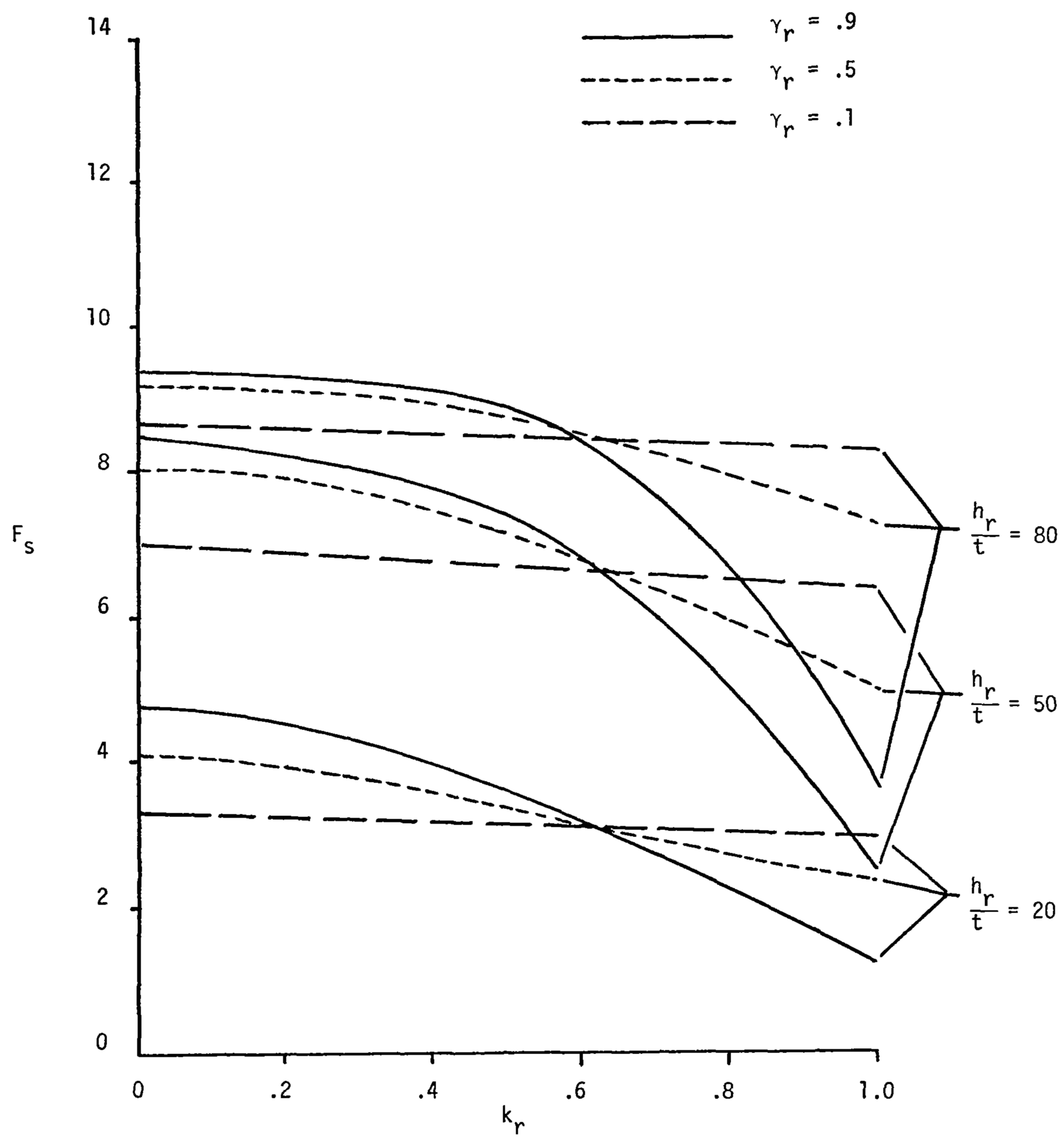

Figure 7. - Calculations to determine shape of ring stiffener. $Z = 10{,}000$; $\bar{S}$ - 0.5; $k_s = 0$; $\gamma_s = 0.5$; $h_s/t = 30$; $\bar{G}_s = 0$; stringers outside; $\bar{R} = 0.05$; $\bar{G}_r = 0$; rings inside.

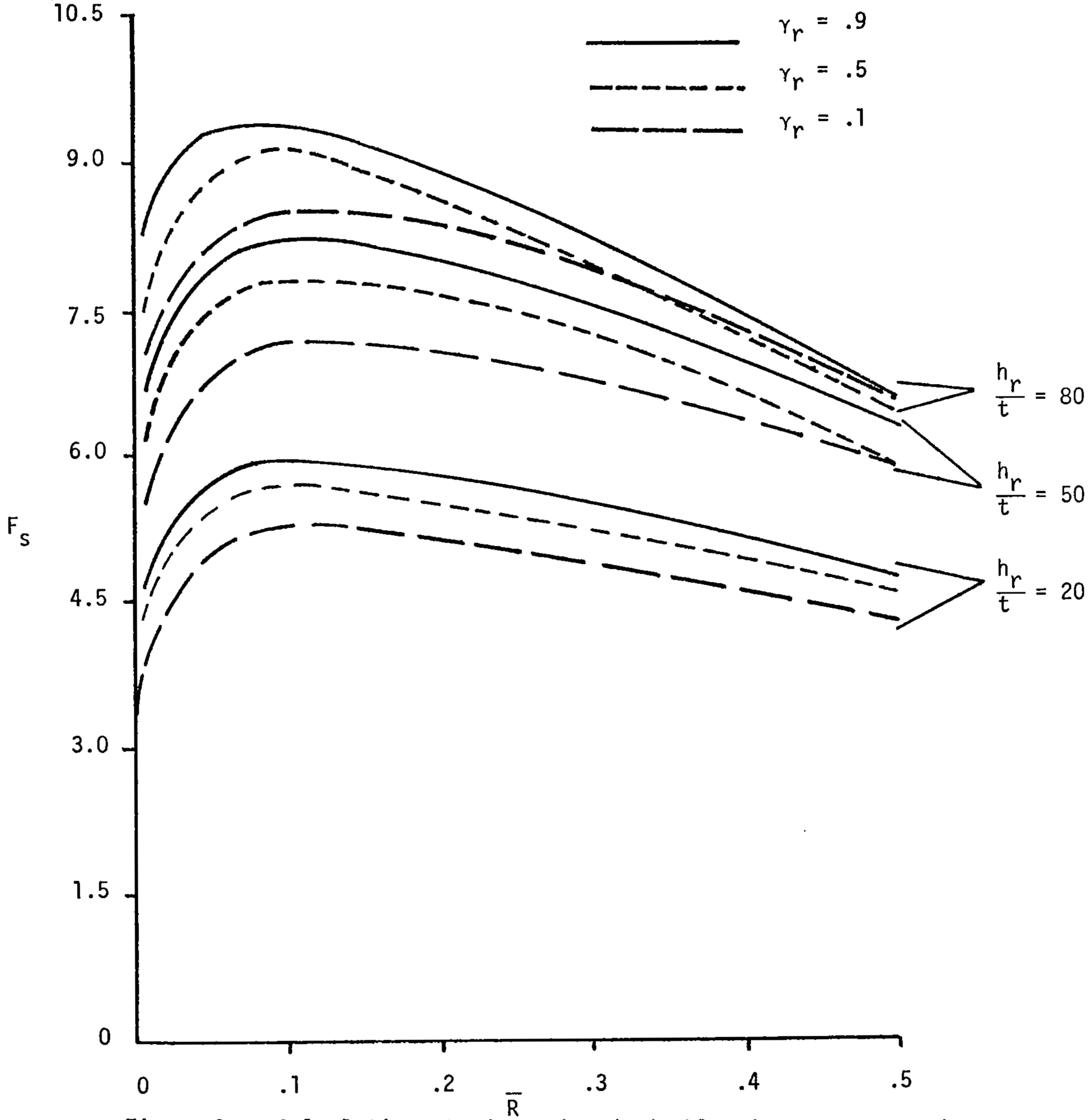

Figure 8. - Calculations to determine desirable ring area parameter. $Z = 500$; $\overline{S} = 0.6$; $k_s = 0$; $\gamma_s = 0.5$; $h_s/t = 30$; $G_s = 0$; stringers outside; $k_r = 0$; $\overline{G}_r = 0$; rings inside.

