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SWINGBY

A LOW THRUST INTERPLANETARY SWINGBY TRAJECTORY OPTIMIZATION PROGRAM

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FOREWORD

This report describes work performed under Contract NAS5-11193 for the NASA Goddard Space Flight Center. It consists of two parts. Part I presents the analytical development and Part II describes the SWINGBY computer program.

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ABSTRACT

SWINGBY is a segmented two-body low thrust interplanetary swingby trajectory and performance optimization program. The program explicitly includes both planetocentric and heliocentric phases by linking together a series of two-body, low thrust trajectories that are alternately planetocentric and heliocentric. At the patch points, the position and velocity are continuous, although the gravitational acceleration is discontinuous. Particular attention is given to the severe sensitivity problem inherent in swingby trajectories, and the program is designed to greatly alleviate the problem. Wide flexibility is provided in selecting the performance index and in specifying boundary conditions. Provisions are also made for generating optimum single leg trajectories. The indirect method of optimization is employed.

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PART I

ANALYSIS AND PROBLEM DEFINITION

NOMENCLATURE

Symbol	Definition
A	Vector of control parameters.
$\bar{a}$, $\bar{b}$, $\bar{c}$	Arbitrary set of unit vectors.
a_i	Coefficients in polynomial expression for power variation with solar distance.
a_o	Reference thrust acceleration, equal to the thrust evaluated at 1 AU divided by the initial spacecraft mass.
b	Coefficient in equation for propulsion system efficiency.
B	Vector of parameters representing arbitrary functions of the boundary values of the state variables.
b_1, b_2, b_3	Coefficients in the expression for the initial spacecraft mass.
c	Jet exhaust speed of the low thrust propulsion system.
d	Constant appearing in the equation for the propulsion system efficiency.
e_e	Eccentricity of the launch hyperbola.
e_t	Eccentricity of the hyperbolic trajectory approaching the target planet prior to the retro maneuver.
$\bar{e}_T$	Unit vector in the direction of thrust.
f	Scalar function defined by equation (88).
F_e	Hyperbolic anomaly of the launch hyperbola at the sphere of influence.
f_r	Scalar function defined by equation (157).
F_t	Hyperbolic anomaly of the target planet approach hyperbola at the sphere of influence.

Symbol	Definition
F_1, F_2	Right hand side of general first order differential equation before and after, respectively, an event which causes the form of the differential equation to change.
G	General form of the vector of discontinuities in state variables.
g_ν	Element of G associated with the mass ratio.
H_p	Planetocentric angular momentum vector at the closest approach point of the swingby planet.
h_ρ	Step function, equal to one inside the swingby planet sphere of influence and equal to zero outside.
h_σ	Step function, equal to one if the switch function is positive and equal to zero if the switch function is negative.
i	Inclination of the heliocentric trajectory relative to the ecliptic plane.
$\bar{i}, \bar{j}, \bar{k}$	Unit vectors along the axes of an inertial Cartesian coordinate system, $\bar{i}$ directed toward the Vernal Equinox, $\bar{k}$ along the ecliptic North Pole, and $\bar{j} = \bar{k} \times \bar{i}$.
i_e	Inclination of the launch hyperbola relative to the equatorial plane of the launch planet.
i_t	Inclination of the approach hyperbola relative to the equatorial plane of the target planet.
j	Variational Hamiltonian.
j_c	Flag set equal to zero for optimal thrusting within swingby planet sphere of influence and equal to one for imposed coasting.
j_{ps}	Flag set equal to one if the low thrust propulsion system is to be jettisoned prior to the retro maneuver and equal to zero otherwise.
j_r	Flag set equal to one if a retro stage is to be employed and equal to zero otherwise.
j_t	Flag set equal to one if the low thrust propellant tankage is to be jettisoned prior to a retro maneuver and equal to zero otherwise.

Symbol	Definition
k_r	Tankage factor of retro stage.
k_s	Structural factor of spacecraft.
k_t	Low thrust propellant tankage factor.
k_x	Swingby planet scientific package mass factor.
$\bar{\ell}_e, \bar{m}_e, \bar{n}_e$	Unit vectors along axes of Cartesian coordinate system associated with launch planet; $\bar{n}_e$ is direction of launch planet North Pole, $\bar{\ell}_e$ along the vector $\bar{k} \times \bar{n}_e$, and $\bar{m}_e = \bar{n}_e \times \bar{\ell}_e$.
$\bar{\ell}_p, \bar{m}_p, \bar{n}_p$	Unit vectors defined as above, except associated with the swingby planet.
$\bar{\ell}_t, \bar{m}_t, \bar{n}_t$	Unit vectors defined as above, except associated with the target planet.
m_i	Inert mass of the retro stage.
m_n	Net spacecraft mass.
m_o	Initial spacecraft mass.
m_p	Low-thrust propellant mass.
m_{pr}	Retro propellant mass.
m_{ps}	Low thrust propulsion system mass.
m_r	Retro stage mass.
m_s	Spacecraft structural mass (proportional to initial mass).
m_t	Low thrust propellant tankage mass.
m_x	Mass of scientific package jettisoned at the swingby planet.

Symbol	Definition
N	Lagrange multiplier vector used to adjoin state variable discontinuities to the performance index.
P	Heliocentric position vector of the swingby planet.
p_c	Net conditioned power.
P_e	Heliocentric position vector of the launch planet.
p_j	Beam power of the propulsion system.
p_o	Reference power of the propulsion system, evaluated at 1 AU.
P_t	Heliocentric position vector of the target planet.
Q	Vector function defined by equation (26).
r	Heliocentric distance of the spacecraft.
R	Heliocentric position vector of the spacecraft.
R_e	Spacecraft position relative to the launch planet at exit from its sphere of influence.
r_{eo}	Radius of the launch parking orbit.
r_{es}	Radius of the launch planet sphere of influence.
r_p	Distance from the center of the swingby planet.
R_p	Planetocentric position vector of the spacecraft relative to the swingby planet.
r_{ps}	Radius of the swingby planet sphere of influence.
R_t	Spacecraft position relative to the target planet at entry into its sphere of influence.
r_{ta}	Apocenter distance of the final orbit about the target planet.

Symbol	Definition
r_{tf}	Pericenter distance of the final orbit about the target planet.
r_{ts}	Radius of the sphere of influence of the target planet.
s	Dummy variable of integration.
s_p	Value of s at which the sphere of influence is crossed.
t	Time.
t_f	Arrival date at the target.
t_o	Date of departure from the launch planet.
t_s	Time at the sphere of influence of a planet.
t_{sw}	Date of swingby.
$t_{\infty a}$	Time within the sphere of influence of the target planet.
$t_{\infty d}$	Time within the sphere of influence of the launch planet.
U	Vector of control variables.
v	Heliocentric speed of the spacecraft.
V	Heliocentric velocity of the spacecraft.
v_c	Characteristic speed required at the launch parking orbit to achieve the specified launch energy.
v_e	Escape speed at the launch parking orbit.
v_{eo}	Same as v_e.
v_{es}	Speed relative to the launch planet at exit from the sphere of influence.
V_f	Final heliocentric velocity vector.

Symbol	Definition
v_p	Speed relative to the swingby planet at closest approach.
v_r	Incremental speed required of the retro stage.
v_{tf}	Speed relative to the target planet at pericenter of the approach hyperbola before the retro maneuver.
v_{ts}	Speed relative to the target planet at entry of the sphere of influence.
$v_{\infty a}$	Hyperbolic excess speed at the target planet.
$V_{\infty a}$	Hyperbolic excess velocity at the target planet.
$v_{\infty d}$	Hyperbolic excess speed at the launch planet.
X	Vector of state variables.
x_f, y_f, z_f	Cartesian components of the final heliocentric position.
$x(s_p^+)$	Limiting value of the arbitrary function $x(s)$ at $s=s_p$ as s approaches s_p from $s > s_p$.
$x(s_p^-)$	Limiting value of the arbitrary function $x(s)$ at $s=s_p$ as s approaches s_p from $s < s_p$.
α	Specific propulsion system mass.
γ	Ratio of instantaneous power to power at 1 AU. Also denotes heliocentric flight path angle in discussion of end conditions.
γ_{es}	Flight path angle relative to the launch planet at exit from the sphere of influence.
γ_p	Flight path angle relative to the swingby planet.
γ_{ts}	Flight path angle relative to the target planet at entry into the sphere of influence.
γ^*	Derivative of the ratio of instantaneous power to reference power with respect to heliocentric distance.

Symbol	Definition
Δm_x	Constant increment of science package jettisoned at the swingby planet.
Δt_m	Total mission duration.
Δt_1	First leg flight time.
Δt_2	Second leg flight time.
η	Propulsion system efficiency.
η^*	Derivative of efficiency with respect to jet exhaust speed.
λ	Magnitude of the vector Λ.
Λ	Vector of variables adjoint to the state variables.
Λ_A	Function defined by equation (66).
λ_{a_o}	Lagrange multiplier adjoint to the reference thrust acceleration.
λ_c	Lagrange multiplier adjoint to the low thrust jet exhaust speed.
Λ_R	Vector of variables adjoint to the heliocentric position vector.
λ_t	Lagrange multiplier adjoint to time.
Λ_V	Vector of variables adjoint to the heliocentric velocity vector.
λ_V	Magnitude of Λ_V .
λ_ν	Lagrange multiplier adjoint to the mass ratio.
μ	Gravitational constant of the sun.
μ_e	Gravitational constant of the launch planet.

Symbol	Definition
μ_p	Gravitational constant of the swingby planet.
μ_t	Gravitational constant of the target planet.
ν	Ratio of instantaneous spacecraft mass to initial mass. Also, Lagrange multiplier used to adjoin function defining discontinuity time to the performance index.
ρ	Function defined by equation (24) or, for the more general problem, by equation (49).
σ	Switch function, defined by equations (81).
τ	Time interval between the swingby passage point and the sphere of influence of the launch or target planet.
ϕ	Performance index.
ϕ^*	Augmented performance index.
ϕ_{a_o}	Partial derivative of ϕ w.r.t. reference thrust acceleration.
ϕ_c	Partial derivative of ϕ w.r.t. jet exhaust speed.
ϕ_t	Partial derivative of ϕ w.r.t. time.
$\phi_{V_{\infty a}}$	Partial derivative of ϕ w.r.t. arrival hyperbolic excess speed.
$\phi_{V_{\infty d}}$	Partial derivative of ϕ w.r.t. departure hyperbolic excess speed.
ϕ_ν	Partial derivative of ϕ w.r.t. mass ratio.
Ψ	General form of the end condition constraints.
ω	Angular position of spacecraft measured in the osculating heliocentric trajectory plane from the line of ascending node positive in the direction of motion.

Symbol	Definition
ω_{es}	Angular position of the spacecraft at the launch planet sphere of influence, measured in the planetocentric trajectory plane from the line of ascending node, positive in the direction of motion.
$\tilde{\omega}_{es}$	$= \omega_{es} - \gamma_{es}$.
ω_p	Angular position of the spacecraft at the swingby passage point measured in the osculating planetocentric trajectory plane from the line of ascending node, positive in the direction of motion.
ω_{ts}	Angular position of the spacecraft at the target planet sphere of influence measured in the planetocentric trajectory plane from the line of ascending node, positive in the direction of motion.
$\tilde{\omega}_{ts}$	$= \omega_{ts} - \gamma_{ts}$.
Ω	Longitude of the ascending node of the heliocentric trajectory on the ecliptic plane measured eastward from $\bar{i}$.
Ω_e	Longitude of the ascending node of the planetocentric launch hyperbola on the launch planet equatorial plane measured eastward from $\bar{\ell}_e$.
Ω_p	Longitude of the ascending node of the osculating planetocentric trajectory relative to the swingby planet on the planet's equatorial plane measured eastward from $\bar{\ell}_p$.
Ω_t	Longitude of the ascending node of the planetocentric arrival hyperbola on the target planet equatorial plane measured eastward from $\bar{\ell}_t$.
$_1(\ ),\ _2(\ )$	Denotes that ( ) is evaluated along first and second trajectory leg, respectively.
$(\dot{\ }),(\ddot{\ }),(\dddot{\ })$	First, second, and third time derivative of ( ), respectively.
$(\ )',(\ )''$	First and second derivative, respectively, of ( ) w.r.t. s.
$(\tilde{\ })$	Denotes prescribed value of ( ) in Appendix A.

INTRODUCTION

The fact that significant performance gains may be achieved with the use of swingby (gravity-assist) trajectories for certain ballistic interplanetary and solar system missions has been known for some time. These gains may either be realized in terms of increased scientific payload for a given mission duration or, conversely, in terms of a reduction in flight time at little or no cost in payload. Swingby missions to nearby planets that appear particularly promising include Mercury probe missions past Venus [1] and short-duration round-trips to Mars employing a swingby past Venus on either the out-bound or homebound legs. [2,3] Because of its great mass, Jupiter has received much attention as a potential swingby planet for such missions as out-of-the-ecliptic, solar, and deep space probes,[4,5] and flybys of the outer planets. [6,7]

The extent to which the use of swingby trajectories are applicable to interplanetary and solar system missions employing electric propulsion can only be conjectured at the present time, because no concerted effort has been made to assess the performance of this particular combination of trajectory profile and propulsion system. The singular contribution to this subject is by Flandro,[8] who showed that significant performance gains may be achieved for Jupiter swingby missions to the outer planets by using solar electric propulsion on the Earth-Jupiter leg. He obtained the low-thrust swingby trajectories by linking the appropriate post-Jupiter ballistic continuation trajectory to an optimum low-thrust Earth-Jupiter flyby trajectory. This is probably not the optimum swingby trajectory; however, for solar electric propulsion, it may be a good approximation to the optimum. Flandro did not compare the performance of low-thrust swingbys with that of low-thrust direct flights to a specific destination. However, his data were compared to the author's unpublished low-thrust flyby trajectory data to the outer planets for the same launch vehicle and specific propulsion system mass. This comparison indicated that a payload gain of 30-50 percent or, conversely, a reduction in mission duration of 25 percent, may be achieved through the use of the Jupiter swingby for low-thrust missions to Uranus or Neptune.

The potential attractiveness of low-thrust swingby trajectories has not been overlooked in the past. The present absence of definitive information on the subject is simply due to the fact that it is an exceedingly difficult problem, and no computer programs capable of treating the problem are currently available. The generation of optimum low-thrust trajectory data for direct flights from Earth to a specific destination is no simple task, and introduction of the dynamics and the effects of an intermediate planetary encounter greatly magnifies the complexity and increases the dimensionality of the problem.

In this report, the necessary groundwork is laid for the development of a patched-conic low-thrust swingby trajectory optimization computer program. The problem to be treated is explicitly stated and formulated for solution by an indirect method of optimization. The Pontryagin Maximum Principle is then applied to yield the necessary conditions that must be satisfied by the solution to the problem. Throughout the formulation of the problem and solution, care is taken to assure maximum flexibility in choosing the type of mission, the mode of operation, and the variety of boundary conditions.

PROBLEM FORMULATION

Desired Program Features

The problem we seek to solve is that of finding an optimum low-thrust trajectory employing a gravitational assist from an intermediate planet. We wish to obtain the solution of this problem within the framework of a patched-conic formulation. That is, two-body motion is assumed throughout the mission with planetocentric motion assumed within a planet's sphere of influence and heliocentric motion outside. At the sphere of influence the trajectories in the two reference frames are patched so as to maintain continuity in both position and velocity.

To provide as much flexibility as possible in the operation of the program, a number of program options and features are desired. Among these are the following:

1) Permit optimal thrusting or imposed coasting within the sphere of influence of the swingby planet.

2) Allow specification of constraints on the swingby trajectory in terms of passage distance, passage speed, and/or inclination to the planet's equator.

3) Permit end conditions compatible with missions to non-specific space points, such as distance or inclination out-of-the-ecliptic, heliocentric distance, etc., as well as flyby and orbiter missions to any of the planets.

4) Incorporate a sufficiently general propulsion system model to permit simulating either nuclear-electric or solar-electric propulsion with high thrust maneuvers at each end.

5) Relate Earth-launch conditions to capabilities of specific launch vehicles.

Thrusting within the spheres of influence of either the launch or target planets will not be permitted; however, motion and time within the spheres are included in the formulation.

Propulsion System Characteristics

The low thrust propulsion system model will assume constant jet exhaust speed (i.e., constant specific impulse) for both nuclear and solar electric systems. For solar electric systems, the net conditioned power will be taken as a function of the heliocentric distance, i.e.,

$$p_c(r) = p_o \, \gamma(r) \tag{1}$$

where $p_c(r)$ is the net conditioned power at a distance r from the sun, p_o is the net conditioned power at 1 AU, and γ represents the variation in power as a function of distance. It will be assumed here that γ is of the form

$$\gamma = \frac{1}{r^2} \sum_{i=0}^{4} a_i \, r^{-i/2} \tag{2}$$

with the a_i being a set of input constants satisfying the constraint

$$\sum_{i=0}^{4} a_i = 1 \tag{3}$$

It is possible that the right side of (2) may be negative for certain values of r. If this occurs, it is understood that $\gamma(r)$ will be set to zero.

For nuclear electric propulsion systems, the same formulation will be employed; however, we will impose the identity

$$\gamma \equiv 1 \tag{4}$$

The overall efficiency, η, of the propulsion system will be taken as a function of the jet exhaust speed, c. The specific form assumed is

$$\eta = \frac{bc^2}{c^2 + d^2} \tag{5}$$

where b and d are input constants. Thus the jet, or beam, power, p_j, of the propulsion system is

$$p_j = \eta p_c = \eta \gamma p_o = \gamma m_o a_o c/2 \tag{6}$$

where a_o represents the thrust acceleration at 1 AU and m_o is the initial spacecraft mass. The mass of the propulsion system, m_{ps}, is assumed to be linearly proportional to the power at 1 AU, i.e.,

$$m_{ps} = \alpha p_o \tag{7}$$

where α denotes the specific mass of the propulsion system.

Spacecraft Mass Components

In addition to the propulsion system, the spacecraft is assumed to be comprised of low-thrust propellant and tankage, structure, retro propulsion system, and a mass package that is jettisoned upon approach of the swingby planet. The masses of these various systems will be denoted m_p and m_t for the propellant and tankage, respectively; m_s for the structure, m_r for the retro propulsion system, and m_x for the package jettisoned at the swingby planet. The spacecraft mass in excess of these items will be termed net spacecraft mass and denoted m_n. Then the initial spacecraft mass is written as the sum of these components as follows:

$$m_o = m_{ps} + m_p + m_t + m_s + m_r + m_x + m_n \tag{8}$$

The initial spacecraft mass is equated to the payload of the specified launch vehicle, which is related to the launch energy (departure hyperbolic excess speed). Since the launch vehicle payload capability may be closely approximated with a simple exponential equation in the characteristic velocity, we assume m_o to be of the form

$$m_o = b_1 e^{-v_c/b_2} - b_3 \tag{9}$$

where

$$v_c = \sqrt{v_{\infty d}^2 + v_e^2} \tag{10}$$

with $v_{\infty d}$ being the departure excess speed and v_e the escape speed from a 185 kilometer circular orbit, and b_1, b_2, and b_3 are input constants representing a specific launch vehicle. The low-thrust propellant and tankage masses are evaluated

$$m_p = \int_{t_o}^{t_f} (-\dot{m})\, dt \tag{11}$$

$$m_t = k_t m_p \tag{12}$$

where t_o and t_f denote initial and final times, respectively, $\dot{m}$ is the propellant flow rate of the propulsion system, and k_t is a specified proportionality factor. The structure mass is linearly proportional to the initial mass, i.e.,

$$m_s = k_s m_o \tag{13}$$

with k_s being a specified constant. The retro propulsion system is comprised of propellant, m_{pr}, and inert mass, m_i, which are evaluated as follows:

$$m_{pr} = (m_o - m_p - m_x - j_{ps} m_{ps} - j_t m_t)\,(1 - e^{-v_r/c_r}) \tag{14}$$

$$m_i = k_r m_{pr} \tag{15}$$

where j_{ps} and j_t are input flags, equal to one if the low thrust propulsion system and tankage are to be jettisoned prior to the retro maneuver and equal to zero otherwise, v_r is the velocity increment to be supplied by the retro stage, c_r is the retro jet exhaust speed, and k_r is a specified constant. The mass of the science package jettisoned at the swingby planet is written

$$m_x = \Delta m_x + k_x m_o \tag{16}$$

where both Δm_x and k_x are specified constants.

Equations of Motion

The two-body equations of motion of the spacecraft outside the sphere of influence of any planet may be written:*

$$\ddot{R} = h_\sigma \frac{a_o \gamma}{\nu} \bar{e}_T - \mu \frac{R}{r^3} \tag{17}$$

$$\dot{\nu} = - h_\sigma \frac{a_o \gamma}{c} \tag{18}$$

where a_o, c, and γ are as defined previously, R is the heliocentric position vector, $r = |R|$, ν is the ratio of instantaneous mass to initial mass, $\bar{e}_T$ is a unit vector in the direction of thrust, μ is the gravitational constant of the sun, and h_σ is a step function, equal to one if the low thrust propulsion system is on and zero if the propulsion system is off. Within the sphere of influence of the swingby planet, two-body motion relative to the planet is assumed and the acceleration may be written

$$\ddot{R}_p = (1-j_c)\, h_\sigma \frac{a_o \gamma}{\nu} \bar{e}_T - \mu_p \frac{R_p}{r_p^3} \tag{19}$$

where we define

$$R_p = R - P \tag{20}$$

*Upper case letters denote vectors, lower case letters with bars denote unit vectors, and all other lower case symbols denote scalars.

$$\dot{R}_p = \dot{R} - \dot{P} \tag{21}$$

with μ_p being the gravitational constant of the swingby planet, $r_p = |R_p|$, and P and $\dot{P}$ are the position and velocity, respectively, of the swingby planet. The flag j_c is input, equal to one if imposed coasting within the sphere of influence is desired and equal to zero otherwise. Finally, we make the assumption that, within the sphere of influence,

$$\ddot{R} = \ddot{R}_p + \ddot{P} \tag{22}$$

where $\ddot{P}$ is the acceleration of the swingby planet as obtained from whatever ephemeris model is being used. P, $\dot{P}$ and $\ddot{P}$ are functions only of time.

The equations (17) and (19) may be combined by introducing the Heaviside step function h_ρ with properties

$$h_\rho = \begin{cases} 1 & \text{if } \rho > 0 \\ 0 & \text{if } \rho < 0 \end{cases} \tag{23}$$

where

$$\rho = r_{ps} - r_p \tag{24}$$

with r_{ps} being a specified value representing the radius of the sphere of influence of the swingby planet. That is

$$\ddot{R} = (1 - j_c h_\rho)\, h_\sigma \frac{a_o \gamma}{\nu}\, \bar{e}_T - \mu \frac{R}{r^3} - h_\rho Q \tag{25}$$

where

$$Q = \mu_p \frac{R_p}{r_p^3} - \mu \frac{R}{r^3} - \ddot{P} \tag{26}$$

Also, we may write

$$\dot{\nu} = - (1-j_c h_\rho) h_\sigma \frac{a_o \gamma}{c} \tag{27}$$

Consequently, equations (25) and (27) represent the complete set of equations of motion and are valid throughout both the heliocentric and planetocentric phases of the trajectory.

Expanded State Equations

Our problem is to determine the optimum direction of $\bar{e}_T$ as a function of time and the conditions under which h_σ switches optimally between zero and one, subject to the condition that the spacecraft pass within the sphere of influence of the swingby planet. We also seek optimum values of selected boundary conditions as well as certain propulsion system parameters. Two optimization criteria are of particular interest—maximum net spacecraft mass and minimum mission duration. This optimization problem begins upon leaving the sphere of influence of the launch planet and terminates upon entry into the sphere of influence of the target planet.

This would be a standard problem in the calculus of variations were it not for the express and implied constraints imposed at an intermediate point of the problem (i.e., at the time of swingby). Variational problems with intermediate point constraints are treated classically using a technique known as the Denbow transformation. The purpose of the Denbow transformation is to convert the original problem with intermediate point constraints into one with only end-point constraints. This is accomplished by separating the path into a series of segments, the end-points of which are the end and the intermediate constraint points of the original problem. A dummy variable of integration is then introduced which is linear in time but scaled such that the time duration of each segment corresponds to the interval $[0,1]$ of the dummy variable. Finally, a complete set of state variables is introduced for each segment, and equations of motion are developed with the dummy variable as the independent variable of integration. The standard variational calculus approach may then be applied to the expanded set of equations of motion.

We will employ this general approach by separating the total mission into two segments, the first corresponding to the launch-to-swingby planet trajectory and the second corresponding to the swingby-to-target planet trajectory. Specifically, the point at which the two segments are separated is the point of closest approach of the swingby planet. A slight deviation from the standard application of the Denbow transformation will be pursued because we wish to commence the integration of both trajectory segments at the swingby planet and proceed in both directions to the launch and target planets. It is thought that this approach will help to reduce the sensitivity of the boundary value problem and thereby facilitate convergence.

In pursuit of our objective, we introduce the notation of pre-subscripts 1 and 2 to distinguish between variables associated with the first and second trajectory segments, respectively. Now, eliminate the second-order differential equations in favor of first-order equations by introducing the velocity vector, V, in the state; i.e.,

$$
{}_1\dot{R} = {}_1V
$$

$$
{}_1\dot{V} = (1-j_c\,{}_1h_\rho)\,{}_1h_\sigma \frac{a_{o1}\gamma}{{}_1\nu}\,{}_1\bar{e}_T - \frac{\mu}{{}_1r^3}\,{}_1R - {}_1h_\rho\,{}_1Q \qquad (28)
$$

$$
{}_1\dot{\nu} = -\,(1-j_c\,{}_1h_\rho)\,{}_1h_\sigma \frac{a_{o1}\gamma}{c}
$$

for the first segment, and

$$
{}_2\dot{R} = {}_2V
$$

$$
{}_2\dot{V} = (1-j_c\,{}_2h_\rho)\,{}_2h_\sigma \frac{a_{o2}\gamma}{{}_2\nu}\,{}_2\bar{e}_T - \frac{\mu}{{}_2r^3}\,{}_2R - {}_2h_\rho\,{}_2Q \qquad (29)
$$

$$
{}_2\dot{\nu} = -\,(1-j_c\,{}_2h_\rho)\,{}_2h_\sigma \frac{a_{o2}\gamma}{c}
$$

for the second segment. To convert these equations to the dummy independent variable, which will be denoted s, it is necessary to specify the relationships between time on each segment and s. Let ${}_1t_s$ and ${}_2t_s$ represent time on the first segment at the sphere of influence of the launch planet, and time on the second segment at the sphere of influence of the target planet, respectively, and denote t_{sw} as the time of closest approach of the swingby planet. Then, since we wish to integrate in both directions from the swingby closest approach point, we specify that $s = 0$ corresponds to the closest approach point of both segments and $s = 1$ corresponds to departing the launch planet sphere of influence on the first segment and entering the target planet sphere of influence on the second segment. Denoting ${}_1t$ and ${}_2t$ as time along the first and second segments, respectively, and defining the parameters

$$
\begin{aligned}
{}_1\tau &= t_{sw} - {}_1t_s \\
{}_2\tau &= {}_2t_s - t_{sw}
\end{aligned}
\tag{30}
$$

then time on each segment is related to s as follows:

$$
\begin{aligned}
{}_1t &= t_{sw} - {}_1\tau\, s \\
{}_2t &= t_{sw} + {}_2\tau\, s
\end{aligned}
\tag{31}
$$

Denoting derivatives with respect to s with the prime, one obtains by inspection

$$
\begin{aligned}
{}_1t' &= -\,{}_1\tau \\
{}_2t' &= {}_2\tau
\end{aligned}
\tag{32}
$$

Furthermore, since

$$
\begin{aligned}
{}_1(\dot{\ }) &= d\,{}_1(\ )/d\,{}_1t \\
{}_2(\dot{\ }) &= d\,{}_2(\ )/d\,{}_2t
\end{aligned}
\tag{33}
$$

the expanded set of equations of motion with s as the independent variable is immediately obtained from the chain rule; i.e.,

$$
\begin{aligned}
{}_1R' &= {}_1\dot{R}\,{}_1t' = -{}_1\tau\,{}_1V \\
{}_1V' &= {}_1\dot{V}\,{}_1t' = -{}_1\tau\left[(1-j_{c\,1}h_\rho)\,{}_1h_\sigma \frac{a_{o1}\gamma}{{}_1\nu}\,{}_1\bar{e}_T - \frac{\mu}{{}_1r^3}\,{}_1R - {}_1h_{\rho 1}Q\right] \\
{}_1\nu' &= {}_1\dot{\nu}\,{}_1t' = {}_1\tau\,(1-j_{c\,1}h_\rho)\,{}_1h_\sigma \frac{a_{o1}\gamma}{c} \\
{}_2R' &= {}_2\dot{R}\,{}_2t' = {}_2\tau\,{}_2V \\
{}_2V' &= {}_2\dot{V}\,{}_2t' = {}_2\tau\left[(1-j_{c\,2}h_\rho)\,{}_2h_\sigma \frac{a_{o2}\gamma}{{}_2\nu}\,{}_2\bar{e}_T - \frac{\mu}{{}_2r^3}\,{}_2R - {}_2h_{\rho 2}Q\right] \\
{}_2\nu' &= {}_2\dot{\nu}_2t' = -{}_2\tau\,(1-j_{c\,2}h_\rho)\,{}_2h_\sigma \frac{a_{o2}\gamma}{c}
\end{aligned}
\qquad (34)
$$

Equations (32) and (34) constitute the complete set of equations of motion; equations (32) are included because time must now be considered as a state variable since it has been replaced as the independent variable of integration but still appears explicitly in (34) through the ephemeris terms in Q.

Boundary Conditions

To maximize the flexibility and usefulness of the program, it is desirable to provide for as many and as varied boundary conditions as possible. The possible variations in the coordinate systems and specific parameters in which one may wish to express boundary conditions are astronomical. Consequently, it is necessary to decide at the outset upon the types of missions that the program is to be capable of treating and to define a reasonable set of parameters that will be of interest for each mission.

We are specifically interested in unmanned scientific missions, originating at the Earth, which swing by a planet enroute to some ultimate destination. Therefore, there are three points along the trajectory where the imposition of boundary conditions will be of general interest. The first point is, of course, at Earth departure, the second is at the swingby point, and the third is at the final destination. The specific boundary conditions provided for are listed in Appendix A. The considerations given in selecting these are indicated in the following paragraphs.

The assumed sequence of events in the Earth-departure phase is as follows:

1) The launch vehicle places the spacecraft in a 185 kilometer altitude circular parking orbit about the Earth.

2) At the appropriate point in the parking orbit, the upper stage of the launch vehicle injects the spacecraft onto an escape trajectory with an excess speed $v_{\infty d}$.

3) The spacecraft coasts to the sphere of influence of the Earth where the electric engines are turned on and the first heliocentric segment then begins.

The initial time of the problem (i.e., launch date) will be taken as the time of injection of the spacecraft onto the hyperbolic escape trajectory, even though the optimization problem does not commence until several hours later when the spacecraft reaches the sphere of influence. We are interested in boundary conditions at the sphere of influence where a total of eight degrees of freedom exist, six to define geocentric position and velocity, one to define mass, and one to define time (time is the one degree of freedom necessary to convert from geocentric to heliocentric position and velocity). Of these eight, two are used in specifying the radii of the parking orbit r_{eo} and the sphere of influence r_{es}; two others, $v_{\infty d}$ and time at the sphere of influence ${}_1t_s$, will be made available as independent parameters of the problem which may be fixed or optimized; and one, the initial mass ratio is, by definition, required to be unity. The remaining three degrees of freedom are left open and will be determined as an output of the solution. The specific equations representing the constraints at Earth are given in (A.4) with μ_e denoting the gravitational constant of Earth and $P_e(1)$ denoting the position of Earth at time ${}_1t(1)$. Note that the second of equations (A.4) is required because we have introduced a new variable $v_{\infty d}$ which is not independent of the state variables. The optional constraints on time at the sphere of influence and departure excess speed are represented by the first of equations (A.8) and (A.9). It may be helpful to note at this point that the required equality of ${}_1t(1)$ and the independent parameter ${}_1t_s$ is not expressly written in the Appendix A. This is not an oversight but simply a recognition that the required equality is trivially satisfied on all trajectories because it is used as the stopping condition of the numerical integration. A similar situation exists at the target between ${}_2t(1)$ and ${}_2t_s$.

If it is desired to fix the launch date (i.e., the date of departure from the parking orbit about Earth), the constraint (A.14) is imposed where $t_{\infty d}$ is the time within the Earth's sphere of influence, starting with injection from the parking orbit, and is computed as follows:

$$t_{\infty d} = (\mu_e/v_{\infty d}^3)\,(e_e \sinh f_e - f_e) \tag{35}$$

with

$$e_e = 1 + r_{eo} v_{\infty d}^2/\mu_e$$

$$f_e = \cosh^{-1}\left[\left(1 + \frac{r_{es} v_{\infty d}^2}{\mu_e}\right)/e_e\right] \tag{36}$$

At the swingby point, there are a total of sixteen degrees of freedom -- eight related to each of the two segments. Half of this total are used in the simple statement that the state is continuous at the swingby point. These are represented by equations (A.1) in Appendix A. Because of the desire to provide the capability of fixing such swingby trajectory parameters as radial distance, speed, and inclination, it is helpful to introduce the polar coordinates of the planetocentric position and velocity at the swingby point. Of the six coordinates required, three are, of course, the passage distance r_p, the passage speed v_p, and the inclination to the swingby planet's equator i_p. A fourth coordinate, whose choice is convenient, is the flight path angle γ_p which, by definition, is zero at the passage point. The fifth and sixth coordinates, which are chosen somewhat arbitrarily, are the angle Ω_p defining the orientation of the line of ascending node of the swingby trajectory and equatorial planes relative to the autumnal equinox of the swingby planet, and the angle ω_p defining the angular position of the spacecraft in the plane of motion relative to the line of ascending node. The statement that γ_p is zero is equivalent to the first of equations (A.3). The latter five equations in (A.3) provide a mathematical definition of the other five polar coordinates. In these equations, the unit vector $\bar{n}_p$ is a specified vector representing the normal to the swingby planet's equatorial plane in the direction of the planet's spin about its polar axis, and $\bar{k}$ is the unit vector normal to the ecliptic in the direction of the celestial north pole. The autumnal equinox of the swingby planet is defined as the direction along $(\bar{k} \times \bar{n}_p)$. Because we have chosen to employ planetocentric rather than heliocentric coordinates as independent parameters of the problem, the six equations represented by (A.2) are required to relate the two coordinate systems. The options of constraining the passage distance, speed and/or inclination are represented by equations (A.6), while similar options for mass ratio and time of swingby are represented by (A.7).

At the destination, there again are eight degrees of freedom and the manner in which these are used as boundary conditions will depend upon the particular type of mission under investigation. For example, if the ultimate destination is a planet, it is most convenient to write the boundary conditions in planetocentric coordinates. On the other hand, for space probe missions, the only meaningful coordinate system in which to express the boundary conditions is heliocentric. Specifically, we are primarily interested in providing boundary conditions that are consistent with four basic types of missions. These are the planetary orbiter, the planetary flyby, the deep space or solar probe, and the out-of-the-ecliptic missions. Associated with each of these mission types are parameters of common interest that will be made available as boundary conditions.

For planetary orbiter and flyby missions, the point at which we must concern ourselves with the eight degrees of freedom is the point of entry of the target planet sphere of influence. The sequence of events within the sphere of influence is basically the mirror image of that at Earth departure. For orbiter missions the spacecraft coasts from the sphere of influence to the pericenter of the planetocentric hyperbolic trajectory, where a chemical retro stage is employed to inject the spacecraft into an elliptic capture orbit. The flyby missions are identical except no retro maneuver is performed. In either case, the parameters of primary interest are the final pericenter distance r_{tf}, the radius of the sphere of influence r_{ts}, and possibly the arrival excess speed $v_{\infty a}$. Since these are direct counterparts of the primary parameters at Earth departure, the equations of constraint at the target for planetary missions are very similar to those for Earth departure and are given in (A.5) where μ_t denotes the gravitational constant of the target planet. The optional constraints on the independent parameters, $v_{\infty a}$ and ${}_2t_s$ are represented by the latter of equations (A.8) and (A.9), respectively, while the optional constraints on final mass ratio and arrival date are given in (A.13) and (A.15), respectively. In (A.15), the parameter $t_{\infty a}$, which is the time within the target planet's sphere of influence to the point of closest approach, is computed in a manner analogous to that of $t_{\infty d}$ in equations (35) and (36). That is, $t_{\infty a}$ is given by

$$t_{\infty a} = (\mu_t / v_{\infty a}^3)\,(e_t \sinh f_t - f_t) \tag{37}$$

where

$$e_t = 1 + r_{tf} v_{\infty a}^2 / \mu_t$$

(38)

$$_t = \cosh^{-1} \left[\left(1 + \frac{r_{ts} v_{\infty a}^2}{\mu_t}\right) / e_t \right]$$

For purposes of computing the retro propellant requirements for planetary orbiter missions, the assumed retro maneuver will be comprised of an impulsive thrust maneuver at the pericenter of the planetocentric approach hyperbola. Thus, the retro velocity increment, v_r, may be written

$$v_r = \sqrt{v_{\infty a}^2 + 2\mu_t / r_{tf}} - \sqrt{\frac{2\mu_t r_{ta}}{r_{tf}(r_{tf} + r_{ta})}} \qquad (39)$$

where r_{ta} is the apocenter distance of the desired final capture orbit. This value of v_r is then used in (14) to evaluate the retro propellant requirement.

Boundary conditions of solar and deep space probes are generally stated in terms of a final solar distance with possible limitations on either time or mass. For extra-ecliptic missions, the boundary condition is more than likely to be inclination to the ecliptic with possible constraints on radial distance, mass, and/or time. For such conditions, it is most convenient to work in a polar coordinate system. However, there also exist many problems which fit in either of these categories for which Cartesian coordinates are more appropriate. Examples include problems for which the final space state (i.e., position and velocity) is completely specified or extra-ecliptic missions for which a specific component of position or velocity normal to the ecliptic plane is to be achieved. Consequently, the capability of expressing end conditions in either Cartesian coordinates, as represented by equations (A.18) - (A.20), or polar coordinates, as represented by equations (A.21), will be provided.

Certain optional constraints are required for all types of missions. These include constraints pertaining to the engine parameters, such as equations (A.10) and (A.11), and to the net spacecraft mass, given by (A.12). The individual segment flight times and the total mission duration are also parameters of interest for most missions and are provided as optional constraints through equations (A.16) and (A.17). Of course, since there is no sphere of influence at the destination for probe and extra-ecliptic missions, the time interval $t_{\infty a}$ is taken to be zero.

Starting Conditions

Given a set of planetocentric polar coordinates of the spatial state at the swingby point, one must then transform to the ecliptic Cartesian coordinate system in which the integration is performed. This is quite easily accomplished after first forming a planetocentric equatorial Cartesian coordinate system with unit vectors $\bar{\ell}_p$, $\bar{m}_p$, and $\bar{n}_p$ along the orthogonal axes. Recall that $\bar{n}_p$ is along the polar axis of the swingby planet and is input in the ecliptic Cartesian coordinate system with unit vectors $\bar{i}$, $\bar{j}$, and $\bar{k}$. Providing $\bar{n}_p$ is not input collinear with $\bar{k}$, then define

$$\bar{\ell}_p = \frac{\bar{k} \times \bar{n}_p}{|\bar{k} \times \bar{n}_p|} \tag{40}$$

That is, $\bar{\ell}_p$ is defined to be directed towards the autumnal equinox of the swingby planet. If $\bar{n}_p$ is input such that

$$|\bar{k} \times \bar{n}_p| = 0 \tag{41}$$

then we will arbitrarily set

$$\bar{\ell}_p = -\bar{i} \tag{42}$$

Finally, the right-handed set is completed by defining

$$\bar{m}_p = \bar{n}_p \times \bar{\ell}_p \tag{43}$$

The planetocentric position and velocity vectors may then be written

$$\begin{aligned} {}_2R_p(0) = r_p\Big[&(\cos\omega_p \cos\Omega_p - \sin\omega_p \sin\Omega_p \cos i_p)\,\bar{\ell}_p + (\cos\omega_p \sin\Omega_p + \sin\omega_p \cos\Omega_p \cos i_p)\,\bar{m}_p \\ &+ \sin\omega_p \sin i_p\, \bar{n}_p\Big] \end{aligned} \tag{44}$$

$$\begin{aligned} {}_2\dot{R}_p(0) = v_p\Big[&-(\sin\omega_p \cos\Omega_p + \cos\omega_p \sin\Omega_p \cos i_p)\,\bar{\ell}_p - (\sin\omega_p \sin\Omega_p - \cos\omega_p \cos\Omega_p \cos i_p)\,\bar{m}_p \\ &+ \cos\omega_p \sin i_p\, \bar{n}_p\Big] \end{aligned} \tag{45}$$

which yield ${}_2R_p(0)$ and ${}_2\dot{R}_p(0)$ in the ecliptic system, since $\bar{\ell}_p$, $\bar{m}_p$, and $\bar{n}_p$ are known in that system. The final step required is to convert from the planetocentric ecliptic to the heliocentric ecliptic by simply adding the position and velocity of the swingby planet, i.e.,

$$
\begin{aligned}
{}_2R(0) &= {}_2R_p(0) + {}_2P(0) \\
{}_2V(0) &= {}_2\dot{R}_p(0) + {}_2\dot{P}(0)
\end{aligned}
\tag{46}
$$

Clearly, continuity considerations at the swingby point coupled with the identities

$$
\begin{aligned}
{}_1P(0) &= {}_2P(0) \\
{}_1\dot{P}(0) &= {}_2\dot{P}(0)
\end{aligned}
\tag{47}
$$

yield the vectors ${}_1R_p(0)$, ${}_1\dot{R}_p(0)$, ${}_1R(0)$, and ${}_1V(0)$ immediately.

NECESSARY CONDITIONS

To outline the necessary conditions that must be satisfied by the solution to the optimization problem at hand, it is helpful to first restate the problem in a general form. To this end, let the state at any point s in the interval $[0, 1]$ be denoted $X(s)$, and require that $X(s)$ satisfy the differential equations

$$X(s)' = F_1[X(s), U(s), A, s] \qquad 0 \le s \le s_p$$
$$X(s)' = F_2[X(s), U(s), A, s] \qquad s_p < s \le 1 \tag{48}$$

where the prime denotes derivatives with respect to s, $U(s)$ is a vector of control variables, and A is a vector of control parameters. The point $s=s_p$ is determined from the constraint

$$\rho[X(s_p^-)] = 0 \tag{49}$$

The superscript minus implies the limit as s approaches s_p from the left. At $s=s_p$, we also admit the possibility of a discontinuity in the state of the form

$$X(s_p^+) - X(s_p^-) = G(B) \tag{50}$$

where B is a vector denoting parameters related to the boundary conditions of the problem. The boundary conditions are expressed in the general form

$$\Psi[X(0), X(1), A, B] = 0 \tag{51}$$

Except at $s=s_p$, $X(s)$ is required to be a continuous and piecewise-differentiable function of s, while $U(s)$ is required to be piecewise-continuous and differentiable, except at a finite number of points. The problem to be solved is that of choosing $U(s)$, A, and B so as to minimize the function

$$\phi[X(1), A, B] \tag{52}$$

subject to the satisfaction of the constraints (51).

