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COMMENTS ON ION-ACOUSTIC SOLITARY WAVES IN PLASMAS

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It was first shown by Sagdeev[1] that ion-sound solitary wave can occur in a plasma with cold ions and hot electrons. Later Washimi and Tanuiti[2] showed that the Korteweg-deVries equation can be derived for the problem of ion-sound wave disturbances moving with velocity slightly greater than the ion-sound speed in a homogeneous plasma free of external field. Consequently steady-state solution for ion-sound solitary wave was again demonstrated by these authors. The theoretical prediction of the existence of ion-sound solitary wave has attracted a great deal of attention in recent years since the subject is highly relevant to the theory of collisionless shock waves. We wish to discuss some basic conceptual difficulties associated with these earlier derivations and to propose a more accurate picture of the essential physics involved.

The basic assumptions adapted by Washimi et al. in deriving the Korteweg-deVries equation for the ion-sound disturbance are: (1) the ions are cold compared with the electrons, i.e. $T_e >> T_i$, and may be described by one-dimensional hydromagnetic equations, (2) electron inertia effects are negligible and (3) the electrons may be described by an isothermal equation of state. From the last two assumptions we obtain

$$0 = e \frac{\partial \Phi_o}{\partial x} - \frac{T_e}{n_e} \frac{\partial n_e}{\partial x} \tag{1}$$

where n_e is the electron density, Φ the electrostatic potential associated with the ion-sound distrubance, and e the absolute value of the electron charge. Integrating (1) one obtains

$$n_e = n_o \exp\left(\frac{e\Phi_o}{T_e}\right) \tag{2}$$

where n_o is the density of the uniform background. Expression (2) resembles the equilibrium Boltzmann distribution. As a matter of fact expression (2) has also been used in the work by Sagdeev[1]. At this point we like to call attention to some of the basic difficulties contained implicitly in justifying Eq. (1) or expression (2). It is not entirely clear that in general expression (2) represents adequately the electron density under the influence of the traveling wave. In the following we shall remark a few points.

1.) From a microscopic or kinetic-theoretical point of view, a steady state (defined with respect to the wave frame) is possible only when the electron distribution is a function of the Hemiltonian H. Here

$$H = \frac{1}{2} mv^2 - e\Phi_o(x) \quad . \tag{3}$$

In (3) v is the particle velocity and Φ denotes the electrostatic potential associated with the solitary wave.

2.) If in the absence of the wave the electrons have Maxwellian distribution, then we expect that

$$F_e \xrightarrow[x \to \pm\infty]{} n_o \left(\frac{m}{2\pi T_e}\right)^{\frac{1}{2}} \exp\left(-\frac{m(v+v_w)}{2T_e}\right)^2 \tag{4}$$

where F_e is defined in the wave frame and v_w is the wave velocity. (Here we have assumed that the potential $\Phi_o(x)$ peaks at $x = 0$).

3.) In order that expression (2) holds true, we must require

$$n_e = \int_{-\infty}^{+\infty} dv\, F_e = n_o \exp\left(\frac{e\Phi_o(x)}{T_e}\right) \qquad (5)$$

From (5), we see that the only permissible distributuion function $F_e(H)$ is

$$F_e(H) = n_o \left(\frac{m}{2\pi T_e}\right)^{1/2} \exp\left(-\frac{H}{T_e}\right)$$

$$= n_o \left(\frac{m}{2\pi T_e}\right)^{1/2} \exp\left(\frac{e\Phi_o(x)}{T_e} - \frac{mv^2}{2T_e}\right) \qquad . \qquad (6)$$

However, from (6)

$$F_e(H) \xrightarrow[x \to \pm\infty]{} n_o \left(\frac{m}{2\pi T_e}\right)^{1/2} \exp\left(-\frac{mv^2}{2T_e}\right) \qquad (7)$$

which is not compatible with (4). The point which we try to make here is that in a laboratory frame the Hamiltonian associated with the traveling wave is not

$$H = \frac{1}{2} mv^2 - e\phi(x - v_w t) \qquad (8)$$

but

$$H = \frac{1}{2} m(v - v_w)^2 - e\Phi(x - v_w t) \qquad . \qquad (9)$$

Therefore from the kinetic theory point of view a steady state solution of the type described by (6) is not conceivable because it would violate the boundary condition described by (4). This is the first difficulty we want to point out.