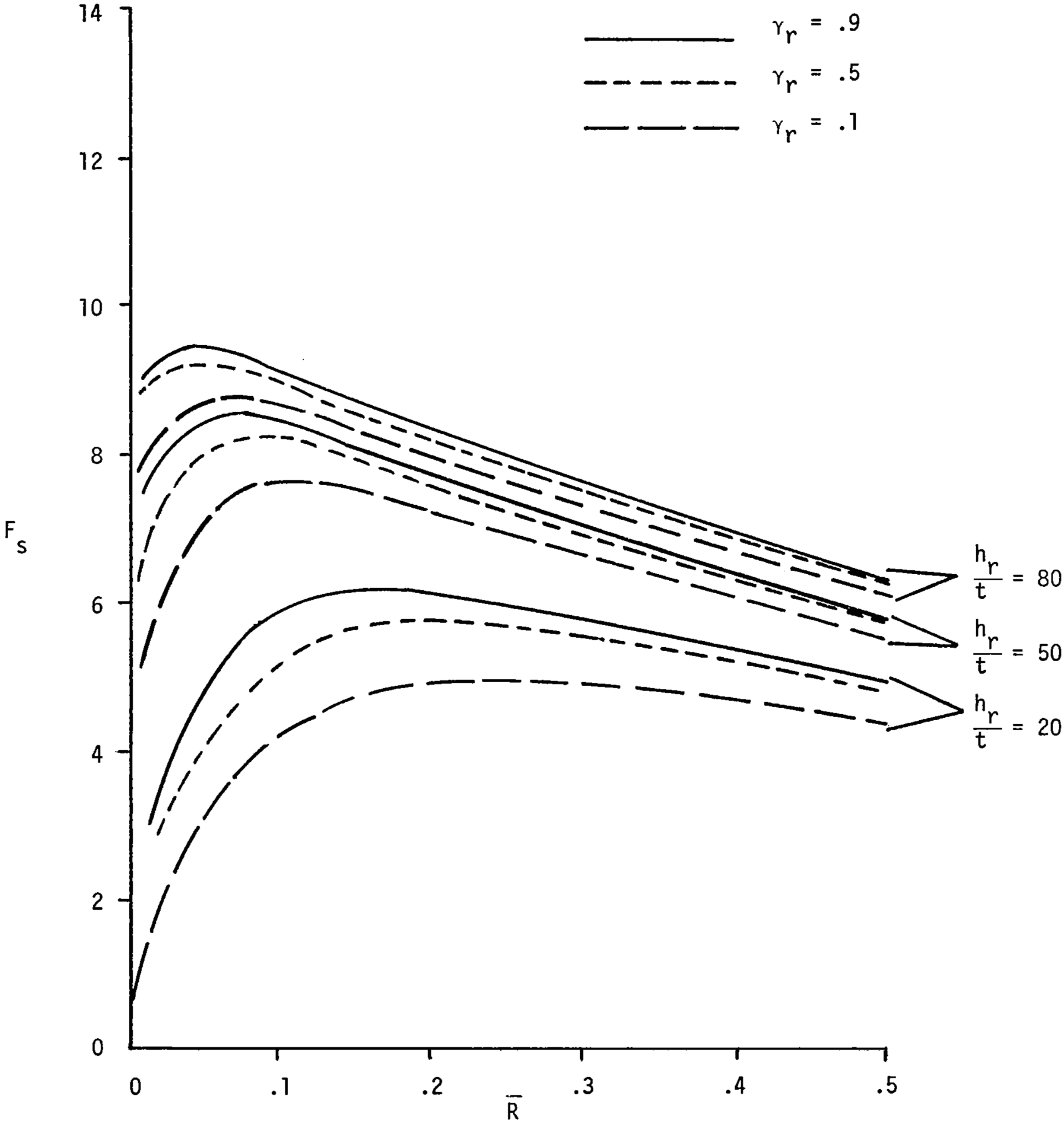

Figure 9. - Calculations to determine desirable ring area parameter. $Z = 10,000$; $\overline{S} = 0.5$; $k_s = 0$; $\gamma_s = 0.5$; $h_s/t = 30$; $\overline{G}_s = 0$; stringers outside; $k_r = 0$; $\overline{G}_r = 0$; rings inside.

$R = 9.55$ in. $L = 38.0$ in.

$E = E_s = E_r = 10.5 \times 10^6$ psi

$\mu = \mu_s = \mu_r = .33$

$\rho = \rho_s = \rho_r = .101$ lb/in.^{3}

$t_{fs}/t_{ws} = t_{fr}/t_{wr} = 1.0$

$\sigma = \sigma_{Ys} = \sigma_{Yr} = 50000$ psi

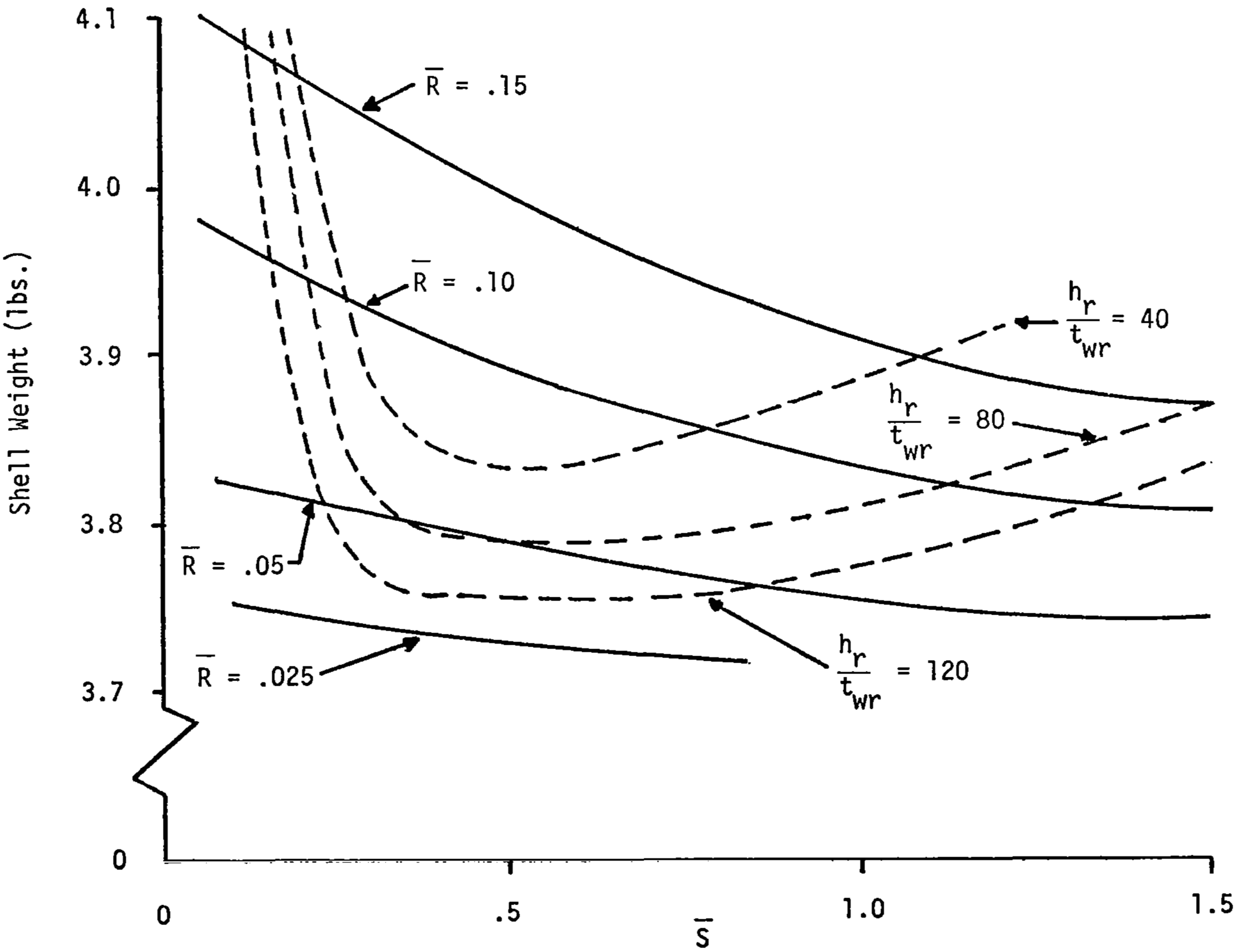

Figure 10. - Calculations to determine minimum weight design of stiffened cylinder. Stringers inside; rings inside; applied axial load, $N_{x_A} = 800$ lb./in.