To verify that the original problem is contained in the more general problem stated above, note the following observations. The sixteen state variables of the problem (i.e., the position and velocity components, mass ratio and time on each of the two segments) are represented by $X(s)$. The dummy variable s and its range are taken to be the same as in the original problem. The control variables, thrust direction and the switch step function h_σ, are contained in $U(s)$, whereas A includes the engine parameters a_o and c. The crossing of the sphere of influence occurs at $s=s_p$ at which time the governing equations of motion switch from F_1 to F_2. The value of s_p is determined by the satisfaction of (49), which is equation (24) in the original problem. The capability of accounting for jettisoning a mass package at the swingby planet is provided through (50). All boundary conditions specified in the preceding section are contained in the general set given by (51), and the general form of ϕ covers both net spacecraft mass and time, which are the primary performance indices of immediate interest. Note that the assumption to minimize ϕ does not restrict the generality of the problem, since ϕ may be equated to the negative of a function which is to be maximized. The vector B is introduced with the intention of using its elements, which would be functions of the initial and/or final values of one or more of the state variables, as independent parameters in the solution of the boundary value problem. It may be argued that, since the intercepts of the swingby planet sphere of influence may not occur at the same value of s on the two segments, there should be one additional equation represented in (48) covering the range $s_{p1} < s < s_{p2}$ where s_{p1} and s_{p2} denote the two values of s where the crossings occur. Technically this argument is correct; however, because the equations of motion for the two segments are completely uncoupled, the corner conditions at the two points of discontinuity are also uncoupled. Consequently, the general equations to be derived for one point of discontinuity may then be applied to the crossings independently.

The general approach to the solution of the problem stated above may be found in most textbooks of optimization theory employing the indirect method. The approach employed here is similar to that of Reference 9. We begin by writing the augmented function ϕ^* by adjoining to ϕ the constraint equations (48) - (50) through the introduction of a set of Lagrange multipliers as follows:

$$\phi^* = \phi + \nu\rho + N \cdot [X(s_p^+) - X(s_p^-) - G(B)]$$

$$+ \int_0^{s_p^-} \Lambda \cdot (X' - F_1)\, ds + \int_{s_p^+}^1 \Lambda \cdot (X' - F_2)\, ds \tag{53}$$

where ν, N, and $\Lambda(s)$ are the multipliers. We then seek the necessary conditions for which ϕ^* is a minimum, i.e., for which

$$d\phi^* = 0 \tag{54}$$

Proceeding formally, one may write

$$
\begin{aligned}
d\phi^* = {} & \frac{\partial \phi}{\partial X(1)} \cdot dX(1) + \frac{\partial \phi}{\partial A} \cdot dA + \frac{\partial \phi}{\partial B} \cdot dB + \nu \frac{\partial \rho}{\partial X(s_p^-)} \cdot dX(s_p^-) \\
& + N \cdot \left[dX(s_p^+) - dX(s_p^-) - \frac{\partial G}{\partial B}\, dB \right] \\
& + \int_0^{s_p^-} \Lambda \cdot \left[\delta X' - \frac{\partial F_1}{\partial X}\, \delta X - \frac{\partial F_1}{\partial U}\, \delta U - \frac{\partial F_1}{\partial A}\, \delta A \right] ds \\
& + \int_{s_p^+}^{1} \Lambda \cdot \left[\delta X' - \frac{\partial F_2}{\partial X}\, \delta X - \frac{\partial F_2}{\partial U}\, \delta U - \frac{\partial F_2}{\partial A}\, \delta A \right] ds
\end{aligned} \tag{55}
$$

and noting that

$$\int_{s_1}^{s_2} (\Lambda \cdot \delta X')\, ds = \Lambda \cdot \delta X \Big|_{s_1}^{s_2} - \int_{s_1}^{s_2} (\Lambda' \cdot \delta X)\, ds \tag{56}$$

$$\delta X = dX - X' ds \tag{57}$$

and

$$\int_{s_1}^{s_2} \Lambda \frac{\partial F}{\partial A}\, \delta A\, ds = \left(\int_{s_1}^{s_2} \left(\Lambda \frac{\partial F}{\partial A}\right) ds \right) dA \tag{58}$$

then (55) may be rewritten

$$d\phi^* = \left[\frac{\partial\phi}{\partial A} - \int_0^{s_p^-} (\Lambda \frac{\partial F_1}{\partial A})\, ds - \int_{s_p^+}^{1} (\Lambda \frac{\partial F_2}{\partial A})\, ds\right] \cdot dA + \left[\frac{\partial\phi}{\partial B} - N \frac{\partial G}{\partial B}\right] \cdot dB$$

$$+ \left[\nu \frac{\partial\rho}{\partial X(s_p^-)} - N + \Lambda(s_p^-)\right] \cdot dX(s_p^-) + \left[N - \Lambda(s_p^+)\right] \cdot dX(s_p^+)$$

$$+ \left[\Lambda(s_p^+) \cdot X'(s_p^+) - \Lambda(s_p^-) \cdot X'(s_p^-)\right] ds_p + \frac{\partial\phi}{\partial X(1)} \cdot dX(1) + \left[\Lambda \cdot dX\right]_0^1$$

$$- \int_0^{s_p^-} \left[\Lambda' + \Lambda \frac{\partial F_1}{\partial X}\right] \cdot \delta X\, ds - \int_0^{s_p^-} \Lambda \frac{\partial F_1}{\partial U}\, \delta U\, ds$$

$$- \int_{s_p^+}^{1} \left[\Lambda' + \Lambda \frac{\partial F_2}{\partial X}\right] \cdot \delta X\, ds - \int_{s_p^+}^{1} \Lambda \frac{\partial F_2}{\partial U}\, \delta U\, ds \qquad (59)$$

Because the variations in $X(s_p^-)$, $X(s_p^+)$ and s_p are independent, the satisfaction of (54) requires that the coefficients of these variations be zero, i.e.,

$$\nu \frac{\partial\rho}{\partial X(s_p^-)} - N + \Lambda(s_p^-) = 0$$

$$N - \Lambda(s_p^+) = 0 \qquad (60)$$

$$\Lambda(s_p^+) \cdot X'(s_p^+) - \Lambda(s_p^-) \cdot X'(s_p^-) = 0$$

Now eliminate N from (60), multiply the first of equations (60) by $X'(s_p^-)$, and add the latter of (60) to obtain

$$\nu \rho'(s_p^-) + \Lambda(s_p^+) \left(X'(s_p^+) - X'(s_p^-) \right) = 0 \tag{61}$$

Solving for ν and substituting into the first equation of (60) then yields the jump conditions in the adjoint variables $\Lambda(s)$ at $s = s_p$.

$$\Lambda(s_p^-) = \Lambda(s_p^+) + \frac{1}{\rho'(s_p^-)} \Lambda(s_p^+) \cdot \left(X'(s_p^+) - X'(s_p^-) \right) \frac{\partial \rho}{\partial X(s_p^-)} \tag{62}$$

A parameter that will be of particular interest shortly is the variational Hamiltonian j defined

$$j(s) = \Lambda(s) \cdot X'(s) \tag{63}$$

The latter of equations (60) indicates that j is continuous at $s = s_p$. The variations in A, B, X(0) and X(1) are not totally independent but are related through the differential form of equations (51), i.e.,

$$d\Psi = \frac{\partial \Psi}{\partial X(0)} dX(0) + \frac{\partial \Psi}{\partial X(1)} dX(1) + \frac{\partial \Psi}{\partial A} dA + \frac{\partial \Psi}{\partial B} dB = 0 \tag{64}$$

Consequently, these constraint equations (64), combined with the collection of remaining terms outside the integrals in (59), i.e.,

$$\left(\frac{\partial \phi}{\partial A} + \Lambda_A\right) \cdot dA + \left(\frac{\partial \phi}{\partial B} - \Lambda(s_p^+)\ \frac{\partial G}{\partial B}\right) \cdot dB + \frac{\partial \phi}{\partial X(1)}\ dX(1) + [\Lambda \cdot dX]_0^1 = 0 \qquad (65)$$

together lead to the particular necessary conditions known as transversality conditions. These conditions are formed by eliminating from (65) as many differentials as there are equations in (64). The differentials remaining after this is done are totally independent and, setting their coefficients to zero individually, produces the transversality conditions. In writing (65) we have employed the relationship

$$\Lambda_A = - \int_0^{s_p^-} \Lambda \frac{\partial F_1}{\partial A}\ ds - \int_{s_p^+}^1 \Lambda \frac{\partial F_2}{\partial A}\ ds \qquad (66)$$

Finally, we are left with the integral terms in (59) and, since the variations in state and control variables are independent, the satisfaction of (54) requires that the individual integrands be zero. This, of course, leads to the well-known necessary conditions known as the Euler-Lagrange, or adjoint, equations.

$$\begin{aligned} \Lambda' &= - \Lambda \frac{\partial F_1}{\partial X} \qquad && 0 \le s \le s_p^- \\ \Lambda' &= - \Lambda \frac{\partial F_2}{\partial X} \qquad && s_p^+ < s \le 1 \end{aligned} \qquad (67)$$

and the optimal control equations

$$\Lambda \frac{\partial F_1}{\partial U} = 0 \qquad 0 \le s \le s_p^-$$

$$\Lambda \frac{\partial F_2}{\partial U} = 0 \qquad s_p^+ < s \le 1 \tag{68}$$

In terms of j, the variational Hamiltonian defined in (63), these equations may equivalently be written

$$\Lambda' = - \frac{\partial j}{\partial X} \tag{69}$$

$$\frac{\partial j}{\partial U} = 0 \tag{70}$$

The satisfaction of the preceding conditions assures the satisfaction of (54). The trouble is that the solution of (54) is not generally unique since any extremum of ϕ^* (and therefore ϕ), whether it be a minimum, maximum or saddlepoint with respect to the control parameters and variables, will satisfy (54). Furthermore, to this point in the formulation of the general solution, the class of admissible control variables has been implicitly limited to be continuous, unconstrained functions of s. In the original problem, h_σ represents a control variable that is both constrained and discontinuous. Hence, it is necessary to admit as possible solutions a larger class of control variables.

Both of these problems are analyzed in detail in both References 9 and 10, and the development of the results will not be repeated here. The answer to both problems is given by what is now known as the Maximum Principle. Basically this principle states that, out of all admissible controls which result in the satisfaction of all end and transversality conditions, the control which minimizes the function ϕ is that which results in the maximum value of the variational Hamiltonian j at every point along the path. This principle does not require that the maximum of j be a stationary maximum; thus in the case of $\dot{h}_\sigma$, one simply selects the particular value which yields the larger value of j.

In summary, the necessary conditions that ϕ be a minimum are that:

1) The control is chosen to maximize the variational Hamiltonian, j, which depends on the adjoint variables, Λ, that are governed by the differential equations (69),

2) The control parameters and open boundary conditions are chosen to satisfy the transversality conditions (65) subject to the constraints (64),

3) The adjoint variables may be discontinuous at points of imposed discontinuity in the derivatives (i.e., at $s=s_p$) and the jumps are given by (62).

It may be helpful to note at this point that the use of the Maximum Principle yields a local minimum of ϕ with respect to the control variables but not necessarily with respect to the control parameters and open boundary conditions. With respect to these parameters, ϕ may be a maximum, a minimum, or a saddle point.

A well-known property of the variational Hamiltonian is that it is a constant of the motion providing the dummy variable s does not appear explicitly in the state equations. This is easily shown as follows. Differentiating (63) yields

$$j' = \Lambda' \cdot X' + \Lambda \cdot X'' \tag{71}$$

But, from (69) and noting that

$$X'' = \frac{\partial X'}{\partial X} X' + \frac{\partial X'}{\partial s}$$

(72)

and

$$\frac{\partial j}{\partial X} = \Lambda \frac{\partial X'}{\partial X}$$

then

$$j' = - \Lambda \frac{\partial X'}{\partial X} X' + \Lambda \frac{\partial X'}{\partial X} X' + \Lambda \frac{\partial X'}{\partial s} = \Lambda \frac{\partial X'}{\partial s} \tag{73}$$

Thus, since X' does not contain s explicitly in the problem of interest, we must have

$$j' = 0 \tag{74}$$

which implies j is a constant.

THE SOLUTION

Optimal Control Equations

To determine the optimum thrust direction along the path and the points at which the engines are switched on or off, one must first consider the variational Hamiltonian, j. Denoting λ_x as the variable adjoint to any state variable x, the variational Hamiltonian for our problem is

$$j = {}_1\Lambda_R \cdot {}_1R' + {}_1\Lambda_V \cdot {}_1V' + {}_1\lambda_{\nu}\, {}_1\nu' + {}_1\lambda_t\, {}_1t' + {}_2\Lambda_R \cdot {}_2R' + {}_2\Lambda_V \cdot {}_2V' + {}_2\lambda_{\nu}\, {}_2\nu' + {}_2\lambda_t\, {}_2t' \tag{75}$$

or, after substituting equations (32) and (34),

$$j = {}_1\tau\, {}_1j + {}_2\tau\, {}_2j \tag{76}$$

where

$$\begin{aligned} {}_1j = &- {}_1\Lambda_R \cdot {}_1V - (1 - j_c\, {}_1h_\rho)\, {}_1h_\sigma \frac{a_o\, {}_1\gamma}{{}_1\nu} \left({}_1\Lambda_V \cdot {}_1\bar{e}_T - \frac{{}_1\nu}{c}\, {}_1\lambda_\nu\right) + \frac{\mu}{{}_1r^3} \left({}_1\Lambda_V \cdot {}_1R\right) \\ &+ {}_1h_\rho \left({}_1\Lambda_V \cdot {}_1Q\right) - {}_1\lambda_t \end{aligned} \tag{77}$$

$$\begin{aligned} {}_2j = &\ {}_2\Lambda_R \cdot {}_2V + (1 - j_c\, {}_2h_\rho)\, {}_2h_\sigma \frac{a_o\, {}_2\gamma}{{}_2\nu} \left({}_2\Lambda_V \cdot {}_2\bar{e}_T - \frac{{}_2\nu}{c}\, {}_2\lambda_\nu\right) - \frac{\mu}{{}_2r^3} \left({}_2\Lambda_V \cdot {}_2R\right) \\ &- {}_2h_\rho \left({}_2\Lambda_V \cdot {}_2Q\right) + {}_2\lambda_t \end{aligned} \tag{78}$$

The division of j into two terms, one term associated with each of the two trajectory segments, serves to emphasize the fact that the equations are completely uncoupled; therefore, the maximization of ${}_1j$ and ${}_2j$ independently is identical to the maximization of j. Because of the particular form of (73) and (74), it is possible to maximize ${}_1j$ with respect to ${}_1\bar{e}_T$ and ${}_1h_\sigma$, and ${}_2j$ with respect to ${}_2\bar{e}_T$ and ${}_2h_\sigma$, simply by inspection. Since the coefficients

$$(1 - j_c\, {}_1h_\rho)\, {}_1h_\sigma \frac{a_{o1}\gamma}{{}_1\nu} \quad \text{and} \quad (1 - j_c\, {}_2h_\rho)\, {}_2h_\sigma \frac{a_{o2}\gamma}{{}_2\nu}$$

are both non-negative, then it is seen that ${}_1j$ is maximized with respect to ${}_1\bar{e}_T$ by choosing ${}_1\bar{e}_T$ diametrically opposed to ${}_1\Lambda_V$, and ${}_2j$ is maximized with respect to ${}_2\bar{e}_T$ by choosing ${}_2\bar{e}_T$ aligned with ${}_2\Lambda_V$. That is to say, we choose ${}_1\bar{e}_T$ such that

$${}_1\Lambda_V \cdot {}_1\bar{e}_T = - {}_1\lambda_V \tag{79}$$

where ${}_1\lambda_V = |{}_1\Lambda_V|$, and choose ${}_2\bar{e}_T$ such that

$${}_2\Lambda_V \cdot {}_2\bar{e}_T = {}_2\lambda_V \tag{80}$$

where ${}_2\lambda_V = |{}_2\Lambda_V|$. After incorporating these results in the expression for j, the conditions for switching ${}_1h_\sigma$ and ${}_2h_\sigma$ between their permissible values of zero and one are also determined by inspection. Defining the switching functions, ${}_1\sigma$ and ${}_2\sigma$

$$
{}_1\sigma = {}_1\lambda_V + \frac{{}_1\nu}{c}\,{}_1\lambda_\nu
$$

$$
{}_2\sigma = {}_2\lambda_V - \frac{{}_2\nu}{c}\,{}_2\lambda_\nu \tag{81}
$$

then the optimal choices for ${}_1h_\sigma$ and ${}_2h_\sigma$ are clearly

$$
{}_1h_\sigma = \begin{cases} 0 & \text{if } {}_1\sigma < 0 \\ 1 & \text{if } {}_1\sigma > 0 \end{cases}
$$

$$
{}_2h_\sigma = \begin{cases} 0 & \text{if } {}_2\sigma < 0 \\ 1 & \text{if } {}_2\sigma > 0 \end{cases} \tag{82}
$$

Upon substituting these results in (73) and (74), one then obtains

$$
{}_1j = (1 - j_c\,{}_1h_\rho)\,{}_1h_\sigma \frac{a_o\,{}_1\gamma}{{}_1\nu}\,{}_1\sigma - {}_1\Lambda_R \cdot {}_1V + \frac{\mu}{{}_1r^3}({}_1\Lambda_V \cdot {}_1R) + {}_1h_\rho({}_1\Lambda \cdot {}_1Q) - {}_1\lambda_t
$$

$$
{}_2j = (1 - j_c\,{}_2h_\rho)\,{}_2h_\sigma \frac{a_o\,{}_2\gamma}{{}_2\nu}\,{}_2\sigma + {}_2\Lambda_R \cdot {}_2V - \frac{\mu}{{}_2r^3}({}_2\Lambda_V \cdot {}_2R) - {}_2h_\rho({}_2\Lambda_V \cdot {}_2Q) + {}_2\lambda_t \tag{83}
$$

Euler-Lagrange Equations

The Euler-Lagrange equations (69) for the problem of interest here may be written by inspection using (72) and (77) with (20), (21), and (26) recalling that P, $\dot{P}$, and $\ddot{P}$ are functions only of time.

$$
{}_1\Lambda'_R = -{}_1\tau\Big\{(1-j_c\,{}_1h_\rho)\,{}_1h_\sigma \frac{a_{o1}\gamma^*}{{}_1\nu_1 r}\,{}_1\sigma_1 R + \frac{\mu}{{}_1r^3}\,{}_1\Lambda_V - \frac{3\mu}{{}_1r^5}({}_1\Lambda_V\cdot{}_1R)\,{}_1R
$$

$$
+\,{}_1h_\rho\Big[\frac{\mu_p}{{}_1r_p^3}\,{}_1\Lambda_V - \frac{3\mu_p}{{}_1r_p^5}({}_1\Lambda_V\cdot{}_1R_p)\,{}_1R_p - \frac{\mu}{{}_1r^3}\,{}_1\Lambda_V + \frac{3\mu}{{}_1r^5}({}_1\Lambda_V\cdot{}_1R)\,{}_1R\Big]\Big\}
$$

$$
{}_1\Lambda'_V = {}_1\tau_1\Lambda_R
$$

$$
{}_1\lambda'_\nu = {}_1\tau(1-j_c\,{}_1h_\rho)\,{}_1h_\sigma \frac{a_{o1}\gamma}{{}_1\nu^2}\,{}_1\lambda_V
$$

$$
{}_1\lambda'_t = {}_1\tau_1 h_\rho\Big[\frac{\mu_p}{{}_1r_p^3}({}_1\Lambda_V\cdot{}_1\dot{P}) - \frac{3\mu_p}{{}_1r_p^5}({}_1\Lambda_V\cdot{}_1R_p)({}_1R_p\cdot{}_1\dot{P}) + {}_1\Lambda_V\cdot{}_1\ddot{P}\Big]
$$

(84)

$$
{}_2\Lambda'_R = -{}_2\tau\Big\{(1-j_c\,{}_2h_\rho)\,{}_2h_\sigma \frac{a_{o2}\gamma^*}{{}_2\nu_2 r}\,{}_2\sigma_2 R - \frac{\mu}{{}_2r^3}\,{}_2\Lambda_V + \frac{3\mu}{{}_2r^5}({}_2\Lambda_V\cdot{}_2R)\,{}_2R
$$

$$
-\,{}_2h_\rho\Big[\frac{\mu_p}{{}_2r_p^3}\,{}_2\Lambda_V - \frac{3\mu_p}{{}_2r_p^5}({}_2\Lambda_V\cdot{}_2R_p)\,{}_2R_p - \frac{\mu}{{}_2r^3}\,{}_2\Lambda_V + \frac{3\mu}{{}_2r^5}({}_2\Lambda_V\cdot{}_2R)\,{}_2R\Big]\Big\}.
$$

$$
{}_2\Lambda'_V = -{}_2\tau_2\Lambda_R
$$

$$
{}_2\lambda'_\nu = {}_2\tau(1-j_c\,{}_2h_\rho)\,{}_2h_\sigma \frac{a_{o2}\gamma}{{}_2\nu^2}\,{}_2\lambda_V
$$

$$
{}_2\lambda'_t = -{}_2\tau_2 h_\rho\Big[\frac{\mu_p}{{}_2r_p^3}({}_2\Lambda_V\cdot{}_2\dot{P}) - \frac{3\mu_p}{{}_2r_p^5}({}_2\Lambda_V\cdot{}_2R_p)({}_2R_p\cdot{}_2\dot{P}) + {}_2\Lambda_V\cdot{}_2\ddot{P}\Big]
$$

where γ^* denotes $d\gamma/dr$. Anticipating a desire to optimize the engine parameters a_o and c, and the time increments ${}_1\tau$ and ${}_2\tau$, it is convenient to introduce at this point the concept of adjoint variables associated with each of these parameters, as implied in equation (66). That is, define

$$\lambda'_{a_o} = -\frac{\partial j}{\partial a_o} \quad ; \quad \lambda'_c = -\frac{\partial j}{\partial_2 \tau}$$

$$ {}_1\lambda'_\tau = -\frac{\partial j}{\partial_1 \tau} \quad ; \quad {}_2\lambda'_\tau = -\frac{\partial j}{\partial_2 \tau} \tag{85}$$

which, using the initial conditions

$$\lambda_{a_o}(0) = \lambda_c(0) = {}_1\lambda_\tau(0) = {}_2\lambda_\tau(0) = 0 \tag{86}$$

lead to the integrals that are needed in the first term of the transversality condition (65). Using (76) and (83), equations (85) become

$$\lambda'_{a_o} = -{}_1\tau(1-j_{c\,1}h_\rho)\,{}_1h_\sigma \frac{{}_1\gamma}{{}_1\nu}\,{}_1\sigma - {}_2\tau(1-j_{c\,2}h_\rho)\,{}_2h_\sigma \frac{{}_2\gamma}{{}_2\nu}\,{}_2\sigma$$

$$\lambda'_c = {}_1\tau(1-j_{c\,1}h_\rho)\,{}_1h_\sigma \frac{a_{o\,1}\gamma}{c^2}\,{}_1\lambda_\nu - {}_2\tau(1-j_{c\,2}h_\rho)\,{}_2h_\sigma \frac{a_{o\,2}\gamma}{c^2}\,{}_2\lambda_\nu \tag{87}$$

$${}_1\lambda'_\tau = -{}_1j$$

$${}_2\lambda'_\tau = -{}_2j$$

Jump Conditions at Sphere of Influence

The equations for the jump conditions are given in general form by (62). It is interesting to note that, in the second term on the right hand side of (62), the quantity

$$f = \frac{1}{\rho'(s_p^-)} \Lambda(s_p^+) \cdot (X'(s_p^+) - X'(s_p^-)) \tag{88}$$

is a scalar. Therefore, it becomes clear that any particular element of Λ is discontinuous at $s = s_p$ only if ρ is an explicit function of the state variable to which that element of Λ is adjoint. For our problem, ρ is a function of R and t; hence, we may expect discontinuities only in Λ_R and λ_t.

To evaluate the scalar f for each of the two segments, it may first be noted from (24) that

$$\begin{aligned} {}_1\rho' &= - {}_1r'_p({}_1s_p) = {}_1\tau \, {}_1\dot{r}_p({}_1s_p) = \frac{{}_1\tau}{r_{ps}} ({}_1R_p({}_1s_p) \cdot {}_1\dot{R}_p({}_1s_p)) \\ {}_2\rho' &= - {}_2r'_p({}_2s_p) = - {}_2\tau \, {}_2\dot{r}_p({}_2s_p) = - \frac{{}_2\tau}{r_{ps}} ({}_2R_p({}_2s_p) \cdot {}_2\dot{R}_p({}_2s_p)) \end{aligned} \tag{89}$$

Then, in writing the discontinuities in the state variable derivatives

$$ {}_1V'({}_1s_p^+) - {}_1V'({}_1s_p^-) = {}_1\tau\left[{}_1h_\sigma a_{o1}\gamma\left(\frac{{}_1\Lambda_V}{{}_1\lambda_V}\right)\left(\frac{1}{{}_1\nu({}_1s_p^+)} - \frac{1-j_c}{{}_1\nu({}_1s_p^-)}\right) - {}_1Q\right] $$

$$ {}_1\nu'({}_1s_p^+) - {}_1\nu'({}_1s_p^-) = {}_1\tau\, j_{c\,1}h_\sigma \frac{a_{o1}\gamma}{c} $$

(90)

$$ {}_2V'({}_2s_p^+) - {}_2V'({}_2s_p^-) = {}_2\tau\left[j_{c\,2}h_\sigma \frac{a_{o2}\gamma}{{}_2\nu}\left(\frac{{}_2\Lambda_V}{{}_2\lambda_V}\right) + {}_2Q\right] $$

$$ {}_2\nu'({}_2s_p^+) - {}_2\nu'({}_2s_p^-) = -\,{}_2\tau\, j_{c\,2}h_\sigma \frac{a_{o2}\gamma}{c} $$

where $${}_1\nu({}_1s_p^+) = {}_1\nu({}_1s_p^-) + m_x/m_o$$ (91)

the expressions for the two scalars ${}_1f$ and ${}_2f$ become

$$ {}_1f = \frac{1}{{}_1\dot{r}_p}\left\{{}_1h_\sigma a_{o1}\gamma\left[\frac{j_{c\,1}\sigma({}_1s_p^-)}{{}_1\nu({}_1s_p^-)} + {}_1\lambda_V\left(\frac{1}{{}_1\nu({}_1s_p^+)} - \frac{1}{{}_1\nu({}_1s_p^-)}\right)\right] - {}_1\Lambda_V \cdot {}_1Q\right\} $$

(92)

$$ {}_2f = -\frac{1}{{}_2\dot{r}_p}\left[j_{c\,2}h_\sigma \frac{a_{o2}\gamma}{{}_2\nu}\, {}_2\sigma + {}_2\Lambda_V \cdot {}_2Q\right] $$

with all variables being evaluated at $s = {}_1s_p^-$ unless otherwise indicated. Finally, upon noting that

$$\frac{\partial\, {}_i\rho}{\partial\, {}_i R} = -\frac{{}_i R_p}{{}_i r_p}$$

$$\frac{\partial\, {}_i\rho}{\partial\, {}_i t} = \frac{{}_i R_p \cdot {}_i \dot{P}}{{}_i r_p} \tag{93}$$

for $i = 1$ or 2, then the discontinuities in the adjoint variables may be written

$${}_1\Lambda_R({}_1s_p^+) = {}_1\Lambda_R({}_1s_p^-) + \frac{{}_1f}{{}_1r_p}\,{}_1R_p$$

$${}_1\lambda_t({}_1s_p^+) = {}_1\lambda_t({}_1s_p^-) - \frac{{}_1f}{{}_1r_p}\,({}_1R_p \cdot {}_1\dot{P})$$

$${}_2\Lambda_R({}_2s_p^+) = {}_2\Lambda_R({}_2s_p^-) + \frac{{}_2f}{{}_2r_p}\,{}_2R_p \tag{94}$$

$${}_2\lambda_t({}_2s_p^+) = {}_2\lambda_t({}_2s_p^-) - \frac{{}_2f}{{}_2r_p}\,({}_2R_p \cdot {}_2\dot{P})$$

It is interesting to note that, although the adjoint variables may be discontinuous, the last of equations (60) indicate that the variational Hamiltonian j is continuous at $s = {}_1s_p$ and $s = {}_2s_p$. It is a simple extension to prove that ${}_1j$ and ${}_2j$ are also continuous functions at the discontinuity points and, like j, are constants of the motion.

Transversality Conditions

Undoubtedly, the most complex and tedious aspect of the application of the necessary conditions is that of developing the transversality conditions. To a large extent, the flexibility of a trajectory optimization program is measured by the variety of combinations available in specifying (or not specifying) boundary conditions. Since each combination of open and fixed boundary conditions generally leads to a different set of transversality conditions, the number of potentially interesting sets is exceedingly large. Consequently, the transversality conditions developed here will be limited to those concomitant with selected problems of the missions discussed previously in the PROBLEM FORMULATION section, for which the specific boundary conditions permitted are listed in Appendix A.

As stated previously, the transversality equations are obtained from the simultaneous solution of equations (64) and (65). Since A and B appear in these equations, it is necessary to first define the elements of these two vectors. In the development of the necessary conditions, the vector A was included to encompass any engine parameters which one may wish to optimize. For the problem at hand, such engine parameters are the initial thrust acceleration a_o and the jet exhaust speed c. The vector B was intended to represent independent parameters of the boundary value problem which are not state variables. Specifically, we shall include in B the hyperbolic excess speeds, v_{∞_d} and v_{∞_a}, at departure from Earth and arrival at the destination, respectively. Additionally, B shall include the polar coordinates of the planetocentric position and velocity at the closest approach point. These are r_p, v_p, i_p, Ω_p, and ω_p. Note that only five coordinates are required since we have defined the point of interest to be the closest approach point, which implies flight path angle (and true anomaly) is zero.

The vector G of discontinuities in the state variables contains, for the problem of interest here, all zero elements except for the one associated with mass ratio on the first segment. Furthermore, this one non-zero element is a function only of one element of B, the departure excess speed $v_{\infty d}$. Denoting the non-zero element of G as g_ν, then

$$g_\nu = \frac{m_x}{m_o} = \frac{\Delta m_x}{m_o} + k_x \tag{95}$$

such that

$$\frac{\partial g_\nu}{\partial v_{\infty d}} = - \frac{\Delta m_x}{m_o^2} \frac{dm_o}{dv_{\infty d}} \tag{96}$$

where the derivative $dm_o/dv_{\infty d}$ is obtained by differentiating the curve fit of the launch vehicle performance data. Finally, upon noting that ${}_1\lambda_\nu$ is continuous at $s = {}_1s_p$, one may write

$$\Lambda(s_p^+) \frac{\partial G}{\partial B} dB = - {}_1\lambda_\nu ({}_1s_p) \frac{\Delta m_x}{m_o^2} \frac{dm_o}{dv_{\infty d}} dv_{\infty d} \tag{97}$$

The performance index ϕ is permitted to represent either the negative of the net spacecraft mass or the total mission duration. Since

$$d\phi = \frac{\partial \phi}{\partial A} \cdot dA + \frac{\partial \phi}{\partial B} \cdot dB + \frac{\partial \phi}{\partial X(1)} \cdot dX(1) \tag{98}$$

the contribution of the performance index to the various terms in (65) is found by forming the total differentials

$$d\phi = -\,dm_n \tag{99}$$

if net spacecraft mass is to be maximized, or

$$d\phi = d\,(\Delta t_m) \tag{100}$$

if mission duration is to be minimized. Considering first the case of maximizing net spacecraft mass, note that the mass of the low-thrust propellant may be written

$$m_p = m_o(1 - {}_2\nu(1)) - m_x \tag{101}$$

Substituting this into (8), rearranging and solving for m_n yields

$$\begin{aligned} m_n = {} & m_o \left[{}_2\nu(1)\,(1+k_t) - k_s - k_t\,(1-k_x) - \frac{\alpha a_o c}{2\eta} \right] + k_t\,\Delta m_x \\ & - j_r(1+k_r)\,(1-e^{-v_r/c_r}) \left\{ m_o \left[{}_2\nu(1)\,(1+j_t k_t) - j_t k_t (1-k_x) - j_{ps}\,\frac{\alpha a_o c}{2\eta} \right] + j_t k_t\,\Delta m_x \right\} \end{aligned} \tag{102}$$

where j_r is input one if a high-thrust retro maneuver is to be performed and zero otherwise. Differentiating (102) and changing signs then gives the desired result

$$- dm_n = m_{ps}\left[1-j_{ps}j_r(1+k_r)(1-e^{-v_r/c_r})\right]\left[\frac{da_o}{a_o} + \left(\frac{1}{c} - \frac{\eta^*}{\eta}\right)dc\right]$$

$$- \frac{1}{m_o}\left\{m_n - k_t\Delta m_x\left[1-j_tj_r(1+k_r)(1-e^{-v_r/c_r})\right]\right\}\frac{dm_o}{dv_{\infty d}}dv_{\infty d}$$

$$+ \frac{j_r(1+k_r)\ v_{\infty a}e^{-v_r/c_r}}{c_r\sqrt{v_{\infty a}^2 + 2\mu_t/r_{tf}}}\left\{m_o\left[{}_2\nu(1)(1+j_tk_t) - j_tk_t(1-k_x) - j_{ps}\frac{\alpha a_o c}{2\eta}\right] + j_tk_t\Delta m_x\right\}dv_{\infty a}$$

$$- m_o\left[(1+k_t) - j_r(1+j_tk_t)(1+k_r)(1-e^{-v_r/c_r})\right]d_2\nu(1) \tag{103}$$

where η^* denotes $d\eta/dc$ which is obtained from the differentiation of (15).

Proceeding to the case of minimization of the mission duration, one immediately obtains from the definition of Δt_m, i.e.,

$$\Delta t_m = {}_2t(1) - {}_1t(1) + t_{\infty d} + t_{\infty a} \tag{104}$$

the differential $d(\Delta t_m)$

$$d(\Delta t_m) = d_2t(1) - d_1t(1) + \frac{\partial t_{\infty d}}{\partial v_{\infty d}}dv_{\infty d} + \frac{\partial t_{\infty a}}{\partial v_{\infty a}}dv_{\infty a} \tag{105}$$

With some alegebraic manipulation, the two partial derivatives indicated in (105) are obtained by differentiating equations (35) - (38) in the form

$$\frac{\partial t_{\infty d}}{\partial v_{\infty d}} = -3\,\frac{t_{\infty d}}{v_{\infty d}} + \frac{2}{v_{\infty d}^{2}\, e_e \sinh f_e}\left[(e_e \cosh f_e - 1)\, r_{es} - (e_e - \cosh f_e)\, r_{eo}\right]$$

$$\frac{\partial t_{\infty a}}{\partial v_{\infty a}} = -3\,\frac{t_{\infty a}}{v_{\infty a}} + \frac{2}{v_{\infty a}^{2}\, e_t \sinh f_t}\left[(e_t \cosh f_t - 1)\, r_{ts} - (e_t - \cosh f_t)\, r_{tf}\right] \tag{106}$$

Denoting ϕ_x as the partial derivative of ϕ with respect to any parameter x, where ϕ represents either of the two permissible performance indices and ϕ_x is the coefficient of the differential dx in the appropriate equations (103) or (105), expand the general equation (65) in terms of the parameters of the problem at hand, i.e.,

$$\Big[{}_1\Lambda_R \cdot d_1R + {}_1\Lambda_V \cdot d_1V + {}_1\lambda_\nu d_1\nu + {}_1\lambda_t d_1 t + {}_2\Lambda_R \cdot d_2R + {}_2\Lambda_V \cdot d_2V + {}_2\lambda_\nu d_2\nu + {}_2\lambda_t d_2 t\Big]_0^1$$

$$+ \left(\phi_{a_o} + \lambda_{a_o}\right) da_o + \left(\phi_c + \lambda_c\right) dc + \left(\phi_{v_{\infty d}} + {}_1\lambda_\nu \left({}_1s_p\right) \frac{\Delta m_x}{m_o^2}\, \frac{dm_o}{dv_{\infty d}}\right) dv_{\infty d}$$

$$+ {}_1\lambda_\tau(1)\, d_1\tau + {}_2\lambda_\tau(1)\, d_2\tau$$

$$+ \phi_{v_{\infty a}}\, dv_{\infty a} + \phi_{{}_2\nu(1)}\, d_2\nu(1) + \phi_{{}_1t(1)}\, d_1t(1) + \phi_{{}_2t(1)}\, d_2t(1) = 0 \tag{107}$$

where, from the solution of the latter two of Equations (87),

$${}_1\lambda_\tau(1) = -{}_1j \quad ; \quad {}_2\lambda_\tau(1) = -{}_2j$$

and where, by definition

$$d_1\tau = d_1t(0) - d_1t(1) \quad ; \quad d_2\tau = d_2t(1) - d_2t(0).$$

We now seek to derive, for all specific problems of interest contained in the class of problems stated in the problem formulation, the various transversality conditions arising from the mutual satisfaction of equation (107) and the differential form of the appropriate (i.e., appropriate for a given problem) set of constraint equations in Appendix A.

To begin, consider the boundary conditions at the swingby point, i.e., at s=0. Letting x represent any one of the eight state variables, it may be seen from (A.1) that

$$d_1 x(0) = d_2 x(0) \tag{108}$$

Since (A.1) is always applicable, we may employ (108) to eliminate the differentials $d_1 x(0)$ from (107). Once this is done, terms of the form

$$\left({}_1\lambda_x(0) + {}_2\lambda_x(0)\right) d_2 x(0) \tag{109}$$

appear in (107), and it is helpful from the standpoint of abbreviating notation to introduce

$$\lambda_x = {}_1\lambda_x(0) + {}_2\lambda_x(0) \tag{110}$$

That is, the lack of a pre-subscript 1 or 2 will imply the sum and it will be understood that we are considering the point s=0. Now remove from equation (107) the terms pertaining explicitly to the point s = 0. Using the notation indicated in (110), these terms are written

$$-\Lambda_R \cdot d_2 R(0) - \Lambda_V \cdot d_2 V(0) - \lambda_\nu d_2 \nu(0) - (\dot{\lambda}_t + {}_1 j - {}_2 j)\, d_2 t_o \tag{111}$$

where the solutions of the differential equations for ${}_1\lambda_\tau$ and ${}_2\lambda_\tau$ have been employed.

In general, this expression may not be equated to zero because it is possible for the variations in ${}_2t(0)$ to be related to variations in other dates through the constraints (A.16). However, we may replace the variations in the heliocentric position and velocity with those referenced to the swingby planet through the use of (A.2), i.e.,

$$d_2R(0) = {}_2\dot{P}(0)\, d_2t(0) + d_2R_p(0)$$

$$d_2V(0) = {}_2\ddot{P}(0)\, d_2t(0) + d_2\dot{R}_p(0) \qquad (112)$$

Upon substituting (112) into (111), collecting terms, and noting that there are no additional constraints involving $R_p(0)$ and $\dot{R}_p(0)$ other than (A.3) and none involving ${}_2\nu(0)$ other than the first of (A.7), one obtains

$$\Lambda_R \cdot d_2R_p(0) + \Lambda_V \cdot d_2\dot{R}_p(0) = 0 \qquad (113)$$

$$\lambda_\nu d_2\nu(0) = 0 \qquad (114)$$

Furthermore, in the absence of both the constraints (A.16), we have the additional condition

$$(\Lambda_R \cdot {}_2\dot{P}(0) + \Lambda_V \cdot {}_2\ddot{P}(0) + \lambda_t + {}_1j - {}_2j)\, d_2t(0) = 0 \qquad (115)$$

If ${}_2\nu(0)$ is specified through the first of equations (A.7), then (114) is identically satisfied, since $d_2\nu(0)$ is zero. But if ${}_2\nu(0)$ is left open, then (114) can only be satisfied if the coefficient of $d_2\nu(0)$ is zero. This leads to the transversality condition

$$\lambda_\nu = {}_1\lambda_\nu(0) + {}_2\lambda_\nu(0) = 0 \qquad (116)$$

Similar arguments lead to the result that, if the swingby date ${}_2t(0)$ is not fixed and if the conditions leading to (115) are met,

$$({}_1\Lambda_R(0) + {}_2\Lambda_R(0)) \cdot {}_2\dot{P}(0) + ({}_1\Lambda_V(0) + {}_2\Lambda_V(0)) \cdot {}_2\ddot{P}(0) + {}_1\lambda_t(0) + {}_2\lambda_t(0) + {}_1j - {}_2j = 0 \qquad (117)$$

The constraint equations (A.3) are equivalent to the equations (44) and (45) which define ${}_2R_p(0)$ and ${}_2\dot{R}_p(0)$ in terms of the five independent polar coordinates r_p, v_p, i_p, Ω_p, and ω_p assuming $\gamma_p=0$. By differentiating (44) and (45) and introducing the angular momentum at the swingby point, defined by

$$H_p = {}_2R_p(0) \times {}_2\dot{R}_p(0) \qquad (118)$$

one obtains

$$d_2R_p(0) = \frac{1}{r_p}\, {}_2R_p(0)\, dr_p + \frac{r_p}{v_p}\, {}_2\dot{R}_p(0)\, d\,\omega_p + (\bar{n}_p \times {}_2R_p(0))\, d\Omega_p + \frac{\bar{n}_p \cdot {}_2R_p(0)}{r_p v_p \sin i_p}\, H_p\, d\,i_p$$

(119)

$$d_2\dot{R}_p(0) = \frac{1}{v_p}\, {}_2\dot{R}_p(0)\, dv_p - \frac{v_p}{r_p}\, {}_2R_p(0)\, d\,\omega_p + (\bar{n}_p \times {}_2\dot{R}_p(0))\, d\,\Omega_p + \frac{\bar{n}_p \cdot {}_2\dot{R}_p(0)}{r_p v_p \sin i_p}\, H_p\, d\,i_p$$

which may be substituted directly into (113). Because of the independence of the resultant differentials, the satisfaction of (107) requires that

$$(\Lambda_R \cdot {}_2R_p(0))\, d\,r_p = 0$$

$$(\Lambda_V \cdot {}_2\dot{R}_p(0))\, d\,v_p = 0$$

$$\left[r_p^2 (\Lambda_R \cdot {}_2\dot{R}_p(0)) - v_p^2 (\Lambda_V \cdot {}_2R_p(0))\right] d\,\omega_p = 0 \tag{120}$$

$$\bar{n}_p \cdot \left[(\Lambda_R x\, {}_2R_p(0)) + (\Lambda_V x\, {}_2\dot{R}_p(0))\right] d\,\Omega_p = 0$$

$$\left[(\bar{n}_p \cdot {}_2R_p(0))\,(\Lambda_R \cdot H_p) + (\bar{n}_p \cdot {}_2\dot{R}_p(0))\,(\Lambda_V \cdot H_p)\right] d\,i_p = 0$$

As before, the satisfaction of (120) is achieved either by fixing the independent parameter, which means the differential is zero, or by leaving the independent parameter open and forcing the coefficient to zero, which yields the transversality condition. Thus, the transversality conditions which must be satisfied if ω_p and/or Ω_p are left unspecified are, respectively,

$$r_p^2 ({}_1\Lambda_R(0) + {}_2\Lambda_R(0)) \cdot {}_2\dot{R}_p(0) - v_p^2 ({}_1\Lambda_V(0) + {}_2\Lambda_V(0)) \cdot {}_2R_p(0) = 0 \tag{121}$$

$$\bar{n}_p \cdot \left[({}_1\Lambda_R(0) + {}_2\Lambda_R(0))\, x\, {}_2R_p(0) + ({}_1\Lambda_V(0) + {}_2\Lambda_V(0))\, x\, {}_2\dot{R}_p(0)\right] = 0$$

The other three transversality conditions

$$({}_1\Lambda_R(0) + {}_2\Lambda_R(0)) \cdot {}_2R_p(0) = 0$$

$$({}_1\Lambda_V(0) + {}_2\Lambda_V(0)) \cdot {}_2\dot{R}_p(0) = 0 \tag{122}$$

$$(\bar{n}_p \cdot {}_2R_p(0))\,({}_1\Lambda_R(0) + {}_2\Lambda_R(0)) \cdot H_p + (\bar{n}_p \cdot {}_2\dot{R}_p(0))\,({}_1\Lambda_V(0) + {}_2\Lambda_V(0)) \cdot H_p = 0$$

are imposed if r_p, v_p, and/or i_p, respectively, are left unspecified.