Let us next discuss the second difficulty. Before going further we see that the first difficulty does not occur if the plasma initially possesses a displaced Maxwellian electron distribution such that the displacement velocity happens to be the wave velocity v_w , as was done in Ref. 3. For this particular case the boundary condition given by (4) reduces to

$$F_e(H) \xrightarrow[x \to \pm \infty]{} n_o \left(\frac{m}{2\pi T_e}\right)^{1/2} \exp\left(-\frac{mv^2}{2T_e}\right) \tag{10}$$

which is indeed compatible with the steady state solution described by (6). However in general two basic time scales should be considered. One is the transit time, say τ_w , which may be defined as

$$\tau_w = \frac{L}{v_w} \tag{11}$$

where L is the typical width of the wave and v_w denotes the wave speed. The second time scale is a relaxation time, τ_r , and its physical meaning may be considered as follows. Let us imagine that in a certain region the plasma is in an equilibrium state when the wave is absent initially. Then let us suppose that the solitary wave is "formed" somewhere else and propagated into this region under consideration. Of course, we expect that the wave and the plasma would interact and consequently the electron distribution function would evolve from an initial form, say

$$F_e = n_o \left(\frac{m}{2\pi T_e}\right)^{1/2} \exp\left(-\frac{mv^2}{2T_e}\right) \tag{12}$$

to a form described by (6). The relaxation time, τ_r, measures such a time-evolution process. A steady state may be argued to exist if we can show that

$$1 >> \frac{\tau_r}{\tau_w} \quad . \tag{13}$$

In general we expect that τ_r is finite and in fact can be very large for the low energy particles such that condition (13) does not hold true.

With the conceptual difficulties just mentioned we cannot justify the density relation described by (2). Therefore, we question the validity of the steady state ion-sound solitary wave.

One may try to sidestep the first difficulty mentioned earlier by arguing that since $v_w \simeq (T_e/M)^{1/2}$ (where M is the ion mass) which is small compared with the electron thermal speed, the boundary condition given by (4) is approximately satisfied by the solution given by (6). This argument is not entirely acceptable. The point is that microscopic processes, namely ion reflection electron trapping, can significantly affect the physical picture, and produce a persistent time-dependent process. To examine the importance of trapping process let us make some order of magnitude estimates.

It is true that for small amplitude waves trapping of particles can be neglected in many cases. The reasons are that usually the population of trapped particles is small and the "trapping time" is long compared to the most important time scales of interest to us. In the following we shall see that neither reasons are true in the case of ion-sound solitary wave. To illustrate this point let us accept the solution for the moment and then check the possible consequence which the wave can produce. We know the solitary wave solution, i.e.,

$$\Phi_o = \frac{3\delta M T_e}{|e|} \operatorname{sech}^2\left(\frac{x}{L}\right) \tag{14}$$

where $L - \lambda_e/(\delta M/2)^{1/2}$, ($\lambda_e$ is the electron Debye length), T_e denotes the electron temperature, and δM is defined by

$$\delta M = \frac{\text{Wave Speed}}{\text{Ion Sound Speed}} - 1 \equiv \frac{v_w}{v_s} - 1 \quad . \tag{15}$$

From (14) we see that particles with velocities within the range

$$\left| v_w - \left(\frac{6\delta M T_e}{m_e}\right)^{1/2} \right| < v < \left| v_w + \left(\frac{6\delta M T_e}{m_e}\right)^{1/2} \right| \tag{16}$$

may be trapped by the wave. Now we define a trapping v_t such that

$$v_t = \left(\frac{6\delta M T_e}{m_e}\right)^{1/2} \equiv (6\delta M)^{1/2}\, v_e \tag{17}$$

where v_e is the electron thermal speed. We know that solution (14) exists[1] if $0 < \delta M < 0.6$. Hence let us consider a very weakly supersonic case, say $\delta M \simeq 0.1$. Even in such case, $v_t \simeq 0.75\ v_e$ which is very large. This result indicates that a large population of electrons can be involved with the trapping process. Thus it is unlikely that trapping is a negligible nonlinear process in a general theory of the ion-sound solitary wave. Furthermore we should point out that the typical bounce period of (or sometimes called the "trapping time") a typical trapped particle is very short. We see easily

$$\tau_b = \frac{L}{v_t} = \frac{1}{\omega_e} \frac{1}{\delta M\sqrt{3}} .$$

For the case $\delta M \simeq 0.1$, τ_b is shorter than ten "electron plasma periods" that is very short compared to the scale [i.e., $(\delta M)^{-3/2} \omega_e^{-1}(m_i/m_e)^{\frac{1}{2}}$] for which the perturbation scheme used by Washimi and Tanuiti is valid.

To conclude: because hydromagnetic theory only described the macroscopic motion of plasma, physical processes associated with discrete particles are absent. Thus in order to take account of the effect of particle trapping, we must use kinetic theory. First, the reflection of ions should be included into a consistent picture. This has recently been done by P.H. Sakanaka[4]. Second, the electron trapping, which is a much faster process than ion Landau damping, must be included. To our knowledge, this has not yet been achieved.

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