$R = 9.55$ in. $L = 38.0$ in.
$E = E_s = E_r = 10.5 \times 10^6$ psi
$\mu = \mu_s = \mu_r = .33$
$\rho = \rho_s = \rho_r = .101$ lb./in.^{3}
$\sigma = \sigma_{Ys} = \sigma_{Yr} = 50{,}000$ psi

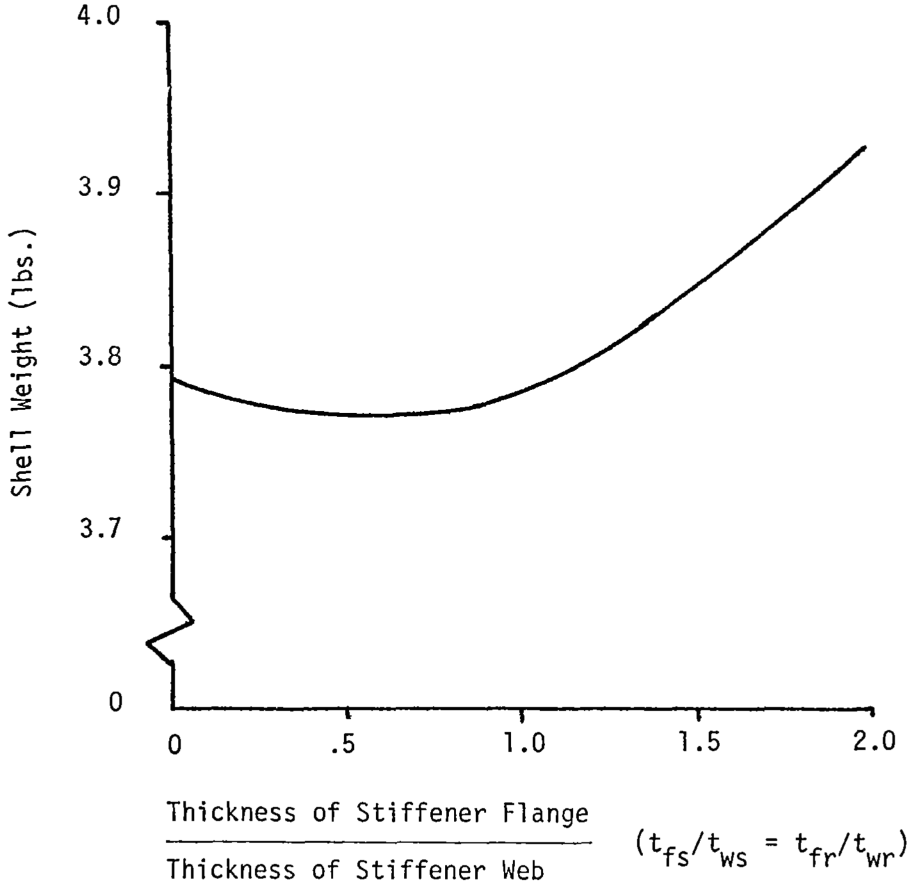

Figure 11. - Effect of stiffener flange to web thickness on minimum weight design. Stringers inside; rings inside; applied axial load, $N_{x_A} = 800$ lb./in.

$R = 200.$ in. $L = 200.$ in.
$E = E_s = E_r = 10.5 \times 10^6$ psi
$\mu = \mu_s = \mu_r = .33$
$\rho = \rho_s = \rho_r = .101$ lb/in.^{3}
$t_{fs}/t_{ws} = t_{fr}/t_{wr} = 1.0$
$\sigma = \sigma_{Ys} = \sigma_{Yr} = 50{,}000$ psi

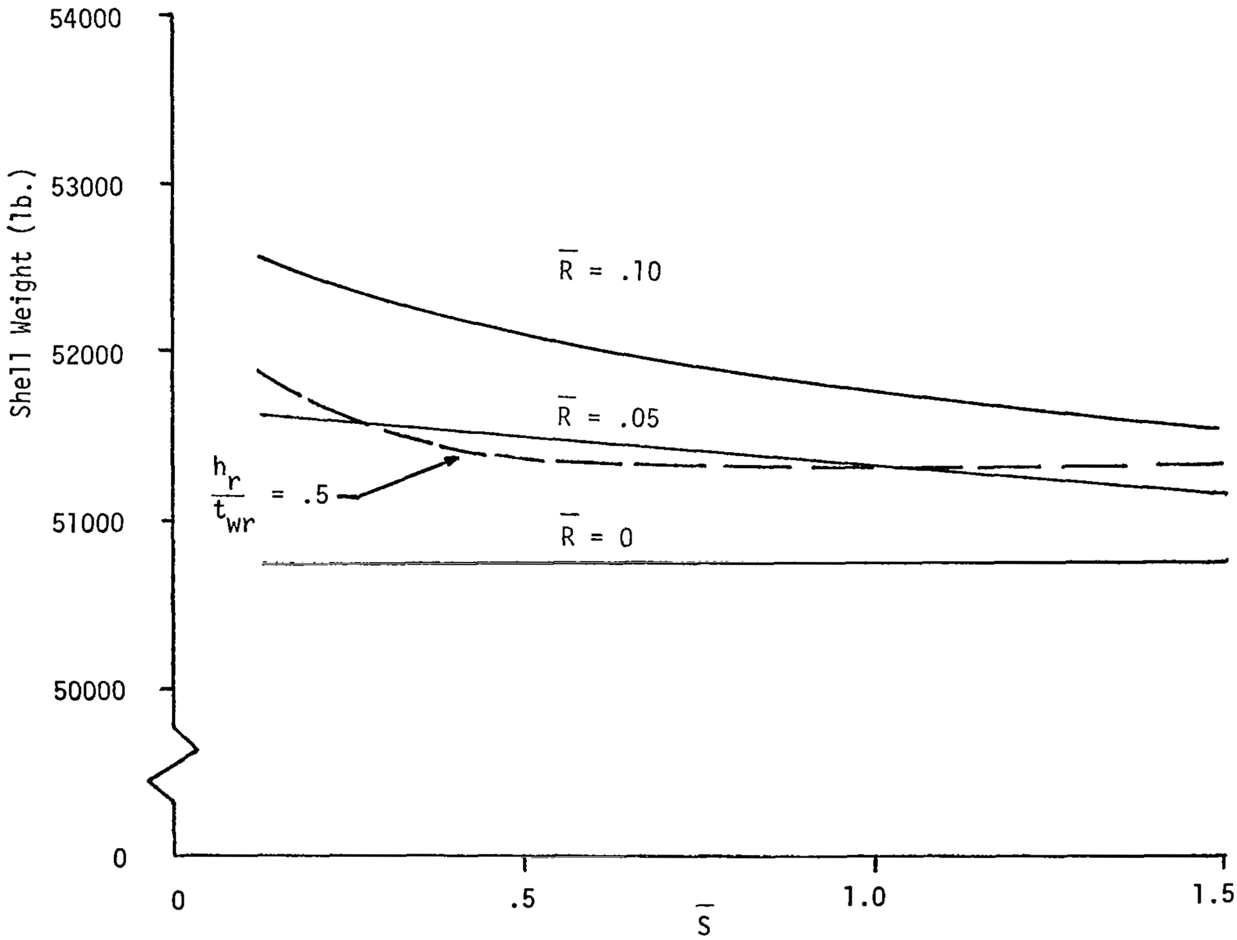

Figure 12. - Calculations to determine minimum weight design of stiffened cylinder. Stringers inside; rings inside; applied axial load, $N_{x_A} = 100{,}000$ lb./in.