Consider now the transversality conditions arising from the fact that the state, at Earth departure, is not entirely specified. Collecting the terms within the square brackets of (107) that pertain to the Earth-departure phase, we have

$$ {}_1\Lambda_R(1) \cdot d_1 R(1) + {}_1\Lambda_V(1) \cdot d_1 V(1) + {}_1\lambda_\nu(1) d_1 \nu(1) + {}_1\lambda_t(1) d_1 t(1) \tag{123} $$

We first note that the third term in (123) may be eliminated immediately because the requirement that ${}_1\nu(1)$ always equal one implies that the differential $d_1\nu(1)$ vanishes. Converting to differentials of geocentric position and velocity with the equations

$$ \begin{aligned} d_1 R(1) &= d R_e(1) + \dot{P}_e(1) d_1 t(1) \\ d_1 V(1) &= d \dot{R}_e(1) + \ddot{P}_e(1) d_1 t(1) \end{aligned} \tag{124} $$

expression (123) may be rewritten

$$ {}_1\Lambda_R(1) \cdot d R_e(1) + {}_1\Lambda_V(1) \cdot d \dot{R}_e(1) + \left[{}_1\Lambda_R(1) \cdot \dot{P}_e(1) + {}_1\Lambda_V(1) \cdot \ddot{P}_e(1) + {}_1\lambda_t(1) \right] d_1 t(1) \tag{125} $$

where $R_e(1)$ represents the geocentric position of the spacecraft and P_e is the heliocentric position of Earth, both evaluated at date ${}_1t(1)$. The bracketed term in (125) may be combined with appropriate terms on the left side of (107) such that, if there are no constraints placed on the launch date, the first segment flight time, or the total mission duration, then one must have

$$\left[\phi_{{}_1t(1)} + {}_1\Lambda_R(1)\cdot \dot{P}_e(1) + {}_1\Lambda_V(1)\cdot \ddot{P}_e(1) + {}_1\lambda_t(1)_1 j\right] d\,{}_1t(1) = 0 \tag{126}$$

Consequently if ${}_1t(1)$ is not fixed, we are left with the transversality condition

$$\phi_{{}_1t(1)} + {}_1\Lambda_R(1)\cdot \dot{P}_e(1) + {}_1\Lambda_V(1)\cdot \ddot{P}_e(1) + {}_1\lambda_t(1) + {}_1 j = 0 \tag{127}$$

Turning attention to the first two terms of (125), we seek to write $R_e(1)$ and $\dot{R}_e(1)$ (and hence $dR_e(1)$ and $d\dot{R}_e(1)$) in terms of the two fixed parameters r_{eo} and r_{es} and the independent parameter $v_{\infty d}$. Of course, three additional parameters are necessary to uniquely define $R_e(1)$ and $\dot{R}_e(1)$. An example of the three additional parameters would be the two angles defining the direction of $\dot{R}_e(1)$ plus an azimuth angle defining the velocity heading. However, since the physical problem is independent of the particular choice of the three additional parameters, the transversality conditions associated with any specific set of three must be equivalent to those for any other set. To make use of this fact, first consider any vector X which is written in terms of its magnitude x and a series of angles α, β, γ,--, which define the orientation of X relative to a fixed coordinate system. Let these angles be defined such that they represent rotations about the unit vectors $\bar{a}$, $\bar{b}$, $\bar{c}$, - - -, respectively. Then, in general, dX may be written

$$dX = \frac{X}{x}\, dx + (\bar{a} \times X)\, d\alpha + (\bar{b} \times X)\, d\beta + (\bar{c} \times X)\, d\gamma + - - - \tag{128}$$

Applying this result to our problem, we first note that if one is given r_{eo}, r_{es}, and $v_{\infty d}$, the dynamics of the motion at the sphere of influence are also known. That is, the speed v_s and flight path angle γ_{es} are computed

$$v_{es} = \sqrt{v_{\infty d}^2 + 2\mu_e/r_{es}}$$

$$\gamma_{es} = \cos^{-1}\left[r_{eo}\sqrt{v_{\infty d}^2 + 2\mu_e/r_{eo}}\;/r_{es}v_{es}\right] \tag{129}$$

Consequently, denoting as α_1, α_2, and α_3 the three independent angular rotations required to specify $R_e(1)$ and $\dot{R}_e(1)$, we may write

$$R_e(1) = R_e\,[\alpha_1,\ \alpha_2,\ \alpha_3\ ]$$

$$\dot{R}_e(1) = \dot{R}_e\,[v_{es},\ \gamma_{es},\ \alpha_1,\ \alpha_2,\ \alpha_3] \tag{130}$$

Note that r_{es} is not included as an argument of $\dot{R}_e$ since it is always fixed. Now, since α_1, α_2, α_3 are completely arbitrary, for convenience we select them to be the necessary rotations about the unit vectors $\bar{i}$, $\bar{j}$, and $\bar{k}$, respectively. Thus, from (128)

$$dR_e(1) = (\bar{i} \times R_e)\,d\alpha_1 + (\bar{j} \times R_e)\,d\alpha_2 + (\bar{k} \times R_e)\,d\alpha_3$$

$$d\dot{R}_e(1) = \frac{\dot{R}_e}{v_{es}}\,dv_{es} + \left(\frac{\dot{R}_e \times R_e}{|\dot{R}_e \times R_e|} \times \dot{R}_e\right)d\gamma_{es} + (\bar{i} \times \dot{R}_e)\,d\alpha_1 + (\bar{j} \times \dot{R}_e)\,d\alpha_2 + (\bar{k} \times \dot{R}_e)\,d\alpha_3 \tag{131}$$

and from (129)

$$dv_{es} = \frac{v_{\infty d}}{v_{es}} \, dv_{\infty d}$$

$$d\gamma_{es} = \frac{r_{eo} v_{\infty d} (v_{eo}^2 - v_{es}^2)}{v_{eo} v_{es}^2 (R_e \cdot \dot{R}_e)} \, dv_{\infty d} \tag{132}$$

where

$$v_{eo} = \sqrt{v_{\infty d}^2 + 2\mu_e / r_{eo}} \tag{133}$$

Upon substituting (131) into the first two terms of (125), collecting and rearranging terms, and setting the coefficients of $d\alpha_1$, $d\alpha_2$, and $d\alpha_3$ to zero, one obtains the vector of three transversality conditions

$$R_e(1) \times {}_1\Lambda_R(1) + \dot{R}_e(1) \times {}_1\Lambda_V(1) = \bar{0} \tag{134}$$

where $\bar{0}$ is the null vector. An additional term involving $dv_{\infty d}$ arises from the first two terms on the right side of the latter of equations (131). Forming the dot product of $\Lambda_V(1)$ with these two terms, employing the relations (132), and adding to the term involving $dv_{\infty d}$ in (107), one obtains an expression which, if there are no constraints on the reference power or net spacecraft mass and no constraints involving $t_{\infty d}$, may be written

$$\left\{ \frac{v_{\infty d}}{v_{eo}^2} \left[{}_1\Lambda_V(1) \cdot \dot{R}_e(1) + \frac{(v_{eo}^2 - v_{es}^2)}{(R_e(1) \cdot \dot{R}_e(1)} \left({}_1\Lambda_V(1) \cdot R_e(1) \right) \right] \right.$$

$$\left. + \, {}_1\lambda_\nu({}_1s_p) \frac{\Delta m_x}{m_o^2} \frac{dm_o}{dv_{\infty d}} + \phi_{v_{\infty d}} \right\} dv_{\infty d} = 0 \tag{135}$$

Then, if $v_{\infty d}$ is left open, one is left with the transversality condition that the coefficient of $dv_{\infty d}$ in (135) must vanish.

Proceeding immediately to consider the transversality conditions at the destination for planetary missions, recall that the independent parameters of interest are direct counterparts at the two ends, and that the options available for specifying or leaving open the spatial end conditions at the two points are identical. Hence, the form of the transversality conditions are identical and we may simply write them down. Because three degrees of freedom are left open in defining the planetocentric spacecraft position and velocity vectors, $R_t(1)$ and $\dot{R}_t(1)$ respectively, at ${}_2t(1)$, we require that

$$R_t(1) \times {}_2\Lambda_R(1) + \dot{R}_t(1) \times {}_2\Lambda_V(1) = \bar{0} \tag{136}$$

Alternate forms of the transversality conditions (134) and (136) are presented in Appendix B which possess properties more amenable to solution by numerical means than those given above. Hence, the conditions as given in Appendix B are recommended.

If $v_{\infty a}$ is not specified and there are no constraints involving $t_{\infty a}$ or net spacecraft mass, then the condition

$$\frac{v_{\infty a}}{v_{tf}^2}\left[{}_2\Lambda_V(1)\cdot\dot{R}_t(1) + \frac{(v_{tf}^2 - v_{ts}^2)}{(R_t(1)\cdot\dot{R}_t(1))}\left({}_2\Lambda_V \cdot R_t(1)\right)\right] + \phi_{v_{\infty a}} = 0 \tag{137}$$

must be satisfied. If the final mass ratio is not fixed and the net spacecraft mass is not constrained, then

$${}_2\lambda_\nu(1) + \phi_{{}_2\nu(1)} = 0 \tag{138}$$

and if there are no constraints involving ${}_2t(1)$

$$\phi_{{}_2t(1)} + {}_2\Lambda_R(1) \cdot \dot{P}_t(1) + {}_2\Lambda_V(1) \cdot \ddot{P}_t(1) + {}_2\lambda_t(1) - {}_2j = 0 \tag{139}$$

For probe and extra-ecliptic missions, the transversality conditions associated with final mass ratio and time are the same as for planetary missions with

$$t_{\infty a} = \frac{\partial t_{\infty a}}{\partial v_{\infty a}} = \dot{P}_t(1) = \ddot{P}_t(1) = 0 \tag{140}$$

whereas those associated with the spatial coordinates are different but are considerably simpler. For example, if the final boundary conditions are expressed in the Cartesian coordinate system, any component of the position or velocity that is left open gives rise to a transversality condition requiring the vanishing of the variable adjoint to the open component. That is, if ${}_2x(1)$ is left open, then

$${}_2\lambda_x(1) = 0 \tag{141}$$

is a transversality condition to be satisfied. If a retro stage is permitted, it will be assumed that the direction that the retro incremental velocity is imparted will be left open. Defining the vector

$$V_{\infty a} = {}_2V(1) - V_f \tag{142}$$

with magnitude $v_{\infty a}$, we see from (128) that

$$d_2V(1) = \frac{V_{\infty a}}{v_{\infty a}} dv_{\infty a} + (\bar{a} \times V_{\infty a})\, d\alpha_1 + (\bar{b} \times V_{\infty a})\, d\alpha_2 \tag{143}$$

where $\bar{a}$ and $\bar{b}$ are unit vectors about which the two independent rotations α_1 and α_2, which define the orientation of $V_{\infty a}$, are made. Then forming the dot product

$$ {}_2\Lambda_V(1) \cdot d_2V(1) = \frac{1}{v_{\infty a}}\left({}_2\Lambda_V(1) \cdot V_{\infty a}\right)dv_{\infty a} + \left(V_{\infty a} \times {}_2\Lambda_V(1)\right) \cdot \left(\bar{a}d\alpha_1 + \bar{b}d\alpha_2\right) \tag{144} $$

and imposing the condition that the coefficients of $d\alpha_1$ and $d\alpha_2$ must vanish independently if (107) is to be satisfied, we see that since $\bar{a}$ and $\bar{b}$ are arbitrary we must have

$$ V_{\infty a} \times {}_2\Lambda_V(1) = 0 \tag{145} $$

which implies $V_{\infty a}$ must be collinear with ${}_2\Lambda_V(1)$. Substituting this result into the first term on the right side of (144) and adding to the term containing $dv_{\infty a}$ in (107) yields for $v_{\infty a}$ unspecified

$$ \phi_{v_{\infty a}} \pm {}_2\lambda_V(1) = 0 \tag{146} $$

The ambiguity in sign arises because (145) requires that $V_{\infty a}$ either be aligned with or opposed to ${}_2\Lambda_V(1)$. The correct sign is dependent upon the sign of $\phi_{v_{\infty a}}$. Since ${}_2\lambda_V(1)$ is non-negative, the correct choice of sign is the opposite of the sign of $\phi_{v_{\infty a}}$. For maximizing net spacecraft mass, $\phi_{v_{\infty a}}$ is positive; hence the negative sign is chosen in (146). For minimum mission duration, $\phi_{v_{\infty a}}$ is zero and the choice of signs is immaterial.

If polar coordinates are employed for end conditions of probe or extra-ecliptic missions, the transversality conditions may be obtained in a manner analogous to that employed for the conditions at Earth. However, because of potential interest in the four parameters r, v, γ, and i at the final point, we will select two specific additional parameters consistent with these to complete the set of six needed to uniquely define ${}_2R(1)$ and ${}_2V(1)$. In particular, we select the osculating elements Ω, defining the longitude of ascending node, and ω, defining the angular position in the plane of motion relative to the ascending node. The four angles involved represent rotations about four unit vectors as follows

$$\begin{aligned}
&\gamma: \quad \frac{{}_2V(1) \times {}_2R(1)}{|{}_2V(1) \times {}_2R(1)|} = -\bar{a} \\
&i: \quad \frac{\bar{k} \times ({}_2R(1) \times {}_2V(1))}{|\bar{k} \times ({}_2R(1) \times {}_2V(1))|} = \bar{b} \\
&\Omega: \quad \bar{k} \\
&\omega: \quad \frac{{}_2R(1) \times {}_2V(1)}{|{}_2R(1) \times {}_2V(1)|} = \bar{a}
\end{aligned} \tag{147}$$

Proceeding as before, using (128), one obtains

$$\left({}_2\Lambda_R(1) \cdot {}_2R(1)\right) d_2 r(1) = 0$$

$$\left({}_2\Lambda_V(1) \cdot {}_2V(1)\right) d_2 v(1) = 0$$

$$\bar{a} \cdot \left({}_2V(1) \times {}_2\Lambda_V(1)\right) d_2 \gamma(1) = 0$$

$$\bar{b} \cdot \left[{}_2R(1) \times {}_2\Lambda_R(1) + {}_2V(1) \times {}_2\Lambda_V(1)\right] d_2 i(1) = 0 \tag{148}$$

$$\bar{k} \cdot \left[{}_2R(1) \times {}_2\Lambda_R(1) + {}_2V(1) \times {}_2\Lambda_V(1)\right] d_2 \Omega(1) = 0$$

$$\bar{a} \cdot \left[{}_2R(1) \times {}_2\Lambda_R(1) + {}_2V(1) \times {}_2\Lambda_V(1)\right] d_2 \omega(1) = 0$$

Thus, the transversality conditions, caused by leaving open any of the six polar parameters r, v, γ, i, Ω, and ω are given by the coefficients of the appropriate differentials in (148).

Providing there are no constraints imposed on either the reference power or the net spacecraft mass, the transversality conditions which result in optimum thrust acceleration and jet exhaust speed are written by inspection of (107), i.e.,

$$\phi_{a_o} + \lambda_{a_o} = 0 \tag{149}$$

yields the optimum a_o, while

$$\phi_c + \lambda_c = 0 \tag{150}$$

yields the optimum c.

The preceding constitutes a complete list of the transversality conditions for the original problem in the absence of any constraints (A.11) - (A.17). The effect of introducing any one or more of these constraints will now be discussed.

It will be assumed that, if the reference power is constrained in the form of (A.11), at least one of the two engine parameters, a_o and c, will be left open. If a_o is open, we eliminate da_o from (107) using the equation

$$da_o = - a_o \left[\left(\frac{1}{c} - \frac{\eta^*}{\eta} \right) dc + \frac{1}{m_o} \frac{dm_o}{dv_{\infty d}} dv_{\infty d} \right] \tag{151}$$

Multiplying this expression by the coefficient of da_o in (107) and adding the two resulting terms to the corresponding terms in (107), it is immediately seen that the effect of fixing the reference power is to eliminate the two conditions (149) and (150) in favor of the one condition

$$\phi_c + \lambda_c - a_o \left(\phi_{a_o} + \lambda_{a_o} \right) \left(\frac{1}{c} - \frac{\eta^*}{\eta} \right) = 0 \tag{152}$$

and to add the term

$$- \frac{a_o}{m_o} \frac{dm_o}{dv_{\infty d}} \left(\phi_{a_o} + \lambda_{a_o} \right) \tag{153}$$

to the coefficient of $dv_{\infty d}$ in (135). If a_o is fixed, then (149) is no longer applicable, and dc is eliminated from (107) using

$$dc = -\left(\frac{1}{c} - \frac{\eta^*}{\eta}\right)^{-1} \frac{1}{m_o} \frac{dm_o}{dv_{\infty d}} dv_{\infty d} \tag{154}$$

and the effect of fixing reference power is to eliminate (150) while adding the term

$$-\left(\frac{1}{c} - \frac{\eta^*}{\eta}\right)^{-1} \left(\phi_c + \lambda_c\right) \frac{1}{m_o} \frac{dm_o}{dv_{\infty d}} \tag{155}$$

to the coefficient of $dv_{\infty d}$ in (135).

The specification of net spacecraft mass is meaningful only if it is not the performance index. Therefore, we will assume that (A.12) will only be employed if mission duration is to be minimized. We will also assume that, if m_n is fixed, the final mass ratio $_2\nu(1)$ will be left open so that $d_2\nu(1)$ may be eliminated from (107) using (103). From (103) it is seen that, if the reference power is not fixed

$$\frac{\partial m_n}{\partial a_o} = -\frac{m_{ps}}{a_o}\left(1 - j_{ps} f_r\right)$$

$$\frac{\partial m_n}{\partial c} = -m_{ps}\left(\frac{1}{c} - \frac{\eta^*}{\eta}\right)\left(1 - j_{ps} f_r\right)$$

$$\frac{\partial m_n}{\partial v_{\infty d}} = \frac{1}{m_o}\left[m_n - k_t \Delta m_x \left(1 - j_t f_r\right)\right]\frac{dm_o}{dv_{\infty d}} \tag{156}$$

$$\frac{\partial m_n}{\partial v_{\infty a}} = -\left(m_o - m_p - m_x - j_t m_t - j_{ps} m_{ps}\right)\frac{j_r (1+k_r) v_{\infty a} e^{-v_r/c_r}}{c_r \sqrt{v_{\infty a}^2 + 2\mu_t / r_{tf}}}$$

$$\frac{\partial m_n}{\partial_2 \nu(1)} = m_o\left[(1+k_t) - (1+j_t k_t) f_r\right]$$

where

$$f_r = j_r (1+k_r)(1-e^{-v_r/c_r}) \tag{157}$$

If the reference power is fixed, the net spacecraft mass is independent of both a_o and c. Consequently, the right-hand side of the first two of equations (156) are then zero and the third becomes

$$\frac{\partial m_n}{\partial v_{\infty d}} = \frac{1}{m_o}\left[m_n + m_{ps} - k_t \Delta m_x - f_r\left(j_{ps} m_{ps} - j_t k_t \Delta m_x\right)\right]\frac{dm_o}{dv_{\infty d}} \tag{158}$$

Thus, solving for $d_2\nu(1)$ yields

$$d_2\nu(1) = -\left[\frac{\partial m_n}{\partial a_o} da_o + \frac{\partial m_n}{\partial c} dc + \frac{\partial m_n}{\partial v_{\infty d}} dv_{\infty d} + \frac{\partial m_n}{\partial v_{\infty a}} dv_{\infty a}\right] / \frac{\partial m_n}{\partial_2 \nu(1)} \tag{159}$$

Since $\phi_{_2\nu(1)}$ is zero when ϕ is mission duration, the only term in (107) containing $d_2\nu(1)$ is the one with the coefficient $_2\lambda_\nu(1)$. Upon multiplying (159) by $_2\lambda_\nu(1)$, substituting into (107) and collecting terms, it is seen that the effect on the transversality conditions of fixing m_n is to add the term

$$-_2\lambda_\nu(1) \frac{\partial m_n}{\partial a_o} / \frac{\partial m_n}{\partial_2 \nu(1)} \tag{160}$$

to the left side of (149); to add the term

$$-_2\lambda_\nu(1) \frac{\partial m_n}{\partial c} / \frac{\partial m_n}{\partial_2 \nu(1)} \tag{161}$$

to the left side of (150); to add the term

$$-_2\lambda_\nu(1) \frac{\partial m_n}{\partial v_{\infty d}} / \frac{\partial m_n}{\partial_2 \nu(1)} \tag{162}$$

to the coefficient of $dv_{\infty d}$ in (135); and to add the term

$$-_2\lambda_\nu(1) \frac{\partial m_n}{\partial v_{\infty a}} / \frac{\partial m_n}{\partial_2 \nu(1)} \tag{163}$$

to the left side of (137). Furthermore, the transversality condition (152) and the additive terms (153) and (155), which arise when reference power is fixed, must be modified when m_n is fixed. The modifications are made by simply replacing ϕ_{a_o} with the term (160) and by replacing ϕ_c with (161). Note that both ϕ_{a_o} and ϕ_c are zero when ϕ is mission duration.

Finally, consider the effects on the transversality conditions of specifying various dates and/or flight times through equations (A.14) - (A.17). First, it may be noted that the only parameters other than times that are involved in equations (A.14) - (A.17) are $v_{\infty d}$ and $v_{\infty a}$. Consequently, only the transversality conditions associated with the dates and the excess speeds can be affected, and these effects are completely independent of those associated with fixing either the net spacecraft mass or the reference power.

If it is desired to prescribe the launch date, then from (A.14)

$$d_1 t(1) - \frac{\partial t_{\infty d}}{\partial v_{\infty d}} \, dv_{\infty d} = 0 \tag{164}$$

and the transversality condition (127) is eliminated in favor of an additional term to be added to the coefficient of $dv_{\infty d}$ in (135). This term is

$$\left[\phi_{{}_1 t(1)} + {}_1\Lambda_R(1) \cdot \dot{P}_e(1) + {}_1\Lambda_V(1) \cdot \ddot{P}_e(1) + {}_1\lambda_t(1) + {}_1 j\right] \frac{\partial t_{\infty d}}{\partial v_{\infty d}} \tag{165}$$

Similarly, prescribing the arrival date as per (A.15) results in the elimination of (139) while adding the term

$$- \left[\phi_{{}_2t(1)} + {}_2\Lambda_R(1) \cdot \dot{P}_t(1) + {}_2\Lambda_V(1) \cdot \ddot{P}_t(1) + {}_2\lambda_t(1) - {}_2j \right] \frac{\partial t_{\infty a}}{\partial v_{\infty a}} \tag{166}$$

to the left side of (137) providing, of course, that the arrival excess speed is left open. Note that (166) is zero for probe and extra-ecliptic missions because of (140).

Fixing the flight time of the first segment implies that

$$d_2t(0) - d_1t(1) + \frac{\partial t_{\infty d}}{\partial v_{\infty d}} \; dv_{\infty d} = 0 \tag{167}$$

where the identity between ${}_1t(0)$ and ${}_2t(0)$ has been employed. If, in addition, the launch date is specified through (A.14), one obtains the simple result

$$d_2t(0) = 0 \tag{168}$$

which is, of course, equivalent to fixing the swingby date. Whenever alternate choices of specifying constraints are available, the preferable choice is the one that employs the specification of independent rather than dependent parameters, because that choice reduces the dimensionality of the boundary value problem. If both launch and swingby date are left open, $d_1t(1)$ may be eliminated from (107) using (167), and the results are 1) to replace equations (117) and (127) in favor of their difference, and 2) to add the term (165) to the coefficient of $dv_{\infty d}$ in (135). Similarly, specification of the second segment flight time with open swingby and arrival dates results in replacing equations (117) and (139) with their difference, and in adding the term (166) to the left side of (137). If both segment flight times are constrained, the three transversality conditions (117), (127), and (139) are replaced with the sum of (127) and (139) less (117), the term (165) is added to the coefficient of $dv_{\infty d}$ in (135), and the term (166) is added to the left side of (137).

Providing both launch and arrival dates are left open, a constraint on total mission duration leads to

$$d_2 t(1) - d_1 t(1) + \frac{\partial t_{\infty d}}{\partial v_{\infty d}} dv_{\infty d} + \frac{\partial t_{\infty a}}{\partial v_{\infty a}} dv_{\infty a} = 0 \tag{169}$$

Therefore, after eliminating $d_1 t(1)$ from (107) using (169), one finds that the effect on the transversality conditions is to eliminate (127) and (139) in favor of their sum, to add the term (165) to the coefficient of $dv_{\infty d}$ in (135), and to add (166) to the left side of (137). Various combinations of fixed mission duration with fixed segment flight times or dates are equivalent to problems treated above, and the transversality conditions are identical to those for the equivalent problem.

APPENDIX A - BOUNDARY CONDITIONS

Planetary Missions

For missions involving the use of an ephemeris to define the position and velocity of the destination as a function of time, the following boundary conditions must be satisfied. At the swingby point (s=0)

$$
\begin{aligned}
{}_1R(0) - {}_2R(0) &= 0 \\
{}_1V(0) - {}_2V(0) &= 0 \\
{}_1\nu(0) - {}_2\nu(0) &= 0 \\
{}_1t(0) - {}_2t(0) &= 0
\end{aligned}
\qquad (A.1)
$$

which assure continuity;

$$
\begin{aligned}
{}_2R(0) - {}_2P(0) - {}_2R_p(0) &= 0 \\
{}_2V(0) - {}_2\dot{P}(0) - {}_2\dot{R}_p(0) &= 0
\end{aligned}
\qquad (A.2)
$$

which relate heliocentric and planetocentric Cartesian components of spacecraft position and velocity; and

$${}_2R_p(0) \cdot {}_2\dot{R}_p(0) = 0$$

$$|{}_2R_p(0)| - r_p = 0$$

$$|{}_2\dot{R}_p(0)| - v_p = 0$$

$$({}_2R_p(0) \times {}_2\dot{R}_p(0)) \cdot \bar{n}_p - r_p v_p \cos i_p = 0 \tag{A.3}$$

$${}_2R_p(0) \cdot \bar{n}_p - r_p \sin i_p \sin \omega_p = 0$$

$${}_2R_p \cdot (\bar{k} \times \bar{n}_p) - r_p |\bar{k} \times \bar{n}_p| (\cos \Omega_p \cos \omega_p - \sin \Omega_p \sin \omega_p \cos i_p) = 0$$

which relate the planetocentric Cartesian coordinates to the polar coordinates used as independent parameters of the problem. All of these equations are satisfied trivially (i.e., inputs are chosen such that the equations are satisfied). At Earth departure, the constraint equations are

$$|{}_1R(1) - P_e(1)| - r_{es} = 0$$

$$|{}_1V(1) - \dot{P}_e(1)| - \sqrt{v_{\infty d}^2 + \frac{2\mu_e}{r_{es}}} = 0$$

$$|({}_1R(1) - P_e(1)) \times ({}_1V(1) - \dot{P}_e(1))| - r_{eo} \sqrt{v_{\infty d}^2 + \frac{2\mu_e}{r_{eo}}} = 0 \tag{A.4}$$

$${}_1\nu(1) - 1 = 0$$

The first three equations of (A. 4) assure compatibility with the assumption of ballistic transfer from the low altitude Earth parking orbit to the sphere of influence, while the latter results from the definition of ν. At the destination, the constraints are

$$|{}_2R(1) - P_t(1)| - r_{ts} = 0$$

$$|{}_2V(1) - \dot{P}_t(1)| - \sqrt{v_{\infty a}^2 + \frac{2\mu_t}{r_{ts}}} = 0 \qquad (A.5)$$

$$|({}_2R(1) - P_t(1)) \times ({}_2V(1) - \dot{P}_t(1))| - r_{tf}\sqrt{v_{\infty a}^2 + \frac{2\mu_t}{r_{tf}}} = 0$$

which assure compatibility with the assumption that the spacecraft coasts from the sphere of influence to the pericenter distance r_{tf} where the high thrust maneuver, if there is one, is performed.

To provide program flexibility, a number of constraints are optional. Among these are most of the independent parameters, such as the polar swingby parameters,

$$r_p - \tilde{r}_p = 0$$

$$v_p - \tilde{v}_p = 0 \qquad (A.6)$$

$$i_p - \tilde{i}_p = 0$$

$$\Omega_p - \tilde{\Omega}_p = 0$$

$$\omega_p - \tilde{\omega}_p = 0$$

the mass ratio and time at swingby

$$
\begin{aligned}
{}_2\nu(0) - \tilde{\nu}_{sw} &= 0 \\
{}_2t(0) - \tilde{t}_{sw} &= 0
\end{aligned}
\tag{A. 7}
$$

the times at the sphere of influence

$$
\begin{aligned}
{}_1t(1) - {}_1\tilde{t}_s &= 0 \\
{}_2t(1) - {}_2\tilde{t}_s &= 0
\end{aligned}
\tag{A. 8}
$$

the departure and arrival excess speeds

$$
\begin{aligned}
v_{\infty d} - \tilde{v}_{\infty d} &= 0 \\
v_{\infty a} - \tilde{v}_{\infty a} &= 0
\end{aligned}
\tag{A.9}
$$

and the engine parameters

$$
\begin{aligned}
a_o - \tilde{a}_o &= 0 \\
c - \tilde{c} &= 0
\end{aligned}
\tag{A. 10}
$$

where the tildes denote the desired values. Of course, being independent parameters, equations (A. 6) - (A. 10) are all satisfied trivially by input.

Other optional constraints which may be of interest in specific problems include the reference power

$$\alpha a_o c\, m_o / 2\eta - \tilde{p}_o = 0 \qquad \text{(A. 11)}$$

the net spacecraft mass

$$m_n - \tilde{m}_n = 0 \qquad \text{(A. 12)}$$

the final mass ratio

$$_2\nu(1) - \tilde{\nu}_f = 0 \qquad \text{(A. 13)}$$

and certain date and flight time parameters such as launch date

$$_1t(1) - t_{\infty d} - \tilde{t}_o = 0 \qquad \text{(A. 14)}$$

arrival date

$$_2t(1) + t_{\infty a} - \tilde{t}_f = 0 \qquad \text{(A. 15)}$$

individual segment flight times

$$\begin{aligned} &_1t(0) - {}_1t(1) + t_{\infty d} - \tilde{\Delta t}_1 = 0 \\ &_2t(1) + t_{\infty a} - {}_2t(0) - \tilde{\Delta t}_2 = 0 \end{aligned} \qquad \text{(A. 16)}$$

and total mission duration

$$ {}_2t(1) - {}_1t(1) + t_{\infty d} + t_{\infty a} - \Delta\tilde{t}_m = 0 \tag{A.17} $$

Certain combinations of the optional boundary conditions above are mutually exclusive because they are not independent. An example would be the combination (A.14), (A.15), and (A.17). A careful examination of equations (A.7) - (A.17) will uncover several other examples. Of course boundary condition (A.12) is not permitted if the net spacecraft mass is to be maximized; likewise, (A.17) is not permitted if mission duration is to be minimized.

Probe and Extra-ecliptic Missions

For any mission for which the destination is not given by an ephemeris, the permissible boundary conditions are identical to those in the preceding paragraphs, with the exception of equations (A.5) which are replaced. It shall be understood, however, that the time interval $t_{\infty a}$ is zero.

In place of equations (A.5), a number of optional final conditions will be permitted. Among these will be the individual Cartesian components of final position

$$ \begin{aligned} {}_2x(1) - x_f &= 0 \\ {}_2y(1) - y_f &= 0 \\ {}_2z(1) - z_f &= 0 \end{aligned} \tag{A.18} $$

and/or velocity (assuming no retro maneuver)

$$
\begin{aligned}
{}_2\dot{x}(1) - \dot{x}_f &= 0 \\
{}_2\dot{y}(1) - \dot{y}_f &= 0 \\
{}_2\dot{z}(1) - \dot{z}_f &= 0
\end{aligned}
\qquad (A.19)
$$

If a retro maneuver is permitted, then (A.19) is replaced by the single equation

$$|{}_2V(1) - V_f| - v_{\infty a} = 0 \qquad (A.20)$$

In the latter case, the retro incremental velocity is taken to be equal to $v_{\infty a}$. The remaining boundary condition possibilities that are of particular interest in this type of problem include the following polar coordinates:

$$
\begin{aligned}
&|{}_2R(1)| - r_f = 0 \\
&|{}_2V(1)| - v_f = 0 \\
&\sin^{-1}\left[|{}_2R(1) \cdot {}_2V(1)| / |{}_2R(1)|\ |{}_2V(1)| \right] - \gamma_f = 0 \\
&\cos^{-1}\left[({}_2R(1) \times {}_2V(1)) \cdot \bar{k} / |{}_2R(1) \times {}_2V(1)| \right] - i_f = 0 \\
&\tan^{-1}\left[\frac{\{[\bar{k} \times ({}_2R(1) \times {}_2V(1))] \times \bar{i}\} \cdot \bar{k}}{[k \times ({}_2R(1) \times {}_2V(1))] \times \bar{i}} \right] - \Omega_f = 0 \\
&\tan^{-1}\left[\frac{\{{}_2R(1) \times [\bar{k} \times ({}_2R(1) \times {}_2V(1))]\} \cdot ({}_2R(1) \times {}_2V(1))}{|{}_2R(1) \times {}_2V(1)|\ [k \times ({}_2R(1) \times {}_2V(1))] \cdot {}_2R(1)} \right] - \omega_f = 0
\end{aligned}
\qquad (A.21)
$$

APPENDIX B - ALTERNATE BOUNDARY AND TRANSVERSALITY CONDITIONS FOR PLANETARY MISSIONS

Consider the vector transversality condition (136) that must be satisfied upon entry into the sphere of influence of the target planet, i.e.,

$$R_t \times {}_2\Lambda_R + \dot{R}_t \times {}_2\Lambda_V = 0 \tag{B.1}$$

with all quantities being evaluated at s=1. An important observation that may be made from this equation is that, if it is to be satisfied, the four vectors R_t, $\dot{R}_t$, ${}_2\Lambda_R$ and ${}_2\Lambda_V$ must all lie in the same plane. Consequently, for the converged solution, the vectors ${}_2\Lambda_R$ and ${}_2\Lambda_V$ define the plane of motion of the hyperbolic approach trajectory relative to the target planet at the time of entry into the target planet's sphere of influence. Thus, if we define the unit vector $\bar{n}_t$ along the planetocentric angular momentum vector, i.e.,

$$\bar{n}_t = \frac{R_t \times \dot{R}_t}{|R_t \times \dot{R}_t|} \tag{B.2}$$

then for the converged solution we will have

$$\bar{n}_t = \pm \frac{{}_2\Lambda_V \times {}_2\Lambda_R}{|{}_2\Lambda_V \times {}_2\Lambda_R|} \tag{B.3}$$

If one is given the vectors ${}_2\Lambda_R$ and ${}_2\Lambda_V$ at the sphere of influence, then the direction of the planetocentric angular momentum vector (and hence the inclination of the hyperbolic trajectory relative to the ecliptic plane) is known except for the ambiguity in signs. Once this ambiguity is resolved, then the inclination of the planetocentric hyperbola relative to the ecliptic plane is

$$i_t = \cos^{-1}(\bar{n}_t \cdot \bar{k}) \qquad (0 \le i_t \le \pi) \tag{B.4}$$

where $\bar{k}$ is the unit vector normal to the ecliptic. Furthermore, the ascending node of the planetocentric hyperbola on the ecliptic plane relative to the vernal equinox direction is given by

$$\Omega_t = \cos^{-1}(\bar{\ell}_t \cdot \bar{i}) = \sin^{-1}(\bar{\ell}_t \cdot \bar{j}) \qquad (0 \leq \Omega_t \leq 2\pi) \tag{B.5}$$

where $\bar{i}$ is the unit vector in the ecliptic plane in the direction of the vernal equinox and $\bar{j}$ completes the right-handed Cartesian coordinate system; $\bar{\ell}_t$ is a unit vector in the ecliptic plane in the direction of the subject ascending node and is given by

$$\bar{\ell}_t = \frac{\bar{k} \times \bar{n}_t}{|\bar{k} \times \bar{n}_t|} \tag{B.6}$$

Thus, except for the ambiguity in the sign of $\bar{n}_t$, the optimal orientation of the planetocentric hyperbolic arrival trajectory is known. There remains only one degree of freedom in specifying the position and velocity of the spacecraft at entry of the sphere of influence. Let this degree of freedom be represented by the angle ω_{ts} between $\bar{\ell}_t$, the direction of the line of nodes, and R_t, the planetocentric position vector at entry of the sphere of influence. The optimum value of this angle may be obtained by solving for the root of the $\bar{n}_t$ - component of the vector equation (B.1). That is, we seek the value of ω_{ts} which results in the satisfaction of

$$\bar{n}_t \cdot (R_t \times {}_2\Lambda_R + \dot{R}_t \times {}_2\Lambda_V) = {}_2\Lambda_R \cdot (\bar{n}_t \times R_t) + {}_2\Lambda_V \cdot (\bar{n}_t \times \dot{R}_t) = 0 \tag{B.7}$$

Rewriting the cross product terms

$$\bar{n}_t \times R_t = r_{ts}(-\sin\omega_{ts}\,\bar{\ell}_t + \cos\omega_{ts}\,\bar{m}_t)$$

$$\bar{n}_t \times \dot{R}_t = -v_{ts}\left[\cos(\omega_{ts} - \gamma_{ts})\,\bar{\ell}_t + \sin(\omega_{ts} - \gamma_{ts})\,\bar{m}_t\right] \tag{B.8}$$

where

$$\bar{m}_t = \bar{n}_t \times \bar{\ell}_t \tag{B.9}$$

and r_{ts}, v_{ts}, and γ_{ts} are the (known) planetocentric radial distance, speed, and flight path angle, respectively, at entry of the sphere of influence. Then, through the use of double angle formulas, substituting (B.8) into (B.7) and rearranging terms, the following expression is obtained for ω_{ts}:

$$\omega_{ts} = \tan^{-1}\left[\frac{r_{ts}({}_2\Lambda_R \cdot \bar{m}_t) - v_{ts}\left[\cos\gamma_{ts}({}_2\Lambda_V \cdot \bar{\ell}_t) - \sin\gamma_{ts}({}_2\Lambda_V \cdot \bar{m}_t)\right]}{r_{ts}({}_2\Lambda_R \cdot \bar{\ell}_t) + v_{ts}\left[\sin\gamma_{ts}({}_2\Lambda_V \cdot \bar{\ell}_t) + \cos\gamma_{ts}({}_2\Lambda_V \cdot \bar{m}_t)\right]}\right] \quad \text{(B.10)}$$

Clearly this equation for ω_{ts} yields two solutions, one differing from the other by π radians. Consequently, there are two ambiguities, one associated with the choice of n_t and the other with ω_{ts}, that must be resolved before a unique set of parameters representing the position and velocity upon entry into the sphere of influence can be defined. Once these ambiguities are resolved (or a choice is made for numerical testing), the planetocentric position and velocity at the sphere of influence may be evaluated as follows:

$$R_t = r_{ts}\Big[(\cos\omega_{ts}\cos\Omega_t - \sin\omega_{ts}\sin\Omega_t\cos i_t)\,\bar{i} + (\cos\omega_{ts}\sin\Omega_t + \sin\omega_{ts}\cos\Omega_t\cos i_t)\,\bar{j} + \sin\omega_{ts}\sin i_t\,\bar{k}\Big] \quad \text{(B.11)}$$

$$\dot{R}_t = v_{ts}\Big[-(\sin\tilde{\omega}_{ts}\cos\Omega_t + \cos\tilde{\omega}_{ts}\sin\Omega_t\cos i_t)\,\bar{i} - (\sin\tilde{\omega}_{ts}\sin\Omega_t - \cos\tilde{\omega}_{ts}\cos\Omega_t\cos i_t)\,\bar{j} + \cos\tilde{\omega}_{ts}\sin i_t\,\bar{k}\Big] \quad \text{(B.12)}$$

where

$$\tilde{\omega}_{ts} = \omega_{ts} - \gamma_{ts} \quad \text{(B.13)}$$

The combination of the three end conditions (A.5) and the three components of (136) may then be replaced with the six equations represented by

$$\begin{aligned} {}_2R(1) - R_t - P_t(1) &= 0 \\ {}_2\dot{R}(1) - \dot{R}_t - \dot{P}_t(1) &= 0 \end{aligned} \quad \text{(B.14)}$$

The equations (B.14) are recommended over the combination of (A.5) and (136). The reason for this is that the functions involved in equations (A.5) are basically quadratic, non-negative quantities, the desired roots of which are frequently difficult to isolate numerically because they lie near the bottom of a trough. The equations (B.14), on the other hand, contain functions which behave linearly over considerably larger changes in the independent parameters in the vicinity of the solution and, hence, are more amenable to numerical solution.

The physical end conditions and associated transversality conditions that must be satisfied at exit from the launch planet sphere of influence for swingby mission are identical in form to those for the target planet. Hence, an alternate set of boundary conditions may be written for departure from the launch planet that are identical in form to those given above for arrival at the target planet. Specifically, the alternate conditions may be written

$$
\begin{aligned}
{}_1R(1) - R_e - P_e(1) &= 0 \\
{}_1\dot{R}(1) - \dot{R}_e - \dot{P}_e(1) &= 0
\end{aligned}
\qquad \text{(B.15)}
$$

where R_e and $\dot{R}_e$ are the planetocentric position and velocity of the spacecraft at departure of the sphere of influence and are given by the equations.

$$
R_e = r_{es}\Big[(\cos\omega_{es}\cos\Omega_e - \sin\omega_{es}\sin\Omega_e\cos i_e)\,\bar{i} + (\cos\omega_{es}\sin\Omega_e + \sin\omega_{es}\cos\Omega_e\cos i_e)\,\bar{j} + \sin\omega_{es}\sin i_e\,\bar{k}\Big] \qquad \text{(B.16)}
$$

$$
\dot{R}_e = v_{es}\Big[-(\sin\tilde{\omega}_{es}\cos\Omega_e + \cos\tilde{\omega}_{es}\sin\Omega_e\cos i_t)\,\bar{i} - (\sin\tilde{\omega}_{es}\cos\omega_{es}\cos\tilde{\omega}_{es}\cos\Omega_e\cos i_e)\,\bar{j} + \cos\tilde{\omega}_{es}\sin i_e\,\bar{k}\Big] \qquad \text{(B.17)}
$$

where

$$
\bar{n}_e = \pm\frac{{}_1\Lambda_V \times {}_1\Lambda_R}{|{}_1\Lambda_V \times {}_1\Lambda_R|}
$$

$$\bar{\ell}_e = \frac{\bar{k} \times \bar{n}_e}{|\bar{k} \times \bar{n}_e|}$$

$$\bar{m}_e = \bar{n}_e \times \bar{\ell}_e$$

$$i_e = \cos^{-1} (\bar{n}_e \cdot \bar{k})$$

$$\Omega_e = \cos^{-1} (\bar{\ell}_e \cdot \bar{i}) = \sin^{-1} (\bar{\ell}_e \cdot \bar{j}) \tag{B.18}$$

$$\omega_{es} = \tan^{-1} \left[\frac{r_{es} ({}_1\Lambda_R \cdot \bar{m}_e) - v_{es} [\cos \gamma_{es} ({}_1\Lambda_V \cdot \bar{\ell}_e) - \sin \gamma_{es} ({}_1\Lambda_V \cdot \bar{m}_e)]}{r_{es} ({}_1\Lambda_R \cdot \bar{\ell}_e) + v_{es} [\sin \gamma_{es} ({}_1\Lambda_V \cdot \bar{\ell}_e) + \cos \gamma_{es} ({}_1\Lambda_V \cdot \bar{m}_e)]} \right]$$

$$\tilde{\omega}_{es} = \omega_{es} - \gamma_{es}$$

Note that again two ambiguities appear in the equations for selecting the optimum position and velocity vectors.

There is insufficient information available at this point to resolve the ambiguities that have arisen above. Before any decisions are made, it is important to understand the reasons for and sources of the ambiguities. Consider first the sign of the vectors $\bar{n}_t$ and $\bar{n}_e$. Clearly these represent uncertainties in the sense of the planetocentric motion at the target and launch planets, respectively. That is, the uncertainty is as to whether the motion is posigrade or retrograde in each of the two cases. It so happens that there will exist a locally optimum solution for each case; hence, the ambiguity in the choice of solutions. At the launch planet, the choice is clear; one must pick the posigrade orbit from launch vehicle payload considerations. It is also likely that posigrade orbits at the target planet would also be desirable. However, this may not necessarily be true, particularly if a notable payload advantage is available with the locally optimum retrograde solution.

Once the direction of motion is chosen, there remains at each terminal the ambiguities in the selections of ω_{ts} and ω_{es}. The important point to remember regarding these choices is that one of the two possible choices for each variable represents a local optimum while the other is merely a saddle point solution. That is to say, given the best possible choice of all other parameters, of the two possible choices of the ω at one end, the one choice is the best one could possibly make and the other is the worst choice. At the present time there is no known mathematical proof available as to which solution is optimum. However, past experiences with many numerical examples have indicated that the appropriate choice is the one that results in a planetocentric velocity vector that is nearly diametrically opposed to the primer vector (the other solution makes them nearly aligned). This information is offered at this point merely as a suggestion rather than a rule; consequently the user should exercise care to investigate this ambiguity for each mission application.

APPENDIX C - ALTERNATE PROCEDURE FOR PROBLEMS

WITH IMPOSED COASTING WITHIN SWINGBY PLANET'S SPHERE OF INFLUENCE

Due to the extremely rapid and large fluctuations that are known to occur in certain of the adjoint variables in the close proximity of a planet, a very sensitive relationship exists between the end conditions at the launch and target planets and the guesses of these adjoint variables at the passage point. Consequently, there exists a correspondingly sensitive boundary value problem that has exhibited extremely poor convergence qualities. A method has been developed, however, which greatly alleviates this difficulty in problems in which coasting flight is imposed within the sphere of influence of the swingby planet.

If thrusting is not permitted within the sphere of influence, a given set of the six passage conditions r_p, v_p, i_p, Ω_p, ω_p, and t_p completely define the spacecraft path within the planetocentric phase. This also implies, of course, that the behavior of the adjoint variables within the sphere have absolutely no effect on the planetocentric path. As a consequence, it is possible to completely disregard the adjoint variables within the sphere by simply moving the point at which the adjoint variables are guessed from the passage point to the crossings of the sphere of influence. That is to say, rather than guess two sets of adjoint variables (one for each leg) at the swingby point, one instead guesses a set for the first leg at entry of the sphere and another set for the second leg at exit of the sphere. The spacecraft states at entry and exit are easily written as explicit functions of the state at passage using the standard conic equations for ballistic motion in an inverse square central force field. Therefore, one may start the optimization problem at the swingby planet's sphere of influence on both legs by simply defining $s = 0$ to represent entry of the sphere on the first leg and exit of the sphere on the second leg. In this way the consideration of the behavior of the adjoint variables in the sphere of influence is completely avoided, and the new independent parameters(same functions, but evaluated at a different point) are considerably more stable and less sensitive.

The implementation of the above technical approach is relatively simple. The state and adjoint equations remain unchanged except for the simplification that results upon recognizing that the step function h_ρ is identically zero throughout the interval $s = 0$ to $s = 1$. The only significant changes are the expressions for the boundary conditions at $s = 0$ and the concomitant changes in the transversality conditions.

To obtain the new boundary conditions at $s = 0$, define

$$v_{\infty p}^2 = v_p^2 - \frac{2\mu_p}{r_p}$$

$$e_p = 1 + \frac{r_p v_{\infty p}^2}{\mu_p}$$

such that the time $t_{\infty p}$ and the travel angle ψ_{ps} between the passage point at a distance r_p and the sphere of influence at a distance r_{ps} are given by

$$t_{\infty p} = \frac{\mu_p}{v_{\infty p}^3}\left(e_p \sinh f_p - f_p\right)$$

$$\psi_{ps} = \cos^{-1}\left[\left(\frac{r_p}{r_{ps}}(e_p+1) - 1\right)/e_p\right]$$

where

$$f_p = \cosh^{-1}\left[\left(1 + \frac{r_{ps} v_{\infty p}^2}{\mu_p}\right)/e_p\right] .$$

Then the speed v_{ps} and flight path angle γ_{ps} at exit of the sphere are obtained from the equations

$$v_{ps}^2 = v_{\infty p}^2 + \frac{2\mu_p}{r_{ps}}$$

$$\gamma_{ps} = \tan^{-1}\left(\frac{e_p \sin\psi_{ps}}{1+e_p \cos\psi_{ps}}\right)$$

The planetocentric positions ${}_i R_p(0)$ and velocities ${}_i \dot{R}_p(0)$, $i = 1$ or 2, may then be written

$$ {}_i R_p(0) = r_{ps} \Big\{ \Big[\cos(\omega_p \mp \psi_{ps}) \cos\Omega_p - \sin(\omega_p \mp \psi_{ps}) \sin\Omega_p \cos i_p\Big] \bar{\ell}_p $$

$$ + \Big[\cos(\omega_p \mp \psi_{ps}) \sin\Omega_p + \sin(\omega_p \mp \psi_{ps}) \cos\Omega_p \cos i_p\Big] \bar{m}_p $$

$$ + \sin(\omega_p \mp \psi_{ps}) \sin i_p \, \bar{n}_p \Big\} $$

$$ {}_i \dot{R}_p(0) = v_{ps} \Big\{ - \Big[\sin(\omega_p \mp \psi_{ps} \pm \gamma_{ps}) \cos\Omega_p + \cos(\omega_p \mp \psi_{ps} \pm \gamma_{ps}) \sin\Omega_p \cos i_p\Big] \bar{\ell}_p $$

$$ - \Big[\sin(\omega_p \mp \psi_{ps} \pm \gamma_{ps}) \sin\Omega_p - \cos(\omega_p \mp \psi_{ps} \pm \gamma_{ps}) \cos\Omega_p \cos i_p\Big] \bar{m}_p $$

$$ + \cos(\omega_p \mp \psi_{ps} \pm \gamma_{ps}) \sin i_p \, \bar{n}_p \Big\} $$

where the upper sign applies for $i=1$ and the lower sign for $i=2$. This convention is also employed throughout the remainder of this appendix. The unit vectors $\bar{\ell}_p$, $\bar{m}_p$, and $\bar{n}_p$ are as defined in equations (40) - (43). The actual boundary conditions for the state are then given by

$$ {}_i V(0) = \dot{P}(t_p \mp t_{\infty p}) + {}_i \dot{R}_p(0) $$

$$ {}_i R(0) = P(t_p \mp t_{\infty p}) + {}_i R_p(0) $$

$$ {}_i t(0) = t_p \mp t_{\infty p} $$

$$ {}_1\nu(0) = {}_1\nu_p + m_x/m_o $$

$$ {}_2\nu(0) = {}_2\nu_p $$

The general form of the transversality condition remains essentially unchanged from (107) except that the second term in the coefficient of $dv_{\infty d}$, which arose due to the discontinuity in mass at $s = {}_1s_p$, is absent. Since the point at which the mass is discontinuous now occurs at $s=0$ rather than ${}_1s_p$, the equivalent of this term arises directly from the quantity ${}_1\lambda_\nu(0)\, d_1\nu(0)$ contained in the bracketed term. We concern ourselves here only with those terms in (107) pertaining to the boundary $s = 0$, since all other terms and conditions remain unchanged from the main text. That is, we desire to derive the appropriate equations which will replace (116), (117), (121), and (122).

The differentials of the boundary conditions above are written

$$ d_iV(0) = \ddot{P}\,(t_p \mp t_{\infty p})\,(dt_p \mp dt_{\infty p}) + d_i\dot{R}_p(0) $$

$$ d_iR(0) = \dot{P}(t_p \mp t_{\infty p})\,(dt_p \mp dt_{\infty p}) + d_iR_p(0) $$

$$ d_it(0) = dt_p \mp dt_{\infty p} $$

$$ d_1\nu(0) = d\nu_p - \frac{\Delta m_x}{m_o^2}\frac{dm_o}{dv_{\infty d}}\,dv_{\infty d} $$

$$ d_2\nu(0) = d\nu_p $$

where

$$ d_iR_p(0) = (\bar{n}_p \times {}_iR_p(0))\,d\Omega_p + \left(\frac{\bar{n}_p \times H_p}{h_p \sin i_p} \times {}_iR_p(0)\right) d\,i_p + \frac{1}{h_p}\,(H_p \times {}_iR_p(0))\,(d\omega_p \mp d\psi_{ps}) $$

$$d_i\dot{R}_p(0) = \frac{{}_i\dot{R}_p(0)}{v_{ps}}\, dv_{ps} + (\bar{n}_p \times {}_i\dot{R}_p(0))\, d\Omega_p + \left(\frac{\bar{n}_p x H_p}{h_p \sin i_p} \times {}_i\dot{R}_p(0)\right) di_p$$

$$+ \frac{1}{h_p} (H_p \times {}_i\dot{R}_p(0))\, (d\omega_p \mp d\psi_{ps} \pm d\gamma_{ps})$$

But since $t_{\infty p}$, v_{ps}, ψ_{ps} and γ_{ps} are functions only of r_p and v_p (assuming r_{ps} is a specified constant), it is possible to write for dx, where x represents any one of the four parameters above,

$$dx = \frac{\partial x}{\partial r_p} dr_p + \frac{\partial x}{\partial v_p} dv_p$$

and thereby eliminate all differentials except those of the state at swingby. The equations for the indicated partial derivatives are:

$$\frac{\partial t_{\infty p}}{\partial r_p} = \frac{\mu_p}{r_p^2 v_{\infty p}^2}\left[\frac{r_p}{v_{\infty p}} (e_p+1) \sinh f_p - 3t_{\infty p} + \frac{r_{ps}}{v_{\infty p}} \frac{(e_p \cosh f_p - 1)}{e_p \sinh f_p}\left(2 - \frac{r_p}{r_{ps}} (e_p+1) \cosh f_p\right)\right]$$

$$\frac{\partial t_{\infty p}}{\partial v_p} = \frac{v_p}{v_{\infty p}^2}\left[\frac{2r_p}{v_{\infty p}} \sinh f_p - 3\, t_{\infty p} + \frac{2r_{ps}}{v_{\infty p} e_p \sinh f_p} (e_p \cosh f_p - 1)\left(1 - \frac{r_p}{r_{ps}} \cosh f_p\right)\right]$$

$$\frac{\partial v_{ps}}{\partial r_p} = \frac{\mu_p}{r_p^2 v_{ps}}$$

$$\frac{\partial v_{ps}}{\partial v_p} = \frac{v_p}{v_{ps}}$$

$$\frac{\partial \psi_{ps}}{\partial r_p} = \left(\cos \psi_{ps} - 2\,\frac{r_p}{r_{ps}}\right) \frac{v_p^2}{\mu_p e_p \sin \psi_{ps}}$$

$$\frac{\partial \psi_{ps}}{\partial v_p} = \frac{2 r_p v_p}{\mu_p e_p \sin \psi_{ps}} \left(\cos \psi_{ps} - \frac{r_p}{r_{ps}}\right)$$

$$\frac{\partial \gamma_{ps}}{\partial r_p} = \frac{\cot \gamma_{ps}}{r_p} \left(\frac{\mu_p}{r_p v_{ps}^2} - 1\right)$$

$$\frac{\partial \gamma_{ps}}{\partial v_p} = \frac{\cot \gamma_{ps}}{v_p} \left(\frac{v_p^2}{v_{ps}^2} - 1\right)$$

Extracting from (107) only those terms pertaining to the boundary $s = 0$, and substituting the above differentials, one may then write

$$-\sum_{i=1}^{2} \left[{}_i\Lambda_V \cdot d_i V + {}_i\Lambda_R \cdot d_i R + {}_i\lambda_\nu d_i \nu + {}_i\lambda_t d_i t \right]_{s=0} + {}_1\lambda_\tau(1) d_1 t(0) - {}_2\lambda_\tau(1) d_2 t(0)$$

$$= \frac{\Delta m_x}{m_o^2} \frac{dm_o}{dv_{\infty d}}\, {}_1\lambda_\nu\, dv_{\infty d} - ({}_1\lambda_\nu + {}_2\lambda_\nu)\, d\nu_p$$

$$- \left[{}_2\Lambda_V \cdot \ddot{P}(t_{ps}^{+}) + {}_2\Lambda_R \cdot \dot{P}(t_{ps}^{+}) + {}_2\lambda_t - {}_2 j + {}_1\Lambda_V \cdot \ddot{P}(t_{ps}^{-}) + {}_1\Lambda_R \cdot \dot{P}(t_{ps}^{-}) + {}_1\lambda_t + {}_1 j \right] dt_p$$

$$- \frac{H_p}{h_p} \cdot \left[{}_2\dot{R}_p \times {}_2\Lambda_V + {}_2R_p \times {}_2\Lambda_R + {}_1\dot{R}_p \times {}_1\Lambda_V + {}_1R_p \times {}_1\Lambda_R \right] d\,\omega_p$$

$$- \bar{n}_p \cdot \left[{}_2\dot{R}_p \times {}_2\Lambda_V + {}_2R_p \times {}_2\Lambda_R + {}_1\dot{R}_p \times {}_1\Lambda_V + {}_1R_p \times {}_1\Lambda_R \right] d\,\Omega_p$$

$$- \frac{\bar{n}_p \times H_p}{h_p \sin i_p} \cdot \left[{}_2\dot{R}_p \times {}_2\Lambda_V + {}_2R_p \times {}_2\Lambda_R + {}_1\dot{R}_p \times {}_1\Lambda_V + {}_1R_p \times {}_1\Lambda_R \right] d\, i_p$$

$$- \left\{ \left[{}_2\Lambda_V \cdot \ddot{P}(t_{ps}^{+}) + {}_2\Lambda_R \cdot \dot{P}(t_{ps}^{+}) + {}_2\lambda_t\, {}_2\bar{j} - {}_1\Lambda_V \cdot \ddot{P}(t_{ps}^{-}) - {}_1\Lambda_R \cdot \dot{P}(t_{ps}^{-}) - {}_1\lambda_t\, {}_1\bar{j} \right] \frac{\partial t_{\infty p}}{\partial v_p} \right.$$

$$+ \frac{1}{v_{ps}} \left[{}_2\Lambda_V \cdot {}_2\dot{R}_p + {}_1\Lambda_V \cdot {}_1\dot{R}_p \right] \frac{\partial v_{ps}}{\partial v_p} - \frac{H_p}{h_p} \cdot \left[{}_2\dot{R}_p \times {}_2\Lambda_V - {}_1\dot{R}_p \times {}_1\Lambda_V \right] \frac{\partial \gamma_{ps}}{\partial v_p}$$

$$\left. + \frac{H_p}{h_p} \cdot \left[{}_2\dot{R}_p \times {}_2\Lambda_V + {}_2R_p \times {}_2\Lambda_R - {}_1\dot{R}_p \times {}_1\Lambda_V - {}_1R_p \times {}_1\Lambda_R \right] \frac{\partial \psi_{ps}}{\partial v_p} \right\} d\,v_p$$

$$- \left\{ \left[{}_2\Lambda_V \cdot \ddot{P}(t_{ps}^{+}) + {}_2\Lambda_R \cdot \dot{P}(t_{ps}^{+}) + {}_2\lambda_t\, {}_2\bar{j} - {}_1\Lambda_V \cdot \ddot{P}(t_{ps}^{-}) - {}_1\Lambda_R \cdot \dot{P}(t_{ps}^{-}) - {}_1\lambda_t\, {}_1\bar{j} \right] \frac{\partial t_{\infty p}}{\partial r_p} \right.$$

$$+ \frac{1}{v_{ps}} \left[{}_2\Lambda_V \cdot {}_2\dot{R}_p + {}_1\Lambda_V \cdot {}_1\dot{R}_p \right] \frac{\partial v_{ps}}{\partial r_p} - \frac{H_p}{h_p} \cdot \left[{}_2R_p \times {}_2\Lambda_V - {}_1R_p \times {}_1\Lambda_V \right] \frac{\partial \gamma_{ps}}{\partial r_p}$$

$$\left. + \frac{H_p}{h_p} \cdot \left[{}_2\dot{R}_p \times {}_2\Lambda_V + {}_2R_p \times {}_2\Lambda_R - {}_1\dot{R}_p \times {}_1\Lambda_V - {}_1R_p \times {}_1\Lambda_R \right] \frac{\partial \psi_{ps}}{\partial r_p} \right\} d\,r_p$$

where all adjoint variables and state variables appearing on the right hand side are evaluated at $s=0$, and where the notation

$$t_{ps}^{-} = t_p - t_{\infty p}$$

$$t_{ps}^{+} = t_p + t_{\infty p}$$

has been employed.

The first term on the right hand side of the equation above is added to the other terms in (107) containing $dv_{\infty d}$ and replaces the similar term containing ${}_1\lambda_\nu({}_1s_p)$ that was discarded earlier. The coefficients of the remaining seven differentials constitute the new transversality conditions that were sought. Note that the condition associated with mass ratio remains unchanged while those associated with t_p, ω_p, Ω_p, and i_p differ slightly due to the fact that the state at $s=0$ on the two legs is no longer equal. The conditions associated with r_p and v_p contain a number of new terms due to the dependence of the state at entry and exit of the sphere of influence on those two parameters.

PART II

SWINGBY PROGRAM USER'S MANUAL

SWINGBY PROGRAM USER'S MANUAL

Introduction

SWINGBY is the name given to the segmented two-body low thrust swingby trajectory optimization program. SWINGBY is a program conceived to yield optimum low thrust trajectory and performance data for missions incorporating a swingby of an intermediate planet enroute to the desired destination. It is designed to accomplish this in a manner which minimizes the effects of the high sensitivities of the problem on the behavior of the boundary value problem. The program is also suitable for generating ballistic swingby trajectories in the patched-conic mode.

This manual provides the user with the necessary instructions and information to operate the program. The manual contains a general program description, a statement of the major program capabilities, features and options, a detailed description of the program inputs and outputs, a summary description of the individual subroutines which comprise the SWINGBY program, a statement of the program machine requirements, and a sample problem.

General Program Description

The SWINGBY program is designed to generate optimal low thrust interplanetary trajectories incorporating a gravitational assist (swingby) of an intermediate planet. With the appropriate choice of inputs, the program can also generate standard trajectories with no swingby maneuver. A patched conic trajectory formulation is employed such that the overall mission trajectory is comprised of a series of appropriate planetocentric and heliocentric arcs which are connected or linked together at the spheres of influence of the planets. Continuity in both position and velocity is maintained at these patch points, although a discontinuity in gravitational acceleration will exist across any sphere of influence.

Swingby trajectories are strongly dependent upon the relative angular positions and motion of the planets involved. An analytic ephemeris of each planet is included in the program and is used to determine the position and velocity of a planet on any particular date of interest. In the ephemeris the elements of the planetary orbits are expressed as quadratic functions of Julian century relative to an ecliptic reference frame of date.

The indirect method is used in generating the optimal trajectories; i.e., the solution is taken to be that which satisfies the Necessary Conditions as derived by the application of the Pontryagin Maximum Principle. To obtain an optimal trajectory it is necessary to solve a set of non-linear ordinary differential equations representing the motion of the spacecraft simultaneously with a like set of equations which are adjoint to these equations of motion. During thrust phases these equations are solved by numerical integration using a fourth-order Runge Kutta technique. The independent variable of integration is the generalized universal anomaly β defined implicitly through the equation

$$\dot{\beta} = \mu/r^n$$

where μ is the gravitational constant of the attracting body, r is the distance of the spacecraft from that body and n is an input constant. For coast phases a closed form solution of the differential equations is used and is evaluated at constant intervals of a universal anomaly defined implicitly by the equation

$$\dot{\beta} = \sqrt{\mu}/r \quad .$$

The technical approach employed in calculating the trajectories takes cognizance of the high sensitivities of the post-encounter leg to the conditions at swingby. To minimize the effects of these sensitivities in solving the boundary value problem, the computation of all trajectories is begun at the closest approach point of the swingby planet. The Earth-to-swingby planet leg is integrated backwards starting in the planetocentric reference frame at the closest approach point. The integration proceeds to the sphere of influence of the swingby planet at which time the motion is switched to the heliocentric reference frame. The backward integration then continues to the time at which exit from the Earth's sphere of influence is desired. The swingby planet-to-target leg is integrated forward in the same manner to the desired time of entry of the target planet sphere of influence (or simply to the final time for area missions). Thus, the dependent parameters of the boundary value problem include conditions at both Earth departure and target arrival, and the independent parameters are basically the conditions at closest approach of the swingby planet.

Because the above formulation still exhibits rather serious sensitivity problems, an alternate formulation is also available in the program as an option. In this alternate formulation, which has proven to greatly alleviate the sensitivity problem, the optimization problem begins at the entry and exit points of the sphere of influence. This is accomplished by prohibiting the use of thrust within the sphere of influence such that the planetocentric motion of the spacecraft is strictly two-body and the behavior of the adjoint variables inside the sphere may be completely ignored. The physical passage conditions are still employed as independent parameters of the boundary value problem, but the adjoint variables at passage are replaced by the same variables evaluated at the sphere of influence.

The mission profile is based on the assumption that the spacecraft departs from a low altitude orbit about the launch planet with a velocity in excess of that necessary for escape from the planet's gravitational field. The initial mass of the spacecraft is a function of the hyperbolic excess speed at departure and is evaluated through a formula that approximates the performance of a specified launch vehicle. The option is provided for including in the mission profile a high thrust retro maneuver at the target. A choice of end conditions representative of a wide variety of planetary orbiter and flyby missions as well as area missions are available.

The low-thrust propulsion system is assumed to be power-limited with constant jet exhaust speed. The propulsion system efficiency is written as a function of the jet exhaust speed while the power is assumed to be a polynomial function of the solar distance. Imposing the condition that power be constant is an input option.

SWINGBY Program Capabilities

In addition to the analytic ephemeris mentioned in the preceding section, the program contains for each planet a set of constants including the gravitational constant, the equatorial radius, the radius of the sphere of influence, and the ecliptic longitude and latitude of the North Pole. The assumed values of these constants are presented for all nine planets of the solar system in Table 1. Because of the uncertainty of many of these parameters, the capability of overriding the built-in values by input is provided. Any combination of the nine planets in the solar system may be assigned as the launch, swingby, and/or target planets.

TABLE 1

Planetary Constants

Planet or Sun	Gravitational Constant (km^3/sec^2)	Planet Radius (km)	Sphere of Influence (10^6km)	Longitude of North Pole (deg)	Latitude of North Pole (deg)
Sun	1.327180×10^{11}	--	--	--	--
Mercury *	2.175620×10^{4}	2500.	.113	90.	90.
Venus	3.248534×10^{5}	6100.	.616	90.	90.
Earth	3.486032×10^{5}	6378.165	.928	90.	66.556
Mars	4.297780×10^{4}	3415.	.577	355.855	64.552
Jupiter	1.267069×10^{8}	69880.	48.188	247.238	87.840
Saturn	3.791794×10^{7}	57540.	54.502	78.957	61.933
Uranus	5.786726×10^{6}	25500.	51.746	77.437	-7.930
Neptune	6.976309×10^{6}	25000.	86.069	312.342	61.218
Pluto	3.317819×10^{5}	6350.	26.958	90.	90.

The program also contains a complete set of constants which characterize the low thrust propulsion system. Those presently stored in the program are representative of expected solar electric propulsion technology in the mid-to late-1970's. To facilitate the study of changes in technology or of other types of propulsion systems, the capability of overriding by input all of the built-in propulsion system constants is provided. The assumed form of the variation in power with solar distance is

$$\gamma = \left(\frac{1}{r^m} \sum_{i=0}^{9} a_i \, r^{-i/2} \right)^n$$

where γ denotes the ratio of power at any distance r to power at $r = 1$ AU, a_i are a set of constant coefficients representative of a particular type (and design) of a power source, and m and n are a pair of exponents introduced to provide more flexibility in writing γ. The efficiency η of the propulsion system is written

$$\eta = \frac{b c^2}{c^2 + d^2} + e$$

where c is the jet exhaust speed, and b, d, and e are a set of coefficients representative of a particular propulsion system.

The spacecraft is divided into a number of mass components including low thrust propellant, propulsion system (proportional to power at 1 AU), tankage (proportional to propellant), structure (proportional to initial mass), swingby planet science package, and retro stage. Any mass remaining is termed net spacecraft mass. A complete set of proportionality factors is built into the program and the option is provided for overriding any or all of them. The retro stage is itself divided into two components, the propellant and the inert mass. The latter is assumed to be proportional to the retro propellant. The swingby planet science package is a mass component that is jettisoned upon entry of the swingby planet sphere of influence. Mathematically, it is expressed as the sum of a constant mass increment and an amount proportional to the initial mass.

The initial mass m_o of the spacecraft is calculated using the equation

$$m_o = b_1 e^{-v_c/b_2} - b_3$$

where v_c is the velocity of the spacecraft at departure of the parking orbit and b_1, b_2, and b_3 are three constants which represent the performance of the prescribed launch vehicle. Currently, there are stored in the program, sets of the three coefficients representative of 24 existing and potential launch vehicles. These coefficients were computed using a least squares curve fit to launch vehicle performance data contained in Reference 11. Of course, the data are valid only for cases for which the Earth is the departure planet. In those cases involving departures from planets other than Earth or for launch vehicles other than those included in the library or for cases in which the initial mass is independent of launch vehicle capability, a feature which permits user specification of the three coefficients is provided.

An option is provided permitting one to employ a high thrust retro stage at the target. The magnitude of the velocity increment cancelled by the stage may be fixed or optimized, but the direction is always optimized. For planetary missions the increment is always imparted at the closest approach point to the target such that the injection point lies on the apsis of the final planetocentric orbit. Input flags are made available which provide the options as to whether or not the low thrust propulsion system and/or tankage are to be jettisoned prior to the retro maneuver.

The switching on and off of the electric propulsion system is generally determined by the switch function, a variable arising in the solution of the optimization problem. The switch function nominally governs the operation of the propulsion system in the heliocentric phases and within the sphere of influence of the swingby planet. However, thrusting is not permitted within the spheres of influence of the launch and target planets. An option is also provided which permits one to override the switch function and impose coasting when within the sphere of influence of the swingby planet. In heliocentric phases coasting is arbitrarily imposed for solar electric propulsion systems when the solar distance is less than about 0.47 AU. This is because the stored coefficients used in the mathematical representation of the power ratio γ lead to negative values of γ at distances less than this critical radius. This is, of course, a physically unrealizable situation and is rectified simply by setting γ equal to zero. With the option of inputting a new set of coefficients for γ, one must also input the value of the critical radius, if any.

The most difficult part of generating optimum swingby trajectories is, of course, the solving of the two point boundary value problem. The basic philosophy of the iterator used to solve this problem is given in Reference 13. This iterator is extremely versatile and has been found to possess very strong convergence properties for low thrust trajectory applications. For boundary value problems in which the number of dependent and independent parameters is equal, the iterator is basically a Newton-Raphson technique. If there are more dependent than independent parameters, the iterator will yield a (weighted) least squares solution. And, if there are more independent than dependent parameters, one has the option of using the degrees of freedom to extremize any specified function. Much of the versatility of this iterator is due to the fact that any function that is available as a dependent parameter is also available as a performance index when operating in this latter (optimize) mode. In this optimize mode, the iterator has been found to have a sizable radius of convergence, but the rate of convergence is somewhat slow since it is a direct parameter optimization technique. Nevertheless, it has frequent utility because it permits one to consider a new or different performance index on a moment's notice with absolutely no reference to any associated transversality conditions. One additional feature that is available with this iterator is the capability to declare an interval constraint for any dependent variable. Unlike a normal constraint for which the variable is driven to the center of a specified tolerance, the interval constraint requires only that the variable be within specified bounds. If this condition is satisfied, the iterator operates as if the dependent variable were not constrained. The partial derivatives required by the iterator are evaluated by perturbing the nominal trajectory and employing finite differences.

A total of 30 independent parameters are made available in the program for possible optimization or specification. These include the spacecraft position and velocity relative to the swingby planet, the mass ratio and the time at swingby, the times of exit from the launch planet's sphere of influence and entry into the target's sphere, the hyperbolic excess speeds at the launch and target planets, and the reference thrust acceleration and jet exhaust speed. The remaining 16 independent parameters consist of two complete sets of adjoint variables (Lagrange multipliers), one set to begin the backward integration of the first leg and the other to begin the forward integration of the second leg. Of course, not all of the 30 available independent parameters would be flagged for any one case. A feature is included which attempts to reduce as much as possible the number of independent parameters (and hence the order of the boundary value problem) for all swingby trajectories. In any case where a degree of freedom is left open in specifying the state of the spacecraft at swingby, there results a transversality condition involving the state and the two sets of Lagrange multipliers, that must be satisfied by the solution. Each such transversality condition is used to eliminate one of the multipliers of the first leg as an independent parameter. This feature is not available when using the alternate formulation discussed in the preceding section. The planetocentric position and velocity of the spacecraft at swingby is expressed in terms of the radius, speed, flight path angle, inclination, node angle, and angular position relative to the node. Of these the flight path angle is always taken to be zero because this is essentially the definition of a swingby or closest approach point. The remaining five position and velocity parameters are optionally available as independent parameters.

Although a maximum of 34 dependent parameters are computed for any one case, a great many more parameters are actually available for use in the boundary value problem. In many instances the same core location is used for several mutually exclusive dependent parameters. The specific choices for a particular case are selected by input flags. The first six dependent parameter possibilities consist of a combination of physical constraints and transversality conditions associated with the exit of the sphere of influence of the launch planet. These conditions are compatible with the specified sphere of influence radius, the specified radius of the circular parking orbit, and the assumption of ballistic transfer from the parking orbit to the sphere of influence along a trajectory with energy defined by the departure hyperbolic excess speed. The second set of six dependent parameters relate to the position, velocity, and/or associated transversality conditions at the target and several forms of the constraints are available. Under one constraint mode setting, the target is assumed to be a planet or other finite body moving along a specified ephemeris, and the available dependent parameters are identical in form to the first set of six associated with the launch planet. There are two other target constraint modes, both of which are included for use in area missions. The two modes differ in that one permits specification of the constraints in Cartesian coordinates and the other in polar coordinates. In both of the latter two modes, one specifies through an input trigger whether the constraint is the coordinate itself or the associated transversality condition. Other potential dependent parameters include the mass ratios at launch and at the target, the launch date, the target arrival date, the first and second leg flight times, the total mission duration, the reference power, the net spacecraft mass, and a host of transversality conditions associated with the following quantities: final mass ratio, launch date, arrival date, launch excess speed, arrival excess speed, reference thrust acceleration, jet exhaust speed, and passage distance speed, inclination, node angle, angular position, mass ratio, and time. The transversality conditions provided are sufficient to treat either maximum net spacecraft mass or minimum mission duration. The choice of either of these performance indices is made by input flag, and the appropriate selection of terms in the transversality conditions are made automatically.

A program feature which facilitates the computation of several optimal trajectories over a range of values of some parameter is available. When this feature is flagged, the entire set of independent parameters that resulted in the solution for one value of the parameter being varied is used as the first guess for the next value of the parameter. The alternative to this feature is to input a set of independent parameters for each case.

INPUT

Inputs to SWINGBY are given through the namelist feature of the IBM Fortran IV programming language. The input namelist is named MINPUT, and every input required or used in the program is declared by name in the list. The general form for assigning an input value to a quantity is, simply

NAME = VALUE

where NAME is the name assigned to the variable and is included in the namelist, and VALUE is a numerical or logical quantity consistent in form (i.e., logical, integer, or real) with NAME. Unless otherwise specified, all MINPUT names commencing with the letters I-N represent integers, whereas all names commencing with the letters A-H or O-Z are double precision floating point numbers. All input data sets must begin with the characters

&MINPUT

commencing in card column 2 and followed by at least one blank, and end with the characters

&END

Preceded by at least one blank if data is contained on the same card. Card Column 1 is ignored on all input cards. Multiple data assignments on a single card is permissible if separated by commas. A comma following the last VALUE on a card is optional. The order of the input data assignments is arbitrary; i.e., they need not be in the same order as listed in the namelist. In fact, there is no requirement that any specific input parameter be represented in the input data set. If no value is included in the inputs for a particular parameter, the default value, if any, is used. If there is no default value, the value used is that which happened to be in the particular core location assigned to the parameter at the time of execution. For other details regarding the namelist feature, the reader is referred to the IBM System/360 Fortran IV Language Manual.

A number of the SWINGBY program inputs relate principally to the control of the program operation. These are:

IPFM — Performance index flag governing the computation of the transversality conditions.

IPFM = 0 Optimize mode (See BY array input description)
= (1)* Net spacecraft mass is to be maximized
= 2 Mission duration is to be minimized

*Possible input values enclosed in parentheses denote default values.

FRWD	Logical constant indicating whether or not there is to be a pre-encounter trajectory leg (i.e., standard or swingby trajectory).	
	FRWD = .TRUE.	No pre-encounter (backward integrated) trajectory
	= (.FALSE.)	Both pre- and post-encounter trajectories
MUPDAT	Flag indicating whether independent parameters at end of one case are to be used as first guesses for next case.	
	MUPDAT = 0	Do not update
	(1)	Update
NSET(I)	NSET(1)	Not used for input
I=1, ---, 5	NSET(2)	Not used for input
	NSET(3)	Maximum number of iterations permitted in attempting to satisfy point and interval constraints. If zero, no upper limit imposed. Default value is 0.
	NSET(4)	Number of nominal trajectories that use the same partial derivative matrix. If NSET(4)=0 or 1, a partial matrix is generated for every nominal. For NSET(4)=n>1, a partial matrix is computed for the first nominal and for every nth nominal thereafter. Default value is zero.
	NSET(5)	Maximum number of iterations permitted after entering optimize mode. If zero, no upper limit is imposed. Setting NSET(5)=1 causes iterator to be bypassed. Default value is zero.
MPRINT	Flag for printing final trajectory as a function of time. If flagged, a standard printout block is printed for each computed point along the path. The printing is done as the point is computed; hence, the pre-encounter trajectory, if any, is printed backward.	
	MPRINT = (0)	Do not print
	1	Print
NPRINT	Print selection flag. Permits selection of amount of printout desired on each case.	
	NPRINT = 0	Print only the case summary
	1	Print switching point summary of final trajectory
	2	Print MINPUT and case setup
	4	Print trajectory summary on each iteration
	8	Print partial derivative matrix each iteration.
	16	Print trajectory summary on every perturbation trajectory.

Combinations of options obtained by summing options desired. Default value is 3.

IPR11	Remote input terminal printout flag (unit 11).	
	IPR11 = (0)	No output on unit 11.
	1	Trajectory summary information and iterator messages written on unit 11. Frequency at which the summary information is printed is dependent upon input value of LONG. Summary data consists of values of independent and dependent parameters.
IPR12	Remote input terminal printout flag (unit 12)	
	IPR12 = (0)	No output on unit 12.
	1	Case summary information, including case number, disposition (whether or not converged), and number of iterations required, is written on unit 12.
	Note that job control cards must either define or dummy out units 11 and 12.	
LONG	Flag defining frequency at which trajectory summary data is to be written on unit 11. Not used if IPR11 = 0.	
	LONG = (0)	Summary data written for final trajectory of case only.
	1	Summary data written for all trajectories except perturbation trajectories.
	2	Summary data written for all trajectories.
MPUNCH	Flag for optional output of all independent parameters of the final trajectory for each case in namelist format.	
	MPUNCH = (0)	Not output
	n	Independent parameters are written on the output class as defined in the job control cards for unit n.
KPART	Flag for initiating an automatic adjustment of the input perturbation step sizes to improve the accuracy of the partial derivative matrix. While successively increasing and/or decreasing the step size of	

KPART (cont.)

a given independent parameter, the algorithm monitors the variation of the elements of the appropriate partial derivative matrix column. The algorithm ultimately selects from all step sizes tested the particular size that appeared to provide the most stability to the least stable element in the column. This adjustment is performed before attempting to converge on an optimum trajectory. The feature has shown only limited success and is retained primarily to provide the interested analyst the framework within which he may test other algorithms with a minimum of effort. The flag also is used for the somewhat unrelated feature of bypassing the primary iterator, MINMX3, in favor of a standard Newton-Raphson search procedure.

KPART = (0)	Normal MINMX3 operation.
$n > 0$	Initiates automatic adjustment of perturbation and allows a maximum of n iterations to select the best value for each independent parameter.
< 0	Invokes Newton-Raphson search procedure. In this mode the input step sizes δx are multiplicative rather than additive (i.e., $x_{pert} = x_{nom}(1 + \delta x)$ rather than $x_{pert} = x_{nom} + \delta x$ as is employed in MINMX3).

ISPHER

Flag for invoking the alternate formulation which initiates the optimization problem at the sphere of influence rather than the passage point (see Appendix C of Part I of this report). This feature is available for both swingby and standard trajectory options.

ISPHER = (0)	Use original formulation which starts problem at passage.
1	Use alternate formulation.

ITOP

Adjoint variable propagation flag. Given a solution to a problem using the ISPHER =1 option, this flag initiates a procedure which propagates the adjoint variables, starting at the sphere of influence, down to the passage point assuming no thrust is permitted. The values of the adjoint variables at passage are then automatically loaded into the BX array to become independent parameters for a problem with ISPHER = 0. Iteration on this problem then begins automatically. Either optimal thrusting or imposed coasting is permitted in this problem. The input value of ISPHER is

ITOP (cont.)	ignored when using this feature and is set to zero internally. Propagation is performed from both entry and exit of the sphere of influence to the passage point if FRWD = .FALSE., and only from exit to the closest approach point otherwise.

ITOP = (0)	Do not propagate.
1	Propagate, ITOP is set to zero upon completion of the propagation.

IMPACT	Flag indicating the manner in which end conditions at the launch and (if applicable) target planets are evaluated.

IMPACT = (0)	End conditions represent a point on the sphere of influence of the appropriate planet and are evaluated as described in Appendix B of Part I of this report. The ambiguities noted there are resolved with the input IPICK array (see below).
1	Desired final position is taken to be that of the planet while the velocity is that of the planet plus the hyperbolic excess velocity applied in the direction diametrically opposed to the primer vector. The sphere of influence radius of the planet is assumed to be zero.
2	Same as IMPACT=1 except the excess velocity direction is left open.

MULAT	Flag which permits SWINGBY to emulate a two-body heliocentric low-thrust trajectory optimization program such as HILTOP (Reference 12). The only difference between SWINGBY operating in the MULAT mode, and HILTOP is that in SWINGBY the adjoint variables must be scaled to satisfy the mass ratio transversality condition, a requirement that is circumvented in HILTOP. This feature is available only for standard trajectories (i.e., FRWD=.TRUE.).

MULAT = (0)	Use segmented two-body formulation.
1	Use heliocentric two-body formulation.

Other features pertaining exclusively to spheres of influence or swingby capabilities are not available with this option.

LOC	Option flag used in conjunction with emulation mode which, given a periapse distance and inclination, permits the computation of the periapse speed, the longitude of node and the argument of periapse of a launch hyperbolic trajectory that is consistent with the launch excess velocity of the converged emulation mode trajectory. These values are then loaded into the appropriate locations of the BX array for use on the next case which is not to use the emulation mode. Two solutions exist for the launch hyperbola for which the difference in performance will generally be insignificant.

LOC = (0)	Bypass computations
± 1	Perform computations. One of the two arbitrary solutions is selected through the choice of sign.

IPICK(6) — Array of flags permitting the user some flexibility in controlling the operation of the program. The first four elements of IPICK are used to resolve the ambiguities arising in the computation of end conditions using the IMPACT = 0 option.

IPICK (1) =	(0)	Posigrade motion about launch planet.
	1	Retrograde motion about launch planet.
IPICK(2) =	(0)	Launch excess velocity nearly aligned with primer vector.
	1	Launch excess velocity nearly diametrically opposed to primer vector (this is generally the correct choice).
IPICK(3) =	(0)	Posigrade motion about target planet.
	1	Retrograde motion about target planet.
IPICK(4) =	(0)	Arrival excess velocity nearly aligned with primer vector.
	1	Arrival excess velocity nearly diametrically opposed to primer vector (this is generally the correct choice).

The fifth element of the array is applicable only if ISPHER = 0, and provides flexibility in the use of the transversality conditions at swingby.

IPICK(5) =	(0)	Any applicable transversality conditions resulting from open passage conditions are treated as end conditions in the boundary value problem.
	1	Those applicable transversality conditions are solved for selected adjoint variables (using subroutine FSTLEG). Both the adjoint variables and the transversality conditions are thereby removed from the boundary value problem, reducing its order by an amount equal to the number of transversality conditions involved.

IPICK(6) is not used at the present time.

ITF — Job time terminator. Prior to commencing the integration of any trajectory, the time remaining for the job is determined using the IBM utility REMTIN. If the remaining time, in seconds, is less than ITF, control is immediately transferred from the iterator to the MAIN program where the last trajectory is then integrated with appropriate flags set to get the desired summary printouts. The default value is 5, which will be adequate to get the summary printouts for most cases.

EN Exponent of r in the differential equation defining the implicit relation between time and the universal anomaly. Default value is 1.5 for which the universal anomaly becomes what is referred to as the regularized variable.

DBETAH Universal anomaly integration interval in heliocentric thrust phases. Default value is 0.03125.

DBETAP Universal anomaly integration interval in planetocentric thrust phases. Default value is 0.00390625.

DZH Universal anomaly step size during heliocentric coast phases. Default value is 0.125.

DZP Universal anomaly step size during planetocentric coast phases. Default value is .0078125.

The following inputs pertain to the planets involved in the mission and their ephemerides.

MOPT1 Launch planet number for swingby missions. Not used if FRWD = .TRUE. Should be set equal to 3 (Earth) if any of the launch vehicles are flagged (see MBOOST below).

MOPT1 =	1	Mercury
	2	Venus
	(3)	Earth
	4	Mars
	5	Jupiter
	6	Saturn
	7	Uranus
	8	Neptune
	9	Pluto
	10	Arbitrary body

MOPT2 Swingby planet number for swingby missions or launch planet number for standard missions (i.e., if FRWD = .TRUE.). Possible settings are identical to those listed above for MOPT1. Default value is 5.

MOPT3 Target planet number. Not used for area missions (i.e., if MODE >1). Possible settings are identical to those listed above for MOPT1. Default value is 6.

SAI
ECI
CNI
OMI
SOI
TPI

These six parameters are the orbital elements of the arbitrary body (planet no. 10). The six elements, in the order listed, are semi-major axis, eccentricity, inclination relative to the ecliptic plane, longitude of ascending node, argument of perigee, and date of perihelion passage. The semi-major axis is expressed in AU, the three angles in degrees, and the date of perihelion passage in days from the reference date (see MYEAR, MONTH, MDAY, and HOUR below). The elements must correspond to elliptic orbits. The default values for these elements, in the order listed, are (1., 0., 0., 0., 0., 0.).