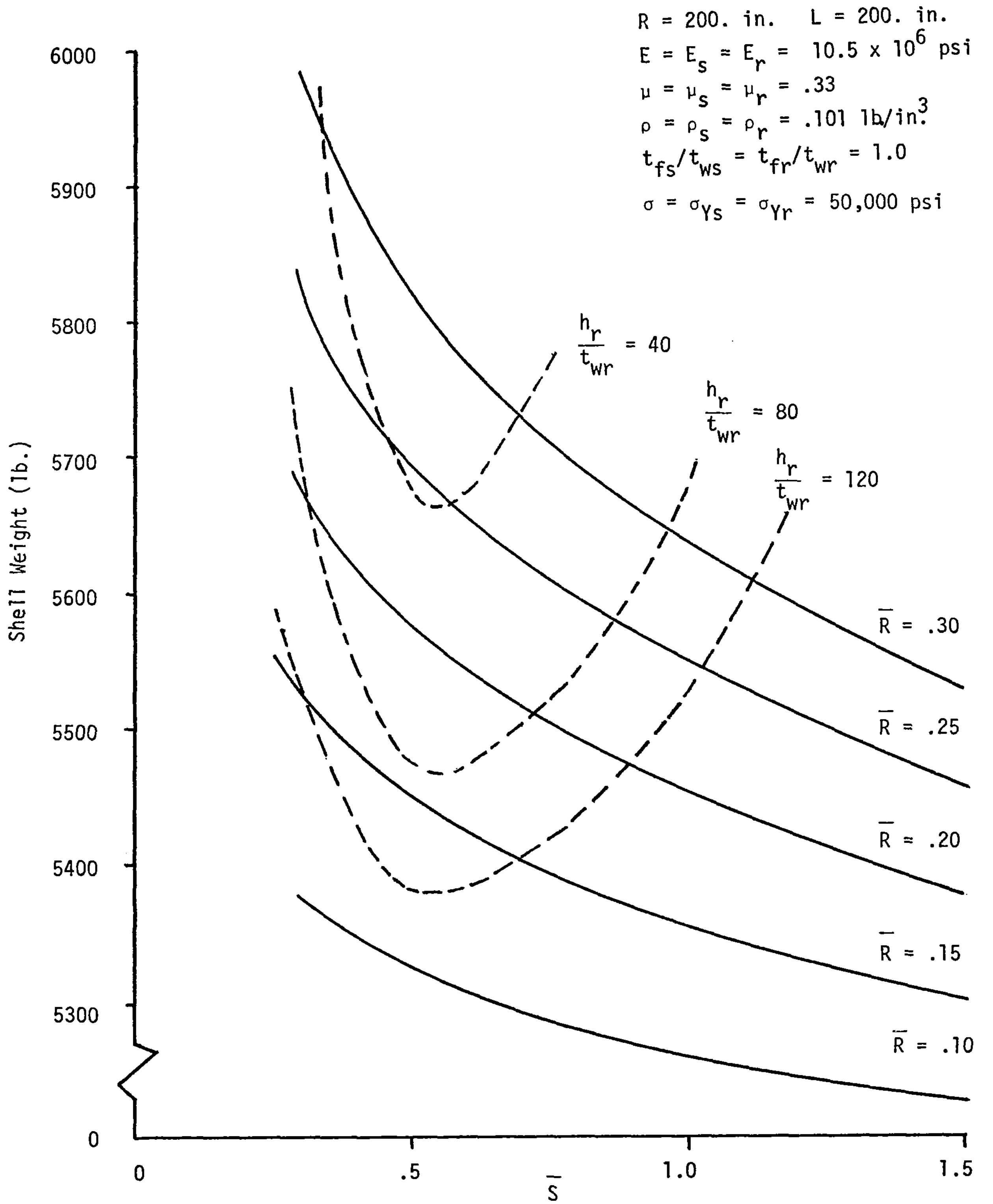

Figure 13. - Calculations to determine minimum weight design of stiffened cylinder. Stringers inside; rings inside; applied axial load, N_{x_A} = 10,000 lb./in.

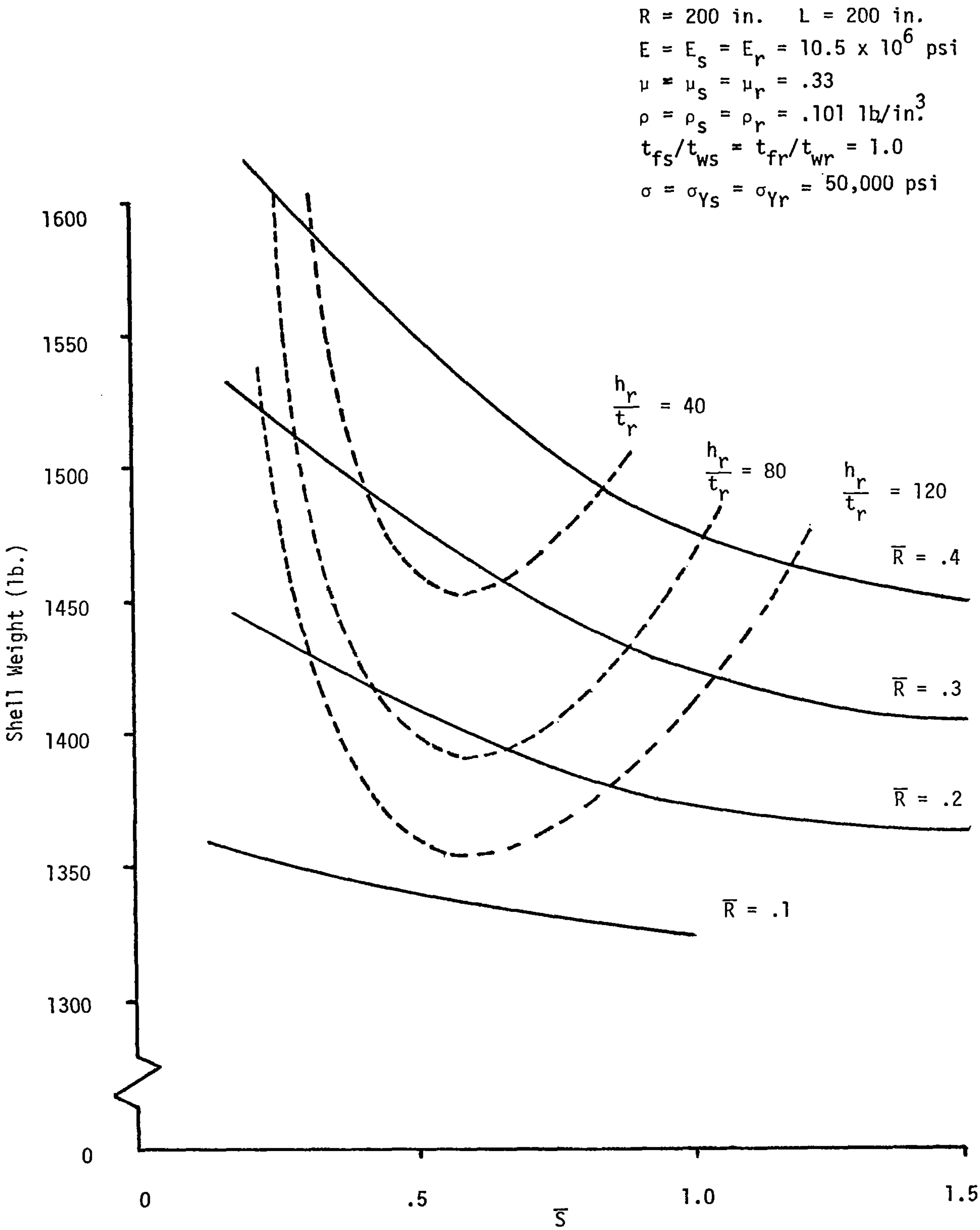

Figure 14. - Calculations to determine mimimum weight design of stiffened cylinder. Stringers inside; rings inside; applied axial load, N_{x_A} = 1,000 lb./in.; yielding skin thickness increased by factor of 2.5.

Table 1. - Stringer Area Parameter $\overline{S}$ for Maximum Structural Efficiency. $k_s = 0$; $\overline{G}_s = 0$ and $\overline{G}_s = 0.5$.