INPFLG(I) I=1, ---, 16	Input flags indicating whether input or built-in solar and planetary constants are to be used. Each flag is checked individually such that any combination of built-in and input constants may be used. A flag setting of zero indicates the built-in value of the associated constant is to be used; a non-zero setting indicates the input value is to be used. Default values of the flags are all zero. The specific solar or planetary constant associated with each of the flags is as follows:

INPFLG(1)	Sun's gravitational constant
INPFLG(2)	Launch planet gravitational constant
INPFLG(3)	Launch planet radius
INPFLG(4)	Launch planet sphere of influence radius
INPFLG(5)	Longitude of launch planet North Pole
INPFLG(6)	Latitude of launch planet North Pole
INPFLG(7)	Swingby planet gravitational constant
INPFLG(8)	Swingby planet radius
INPFLG(9)	Swingby planet sphere of influence radius
INPFLG(10)	Longitude of swingby planet North Pole
INPFLG(11)	Latitude of swingby planet North Pole
INPFLG(12)	Target planet gravitational constant
INPFLG(13)	Target planet radius
INPFLG(14)	Target planet sphere of influence radius
INPFLG(15)	Longitude of target planet North Pole
INPFLG(16)	Latitude of target planet North Pole

Of course, if FRWD= .TRUE., INPFLG(2) - INPFLG(6) and the associated constants are not used, and INPFLG(7) - INPFLG(11) pertain to the launch planet. If the arbitrary body (planet no. 10) is assigned as the launch, swingby, or target planet, then the elements of the corresponding set of five input flags should each be set to one and the associated constants should be input.

Following are the input solar and planetary constants associated with the input flags INPFLG(I). The correct input units of each parameter is stated in parentheses. The default values of all 16 constants are zero.

EMSUN	Sun's gravitational constant (m^3/sec^2)
EMLNCH	Launch planet gravitational constant (m^3/sec^2)
RLNCH	Radius of launch planet (km)
RSLNCH	Radius of launch planet sphere of influence (km)
ELOLNC	Ecliptic longitude of launch planet North Pole (deg)

ELALNC	Ecliptic latitude of launch planet North Pole (deg)
EMSWBY	Swingby planet gravitational constant (m^3/sec^2)
RSWBY	Radius of swingby planet (km)
RSSWBY	Radius of swingby planet sphere of influence (km)
ELOSWB	Ecliptic longitude of swingby planet North Pole (deg)
ELASWB	Ecliptic latitude of swingby planet North Pole (deg)
EMTARG	Target planet gravitational constant (m^3/sec^2)
RTARG	Radius of target planet (km)
RSTARG	Radius of target planet sphere of influence (km)
ELOTRG	Ecliptic longitude of target planet North Pole (deg)
ELATRG	Ecliptic latitude of target planet North Pole (deg)

Virtually all the program computations are performed using internal units with distance expressed in AU and time in taus. The conversion constants between the internal units and the MKS system depend upon two parameters - the sun's gravitational constant and the length of the AU in meters (or kilometers). Since the capability of overriding the sun's gravitational constant is provided (thru EMSUN above), it is necessary to compute the conversion constants using the input values. For completeness, the capability of specifying the AU is also available as follows:

AUKM	Factor for converting distances from AU to kilometers. Default value is 1.49598×10^8.

The default value of the sun's gravitational constant is given in Table 1. Additional reference quantities used in conversion constants which are available as inputs are:

ER	Equatorial radius of the Earth, in kilometers. Default value is 6378.165.
GRAV	Reference acceleration of gravity at the Earth's surface. in m/sec^2. Default value is 9.80665.

All dates input to the program are expressed in days measured from an input reference date. This reference date may either be input as a calendar date in terms of year, month, day, and hour or as a modified Julian date. The modified Julian date is simply the actual Julian date less the number 2400000. For example, the Julian date corresponding to noon of 1 January 1970 is 2440588.0; hence, the modified Julian date is 40588.0. The means of inputting the reference date are as follows:

MYEAR	Year (1970)
MONTH	Month (1)
MDAY	Day (1)
HOUR	Hour (12.)

30

The numbers in parentheses are the default values. If MYEAR is input equal to zero, the reference date is assumed to be given in terms of the modified Julian date, the value of which is given by HOUR. Thus, a set of inputs that is equivalent to the default reference date is MYEAR = 0 with HOUR = 40588.0. In the latter case, MONTH and MDAY are not used.

A sizable number of inputs relate to the specification of the launch vehicle, propulsion system, and spacecraft performance and characteristics.

MBOOST — Launch vehicle identification number. If input equal to zero, parameters B1, B2, and B3 must be input (see specifications below). Permissible settings and corresponding launch vehicles are:

MBOOST = 0	User specified vehicle
1	Atlas (SLV3C)/Centaur
2	Atlas (SLV3C)/Centaur/Burner II (2336)
3	Atlas (SLV3X)/Centaur
4	Atlas (SLV3X)/Centaur/Burner II (2336)
5	Atlas (SLV3X)/Centaur I
6	Atlas (SLV3X)/Centaur I/kick
7	Titan III C
8	Titan III C/Burner II (2336)
9	Titan IIIX/Centaur
(10)	Titan III X (1205)/Centaur
11	Titan III X (1205)/Centaur/Burner II (2336)
12	Titan III X (1207)
13	Titan III X (1207)/Centaur
14	Titan III X (1207)/Centaur/Burner II (2336)
15	Titan III X (1207)/Centaur I
16	Titan III X (1207)/Centaur I/kick
17	SIB/Centaur
18	SIC/SIV B/Centaur
19	SIC/SIV B/Centaur I
20	SIC/SIV B/Centaur I/kick
21	Saturn V
22	Saturn V/Centaur
23	Saturn V/Centaur I
24	Saturn V/Centaur I/kick

The formula used to estimate the launch vehicle performance (see preceding section) has been found to be about as accurate as one can read the performance data from the graphs in Reference 4. The vehicles with the Burner II and Kick upper stages have been included because they represent the most probable alternates to electric propulsion upper stages. With the proper inputs, the program can be forced to operate in the coasting mode throughout the mission, thereby yielding the performance requirements for an all high thrust vehicle in the same reference frame that low thrust performance is evaluated. If MBOOST is entered zero, then also input the following

B1
B2
B3

Coefficients corresponding to b_1, b_2, and b_3, respectively, appearing in the equation for launch vehicle performance given in the preceding section. B1 and B3 have units of kilograms while B2 has units of meters per second. Note that setting B1 to zero makes the initial mass independent of the launch excess speed and equal to -B3. Default values of B1, B2, and B3 are all zero.

The built-in representation of the power variation with distance is, for distances greater than 1 AU, characterized by an exponential decay asymptotically approaching zero at large distances. At distances less than 1 AU, γ peaks with a value of about 1.4, and abruptly drops off to zero as the distance is decreased. Theoretically, this abrupt drop in γ may be eliminated by rotating the solar arrays to reduce the planform area exposed to the sun. An option is available to simulate this operational procedure. The inputs available for specification of the power supply characteristics are

MSOLAR — Flag indicating whether built-in power profile curve is to be used or overriden.

MSOLAR =	(0)	Use built-in values.
	1	Use input values. (Note: BJ(I), RPOI, RPMI, and PMI must all be input).

BJ(I) I=1, ---, 10 — Coefficients corresponding to the a_i in the equation for γ given in the preceding section. Used only if MSOLAR = 1. Default value of BJ(1) is 1.; all other BJ(I) are 0. (Note: the built-in values of the 10 coefficients used in evaluating γ if MSOLAR=0 are .627, 5.3054, -10.0376, 7.1073, -2.0021, 0., 0., 0., 0., 0., respectively.)

PMI — Peak value of power ratio. Used only if MSOLAR = 1. Default value of PMI is 1. When MSOLAR = 0, the peak power ratio value is 1.396328511.

RPMI — Solar distance in AU at which the input power profile attains its peak value of PMI. Used only if MSOLAR = 1. Default value is 0. The radius at which the power ratio peaks when MSOLAR = 0 is 0.6652595436 AU.

RPOI — Solar distance in AU below which the function γ, as evaluated using the coefficients BJ(I), is negative. Used only if MSOLAR = 1. Default value is 0. The distance at which γ is zero when MSOLAR= 0 is 0.469382496793 AU.

MSHLD — Flag indicating that at solar distances below the distance at which the peak power ratio is attained, the solar arrays are to be tilted so as to maintain the peak value. If this option is flagged, the distance at which the power goes to zero is set to zero.

MSHLD =	(0)	Do not tilt arrays; allow power degradation
	1	Tilt solar arrays; maintain peak power at small solar distances.

FM — Parameter corresponding to the exponent m in the equation for γ given in the preceding section. Default value is 2.

ENN — Parameter corresponding to the exponent n in the equation for γ given in the preceding section. The prefferable method for setting power equal to a constant is to set ENN = 0. To get a solution for solar electric propulsion given a constant power solution, or vice-versa, one may simply generate a sequence of solutions for various values of ENN between zero and one. Default value is 1.

MEFFIC — Flag indicating whether built-in or input coefficients are to be used in evaluating the propulsion system efficiency. The built-in values of the coefficients b, d, and e appearing in the equation for η in the preceding section are 0.769, 14300. m/sec, and 0., respectively.

MEFFIC =	(0)	Use built-in coefficients
=	1	Use input coefficients

BI, DI, EI — Input coefficients corresponding to b, d, and e, respectively, in the equation for η. Used only if MEFFIC = 1. The units of DI are m/sec. Default values are 1., 20000., and 0., respectively.

Most of the mass components comprising the spacecraft are evaluated through a set of linear scaling laws. The inputs used in these computations are:

CALPHA — Specific propulsion system mass in kg/kw; i.e., the ratio of propulsion system mass to reference power. Default value is 30 kg/kw.

EKS — Structure factor, the ratio of structure mass to initial spacecraft mass. Default value is 0.

EKT — Tankage factor, the ratio of tankage mass to low thrust propellant mass. Default value is 0.03.

DELMX — Mass increment in kg comprising a portion of the scientific package jettisoned upon entry of the swingby planet sphere of influence. Default value is 0.

EKX — Swingby planet scientific package proportionality factor, ratio of remainder of scientific package (in excess of DELMX) to the initial spacecraft mass. Default value is 0.

EKR	Retro inert factor, ratio of retro stage inert weight to retro propellant weight. Default value is 0.1.

The following inputs are used in the computation of the retro stage propellant requirements.

JR — Flag indicating whether a high thrust retro stage is to be used in attaining the specified end conditions at the target. A retro stage is required for planetary orbiter missions.

JR = (0)	No high thrust retro stage
= 1	Retro stage is to be used

The following parameters are used only if JR = 1.

SPIRET — Specific impulse of retro stage, in seconds. Default value is 300.

RTF — Planetocentric distance, in target planet radii, to the point of injection into the capture orbit about the target planet. Used only if MODE=1 (See MODE below). Default value is 2.

RTA — Planetocentric distance, in target planet radii, of the apsis of the capture orbit 180 degrees from the injection point. Used only if MODE =1. Default value is 38.

JPS — Propulsion system jettison flag

JPS = (0)	Electric propulsion system is retained as part of the spacecraft through the retro maneuver.
= 1	Electric propulsion system is jettisoned prior to the retro maneuver.

JT — Tankage jettison flag

JT = (0)	Low thrust propellant tankage is retained as part of the spacecraft through the retro maneuver.
= 1	Low thrust tankage is jettisoned prior to the retro maneuver.

Two additional, and very important, option flags are

JC — Planetocentric phase coasting flag

JC = (0)	Thrusting permitted, if indicated optimal by the switching function, within the swingby planet sphere of influence.

JC (cont.)

= 1 — Thrusting within the swingby planet sphere of influence not permitted.

If FRWD = .TRUE. this option is available for launch planet. This flag is ignored if ISPHER = 1.

MODE — Target type flag

MODE = (1)	Target is a planet or other body with a prescribed ephemeris.
= 2	Target is represented as a point or area in Cartesian ecliptic coordinates.
= 3	Target is represented as a point or area in polar ecliptic coordinates.

All input information pertaining to the individual independent parameters is contained in a single array named BX.

BX(I, J) I-1, ---, 5 J=1, ---, 30 — For each independent parameter, five pieces of information are required by the iterator. The subscript I relates to these five items; J relates to the individual independent parameters. Default values of all elements of BX are zero.

BX(1,J) — Trigger indicating whether Jth parameter is to be an independent parameter in boundary value problem.

BX(1,J)=0. Not an independent parameter.
=1. Use as independent parameter

BX(2,J) — Input value of Jth parameter. Must be input regardless of trigger setting except as noted for individual parameters below. If trigger is on (i.e., B(1,J)=1), input value is used as initial guess of independent parameter and is varied at each subsequent iteration. If trigger is off, the parameter is not used as an independent parameter.

BX(3,J) — Perturbation increment used to compute partial derivatives. Used only if trigger is on. Units are same as that of the parameter.

BX(4,J) — Maximum change to Jth parameter permitted in a single iteration. Should be a positive quantity. Used only if trigger is on. Units are same as that of the parameter.

BX(I, J) (cont.)	BX(5, J)	Weighting factor. Should be a positive quantity. A guideline for selecting these weights is to estimate the uncertainty in how well you think you know a given independent variable. Then set the weighting factor equal to the inverse square of the uncertainty, where the uncertainty is expressed in the same units as the variable. The smaller the value of the weighting factor, the more the importance given to the associated variable by the iterator.

The specific parameters associated with the various values of J are as follows:

J = 1	Swingby planet passage distance, in kilometers. If FRWD = . TRUE., radius of parking orbit about launch planet. In latter case, if launch planet is Earth and built-in launch vehicle performance is used, then set BX(2, 1)=6563.365.
J = 2	Planetocentric speed at swingby, in m/sec. If FRWD = . TRUE., planetocentric speed at departure of launch planet parking orbit.
J = 3	Flight path angle at swingby, in degrees. If FRWD = . FALSE. and IPFM > 0, both BX(1, 3) and BX(2, 3) must be set to zero. For other settings of FRWD and/or IPFM, the flight path angle at swingby (launch if FRWD=. TRUE.) may be an independent parameter. It must be recognized, however, that if the flight path angle at swingby (launch) is not zero, the other position and velocity inputs no longer apply to the closest approach point.
J = 4	Inclination of planetocentric swingby trajectory to equator of swingby planet, in degrees. Input value should lie between limits of 0 and 180 degrees. If FRWD = . TRUE., parameter is inclination of departure trajectory relative to launch planet equator.
J = 5	Longitude of ascending node of planetocentric swingby trajectory on swingby planet equatorial

BX(I, J)
(cont.)

plane measured eastward along the equator from the planet's autumnal equinox, in degrees. Input value may lie in any of the four quadrants. If FRWD=. TRUE., parameter refers to the launch planet.

J = 6 — Angular position of spacecraft at swingby (launch if FRWD=. TRUE.) measured in the osculating orbit plane in the direction of motion from the ascending node, in degrees. If the input flight path angle is zero (i.e., BX(2, 3)=0.), this angle is also the planetocentric argument of periapse. Input value may lie in any of the four quadrants.

J = 7 — Mass ratio at the time of swingby. Should be a positive value less than or equal to one. If FRWD=. TRUE., mass ratio at departure of launch planet parking orbit and should be a fixed value equal to one (i.e., BX(1, 7)=0., BX(2, 7)=1.).

J = 8 — Time of swingby in days from reference date. If FRWD=. TRUE., time of departure from launch planet parking orbit.

J = 9 — Time of exit from sphere of influence of launch planet, in days from reference date. Backward integration is terminated when this time is reached. Not used if FRWD=. TRUE.

J = 10 — Time of entry into sphere of influence of target planet (if MODE = 1), in days from reference date. If MODE > 1, final time of problem. Forward integration is terminated when this time is reached.

J = 11 — Hyperbolic excess speed at launch, in m/sec. Not used if FRWD=. TRUE.

J = 12 — Hyperbolic excess speed at target, in m/sec, if MODE = 1. If MODE=2 and JR=1, velocity increment of retro stage, in m/sec. Not used if MODE=3.

J = 13 — Reference thrust acceleration in m/sec/sec. Equal to thrust generated at 1 AU (or simply the thrust for constant power systems) divided by initial spacecraft mass.

BX(I, J) (cont.)	J = 14	Jet exhaust speed of low thrust propulsion system, in m/sec.
	J = 15 - 22	Initial Lagrange multipliers for starting the backward integration of the adjoint equations for the first leg. Not used if FRWD=.TRUE. Order of the multipliers in terms of increasing J, is as follows: x, y, and z components of the primer (i.e., the adjoints to the velocity); x, y, and z components of the time derivative of the primer (i.e., the negative of the adjoints to the position); the mass ratio multiplier; and the time multiplier. If IPICK(5) = 1, IPFM > 0, and ISPHER = 0, certain of the initial multipliers for the first leg are computed internally to satisfy a like number of transversality conditions involving the state variables and the two sets of Lagrange multipliers at the swingby point. The specific multipliers computed are as follows: If BX(1, 6)≠0., compute BX(2, 15) and set BX(1, 15)=0.; If BX(1, 5)≠0., compute BX(2, 18) and set BX(1, 18)=0.; If BX(1, 2)≠0., compute BX(2, 16) and set BX(1, 16)=0.; If BX(1, 1)≠0., compute BX(2, 19) and set BX(1, 19)=0.; If BX(1, 4)≠0., compute BX(2, 17) and set BX(1, 17)=0.; If BX(1, 7)≠0., compute BX(2, 21) and set BX(1, 21)=0.; If BX(1, 8)≠0., compute BX(2, 20) and set BX(1, 20)=0. No inputs are necessary for these multipliers which are computed internally. If inputs are made, the value of the multiplier and the trigger setting will be overridden.
	J = 23 - 30	Initial Lagrange multipliers for starting the forward integration of the adjoint equations on the second leg (or the only leg in the case of FRWD=.TRUE.). The order of these multipliers, in terms of ascending values of J, is the same as that stated above for the first leg. It should be noted that there is no conversion from input to internal units for the Lagrange multipliers. Hence the input values of the multipliers are consistent with the internal units of the state variables; i.e., mass in kilograms, distance in AU, and time in taus (one tau is the length of time required for a massless particle to travel one radian in a circular orbit of radius 1 AU about the sun).

All input information pertinent to the individual dependent parameters is also contained in a single array named BY.

BY(K, L)
K=1, ---, 5
L=1, ---, 34

For each dependent parameter, corresponding to a specific value of L, the iterator requires up to five input quantities corresponding to the five locations indicated by the subscript K. These inputs are:

BY(1, L) Trigger. If off (i.e., equal to zero), the parameter is ignored and is not considered a dependent parameter. Then the other four inputs pertaining to the Lth parameter need not be input. If trigger is on (i.e., not equal to zero), the Lth parameter is considered to be a dependent parameter or constraint. Certain of the L parameters may have either of two non-zero trigger settings. These will be discussed individually below.

BY(2, L) Minimum acceptable value of the dependent parameter.

BY(3, L) Maximum acceptable value of the dependent parameter.

BY(4, L) Importance factor for interval constraints. Used only if interval constraint is declared on Lth parameter.

BY(5, L) Constraint type indicator, as follows:

BY(5, L)=1. Point, or normal, constraint. Iterator attempts to drive parameter to the center of the tolerance specified.

=0. Interval constraint. Once within the specified interval, the constraint is ignored unless the boundaries are subsequently violated.

=-1. Denotes performance index to be used in optimize mode. Only one such variable permitted in a case. Maximization or minimization is achieved by setting both acceptable upper and lower bounds to unattainably high (maximize) or low (minimize) values.

It should be noted that the transversality conditions, which comprise some of the L parameters, are developed under the assumption that all constraints are of the point constraint type. The use of the latter two types of constraints in conjunction with constraints of any type on transversality conditions is permitted.

BY(K, L) (cont.) — BY(5, L) (cont.)

However, the user should exercise caution in so doing to assure that the results are meaningful and consistent with the assumptions made in deriving the transversality conditions originally.

The default values of all elements of the BY array are zero. Because it is frequently desirable to know the value of a function that is available as a dependent parameter, even though the function may not be constrained on a particular case, a complete set dependent parameters is evaluated on all trajectories regardless of the trigger setting. That is, the trigger setting is used only to indicate to the iterator whether a particular dependent parameter is to be constrained, not whether it is to be evaluated. If a particular value of L is used for two or more mutually exclusive constraints, the function evaluated is that particular function that would be constrained had the trigger been set equal to one with all other inputs unchanged. The only instance in which a full set of 34 dependent parameters is not evaluated is when FRWD=. TRUE. for which case all functions pertaining only to the pre-swingby leg are ignored. The specific constraint equations associated with the several possible values of L are as follows:

L = 1-3 — Actual final integrated heliocentric x, y, z position components respectively, on pre-encounter leg, less the desired values, in AU (i.e., position error). The desired values are computed consistent with the input value of IMPACT. That is, if IMPACT=0, the desired point lies on the sphere of influence of the launch planet, while if IMPACT≠0, the desired point is coincidental with the location of the launch planet as obtained from the ephemeris. Not used if FRWD=. TRUE.

L = 4 - 6 — Actual final integrated heliocentric velocity Cartesian components on pre-encounter leg less the desired values, in AU/tau. If IMPACT=0, desired values are those at the sphere of influence of the launch planet. If IMPACT=1, the desired values are those of the launch planet plus the excess velocity directed opposite the primer vector. If IMPACT=2, a single condition on the velocity is imposed and is associated with L=4. L=5 and 6 are not used. The single condition is the magnitude of the difference of the final integrated heliocentric velocity and the velocity of the launch planet less the launch excess speed. Not used if FRWD=. TRUE.

BY(K, L)	L=4-6	
(cont.)	(cont.)	Note: All of the constraints above, as well as all transversality conditions to follow, have been formulated such that under normal circumstances the desired value of the constraint is zero. This is done to make the inputs as simple as possible.
	L=7-12	Constraints on the position, velocity, and/or associated transversality conditions upon arrival at the target. The form of the constraint depends, in part, on the value assigned to MODE as follows:

MODE=1 — Constraints 7-12 are identical in form to constraints 1-6, respectively, above except that constraints 7-12 refer to the specified target planet rather than the launch planet. Note that the only difference in inputs for orbiter and flyby missions is the flag JR which indicates whether a retro stage is to be used.

MODE=2 — Constraints 7-9 represent the ecliptic Cartesian x, y, and z coordinates, respectively, of the desired heliocentric final position while constraints 10-12 represent the corresponding coordinates of final heliocentric velocity. If JR=1, the latter constraints are of the velocity after completion of the retro maneuver. Two non-zero trigger settings are available with each of the six constraints. A trigger equal to 1. implies that the coordinate itself is constrained whereas a trigger equal to 2. implies that the coordinate is left open and the constraint is on the transversality condition that arises because the spatial coordinate is left unspecified. Units are consistent with distance in AU and velocity in EMOS.

MODE=3 — Constraints 7-12 represent heliocentric polar coordinates of the target in the following order: radial distance in AU; speed, in EMOS; flight path angle, in degrees; inclination to the ecliptic, in degrees; longitude of ascending node, in degrees; and argument of position in the plane of motion relative to the ascending

BY(K, L) (cont.)	L=12 (cont.	node, in degrees. The use of a retro stage in conjunction with this MODE setting is not available. Again, two non-zero trigger settings are available--a trigger equal to one implies the coordinate is constrained while a setting equal to two implies the associated transversality condition is constrained.
	L=13	Mass ratio at exit from launch planet sphere of influence. Under normal circumstances, this should be one of the dependent parameters and the desired value is one. Not used if FRWD=. TRUE.
	L=14	Mass ratio upon entry into target planet sphere of influence (or at the final time for area missions). This is the desired value prior to any retro maneuver and before any systems are jettisoned. Two non-zero trigger settings are available with this constraint. A setting of one implies the mass ratio is constrained; a setting of two implies the transversality condition arising as a result of not specifying final mass ratio is constrained.
	L=15	Launch date or date of departure from launch planet parking orbit, in days from reference date. Not used if FRWD=. TRUE.
	L=16	Arrival date, in days from reference date. Date of closest approach for planetary missions; date at target for area missions.
	L=17	First leg flight time, in days. Measured from departure from launch planet parking orbit to point of closest approach of swingby planet. Not used if FRWD=. TRUE.
	L=18	Second leg flight time, in days. Measured from date of swingby closest approach to the target arrival date.
	L=19	Transversality condition associated with launch date. Desired value is zero. Not used if FRWD=. TRUE., if the total mission duration is constrained, or if both leg flight times are constrained.
	L=20	Transversality condition associated with arrival date. Also, transversality condition associated with open launch and arrival dates with fixed time between. Desired value is zero.

BY(K, L) (cont.)	L=21	Transversality condition associated with launch excess speed. Desired value is zero. Not used if FRWD=. TRUE.
	L=22	Transversality condition associated with the hyperbolic excess speed at target planet if MODE=1. Also, transversality condition associated with retro velocity increment if MODE=2. Not to be used if MODE=3. Desired value is zero.
	L=23	Total mission duration, in days. Time between launch and arrival dates. If FRWD=. TRUE., use second leg flight time rather than mission duration if constraint is desired.
	L=24	Reference power, in kilowatts. Power at 1 AU for solar electric propulsion, otherwise the actual constant value of the power input to the propulsion system. If power is constrained, either the reference thrust acceleration or the jet exhaust speed, or both must be declared as independent parameters.
	L=25	Transversality condition associated with the reference thrust acceleration. Desired value is zero.
	L=26	Transversality condition associated with jet exhaust speed. Desired value is zero. Not used if reference power is constrained.
	L=27	Net spacecraft mass, in kilograms.
	L=28-34	Transversality conditions associated with open passage conditions (launch conditions if FRWD=. TRUE.), in the following order: distance, speed, inclination, longitude of node, argument of position, mass ratio, and time. Desired values are zero.

The five-element array HDNG is provided to permit the writing of an arbitrary alpha-numeric information message at selected points throughout the normal printout. Each element provides for eight characters; therefore, the total message may be up to 40 characters in length. The input is written as follows:

HDNG = ' - - message - -'

That is, the message is enclosed within apostrophes. No delineators are required to separate the individual elements. The default setting of each element is blank.

Two parameters, KOUNT and IDATE, which appear in the namelist MINPUT, are not used as inputs but are used for output purposes. The two parameters are initialized automatically and need not be included in the input data set.

OUTPUT

The printout of the SWINGBY program is controlled through two input integer variables, MPRINT and NPRINT. As stated above in the description of inputs, setting MPRINT=1 results in the printing of a group of trajectory and spacecraft parameters at each computed point along the final trajectory. The group of numbers at each point is arranged in an array consisting of six lines while the spacecraft is within the swingby planet (launch planet if FRWD=. TRUE.) sphere of influence and five lines in heliocentric space. Each line contains eight parameters. As an aid in locating each parameter in the standard printout block, a title block is printed at the top of each page. The titles listed and their definitions are given below. The titles in each line are listed in the order they appear reading from left to right across the page. The asterisk beside the title below implies the parameter is evaluated in the planetocentric system if the spacecraft is within the sphere of influence and in the heliocentric system otherwise.

Line 1

TIME — Current time, in days, measured from departure of the launch planet parking orbit.

MASS RATIO — Ratio of current mass to initial mass.

*SEMI AXIS — Semi-major axis, in kilometers when planetocentric and in AU when heliocentric.

*ECCENTRICITY — Instantaneous eccentricity of osculating trajectory.

*FLT PATH ANGLE — Elevation of velocity vector above local horizontal, in degrees

*INCLINATION — Instantaneous inclination of osculating orbit, in degrees. Relative to equator when planetocentric and to ecliptic when heliocentric.

*NODE — Longitude of ascending node of osculating orbit on equatorial plane from autumnal equinox when planetocentric and on ecliptic from x-axis when heliocentric, in degrees.

*ARG POS — Angular position of spacecraft in the osculating plane of motion from the ascending node, in degrees.

Line 2

LAMBDA T — Lagrange multiplier associated with time.

Line 2 (cont.)	LAMBDA NU	Lagrange multiplier associated with mass ratio.
	LAMBDA A	Lagrange multiplier associated with thrust acceleration.
	LAMBDA C	Lagrange multiplier associated with jet exhaust speed.
	SWITCH FNCT	Switch function which governs the switching of the low thrust engine.
	POWER FACTOR	Ratio of power generated at current solar distance to that generated at 1 AU.
	*THRUST ACCEL	Ratio of instantaneous thrust acceleration to instantaneous gravitational attraction on the spacecraft.
	*ANGULAR MOM	Magnitude of the angular momentum vector, in m^2/sec when planetocentric and AU·EMOS when heliocentric. If the angular momentum vector has a negative component along the North Pole, the magnitude is expressed as a negative number.

Line 3 (This line of data deleted when in the heliocentric phase, exept at a sphere of influence.)

	XP, YP, ZP	Planetocentric ecliptic Cartesian coordinates of spacecraft position relative to the swingby planet, in kilometers.
	XP DOT, YP DOT, ZP DOT	Planetocentric ecliptic Cartesian coordinates of spacecraft velocity relative to the swingby planet, in m/sec.
	RP	Magnitude of planetocentric position vector, in kilometers.
	VP	Magnitude of planetocentric velocity vector, in m/sec.

Line 4

	X, Y, Z	Heliocentric ecliptic Cartesian coordinates of spacecraft position, in AU.
	X DOT, Y DOT, Z DOT	Heliocentric ecliptic Cartesian coordinates of spacecraft velocity, in EMOS.
	R	Magnitude of heliocentric position vector, in AU.
	V	Magnitude of heliocentric velocity vector, in EMOS.

Line 5

LAMBDA X / LAMBDA Y / LAMBDA Z — Ecliptic Cartesian components of the primer vector (i.e., the Lagrange multipliers adjoint to the x, y, and z components, respectively, of the velocity).

LAMBDA DOT X / LAMBDA DOT Y / LAMBDA DOT Z — Ecliptic Cartesian components of the time derivative of the primer vector (i.e., the negatives of the Lagrange multipliers adjoint to the x, y, and z components, respectively, of the position).

LAMBDA — Magnitude of the primer vector

LAMBDA DOT — Magnitude of the time derivative of the primer vector.

Line 6

LONGITUDE — Heliocentric ecliptic longitude of spacecraft, in degrees.

LATITUDE — Heliocentric ecliptic latitude of spacecraft, in degrees.

PHI — Angle between heliocentric position vector and thrust (primer) vector, in degrees.

THETA OSC — Angle between the heliocentric position vector and the projection of the thrust (primer) vector in the instantaneous heliocentric osculating plane, in degrees. Measured positive in the direction of motion.

PSI OSC — Angle between the thrust (primer)vector and its projection in the instantaneous heliocentric osculating plane, in degrees. Positive if projection of thrust vector on heliocentric angular momentum vector is positive.

THETA I — Angle between the projection of the thrust (primer) vector in the ecliptic plane and the x-axis (vernal equinox), in degrees.

PSI I — Angle between the thrust (primer)vector and its projection in the ecliptic plane, in degrees.

HAMILTONIAN — The variational Hamiltonian.

Below the title block, several data blocks are printed to fill the page. The data blocks are separated by blank lines, for easier reading, and appropriate comments are included to indicate discontinuity points, such as engine switch points or crossing a sphere of influence. Since the printing of a point is accomplished at the time it is computed, the first leg of a swingby mission is printed backward in time.

The print control parameter NPRINT provides for the printing of various levels of trajectory and case summaries. Selected output is considered minimal for each case, i.e., it may not be suppressed. This includes messages originating in the iterator stating success or failure and an indication of how hard it worked to come to that conclusion, messages from other subroutines if numerical difficulties are encountered, and a case summary. The latter is a concise description of the mission, the spacecraft and the propulsion system requirements, and includes most of the information required by a mission analyst for preliminary performance studies. The case summary page is divided into a number of sections or groups of numbers reading down the page. Each member of each group of numbers is clearly titled and the units specified. At the top of the page the case number is printed along with a simple message indicating whether the iterator converged in obtaining the data printed. This is followed by a mission itinerary naming the launch, swingby (if any) and target planets, and the title of the launch vehicle. A mass breakdown giving the initial mass and the seven basic components is then printed followed by a description of the propulsion system in terms of the reference power, reference thrust, reference thrust acceleration, jet exhaust speed, efficiency, and total propulsion time. The next three groups of data present the date and several parameters representing the planetocentric state of the pericenter points of the launch, swingby, and target planets. In addition to the Julian date, the specific parameters include the radial distance, speed, flight path angle, inclination, node, position angle, hyperbolic excess speed, and time within the sphere of influence. Of course, the group pertaining to the swingby planet is deleted if FRWD=. TRUE. Exceptions to the particular parameters printed will occur for the target planet if MODE $\neq 1$. For MODE = 2, the heliocentric Cartesian coordinates of final position and velocity are printed rather than planetocentric polar coordinates, and the time within the sphere of influence is deleted. If MODE = 3 heliocentric polar rather than planetocentric polar coordinates are printed, and both excess speed and time within the sphere of influence are deleted. Next is printed a trajectory schedule, in days from launch, of certain events of interest including crossings of spheres of influence, swingby planet passage, and arrival at target. Finally, for orbiter missions, data are printed to provide information regarding the capture orbit and the retro stage. Included are the periapse and apoapse distances, the capture orbit speed at the injection point, the incremental velocity imparted by the retro stage, the specific impulse, the inert mass, and the propellant mass of the retro stage.

If the integer one is summed in NPRINT there results the printing of the namelist MINPUT followed by the case setup. The printing of the namelist is done in the same general format that it is input. Every name included in the namelist is printed along with the value assigned to it. The order of printing is that in which the parameters were listed in the namelist. The case set-up breaks out from the total input array the information pertaining to the individual independent and dependent parameters that are to be used for the case. Only those parameters which have non-zero trigger settings are included. The information printed for each parameter is comprised of the appropriate five inputs in the BX or BY array. The independent parameters are printed first preceded by the heading "INDEPENDENT PARAMETERS". After the independent parameters is printed the heading "DEPENDENT PARAMETERS" followed by the pertinent information.

By summing the integer two in NPRINT a trajectory summary and a discontinuity point summary are printed for the last trajectory computed. The trajectory summary

Brief Subroutine Descriptions

Excluding the IBM System/360 library routines, the SWINGBY program is comprised of 53 subroutines and function subprograms, plus a BLOCK DATA subprogram. The latter is an IBM System/360 feature which permits the assignment of values to variables in common arrays through the use of DATA initialization statements. Many of the subroutines listed below are actually entry points within other subroutines, A brief description of the 53 routines is given below, and each routine that is an entry point is so indicated.

Identification	Purpose
AMAINT	Driver routine of integration package. This is the routine called by program when referencing the integrator whether initializing or integrating. Calls EXEC1 and RK9CYL.
BOOSTER	Initializes LAUNCH subroutine by assigning built-in or input performance constants for specified launch vehicle. Also prints launch vehicle name. Entry point in LAUNCH.
CHECK	Monitors trajectory at each computed point to determine if final time, sphere of influence, radius where power goes to zero, or engine switching points are passed. If so, initiates iteration to isolate the point. Calls AMAINT, DPCNV, GETB, INTERP, RESTOR, STORE, VDOT and VMAG.
COAST	Generates solution of state and adjoint equations in closed form during all coast phases. Solution is obtained through the use of f and g series. Calls EPH, GCOMP, VADD, VDOT, VMAG and VSCAL.
COEFF	Initializes SOLAR subroutine by assigning built-in or input values to the coefficients and parameters used in the representation of power variation with solar distance. Entry point in SOLAR.
DATE 1	Computes Julian date given calendar date.
DERIV	Evaluates the derivatives of the state and adjoint variables for use by the integrator. Calls EPH, SOLAR, and VDOT. Entry point in DRVINT.
DPCNV	Converts first time derivatives of position and primer in thrust phases to derivatives with respect to universal anomaly. Calls VMAG. Entry point in DRVINT.

DRVINT	Initializes for forward and backward integration at start of planetocentric and heliocentric phases. Calls EPH, VADD, VDOT, VMAG, and VSCAL.
DSCONT	Computes discontinuities in adjoint variables and mass at crossings of sphere of influence. Calls EPH, VDOT and VMAG. Entry point in DRVINT.
EFFIC	Computes efficiency and derivative of efficiency with respect to jet exhaust speed, given the jet exhaust speed.
ELEM	Computes the planetocentric passage distance, speed, inclination, longitude of ascending node, and position angle given the planetocentric ecliptic Cartesian coordinates of the position and velocity at the sphere of influence and the direction of the planets North Pole. Calls VCROSS, VDOT, VMAG and VSCAL.
EPH	Computes position, velocity, and acceleration of a planet in heliocentric ecliptic Cartesian coordinates given a specific date and planet number. Uses built-in time varying osculating elements for each planetary orbit.
ETAINT	Initializes EFFIC by assigning either built-in or input constants to the coefficients representing the propulsion system efficiency. Entry point in EFFIC.
EXEC1	Interface between the integrator RK9CYL and the derivative routine DERIV. Calls DERIV.
FSTLEG	Computes initial values of certain of the Lagrange multipliers for the first leg of a swingby mission. Values are selected to satisfy transversality conditions arising when the position and velocity at swingby are not completely specified. Calls FX, MATINV, VCROSS, VDOT, VMAG, and VSUB.
FX	Function subroutine which evaluates the transversality conditions used by FSTLEG. Calls VCROSS and VDOT.
GCOMP	Computes coefficients for f and g series coast phase solution using series expression and recursive formula.
GETB	Evaluates at each computed point, the functions being monitored by CHECK which dictate changes in the mode of operation.

HEADER	Prints the case setup page which denotes the independent and dependent parameters of the case.
INCOND	Computes planetocentric ecliptic Cartesian coordinates of spacecraft position and velocity at the swingby point given the input planetocentric polar coordinates. Calls VCROSS and VSCAL.
INTERP	Iteratively isolates a point of interest once initiated by CHECK. Calls AMAINT, DPCNV, COAST, GETB and RESTOR.
LAUNCH	Evaluates the initial spacecraft mass for a given launch vehicle and departure speed. Also computes the derivative of the mass with respect to the departure speed.
LOCATE	Computes the longitude of ascending node and the argument of periapse of a planetocentric hyperbola consistent with a specified hyperbolic excess velocity vector, passage distance, and inclination.
MAIN	Driver routine of the SWINGBY program. Reads the inputs, initializes for each case, transfers control to the iterator, and calls the summary print routines after the iterator has completed its work. Calls BOOSTER, COEFF, DATE 1, ETAINT, HEADER, MINMX3, PLNVAL, PRINT, PRINTT, PROPGT, TMESET, TRAJ and TRAJSM.
MATINV	Matrix inversion routine.
MINMX3	Generalized iterator routine. Basically, all decisions regarding the solution of the two point boundary value problem are made in this routine, including the calling of the trajectory generator routine, the evaluation of the partial derivative matrix, the selection of the changes in the independent parameters, and the printing of selected information on each iteration if requested. Calls MATINV, PARINC, PMPINT, PMPRNT, SIMEQ, SMQINT, TRAJ and TRAJSM.
OPTPV	Computes desired planetocentric position and velocity of spacecraft at sphere of influence of launch or target planet which satisfy transversality conditions associated with open inclination nodal angle and argument of position. Calls VCROSS, VDOT and VSCAL.
PARINC	Algorithm for selecting best perturbation step size for each independent parameter. Calls TRAJ.

PDATE	Determines calendar date in terms of year, month, day, and hour (GMT) given Julian date.
PLNVAL	Given a planet number, this routine returns to the calling program the built-in values of the gravitational constant, the planet radius, the radius of the sphere of influence, and the latitude and longitude of the North Pole of the planet.
PMPINT	Initializes subroutine PMPRNT by assigning dimensions of the partial derivative matrix for purposes of printing.
PMPRNT	Prints the partial derivative matrix. Entry point in PMPINT.
PRINT	Evaluates and prints the data in the standard block print-out appearing in the discontinuity point summary and trajectory printout resulting from setting MPRINT=1. Calls EPH, SOLAR, VCROSS, VDOT, VMAG and VSCAL.
PRINTT	Prints the case summary page. Calls BOOSTR, ELEM, VDOT and VMAG.
PROPGT	Propagates adjoint and state variables from sphere of influence to closest approach point assuming no thrust is permitted. Calls INCOND, EPH, VADD, VSUB, VSCAL, COEFF, SOLAR VMAG, BOOSTR, LAUNCH, VDOT and COAST.
RESTOR	Sets all parameters required for continuing integration to values saved from previously obtained point. RESTOR is used after integrating using a non-standard integration interval to reset the necessary quantities to their appropriate values at the end of the last standard interval. Entry point in STORE.
RK9CYL	Fourth-order Runge-Kutta integration routine. Calls EXEC1.
SIMEQ	Solves for the changes in the independent parameters given the partial derivative matrix and the desired changes in the dependent parameters. Entry point in SMQINT.
SMQINT	Initializes SIMEQ by assigning limits to ranges of DO loops.

SOLAR	Computes the power ratio, γ, and its derivative with respect to solar distance given the solar distance.
STORE	Temporarily saves all information at each point along a trajectory such that, if necessary, the integration could proceed from that point after digressing to integrate over a non-standard interval. Also permanently stores for all discontinuity points the information necessary to compute all the parameters appearing in the standard block printout.
TAP	This subroutine controls the computation of one leg of the trajectory by making the appropriate calls to the integration and analytic coast phase solution routines and by instituting such features as checking for discontinuity points, printing, and storing for summary prints. Calls AMAINT, CHECK, COAST, DPCNV, DRVINT, DSCONT, EPH, PRINT, SOLAR, STORE, UPDER, VDOT, and VMAG.
TMESET	Carries the input reference date to EPH and defines the Julian date of perihelion passage for the oddball planet. Entry point in EPH.
TRAJ	This routine controls the computation of a complete swingby trajectory through the appropriate initialization and calls to TAP and then computes the appropriate dependent parameters. Calls EFFIC, EPH, FSTLEG, INCOND, LAUNCH, OPTPV, SOLAR, TAP, VADD, VCROSS, VDOT, VMAG, VSCAL and VSUB.
TRAJSM	Prints the trajectory summary.
UPDER	Updates an array of values of the functions being monitored by subroutine CHECK. Entry point in CHECK.
VADD	General vector addition subroutine. Entry point in VSCAL.
VCROSS	General vector cross product subroutine. Entry point in VSCAL.
VDOT	General vector dot product function subprogram. Entry point in VMAG.

VMAG	General function subprogram which evaluates the magnitude of an input vector.
VSCAL	General subroutine which evaluates the product of a scalar and a vector.
VSUB	General vector subtraction subroutine. Entry point in VSCAL.

SWINGBY Program Machine Requirements

When compiled by the IBM 360/Model 91 computer at the Goddard Space Flight Center under their Fortran H, Level 18 compiler with compiler optimization level equal to two, the SWINGBY program occupies about 44000 (hexidecimal) bytes in core. This includes the core requirements for the following IBM library subroutines which must be accessible to the program:

IHCLASCN	IHCFCVTH
IHCLATN2	IHCEFNTH
IHCLSCN	IHCLEXP
IHCLSCNH	IHCLLOG
IHCLSQRT	IHCEFIOS
IHCFDXPD	IHCERRM
IHCNAMEL	IHCUOPT
IHCECOMH	IHCETRCH
IHCCOMH2	IHCUATBL
IHCFEXIT	REMTIM

The program is written entirely in double precision Fortran IV using the non-standard Fortran statement IMPLICIT REAL *8 (A-H, O-Z). This results in the assignment of an 8-byte word location to each real variable name commencing with the letters A-H or O-Z, unless the name is specifically declared to be of another type. An 8-byte word contains 15 hexadecimal digits. As in standard Fortran IV, names commencing with the letters I-N represent integer variables of 4-byte word length.