Z	h_s/t	STRINGERS OUTSIDE γ_s .1	.5	.9	STRINGERS INSIDE γ_s .1	.5	.9
1	10	.60	.55	.50	.60	.55	.50
1	20	.60	.55	.50	.60	.55	.50
1	30	.60	.55	.50	.60	.55	.50
1	40	.60	.55	.50	.60	.55	.50
10	10	.55	.55	.50	.55	.55	.50
10	20	.60	.55	.50	.60	.55	.50
10	30	.60	.55	.50	.60	.55	.50
10	40	.60	.55	.50	.60	.55	.50
100	10	.55	.60	.60	.35	.45	.45
100	20	.65	.60	.50	.55	.55	.50
100	30	.60	.50	.40	.60	.55	.50
100	40	.50	.40	.40	.60	.55	.50
500	10	.25	.40	.45	0	0	0
500	20	.60	.65	.70	.25	.40	.45
500	30	.70	.75	.75	.50	.50	.50
500	40	.75	.80	.75	.55	.55	.50
1,000	10	.05	.20	.30	0	0	0
1,000	20	.45	.55	.60	0	.05	.25
1,000	30	.60	.70	.75	.30	.45	.45
1,000	40	.70	.75	.80	.50	.50	.50
5,000	10	0	0	0	0	0	0
5,000	20	0	.15	.25	0	0	0
5,000	30	.20	.40	.50	0	0	0
5,000	40	.40	.55	.60	0	0	.05
10,000	10	0	0	0	0	0	0
10,000	20	0	0	0	0	0	0
10,000	30	0	.20	.30	0	0	0
10,000	40	.20	.35	.45	0	0	0

Table 2. - Ring Area Parameter $\overline{R}$ for Maximum Structural Efficiency.
$\overline{S} = 0.3$; $\gamma_s = 0.1$; $k_s = k_r = 0$; $\overline{G}_s = \overline{G}_r = 0$; Rings Inside.

Z	h_r/t	h_s/t	STRINGERS OUTSIDE $\gamma_r = .1$	$= .5$	$= .9$	STRINGERS INSIDE $\gamma_r = .1$	$= .5$	$= .9$
10	20	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
	50	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
	80	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
100	20	10	.10	.10	.05	.15	.15	.15
		30	.05	.05	.05	.05	.10	.10
	50	10	.05	.05	.05	.10	.05	.05
		30	.05	.05	.05	.10	.10	.05
	80	10	.05	.05	.05	.05	.05	.05
		30	.05	.05	.05	.05	.05	.05
500	20	10	.05	.05	.05	.10	.10	.10
		30	.15	.15	.15	.20	.20	.20
	50	10	.05	.05	.05	.10	.05	.05
		30	.10	.05	.05	.15	.10	.10
	80	10	.05	.05	.05	.05	.05	.05
		30	.05	.05	.05	.10	.10	.05
1,000	20	10	.05	.05	.05	.10	.10	.10
		30	.20	.15	.10	.20	.15	.15
	50	10	.05	.05	.05	.10	.05	.05
		30	.10	.05	.05	.15	.15	.15
	80	10	.05	.05	.05	.05	.05	.05
		30	.05	.05	.05	.15	.10	.10
5,000	20	10	.05	.05	.05	.15	.10	.10
		30	.15	.15	.10	.20	.15	.15
	50	10	.05	.05	.05	.10	.05	.05
		30	.10	.05	.05	.15	.10	.10
	80	10	.05	.05	.05	.05	.05	.05
		30	.05	.05	.05	.10	.10	.10
10,000	20	10	.05	.05	.05	.15	.10	.10
		30	.15	.15	.10	.20	.20	.15
	50	10	.05	.05	.05	.10	.05	.05
		30	.10	.05	.05	.15	.10	.10
	80	10	.05	.05	.05	.05	.05	.05
		30	.05	.05	.05	.10	.10	.10
50,000	20	10	.05	.05	.05	.15	.10	.10
		30	.15	.15	.10	.20	.20	.15
	50	10	.05	.05	.05	.10	.05	.05
		30	.10	.05	.05	.15	.10	.10
	80	10	.05	.05	.05	.05	.05	.05
		30	.05	.05	.05	.10	.10	.10
100,000	20	10	.05	.05	.05	.15	.10	.10
		30	.15	.15	.10	.20	.20	.15
	50	10	.05	.05	.05	.10	.05	.05
		30	.10	.05	.05	.15	.10	.10
	80	10	.05	.05	.05	.05	.05	.05
		30	.05	.05	.05	.10	.10	.10

Table 3. - Ring Area Parameter $\overline{R}$ for Maximum Structural Efficiency.
$\overline{S} = 0.3$; $\gamma_s = 0.5$; $k_s = k_r = 0$; $\overline{G}_s = \overline{G}_r = 0$; Rings Inside.

Z	h_r/t	h_s/t	STRINGERS OUTSIDE γ_r=.1	=.5	=.9	STRINGERS INSIDE γ_r=.1	=.5	=.9
10	20	10	.00	.00	.00	.00	.05	.05
		30	.00	.00	.00	.00	.00	.00
	50	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
	80	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
100	20	10	.10	.10	.10	.15	.15	.10
		30	.00	.05	.05	.05	.05	.05
	50	10	.05	.05	.05	.10	.10	.10
		30	.05	.05	.05	.05	.05	.05
	80	10	.05	.05	.05	.10	.10	.05
		30	.05	.05	.05	.05	.05	.05
500	20	10	.10	.05	.05	.15	.15	.15
		30	.15	.15	.15	.20	.20	.20
	50	10	.05	.05	.05	.10	.05	.05
		30	.15	.10	.10	.20	.15	.15
	80	10	.05	.05	.05	.05	.05	.05
		30	.05	.05	.05	.15	.10	.10
1,000	20	10	.10	.05	.05	.15	.15	.10
		30	.20	.20	.15	.25	.25	.20
	50	10	.05	.05	.05	.10	.05	.05
		30	.10	.05	.05	.15	.15	.10
	80	10	.05	.05	.05	.05	.05	.05
		30	.05	.05	.05	.10	.10	.10
5,000	20	10	.10	.05	.05	.15	.15	.10
		30	.20	.15	.15	.20	.20	.20
	50	10	.05	.05	.05	.10	.05	.05
		30	.10	.10	.05	.15	.15	.15
	80	10	.05	.05	.05	.05	.05	.05
		30	.05	.05	.05	.15	.10	.10
10,000	20	10	.10	.05	.05	.15	.15	.10
		30	.20	.15	.15	.25	20	.20
	50	10	.05	.05	.05	.10	.10	.05
		30	.10	.10	.05	.15	.15	.10
	80	10	.05	.05	.05	.05	.05	.05
		30	.05	.05	.05	.15	.10	.10
50,000	20	10	.10	.05	.05	.15	.15	.10
		30	.20	.15	.15	.25	.20	.20
	50	10	.05	.05	.05	.10	.10	.05
		30	.10	.10	.05	.15	.15	.10
	80	10	.05	.05	.05	.05	.05	.05
		30	.05	.05	.05	.15	.10	.10
100,000	20	10	.10	.05	.05	.15	.15	.10
		30	.20	.15	.15	.25	.20	.20
	50	10	.05	.05	.05	.10	.10	.05
		30	.10	.10	.05	.15	.15	.10
	80	10	.05	.05	.05	.05	.05	.05
		30	.05	.05	.05	.15	.15	.10

Table 4. - Ring Area Parameter $\overline{R}$ for Maximum Structural Efficiency.
$\overline{S} = 0.3$; $\gamma_s = 0.9$; $k_s = k_r = 0$; $\overline{G}_s = \overline{G}_r = 0$; Rings Inside.