The only peripheral equipment referenced by the SWINGBY program are the card reader, assigned to UNIT 5, the high-speed printer, assigned to UNIT 6, the card punch assigned to UNIT 7, and two arbitrary output devices, assigned to UNITS 11 and 12, being used for remote terminal output at GSFC. No magnetic tapes are employed by the program for either input or output. Of course, temporary data storage assignments are made as required on the disk and drum storage areas. The linkage editor step space requirements are approximately 160,000 bytes LCS and the execution step requirements are about 380,000 bytes LCS.

Example Cases

In this section are presented several sample cases generated with the SWINGBY program which illustrate many of its most useful features. A total of five individual cases are presented, the five cases being obtained with two separate job submissions. The program is currently stored in object module form on a disc pack which is accessible to the IBM 360/91 computer at GSFC. A complete set of control cards required to access the program module and to execute a job is as follows:

```
// JOB CARD
// EXEC LOADER,REGION.GO=400K,PARM='EP=MAIN,SIZE=400K,LET'
//GO.FT07F001 DD DSNAME=DECK,SYSOUT=B
//GO.SYSLIN DD  DSN=MMLOAD(M7JLHSWB),DISP=SHR,DCB=RECFM=
//GO.FT11F001 DD SYSOUT=R,DCB=(RECFM=FB,LRECL=80,BLKSIZE=80)
//GO.FT12F001 DD SYSOUT=R,DCB=(RECFM=FB,LRECL=80,BLKSIZE=80)
//GO.DATA5 DD *
```

The input data for the job follows immediately after the last of the control cards.

The first job includes two cases which were selected to illustrate the operation of the program for a swingby mission. The specific mission chosen involves a 1400-day Earth-Jupiter-Saturn trajectory with orbiter end conditions at Saturn. Although swingby trajectories will generally not prove attractive, from a performance standpoint, for orbiter missions, the orbiter end conditions were chosen for this example case to illustrate the retro stage feature. The example cases involve an August 1977 launch from Earth employing the Titan III X(1205)/Centaur launch vehicle. The launch, swingby, and arrival dates are all optimized, subject to the total mission duration constraint, to yield maximum net spacecraft mass. Other trajectory and propulsion system parameters that are optimized include the Jupiter passage conditions, the launch and target hyperbolic excess speeds, the reference thrust acceleration, and the jet exhaust speed. Scaling parameters representative of near-term propulsion system technology are assumed. A retro stage with specific impulse of 300 seconds is specified for inserting the spacecraft into a loose elliptical capture orbit about Saturn with periapse and apoapse distances of 2 and 38 Saturn radii, respectively. Both the low thrust propulsion system and the tankage are jettisoned prior to the retro maneuver.

For the first of the two cases of this job, the option which assumes no thrust within the swingby planet sphere of influence and which commences the optimization problem at the sphere of influence is imposed (i.e., ISPHER=1). This particular problem results in a two-point boundary value problem of order 27. The input values of the independent parameters are those which yield a nearly converged solution to the same problem for a 1410 day mission duration. Using the MUPDAT feature the converged independent variables from this first case are stored for use in the next case. In the second case, the ITOP option is invoked which, commencing with the converged values of the state and adjoint variables at the entry and exit points of the swingby planet sphere of influence, propagates the trajectory and the adjoint variables along a coasting path to the passage point. Once this is accomplished, ISPHER is set to zero (automatically), and the passage values of the adjoint variables become the independent parameters of the new boundary value problem. The flag JC is set to zero to permit thrusting within the sphere of influence if thrusting is optimum. If thrusting is not optimum, then the solution to the first case will also be a solution for this case. One additional feature which permits the reduction of the order of the boundary value problem is also invoked for this case by setting IPICK(5) equal to one. This causes the program to solve for the seven passage values of the adjoint variables required for the backward integration using the seven transversality conditions arising because the passage conditions are left open. As a consequence, the order of the boundary value problem for this second case is 20. A listing of the input data set required to run these two cases is presented on a subsequent page. For any namelist parameter not included in this data set, the default value is used.

Following the input data set are presented 17 pages of computer printout (photographically reduced) which represent the total output obtained for these two cases using NPRINT setting of 3. For the first case, the trajectory counters indicate that the iterator required four partial derivative matrix evaluations plus nine additional trajectories for convergence. The CPU time required to accomplish this on the IBM 360/91 computer was 53 seconds. An important input in achieving convergence in this case is the specified tolerance on the mission duration constraint. This allowable tolerance is reflected through the input minimum and maximum allowable values BY (2,23) and BY (3,23), respectively. In addition to being an acceptable tolerance on the accuracy to which the end condition is to be satisfied, the tolerance is also used internally to generate a weight for the end condition. The smaller the value of the tolerance, the more importance is given to meeting that end condition relative to other end conditions. For the first case, the inputs yielded a trajectory which essentially satisfied all end conditions except the mission duration, which was in error by ten days. By specifying a very small tolerance for the mission duration, primary emphasis is given to satisfying that end condition. The input tolerance of $\pm 10^{-7}$ days is nearly optimum for this case. The importance of the input can be observed by specifying a tolerance of $\pm 10^{-6}$ days for which the iterator fails to converge.

The second case is seen to require 23 partial derivative matrix evaluations plus 65 additional trajectories. Since the ITOP feature in conjunction with starting conditions from the first case yields very close initial guesses of all independent parameters for the second case, this relatively large number of iterations attests to the extreme sensitivity of a problem which commences at the passage point (i.e., with ISPHER=0). In addition to the large number of iterations, the use of the ISPHER=0 option is costly in terms of CPU time. For the second case presented here, the CPU time required for convergence was 1081 seconds. This corresponds to approximately 2.06

seconds of CPU time per individual trajectory as compared to 0.45 seconds per trajectory for the first case. The primary cause for this large time difference is that the computing interval required to maintain sufficient accuracy during the thrusting maneuver near the pericenter point is extremely small. This integration sensitivity is caused by the large fluctuations (several orders of magnitude) in certain of the adjoint variables in the vicinity of the passage point. From the discontinuity point summary pages of the second case, it is seen that the optimal solution does contain a thrusting period at swingby; however, the total duration of the period is only about 1.6 days and is nearly centered about the passage point. This is insufficient time to have any significant effect on the net spacecraft mass -- the additional thrust period permitting an increase in net mass of only 16 grams out of about 470 kilograms (.0034 percent).

NAMELIST INPUT DATA FOR SWINGBY TRAJECTORY CASES

```
&MINPUT
BX(1,1)=1.D0,6.8265567D5,1.D-2,2.D4,1.D-12
BX(1,2)=1.D0,20976.0400D0,1.D-3,1.D3,1.D-9
BX(1,4)=1.D0, 6.1394171D0,1.D-5,1.D1,1.D-3
BX(1,5)=1.D0,146.21429D0,1.D-5,1.D1,1.D-3
BX(1,6)=1.D0,293.76273D0,1.D-5,1.D1,1.D-3
BX(1,7)=1.D0,.829983770D0,1.D-7,1.D-1,1.D0
BX(1,8)=1.D0,655.37049D0,1.D-5,1.D1,1.D-5
BX(1,9)=1.D0,-1.1880956D1,1.D-5,1.D1,1.D-5
BX(1,10)=1.D0,1342.14030D0,1.D-5,1.D1,1.D-5
BX(1,11)=1.D0,5704.4943D0,1.D-3,1.D3,1.D-9
BX(1,12)=1.D0,11352.844D0,1.D-3,1.D3,1.D-9
BX(1,13)=1.D0,3.9751227D-4,1.D-10,.5D-4,1.D8
BX(1,14)=1.D0,27501.870D0,1.D-2,1.D3,1.D-9
BX(1,15)=1.D0,7.61296700D0,1.D-5,1.D3,1.D-5
BX(1,16)=1.D0, 2.70394320D2,1.D-5,1.D3,1.D-5
BX(1,17)=1.D0,-3.94228150D1,1.D-5,1.D3,1.D-5
BX(1,18)=1.D0,-8.99500710D1,1.D-5,1.D3,1.D-5
BX(1,19)=1.D0,1.48706100D2,1.D-5,1.D3,1.D-5
BX(1,20)=1.D0,2.36815410D1,1.D-5,1.D3,1.D-5
BX(1,21)=1.D0,-8.95215810D2,1.D-5,1.D3,1.D-5
BX(1,23)=1.D0, 5.42408940D2,1.D-5,1.D3,1.D-4
BX(1,24)=1.D0, 1.89315560D2,1.D-5,1.D3,1.D-4
BX(1,25)=1.D0,-3.94228150D1,1.D-5,1.D3,1.D-4
BX(1,26)=1.D0, 1.50020018D2,1.D-5,1.D3,1.D-4
BX(1,27)=1.D0, 7.43438720D1,1.D-5,1.D3,1.D-4
BX(1,28)=1.D0,-7.12509950D0,1.D-5,1.D3,1.D-4
BX(1,29)=1.D0,8.95215810D2,1.D-5,1.D3,1.D-4
BY(1, 1)=1.D0,-1.D-8,1.D-8,2*1.D0,BY(1, 2)=1.D0,-1.D-8,1.D-8,2*1.D0
BY(1, 3)=1.D0,-1.D-8,1.D-8,2*1.D0,BY(1, 4)=1.D0,-1.D-8,1.D-8,2*1.D0
BY(1, 5)=1.D0,-1.D-8,1.D-8,2*1.D0,BY(1, 6)=1.D0,-1.D-8,1.D-8,2*1.D0
BY(1, 7)=1.D0,-1.D-8,1.D-8,2*1.D0,BY(1, 8)=1.D0,-1.D-8,1.D-8,2*1.D0
BY(1, 9)=1.D0,-1.D-8,1.D-8,2*1.D0,BY(1,10)=1.D0,-1.D-8,1.D-8,2*1.D0
BY(1,11)=1.D0,-1.D-8,1.D-8,2*1.D0,BY(1,12)=1.D0,-1.D-8,1.D-8,2*1.D0
BY(1,13)=1.D0,-1.D-6,1.D-6,2*1.D0,BY(1,14)=2.D0,-1.D-4,1.D-4,2*1.D0
BY(1,20)=1.D0,-1.D-4,1.D-4,2*1.D0,BY(1,28)=1.D0,-1.D-4,1.D-4,2*1.D0
BY(1,21)=1.D0,-1.D-4,1.D-4,2*1.D0,BY(1,22)=1.D0,-1.D-4,1.D-4,2*1.D0
BY(1,23)=1.D0,1399.9999999D0,1400.0000001D0,2*1.D0
BY(1,25)=1.D0,-1.D-6,1.D-6,2*1.D0,BY(1,26)=1.D0,-1.D-6,1.D-6,2*1.D0
BY(1,29)=1.D0,-1.D-4,1.D-4,2*1.D0,BY(1,30)=1.D0,-1.D-6,1.D-6,2*1.D0
BY(1,31)=1.D0,-1.D-4,1.D-4,2*1.D0,BY(1,32)=1.D0,-1.D-4,1.D-4,2*1.D0
BY(1,33)=1.D0,-1.D-4,1.D-4,2*1.D0,BY(1,34)=1.D0,-1.D-4,1.D-4,2*1.D0
EKR=.1111111111D0,DBETAH=.15625D-1,MYEAR=1977,MONTH=8,MDAY=24
JR=1,JPS=1,JT=1,ISPHER=1,IPICK(3)=1,1,IDATE=71,03,15
HDNG='  SWINGBY - STARTING AT SPH. OF INFL.'
&END
&MINPUT  ITOP=1,IPICK(5)=1,BY(1,28)=0.D0,BY(1,29)=0.D0,BY(1,30)=0.D0
BY(1,31)=0.D0,BY(1,32)=0.D0,BY(1,33)=0.D0,BY(1,34)=0.D0,
DBETAP=.244140625D-3,DZP=.625D-1
HDNG=' SWINGBY - STARTING AT CLOSEST APPROACH'
&END
```

```
          CASE  1                    3/15/71          SWINGBY - STARTING AT SPH. OF INFL.          CPU - I/O TIMES REMAINING (SEC)11971187
&MINPUT
BX=  1.0000000000000000    ,  682655.6700000000    , 0.99999999999999990D-02,  20000.0000000000    , 0.99999999999999999D-12,
  1.0000000000000000       ,  20976.0400000000     , 0.99999999999999990D-03,  1000.00000000000000 , 0.10000000000000000D-08,
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  1.0000000000000000       ,  6.1394171000000000   , 0.99999999999999998D-05,  10.000000000000000  , 0.39999999999999999D-03,
  1.0000000000000000       ,  146.21429000000000   , 0.99999999999999998D-05,  10.000000000000000  , 0.99999999999999999D-03,
  1.0000000000000000       ,  293.76273000000000   , 0.99999999999999998D-05,  10.000000000000000  , 0.09999999999999999D-03,
  1.0000000000000000       , 0.82978377000000000   , 0.10000000000000000D-06, 0.10000000000000000D 00,  1.0000000000000000   ,
  1.0000000000000000       ,  655.37049000000000   , 0.99999999999999998D-05,  10.000000000000000  , 0.99999999999999998D-05,
  1.0000000000000000       , -11.880556000000000   , 0.99999999999999998D-05,  10.000000000000000  , 0.99999999999999998D-05,
  1.0000000000000000       ,  1342.1403000000000   , 0.99999999999999998D-05,  10.000000000000000  , 0.99999999999999998D-05,
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  1.0000000000000000       , 0.39751226999999999D-03, 0.10000000000000000D-09, 0.50000000000000000D-04, 99999999.999999999   ,
  1.0000000000000000       ,  27501.870000000000   , 0.99999999999999999D-02,  1000.0000000000000  , 0.10000000000000000D-08,
  1.0000000000000000       ,  7.6129670000000000   , 0.99999999999999998D-05,  1000.0000000000000  , 0.99999999999999998D-05,
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  1.0000000000000000       , -39.422815000000000   , 0.99999999999999998D-05,  1000.0000000000000  , 0.99999999999999998D-05,
  1.0000000000000000       , -89.950071000000000   , 0.99999999999999998D-05,  1000.0000000000000  , 0.99999999999999998D-05,
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  1.0000000000000000       ,  185.31556000000000   , 0.99999999999999998D-05,  1000.0000000000000  , 0.99999999999999999D-04,
  1.0000000000000000       , -39.422815000000000   , 0.99999999999999998D-05,  1000.0000000000000  , 0.99999999999999999D-04,
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 0.0                  ,CNI= 0.0                ,OMI= 0.0                ,SOI= 0.0                ,TPI=
 0.0                  ,EMSUN= 0.0              ,EMLNCH= 0.0             ,PLNCH= 0.0              ,RSLNCH=
 0.0                  ,ELOLNC= 0.0             ,FLALNC= 0.0             ,EMSWBY= 0.0             ,RSWBY=
 0.0                  ,RSSWEY= 0.0             ,FLOSWB= 0.0             ,FLASWB= 0.0             ,EMTARG=
 0.0                  ,RTARG= 0.0              ,RSTARG= 0.0             ,FLOTRG= 0.0             ,FLATRG=
 0.0                  ,B1= 0.0                 ,B2= 0.0                 ,B3= 0.0                 ,BJ=  1.0000000000000000   ,
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 0.0                  , 0.0                   , 0.0                    , 0.0                   ,PMI=  1.0000000000000000   ,RPMI=
 0.0                  ,PPOI= 0.0               ,EM=  2.0000000000000000 ,ENN=  1.0000000000000000   ,BI=
 1.0000000000000000   ,DI=  20000.000000000000  ,EI= 0.0                ,CALPHA=  30.000000000000000   ,EKS=
 0.0                  ,EKT= 0.29999999999999999D-01,DFLMX= 0.0         ,FKX= 0.0                ,EKP=
 0.11111111111000000  ,SPIPET=  300.00000000000000  ,RTF=  2.0000000000000000  ,RTA=  38.000000000000000   ,DZP=
 0.78125000000000000D-02,DZH= 0.12500000000000000  ,IPEM=          1,MUBDAT=          1,NSET=          0,          0,          0,
          0,          0,MPPINT=          0,NPPINT=          3,MOPT1=          3,MOPT2=          5,MOPT3=          6,INPFLG=
          0,          0,          0,          0,          0,          0,          0,          0,          0,          0,
          0,          0,          0,          0,          0,MYEAR=       1977,MONTH=          8,MDAY=         24,HOUR=
 12.000000000000000   ,MBOOST=          10,MSOLAR=          0,MSHLD=          0,MEFFIC=          0,JR=          1,JPS=          1,
JT=          1,JC=          0,MODE=          1,IPICK=          0,          0,          1,          1,          0,          0,FRWD=F,
IPR11=          0,IPR12=          0,ITOP=          0,LONG=          0,KPART=          0,IMPACT=          0,MULAT=          0,
ISPHER=          1,ITE=          5,AUKM=  149598000.0000000   ,GRAV=  9.8066500000000000   ,FR=  6378.1650000000000   ,MPUNCH=
          0,KOUNT=          1,IDATE=         71,          3,         15,          0,          0,          0,HDNG=
 0.25346224238981480  ,-0.36752193326464070 48,-0.10988946543155410 43,-0.73426345224513580 41, -57385235636.01568   ,LCC=
          0
&END
```

INDEPENDENT PARAMETERS

		NAME	VALUE	DELTA	MAX STEP	WEIGHT
1	1	PASSDIST	6.8265567000000D 05	1.0000000000D-02	2.0000000000D 04	1.0000000000D-12
2	2	PASSVELO	2.0976040000000D 04	1.0000000000D-03	1.0000000000D 03	1.0000000000D-09
3	4	PASSINCL	6.1394171000000D 00	1.0000000000D-05	1.0000000000D 01	1.0000000000D-03
4	5	PASSNODE	1.4621429000000D 02	1.0000000000D-05	1.0000000000D 01	1.0000000000D-03
5	6	PASSARGP	2.9376273000000D 02	1.0000000000D-05	1.0000000000D 01	1.0000000000D-03
6	7	PASSMASS	8.2998377000000D-01	1.0000000000D-07	1.0000000000D-01	1.0000000000D 00
7	8	PASSTIME	6.5537049000000D 02	1.0000000000D-05	1.0000000000D 01	1.0000000000D-05
8	9	LAUNTIME	-1.1890956000000D 01	1.0000000000D-05	1.0000000000D 01	1.0000000000D-05
9	10	TARGTIME	1.7421403000000D 03	1.0000000000D-05	1.0000000000D 01	1.0000000000D-05
10	11	LAUN V00	5.7044942999999D 03	1.0000000000D-03	1.0000000000D 03	1.0000000000D-09
11	12	TARG V00	1.1352844000000D 04	1.0000000000D-03	1.0000000000D 03	1.0000000000D-09
12	13	THR ACCL	3.9751226999999D-04	1.0000000000D-10	5.0000000000D-05	1.0000000000D 08
13	14	JETSPEED	2.7501879000000D 04	1.0000000000D-02	1.0000000000D 03	1.0000000000D-09
14	15	LEG1 P1	7.6129567000000D 00	1.0000000000D-05	1.0000000000D 03	1.0000000000D-05
15	16	LEG1 P2	2.7039432000000D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-05
16	17	LEG1 P3	-3.9422815000000D 01	1.0000000000D-05	1.0000000000D 03	1.0000000000D-05
17	18	LEG1 PD1	-8.9950071000000D 01	1.0000000000D-05	1.0000000000D 03	1.0000000000D-05
18	19	LEG1 PD2	1.4870610000000D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-05
19	20	LEG1 PD3	2.3681541000000D 01	1.0000000000D-05	1.0000000000D 03	1.0000000000D-05
20	21	LEG1 PMA	-8.9521581000000D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-05
21	23	LEG2 P1	5.4240894000000D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
22	24	LEG2 P2	1.8931556000000D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
23	25	LEG2 P3	-3.9422815000000D 01	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
24	26	LEG2 PD1	1.5020018000000D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
25	27	LEG2 PD2	7.4343872000000D 01	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
26	28	LEG2 PD3	-7.1250995000000D 00	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
27	29	LEG2 PMA	8.9521581000000D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04

DEPENDENT PARAMETERS

		NAME	TRIG	LOW	HIGH	WEIGHT	TYPE
1	1	DELTA XL	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
2	2	DELTA YL	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
3	3	DELTA ZL	1	-1.0000000000D-08	1.0000000000D-09	1.0000000000D 00	1
4	4	DELT XDL	1	-1.0000000000D-09	1.0000000000D-08	1.0000000000D 00	1
5	5	DELT YDL	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
6	6	DELT ZDL	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
7	7	DELTA XT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
8	8	DELTA YT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
9	9	DELTA ZT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
10	10	DELT XDT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
11	11	DELT YDT	1	-1.0000000000D-08	1.0000000000D-06	1.0000000000D 00	1
12	12	DELT ZDT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
13	13	LAUNMASS	1	-1.0000000000D-06	1.0000000000D-06	1.0000000000D 00	1
14	14	TARGMASS*	2	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
15	20	T(TG DT)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
16	21	T(LNV00)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
17	22	T(TGV00)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
18	23	DURATION	1	1.3959999999D 03	1.4000000001D 03	1.0000000000D 00	1
19	25	T(THRAC)	1	-1.0000000000D-06	1.0000000000D-06	1.0000000000D 00	1
20	26	T(JETVL)	1	-1.0000000000D-06	1.0000000000D-06	1.0000000000D 00	1
21	28	T(SWB R)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
22	29	T(SWB V)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
23	30	T(SWINC)	1	-1.0000000000D-06	1.0000000000D-05	1.0000000000D 00	1
24	31	T(SWNOD)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
25	32	T(SWARG)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
26	33	T(SW MR)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
27	34	T(SWTIM)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1

NOTE THE ABOVE INDICATED DEPENDENT AND INDEPENDENT PARAMETERS MAY BE ALTERED INTERNALLY TO AUTOMATICALLY SATISFY TRANSVERSALITY CONDITIONS AT THE SWINGBY CLOSEST APPROACH POINT. THEIR TRIGGERS ARE SET TO ZERO. SEE BOXED QUANTITIES ON TRAJECTORY SUMMARY PAGE FOR DISPLAY OF ACTUAL DEPENDENT AND INDEPENDENT PARAMETERS

* NAME APPLIES TO TRIGGER SETTING OF 1

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THIS CASE IS CCNVERGED.
   9 TRAJECTCRIES WITHOUT PAPTIALS AND   4 TRAJECTORIES WITH PARTIALS.      INHIBITOR =  0.47683716D-06
```

SWINGBY - STARTING AT SPH. OF INFL.

INDEPENDENT PARAMETERS

PASS DIST	PASS SPEED	PASS GAMMA	PASS INCL	PASS NODE	PASS ARGP	PASS MASS	PASS TIME
6.62636220D 05	2.12807490D 04	0.0	6.11803880D 00	1.46770600D 02	2.93763930D 02	9.30089070D-01	6.50995550D 02
LAUN TIME	**TARG TIME**	**LAUN V00**	**TARG V00**	**THR ACCEL**	**JET SPEED**	**LEG1 P1**	**LEG1 P2**
-1.15317790D 01	1.33309000D 03	5.73057300D 03	1.14840990D 04	3.98702640D-04	2.74627700D 04	4.41135610D 00	2.71829650D 02
LEG1 P3	**LEG1 PD1**	**LEG1 PD2**	**LEG1 PD3**	**LEG1 PMASS**	**LEG1 PTIME**	**LEG2 P1**	**LEG2 P2**
-2.88933460D 01	-8.77715270D 01	1.47219570D 02	2.29764320D 01	-8.67410030D 02	0.0	5.39304280D 02	1.88555590D 02
LEG2 P3	**LEG2 PD1**	**LEG2 PD2**	**LEG2 PD3**	**LEG2 PMASS**	**LEG2 PTIME**		
-3.90258360D 01	1.49052180D 03	7.33236920D 01	-7.01758060D 00	9.67418830D 02	0.0		

DEPENDENT PARAMETERS (REFERENCED TO ZERO)

DELTA XL	DELTA YL	DELTA ZL	DELTA XDL	DELTA YDL	DELTA ZDL	DELTA XT	DELTA YT
6.92436390D-12	2.53025380D-11	6.34949720D-13	-1.53167620D-11	6.56230630D-12	-3.09083730D-12	2.07820430D-11	5.95501430D-12
DELTA ZT	**DELTA XDT**	**DELTA YDT**	**DELTA ZDT**	**LAUN MASS**	**TARG MASS**	**LAUN DATE**	**TARG DATE**
-2.46076350D-11	2.29930000D-12	1.38472570D-13	-1.61801680D-12	3.01759730D-10	3.17623970D-09	-1.3317217D 01	1.3866828D 03
LEG1 TIME	**LEG2 TIME**	**T(LAUN DATE)**	**T(TARG DATE)**	**T(LAUN V00)**	**T(TARG V00)**	**TOTAL TIME**	**REF POWER**
6.6431277D 02	7.3568723D 02	8.8371653D 01	5.94855540D-06	-9.15229770D-07	3.13077380D-06	7.38964450D-13	2.8922628D 01
T(THR ACC)	**T(JET VEL)**	**NET MASS**	**T(SWB DIST)**	**T(SWB VEL)**	**T(SWB INC)**	**T(SWB NODE)**	**T(SWB ARG)**
4.19954630D-08	-3.34900640D-09	4.6994599D 02	-6.95825670D-06	-1.02113120D-07	-7.89095040D-09	1.13275880D-07	-1.13798120D-07
T(SWB MASS)	**T(SWB TIME)**						
7.21229300D-10	5.37180040D-08						

DISCONTINUITY POINTS

LAUNCH	LSOI	OFF	SSOIN	SWINGBY	SSOIX	ON	TSOI	ARRIVE
0.0	1.785	258.337	604.548	664.313	723.977	797.253	1346.407	1400.000

SPACECRAFT PARAMETERS

NET S/C MASS	INITIAL MASS	PROP SYSTEM	PROPELLANT	RETRO SYSTEM	SWB SCI PKG	POWER	EFFICIENCY
4.69945985D 02	3.19601950D 03	8.67678851D 02	6.07694380D 02	1.23246949D 03	0.0	2.89226284D 01	6.04971801D-01

SWINGBY - STARTING AT SPH. OF INFL.

TIME	MASS RATIO	SEMI-AXIS	ECCENTRICITY	FLT PTH ANGLE	INCLINATION	NODE	ARG POS
LAMBDA T	LAMBDA NU	LAMBDA A	LAMBDA C	SWITCH FNCT	POWER FACTOR	THRUST ACCEL	ANGULAR MOM
XP	YP	ZP	XP DOT	YP DOT	ZP DOT	RP	VP
X	Y	Z	X DOT	Y DOT	Z DOT	R	V
LAMBDA X	LAMBDA Y	LAMBDA Z	LAMBDA DOT X	LAMBDA DOT Y	LAMBDA DOT Z	LAMBDA	LAMBDA DOT
LONGITUDE	LATITUDE	PHI	THETA OSC	PSI OSC	THETA I	PSI I	HAMILTONIAN

EARTH SPHERE OF INFLUENCE (HELIOCENTRIC) THRUST ON

1.78543814D 00	1.00000000D 00	1.73178378D 00	4.15036055D-01	-3.79346460D-01	1.64910544D-02	3.19442333D 02	1.01811536D 00
0.0	-4.96413494D 02	-3.32615506D 03	-1.29558596D 02	2.15314002D 03	9.87471241D-01	6.78645490D-02	1.19727837D 00
5.98095637D 05	7.17887311D 05	7.75100999D 02	3.61045708D 03	4.54565785D 03	1.01285680D 01	9.28019004D 05	5.80504162D 03
7.81293060D-01	-6.44954649D-01	5.18122568D-06	7.46305298D-01	9.16359410D-01	3.40052544D-04	1.01310678D 00	1.18181483D 00
-1.62703326D 03	-2.14407478D 03	-3.27423869D 00	1.37959334D 03	-1.31242306D 03	-1.12626198D 02	2.69153531D 03	1.87878498D 03
3.20460443D 02	2.93021792D-04	8.76535405D 01	8.76535314D 01	-1.59674649D-01	1.27193081D 02	-1.76137277D-01	4.67214486D 01

SWITCH THRUST OFF

2.88332857D 02	8.30089068D-01	4.14371938D 00	7.27025913D-01	4.46558798D 01	8.39526890D-01	7.38890351D 01	3.03248515D 01
0.0	-8.67418831D 02	0.0	0.0	3.97903932D-13	1.59451886D-01	0.0	1.39767213D 00
- - - - (PLANETOCENTRIC PARAMETERS UNAVAILABLE) - - - -							
-7.51294496D-01	2.96686458D 00	2.26417215D-02	-5.53408663D-01	3.25260553D-01	9.11357794D-03	3.06059496D 00	6.41980244D-01
4.81511355D 02	-5.98541244D 02	-1.40514618D 02	-5.47961832D 01	2.11506719D 02	1.48434597D 01	7.80928399D 02	2.32253605D 02
1.04210206D 02	4.23967559D-01	1.53593721D 02	1.55311922D 02	-9.67838936D 00	5.11841986D 01	-1.03659445D 01	4.67214439D 01

JUPITER SPHERE OF INFLUENCE (HELIOCENTRIC) THRUST OFF

6.04548380D 02	8.30089068D-01	4.14371938D 00	7.27025913D-01	4.53767858D 01	8.39526890D-01	7.38890351D 01	5.76331078D 01
0.0	-8.67418831D 02	0.0	0.0	-5.07531349D 02	6.14579314D-02	0.0	1.39767213D 00
1.03988203D 07	-4.70514595D 07	3.76145647D 05	-1.59152668D 03	8.55307925D 03	-9.06254192D 01	4.81893470D 07	8.70036406D 03
-3.32630901D 00	3.75729197D 00	6.21070331D-02	-3.95542496D-01	2.66483340D-02	5.67681083D-03	5.01850144D 00	3.96479833D-01
4.41135609D 00	2.71829551D 02	-2.88938461D 01	-8.77715275D 01	1.47217565D 02	2.29764293D 01	2.73396550D 02	1.72931656D 02
1.31518358D 02	7.09088512D-01	4.29126527D 01	4.25385737D 01	-6.28586653D 00	8.90702637D 01	-6.06661938D 00	4.67214439D 01

JUPITER SPHERE OF INFLUENCE (HELIOCENTRIC) THRUST OFF

7.23977161D 02	8.30089068D-01	-8.59712818D 00	1.59320101D 00	1.81538314D 01	2.55017017D 00	1.17624876D 02	2.43523070D 01
0.0	8.67418831D 02	0.0	0.0	-2.09280781D 02	5.04889481D-02	0.0	3.63660166D 00
-4.69199481D 07	-1.03247264D 07	3.74636505D 06	-8.40350139D 03	-2.14881171D 03	6.78312114D 02	4.81893470D 07	8.70036406D 03
-4.36143225D 00	3.41293815D 00	1.01624130D-01	-5.73329324D-01	-3.84283653D-01	3.05629339D-02	5.53900402D 00	6.90936796D-01
5.39304275D 02	1.88555503D 02	-3.90258359D 01	1.49052176D 02	7.33236225D 01	-7.01758065D 00	5.72647318D 02	1.66259320D 02
1.41955863D 02	1.05126489D 00	1.22678319D 02	1.22689053D 02	-1.38455917D 00	1.92710208D 01	-3.90772627D 00	1.23935393D 02

SWITCH THRUST ON

7.97253065D 02	8.30089068D-01	-8.59712818D 00	1.59320101D 00	2.32281795D 01	2.55017017D 00	1.17624876D 02	3.24815779D 01
0.0	8.67418831D 02	0.0	0.0	3.97903932D-13	4.53396379D-02	0.0	3.63660166D 00
- - - - (PLANETOCENTRIC PARAMETERS UNAVAILABLE) - - - -							
-5.06380581D 00	2.91408288D 00	1.39642639D-01	-5.41080802D-01	-4.06067276D-01	2.97375169D-02	5.84410031D 00	6.77158317D-01
7.29505349D 02	2.77210505D 02	-4.77849494D 01	1.51493315D 02	6.72098568D 01	-6.89427736D 00	7.80928399D 02	1.65876220D 02
1.50080741D 02	1.36919201D 00	1.29255059D 02	1.29261851D 02	-9.75806243D-01	2.08328037D 01	-3.50811591D 00	1.23935393D 02

SWINGBY - STARTING AT SPH. OF INFL.   CASE 1   3/15/71

TIME	MASS RATIO	SEMI-AXIS	ECCENTRICITY	FLT PTH ANGLE	INCLINATION	NODE	ARG POS
LAMBDA T	LAMBDA NU	LAMBDA A	LAMBDA C	SWITCH FNCT	POWER FACTOR	THRUST ACCEL	ANGULAR MOM
XP	YP	ZP	XP DOT	YP DOT	ZP DOT	RP	VP
X	Y	Z	X DOT	Y DOT	Z DOT	R	V
LAMBDA X	LAMBDA Y	LAMBDA Z	LAMBDA DOT X	LAMBDA DOT Y	LAMBDA DOT Z	LAMBDA	LAMBDA DOT
LONGITUDE	LATITUDE	PHI	THETA OSC	PSI OSC	THETA I	PSI I	HAMILTONIAN

SATURN SPHERE OF INFLUENCE (HELIOCENTRIC)   THRUST ON

1.34640720D 03	8.09858991D-01	-1.10365460D 01	1.46518667D 00	4.58756631D 01	2.53603250D 00	1.17274192D 02	6.99490091D 01
0.0	9.08717801D 02	-2.36473430D 02	-1.93962413D 01	1.73766021D 03	1.78668377D-02	1.01921893D-01	3.55758368D 00
5.20768951D 07	1.58774913D 07	-2.52521362D 06	-1.10481381D 04	-3.70649530D 03	5.30808678D 02	5.45020590D 07	1.15445207D 04
-9.13073971D 00	-1.15430827D 00	3.82877440D-01	-3.46043753D-01	-4.32992359D-01	2.24108400D-02	9.21137505D 00	5.54734989D-01
2.42075923D 03	7.45949095D 02	-1.18009579D 02	2.12591851D 02	4.13448919D 01	-8.49291871D 00	2.53583153D 03	2.16856631D 02
1.87205112D 02	2.38222665D 00	1.70084166D 02	1.70085624D 02	-1.70910823D-01	1.71265119D 01	-2.66732771D 00	1.23935392D 02

CASE 1 (CONVERGED) CASE SUMMARY 3/15/71

SWINGBY - STARTING AT SPH. OF INFL.

LAUNCH	SWINGBY	TARGET
EARTH	JUPITER	SATURN

LAUNCH VEHICLE IS TITAN III X(1205)/CENTAUR

MASS BREAKDOWN(KG)

INITIAL	PROPULSION	PROPELLANT	TANKAGE	STRUCTURE	SWB SCI PKG	RETRO STAGE	NET S/C
3196.02	867.68	607.69	18.23	0.0	0.0	1232.47	469.95

PROPULSION SYSTEM PARAMETERS

REF POWER (KW)	REF THRUST (N)	THR ACCEL (M/SEC/SEC)	EXHAUST SPEED (M/SEC)	EFFICIENCY	PROP TIME (DAYS)
28.9226	1.274261	0.398703D-03	27462.7703	0.604972	835.7016

LAUNCH CONDITIONS (PLANETOCENTRIC)

DATE (JULIAN)	RADIUS (RADII)	SPEED (M/SEC)	FLT PATH (DEG)	INCLINATION (DEG)	NODE (DEG)	ARG POS (DEG)	EXCESS SPEED (M/SEC)	T INF (DAYS)
43366.6828	1.02901	12422.0067	0.0	25.7921	173.9353	84.4358	5730.5730	1.7854

SWINGBY CONDITIONS (PLANETOCENTRIC)

DATE (JULIAN)	RADIUS (RADII)	SPEED (M/SEC)	FLT PATH (DEG)	INCLINATION (DEG)	NODE (DEG)	ARG POS (DEG)	EXCESS SPEED (M/SEC)	T INF (DAYS)
44030.9956	9.48249	21280.7490	0.0	6.1180	146.3706	293.7639	8392.7061	119.3288

ARRIVAL CONDITIONS (PLANETOCENTRIC)

DATE (JULIAN)	RADIUS (RADII)	SPEED (M/SEC)	FLT PATH (DEG)	INCLINATION (DEG)	NODE (DEG)	ARG POS (DEG)	EXCESS SPEED (M/SEC)	T INF (DAYS)
44766.6828	2.00000	28122.3850	0.0	23.3518	178.7567	108.7094	11484.0989	53.5928

TRAJECTORY SCHEDULE (DAYS)

LAUNCH	LAUNCH SPHERE	ENTER SWB SPH	SWINGBY	EXIT SWB SPH	TARGET SPHERE	ARRIVE
0.	1.7354	604.6484	664.3128	723.9772	1346.4072	1400.0000

CAPTURE ORBIT AND RETRO STAGE

RPER (RADII)	RAP (RADII)	VORB (M/SEC)	INC VEL (M/SEC)	SPEC IMP (SEC)	INERT (KG)	PROPELLANT (KG)
2.0000	38.0000	25020.6877	3101.6973	300.00	123.25	1109.22