Z	h_r/t	h_s/t	STRINGERS OUTSIDE $\gamma_r=.1$	$=.5$	$=.9$	STRINGERS INSIDE $\gamma_r=.1$	$=.5$	$=.9$
10	20	10	.00	.00	.00	.00	.00	.00
		30	.00	.00	.00	.00	.00	.00
	50	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
	80	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
100	20	10	.05	.05	.05	.15	.15	.10
		30	.00	.00	.00	.05	.05	.05
	50	10	.05	.05	.05	.10	.10	.10
		30	.00	.00	.00	.05	.05	.05
	80	10	.05	.05	.05	.10	.10	.05
		30	.00	.00	.00	.05	.05	.05
500	20	10	.10	.10	.05	.15	.15	.15
		30	.10	.10	.10	.20	.20	.20
	50	10	.05	.05	.05	.10	.10	.05
		30	.10	.10	.10	.20	.15	.15
	80	10	.05	.05	.05	.05	.05	.05
		30	.10	.05	.05	.15	.15	.15
1,000	20	10	.10	.10	.05	.15	.15	.10
		30	.20	.20	.15	.25	.25	.25
	50	10	.05	.05	.05	.10	.10	.05
		30	.10	.10	.05	.15	.10	.10
	80	10	.05	.05	.05	.05	.05	.05
		30	.05	.05	.05	.05	.05	.05
5,000	20	10	.10	.10	.05	.15	.15	.10
		30	.25	.20	.15	.25	.20	.20
	50	10	.05	.05	.05	.10	.10	.10
		30	.10	.10	.10	.20	.15	.15
	80	10	.05	.05	.05	.10	.05	.05
		30	.10	.05	.05	.15	.10	.10
10,000	20	10	.10	.10	.05	.15	.15	.15
		30	.25	.20	.15	.25	.20	.20
	50	10	.05	.05	.05	.10	.10	.10
		30	.10	.10	.05	.15	.15	.15
	80	10	.05	.05	.05	.10	.05	.05
		30	.05	.05	.05	.15	.10	.10
50,000	20	10	.10	.05	.05	.15	.15	.10
		30	.25	.20	.15	.25	.20	.20
	50	10	.05	.05	.05	.10	.10	.10
		30	.10	.10	.05	.15	.15	.15
	80	10	.05	.05	.05	.10	.05	.05
		30	.05	.05	.05	.15	.10	.10
100,000	20	10	.10	.05	.05	.15	.15	.10
		30	.25	.20	.15	.25	.20	.20
	50	10	.05	.05	.05	.10	.10	.05
		30	.10	.05	.05	.15	.15	.15
	80	10	.05	.05	.05	.10	.05	.05
		30	.05	.05	.05	.15	.10	.10

Table 5. - Ring Area Parameter $\overline{R}$ for Maximum Structural Efficiency.
$\overline{S} = 0.5$; $\gamma_s = 0.1$; $k_s = k_r = 0$; $\overline{G}_s = \overline{G}_r = 0$; Rings Inside.

Z	h_r/t	h_s/t	STRINGERS OUTSIDE $\gamma_r = .1$	$= .5$	$= .9$	STRINGERS INSIDE $\gamma_r = .1$	$= .5$	$= .9$
10	20	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
	50	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
	80	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
100	20	10	.10	.10	.10	.15	.15	.15
		30	.00	.00	.00	.05	.05	.05
	50	10	.10	.05	.05	.15	.10	.10
		30	.05	.05	.05	.10	.10	.05
	80	10	.05	.05	.05	.10	.10	.05
		30	.05	.05	.05	.10	.05	.05
500	20	10	.10	.05	.05	.20	.15	.15
		30	.15	.15	.15	.25	.25	.20
	50	10	.05	.05	.05	.10	.10	.05
		30	.15	.10	.05	.20	.20	.15
	80	10	.05	.05	.05	.10	.05	.05
		30	.05	.05	.05	.15	.10	.10
1,000	20	10	.10	.05	.05	.15	.15	.10
		30	.20	.20	.15	.30	.25	.20
	50	10	.05	.05	.05	.10	.10	.10
		30	.10	.10	.05	.15	.15	.15
	80	10	.05	.05	.05	.10	.05	.05
		30	.10	.05	.05	.15	.15	.10
5,000	20	10	.10	.05	.05	.15	.15	.10
		30	.20	.15	.15	.25	.25	.20
	50	10	.05	.05	.05	.10	.10	.05
		30	.10	.10	.05	.15	.15	.15
	80	10	.05	.05	.05	.05	.05	.05
		30	.05	.05	.05	.15	.10	.10
10,000	20	10	.10	.05	.05	.15	.15	.10
		30	.20	.15	.15	.25	.20	.20
	50	10	.05	.05	.05	.10	.10	.05
		30	.10	.10	.05	.15	.15	.15
	80	10	.05	.05	.05	.05	.05	.05
		30	.05	.05	.05	.15	.10	.10
50,000	20	10	.10	.05	.05	.15	.15	.10
		30	.20	.15	.15	.25	.25	.20
	50	10	.05	.05	.05	.10	.10	.05
		30	.10	.10	.05	.15	.15	.15
	80	10	.05	.05	.05	.05	.05	.05
		30	.05	.05	.05	.15	.10	.10
100,000	20	10	.10	.05	.05	.15	.15	.10
		30	.20	.15	.15	.25	.20	.20
	50	10	.05	.05	.05	.10	.10	.05
		30	.10	.10	.05	.15	.15	.15
	80	10	.05	.05	.05	.10	.05	.05
		30	.05	.05	.05	.15	.10	.10

Table 6. - Ring Area Parameter $\overline{R}$ for Maximum Structural Efficiency.
$\overline{S} = 0.5$; $\gamma_s = 0.5$; $k_s = k_r = 0$; $\overline{G}_s = \overline{G}_r = 0$; Rings Inside.

Z	h_r/t	h_s/t	STRINGERS OUTSIDE $\gamma_r=.1$	$=.5$	$=.9$	STRINGERS INSIDE $\gamma_r=.1$	$=.5$	$=.9$
10	20	10	.00	.00	.00	.00	.00	.05
		30	.00	.00	.00	.00	.00	.00
	50	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
	80	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
100	20	10	.05	.05	.05	.15	.15	.15
		30	.00	.00	.00	.05	.05	.05
	50	10	.05	.05	.05	.10	.10	.10
		30	.00	.00	.00	.05	.05	.05
	80	10	.05	.05	.05	.10	.10	.10
		30	.00	.00	.00	.05	.05	.05
500	20	10	.10	.10	.10	.15	.15	.15
		30	.10	.10	.10	.20	.20	.20
	50	10	.05	.05	.05	.15	.10	.10
		30	.10	.10	.10	.20	.20	.20
	80	10	.05	.05	.05	.10	.05	.05
		30	.10	.05	.05	.20	.20	.15
1,000	20	10	.10	.10	.05	.15	.15	.15
		30	.20	.20	.20	.30	.25	.25
	50	10	.05	.05	.05	.15	.10	.10
		30	.10	.10	.05	.20	.15	.25
	80	10	.05	.05	.05	.10	.05	.05
		30	.05	.05	.05	.15	.15	.10
5,000	20	10	.10	.10	.05	.20	.15	.15
		30	.20	.20	.20	.30	.25	.20
	50	10	.05	.05	.05	.10	.10	.10
		30	.10	.10	.10	.20	.15	.15
	80	10	.05	.05	.05	.10	.05	.05
		30	.10	.05	.05	.15	.15	.10
10,000	20	10	.10	.10	.05	.20	.15	.15
		30	.25	.20	.15	.25	.25	.25
	50	10	.05	.05	.05	.10	.10	.10
		30	.10	.10	.05	.20	.20	.15
	80	10	.05	.05	.05	.10	.05	.05
		30	.10	.05	.05	.15	.15	.15
50,000	20	10	.10	.10	.05	.20	.15	.15
		30	.25	.20	.15	.25	.25	.20
	50	10	.05	.05	.05	.10	.10	.10
		30	.10	.10	.05	.20	.15	.15
	80	10	.05	.05	.05	.10	.05	.05
		30	.10	.05	.05	.15	.15	.10
100,000	20	10	.10	.10	.05	.20	.15	.15
		30	.25	.20	.15	.25	.25	.20
	50	10	.05	.05	.05	.10	.10	.10
		30	.10	.10	.10	.20	.15	.15
	80	10	.05	.05	.05	.10	.05	.05
		30	.05	.05	.05	.15	.15	.10

Table 7. - Ring Area Parameter $\overline{R}$ for Maximum Structural Efficiency.
$\overline{S} = 0.5$; $\gamma_s = 0.9$; $k_s = k_r = 0$; $\overline{G}_s = \overline{G}_r = 0$; Rings Inside.

Z	h_r/t	h_s/t	STRINGERS OUTSIDE γ_r=.1	=.5	=.9	STRINGERS INSIDE γ_r=.1	=.5	=.9
10	20	10	.00	.00	.00	.00	.00	.00
		30	.00	.00	.00	.00	.00	.00
	50	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
	80	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
100	20	10	.05	.05	.05	.15	.15	.15
		30	.00	.00	.00	.00	.05	.05
	50	10	.05	.05	.05	.10	.10	.10
		30	.00	.00	.00	.05	.05	.05
	80	10	.05	.05	.05	.10	.10	.10
		30	.00	.00	.00	.05	.05	.05
500	20	10	.10	.10	.05	.15	.15	.15
		30	.10	.10	.10	.20	.20	.20
	50	10	.05	.05	.05	.15	.10	.10
		30	.10	.10	.10	.20	.20	.20
	80	10	.05	.05	.05	.10	.10	.05
		30	.10	.10	.10	.20	.15	.15
1,000	20	10	.10	.10	.05	.25	.15	.15
		30	.20	.15	.15	.30	.25	.25
	50	10	.05	.05	.05	.15	.10	.10
		30	.15	.10	.10	.20	.20	.15
	80	10	.05	.05	.05	.10	.10	.05
		30	.10	.05	.05	.15	.10	.10
5,000	20	10	.10	.10	.05	.20	.15	.15
		30	.30	.20	.20	.30	.30	.25
	50	10	.05	.05	.05	.10	.10	.10
		30	.10	.10	.10	.20	.20	.20
	80	10	.05	.05	.05	.10	.10	.05
		30	.10	.05	.05	.20	.15	.10
10,000	20	10	.10	.10	.05	.20	.15	.15
		30	.25	.20	.20	.30	.30	.25
	50	10	.05	.05	.05	.15	.10	.10
		30	.15	.10	.10	.20	.20	.15
	80	10	.05	.05	.05	.10	.10	.05
		30	.10	.05	.05	.15	.15	.15
50,000	20	10	.10	.10	.05	.20	.15	.15
		30	.30	.20	.20	.30	.25	.25
	50	10	.05	.05	.05	.15	.10	.10
		30	.10	.10	.10	.20	.20	.15
	80	10	.05	.05	.05	.10	.10	.05
		30	.10	.05	.05	.15	.15	.15
100,000	20	10	.10	.10	.05	.20	.15	.15
		30	.30	.20	.20	.30	.25	.25
	50	10	.05	.05	.05	.15	.10	.10
		30	.10	.10	.10	.20	.20	.15
	80	10	.05	.05	.05	.10	.10	.05
		30	.10	.05	.05	.20	.15	.15

Table 8. - Ring Area Parameter $\overline{R}$ for Maximum Structural Efficiency. $\overline{S} = 0.7$; $\gamma_s = 0.1$; $k_s = k_r = 0$; $\overline{G}_s = \overline{G}_r = 0$; Rings Inside.

Z	h_r/t	h_s/t	STRINGERS OUTSIDE $\gamma_r = .1$	$= .5$	$= .9$	STRINGERS INSIDE $\gamma_r = .1$	$= .5$	$= .9$
10	20	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
	50	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
	80	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
100	20	10	.10	.10	.10	.15	.15	.15
		30	.00	.00	.00	.05	.05	.05
	50	10	.10	.05	.05	.15	.10	.10
		30	.00	.00	.00	.10	.10	.05
	80	10	.05	.05	.05	.10	.10	.10
		30	.00	.00	.00	.10	.05	.05
500	20	10	.10	.10	.05	.20	.20	.15
		30	.15	.15	.15	.25	.25	.25
	50	10	.05	.05	.05	.10	.10	.10
		30	.15	.10	.10	.20	.20	.20
	80	10	.05	.05	.05	.10	.05	.05
		30	.10	.05	.05	.20	.20	.15
1,000	20	10	.10	.05	.05	.20	.15	.15
		30	.20	.20	.15	.30	.30	.30
	50	10	.05	.05	.05	.10	.10	.10
		30	.10	.05	.05	.15	.15	.15
	80	10	.05	.05	.05	.10	.05	.05
		30	.05	.05	.05	.15	.15	.10
5,000	20	10	.10	.05	.05	.20	.15	.15
		30	.25	.15	.15	.25	.25	.25
	50	10	.05	.05	.05	.10	.10	.10
		30	.10	.10	.05	.20	.15	.15
	80	10	.05	.05	.05	.10	.05	.05
		30	.05	.05	.05	.15	.15	.10
10,000	20	10	.10	.05	.05	.20	.15	.15
		30	.25	.15	.15	.30	.25	.25
	50	10	.05	.05	.05	.10	.10	.10
		30	.10	.10	.05	.20	.15	.15
	80	10	.05	.05	.05	.10	.05	.05
		30	.05	.05	.05	.15	.15	.10
50,000	20	10	.10	.05	.05	.20	.15	.15
		30	.20	.20	.15	.30	.25	.25
	50	10	.05	.05	.05	.10	.10	.10
		30	.10	.10	.05	.20	.20	.15
	80	10	.05	.05	.05	.10	.05	.05
		30	.05	.05	.05	.15	.15	.15
100,000	20	10	.10	.05	.05	.20	.15	.15
		30	.20	.20	.15	.30	.25	.25
	50	10	.05	.05	.05	.10	.10	.10
		30	.10	.10	.05	.20	.20	.15
	80	10	.05	.05	.05	.10	.05	.05
		30	.05	.05	.05	.15	.15	.10

Table 9. - Ring Area Parameter $\overline{R}$ for Maximum Structural Efficiency.
$\overline{S} = 0.7$; $\gamma_s = 0.5$; $k_s = k_r = 0$; $\overline{G}_s = \overline{G}_r = 0$; Rings Inside.

Z	h_r/t	h_s/t	STRINGERS OUTSIDE γ_r=.1	=.5	=.9	STRINGERS INSIDE γ_r=.1	=.5	=.9
10	20	10	.00	.00	.00	.00	.00	.05
		30	.00	.00	.00	.00	.00	.00
	50	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
	80	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
100	20	10	.05	.05	.05	.15	.15	.15
		30	.00	.00	.00	.00	.05	.05
	50	10	.05	.05	.05	.15	.10	.10
		30	.00	.00	.00	.05	.05	.05
	80	10	.05	.05	.05	.10	.10	.10
		30	.00	.00	.00	.05	.05	.05
500	20	10	.10	.10	.10	.20	.15	.15
		30	.10	.10	.10	.20	.20	.20
	50	10	.05	.05	.05	.15	.15	.10
		30	.10	.10	.10	.20	.20	.20
	80	10	.05	.05	.05	.15	.10	.05
		30	.10	.10	.10	.20	.20	.15
1,000	20	10	.10	.10	.05	.20	.15	.15
		30	.20	.20	.20	.30	.30	.30
	50	10	.05	.05	.05	.15	.15	.10
		30	.15	.10	.05	.25	.20	.15
	80	10	.05	.05	.05	.10	.10	.05
		30	.05	.05	.05	.15	.15	.10
5,000	20	10	.10	.10	.05	.20	.20	.15
		30	.25	.20	.20	.35	.30	.25
	50	10	.05	.05	.05	.15	.10	.10
		30	.15	.10	.10	.25	.20	.20
	80	10	.05	.05	.05	.10	.10	.05
		30	.10	.05	.05	.20	.15	.15
10,000	20	10	.10	.10	.05	.20	.20	.15
		30	.30	.20	.20	.30	.25	.25
	50	10	.05	.05	.05	.15	.10	.10
		30	.10	.10	.10	.25	.20	.15
	80	10	.05	.05	.05	.10	.10	.10
		30	.10	.05	.05	.20	.15	.15
50,000	20	10	.10	.10	.05	.20	.20	.15
		30	.25	.20	.20	.30	.25	.25
	50	10	.05	.05	.05	.15	.10	.10
		30	.10	.10	.10	.20	.20	.20
	80	10	.05	.05	.05	.10	.10	.10
		30	.10	.05	.05	.20	.15	.15
100,000	20	10	.10	.10	.05	.20	.20	.15
		30	.25	.20	.20	.30	.25	.25
	50	10	.05	.05	.05	.15	.10	.10
		30	.10	.10	.10	.20	.20	.20
	80	10	.05	.05	.05	.10	.10	.10
		30	.10	.05	.05	.20	.15	.15

Table 10. - Ring Area Parameter $\overline{R}$ for Maximum Structural Efficiency.
$\overline{S} = 0.7$; $\gamma_s = 0.9$; $k_s = k_r = 0$; $\overline{G}_s = \overline{G}_r = 0$; Rings Inside.