```
          CASE  2              3/15/71      SWINGBY - STARTING AT CLOSEST APPROACH      CPU - I/O TIMES REMAINING (SEC)11441186
&MINPUT
BX=  1.0000000000000000    .  552636.2224438632    . 0.9999999999999999D-02,  20000.0000000000    . 0.9999999999999999D-12,
 1.000000000000000   .  21290.74396542015   . 0.9999999999999999D-03,  1000.000000000000   . 0.1000000000000000D-08,
0.0                  . 0.0                  . 0.0                     . 0.0                   . 0.0                    .
 1.000000000000000   .  6.118038816806920   . 0.9999999999999998D-05,  10.00000000000000   . 0.9999999999999999D-03,
 1.000000000000000   .  146.3705981814239   . 0.9999999999999998D-05,  10.00000000000000   . 0.9999999999999999D-03,
 1.000000000000000   .  293.7539314532777   . 0.9999999999999998D-05,  10.00000000000000   . 0.9999999999999999D-03,
 1.000000000000000   . 0.8300390677776787   . 0.1000000000000000D-06, 0.1000000000000000D 00,  1.000000000000000   .
 1.000000000000000   .  650.9955529217739   . 0.9999999999999998D-05,  10.00000000000000   . 0.9999999999999998D-05,
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 1.000000000000000   .  1333.089980763558   . 0.9999999999999998D-05,  10.00000000000000   . 0.9999999999999998D-05,
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 1.000000000000000   . 0.3987026426729816D-03, 0.1000000000000000D-09, 0.5000000000000000D-04,  99999999.99999999  .
 1.000000000000000   .  27462.77031680362   . 0.9999999999999999D-02,  1000.000000000000   . 0.1000000000000000D-08,
 1.000000000000000   .  4.411356077311865   . 0.9999999999999998D-05,  1000.000000000000   . 0.9999999999999998D-05,
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 1.000000000000000   . -28.89338451239347   . 0.9999999999999998D-05,  1000.000000000000   . 0.9999999999999998D-05,
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 1.000000000000000   . -867.4188314859622   . 0.9999999999999998D-05,  1000.000000000000   . 0.9999999999999998D-05,
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 1.000000000000000   .  539.3042750835909   . 0.9999999999999998D-05,  1000.000000000000   . 0.9999999999999990D-04,
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0.0                  . 0.0                  . 0.0                     . 0.0                   . 0.0                    .
0.0                  . 0.0                  . 0.0                     . 0.0                   . 0.0                    ,BY=
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 1.000000000000000   ,-0.1000000000000000D-07, 0.1000000000000000D-07,  1.000000000000000   .  1.000000000000000   .
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 1.000000000000000   ,-0.9999999999999999D-06, 0.9999999999999999D-06,  1.000000000000000   .  1.000000000000000   .
 2.000000000000000   ,-0.9999999999999999D-04, 0.9999999999999999D-04,  1.000000000000000   .  1.000000000000000   .
0.0                  . 0.0                  . 0.0                     . 0.0                   . 0.0                    .
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 0.0                  . 0.0                   . 0.0                   . 0.0                  . 0.0                  .
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  1.000000000000000   .-0.9999999999999999D-04. 0.9999999999999999D-04.  1.000000000000000   .  1.000000000000000  .
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 0.0                  .ELOLNC= 0.0                   .FLALNC= 0.0                  .EMSWRY= 0.0                  .RSWRY=
 0.0                  .RSSWRY= 0.0                   .ELOSWP= 0.0                  .FLASWR= 0.0                  .EMTARG=
 0.0                  .RTARG= 0.0                    .RSTARG= 0.0                  .ELOTRG= 0.0                  .ELATRG=
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 0.0                  .RPOI= 0.0                 .FM=  2.000000000000000      .FNN=  1.000000000000000    .BI=
  1.000000000000000   .DI=  20000.00000000000    .EI= 0.0               .CALPHA=  30.00000000000000    .EKS=
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 0.1111111111000000   .SPIRET=  300.0000000000000    .RTF=  2.000000000000000     .RTA=  38.00000000000000    .DZP=
 0.6250000000000000D-01.DZH= 0.1250000000000000       .IPEM=           1.MUPDAT=          1.NSET=          27.        27.        0.
          0.          0.MPRINT=          0.NPRINT=          3.NCPT1=          3.MOPT2=          5.MOPT3=          6.INPFLG=
          0.          0.          0.          0.          0.          0.          0.          0.          0.          0.          0.
          0.          0.          0.          0.          0.MYEAR=          1977.MONTH=          8.MDAY=          24.HOUR=
  12.00000000000000   .MBOOST=          10.MSOLAR=          0.MSHLD=          0.MEFFIC=          0.JR=          1.JPS=          1.
JT=          1.JC=          0.MODE=          1.IPICK=          0.          0.          1.          1.          1.          0.FRWD=F.
IPR11=          0.IPR12=          0.ITOP=          1.LONG=          0.KPART=          0.IMPACT=          0.MIKLAT=          0.
ISPHER=          1.ITF=          5.AUKM=  149598000.0000000    .GRAV=  9.806650000000000    .ER=  6378.165000000000    .MPUNCH=
          0.KOUNT=          2.IDATE=          71.          3.          15.          0.          0.          0.HDNG=
 0.8863340517925838   . 0.3759900861785819   . -5738568398.20331   .-0.6342310316899999D 23.-0.4175164260950131D 28.LOC=
          0
&END
```

SETUP FOR CASE 2 3/15/71

SWINGBY - STARTING AT CLOSEST APPROACH

INDEPENDENT PARAMETERS

		NAME	VALUE	DELTA	MAX STEP	WEIGHT
1	1	PASSDIST	6.626362224438632D 05	1.0000000000D-02	2.0000000000D 04	1.0000000000D-12
2	2	PASSVELO	2.128074896542015D 04	1.0000000000D-03	1.0000000000D 03	1.0000000000D-09
3	4	PASSINCL	6.119038816806920D 00	1.0000000000D-05	1.0000000000D 01	1.0000000000D-03
4	5	PASSNODE	1.463705991814239D 02	1.0000000000D-05	1.0000000000D 01	1.0000000000D-03
5	6	PASSARGP	2.937539314532777D 02	1.0000000000D-05	1.0000000000D 01	1.0000000000D-03
6	7	PASSMASS	8.703890677776787D-01	1.0000000000D-07	1.0000000000D-01	1.0000000000D 00
7	8	PASSTIME	6.509555292177739D 02	1.0000000000D-05	1.0000000000D 01	1.0000000000D-05
8	9	LAUNTIME	-1.153177927070960D 01	1.0000000000D-05	1.0000000000D 01	1.0000000000D-05
9	10	TARGTIME	1.333089980763558D 03	1.0000000000D-05	1.0000000000D 01	1.0000000000D-05
10	11	LAUN VOO	5.730573023497379D 03	1.0000000000D-03	1.0000000000D 03	1.0000000000D-09
11	12	TARG VOO	1.143409886589280D 04	1.0000000000D-03	1.0000000000D 03	1.0000000000D-09
12	13	THR ACCL	3.987026426729816D-04	1.0000000000D-10	5.0000000000D-05	1.0000000000D 08
13	14	JETSPEED	2.746277031680362D 04	1.0000000000D-02	1.0000000000D 03	1.0000000000D-09
14	15	LEG1 P1	-8.604790956962276D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-05
15	16	LEG1 P2	5.459498224064178D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-05
16	17	LEG1 P3	5.099537681715588D 01	1.0000000000D-05	1.0000000000D 03	1.0000000000D-05
17	18	LEG1 PD1	-3.759782953105167D 04	1.0000000000D-05	1.0000000000D 03	1.0000000000D-05
18	19	LEG1 PD2	-5.836743913856011D 04	1.0000000000D-05	1.0000000000D 03	1.0000000000D-05
19	20	LEG1 PD3	4.252293901771438D 03	1.0000000000D-05	1.0000000000D 03	1.0000000000D-05
20	21	LEG1 PMA	-8.674188314855622D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-05
21	23	LEG2 P1	8.604790957242931D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
22	24	LEG2 P2	-5.459498222973430D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
23	25	LEG2 P3	-5.099537959359144D 01	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
24	26	LEG2 PD1	3.759782956367427D 04	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
25	27	LEG2 PD2	5.836743913871223D 04	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
26	28	LEG2 PD3	-4.252293725677786D 03	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
27	29	LEG2 PMA	8.674188314866834D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04

DEPENDENT PARAMETERS

		NAME	TRIG	LOW	HIGH	WEIGHT	TYPE
1	1	DELTA XL	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
2	2	DELTA YL	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
3	3	DELTA ZL	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
4	4	DELT XDL	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
5	5	DELT YDL	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
6	6	DELT ZDL	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
7	7	DELTA XT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
8	8	DELTA YT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
9	9	DELTA ZT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
10	10	DELT XDT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
11	11	DELT YDT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
12	12	DELT ZDT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
13	13	LAUNMASS	1	-1.0000000000D-06	1.0000000000D-06	1.0000000000D 00	1
14	14	TARGMASS*	2	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
15	20	T(TG DT)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
16	21	T(LNVOO)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
17	22	T(TGVOO)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
18	23	DURATION	1	1.3999999999D 03	1.4000000001D 03	1.0000000000D 00	1
19	25	T(THRAC)	1	-1.0000000000D-06	1.0000000000D-06	1.0000000000D 00	1
20	26	T(JETVL)	1	-1.0000000000D-06	1.0000000000D-06	1.0000000000D 00	1

NOTE THE ABOVE INDICATED DEPENDENT AND INDEPENDENT PARAMETERS MAY BE ALTERED INTERNALLY TO AUTOMATICALLY SATISFY TRANSVERSALITY CONDITIONS AT THE SWINGBY CLOSEST APPROACH POINT. THEIR TRIGGERS ARE SET TO ZERO. SEE BOXED QUANTITIES ON TRAJECTORY SUMMARY PAGE FOR DISPLAY OF ACTUAL DEPENDENT AND INDEPENDENT PARAMETERS

* NAME APPLIES TO TRIGGER SETTING OF 1

THIS CASE IS CONVERGED.
65 TRAJECTORIES WITHOUT PARTIALS AND 23 TRAJECTORIES WITH PARTIALS. INHIBITOR = 0.22204460D-15

SWINGBY - STARTING AT CLOSEST APPROACH

INDEPENDENT PARAMETERS

PASS DIST	PASS SPEED	PASS GAMMA	PASS INCL	PASS NODE	PASS ARGP	PASS MASS	PASS TIME
6.62514520 05	2.12827460 04	0.0	6.11832290 00	1.46371240 02	2.93757590 02	8.30043270-01	6.50785380 02
LAUN TIME	TARG TIME	LAUN V00	TARG V00	THR ACCEL	JET SPEED	LEG1 P1	LEG1 P2
-1.15273660 01	1.33208420 03	5.73148310 03	1.14817650 04	3.98730220-04	2.74679840 04	-8.57202920 02	5.43869570 02
LEG1 P3	LEG1 PD1	LEG1 PD2	LEG1 PD3	LEG1 PMASS	LEG1 PTIME	LEG2 P1	LEG2 P2
5.08030610 01	-3.74756370 04	-5.81544570 04	4.23832460 03	-8.67639010 02	2.87932930 04	8.57202920 02	-5.43869570 02
LEG2 P3	LEG2 PD1	LEG2 PD2	LEG2 PD3	LEG2 PMASS	LEG2 PTIME		
-5.08030610 01	3.74756370 04	5.81544570 04	-4.23832460 03	8.67639010 02	-2.87160790 04		

DEPENDENT PARAMETERS (REFERENCED TO ZERO)

DELTA XL	DELTA YL	DELTA ZL	DELTA XDL	DELTA YDL	DELTA ZDL	DELTA XT	DELTA YT
-5.03596980-10	4.90345420-10	4.79557240-11	6.88010400-10	-9.80738730-11	6.84798290-11	-1.73536740-10	4.34797090-10
DELTA ZT	DELTA XDT	DELTA YDT	DELTA ZDT	LAUN MASS	TARG MASS	LAUN DATE	TARG DATE
5.10293120-12	3.53646760-10	-4.52097900-10	-1.55848820-11	4.30160130-10	-8.75877500-09	-1.33125440 01	1.38668750 03
LEG1 TIME	LEG2 TIME	T(LAUN DATE)	T(TARG DATE)	T(LAUN V00)	T(TARG V00)	TOTAL TIME	REF POWER
6.64097930 02	7.35902070 02	8.83565430 01	8.71227830-06	-8.85027330-05	3.52123010-06	-5.69434190-14	2.89227580 01
T(THR ACC)	T(JET VEL)	NET MASS	T(SWB DIST)	T(SWB VEL)	T(SWB INC)	T(SWB NODE)	T(SWB ARG)
5.84183350-07	-2.94351760-07	4.69961980 02	0.0	0.0	0.0	0.0	0.0
T(SWB MASS)	T(SWB TIME)						
0.0	0.0						

DISCONTINUITY POINTS

LAUNCH	LSOI	OFF	SSOIN	ON	SWINGBY	OFF	SSOIX	ON	TSOI
0.0	1.785	288.428	604.463	663.285	664.098	664.960	723.783	797.589	1346.397
ARRIVE									
1400.000									

SPACECRAFT PARAMETERS

NET S/C MASS	INITIAL MASS	PROP SYSTEM	PROPELLANT	RETRO SYSTEM	SWB SCI PKG	POWER	EFFICIENCY
4.699619850 02	3.195464980 03	8.676827510 02	6.078462660 02	1.231738600 03	0.0	2.892275840 01	6.050207840-01

DISCONTINUITY POINT SUMMARY CASE 2 3/15/71

SWINGBY - STARTING AT CLOSEST APPROACH

TIME	MASS RATIO	SEMI-AXIS	ECCENTRICITY	FLT PTH ANGLE	INCLINATION	NODE	ARG POS
LAMBDA T	LAMBDA NU	LAMBDA A	LAMBDA C	SWITCH FNCT	POWER FACTOR	THRUST ACCEL	ANGULAR MOM
XP	YP	ZP	XP DOT	YP DOT	ZP DOT	RP	VP
X	Y	Z	X DOT	Y DOT	Z DOT	R	V
LAMBDA X	LAMBDA Y	LAMBDA Z	LAMBDA DOT X	LAMBDA DOT Y	LAMBDA DOT Z	LAMBDA	LAMBDA DOT
LONGITUDE	LATITUDE	PHI	THETA OSC	PSI OSC	THETA I	PSI I	HAMILTONIAN

EARTH SPHERE OF INFLUENCE (HELIOCENTRIC) THRUST ON

1.785177930 00	1.000000000D 00	1.731594840 00	4.151091880-01	-3.800473820-01	1.667906680-02	3.154339470 02	1.030738340 00
1.022235230 02	-4.964369190 02	-3.327703900 03	-1.295974750 02	2.153796610 03	9.834727450-01	6.786918590-02	1.193710801D 00
5.879528210 05	7.179715130 05	7.936553980 02	3.610373430 03	4.546871330 03	1.024425080 01	9.280190090 05	5.805940070 03
7.813398280-01	-6.448961000-01	5.305254070-06	7.462457370-01	9.166475830-01	3.439364310-04	1.013105580 00	1.181845590 00
-1.627091210 03	-2.144758410 03	-9.327240410 00	1.379803080 03	-1.312709290 03	-1.126191730 02	2.692115120 03	1.879085160 03
3.204646850 02	3.000365160-04	9.764996520 01	8.764995600 01	-1.605773340-01	1.271852680 02	-1.772273730-01	-5.549926530 01

SWITCH THRUST OFF

2.884277780 02	8.300990800-01	4.145559280 00	7.271510830-01	4.466561550 01	8.392682300-01	7.388379450 01	3.034310520 01
1.022235230 02	-8.675713170 02	-1.155421650-01	-5.250777450-02	6.252776070-13	1.593559460-01	0.0	1.397712490 00
- (PLANETOCENTRIC PARAMETERS UNAVAILABLE) -							
-7.522626760-01	2.967637560 00	2.265437760-02	-5.533634260-01	3.251783630-01	9.109971040-03	3.061582120 00	6.418995370-01
4.813369830 02	-5.987037120 02	-1.404112560 02	-9.486572480 01	2.113782790 02	1.486453850 01	7.809268510 02	2.321664000 02
1.042242200 02	4.239677450-01	1.536256940 02	1.553437270 02	-9.671026370 00	5.120192750 01	-1.035815600 01	-5.549926990 01

JUPITER SPHERE OF INFLUENCE (HELIOCENTRIC) THRUST OFF

6.044634260 02	8.300990800-01	4.145559280 00	7.271510830-01	4.539474180 01	8.392682300-01	7.388379450 01	5.762187380 01
1.022235230 02	-8.675713170 02	-1.155421650-01	-5.250777450-02	-5.089005400 02	6.146064960-02	0.0	1.397712490 00
1.040278840 07	-4.705058370 07	3.759695880 05	-1.593302230 03	8.557722950 03	-9.063340910 01	4.918834700 07	8.705254080 03
-3.325220500 00	3.758096320 00	6.207880030-02	-3.956728020-01	2.688963210-02	5.677782880-03	5.018389500 00	3.966260900-01
4.465008540 00	2.704550710 02	-2.885190110 01	-8.776721770 01	1.471284990 02	2.297722020 01	2.720263110 02	1.728520540 02
1.315028880 02	7.087819660-01	4.291610390 01	4.253947880 01	-6.307418610 00	8.905417600 01	-6.088408580 00	-5.549926990 01

JUPITER SPHERE OF INFLUENCE (PLANETOCENTRIC) THRUST OFF

6.044634260 02	8.300990800-01	-1.796684280 06	1.368755400 00	-8.807360070 01	6.118323210 00	1.463712420 02	1.587768770 02
1.003386490 02	-8.675713170 02	-1.155421650-01	-5.250777450-02	-5.089005400 02	6.146064960-02	0.0	1.410150030 10
1.040278840 07	-4.705058370 07	3.759695880 05	-1.593302230 03	8.557722950 03	-9.063340910 01	4.918834700 07	8.705254080 03
-3.325220500 00	3.758096320 00	6.207880030-02	-3.956728020-01	2.688963210-02	5.677782880-03	5.018389500 00	3.966260900-01
4.465008540 00	2.704550710 02	-2.885190110 01	-9.002184480 01	1.573259120 02	2.289573520 01	2.720263110 02	1.827008200 02
1.315028880 02	7.087819660-01	1.654739450 02	1.662081660 02	-4.604657490 00	8.905417600 01	-6.088408580 00	-5.549926590 01

SWITCH THRUST ON

6.632854770 02	8.300990800-01	-1.796684280 06	1.368755400 00	-4.732337430 01	6.118323210 00	1.463712420 02	2.139403450 02
3.585726040 03	-8.675713170 02	-1.155421650-01	-5.250777450-02	-1.136868380-13	5.486022180-02	2.755956320-04	1.410150030 10
1.165469110 06	-4.821153430 05	-7.500067300 04	-6.936262120 03	1.493172120 04	1.513702350 02	1.263478750 06	1.646483960 04
-3.721485720 00	3.791978050 00	6.767870960-02	-5.509744310-01	2.142877150-01	1.337100670-02	5.313486040 00	5.913297160-01
-3.334229670 02	7.061315420 02	7.356426660 00	-1.749057850 04	7.374609950 03	1.156719450 03	7.809268510 02	1.901691900 04
1.344624590 02	7.298051130-01	1.376827670 02	1.376827690 02	-1.494190650-02	1.152758620 02	5.397412490-01	-5.549926990 01

SWINGBY - STARTING AT CLOSEST APPROACH CASE 2 3/15/71

TIME	MASS RATIO	SEMI-AXIS	ECCENTRICITY	FLT PTH ANGLE	INCLINATION	NODE	ARG POS
LAMBDA T	LAMBDA NU	LAMBDA A	LAMBDA C	SWITCH FNCT	POWER FACTOR	THRUST ACCEL	ANGULAR MOM
XP	YP	ZP	XP DOT	YP DOT	ZP DOT	RP	VP
X	Y	Z	X DOT	Y DOT	Z DOT	R	V
LAMBDA X	LAMBDA Y	LAMBDA Z	LAMBDA DOT X	LAMBDA DOT Y	LAMBDA DOT Z	LAMBDA	LAMBDA DOT
LONGITUDE	LATITUDE	PHI	THETA OSC	PSI OSC	THETA I	PSI I	HAMILTONIAN

JUPITER CLOSEST APPROACH T-MINUS THRUST ON

6.64097928D 02	8.30043268D-01	-1.79847841D 06	1.36837502D 00	5.82788900D-16	6.11832288D 00	1.46371244D 02	2.93757591D 02
2.87932928D 04	-8.67639011D 02	0.0	0.0	2.35515372D 02	5.46752248D-02	7.55197021D-05	1.41001285D 10
3.55153727D 05	5.57838773D 05	-4.00901954D 04	-1.78967259D 04	1.14697544D 04	1.05224774D 03	6.62514521D 05	2.12827463D 04
-3.73134952D 00	3.79491214D 00	6.80279290D-02	-9.18610327D-01	9.77048805D-02	4.36103793D-02	5.32249520D 00	9.24820546D-01
-8.57202924D 02	5.43869574D 02	5.08030615D 01	-3.74756372D 04	-5.81544572D 04	4.23832455D 03	1.01645065D 03	6.93132575D 04
1.34516123D 02	7.32329331D-01	9.02623943D 01	9.02623943D 01	1.51086667D-02	1.47606097D 02	2.86488506D 00	-5.54980703D 01

JUPITER CLOSEST APPROACH T-PLUS THRUST ON

6.64097928D 02	8.30043268D-01	-1.79847841D 06	1.36837502D 00	5.82788900D-16	6.11832288D 00	1.46371244D 02	2.93757591D 02
-2.87160789D 04	8.67679011D 02	0.0	0.0	2.35515372D 02	5.46752248D-02	7.55197021D-05	1.41001285D 10
3.55153727D 05	5.57838773D 05	-4.00901954D 04	-1.78967259D 04	1.14697544D 04	1.05224774D 03	6.62514521D 05	2.12827463D 04
-3.73134952D 00	3.79491214D 00	6.80279290D-02	-9.18610327D-01	9.77048805D-02	4.36103793D-02	5.32249520D 00	9.24820546D-01
8.57202924D 02	-5.43869574D 02	-5.08030615D 01	3.74756372D 04	5.81544572D 04	-4.23832455D 03	1.01645065D 03	6.93132575D 04
1.34516123D 02	7.32329331D-01	8.97376057D 01	8.97376057D 01	-1.51086667D-02	3.23939031D 01	-2.86488506D 00	2.17158702D 01

SWITCH THRUST OFF

6.64960032D 02	8.29984249D-01	-1.80036419D 06	1.36797439D 00	4.86342843D 01	6.11832006D 00	1.46371247D 02	1.56582061D 01
7.83609849D 02	8.67710602D 02	-1.22131345D-01	-5.55298156D-02	1.13686839D-12	5.45108192D-02	2.96821580D-04	1.40986528D 10
-9.95762977D 05	8.57891836D 05	5.31729635D 04	-1.61719360D 04	-1.82544108D 02	1.20876858D 03	1.31542756D 06	1.62180751D 04
-3.74509351D 00	3.79264938D 00	6.87741954D-02	-8.60334959D-01	-2.93877977D-01	4.88586173D-02	5.33053886D 00	9.10454651D-01
7.78569863D 02	1.58466110D 01	-5.87508137D 01	-1.32148002D 04	1.17171094D 04	6.99520400D 02	7.80944175D 02	1.76751567D 04
1.34638524D 02	7.39246175D-01	1.38115631D 02	1.38115639D 02	-2.80958159D-02	1.16602258D 00	-4.31446585D 00	2.17146411D 01

JUPITER SPHERE OF INFLUENCE (PLANETOCENTRIC) THRUST OFF

7.23782844D 02	8.29984249D-01	-1.80036419D 06	1.36797439D 00	8.80721548D 01	6.11832006D 00	1.46371247D 02	6.87594788D 01
-9.59460666D 01	8.67710602D 02	-1.22131345D-01	-5.55298156D-02	-2.09853528D 02	5.04915530D-02	0.0	1.40986528D 10
-4.69190368D 07	-1.03287828D 07	3.74660040D 06	-8.40002074D 03	-2.14875537D 03	6.78088478D 02	4.81883470D 07	8.69697085D 03
-4.36047500D 00	3.41393014D 00	1.01600250D-01	-5.73368256D-01	-3.84207080D-01	3.05570672D-02	5.53896124D 00	6.90868998D-01
5.37745788D 02	1.88309751D 02	-3.89029213D 01	1.31790961D 02	6.94754595D 01	-5.63818709D 00	5.71090651D 02	1.49088852D 02
1.41941677D 02	1.05104493D 00	1.73111821D 02	1.73135737D 02	5.74884201D-01	1.92994167D 01	-3.90603609D 00	2.17146411D 01

JUPITER SPHERE OF INFLUENCE (HELIOCENTRIC) THRUST OFF

7.23782844D 02	8.29984249D-01	-8.60474716D 00	1.59271076D 00	1.81435364D 01	2.54996440D 00	1.17614019D 02	2.43489725D 01
-1.02193502D 02	8.67710602D 02	-1.22131345D-01	-5.55298156D-02	-2.09853528D 02	5.04915530D-02	0.0	3.63636525D 00
-4.69190368D 07	-1.03287828D 07	3.74660040D 06	-8.40002074D 03	-2.14875537D 03	6.78088478D 02	4.81883470D 07	8.69697085D 03
-4.36047500D 00	3.41393014D 00	1.01600250D-01	-5.73368256D-01	-3.84207080D-01	3.05570672D-02	5.53896124D 00	6.90868998D-01
5.37745788D 02	1.88309751D 02	-3.89029213D 01	1.49112724D 02	7.32886321D 01	-7.02137246D 00	5.71090651D 02	1.66298331D 02
1.41941677D 02	1.05104493D 00	1.22635868D 02	1.22646558D 02	-1.39281101D 00	1.92994167D 01	-3.90603609D 00	2.17146411D 01

SWINGBY - STARTING AT CLOSEST APPROACH CASE 2 3/15/71

TIME	MASS RATIO	SEMI-AXIS	ECCENTRICITY	FLT PTH ANGLE	INCLINATION	NODE	ARG POS
LAMBDA T	LAMBDA NU	LAMBDA A	LAMBDA C	SWITCH FNCT	POWER FACTOR	THRUST ACCEL	ANGULAR MOM
XP	YP	ZP	XP DOT	YP DOT	ZP DOT	RP	VP
X	Y	Z	X DOT	Y DOT	Z DOT	R	V
LAMBDA X	LAMBDA Y	LAMBDA Z	LAMBDA DOT X	LAMBDA DOT Y	LAMBDA DOT Z	LAMBDA	LAMBDA DOT
LONGITUDE	LATITUDE	PHI	THETA OSC	PSI OSC	THETA I	PSI I	HAMILTONIAN

SWITCH THRUST ON

7.97589318D 02	8.29984249D-01	-8.60474716D 00	1.59271076D 00	2.32521482D 01	2.54996440D 00	1.17614019D 02	3.25340322D 01
-1.02193502D 02	8.67710602D 02	-1.22131345D-01	-5.55298156D-02	5.68434189D-14	4.53066333D-02	0.0	3.63636525D 00
- (PLANETOCENTRIC PARAMETERS UNAVAILABLE) -							
-5.06774998D 00	2.91145910D 00	1.39882762D-01	-5.40821978D-01	-4.06133586D-01	2.97253499D-02	5.84621682D 00	6.76990766D-01
7.28399304D 02	2.77543001D 02	-4.77299513D 01	1.51567570D 02	6.71422127D 01	-6.89774279D 00	7.80944179D 02	1.65916798D 02
1.50122320D 02	1.37105021D 00	1.29271188D 02	1.29277926D 02	-9.71706003D-01	2.08584112D 01	-3.50400230D 00	2.17146411D 01

SATURN SPHERE OF INFLUENCE (HELIOCENTRIC) THRUST ON

1.34639677D 03	8.09778462D-01	-1.10455247D 01	1.46483409D 00	4.59714496D 01.	2.53589894D 00	1.17264647D 02	6.99583742D 01
-1.02193502D 02	9.09972763D 02	-2.36312102D 02	-1.94309000D 01	1.73690212D 03	1.78668472D-02	1.01928945D-01	3.55742709D 00
5.20771196D 07	1.58768270D 07	-2.52475881D 06	-1.10459666D 04	-3.30566895D 03	5.30621687D 02	5.45020590D 07	1.15422001D 04
-9.13074068D 00	-1.15428071D 00	3.82880025D-01	-3.45971922D-01	-4.32965094D-01	2.24046073D-02	9.21137267D 00	5.54668649D-01
2.42003379D 03	7.45711595D 02	-1.17951997D 02	2.12603133D 02	4.19728088D 01	-8.49684934D 00	2.52506647D 03	2.16965509D 02
1.87204941D 02	2.38224336D 00	1.70084309D 02	1.70085760D 02	-1.70473095D-01	1.71262099D 01	-2.66683043D 00	2.17146397D 01

CASE 2 (CONVERGED) CASE SUMMARY 3/15/71

SWINGBY - STARTING AT CLOSEST APPROACH

LAUNCH	SWINGBY	TARGET
EARTH	JUPITER	SATURN

LAUNCH VEHICLE IS TITAN III X(1205)/CENTAUR

MASS BREAKDOWN(KG)

INITIAL	PROPULSION	PROPELLANT	TANKAGE	STRUCTURE	SWB SCI PKG	RETRO STAGE	NET S/C
3195.46	867.68	607.85	18.24	0.0	0.0	1231.74	469.96

PROPULSION SYSTEM PARAMETERS

REF POWER (KW)	REF THRUST (N)	THR ACCEL (M/SEC/SEC)	EXHAUST SPEED (M/SEC)	EFFICIENCY	PROP TIME (DAYS)
28.9228	1.274128	0.398730D-03	27467.9838	0.605021	837.1246

LAUNCH CONDITIONS (PLANETOCENTRIC)

DATE (JULIAN)	RADIUS (RADII)	SPEED (M/SEC)	FLT PATH (DEG)	INCLINATION (DEG)	NODE (DEG)	ARG POS (DEG)	EXCESS SPEED (M/SEC)	T INF (DAYS)
43366.6875	1.02899	12422.4875	0.0	25.7920	173.9377	84.4202	5731.4831	1.7852

SWINGBY CONDITIONS (PLANETOCENTRIC)

DATE (JULIAN)	RADIUS (RADII)	SPEED (M/SEC)	FLT PATH (DEG)	INCLINATION (DEG)	NODE (DEG)	ARG POS (DEG)	EXCESS SPEED (M/SEC)	T INF (DAYS)
44030.7854	9.48075	21282.7463	0.0	6.1183	146.3712	293.7576	8393.5855	119.3194

ARRIVAL CONDITIONS (PLANETOCENTRIC)

DATE (JULIAN)	RADIUS (RADII)	SPEED (M/SEC)	FLT PATH (DEG)	INCLINATION (DEG)	NODE (DEG)	ARG POS (DEG)	EXCESS SPEED (M/SEC)	T INF (DAYS)
44766.6875	2.00000	28121.4329	0.0	23.3664	178.7391	108.3330	11481.7661	53.6032

TRAJECTORY SCHEDULE (DAYS)

LAUNCH	LAUNCH SPHERE	ENTER SWB SPH	SWINGBY	EXIT SWB SPH	TARGET SPHERE	ARRIVE
0.	1.7852	604.4634	664.0979	723.7928	1346.3968	1400.0000

CAPTURE ORBIT AND RETRO STAGE

RPER (RADII)	RAP (RADII)	VORB (M/SEC)	INC VEL (M/SEC)	SPEC IMP (SEC)	INERT (KG)	PROPELLANT (KG)
2.0000	38.0000	25020.6877	3100.7448	300.00	123.17	1108.56

The second job contains a series of three cases which illustrate the use of the SWINGBY program for standard, single-leg missions. The mission is idential to that of the first job described earlier except the spacecraft proceeds from Earth directly to Saturn without an intermediate encounter of Jupiter. The series of three cases are intended to show how one can make use of a solution from the two-body, heliocentric program HILTOP to assist in obtaining a segmented two-body solution with SWINGBY.

The first case invokes the HILTOP emulation mode (MULAT=1), which generates a converged trajectory identical to one generated by the HILTOP program. This is accomplished by loading into SWINGBY the independent parameters from a converged case from HILTOP after scaling all 7 of the initial adjoint variables from HILTOP by the factor

$$m_o \left[1+k_t - j_r(1+j_t k_t)\,(1+k_{rt})\,(1 - e^{-v_r/c_r})\right] / \lambda_{\nu_a}$$

where λ_{ν_a} is the final value of the mass ratio multiplier from HILTOP and all other parameters are as defined in Part I of this report. Because of slightly different conversion constants in the two programs it will generally be necessary to allow the SWINGBY program to iterate a few times to satisfy all end conditions to the desired tolerances. An option is employed (LOC=1) which computes estimates of the launch speed, longitude of ascending node and argument of periapse of an hyperbolic trajectory consistent with the launch excess velocity. These quantities are required for the boundary value problem of case 2.

The second case progresses from case 1 by explicitly including the Earth sphere of influence phase which is achieved by setting MULAT equal to zero and ISPHER equal to one. To the initial value of each of the adjoint variables from the converged case 1 is added the product of its time derivative at the initial time and an estimate of the time within the sphere of influence. These adjusted values then become estimates of the adjoint variables at exit from Earth's sphere of influence in case 2. The LOC=1 feature employed in case 1 makes all the necessary adjustments in the independent parameters associated with the launch conditions and the initial adjoint variables. The target conditions for this second case are identical to those of the emulation mode. This is achieved by setting IMPACT=1. This, in essence, is equivalent to assuming the radius of the target planet sphere of influence is zero.

The third case is identical to the second except the sphere of influence of Saturn is finally taken into account. This is accomplished by setting IMPACT=0. Also to account for the time within the sphere of influence, the estimated time of entry into Saturn's sphere of influence is input about 80 days prior to the final time of case 2. The input data set required for the three cases follows below.

The computer output for the three cases that results with NPRINT set to 3 follows the input data set. The first case, operating in the emulation mode, is seen to converge after 2 partial derivative matrix evaluations and 5 additional trajectories.

The CPU time required to achieve the converged trajectory was 15 seconds. Case 2, which accounts only for Earth's sphere of influence, required 5 partial derivative matrix evaluations plus 12 additional trajectories for convergence while consuming 40 seconds of CPU time. The third case, accounting for both Earth's and Saturn's spheres of influence, required 4 partial derivative matrix evaluations plus 10 additional trajectories and used 31 seconds of CPU time.

NAMELIST INPUT DATA FOR SINGLE LEG TRAJECTORY CASES

```
&MINPUT
BX(1,1)=0.D0,6563.3650D0,1.D-2,2.D4,1.D-12
BX(1,2)=1.D0,5684.21540D0,1.D-3,1.D3,1.D-4
BX(2,4)=28.5D0
BX(1,5)=2*0.D0,1.D-4,2.D1,1.D-0
BX(1,6)=2*0.D0,1.D-4,2.D1,1.D-0
BX(1,7)=0.D0,1.D0,1.D-7,1.D-1,1.D0
BX(1,8)=1.D0,39.638506706D0,1.D-5,10.D0,1.D-1
BX(1,10)=1.D0,1439.6385567D0,1.D-5,1.D1,1.D-1
BX(1,12)=1.D0,7507.0318898D0,1.D-3,1.D3,1.D-4
BX(1,13)=1.D0,4.6479548141D-4,1.D-10,.5D-4,1.D10
BX(1,14)=1.D0,27417.240606D0,1.D-2,1.D3,1.D-4
BX(1,23)=1.D0,-294.95941D0,1.D-5,1.D3,1.D-4
BX(1,24)=1.D0, 4.0546758D03,1.D-5,1.D3,1.D-3
BX(1,25)=1.D0,- 86.560901D0,1.D-5,1.D3,1.D-2
BX(1,26)=1.D0,-2793.85770D0,1.D-5,1.D3,1.D-4
BX(1,27)=1.D0, 8.29830090D2,1.D-5,1.D3,1.D-4
BX(1,28)=1.D0,348.390910D00,1.D-5,1.D3,1.D-1
BX(1,29)=1.D0,771.838220D0,1.D-5,1.D3,1.D-2
BY(1, 7)=1.D0,-1.D-8,1.D-8,2*1.D0,BY(1, 8)=1.D0,-1.D-8,1.D-8,2*1.D0
BY(1, 9)=1.D0,-1.D-8,1.D-8,2*1.D0,BY(1,10)=1.D0,-1.D-8,1.D-8,2*1.D0
BY(1,11)=1.D0,-1.D-8,1.D-8,2*1.D0,BY(1,12)=1.D0,-1.D-8,1.D-8,2*1.D0
BY(1,14)=2.D0,-1.D-8,1.D-8,2*1.D0,BY(1,20)=1.D0,-1.D-7,1.D-7,2*1.D0
BY(1,18)=1.D0,1399.999999D0,1400.000001D0,2*1.D0
BY(1,21)=1.D0,-1.D-4,1.D-4,2*1.D0,BY(1,22)=1.D0,-1.D-4,1.D-4,2*1.D0
BY(1,25)=1.D0,-1.D-4,1.D-4,2*1.D0,BY(1,26)=1.D0,-1.D-6,1.D-6,2*1.D0
BY(1,29)=0.D0,-1.D-4,1.D-4,2*1.D0,BY(1,30)=0.D0,-1.D-6,1.D-6,2*1.D0
BY(1,31)=0.D0,-1.D-4,1.D-4,2*1.D0,BY(1,32)=0.D0,-1.D-4,1.D-4,2*1.D0
BY(1,33)=0.D0,-1.D-4,1.D-4,2*1.D0,BY(1,34)=0.D0,-1.D-4,1.D-4,2*1.D0
DBETAH=.15625D-1,EKR=.1111111111D0,MYEAR=1977,MONTH=08,MDAY=09
JR=1,JPS=1,JT=1,JC=1,MOPT2=3,MULAT=1,IPICK(3)=1,1,LOC=1,FRWD=T
HDNG =' EARTH-SATURN MISSION, EMULATION MODE',IDATE=71,03,15
&END
&MINPUT
ISPHER=1,MULAT=0,BX(1,5)=1.D0,BX(1,6)=1.D0,BY(1,21)=0.D0
BY(1,29)=1.D0,BY(1,31)=1.D0,BY(1,32)=1.D0
HDNG='EARTH-SATURN MISSION, IMPACT=1 END COND.'
&END
&MINPUT
IMPACT=0,BX(2,10)=1360.D0
HDNG=' EARTH-SATURN MISSION, SEGMENTED 2-BODY '
&END
```

CASE 1 3/15/71 EARTH-SATURN MISSION, EMULATION MODE CPU - I/O TIMES REMAINING (SEC) 99 65

SWINFUT

BX= 0.0	6563.365000000000	0.9999999999999999D-02	20000.0000000000	0.9999999999999999D-12	
1.000000000000000	5684.215400000000	0.9999999999999999D-03	1000.000000000000	0.9999999999999999D-04	
0.0	0.0	0.0	0.0	0.0	
0.0	28.50000000000000	0.0	0.0	0.0	
0.0	0.0	0.9999999999999999D-04	20.00000000000000	1.000000000000000	
0.0	0.0	0.9999999999999999D-04	20.00000000000000	1.000000000000000	
0.0	1.000000000000000	0.1000000000000000D-06	0.1000000000000000D 00	1.000000000000000	
1.000000000000000	39.63850670600000	0.9999999999999998D-05	10.00000000000000	0.1000000000000000D 00	
0.0	0.0	0.0	0.0	0.0	
1.000000000000000	1439.638556700000	0.9999999999999998D-05	10.00000000000000	0.1000000000000000D 00	
0.0	0.0	0.0	0.0	0.0	
1.000000000000000	7507.031889799999	0.9999999999999999D-03	1000.000000000000	0.9999999999999999D-04	
1.000000000000000	0.4647954814099999D-03	0.1000000000000000D-09	0.5000000000000000D-04	1000000000.00000	
1.000000000000000	27417.24060600000	0.9999999999999999D-02	1000.000000000000	0.9999999999999999D-04	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
1.000000000000000	-294.9594100000000	0.9999999999999998D-05	1000.000000000000	0.9999999999999999D-04	
1.000000000000000	4054.675800000000	0.9999999999999998D-05	1000.000000000000	0.9999999999999999D-03	
1.000000000000000	-86.56090100000000	0.9999999999999998D-05	1000.000000000000	0.9999999999999999D-02	
1.000000000000000	-2793.857700000000	0.9999999999999998D-05	1000.000000000000	0.9999999999999999D-04	
1.000000000000000	829.8300900000000	0.9999999999999998D-05	1000.000000000000	0.9999999999999999D-04	
1.000000000000000	348.3909100000000	0.9999999999999998D-05	1000.000000000000	0.1000000000000000D 00	
1.000000000000000	771.8382200000000	0.9999999999999998D-05	1000.000000000000	0.9999999999999999D-02	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	BY=
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
0.0	0.0	0.0	0.0	0.0	
1.000000000000000	-0.1000000000000000D-07	0.1000000000000000D-07	1.000000000000000	1.000000000000000	
1.000000000000000	-0.1000000000000000D-07	0.1000000000000000D-07	1.000000000000000	1.000000000000000	
1.000000000000000	-0.1000000000000000D-07	0.1000000000000000D-07	1.000000000000000	1.000000000000000	
1.000000000000000	-0.1000000000000000D-07	0.1000000000000000D-07	1.000000000000000	1.000000000000000	
1.000000000000000	-0.1000000000000000D-07	0.1000000000000000D-07	1.000000000000000	1.000000000000000	
1.000000000000000	-0.1000000000000000D-07	0.1000000000000000D-07	1.000000000000000	1.000000000000000	
0.0	0.0	0.0	0.0	0.0	

```
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&END
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SETUP FOR CASE 1 3/15/71

EARTH-SATURN MISSION, EMULATION MODE

INDEPENDENT PARAMETERS

		NAME	VALUE	DELTA	MAX STEP	WEIGHT
1	2	PASSVELO	5.684215400000000D 03	1.0000000000D-03	1.0000000000D 03	1.0000000000D-04
2	8	PASSTIME	3.963850670600000D 01	1.0000000000D-05	1.0000000000D 01	1.0000000000D-01
3	10	TARGTIME	1.439636556700000D 03	1.0000000000D-05	1.0000000000D 01	1.0000000000D-01
4	12	TARG V00	7.507031889799999D 03	1.0000000000D-03	1.0000000000D 03	1.0000000000D-04
5	13	THR ACCL	4.647954814099999D-04	1.0000000000D-10	5.0000000000D-05	1.0000000000D 10
6	14	JETSPEED	2.741724060600000D 04	1.0000000000D-02	1.0000000000D 03	1.0000000000D-04
7	23	LEG2 P1	-2.949594100000000D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
8	24	LEG2 P2	4.054675800000000D 03	1.0000000000D-05	1.0000000000D 03	1.0000000000D-03
9	25	LEG2 P3	-8.656030100000000D 01	1.0000000000D-05	1.0000000000D 03	1.0000000000D-02
10	26	LEG2 PD1	-2.793857700000000D 03	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
11	27	LEG2 PD2	8.298300900000000D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
12	28	LEG2 PD3	3.483909100000000D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-01
13	29	LEG2 PMA	7.718382200000000D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-02

DEPENDENT PARAMETERS

		NAME	TRIG	LOW	HIGH	WEIGHT	TYPE
1	7	DELTA XT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
2	8	DELTA YT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
3	9	DELTA ZT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
4	10	DELT XDT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
5	11	DELT YDT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
6	12	DELT ZDT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
7	14	TARGMASS*	2	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
8	18	LEG2TIME	1	1.3999999990D 03	1.4000000010D 03	1.0000000000D 60	1
9	20	T(TG DT)	1	-1.0000000000D-07	1.0000000000D-07	1.0000000000D 00	1
10	21	T(LNV0C)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
11	22	T(TGV0C)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
12	25	T(THRAC)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
13	26	T(JETVL)	1	-1.0000000000D-06	1.0000000000D-06	1.0000000000D 00	1

NOTE THE ABOVE INDICATED DEPENDENT AND INDEPENDENT PARAMETERS MAY BE ALTERED INTERNALLY TO AUTOMATICALLY SATISFY TRANSVERSALITY CONDITIONS AT THE SWINGBY CLOSEST APPROACH POINT. THEIR TRIGGERS ARE SET TO ZERO. SEE BOXED QUANTITIES ON TRAJECTORY SUMMARY PAGE FOR DISPLAY OF ACTUAL DEPENDENT AND INDEPENDENT PARAMETERS

* NAME APPLIES TO TRIGGER SETTING OF 1

THIS CASE IS CONVERGED.
5 TRAJECTORIES WITHOUT PARTIALS AND 2 TRAJECTORIES WITH PARTIALS. INHIBITOR = 0.454747350D-12

TRAJECTORY SUMMARY CASE 1 3/15/71

EARTH-SATURN MISSION, EMULATION MODE

INDEPENDENT PARAMETERS

PASS DIST	PASS SPEED	PASS GAMMA	PASS INCL	PASS NODE	PASS APGP	PASS MASS	PASS TIME
6.5633650D 03	5.68427060D 03	0.0	2.85000000D 01	0.0	0.0	1.00000000D 00	3.96363020D 01
LAUN TIME	**TARG TIME**	**LAUN VOO**	**TARG V20**	**IMP ACCEL**	**JET SPEED**	**LEG1 P1**	**LEG1 P2**
0.0	1.43963690D 03	0.0	7.50599950D 03	4.64904330D-04	2.74170830D 04	0.0	0.0
LEG1 P3	**LEG1 PD1**	**LEG1 PD2**	**LEG1 PD3**	**LEG1 PMASS**	**LEG1 PTIME**	**LEG2 P1**	**LEG2 P2**
0.0	0.0	0.0	0.0	0.0	0.0	-2.94642790D 02	4.05450210D 03
LEG2 P3	**LEG2 PD1**	**LEG2 PD2**	**LEG2 PD3**	**LEG2 PMASS**	**LEG2 PTIME**		
-8.65638460D 01	-2.79372460D 03	9.29878040D 02	3.49363830D 02	7.71779840D 02	0.0		

DEPENDENT PARAMETERS (REFERENCED TO ZERO)

DELTA XL	DELTA YL	DELTA ZL	DELTA XDL	DELTA YDL	DELTA ZDL	DELTA XT	DELTA YT
0.0	0.0	0.0	0.0	0.0	0.0	-1.85594880D-12	-1.32462930D-11
DELTA ZT	**DELTA XDT**	**DELTA YDT**	**DELTA ZDT**	**LAUN MASS**	**TARG MASS**	**LAUN DATE**	**TARG DATE**
3.42877680D-12	8.57212860D-13	4.19225270D-12	-3.56324520D-15	0.0	3.92134690D-09	0.0	1.43963690D 03
LEG1 TIME	**LEG2 TIME**	**T(LAUN DATE)**	**T(TARG DATE)**	**T(LAUN VOO)**	**T(TARG VOO)**	**TOTAL TIME**	**REF POWER**
0.0	-3.41060510D-13	0.0	1.29925140D-07	-4.83566960D-09	-5.27154500D-09	1.40000000D 03	3.39829110D 01
T(IMP ACC)	**T(JET VEL)**	**NET MASS**	**T(SWB DIST)**	**T(SWB VEL)**	**T(SWB INC)**	**T(SWB NODE)**	**T(SWB ARG)**
-2.55574890D-08	-1.22568260D-09	7.22734980D 02	0.0	0.0	0.0	0.0	0.0
T(SWB MASS)	**T(SWB TIME)**						
0.0	-5.62826330D 01						

DISCONTINUITY POINTS

LAUNCH	OFF	ON	TSOI	ARRIVE
0.0	437.884	1070.842	1400.000	1400.000

SPACECRAFT PARAMETERS

NET S/C MASS	INITIAL MASS	PROP SYSTEM	PROPELLANT	RETRO SYSTEM	SWB SCI PKG	POWER	EFFICIENCY
7.227349820 02	3.224225600 03	1.019487340 03	7.566532440 02	7.026504300 02	0.0	3.398291150 01	6.045416890-01

DISCONTINUITY POINT SUMMARY CASE 1 3/15/71

EARTH-SATURN MISSION, EMULATION MODE

TIME LAMBDA T XP X LAMBDA X LONGITUDE	MASS RATIO LAMBDA NU YP Y LAMBDA Y LATITUDE	SEMI-AXIS LAMBDA A ZP Z LAMBDA Z PHI	ECCENTRICITY LAMHDA C XP DOT X DOT LAMHDA DOT X THETA OSC	FLT PTH ANGLE SWITCH FNCT YP DOT Y DOT LAMBDA DOT Y PSI OSC	INCLINATION POWER FACTOR ZP DOT Z DOT LAMBDA DOT Z THETA I	NODE THRUST ACCEL RP R LAMBDA PSI I	ARG POS ANGULAR MOM VP V LAMBDA DOT HAMILTONIAN
			EARTH LAUNCH.		T-PLUS		THRUST ON
0.0	1.00000000D 00	1.70178560D 00	4.11114022D-01	-2.22474987D 00	1.96681779D-01	1.75104172D 02	1.80000000D 02
0.0	7.71779836D 02	0.0	0.0	3.22768574D 03	9.93965996D-01	7.86478947D-02	1.18918442D 00
- - - - - - (PLANETOCENTRIC PARAMETERS UNAVAILABLE) - - - - - -							
1.00109298D 00	-8.57505134D-02	0.0	5.51971693D-02	1.18315106D 00	-4.06282410D-03	1.00475883D 00	1.18444488D 00
-2.94842795D 02	4.05450214D 03	-8.65638478D 01	-2.79372461D 03	8.29878039D 02	3.48363893D 02	4.06613000D 03	2.93512388D 03
3.55104172D 02	0.0	9.90529779D 01	9.90544407D 01	-1.02563128D 00	9.41592193D 01	-1.21986209D 00	5.89515451D 01
			SWITCH THRUST OFF				
4.37884216D 02	7.74550694D-01	7.46605599D 00	8.33385528D-01	5.27115660D 01	2.92362269D 00	1.34914668D 02	2.47122195D 01
0.0	1.65664252D 03	-6.52448962D 03	-3.04648319D 02	3.97903932D-13	7.93325083D-02	1.20839051D-01	1.51018058D 00
- - - - - - (PLANETOCENTRIC PARAMETERS UNAVAILABLE) - - - - - -							
-4.13054643D 00	1.53640347D 00	9.40008515D-02	-5.40530260D-01	-1.64064674D-01	2.54650973D-02	4.40841130D 00	5.65454375D-01
-1.22186683D 03	-3.52929301D 02	5.70691108D 02	2.33372313D 02	-8.89478948D 01	-2.40373430D 01	1.39398923D 03	2.50902687D 02
1.55565555D 02	1.22121437D 00	4.21108138D 01	3.70732490D 01	2.15975154D 01	1.63808938D 02	2.41667852D 01	5.89515423D 01
			SWITCH THRUST ON				
1.07084228D 03	7.74550694D-01	7.46605599D 00	8.33385528D-01	5.62526985D 01	2.92362270D 00	1.34914668D 02	4.94329195D 01
0.0	1.65664252D 03	-6.52448962D 03	-3.04648319D 02	-2.84217094D-13	2.26191294D-02	0.0	1.51018058D 00
- - - - - - (PLANETOCENTRIC PARAMETERS UNAVAILABLE) - - - - - -							
-8.19048393D 00	-6.17385016D-01	3.18483539D-01	-2.59967092D-01	-2.03738234D-01	1.67486914D-02	8.21989192D 00	3.30715400D-01
9.85791964D 02	-9.73369798D 02	1.54827055D 02	2.05554183D 02	-4.21907585D 01	-4.43716861D 01	1.39398923D 03	2.14479436D 02
1.84310708D 02	2.22050747D 00	1.30383035D 02	1.30688519D 02	6.39140530D 00	4.46367180D 01	6.37686237D 00	5.89515423D 01
			SATURN SPHERE OF INFLUENCE	(HELIOCENTRIC)			THRUST ON
1.40000000D 03	7.65322487D-01	7.39344813D 00	8.14672511D-01	5.25850759D 01	2.99743247D 00	1.36587332D 02	5.40695573D 01
0.0	1.68387331D 03	-6.59533254D 03	-3.21379373D 02	1.10376002D 03	1.63987645D-02	1.18425588D-01	1.57685793D 00
-2.77631270D-04	-1.98161893D-03	6.62536150D-04	-6.62024174D 03	3.52598501D 03	3.05808248D 02	2.10780617D-03	7.50590946D 03
-9.42602767D 00	-1.76737237D 00	4.06434088D-01	-1.80409806D-01	-2.00885460D-01	1.41334684D-02	9.59889532D 00	2.70374594D-01
2.20805086D 03	-1.17602265D 03	-1.01996300D 02	2.27388052D 02	-2.89022217D 01	-4.60780098D 01	2.50378136D 03	2.33803010D 02
1.90619609D 02	2.42672943D 00	1.41374915D 02	1.41400747D 02	-1.53746086D 00	2.80400632D 01	-2.33469870D 00	5.89515422D 01

CASE 1 (CONVERGED) CASE SUMMARY 3/15/71

EARTH-SATURN MISSION, EMULATION MODE

LAUNCH	TARGET
EARTH	SATURN

LAUNCH VEHICLE IS TITAN III X(1205)/CENTAUR

MASS BREAKDOWN(KG)

INITIAL	PROPULSION	PROPELLANT	TANKAGE	STRUCTURE	SWB SCI PKG	RETRO STAGE	NET S/C
3224.23	1019.49	756.65	22.70	0.0	0.0	702.65	722.73

PROPULSION SYSTEM PARAMETERS

REF POWER (KW)	REF THRUST (N)	THR ACCEL (M/SEC/SEC)	EXHAUST SPEED (M/SEC)	EFFICIENCY	PROP TIME (DAYS)
33.9829	1.498634	0.4648C40-03	27417.0833	0.604542	767.0419

LAUNCH CONDITIONS (PLANETOCENTRIC)

DATE (JULIAN)	RADIUS (RADII)	SPEED (M/SEC)	FLT PATH (DEG)	INCLINATION (DEG)	NODE (DEG)	ARG POS (DEG)	EXCESS SPEED (M/SEC)	T INF (DAYS)
43404.6369	0.0	0.0	0.0	0.0	0.0	0.0	5684.2706	0.0

ARRIVAL CONDITIONS (PLANETOCENTRIC)

DATE (JULIAN)	RADIUS (RADII)	SPEED (M/SEC)	FLT PATH (DEG)	INCLINATION (DEG)	NODE (DEG)	ARG POS (DEG)	EXCESS SPEED (M/SEC)	T INF (DAYS)
44804.6369	0.0	0.0	0.0	0.0	0.0	0.0	7506.9095	0.0

TRAJECTORY SCHEDULE (DAYS)

LAUNCH	LAUNCH SPHERE	TARGET SPHERE	ARRIVE
0.	0.0	1400.0000	1400.0000

CAPTURE ORBIT AND RETRO STAGE

RPER (RADII)	RAP (RADII)	VORB (M/SEC)	INC VEL (M/SEC)	SPEC IMP (SEC)	INERT (KG)	PROPELLANT (KG)
2.0000	38.0000	25020.6877	1725.1102	300.00	70.27	632.39

* * * * * HILTOP EMULATION MODE * * * * *

```
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          1
&END
```

SETUP FOR CASE 2 3/15/71

EARTH-SATURN MISSION. IMPACT=1 END COND.