Z	h_r/t	h_s/t	STRINGERS OUTSIDE γ_r=.1	=.5	=.9	STRINGERS INSIDE γ_r=.1	=.5	=.9
10	20	10	.00	.00	.00	.00	.00	.00
		30	.00	.00	.00	.00	.00	.00
	50	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
	80	10	.00	.00	.00	.05	.05	.05
		30	.00	.00	.00	.00	.00	.00
100	20	10	.05	.05	.05	.15	.15	.15
		30	.00	.00	.00	.00	.00	.05
	50	10	.05	.05	.05	.15	.10	.10
		30	.00	.00	.00	.05	.05	.05
	80	10	.05	.05	.05	.10	.10	.10
		30	.00	.00	.00	.05	.05	.05
500	20	10	.10	.10	.05	.20	.15	.15
		30	.10	.10	.10	.20	.20	.20
	50	10	.05	.05	.05	.15	.15	.10
		30	.10	.10	.10	.20	.20	.20
	80	10	.05	.05	.05	.15	.10	.10
		30	.10	.10	.10	.20	.20	.15
1,000	20	10	.10	.10	.05	.25	.25	.20
		30	.15	.15	.15	.30	.30	.25
	50	10	.05	.05	.05	.15	.15	.10
		30	.15	.10	.10	.25	.25	.25
	80	10	.05	.05	.05	.10	.10	.10
		30	.10	.05	.05	.25	.15	.10
5,000	20	10	.10	.10	.05	.25	.20	.20
		30	.30	.25	.20	.30	.30	.30
	50	10	.05	.05	.05	.15	.15	.10
		30	.15	.10	.10	.25	.20	.20
	80	10	.05	.05	.05	.10	.10	.10
		30	.10	.05	.05	.20	.20	.15
10,000	20	10	.10	.10	.05	.25	.20	.15
		30	.30	.25	.20	.30	.30	.30
	50	10	.05	.05	.05	.15	.15	.10
		30	.15	.10	.10	.25	.20	.20
	80	10	.05	.05	.05	.10	.10	.10
		30	.10	.05	.05	.20	.15	.15
50,000	20	10	.10	.10	.05	.25	.20	.20
		30	.30	.25	.20	.35	.30	.30
	50	10	.05	.05	.05	.15	.15	.10
		30	.15	.10	.10	.25	.20	.20
	80	10	.05	.05	.05	.10	.10	.10
		30	.10	.05	.05	.20	.15	.15
100,000	20	10	.10	.10	.05	.20	.20	.20
		30	.30	.25	.20	.30	.30	.25
	50	10	.05	.05	.05	.15	.15	.10
		30	.15	.10	.10	.25	.20	.20
	80	10	.05	.05	.05	.10	.10	.10
		30	.10	.05	.05	.20	.15	.15

Table 11. - Calculated Design Variables and Load Ratios. Stringers Inside; Rings Inside; Applied Axial Load, N_{x_A} = 800 lb./in.

$R = 9.55$ in. $L = 38.0$ in.

$E = E_s = E_r = 10.5 \times 10^6$ psi

$\mu = \mu_s = \mu_r = .33$

$\rho = \rho_s = \rho_r = .101$ lb./in.^{3}

$t_{fs}/t_{ws} = t_{fr}/t_{wr} = 1.0$

$\sigma = \sigma_{Ys} = \sigma_{Yr} = 50{,}000$ psi

Variable	Value	Failure Mode	N_{x_A}/N_x
t	.0107 (in.)	General Instability	.99
t_{ws}	.0044 (in.)	Panel Instability	.98
t_{wr}	.0030 (in.)	Skin Yielding	.99
h_{ws}	.144 (in.)	Stringer Yielding	1.0
h_{wr}	.247 (in.)	Ring Yielding	.31
c_{fs}	.070 (in.)	Local Skin Buckling	1.0
c_{fr}	.120 (in.)	Local Stringer Buckling	1.0
d	.246 (in.)	Local Ring Buckling	-
l	2.71 (in.)		
W	3.78 (lbs.)		

Table 12. - Stiffener Parameters and Ratios for Minimum Weight Design. Stringers Inside; Rings Inside; Applied Axial Load, N_{x_A} = 800 lb./in.

R = 9.55 in. L = 38.0 in.

$E = E_s = E_r = 10.5 \times 10^6$ psi

$\mu = \mu_s = \mu_r = .33$

$\rho = \rho_s = \rho_r = .101$ lb./in.^{3}

$\sigma = \sigma_{Ys} = \sigma_{Yr}$ = 50,000 psi

t_{fs}/t_{ws}, t_{fr}/t_{wr}	0.0	0.5	1.0	1.5	2.0
$\overline{S}$	0.34	0.40	0.48	0.50	0.60
$\overline{R}$	0.048	0.040	0.050	0.070	0.125
h_s/t	16.0	15.6	13.4	11.0	9.3
h_r/t	27.5	26.0	23.0	20.0	21.0
γ_s, γ_r	0.0	0.19	0.49	0.68	0.79
h_s/t_{ws}	32.	32.	32.	32.	32.
h_r/t_{wr}	81.	82.	82.	81.	81.
W (lbs.)	3.79	3.77	3.78	3.85	3.92

Table 13. - Calculated Design Variables and Load Ratios. Stringers Inside; Rings Inside; Applied Axial Load, N_{x_A} = 100,000 lb./in.

$R = 200$ in. $L = 200$ in.

$E = E_s = E_r = 10.5 \times 10^6$ psi

$\mu = \mu_s = \mu_r = .33$

$\rho = \rho_s = \rho_r = .101$ lb./in.^{3}

$t_{fs}/t_{ws} = t_{fr}/t_{wr} = 1.0$

$\sigma = \sigma_{Ys} = \sigma_{Yr} = 50{,}000$ psi

Variable	Value	Failure Mode	N_{x_A}/N_x
t	1.11 (in.)	General Instability	-
t_{ws}	.567 (in.)	Panel Instability	.28
t_{wr}	-	Skin Yielding	1.0
h_{ws}	18.3 (in.)	Stringer Yielding	1.0
h_{wr}	-	Ring Yielding	.33
c_{fs}	8.89 (in.)	Local Skin Buckling	1.0
c_{fr}	-	Local Stringer Buckling	1.0
d	23.0 (in.)	Local Ring Buckling	-
l	-		
W	50768. (lbs.)		

Table 14. - Calculated Design Variables and Load Ratios. Stringers Inside; Rings Inside; Applied Axial Load, N_{x_A} = 10,000 lb./in.

$R = 200$ in. $L = 200$ in.

$E = E_s = E_r = 10.5 \times 10^6$ psi

$\mu = \mu_s = \mu_r = .33$

$\rho = \rho_s = \rho_r = .101$ lb./in.^{3}

$t_{fs}/t_{ws} = t_{fr}/t_{wr} = 1.0$

$\sigma = \sigma_{Ys} = \sigma_{Yr} = 50{,}000$ psi

Variable	Value	Failure Mode	N_{x_A}/N_x
t	.132 (in.)	General Instability	.99
t_{ws}	.0556 (in.)	Panel Instability	.96
t_{wr}	.0656 (in.)	Skin Yielding	.99
h_{ws}	1.80 (in.)	Stringer Yielding	1.0
h_{wr}	5.28 (in.)	Ring Yielding	.28
c_{fs}	.872 (in.)	Local Skin Buckling	1.0
c_{fr}	2.55 (in.)	Local Stringer Buckling	1.0
d	2.98 (in.)	Local Ring Buckling	-
l	33.3 (in.)		
W	5460. (lbs.)		

Table 15. - Calculated Design Variables and Load Ratios. Stringers Inside; Rings Inside; Applied Axial Load, N_{x_A} = 1,000 lb./in.

R = 200 in. L = 200 in.

$E = E_s = E_r = 10.5 \times 10^6$ psi

$\mu = \mu_s = \mu_r = .33$

$\rho = \rho_s = \rho_r = .101$ lb./in.^{3}

$t_{fs}/t_{ws} = t_{fr}/t_{wr} = 1.0$

$\sigma = \sigma_{Ys} = \sigma_{Yr} = 50{,}000$ psi

Variable	Value	Failure Mode	N_{x_A}/N_x
t	.0309 (in.)	General Instability	.99
t_{ws}	.0140 (in.)	Panel Instability	.84
t_{wr}	.0271 (in.)	Skin Yielding	.39
h_{ws}	.718 (in.)	Stringer Yielding	.40
h_{wr}	2.19 (in.)	Ring Yielding	.11
c_{fs}	.347 (in.)	Local Skin Buckling	1.0
c_{fr}	1.06 (in.)	Local Stringer Buckling	1.0
d	1.07 (in.)	Local Ring Buckling	-
l	20.0 (in.)		
W	1389. (lbs.)		