INDEPENDENT PARAMETERS

		NAME	VALUE	DELTA	MAX STEP	WEIGHT
1	2	PASSVELO	1.240056422547545D 04	1.0000000000D-03	1.0000000000D 03	1.0000000000D-04
2	5	PASSNODE	2.258925944108320D 02	1.0000000000D-04	2.0000000000D 01	1.0000000000D 00
3	6	PASSARGP	2.814824654341033D 02	1.0000000000D-04	2.0000000000D 01	1.0000000000D 00
4	8	PASSTIME	3.963690177814745D 01	1.0000000000D-05	1.0000000000D 01	1.0000000000D-01
5	10	TARGTIME	1.439636901778147D 03	1.0000000000D-05	1.0000000000D 01	1.0000000000D-01
6	12	TARG V00	7.506909459035281D 03	1.0000000000D-03	1.0000000000D 03	1.0000000000D-04
7	13	THR ACCL	4.648043257322899D-04	1.0000000000D-10	5.0000000000D-05	1.0000000000D 10
8	14	JETSPEED	2.741708333466695D 04	1.0000000000D-02	1.0000000000D 03	1.0000000000D-04
9	23	LEG2 P1	-3.856545245319564D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
10	24	LEG2 P2	4.081477835472121D 03	1.0000000000D-05	1.0000000000D 03	1.0000000000D-03
11	25	LEG2 P3	-7.524006608617469D 01	1.0000000000D-05	1.0000000000D 03	1.0000000000D-02
12	26	LEG2 PD1	-2.855916939634685D 03	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
13	27	LEG2 PD2	7.060842865432188D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
14	28	LEG2 PD3	3.511379120781269D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-01
15	29	LEG2 PMA	7.820766541438082D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-02

DEPENDENT PARAMETERS

		NAME	TRIG	LOW	HIGH	WEIGHT	TYPE
1	7	DELTA XT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
2	8	DELTA YT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
3	9	DELTA ZT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
4	10	DELT XDT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
5	11	DELT YDT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
6	12	DELT ZDT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
7	14	TARGMASS*	2	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
8	18	LEG2TIME	1	1.3999999990D 03	1.4000000010D 03	1.0000000000D 00	1
9	20	T(TG DT)	1	-1.0000000000D-07	1.0000000000D-07	1.0000000000D 00	1
10	22	T(TGV00)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
11	25	T(THRAC)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
12	26	T(JETVL)	1	-1.0000000000D-06	1.0000000000D-06	1.0000000000D 00	1
13	29	T(SWB V)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
14	31	T(SWNOD)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
15	32	T(SWARG)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1

NOTE THE ABOVE INDICATED DEPENDENT AND INDEPENDENT PARAMETERS MAY BE ALTERED INTERNALLY TO AUTOMATICALLY SATISFY TRANSVERSALITY CONDITIONS AT THE SWINGBY CLOSEST APPROACH POINT. THEIR TRIGGERS ARE SET TO ZERO. SEE BOXED QUANTITIES ON TRAJECTORY SUMMARY PAGE FOR DISPLAY OF ACTUAL DEPENDENT AND INDEPENDENT PARAMETERS

* NAME APPLIES TO TRIGGER SETTING OF 1

THIS CASE IS CONVERGED.
12 TRAJECTORIES WITHOUT PARTIALS AND 5 TRAJECTORIES WITH PARTIALS. INHIBITOR = 0.27755576D-16

TRAJECTORY SUMMARY CASE 2 3/15/71

EARTH-SATURN MISSION, IMPACT=1 END COND.

INDEPENDENT PARAMETERS

PASS DIST	PASS SPEED	PASS GAMMA	PASS INCL	PASS NODE	PASS ASGP	PASS MASS	PASS TIME
6.5633650D 03	1.2426671D 04	0.0	2.8500000D 01	2.2592055D 02	2.8161866D 02	1.0000000D 00	3.9821026D 01
LAUN TIME	TARG TIME	LAUN VOO	TARG VOO	THR ACCEL	JET SPEED	LEG1 P1	LEG1 P2
0.0	1.4398210D 03	0.0	7.5269234D 03	4.6440676D-04	2.7328366D 04	0.0	0.0
LEG1 P3	LEG1 PD1	LEG1 PD2	LEG1 PD3	LEG1 PMASS	LEG1 PTIME	LEG2 P1	LEG2 P2
0.0	0.0	0.0	0.0	0.0	0.0	-3.9607686D 02	4.1210102D 03
LEG2 P3	LEG2 PD1	LEG2 PD2	LEG2 PD3	LEG2 PMASS	LEG2 PTIME		
-7.5109977D 01	-2.8817375D 03	7.0612333D 02	3.5372221D 02	7.7454971D 02	0.0		

DEPENDENT PARAMETERS (REFERENCED TO ZERO)

DELTA XL	DELTA YL	DELTA ZL	DELTA XDL	DELTA YDL	DELTA ZDL	DELTA XT	DELTA YT
0.0	0.0	0.0	0.0	0.0	0.0	-1.6964208D-13	2.1382416D-13
DELTA ZT	DELTA XDT	DELTA YDT	DELTA ZDT	LAUN MASS	TARG MASS	LAUN DATE	TARG DATE
3.5344619D-14	3.6623482D-14	2.0033139D-13	-1.0554225D-14	0.0	-5.5218141D-11	0.0	1.4398210D 03
LEG1 TIME	LEG2 TIME	T(LAUN DATE)	T(TARG DATE)	T(LAUN VOO)	T(TARG VOO)	TOTAL TIME	REF POWER
0.0	-1.1758684D-13	0.0	1.2391345D-07	0.0	3.7308619D-09	1.4000000D 03	3.3814971D 01
T(THR ACC)	T(JET VEL)	NET MASS	T(SWB DIST)	T(SWB VEL)	T(SWB INC)	T(SWB NODE)	T(SWB ARG)
7.0031092D-11	4.3257842D-11	7.2562546D 02	3.3391564D 07	-5.0295057D-10	-4.5894982D-02	8.3487557D-13	-2.9619441D-12
T(SWB MASS)	T(SWB TIME)						
7.7454971D 02	-5.5607774D 01						

DISCONTINUITY POINTS

LAUNCH	LSOI	OFF	ON	TSOI	ARRIVE
0.0	1.795	441.268	1070.954	1400.000	1400.000

SPACECRAFT PARAMETERS

NET S/C MASS	INITIAL MASS	PROP SYSTEM	PROPELLANT	RETRO SYSTEM	SWB SCI PKG	POWER	EFFICIENCY
7.256254610 02	3.216317240 03	1.014449130 03	7.483233730 02	7.054695680 02	0.0	3.381497110 01	6.037970040-01

DISCONTINUITY POINT SUMMARY CASE 2 3/15/71

EARTH-SATURN MISSION, IMPACT=1 END COND.

TIME	MASS RATIO	SEMI-AXIS	ECCENTRICITY	FLT PTH ANGLE	INCLINATION	NODE	ARG POS
LAMBDA T	LAMBDA NU	LAMBDA A	LAMBDA C	SWITCH FNCT	POWER FACTOR	THRUST ACCEL	ANGULAR MOM
XP	YP	ZP	XP DOT	YP DOT	ZP DOT	RP	VP
X	Y	Z	X DOT	Y DOT	Z DOT	R	V
LAMBDA X	LAMBDA Y	LAMBDA Z	LAMBDA DOT X	LAMBDA DOT Y	LAMBDA DOT Z	LAMBDA	LAMBDA DOT
LONGITUDE	LATITUDE	PHI	THETA OSC	PSI OSC	THETA I	PSI I	HAMILTONIAN

EARTH SPHERE OF INFLUENCE (HELIOCENTRIC) THRUST ON

1.79492130D 00	1.00000000D 00	1.72358669D 00	4.18523536D-01	-1.59496348D 00	1.96477667D-01	1.74680459D 02	1.82708581D 02
0.0	7.74549711D 02	0.0	0.0	3.29680514D 03	9.95503918D-01	7.85118750D-02	1.19234219D 00
-5.52548266D 04	9.26053082D 05	-2.43281441D 04	-4.25920797D 02	5.75546656D 03	-1.21059440D 02	9.28019000D 05	5.77247427D 03
1.00250131D 00	-4.57157602D-02	-1.62623458D-04	2.10749555D-02	1.18839916D 00	-4.06440185D-03	1.00354314D 00	1.18859297D 00
-3.96076863D 02	4.12101023D 03	-7.51009770D 01	-2.88173754D 03	7.06123329D 02	3.53720205D 02	4.14068139D 03	2.98799923D 03
3.57389024D 02	-9.28474071D-03	9.80993900D 01	9.81002775D 01	-8.46258820D-01	9.54899259D 01	-1.03925044D 00	5.92989504D 01

SWITCH THRUST OFF

4.41267579D 02	7.76578439D-01	7.46667459D 00	8.33282036D-01	5.27744187D 01	2.93074933D 00	1.35120899D 02	2.47630870D 01
0.0	1.65258162D 03	-6.49733061D 03	-3.02859516D 02	-5.68434189D-13	7.84009961D-02	1.20767252D-01	1.51066946D 00
- - - - - - - -	- - - - - - - -	- - - - - - - -	(PLANETOCENTRIC	PARAMETERS	UNAVAILABLE)	- - - - - - - -	- - - - - - - -
-4.16283861D 00	1.52705172D 00	9.49831589D-02	-5.37667862D-01	-1.65187034D-01	2.54155100D-02	4.43510249D 00	5.63044789D-01
-1.22316832D 03	-3.60525244D 02	5.73515316D 02	2.34276984D 02	-8.84361923D 01	-2.45764887D 01	1.39822709D 03	2.51616115D 02
1.59855476D 02	1.22715320D 00	4.21745634D 01	3.71285185D 01	2.16376061D 01	1.63577275D 02	2.42157170D 01	5.92989477D 01

SWITCH THRUST ON

1.07095404D 03	7.76578439D-01	7.46667450D 00	8.33282036D-01	5.62419563D 01	2.93074933D 00	1.35120899D 02	4.92340382D 01
0.0	1.65258162D 03	-6.49733061D 03	-3.02859516D 02	0.0	2.26159640D-02	0.0	1.51066946D 00
- - - - - - - -	- - - - - - - -	- - - - - - - -	(PLANETOCENTRIC	PARAMETERS	UNAVAILABLE)	- - - - - - - -	- - - - - - - -
-8.19096918D 00	-6.18448169D-01	3.18330179D-01	-2.59695931D-01	-2.03812971D-01	1.67823843D-02	8.22044940D 00	3.30707227D-01
9.87583957D 02	-9.77462976D 02	1.55830243D 02	2.06901244D 02	-4.22373227D 01	-4.46977810D 01	1.39822709D 03	2.15847186D 02
1.84317849D 02	2.21928709D 00	1.30305992D 02	1.30611402D 02	6.39932185D 00	4.47049005D 01	6.39881913D 00	5.92989477D 01

SATURN SPHERE OF INFLUENCE (HELIOCENTRIC) THRUST ON

1.40000000D 03	7.67335335D-01	7.39453410D 00	8.14634503D-01	5.25836664D 01	3.00428226D 00	1.36771485D 02	5.38917561D 01
0.0	1.67973257D 03	-6.56831589D 03	-3.19624554D 02	1.10953084D 03	1.63985629D-02	1.18324237D-01	1.57711891D 00
-2.53781156D-05	3.28852545D-05	8.27944413D-06	-6.62111464D 03	3.52436582D 03	3.06898301D 02	4.23560847D-05	7.50696344D 03
-9.42569445D 00	-1.76838738D 00	4.06446364D-01	-1.80405228D-01	-2.00933470D-01	1.41686054D-02	9.59895196D 00	2.70409052D-01
2.21717520D 03	-1.18018444D 03	-1.02769298D 02	2.28790779D 02	-2.89019426D 01	-4.64089553D 01	2.51381438D 03	2.35232510D 02
1.90625716D 02	2.42678844D 00	1.41382992D 02	1.41409340D 02	-1.55252859D 00	2.80260120D 01	-2.34300850D 00	5.92989475D 01

CASE 2 (CONVERGED) CASE SUMMARY 3/15/71

EARTH-SATURN MISSION, IMPACT=1 END COND.

LAUNCH	TARGET
EARTH	SATURN

LAUNCH VEHICLE IS TITAN III X(1205)/CENTAUR

MASS BREAKDOWN(KG)

INITIAL	PROPULSION	PROPELLANT	TANKAGE	STRUCTURE	SWB SCI PKG	RETRO STAGE	NET S/C
3216.32	1034.45	748.32	22.45	0.0	0.0	705.47	725.63

PROPULSION SYSTEM PARAMETERS

REF POWER (KW)	REF THRUST (N)	THR ACCEL (M/SEC/SEC)	EXHAUST SPEED (M/SEC)	EFFICIENCY	PROP TIME (DAYS)
33.8150	1.493679	0.4644070D-03	27338.3660	0.603797	768.5186

LAUNCH CONDITIONS (PLANETOCENTRIC)

DATE (JULIAN)	RADIUS (RADII)	SPEED (M/SEC)	FLT PATH (DEG)	INCLINATION (DEG)	NODE (DEG)	ARG POS (DEG)	EXCESS SPEED (M/SEC)	T INF (DAYS)
43404.8210	1.02904	12406.6707	0.0	28.5000	225.9205	281.6187	5697.5800	1.7949

ARRIVAL CONDITIONS (PLANETOCENTRIC)

DATE (JULIAN)	RADIUS (RADII)	SPEED (M/SEC)	FLT PATH (DEG)	INCLINATION (DEG)	NODE (DEG)	ARG POS (DEG)	EXCESS SPEED (M/SEC)	T INF (DAYS)
44804.8210	0.0	0.0	0.0	0.0	0.0	0.0	7506.9634	0.0

TRAJECTORY SCHEDULE (DAYS)

LAUNCH	LAUNCH SPHERE	TARGET SPHERE	ARRIVE
0.	1.7949	1400.0000	1400.0000

CAPTURE ORBIT AND RETRO STAGE

RPER (RADII)	RAP (RADII)	VORB (M/SEC)	INC VEL (M/SEC)	SPEC IMP (SEC)	INERT (KG)	PROPELLANT (KG)
2.0000	38.0000	25020.6877	1725.1253	300.00	70.55	634.92

```
           CASE  3            3/15/71       EARTH-SATURN MISSION, SEGMENTED 2-BODY       CPU - I/O TIMES REMAINING (SEC)  44   64
&MINPUT
AX= 0.0                     ,  6563.3649999999998   , 0.99999999999999999D-02,  20000.000000000000    , 0.99999999999999999D-12,
  1.0000000000000000        ,  12406.670732521810   , 0.99999999999999999D-03,  1000.0000000000000    , 0.99999999999999999D-04,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        ,  28.499999999999999   , 0.0                    , 0.0                    , 0.0                    ,
  1.0000000000000000        ,  225.92054935790650   , 0.99999999999999999D-04,  20.000000000000000    ,  1.0000000000000000    ,
  1.0000000000000000        ,  281.61866053741980   , 0.99999999999999999D-04,  20.000000000000000    ,  1.0000000000000000    ,
 0.0                        ,  1.0000000000000000   , 0.10000000000000000D-06, 0.10000000000000000D 00,  1.0000000000000000    ,
  1.0000000000000000        ,  39.821026476189735   , 0.99999999999999998D-05,  10.000000000000000    , 0.10000000000000000D 00,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
  1.0000000000000000        ,  1360.0000000000000   , 0.99999999999999998D-05,  10.000000000000000    , 0.10000000000000000D 00,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
  1.0000000000000000        ,  7506.963435232433    , 0.99999999999999999D-03,  1000.0000000000000    , 0.99999999999999999D-04,
  1.0000000000000000        , 0.46440676465644260D-03, 0.10000000000000000D-09, 0.50000000000000000D-04,  10000000000.00000   ,
  1.0000000000000000        ,  27338.366037330250   , 0.99999999999999999D-02,  1000.0000000000000    , 0.99999999999999999D-04,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
  1.0000000000000000        , -396.07686297957160   , 0.99999999999999998D-05,  1000.0000000000000    , 0.99999999999999999D-04,
  1.0000000000000000        ,  4121.0102333262190   , 0.99999999999999998D-05,  1000.0000000000000    , 0.99999999999999999D-03,
  1.0000000000000000        , -75.100976978978920   , 0.99999999999999998D-05,  1000.0000000000000    , 0.99999999999999999D-02,
  1.0000000000000000        , -2881.7375435447770   , 0.99999999999999998D-05,  1000.0000000000000    , 0.99999999999999999D-04,
  1.0000000000000000        ,  706.12332936319040   , 0.99999999999999998D-05,  1000.0000000000000    , 0.99999999999999999D-04,
  1.0000000000000000        ,  353.72020502300040   , 0.99999999999999998D-05,  1000.0000000000000    , 0.10000000000000000D 00,
  1.0000000000000000        ,  774.54971103446530   , 0.99999999999999998D-05,  1000.0000000000000    , 0.99999999999999999D-02,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
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 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
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 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
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 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
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 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    , BY=
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0                    ,
  1.0000000000000000        ,-0.10000000000000000D-07, 0.10000000000000000D-07,  1.0000000000000000   ,  1.0000000000000000    ,
  1.0000000000000000        ,-0.10000000000000000D-07, 0.10000000000000000D-07,  1.0000000000000000   ,  1.0000000000000000    ,
  1.0000000000000000        ,-0.10000000000000000D-07, 0.10000000000000000D-07,  1.0000000000000000   ,  1.0000000000000000    ,
  1.0000000000000000        ,-0.10000000000000000D-07, 0.10000000000000000D-07,  1.0000000000000000   ,  1.0000000000000000    ,
  1.0000000000000000        ,-0.10000000000000000D-07, 0.10000000000000000D-07,  1.0000000000000000   ,  1.0000000000000000    ,
  1.0000000000000000        ,-0.10000000000000000D-07, 0.10000000000000000D-07,  1.0000000000000000   ,  1.0000000000000000    ,
 0.0                        , 0.0                   , 0.0                    , 0.0                    , 0.0
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 2.0000000000000000   ,-0.10000000000000000D-07, 0.10000000000000000D-07,  1.0000000000000000  ,  1.0000000000000000   ,
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0.0                   , 0.0                     , 0.0                     , 0.0                  , 0.0                   ,
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0.0                   , 0.0                     , 0.0                     , 0.0                  , 0.0                   ,
 1.0000000000000000   ,-0.10000000000000000D-06, 0.10000000000000000D-06,  1.0000000000000000  ,  1.0000000000000000   ,
0.0                   ,-0.99999999999999999D-04, 0.99999999999999999D-04,  1.0000000000000000  ,  1.0000000000000000   ,
 1.0000000000000000   ,-0.99999999999999999D-04, 0.99999999999999999D-04,  1.0000000000000000  ,  1.0000000000000000   ,
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0.0                   , 0.0                     , 0.0                     , 0.0                  , 0.0                   ,
 1.0000000000000000   ,-0.99999999999999999D-04, 0.99999999999999999D-04,  1.0000000000000000  ,  1.0000000000000000   ,
 1.0000000000000000   ,-0.99999999999999999D-06, 0.99999999999999999D-06,  1.0000000000000000  ,  1.0000000000000000   ,
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0.0                   , 0.0                     , 0.0                     , 0.0                  , 0.0                   ,
 1.0000000000000000   ,-0.99999999999999999D-04, 0.99999999999999999D-04,  1.0000000000000000  ,  1.0000000000000000   ,
0.0                   ,-0.99999999999999999D-06, 0.99999999999999999D-06,  1.0000000000000000  ,  1.0000000000000000   ,
 1.0000000000000000   ,-0.99999999999999999D-04, 0.99999999999999999D-04,  1.0000000000000000  ,  1.0000000000000000   ,
 1.0000000000000000   ,-0.99999999999999999D-04, 0.99999999999999999D-04,  1.0000000000000000  ,  1.0000000000000000   ,
0.0                   ,-0.99999999999999999D-04, 0.99999999999999999D-04,  1.0000000000000000  ,  1.0000000000000000   ,
0.0                   ,-0.99999999999999999D-04, 0.99999999999999999D-04,  1.0000000000000000  ,  1.0000000000000000   ,
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0.0                   , 0.0                     , 0.0                     , 0.0                  , 0.0                   ,
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0.0                   , 0.0                     , 0.0                     , 0.0                  , 0.0                   ,
0.0                   , 0.0                     , 0.0                     , 0.0                  , 0.0                   ,
0.0                   , 0.0                     , 0.0                     , 0.0                  , 0.0                   ,
0.0                   , 0.0                     , 0.0                     , 0.0                  , 0.0                   ,
0.0                   , 0.0                     , 0.0                     , 0.0                  , 0.0                   ,
0.0                   , 0.0                     , 0.0                     , 0.0                  , 0.0                   ,
0.0                   , 0.0                     , 0.0                     , 0.0                  , 0.0                   ,
0.0                   , 0.0                     , 0.0                     , 0.0                  , 0.0                   ,
0.0                   , 0.0                     , 0.0                     , 0.0                  , 0.0                   ,
0.0                   , 0.0                     , 0.0                     , 0.0                  , 0.0                   ,EN=
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0.0                   ,CNI= 0.0                 ,CMI= 0.0                 ,SOI= 0.0                 ,TPI=
0.0                   ,EMSUN= 0.0               ,EMLNCH= 0.0              ,RLNCH= 0.0               ,RSLNCH=
0.0                   ,ELOLNC= 0.0              ,ELALNC= 0.0              ,EMSWBY= 0.0              ,RSWBY=
0.0                   ,RSSWBY= 0.0              ,ELOSWB= 0.0              ,ELASWB= 0.0              ,EMTARG=
0.0                   ,RTARG= 0.0               ,RSTARG= 0.0              ,ELOTRG= 0.0              ,ELATRG=
0.0                   ,B1= 0.0                  ,B2= 0.0                  ,B3= 0.0                  ,BJ=  1.0000000000000000   ,
0.0                   , 0.0                     , 0.0                     , 0.0                     , 0.0                  ,
0.0                   , 0.0                     , 0.0                     , 0.0                     ,PMI=  1.0000000000000000   ,RPMI=
0.0                   ,RPOI= 0.0                ,FM=  2.0000000000000000  ,ENN=  1.0000000000000000   ,BI=
 1.0000000000000000   ,DI=  20000.000000000000  ,EI= 0.0                  ,CALPHA=  29.999999999999999   ,EKS=
0.0                   ,EKT= 0.29999999999999999D-01,DELMX= 0.0            ,EKX= 0.0                 ,EKR=
0.1111111111110000    ,SPIRET=  300.00000000000000 ,RTF=  2.0000000000000000  ,RTA=  37.999999999999999  ,DZP=
0.78125000000000000D-02,DZH= 0.12500000000000000   ,IFFM=        1,MLPCAT=        1,NSET=       15,        15,        0,
        0,            0,MPRINT=          0,NFRINT=          3,MOPT1=          3,MOPT2=          3,MOPT3=          6,INPFLG=
        0,            0,          0,          0,          0,          0,          0,          0,          0,          0,          0,
        0,            0,          0,          0,          0,MYEAR=       1977,MONTH=          8,MDAY=          9,HOUR=
 12.000000000000000   ,MBOOST=        10,MSCLAR=          0,MSHLD=          0,MEFFIC=          0,JR=          1,JPS=          1,
JT=          1,JC=          0,MODE=          1,IPICK=          0,          0,          1,          1,          0,          0,FRWD=T,
IPR11=          0,IPR12=          0,ITOP=          0,LONG=          0,KPART=          0,IMPACT=          0,MULAT=          0,
ISPHER=          1,ITF=          5,AUKM=  149598000.00000000  ,GRAV=  9.8066500000000000  ,ER=  6378.1650000000000   ,MPUNCH=
        0,KOUNT=          3,IDATE=         71,          3,         15,          0,          0,          0,HONG=
 0.77248918353177800   , -14.243371804256910   ,-0.77172105231463370 41, -818508.38471917390   , 0.94678895705593020   ,LOC=
        1
$END
```

SETUP FOR CASE 3 3/15/71

EARTH-SATURN MISSION, SEGMENTED 2-BODY

INDEPENDENT PARAMETERS

		NAME	VALUE	DELTA	MAX STEP	WEIGHT
1	2	PASSVELO	1.240667073252181D 04	1.0000000000D-03	1.0000000000D 03	1.0000000000D-04
2	5	PASSNODE	2.259205493578065D 02	1.0000000000D-04	2.0000000000D 01	1.0000000000D 00
3	6	PASSARGP	2.816186605374198D 02	1.0000000000D-04	2.0000000000D 01	1.0000000000D 00
4	8	PASSTIME	3.982102647618735D 01	1.0000000000D-05	1.0000000000D 01	1.0000000000D-01
5	10	TARGTIME	1.360000000000000D 03	1.0000000000D-05	1.0000000000D 01	1.0000000000D-01
6	12	TARG VOO	7.506963435232433D 03	1.0000000000D-03	1.0000000000D 03	1.0000000000D-04
7	13	THR ACCL	4.644067646564426D-04	1.0000000000D-10	5.0000000000D-05	1.0000000000D 10
8	14	JETSPEED	2.733836603733025D 04	1.0000000000D-02	1.0000000000D 03	1.0000000000D-04
9	23	LEG2 P1	-3.960768629795716D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
10	24	LEG2 P2	4.121010233326219D 03	1.0000000000D-05	1.0000000000D 03	1.0000000000D-03
11	25	LEG2 P3	-7.510097697897892D 01	1.0000000000D-05	1.0000000000D 03	1.0000000000D-02
12	26	LEG2 PD1	-2.881737543544777D 03	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
13	27	LEG2 PD2	7.061233293631904D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-04
14	28	LEG2 PD3	3.537202050230004D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-01
15	29	LEG2 PMA	7.745497110344653D 02	1.0000000000D-05	1.0000000000D 03	1.0000000000D-02

DEPENDENT PARAMETERS

		NAME	TRIG	LOW	HIGH	WEIGHT	TYPE
1	7	DELTA XT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
2	8	DELTA YT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
3	9	DELTA ZT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
4	10	DELT XDT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
5	11	DELT YDT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
6	12	DELT ZDT	1	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
7	14	TARGMASS*	2	-1.0000000000D-08	1.0000000000D-08	1.0000000000D 00	1
8	18	LEG2TIME	1	1.3999999990D 03	1.4000000010D 03	1.0000000000D 00	1
9	20	T(TG DT)	1	-1.0000000000D-07	1.0000000000D-07	1.0000000000D 00	1
10	22	T(TGVOO)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
11	25	T(THRAC)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
12	26	T(JETVL)	1	-1.0000000000D-06	1.0000000000D-06	1.0000000000D 00	1
13	29	T(SWB V)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
14	31	T(SWNOD)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1
15	32	T(SWARG)	1	-1.0000000000D-04	1.0000000000D-04	1.0000000000D 00	1

NOTE THE ABOVE INDICATED DEPENDENT AND INDEPENDENT PARAMETERS MAY BE ALTERED INTERNALLY TO AUTOMATICALLY SATISFY TRANSVERSALITY CONDITIONS AT THE SWINGBY CLOSEST APPROACH POINT. THEIR TRIGGERS ARE SET TO ZERO. SEE BOXED QUANTITIES ON TRAJECTORY SUMMARY PAGE FOR DISPLAY OF ACTUAL DEPENDENT AND INDEPENDENT PARAMETERS

* NAME APPLIES TO TRIGGER SETTING OF 1

THIS CASE IS CONVERGED.
10 TRAJECTORIES WITHOUT PARTIALS AND 4 TRAJECTORIES WITH PARTIALS. INHIBITOR = 0.88817842D-15

TRAJECTORY SUMMARY　　CASE 3　　3/15/71

EARTH-SATURN MISSION, SEGMENTED 2-BODY

INDEPENDENT PARAMETERS

PASS DIST	PASS SPEED	PASS GAMMA	PASS INCL	PASS NODE	PASS ARGP	PASS MASS	PASS TIME
6.5633650D 03	1.2405694D 04	0.0	2.8500000D 01	2.2587922D 02	2.3160310D 02	1.0000000D 00	3.9804384D 01
LAUN TIME	TARG TIME	LAUN VOO	TARG VOO	THR ACCEL	JET SPEED	LEG1 P1	LEG1 P2
0.0	1.3594028D 03	0.0	7.4592655D 03	4.6406104D-04	2.7268243D 04	0.0	0.0
LEG1 P3	LEG1 PD1	LEG1 PD2	LEG1 PD3	LEG1 PMASS	LEG1 PTIME	LEG2 P1	LEG2 P2
0.0	0.0	0.0	0.0	0.0	0.0	-3.9744066D 02	4.1657754D 03
LEG2 P3	LEG2 PD1	LEG2 PD2	LEG2 PD3	LEG2 PMASS	LEG2 PTIME		
-7.6101259D 01	-2.9129733D 03	7.1279238D 02	3.5646228D 02	7.8298813D 02	0.0		

DEPENDENT PARAMETERS (REFERENCED TO ZERO)

DELTA XL	DELTA YL	DELTA ZL	DELTA XDL	DELTA YDL	DELTA ZDL	DELTA XT	DELTA YT
0.0	0.0	0.0	0.0	0.0	0.0	-1.0036416D-13	2.0561330D-13
DELTA ZT	DELTA XDT	DELTA YDT	DELTA ZDT	LAUN MASS	TARG MASS	LAUN DATE	TARG DATE
-5.4817262D-15	2.6295428D-14	1.2423392D-13	-1.1312197D-14	0.0	-1.0972790D-10	0.0	1.4398044D 03
LEG1 TIME	LEG2 TIME	T(LAUN DATE)	T(TARG DATE)	T(LAUN VOO)	T(TARG VOO)	TOTAL TIME	REF POWER
0.0	-1.1368684D-13	0.0	1.0140712D-07	0.0	5.1797739D-07	1.4000000D 03	3.3754005D 01
T(THR ACC)	T(JET VEL)	NET MASS	T(SWB DIST)	T(SWB VEL)	T(SWB INC)	T(SWB NODE)	T(SWB ARG)
3.8386676D-10	5.8346625D-11	7.3377285D 02	3.3764774D 07	-6.6211214D-10	-4.6438907D-02	4.7100431D-13	-9.0875224D-14
T(SWB MASS)	T(SWB TIME)						
7.8298813D 02	-5.7477608D 01						

DISCONTINUITY POINTS

LAUNCH	LSOI	OFF	ON	TSOI	ARRIVE
0.0	1.796	439.396	1066.486	1319.598	1400.000

SPACECRAFT PARAMETERS

NET S/C MASS	INITIAL MASS	PROP SYSTEM	PROPELLANT	RETRO SYSTEM	SWB SCI PKG	POWER	EFFICIENCY
7.337728480 02	3.217612850 03	1.012620150 03	7.435024430 02	7.054123420 02	0.0	3.375400480 01	6.031297470-01

DISCONTINUITY POINT SUMMARY CASE 3 3/15/71

EARTH-SATURN MISSION, SEGMENTED 2-BODY

TIME	MASS RATIO	SEMI-AXIS	ECCENTRICITY	FLT PTH ANGLE	INCLINATION	NODE	ARG POS
LAMBDA T	LAMBDA NU	LAMBDA A	LAMBDA C	SWITCH FNCT	POWER FACTOR	THRUST ACCEL	ANGULAR MOM
XP	YP	ZP	XP DOT	YP DOT	ZP DOT	RP	VP
X	Y	Z	X DOT	Y DOT	Z DOT	R	V
LAMBDA X	LAMBDA Y	LAMBDA Z	LAMBDA DOT X	LAMBDA DOT Y	LAMBDA DOT Z	LAMBDA	LAMBDA DOT
LONGITUDE	LATITUDE	PHI	THETA OSC	PSI OSC	THETA I	PSI I	HAMILTONIAN

EARTH SPHERE OF INFLUENCE (HELIOCENTRIC) THRUST ON

1.79553599D 00	1.000000000D 00	1.72315629D 00	4.18370048D-01	-1.59049508D 00	1.96769029D-01	1.74665222D 02	1.82708170D 02
0.0	7.32988132D 02	0.0	0.0	3.33011976D 03	9.95494815D-01	7.84538336D-02	1.19228613D 00
-5.45890467D 04	9.26091711D 05	-2.43606965D 04	-4.21661665D 02	5.75367104D 03	-1.21232907D 02	9.28019000D 05	5.77037489D 03
1.00249597D 00	-4.59898829D-02	-1.62841058D-04	2.14909879D-02	1.18832471D 00	-4.07022578D-03	1.00355033D 00	1.18852600D 00
-3.97440660D 02	4.16577535D 03	-7.61012593D 01	-2.91297327D 03	7.12792384D 02	3.56469285D 02	4.18538347D 03	3.02002593D 03
3.57373376D 02	-9.29709763D-03	9.80749842D 01	9.80758758D 01	-8.48551322D-01	9.54498751D 01	-1.04184510D 00	6.02041745D 01

SWITCH THRUST OFF

4.35355594D 02	7.76350074D-01	7.43131226D 00	8.32576371D-01	5.27048071D 01	2.91626290D 00	1.35018652D 02	2.47188659D 01
0.0	1.66912878D 03	-6.55367866D 03	-3.07053436D 02	-3.41060513D-13	7.89600317D-02	1.20658492D-01	1.50998340D 00
- - - - - - - - - - - - (PLANETOCENTRIC PARAMETERS UNAVAILABLE) - - - - - - - - - - - -							
-4.14386591D 00	1.53209722D 00	9.40139489D-02	-5.38743355D-01	-1.64730246D-01	2.53359335D-02	4.41902912D 00	5.63934718D-01
-1.23912049D 03	-3.63471699D 02	5.79609261D 02	2.37906609D 02	-9.04908759D 01	-2.45273167D 01	1.41544275D 03	2.55714181D 02
1.59709327D 02	1.21904807D 00	4.22173753D 01	3.71958303D 01	2.16068060D 01	1.63651984D 02	2.41727842D 01	6.02041720D 01

SWITCH THRUST ON

1.06649583D 03	7.76350074D-01	7.43131226D 00	8.32576371D-01	5.61634636D 01	2.91626290D 00	1.35018652D 02	4.92540570D 01
0.0	1.66912878D 03	-6.55367866D 03	-3.07053436D 02	1.70530257D-13	2.27856072D-02	0.0	1.50998340D 00
- - - - - - - - - - - - (PLANETOCENTRIC PARAMETERS UNAVAILABLE) - - - - - - - - - - - -							
-8.16228594D 00	-6.04555796D-01	3.15707971D-01	-2.60203118D-01	-2.04028040D-01	1.67217397D-02	8.19073080D 00	3.31078118D-01
9.96348370D 02	-9.92367773D 02	1.61165427D 02	2.10089171D 02	-4.32818198D 01	-4.50692658D 01	1.41544275D 03	2.19184887D 02
1.84235990D 02	2.20898681D 00	1.30190534D 02	1.30507768D 02	6.53438467D 00	4.48853174D 01	6.53800292D 00	6.02041720D 01

SATURN SPHERE OF INFLUENCE (HELIOCENTRIC) THRUST ON

1.31955644D 03	7.68927314D-01	7.37645403D 00	8.17760349D-01	5.34345008D 01	3.00197196D 00	1.36978805D 02	5.23202044D 01
0.0	1.68972476D 03	-6.59773194D 03	-3.20653324D 02	8.45738744D 02	1.75925497D-02	1.18560329D-01	1.56319764D 00
4.82336392D 07	-2.52638387D 07	-2.39352002D 06	-6.66063146D 03	3.54597325D 03	3.06913132D 02	5.45020590D 07	7.55196222D 03
-9.15135816D 00	-1.49218716D 00	3.84644631D-01	-1.96628894D-01	-2.02643105D-01	1.48047955D-02	9.28019020D 00	2.82747823D-01
1.94543506D 03	-1.15920998D 03	-3.87220684D 01	2.26686962D 02	-3.30762734D 01	-4.65661881D 01	2.26494697D 03	2.33772172D 02
1.85260541D 02	2.37547167D 00	1.39944541D 02	1.39945759D 02	-3.42548705D-01	3.07890889D 01	-9.79589905D-01	6.02041719D 01

CASE 3 (CONVERGED) CASE SUMMARY 3/15/71

EARTH-SATURN MISSION, SEGMENTED 2-BODY

LAUNCH	TARGET
EARTH	SATURN

LAUNCH VEHICLE IS TITAN III X(1205)/CENTAUR

MASS BREAKDOWN(KG)

INITIAL	PROPULSION	PROPELLANT	TANKAGE	STRUCTURE	SWB SCI PKG	RETRO STAGE	NET S/C
3217.61	1012.62	743.50	22.31	0.0	0.0	705.41	733.77

PROPULSION SYSTEM PARAMETERS

REF POWER (KW)	REF THRUST (N)	THR ACCEL (M/SEC/SEC)	EXHAUST SPEED (M/SEC)	EFFICIENCY	PROP TIME (DAYS)
33.7540	1.493169	0.4640610D-03	27268.2432	0.603130	690.7127

LAUNCH CONDITIONS (PLANETOCENTRIC)

DATE (JULIAN)	RADIUS (RADII)	SPEED (M/SEC)	FLT PATH (DEG)	INCLINATION (DEG)	NODE (DEG)	ARG POS (DEG)	EXCESS SPEED (M/SEC)	T INF (DAYS)
43404.8044	1.02904	12405.6941	0.0	28.5000	225.6792	281.6031	5695.4530	1.7955

ARRIVAL CONDITIONS (PLANETOCENTRIC)

DATE (JULIAN)	RADIUS (RADII)	SPEED (M/SEC)	FLT PATH (DEG)	INCLINATION (DEG)	NODE (DEG)	ARG POS (DEG)	EXCESS SPEED (M/SEC)	T INF (DAYS)
44804.8044	2.00000	26732.4655	0.0	10.0406	110.6671	65.3699	7459.2695	80.4016

TRAJECTORY SCHEDULE (DAYS)

LAUNCH	LAUNCH SPHERE	TARGET SPHERE	ARRIVE
0.	1.7955	1319.5984	1400.0000

CAPTURE ORBIT AND RETRO STAGE

RPER (RADII)	RAP (RADII)	VORB (M/SEC)	INC VEL (M/SEC)	SPEC IMP (SEC)	INERT (KG)	PROPELLANT (KG)
2.0000	38.0000	25020.6877	1711.7779	300.00	70.54	634.87

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